

Imperfect Risk-Sharing and the Business Cycle^{*}

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Abstract

This paper studies the aggregate implications of imperfect risk-sharing implied by a class of New Keynesian models with idiosyncratic income risk and incomplete financial markets. The models in this class can be equivalently represented as an economy with a representative household that has state-dependent preferences. These preference “shocks” are functions of households’ consumption shares and relative wages in the original economy with heterogeneous agents, and they summarize all the information from the cross-section that is relevant for aggregate fluctuations. Our approach is to use this representation as a measurement device: we use the Consumption Expenditure Survey to measure the preference shocks, and feed them into the equivalent representative-agent economy to perform counterfactuals. We find that deviations from perfect risk-sharing were an important determinant of the behavior of aggregate demand during the US Great Recession.

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1 Introduction

A classic question in macroeconomics is to what extent household heterogeneity and deviations from perfect risk-sharing are important for aggregate fluctuations. For a long time, most business cycle studies relied on the representative-agent paradigm, an approach that was partly justified by the influential result in [Krusell and Smith \(1998\)](#) that distributional issues play a limited role for macroeconomic dynamics in standard real business cycle models. More recently, a new line of research is reevaluating this question. Studies have shown that in environments where risk-sharing is limited, time-varying precautionary saving motives of households can be quite relevant for the behavior of aggregate demand, and can thus play an important role for the business cycle if output is partly demand-determined, see [Krueger, Mitman, and Perri \(2016\)](#) and [Kaplan and Violante \(2018\)](#) for surveys of this literature. The answers to this question given so far, however, depend on specific features of the structural model that are hard to discipline empirically, such as the precise set of financial assets and risk-sharing mechanisms that are available to households and the nature of idiosyncratic risk they face.¹

The main contribution of our paper is to demonstrate that one does not need to take a stand on these modeling features to address this question. Our approach builds on research by [Nakajima \(2005\)](#), [Krueger and Lustig \(2010\)](#) and [Werning \(2015\)](#), who show that models where households face idiosyncratic income risk and incomplete financial markets can be equivalently represented as an economy with a representative agent. In this equivalent representation, the stand-in household has state-dependent preferences, which are functions of the joint distribution of households' consumption shares and relative wages in the original heterogeneous agent economy. These preference "shocks" capture deviations from perfect risk-sharing, and they summarize all the information from the cross-section that is relevant for aggregate fluctuations. The key idea in this paper is to use this representation as a measurement device: we first measure the preference shocks using household-level data, and show how to combine these series with the equivalent representative-agent economy to quantify the role of imperfect risk-sharing for business cycle fluctuations. In our application, we find that deviations from perfect risk-sharing contributed significantly to the output decline observed during the US Great Recession but this contribution was short-lived.

We begin by describing a class of New Keynesian models where households face idiosyn-

¹A prominent example is [Kaplan and Violante \(2014\)](#), which shows that the consumption response to fiscal transfers is very different if households can trade one liquid asset or one liquid and one illiquid asset. In addition, researchers have shown that many other modeling choices, which are inconsequential in representative agent economy, matter in heterogeneous agent economies. These include the timing and distribution of the fiscal transfers ([Kaplan, Moll, and Violante, 2018](#)), how profits get distributed across households ([Broer, Hansen, Krusell, and Oberg, 2018](#)), and the cyclicalities of idiosyncratic risk and access to liquidity ([Werning, 2015](#)).

cratic income risk and have isoelastic preferences over consumption and hours worked. The models in this class share the same specification for households' preferences, technology, goods and labor market structure, and the conduct of monetary policy. They can differ in the nature of idiosyncratic risk faced by households, the set of assets they can trade, and their financial constraints. Each of these potential specifications imposes a particular set of restrictions on the joint distribution of households' consumption and labor choices. However, as we show in the paper, the law of motion for aggregate variables for all the models in this class has the same representation: that of an economy with a representative household whose preferences involve a time-varying and stochastic rate of time preference and disutility of labor. Importantly, the mapping between these preference shocks and households' choices is the same for all the models in this class: conditional on observing their consumption and wages, we do not need to take a stance on the details of the risk-sharing mechanisms available and how they are modeled.

Frictions that impede risk-sharing are summarized in this equivalent representation by shocks to the discount factor of the stand-in household. This discount factor is the product of the "true" rate of time-preference of households and the expectation of the inverse of the change in consumption shares for the savers, that is the households that are on their Euler equation for liquid assets. To understand this result, consider first the case of complete financial markets. With our preferences, the allocation features constant consumption shares, and the discount factor of the stand-in household coincides at every point in time with the households' rate of time preferences. When risk-sharing is not perfect, consumption shares are state dependent, and the two no longer coincide.

The discount factor for the stand-in household is typically higher than the true rate of time preference, capturing the fact that a household facing uninsurable idiosyncratic risk has more incentives to save than a stand-in household that only needs to smooth consumption and hours over aggregate shocks.² Thus, the stand-in household must be more "patient" to be on his Euler equation at the interest rate implied by the heterogeneous agent economy. The wedge between the true discount factor and the one in the equivalent representative-agent economy can be time varying, reflecting movements in the expected growth rate of the savers' consumption shares and their volatility.

In addition to a time-varying discount factor, the stand-in household in the equivalent representation also features a state-dependent preference for leisure. This preference shock is a function of the joint distribution of idiosyncratic labor productivity and consumption shares, and it captures compositional changes in hours worked that occur in the original economy with heterogeneous households. As an example, consider the case in which house-

²Mechanically, this follows from Jensen's inequality: if the consumption share is constant in expectation but volatile, the expectation of the inverse of the change in consumption share is greater than one.

holds with relatively high productivity also have relatively high consumption. An increase in the cross-sectional dispersion in consumption shares induces high productivity households to work less and low productivity households to work more because of wealth effects, a compositional change that reduces worked hours in efficiency units. In the equivalent representative-agent economy, these effects are captured by an increase in the disutility of labor of the stand-in household – a reduction in labor supply.

After describing this equivalent representation, we turn to measurement. We use the Consumer Expenditure Survey to compute changes in consumption shares and labor productivity at the household level for the 1996-2012 period. We divide households into groups, depending on their income and net worth, and compute cross-sectional averages of the inverse change in consumption shares to measure the discount factor for different sub-groups of the population. In the cross-section, we find that high income households have typically higher implicit discount rates which, through the lens of our framework, identifies them as the savers. In the time series, we find that the implicit discount factor of high income households increases substantially during the Great Recession, which signals an increase in their self-insurance motives. It is well known in the literature that an increase in the discount factor can induce sizable output drops in New Keynesian models, especially when nominal interest rates cannot fall because of the zero lower bound as during the Great Recession ([Christiano, Eichenbaum, and Rebelo, 2011](#)). An important question is whether these movements are large enough to be an important source of business cycle fluctuations.

To address this question, we estimate the parameters of the equivalent representative-agent economy and of the process governing the preference shocks and construct the counterfactual path for aggregate output, inflation and nominal interest rates that would have emerged in an economy with complete financial markets—that is, an economy with time-invariant consumption shares and a constant discount factor. The difference between the observed path and this counterfactual isolates the aggregate implications of imperfect risk-sharing. Our main finding is that imperfect risk-sharing through the implied movements in the discount factor had sizable albeit transitory macroeconomic effects, accounting for roughly one fifth of the output decline observed in 2009 and 2010. We show that the presence of a binding zero lower bound plays a key role in amplifying these shocks, and in reversing the positive impact that a higher propensity to save would otherwise have on aggregate investment.

It is important to stress that our approach is not designed to identify the primitive frictions driving the observed deviations from perfect risk-sharing. As a last exercise, we investigate what features of the data are responsible for the increase in the measured discount factor during the Great Recession. Mechanically, this increase can happen for two mutually non-exclusive reasons: the consumption shares of savers fell, or there was an increase in the

cross-sectional dispersion of their consumption shares. We show that the observed increase in the discount factor during the Great Recession is driven mostly by the latter effect. This finding suggests that models aimed at capturing these patterns in the data should focus on frictions that generate uninsured idiosyncratic risk within the group of savers.³ In addition, we show that the increase in the cross-sectional dispersion is driven by a reduction in the ability to smooth negative income shocks rather than by an increase in the volatility of their labor income. These findings provide support for research that emphasizes the importance of households' credit constraints over this episode or, alternatively, to the view that income shocks occurring at the time were perceived to be particularly persistent.

Related Literature. Our research contributes to a growing literature that introduces heterogeneous agents and incomplete financial markets in New Keynesian models of the business cycle. Researchers have used these environments to study how frictions impeding risk-sharing across households affect the transmission mechanism of monetary and fiscal policy,⁴ and more generally the business cycle. In this respect, the literature has stressed the interactions between households' precautionary savings and aggregate demand: when the former increase, the latter falls, resulting in lower levels of economic activity. These changes in households' precautionary behavior may occur via different mechanisms. [Guerrieri and Lorenzoni \(2017\)](#) and [Jones, Midrigan, and Philippon \(2018\)](#) show that a tightening of individual borrowing constraints can induce households to save more because of self-insurance. Other researchers highlight the importance of time-varying labor income risk, see for example [McKay \(2017\)](#), [Challe, Matheron, Ragot, and Rubio-Ramirez \(2017\)](#), [Den Haan, Rendahl, and Riegler \(2017\)](#) and [Bayer, Lüttinge, Pham-Dao, and Tjaden \(2019\)](#) for quantitative analyses and [Heathcote and Perri \(2018\)](#) and [Ravn and Sterk \(2017\)](#) for more stylized frameworks.

All these papers consider specific departures from perfect risk-sharing by imposing a given asset structure, income process and set of borrowing constraints. We instead take a more agnostic approach about the amount of risk-sharing available to households and infer it from their observed choices. We think that these two approaches are complementary. We identify a set of cross-sectional moments that are informative about the macroeconomic effects of imperfect risk-sharing in this class of models and, consistent with some of the above-mentioned papers, we document that their behavior points toward an important role for micro-level frictions in explaining the decline in aggregate demand during the Great

³This effect is, by construction, absent in models with simple form of heterogeneity, such as the “two-agent” New Keynesian model studied in [Galí, López-Salido, and Vallés \(2007\)](#), [Bilbiie \(2008\)](#), and [Debortoli and Galí \(2017\)](#).

⁴See [Kaplan, Moll, and Violante \(2018\)](#); [Auclert \(2017\)](#); [McKay, Nakamura, and Steinsson \(2016\)](#); [McKay and Reis \(2016\)](#); [Hagedorn, Manovskii, and Mitman \(2019\)](#); [Gornemann, Kuester, and Nakajima \(2016\)](#) for recent contributions.

Recession.⁵ However, our approach is mostly silent about the set of underlying frictions and shocks that can replicate the observed patterns of the preferences shocks. Identifying these frictions is important because we cannot use our framework for policy evaluation, and so a fully specified structural model is needed.

The counterfactuals that we perform are related to the business cycle accounting methodology of [Chari, Kehoe, and McGrattan \(2007\)](#). The time varying preference shocks of the stand-in household in our approach can also be interpreted as “wedges” in the Euler equation and in the labor supply condition of the representative-agent economy. Beside the different focus, there are two main differences between these procedures. First, in our approach the preference shocks are measured using household-level observations, rather than being chosen to replicate the observed path of aggregate data. Second, our main quantitative experiment constructs the path for macroeconomic variables in a counterfactual economy with complete financial markets. This is not equivalent to the approach of [Chari, Kehoe, and McGrattan \(2007\)](#), which assesses the effects of specific wedges on the business cycle.

Our approach builds on a large literature that uses data on household consumption, labor supply, and earnings to measure the degree of risk-sharing in the data without explicitly specifying the mechanisms through which households share risk. See for example [Blundell, Pistaferri, and Preston \(2008\)](#) and the survey in [Jappelli and Pistaferri \(2010\)](#). The paper that is closer to our approach is [Heathcote, Storesletten, and Violante \(2014\)](#) who use households’ optimality conditions and PSID and CEX data to measure the extent of risk-sharing present in the U.S. economy. The contribution of our paper relative to this literature is to study how the measured degree of partial risk-sharing affects aggregate dynamics.

Finally, our paper is related to the literature that evaluates asset pricing models where aggregation does not hold using households’ consumption data. See for example [Brav, Constantinides, and Geczy \(2002\)](#), [Vissing-Jørgensen \(2002\)](#), [Krueger, Lustig, and Perri \(2008\)](#), and [Kocherlakota and Pistaferri \(2009\)](#). The goal of these papers is to estimate the stochastic discount factor with micro data given a particular form of market incompleteness. This is similar to the construction of the discount factor in the equivalent representative-agent economy in our approach. Clearly the scope of our analysis differs from these papers.

Layout. The paper is organized as follows. Section 2 introduces the class of heterogeneous agents economies at the center of our application, derives the equivalent representative-agent representation and explains the nature of our counterfactuals. In Section 3 we use the CEX to

⁵The literature emphasizes the role of the distribution of marginal propensities to consume (MPCs) as a critical statistic to discipline these structural models. [Auclert, Rognlie, and Straub \(2018\)](#) show that the distribution of MPCs at different time horizons – what they term intertemporal MPCs – is a sufficient statistic for the response of output to fiscal shocks in a class of New Keynesian models. In Appendix A we derive formally the relation between the cross-sectional moments we identify and the intertemporal MPCs.

measure the preference shocks. Section 4 combines the measured preference shocks and the equivalent representative-agent economy to perform the main counterfactual of the paper. Section 5 discusses possible models that are consistent with the pattern we identify in the micro data. Section 6 concludes.

2 New Keynesian models with heterogeneous agents

We start in Section 2.1 by introducing a class of New Keynesian models with isoelastic preferences, idiosyncratic income risk, and incomplete financial markets. The models in this class share the same specification for households' preferences, technology, market structure and the conduct of monetary policy. However, they can differ in the cyclical risk faced by households, the set of assets they can trade, their financial constraints, as well as the timing and distribution of fiscal transfers.

After defining an equilibrium, we show in Section 2.2 that all the models in this class admit an equivalent representation: that of a representative-agent economy where the stand-in household has a state-dependent rate of time preference and disutility of labor. Section 2.3 derives analytically this representation for specific models in this class. Section 2.4 discusses at a conceptual level how we can use this representation to measure the macroeconomic effects of imperfect risk-sharing.

2.1 The model

Time is discrete and indexed by $t = 0, 1, \dots$. The economy is populated by a continuum of households, final good producers, intermediate good firms, and the monetary authority. Households are divided into a finite number of types $i \in I$. Let λ_i be the measure of type i households in the economy. There are two types of states: aggregate and idiosyncratic. We denote the aggregate state by z_t and the idiosyncratic state by v_t , both of which are potentially vector valued. Let $z^t = (z_0, z_1, \dots, z_t)$ be a history of realized aggregate states up to period t and $v^t = (v_0, v_1, \dots, v_t)$ be a history of idiosyncratic states up to period t . We also let $s_t = (z_t, v_t)$ and $s^t = (z^t, v^t)$. Let $\Pr_i(s^t | s^{t-1})$ be the probability of a history s^t . We assume that $\Pr_i(s^t | s^{t-1}) = \Pr_i(v^t | v^{t-1}, z^t) \Pr(z^t | z^{t-1})$ so we allow for the possibility that the aggregate state affects the cross-sectional distribution of the idiosyncratic state and that the agent's type affects the probability of drawing a given v_t .

Households are infinitely lived and have preferences over consumption, $c_i(s^t)$, and hours

worked, $l_i(s^t)$, given by

$$\sum_{t=0}^{\infty} \sum_{s^t} \beta^t \Pr_i(s^t | s_0) \tilde{\theta}(z_t) U(c_i(s^t), l_i(s^t)), \quad (1)$$

where β is the discount factor and $\tilde{\theta}(z_t)$ is a shock to the marginal utility of consumption and disutility of labor. This preference shock is commonly used in the literature to obtain a binding zero lower bound constraint, a feature that will be important in the quantitative analysis. We further assume that the period utility is given by

$$U(c, l) = \frac{c^{1-\sigma} - 1}{1-\sigma} - \chi \frac{l^{1+\psi}}{1+\psi}, \quad (2)$$

with $\sigma > 0$ and $\psi > 0$.

The final good is produced combining differentiated intermediate goods according to the technology

$$Y(z^t) = \left(\int_0^1 y_j(z^t)^{\frac{1}{\mu}} dj \right)^{\mu}, \quad (3)$$

where μ is related to the (constant) elasticity of substitution across varieties, ε , by the following, $\mu = \varepsilon / (\varepsilon - 1)$. The intermediate inputs are produced using labor

$$y_j(z^t) = A(z_t) n_j(z^t), \quad (4)$$

where $A(z_t)$ is an aggregate technology shock, common across firms, and $n_j(z^t)$ is labor in *efficiency units* utilized by the producer of intermediate good j . Feasibility requires that

$$\int n_j(z^t) dj = \sum_i \lambda_i \sum_{v^t} \Pr_i(v^t | z^t) e(v_t) l_i(v^t, z^t). \quad (5)$$

That is, each individual v^t is associated to a particular level of efficiency $e(v_t)$: hiring more high-efficiency types, holding total hours worked fixed, results in higher output produced by the firm. This individual-specific productivity shock $e(v_t)$ generates idiosyncratic income risk for households.

We now define the market structure for this economy with a particular emphasis on the households side.

Households. Households enter the period with financial assets and they work for intermediate good producers. They choose consumption, new financial positions and labor in order to maximize their expected life-time utility.

We model financial markets in a flexible way. First, we assume that households can trade a risk-free nominal bond. We denote by $b_i(s^t)$ the position taken today by a household of type i and by $1 + i(z^t)$ the nominal return on the bond. We also assume that the household can trade a set \mathcal{K} of possible assets, with the nominal payout of a generic asset $k \in \mathcal{K}$ given by $R_k(s^t, s_{t+1})$. We let $q_k(z^t, v^t)$ be the price of the asset. This formulation allows for different types of financial assets: individual Arrow securities, shares of the intermediate good firms, complex financial derivatives, etc. We let $a_{k,i}(z^{t-1}, v^{t-1})$ be the holdings of assets k that a household of type i with history v^{t-1} has accumulated after an aggregate history z^{t-1} . Trades in these additional financial assets potentially require transaction costs $\mathcal{T}(\{a_{k,i}(s^{t-1})\}_{k \in \mathcal{K}}, \{a_{k,i}(s^t)\}_{k \in \mathcal{K}}, s^t)$ that can depend on the inherited portfolio $\{a_{k,i}(s^{t-1})\}_{k \in \mathcal{K}}$, the new portfolio $\{a_{k,i}(s^t)\}_{k \in \mathcal{K}}$, and s^t .

In addition, we allow for a number of constraints that potentially restricts the financial positions that households can choose,

$$H_i(b_i(s^t), \{a_{k,i}(s^t)\}_{k \in \mathcal{K}}, s^t) \geq 0 \quad (6)$$

for some vector-valued function H_i . We refer to the set of constraints in (6) as *trading restrictions*.

The set of assets \mathcal{K} , the transaction costs \mathcal{T} , and the trading restrictions in (6) are a flexible way of representing different sets of risk-sharing mechanisms available to households. The only restriction that we impose is that purchasing risk-free nominal bonds weakly relaxes these constraints, $\partial H_i(b, \{a_k\}_{k \in \mathcal{K}}, s^t) / \partial b \geq 0$, and does not require a transaction cost. By doing so we are ruling out limited participation economies where agents must pay a fixed cost to have access to the risk-free nominal bond. Our formulation nests the complete financial market case, when the set of tradable assets spans all possible aggregate and idiosyncratic histories and there are no transaction costs or trading restrictions. In addition, it encompasses as special cases a large class of models with incomplete financial markets: the Bewley-Huggett-Aiyagari economy, the two-assets economy in [Kaplan and Violante \(2014\)](#) and [Kaplan, Moll, and Violante \(2018\)](#), the endogenous debt limits in [Alvarez and Jermann \(2000\)](#), or the various forms of restrictions on asset trading in [Chien, Cole, and Lustig \(2011, 2012\)](#). Note, also, that the H_i function can depend on s^t , which implies that we are allowing for aggregate and idiosyncratic shocks to affect the financial constraints of households.

The problem of households is to choose $[c_i(s^t), l_i(s^t), b_i(s^t), \{a_{k,i}(s^t)\}_{k \in \mathcal{K}}]$ to maximize util-

ity (1) subject to the nominal budget constraint,

$$\begin{aligned} P(z^t) c_i(s^t) + \frac{b_i(s^t)}{1+i(z^t)} + \sum_{k \in \mathcal{K}} q_k(s^t) a_{k,i}(s^t) + \mathcal{T}(\{a_{k,i}(s^{t-1})\}_{k \in \mathcal{K}}, \{a_{k,i}(s^t)\}_{k \in \mathcal{K}}, s^t) \\ \leq W(z^t) e(v_t) l_i(v^t, z^t) - T_i(s^t) + b_i(s^{t-1}) + \sum_{k \in \mathcal{K}} R_k(s^{t-1}, s_t) a_{k,i}(s^{t-1}), \end{aligned}$$

where $W(z^t)$ is the nominal wage per efficiency units and $T_i(s^t)$ are lump-sum taxes, and the trading restrictions in (6) given initial asset holdings.

Because of the assumption that $\partial H_i / \partial b \geq 0$, a necessary condition for optimality is

$$\frac{1}{1+i(z^t)} \geq \beta \sum_{s_{t+1}} \left\{ \frac{\Pr_i(s^{t+1}|s^t) \theta(z^{t+1})}{1+\pi(z^{t+1})} \left[\frac{c_i(s^t, s_{t+1})}{c_i(s^t)} \right]^{-\sigma} \right\}, \quad (7)$$

where $\theta(z^{t+1}) = \tilde{\theta}(z_{t+1}) / \tilde{\theta}(z_t)$ and $\pi(z^{t+1}) = P(z^{t+1}) / P(z^t) - 1$ is the net inflation rate. The condition must hold with equality if the trading restrictions on the nominal bond do not bind. For the rest of the paper, we assume that there always exist an agent for which equation (7) holds as an equality.⁶ Because we have assumed that $\partial H_i / \partial b \geq 0$, equation (7) holds with equality for the agents with the highest valuation for the risk-free bond.⁷ Moreover, labor supply must satisfy

$$c_i(s^t)^{-\sigma} w(z^t) e(v_t) = \chi l_i(s^t)^\psi \quad (8)$$

where $w(z^t) = W(z^t) / P(z^t)$ is the real wage per efficiency unit.

