Banker Compensation, Relative Performance, and Bank Risk

Arantxa Jarque
Federal Reserve Bank of Richmond

Edward Simpson Prescott*
Federal Reserve Bank of Cleveland

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Abstract

A multi–agent, moral–hazard model is used to analyze the connection between compensation of bank employees (below a CEO) and bank risk. Optimal contracts consist of heavy use of relative performance and under some conditions under provide effort. Unlike in the single–agent model, pay for performance does not necessarily create risk. If employee returns are uncorrelated, pay is irrelevant for risk. If returns are perfectly correlated, effort is underprovided and a low wage increases risk. For intermediate levels of correlations, optimal contracts are characterized and described as relative performance schemes that depend on individual performance and bank revenue. Comovement of compensation with bank performance are derived for a simple production function and used to analyze the role of bonuses. Connections to compensation regulations are discussed.

Keywords: incentive compensation, relative performance, bank regulation, controls

JEL Codes: D82, G21, G28, J33

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1 Introduction

This paper uses organizational economics to analyze the connection between banker compensation and bank risk. In a framework where a bank operates under limited liability with deposit insurance, a multi-agent, moral-hazard model is developed to analyze bank risk. Optimal contracts are solved for and linked to the deposit insurance distortion. How these contracts change with risk at the bank level is analyzed. The results are then used to discuss implications for regulation.

Controlling bank risk via regulation of compensation arrangements is a new focus of bank regulation. The Federal Reserve Board in 2010 issued supervisory guidance to banks that their compensation arrangements “Provide employees incentives that appropriately balance risk and reward” (Federal Register, 2010). Similarly, the Dodd-Frank law requires that regulations be written that prohibit incentive-based compensation that encourages inappropriate risks. These regulations are motivated by the belief that bank compensation practices were a significant contributory factor to the recent financial crisis (e.g., Financial Stability Forum (2009)).

Conceptually, there are two classes of people in a bank who could materially contribute to the risk of a bank. The first is an individual, like a CEO or some traders, whose individual decisions can materially affect the bank’s performance. The second is a group of individuals like loan officers whose decisions together can have a significant impact on the bank’s performance.

This paper analyzes the second class of people. We do this for two reasons. First, a CEO is limited in his ability to directly control the actions of his subordinates. Instead, he has to rely on indirect methods, such as delegation of authority, usage of internal controls,
and compensation to direct the actions of subordinates. In the end, a bank’s risk profile is determined by the actions of its lending officers and other employees. Second, despite the high level of CEO pay, by far most labor compensation paid out by a bank goes to its other employees, so compensation regulations have the largest effect on them. For example, in 2012 the largest bank holding company in the United States is J.P. Morgan. As of December 31, 2012, it had 248,633 employees, measured at full time equivalents, and paid them 31 billion dollars in salaries and benefits (source: FR Y-9C). Meanwhile, its CEO was paid 18.7 million dollars (source: Execucomp), a very small fraction of total compensation.

Our approach is to develop a multi–agent, moral–hazard model where bank equity owners may like risk, but risk–averse employees are the agents that make the decisions that determine bank risk. We solve the model for compensation contracts that are optimal from the bank’s perspective and then evaluate the connections between the contracts and bank risk. We also discuss the implications for bank regulation.

This class of multi-agent models has three features that characterize large banks. First, large banks benefit from explicit and implicit government insurance of their liabilities, so the equity owners do not bear all the costs of a bank failure. Second, in large banks there are many employees, most of whom alone have a minuscule effect on the performance and risk of the bank. Third, in practice, a significant fraction of compensation is paid through bonus pools that are tied to bank profitability. As we will show, this type of compensation can be interpreted as a relative performance contract.

Modeling a bank as an organization has strong implications for the connection between employee compensation and bank risk. The first implication is that because each loan officer has an infinitesimal effect on the performance of a bank, bank risk is determined by the correlation of loan officers’ returns, not the variance of an individual loan officer’s project. The second implication is that compensation contracts will make heavy use of relative performance because comparing loan officers’ return has a high signal value about
loan officer effort. Both of these first two features are absent from the single-agent CEO model. The third implication is that when the loan officers are risk averse, as we assume, they do not like bankruptcy because their compensation is limited in that event. A bank that is trying to exploit limited liability will have to pay its employees more to make up for this risk, which raises the cost to the bank from taking advantage of the limited liability distortion.

Correlation is so important for determining bank risk that when it is exogenous, that is, when loan officer actions do not affect the correlation of their returns, there are some surprising connections between compensation and bank risk. For example, we show that when loan officer returns are perfectly uncorrelated, there is no bank risk because the loan officer risk is entirely idiosyncratic and averages out. Consequently, compensation is irrelevant for bank risk, though it may matter for bank profits and it certainly matters for the risk to a loan officer. We also show that when loan officer returns are perfectly correlated, loan officer effort can be perfectly inferred from bank output, so there is no moral hazard problem and the officer can be paid a wage. Here, the correlation in returns means that there is a lot of risk for the bank and it can be shown, under reasonable conditions, that a low wage creates more risk than a high wage.

For the more general case, with partial correlation, we analyze the effort distortion from limited liability and deposit insurance. We also solve for optimal contracts in several examples where the degree of correlation is parameterized. We show how compensation depends on both individual and bank performance and how these contracts can be implemented with a bonus scheme.

Two features of compensation that we do not discuss in this paper are multi-period contracts and monitoring. Multi-period contracts can be used to study “claw backs”, that is, compensation that is reduced if the loan or project performs badly in the long run. We leave this feature out, however, to focus on the connection between compensation and
correlation of returns. For work addressing the timing question using dynamic moral–hazard models with persistence, see Jarque and Prescott (2013). Lending and other activities, like trading, are typically monitored by a bank and subject to limits and other controls. We leave these features out to focus on relative performance. However, we discuss later how the model can be extended to address these institutional features.

2 Literature

The multi-agent, principal agent model we use is based on the relative–performance model of Holmström (1982). This model is characterized by multiple agents and joint production of either physical output or information relevant to contracting. It has been adapted to consider many aspects of organizational design such as monitoring, task assignment, and job rotation.

