The limits of forward guidance*

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Abstract

The viability of forward guidance as a monetary policy tool depends on the horizon over which it can be communicated and its influence on expectations over that horizon. This paper develops and estimates a model of imperfect central bank communications and uses it to measure how effectively the Fed has managed expectations about future interest rates and the influence of its communications on macroeconomic outcomes. Standard models assume central banks have perfect control over expectations about the policy rate up to an arbitrarily long horizon, and this is the source of the so-called "forward guidance puzzle." Our estimated model suggests that the Fed has limited ability to affect expectations at horizons that are sufficiently long to give rise to the forward guidance puzzle. Additionally, imperfect communication has a significant impact on the propagation of forward guidance. Finally, we develop a novel decomposition of the response of the economy to forward guidance. The decomposition shows that empirically plausible imperfect forward guidance has a quantitatively important role bringing forward the effects of future rate changes and that poor communications have been a source of macroeconomic volatility.

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1 Introduction

Since the onset of the financial crisis, informing the public about its future intentions has become central to Fed communications. Even before the crisis, post-meeting statements and speeches by Fed officials included forward-looking language intended to clarify its intentions about policy in the future. Such communications have become known as *forward guidance*. The Fed's forward guidance when the policy rate was set at its effective lower bound (ELB) is widely viewed to have been explicitly designed to lower expectations of future short term rates. By doing so far enough into the future, such communications may have had a material effect on the interest rates that govern longer term investment projects and boosted economic activity more broadly.

This mechanism is present within the standard New Keynesian (NK) framework. In NK models, forward guidance allows the central bank to influence private-sector expectations about future interest rates and thereby to potentially improve macroeconomic outcomes. However the practical viability of this policy tool ultimately depends on power of the Fed's communications to influence expectations and horizon over which it is able to influence expectations. This paper develops an empirically tractable NK model with imperfect central bank communications and uses it to measure how effectively the Fed influenced private-sector expectations about future interest rates and the macroeconomic consequences of this influence over the period 1993-2016.

Standard NK models assume the central bank has perfect control over private-sector expectations about the policy rate up to an arbitrarily long horizon. Eggertsson and Woodford (2003) and Krugman (1998) rely on this assumption to formulate policies to combat the severe consequences of monetary policy being constrained by the ELB. However, such perfect control gives rise to implausible implications. In particular, it is the source of the "forward guidance puzzle" highlighted in an empirical context by Del Negro et al. (2015). As Carlstrom et al. (2015) demonstrated, the forward guidance puzzle arises from a theoretical implication common to all standard NK models: the near term effects of a commitment to maintain an interest rate peg for an additional period increases without bound with the horizon of the peg. This is clearly implausible and so it has inspired a large and growing literature exploring modifications to the NK framework that dampen the effects of forward guidance as traditionally conceived.[1]

The strong effects of forward guidance in these models derive from the assumption that the central bank can clearly communicate credible commitments to particular interest rate paths. Is it plausible that the central bank can do so? If there are failures to communicate clearly and credibly, what are their implications for the conduct of policy? Our framework allows us to address these questions.

Consistent with the previous literature forward guidance is conceived of as communications about future deviations from the central bank's policy rule. The central bank cannot perfectly communicate these deviations in advance but instead is limited to sending noisy signals about them. The public's expectations are influenced as they learn from the signals about future policy deviations. Using Campbell et al. (2012)'s terminology the central bank wants to communicate Odyssean forward guidance but is limited in doing so by its inability to send perfectly understood signals. Our framework abstracts from Delphic forward guidance.

We interpret the noise as reflecting two key challenges to the communication of forward guidance. The most obvious challenge is that the words used by central bankers can confuse the public. One example is the "taper tantrum" episode in May 2013 when bond markets seemingly over-reacted to remarks by Chairman Bernanke about winding down the Fed's bond purchases. This reaction was puzzling to many observers since earlier Fed communications should have prepared markets for this eventuality. Another challenge is that the central bank simply does not know how it might want to deviate from its established rule in the future. For example, the policy rule assumed in standard models does not address risk management. Even when the policy rate is far from the ELB central banks might want to deviate from their implicit rules to guard against newly perceived risks to the economic

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[1] Key contributions include Del Negro et al. (2015) who emphasize finite lifetimes, Gabaix (2016) and Angeletos and Lian (2018) who consider bounded rationality, McKay et al. (2015) and Hagedorn et al. (2019) who study incomplete markets, Campbell et al. (2017) and Michaillat and Saez (2018) who focus on preferences for government bonds, Campbell and Weber (2019) who study imperfect credibility, Farhi and Werning (2017) who explore the interaction of bounded rationality and incomplete markets, and Kaplan et al. (2018) who study agent heterogeneity. In most of these papers the attenuated effects of forward guidance arise due to particular forms of discounting. Nakata et al. (2019) study optimal monetary policy in models with such discounting.

outlook. The history of Fed communications is replete with examples of such behavior.[2]

The communications framework is embedded within an otherwise familiar medium-scale NK model which is then estimated using a rich array of macroeconomic data. An important feature of our empirical strategy is that expected future interest rates and forward guidance in the model reconcile data on aggregate activity with interest rate futures' prices. Our estimated model suggests that the Fed's ability to affect expectations at horizons that are sufficiently long to give rise to the forward guidance puzzle is substantially limited. The difficulties inherent in communicating complicated decisions about the future path of interest rates may be too great for forward guidance to be the powerful tool predicted by standard NK models. We also show that imperfect communications heavily influence the propagation of monetary shocks. The response of the economy to a shock that changes the anticipated path of policy deviations is delayed and has greater amplitude when communications are imperfect compared with when they are not.

Another contribution of the paper is a novel decomposition of the dynamic effects of forward guidance into the part solely due to the change in expectations it triggers and the part arising from a scenario in which the central bank opts not to communicate its future policy deviations. Two new results emerge from this decomposition. First, private sector expectations of future policy deviations pull forward the effects of those deviations significantly compared with not communicating at all. This suggests that forward guidance, even if it is imperfectly communicated, could be a valuable tool for a central bank seeking to guard against perceived risks to the economic outlook. Second, unintended communication that contains no information about future policy deviations leads to sizable macroeconomic volatility. This result highlights the benefits that might accrue to a central bank that invests in developing effective communications. It also helps to explain why central bankers are typically wary of what they say in public.

To solve and estimate our model we exploit the observational equivalence established by Chahrour and Jurado (2018). In particular there is an observationally equivalent version of our model in which the central bank perfectly communicates *news* about future interest

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[2] See Evans et al. (2015) for examples of risk management considerations in statements and minutes of the Federal Open Market Committee.

rates. Another of our contributions is to develop a simpler demonstration of this link. The literature thus far has almost exclusively modeled forward guidance as news. Why focus on signal extraction instead? If one wants to evaluate the effectiveness of forward guidance as a policy tool our model seems more appropriate. The reason is that by focusing on news one abstracts from how effectively central bank announcements affect private expectations. Our model addresses the communication challenges inherent in forward guidance and allows us to shed light on the ability of the central bank to exploit forward guidance. Of course, observational equivalence means that the data cannot select one approach over the other. Nevertheless the signaling approach seems better suited to interpreting the evolution of expectations and therefore the nature of the communications challenges faced by central banks.

The importance of communications in the transmission of monetary policy has long been acknowledged. For example, Woodford (2003) emphasizes that successful monetary policy is not so much a matter of effective control of overnight interest rates as it is the shaping of market expectations about how interest rates, inflation, and output evolve in the future. One of our objectives is to evaluate the Fed's ability to influence these expectations. There are several papers that study the central bank's ability to shape expectations. Two key contributions include Eusepi and Preston (2010) and Andrade et al. (2018). The former develop a model with learning to study the link between central bank communications and the anchoring of inflation expectations. The latter study the normative implications of forward guidance in a NK model where agents have heterogeneous beliefs about the strength of the central bank's commitment. Melosi (2017) studies the Delphic effects of monetary policy in a model with heterogeneous beliefs in which the central bank reveals information about the state of the economy by setting the current policy rate but does not send any signals about its future path.

There are a few papers that use structural models to study the quantitative implications of forward guidance beginning with Del Negro et al. (2015). Campbell et al. (2017) quantify the macroeconomic effects of forward guidance after the onset of the Great Recession. Their analysis focuses on news about future interest rates and therefore has little to say about the ability of the central bank to communicate its future intentions. Bianchi and Melosi (2017) use a NK model to evaluate the welfare implications of forward looking communications. Their empirical framework does not exploit the information contained in interest rate futures data which is central to our analysis. Levin et al. (2010) use a simple NK model to show that the effects of forward guidance are sensitive to the nature of the shocks. Finally, Nakamura and Steinsson (2018) develop a stylized NK model which they use to estimate the strength of Delphic forward guidance. None of the papers in this literature empirically evaluate the central bank's ability to steer private-sector expectations at different horizons.

Our work is also related to the reduced form empirical literature launched by Kuttner (2001) which uses event studies to identify unexpected FOMC policy actions and their effects on macroeconomic outcomes and asset prices. Gürkaynak et al. (2005) extended that methodology to identify forward guidance shocks which leave the current policy rate unchanged while impacting current expectations of its future values. They found substantial impacts of near-term (one to six quarters out) forward guidance shocks on two, five, and ten year Treasury bond yields. Campbell et al. (2012) verified that this pattern continued while the FOMC set its policy rate at the ELB. The large and growing event study literature has not settled on an interpretation of these long horizon effects. Our analysis strongly suggests that they are not due to the ability of the Fed to shape expectations over such long horizons.

The remainder of the paper begins by presenting the central bank communications environment. It then describes the model in which this environment is embedded and our estimation of this model. Three sets of findings are described next: the comparison of the noise identified from the model to the historical record of forward guidance; the information flows implied by our estimates; and the propagation of forward guidance shocks under various assumptions about communication. The penultimate section examines a forward guidance experiment analogous to the one in Del Negro et al. (2015) but tailored to the conditions prevailing in the U.S. economy in 2016q4. The last section offers concluding remarks.

2 Central bank communications

The central bank sets the quarter t interest rate on one-period government bonds, R_t, according to

$$R_t = g_t(R_{t-1}, \pi_t^{gap}, y_t^{gap}) + \theta_t, \tag{1}$$

where g_t is the possibly time-varying *policy rule* which depends on the lagged policy rate and the central bank's measures of the inflation and output gaps, π_t^{gap} and y_t^{gap}. The private sector observes R_t, understands that policy is set according to (1), and the central bank perfectly communicates its measures of the gaps. Deviations of the policy rate from the rule, θ_t, are exogenous, stationary, and zero mean Gaussian random variables. These are allowed to be serially correlated up to the H-th lag. This is to be consistent with the fact that changes in interest rate futures before and after the release of FOMC statements are correlated across their term structure.

The policy deviations capture two key aspects of monetary policy that the central bank may want to implement contemporaneously or communicate about in advance. During "normal times" when the policy rate is relatively far from the ELB they represent the central bank implementing a modestly different policy than what its rule would otherwise stipulate, along the lines discussed by Leeper and Zha (2003) and Laséen and Svensson (2011).[3] A natural interpretation of these deviations is that they capture *risk management* behavior by the central bank. That is, the central bank's responses to changes it perceives in the risk environment. In the linearized model studied below risk has no impact on agents' decisions. Yet the interest rate futures data used to estimate the model embeds responses by the Fed to changes in risk. If such shocks to the risk environment are orthogonal to the other shocks which drive fluctuations and if the Fed's responses to them are systematic, then our assumptions about the deviations do not seem particularly restrictive. The policy deviations also capture monetary policy decisions when the policy rate is near the ELB or constrained by it. To some extent these deviations are similar to risk management. But they also incorporate the Krugman-Eggertson-Woodford policy of keeping rates "lower for

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[3] The modifier "modestly" indicates that the deviations from the rule are not sufficiently large to lead agents to change their view that the central bank is committed to the policy rule g_t.

longer."

Forward guidance consists of communicating to the private sector about future deviations of the policy rate from the rule up to H periods ahead, θ_{t+h}, $h = 0, 1, 2, \ldots, H$. Communication at date t consists of sending an $(H+1) \times 1$ vector of noisy signals $s_t = [s_t^h]$ about the future deviations $\boldsymbol{\theta}_t = [\theta_{t+h}]$. The signals are specified as:

$$s_t = \boldsymbol{\theta}_t + v_t, \tag{2}$$

where the vector $v_t = [v_t^h]$ denotes noise. The noise is Gaussian with mean zero and variance-covariance matrix Ξ_v.

Since agents observe current and lagged policy rates as well as the central bank's gaps, they observe θ_t as well so that $v_t^0 = 0$. The remaining noise should be interpreted as comprising two components. The first component represents the central bank's imperfect knowledge about what it will do in the future. For example, unforseen events such as the near failure of Long-Term Capital Management in September 1998 and the September 11, 2001 terrorist attacks led the Fed to reassess risks to the macroeconomic outlook and lower the federal funds rate seemingly below the level indicated by its contemporaneous measures of the output and inflation gaps. The second component represents miscommunication to the private sector about the central bank's true intentions.

The private sector observes the signals, knows the stochastic processes governing $\boldsymbol{\theta}_t$ and v_t, and updates their beliefs about $\boldsymbol{\theta}_t$ in a Bayesian fashion. The Gaussian structure of the shocks implies that agents use the Kalman filter. Specifically, expectations following the release of date t signals are updated as

$$E_t\boldsymbol{\theta}_t = E_{t-1}\boldsymbol{\theta}_t + \kappa \cdot \left(s_t - E_{t-1}\boldsymbol{\theta}_t\right), \tag{3}$$

where $\kappa = \Xi_\theta \left[\Xi_\theta + \Xi_v\right]^{-1}$ is the Kalman gain and Ξ_θ denotes the variance-covariance matrix of current and future deviations from the rule, $\boldsymbol{\theta}_t$, conditional on receiving signals up to date $t-1$, i.e. Ξ_θ is the variance-covariance matrix of $\boldsymbol{\theta}_t - E_{t-1}\boldsymbol{\theta}_t$. We have assumed that the variance-covariance matrix of noise is stationary and made assumptions such that the conditional variance-covariance matrix Ξ_θ is as well.[4] Therefore the Kalman gain is stationary as well. Notice that in this setup, even if communication is perfect in some periods, it will not be interpreted that way unless the Kalman gain is the identity matrix. This captures another conundrum for central bankers, namely past imperfect communication hampers the ability to communicate effectively going forward.

In principle one would like a communications framework with time-varying second moments. There are plenty of reasons to think that the signal extraction problem is non-stationary. For example, Fed communications have evolved from there being no statements following FOMC meetings to there being detailed statements, an explicit inflation target, published FOMC participants' economic projections, and press conferences. Furthermore each Fed Chair has their own style of communications and the composition of the FOMC changes. In our empirical analysis we go one step in the direction of allowing for a time-varying signal extraction problem by assuming a sample split in which the horizon and stochastic process of forward guidance is allowed to change. This approach seems like a natural start, but clearly more can be done.

Our estimated general equilibrium model along with interest rate futures data and realized interest rates allow us to identify the signals and noise. Changes in interest rate futures and the path of interest rates implied by the estimated policy rule identify the revision of agents' expectations about future deviations of the policy rate from the rule, $E_t\boldsymbol{\theta}_t - E_{t-1}\boldsymbol{\theta}_t$.[5] Equation (3) shows that these revisions of expectations are triggered by the realization of the signals via our estimate of κ. The noise is then identified as the residual in equation (2) once the signals and the actual future deviations from the rule are fully realized. The actual future policy deviations are simply the difference between the policy rate implied by the rule in future periods and the observed federal funds rate in those same future periods.

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[4] To see this notice that in every period t agents have received one signal about the H-period-ahead deviation θ_{t+H}; two signals about the $(H-1)$-period-ahead deviation θ_{t+H-1} (one today and the other yesterday), and so on. Given that the noise is stationary, it follows that the horizon of each central bank announcement is a sufficient statistic for tracking agents' beliefs about θ_{t+h} over time. Therefore the variance-covariance matrix of the vector of deviations $\boldsymbol{\theta}_t$ conditional on publicly available information at date $t-1$ is time-invariant.

[5] Note that all the variables of the monetary policy rule are observed in estimation.

3 Model and estimation

Our empirical investigation of forward guidance embeds the communications environment just described within Campbell et al. (2017)'s medium-scale NK model and estimates it using Bayesian methods. Since much of the model is familiar – it is a variant of Christiano et al. (2005) and Smets and Wouters (2007) – this section provides just a brief overview, highlighting key features that distinguish our model from the literature. Similarly it highlights key features of the estimation leaving the details to the Appendix.

3.1 Model overview

The representative household's preferences are non-separable over consumption and labor and separable with respect to real government bonds. Including preferences for government bonds has important implications for our measurement. As discussed by Campbell et al. (2017) they allow for an empirically plausible spread between interest rates on private and government bonds that is otherwise absent. This brings discounting into the household's linearized inter-temporal Euler equation for consumption which mitigates the forward guidance puzzle. Preferences are buffeted by shocks to the discount factor and to the preference for government bonds. Fisher (2015) showed the latter shock provides a simple micro-foundation for Smets and Wouters (2007)'s ad hoc shock to the consumption Euler equation.[6] The two preference shocks follow AR(1) processes.