Final good producers. The final good is produced by competitive firms that operate the production function in (3). From their decision problem, we can derive the demand function for the j -th variety

$$y_j(z^t) = \left(\frac{P_j(z^t)}{P(z^t)} \right)^{\mu/(1-\mu)} Y(z^t) \quad (9)$$

where $P_j(z^t)$ is the price of variety j and $P(z^t) = \left[\int P_j(z^t)^{1/(1-\mu)} dj \right]^{1-\mu}$ is the price index.

⁶This assumption implicitly imposes restrictions on the set of additional assets available, the trading restrictions, and the shocks. As an example, the assumption is automatically satisfied in a Huggett economy where the only asset available is the nominal risk-free bond in zero net supply and the trading restrictions are a debt limit of the form $b \geq -\phi$ with $\phi > 0$.

⁷To see this, simply note that the agents that attains the maximum in the right side of (7) are the ones with the lowest multipliers on the trading restriction constraints.

Intermediate good producers. Each intermediate good is supplied by a monopolistic competitive firm. The monopolist of variety j operates the technology (4). As in Rotemberg (1982), we assume that the firm faces quadratic costs to adjust its price,

$$\frac{\kappa}{2} \left[\frac{P_j(z^t)}{P_j(z^{t-1})(1+\bar{\pi})} - 1 \right]^2, \quad (10)$$

where $\bar{\pi}$ is the inflation target of the monetary authority.

The problem of firm j is to choose its price $P_j(z^t)$ given its previous price $P_j(z^{t-1})$ to maximize the present discounted value of real profits. We assume that the firm discounts future profits using the real state price

$$Q(z^{t+1}) = \beta \max_{i, v^t} \left\{ \Pr(z^{t+1}|z^t) \theta(z^{t+1}) \sum_{v_{t+1}} \Pr_i(v^{t+1}|z^{t+1}, v^t) \left[\frac{c_i(z^{t+1}, v^{t+1})}{c_i(z^t, v^t)} \right]^{-\sigma} \right\}. \quad (11)$$

That is, firms discount future profits using the marginal rate of substitution of the agent that values dividends in the aggregate state the most.⁸ The firm's problem can be written recursively as

$$V(P_j, z^t) = \max_{p_j, y_j, n_j} \frac{p_j y_j}{P(z^t)} - w(z^t) n_j(z^t) - \frac{\kappa}{2} \left[\frac{p_j}{P_j(1+\bar{\pi})} - 1 \right]^2 + \sum_{z^{t+1}} Q(z^{t+1}|z^t) V(p_j, z^{t+1}) \quad (12)$$

subject to the production function (4) and the demand function (9).

The solution to the firm's problem together with symmetry across firms requires that the following version of the New Keynesian Phillips curve holds in equilibrium

$$\tilde{\pi}(z^t) = \frac{1}{\kappa(\mu-1)} Y(z^t) \left[\mu \frac{w(z^t)}{A(z_t)} - 1 \right] + \sum_{z^{t+1}} Q(z^{t+1}|z^t) \tilde{\pi}(z^{t+1}) \quad (13)$$

where we define $\tilde{\pi}(z^t) = [(\pi(z^t) - \bar{\pi}) / (1 + \bar{\pi})] \times [(\pi(z^t) + 1) / (1 + \bar{\pi})]$ and $w(z^t) / A(z_t)$ is the real marginal cost for producing a unit of the final good.

Monetary policy and market clearing. We assume that the monetary authority follows a standard Taylor rule

$$1 + i(z^t) = \max \left\{ [1 + i(z^{t-1})]^{\rho_i} \left[(1 + \bar{i}) \left(\frac{1 + \pi(z^t)}{1 + \bar{\pi}} \right)^{\gamma_\pi} \left(\frac{Y(z^t)}{Y^{\text{Pot}}(z^t)} \right)^{\gamma_y} \right]^{1-\rho_i} \exp\{\varepsilon_m(z_t)\}, 1 \right\}, \quad (14)$$

⁸If all agents could trade Arrow securities contingent on the aggregate state then this would be the equilibrium state price.

where $(1 + \bar{i}) = (1 + \bar{\pi})/\beta$ is the nominal interest in a deterministic steady state of the model, $Y^{\text{pot}}(z^t)$ is potential output and $\varepsilon_m(z_t)$ is a monetary shock.⁹ Note that we allow for the possibility of a binding zero lower bound constraint.

The evolution of the aggregate supply of the nominal bond, $B(z^t)$, and taxes, $T_i(s^t)$, must satisfy the government budget constraint,

$$B(z^{t-1}) = \frac{B(z^t)}{1 + i(z^t)} + \sum_i \lambda_i \sum_{v^t} \Pr_i(v^t | z^t) T_i(z^t, v^t) \quad (15)$$

In equilibrium, the labor market, goods markets, and financial markets clear. Specifically, market clearing in the nominal bond market requires that

$$\sum_i \lambda_i \sum_{v^t} \Pr_i(v^t | z^t) b_i(z^t, v^t) = B(z^t). \quad (16)$$

Since firms' equity is the only asset in positive net supply other than the nominal risk-free bond, market clearing in all the other assets requires that the value of inherited assets must equal the nominal value of the firm cum-dividend,

$$\sum_i \lambda_i \sum_{v^t} \Pr_i(v^t | z^t) \sum_{k \in \mathcal{K}} R_k(s^{t-1}, s_t) a_{k,i}(s^{t-1}) = P(z^t) V(P(z^{t-1}), z^t), \quad (17)$$

and the total value of new asset positions must equal to the nominal value of the firm ex-dividend,

$$\sum_i \lambda_i \sum_{v^t} \Pr_i(v^t | z^t) \sum_{k \in \mathcal{K}} q_k(s^t) a_{k,i}(s^t) = P(z^t) \sum_{z^{t+1}} Q(z^{t+1} | z^t) V(P(z^t), z^{t+1}). \quad (18)$$

We can then define an equilibrium for this economy.

Definition 1. *Given an asset structure $(\mathcal{K}, \mathcal{T}, R_k, H)$, the distribution of initial assets and lagged prices, an equilibrium is a set of households' allocations $\{c_i(s^t), l_i(s^t), b_i(s^t), a_{k,i}(s^t)\}$, a fiscal policy $\{B(z^t), T_i(s^t)\}$, prices $\{P(z^t), W(z^t), 1 + i(z^t), Q(z^t), q_k(z^t)\}$, and aggregates $\{C(z^t), Y(z^t)\}$ such that i) the households' allocation solves the households' decision problem, ii) the price for the final good solve (12) with $P(z^t) = P_j(z^t)$, iii) the state price is given by (11), iv) the nominal interest rate satisfies the Taylor rule (14), v) the government budget constraint (15) is satisfied, and vi) markets clear in that (16)–(18) hold and*

$$Y(z^t) = C(z^t) + \frac{\kappa}{2} \left[\frac{\pi(z^t) - \bar{\pi}}{1 + \bar{\pi}} \right]^2 + \mathcal{T}(z^t)$$

⁹Potential output is the level of output that would prevail in an economy with no price-adjustment costs.

where aggregates are given by

$$\begin{aligned} Y(z^t) &= A(z_t) \sum_i \lambda_i \sum_{v^t} \Pr_i(v^t|z^t) e(v_t) l_i(v^t, z^t), \\ C(z^t) &= \sum_i \lambda_i \sum_{v^t} \Pr_i(v^t|z^t) c_i(z^t, v^t), \end{aligned}$$

and $\mathcal{T}(z^t)$ are the aggregate transaction costs,

$$\mathcal{T}(z^t) = \sum_i \lambda_i \sum_{v^t} \Pr_i(v^t|z^t) \mathcal{T}(\{a_{k,i}(s^{t-1})\}_{k \in \mathcal{K}}, \{a_{k,i}(s^t)\}_{k \in \mathcal{K}}, s^t).$$

2.2 Equilibrium representation

We now show that the law of motion for aggregate variables in this class of New Keynesian models can be equivalently derived from the equilibrium conditions of a fictitious representative agent economy where the stand-in household has a time-varying rate of time preference and disutility of labor. These “preference shocks” summarize all the implications that household-level heterogeneity has on aggregate variables. To this end, we define

$$\beta_i(v^t, z^{t+1}) \equiv \sum_{v_{t+1}} \Pr_i(v_{t+1}|v^t, z^{t+1}) \left(\frac{c_i(z^{t+1}, v_{t+1})/C(z^{t+1})}{c_i(z^t, v^t)/C(z^t)} \right)^{-\sigma} \quad (19)$$

$$\omega(z^t) \equiv \left[\sum_i \lambda_i \sum_{v^t} \Pr_i(v^t|z^t) \left(\frac{c_i(z^t, v^t)}{C(z^t)} \right)^{\frac{-\sigma}{\psi}} e(v_t)^{\frac{1+\psi}{\psi}} \right]^{-\psi}. \quad (20)$$

We have the following proposition where we assume that the aggregate transaction costs are negligible, $\mathcal{T}(z^t)$.

Proposition 1. *Given $\{\beta_i(v^t, z^{t+1}), \omega(z^t)\}$ defined in (19) and (20), the equilibrium aggregate consumption, output, inflation, and nominal interest rate, $\{C(z^t), Y(z^t), \pi(z^t), i(z^t), Q(z^{t+1})\}$ must satisfy the aggregate Euler equation,*

$$\frac{1}{1+i(z^t)} = \beta \max_{i, v^t} \sum_{z_{t+1}} \left\{ \frac{\Pr(z^{t+1}|z^t) \theta(z_{t+1}) \beta_i(v^t, z^{t+1})}{1 + \pi(z^{t+1})} \left(\frac{C(z^{t+1})}{C(z^t)} \right)^{-\sigma} \right\}, \quad (21)$$

the Phillips curve,

$$\tilde{\pi}(z^t) = \frac{Y(z^t)}{\kappa(\mu - 1)} \left[\mu \frac{\chi Y(z^t)^\psi C(z^t)^\sigma \omega(z^t)}{A(z_t)^{1+\psi}} - 1 \right] + \sum_{z^{t+1}} Q(z^{t+1}|z^t) \tilde{\pi}(z^{t+1}) \quad (22)$$

the Taylor rule (14), the resource constraint

$$Y(z^t) = C(z^t) + \frac{\kappa}{2} \left[\frac{\pi(z^t) - \bar{\pi}}{1 + \bar{\pi}} \right]^2, \quad (23)$$

and

$$Q(z^{t+1}|z^t) = \beta \max_{i, v^t} \left\{ \beta_i(v^t, z^{t+1}) \Pr(z^{t+1}|z^t) \theta(z^{t+1}) \left(\frac{C(z^{t+1})}{C(z^t)} \right)^\sigma \right\}. \quad (24)$$

The proof for this result is straightforward. The aggregate Euler equation (21) is obtained by using (19) to substitute for the marginal rate of substitution $(c_i(s^{t+1})/c_i(s^t))^{-\sigma}$ in the individual Euler equation (7) and noting that under our assumptions it holds with equality for the agent with the highest marginal valuation of the bond—the "max" in equation (21). The Phillips curve (22) can be derived by substituting for the wage in (13) using the individual labor supply decisions. Indeed, multiplying both side of equation (8) by $e(v_t)/C(z^t)^{-\sigma/\psi}$ and averaging both sides across individuals we obtain

$$w(z^t)^{\frac{1}{\psi}} \left[\sum_i \lambda_i \sum_{v^t} \Pr_i(v^t|z^t) \left(\frac{c_i(z^t, v^t)}{C(z^t)} \right)^{\frac{-\sigma}{\psi}} e(v_t)^{\frac{1+\psi}{\psi}} \right] = \chi^{\frac{1}{\psi}} \left[\sum_i \lambda_i \sum_{v^t} \Pr_i(v^t|z^t) e(v_t) l_i(s^t) \right] C(z^t)^{\frac{\sigma}{\psi}}.$$

We can then use the production function (4) to express the real wage as

$$w(z^t) = \chi \left[\frac{Y(z^t)}{A(z^t)} \right]^\psi C(z^t)^\sigma \omega(z^t),$$

and substitute it in equation (13) to obtain the Phillips curve (22). To obtain (23) we simply substitute the adjustment costs (10) in the resource constraint.

Equations (14), (21), (22), (23) and (24) are equivalent to those of a representative agent economy with "shocks" to the rate of time preferences and to the disutility of labor. Thus, the effects that micro heterogeneity on macroeconomic variables can be represented in this class of models *as if* the stand-in household in a representative-agent economy becomes more/less patient or more/less inclined to work. See Nakajima (2005), Krueger and Lustig (2010), and Werning (2015) for related results.

Proposition 1 has two main implications. The first implication is that $\beta_i(v^t, z^{t+1})$ and $\omega(z^t)$ defined in equations (19) and (20) summarize all the information from the "micro block" of the model that is needed to characterize the behavior of aggregate variables. That is, we do not need to know the specifics of the model regarding the set of assets traded, the transaction costs and trading restrictions faced by households, the fiscal policy $\{B(z^t), T_i(s^t)\}$, and the nature of their idiosyncratic income risk to characterize the behavior of macro aggregates, as long as we know how $\{\beta_i(v^t, z^{t+1}), \omega(z^t)\}$ evolve. Of course, all these elements are important

determinants of $\{\beta_i(v^t, z^{t+1}), \omega(z^t)\}$ but knowing this process is enough for the evolution of aggregates. Second, the mapping between individual allocations and the preference shock defined in (19) and (20) is invariant to the specifics of the model considered. Thus, as long as we observe households' choices, we do not need to take a stand on the specifics of the model to measure $\{\beta_i(v^t, z^{t+1}), \omega(z^t)\}$.

2.3 Examples

Before moving to show how to combine the measured $\{\beta_i(v^t, z^{t+1}), \omega(z^t)\}$ with the representation in Proposition 1 to quantify the contribution of imperfect risk-sharing to business cycle fluctuations, we further illustrate the main result of this section with three examples. We start by considering an economy with complete markets as a benchmark. We then study two simple examples of economies with incomplete markets where we can derive explicit formulas for $\beta_i(v^t, z^{t+1})$ and $\omega(z^t)$. Both examples are motivated by recent research suggesting that microeconomic frictions might have been important factors behind the Great Recession. The first of these two example isolates the implications of a rise in idiosyncratic labor income risk on precautionary saving motives of households, a mechanism studied by [Ravn and Sterk \(2017\)](#) and [Heathcote and Perri \(2018\)](#) among others. The second shows how a tightening of credit constraints at the micro level can lead to a fall in aggregate demand, see [Eggertsson and Krugman \(2012\)](#) and [Guerrieri and Lorenzoni \(2017\)](#). This second example also shows how the behavior of households that are not on their Euler equation indirectly affects the discount factor for the stand-in household in Proposition 1.

These two examples illustrate the two main forces that can generate fluctuations in the discount factor of the stand-in household in our equivalent representation. Broadly speaking, as shown in [Krueger and Lustig \(2010\)](#), if there is no time variation in idiosyncratic labor income risk or in the household's ability to smooth income shocks (borrowing constraints, less assets available, etc.) then there is no variation in $\beta(v^t, z^{t+1})$ and the heterogeneous agent economy is equivalent to a representative agent economy with a different time-invariant discount factor.

Example 1: Complete markets. We start by considering the complete markets benchmark. The set of assets \mathcal{K} contains Arrow securities contingent on the realizations of the aggregate and idiosyncratic state and there are no trading restrictions other than a non-binding no-Ponzi condition. Clearly, the equilibrium outcome in this economy is Pareto efficient and both aggregate and idiosyncratic risk are shared efficiently: the ratio of the marginal utility between any two individuals is constant for all histories: for all $v^{t+1}, \tilde{v}^{t+1}, i$, and j it must be that $U_{c,i}(z^{t+1}, v^{t+1}) / U_{c,i}(z^t, v^t) = U_{c,j}(z^{t+1}, \tilde{v}^{t+1}) / U_{c,j}(z^t, \tilde{v}^t)$. Given our isoelastic pref-

ferences in (2), this implies that individual consumption is a constant fraction of aggregate consumption,

$$c_i(z^t, v^t) = \varphi_i(v_0)C(z^t)$$

for some consumption share $\varphi_i(v_0)$ constant over time and histories. Thus the discount factor in (19) for the stand-in household in the equivalent representation equals 1 for all histories,

$$\beta_i^{\text{cm}}(v^t, z^{t+1}) = 1. \quad (25)$$

Moreover, the disutility of labor for the stand-in household in (20) is given by

$$\omega^{\text{cm}}(z^t) = \left[\sum_i \lambda_i \sum_{v^t} \Pr_i(v^t | z^t) \varphi_i(v_0)^{-\frac{\sigma}{\psi}} e(v_t)^{\frac{1+\psi}{\psi}} \right]^{-\psi}. \quad (26)$$

The above expression can be time-varying even in presence of complete markets. To understand this expression, suppose that $\psi = 1$ and households have the same initial wealth, so that $\varphi_i(v_0) = 1$ for all i . In this case, $\omega(z^t)$ equals the inverse of the cross-sectional variance in households' idiosyncratic productivity. This expression reflects compositional change in the labor force that take place in the heterogeneous agent economy: when the cross-sectional variance of $e(v_t)$ increases, the labor supplied by high productivity households increases and the one supplied by the low productivity households decline because of a substitution effect. This development is captured in the equivalent representative-agent economy by a decline in $\omega(z^t)$, that is, by an increase in the willingness to work of the stand-in household.

Example 2: Labor income risk and aggregate demand. Suppose there is only one type of agent and drop the subscript i from allocations to economize on notation. Let $\sigma = 1$ and the idiosyncratic productivity shocks evolve according to

$$\Delta \log[e(v_t)] = -\frac{\sigma_e(z_t)^2}{2} + \varepsilon_t \quad \varepsilon_t | z^t \sim \mathcal{N}\left(0, \sigma_e(z_t)^2\right).$$

That is, idiosyncratic productivity is a random walk with Gaussian shocks. The standard deviation of individual productivity growth varies over time with the aggregate state z^t : when $\sigma_e(z^{t+1})^2$ is high, households face higher idiosyncratic risk.

To obtain analytical expressions for the $\beta(v^t, z^{t+1})$ and the $\omega(z^t)$ implied by this model, we assume that households can only trade the risk-free bond and that they face the borrowing limit $b(s^t) \geq 0$. Because households cannot trade stocks of the firms, we also assume that the government levies taxes on the intermediate good producers and transfers the profits to the households in proportion to the realization of idiosyncratic productivity, $e(v_t)T(z^t)$.

The tight borrowing limits, coupled with the fact that bonds are in zero net-supply, implies that households in equilibrium cannot save.¹⁰ Thus, every household is hand-to-mouth and every period every period consumes all their cash on hand,

$$c(s^t) = e(v_t) [w(z^t)l(s^t) + T(z^t)] .$$

Furthermore, we can verify from the labor supply condition (8) and $\sigma = 1$ that $l(s^t)$ is the same across individuals. So, it must also be that $c(s^t) = e(v_t)C(z^t)$ from the aggregate resource constraint.

Given the equilibrium consumption function, the relative marginal rate of substitution of the households are just functions of the idiosyncratic income process,

$$\left(\frac{c_i(z^{t+1}, v^t, v_{t+1})/C(z^{t+1})}{c_i(z^t, v^t)/C(z^t)} \right)^{-\sigma} = \frac{e(v_t)}{e(v_{t+1})},$$

and the consumption share of an individual with history s^t is $c(s^t)/C(z^t) = e(v_t)$.

Substituting these expressions in equation (19) and (20) we can compute the implied $\beta(v^t, z^{t+1})$ and $\omega(z^t)$ in this specific model:

$$\begin{aligned} \beta(v^t, z^{t+1}) &= \sum_{v^{t+1}} \Pr(v^{t+1}|v^t, z^{t+1}) \exp \{ -\Delta \log[e(v_{t+1})] \} \\ &= \exp \{ \sigma_e (z^{t+1})^2 \} \end{aligned}$$

and $\omega(z^t) = 1$. Note that in this example $\beta(v^t, z^{t+1})$ does not depend on individual histories and $\omega(z^t)$ does not vary over time. These two expressions, coupled with equations (14), (21), (22) and (23), are enough to characterize the law of motion for aggregate variables in this specific example.

This representation is useful to understand how the interaction between idiosyncratic risk and incomplete financial markets can affect aggregate variables in this class of models. Suppose that households face today higher idiosyncratic risk, that is they expect higher $\sigma_e(z^{t+1})$. If financial markets were complete, this shocks would not have any effects on the allocation. Because of incomplete financial markets, however, households have a precautionary motive to save in the risk-free bond. This increase in the propensity to save at the micro level can be represented as an increase in the discount factor in the equivalent representative agent New Keynesian model.

¹⁰The literature refers to this example with tight borrowing limits and bonds in zero net supply as the *zero liquidity limit*. See [Werning \(2015\)](#) and [Ravn and Sterk \(2017\)](#) for example.

Example 3: Credit constraints and aggregate demand. Consider now an economy where the debt limit depends on aggregate conditions. For simplicity, assume that there are only two types of agents, $i = 1, 2$ of equal measure. When the aggregate state z^t realizes, one the types samples a high efficiency labor units, e_H , while households in the other type draw e_L . To simplify the algebra, we further assume that households within types have no further idiosyncratic shocks to their efficiency of labor and that profits from the monopolistic competitive firms are distributed to households so that $T_i(z^t) + w(z^t)e_i(z_t)l_i(z^t) = e_i(z_t)C(z^t)$.