In the banking literature, the bank’s investment decisions is usually modeled as being chosen by a single agent. Usually, the single agent represents equity owners who maximize profits while enjoying limited liability and funding a portion of the investment with insured deposits. While keeping the limited liability and insured deposit assumptions, a smaller part of the banking literature follows the Jensen and Murphy (1990) approach where the equity owners are the principal and the agent is typically a CEO with private information who makes the investment decisions. John, Saunders, and Senbet (2000), Phelan (2009), and Bolton, Mehren, and Shapiro (2010) consider this problem and examine how to regulate compensation to limit the distortion caused by limited liability. Thanassoulis (2012, 2013) also looks at CEO compensation and bank risk, but considers a market assignment problem

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3 For examples, see Prescott and Townsend (2002, 2006) and the surveys in Bolton and Dewatripont (2005), Gibbons and Roberts (2013), and Mookherjee (2013).

4 See, for example, Kareken and Wallace (1978), Kim and Santomero (1988), Flannery (1989), and Furlong and Keeley (1990). The savings and loan crisis in the United States during the 1980s is often viewed as evidence for this model (White (1991)).
where, in the absence of pay caps, compensation of CEOs is bid up to levels that makes banks inefficiently risky. These models feature a single agent that chooses bank risk and are appropriate for studying CEO compensation. In this paper, instead, our focus is on compensation of lower level employees like loan officers.

There are also a few theoretical papers in the banking literature that consider how organizational features affect compensation. Ang, Lauterbach and Schreiber (2001) consider a model where bank executives monitor each other. Lóránth and Morrison (2010) looks at internal reporting systems and loan officer incentives. Heider and Inderst (2012) study a multi-task problem where loan officers generate soft information about borrowers. Kupiec (2013) studies incentive compensation in a model where a loan officer determines the risk of a loan and a risk manager determines the losses in case of default.\(^5\)

Most of the empirical literature on banker compensation and bank risk looks at CEO compensation, mainly because of data availability.\(^6\) There are very few studies of compensation of lower level bank employees because this data is proprietary. One exception is Agarwal and Ben–David (2012) who studied the results of an experiment that was run at a bank, which for a period of time paid half of its small business loan officers a wage and paid the other half with a wage plus an incentive. They found that the incentive plan increased the loan origination rate by 31 percent and the size of loans by 15 percent. Unfortunately for the bank, the plan also increased the default rate by 28 percent, so the plan was dropped.

Berg, Puri, and Rocholl (2012) studied the data input behavior by loan officers who are paid based on volume. These loan officers entered hard information, that is, non-judgmental information, into a bank’s loan scoring system that determined approval. They

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\(^5\)While not explicitly concerned with organizational economics, Acharya and Yorulmazer (2008) study a model with a systemic shock in which inference of this shock and idiosyncratic shocks from bank performance by the market leads to herding, that is, banks choosing to make correlated investments.

find evidence of selective entering of hard information into the scoring system to improve a borrower’s chance of approval. Cole, Kanz, and Klapper (2011) ran laboratory experiments on commercial bank loan officers where they varied the connection between compensation and incentives. They found that the compensation structure had a large effect on lending and the quality of the loans.

Finally, Hertzberg, Liberti, and Paravisini (2010) examined the connection between pay, organizational structure, and reporting of information. They examined the use of loan officer rotation at a large international bank and argued that it alleviates incentives to hide the quality of poorly performing loans. In their analysis, the bank’s policies effectively tie pay to a loan officer’s revenues, which gives him an incentive to continue lending even to weak borrowers. Furthermore, lending is tied to the loan officer’s ongoing assessment of borrower’s in his portfolio. They argue that the rotation along with career concerns mitigate the incentive loan officers have to under report the risk in their portfolio of loans.

3 The Model

There is a bank that consists of depositors, equity holders, and a continuum of loan officers of measure one, each of whom has an infinitesimally small effect on the performance of the bank. Each loan officer takes an action $a \in A \subset \mathbb{R}_+$ that produces a return $r$ as a function of an idiosyncratic shock and a common shock $\theta$. Both shocks occur after the action is taken. There is a finite number of possible returns for each loan officer. For most of the analysis there is also a finite number of actions, though in one subsection we allow for a continuum of actions. The common shock can take on a continuum of values over the interval $[0, \Theta]$ and is drawn according to the probability density function $h(\theta)$ with cumulative distribution function $H(\theta)$. The probability of a loan officer’s return is written $f(r|\theta, a)$ with $\sum_r f(r|\theta, a)r \geq \sum_r f(r|\theta, \hat{a})r$ for all $\theta$ and all $\hat{a} < a$, that is, a loan officer’s expected return is increasing in his action. Furthermore, we also assume that given $a$ the expected return is
continuous and increasing in the common shock, that is, \( \forall a, \sum_r f(r|\theta', a)r \geq \sum_r f(r|\theta, a)r \) if \( \theta' > \theta \).

A loan officer’s action and idiosyncratic shock are private information, while the common shock is observed by the bank.\(^7\) A loan officer receives utility from consumption, \( c \geq 0 \), and action, \( a \), of \( U(c) - V(a) \), where \( U \) is concave and increasing, \( U(0) \geq 0 \), and \( V \) is increasing and weakly convex. Each loan officer has an ex ante reservation utility level of \( \bar{U} \).

The bank finances the loan officers’ investment projects with an investment of size one. The investment is financed by government insured deposits, \( 0 \leq D \leq 1 \), and equity \( 1 - D \). Because of deposit insurance, depositors receive the face value of deposits at the end of the period no matter how the bank performs. For simplicity, we take the level of deposits as given.\(^8\)

The bank operates in the best interest of the equity holders, so we will often refer to the bank and the equity holders interchangeably. The equity holders are treated as a single risk–neutral principal with limited liability. The bank receives a total return of \( \bar{r}(\theta) \), which is the sum of the loan officers’ returns, and pays out funds to depositors and compensation to loan officers. The total compensation bill is \( \bar{c}(\theta) \).

The bank’s expected profit is

\[
\int_0^\Theta \max\{\bar{r}(\theta) - \bar{c}(\theta) - D, 0\} h(\theta)d\theta.
\]

\(^7\)We could assume that \( \theta \) is not observed by anyone, but as long as the mapping from \( a \) to the total return is an invertible function, then \( \theta \) can be identified from the contract and the total return. For that reason, we simply assume that \( \theta \) is public information. However, the contract can also be interpreted as a relative performance contract where an individual loan officer’s return is compared with the total return. Sometimes, we will use the relative performance language.

