Market-wide Events and Time Fixed Effects*

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Abstract

Market-wide events (e.g., financial crises) and regulatory changes empirically have group heterogenous impact on firm outcomes. Inappropriate modelling of the heterogeneity by existing econometric models such as time-fixed effect (assuming a homogenous response to shocks) and industry-year interacted fixed effect (assuming a heterogenous responses to shocks based on industry) is likely to result in biased estimates. This paper investigates the effect of heterogenous responses to common shocks for existing panel studies. We demonstrate theoretically and empirically that ignoring time-varying unobserved heterogeneity that is correlated with regressors in current empirical practices leads to biased estimates and standard errors. To overcome the bias, we propose the use of the "group fixed effect, GFE" class of models, which produce consistent estimates even under the two-way fixed effect and interacted fixed effect data generating processes. We study the finite sample properties of GFE through simulations and demonstrate its economic importance with two empirical applications. We also extend the GFE class of models to accommodate two-stage least squares estimators. Finally, we provide researchers with guidance and user-written functions in statistical packages to overcome the limitations of existing approaches.

Keywords: Time-varying unobserved heterogeneity; clustering; common shocks; group fixed effects; fixed effects; heterogeneity bias.
JEL Classification: G10; G20; G14.

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1 Introduction

Market-wide events like the introduction of financial regulations (e.g. Sarbanes-Oxley Act, SOX) and financial crises affect firms' outcomes and policies. A salient empirical finding of the literature on market-wide events is that these shocks have heterogenous group effects on firm outcomes, i.e. firms in the same group respond similarly to the shocks, while the responses differ across groups.[1] Response heterogeneity across groups may be due to differences in firms' time-varying corporate culture, attitude towards risk, product markets, and business strategies, and may be captured by observable and unobservable managerial and firm attributes (e.g., profitability, industry, credit rating, governance, managerial qualities, etc.). Given the empirical findings, several questions arise: how should one account for heterogenous effects of common shocks in panel regressions? Does ignoring the heterogeneity in econometric specifications affect slope estimates of interest, statistically and economically? What determines the degree of bias? How should one account for the endogenous grouping of firms with heterogenous responses in econometric models? In this paper, we provide answers to these questions through econometric derivations, simulations and empirical examples.

While most finance scholars regularly observe heterogenous responses to common (regulatory) shocks in financial markets, they routinely use time fixed effect models with homogenous impact across firms, λ_t, to control for these shocks specified as:

$$y_{it} = \alpha_i + \lambda_t + X_{it}'\beta + \epsilon_{it} \quad i = 1, \dots, N \quad t = 1, \dots, T,$$

where the individual and time fixed effects α_i and λ_t can be correlated with covariates X_{it}, but not with the idiosyncratic error ϵ_{it}. As an example, we survey recently published empirical papers in the top three finance journals for the use of fixed effects. In 2017 and 2018, there are 389 empirical papers using panel data in the Journal of Finance (76), the Review of Financial Studies (156), and the Journal of Financial Economics (157). There are 383 papers that use panel regressions and 6 that do not. Among those using panel regressions, 359 papers use fixed effect models: 95 papers only use one-way fixed effects and 264 use two-way (individual and time) fixed effect (TFE) models.

Only 81 papers employ an interacted fixed effects (IFE) model, relaxing the homogeneity assumption. Although interacted fixed effects models in finance mostly include time dummies interacted with industry or state dummies, the grouping of firms should depend on the context of the research

[1] For example see Mitton (2002), Joh (2003), Lemmon and Lins (2003), Linck et al. (2009), Duchin et al. (2010a), Duchin et al. (2010b), Banerjee et al. (2015), Thakor (2015), among many others.

question and the economic outcomes of interest. The assumption that heterogenous responses are driven by one covariate, e.g. industry or state, is unlikely to be correct, because: (1) it is inconsistent with the empirical evidence that the group heterogeneity is related to observable and to unobservable corporate strategies, risk attitude, managerial and firm characteristics, and not (at least only) to industry or states;[2] (2) the pattern of group heterogeneity typically depends on the outcome variable of interest, and it is difficult for a researcher to know the exact grouping of the heterogeneity, ex ante;[3] (3) few categorizations of individuals and firms in finance are intrinsically exogenous (gender is one of the few cases). Categorization and grouping of economic agents and firms is probably endogenously determined through both observable and unobservable determinants.

We study the econometric and economic implications of heterogenous effects of market-wide shocks for the current econometric models, practices, and findings in the finance literature. We theoretically derive the bias in the coefficient estimates and standard errors of current practices under the data structure with time-varying group heterogeneity. We call this ***heterogeneity bias*** and find that heterogeneity bias can be economically substantial in the estimates of TFE and IFE models. The degree of bias depends on the correlation between the grouped time fixed effect and the explanatory variables, as well as on the size of the heterogeneity. Particularly, the bias of coefficient estimates is high when heterogenous responses to market-wide shocks are large and highly correlated with the covariates.[4]

To overcome the problem of heterogeneity bias, we introduce a new class of models, "group fixed effect" (GFE) models, which leave the groups unrestricted and identify them *endogenously.* GFE

[2]Two recent examples of the heterogeneous impact of common shocks on firm outcomes are the implementation of the SOX Act and the 2008 financial crisis. The Securities and Exchange Commission SOX Act in 2002 affected a myriad of market participants, who are regularly expected to make firm-related decisions, e.g. firm managers and executive boards, and who affect firms and managers decisions, e.g. auditors and institutional investors. SOX was implemented to strengthen the independence of auditing firms, to improve the quality and transparency of financial statements and corporate disclosure, to enhance corporate governance, to improve the objectivity of research, and to strengthen the enforcement of the federal securities laws. Several recent papers show the heterogenous effect of SOX on corporate governance, due to the heterogeneity in observable and unobservable managers and firm characteristics, see Engel et al. (2007), Chhaochharia and Grinstein (2007), Heron and Lie (2007), Koh et al. (2008), Cohen et al. (2008), Linck et al. (2009), Bargeron et al. (2010), Carter et al. (2009), Duchin et al. (2010a), Duchin et al. (2010b), Brochet (2010), and Banerjee et al. (2015). Work on recent financial crises finds that observable firm characteristics (e.g., industry, profitability, and credit rating), as well as unobservable characteristics (e.g., managerial qualities and characteristics) affect firms' outcome variables in the cross-section (see Mitton, 2002; Joh, 2003; Lemmon and Lins, 2003; Campello et al., 2010, 2011; Thakor, 2015; Ho et al., 2016).

[3]In practice, the group membership that leads to heterogeneity is typically unknown, making such an interacted fixed effect estimator an "infeasible" model or impossible to specify correctly, especially when unobservables are driving the heterogenous responses. When the pre-specified group structure (e.g. industry or state) does not coincide with the underlying heterogeneity pattern, the coefficient estimates produced by IFE are biased, and therefore, IFE does not capture the heterogenous response to shocks.