Households can trade only a non-contingent bond in zero-net supply, subject to the debt limit $\phi(z^t)$. Thus, asset holdings must be such that

$$b_i(z^t) \geq -\phi(z^t)$$

In what follows, we assume that $\phi(z^t)$ is sufficiently small so that the debt limit is always binding for the agents with a low realization of the individual productivity shock, e_L . Thus the individual consumption allocations are given by

$$c_i(z^t) = \begin{cases} e_L C(z^t) + \frac{b_i(z^{t-1})}{1+\pi(z^t)} + \frac{\phi(z^t)}{1+i(z^t)} & \text{if } e_i(z_t) = e_L \\ e_H C(z^t) + \frac{b_i(z^{t-1})}{1+\pi(z^t)} - \frac{\phi(z^t)}{1+i(z^t)} & \text{if } e_i(z_t) = e_H \end{cases}$$

where $b_i(z^{t-1})$ depends on the particular history:

$$b_i(z^{t-1}) = \begin{cases} \phi(z^{t-1}) & \text{if } e_i(z_{t-1}) = e_H \\ -\phi(z^{t-1}) & \text{if } e_i(z_{t-1}) = e_L \end{cases}$$

We can then express the $\beta_i(z^{t+1})$ for the type that attains the maximum in the aggregate Euler equation (21) – the agent with a high realization of the individual productivity in z^t – as

$$\beta_i(z^{t+1}) = \left(\frac{e_i(z_{t+1}) + \left(\frac{\phi(z_t)}{1+\pi(z^{t+1})} - \frac{b_i(z^{t+1})}{1+i(z^{t+1})} \right) / C(z^{t+1})}{e_H(z_t) + \left(\frac{b_i(z^{t-1})}{1+\pi(z^t)} - \frac{\phi(z_t)}{1+i(z^t)} \right) / C(z^t)} \right)^{-\sigma}.$$

From the expression above, it is evident how a tightening of the debt limit, a reduction in $\phi(z^t)$, increases $\beta_i(z^t, z_{t+1})$ for the marginal agent. Intuitively, a reduction in $\phi(z^t)$ means that agent with a low income shock can borrow less to smooth their consumption. In equilibrium, this implies that the agent with currently higher income must save less and consume more to clear the asset market. The increase in current consumption share makes this agent more willing to save and thus the measured $\beta_i(s^t)$ increases for the marginal agent.

This example illustrates two properties of our equivalent representation. First, the behavior of non-marginal households is not irrelevant for the dynamics of aggregates despite only the consumption profile for agents on their individual Euler equation for the nominal bond is appearing in the aggregate Euler equation (21). Here the behavior of borrowing constrained households affects the discount factor of the stand-in household through a general equilibrium relationship. This is important because the literature so far has emphasized the role of these agents with high marginal propensity to consume as critical for the propagation of aggregate shocks. Our representation does not contradict this intuition.

Second, the above expression for $\beta_i(z^t, z_{t+1})$ shows how the “micro-block” is not independent from the “macro-block” that determines the dynamics of aggregates as the $\beta_i(z^t, z_{t+1})$ depends on aggregate consumption, the inflation rate, and the policy rate. Conversely, from Proposition 1, the evolution of these aggregates is affected by $\beta_i(z^t)$. Despite this caveat, we can still use the representation to assess the role of imperfect risk sharing in accounting for aggregate fluctuations as we next show.

2.4 Counterfactuals at a conceptual level

We now explain how to use the representation in Proposition 1 and $\{\beta_i(v^t, z^{t+1}), \omega(z^t)\}$ to evaluate the macroeconomic implications of imperfect risk sharing over the business cycle. For now, we assume that we know the probability distribution of z^t and the process for $\{\theta(\cdot), A(\cdot), \varepsilon_m(\cdot), \beta_i(\cdot), \omega(\cdot)\}$. Thus, given a realization of z^t , we can use the representation in Proposition 1 to obtain the underlying equilibrium path for aggregate variables— output, inflation and nominal interest rates.

Our approach involves comparing these benchmark paths to those that would arise in an economy with complete financial markets. In the previous section we have seen that the economy with complete financial markets features a different stochastic process for $\beta_i(v^t, z^{t+1})$ and $\omega(z^t)$ given by (25) and (26). Because of this difference, it also features a different behavior for the aggregate variables. We label these the *complete markets* paths. The comparison between the benchmark and the complete markets paths isolates the impact that imperfect risk-sharing across households has for macroeconomic aggregates over the particular history z^t .

Our main application will be to quantify the importance of imperfect risk sharing during the Great Recession. As we mentioned earlier, several papers in the literature have suggested that the deep decline in real economic activity during the Great Recession was partly induced by an increase in households’ propensity to save, either because of an increase in precautionary motives or because of a tightening of individual’s borrowing constraints. If these mechanisms were important, we should observe the output trajectory in the bench-

mark to be substantially below its complete market counterfactual when feeding the history z^t that led to the Great Recession.

In this discussion we have assumed that we know $\Pr_i(s^t|s^{t-1})$, the realization of z^t in a particular event, and how it affects $\{\theta(\cdot), A(\cdot), \varepsilon_m(\cdot), \beta_i(\cdot), \omega(\cdot)\}$. In practice, however, we need to estimate these stochastic processes, and we need a procedure to retrieve z^t from the data. In the following sections we discuss how to use survey data to obtain the time path for the preference shocks and how to implement in practice the counterfactuals. Before getting there, it is important to emphasize two points.

First, while we are agnostic about the “micro” aspect of the model, we did make specific assumptions about technology, aggregate structural shocks, monetary policy rule etc, as we carried the analysis within the context of the standard three equations New Keynesian model. This is a deliberate choice because it keeps the analysis transparent. The result in proposition 1 is more general, and can be derived in richer versions of the model. In the robustness analysis of Section 4.5, for example, we introduce capital accumulation in the model. Similarly, we could accommodate different types of nominal rigidities and different preferences for households. As it will become apparent from Section 3, however, a critical assumption for the measurement of the preference shock in the data is that households have the same preferences and there are no idiosyncratic shock to the marginal utility of consumption. This assumption is shared by most of the quantitative heterogeneous agent models.

Second, our approach is silent about the forces behind the fluctuations of the preference shocks, and cannot be used to understand how imperfect risk-sharing affects the propagation of specific structural shocks.¹¹ In this respect, we differ in objectives from recent papers that identify statistics that are sufficient to measure the response of macroeconomic variables to aggregate shocks in this class of models. [Auclert, Rognlie, and Straub \(2018\)](#) show that the distribution of households’ marginal propensity to consume (MPCs) at different time horizons – what they term *intertemporal MPCs* – is a sufficient statistic for the response of output to fiscal shocks in a class of New Keynesian models.¹² Our objective is, instead, to measure the overall macroeconomic impact of imperfect risk-sharing. As we have shown, there is no need to estimate intertemporal MPCs to address this question.

¹¹In principle, one could estimate the impulse response function of the preference shocks to identified shocks and use the equivalent representative-agent formulation to address these questions. This is an interesting avenue for future research, but it is outside the scope of this paper.

¹²In Appendix A we formally derive the mapping between the iMPCs and the preference shocks in the equivalent representative-agent economy.

3 Measuring the preference shocks

We now use household-level observations to measure the preference shocks defined in equation (19)-(20). In order to compute these objects, we need information on the change in consumption shares, $(c_{it-1}/C_{t-1})/(c_{it}/C_t)$, and on the relative wage per hour worked, $e_{it} = W_{it}/W_t$. Section 3.1 describes the data sources. Section 3.2 and 3.3 present the behavior of the measured preference shocks over the sample. Section 3.4 discusses issues related to the presence of measurement errors in the survey.

3.1 Data description

The main data set is the Consumer Expenditure Survey (CEX), which collects information on income, expenditures, employment outcomes, wealth and demographic characteristics for a panel of US households selected to be representative of the population. Households report information on consumption expenditures for a maximum of four consecutive quarters, income and employment information is collected in the first and last interview, and wealth information in the last interview only.¹³

Our measure of consumption is dollar spending on non-durables and services by the household. We measure wage per hour worked by scaling total labor income by total hours. Labor income is a pre-tax measure, and it includes wages and salaries, bonuses, overtime, tips plus income from a business, while total hours worked include hours worked by the head of household and the spouse over the entire year in all jobs. In addition to the variables that are necessary for the calculation of the preference shocks, we obtain socio-demographics indicators about the households (education, sex, family size, etc.) and information on assets and liabilities. Specifically, we use in our analysis an indicator of net-worth (assets minus liabilities) at the household level. The precise definition of the variables used in the analysis is in Appendix B.

The baseline sample includes all households where the head of the household is between the ages of 22 and 64. We only use households who participate in all four interviews in the CEX. We restrict the sample to those which the CEX labels as "complete income reporters," corresponding to households with at least one non-zero response to any of the income and benefits questions. We use the assigned "replicate" or sample weights, designed to map the CEX into the national population in all calculations. We use the CPI-U to express all monetary variables in constant 2000 dollars. To eliminate outliers and mitigate any impact

¹³The CEX asks questions about how assets and liabilities have changed in the preceding year, which allows us to back-date wealth information. See <https://www.bls.gov/opub/mlr/2012/05/art3full.pdf> for more details.

of time-varying top-coding, we drop observations in the top and bottom one percent of the consumption, hours, labor income, wage per hour, total assets, total liabilities and net-worth distribution. Further details about sample selection are discussed in Appendix B.

The model of Section 2 abstracts from important features of the micro data, such as demographics and life-cycle dynamics. In order to have a clear mapping between model and data, we use panel regressions to partial out the effects of these possible confounders. Let \tilde{c}_{it} be the log of consumption expenditures and \tilde{y}_{it} the log of labor income. We estimate the following linear equation

$$\tilde{c}_{it} = \alpha + \gamma' X_i + \gamma_y \tilde{y}_{it} + e_{it},$$

where X_i includes dummies for the sex, race, education, age of the head of household and the state of residence. After estimating this regression, we predict consumption only using labor income and the residual,

$$\tilde{c}_{it}^p = \alpha + \gamma_y \tilde{y}_{it} + e_{it}.$$

We repeat this procedure for all variables used in the analysis.¹⁴ After estimating these relationships, we divide all variables in levels, with the exception of wage per hour, by the number of family members in order to obtain per-capita figures.

Appendix B presents summary statistics of the underlying micro data and some comparisons with previous studies in the literature. Households' characteristics in our sample are comparable with the ones reported in Heathcote and Perri (2018). In line with their findings, we also verify that the behavior of aggregate consumption expenditures, labor income and hours worked implied by the CEX tracks the corresponding national statistics reasonably well. We finally compute a set of cross-sectional statistics and show that their behavior over time closely mirrors results reported in previous papers in a large body of work on consumption and income inequality (Blundell, Pistaferri, and Preston, 2008; Krueger and Perri, 2006; Aguiar and Bils, 2015; Attanasio and Pistaferri, 2016).

3.2 Measuring β_{it}

We now use the data to obtain an empirical counterpart to β_{it} . For the subsequent analysis, we will set $\sigma = 1$. From equation (19) we can then see that β_{it} is the conditional expectations, across idiosyncratic histories, of the inverse change in the consumption share of household i between time $t - 1$ and time t . Because we observe only one realization of the change in consumption share for each i , we need to estimate this conditional expectation. Our approach consists in grouping households with similar observable characteristics at time

¹⁴We do not include \tilde{y}_{it} when predicting labor income.

$t - 1$ and computing

$$\beta_{it} \equiv \frac{1}{N_i} \sum_{j=1}^{N_i} \left[\frac{c_{jt-1}/C_{t-1}}{c_{jt}/C_t} \right], \quad (27)$$

where N_i is the number of households in the group at time $t - 1$.

The logic of equation (27) builds on two premises. The first is that, by grouping individuals along certain observable characteristics, we are effectively proxying for an individual history v^{t-1} and type. The second is that the size of the groups is large enough, so that β_{it} in equation (27) approximates the conditional expectation in equation (19). That is, the cross-sectional average is intended to proxy for an expectation over realizations of v^t , for households with a common v^{t-1} .

We partition households into different groups following the structure of baseline incomplete markets models. In the basic version of these models, the current level of income and net worth are sufficient statistics for individual histories. We follow this insight and group households according to whether their income at date $t - 1$ is above or below median income and, within each of these two groups, whether the level of their net worth is above or below the group median. Thus, for each $t - 1$, we end up with four different groups of households of approximately equal size: *low income/low net worth*, *low income/high net worth*, *high income/low net worth* and *high income/high net worth*. For each group i , we use equation (27) to construct β_{it} .¹⁵

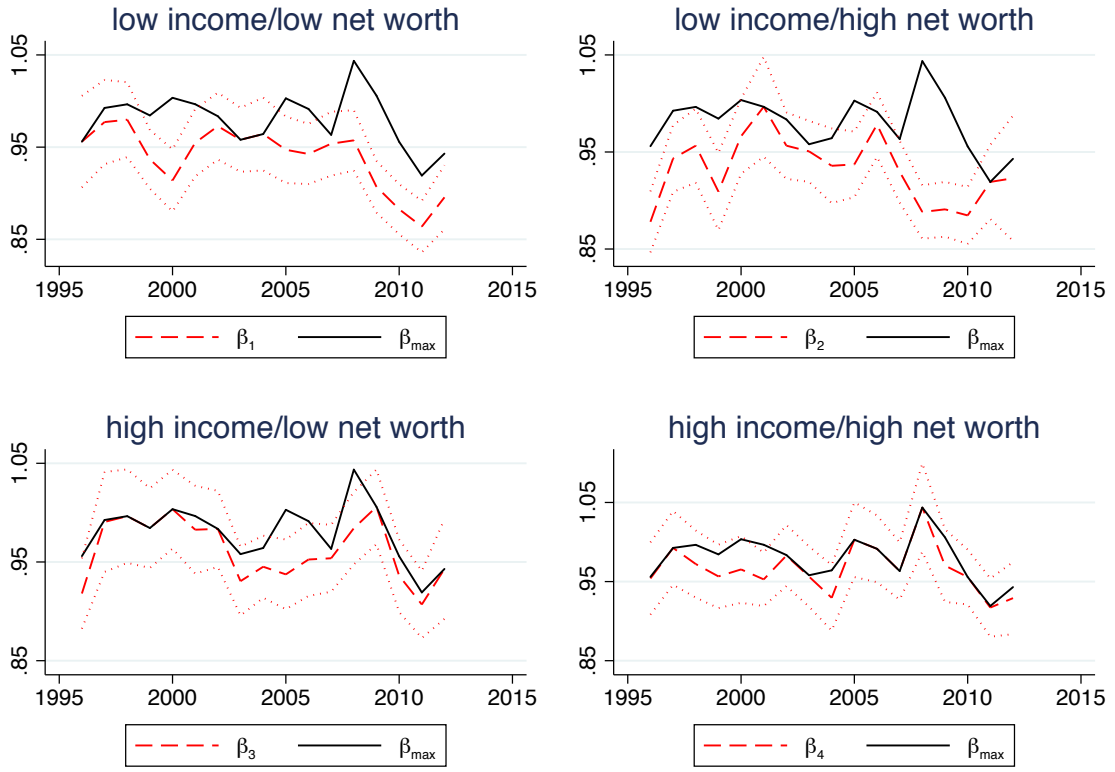
We measure β_{it} at annual frequency. Following [Vissing-Jørgensen \(2002\)](#), we compute the semi-annual consumption change for each household in our dataset,

$$\frac{c_m + c_{m+1} + c_{m+2} + c_{m+3} + c_{m+4} + c_{m+5}}{c_{m+6} + c_{m+7} + c_{m+8} + c_{m+9} + c_{m+10} + c_{m+11}}$$

and scale it by an equivalent semi-annual change in aggregate consumption over the same horizon constructed using monthly data of aggregate consumption of non-durable and services from NIPA. We square the resulting ratio to obtain an annualized change. In order to aggregate up to an annual frequency we must allocate this to a given year. This is easiest when the CEX interview aligns perfectly with the calendar year (that is, $m = 1$). In this case we assign this observation solely to this year. However, this case only happens 1/12 of the time. For the rest of the cases we assign the observation to a given year in proportion to its time in that year. For example, suppose that a particular household's last interview was in month 7 of year t . In that case, this observation would receive a weight of (7/12) in year t

¹⁵In principle, one could consider finer partitions of the joint distribution of income and net worth. However, given the sample size in the CEX, this would produce substantially noisier estimates of β_{it} . With our partitions, we have roughly 600 households per year within each group. However, in [Appendix B.5](#) we consider alternative partitions including four income groups, two total asset and two income groups, two liquid asset and two income groups. See [Figure A-3](#) for details.

Figure 1: Changes in consumption shares by group



Notes: Each panel shows an estimate of β_{it} for four groups of roughly equal size: low income/low net worth, low income/high net worth, high income/low net worth and high income/high net worth (red dashed line) along with the year-by-year value of the maximum across the four groups (solid black line) over the periods 1996-2012. The red dotted lines are 90% confidence bands.

and a weight of $(5/12)$ in year $t - 1$.¹⁶

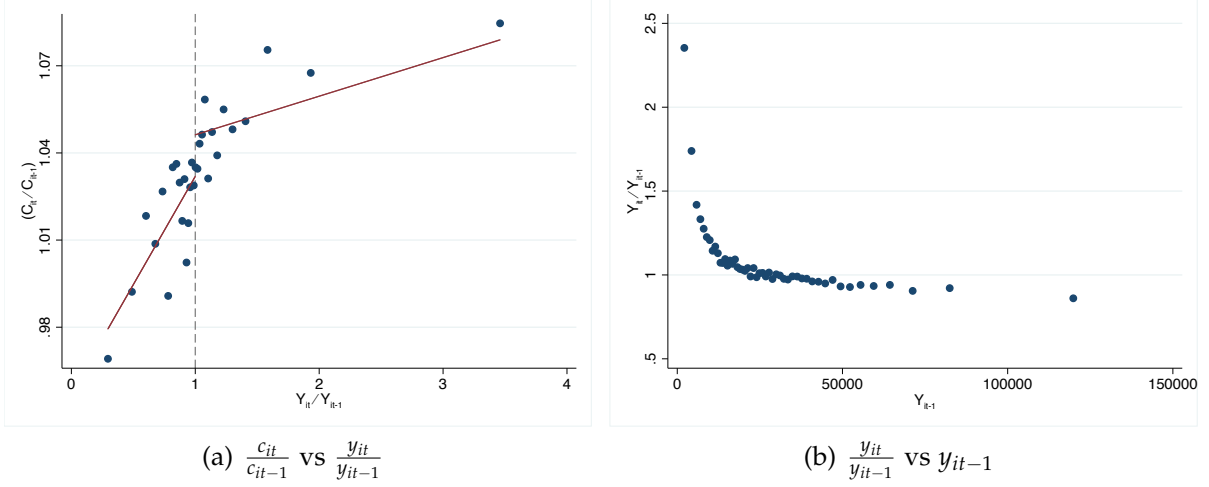
Figure 1 plots the time path of β_{it} for each group along with the year-by-year value of the maximum across the four groups. The path for $\max \beta_{it}$ is normalized to have a mean of 1, and we normalize the other β_{it} relative to this value.¹⁷ This statistic plays an important role in our analysis because, up to a first-order approximation, households with the highest β_{it} are the ones on their Euler equation, see equation (21).

There are two important facts about Figure 1 that we want to emphasize. First, the high income groups have higher implicit discount factors relative to low income households in most years. This is shown in the bottom two panels of Figure 1, as the β_{it} for the two high income groups is equal to the year-by-year max value across all groups (solid line) for most of the years. Second, the discount factor measured in our approach displays a substantial increase during the Great Recession. Figure A-3 shows that both of these results are robust

¹⁶Our results are robust to only using observations that fall mostly within one calendar year.

¹⁷As we show in Section 3.4, the level of β_{it} is affected by the presence of classical measurement errors.

Figure 2: Changes in consumption vs. changes in income



Notes: Panel (a) displays a binned scatter plot of changes in households' consumption (c_{it}/c_{it-1}) and income (y_{it}/y_{it-1}). Panel (b) displays a binned scatter plot of households' changes in income $t - 1$ and t and the level of income at $t - 1$.

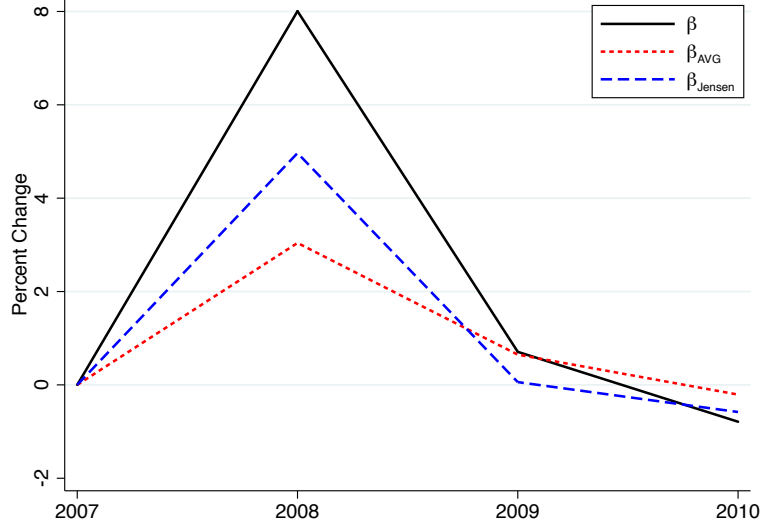
when we consider different partitions of the data.

Why are households with high income at $t - 1$ the ones with the highest β_{it} ? Figure 2 shows that this is due to two features of the data: sensitivity of consumption to negative income changes and the presence of mean-reversion in households' income. Panel (a) of the figure reports a binned scatter plot of the change in households' consumption and the change in their income. The figure displays a non-linear relation: households whose income falls between year $t - 1$ and year t experience on average a large decline in their consumption expenditures. Because of that, households with negative income changes between time $t - 1$ and time t tend to have on average a high measured β_{it} , the average of the inverse change in consumption shares. Panel (b) of the figure plots the relation between y_{it}/y_{it-1} and y_{it-1} : households that at time $t - 1$ have relatively high income experience, on average, a fall in their income between $t - 1$ and t . Taken together, these facts explain why the high income group is the one with the highest measured β_{it} : they are the group whose income is expected to fall the most between $t - 1$ and t , and so they are more likely to experience a decline of their consumption expenditures in relative terms.