The remainder of the specification of the private sector is relatively standard. It includes Calvo-style sticky wages and prices with mark-up shocks and partial indexing; variable capacity utilization with capital depreciation an increasing function of utilization; stochastic investment adjustment costs; balanced growth driven by neutral and investment-specific technical change subject to permanent shocks; and government spending shocks. The shocks to the growth rates of the two technologies, investment, and government spending all follow AR(1) processes. The mark-up shocks follow ARMA(1,1) processes.

The central bank sets its policy rate, communicates with the public, and the public

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[6] This shock is crucial to the identification of empirical NK models since it is one of the few sources of co-movement between consumption, investment and hours.

updates its expectations about policy deviations as in Section 2.[7] The monetary policy rule in (1) is assumed to be the same as in Campbell et al. (2017). This specification includes gap terms that depend on publicly observable variables and an inflation drift term to address inflation's low-frequency movements during our sample.[8] The inflation drift is AR(1).

Our empirical strategy involves the solution to the model's log-linearized equilibrium conditions and applies econometric techniques that rely on linearity to estimate the model's parameters. Without forward guidance shocks agents' expectations of future policy rates could violate the ELB constraint. However, our estimation prevents this from happening because it matches data on expected future funds rates, which of course do not violate the ELB, with the model's private expectations of future policy.[9] This approach has the added benefit of allowing the model to explain strategic deviations from the rule, such as a policy of "lower-for-longer" in which lift-off is delayed and slower than otherwise predicted by the rule.[10]

While our approach to addressing the ELB simplifies the analysis considerably, it relies on certainty equivalence. As such, our solution method does not take into account that the probability distributions of future outcomes are non-symmetric in models with occasionally binding constraints and that this asymmetry affects agents' beliefs and thereby equilibrium outcomes. This limitation also characterizes quasi-linear solution methods, e.g. Guerrieri and Iacoviello (2015). Indeed, Gust et al. (2017) find in a model similar to ours that along some dimensions the quasi-linear method can lead to biased results. It is unclear whether their findings apply to our case since they abstract from forward guidance and do not use interest rate futures data in estimation. For example, the communication of lower-for-longer policies and other strategic deviations from the rule are excluded from their analysis.

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[7] Our model abstracts from the Fed's large scale asset purchases. To the extent that the macroeconomic effects of these policies were through signalling or establishing a commitment to keep rates lower for longer they can be viewed through the lens of forward guidance communications we consider in this paper.

[8] Since the policy rule is perfectly communicated so is the drift term. Clearly this is not an innocuous assumption since the drift influences agents' inflation expectations and the Fed's communications about its inflation objectives were almost surely imperfect over much of our sample period.

[9] Since the ELB is not imposed explicitly, distributions of interest rates over states on given dates include negative values. Our model solution is certainty equivalent so this does not influence agents' decisions.

[10] The optimality of this kind of policy under discretion is studied by Evans et al. (2015).

3.2 Estimation

Chahrour and Jurado (2018) show that models like ours have an observationally equivalent *news representation*. In our context this involves the central bank perfectly communicating news about future policy deviations. The news representation is easier to solve so we estimate it and use it to identify the key communications parameters Ξ_θ and Ξ_v. This approach is similar to Blanchard et al. (2013).

Let ε_t^h denote news about the policy deviation h periods hence revealed to the public through the policy signals at date t, s_t. By the definition of news $\varepsilon_t^h = E_t\theta_{t+h} - E_{t-1}\theta_{t+h}$. The forecast errors of policy deviations for each $h = 0, 1, \ldots, H$ periods ahead are related to news as follows:

$$\theta_{t+h} - E_{t-1}\theta_{t+h} = \sum_{j=0}^{h} \varepsilon_{t+h-j}^{j}. \tag{4}$$

This equation states that the error in the forecast of θ_{t+h} made in date $t-1$ equals the sum of all news about it revealed from date t to date $t+h$. The variance-covariance matrix of the $H+1$ random variables $\theta_{t+h} - E_{t-1}\theta_{t+h}$ corresponds to Ξ_θ. Therefore given a stochastic process for news Ξ_θ is obtained using the relationship between news and the forecast errors of policy deviations given by (4).

The date t revisions of private sector expectations about future policy deviations are given by equation (3). It and the definition of news imply

$$\kappa\left(\boldsymbol{\theta}_t + v_t - E_{t-1}\boldsymbol{\theta}_t\right) = \boldsymbol{\varepsilon}_t, \tag{5}$$

where news about policy deviations up to H periods ahead is collected in the vector $\boldsymbol{\varepsilon}_t$. Using (5) the communications technology's variance-covariance matrices and the variance-covariance matrix of news, $\Sigma_\varepsilon \equiv E(\boldsymbol{\varepsilon}_t\boldsymbol{\varepsilon}_t')$, satisfy $\Xi_\theta\left[\Xi_\theta + \Xi_v\right]^{-1}\Xi_\theta = \Sigma_\varepsilon$. With Ξ_θ in hand this relationship is used to solve for Ξ_v.

The foregoing demonstrates that given a stochastic process of news one can obtain the parameters of the communications technology. The stochastic process for news is obtained by estimating the news representation of the model. In the news representation each period t the central bank perfectly communicates the $(H+1)\times 1$ vector $\boldsymbol{\varepsilon}_t$. At the time this information is communicated agents believe it represents credible commitments to deviate from the rule up to H periods in the future with the full knowledge that more news may come along that changes expectations about future deviations. Any given period's policy deviation is communicated through news for up to H periods before the policy deviation is realized. It follows that the policy deviation at quarter t, θ_t, is related to previous news according to

$$\theta_t = \sum_{j=0}^{H} \varepsilon_{t-j}^{j}. \tag{6}$$

The news representation is obtained simply by dropping central bank communications from the model, replacing θ_t in equation (1) with $\sum_{j=0}^{H} \varepsilon_{t-j}^{j}$, and specifying a stochastic process for news. News' stochastic process is the factor structure in Campbell et al. (2017) and is described in the appendix. This allows for the possibility of serially correlated policy deviations and so is consistent with our model.[11]

Our estimation of the news representation follows Campbell et al. (2017) except that here the well-known evidence of secular declines in economic growth and nominal risk free interest rates is addressed. Our discussion here is brief and more details are in the Appendix. The sample period of our estimation is 1993q1–2016q4.[12] Parameters that appear in the corresponding neoclassical growth model are calibrated to match sample averages from the U.S. economy. These parameters determine the model's steady state. The secular declines in growth and interest rates are addressed by imposing a change in steady state in 2008q4. The steady state growth rates of the two technologies are cut to reduce steady state GDP growth from 3% to 2% and the steady state demand for government bonds is increased to reduce the steady state risk free rate from 2% to 1%.[13] The date for the sample break is justified by the evidence that points to lower interest rates and trend economic growth later in the sample, the apparent increase in the horizon over which forward guidance was communicated during

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[11] The Appendixs explain in detail how to obtain estimates of Ξ_θ and Ξ_ν as well as the time series of noise, v_t, from the estimated model. Equation (6) is used obtain the time series for $\boldsymbol{\theta}_t$.

[12] The start date is based on the availability and reliability of our interest rate futures data.

[13] These adjustments leave the other calibrated parameters unchanged but do change the steady state values of the endogenous variables and therefore the point at which the economy is log-linearized. Our re-calibration changes the return on private assets by a little. This small change is consistent with Yi and Zhang (2017) who showed that rates of return on private capital appear stationary in the face of declines in risk free rates.

the ELB period, and the halt to the downward trends in inflation and inflation expectations in the mid-2000s.

Given the calibrated parameters standard Bayesian methods are applied to estimate the remainder of the model plus some auxiliary parameters which are used to account for chain-weighting when mapping the model into the data. The full suite of non-calibrated structural parameters is estimated using the 1993q1–2008q3 sample under the assumption that forward guidance extends for $H = 4$ quarters. Starting in 2008q4 the model environment is assumed to change in three ways and then the model is re-estimated using the 2008q4–2016q4 sample. First, the change in steady state described above is imposed. Second, forward guidance is lengthened to $H = 10$ quarters.[14] Third, the inflation drift from the first sample becomes a constant equal to the steady state rate of inflation, 2% at an annual rate. All three changes are assumed to be unanticipated and permanent.

Twenty-one time series are used to estimate the model in the first sample. In addition to the real quantities and federal funds rate that are standard in the literature our estimation includes multiple measures of wage and consumer price inflation, two measures each of average inflation expected over the next ten years and over one quarter, and $H = 4$ quarters of interest rate futures. The second sample estimation is restricted to estimating the parameters of news' stochastic process with $H = 10$ plus the processes driving two auxillary variables used to map the model to the data, holding fixed the remaining parameters at their values estimated using the first sample. Six additional quarters of interest rate futures data are used for this estimation.[15] Because our estimation forces data on real activity, wages and prices to coexist with the interest rate futures data, we expect the estimation to mitigate the forward guidance puzzle.

The estimated model shares the desirable features of Campbell et al. (2017)'s model. Mark-up shocks contribute very little to real fluctuations and in the first sample monetary policy shocks contribute little as well. The model continues to provide a plausible interpreta-

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[14] Extending the horizon seems justified on *a priori* grounds – it appears the Fed attempted to shape expectations over a longer horizon in the second sample. However it also guarantees the ELB is never violated in expectation. Results presented below rationalize our assumptions for H in the sense that they show that at the limit of each horizon very little new information is transmitted.

[15] Our identification of news' stochastic process with only 33 observations in the second sample relies on our priors. These are informed by estimating a factor model over the second sample using Gürkaynak et al. (2005)'s high-frequency strategy.

tion of the two recessions in our sample. The 2001 recession is explained by tighter financial conditions, i.e. an increase in demand for government bonds, and to a lesser extent weaker technology growth. During this recession the sharp drop in the funds rate is larger than stipulated by the policy rule and this lifts aggregate activity and inflation. Tighter financial conditions are the main driver of the Great Recession and the ELB led to monetary policy being a substantial drag on real activity. Finally, the estimated model does not exhibit Del Negro et al. (2015)'s forward guidance puzzle. In particular, their experiment in which the low interest rate peg anticipated by markets in 2012q3 is extended by one quarter yields small effects on real activity and inflation in our estimated model.

4 Assessing the plausibility of the estimated communication

To build confidence in our estimated model the identified noise is now compared to the historical record of forward guidance. We focus on the second sample during which forward guidance was most prominent and there are a greater number of easily identifiable episodes when it changed. Figure 1 shows the contribution of all the contemporaneous noise to the expectation of the six-quarter-ahead funds rate at each indicated date. The dashed and solid lines are the market-expected and realized funds rate six quarters out. The bars show the contribution of noise to the expected rates. The difference between the bars and the expected funds rates reflects the contributions of expectations of the arguments of the policy rule, which include the effects of past communication, and the future deviations from the rule. When forward guidance is successful the effect of noise should be small.

The seven vertical lines indicate notable changes in forward guidance and other unconventional monetary policies. Following the December 2008 FOMC meeting the statement introduced the forward guidance that rates would stay exceptionally low "for some time." In the statement following the March 2009 meeting "some time" was replaced by "extended period." The announcement of "QE2," that is the second round of large scale asset purchases (LSAPs), following the November 2010 meeting is included because Krishnamurthy and Vissing-Jorgensen (2011) and others have hypothesized it to have had a signalling effect. In particular, it may have had a role in cementing expectations about lower rates for a longer period than otherwise. The statement following the August 2011 FOMC meeting replaced "extended period" with language indicating that the FOMC anticipated conditions would warrant rates at the ELB "at least through mid-2013." This was the first instance of *date-based forward guidance.* The September 2012 statement announced the open-ended LSAPs known as "QE3" which also may have had a signalling effect. In December 2012 the statement replaced date-based guidance with *threshold-based forward guidance.* This involved language indicating that rates would stay at the ELB *at least as long as* the unemployment rate exceeded 6.5%, inflation was projected not to exceed 2.5%, and inflation expectations remained well-anchored. The final episode we examine is the replacement of threshold-based guidance with language that indicated the FOMC would consider a broader array of economic statistics when deciding whether to lift rates from the ELB. This *state-based forward guidance* was first used in the March 2014 statement.

Figure 1 shows that for almost the entire ELB period noise contributes positively to the expected funds rate 6 quarters ahead. This suggests the FOMC was, at least in hindsight, more hawkish than it intended. Throughout most of the ELB period expected rates were substantially higher than they turned out to be. This was not all due to the noise of the day. Much of it is due to structural shocks feeding through to the policy rule. Indeed early on when the "some time" and "extended period" language was first used, it was apparently quite effective – the noise was small. However the longer the "extended period" language was used, the rise in noise indicates its meaning became less clear to market participants. The announcement of QE2 did not bring much clarity initially, but the longer it was in place noise fell. This may have been due to the signalling effect. QE3 appears not to have been as helpful in cementing expectations.

The most striking reduction in noise came with the introduction of date-based guidance. Essentially all of the drop in the expected funds rate at that time was due to the reduction in noise. This seems to accord well with the intentions of the FOMC at the time. The meeting transcripts leading up to the change in guidance clearly indicate the FOMC was concerned that market participants did not appreciate that they intended to keep rates lower than the market was expecting. The date-based guidance was introduced precisely because of this concern and appears to have had the desired effect.[16] In the lead up to threshold-based guidance the transcripts indicated that many of the FOMC participants were concerned that the date-based guidance was being interpreted by market participants as representing a commitment to keep rates low for the period specified in the statement even though the language of the statement indicated the FOMC intended for markets to appreciate that the setting of rates would depend on economic developments. Evidently the threshold-based guidance did not improve private forecasts of FOMC actions.

Following the introduction of threshold-based guidance the unemployment rate fell much faster than anticipated. As the rate approached the 6.5% threshold, futures rates began to climb. The transcripts from that time suggest that the FOMC was concerned that the market was not absorbing the "at least" part of this guidance. This, and the fact that the unemployment threshold was fast becoming obsolete, was the motivation for the change to state-based guidance. It had the desired effect, leading to a reduction in the futures rate which can be attributed almost entirely to the reduction in noise at that time. Soon after the futures rate started rising again as the unemployment rate continued to fall. The rise in the contribution of noise to the futures rates after the state-based guidance was introduced indicates that with the benefit of hindsight the FOMC was much more hawkish in its communications than it intended. Indeed noise lead to a more than 150 basis points increase in the futures rate beyond what the rule stipulated and the FOMC intended to communicate.

Our estimated noise measures the impact of FOMC communications on market outcomes. These communications may or may not have been effective, and the plausibility of our estimates of noise do not rest on them responding to every change in forward guidance. However, the fact that for some of the dates noise simultaneously fell when guidance changed lends credence to our estimates.

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[16] This finding is consistent with Campbell et al. (2017)'s conclusion that it was only after date-based guidance was introduced that forward guidance was effective in improving macroeconomic outcomes.

5 The information flows from Fed communications

This section quantifies the information content of our estimated policy signals. This is accomplished in two ways. First, we measure the contribution of each signal emitted at an arbitrary date to reducing uncertainty about the path of interest rates from that date. Second, we measure how agents learn about a given future deviation as they receive more signals about it over time. These exercises offer evidence on the limits of forward guidance if the central bank tries to steer private sector expectations over many periods.

5.1 Information about the policy path embedded in s_t

The information about the policy path embedded in s_t is measured starting from a situation in which agents have received signals up to $t-1$ and do not have any time t signals. Suppose that at time t the central bank sends signals within the vector s_t sequentially starting with the longest horizon H. For each new signal the reduction in uncertainty is measured relative to not having received any time t signals. This allows us to quantify the role of the individual signals in reducing agents' uncertainty about the monetary policy path.

In our model agents' uncertainty about future policy is encoded in the conditional and unconditional variance-covariance matrices of the signals. The concept of entropy, defined as the average uncertainty of random vectors, allows one to summarize this uncertainty with a scalar while preserving the estimated correlation structure of the signals. For a Gaussian distributed random vector, $x \sim N(\mu, \Sigma)$, the entropy is given by $\xi(x) = \frac{1}{2}\log_2|\Sigma| + \frac{n}{2}(\log_2 2\pi e)$, where n is the dimension of x. The role of individual signals in reducing agents' uncertainty about the path of monetary policy is quantified using the reduction in entropy due to the signals.[17]

Our measurement focuses on the future path of monetary policy, $\boldsymbol{\theta}_t^p$, the $H \times 1$ vector of future policy deviations that excludes the perfectly revealing contemporaneous signal $s_t^0 = \theta_t$. The *information gains*, or reduction in uncertainty, induced by the revelation of signals is measured by taking the difference between the posterior entropy after receiving the signals

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[17] Reduction in entropy is a widely used metric in the engineering literature to measure information flows, e.g. Cover and Thomas (1991). In economics, the rational inattention literature uses this measure to characterize the information-processing constraint, e.g. Sims (2003) and Maćkowiak and Wiederholt (2009).

of policy deviations H to $H-h$ quarters out, $h = 0, 1, \ldots, H-1$, minus the prior entropy before receiving any new signals. Specifically, the information gains of receiving the $h+1$ signals for horizons H to $H-h$ is defined as

$$\mathcal{G}(h) = 1 - \exp\left[\xi(\boldsymbol{\theta}_t^p \mid s^{t-1}, s_t^H, s_t^{H-1}, \ldots, s_t^{H-h}) - \xi(\boldsymbol{\theta}_t^p \mid s^{t-1})\right].$$

Here s^{t-1} denotes the history of all signals received up to and including period $t-1$. The variance-covariance matrices of the posterior and prior distributions of $\boldsymbol{\theta}_t^p$ which are necessary for calculating $\mathcal{G}(h)$ are derived in the Appendix.