[4]If the heterogeneous time fixed effect is independent from the covariates, the slope coefficients can still be consistently estimated using standard two-way fixed effect models.

models, first proposed by Bonhomme and Manresa (2015a), are analogous to the two-way fixed effect models but allow for heterogenous responses to market shocks. GFE allows the heterogeneity to be characterized by a latent group structure, which is endogenously recovered from the specified model and the data rather than exogenously imposed by researchers, such as industry grouping. The grouping of firms allows for a simple iterative procedure for the GFE estimation with desirable asymptotic and finite-sample properties (Bonhomme and Manresa, 2015a). We propose a method to evaluate whether GFE is the appropriate model in comparison to TFE and IFE, based on a Hausman-type specification test. In addition, we extend GFE with a new methodology, two-stage least squares "group fixed effect" (TSLS-GFE), to address heterogeneity bias and endogeneity concerns.

Generally, GFE allows for heterogenous responses to market-wide shocks across groups through $\theta_{g_i,t}$:

$$y_{it} = \alpha_i + \theta_{g_i,t} + X_{it}'\beta + \epsilon_{it}, \quad g_i = 1, 2, ..., G,$$

where g_i is the group membership of firm i, G is the number of groups, and the group specific time effects $\theta_{g_i,t}$ can be regarded as group interacted with time dummies. Intuitively, GFE groups firms whose time profiles of outcomes conditional on covariates, i.e. residuals, are most similar. More specifically, it employs the "kmeans" method (Forgy, 1965; Steinley, 2006) that estimates group membership using least squares.

Asymptotically, the estimated group membership of each unit converges to the true population membership, as the time dimension increases. The probability of misclassifying a unit into the wrong group tends to zero at an exponential rate and each unit converges to the true population membership, as the time series increases.[5] We examine the finite sample properties of the group membership estimator via Monte Carlo simulations under different data generating processes (DGPs) and shock characteristics. Empirically, we show the reliability of the group estimator using the setting of natural disasters (see Barrot and Sauvagnat, 2016), where we ex-ante know the observable determinants of group memberships. Specifically, we back out group memberships from a GFE regression of sales growth on several determinants. We find that GFE groups cluster according to the location and magnitude of natural disasters, differently from TFE and IFE estimates.

We conduct horse races via Monte Carlo simulations to evaluate the efficacy of the two-way fixed

[5]The standard convergence rate of least square estimates of β in fixed effects models is $\sqrt{NT}$, which is slower than the exponential convergence rate of group membership estimates. The GFE procedure with TSLS estimation of slope coefficients also leads to accurate estimates of group memberships, in the presence of endogenous variables.

effect and interacted fixed effect models with respect to the GFE model, under different DGPs. The coefficient estimates produced by GFE are consistent, while those produced by TFE and IFE can be severely biased, when time effects are heterogeneous and the heterogeneity pattern does not coincide with the specified grouping of IFE. These results show that GFE can capture heterogenous responses and overcome the risk of misspecifying the group pattern in IFE models. We also show that GFE provides consistent coefficient estimates not only under the assumption of time-varying unobserved group heterogeneity, but also when units respond homogeneously (or heterogeneously based on industry) to market-wide shocks under the DGP of TFE and IFE. In other words, GFE is a ***robust method*** to incorporate time effects, providing consistent estimates regardless of whether the time effect is heterogeneous or homogeneous.

Finally, we highlight the economic importance of GFE empirically by revisting the evidence in Sunder et al. (2017). We examine the relation between CEO attributes and corporate innovation using different estimators and find that the difference between GFE and competing models is economically substantial. The results show that TFE and IFE can result in large biases in slope estimates by ignoring unobserved time-varying variables like corporate strategies and culture.

Overall, our work is closely related to Gormley and Matsa (2014), which provides practical guidance on how to handle unobserved (group) heterogeneity in empirical research. Complementing their work, our results challenge the standard assumptions of empirical estimations in the presence of unobserved group heterogeneity in empirical finance research, by showing the impact of heterogenous responses to market-wide events on empirical studies. We propose a new robust method to overcome the heterogeneity bias.

We contribute to the econometric literature of panel group structure models by complementing the work of Bonhomme and Manresa (2015a), first by proposing a method to jointly estimate coefficients and group membership using two-stage least squares (TSLS) in the presence of endogenous regressors. Second, furthering the applicability of their work, we study GFE in general settings that are commonly encountered by finance empiricists and compare GFE to various popular models used in the finance literature. Currently, the literature does not inform us on the properties of GFE estimates under TFE and IFE DGPs and on the magnitude of the bias, when using TFE and IFE under time-varying heterogeneity. This is the first paper to explicitly derive the heterogeneity bias, which is determined by the shock size and the correlation between heterogeneous responses and regressors. We show that GFE is a robust estimator for TFE and IFE DGPs both theoretically and in simulations. Third, we consider the case of time-varying group memberships and discuss under which situations GFE can capture the dynamics of the group membership structure. Finally, we propose a specification test to evaluate GFE against TFE and IFE models. To facilitate and encourage the use of GFE estimators, we provide guidance on its use and STATA user-written functions.

Our paper is in line with the burgeoning work on econometric challenges faced by researchers in empirical methods. Bertrand et al. (2004), Petersen (2009), and Thompson (2011) discuss methods on appropriately computing standard errors in the presence of cross-sectional and time series dependence across residuals. Roberts and Whited (2013) and Grieser and Hadlock (2019) discuss and investigate limitations of IV techniques using lagged variables and the violation of the strict-exogeneity assumption in finance panel-data applications, respectively. Flannery and Hankins (2013) discuss the use of dynamic panel model in finance applications. Erickson and Whited (2012) and Koh et al. (2018) discuss issues on measurement error in investment regressions and missing data problem in R&D and patents.

The paper is organised as follows. The next section introduces the three main models we investigate, TFE, IFE and GFE, derives the coefficient bias of different econometric models under a GFE DGP, discusses the GFE model and methodology, and proposes the specification test. Section 3 conducts Monte Carlo simulations based on simulated data and the empirical distribution of Compustat data. Section 4 provides evidence on the economic importance of GFE by replicating current work in finance. Section 5 provides the two stage least square extensions and discusses issues related to standard errors. Section 6 concludes.

2 Unobserved Heterogeneity

In this section, we analyze issues related to unobserved heterogeneity. First, we discuss the two-way (individual and time) fixed effects model, the most widely used empirical model to address unobserved heterogeneity. We then derive the bias of TFE estimates when the outcome variable is characterized by time-varying group heterogeneity, i.e. the DGP of GFE. Next, we discuss the interacted fixed effects model and derive its bias under the GFE DGP. Finally, we present the GFE model and discuss its properties.