\(^8\)The model can be extended to include franchise value, the value of a bank being a continuing concern. A positive franchise value reduces risk-taking incentives because it is lost in the event of failure. The empirical banking literature finds that franchise value has a significant impact on risk taking. Keeley (1990) argued that the low rate of bank failure pre-1980, before deregulation, was due to banks’ incentive to preserve the positive franchise value that came with monopoly profits. The second half of the savings and loan crises is often attributed to savings and loans institutions gambling for resurrection when they had negative franchise value (e.g., White (1991)). Demsetz, Saidenberg, and Strahan (1996) find that franchise value is negatively correlated with risk taking in bank data from the 1990s. We leave franchise value out to keep the problem simpler.
The total return to the bank is the sum of the individual loan officers’ returns, which is
\[ \forall \theta, \quad \bar{r}(\theta) = \sum_r f(r|\theta, a)r. \] (1)

The bank gives each loan officer the same compensation schedule, \( c(r, \theta) \), where \( r \) is the return produced by a loan officer. The total compensation bill is then
\[ \forall \theta, \quad \bar{c}(\theta) = \sum_r f(r|\theta, a)c(r, \theta). \] (2)

Finally, we assume that in the event of bankruptcy, depositors are paid before loan officers, so if \( \bar{r}(\theta) < D \) then \( c(r, \theta) = 0 \).

The problem for the bank is:

**Bank Program**

\[
\begin{align*}
\max_{a,c(r,\theta)\geq 0, \bar{c}(\theta)\geq 0, \bar{r}(\theta)} & \int_0^\Theta \max\{\bar{r}(\theta) - \bar{c}(\theta) - D, 0\} h(\theta) d\theta \\
\text{subject to} \quad & \forall \theta, \quad \bar{c}(\theta) \leq \max\{\bar{r}(\theta) - D, 0\}, \\
& \int_0^\Theta \sum_r f(r|\theta,a)U(c(r,\theta))h(\theta)d\theta - V(a) \geq \bar{U}, \\
& \int_0^\Theta \sum_r f(r|\theta,\hat{a})U(c(r,\theta))h(\theta)d\theta - V(\hat{a}), \quad \forall \hat{a}. \quad (3)
\end{align*}
\]

Equation (4) limits total compensation to be less than bank revenue, net of payments to depositors. Equation (5) is the participation constraint for a loan officer, and equation (6) is the incentive constraint.

The piecewise linear objective function and the piecewise linear constraint, (4), make this optimization problem non–differentiable. In order to derive results about compensation from
first-order conditions, we consider the subproblem of implementing a given action. Because
we assumed that the expected return is continuous and increasing in $\theta$, for each $a$, there is a
$\theta(a)$ such that for all $\theta < \theta(a)$, $\bar{r}(\theta) < D$, that is, the bank is bankrupt and limited liability
binds. Note that in these states $c(r, \theta) = \bar{c}(\theta) = 0$. Furthermore, because the expected value
of a loan officer’s return increases with $a$, $\theta(a)$ is increasing in $a$.

Now consider the subproblem of implementing action $a$ and choosing $c(r, \theta)$ for $\theta \geq \theta(a)$.
This subproblem is

**Bank Subprogram**

$$\max_{\forall \theta \geq \theta(a), c(r, \theta) \geq 0, \bar{c}(\theta) \geq 0, \bar{r}(\theta)} \int_{\theta(a)}^{\theta} (\bar{r}(\theta) - \bar{c}(\theta) - D) h(\theta) d\theta \quad \text{(7)}$$

subject to

$$\forall \theta \geq \theta(a), \bar{r}(\theta) = \sum_r f(r|\theta, a)r, \quad \text{(8)}$$

$$\forall \theta \geq \theta(a), \bar{c}(\theta) = \sum_r f(r|\theta, a)c(r, \theta), \quad \text{(9)}$$

$$\forall \theta \geq \theta(a), \bar{c}(\theta) \leq \bar{r}(\theta) - D, \quad \text{(10)}$$

$$H(\theta(a))U(0) + \int_{\theta(a)}^{\theta} \sum_r f(r|\theta, a)U(c(r, \theta))h(\theta)d\theta - V(a) \geq \bar{U}, \quad \text{(11)}$$

$$\int_{\theta(a)}^{\theta} \sum_r f(r|\theta, a)U(c(r, \theta))h(\theta)d\theta - V(a) \geq \int_{\theta(a)}^{\theta} \sum_r f(r|\theta, \hat{a})U(c(r, \theta))h(\theta)d\theta - V(\hat{a}), \forall \hat{a}. \quad \text{(12)}$$

Note that in the incentive constraint, the bankruptcy states on the right-hand side of (12)
are a function of $a$ and not the deviating action $\hat{a}$. The bankruptcy states are not influenced
by a loan officer’s deviating action because in equilibrium all other loan officers, who are of
measure one, choose the recommended action $a$ and that determines the aggregate return.
and thus whether there is bankruptcy in state \( \theta \). This is one difference from the single-agent problem in which the agent’s deviating action does affect the probability of default.

The objective function and constraints in the subproblem are differentiable, so we can use the Lagrangian multipliers to characterize an optimal compensation contract. Let \( \nu(\theta) \) be the multiplier on (10), \( \lambda \) on (11), and \( \mu(\hat{a}) \) on (12). The first–order condition on \( c(r, \theta) \) gives

\[
\frac{h(\theta) + \nu(\theta)}{h(\theta)U''(c(r, \theta))} = \lambda + \sum_{\hat{a} \neq a} \mu(\hat{a}) \left( 1 - \frac{f(r|\theta, \hat{a})}{f(r|\theta, a)} \right),
\]

(13)

where \( \lambda \geq 0 \) and \( \mu(\hat{a}) \geq 0 \), and when \( c(r, \theta) > 0 \).

There are two cases to consider. First, when \( 0 < \bar{c}(\theta) < \bar{r}(\theta) - D, \nu(\theta) = 0 \) and the first–order condition is the same as in the standard moral hazard problem where consumption increases as the likelihood ratio decreases. Second, when \( \bar{c}(\theta) = \bar{r}(\theta) - D \geq 0, \nu(\theta) > 0 \), so the upper bound on the total compensation bill reduces what the bank would pay out if this constraint did not bind. Note that \( \nu \) is not a function of \( r \), so the resource constraint (10) will distort consumption in states where the payment is not zero but \( \nu(\theta) > 0 \). In this case compensation is lower for all values of \( r \) than it would be if \( \nu(\theta) = 0 \).