The information gains $\mathcal{G}(h)$ induced by signals $H-h$ to H indicate the fraction of the reduction in the entropy of the monetary policy path $\boldsymbol{\theta}_t^p$ that is attributable to these signals alone. When the difference between the posterior and prior entropy (the argument in the exponential) is close to zero, the reduction in uncertainty is small and hence so are the information gains. Conversely, when the reduction in the posterior entropy is sizable, the argument in the exponential is negative and information gains become large. If all uncertainty is resolved after receiving $h+1$ signals then $\mathcal{G}(h) = 1$.

Figure 2 reports the information gains in the first and second samples. The first sample plot shows close to zero reduction in uncertainty about the interest rate path after receiving s_t^4. The reduction in uncertainty is just 20% when agents have received s_t^3 and s_t^2 as well. It is only when they receive the signal about next quarter's policy rate (s_t^1) that a substantial reduction in uncertainty can be seen. Even then 40% of the original entropy remains. So in normal times it appears the Fed has a very limited ability to use forward guidance to influence private sector expectations.

The second sample plot shows virtually zero reduction in uncertainty 10 quarters out. There is a discrete jump 7 quarters out and another 6 quarters out that lifts the information gain above 50%. By 2 quarters out about 80% of uncertainty is reduced. These results cast doubt on the power of forward guidance to affect agents' uncertainty about future deviations at horizons very far from the policy implementation. However the larger information gains at longer horizons in the second sample compared to the first sample suggest that the tool of forward guidance may have some bite during periods of extreme economic distress.

Nevertheless there are limits to the power of this tool even then. It seems very difficult to convince the public that an interest rate peg can be extended for very long. From this perspective there is no forward guidance puzzle. In particular, Del Negro et al. (2015) frame the forward guidance puzzle as implausibly large effects on aggregate activity from extending the 10 quarter low interest rate peg expected by markets in 2012q3 to 11 quarters. Figure 2 suggests it was not possible for the Fed to communicate such an extension.

The relatively large information gains at longer horizons in the second sample might arise from two factors. First, it is possible that the extreme economic conditions during the second sample may have led agents to pay more attention to the Fed than they did during more normal times. This should be reflected in the Kalman gain matrix.[18] Second, it might be easier to communicate when policy is constrained by the ELB. Communicating policy deviations while being constrained by the ELB boils down to explaining how long the policy rate will remain near zero.

5.2 Accumulation of knowledge about a future policy deviation

The rate at which agents' uncertainty about the policy deviation at a fixed future date resolves over time is now examined. We call this the *accumulation of knowledge* about a future policy deviation. The accumulation of knowledge is quantified by the reduction in entropy of the future policy deviation as its implementation approaches. Specifically, the reduction in entropy after receiving $h+1$ vectors of signals that include information about the policy deviation at $t+H$ is given by

$$\mathcal{K}\left(h\right)=1-\exp\left[\xi\left(\theta_{t+H}|s^{t+h}\right)-\xi\left(\theta_{t+H}|s^{t-1}\right)\right],\quad h=0,1,2,\ldots,H.$$

Here $\xi\left(\theta_{t+H}|s^{t-1}\right)$ is the prior entropy of θ_{t+H}, which measures agents' uncertainty about θ_{t+H} before having received any signals about it, and $\xi\left(\theta_{t+H}|s^{t+h}\right)$ is the posterior entropy conditional on receiving $h+1$ vectors of signals.[19] Since the last signal is perfectly revealing

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[18] Our analysis has not been framed in terms of the optimal allocation of attention. However there is a close connection between the Kalman gain and this allocation problem. See for example Maćkowiak and Wiederholt (2009) and Melosi (2014).

[19] The variance used in the posterior entropy $\xi\left(\theta_{t+H}|s^{t+h}\right)$ corresponds to the $(H+1-h)\times(H+1-h)$ element of the variance-covariance matrix of the posterior distribution of the entire state $\boldsymbol{\theta}_t|s^{t+h}$ which is

$\mathcal{K}(H) = 1$.

Figure 3 reports the accumulation of knowledge, $\mathcal{K}(h)$, in the first and second samples. The x-axis in the plots indicates the quarters before the policy deviation is implemented. For example the left-most bars correspond to the case $h = 0$ in which only one vector of signals about the policy deviation at $t + 4$ has been communicated. Knowledge accumulates at a fairly steady pace both in the first and second sample. In the first sample the reduction in uncertainty is sizable only one period away from the policy implementation, while in the second sample the size of uncertainty is reduced by half more than a year before the policy is implemented. Figure 3 reinforces the main lessons from Figure 2. In the first sample it was hard to communicate much more than 1 quarter before a policy decision. In the second sample the communication horizon was substantially longer, but limited.

6 The dynamic effects of a forward guidance shock

This section explores the role of communication in the dynamic response of the economy to a hypothetical *forward guidance shock*, defined as an orthogonalized shock to $\boldsymbol{s}_t$ that gives no signal about the current deviation. The forward guidance shock is constructed by first decomposing the signal equation (2): $s_t = \Phi u_t$, where $u_t \sim \mathcal{N}(\mathbf{0}, \mathbf{I})$ is a $(H+1) \times 1$ random vector of shocks and the matrix Φ is lower triangular with $E(\Phi u_t u_t' \Phi') = E_{t-1}(\boldsymbol{s}_t \boldsymbol{s}_t') = \Xi_\theta + \Xi_\nu$. A forward guidance shock corresponds to a unit innovation to the second element of the vector u_t fixing all other elements equal to zero.[20]

The dynamic response of the model economy to a forward guidance shock so constructed is used to make three main points. First, imperfect communication delays and later amplifies the response of the economy to forward guidance compared to perfect communication. Second, even with imperfect communication, forward guidance can substantially shift the effects of future interest rate changes into the present. Finally, miscommunication in the form of noise is a source of macroeconomic volatility.

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described in the Appendix.

[20] Other shocks are potentially interesting to study. The h^{th} shock in u_t is the shock that does not affect signals concerning deviations that will occur from period t through period $t + h - 1$. While these shocks certainly belong to the forward guidance class, they do not affect *all* the signals concerning future deviations from the rule. Therefore, the second shock seems like the most natural candidate to study.

6.1 The role of imperfect communication

To study the role of imperfect communication in forward guidance propagation, suppose that the central bank announces a vector of signals in an arbitrary quarter t that contains no noise and comprises its actual policy deviations from quarter $t+1$ through quarter $t+H$. The announcement comes when the economy is initially at steady state, and so before it is made, agents expect that all future deviations are zero. However, to be consistent with our learning environment, suppose that before the announcement agents have received $H - h$ signals equal to zero about the hth quarter ahead policy deviation, and this information is embedded in their prior uncertainty. Further, over the period $t + 1$ to $t + H$ the central bank continues to send accurate signals about its policy deviations but does not send non-zero signals beyond the horizon of the initial forward guidance. Since the signals are the only source of information for the private sector to learn about future policy deviations, agents' expectations about the deviations from time $t + H$ onward equal zero, which is the unconditional mean of the policy deviations. No more deviations will be carried out after period $t + H$. These two assumptions imply that agents correctly anticipate the model economy is not hit by any shock after period $t + H$ and will transition back to the steady state.

Given this setup we consider two ways in which the forward guidance shock is communicated to private agents. In the *perfect communication* case agents learn with the Kalman gain matrix κ set equal to the identity matrix. That is, they believe that the central bank means exactly what it says. In the *imperfect communication* case agents doubt the central bank and so learn with the estimated Kalman gain matrix. These two cases allow us to measure how imperfect communication influences the response to forward guidance.

Figure 4 shows the influence of imperfect communication on agents' expectations of future policy deviations in the most recent sample. The forward guidance shock is normalized so that it causes the annualized interest rate to deviate from its rule-consistent value by 100 basis points after 10 quarters.[21] The red stars indicate the actual future deviations from the rule looking ahead from each date following the forward guidance shock. They also correspond to the signals sent by the central bank because these signals do not contain any noise. Since the

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[21] This requires us to re-scale the forward guidance shock by a factor of 4.

signals perfectly reveal the current deviation the stars associated with horizon 0 on the x-axis of each plot correspond to the actual deviation in the indicated period following the shock. The black lines show private agents' expectations about future policy deviations when they believe that noise contaminates central bank announcements. The expectations are over the next 10 quarters, $E_t\boldsymbol{\theta}_t$, at time t, which is when the first announcement is made, and in the following periods. With perfect communication the analogous expectations are just the red stars.

Figure 4 shows that at time t, the central bank spectacularly fails to communicate its future deviations from the rule. Expectations about future deviations hardly budge. The second announcement at time $t+1$ does not seem to materially move agents' expectations either. At time $t+2$, the third announcement has some impact and lifts private expectations toward the truth (the red stars) a bit. From period $t+3$ on, agents' expectations quickly rise, overshoot the true deviations in periods $t+6$ and $t+7$, and finally line up with the truth in period $t+8$ and in subsequent periods. These patterns seem very much in line with the knowledge accumulation shown in Figure 3 and suggest it takes four or five periods of forward guidance for agents to have gathered enough information to start adjusting their expectations in line with what the future deviations will actually be.

The findings in the top row of Figure 4 are striking and call for caution in using the news representation of a model to study the macroeconomic effects of forward guidance. In the news representation future deviations are perfectly communicated. This plot shows that the central bank's ability to steer expectations about future monetary policy is substantially limited. Therefore, using the news representation can lead to predictions like those underlying the forward guidance puzzle that would not arise if one takes into account the imperfect ability of the central bank to communicate.

Figure 5 shows the response of hours to the forward guidance shock in the most recent sample. The yellow and blue bars in this figure are discussed below. The focus here is on the black line and red stars. The black line and red stars correspond to the imperfect and perfect communication cases, respectively. Hence, the difference between them captures the effects of the central bank not being able to perfectly communicate the future course of policy. Imperfect communication delays the response of real activity to forward guidance and amplifies it in the medium term. At the time of the first forward guidance (time t), hours barely adjust because private sector expectations fail to react to the announcement (see Figure 4). As time goes by and more guidance is provided, expectations adjust, and consequently economic activity quickly deteriorates and contracts more than in the case of perfect information.

There are two reasons why imperfect communication triggers a deeper recession compared to perfect communication. First, the overshooting of expectations in periods $t+6$ and $t+7$ (see Figure 4) deepens the recession. Second, the delayed revisions of agents' expectations contribute to lower hours. In the perfect communication case, agents never revise their expectations about future deviations after the first announcement is made. Under imperfect communication agents largely fail to anticipate the path of policy initially and then revise their expectations slowly over time. Consequently, when agents finally learn the future policy deviations, they have less time to adjust to the consequences of them relative to the case of perfect communication. As in all standard NK models a shorter anticipation time for future deviations boosts the response of real activity. When firms anticipate a monetary shock far in advance, the effects will be relatively small because fewer firms are constrained by the Calvo lottery before the anticipated policy deviation is realized.[22] This is a manifestation of the vertical long-run Phillips curve. More flexible prices imply a smaller response of real activity. Thus, by slowing down the information flow from the central bank to the private sector, imperfect communication magnifies the effects of forward guidance on real activity.

Figure 6 shows the expected deviations from the rule following a forward guidance shock in the first sample. The figure is constructed analogously to Figure 4 and the scales are comparable so notice that forward guidance shocks in the first sample are only 8% as volatile as in the second sample.[23] Recall that forward guidance in the first sample extends out only $H = 4$ quarters. Figure 6 indicates the central bank poorly communicates its future policy deviations in the first three periods. This is reflected in the black lines being far from the

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[22] This result also holds in presence of Rotemberg-style price adjustment. A long anticipation horizon allows firms to smooth out the price changes over a longer period and hence lowers the cost for firms to change their price relative to a surprise monetary shock.

[23] The extremely small shocks in the first sample indicate two important features of our estimation. First, the small size of the deviations indicate the estimated policy rule is an excellent summary of the Fed's behavior. Second, the Fed did not do much forward guidance in the first sample. These features are likely influenced by our assumption that the inflation drift shock is perfectly communicated.

red stars in period t, when the first announcement is made, and in the following two periods. Consistent with the accumulation of knowledge shown in Figure 3, only after the central bank has made four announcements do private sector expectations move close to the actual deviations.

The response of hours to a forward guidance shock in the first sample under perfect and imperfect communication are shown in Figure 7. These are qualitatively similar to those in the second sample. Imperfect communication initially delays and later amplifies the response of hours. Compared with the second sample the effects are smaller due to the small size of the shock and less persistent due to the shorter duration of the guidance.

While forward guidance is exogenous and risk has no impact on agents' decisions in our linearized model, we think the dynamic responses of hours shown in Figures 5 and 7 are informative about the use of forward guidance as a risk management tool, for the reasons discussed in Section 2. The delayed response of the economy with our estimated communications technology suggests that imperfect communication hampers the use of forward guidance as a risk management tool. If a central bank wants to use forward guidance as a risk management tool, its effects may come too late to have the desired impact on the economy. Nevertheless, the extent of the communication imperfections do not render forward guidance completely impotent, which is demonstrated next.

6.2 The expectations channel of forward guidance

The Appendix shows how to decompose the effects of forward guidance into two interesting additive components. The first represents the sole effects of the change in expectations triggered by forward guidance. We call this component the *expectations channel* of forward guidance. The second component captures the effects of implementing the deviations by taking the private sector by surprise in every period. This is akin to the central bank opting not to communicate how it will deviate from its rule in the future.

To evaluate the expectations channel, consider the policy experiment in which the path of signals generated by the forward guidance shock is just noise. In this case expectations about the future deviations of monetary policy are the same as those discussed above under imperfect communication. This can be seen by inspecting equations (2) and (3). In addition, when forward guidance is noise-driven, the perfectly revealing signal for the contemporaneous deviation must be always equal to zero. This experiment captures the expectations channel of forward guidance because there are no actual policy deviations and so the only effects of forward guidance arise due to expectations that deviations will occur. The difference between the response of hours (or any other endogenous variable) due to the expectations channel and the overall response (the black lines in Figures 5 and 7) corresponds to the case where the central bank opts not to communicate its future deviations and conducts policy by surprising agents every time the deviations are implemented.

The yellow bars and blue bars in Figures 5 and 7 show the decomposition of the imperfect communication response of hours to the forward guidance shock into the expectations channel and the no communication components, respectively. The expectations channel heavily affects the propagation of forward guidance shocks and is most potent early on. In the second sample its effects dominate the response of hours in the first year and a half after the shock, but its intensity quickly diminishes thereafter. In the first sample the dynamics are similar but obviously more short-lived. The expectations channel dominates the response of hours over the first three quarters of guidance. Comparing the yellow with the blue bars demonstrates that the expectations channel makes the effects of monetary shocks more front-loaded. This suggests that forward-looking communication, even if imperfect, might be a useful tool to provide a buffer against perceived risks to the outlook.

6.3 The effects of miscommunication

Since the yellow bars in Figures 5 and 7 correspond to the effects of forward guidance driven by noise only, they capture situations in which either the central bank's communication is misinterpreted by the private sector or the central bank changes its mind about future deviations from the rule after it has announced them. The magnitude of the yellow bars suggests that such communication feeds macroeconomic volatility and can challenge the central bank's ability to stabilize the economy. This highlights the benefits that might accrue to a central bank that invests in developing effective communications. It also helps to explain why central bankers are typically wary of what they say in public.

7 Imperfect communication and the forward guidance puzzle

The empirical forward guidance puzzle examined by Del Negro et al. (2015) involved the extension of a low interest rate peg. As has already been emphasized, conducting their experiment in our model yields small effects. Our model exhibits seemingly powerful and implausible effects of forward guidance under some circumstances, but these effects require perfect communication and disappear once we account for imperfect communication. We demonstrate this with an experiment based on prevailing conditions in the U.S. economy as of 2016q4.

Figure 8 reports various forecasts of output growth (bottom panel) assuming different paths of the federal funds rate expected by agents in the model (top panel) as of 2016q4. The solid (blue) lines are based on the path of the funds rate expected by markets at the end of 2016q4.[24] Market participants were expecting a gradual increase of the federal funds rate over the next ten quarters, from a little more than 50 basis points at the beginning of 2017 to 2.25% by 2019q2. From then onward, the policy rule takes over. In this case output growth converges gradually to its long run potential of 2% from below over the same forecasting period.