2.1 Two-way fixed effects model

We consider the following two-way fixed effect model:

$$y_{it} = \alpha_i + \lambda_t + X_{it}'\beta + \epsilon_{it} \quad i = 1, \ldots, N \quad t = 1, \ldots, T \tag{1}$$

where y_{it} is the dependent variable for unit i at time t, α_i models the unit fixed effects, λ_t models the time fixed effects, X_{it} is a $K \times 1$ vector of regressors, and ϵ_{it} is the error term.[6] Both y_{it} and X_{it} are stationary. With strict exogeneity, it is typically assumed that

$$\begin{aligned}
&\epsilon_{it} \sim \text{i.i.d } (0, \sigma_\epsilon^2),\\
&\text{var}(X_{it}) = \sigma_X^2,\\
&\text{cov}(\epsilon_{it}, X_{it}) = 0,\\
&\text{cov}(\alpha_i, X_{it}) \neq 0,\ \text{cov}(\lambda_t, X_{it}) \neq 0,\\
&\text{cov}(\epsilon_{it}, \alpha_i) = 0,\ \text{cov}(\epsilon_{it}, \lambda_t) = 0.
\end{aligned}$$

These assumptions imply that the regressors, unit and time fixed effects are uncorrelated with errors. However, regressors can be arbitrarily correlated with unit and time fixed effects as in Gormley and Matsa (2014). As pointed out by Gormley and Matsa (2014), ignoring the nonzero covariance of the unobserved characteristics and the explanatory variable of interest results in inconsistent estimates.

To estimate this model, we define $y_i = (y_{i1}, \ldots, y_{iT})'$ and $X_i = (X_{i1}, \ldots, X_{iT})'$ for unit i. We define $y = (y_1, \ldots, y_N)'$ and $X = (X_1, \ldots, X_N)'$ across N cross-sectional units. Two-way fixed effects estimators can be obtained by applying least squares on the transformed data, data that are demeaned both in the cross-section and in the time-series dimension. In particular, we define the transformation matrix M as:

$$M = I_N \otimes I_T - I_N \otimes \bar{J}_T - \bar{J}_N \otimes I_T + \bar{J}_N \otimes \bar{J}_T, \tag{2}$$

where I_N is an identity matrix of dimension N, I_T is an identity matrix of dimension T, J_T is a matrix of ones of dimension T, J_N is a matrix of ones of dimension N, $\bar{J}_N = J_N/N$, $\bar{J}_T = J_T/T$, and $\otimes$ is the Kronecker product. The transformed data $\widetilde{y} = My$ and $\widetilde{X} = MX$ have typical elements as $\widetilde{y}_{it} = (y_{it} - \bar{y}_{i\cdot} - \bar{y}_{\cdot t} + \bar{y}_{\cdot\cdot})$ and $\widetilde{X}_{it} = (X_{it} - \bar{X}_{i\cdot} - \bar{X}_{\cdot t} + \bar{X}_{\cdot\cdot})$, respectively, where $\bar{y}_{i\cdot} = \sum_t y_{it}/T$, $\bar{y}_{\cdot t} = \sum_i y_{it}/N$, and $\bar{y}_{\cdot\cdot} = \sum_i \sum_t y_{it}/(NT)$. Similar notation applies to X. The slope coefficient can then be estimated by applying least squares on the transformed data: $\widehat{\beta}_{\text{TFE}} = (X'MX)^{-1}X'My$.

In the two-way fixed effects model, the effect of regressors of interest β can be consistently

[6] A unit can be thought of as a firm or an individual.

estimated by within individual-time transformation, if the DGP coincides with model (1). A key assumption of model (1) is that all individual units are exposed and respond homogeneously to the same time-varying shock, λ_t. This assumption may not hold in practice due to heterogeneity in individual units. In particular, when the dependent variable y_{it} is influenced by some shocks, units may respond to shocks differently. A group pattern of heterogeneity often exists, i.e. the individual's behavior and responses to shocks are similar within a group, even though there are differences across groups. In this case, imposing the homogeneous time fixed effects as in model (1) leads to biased estimates of the coefficients of interest, because the within transformation cannot completely eliminate the heterogeneous time fixed effects.

2.1.1 Bias in two-way fixed effects models

To illustrate the mechanism behind the bias and to quantify the size of the bias, we assume that there are G groups of units. Within the group, individual units have the same response to shocks, while the response differs across the groups. Then the data generating process can be written as:

$$y_{it} = \alpha_i^0 + \theta^0_{g_i^0,t} + X_{it}'\beta^0 + \epsilon_{it}, \quad g_i^0 = 1, \dots, G, \tag{3}$$

where g_i^0 is the group membership of unit i, and the superscript "0" denotes the true value of the parameter. For example, if unit i belongs to group 1, then $g_i^0 = 1$. In model (3), the outcome variable is affected by heterogeneous time-varying shocks $\theta^0_{g,t}$ for group g. Intuitively, the within individual-time transformation can only eliminate the common component of $\theta^0_{g_i^0,t}$, while the group-specific components remain for each unit, which influence the coefficient estimates as omitted variables. To quantify this bias, we define $\widetilde{\theta}_{g_i^0,t} = \theta^0_{g_i^0,t} - \sum_t \theta^0_{g_i^0,t}/T - \sum_i \theta^0_{g_i^0,t}/N + \sum_i \sum_t \theta^0_{g_i^0,t}/(NT)$, the within individual-time transformed group time effects . Then the bias of the two-way fixed effects estimator can be obtained by:

$$\begin{aligned} b_{\mathrm{TFE}} = E\left[\widehat{\beta}_{\mathrm{TFE}} - \beta^0\right] &= \left[\sum_{i=1}^{N}\sum_{t=1}^{T}\widetilde{X}_{it}'\widetilde{X}_{it}\right]^{-1}\left[\sum_{i=1}^{N}\sum_{t=1}^{T}\widetilde{X}_{it}'\left(\widetilde{\epsilon}_{it} + \widetilde{\theta}_{g_i^0,t}\right)\right] \\ &= \left[\sum_{i=1}^{N}\sum_{t=1}^{T}\widetilde{X}_{it}'\widetilde{X}_{it}\right]^{-1}\underbrace{\left[\sum_{i=1}^{N}\sum_{t=1}^{T}\widetilde{X}_{it}'\underbrace{\widetilde{\theta}_{g_i^0,t}}_{\text{het. size}}\right]}_{\text{covariance}}, \end{aligned} \tag{4}$$

where $\widetilde{\epsilon}_{it}$ is the error term after applying transformation (2).

The degree of bias depends on two components: on the covariance between the grouped time fixed effects and the regressors and on the relative size of the heterogeneity with respect to the variation of $\widetilde{X}_{it}$. Particularly, the bias of coefficient estimates is high when the heterogeneity is large and highly correlated with $\widetilde{X}_{it}$. If the heterogeneous time fixed effects are independent from $\widetilde{X}_{it}$, the slope coefficients can still be consistently estimated. However, existing empirical findings on firms' heterogeneous responses to market-wide events suggest that the responses are often correlated with regressors and such bias is likely to exist. For example, Campello et al. (2010, 2011) find heterogeneous impact of the 2008 financial crisis on firms due to unobservable characteristics, such as managerial liquidity and credit management qualities. These findings imply that the heterogeneous time effects are unlikely to be independent from the regressors, especially when some regressors are likely to be a function of managerial characteristics.