Why is the β_{it} of high income households increasing during the Great Recession? To answer this question, it is useful to decompose β_{it} as follows

$$\beta_{it} = \underbrace{\left[\frac{C_t/C_{t-1}}{\frac{1}{N_i} \sum_{j=1}^{N_i} c_{jt}/c_{jt-1}} \right]}_{\beta_{AVG,it}} \underbrace{\left[\sum_{j=1}^{N_i} \left[\frac{\sum_{j=1}^{N_i} c_{jt}/c_{jt-1}}{c_{jt}/c_{jt-1}} \right]}_{\beta_{JEN,it}}. \quad (28)$$

Figure 3: Decomposition of equation (28)



Notes: This figure plots the decomposition described in equation (28) for the marginal group during the Great Recession (the high income/high net worth group). In particular, it shows how to decompose β_{it} (solid black line) into the average the consumption growth of the group ($\beta_{AVG,it}$: red dotted line) and a component that measures the increase in dispersion of consumption growth within the group ($\beta_{JEN,it}$: dashed blue line).

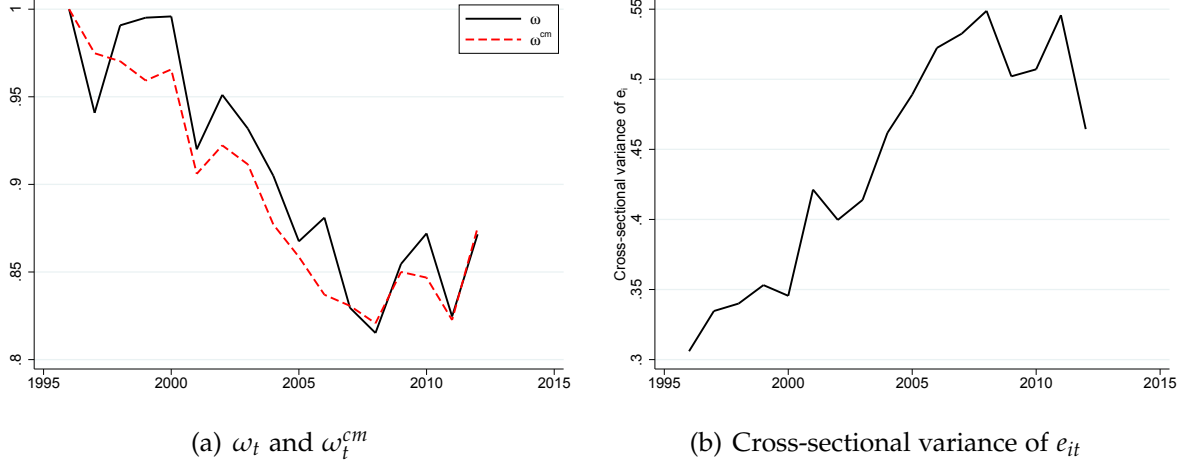
Mechanically, β_{it} can increase for two reasons. First, if the average change in consumption between time $t - 1$ and time t for a given group i decreases relative to the change in aggregate consumption over the same period. This effect is captured by the term $\beta_{AVG,it}$ in the above expression. Second, because of Jensen's inequality, β_{it} can increase because of an increase in the cross-sectional dispersion of c_{jt}/c_{jt-1} , an effect that is captured by $\beta_{JEN,it}$ in equation (28). This decomposition is shown in Figure 3 for the high income/high net worth group, the one with the highest β_{it} during the Great Recession. While both components increase, most of the increase is due to an increase in $\beta_{JEN,it}$. In Section 5 we explore possible explanations for the increase in the cross-sectional dispersion of c_{jt}/c_{jt-1} for the high income/high net worth group during the Great Recession.

3.3 Measuring ω_t

We now turn to the measurement of ω_t in equation (20). For each household in our panel, we compute the consumption share, $\varphi_{it} = c_{it}/C_t$ and combine it with the relative wage, $e_{it} = \frac{w_{it}}{W_t}$, to construct ω_t .¹⁸ In particular, we compute for each household $\varphi_{it}^{-\sigma/\psi} e_{it}^{(1+\psi)/\psi}$, take the cross-sectional average for each t , and raise it to $-1/\psi$ power. We set $\psi = 1$, a common value used in the business cycle literature. In order to compute the value of ω_t

¹⁸We assign a productivity level of zero for households that report zero hours worked for the entire year. These cases are rare in our dataset.

Figure 4: The evolution of ω_t



that would prevail in a world with complete financial markets, we need to know the initial distribution of consumption shares and its correlation with e_{it} for every t , see equation (26). We assume that the moments of the initial distribution are those of the first year in our sample, 1996. That is, we compute ω_t^{cm} as follows

$$\omega_t^{cm} = \left[\frac{1}{N} \sum_{j=1}^N \varphi_{i1996}^{-1} \times \frac{1}{N} \sum_{j=1}^N e_{it}^2 + \text{cov} \left(\varphi_{i1996}^{-1}, e_{i1996}^2 \right) \right]^{-1}.$$

Panel (a) in Figure 4 plots the time series for ω_t and ω_t^{cm} . We can see that both series display a downward trend. This pattern is explained almost entirely by the increase in the cross-sectional variance of e_{it} , see panel (b) of the figure.¹⁹ As explained in Section 2.3, an increase in the cross-sectional variance of individual productivity induces compositional change in the labor force that are captured by a lower disutility of labor in the equivalent representative agent economy. We can also observe from the figure that the deviations between ω_{it} and ω_{it}^{cm} are typically small relative to the overall variability in the series. Throughout the Great Recession, ω_t lies above ω_t^{cm} . Through the lens of our framework, this means that the impact of imperfect risk-sharing on labor supply during the Great Recession can be interpreted as an increase in the marginal disutility of labor in the equivalent representative agent economy.

¹⁹Heathcote, Perri, and Violante (2010) also finds that the cross-sectional variance of log wages are increasing in the CEX, PSID and CPS, see Figure 16 in their paper for more details.

3.4 Measurement error

A possible concern with our analysis is that the fluctuations we measure in β_{it} and ω_t are due to measurement error. While we cannot rule out arbitrary measurement error, in this section we try to gauge whether our results can be attributed to specific forms of measurement error.

One form of measurement error is simply recording errors in the CEX that create extreme outliers. This could be particularly relevant for our analysis because errors in (log) levels of consumption are magnified when they are differenced. When selecting the sample, we remove the top and bottom 1% (year by year) of the observations for all the variables used in the analysis. In addition, we follow [Vissing-Jørgensen \(2002\)](#) and remove observations in consumption growth that are less 0.20 and greater than 5 (this removes 10 observations). Trimming more aggressively (top/bottom 2%) led to similar estimates of β_{it} and ω_t .

The second way we address potential measurement error in consumption growth is by adopting the approach in [Vissing-Jørgensen \(2002\)](#) of using semi-annual changes in order to minimize time aggregation and category switching concerns due to the fact that households may only purchase certain categories of goods infrequently. This aggregation from quarterly to semi-annual consumption changes removes a significant amount of the variation in consumption growth: the standard deviation of consumption growth falls from 0.311 to 0.248. This suggests that time aggregation and category switching are a significant phenomena in the micro data that the [Vissing-Jørgensen \(2002\)](#) procedure helps to remove.

Third, it is worth noting that β_{it} and ω_t are cross-sectional averages of household-level observations, an operation that tends to reduce the impact of measurement errors. To see why, suppose that the observed consumption of an household is related to the true consumption, \tilde{c}_{jt} , as follows $c_{jt} = \tilde{c}_{jt} \times \exp\{\eta_{jt}\}$ where η_{jt} is a Gaussian iid measurement error with mean $-\sigma_\eta^2/2$ and variance σ_η^2 . Assume further that η_{jt} is independent from the true level of consumption \tilde{c}_{jt} . Then, we have

$$\begin{aligned} \frac{1}{N_i} \sum_{j=1}^{N_i} \frac{c_{jt-1}}{c_{jt}} &= \frac{1}{N_i} \sum_{j=1}^{N_i} \exp\{-\Delta \log(\tilde{c}_{jt})\} \times \frac{1}{N_i} \sum_{j=1}^{N_i} \exp\{-\Delta \eta_{jt}\} \\ &+ \text{Cov}(\exp\{-\Delta \log(\tilde{c}_{jt})\}, \exp\{-\Delta \eta_{jt}\}). \end{aligned}$$

If $N_i \rightarrow \infty$, then the covariance term goes to zero and the measured β_{it} becomes

$$\beta_{it} = \tilde{\beta}_{it} \times \exp\{\sigma_\eta^2\},$$

where $\tilde{\beta}_{it}$ is the statistics computed using the true consumption. That is, in the case of classical measurement errors, our β_{it} statistics is off relative to the truth by a time-invariant

factor.²⁰ A similar derivation can be done for ω_t . Because our analysis is not focused on the levels of these variables but rather on their changes over time, it is robust to the presence of classical multiplicative measurement errors in consumption.

In the discussion above, we have assumed that the distribution of the measurement errors is uncorrelated with households' characteristics. Aguiar and Bils (2015) provide evidence that rich households systematically underreport consumption. Mean differences in measurement error ($\mu_{\eta_{Y_{high}}} > \mu_{\eta_{Y_{low}}}$) is not a problem for our approach because the difference in means cancels in the above calculation. Furthermore, Figure A-3 in Appendix B.5 shows that we find a similar time-series pattern and relative ranking for β_{it} 's when we measure β_{it} in the PSID, a data set where there is no concern about differential measurement error.

Fourth, in the implementation of the counterfactual in the next section, we will add Gaussian measurement errors on both β_{it} and ω_t . This feature is intended to mitigate the impact that non-classical errors have on the measured preference shocks.

4 Imperfect risk-sharing and the US Great Recession

In this section we exploit the representation in Proposition 1 along with the time series for β_{it} , ω_t and ω_t^{cm} to quantify the macroeconomic effects of imperfect risk-sharing during the US Great Recession. We start in Section 4.1 by detailing how we implement in practice the counterfactuals that we discussed in Section 2.4. As we explain there, a preliminary step consists in choosing the structural parameters of the model, which we estimate in Section 4.2. Section 4.3 presents the impulse response functions to β_{it} and ω_t , an exercise that is useful to understand how changes in these preference shocks feed back to macroeconomic variables in the estimated model. Section 4.4 reports the results of the counterfactual. In Section 4.5 we perform a sensitivity analysis of our results by introducing capital accumulation in the model.

4.1 Counterfactuals in practice

In order to implement the counterfactuals discussed in Section 2.4 we need a procedure to estimate the stochastic processes $\{\theta(z_t), A(z_t), \varepsilon_m(z_t), \beta_i(v^{t-1}, z_t), \omega(z_t)\}$ and to retrieve the realization of z^t from the data.

We start by assuming a Markovian structure for the states, $\Pr_i(s_t|s^{t-1}) = \Pr_i(s_t|s_{t-1})$. Without loss of generality, we can then set $z_{1,t} = \hat{\theta}_t$, $z_{2,t} = \hat{A}_t$ and $z_{3,t} = \varepsilon_{m,t}$, where $\hat{\theta}_t$

²⁰In our applications, the group have a size of roughly 600 observations in each year. Through Monte Carlo simulations we have verified that this is enough for the law of large numbers to apply when η_{jt} is iid.

and \hat{A}_t are the logarithm of these two variables. The vector z_t could potentially incorporate other aggregate shocks (for example, shocks affecting financial constraints), and we leave that unrestricted. Following much of the existing literature, we assume that the aggregate preference shock and the technology shocks follow AR(1) processes,

$$\begin{aligned}\hat{\theta}_t &= \rho_\theta \hat{\theta}_{t-1} + \varepsilon_{\theta,t} & \varepsilon_{\theta,t} &\sim \mathcal{N}(0, \sigma_\theta^2) \\ \hat{A}_t &= \rho_a \hat{A}_{t-1} + \varepsilon_{A,t} & \varepsilon_{A,t} &\sim \mathcal{N}(0, \sigma_A^2),\end{aligned}$$

and $\varepsilon_{m,t} \sim \mathcal{N}(0, \sigma_m^2)$.

Making assumptions about $\beta_i(v^t, z^{t+1})$ and $\omega(z^t)$ is conceptually more problematic, because those are not fundamental shocks and do not necessarily inherit the Markov structure of s_t .²¹ To explain the nature of the problem, suppose that households can only save/borrow in a risk-free nominal bond and they face a borrowing limit $b_i(s^t) \geq -\phi$. In a recursive competitive equilibrium, the distribution of assets is an aggregate state variable that itself follows a Markov process. The implied process for $\beta_i(v^{t-1}, z^t)$ and $\omega(z^t)$, however, will be a vector moving average of order infinity, which is not a feasible process for estimation. In time series analysis, these processes are approximated with finite-order ones. We follow this strategy and approximate their law of motion using independent AR(1) processes. Specifically, let $\beta_t \equiv \max_i \beta_{it-1}$ and define $\mathbf{T}_t = [\hat{\beta}_t, \hat{\omega}_t]'$ to be the de-meaned logarithm of these processes.²² We assume that \mathbf{T}_t follows the process

$$\mathbf{T}_t = \Phi \mathbf{T}_{t-1} + \varepsilon_{T,t}, \quad (29)$$

where $\varepsilon_{T,t} \sim \mathcal{N}(\mathbf{0}, \Sigma)$, and the restriction is that Φ and Σ are diagonal matrices. In our application, we choose a parsimonious stochastic process due to the limited length of the sample.

Let us emphasize two features of the system in (29). First, the process for β_{it} is individual-specific. Given our strategy of grouping households, we should in principle include in \mathbf{T}_t the measured β_{it} for each group. In our application, we only include the “max” in \mathbf{T}_t . This simplification allows us to economize on the number of state variables when solving the equivalent representative-agent economy with global methods. In addition, it is worth noting that in our sample the max is achieved in almost every period by the income-rich

²¹A similar issue arises in the accounting procedure of Chari, Kehoe, and McGrattan (2007), see their discussion in Section 2.

²²To understand why we lag β_{it} by one period, consider β_{i2008} defined in equation (27). This variable measures the cross-sectional average of the inverse changes in consumption shares for group i within the 2008 calendar year. This corresponds in the model to *realization* of the discount factor for the Euler equation that holds at the beginning of 2008, see equation (21). The *realization* of this variable is relevant for forecasting the discount factor in the Euler equation at the beginning of 2009, explaining why $\hat{\beta}_{i2008}$ is a state variable for 2009.

households which have similar stochastic properties for β_{it} .²³ Therefore, we do not think we are losing much in practice by using $\max_i \beta_{it}$ in our application. Second, we are not allowing the structural shocks to load on \mathbf{T}_t . This is mostly an ex-post result, rather than an actual restriction: when estimating the model, we found that these loadings were imprecisely estimated, so we decided to set them to zero.

Given a stochastic process for $\mathbf{X}_t = [\hat{\theta}_t, \hat{A}_t, \varepsilon_{m,t}, \mathbf{T}_t]'$ and numerical values for the model parameters, we can then solve for the aggregate variables as a function of the state $\mathbf{S}_t = [i_{t-1}, \mathbf{X}_t]$ using the system of equations in Proposition 1. For this purpose, we employ a variant of the algorithm developed in Gust, Herbst, López-Salido, and Smith (2017) to solve the representative agent New Keynesian model with an occasionally binding zero lower bound constraint, see Appendix C for a detailed description of the algorithm. We can then implement the counterfactuals described in Section 2.4 in three steps.

In the first step, we estimate the parameters of the equivalent representative-agent economy using the measured preference shocks and macroeconomic variables that are routinely used to estimate the three-equations representative agent New Keynesian model: nominal interest rates, inflation and real GDP.

In the second step, we apply a non-linear filter to the estimated model and retrieve the sequence of shocks that rationalizes the path of the observed variables in the sample. This step provides us with an estimate of the aggregate shocks, $\{\hat{\theta}_t, \hat{A}_t, \varepsilon_{m,t}\}$.

In the third and final step, we obtain the paths for macroeconomic variables in a counterfactual economy with complete financial markets. To do so, we first solve for the policy function of the system in Proposition 1 under the assumption that $\hat{\beta}_t = 0, \forall t$. We then obtain the counterfactual path for the aggregate variables by feeding these policy functions with the time path of $\{\hat{\theta}_t, \hat{A}_t, \varepsilon_{m,t}\}$ estimated in step 2 and ω_t^{cm} measured in Section 3.

4.2 Estimation

The model is estimated at an annual frequency on the 1997-2012 period. We map the log of output in the model, \hat{Y}_t , to the percentage deviations of log real GDP from a linear deterministic trend.²⁴ The inflation rate π_t is the annual percent change in the consumer price index, and i_t is mapped to the annual effective federal funds rate.

The model parameters are the ones governing preferences, $[\beta, \sigma, \psi, \chi]$, the importance of price adjustment costs κ , the elasticity of substitution across varieties μ , the behavior of the monetary authority $[\rho_i, \gamma_\pi, \gamma_y, \pi^*]$ and the stochastic process for \mathbf{X}_t .

²³For β_{3t} , the first-order autocorrelation in the sample is 0.39 and its standard deviation is 0.043. For β_{4t} these statistics are 0.33 and 0.043.

²⁴In the model, output is net of adjustment costs.

We fix a subset of these parameters to conventional values in the literature. Consistent with the measurement of the preference shocks, we set $\sigma = \psi = 1$. We let $\mu = 1.2$ and set χ to $1/\mu$, so that consumption and output equal 1 in a deterministic steady state of the model. Finally, we set the target inflation rate to 2%, and $\beta = 0.99$, values that guarantee that the model matches the average inflation and nominal interest rate in our sample in a deterministic steady state.

The remaining parameters, $[\kappa, \rho_i, \gamma_\pi, \gamma_y]$ and $[\rho_\theta, \sigma_\theta, \rho_A, \sigma_A, \sigma_m, \Phi, \Sigma]$ are estimated. Let $\mathbf{Y}_t = [\hat{Y}_t, i_t, \pi_t, \mathbf{T}_t]$ be the vector of observable variables, and denote by \mathbf{Y}^T all the observations in our sample. The state vector is $\mathbf{S}_t = [i_{t-1}, \hat{\theta}_t, \hat{A}_t, \varepsilon_{m,t}, \mathbf{T}_t]$. The model of Section 2 defines the non-linear state space model

$$\begin{aligned}\mathbf{Y}_t &= \mathbf{g}(\mathbf{S}_t; \phi) + \eta_t \\ \mathbf{S}_t &= \mathbf{f}(\mathbf{S}_{t-1}, \varepsilon_t; \phi),\end{aligned}\tag{30}$$

where $\mathbf{g}(\cdot)$ and $\mathbf{f}(\cdot)$ represent the policy functions of the model, ϕ the vector of parameters to be estimated, ε_t collects the innovations to the stochastic variables of the model. The vector η_t collects Gaussian measurement errors that capture deviations between the data \mathbf{Y}_t and $\mathbf{g}(\mathbf{S}_t; \phi)$. We introduce measurement errors only for the preference shocks, as those are measured using survey data, and fix their variance to 10% of the unconditional variance of these series. In Appendix D.3 we repeat the analysis with a larger variance for the measurement errors.

Given this representation, we can apply filtering techniques to the state-space system and evaluate the likelihood of the model, $\mathcal{L}(\phi|\mathbf{Y}^T)$. We can then combine the likelihood function with a prior for the structural parameters, $p(\phi)$, and apply the Metropolis-Hastings algorithm to sample from the posterior distribution of ϕ (An and Schorfheide, 2007),

$$p(\phi|\mathbf{Y}^T) \propto p(\phi)\mathcal{L}(\phi|\mathbf{Y}^T).$$

Appendix D describes the algorithm for estimating the model and its in-sample fit. For the purpose of estimation, we solve for the policy functions with a first-order perturbation. The first-order perturbation solution is much faster and numerically more stable than the global approximation discussed in Appendix C, and it allows us to use the Kalman filter for the evaluation of the likelihood function. The main drawback is that, by using perturbation methods, we do not account for the possibility of a binding zero lower bound constraint on nominal interest rates when estimating the model parameters. The non-linear model, however, fits the data remarkably well once we apply the parameters estimates that we obtain here (see the model-fit analysis in Appendix D), so we believe we would obtain very

Table 1: Prior and posterior distribution of model parameters

Parameter	Distribution	Prior		Posterior	
		Mean	St. dev.	Mean	90% Interval
$4 \times \kappa$	Gamma	85.00	15.00	73.71	[52.17, 93.81]
ρ_i	Beta	0.50	0.25	0.57	[0.34, 0.80]
γ_π	Normal	1.50	2.00	3.72	[1.91, 5.41]
γ_y	Normal	1.00	2.00	0.18	[0.00, 0.42]
ρ_θ	Beta	0.50	0.28	0.69	[0.49, 0.90]
ρ_A	Beta	0.50	0.28	0.91	[0.83, 0.99]
$\Phi_{\beta,\beta}$	Beta	0.50	0.25	0.33	[0.06, 0.55]
$\Phi_{\omega,\omega}$	Beta	0.50	0.25	0.86	[0.74, 0.99]
$100 \times \sigma_\theta$	InvGamma	1.00	5.00	2.48	[1.02, 4.02]
$100 \times \sigma_A$	InvGamma	1.00	5.00	2.18	[0.73, 2.89]
$100 \times \sigma_m$	InvGamma	1.00	5.00	1.94	[1.15, 2.69]
$100 \times \sigma_\beta$	InvGamma	1.00	5.00	2.24	[1.51, 2.96]
$100 \times \sigma_\omega$	InvGamma	1.00	5.00	2.28	[1.29, 3.25]

Notes: The prior on γ_π and γ_y is truncated at zero. The posterior statistics reports the mean, fifth and ninety-fifth percentile of the posterior distribution estimated by pooling 2 Markov chains with 100,000 draws each (including a 100,000 draw burn-in period for each chain).

similar parameter estimates if we were to estimate it with non-linear methods.