The subsequent analysis will make frequent use of the likelihood ratio in (13). Let \( LR(r, \theta, \hat{a}; a) \) be the likelihood ratio corresponding to the incentive constraint where \( a \) is recommended and \( \hat{a} \) is the deviating action, that is,

\[
LR(r, \theta, \hat{a}; a) = \frac{f(r|\theta, \hat{a})}{f(r|\theta, a)}.
\]

In the second case, when the resource constraint binds, an important property of the optimal contract in unconstrained states still holds. For example, the relative levels of marginal utility for any two returns, \( r_1 \) and \( r_2 \), will be the same as they would be if it did

\[9\]This also allows us to drop the agent’s utility in bankruptcy states because the \( U(0) \) term cancels out on both sides of (12).
not bind because
\[
\frac{U'(c(r_1, \theta))}{U'(c(r_2, \theta))} = \frac{\lambda + \sum_{\hat{a} \neq a} \mu(\hat{a})(1 - LR(r_1, \theta))}{\lambda + \sum_{\hat{a} \neq a} \mu(\hat{a})(1 - LR(r_2, \theta))}, \forall \theta \geq \theta(a).
\]

The binding resource constraint does not change relative marginal utilities between any two states, but it will, as we discussed above, shift the levels of consumption.\(^{10}\)

The distortion in the contract can also be viewed as the bank being unable to fully use the information contained in the likelihood ratios due to the limits imposed by the resource constraint. To see this, take the expectation of (13) over \(r\) for each \(\theta\). For \(\theta\) where neither the bankruptcy constraint nor the resource constraint binds, this gives
\[
E\left[\frac{1}{U'(c(r, \theta))}\right] = \lambda.
\]

For \(\theta\) where the resource constraint binds, this gives
\[
E\left[\frac{1}{U'(c(r, \theta))}\right] = \frac{1}{1 + \nu(\theta)/h(\theta)} \lambda.
\]

The inverse of marginal utility is the marginal increase in consumption that the bank would receive if the bank could pay the employees one less util. In states where the resource constraint does not bind, the weighted sum of these payments equals \(\lambda\), the shadow price of relaxing the participation constraint. The cost to the bank of paying compensation to the loan officers in each unconstrained state is the same in expectation.\(^{11}\) In the constrained states, the right-hand side is lower because \(1/(1 + \nu(\theta)/h(\theta)) < 1\). This means that relaxing the participation constraint corresponds to a smaller drop in consumption to the loan officers, and thus a smaller gain to the bank in this state, than if the resource constraint did not bind.

\(^{10}\)One caveat to this analysis is that it is possible that \(c(r, \theta) = 0\), which is the lower bound on consumption. In this case, the first-order condition will not hold at equality for that \(r\). Still, the relative ordering will hold except that lower levels of consumption may be constrained to be zero.

\(^{11}\)If there was no incentive constraint then the corresponding equation would hold for each \(r\).
In general, the analysis shows that the bankruptcy states and resource constraints restrict consumption to the loan officers in these states. Consequently, actions that raise the probability of these states will incur an additional cost to implement.

### 3.1 Welfare

In this partial equilibrium model, there is the bank, bank employees, depositors, and an unmodeled deposit insurer. Depositors always receive their deposits, while bank employees always receive their reservation utility, so welfare depends on bank profits and the cost of the transfer from the deposit insurer. We assume that this cost is simply equal to the size of the transfer, representing a loss to the rest of the economy, so welfare can be evaluated by just considering the present value of a bank’s investment, net of compensation costs and payments to depositors, that is,

$$\int_0^\Theta (\bar{r}(\theta) - \bar{c}(\theta))h(\theta) d\theta - D.$$  

The only difference from the bank’s objective function, (3), is the absence of limited liability.

The existence of limited liability and deposit insurance will distort the bank’s decision from the social optimum. One way to express the distortion is to explicitly describe the implicit transfer from deposit insurance as a function of the chosen action. This illustrates the basics of the distortion and will be useful for some of the analysis later.

Let $c^*(r, \theta)$ be the optimal compensation contract for a given $a$. Then, substituting for $\bar{r}(\theta)$ and $\bar{c}(\theta)$ into the objective function, (7), gives expected bank profits conditional on action $a$ as

$$\int_0^\Theta \sum_r f(r|\theta, a) (r - c^*(r, \theta) - D) h(\theta) d\theta$$

$$= \int_0^\Theta \sum_r f(r|\theta, a) rh(\theta) d\theta - \int_0^\Theta \left( \sum_r f(r|\theta, a)r - D \right) h(\theta) d\theta - \int_0^\Theta \sum_r f(r|\theta, a)c^*(r, \theta) h(\theta) d\theta - D.$$  

To simplify the notation, let $E(\bar{r}|a)$ be the expected return produced by the bank conditional on $a$, let $E(\bar{c}|a)$ be the expected compensation paid out by the bank conditional on $a$,
and let \( z(a) \) be the expected value of the implicit transfers from the deposit insurer to the bank conditional on \( a \). Then,

\[
E(\bar{r}|a) = \int_0^\Theta \sum_r f(r|\theta, a) r h(\theta) d\theta,
\]

\[
E(\bar{c}|a) = \int_{\Theta(a)}^\Theta \sum_r f(r|\theta, a) c^*(r, \theta) h(\theta) d\theta,
\]

\[
z(a) = \int_0^{\Theta(a)} \left( D - \sum_r f(r|\theta, a) r \right) h(\theta) d\theta,
\]

so a bank’s expected profits are \( E(\bar{r}|a) - E(\bar{c}|a) - D + z(a) \). The term \( z(a) \) is sometimes referred to as the value of the deposit insurance put option because the bank gets to put its losses onto the deposit insurer.

In terms of this notation, the bank’s problem is

\[
\max_a E(\bar{r}|a) - E(\bar{c}|a) - D + z(a). \tag{14}
\]

At a social optimum, society takes into account that \( z(a) \) is a transfer. The social optimum is the solution to

\[
\max_a E(\bar{r}|a) - E(\bar{c}|a) - D.
\]

The distortion from limited liability and deposit insurance, as represented here by \( z(a) \), leads to the well-known risk shifting problem in banking. However, in this model, the bank needs to give the employees incentives to take on the risk. Bankruptcy and the resource constraint limit the compensation that can be paid to employees in some states. Because employees are risk averse, they do not like the additional variation this creates in their compensation contract, so the bank needs to raise compensation levels in other states to make sure that the participation constraint holds. This additional cost will be reflected in the functional form of \( E(\bar{c}|a) \). This is one feature of the model that can make risk taking costly to the bank.
4 The Importance of Correlation

In general there is not a direct mapping from the form of loan officer compensation to bank risk. The mapping will depend on the production technology. Nevertheless, we can provide some general results for the two extreme cases of uncorrelated loan officer returns and perfectly correlated returns.