Table 1
TFE transformation bias under GFE

The table presents a simple numerical example of how heterogeneity bias is caused by ignoring heterogeneity in responses to shocks, when using a two-way fixed effects model. Columns (1)-(3) present the basic data structure: (1) is the individual unique identification, (2) is the time period denomination, and (3) is the group identification. Columns (4)-(6) present the data: (4) is the individual fixed effects, (5) is the time effects, and (6) is the total value of fixed effects, i.e. $(6) = (4) + (5)$. Column (7) presents the time-series average for each unit, column (8) presents the cross-sectional average for each time period, column (9) is the total average for the sample. Column (10) shows the demeaned cross-sectional fixed effects, $\widetilde{\alpha}_i = \alpha_i - \sum_{t=1}^{T} \alpha_i - \sum_{i=1}^{N} \alpha_i + \sum_{t=1}^{T}\sum_{i=1}^{N} \alpha_i = 0$, column (11) shows the demeaned time fixed effect $\widetilde{\theta}_{g,t} = \theta_{g,t} - \sum_{t=1}^{T} \theta_{g,t} - \sum_{i=1}^{N} \theta_{g,t} + \sum_{t=1}^{T}\sum_{i=1}^{N} \theta_{g,t}$, and column (12) shows the demeaned total fixed effects, i.e. $(12) = (10) + (11)$, or alternatively, (12)=(6)−time series avg. (7)−cross sec. avg. (8)+total avg. (9), as in equation (2).

Unit & Time			Data						Transformed data		
Indiv. ID (1)	Time (2)	Group ID (3)	α_i (4)	$\theta^0_{g,t}$ (5)	Total FE (6)	TS ave. (7)	CS ave. (8)	Total ave. (9)	$\widetilde{\alpha}_i$ (10)	$\widetilde{\theta}_{g,t}$ (11)	Total FE (12)
1	1	1	2	3	5	6	8	10	0	1	1
1	2	1	2	4	6	6	10	10	0	0	0
1	3	1	2	5	7	6	12	10	0	-1	-1
2	1	1	4	3	7	8	8	10	0	1	1
2	2	1	4	4	8	8	10	10	0	0	0
2	3	1	4	5	9	8	12	10	0	-1	-1
3	1	2	6	3	9	12	8	10	0	-1	-1
3	2	2	6	6	12	12	10	10	0	0	0
3	3	2	6	9	15	12	12	10	0	1	1
4	1	2	8	3	11	14	8	10	0	-1	-1
4	2	2	8	6	14	14	10	10	0	0	0
4	3	2	8	9	17	14	12	10	0	1	1

Table 1 provides a numerical example of how heterogeneity bias arises. In this example, there are four individual units, and they are realizations of two population groups. Individuals 1 and 2 belong to the same group as their response to shocks is homogeneous $\theta^0_{1,t}$, while individuals 3 and 4 belong to another group, sharing a common $\theta^0_{2,t}$ that is different from $\theta^0_{1,t}$. The data is presented in columns (4)-(6). If we implement the TFE transformation in equation (2), i.e. first demean in the time series and cross-sectionally as in columns (7) and (8) and then add the total average given in column (9), the resulting data are given in column (12) of Table 1. It is clear that the transformed total fixed effects is not a zero vector, because the time fixed effect is still not completely eliminated by the transformation as shown by $\widetilde{\theta}_{g,t}$ in column (11). If we estimate the slope parameters simply using the transformed $\widetilde{X}$ and $\widetilde{y}$, we fail to incorporate the effect of $\widetilde{\theta}_{g,t}$, which is non-zero and possibly correlated with $\widetilde{X}_{it}$, and thus the slope estimates suffer from heterogeneity bias.

Note that if the heterogeneous time fixed effects only differ in a constant shift, then the within individual-time transformation can successfully eliminate both the individual and time fixed effects (see Table A.1 in the Appendix). This means that if the cross-sectional difference in a unit's response to shocks can be explained by the individual fixed effects, i.e., the cross-sectional difference of response is time-invariant, then the TFE coefficient estimates are still consistent. However, such time-invariant heterogeneity does not seem plausible in practice, since individuals experience different types of shocks over time and the degree of heterogeneity in responses often depends on shock features.

2.2 Interacted fixed effects model

A recent approach to control for time-varying heterogeneity is to include interactions of group and time fixed effects (see e.g. Gormley and Matsa, 2014). We refer to this as an interacted fixed effects model, as in Gormley and Matsa (2014). This model requires a priori knowledge of the group structure, i.e. which group each unit belongs to. Typically, the grouping is based on observed characteristics, such as industry, firm size, etc. Given the group pattern, we can construct the (time-invariant) group dummies for each unit i as $GD_i = \{GD_{i1}, \ldots, GD_{iG}\}'$ with $GD_{ig} = 1$ if unit i belongs to group g and zero otherwise. Let $TD_t = \{TD_{t1}, \ldots, TD_{tT}\}'$ be the (individual-invariant) time dummies for each period t with $TD_{ts} = 1$ if $t = s$ and zero otherwise. Then the interacted fixed effects model can be written as

$$y_{it} = \alpha_i + X_{it}'\beta + \sum_{g=1}^{G}\sum_{s=1}^{T}(GD_{ig} \times TD_{ts})\theta_{g,s} + \epsilon_{it}, \tag{5}$$

where $\theta_{g,s}$ is the heterogeneous time effect of group g at time s. For example, for industry-based grouping, GD is a group dummy, and then $\sum_{g=1}^{G}\sum_{s=1}^{T}(GD_{ig} \times TD_{ts})$ is an industry-time interaction dummy. If the postulated grouping is correct, we can obtain consistent estimates of homogeneous slope parameter β by applying least squares on the following within-transformation regression:

$$\dot{y}_{it} = \dot{X}_{it}'\beta + \sum_{g=1}^{G}\sum_{s=1}^{T}(GD_{ig} \times TD_{ts})\dot{\theta}_{g,s} + \dot{\epsilon}_{it},$$

where $\dot{y}_{it} = y_{it} - 1/T\sum_{t=1}^{T} y_{it}$, $\dot{X}_{it} = X_{it} - 1/T\sum_{t=1}^{T} X_{it}$, $\dot{\epsilon}_{it} = \epsilon_{it} - 1/T\sum_{t=1}^{T}\epsilon_{it}$, and $\dot{\theta}_{g,s}$ represents the transformed time effects.

However, in practice, the group membership is typically unknown and depends on the outcome variable of interest. The group structure may be driven by unobserved characteristics or a mixture of observables and unobservables. Hence, the precise group dummies are difficult to know, and the estimator obtained from the "infeasible" model (5) with correctly specified group dummies is called an infeasible estimator. In real applications, when we misspecify the group structure, IFE leads to inconsistent slope coefficient estimates, if group-specific heterogeneity is correlated with regressors.