Table 1 reports posterior statistics for the model parameters. The structural parameters defining the behavior of the monetary authority and the price adjustment costs are in line with previous estimates reported in the literature. For instance, they are comparable to the estimates reported in the working paper version of [Gust et al. \(2017\)](#), who used aggregate data to estimate a similar version of our model – a representative agent three-equations New Keynesian with technology shocks, discount factor shocks and monetary policy shocks. An important parameter for the analysis that follows is the persistence of $\hat{\beta}_t$, $\Phi_{\beta,\beta}$, because this parameter influences the effect that a high realization of $\hat{\beta}_t$ has on the discount factor in the Euler equation of the model. When $\Phi_{\beta,\beta} = 0$, for example, a high value of $\hat{\beta}_t$ has no impact on this discount factor, and so it does not affect the behavior of aggregate variables. The posterior mean of this parameter is 0.33, which is close to the OLS coefficient estimated by fitting an AR(1) process on $\hat{\beta}_t$ (0.27).

4.3 Impulse response functions

Before moving to our counterfactual, it is useful to study how changes in $\hat{\beta}_t$ and $\hat{\omega}_t$ —the preference shifters measured using households-level data—affect aggregate variables in the estimated model.

For this purpose, we fix the model parameters at the posterior mean of Table 1, solve the model numerically, and compute impulse response functions (IRFS) to a 2 standard deviations increase in $\hat{\beta}_t$ and a 0.14 standard deviation increase in $\hat{\omega}_t$. We chose a 0.14 standard deviation shock to mimic the typical deviation between $\hat{\omega}_t$ and $\hat{\omega}_t^{\text{cm}}$ in Figure 4.²⁵ Because the model is non-linear, the IRFs are potentially state-dependent. The solid line in Figure 5 reports the IRFs when the initial state S_0 is at the ergodic mean while the circled line reports IRFs when S_0 is such that the implied level of the nominal interest rate absent the shock equals zero.²⁶

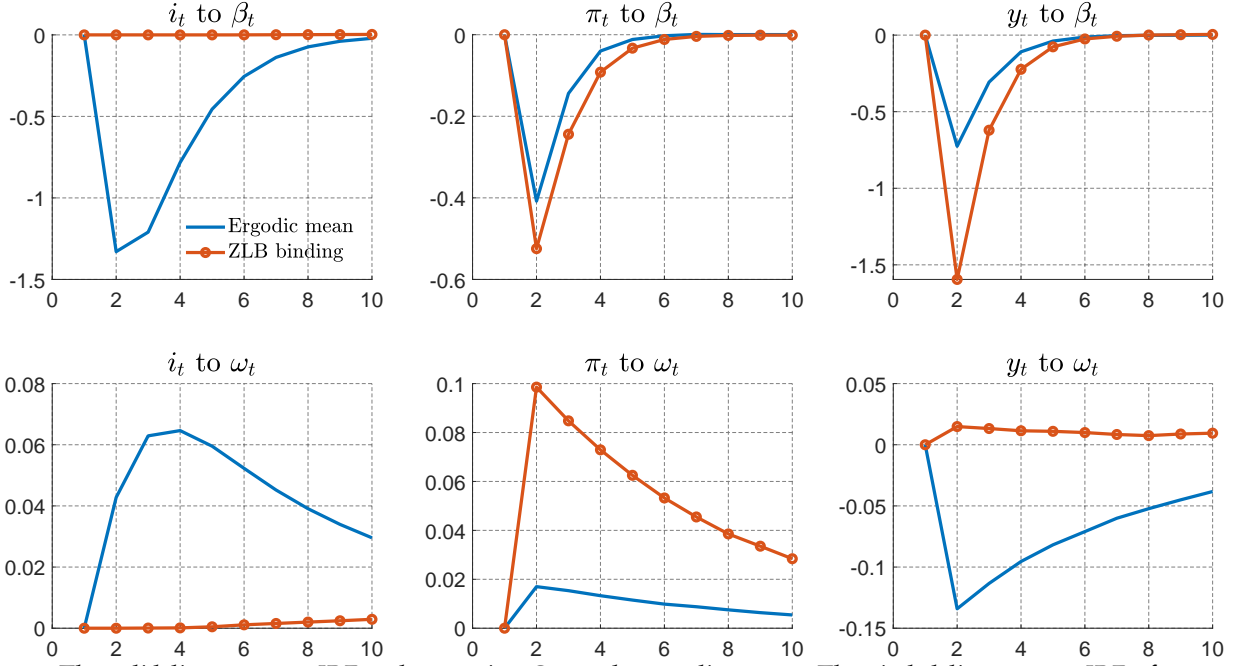
Starting with the top panels of the figure, we can study the effects of an increase in β_t , as we have seen for example during the 2008-2010 recession. This increase in patience induces a decline in aggregate consumption for a given level of the real interest rate. This decline in aggregate demand lowers inflation which, for a given level of the nominal interest rate, increases real interest rates and further depresses aggregate consumption. When nominal interest rates are positive, as in the case described by the solid line in the figure, the central bank responds to the shock by cutting nominal interest rates by 130 basis points, given the estimated Taylor rule. This mitigates the aggregate effects of the β_t shock: a two standard deviations increase in β_t results in a 0.6% (0.10 standard deviations) decline in aggregate output and 0.4% (0.23 standard deviations) decline in inflation. When the economy is at the zero lower bound, instead, the central bank cannot cut nominal interest rates further and the shock has larger effects on output (-1.5%) and inflation (-0.5%).

The bottom panels in Figure 5 reports the response to an increase in $\hat{\omega}_t$, in line with the behavior of $\hat{\omega}_t - \hat{\omega}_t^{\text{cm}}$ during the 2008-2010 recession. This shock reduces labor supply and it increases the marginal cost for firms, leading to an increase in inflation and a reduction in output. The central bank responds by increasing nominal and real interest rates, which further depress output and mitigate the rise in inflation. At the zero lower bound, the central bank cannot respond to the shock. Because expected inflation increase, real interest rates on nominal bonds fall, thus mitigating the impact of the shock on output: given our parametrization, output slightly increases at the zero lower bound. We can see from the figure, however, that these changes have quantitatively small effects on aggregate variables in the estimated model, implying that the measured differences between $\hat{\omega}_t$ and $\hat{\omega}_t^{\text{cm}}$ are not large enough to induce sizable movements in aggregate variables.

²⁵The ratio between $\text{Stdev}(\hat{\omega}_t^{\text{cm}} - \hat{\omega}_t)$ and $\text{Stdev}(\hat{\omega}_t)$ is 0.07. Thus, 0.14 corresponds to a two standard deviations change in the value of leisure due to imperfect risk-sharing.

²⁶We compute non-linear IRFs following [Koop, Pesaran, and Potter \(1996\)](#). Consider the IRFs to $\hat{\beta}_t$. Given an initial condition S_0 , we compute $2 \times M$ simulations of the model of length T . In the first M simulations, we restrict the innovations to $\hat{\beta}_t$ at $t = 1$ to equal $2 \times \sigma_\beta$. The innovations in the second M simulations are the same as in the first, with the exception that $\varepsilon_{\beta,1} = 0$. To obtain the IRFs, we average the first and second sets of simulations across M and take the difference between the two paths. The IRFs to ω_t are computed in a similar fashion. In the figure, we set $M = 10,000$ and $T = 10$.

Figure 5: IRFs to preference shocks



Notes: The solid lines reports IRFs when setting \mathbf{S}_0 to the ergodic mean. The circled lines report IRFs for an economy that is at the zero lower bound. Specifically, we set the $i_0 = 0$ and $\theta_0 = 0.08$ while keeping the other state variables at their ergodic mean. These conditions guarantee that the economy is at the zero lower bound. The IRFs are computed as explained in footnote 26, and they are reported in percentages.

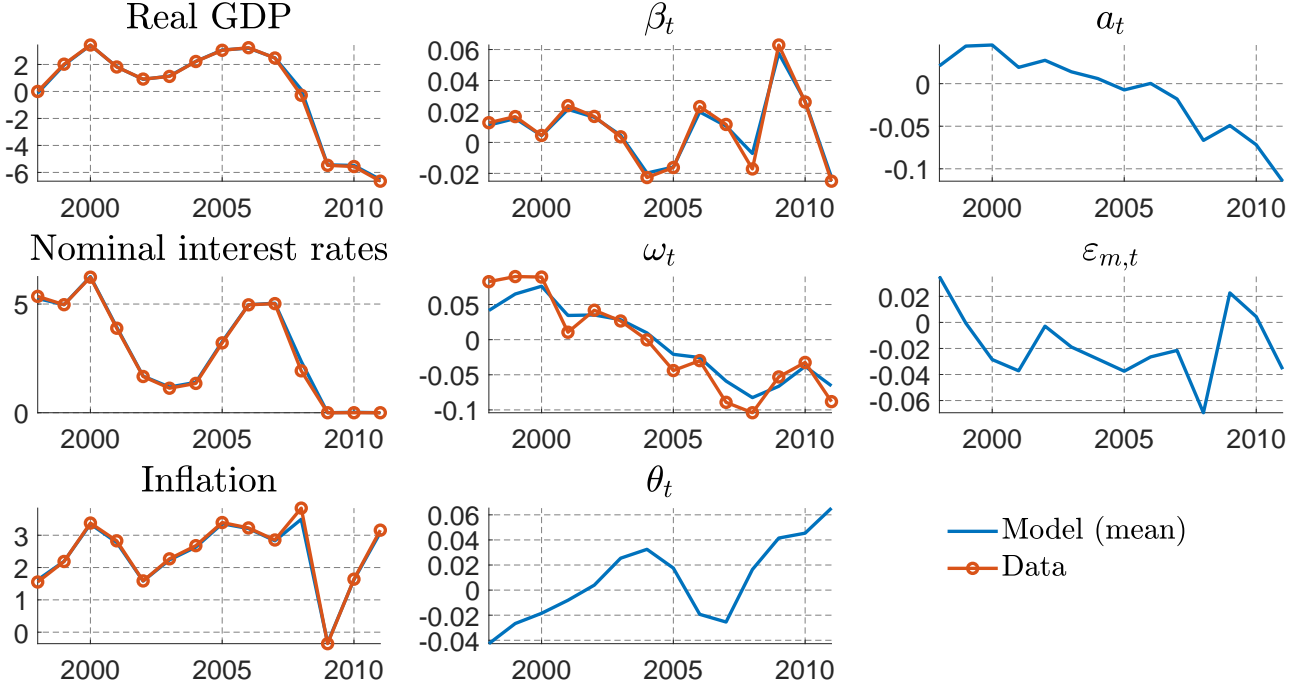
4.4 Counterfactuals

We can now use the estimated model to measure the macroeconomic effects of imperfect risk-sharing during the Great Recession. We start by applying the particle filter to the state-space representation (30) and obtain an estimate for the latent variables \mathbf{S}_t . Because i_t and \mathbf{T}_t are elements of \mathbf{Y}_t , the truly latent states are $[\hat{\theta}_t, \hat{A}_t, \varepsilon_{m,t}]$. Consistent with the estimation of the model, we set a measurement error for \mathbf{T}_t equal to 10% of its unconditional variance, while we introduce a small measurement error for the macroeconomic variables (1% of their unconditional variance) in order to increase the stability of the filter.

Figure 6 reports the data used in the experiment along with the mean of the filtered states and endogenous variables implied by the model. By construction, the model tracks almost perfectly the real GDP, inflation and nominal interest rates over the sample because the measurement errors on these variables are small, and it follows closely the behavior of $\hat{\beta}_t$ and $\hat{\omega}_t$. As for the structural shocks, the model requires a persistent decline in technology shocks and a persistent increase in $\hat{\theta}_t$ in order to replicate the dynamics of the macroeconomic variables during the Great Recession.

Equipped with the path \mathbf{S}_t , we can then construct the trajectories for real GDP, nominal interest rates and inflation that would prevail in an economy with complete financial

Figure 6: Data and model objects



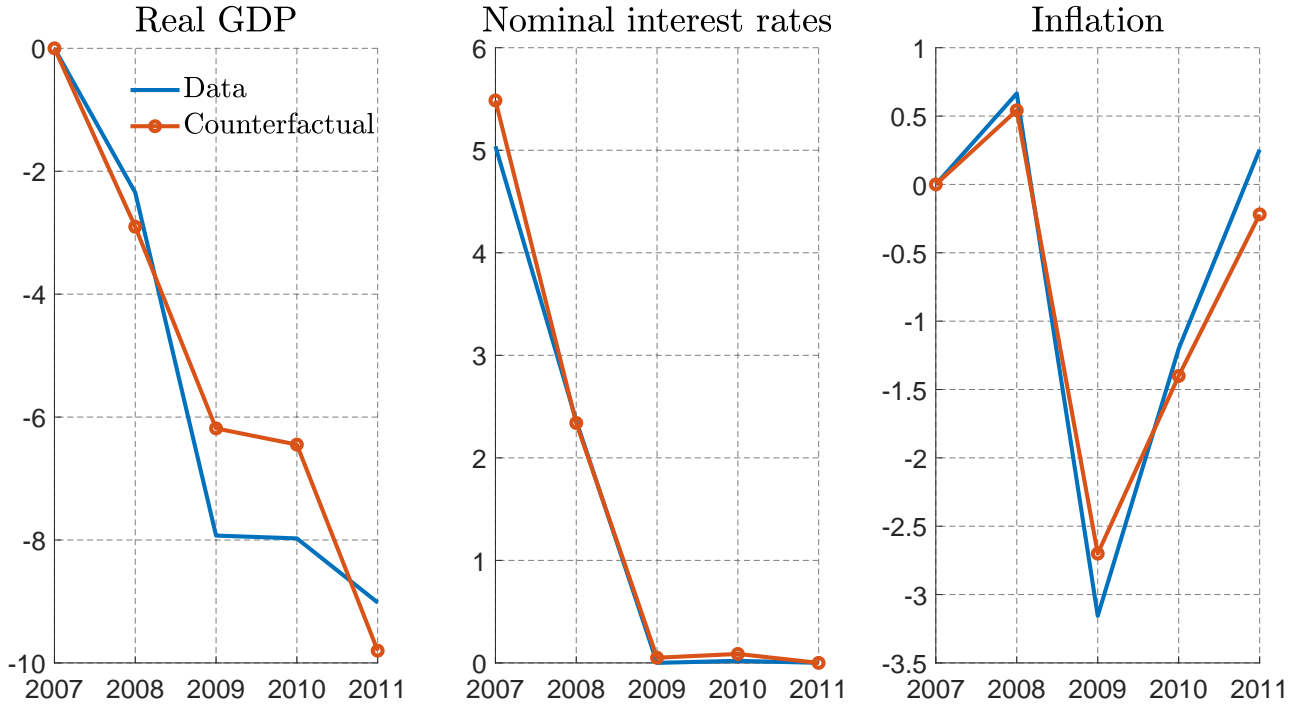
Notes: The circled line reports the data used in the experiment. The solid lines report the filtered series for \mathbf{Y}_t and \mathbf{S}_t in the model.

markets. For that purpose, we solve numerically for the policy functions of the equivalent representative-agent economy with no shocks to $\hat{\beta}_t$. After obtaining these policy functions, we construct the counterfactual path for the macroeconomic variables by feeding these policy functions with the estimated path for $[\hat{\theta}_t, \hat{A}_t, \varepsilon_{m,t}]$ and $\hat{\omega}_t^{\text{cm}}$, see Appendix D for a detailed description of this experiment.

Figure 7 compares the trajectories for output, inflation and nominal interest rates in this counterfactual (circled lines) with the actual trajectories in US data (solid lines) during the 2007-2011 period. From peak to trough, de-trended real GDP in the US fell by 8% in 2009-2010. The counterfactual economy with $\beta_t = 1$ displays a smaller decline in real economic activity during this period, respectively 6.1% and 6.4%. Thus, our findings are consistent with the view that deviations from perfect risk-sharing was an important dimension of the US Great Recession, accounting for roughly 20% of the observed output declines.

Why do we observe these differences between the baseline and the counterfactual economy? From Figure 6 we can see that $\hat{\beta}_t$ was sensibly above the mean in those years. As we have seen from the impulse response analysis, a higher rate of time preference contributes to a decline in aggregate demand and inflation, amplified by the fact that nominal interest rates were at zero in 2009. In the counterfactual economy with $\beta_t = 1$, these developments are absent, explaining the differences between the data and the counterfactual.

Figure 7: Imperfect risk-sharing and the US Great Recession



Notes: The solid line reports real GDP, nominal interest rates and inflation during the 2007-2011 period. We normalize real GDP and inflation to 0 in 2007. The circled line reports the same variable in the counterfactual economy with complete financial markets.

While this exercise detects an important role for imperfect risk-sharing in 2009-2010, it also points toward fairly transitory effects. This is due to two features. First, by 2011 the measured $\hat{\beta}_t$ falls back to its unconditional mean. Second, the model of Section 2 lacks an internal propagation mechanism, as nominal interest rates are the only endogenous variable and they are equal to zero during this period.

4.5 Adding capital accumulation

So far, we have conducted our analysis in the context of the standard three-equations New Keynesian model. While we have deliberately chosen this framework for its simplicity, a natural question is whether the results of the counterfactual would be sensibly different if we were to consider, instead, a New Keynesian model closer to the ones used in quantitative analyses. Introducing capital accumulation might be particularly relevant in this respect. In a model with physical capital, an increase in households' propensity to save – a higher β_t – tends to increase aggregate investment, a feature that counteracts the negative effects that this shock has on consumption. In the context of our counterfactual, this could potentially reduce the output losses displayed in Figure 7. We argue, however, that this mechanism

is not particularly relevant for our application. As we show next, when the zero lower bound constraint binds, an increase in β_t reduces not only aggregate consumption but also aggregate investment.

We introduce physical capital in the model of Section 2 by assuming that the intermediate good producers have a Cobb-Douglas production function,

$$y_j(z^t) = A(z_t)k_j(z^{t-1})^\alpha n_j(z^t)^{1-\alpha}.$$

The intermediate good producers rent capital from a capital good producing firm that discount dividend using the state price (24). Physical capital depreciates at the rate δ , and it is accumulated following the law of motion

$$K(z^t) = (1 - \delta)K(z^{t-1}) + I(z^t).$$

In addition, we assume that capital good producers pay a quadratic cost when adjusting the capital stock,

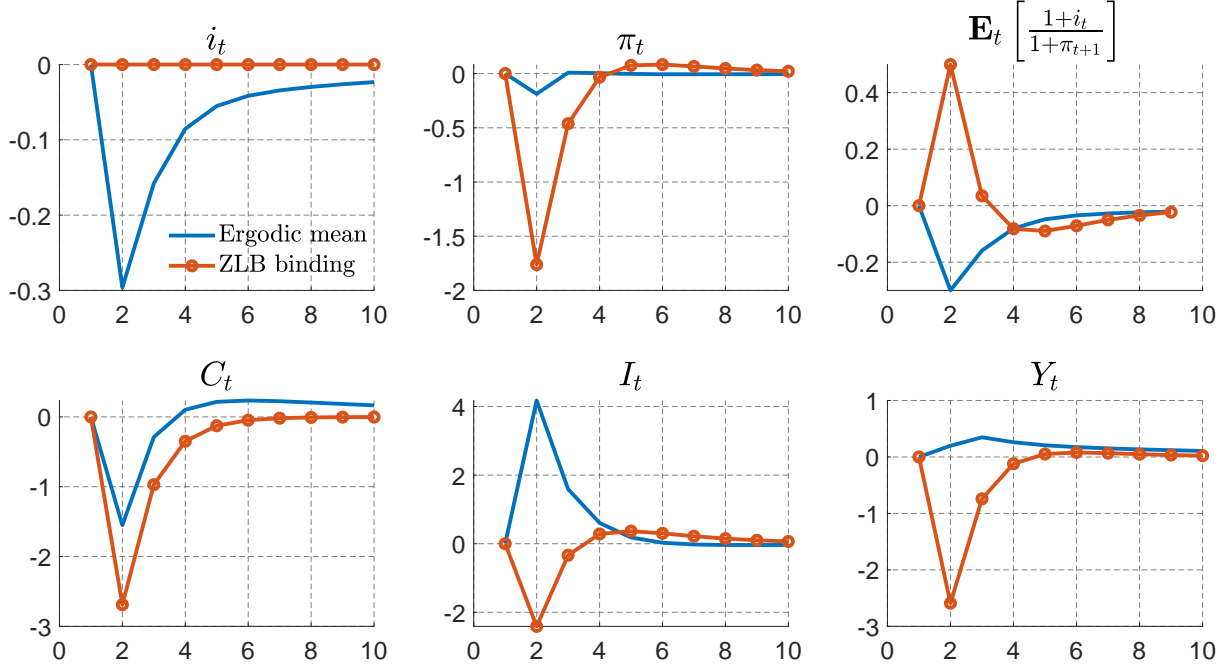
$$\frac{\xi}{2} \left(\frac{I(z^t)}{K(z^{t-1})} - \delta \right)^2 K(z^{t-1}).$$

These features change the decision problem of the firms presented in Section 2.1, while they leave the decision problem of households mostly unaffected. The only difference is that that the assets traded by the households must include claims on the capital stock. Appendix E discusses the details of this extension, and formally states a version of Proposition 1 in this environment. In addition to the resource constraint, the Phillips curve, the Euler equation for nominal bonds and the Taylor rule, the equivalent representative-agent economy now features also an Euler equation for capital.

To explore how capital accumulation affects our results, we set $\alpha = 0.33$, $\delta = 0.10$ and $\xi = 1$, standard values in the business cycle literature, while we keep the remaining parameters at the posterior mean reported in Table 1. We then apply our global solution algorithm to obtain the model's policy functions. Figure 8 reports IRFs to a two standard deviations increase in $\hat{\beta}_t$. Similarly to the previous analysis, we report the IRFs at two different points in the state space: the ergodic mean (solid line), where nominal interest rates are positive, and an initial state such that nominal interest rates are at the zero lower bound (circled line).