The dashed-dotted lines in Figure 8 depict two alternative scenarios. We suppose that in 2017q1 the FOMC announces a much softer path for the federal funds rate increases, so that by 2019q2 the funds rate is only about 1.25%, 100 basis points lower than markets were expecting. The case when this policy is perfectly communicated is represented by the line with asterisks. In this alternative simulation, there is a strong increase in economic activity. Forward guidance adds 1.6 (0.2) percent of output growth relative to the baseline case in 2017 (2018). These numbers seem quite large, and they echo the forward guidance puzzle discussed in Del Negro et al. (2015).

Such strong responses of real activity rely on the assumption that the Fed can perfectly communicate its intentions at any horizon. In our model, agents know that the signals they receive do not match the central bank's intended forward guidance. As a consequence, their expectations about future policy deviations will not adjust one-for-one in response to a

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[24] The path of the funds rate is based on the New York Fed's Survey of Market Participants conducted on December 12 of 2016.

change in forward guidance. Our empirical analysis thus far suggests that the Fed's ability to steer expectations is fairly limited.

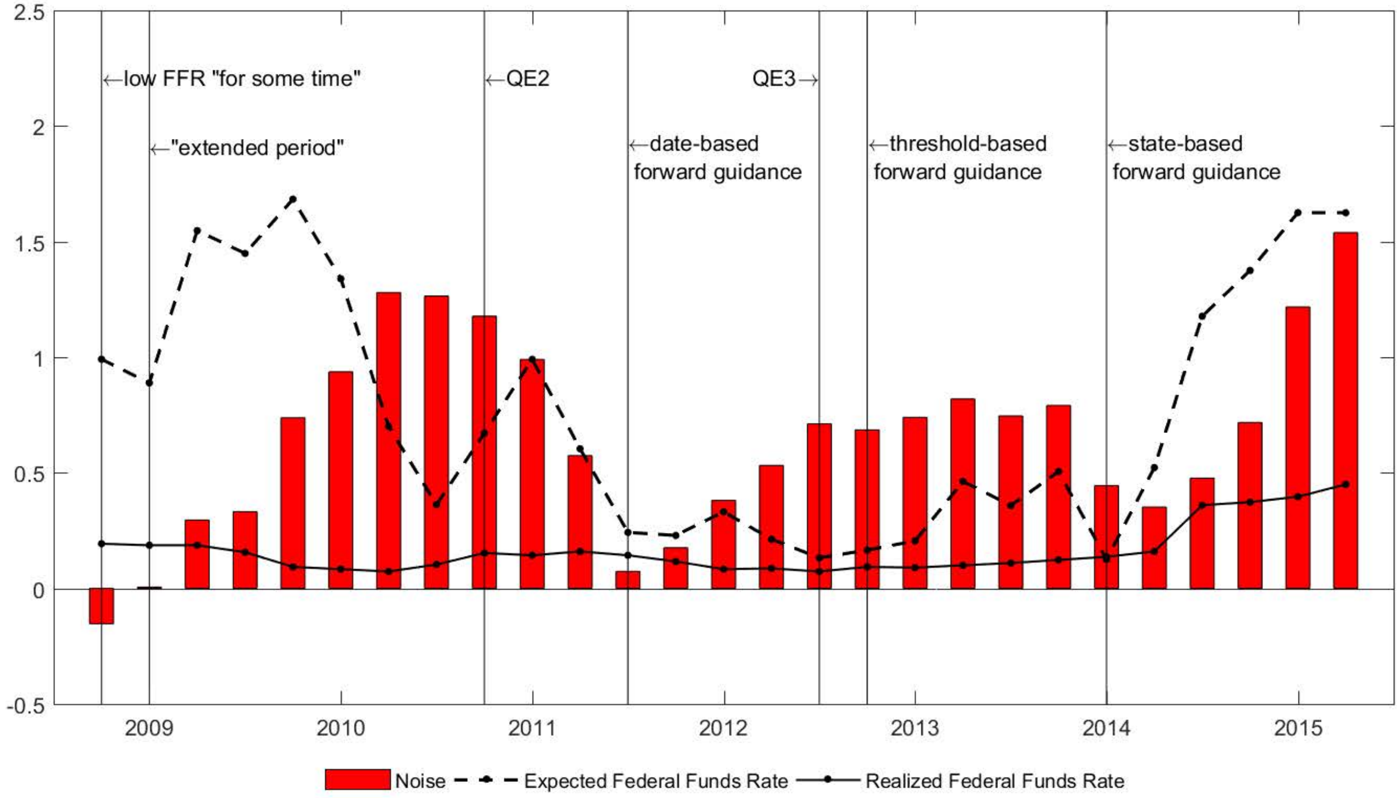
We illustrate the quantitative implications of this limited ability to steer expectations by considering the case where the central bank announces the very same forward guidance corresponding to the line with asterisks but without any noise. Agents know that there has been noise in the past and believe that the signals they receive are only partially related to the true intentions of the central bank. In particular they update their expectations about the federal funds rate using the Kalman gain matrix estimated from the second sample. In this scenario, the imperfect communication leads agents' to expect the funds rate to be about 1.75% (instead of 1.25%) by 2019q2. This is shown by the dashed-dotted (red) line with circles in the top panel of Figure 8. When this path is expected, just 65 (10) basis points of growth is added in 2017 (2018) compared to the baseline case with no changes to forward guidance. Relative to the case where the central bank perfectly communicates the lower interest rate policy, this increment to growth is significantly smaller. From this perspective there is no echo of the forward guidance puzzle. Empirically plausible imperfect communication renders it mute.

8 Conclusion

Forward guidance is now a key component of the policy toolbox of central banks. We have measured the imperfect ability of the Fed to communicate forward guidance and the influence of its imperfect communication on private-sector expectations and macroeconomic outcomes. While our findings show clearly that the Fed has a limited ability to shape expectations, they do not suggest the Fed should eliminate forward guidance from its policy toolbox. The power of forward guidance was extremely limited in the period when the funds rate was far from the ELB, but in the period after 2008 the horizon over which the Fed could influence expectations grew substantially. We conjectured that this may be due to two factors. First, it is possible that the extreme economic conditions after 2008 may have led the public to pay more attention to the Fed than they did during more normal times. Second, it just might be easier to communicate when policy is constrained by the ELB.

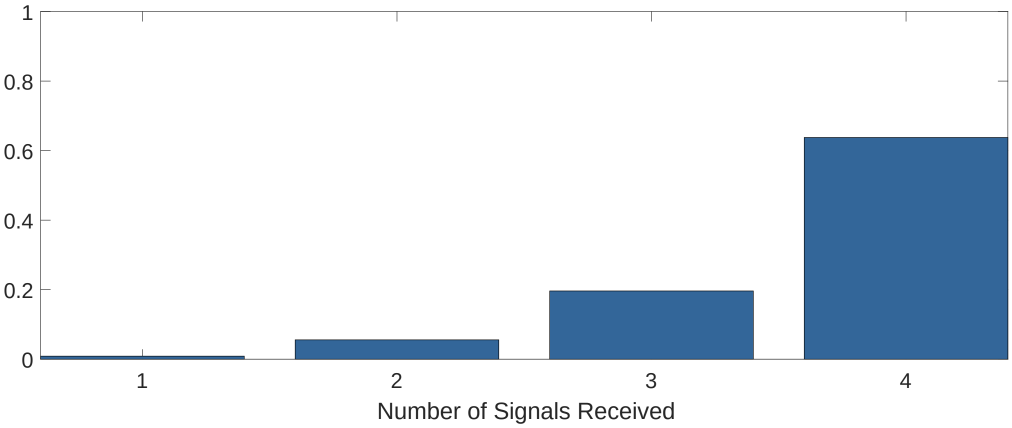
Imperfect central bank communication seems ripe for further study. Perhaps the most important tasks ahead are endogenizing forward guidance and incorporating an explicit role for risk management. Still our framework has useful applications that seem worth pursuing, including investigating central bank communication in other economies, and determining how forward guidance shaped inflation and inflation expectations during their gradual decline. We have explored one strategy for empirically evaluating central bank communication, but clearly there is room for other approaches. Recently Coibion et al. (2019) have opened up a new and exciting area of research which uses an experimental approach to understand the effects of central bank communications. Finally, the welfare implications of trying to communicate forward guidance are an open question. Are such communications advisable? Some of our findings hint at an answer to this question but there is much more to be done here as well.

Figure 1: Contribution of noise to six-quarter ahead expected federal funds rates

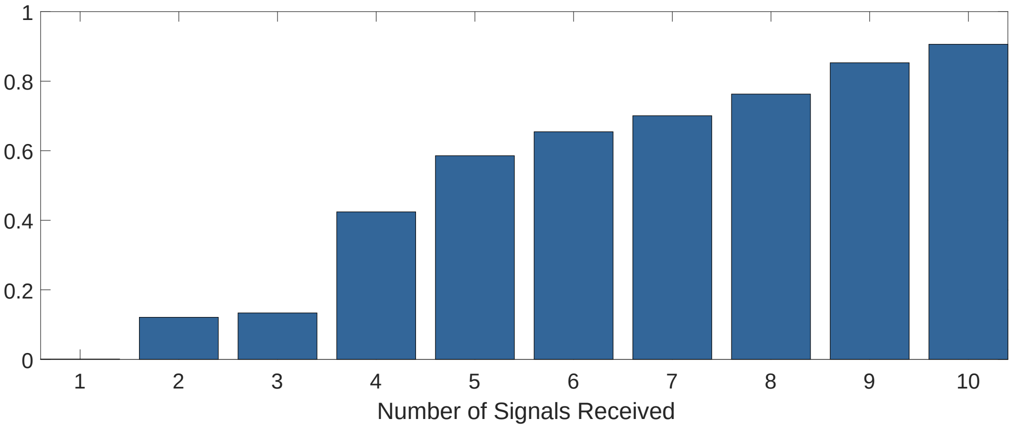


Note: Contribution of identified noise to six-quarter ahead expected funds rate. The dashed and solid lines indicate the market expected and realized funds rate six quarters out from the indicated date. The bars indicate the contribution of noise to the expected funds rate at each date. The vertical lines indicate key dates in the history of forward guidance. Units are percentage points at an annual rate.

Figure 2: Cumulative information gains



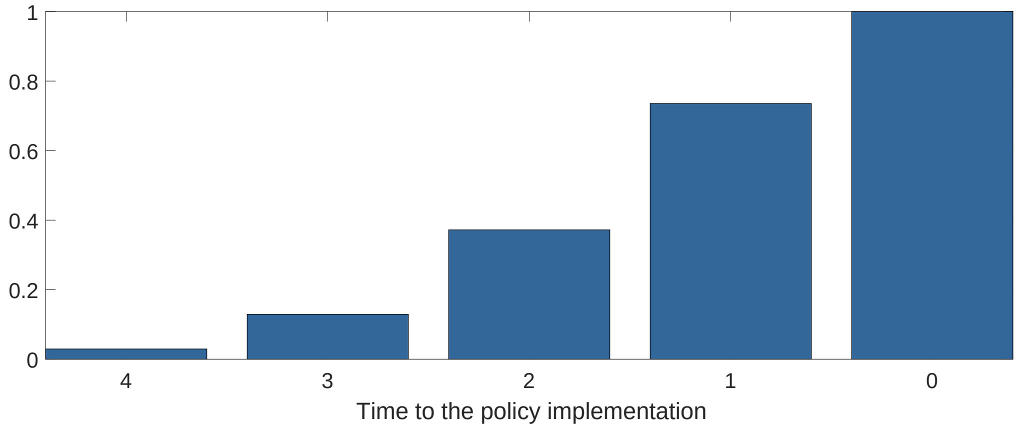
(a) First Sample



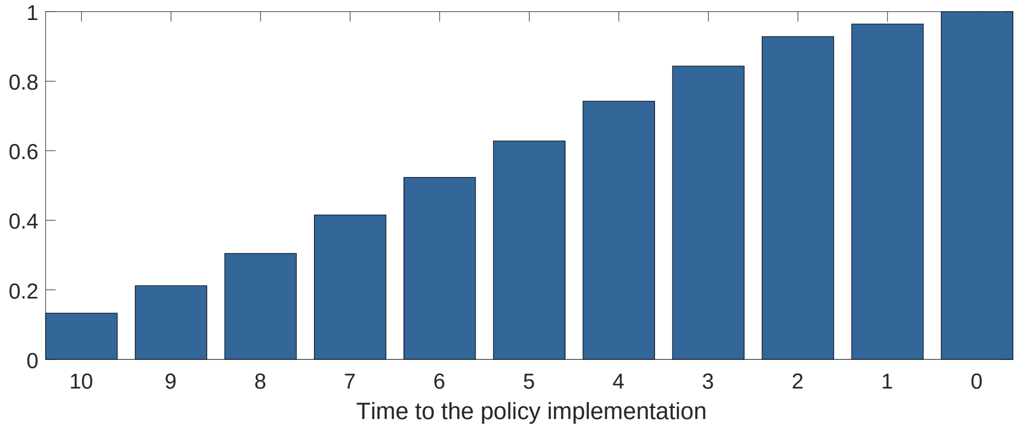
(b) Second Sample

Note: Each bar represents $\mathcal{G}(h)$ where the x-axis indicates the number of signals, $h+1$, received and $h=0,1,\ldots,H-1$. The first signals received give information on the longest horizons.

Figure 3: Dynamic accumulation of knowledge



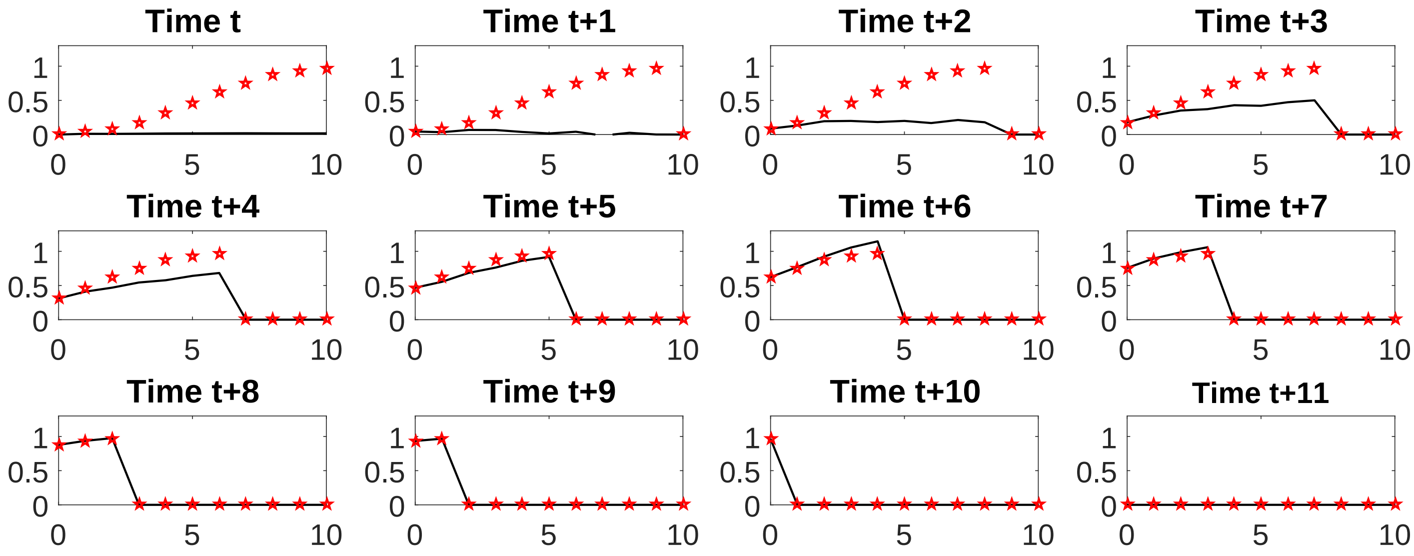
(a) First Sample



(b) Second Sample

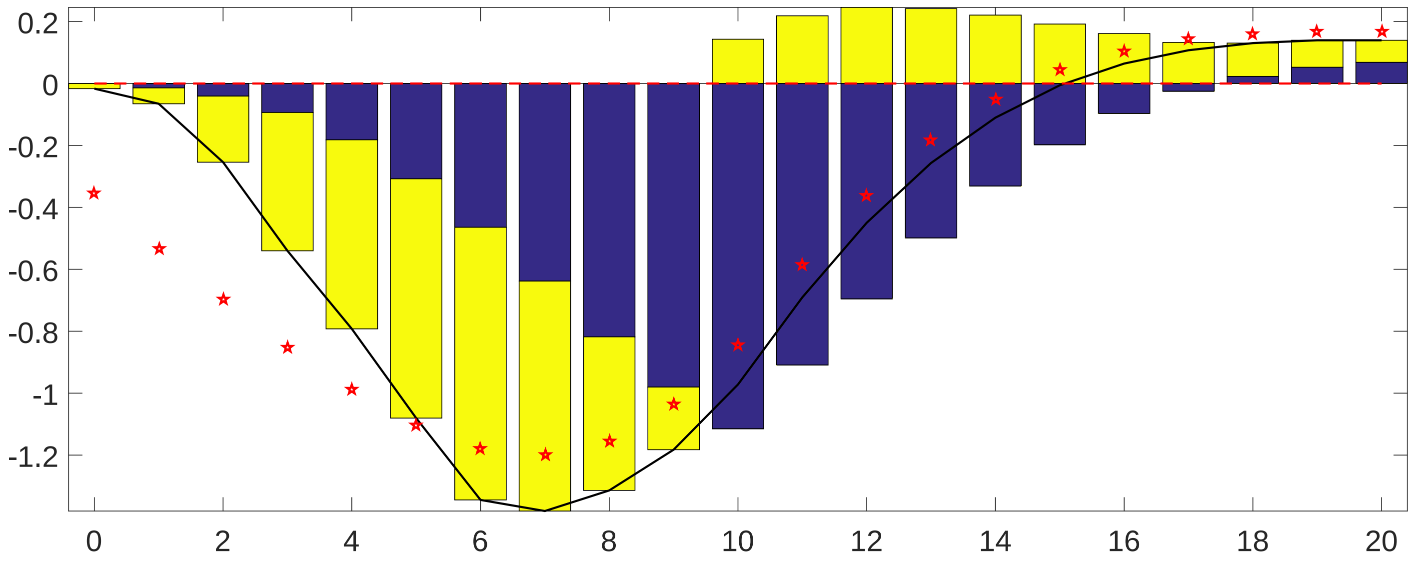
Note: Each bar represents $\mathcal{K}(h)$ where the x-axis indicates the time to the implementation of the deviation from the policy rule, $H - h$, $h = 0, 1, \ldots, H$.