2.2.1 Bias in interacted fixed effects models

The bias of IFE estimates depends on how the IFE group structure is misspecified and there are infinitely many misspecifications. To derive the bias in this case, we denote g_i^{IFE} as the postulated group membership of unit i based on some observables specified by the econometrician, and potentially $g_i^{IFE} \neq g_i^0$ for some unit $i \in \{1, \ldots, N\}$. Misclassification can be caused by, e.g. underspecification of the number of groups, incorrect choice of the grouping variables, or unavailability of the grouping variables. Misclassification can be regarded as a measurement error in the group dummies. In particular, the group dummies used in the IFE model (5) are now $GD_i = GD_i^0 + m_i$, where $GD_i^0 = \{GD_{i1}^0, \ldots, GD_{iG}^0\}'$ is the group dummy based on the true group membership g_i^0 and $m_i = \{m_{i1}, \ldots, m_{iG}\}$ is an error vector of unit i that is caused by possible deviation of the postulated group membership from the true membership.[7] This error may often be correlated with X_{it} in practice, because the group pattern is likely to be correlated with other variables also included in the regression model. In this case, we can rewrite equation (5) with measurement errors as:

$$\begin{aligned} y_{it} &= \alpha_i + X_{it}'\beta + \sum_{g=1}^{G}\sum_{s=1}^{T}(GD_{ig} \times TD_{ts})\theta_{g,s} + \epsilon_{it}, \\ &= \alpha_i + X_{it}'\beta + \sum_{g=1}^{G}\sum_{s=1}^{T}(GD_{ig}^0 \times TD_{ts})\theta_{g,s} + \sum_{g=1}^{G}\sum_{s=1}^{T}(m_i \times TD_{ts})\theta_{g,s} + \epsilon_{it}. \end{aligned} \quad (6)$$

[7] Since GD_i is a $G \times 1$ vector of dummy variables, m_i is also a vector of the same length whose elements only take the value -1, 0, or 1.

Obviously, the error term in equation (6) now contains two components, the idiosyncratic noise and the measurement error of the group memberships. We stack the (transformed) explanatory variables and all dummy interaction terms in a vector as $Z_{it} = \{\dot{X}'_{it}, GD^0_{i1}TD_{t1}, \ldots, GD^0_{iG}TD_{tT}\}'$ and let $\theta = \{\theta_{1,1}, \ldots, \theta_{1,T}, \theta_{2,1}, \ldots, \theta_{G,T}\}'$. Then we can obtain the bias of IFE estimators as:

$$E\begin{pmatrix} \widehat{\beta}_{\mathrm{IFE}} - \beta^0 \\ \widehat{\theta}_{\mathrm{IFE}} - \theta^0 \end{pmatrix} = \left[\sum_{i=1}^{N}\sum_{t=1}^{T} Z'_{it}Z_{it}\right]^{-1}\left[\sum_{i=1}^{N}\sum_{t=1}^{T}\sum_{g=1}^{G}\sum_{s=1}^{S} Z'_{it}(m_i \times TD_{ts})\dot{\theta}_{g,s}\right]. \tag{7}$$

If the transformed group-time effects and the difference between the postulated and the true group structure are both uncorrelated with $\dot{X}_{it}$, we can still obtain unbiased and consistent slope coefficient estimates under IFE. However, neither of the two conditions are likely to be satisfied in practice. Empirical work typically classifies firms based on one or two popular observables, e.g. industry or firm size, whereas in many applications the true group structure is driven by several observable and even unobservable variables. These determinants of group structure are often closely related with covariates. Moreover, these determinants typically vary across settings, since the group pattern of heterogeneity in firms' responses to shocks depends on the features of shocks, the samples, and the variables of interest. The bias is large when the size of the heterogeneous response is large and when the degree of misspecification is large and highly correlated with explanatory variables, as in TFE.

2.3 Time-varying heterogeneity: Group fixed effects model

To identify the group structure, we consider a linear panel data model, where the additive time fixed effects have an unrestricted and flexible group pattern of heterogeneity, namely grouped fixed effects (GFE). The group pattern is unrestricted and flexible, since it can depend on both observables and unobserables, which we do not need to specify and may vary across applications. This model class was proposed by Bonhomme and Manresa (2015a) as an effective way to avoid the issue of "incidental parameter" (Neyman and Scott, 1948), while still allowing for unobserved heterogeneity, which may be correlated with regressors. The idea of "grouped fixed effects" fits the argument in the empirical finance literature that heterogeneous responses to market-wide shocks are due to unobserved time-varying heterogeneity, such as unobservable managerial characteristics and corporate culture.

2.3.1 GFE model and estimation method

The GFE model precisely coincides with the DGP in model (3) and can be written as

$$y_{it} = \alpha_i + \theta_{g_i,t} + X_{it}'\beta + \epsilon_{it}, \quad g_i = 1, \ldots, G. \tag{8}$$

This model differs from the standard fixed effects model, because it includes a time-varying group specific variable $\theta_{g_i,t}$, in addition to individual fixed effects α_i. One may think of $\theta_{g_i,t}$ as interactions of group indicators with time dummies. The individual fixed effects capture the individual firm's characteristics, while the grouped-fixed effects parameter $\theta_{g_i,t}$ models how firms respond to time-varying common shocks. This response is common within a group but differs across groups. There are two types of parameters to estimate, the group membership parameter g_i for all units $i = 1, \ldots, N$ and the standard regression parameters $\theta_{g_i,t}$, and β.[8]

To identify the latent group structure, we need the group separation condition:

$$\text{plim}_{T\to\infty} \frac{1}{T} \sum_{t=1}^{T} (\theta^0_{g,t} - \bar{\theta}^0_{g,t} - \theta^0_{\tilde{g},t} + \bar{\theta}^0_{\tilde{g},t})^2 > 0, \quad \text{for} \quad g \neq \tilde{g},$$

where $\bar{\theta}^0_{g,t} = 1/T \sum_{t=1}^{T} \theta^0_{g,t}$ and $\theta^0_{g,t}$ denotes the true value of $\theta_{g,t}$. This implies that group heterogeneity exists after controlling for individual fixed effects, i.e. the group specific responses to shocks are time-varying. Time-varying heterogeneity is consistent with the empirical observations in the financial crises and market wide event literature.

To consistently estimate parameters in equation (8) without a priori knowledge of the group membership, one can employ the least squares technique and jointly estimate the group and coefficient parameters by solving the following minimization problem:

$$Q_{NT} = \min_{\theta,\beta,g} \sum_{i=1}^{N} \sum_{t=1}^{T} (\dot{y}_{it} - \dot{X}_{it}'\beta - \theta_{g_i,t})^2. \tag{9}$$

Since the group membership parameters appear in the objective function in a nonlinear way, we cannot analytically solve this optimization problem. An exhaustive search of the optimal group partition is also virtually infeasible due to a large number of possible partitions. To solve the optimization problem, Bonhomme and Manresa (2015a) suggest the following iterative procedure.

[8] Bonhomme et al. (2017) extended this model by allowing the number of groups to increase to N, and thus incorporating individual heterogeneity. Alternatively, Bai (2009) proposes to model individual-specific responses to shocks using an unobserved factor structure.