At the ergodic mean, an increase in $\hat{\beta}_t$ leads to a reduction in aggregate consumption, inflation and nominal interest rates, as it was the case for the economy without capital. Because nominal interest rates decline by more than expected inflation, the real interest rate on nominal bonds decreases. This reduction in real interest rates induces an increase in aggregate investment through the Euler equation for capital. Thus, away from the zero lower

Figure 8: IRFs to $\hat{\beta}_t$ in the model with capital



Notes: The solid lines reports IRFs when setting S_0 to the ergodic mean. The circled lines report IRFs for an economy that is at the zero lower bound. The initial conditions are set at their long run average under the assumption that $\theta_t = 0.10 \forall t$. These conditions guarantee that the economy is at the zero lower bound. The IRFs are computed as explained in footnote 26, and they are reported in percentages.

bound, the increase in patience moves consumption and investment in opposite directions. Given our calibration, the overall effects of the shocks on output are small and positive: that is, an increase in households' incentives to save in the model with capital are typically expansionary.

When the economy is at the zero lower bound, however, the dynamics are different. The increase in $\hat{\beta}_t$ still reduces aggregate consumption and inflation, but now the monetary authority cannot reduce nominal interest rates. Because the decline in inflation is persistent, the real interest rate on nominal bonds *increases*. The increase in real interest rates reduces the incentives to accumulate capital, and so also aggregate investment falls. Hence, at the zero lower bound an increase in households' incentives to save generates a fall in aggregate consumption and investment, which implies that the overall effects are recessionary as in our benchmark model.

Comparing the IRFs in Figure 8 with those in the top panel of Figure 5, we can see that the overall output effects of a two standard deviations increase in $\hat{\beta}_t$ are quantitatively comparable across the two models when the zero lower bound constraint binds: if anything, the model with capital produces slightly larger output losses. We conclude that the results of the main counterfactual would not be sensibly different in more quantitatively plausible

versions of the model, because the measured increase in $\hat{\beta}_t$ took place when nominal interest rates were at zero.

5 Inspecting the mechanism

The dynamics of $\{\beta_{it}, \omega_t\}$ summarize all the information from cross-sectional data that is needed to assess the aggregate implications of imperfect risk-sharing in the class of models studied in this paper. That is, two economies with different primitives that generate the same stochastic process for $\{\beta_{it}, \omega_t\}$ imply the same pattern for macroeconomic aggregates. While our approach is not designed to discriminate between alternative models, we can use the information in the CEX to evaluate different mechanisms that have been proposed in the literature.

From the analysis in Section 3, we know that the Jensen term in the decomposition of equation (28) accounts for 5/8 of the increase in 2008 in β_{it} for the savers. That is, most of the increase in the measured discount factor during the Great Recession is due to an increase in the *dispersion* of consumption changes for the high income group. This observation casts some doubts on the empirical relevance of the so-called “two-agent” New Keynesian models studied for example in Galí, López-Salido, and Vallés (2007), Bilbiie (2008), and Debortoli and Galí (2017). These models emphasize differential consumption growth across two groups of households, hand-to-mouth and savers, as the key mechanism through which imperfect risk-sharing impact the aggregate. By construction, they do not generate dispersion within the group of savers. Our analysis, instead, shows that idiosyncratic risk within this group is of first-order importance for understanding deviations from perfect risk-sharing during the Great Recession.

There are two mutually non exclusive channels that have been proposed in the literature that can explain this increase in dispersion. One channel emphasizes a deterioration of the risk-sharing mechanisms available during the Great Recession, such as a tightening of borrowing constraints. A tightening of individual borrowing constraints can generate higher sensitivity of consumption to negative income changes, leading to an increase in β_{it} in the equivalent representative-agent economy. A second force that can lead to an increase in the cross-sectional dispersion of consumption shares would be an increase in idiosyncratic labor income risk, see our example in Section 2.3.

We can use the household-level data to investigate which of these two mechanisms better account for the dynamics of β_{it} . We first check whether there has been a change in the distribution of y_t/y_{t-1} during the Great Recession for the high income groups, the one we identify as savers. The top panel of the Table 2 displays the percentiles of y_t/y_{t-1} before

and during the Great Recession. The two distributions appear remarkably similar, suggesting that changes in the distribution of labor income for this group of households did not contribute to the increase in their measured β_{it} .²⁷

Second, we check if there has been an increase in the sensitivity of consumption to income changes during the Great Recession. Specifically, we estimate the following linear relation

$$\left(\frac{c_{jt-1}/C_{t-1}}{c_{jt}/C_t} \right) = \alpha + \beta \frac{y_{jt}}{y_{jt-1}} + \delta \text{rec}_t + \gamma \frac{y_{jt}}{y_{jt-1}} \times \text{rec}_t + e_{jt},$$

where rec_t is an indicator function equal to 1 for $t = 2008$ or $t = 2009$ and we condition only on households experiencing negative income changes ($y_{it}/y_{it-1} < 1$).²⁸ A negative value of γ implies that, for a given negative change in income, households were cutting their consumption expenditures more in 2008-2009 than they did in 2006-2007. The bottom panel of Table 2 reports the estimates for all households, and when partitioning individuals along the income and net worth dimension. We can detect an increase in the sensitivity of consumption to income during the Great Recession for households with high income and high net worth, the group with the highest measured β_{it} over this period.

The evidence in Table 2 suggests that structural models that emphasize a reduction in the ability of savers to smooth income shocks during the Great Recession have a better chance of being consistent with the micro data than structural models emphasizing a pure increase in idiosyncratic labor income risk. A labor-income risk explanation can however be consistent with the evidence of Table 2 if the Great Recession was associated with a change in the nature of idiosyncratic income shocks affecting high income individuals. For instance, consider a shift in the composition between a permanent and a transitory component of idiosyncratic income shocks.²⁹ If the permanent component increases while the transitory decreases, this could rationalize why in Table 2 we detect an increase in the responsiveness to negative income shock in absence of significant shifts in the distribution of income changes. This is because permanent shocks are harder to smooth than transitory ones in baseline incomplete

²⁷These results are consistent with previous papers that examined the cyclical behavior of income changes. While Storesletten, Telmer, and Yaron (2004) found evidence of countercyclical income risk, their estimates are obtained from an estimated income process for the entire population, and it is not comparable to the results in Table 2 that condition on sub-group of the population. The work by Heathcote, Perri, and Violante (2010) finds little cyclical variation in earnings growth for households between the 50th and 90th percentiles of the earnings distribution, which corresponds roughly to the sub-group displayed in Table 2 (households with income above the median). The authors document significant cyclical variation for poorer households due to a higher incidence of unemployment during recessions, a result that is confirmed by Guvenen, Ozkan, and Song (2014) with Social Security Administration data.

²⁸This sample restriction is motivated by the fact that sharp non-linearity in the relationship between relative consumption shares and income growth occurs when individual income growth falls. See panel (a) of Figure 2 for a graphical illustration.

²⁹This hypothesis is similar to the mechanism in Blundell, Pistaferri, and Preston (2008) to account for the changes in the consumption distribution relative to the income distribution over the 1970s to 1990s.

Table 2: Explaining the rise in $\beta_{JEN,it}$ during the Great Recession

Distribution of Income Growth Rates									
Y_H Households	p1	p5	p10	p25	p50	p75	p90	p95	p99
2006-2007	0.25	0.44	0.69	0.80	0.94	1.06	1.23	1.41	2.00
2008-2009	0.21	0.45	0.69	0.81	0.95	1.07	1.23	1.38	1.89
Y_H, NW_H Households									
2006-2007	0.25	0.44	0.61	0.80	0.94	1.05	1.23	1.45	2.06
2008-2009	0.23	0.44	0.58	0.79	0.95	1.06	1.24	1.43	1.93

Consumption Response to Income Changes in 2006-2009							
	All Groups		(Y_H)	(Y_L, NW_L)	Separate Groups		
					(Y_L, NW_H)	(Y_H, NW_L)	(Y_H, NW_H)
ΔY	-0.431*** (-4.04)	-0.247** (-2.47)	-0.146 (-1.07)	-0.579** (-2.28)	-0.213 (-1.39)	-0.203 (-0.90)	-0.0909 (-0.59)
Recession		0.287 (1.59)	0.630** (2.07)	-0.240 (-0.88)	-0.0274 (-0.16)	0.179 (0.73)	1.078** (1.96)
Recession x ΔY		-0.369* (-1.74)	-0.733** (-2.05)	0.220 (0.69)	-0.0584 (-0.28)	-0.198 (-0.68)	-1.272** (-1.96)
_cons	1.611*** (17.79)	1.468*** (17.49)	1.377*** (12.14)	1.726*** (7.95)	1.472*** (11.77)	1.438*** (7.66)	1.318*** (10.22)
N	9016	9016	4818	2032	2166	2305	2513
Weights	Y	Y	Y	Y	Y	Y	Y

Notes: The top panel displays the distribution in income growth for two time periods (2006/2007 and 2008/2009). The first two rows reports the distribution for households with income above the median and the third and fourth rows report it for households with income and net worth above the median. The bottom panel reports the OLS estimates of the following relation over the 2006-2009 period

$$\left(\frac{c_{it-1}/C_{t-1}}{c_{it}/C_t} \right) = \alpha + \beta \frac{y_{it}}{y_{it-1}} + \delta rec_t + \gamma \frac{y_{it}}{y_{it-1}} \times rec_t + e_{it},$$

where rec_t is an indicator function equal to 1 for $t = 2008$ or $t = 2009$. All t statistics are reported in parentheses and all regressions use robust standard errors. The first and second column reports the estimates when using all households. The third column when using households with y_{it-1} above median. The remaining columns report the estimates separately for each of the group.

markets model, and so consumption is more sensitive to the former than the latter.

6 Conclusion

This paper has proposed a simple approach to assess the macroeconomic implications of imperfect risk-sharing implied by a class of New Keynesian models with isoelastic preferences, idiosyncratic income risk and incomplete financial markets. In this class of models, households' inability to insure idiosyncratic risk is reflected in time-variation in their consumption shares. Leveraging this insight, we use households' consumption choices to directly measure the degree of imperfect risk-sharing for the US economy. We document a deterioration of risk-sharing during the US Great Recession, as the cross-sectional dispersion of households' consumption shares increases throughout this period. In addition, we show that households' self-insurance motives associated to this shift contributed to a decline in aggregate demand and they accounted for roughly 20% of the output losses observed in 2009 and 2010.

Our approach is silent about the shocks and frictions that contribute to the observed deviations from perfect risk-sharing. As a result, it is not designed to address important questions in the literature, such as the transmission mechanisms of monetary policy when financial markets are incomplete or the effects of specific shocks and frictions on households' precautionary savings. To address these questions, one needs fully specified structural models. Our paper clarifies that different model ingredients matter for aggregate fluctuations only through their impact on two summary statistics of the distribution of households' consumption shares and relative wages – what we labeled preference shocks. We believe that structural models with heterogeneous agents interested in aggregate fluctuations should be calibrated/estimated to match the statistical properties of these two summary statistics.

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APPENDIX TO “IMPERFECT RISK-SHARING AND THE BUSINESS CYCLE”

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A Relation with [Auclert, Rognlie, and Straub \(2018\)](#)

There is a large and growing literature that has emphasized the role of the distribution of marginal propensities to consume (MPCs) as a critical statistic to discipline structural New Keynesian models with heterogeneous agents. This is because MPCs are informative about the response of aggregate variables to redistributive policies or to shocks like the tightening of borrowing constraints in partial equilibrium. [Auclert, Rognlie, and Straub \(2018\)](#) show that a summary statistic for the distribution of MPCs at different time horizons – what they term *intertemporal MPCs* – is a sufficient statistic for the response of output to fiscal shocks in a class of New Keynesian models. Here we show how our work is connected to this line of research. In particular, we show that there is a mapping between the relevant statistic of the distribution of intertemporal MPCs and the impulse response function to aggregate shocks of the discount factor and disutility of leisure of the stand-in household in our framework. We do so within the context of a simple two-period economy with shocks to fiscal policy that satisfies the conditions in [Auclert, Rognlie, and Straub \(2018\)](#).

Let $t = 1, 2$ and assume that wages are sticky in period 1 and flexible in period 2. Assume there are two types of agents, $i \in \{L, H\}$. Agents differ only in their endowment of efficiency unit of labor in period 1, $e_H > e_L$. Since wages are sticky in period 1, we need to postulate a mechanism for the allocation of labor between the two types of agents. We assume each agent works the same amount of hours so that the real labor income of type i agent is $e_i Y_1$. The resource constraints are then

$$Y_1 = \sum_i \lambda_i e_i l_{i1}, \quad Y_2 = \sum_i \lambda_i l_{i2}$$

Fiscal policy consists of a lump-sum transfer in period 1 financed by issuing debt to be repaid in period 2 with lump-sum taxes. For simplicity we allow for taxes in period 2 to depend on the household's type.

The problem for a household of type i is

$$\max_{c_{1i}, a_i, c_{2i}, l_{2i}} \sum_{t=1}^2 \beta^{t-1} \left[\log c_{it} - \chi \frac{l_{it}^{1+\psi}}{1+\psi} \right]$$

subject to

$$\begin{aligned} c_{1i} + a_i &\leq e_i Y_1 + T \\ c_{2i} &\leq w_2 l_{2i} - \tau_{2i}(T) + (1+r)a_i \\ a_i &\geq 0 \end{aligned}$$

where a_i are the holdings of government debt, T are lump-sum transfers in period 1, and $\tau_i(T)$ are lump-sum taxes in period 2.

The government budget constraint in period 1 is

$$T = \sum_i \lambda_i a_i$$

and in period 2 is

$$\sum_i \lambda_i (1+r)a_i = \sum_i \lambda_i \tau_i(T)$$

To simplify the algebra, we assume that taxes in period 2 are given by $\tau_L(T) = 0$ and $\tau_H(T) = T/\lambda_H$ so that there are no wealth effect in period 2 and both types of agents consume and work the same amount. Finally, we assume that monetary policy targets a real rate r as [Auclert, Rognlie, and Straub \(2018\)](#).

We can then characterize the equilibrium in this economy as a function of T . In period 2, the output is efficient and it solves $\frac{1}{Y_2} = \chi Y_2^\psi$. We normalize χ to one so that $Y_2 = c_{2H} = c_{2L} = 1$.

If T is small enough, the debt limit $a_i \geq 0$ is binding for the type L agents and the allocations in period 1 are given by

$$\begin{aligned} Y_1 &= \frac{1}{\beta(1+r)e_H} + \frac{1-\lambda_H}{e_H\lambda_H} T \\ c_{1H} &= e_H Y_1 - \frac{1-\lambda_H}{\lambda_H} T \\ c_{1L} &= e_L Y_1 + T \end{aligned}$$

Thus the effect of an increase in transfers in period 1 is expansionary and given by

$$\frac{\partial Y_1}{\partial T} = \frac{1-\lambda_H}{e_H\lambda_H}.$$

Consider now our representative agent formulation for this economy. The taste shock for

the marginal agent, β_H , is given by

$$\beta_H(T) = \beta \left(\frac{c_{1H}}{c_{2H}} / \frac{Y_{1H}}{Y_2} \right)^{-1} = \beta \left(e_H - T \frac{1 - \lambda_H}{\lambda_H} / Y_1(T) \right)$$

and the aggregate Euler equation is

$$\frac{1}{Y_1(T)} = \beta_H(T)(1+r) \frac{1}{Y_2}$$

so

$$\frac{\partial Y_1(T)}{\partial T} = - \frac{Y_1(T)}{\beta_H(T)} \frac{\partial \beta_H(T)}{\partial T} \quad (\text{A.1})$$

where

$$\frac{\partial \beta_H(T)}{\partial T} = - \frac{1 - \lambda_H}{e_H \lambda_H} \frac{\beta_H(T)}{Y_1(T)}$$

Thus, if we know how a fiscal policy shock in the detailed economy affect the discount factor in our representative agent formulation, $\beta_H(T)/\partial T$, we can calculate the response of output to T by calculating the response of output to the change in the discount factor in the representative agent formulation.

We now show how we can use the logic in [Auclert, Rognlie, and Straub \(2018\)](#) to express the change in output as a function of the intertemporal MPCs. Let $x = (Y_1, T)$ and note that the solution to the household problem can be expressed as functions $c_{1i}(x), c_{2i}(x)$. Market clearing in the consumption good market requires

$$Y_1(T) = \sum_i \lambda_i c_{1i}(Y_1, T)$$

totally differentiating the expression above we obtain

$$dY_1 = \sum_i \lambda_i \left(\frac{\partial c_{1i}}{\partial Y_1} dY_1 + \frac{\partial c_{1i}}{\partial T} dT + \frac{\partial c_{1i}}{\partial \tau_i} \frac{\partial \tau_i}{\partial T} dT \right)$$

Letting $MPC_i(t, j)$ be agent i 's marginal propensity to consume in period t income earned in period j , we can write

$$\frac{\partial c_{1i}}{\partial T} = MPC_i(1, 1) = \begin{cases} \frac{1}{1+\beta} & \text{if } i = H \\ 1 & \text{if } i = L \end{cases}$$

$$\frac{\partial c_{1i}}{\partial \tau} = MPC_i(1, 2) = \begin{cases} \frac{1}{(1+\beta)(1+r)} & \text{if } i = H \\ 0 & \text{if } i = L \end{cases}$$

and

$$\frac{\partial c_{1i}}{\partial Y} = e_i MPC_i(1, 1)$$

Thus we can combine the expressions above to obtain

$$\frac{\partial Y_1}{\partial T} \left(1 - \sum_i \lambda_i e_i MPC_i(1, 1) \right) = \sum_{t=1}^2 \sum_i \lambda_i MPC_i(1, t) dI_{t,i}$$

where $dI_{t,i}$ is the direct income change induced by the fiscal policy to agent i in period t :

$$dI_{t,i} = \begin{cases} 1 & \text{if } t = 1 \\ -\frac{1}{\lambda_H} & \text{if } t = 2, i = H \\ 0 & \text{if } t = 2, i = L \end{cases}$$

Thus

$$\frac{\partial Y_1}{\partial T} = \frac{\sum_{t=1}^2 \sum_i \lambda_i MPC_i(1, t) dI_{t,i}}{(1 - \sum_i \lambda_i e_i MPC_i(1, 1))} = \frac{1 - \lambda_H}{e_H \lambda_H} \quad (\text{A.2})$$

Comparing (A.1) with (A.2) we have:

$$\frac{\sum_{t=1}^2 \sum_i \lambda_i MPC_{i,1t} dI_{t,i}}{(1 - \sum_i \lambda_i e_i MPC_{i,11})} = -\frac{Y_1(T)}{\beta_H(T)} \frac{\partial \beta_H(T)}{\partial T}$$

Our approach has the advantage that it can be more easily implemented without the need for natural experiments that are necessary for estimating intertemporal MPCs. However, while we can measure β_i from the data, without knowledge of the impulse response functions (e.g. $\partial \beta_H(T)/\partial T$), we cannot study how imperfect risk-sharing affects the propagation of specific structural shocks (eg. a fiscal policy shock).

B Data

In this appendix we give more details about sample selection and variables definition. We also present some summary statistics of the raw data and show that our sample both aggregates reasonably and is consistent with recent work on consumption inequality.

B.1 Definition of variables and sample selection in the CEI

Consumption expenditures. Our measure of consumption expenditure is close to the NIPA definition of nondurable and services expenditures. It is constructed by aggregating up the

following expenditure sub-categories: food, tobacco, domestic services, adult and child care, utilities, transportation, pet expenses, apparel, education, work-related and training, health-care, insurance, furniture rental and small textiles, housing related expenditures excluding rent.

Total hours worked. We compute total hours worked for the head of household by multiplying the number of weeks worked full or part time over the last year (*INCWEEK1*) multiplied by the numbers of hours usually worked per week (*INC_HRS1*). We obtain the same indicator for the spouse and add the two.

Labor income. We compute labor income as the sum of total household (CU) income from earnings before taxes (*FSALARYI*), plus the total income received from farm (*FFRMINCI*) and nonfarm business (*FNONFRMI*).

Liquid assets. It includes the total amount the households held in savings accounts in financial institutions (*SAVACCTI*), checking and brokerage accounts (*CKBKACTI*). In the CEI, these amounts are only reported in the last interview. Thus they represent end of period values for the household. In order to define beginning of period values for these assets, we use the following variables (*COMPSAVI* and *COMPCKGI*), which report the total change in savings and checking accounts over the previous year, respectively. Then beginning of period values are defined as end of period values minus the change in value.

Illiquid Assets. It includes the value of owned automobiles (*NETPURI*), residential housing (*PROPVALI*), U.S. savings bonds (*USBNDI*), the value of all securities directly held by the household (include stocks, mutual funds and non U.S. savings bonds) (*SECESTI*), and money owned to the household by individuals outside of the household (*MONYOWDI*). The value of U.S. savings bonds and total securities are only reported in household interview. In order to define beginning of period values for these assets, we use the following variables (*COMPBNDI* and *COMPSECI*), which report the total change in U.S. savings bonds and all securities over the previous year, respectively. Then beginning of period values are defined as end of period values minus the change in value.

Total assets. It is the value of liquid assets plus illiquid assets each household owns.

Liabilities. It is the current value of the household's home mortgage (*QBLNCM3I*) plus the outstanding principal balance on auto debt. (*QBALNM3I*).

Net worth. Net Worth is total assets minus liabilities.

Taking 2006 as the year of reference, we have 5180 households that report full consumption information in all four interviews. We next keep households whose head is in the age bracket 22-64, leaving us with 3890 households that reported income and consumption in 2006. Within this group, we keep households that are considered “full income responders” (3163), and drop any household that observed a change in family size between the first and the last interview (2736). We then drop observations on consumption, labor income, total hours, wage per hour, disposable income, liquid assets and net worth that fall below the 1st percentile or above the 99th percentile of the distribution of these variables in each year, leaving us with 2508 households for 2006. Finally, we are not able to run our Mincer regressions for households that do not report information on their education, sex, marital status, race or state of residence. This leaves us with 2328 households in 2006.