Figure 4: Expectations about future policy deviations after a forward guidance shock in the second sample



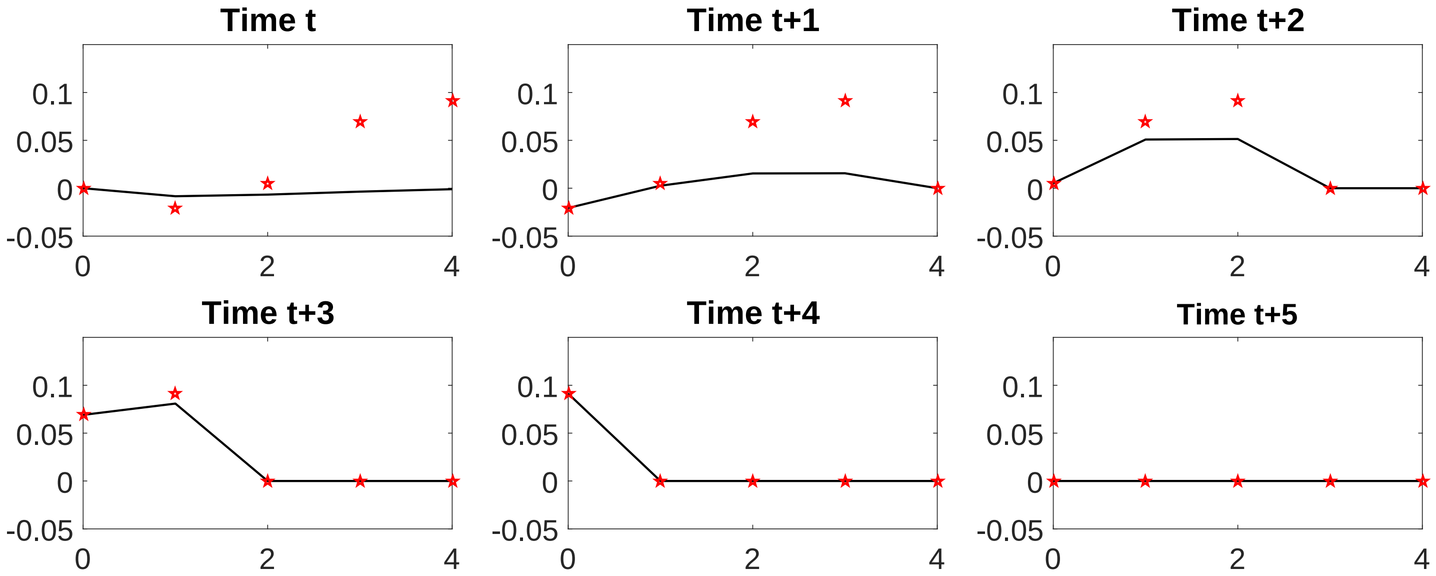
Note: Response of the private sector's expectations about future deviations from the policy rule to a forward guidance shock. The black solid line denotes the case of imperfect communication and the red stars mark the case of perfect communication. The units are percentage points at annualized rates.

Figure 5: Response of hours to a forward guidance shock in the second sample



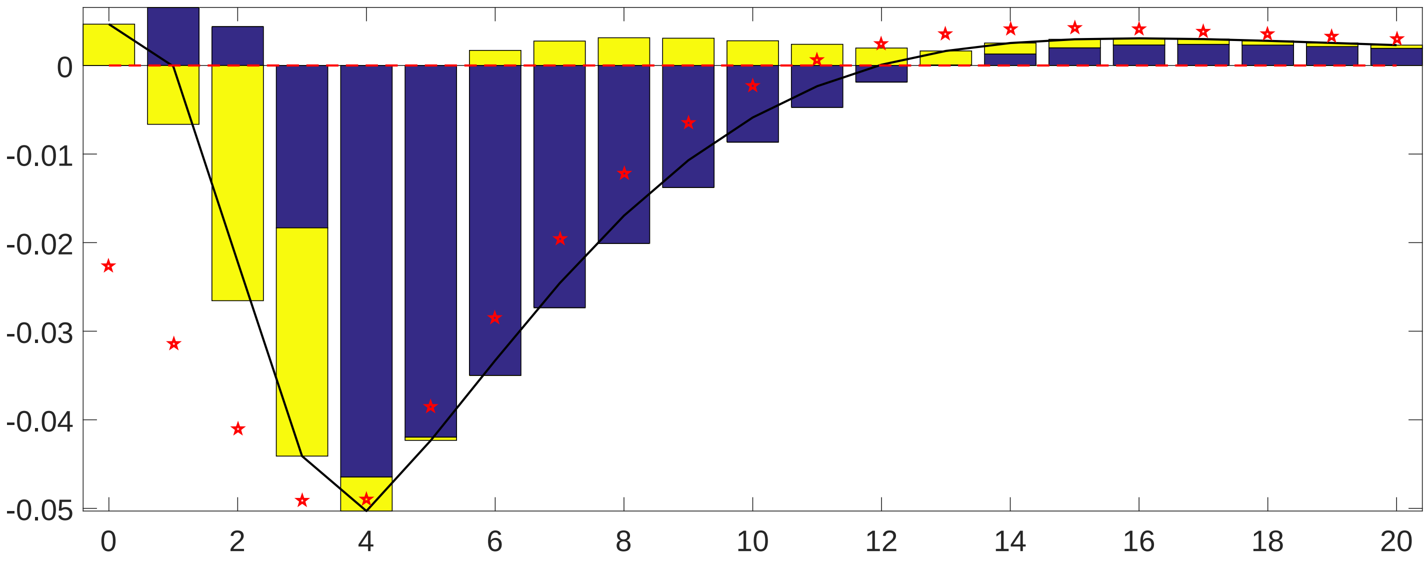
Note: Response of hours to a forward guidance shock that causes the interest rate to deviate from its rule-consistent value by 100 basis points in 10 quarters. The black solid line denotes the case of imperfect communication, the red stars mark the case of perfect communication, the yellow bars indicate the effect on the imperfect communication response due to expectations about future policy deviations alone, and the blue bars show the response when the future policy deviations are not communicated and only implemented as surprises. The units are percentage point deviations from steady state.

Figure 6: Expectations about future policy deviations after a forward guidance shock in the first sample



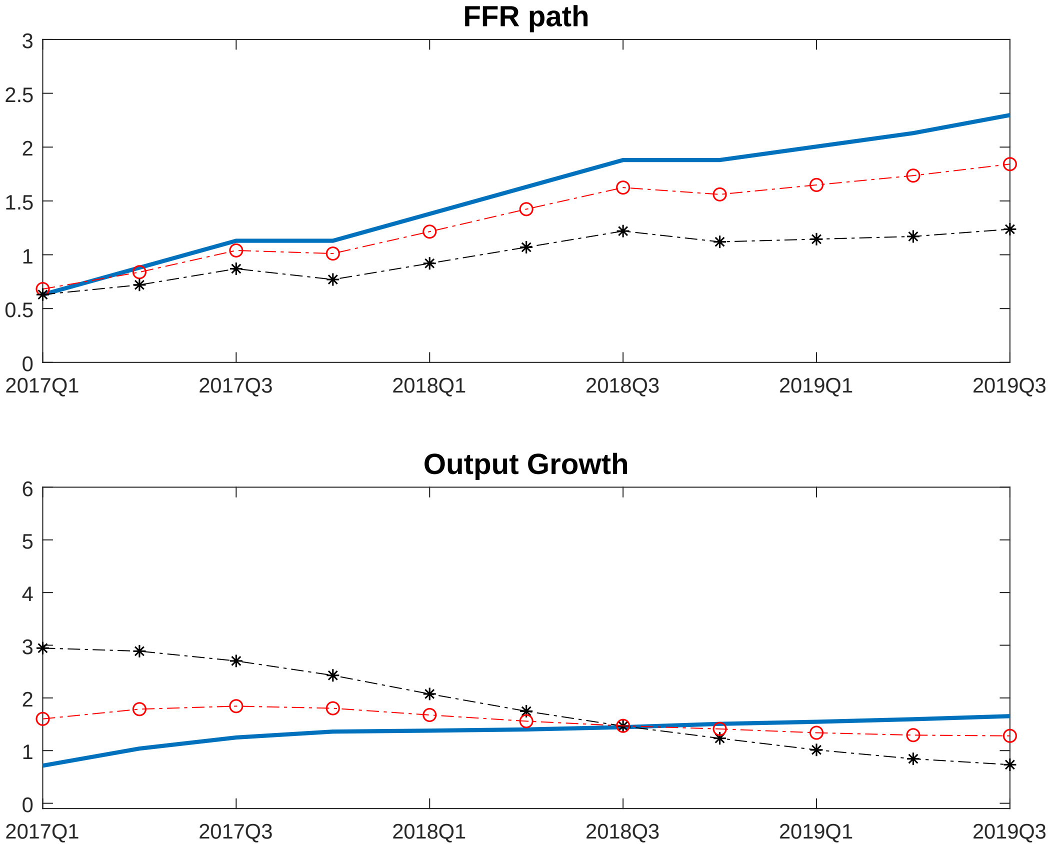
Note: Response of the private sector's expectations about future deviations from the policy rule to a forward guidance shock. The black solid line denotes the case of imperfect communication and the red stars mark the case of perfect communication. The units are percentage points at annualized rates.

Figure 7: Response of hours to a forward guidance shock in the first sample



Note: Response of hours to a forward guidance shock that causes the interest rate to deviate from its rule-consistent value by 8 basis points in 4 quarters. The black solid line denotes the case of imperfect communication, the red stars mark the case of perfect communication, the yellow bars indicate the effect on the imperfect communication response due to expectations about future policy deviations alone, and the blue bars show the response when the future policy deviations are not communicated and only implemented as surprises. The units are percentage point deviations from steady state.

Figure 8: Forward guidance in the first quarter of 2017



Note: Forecasts of output conditioned on 2016q4 data under three scenarios. The solid (blue) line in the top panel depicts the path of the funds rate expected by markets as of 2016q4 and in the bottom panel shows the expected path of output conditional on this path and the state of the economy in 2016q4. The dashed and dotted (black) line with asterisks depicts an alternative scenario in which a different path of the funds rate is perfectly communicated to the public in 2017q1. The dashed and dotted (red) line with open circles displays the same alternative scenario under the imperfect communication technology estimated from the second sample.

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Appendix to "The limits of forward guidance"

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Abstract

This Appendix includes more details on the general equilibrium model and the estimation. It also reports the priors and posterior modes corresponding to our estimation. Finally, detailed derivations of the moments of the communications technology are provided.

[⋆] We thank May Tysinger for her assistance in preparing this document. The views expressed herein are the authors' and do not necessarily represent those of the Federal Reserve Bank of Chicago or the Federal Reserve System.

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1. General equilibrium model

Our empirical investigation of the communication of forward guidance involves embedding the communications environment of Section 2 within Campbell, Fisher, Justiniano and Melosi (2017)'s medium-scale NK model. Much of the model is familiar and described in that paper so here our discussion here is brief and emphasizes the model's shocks and other key features that are integral to our measurement of forward guidance.

1.1. Households

The economy consists of a large number of identical, infinitely lived households with preferences described by the lifetime utility function

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[\varepsilon_t^b \frac{\left[\left(C_t - \varrho \bar{C}_{t-1}\right)\left(1 - \vartheta H_t^{1+\gamma_H}\right)\right]^{1-\gamma_C} - 1}{1-\gamma_C} + \varepsilon_t^s L\left(\frac{B_{t+1}}{P_t R_t}\right)\right], \ \gamma_C, \gamma_H, \varrho > 0.$$

Here C_t denotes the household's consumption purchased in the competitive final goods market at nominal price P_t, $\bar{C}_t$ denotes *aggregate* per capita consumption (which equals C_t in equilibrium) and H_t denotes hours worked.[1] We set ϑ to normalize hours to equal 1 in steady state. The argument of the increasing and concave function L, $B_{t+1}/\left(P_t R_t\right)$, is the consumption value of one-period nominal government bonds purchased by the household at date t and carried into date $t+1$, B_{t+1}.

The *discount factor shock* ε_t^b has been shown by Justiniano, Primiceri and Tambalotti (2010a) and others to be a key driver of consumption fluctuations. Eggertsson and Woodford (2003) and many others use this shock to drive the policy rate to the ELB and so it is particularly relevant for our analysis. We assume ε_t^b evolves according to the stationary process given by

$$\ln \varepsilon_t^b = \rho_b \ln \varepsilon_{t-1}^b + \eta_t^b, \eta_t^b \sim N(0, \sigma_b^2).$$

Including preferences for government bonds has important implications for our measurement.[2] First, they allow for an empirically plausible spread between interest rates on private and government bonds that is otherwise absent. Second, as discussed by Campbell et al. (2017) and Fisher (2015), the interest rate spread brings discounting into the household's linearized inter-temporal Euler equation for consumption. This discounting mitigates the forward guidance puzzle. Finally, Fisher (2015) shows the exogenous shock to these preferences, ε_t^s, provides a simple micro-foundation for Smets and Wouters (2007)'s ad hoc shock to the consumption Euler equation. This shock is crucial to the identification of empirical NK models since it is one of the few sources of co-movement between consumption, investment and hours. We assume the *liquidity preference shock* to the preference for "safe and liquid" government bonds, ε_t^s, evolves according to the

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[1] Campbell et al. (2017) work with a more general specification of preferences. It turns out that this generality is not important for our empirical results and so we abstract from it here.

[2] These preferences were first introduced into an empirical NK model by Campbell et al. (2017). Krishnamurthy and Vissing-Jorgensen (2011) used them to study the market for government securities. They are beginning to get wider attention in the literature. See, for example, Auclert, Rognlie and Straub (2018) and Michaillat and Saez (2018).

stationary process given by

$$\ln \varepsilon_t^s = (1-\rho_s)\varepsilon_*^s + \rho_s \ln \varepsilon_{t-1}^s + \eta_t^s, \eta_t^s \sim N(0,\sigma_s^2).$$

The parameter ε_*^s determines the steady state spread between the rates of return on government and private bonds.

Households own the installed capital stock K_t which they accumulate using the technology

$$K_{t+1} = \left[1-\delta(U_t)\right] K_t + \varepsilon_t^i \left[1 - S\left(\frac{I_t}{q_t I_{t-1}}\right)\right] I_t,$$

where I_t denotes gross investment purchased from investment good producers described below and S is an adjustment cost function that has the usual properties. The term q_t, defined below, corresponds to the growth rate of investment's stochastic trend. The owners of installed capital control the intensity with which it is utilized, U_t, so that the effective supply of capital services in period t is $K_t^e = U_t K_t$. Increasing capacity utilization induces faster depreciation via the function δ which we specify as in Campbell et al. (2017):

$$\delta(U_t) = \delta_0 + \delta_1(U_t - 1) + \frac{\delta_2}{2}(U_t-1)^2,$$

with $\delta_0, \delta_1, \delta_2 > 0$. The parameter δ_2 determines the sensitivity of capacity utilization to variation in the rental rate of capital; the parameter δ_1 governs the steady state utilization rate, which we normalize to unity; and the parameter δ_0 corresponds to the rate of depreciation along the non-stochastic growth path or steady state. The technology for transforming investment goods into installed capital is subject to the shock ε_t^i. We assume this *investment-demand shock* evolves according to the stationary process given by

$$\ln \varepsilon_t^i = \rho_i \ln \varepsilon_{t-1}^i + \eta_t^i, \eta_t^i \sim N(0,\sigma_i^2).$$

Justiniano, Primiceri and Tambalotti (2010b) find that this shock explains a substantial fraction of business cycle fluctuations in investment.