Algorithm 1

1. Let $g^{(0)}$ be an initial value of grouping. Set $s = 0$.

2. For the given $g^{(s)}$, compute:

$$(\theta^{(s+1)}, \beta^{(s+1)}) = \arg\min_{\beta,\theta} \sum_{i=1}^{N} \sum_{t=1}^{T} (\dot{y}_{it} - \dot{X}_{it}'\beta - \theta_{g_i^{(s)},t})^2.$$

3. Compute for all $i \in \{1, \ldots, N\}$:

$$g_i^{(s+1)} = \arg \min_{g\in\{1,\ldots,G\}} \sum_{t=1}^{T} (\dot{y}_{it} - \dot{X}_{it}'\beta^{(s+1)} - \theta_{g_i,t}^{(s+1)})^2.$$

4. Set $s = s + 1$ and go to Step 2 (until numerical convergence).

This algorithm iterates between estimating the coefficient parameters and group membership parameters and can be viewed as an EM-type algorithm (see, for example, Dempster et al., 1977). Specifically, Step 2 estimates the coefficient parameters for a given group structure as in the usual least squares problem.[9] Then Step 3 finds the optimal group in terms of minimum sum of squared residuals over time for each unit, based on the estimated coefficient parameters from the previous step. Put it differently, in step $s+1$, firm i will be classified into group g if its time-series summation of squared residuals computed using the estimated coefficient parameters $\theta_{g,t}^{(s+1)}$ is less than that computed using $\theta_{g',t}^{(s+1)}$ for any $g' \neq g$.[10] Since the objective function decreases at each iteration step, we can obtain the parameters that minimize equation (9) when the algorithm converges. Clearly, this algorithm depends on the chosen initial values $g^{(0)}$. Certain initial values may lead to local optima of the least squares objective function. To avoid local optima, one should try a large number of initial values and select the one with the lowest sum of squared residuals.

2.3.2 Asymptotic properties of group membership and coefficient estimates

Bonhomme and Manresa (2015a) show that the group membership and coefficient estimates pro-

[9] In this step, group-specific time effects are captured by an interaction of group and time dummy variables. Considering the computational issues highlighted by Gormley and Matsa (2014), in each iteration one can apply within group transformation for each time period to compute the slope coefficient estimates for some given group structure. This leads to the same results as interacting group and time dummies, but may be computationally less costly.

[10] Whenever a group only contains a single unit, we reinitialize the group structure and restart the iterative algorithm, so that we rule out the singleton group in this algorithm. In practice, singleton groups are rare in typical financial data, such as CRSP/Compustat.

duced by this "grouped fixed effects" approach are consistent and have the "oracle" property. In particular, note that in step 3, the estimated g_i minimizes the time series summation of squared residuals. Therefore, one can show that the GFE estimate of group membership converges to the true population membership, as the time-series dimension increases. Increases in the cross-sectional dimension (N) do not contribute to the convergence of group membership estimates *asymptotically*, but they may improve the membership estimates indirectly in finite samples by improving the estimation accuracy of coefficient parameters.

To more precisely quantify the asymptotic properties of group membership estimates, we consider the probability of misclassification uniform across units, i.e.

$$\text{Prob}\left(\sup_{i\in\{1,\dots,N\}} |\widehat{g}_i - g_i^0| > 0\right),$$

where $\widehat{g}_i$ is the GFE estimate and g_i^0 is the true value of the group membership of unit i. Bonhomme and Manresa (2015a) show that the probability of misclassification uniform across units converges to zero with the rate $1/T^{\delta}$, where δ is an arbitrarily large positive number. This convergence rate is much faster than the usual convergence rate of least squares estimates of β, i.e. $1/\sqrt{NT}$, and we often refer to it as "super-consistency". Our generated and real data based simulations in Section 3 show that the GFE group membership estimates are accurate in finite samples, even when the sample time series is moderately short.

Due to the super-consistent group membership estimates, the GFE estimates of coefficient parameters $\theta_{g,t}$ and β have the asymptotic "oracle" property that they are consistent and asymptotically equivalent to the infeasible estimate from model (5), as if we knew the true group membership. The asymptotic equivalence further implies that if a large number of time series observations is available, we can ignore the error caused by estimating unknown group memberships and estimate standard errors of coefficient estimates using the sample analogue of the usual asymptotic robust variance, namely the robust variance of the least squares estimator for each group. To compute this variance, let $\bar{\dot{X}}_{\widehat{g}_i,t}$ be the mean of $\dot{X}_{it}$ in group $\widehat{g}_i = g$ and define:

$$\widehat{\Sigma}_{\beta} = \text{plim}_{(N,T)\to\infty} \frac{1}{NT} \sum_{i=1}^{N} \sum_{t=1}^{T} (\dot{X}_{it} - \bar{\dot{X}}_{\widehat{g}_i,t})(\dot{X}_{it} - \bar{\dot{X}}_{\widehat{g}_i,t})',$$

$$\widehat{\Omega}_{\beta} = \lim_{(N,T)\to\infty} \frac{1}{NT} \sum_{i=1}^{N} \sum_{j=1}^{N} \sum_{t=1}^{T} \sum_{s=1}^{T} E\left[\widehat{\epsilon}_{it}\widehat{\epsilon}_{js}(\dot{X}_{it} - \bar{\dot{X}}_{\widehat{g}_i,t})(\dot{X}_{js} - \bar{\dot{X}}_{\widehat{g}_j,s})'\right],$$

where $\widehat{\epsilon}_{it}$ is the residual from the GFE model (8). Then, we can obtain the asymptotic variance of $\widehat{\beta}_{\text{gfe}}$ as:

$$var(\widehat{\beta}_{\text{gfe}}) = \frac{1}{NT}\widehat{\Sigma}_{\beta}^{-1}\widehat{\Omega}_{\beta}\widehat{\Sigma}_{\beta}^{-1}.$$

This variance is robust to heteroscedasticity and correlation across groups and time, but it does not take into account the uncertainty caused by estimating the group membership. Given that the group membership estimator converges very quickly as the number of time periods increases, the asymptotic variance can often be a good approximate of variance. However, if the panel has small T, one should account for errors caused by the group estimation, because misclassification may increase the small-sample dispersion of the estimator. A possible way to account for the uncertainty caused by the group estimation is to use a bootstrapped standard error estimate based on re-sampling unit blocks of (y_i, X_i) from the sample. This is easy to implement but computationally more intense.[11]

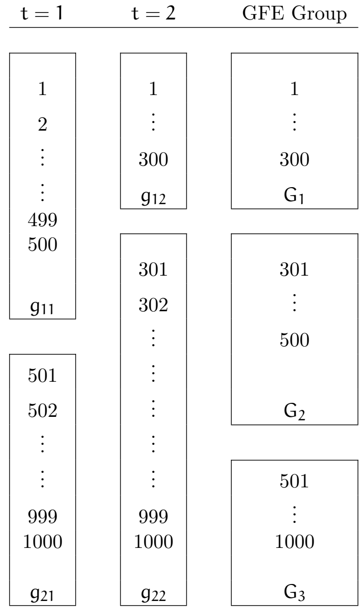
2.3.3 Time-varying group membership

Although the group membership index g in model (8) does not explicitly depend on time, the GFE specification can implicitly model time-varying group memberships provided that the time variation of memberships and the number of individuals that change their memberships are both limited. Particularly, since classification is based on the entire time path of $\theta_{g,t}$, individuals that change their group memberships can always be segmented to form new groups, so that no membership change occurs in any group during the sample period.

For example, suppose at time $t = 1$ there are two groups (g_{11} and g_{21}) with 500 firms each, where firms within each group respond similarly to the shock at time t_1; see Table 2. At time $t = 2$, 200 firms in group 1 respond to the shock in a similar way to firms in group 2, (g_{12} and g_{22}). For sample periods 1 to 2, these firms are equivalent to three groups with time-invariant memberships: the first group contains the 300 firms from g_{11} that do not change memberships, G_1; the second group contains the 200 firms that change their membership into g_{22}, G_2; the final group contains the 500 firms from the original g_{21} that do not change membership, G_3. Thus, time-varying memberships can be accommodated in the GFE procedure by allowing for more groups.