B.2 Summary statistics

Table A-1 reports selected households’ characteristics for 2006. In the CEI, the average age for the head of household was 44 years, and roughly 34% of the households’ head held a college degree. The average size of the household was 2.7. On average, households spent roughly 10000 dollars per person in non-durables and services, and the average income per person was 26000 dollars. Households worked 1300 hours per year per person on average, earning an average wage of 19.80 dollars per hour. The mean net worth for the household was 142000 dollars, with 14000 dollars in liquid assets. As a comparison with previous papers, the average characteristics of the household in our sample are very close to those reported in [Heathcote and Perri \(2018\)](#), see Table 1 in their paper.

B.3 Aggregation

In this section we examine whether the dynamics of aggregate consumption, income, and total hours per capita in our cross sectional data capture the broad contours of national income and product accounts (NIPA) aggregates. The results are shown in figure A-1. Each graph is normalize to 1 in 2004.³⁰

The top left panel of figure A-1 shows the dynamics of average per capita expenditures in the CEI and the equivalent measure in the NIPA. The top left panel shows average per capita disposable income in the CEI and NIPA. The bottom panel shows average total hours worker per capita in the CEI as well as its aggregate counterpart obtained from the BLS. While the fit is not perfect, it is clear that both datasets capture the broad contour of each

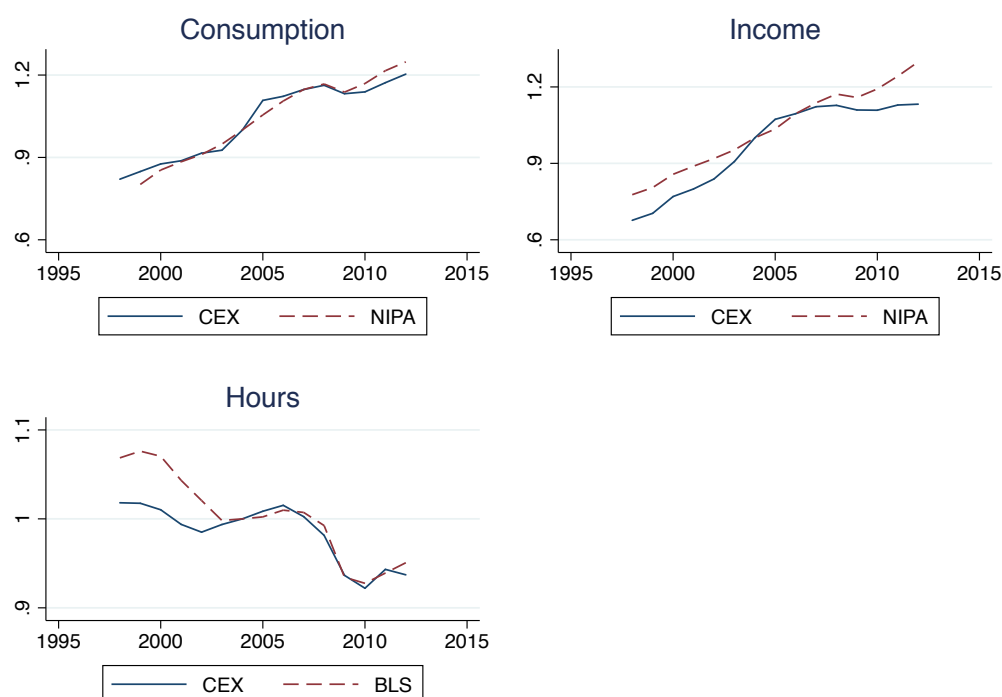
³⁰These figures are constructed before any sample selection.

Table A-1: Average characteristics of households in 2006

	CEI
Age of head	44.10
Household size	2.71
Head with college (%)	34.25
Consumption expenditures per person	10330.98
Labor income per person	26456.95
Disposable income per person	26492.00
Hours worked per person	1301.17
Wage per hour	21.69
Household's net worth	142174.40
Liquid assets	14296.21

Notes: The sample size is 2328 households. All statistics are computed using sample weights. All monetary variables are expressed in 2000 U.S. dollars.

Figure A-1: Comparing aggregates across micro data sets

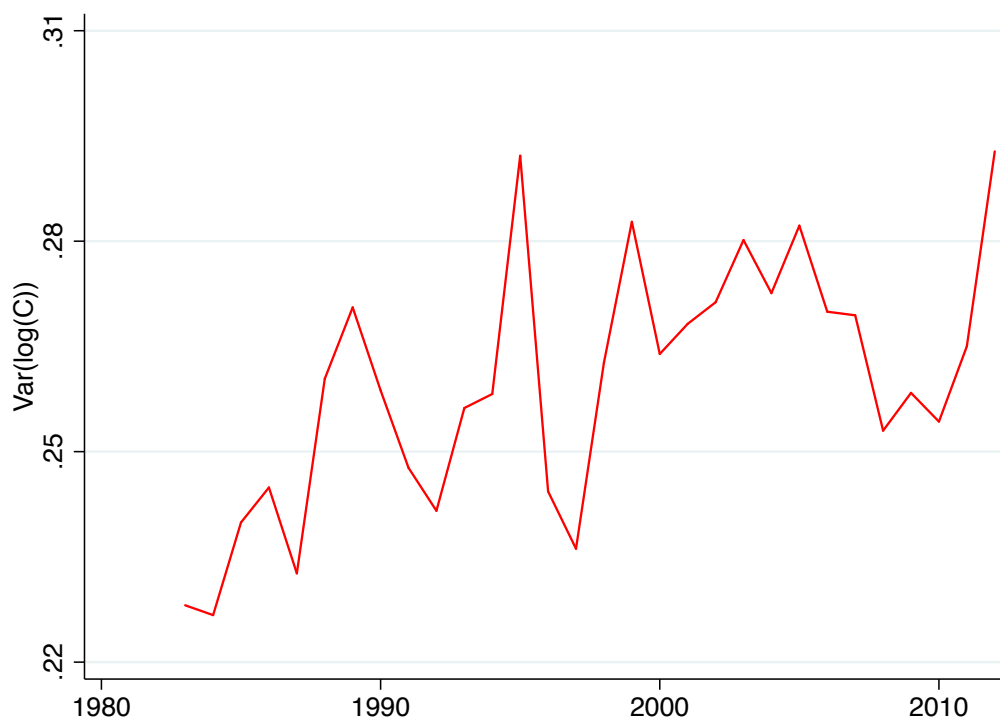


Top panel compares the behavior of nominal consumption and disposable income per capita in the CEI and NIPA. The bottom panel shows the behavior of total hours worker per capita in the two data sets (CEI and BLS). Each graph is normalized to 1 in 2004 because the levels vary somewhat across data sets.

aggregate series during the Great Recession.³¹

³¹Our figure A-1 is very similar to the relevant panels in figure 13 of [Heathcote and Perri \(2018\)](#) giving us further confidence.

Figure A-2: Consumption Inequality in the CEI



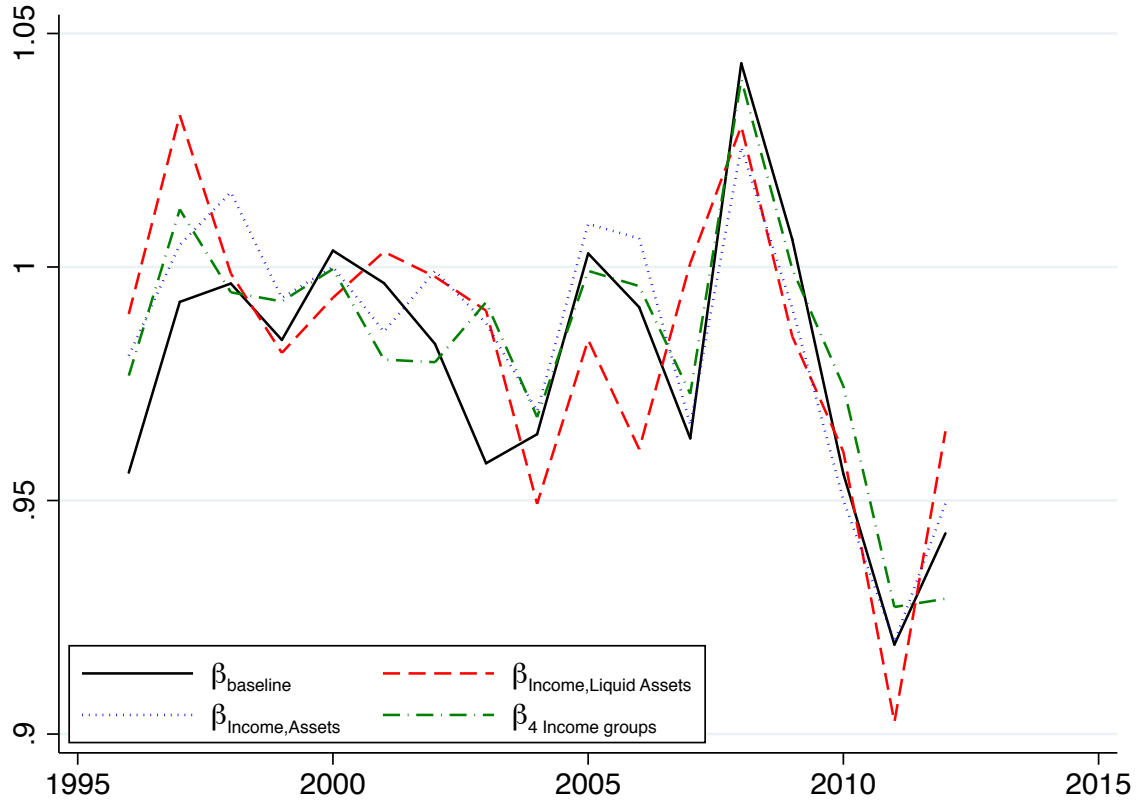
B.4 Trends in inequality

A large literature has documented that consumption inequality has increased in the U.S. since 1980 ([Blundell, Pistaferri, and Preston \(2008\)](#); [Krueger and Perri \(2006\)](#); [Aguiar and Bils \(2015\)](#); [Attanasio and Pistaferri \(2016\)](#)). Consistent with this literature, we find that the variance on log consumption has increased significantly in the CEI. These results are displayed in figure A-2. There is clear visual evidence that consumption volatility has increased. Moreover, the levels of consumption inequality that we find are very similar to previous work in the literature. In particular, we find that the variance of log consumption has increased from 0.23 to 0.28 in the CEI over the period 1985 to 2005, which is almost the exact same increase in both levels and changes that [Heathcote, Perri, and Violante \(2010\)](#) find over the same time period (see figure 1 in the recent survey by [Attanasio and Pistaferri \(2016\)](#) for more details). Overall, this suggests that our sample selection procedure is reasonable.

B.5 β measurement robustness

Figure 1 shows the results of our baseline measurement of the (max) β_{it} where we grouped households according to whether their income at date $t - 1$ is above or below median income and, within each of these two groups, whether the level of their net worth is above or below

Figure A-3: β measurement robustness

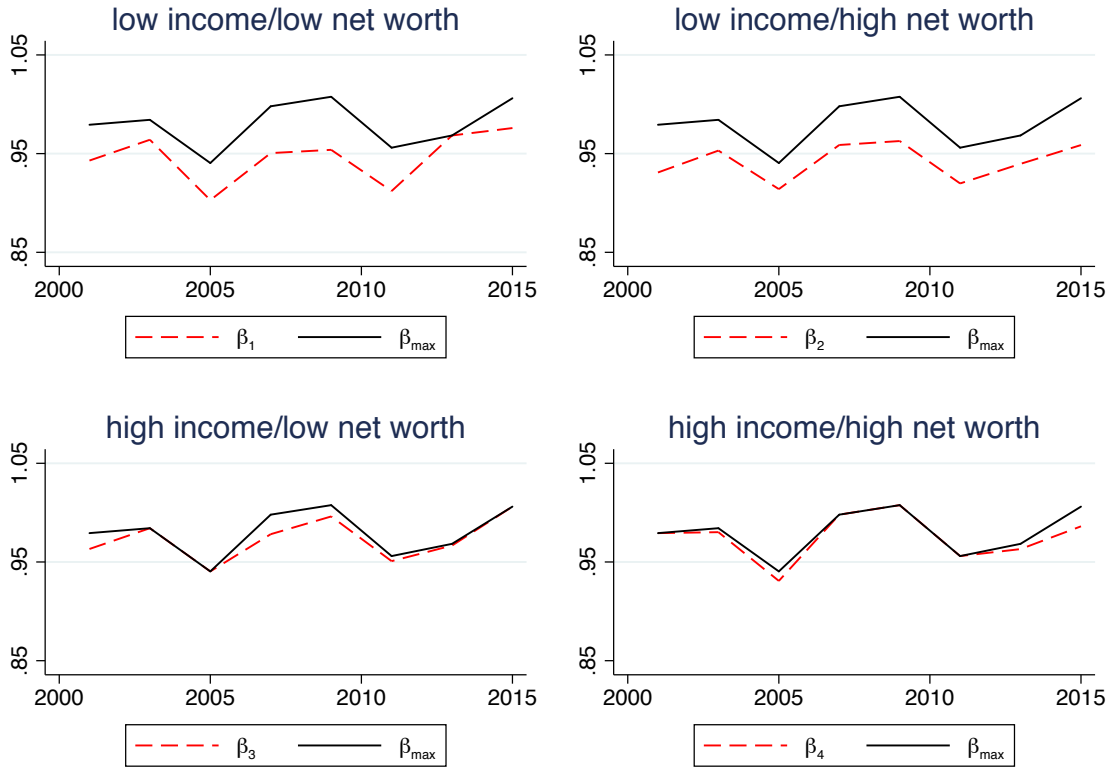


Notes: This figure shows an estimate of the $\max \beta_{it}$ for four different partitions of the data: our baseline with two income and two net worth groups (solid black line), two income and two liquid asset groups (red dashed line), two income and two total asset groups (blue dotted line), and four income groups (green dash-dotted line), all over the period 1996-2012.

the group median. Figure A-3 reports results when we use partition households using the following different $t - 1$ state variables: two income and two liquid asset groups (red dashed line), two income and two total asset groups (blue dotted line), and four income groups (green dash-dotted line). Our baseline group is shown in the black dotted line. The main takeaway from this exercise is that all four time-series look very similar with each showing a similarly large increase during the Great Recession. This suggests that our results are robustness to using other natural partitions of the CEI data.

Figure A-4 shows a different robustness check on our measurement of β_{it} . Here we report results for a baseline partition where we group households according to whether their income and net worth at date $t - 1$ in the bi-annual Panel Survey of Income Dynamics (PSID) over the period 1999-2015. Concretely, at each $t - 1$, we end up with four different groups of households of approximately equal size: *low income/low net worth*, *low income/high net worth*, *high income/low net worth* and *high income/high net worth*. For each group i , we use equation (27) to construct β_{it} in the PSID. Comparing Figure A-4 with Figure 1, we see

Figure A-4: Measured β in the PSID



Notes: Each panel shows an estimate of β_{it} in the PSID for four groups of roughly equal size: low income/low net worth, low income/high net worth, high income/low net worth and high income/high net worth (red dashed line) along with the year-by-year value of the maximum across the four groups (solid black line) over the periods 1999-2015. Over this period the PSID is bi-annual.

there are many broad similarities. In particular, the high income groups have higher implicit discount factors relative to low income households in most years (particularly so for the high income/ high net worth group). That is, we find similar relative rankings of groups in both the CEI and the PSID. This suggests that our measurement approach is capturing relevant variation in the data and not just measurement error.

C Numerical solution

Let the state vector be $\mathbf{S}_t = [i_{t-1}, \hat{\theta}_t, \hat{A}_t, \varepsilon_{m,t}, \hat{\beta}_t, \hat{\omega}_t]$. The equilibrium conditions of the model can be summarized by the following equations

$$Y(\mathbf{S}_t) = C(\mathbf{S}_t) + \frac{\kappa}{2} \left(\frac{\pi(\mathbf{S}_t) - \pi^*}{1 + \pi^*} \right)^2 \quad (\text{A.3})$$

$$Y^{\text{pot}}(\mathbf{S}_t) = \left[\frac{\exp\{\hat{A}_t\}^{1+\psi}}{\exp\{\hat{\omega}_t\} \mu \chi} \right]^{\frac{\psi}{1+\psi\sigma}} \quad (\text{A.4})$$

$$1 + i(\mathbf{S}_t) = \max \left\{ (1 + i_{t-1})^{\rho_i} \left[(1 + \tilde{i}) \left(\frac{1 + \pi(\mathbf{S}_t)}{1 + \pi^*} \right)^{\psi_\pi} \left(\frac{Y(\mathbf{S}_t)}{Y^{\text{pot}}(\mathbf{S}_t)} \right)^{\phi_y} \right]^{(1-\rho_i)}, 1 \right\} \quad (\text{A.5})$$

$$1 = [1 + i(\mathbf{S}_t)] \beta \mathbb{E}_t \left[\exp\{\hat{\theta}_{t+1} + \hat{\beta}_{t+1}\} \left(\frac{C(\mathbf{S}_{t+1})}{C(\mathbf{S}_t)} \right)^{-\sigma} \frac{1}{1 + \pi(\mathbf{S}_{t+1})} \right] \quad (\text{A.6})$$

$$\begin{aligned} \frac{\pi(\mathbf{S}_t) - \pi^*}{1 + \pi^*} \frac{1 + \pi(\mathbf{S}_t)}{1 + \pi^*} &= \frac{1}{\kappa(\mu - 1)} Y(\mathbf{S}_t) \left(\mu \chi \frac{Y(\mathbf{S}_t)^\psi C(\mathbf{S}_t)^\sigma \exp\{\hat{\omega}_t\}}{\exp\{\hat{A}_t\}^{1+\psi}} - 1 \right) \\ &+ \beta \mathbb{E}_t \left[\exp\{\hat{\theta}_{t+1} + \hat{\beta}_{t+1}\} \left(\frac{C(\mathbf{S}_{t+1})}{C(\mathbf{S}_t)} \right)^{-\sigma} \frac{\pi(\mathbf{S}_{t+1}) - \pi^*}{1 + \pi^*} \frac{1 + \pi(\mathbf{S}_{t+1})}{1 + \pi^*} \right]. \end{aligned} \quad (\text{A.7})$$

Given policy functions for $C(\mathbf{S}_t)$ and $\pi(\mathbf{S}_t)$, we can use equations (A.3)-(A.5) to solve for $Y(\mathbf{S}_t)$ and $i(\mathbf{S}_t)$. Thus, the numerical solution of the model can be equivalently expressed as approximating $C(\mathbf{S}_t)$ and $\pi(\mathbf{S}_t)$.

Due to the max operator in equation (A.5), $C(\mathbf{S}_t)$ and $\pi(\mathbf{S}_t)$ may have kinks in a region of \mathbf{S}_t where the zero lower bound constraint starts binding, a feature that makes it challenging to approximate these functions with smooth polynomials. We approach this feature following [Gust et al. \(2017\)](#). Specifically, we approximate these variables using a piece-wise smooth function,

$$x(\mathbf{S}_t) = \mathbb{1}(1 + \tilde{i}(\mathbf{S}_t) > 1) \gamma_x^{\text{no zlb}} \mathbf{T}(\mathbf{S}_t) + \mathbb{1}(1 + \tilde{i}(\mathbf{S}_t) \leq 1) \gamma_x^{\text{zlb}} \mathbf{T}(\mathbf{S}_t), \quad (\text{A.8})$$

where $x = \{C, \pi\}$, $1 + \tilde{i}(\mathbf{S}_t)$ is the “notional” interest rate at \mathbf{S}_t (the first term inside the max operator of equation (A.5)), $\mathbf{T}(\mathbf{S}_t)$ is a vector collecting Chebyshev’s polynomials evaluated at \mathbf{S}_t and $\{\gamma_x^{\text{no zlb}}, \gamma_x^{\text{zlb}}\}$ a set of coefficients.

The numerical solution of the model consists in choosing $\Gamma = \{\gamma_x^{\text{no zlb}}, \gamma_x^{\text{zlb}}\}_{x=C, \pi}$ so that equations (A.6) and (A.7) are satisfied for a set of collocation points $\tilde{\mathbf{S}}^i \in \mathcal{S}$. The choice of collocation points and the associated Chebyshev’s polynomials follows the method of Smolyak. Conditional expectations in equations (A.6) and (A.7) are evaluated using Gauss-Hermite quadrature.

The algorithm for the numerical solution of the model is as follows:

Step 0.A: Defining the grid and the polynomials. Set upper and lower bounds on the state variables $\tilde{\mathbf{S}} = [i, \hat{\theta}, \hat{A}, \varepsilon_m, \hat{\beta}, \hat{\omega}]$. Given these bounds, construct a Smolyak grid and

the associated Chebyshev's polynomials.

Step 1: Equilibrium conditions at the candidate solution. Start with a guess for the model's policy functions Γ^c . For each $\tilde{\mathbf{S}}^i$, compute $C^{\text{no zlb}}(\tilde{\mathbf{S}}^i)$, $C^{\text{zlb}}(\tilde{\mathbf{S}}^i)$, $\pi^{\text{no zlb}}(\tilde{\mathbf{S}}^i)$ and $\pi^{\text{zlb}}(\tilde{\mathbf{S}}^i)$ using the coefficients in Γ^c . Evaluate equation (A.3) using $C^{\text{no zlb}}(\tilde{\mathbf{S}}^i)$ and $\pi^{\text{no zlb}}(\tilde{\mathbf{S}}^i)$ to obtain $Y^{\text{no zlb}}(\tilde{\mathbf{S}}^i)$, and similarly obtain a value for $Y^{\text{zlb}}(\tilde{\mathbf{S}}^i)$. Use equation (A.4) and (A.5) along with $Y^{\text{no zlb}}(\tilde{\mathbf{S}}^i)$ and $\pi^{\text{no zlb}}(\tilde{\mathbf{S}}^i)$ to obtain the notional interest rate $1 + \tilde{i}(\tilde{\mathbf{S}}^i)$. Compute the actual interest rate $1 + i(\tilde{\mathbf{S}}^i) = \max\{1 + \tilde{i}(\tilde{\mathbf{S}}^i), 1\}$.