1.2. Goods and labor markets

Final goods are produced using differentiated intermediate inputs with the usual Dixit-Stiglitz technology that is subject to shocks to the elasticity of substitution. Intermediate goods producers are monopolistic competitors who maximize profits subject to a standard Calvo pricing scheme with indexing. Each firm is subject to an exogenous probability of having the opportunity to adjust its price, $\zeta_p \in (0,1)$. Absent this opportunity firms index the previously set price using the exogenous formula $\pi_{t-1}^{\iota_p}\pi_*^{1-\iota_p}$, where π_* is the the central bank's inflation target (corresponding to steady state inflation), and $\iota_p \in [0,1]$. The Calvo pricing scheme plus shocks to the elasticity of substitution translate to a *price mark-up shock*, λ_t^p, which evolves according to the stationary process given by

$$\ln \lambda_t^p = (1-\rho_{\lambda_p}) \ln \lambda_*^p + \rho_{\lambda_p} \ln \lambda_{t-1}^p - \phi_p \eta_{t-1}^p + \eta_t^p, \eta_t^p \sim N(0,\sigma_{\lambda_p}^2).$$

The parameter λ_*^p denotes the steady state mark-up. Notice that we allow for innovations to price markups to be a first-order moving average process.

Intermediate goods producer i produces its output Y_{it} using the technology:

$$Y_{it} = \left(K_{it}^{e}\right)^{\alpha} \left[A_t^Y H_{it}\right]^{1-\alpha} - A_t \Phi,\ \alpha \in (0,1), \Phi > 0,$$

where H_{it} is labor purchased at consumption wage W_t in a competitive market and Φ is a fixed cost of production in units of the final good. The term A_t^Y is the level of the *neutral technology.* This is a non-stationary process with growth rate $\nu_t \equiv \ln\left(A_t^Y / A_{t-1}^Y\right)$ that evolves according to the stationary process given by

$$\nu_t = (1-\rho_\nu)\,\nu_* + \rho_v \nu_{t-1} + \eta_t^\nu, \eta_t^\nu \sim N(0, \sigma_\nu^2),$$

where ν_* is the steady state growth rate of the neutral technology. We refer to ν_t as the *neutral technology shock.* The term A_t in the expression for Y_{it} above is the stochastic trend of equilibrium consumption and output measured in consumption units which equals $A_t^Y \left(A_t^I\right)^{\alpha/(1-\alpha)}$, where A_t^I is the level of the investment-specific technology described below with log growth rate denoted ω_t. The log growth rate of A_t is $z_t = \nu_t + \alpha\omega_t/\left(1-\alpha\right)$.

Perfectly competitive firms produce the investment goods supplied to households using a linear technology that transforms final goods into investment goods at rate A_t^I. The growth rate of A_t^I, which we call the *investment-specific technology shock* evolves according to the stationary process given by

$$\omega_t = (1-\rho_\omega)\omega_* + \rho_\omega \omega_{t-1} + \eta_t^\omega, \eta_t^\omega \sim N(0, \sigma_\omega^2).$$

The parameter ω_* is the mean growth rate of the investment-specific technology. In equilibrium investment has a stochastic trend with log growth rate $q_t = \nu_t + \omega_t/\left(1-\alpha\right)$.

We adopt Smets and Wouters (2007)'s approach to modeling the labor market when preferences are non-separable in consumption and labor. This approach involves Calvo-style sticky wages with a *wage mark-up shock.* The exogenous probability of having the opportunity to adjust wages is $\zeta_w \in (0,1)$. Absent this opportunity wages are indexed to their previously set value using the exogenous formula $\left(\pi_{t-1} z_{t-1}\right)^{\iota_w} \left(\pi_* z_*\right)^{1-\iota_w}$, where $\iota_w \in [0,1]$. We assume the wage markup shock, λ_t^w, follows a stationary process similar to λ_t^p:

$$\ln \lambda_t^w = (1-\rho_{\lambda_w}) \ln \lambda_*^w + \rho_{\lambda_w} \ln \lambda_{t-1}^w - \phi_w \epsilon_{t-1}^w + \eta_t^w, \eta_t^w \sim N(0, \sigma_{\lambda_w}^2).$$

The parameter λ_*^w denotes the steady state mark-up.

1.3. Central bank and government

The central bank sets its policy rate, communicates with the public, and the public updates is expectations about policy deviations in the way described in Section 2 of the main text. Our parametric specification of the monetary policy rule in equation (1) in the main text is

$$g_t(R_{t-1}, \pi_t^{gap}, y_t^{gap}) = \rho_R \ln R_{t-1} + (1-\rho_R) \ln R_t^n,\ \rho_R \in [0,1),$$

where R_t^n is the *notional* target interest rate given by

$$\ln R_t^n = \ln r_* + \ln \pi_t^* + \psi_1 \pi_t^{gap} + \psi_2 y_t^{gap},\ \psi_1, \psi_2 > 0.$$

The constant r_* corresponds to the steady state real interest rate on government bonds and π_t^* is an exogenous *inflation drift shock* that could be interpreted as the central bank's intermediate target for inflation.

The drift term is included in the rule to address inflation's low-frequency movements during our sample. Since we have assumed the policy rule is perfectly communicated to the public so is π_t^*. Clearly this is not an innocuous assumption since the drift influences agents' inflation expectations and the Fed's communications about its inflation objectives were almost surely imperfect over much of the sample period we study. The *inflation drift shock* evolves according to the stationary process

$$\ln \pi_t^* = (1-\rho_\pi)\pi_* + \rho_\pi \ln \pi_{t-1}^* + +\eta_t^\pi, \eta_t^\pi \sim N(0, \sigma_\pi^2),$$

where π_* is steady state inflation.

The gaps in the policy rule are four-quarter moving averages of variables observed by private agents. We assume the central bank measures the inflation gap using the deviation of inflation from the contemporaneous value of the drift term:

$$\pi_t^{gap} = \frac{1}{4} E_t \sum_{j=-2}^{1} \left(\ln \pi_{t+j} - \ln \pi_t^* \right).$$

It measures the output gap using the difference between the log level of aggregate output and its stochastic trend:

$$y_t^{gap} = \frac{1}{4} E_t \sum_{j=-2}^{1} \left(\ln Y_{t+j} - \ln y^* - \ln A_{t+j} \right).$$

The constant y_* denotes steady state output. This is included to ensure that both gaps are closed in steady state with the nominal interest rate on government bonds $R_* = r_* \pi_*$.

The government issues bonds B_{t+1} and collects lump sum taxes to pay for government spending $A_t g_t$ purchased in the final goods market. We assume the government balances its budget every period and has no legacy debt, so government bonds are in zero net supply. The *government spending shock* g_t evolves according to the stationary process

$$\ln g_t = (1-\rho_g) \ln s_*^g + \rho_g \ln g_{t-1} + \eta_t^g, \eta_t^g \sim N(0, \sigma_g^2),$$

where s_*^g is the government's share of output in steady state.

1.4. Equilibrium

Equilibrium is defined in the usual way and is described in more detail in Campbell et al. (2017). We study the solution to the log linearized equilibrium conditions of the detrended economy and apply econometric techniques that rely on linearity to estimate parameters and to study central bank communications. One may question how such an approach can be squared with the ELB. Without forward guidance it is possible that at some dates agents' expectations of future policy rates would violate the ELB constraint even if the contemporaneous rate did not. We use data on expected future funds rates, which of course do not violate the ELB, in our list of observables when we estimate our model. Forward guidance gives our model the flexibility to fit these data and thereby respect the ELB.

2. Estimating the news representation

Our estimation of the model's news representation proceeds in two steps. The first step is to calibrate the parameters that appear in the corresponding neoclassical growth model to match sample averages from the U.S. economy. Our calibration procedure is described in Section 3 below. With the exceptions noted below to address low frequency movements in growth and interest rates these parameters are held fixed throughout the analysis. The second step takes the calibrated parameters as given and applies standard Bayesian methods to estimate the remainder of the model plus some auxiliary parameters which are used to map the model into the data.

Our calibration strategy is the same as in Campbell et al. (2017) except that we address the well-known evidence of secular declines in economic growth and rates of return on nominally risk free assets. We address these developments by imposing a change in steady state in 2008q4 (the choice of this date is motivated below.) Steady state GDP growth is governed by the mean growth rates of the neutral and investment-specific technologies, ν_* and ω_*. We adjust ω_* down to account for the slower decline in the relative price of investment since 2008q4. Given this change we then lower ν_* so that steady state GDP growth is reduced to 2%. To match a lower real risk-free rate of 1% we increase the steady state marginal utility of government bonds using ε^s_*.[3] These adjustments leave the other calibrated parameters unchanged but do change the steady state values of the endogenous variables and therefore the point at which the economy is log-linearized.[4]

Our Bayesian estimation uses the same split-sample strategy as in Campbell et al. (2017) except that we incorporate the change in steady state described above and one other change noted below. As in Campbell et al. (2017) our sample begins in 1993q1. This date is based on the availability and reliability of the overnight interest rate futures data. The sample period ends in 2016q4 but we impose a sample break in 2008q4. Our choice of this latter date is motivated by three main considerations. First, there is the evidence that points to lower interest rates and economic growth later in the sample. Second, it seems clear that the horizon over which forward guidance was communicated by the Fed lengthened substantially during the ELB period. Finally, the downward trends in inflation and inflation expectations from the early 1990s appear to come to an end in the mid-2000s. Splitting the sample in 2008q4 and assuming some parameters change at that date is our way of striking a balance between parsimony and addressing the multiple structural changes that seem to occur around the same time.

We estimate the full suite of non-calibrated structural parameters in the first sample under the assumption that forward guidance extends for $H = 4$ quarters. Starting in 2008q4 we assume the model environment changes in three ways. First we assume the change in the steady state described above. Second, forward guidance lengthens to $H = 10$ quarters. Third, the time-varying inflation target from the first sample becomes a constant equal to the steady state rate of inflation, 2% at an annual rate. All three changes are assumed to be unanticipated and permanent.

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[3] The targets for steady state GDP growth and risk-free rate reflect a variety of evidence including the Fed's Summary of Economic Projections.

[4] Our re-calibration changes the return on private assets by a little. This small change is consistent with Yi and Zhang (2017) who show that rates of return on private capital have stayed roughly constant in the face of declines in risk free rates.

The measurement equations for the first sample estimation are as follows:

$$\begin{aligned}
\Delta \ln Q_t^{obs} &= f\left(\hat{c}_t, \hat{c}_{t-1}, \hat{i}_t, \hat{i}_{t-1}, \hat{g}_t, \hat{\omega}_t, \hat{\pi}_t^{g,obs}\right);\\
\Delta \ln C_t^{obs} &= z_* + \Delta\hat{c}_t + \hat{z}_t;\\
\Delta \ln I_t^{obs} &= z_* + \omega_* + \Delta\hat{\imath}_t + \hat{z}_t + \hat{\omega}_t;\\
\log H_t^{obs} &= \hat{H}_t;\\
\pi_t^{i,obs} &= \omega_* + \hat{\omega}_t + u_t^i;\\
R_t^{obs} &= R_* + \hat{R}_t;\\
R_t^{j,obs} &= R_* + E_t\hat{R}_{t+j},\ j = 1, 2, \ldots, H;\\
\pi_t^{l,j,obs} &= \pi_* + \pi_*^{l,j} + \frac{\beta^{l,j}}{l}\sum_{i=1}^{l} E_t\hat{\pi}_{t+i} + u_t^{l,j,\pi},\ j = 1, 2,\ l = 1, 40;\\
\pi_t^{j,obs} &= \pi_* + \pi_*^j + \beta^{\pi,j}\hat{\pi}_t + \gamma^{\pi,j}\pi_t^{d,obs} + u_t^{j,p},\ \text{ with } \beta^{\pi,1} = 1, j = 1, 2, 3;\\
\Delta \ln w_t^{j,obs} &= z_* + w_*^j + \beta^{w,j}\left(\hat{w}_t - \hat{w}_{t-1} + \hat{z}_t\right) + u_t^{j,w},\ \text{ with } \beta^{w,1} = 1, j = 1, 2;\\
\pi_t^{d,obs} &= \pi_*^d + \beta_{1,1}\pi_{t-1}^{d,obs} + \beta_{1,2}\pi_{t-2}^{d,obs} + u_t^d;\\
\pi_t^{g,obs} &= \pi_*^g + \beta_{2,1}\pi_{t-1}^{g,obs} + \beta_{2,2}\pi_{t-2}^{g,obs} + u_t^g.
\end{aligned}$$

The "hat" notation denotes log deviations from steady state; the de-trended counterparts of the upper case endogenous variables described in Section 1 are denoted with their corresponding lower case; and"Δ" is the first difference operator. The left hand side variables represent data (Q denotes chain-weighted GDP). These data are described in Campbell et al. (2017).[5] The function f in the first equation represents the linear approximation to the chain-weighted GDP formula discussed in Campbell et al. (2017). Two variables are included to complete the mapping from model to data but are not endogenous to the model. Specifically, the consumption price of government consumption plus net exports, $\pi_t^{g,obs}$, helps map model GDP to our model-consistent measure of chain-weighted GDP, and inflation in the consumption price of consumer durable goods, $\pi_t^{d,obs}$, is used to complete the mapping from model inflation to measured inflation.

The measurement equations introduce some additional notation. The variables $\pi_t^{j,obs}$ and $w_t^{j,obs}$ represent the inflation and wage indicators discussed in Campbell et al. (2017). The variables $\pi_t^{l,j,obs}$ denote measures of inflation expectations. The variables u_t^i, $u_t^{l,j,\pi}$, $u_t^{j,p}$ and $u_t^{j,w}$ denote AR(1) measurement errors, and u_t^d and u_t^g denote AR(1) regression residuals. The constants π_*^j and $\pi_*^{l,j}$ account for the average differences between the observable measures of inflation and inflation expectations and steady state inflation. The coefficients $\beta^{\pi,j}$ and $\gamma^{\pi,j}$ denote the factor loadings relating observable inflation to model inflation and observed consumer durable inflation, and $\beta^{l,j}$ are factor loadings relating observable inflation expectations to their model counterparts. The $\beta_{i,j}$ are regression coefficients.

The measurement equations indicate we use 21 time series to estimate the model in the first sample. In addition to the real quantities and federal funds rate that are standard in the literature our estimation includes multiple measures of wage and consumer price inflation, two measures each of average inflation expected over the next ten years and over

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[5] We use three additional time series, all measures of expected inflation from the Survey of Professional Forecasters: PCE expected inflation over the next 10 years and both CPI and PCE expected inflation one quarter out.

one quarter, and $H = 4$ quarters of interest rate futures. Our second sample estimation is restricted to estimating the parameters of the stochastic process for forward guidance news with $H = 10$ plus the processes driving $\pi_t^{g,obs}$ and $\pi_t^{d,obs}$. This estimation uses the measurement equations involving the current federal funds rate and 10 quarters of expected future policy rates plus the last two equations. We take into account the change in steady state but keep the remaining structural parameters at their first sample values. Because our estimation forces data on real activity, wages and prices to coexist with the interest rate futures data, we expect the estimation to mitigate the forward guidance puzzle. Finally, it is worth reiterating that our estimation respects the ELB in the second sample. This is because we measure expected future rates in the model, the $E_t\hat{R}_{t+j}$, using the corresponding empirical futures rates, $R_t^{j,obs}$, and we use futures rates extending out 10 quarters.

3. Calibration

We observe the long-run average of the following aggregates: nominal federal funds rate, labor share, government spending share, investment spending share, the capital-output ratio, real per-capita GDP growth (g_y), inflation in price of government, net exports and inventory investment relative to non-durables and services consumption, and the growth rate of the consumption-investment relative price.