4.1 Uncorrelated Returns

Consider the extreme case where there is no correlation in loan officer returns, that is, $f(r|a, \theta) = f(r|a)$. All risk is idiosyncratic, so the gross return of the bank is a constant $\bar{r}(a) = \sum_r f(r|a)r$, which depends only on the loan officers’ action. Similarly, the total compensation bill is a constant $\bar{c}(a)$, which will only depend on the chosen action. Also, because the bank does not fail, the value of the deposit insurance option is $z(a) = 0$.

The bank’s optimization problem is to choose an action $a$ that solves

$$\max_a \bar{r}(a) - \bar{c}(a) - D.$$ 

As long as there exists an $a$ such that bank profits are non-negative, the action chosen by the bank will be the same as the one preferred by society. Basically, when there is no variation in a bank’s total return, limited liability does not distort bank decisions, so compensation is socially optimal and there is no reason to regulate it.

4.2 Perfectly Correlated Returns

Now consider the other extreme case, where loan officer returns are perfectly correlated. In this case, the bank’s gross return does vary with $\theta$ and the bank may want to encourage its loan officers to take on risk due to the limited liability distortion. Interestingly, loan officer compensation matters for risk, but in a surprising way.

When returns are perfectly correlated, there is no idiosyncratic risk, so the bank can infer
a loan officer’s action from the common shock, $\theta$, and the loan officer’s return $r$. Since the bank essentially knows the action, it can pay each loan officer a wage if his return is what it is supposed to be and zero otherwise. We assume that the zero payment penalty is enough to induce the loan officer to take the recommended action. An alternative way of viewing this contract — and the way we view it — is as a relative performance contract. Each loan officer’s return is compared with that of everyone else’s. If his return is the same, he is paid a wage. If it differs, he is paid zero. When returns are perfectly correlated, there is no idiosyncratic risk, so the bank can infer a loan officer’s action from the common shock, $\theta$, and the loan officer’s return $r$. Since the bank essentially knows the action, it can pay each loan officer a wage if his return is what it is supposed to be and zero otherwise. We assume that the zero payment penalty is enough to induce the loan officer to take the recommended action. An alternative way of viewing this contract — and the way we view it — is as a relative performance contract. Each loan officer’s return is compared with that of everyone else’s. If his return is the same, he is paid a wage. If it differs, he is paid zero.

The contract has strong incentives in it, but the incentives are not directly tied to his own performance, but instead to how his performance compares with others. In equilibrium, loan officers do not deviate, so what is observed is a compensation contract that is a wage $\bar{c}$ that does not vary with a loan officer’s return, though it may vary with the aggregate return if constraint (10) binds. In this case where there is a wage paid, there is a threshold state $\tilde{\theta}(a) > \bar{\theta}(a)$ where the resource constraint stops binding. In this interval, $c = \bar{r}(\theta) - D$.

The connection between effort and the wage level comes directly from the participation constraint, (11),

$$ a = V^{-1} \left( H(\bar{\theta}(a))U(0) + \int_{\tilde{\theta}(a)}^{\bar{\theta}(a)} U(\bar{r}(\theta) - D)h(\theta)d\theta + \int_{\bar{\theta}(a)}^{\Theta} U(\bar{c})h(\theta)d\theta - \bar{U} \right). $$

The higher the compensation, the harder the loan officer works. Which effort level gives the bank the best opportunity to exploit the safety net depends on the tradeoff between the
aggregate return and the aggregate wage bill. Indeed, it is possible that a bank pays a low wage to increase its probability of failure as Figure 1 illustrates. The idea in that figure is that the savings in wage payments from lowering $a$ increase the bank’s profits when it is successful and this benefit outweighs the higher probability of failure, the cost of which is borne by the deposit insurer.

In the rest of this subsection, we make two assumptions. First, we assume that $a$ is chosen from a continuum. This assumption is not essential, but simplifies the analysis. Second, we assume that for all $\theta$, $\sum_r f(r|a, \theta)r$ is differentiable, increasing and concave in $a$, that is, there is diminishing returns in expected production given $\theta$. This assumption means that $E(\bar{r}|a)$ is differentiable, increasing and concave and that $z'(a) < 0$.\(^\text{12}\)

Using the objective function defined earlier, the bank will choose an $a$ that satisfies

$$\frac{\partial E(\bar{r}|a)}{\partial a} + z'(a) = \frac{\partial E(\bar{c}|a)}{\partial a}.$$

while at a social optimum, society takes into account that $z(a)$ is a transfer. The social optimum is the solution to

$$\max_a E(\bar{r}|a) - E(\bar{c}|a) - D.$$

so

$$\frac{\partial E(\bar{r}|a)}{\partial a} = \frac{\partial E(\bar{c}|a)}{\partial a}.$$

**Proposition 1** When loan officer returns are perfectly correlated, if $E(\bar{c}|a)$ is increasing and convex in $a$, then the bank chooses an $a$ that is less than the social optimum.

\(^{12}\)To see this, use Leibniz’s rule to get

$$z'(a) = \left[ h(\theta(a))(D - \sum_r f(r|\theta(a), a)r) \right] - \int_0^{\theta(a)} \sum_r \frac{\partial f(r|\theta, a)}{\partial a} h(\theta) d(\theta).$$

By definition of $\theta(a)$, the term in the brackets is zero. Furthermore, $\forall \theta$, $\sum_r \frac{\partial f(r|\theta, a)}{\partial a} > 0$ by assumption, so $z'(a) < 0$. 

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Figure 1: Example of a bank that pays a low wage to increase bank risk when loan officer returns are perfectly correlated. The variable wage(a) is the wage paid to loan officers if a is taken and the bank has produced a high enough return to pay the full wage. The solid line that intercepts the x–axis is profits for the bank if a_l is taken and if r(θ) ≥ D + wage(a_l). (For lower values of r(θ), either all the return net of deposits is paid to loan officers or limited liability binds and the bank receives zero profit.) The dashed line that intercepts the x–axis is profits if a_h is taken and r(θ) ≥ D + wage(a_h). The solid curve is the density function of r(θ) when a_l is taken and the dashed curve is the corresponding density when a_h is taken. For each density, the area under the curve to the left of D is the probability of failure; it is much higher for a_l. In this figure, the wage to implement a_h is so large that the bank receives little profit if its return exceeds D. Consequently, the bank prefers to take a_l. It pays a low wage and the bank fails more frequently.
Proof: Follows directly from $z'(a) < 0$.