[11] Alternatively, one can also compute an analytical version of finite-sample standard errors using the formulas provided in Bonhomme and Manresa (2015b).

Table 2
Time-varying group memberships



2.3.4 Determining the number of groups

So far in this section, we have assumed that the number of groups G is known. However, the number of groups G is unknown a priori, in practice. We first consider any issues that might arise due to the misspecification of the number of groups, and then discuss how to appropriately determine this number. When the postulated number of groups is less than the true number, the GFE estimator of slope coefficients $\widehat{\beta}_{\text{gfe}}$ is inconsistent, if the heterogeneous responses of misclassified units are correlated with regressors. However, if one over-specifies the number of groups, one can still achieve a consistent estimator of β, but probably with less efficiency (Liu et al., 2018; Bonhomme and Manresa, 2015b).

Using these properties, we can determine the number of groups by examining the values of $\widehat{\beta}_{\text{gfe}}$ for different choices of G. In particular, since $\widehat{\beta}_{\text{gfe}}$ remains consistent for an over-specified G but not for an under-specified G, we should expect that $\widehat{\beta}_{\text{gfe}}$ converges to a certain value and remains relatively stable as G increases. Thus, we can choose G, when $\widehat{\beta}_{\text{gfe}}$ is stable. To determine G, one can use information criteria as suggested by Bonhomme and Manresa (2015a) and Su et al. (2016):

$$\text{BIC}(G) = \frac{1}{NT}\sum_{t=1}^{T}\sum_{i=1}^{N}(\dot{y}_{it} - \dot{X}_{it}'\widehat{\beta}_{\text{gfe}}^{(G)} - \widehat{\theta}_{g_i,t}^{(G)})^2 + \widehat{\sigma}^2\frac{GT+K+N}{NT}\log(NT), \tag{10}$$

where $\widehat{\beta}^{(G)}_{gfe}$ and $\widehat{\theta}^{(G)}_{g_i,t}$ are the GFE estimators with G groups and $\hat{\sigma}^2$ is a consistent estimate of the variance of ϵ_{it}, e.g. the sum of squared residuals obtained from $G_{\max}$ groups.

GFE as a robust estimator

Overall, the results show that GFE is a robust method compared to TFE and IFE approaches. In particular, if the DGP is TFE, GFE can still produce consistent coefficient estimates, although it may be potentially less efficient. This is a direct outcome of the consistency of GFE under overspecification of the number of groups. If industry is the true group structure (as specified in IFE models), GFE can consistently identify the industry group structure and produce consistent coefficient estimates. However, if a non-industry related group pattern of heterogeneity in responses exists, GFE still produces consistent estimates, but TFE and IFE coefficient estimates will be biased. This robust property of GFE also facilitates the selection between GFE and TFE/IFE using a specification test discussed in the next subsection.

2.3.5 Specification test for group fixed effects

To evaluate which of the two models, two-way or group fixed effects, provides a better description of the data, we can perform specification tests. The null hypothesis is the TFE model

$$y_{it} = \alpha_i + \lambda_t + X_{it}'\beta + \epsilon_{it},$$

and the alternative hypothesis is the group fixed effects model

$$y_{it} = \alpha_i + \theta_{g_i,t} + X_{it}'\beta + \epsilon_{it}.$$

The alternative is the encompassing general model that nests the null model by setting $\theta_{g_i,t}$ homogeneously across units at each time t.

The GFE estimator is consistent in both null and alternative models, but it is less efficient under the null, because it unnecessarily divides the sample into sub-groups and needs to estimate group memberships. The TFE estimator is consistent under the null, but not under the alternative model due to neglecting the heterogeneous time effects (see Section 2.2.1). These observations suggest that we can use a Hausman test statistic to evaluate the two models. With a similar argument to Bai (2009), we can show that $\mathrm{var}(\widehat{\beta}_{GFE} - \widehat{\beta}_{TFE}) = \mathrm{var}(\widehat{\beta}_{GFE}) - \mathrm{var}(\widehat{\beta}_{TFE})$, and thus a Hausman-type test statistic can be obtained by

$$\mathsf{H} = (\widehat{\beta}_{\mathsf{GFE}} - \widehat{\beta}_{\mathsf{TFE}})' \left[\text{var}(\widehat{\beta}_{\mathsf{GFE}}) - \text{var}(\widehat{\beta}_{\mathsf{TFE}})\right]^{-1} (\widehat{\beta}_{\mathsf{GFE}} - \widehat{\beta}_{\mathsf{TFE}}) \sim \chi^2_{\mathsf{K}}.$$

When the difference between the two estimates is statistically large, i.e. large H, it suggests that the TFE estimator is likely to be inconsistent, and one should use GFE for more reliable results. On the contrary, when the two estimates are not statistically different, TFE yields consistent estimates.

The above discussion also applies to testing interacted versus group fixed effects, as the tradeoffs between consistency and efficiency are similar, depending on whether the interacted grouping is the correct specification. In particular, when firms respond to shocks heterogeneously according to their interacted groups, both interacted and group fixed effects estimators are consistent, with the latter being potentially less efficient since the group memberships need to be estimated. On the contrary, if interacted grouping is misspecified, the group fixed effects remain consistent, while interacted fixed effects are not. Thus, a similar Hausman-type test statistic can be used to compare the two models.

3 Simulation

In this section, we investigate the finite sample properties of grouped fixed effects estimates and compare them with the two-way and interacted fixed effects estimates via Monte Carlo simulations. The purpose of the simulation study is twofold. First, we compare GFE with two widely used models in finance applications and evaluate their relative advantages in different DGPs. We use data generated from an extensive set of simulation designs, including heterogeneous shocks that occur at different frequency, homogeneous shocks, shocks with different degrees of correlation with the regressors, and shocks of different sizes. We also consider the case when the number of groups is relatively large and the case when the number of groups is misspecified. The second purpose of this simulation study is to examine the performance of GFE in typical finance data sets, especially the accuracy of group membership estimation. We conduct a natural experiment using data on natural disasters, as well as a real-data-based experiment that generates the group structure and outcome variable based on natural disasters.