- The labor share can be used to calibrate the parameter α.
- The government spending share determines s_*^g.
- The government price growth rate pins down π_*^g.
- The growth rate of the consumption-investment relative price pins down ω_*.
- The investment share pins down i_*/y_*.
- The capital output ratio pins down k_*/y_*.
- Calculate the consumption-output share

$$\frac{c_*}{y_*} = \left(1 - \frac{i_*}{y_*} - \frac{g_*}{y_*}\right). \tag{1}$$

- The growth rate of real chain-weighted GDP is used to pin down the growth rate of the common trend z_*. First

$$g_y = e^{z_*}\sqrt{\frac{\frac{c_*}{y_*} + e^{\omega}\frac{i_*}{y_*} + \left(\pi_*^g\right)^{-1}\frac{g_*}{y_*}}{\frac{c_*}{y_*} + e^{-\omega}\frac{i_*}{y_*} + \pi_*^g\frac{g_*}{y_*}}}$$

  All the variables in this equation are known except for z_*. So we can solve for z_*:

$$z_* \;=\; g_y - \frac{1}{2}\ln\left(\frac{\frac{c_*}{y_*} + e^{\omega}\frac{i_*}{y_*} + \left(\pi_*^g\right)^{-1}\frac{g_*}{y_*}}{\frac{c_*}{y_*} + e^{-\omega}\frac{i_*}{y_*} + \pi_*^g\frac{g_*}{y_*}}\right) \tag{2}$$

- The growth rate of the labor-augmenting technology ν_* can be easily obtained by exploiting the following equation:

$$z_* = v_* + \frac{\alpha}{1-\alpha}\omega_*. \tag{3}$$

- We are now in a position to identify the depreciation rate δ_0 using the steady-state equation pinning down the investment capital ratio:

$$\begin{aligned} \frac{i_*}{k_*} &= 1-(1-\delta_0)e^{-z_*-\omega_*} \\ &\Rightarrow \delta_0 = 1+\left(\frac{i_*}{k_*}-1\right)e^{z_*+\omega_*} \end{aligned}$$

  where the investment capital ratio is obtained combining the investment share and the capital output ratio:

$$\frac{i_*}{k_*} = \frac{i_*/y_*}{k_*/y_*}. \tag{4}$$

- From the steady-state equilibrium we have that

$$\frac{y_*}{k_*} = e^{-z_*-\omega_*}\frac{\delta_1}{\alpha}. \tag{5}$$

  Therefore

$$\delta_1 = \alpha\left(\frac{k_*}{y_*}\right)^{-1}e^{z_*+\omega_*} \tag{6}$$

  where the capital output ratio is given above.

- In steady state, the real rate of return on private bonds is derived from the first order condition for private bonds:

$$r_*^p \equiv \frac{R_*^P}{\pi_*} = \frac{e^{\gamma_c z_*}}{\beta}. \tag{7}$$

  In steady state the real rental rate of capital is derived from the first order condition for capital:

$$r_*^k = \left[\frac{e^{\gamma_c z_*}}{\beta}\right]e^{\omega_*}-(1-\delta_0) \tag{8}$$

  Combining these last two equations yields

$$r_*^k = r_*^p e^{\omega_*}-(1-\delta_0)$$

  and hence

$$r_*^p = \left[r_*^k+1-\delta_0\right]e^{-\omega_*}.$$

  Note that $r_*^k = \delta_1$ from the first order condition for capacity utilization. It follows that

$$r_*^p = (1-\delta_0+\delta_1)\,e^{-\omega_*}$$

- The liquidity premium in steady state (i.e., $\frac{R_*/\pi_*}{r_*^p}$) can be computed now by assuming a *nominal* average federal funds rate, R_*, and an annualized average inflation rate.

- Using equation (8) and the fact that $r_*^k = \delta_1$, we can calibrate the discount factor β :

  $$\beta = \left(1 - \delta_0 + \delta_1\right)^{-1} e^{\omega_*} e^{\gamma_c z_*}$$

  where γ_c is a parameter of the utility function to be estimated.

4. The factor structure of news

Since the deviations from the rule $\boldsymbol{\theta}_t$ are possibly correlated up to the H-th lag in our model, we allow news shocks in the observationally equivalent news representation to be correlated across horizons at a point in time. To capture this, we follow Campbell et al. (2017) and assume that a factor structure determines the cross-correlations among news shocks. Specifically, we assume

$$\varepsilon_{R,t}^{j} = \alpha_j f_t^{\alpha} + \beta_j f_t^{\beta} + \psi_i \eta_t,$$

where the factors f_t^{α} and f_t^{β} and factor loadings α_i and β_i are scalars, η_t is an $H \times 1$ column vector of shocks, and ψ_i is a $1 \times H$ vector of coefficients that depend on the model's structural parameters and is described in Campbell et al. (2017). The factors and shocks have zero means and are independent and normally distributed. In matrix notation, we have

$$\boldsymbol{\varepsilon}_{R,t} = \alpha f_t^{\alpha} + \beta f_t^{\beta} + \psi \eta_t,$$

where $\alpha = [\alpha_0, \ldots, \alpha_H]'$, $\beta = [\beta_0, \ldots, \beta_H]'$ and $\psi = [\psi_0', \ldots, \psi_H']'$. Let $\Sigma_\eta = E\left(\eta_t \eta_t'\right)$ denote the variance-covariance matrix of the idiosyncratic shocks, and σ_α^2 (σ_β^2) denote the variance of f_t^{α} (f_t^{β}). Recall $\Sigma_\varepsilon \equiv E(\boldsymbol{\varepsilon}_{R,t}\boldsymbol{\varepsilon}_{R,t}')$. Our estimate of the variance-covariance matrix of news is then given by

$$\begin{aligned} \Sigma_\varepsilon &= E(\alpha f_t^{\alpha} + \beta f_t^{\beta} + \psi \eta_t)(\alpha f_t^{\alpha} + \beta f_t^{\beta} + \psi \eta_t)' \\ &= \alpha \alpha' \sigma_\alpha^2 + \beta \beta' \sigma_\beta^2 + \psi \Sigma_\eta \psi'. \end{aligned} \tag{9}$$

5. Prior and parameter estimates

Table 1: First Sample Calibration Targets

Description	Expression	Value
Fixed Interest Rate (quarterly, gross)	R^*	1.011
Per-Capita Steady-State Output Growth Rate (quarterly)	Y_{t+1}/Y_t	1.005
Investment to Output Ratio	I_t/Y_t	0.260
Capital to Output Ratio	K_t/Y_t	10.763
Fraction of Final Good Output Spent on Public Goods	G_t/Y_t	0.153
Growth Rate of Relative Price of Consumption to Investment	P_C/P_I	0.371

Table 2: First Sample Calibrated Parameters

Parameter	**Symbol**	**Value**
Discount Factor	β	0.986
Steady-State Measured TFP Growth (quarterly)	z_*	0.489
Investment-Specific Technology Growth Rate	ω_*	0.371
Elasticity of Output w.r.t Capital Services	α	0.401
Steady-State Wage Markup	λ_*^w	1.500
Steady-State Price Markup	λ_*^p	1.500
Steady-State Scale of the Economy	H_*	1.000
Steady-State Inflation Rate (quarterly)	π_*	0.500
Steady-State Depreciation Rate	δ_0	0.016
Steady-State Marginal Depreciation Cost	δ_1	0.039
Core PCE, 1Q Ahead and 10Y Ahead Expected PCE		
Constant	$\pi_*^1, \pi_*^{l,1}$	0.000
Loading 1	$\beta^{\pi,1}, \beta^{l,1}$	1.000
Core CPI, 1Q Ahead and 10Y Ahead Expected CPI		
Constant	$\pi_*^2, \pi_*^{l,2}$	0.122
10Y Ahead Expected CPI and PCE		
Standard Deviation of $u_t^{40,j,\pi}$		0.010
PCE Durable Goods Inflation		
1st Lag Coefficient	$\beta_{1,1}$	0.418
2nd Lag Coefficient	$\beta_{1,2}$	0.379
Inflation in Relative Price of Government, Inventories and Net Exports to Consumption		
1st Lag Coefficient	$\beta_{2,1}$	0.311
2nd Lag Coefficient	$\beta_{2,2}$	0.006
Compensation		
Constant	w_*^1	-0.202
Loading	$\beta^{w,1}$	1.000
Earnings Constant	w_*^2	-0.237
Loading 0 Factor A	α_0	0.981
Loading 0 Factor B	β_0	0.000
Loading 4 Factor B	β_4	0.951

Table 3: First Sample Estimated Parameters

			Prior		Posterior
Parameter	**Symbol**	**Density**	**Mean**	**Std.Dev**	**Mode**
Depreciation Curve	$\frac{\delta_2}{\delta_1}$	G	1.0000	0.150	0.474
Active Price Indexation Rate	ι_p	B	0.5000	0.150	0.409
Active Wage Indexation Rate	ι_w	B	0.5000	0.150	0.077
External Habit Weight	λ	B	0.7500	0.025	0.780
Labor Supply Elasticity	γ_H	N	0.6000	0.050	0.589
Price Stickiness Probability	ζ_p	B	0.8000	0.050	0.831
Wage Stickiness Probability	ζ_w	B	0.7500	0.050	0.914
Adjustment Cost of Investment	φ	G	3.0000	0.750	5.354
Elasticity of Intertemporal Substitution	γ_c	N	1.5000	0.375	1.319
Interest Rate Response to Inflation	ψ_1	G	1.7000	0.150	1.791
Interest Rate Response to Output	ψ_2	G	0.2500	0.100	0.398
Interest Rate Smoothing Coefficient	ρ_R	B	0.8000	0.100	0.801
Autoregressive Coefficients of Shocks					
Discount Factor	ρ_b	B	0.5000	0.250	0.813
Inflation Drift	ρ_π	B	0.9900	0.010	0.998
Exogenous Spending	ρ_g	B	0.6000	0.100	0.887
Investment-Demand	ρ_i	B	0.5000	0.100	0.791
Liquidity Preference	ρ_s	B	0.6000	0.200	0.887
Price Markup	ρ_{λ_p}	B	0.6000	0.200	0.136
Wage Markup	ρ_{λ_w}	B	0.5000	0.150	0.469
Neutral Technology	ρ_ν	B	0.3000	0.150	0.492
Investment Specific Technology	ρ_ω	B	0.3500	0.100	0.303
Moving Average Coefficients of Shocks					
Price Markup	θ_{λ_p}	B	0.4000	0.200	0.307
Wage Markup	θ_{λ_w}	B	0.4000	0.200	0.391
Standard Deviations of Innovations					
Discount Factor	σ_b	U	0.5000	2.000	1.768
Inflation Drift	σ_π	I	0.0150	0.0075	0.077
Exogenous Spending	σ_g	U	1.0000	2.000	4.139
Investment-Demand	σ_i	I	0.2000	0.200	0.549
Liquidity Preference	σ_s	U	0.5000	2.000	0.341
Price Markup	σ_{λ_p}	I	0.1000	1.000	0.101
Wage Markup	σ_{λ_w}	I	0.1000	1.000	0.035
Neutral Technology	σ_ν	U	0.5000	0.250	0.530
Investment Specific Technology	σ_ω	I	0.2000	0.100	0.259
Relative Price of Cons to Inv	$\sigma_{\frac{c}{i}}$	I	0.0500	2.000	0.675
Monetary Policy					
Unanticipated	σ_{η_0}	N	0.0050	0.0025	0.012
1Q Ahead	σ_{η_1}	N	0.0050	0.0025	0.012
2Q Ahead	σ_{η_2}	N	0.0050	0.0025	0.008
3Q Ahead	σ_{η_3}	N	0.0050	0.0025	0.009
4Q Ahead	σ_{η_4}	N	0.0050	0.0025	0.012
Compensation					

Notes: Distributions (**N**) Normal, (**G**) Gamma, (**B**) Beta, (**I**) Inverse-gamma-1, (**U**) Uniform

First Sample Estimated Parameters (Continued)

Parameter	**Symbol**	**Density**	**Prior Mean**	**Std.Dev**	**Posterior Mode**
Standard Deviation of $u_t^{1,w}$		I	0.0500	0.100	0.194
AR(1) Coefficient of $u_t^{1,w}$		B	0.4000	0.100	0.458
Earnings					
Loading 1	$\beta^{w,2}$	N	0.8000	0.100	0.904
Standard Deviation of $u_t^{2,w}$		I	0.0500	0.100	0.143
AR(1) Coefficient of $u_t^{2,w}$		B	0.4000	0.100	0.674
Core PCE					
Loading 2	$\gamma^{\pi,1}$	N	0.0000	1.000	0.045
Standard Deviation of $u_t^{1,p}$		I	0.0500	0.100	0.046
AR(1) Coefficient of $u_t^{1,p}$		B	0.2000	0.100	0.108
Core CPI					
Loading 1	$\beta^{\pi,2}$	N	1.0000	0.100	0.808
Loading 2	$\gamma^{\pi,2}$	N	0.0000	1.000	0.087
Standard Deviation of $u_t^{2,p}$		I	0.1000	0.100	0.077
AR(1) Coefficient of $u_t^{2,p}$		B	0.4000	0.200	0.586
Market-Based Core PCE					
Constant	π_*^3	N	-0.1000	0.100	-0.037
Loading 1	$\beta^{\pi,3}$	N	1.0000	0.100	1.121
Loading 2	$\gamma^{\pi,3}$	N	0.0000	1.000	0.015
Standard Deviation of $u_t^{3,p}$		I	0.0500	0.100	0.035
AR(1) Coefficient of $u_t^{3,p}$		B	0.2000	0.100	0.144
1Q Ahead Expected PCE					
Standard Deviation of $u_t^{1,1,\pi}$		I	0.0500	0.100	0.026
AR(1) Coefficient of $u_t^{1,1,\pi}$		B	0.2000	0.100	0.196
1Q Ahead Expected CPI					
Loading	$\beta^{1,2}$	N	1.0000	0.100	0.980
Standard Deviation of $u_t^{1,2,\pi}$		I	0.0500	0.100	0.062
AR(1) Coefficient of $u_t^{1,2,\pi}$		B	0.2000	0.100	0.198
10Y Ahead Expected PCE					
AR(1) Coefficient of $u_t^{40,1,\pi}$		B	0.2000	0.100	0.271
10Y Ahead Expected CPI					
Loading	$\beta^{40,2}$	N	1.0000	0.100	1.021
AR(1) Coefficient of $u_t^{40,2,\pi}$		B	0.2000	0.100	0.213
PCE Durable Goods Inflation					
Constant	π_*^d	N	-0.3500	0.100	-0.360
Standard Deviation of u_t^d		I	0.2000	2.000	0.286

Notes: Distributions (**N**) Normal, (**G**) Gamma, (**B**) Beta, (**I**) Inverse-gamma-1, (**U**) Uniform

First Sample Estimated Parameters (Continued)

Parameter	**Symbol**	**Density**	**Prior** **Mean**	**Std.Dev**	**Posterior** **Mode**
Inflation in Relative Price of Government, Inventories and Net Exports to Consumption					
Constant	π_*^g	N	0.1980	1.000	-0.666
Standard Deviation of u_t^g		I	0.5000	2.000	1.861
Factor A					
Loading 1	α_1	N	0.6839	0.200	1.305
Loading 2	α_2	N	0.5224	0.200	0.877
Loading 3	α_3	N	0.4314	0.200	0.306
Loading 4	α_4	N	0.3243	0.200	-0.012
Standard Deviation	σ_α	N	0.1000	0.0750	0.040
Factor B					
Loading 1	β_1	N	0.3310	0.200	0.656
Loading 2	β_2	N	0.6525	0.200	1.104
Loading 3	β_3	N	0.8059	0.200	1.162
Standard Deviation	σ_β	N	0.1000	0.0750	0.078

Notes: Distributions (**N**) Normal, (**G**) Gamma, (**B**) Beta, (**I**) Inverse-gamma-1, (**U**) Uniform

Table 4: Second Sample Calibration Targets (Different from First Sample)

Description	**Expression**	**Value**
Fixed Interest Rate (quarterly, gross)	R^*	1.007
Per-Capita Steady-State Output Growth Rate (quarterly)	Y_{t+1}/Y_t	1.003
Growth Rate of Relative Price of Consumption to Investment	P_C/P_I	0.171

Table 5: Second Sample Calibrated Parameters (Different from First Sample)

Parameter	**Symbol**	**Value**
Steady-State Measured TFP Growth (quarterly)	z_*	0.415
Investment-Specific Technology Growth Rate	ω_*	0.171
Steady-State Marginal Depreciation Cost	δ_1	0.038
Core CPI, 1Q Ahead and 10Y Ahead Expected CPI		
Constant	$\pi_*^2, \pi_*^{l,2}$	0.060
10Y Ahead Expected CPI and PCE		
Standard Deviation of $u_t^{40,j,\pi}$		0.020
PCE Durable Goods Inflation		
1st Lag Coefficient	$\beta_{1,1}$	0.000
2nd Lag Coefficient	$\beta_{1,2}$	0.000
Inflation in Relative Price of Government, Inventories and Net Exports to Consumption		
1st Lag Coefficient	$\beta_{2,1}$	0.320
2nd Lag Coefficient	$\beta_{2,2}$	-0.240
Compensation Loading	$\beta^{w,1}$	1.000
Loading 5 Factor A	α_5	0.932
Loading 8 Factor B	β_8	0.210
Loading 10 Factor B	β_{10}	0.000