Normally, the monotonicity and convexity of expected pay would follow from standard assumptions on preferences. Here, we assume these properties because in this model, bankruptcy has an effect on the expected wage bill that could violate these assumptions in certain extreme situations. In the absence of bankruptcy, where loan officers receive a constant wage, the level of the wage is a convex function of the effort level. However, with bankruptcy, it is possible that as $a$ increases and the measure of bankruptcy states declines, the wage bill will drop (despite the higher effort) because there are fewer states where the loan officers receive zero. This possibility would seem mainly to be an issue when there is a high level of failure and increases in $a$ lead to a substantial marginal decrease in the probability of failure. While we do not think this possibility is the most likely case, it does illustrate that it is possible that the risk aversion of the loan officers can mitigate the limited liability distortion.

The inability to pay employees when there is failure is not in the traditional corporate finance model of risk shifting and the difference illustrates an important point. As long as bank employees do not like risk, they have to be compensated to bear it, and that can make it more expensive for a bank to take risk, which in turn reduces its incentive to exploit risk shifting. Nevertheless, despite these costs, the bank will still not take the socially optimal $a$ because of the deposit insurance safety net factor $z(a)$. In the case that we think is relevant most of the time, namely, that the assumptions on expected compensation in Proposition 1 hold, loan officers work less than is socially optimal and a bank fails more frequently than is socially optimal. In this case, the compensation arrangement that encourages excessive risk is a low wage.
4.3 Partial Correlation

For intermediate levels of correlation, optimal compensation will depend on the specification of \( f(r|a, \theta) \). A useful way to represent correlation and see its effect on compensation is to consider the following specification. Let \( f_1(r|a) \) be a probability distribution of \( r \) that only depends on \( a \) and let \( f_2(r|a, \theta) \) be the conditional probability distribution of a deterministic function \( r = r(a, \theta) \), that is, \( f_2(r|a, \theta) = 1 \) if \( r = r(a, \theta) \) and 0 otherwise. Furthermore, let \( \alpha \in [0, 1] \) index the correlation in return. In particular, let

\[
f(r|a, \theta) = \alpha f_1(r|a) + (1 - \alpha) f_2(r|a, \theta)\]

If \( \alpha = 1 \), loan officer returns are uncorrelated, while if \( \alpha = 0 \), loan officer returns are perfectly correlated.

The bank’s gross return is

\[
\bar{r}(\theta) = \alpha \sum_r f_1(r|a) r + (1 - \alpha) \sum_r f_2(r|a, \theta) r.
\]

The first term is a constant, while the second varies with \( \theta \). More correlation in loan officer returns (a lower \( \alpha \)) increases the variation of bank gross returns.

The effect of \( \alpha \) on compensation can be seen from the likelihood ratios. These are

\[
LR(r, \theta) = \frac{\alpha f_1(r|\hat{a}) + (1 - \alpha) f_2(r|\hat{a}, \theta)}{\alpha f_1(r|a) + (1 - \alpha) f_2(r|a, \theta)}.
\]

For small \( \alpha \), the first term in the numerator and denominator are not that important, so a deviation leads to a likelihood ratio that will be very high if a loan officer’s return differs from those of other loan officers and thus he will receive very low compensation in that case. Incentives will make heavy use of relative performance performance incentives.\(^{13}\)

\(^{13}\)The comparative statics of changing \( \alpha \) are difficult to derive because not only do the likelihood ratios change, but so do the Lagrangian multipliers in (13).
5 A Simplification

In this section, we simplify the model by shutting down the effort distortion and by specifying the production function. The purpose of this simplification is to be able to illustrate what relative performance contracts could look like. In particular, we assume that the loan officer can only take two effort levels and produce two returns. The two possible returns can be interpreted as a loan that either repays or does not. With this simplification, we can also examine how optimal compensation changes with the complementarity between a loan officer’s effort and the variance of the bank’s aggregate return.

Formally, each loan officer can take either $a_1$ or $a_h$, with $0 < a_1 < a_h < 1$. There are only two possible returns, failure ($r = 0$) or success ($r = 1$). As before, $\theta$ is the common shock, though now it is restricted to take on values between 0 and 1. Its mean is $\bar{\theta}$. We assume that the bank implements $a_h$.

5.1 Effort only Increases the Mean

We first consider the case where the marginal effect of the loan officers’ actions is to only increase the bank’s mean return. There are no complementarities in production between the action and $\theta$. The probability of success for a loan officer is

$$f(r = 1|\theta, a) = a + (\alpha \bar{\theta} + (1 - \alpha)\theta).$$

(15)

The parameter $(1 - \alpha)$ measures the importance of the common shock. For low values of $\alpha$, the return of the bank will vary more with the realization of $\theta$ than for high values of $\alpha$.\textsuperscript{14} Notice that a loan officer’s expected return is $a + \bar{\theta}$, which does not depend on $\alpha$.

\textsuperscript{14}To link this production function to the earlier analysis, consider the two extreme values of $\alpha$. If $\alpha = 1$ then loan officer returns are uncorrelated. The probability of each loan officer’s return being successful, paying $r = 1$, is $a + \bar{\theta}$. Some loan officers are successful and others are not, but there is no variation in the bank’s aggregate return; the bank produces $a + \bar{\theta}$ no matter what. If instead $\alpha = 0$ then the probability of each loan officer’s return being successful is $a + \theta$, so the bank’s aggregate return depends on the realization of the common shock $\theta$. However, while loan officer returns are correlated, they are not perfectly correlated because as long as $0 < a + \theta < 1$ when some loan officers succeed, others will fail.
Furthermore, for the bank, $E(\bar{r}) = a + \bar{\theta}$ and $Var(\bar{r}) = (1 - \alpha)^2 Var(\theta)$. In this example, effort only affects the bank’s mean return, not its variance.

Compensation is determined by the likelihood ratios. When the recommended action is $a_h$, these are

$$LR(r = 1, \theta, a_l; a_h) = \frac{a_l + (\alpha \bar{\theta} + (1 - \alpha)\theta)}{a_h + (\alpha \bar{\theta} + (1 - \alpha)\theta)},$$

$$LR(r = 0, \theta, a_l; a_h) = \frac{1 - a_l - (\alpha \bar{\theta} + (1 - \alpha)\theta)}{1 - a_h - (\alpha \theta + (1 - \alpha)\theta)}.$$

**Proposition 2** For the technology specified in (15), at an interior solution, consumption for $r = 1$ decreases with $\theta$ and consumption for $r = 0$ decreases with $\theta$.