Step 3: Evaluate residual equations. For each $\tilde{\mathbf{S}}^i$, compute the residual equations

$$\begin{aligned}\mathcal{R}^1(\tilde{\mathbf{S}}^i) &\equiv \left[\frac{1}{1 + \tilde{i}(\tilde{\mathbf{S}}^i)} \right] - \beta \mathbb{E} \left[\exp\{\theta' + \beta'\} \left(\frac{C(\mathbf{S}')}{C^{\text{no zlb}}(\tilde{\mathbf{S}}^i)} \right)^{-\sigma} \frac{1}{1 + \pi(\mathbf{S}')} \right] \\ \mathcal{R}^2(\tilde{\mathbf{S}}^i) &\equiv 1 - \beta \mathbb{E} \left[\exp\{\theta' + \beta'\} \left(\frac{C(\mathbf{S}')}{C^{\text{zlb}}(\tilde{\mathbf{S}}^i)} \right)^{-\sigma} \frac{1}{1 + \pi(\mathbf{S}')} \right].\end{aligned}$$

Similarly, compute $\mathcal{R}^3(\tilde{\mathbf{S}}^i)$ and $\mathcal{R}^4(\tilde{\mathbf{S}}^i)$ using equation (A.7).

Step 4: Iteration. Let $\mathcal{R}(\Gamma^c)$ the vector collecting all the computed residuals at the collocation point, and let r be its Euclidean norm. If $r \leq 10^{-10}$, stop the algorithm. If not, update the guess and repeat Step 1-4. \square

The specifics for the algorithm are as follows. The bounds on $[\hat{\theta}, \hat{A}, \varepsilon_m, \hat{\beta}, \hat{\omega}]$ are +/- 3 standard deviations from their mean. The bounds on i is set to $[0, 0.20]$, wide enough to span the ergodic distribution of nominal interest rates. We consider a second-order Smolyak grid, and use 243 points for Gauss-Hermite quadrature (three points for each shock and tensor multiplications). Finally, we use a Newton algorithm to find the zeros of $\mathcal{R}(\Gamma^c)$ at the collocation points.

D Quantitative analysis

In this Appendix we present additional details regarding the quantitative experiments of Section 4. We start with the estimation of the model and a discussion of model fit. We then present some details of the counterfactual of Section 4.4.

D.1 Model estimation

We estimate a first-order approximation of the New Keynesian model, ignoring the presence of the zero lower bound constraint on nominal interest rates. This has two advantages.

Table A-2: Sample statistics: model vs. data

	Data	Model (linear)	Model (non-linear)
Mean(π_t)	2.69	2.00	1.87
Mean(i_t)	3.87	3.00	3.57
Stdev(Y_t)	4.15	3.42	5.85
Stdev(π_t)	1.23	1.59	1.50
Stdev(i_t)	3.02	2.68	3.16
Corr(Y_t, Y_{t-1})	0.93	0.85	0.81
Corr(i_t, i_{t-1})	0.90	0.72	0.42
Corr(π_t, π_{t-1})	0.51	0.14	0.17
Corr(Y_t, i_t)	0.11	-0.08	0.04
Corr(Y_t, π_t)	0.14	0.12	0.14
Corr(i_t, π_t)	0.71	0.52	0.68

Notes: Model statistics are computed on a long simulation ($T = 100,000$) from the model solved using a first-order approximation ("Model (linear)") and global methods ("Model (non-linear)"). Data statistics are computed over the 1984-2017 period.

First, the solution of the model is fast and numerically more stable than the global solution described in Appendix C. Second, the likelihood function can be evaluated with the Kalman filter. These two features make the estimation sensibly more tractable than the alternative of estimating the model with non-linear methods.

Draws from the posterior distribution of the model parameters are generated using the random walk Metropolis Hastings described in An and Schorfheide (2007). The proposal distribution is a multivariate normal, with variance-covariance matrix given by $c\Sigma$, where Σ is the negative of the inverse hessian of the log-posterior evaluated at the posterior mode and c is a constant that we set to obtain roughly a 30% acceptance rate in Markov chain. We generate 2 Markov chains of 200,000 each, and discard the first 100,000 draws in each chain. The statistics of the posterior distribution of model parameters reported in Table 1 are computed by combining the last 100,000 draws for each chain.

The purpose of the estimation step is to have a parametrized model that fits US data reasonably well in the sample, an important pre-condition for carrying out the proposed counterfactuals. Table A-2 reports a set of summary statistics for output, inflation and nominal interest rates in the data, in the first-order approximation of the model ("Model (linear)") and in the model solved using the global method described in C.

We can verify that the estimated model replicates key features of US data. We can also see that the model solved with non-linear methods implies statistics that are close to both US data and those derived in the first-order approximation of the model.

D.2 Counterfactuals

We now detail the counterfactual experiment of Section 4. We first explain how we use the particle filter to obtain an estimate of the structural shocks. Next, we discuss how we generate the decomposition of Figure 7.

The equivalent representative-agent economy has the state space representation

$$\begin{aligned}\mathbf{Y}_t &= \mathbf{g}(\mathbf{S}_t) + \eta_t \\ \mathbf{S}_t &= \mathbf{f}(\mathbf{S}_{t-1}, \varepsilon_t).\end{aligned}$$

The first set of equations describe the evolution of the observables \mathbf{Y}_t , with η_t being a vector of iid Gaussian errors with a diagonal variance-covariance matrix equal to \mathbf{H} . The second equation describes the evolution of the state variables \mathbf{S}_t . The vector ε_t collects the innovations to the structural shocks $\hat{\theta}_t$, \hat{A}_t , and $\varepsilon_{m,t}$ and the preference “shocks” $\hat{\beta}_t$ and $\hat{\omega}_t$. The functions $g(\cdot)$ and $f(\cdot)$ are generated using the numerical algorithm described previously and they depend implicitly on the structural parameters of the model. For these experiments, we set the estimated parameters at their posterior mean reported in Table 1

Let $\mathbf{Y}^t = [\mathbf{Y}_1, \dots, \mathbf{Y}_t]$, and denote by $p(\mathbf{S}_t | \mathbf{Y}^t)$ the conditional distribution of the state vector given observations up to period t . Although the conditional density of \mathbf{Y}_t given \mathbf{S}_t is known and Gaussian, there is no analytical expression for the density $p(\mathbf{S}_t | \mathbf{Y}^t)$. We use the particle filter to approximate this density for each t . The approximation is done via a set of pairs $\{\mathbf{S}_t^i, \tilde{w}_t^i\}_{i=1}^N$, in the sense that

$$\frac{1}{N} \sum_{i=1}^N f(\mathbf{S}_t^i) \tilde{w}_t^i \xrightarrow{\text{a.s.}} \mathbb{E}[f(\mathbf{S}_t) | \mathbf{Y}^t].$$

We refer to \mathbf{S}_t^i as a particle and to \tilde{w}_t^i as its weight. The algorithm used to approximate $\{p(\mathbf{S}_t | \mathbf{Y}^t)\}_t$ builds on Kitagawa (1996), and it goes as follows:

Step 0: Initialization. Set $t = 1$. Initialize $\{\mathbf{S}_0^i, \tilde{w}_0^i\}_{i=1}^N$ and set $\tilde{w}_0^i = 1 \forall i$.

Step 1: Prediction. For each $i = 1, \dots, N$, obtain a realization for the state vector $\mathbf{S}_{t|t-1}^i$ given \mathbf{S}_{t-1}^i by simulating the model forward.

Step 2: Filtering. Assign to each particle $\mathbf{S}_{t|t-1}^i$ the weight

$$w_t^i = p(\mathbf{Y}_t | \mathbf{S}_{t|t-1}^i) \tilde{w}_{t-1}^i.$$

Step 3: Resampling. Rescale the weights $\{w_t^i\}$ so that they add up to one, and denote these rescaled values by $\{\tilde{w}_t^i\}$. Sample N values for the state vector with replacement

from $\{\mathbf{S}_{t|t-1}^i, \tilde{w}_t^i\}_{i=1}^N$, and denote these draws by $\{\mathbf{S}_t^i\}_i$. Set $\tilde{w}_t^i = 1 \forall i$. If $t < T$, set $t = t + 1$ and go to Step 1. If not, stop. \square

In our exercise, the measurement equation includes nominal interest rates, linearly detrended real GDP, inflation, $\hat{\beta}_t$ and $\hat{\omega}_t$. The variance on the measurement errors on the first three variables is set to 1% of their unconditional variance, while we set the variance of the measurement errors on $\hat{\beta}_t$ and $\hat{\omega}_t$ to 10% of the unconditional variance of these series. We set N to 5,000,000. Figure 6 reports the (mean) \mathbf{Y}_t and \mathbf{S}_t obtained when applying the particle filter to the model over our sample. That is, for each variable $y_t = g_y(\mathbf{S}_t)$ in the observation equations, we compute

$$y_t^{\text{model}} = \sum_{i=1}^N g_y(\mathbf{S}_t^i) \tilde{w}_t^i.$$

We repeat this for every variable in the state equations, using the policy functions $\mathbf{f}(\cdot)$.

In order to generate the counterfactual of Figure 7, we first solve the model without the $\hat{\beta}_t$ shock, obtaining the policy function $\mathbf{g}^{\text{no } \beta}(\cdot)$ and $\mathbf{f}^{\text{no } \beta}(\cdot)$. We then compute the counterfactual value of a variable y_t as

$$y_t^{\text{counterfactual}} = \sum_{i=1}^N g_y^{\text{no } \beta}(\mathbf{S}_t^i) \tilde{w}_t^i,$$

where $\mathbf{S}_t^i = [\hat{\theta}_t^i, \hat{A}_t^i, \varepsilon_{m,t}^i, \hat{\omega}_t^i \times (\hat{\omega}_t^{\text{cm}} / \hat{\omega}_t)]$.

D.3 Sensitivity to the measurement errors

In this section we study the sensitivity of our results to the measurement errors we added on \mathbf{T}_t . In our benchmark, we set the variance of these measurement errors to be equal to 10% of the unconditional variance on $\hat{\beta}_t$ and $\hat{\omega}_t$. We now re-estimate the model and perform the counterfactual by doubling the variance on these measurement errors.

Table A-3 reports prior and posterior distribution of the model parameters in this scenario. By and large, the posterior distribution is very similar to that reported in our benchmark, with the exception that the credible sets on the parameters of the preference shocks are now larger. In Figure A-5 we report the main counterfactual of the paper in this specification. We can again see that the results are fairly similar to those reported in the main text.

E The model with capital

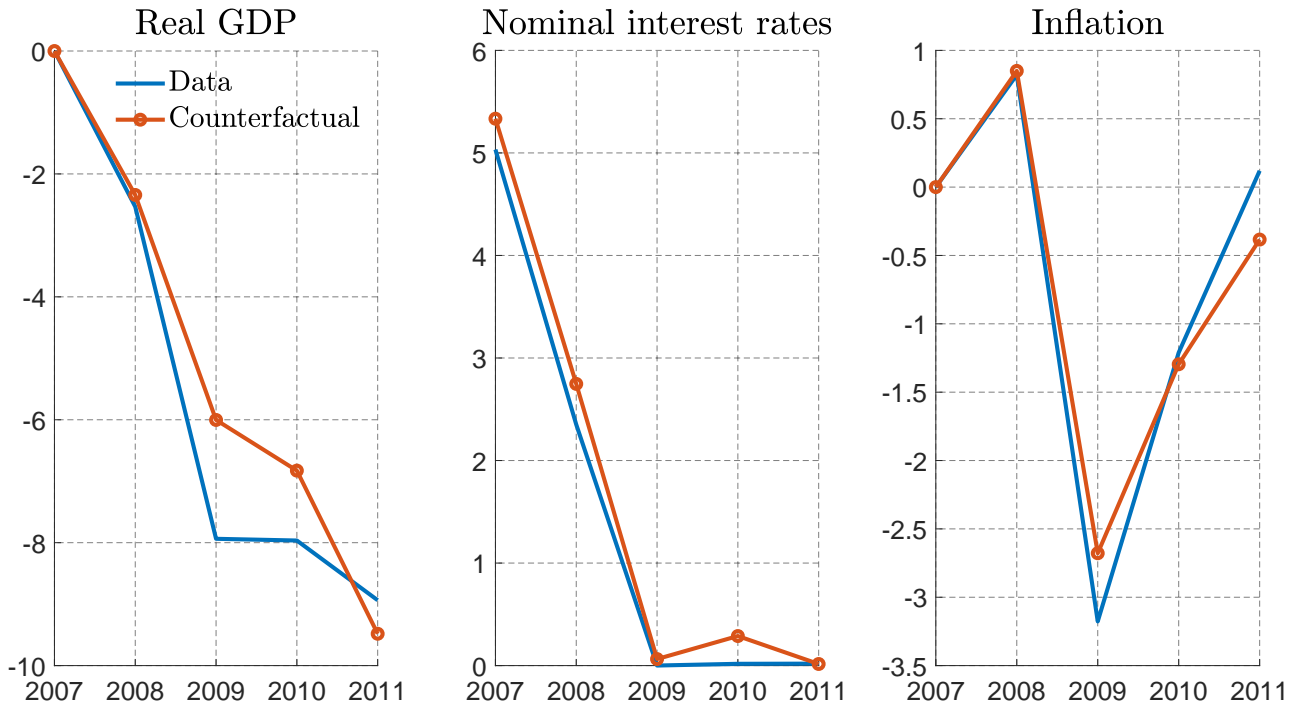
In this section, we describe in more detail the model with capital in Section 4.5.

Table A-3: Prior and posterior distribution of model parameters: larger measurement errors

Parameter	Distribution	Prior		Posterior	
		Mean	St. dev.	Mean	90% Interval
$4 \times \kappa$	Gamma	85.00	15.00	73.53	[53.10, 93.57]
ρ_i	Beta	0.50	0.25	0.57	[0.34, 0.81]
γ_π	Normal	1.50	2.00	3.67	[1.90, 5.38]
γ_y	Normal	1.00	2.00	0.23	[0.00, 0.53]
ρ_θ	Beta	0.50	0.28	0.69	[0.49, 0.90]
ρ_A	Beta	0.50	0.28	0.92	[0.84, 0.99]
$\Phi_{\beta,\beta}$	Beta	0.50	0.25	0.39	[0.09, 0.69]
$\Phi_{\omega,\omega}$	Beta	0.50	0.25	0.85	[0.72, 0.99]
$100 \times \sigma_\theta$	InvGamma	1.00	5.00	2.46	[0.94, 3.99]
$100 \times \sigma_A$	InvGamma	1.00	5.00	2.01	[1.30, 2.68]
$100 \times \sigma_m$	InvGamma	1.00	5.00	1.95	[1.15, 2.72]
$100 \times \sigma_\beta$	InvGamma	1.00	5.00	1.85	[0.97, 2.69]
$100 \times \sigma_\omega$	InvGamma	1.00	5.00	2.09	[1.16, 3.03]

Notes: See the notes to Table 1.

Figure A-5: Counterfactual with larger measurement errors



Notes: See the notes to Figure 7.

Capital is accumulated by a financial intermediary that rents it out to monopolistic com-

petitive firms. We assume that the financial intermediary discounts the future dividends using the real state price $Q(z^t)$. The problem for the representative intermediary is

$$\Omega(K(z^{t-1}), z^t) = \max_{K(z^t), I(z^t), d(z^t)} \sum_t \sum_{z^t} \prod_{z^j \preceq z^t} Q(z^j) d(z^t) \quad (\text{A.9})$$

subject to its budget constraint,

$$d(z^t) + I(z^t) + \frac{\xi}{2} \left(\frac{I(z^t)}{K(z^{t-1})} - \delta \right)^2 K(z^{t-1}) \leq r(z^t) K(z^{t-1}),$$

and the law of motion of capital,

$$K(z^t) \leq (1 - \delta) K(z^{t-1}) + I(z^t), \quad (\text{A.10})$$

and $r(z^t) = R(z^t) / P(z^t)$ is the real rental rate of capital.

The optimality conditions are

$$q(z^t) = \sum_{z^{t+1}} Q(z^{t+1}) \left[r(z^{t+1}) + q(z^{t+1}) (1 - \delta) - \frac{\xi}{2} \left(\frac{I(z^{t+1})}{K(z^t)} - \delta \right)^2 + \xi \left(\frac{I(z^{t+1})}{K(z^t)} - \delta \right) \frac{I(z^{t+1})}{K(z^t)} \right], \quad (\text{A.11})$$

where the price of capital $q(z^t)$ – the normalized multiplier on (A.10) – is

$$q(z^t) = 1 + \xi \left(\frac{I(z^t)}{K(z^{t-1})} - \delta \right) \quad (\text{A.12})$$

A monopolistic competitive firm produces the intermediate input using the technology $y_i(z^t) = A(z_t) k_i(z^{t-1})^\alpha n_i(z^t)^{1-\alpha}$. The problem for a monopolistic competitive firm can be split in two subproblems. First, the firm chooses the optimal input mix to minimize its marginal cost:

$$mc(z^t) = \min_{k,n} \frac{W(z^t)}{P(z^t)} n + \frac{R(z^t)}{P(z^t)} k$$

subject to

$$A(z_t) k^\alpha n^{1-\alpha} \geq 1$$

Second, given the optimal factor allocation, the firm chooses its price to solve:

$$V(p_j, z^t) = \max_{p_j, y_j} \frac{p_j y_j}{P(z^t)} - mc(z^t) y_j - \frac{\kappa}{2} \left[\frac{p_j}{P_j(1 + \bar{\pi})} - 1 \right]^2 + \sum_{z^{t+1}} Q(z^{t+1} | z^t) V(p_j, z^{t+1}) \quad (\text{A.13})$$

subject to the demand function (9).

The solution to the firm's problem together with symmetry across firms requires that the following version of the New Keynesian Phillips curve holds in equilibrium

$$\tilde{\pi}(z^t) = \frac{1}{\kappa(\mu - 1)} Y(z^t) [\mu mc(z^t) - 1] + \sum_{z^{t+1}} Q(z^{t+1}|z^t) \tilde{\pi}(z^{t+1}) \quad (\text{A.14})$$

and the marginal cost is given by

$$mc(z^t) = \frac{r(z^t)^\alpha w(z^t)^{1-\alpha}}{(1-\alpha)^{1-\alpha} \alpha^\alpha} \frac{1}{A(z^t)} \quad (\text{A.15})$$

The problem for the household is unchanged and so is the monetary policy rule. Market clearing in assets markets now must account for the stock of capital. Specifically, equilibrium in financial markets requires that the value of inherited assets must equal the nominal value of the monopolistic competitive and capital firms cum-dividend and ,

$$\sum_i \lambda_i \sum_{v^t} \Pr_i(v^t|z^t) \sum_{k \in \mathcal{K}} R_k(s^{t-1}, s_t) a_{k,i}(s^{t-1}) = P(z^t) [V(P(z^{t-1}), z^t) + \Omega(K(z^{t-1}), z^t)], \quad (\text{A.16})$$

and the total value of new asset positions must equal to the nominal value of the two firms ex-dividend,

$$\sum_i \lambda_i \sum_{v^t} \Pr_i(v^t|z^t) \sum_{k \in \mathcal{K}} q_k(s^t) a_{k,i}(s^t) = P(z^t) \sum_{z^{t+1}} Q(z^{t+1}|z^t) [V(P(z^t), z^{t+1}) + \Omega(K(z^t), z^{t+1})]. \quad (\text{A.17})$$

We then have the analog of Proposition 1:

Proposition 2. *Given $\{\beta_i(v^t, z^{t+1}), \omega(z^t)\}$ defined in (19) and (20) and the initial capital stock, the equilibrium aggregate consumption, investment, capital, gross output, inflation, price of capital, and nominal interest rate, $\{C(z^t), I(z^t), K(z^t), Y(z^t), \pi(z^t), q(z^t), i(s^t)\}$ must satisfy the aggregate Euler equation (21), the Euler equation for capital*

$$q(z^t) = \sum_{z^{t+1}} Q(z^{t+1}) \left[\alpha \frac{Y(z^t)}{K(z^{t-1})} + q(z^{t+1}) (1 - \delta) - \frac{\xi}{2} \left(\frac{I(z^{t+1})}{K(z^t)} - \delta \right)^2 + \xi \left(\frac{I(z^{t+1})}{K(z^t)} - \delta \right) \frac{I(z^{t+1})}{K(z^t)} \right] \quad (\text{A.18})$$

and (A.12), the Phillips curve (A.14) the Taylor rule (14), the resource constraint

$$Y(z^t) = C(z^t) + I(z^t) + \frac{\kappa}{2} \left[\frac{\pi(z^t) - \bar{\pi}}{1 + \bar{\pi}} \right]^2 + \frac{\xi}{2} \left[\frac{I(z^t)}{K(z^{t-1})} - \delta \right]^2 K(z^{t-1}), \quad (\text{A.19})$$

and the law of motion for the capital stock (A.10), where the real state prices is given by (24), and the marginal cost is

$$mc(z^t) = \left(\frac{Y(z^t)}{K(z^{t-1})} \right)^\alpha \frac{\left[\omega(z^t)^{-\psi} \chi \left(\frac{Y(z^t)}{A(z_t)K(z^{t-1})^\alpha} \right)^{\psi/(1-\alpha)} C(z^t)^\sigma \right]^{1-\alpha}}{(1-\alpha)^{1-\alpha}} \frac{1}{A(z_t)} \quad (\text{A.20})$$

The critical assumption for the equivalent representation for the model with capital is that the firm that does the capital accumulation process uses the aggregate state price (24) to discount dividends in (A.9). This assumption mirrors the one for the monopolistic competitive firms in (12).³² Our implicit assumption is that firms value dividends in a given state z^t using the valuation of the agent with the highest valuation. This choice is not innocuous but in any model with incomplete markets there is a degree of freedom in choosing the firms' discount factor. With richer data that would allow us to identify the households that actively trade shares in the firms, we could measure the wedge between the aggregate state price in (24) and the stochastic discount factor of the marginal agent pricing capital.

³²This assumption allows us to consolidate the two problems and consider one firm that produces the intermediate good and makes investment decisions.