Table 6: Second Sample Estimated Parameters

		Prior		**Posterior**
Parameter	**Symbol**	**Mean**	**Std.Dev**	**Mode**
Compensation				
Constant	w^1_*	-0.2023	0.100	-0.129
Standard Deviation of $u^{1,w}_t$		0.1941	0.100	0.267
AR(1) Coefficient of $u^{1,w}_t$		0.4579	0.100	0.388
Earnings				
Constant	w^2_*	-0.2370	0.100	-0.131
Loading 1	$\beta^{w,2}$	0.9039	0.100	0.721
Standard Deviation of $u^{2,w}_t$		0.1434	0.100	0.255
AR(1) Coefficient of $u^{2,w}_t$		0.6741	0.100	0.600
Core PCE				
Loading 2	$\gamma^{\pi,1}$	0.0449	0.100	0.211
Standard Deviation of $u^{1,p}_t$		0.0457	0.100	0.247
AR(1) Coefficient of $u^{1,p}_t$		0.1081	0.150	0.180
Core CPI				
Loading 1	$\beta^{\pi,2}$	0.8083	0.150	0.192
Loading 2	$\gamma^{\pi,2}$	0.0868	0.100	0.252
Standard Deviation of $u^{2,p}_t$		0.0770	0.100	0.096
AR(1) Coefficient of $u^{2,p}_t$		0.5856	0.150	0.625
Market PCE				
Constant	π^3_*	-0.0367	0.100	-0.120
Loading 1	$\beta^{\pi,3}$	1.1213	0.150	0.292
Loading 2	$\gamma^{\pi,3}$	0.0153	0.100	0.245
Standard Deviation of $u^{3,p}_t$		0.0349	0.100	0.096
AR(1) Coefficient of $u^{3,p}_t$		0.1436	0.150	0.196
1Q Ahead Expected PCE				
Standard Deviation of $u^{1,1,\pi}_t$		0.0259	0.020	0.070
AR(1) Coefficient of $u^{1,1,\pi}_t$		0.1960	0.050	0.256
1Q Ahead Expected CPI				
Loading	$\beta^{1,2}$	0.9803	0.080	0.993
Standard Deviation of $u^{1,2,\pi}_t$		0.0622	0.020	0.101
AR(1) Coefficient of $u^{1,2,\pi}_t$		0.1982	0.050	0.220
10Y Ahead Expected PCE				
AR(1) Coefficient of $u^{40,1,\pi}_t$		0.2711	0.050	0.310
10Y Ahead Expected CPI				
Loading	$\beta^{40,2}$	1.0207	0.100	1.062
AR(1) Coefficient of $u^{40,2,\pi}_t$		0.2133	0.050	0.212
PCE Durable Goods Inflation				
Constant	π^d_*	-0.4500	0.200	-0.451
Standard Deviation of u^d_t		0.5000	0.150	0.316
Inflation in Relative Price of Government, Inventories and Net Exports to Consumption				
Constant	π^g_*	0.8900	0.400	0.067

Second Sample Estimated Parameters (Continued)

		Prior		**Posterior**
Parameter	**Symbol**	**Mean**	**Std.Dev**	**Mode**
Standard Deviation of u_t^g		0.8143	0.150	1.267
Factor A				
Loading 0	α_0	0.0180	0.250	0.135
Loading 1	α_1	0.0574	0.250	0.120
Loading 2	α_2	0.1941	0.250	0.284
Loading 3	α_3	0.3996	0.250	0.460
Loading 4	α_4	0.6520	0.250	0.760
Loading 6	α_6	1.2266	0.250	1.127
Loading 7	α_7	1.5237	0.250	1.465
Loading 8	α_8	1.8139	0.250	1.697
Loading 9	α_9	2.0914	0.250	1.919
Loading 10	α_{10}	2.3523	0.250	2.742
Standard Deviation	σ_α	0.0442	0.100	0.055
Factor B				
Loading 0	β_0	-0.0181	0.300	0.029
Loading 1	β_1	0.2211	0.300	0.033
Loading 2	β_2	0.3679	0.300	0.070
Loading 3	β_3	0.4424	0.300	0.103
Loading 4	β_4	0.4612	0.300	0.126
Loading 5	β_5	0.4370	0.300	0.137
Loading 6	β_6	0.3817	0.300	0.162
Loading 7	β_7	0.3032	0.300	0.179
Loading 9	β_9	0.1074	0.300	0.212
Standard Deviation	σ_β	0.0334	0.100	0.439
Standard Deviations of Monetary Policy Innovations				
Unanticipated	σ_{η_0}	0.0061	0.005	0.011
1Q Ahead	σ_{η_1}	0.0021	0.005	0.010
2Q Ahead	σ_{η_2}	0.0004	0.005	0.009
3Q Ahead	σ_{η_3}	0.0019	0.005	0.010
4Q Ahead	σ_{η_4}	0.0001	0.005	0.010
5Q Ahead	σ_{η_5}	0.0025	0.005	0.000
6Q Ahead	σ_{η_6}	0.0019	0.005	0.010
7Q Ahead	σ_{η_7}	0.0011	0.005	0.010
8Q Ahead	σ_{η_8}	0.0001	0.005	0.000
9Q Ahead	σ_{η_9}	0.0014	0.005	0.003
10Q Ahead	$\sigma_{\eta_{10}}$	0.0028	0.005	0.009

6. Deriving the moments of the policy deviations from the news representation

We now derive the first and second moments of $\boldsymbol{\theta}_t$, $\boldsymbol{\theta}_t|s_{t-1}$, $\boldsymbol{\theta}_t|s_t$ and $\boldsymbol{\theta}_t^p$. To this aim, it is useful to write the vector of signals in matrix notation as follows

$$\boldsymbol{\theta}_t = \begin{bmatrix} \theta_t \\ \theta_{t+1} \\ \vdots \\ \theta_{t+H} \end{bmatrix} = \begin{bmatrix} \varepsilon^H_{R,t-H}+ & \varepsilon^{H-1}_{R,t-H+1}+\dots & +\varepsilon^1_{R,t-1}+ & \varepsilon^0_{R,t}+ & 0+ & \dots & +0 \\ 0+ & \varepsilon^H_{R,t-H+1}+\dots & +\varepsilon^2_{R,t-1}+ & \varepsilon^1_{R,t}+ & \varepsilon^0_{R,t+1}+ & \dots & +0 \\ & & & \vdots & & & \\ 0+ & \dots & +\varepsilon^H_{R,t-1}+ & \varepsilon^{H-1}_{R,t}+ & \varepsilon^{H-2}_{R,t+1}+ & \dots & +0 \\ 0+ & \dots & +0 & \varepsilon^H_{R,t}+ & \varepsilon^{H-1}_{R,t+1}+ & \dots & +\varepsilon^0_{R,t+H} \end{bmatrix}$$

or equivalently

$$\boldsymbol{\theta}_t = J_H\boldsymbol{\varepsilon}_{R,t+H} + \dots + J_1\boldsymbol{\varepsilon}_{R,t+1} + \boldsymbol{\varepsilon}_{R,t} + J_1'\boldsymbol{\varepsilon}_{R,t-1} + \dots + J_H'\boldsymbol{\varepsilon}_{R,t-H}, \tag{10}$$

where J_k is a $(H+1)\times(H+1)$ matrix of zeros with ones on the kth lower diagonal; J_0 coincides with the identity matrix. For example, for when $H=4$ and $k=2$

$$J_2 = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}.$$

We have that the first moments of $\boldsymbol{\theta}_t$, $\boldsymbol{\theta}_t|s_{t-1}$, and $\boldsymbol{\theta}_t|s_t$ are

$$\begin{aligned} E(\boldsymbol{\theta}_t) &= 0 \\ E_{t-1}(\boldsymbol{\theta}_t) &= J_1'\boldsymbol{\varepsilon}_{R,t-1} + \dots + J_H'\boldsymbol{\varepsilon}_{R,t-H} \\ E_t(\boldsymbol{\theta}_t) &= E_{t-1}(\boldsymbol{\theta}_t) + \kappa(s_t - E_{t-1}(\boldsymbol{\theta}_t)) \\ &= E_{t-1}(\boldsymbol{\theta}_t) + \kappa(\boldsymbol{\theta}_t + v_t - E_{t-1}(\boldsymbol{\theta}_t)). \end{aligned} \tag{11}$$

The corresponding second moments are given by

$$\begin{aligned} \Sigma_\theta &= E(\boldsymbol{\theta}_t\boldsymbol{\theta}_t') \\ &= J_H\Sigma_\varepsilon J_H' + \dots + J_1\Sigma_\varepsilon J_1' + \Sigma_\varepsilon + J_1'\Sigma_\varepsilon J_1 + \dots + J_H'\Sigma_\varepsilon J_H \\ \Xi_\theta &= E(\boldsymbol{\theta}_t - E_{t-1}(\boldsymbol{\theta}_t))(\boldsymbol{\theta}_t - E_{t-1}(\boldsymbol{\theta}_t))' \\ &= J_H\Sigma_\varepsilon J_H' + \dots + J_1\Sigma_\varepsilon J_1' + \Sigma_\varepsilon \\ \tilde{\Xi}_\theta &= E(\boldsymbol{\theta}_t - E_t(\boldsymbol{\theta}_t))(\boldsymbol{\theta}_t - E_t(\boldsymbol{\theta}_t))' \\ &= E[\boldsymbol{\theta}_t - E_{t-1}(\boldsymbol{\theta}_t) - \kappa(\boldsymbol{\theta}_t + v_t - E_{t-1}(\boldsymbol{\theta}_t))][\boldsymbol{\theta}_t - E_{t-1}(\boldsymbol{\theta}_t) - \kappa(\boldsymbol{\theta}_t + v_t - E_{t-1}(\boldsymbol{\theta}_t))]' \\ &= E[\boldsymbol{\theta}_t - E_{t-1}(\boldsymbol{\theta}_t)][\boldsymbol{\theta}_t - E_{t-1}(\boldsymbol{\theta}_t)]' + E[\kappa(\boldsymbol{\theta}_t + v_t - E_{t-1}(\boldsymbol{\theta}_t))][\kappa(\boldsymbol{\theta}_t + v_t - E_{t-1}(\boldsymbol{\theta}_t))]' \\ &\quad -2\times E[\boldsymbol{\theta}_t - E_{t-1}(\boldsymbol{\theta}_t)][\kappa(\boldsymbol{\theta}_t + v_t - E_{t-1}(\boldsymbol{\theta}_t))]' \\ &= \Xi_\theta + \kappa(\Xi_\theta + \Xi_v)\kappa' - 2\Xi_\theta\kappa' \\ &= \Xi_\theta + \Xi_\theta(\Xi_\theta + \Xi_v)^{-1}(\Xi_\theta + \Xi_v)(\Xi_\theta + \Xi_v)^{-1}\Xi_\theta' - 2\Xi_\theta(\Xi_\theta + \Xi_v)^{-1}\Xi_\theta' \\ &= \Xi_\theta - \Xi_\theta(\Xi_\theta + \Xi_v)^{-1}\Xi_\theta'. \end{aligned} \tag{12}$$

The last derivation uses the property of noise that $E[\boldsymbol{\theta}_t - E_{t-1}(\boldsymbol{\theta}_t)]v_t' = 0$.

Notice that we now have all the ingredients necessary to back out times series for $\boldsymbol{\theta_t}$ and v_t from the monetary policy news derived from the estimated model. We can obtain $\boldsymbol{\theta_t}$ from (10). We can obtain v_t using the main text's equation (5) with (11), our time series for $\boldsymbol{\theta}_t$, and our estimate of $\kappa = \Xi_\theta \left[\Xi_\theta + \Xi_v\right]^{-1}$ that uses $\Xi_\theta \left[\Xi_\theta + \Xi_v\right]^{-1} \Xi_\theta = \Sigma_\varepsilon$, (9), and (12).

Recall

$$\begin{aligned}\mathcal{G}(h) &= 1 - \exp\left[\xi(\boldsymbol{\theta}_t^p \mid s^{t-1}, s_t^H, s_t^{H-1}, \ldots, s_t^{H-h}) - \xi(\boldsymbol{\theta}_t^p \mid s^{t-1})\right] \\ &= 1 - \exp\left[\frac{1}{2}\log_2 |\Xi_{\theta,h}^p| - \frac{1}{2}\log_2 |\Xi_\theta^p|\right].\end{aligned}$$

The derivations of the moments of the path of monetary policy, $\boldsymbol{\theta}_t^p = [\theta_{t+1}, ..., \theta_{t+H}]'$, are quite straightforward as we just apply the previous calculations to a linear transformation of $\boldsymbol{\theta}_t$. In particular, we have the future path of monetary policy is given by

$$\boldsymbol{\theta}_t^p = P\boldsymbol{\theta}_t,$$

where P is the $H \times (H+1)$ matrix that removes the first element of the vector $\boldsymbol{\theta}_t$. First and second moments of $\boldsymbol{\theta}_t^p$ are then easy to derive. In particular,

$$\Xi_\theta^p = E(P\boldsymbol{\theta}_t - E_{t-1}(P\boldsymbol{\theta}_t))(P\boldsymbol{\theta}_t - E_{t-1}(P\boldsymbol{\theta}_t))' = P\Xi_\theta P'.$$

The expected value of $\boldsymbol{\theta}_t^p$ conditional of having received the signals about the deviations from the policy rule from H and until $H-h$ quarters out is given by

$$\begin{aligned}E(\boldsymbol{\theta}_t^p | s^{t-1}, s_t^H, s_t^{H-1} ..., s_t^{H-h}) &= E_{t-1}(\boldsymbol{\theta}_t^p) + \kappa_h S_h(s_t - E_{t-1}(\boldsymbol{\theta}_t^p)) \\ &= E_{t-1}(\boldsymbol{\theta}_t^p) + \kappa_h S_h(\boldsymbol{\theta}_t^p + v_t - E_{t-1}(\boldsymbol{\theta}_t^p))\end{aligned}$$

where $\kappa_h = \Xi_\theta S_h'(S_h(\Xi_\theta + \Xi_v)S_h')^{-1}$ and S_h is a $(h+1) \times H$ selection matrix that has ones on the right most diagonal and zeros elsewhere. For example, at the longest horizon when $h = 0$, $S_0 = [0, 0, ..., 1]$. When $h = 1$,

$$S_1 = \begin{bmatrix} 0 & \ldots & 1 & 0 \\ 0 & \ldots & 0 & 1 \end{bmatrix}.$$

We can derive the covariance matrix of $\boldsymbol{\theta}_t^p$ conditional of having received a subset of signal signals in the same way we derived $\tilde{\Xi}_\theta$. Doing so we obtain

$$\Xi_{\theta,h}^p = \Xi_\theta^p - \Xi_\theta^p S_h'(S_h(\Xi_\theta^p + \Xi_v)S_h')^{-1} S_h \Xi_\theta^p.$$

7. Decomposing Forward Guidance

In the main text we claim that our model allows us to decompose the dynamic response of an endogenous variable to a forward guidance shock into two additive components. The first component represents the sole effects of the change in expectations triggered by forward guidance (the yellow bars in Figures 5 and 7. The second component captures the effects of implementing the deviations implied by the forward guidance shocks without announcing them in advance (the blue bars in Figures 5 and 7. In this section we sketch the proof of this claim.

It is easier to work with the news representation of the model. Recall that the actual deviation from the rule at time t is given by all the news about it received up until that date plus the contemporaneous news:

$$\theta_t = \sum_{j=0}^{H} \varepsilon^{j}_{R,t-j}. \tag{13}$$

The news are obtained by using (5) in the main text.

The response of hours in the baseline case of truthful forward guidance (the black line in Figures 5 and 7) is obtained by assuming that the forward guidance shock is driven by actual future deviations from the rule (zero noise). The alternative scenario is obtained by simulating the news representation with the contemporaneous news $\varepsilon^{0}_{R,t}$ replaced by $\tilde{\varepsilon}^{0}_{R,t}$ where

$$\tilde{\varepsilon}^{0}_{R,t} \equiv -\sum_{j=1}^{H} \varepsilon^{j}_{R,t-j}, \tag{14}$$

for every t. In the alternative scenario with forward guidance driven by noise the contemporaneous news shock must neutralize the effects of all the news about the date t deviation in the previous H periods. Imposing (14) guarantees this. The remaining news shocks are the same in both the baseline case and in the alternative scenario.

The infinite moving average representation of hours in our model, h_t, can be expressed as

$$h_t = \Phi\left(L\right)\xi_t, \tag{15}$$

where ξ_t is the column vector containing all the shocks realized at time t. We order the shocks in ξ_t such that the news shocks realized at date t are ordered first. The blue bars in Figures 5 and 7 are constructed by taking the difference between the response of hours to news shocks in the baseline (h_t), the black line, and in the alternative scenario of noise-driven forward guidance ($\tilde{h}_t$), the yellow bars. Specifically,

$$\begin{aligned} h_t - \tilde{h}_t &= \Phi_h\left(L\right)\begin{bmatrix} \varepsilon^{0}_{R,t} - \tilde{\varepsilon}^{0}_{R,t} \\ \mathbf{0} \end{bmatrix} && (16) \\ &= \Phi_h\left(L\right)\begin{bmatrix} \varepsilon^{0}_{R,t} + \sum_{j=1}^{H} \varepsilon^{j}_{R,t-j} \\ \mathbf{0} \end{bmatrix} && (17) \\ &= \Phi_h\left(L\right)\begin{bmatrix} \theta_t \\ \mathbf{0} \end{bmatrix}, && (18) \end{aligned}$$

where in the first line we use the infinite moving average representation of hours, the second line is obtained by plugging equation (14), and the third line stems from equation (13).

It follows that the difference in hours between the baseline case and the alternative scenario, $h_t - \tilde{h}_t$, can be obtained by simulating the news representation using the actual deviations from the rule θ_t as contemporary news shocks and without any other news. This scenario is tantamount to the case of no communication in which the central bank does not say anything about its future deviations and it just implements them over time.

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