**Proof:** Likelihood ratios comove with $\theta$ such that

$$\frac{\partial LR(r = 1, \theta, a_l; a_h)}{\partial \theta} > 0 \Rightarrow \frac{\partial c(r = 1, \theta)}{\partial \theta} < 0$$

$$\frac{\partial LR(r = 0, \theta, a_l; a_h)}{\partial \theta} > 0 \Rightarrow \frac{\partial c(r = 0, \theta)}{\partial \theta} < 0.$$ 

Furthermore, it is not hard to show that $LR(r = 1, \theta, a_l; a_h) < 1, \forall \theta$ and $LR(r = 0, \theta, a_l; a_h) > 1, \forall \theta$. This implies that $c(r = 0, \theta) < c(r = 1, \theta'), \forall \theta, \theta'$, that is, the lowest level of consumption for $r = 1$ is more than the highest level of consumption for $r = 0$.

Figure 2 illustrates the comovement of consumption with $\theta$. Both successful and unsuccessful loan officers receive a level of pay that decreases with bank’s return. The reason for the decrease is that the likelihood ratio for $r = 1$ is well below one when $\theta$ is low, which means $r = 1$ has a high value as a signal that $a_h$ was taken. Consequently, it is efficient to

An example of a production function that generates perfect correlation of returns, like the case studied in Section 4.2, is

$$f(r = 1|\theta, a) = \begin{cases} 
1 & \text{if } a = a_h, \theta \geq \theta_1 \\
0 & \text{if } a = a_h, \theta < \theta_1 \\
1 & \text{if } a = a_l, \theta < \theta_1 \\
0 & \text{if } a = a_l, \theta \geq \theta_1 
\end{cases},$$

where $\theta_1$ is a parameter.
reward the loan officer with high consumption. As $\theta$ increases, the common shock becomes proportionally more important for determining a loan officer’s success, so the likelihood ratio increases and gets closer to one, which means the signal value of $r = 1$ declines and consumption declines. For $r = 0$ the likelihood ratio is at its lowest for the lowest value of $\theta$. At that point, the common shock is proportionally more important than individual effort in determining failure, so the signal value of $r = 0$ is low and the loan officer is not punished that much for failure. As $\theta$ increases, the likelihood ratio increases and moves away from one, so the signal value of $r = 0$ increases. Consequently, it is efficient for the bank to punish the loan officer with low consumption.

In investment banking and some parts of traditional commercial banking, a substantial
fraction of a firm’s total compensation bill is often directly tied to performance of the bank or a line of business. For example, investment banks often decide on and report on total compensation as a percentage of revenue.

In this example, the fraction of successful loan officers is \( f(r = 1|a, \theta) = \bar{r}(\theta) \). Therefore, the share of revenue distributed to loan officers — the only employees in this problem — is

\[
WS(\theta) = \frac{\bar{r}(\theta)c(r = 1, \theta) + (1 - \bar{r}(\theta))c(r = 0, \theta)}{\bar{r}(\theta)}
\]

for the range of consumption that is interior.

Proposition 3 describes the relationship for this technology.

**Proposition 3** For the technology specified in (15) and where consumption is interior

\[
\frac{\partial WS(\theta)}{\partial \bar{r}(\theta)} < 0.
\]

**Proof:** See the appendix.

In this example, at an optimum the employees’ share of income decreases with bank performance.\textsuperscript{15} This compensation arrangement can also be interpreted as a relative performance contract, where \( \theta \) is replaced by the average loan officer’s return.\textsuperscript{16}

### 5.2 Effort Increases the Variance of the Return

In this specification, loan officer effort affects the mean of the return and the variance of the bank’s return. We introduce this complementarity by making effort and \( \theta \) complements in the probability of success. In particular, the probability of success for a loan officer is

\[
f(r = 1|\theta, a) = a(\alpha \theta + (1 - \alpha)\theta).
\]

\textsuperscript{15}There are other proposed explanations for this behavior, like sorting and retention of workers. See Oyer and Schaefer (2005).

\textsuperscript{16}These kind of contracts exist in other industries. See Tsoulouhas and Vukina (1999) and Hueth and Ligon (2001) for applications to agriculture. The latter paper describe how agricultural production contracts with compensation that depend on market prices can be interpreted as relative performance contracts. For recent work on relative performance models see Celentani and Loveira (2006) and Fleckinger (2012).
Notice that, as with the previous example, a loan officer’s expected return is $a\bar{\theta}$, which does not depend on $\alpha$. However, the bank’s mean return is $E(\bar{r}) = a\theta$ and $Var(\bar{r}) = (1-\alpha)^2a^2Var(\theta)$. In this example, effort increases the bank’s mean return and increases the variance of its return.

Compensation is determined by the likelihood ratios. When the recommended action is $a_h$, these are

\[
LR(r = 1, \theta, a_l; a_h) = \frac{a_l}{a_h},
\]
\[
LR(r = 0, \theta, a_l; a_h) = \frac{1 - a_l(a\bar{\theta} + (1-\alpha)\theta)}{1 - a_h(a\bar{\theta} + (1-\alpha)\theta)}.
\]

**Proposition 4** For the technology specified in (16), at an interior solution, consumption for $r = 1$ does not vary with $\theta$ and consumption for $r = 0$ decreases with $\theta$.

**Proof:** Likelihood ratios comove with $\theta$ such that

\[
\frac{\partial LR(r = 1, \theta, a_l; a_h)}{\partial \theta} = 0 \Rightarrow \frac{\partial c(r = 1, \theta)}{\partial \theta} = 0
\]
\[
\frac{\partial LR(r = 0, \theta, a_l; a_h)}{\partial \theta} > 0 \Rightarrow \frac{\partial c(r = 0, \theta)}{\partial \theta} < 0.
\]

Figure 3 illustrates the comovement of consumption with $\theta$. Successful loan officers receive a constant level of pay, while unsuccessful ones see their pay drop with the success of the bank. The reason their pay drops is that failure is less likely when there is a high value of $\theta$.

As with the previous technology, the worker’s share of revenue declines with revenue.

**Proposition 5** For the technology specified in (16) and where consumption is interior

\[
\frac{\partial WS(\theta)}{\partial \bar{r}(\theta)} < 0.
\]

**Proof:** See the appendix.
Figure 3: Optimal compensation in example where effort is complementary with $\theta$ so increases the mean and variance of the bank’s return, where $a_h$ is implemented, and under the assumption of an interior solution. Note that $r(\theta)$ is linear in $\theta$, so the $x$-axis is proportional to gross return of the bank.
6 Discussion of Regulation and Extensions

Our analysis of the general case identified a distortion that generally leads to underprovision of effort. Lower effort usually lowers the wage bill of the bank, which suggests that low compensation may indicate an inefficiency. We were able to show this most starkly in the perfect correlation case, but the analysis implies that it applies more generally. Effort in our model increased the mean return, so a regulator would want to look for compensation practices that did not compensate employees sufficiently for working hard to keep production up.

The implications of the general model about the form of compensation and bank risk show that the use of incentives, measured by something like the size of a bonus, does not directly correspond to bank risk. In the idiosyncratic risk case, compensation was incentive based, but it did not matter for bank risk. In contrast, for perfect correlation the compensation contract that prevailing wisdom would think is the safest, namely a wage, was associated with bank risk.

The simplified models can be used to think about how bonuses may operate in practice. Each of the optimal contracts shown in Figures 2 and 3 can be implemented via a bonus arrangement. Simply, set the wage to be the lowest level $c(r = 0, \theta)$ and then pay a bonus that depends on $r$ and $\theta$. In practice, employees of large banks and investment banks are paid with discretionary bonus schemes in which bonus pools are determined by bank performance and division performance and then each division allocates bonuses among its employees, presumably, with some connection to performance. Implicitly, these arrangements are relative performance type schemes and could be used to implement the types of contracts studied here. In these two models, bonuses are a natural feature of the optimal contract. Thus, banning them is not beneficial.

Our analysis emphasized underprovision of effort that lowered returns. In the perfect...
correlation case, lower effort simply shifted the distribution of returns down. That analysis left out a dimension of bank risk in that we did not consider the effects of actions that increase the correlation of loan officer returns. The first thing to note about this source of bank risk is that regulatory practices typically manage these risk through loan concentration rules that limit lending to a single borrower or to an industry. Whether compensation rules can affect correlation in loan officer returns would seem to depend on how much ability a loan officer has to control the correlation in a loan that they make.\textsuperscript{17} It is possible that a loan officer could do this, but that is less intuitive than purely affecting the success probability. Instead, correlation would seem to be more controlled by decisions made by risk managers and other control functions within a bank.

Along these lines, one direction to extend our approach is to model the bank as more than just isolated loan officers, but to add departments like loan review, risk management, auditing, etc. In practice, large banks have large numbers of employees who do these functions. Large loans are reviewed by a loan committee, risk management measures department risk, and auditing randomly checks departments to see if they are complying with bank rules. The existence of these functions shows that there is a lot of monitoring going on internally within a bank. This monitoring will affect the private information of loan officers, put limits on their choices, and thus impact their compensation. Furthermore, with a very different role than a loan officer, it would interesting to study their compensation.\textsuperscript{18}

7 Conclusion

This paper modeled a bank as a large number of independent loan officers subject to a common shock and then analyzed optimal compensation. The optimal contract made heavy

\textsuperscript{17}For an analysis along these lines see the earlier working paper version of this paper, Jarque and Prescott (2013).
\textsuperscript{18}In the earlier working paper version, Jarque and Prescott (2013), there is an analysis of the loan review function. Also, see the discussion in Prescott (2016).
use of relative performance. We showed how information contained in the common shock impacted the signal value of a loan officer’s return and compensation. As part of the optimal contract, compensation was tied to total bank performance.

The paper showed that deposit insurance led to underprovision of effort relative to the social optimum. However, relative to the standard model we also showed that risk averse loan officers added an additional cost to exploiting the safety net because they need to be compensated for the extra risk. The mapping from bank risk to the optimal contract suggests that the commonly held perception that high bonuses create risk is not necessarily true. We showed that with only idiosyncratic risk, compensation contracts are irrelevant for bank risk and with perfect correlation, a low wage is what creates an inefficient amount of risk. An additional analysis of simpler technologies illustrated how individual compensation can change with bank performance.

The analysis demonstrated that the connection between compensation and bank risk is not straightforward and depends on the production technology. Evaluation of bank risk requires a detailed understanding of the production technology to identify precise effects. Nevertheless, the analysis showed the importance of relative performance schemes in compensation and suggests that identifying ways that relative performance can increase correlation in returns is a way to study these issues.

A Proofs

Proof of Proposition 3

Let $WS(\theta)$ be the loan officers’ share of total revenue. Also, recall that $\bar{r}(\theta)$ is not only the total revenue, but also the fraction of loan officers who produce the high return of one. First,

\[
WS(\theta) = \frac{\bar{c}(\theta)}{\bar{r}(\theta)} = \frac{\bar{r}(\theta)c(r = 1, \theta) + (1 - \bar{r}(\theta))c(r = 0, \theta)}{\bar{r}(\theta)}
\]
\[ c(r = 1, \theta) + \frac{c(r = 0, \theta)}{\bar{r}(\theta)} - c(r = 0, \theta). \]

Differentiating with respect to total revenue gives

\[
\frac{dWS(\theta)}{d \bar{r}(\theta)} = \frac{\partial c(r = 1, \theta)}{\partial \bar{r}(\theta)} + \frac{\bar{r}(\theta) \frac{\partial c(r = 0, \theta)}{\partial \bar{r}(\theta)}}{\bar{r}(\theta)^2} - \frac{c(r = 0, \theta)}{\partial \bar{r}(\theta)}.
\]

Rearranging terms we get

\[
\frac{dWS(\theta)}{d \bar{r}(\theta)} = \frac{\partial c(r = 1, \theta)}{\partial \bar{r}(\theta)} + \frac{\partial c(r = 0, \theta)}{\partial \bar{r}(\theta)} \left( \frac{1}{\bar{r}(\theta)} - 1 \right) - \frac{c(r = 0, \theta)}{\bar{r}(\theta)^2} < 0. \tag{17}
\]

The inequality holds because all three terms are negative. The first term is negative by Proposition 2. The term \( \frac{\partial c(r = 0, \theta)}{\partial \bar{r}(\theta)} \) is negative because \( \frac{\partial c(r = 0, \theta)}{\partial \theta} < 0 \) and \( \frac{\partial \theta}{\partial \bar{r}(\theta)} > 0 \). Furthermore, \( \bar{r}(\theta) < 1 \), so the second term is negative. The third term is also negative, so the sum is negative.

**Proof of Proposition 5**

The proof follows that of Proposition 3 except that the first term in (17) is zero. Since the other two terms in (17) are negative, \( \frac{dWS(\theta)}{d \bar{r}(\theta)} < 0. \)

**References**