3.1 Simulation with generated data

3.1.1 Data generating process

We generate the data from the following linear panel data model with time-varying and potentially heterogeneous shocks:

$$y_{it} = \alpha_i + \theta_{g_i,t} + \sum_{l=1}^{2} X_{l,it}\beta_l + \epsilon_{it}, \quad i = 1,\dots,N; \quad t = 1,\dots,T; \quad \epsilon_{it} \sim N(0,\sigma_\epsilon^2), \tag{11}$$

where $\alpha_i = 1/T \sum_{t=1}^{T} X_{1,it}$, $X_{it} = (X_{1,it}, X_{2,it})'$ is a vector of regressors with loadings $\beta = (\beta_1, \beta_2)' = (1,2)'$. The major difference between model (11) and standard panel data models lies in $\theta_{g_i,t}$ for $g = 1,\dots,G$ with $G = 5$ that captures the group-wise heterogenous response to time-varying shocks. To generate the correlation between $\theta_{g_i,t}$ and X_{it}, they are both driven by a group structure, i.e.

$$X_{l,it} = c_\tau \tau_{l,g_i,t} + \nu_{it}, \quad \text{for } l = 1,2$$

and

$$\theta_{g_i,t} = g_i^2(\tau_{1,g_i,t} + \tau_{2,g_i,t})/c_\theta,$$

where $\tau_{l,g_i,t}$ is the group component drawn from $G(=5)$ different normal distributions with mean g and variance $2g$ for $g = 1,\dots,G$, and ν_{it} is the idiosyncratic component drawn from the normal distribution with mean 1 and variance $\sigma_\nu^2 = 5$. One may think of $\tau_{l,g_i,t}$ as an unobserved group specific determinant of X, like corporate risk culture, which also drives the heterogeneous response to a shock. The constant c_τ reflects the importance of the group component in X and determines the degree of correlation between X and θ. The constant c_θ controls the size of shocks. To make the variation of error terms comparable to that of the regressors, we set $\sigma_\epsilon^2 = 5$.[12]

Since the nature and the characteristics of shocks are important for the consistency of each econometric model, we consider three different types of shocks:

DGP1 (Frequent shocks): Shocks occur every time period t.

DGP2 (Sparse shocks): Shocks occur only 25% of the time.

DGP3 (Homogeneous shocks): Common shocks with homogeneous impact on all units at each

[12]Setting these variances to 5 is not essential. However, we need to ensure that the variation of the regressors and the error is of comparable size, in order to generate a reasonable signal to noise ratio as well as realistic shock sizes.

time period.

DGP1 and DGP2 allow for heterogeneous responses to shocks. DGP2 contains less shocks than DGP1, and from theory we should expect the bias of TFE in DGP2 to be smaller than that in DGP1. DGP3 assumes that the effect of shocks is homogeneous across units, and therefore it is the same data generating process as the TFE model with individual and time fixed effects. Hence, GFE and TFE are expected to produce similar results under DGP3, see robust estimator discussion in Section 2.3.4.

Within each DGP, we also consider different choices of key parameters, i.e. c_τ, c_θ, and the number of groups G. These extensions help to better understand how estimation bias is affected by the correlation between regressors and heterogenous responses, the size of shocks, and the number of groups. In addition, we also consider the case of misspecifying G for GFE.

To mimic typical empirical data in corporate finance studies, we consider panels with large N and moderate T. In particular, we conduct simulations under three different sample sizes: ($N = 1000$, $T = 15$), ($N = 1000$, $T = 30$), and ($N = 2000$, $T = 15$). These sample sizes allow us to examine how the dimension of N and T affects the relative performance of various models.

3.1.2 Model comparison and evaluation

We compare three models: grouped fixed effects, two-way fixed effects, and interacted fixed effects. The major difference between GFE and IFE lies in that the grouping in IFE is specified exogenously by researchers, while the group pattern in GFE is determined by the data. The group pattern produced by GFE is case specific and depends on the nature of the shock and outcome variable.

The validity of IFE obviously depends on the specification of groups and shocks, and we consider four scenarios for IFE. Scenario 1 correctly specifies the group memberships of all units and the time of all shocks (denoted as IFE_{gt}), which is impossible in practice. We refer to this as an infeasible estimator. Scenario 2 exactly specifies the groups but not the shocks (denoted as IFE_g). In IFE_g, when the shocks are misspecified we generate a time dummy variable every 1/3 of the time. Scenario 3 correctly specifies the shocks but not the groups (denoted as IFE_t). In IFE_t, when the groups are misspecified we assign 30% of units in one group to the closest neighboring groups. Scenario 4 misspecifies both groups and shocks (denoted as IFE_{null}).

We evaluate all models based on the accuracy of grouping and of the slope coefficient estimates. We measure the accuracy of grouping by the average of the misclassification frequency across replications, defined as $\mathsf{AMF} = 1/\mathsf{R}\sum_{r=1}^{R} \mathrm{MF}_r$, where $\mathsf{R} = 1000$ is the number of replications, and

$$\mathrm{MF}_r = \frac{1}{N}\sum_{i=1}^{N} I(\widehat{g}_i \neq g_i^0).$$

The misclassification variable MF counts the number of units that are assigned wrongly (misclassified) as a proportion of the total number of units, for each replication. We compute the bias, the estimated standard deviation, and the root mean square error (RMSE) as:

$$\mathrm{Bias}(\widehat{\beta}) = \frac{1}{R}\sum_{r=1}^{R} \widehat{\beta}^r - \beta^0, \quad \mathrm{Std}(\widehat{\beta}) = \sqrt{\frac{1}{R-1}\sum_{r=1}^{R} (\widehat{\beta}^r - \bar{\widehat{\beta}})^2}$$

and

$$\mathrm{RMSE}(\widehat{\beta}) = \sqrt{\frac{1}{R}\sum_{r=1}^{R} ||\widehat{\beta}^r - \beta^0||^2},$$

where $\widehat{\beta}^r$ is the estimator in the r-th replication, β^0 is the true value, and $\bar{\widehat{\beta}} = 1/R\sum_{r=1}^{R} \widehat{\beta}^r$ is the sample average across replications. We report the *empirical* standard deviation across replications, because it can capture the uncertainty caused by estimating the unknown group structure.[13,14]

3.1.3 Results

In this section, we present the simulation results, in line with our theoretical discussion in Section 2. We first present the detailed results for DGP1 – DGP3 under a benchmark parametrization. Then, we consider alternative parametrizations in order to examine how the performance of different models is affected by the shock size, the correlation between responses and regressors, and the number of groups. We also examine how GFE performs when the number of groups is misspecified.

Frequent shocks

Table 3 presents the bias and the mean square error of estimated coefficients under DGP1. We consider the parameterization $c_\tau = 0.5$, $c_\theta = 15$, and $G = 5$ as a benchmark. This translates to

[13]This standard deviation differs from the quantity $\frac{1}{R}\sum_{r=1}^{R} \widehat{\sigma}_{\widehat{\beta}}$, where $\widehat{\sigma}_{\widehat{\beta}}$ is the estimated asymptotic standard error of $\widehat{\beta}$, which does not take into account the grouping uncertainty. With a moderate time-series dimension, these two standard deviation estimates are equivalent, because the group estimation is super-consistent with the convergence rate of an arbitrarily large exponential order of T (see Theorem 2 of Bonhomme and Manresa (2015a)). However, they differ slightly from each other when T is small. Our unreported simulation results (available upon request) suggest that when T is larger than 15, the difference between these two estimates is small.

[14]Note that $\mathrm{RMSE}(\widehat{\beta})$ is not precisely the square root of the summation of squared $\mathrm{Bias}(\widehat{\beta})$ and $\mathrm{Std}(\widehat{\beta})$, since $\mathrm{Std}(\widehat{\beta})$ is the *estimated* standard deviation of $\widehat{\beta}$ and not the true one.

moderate shocks, occurring 35% of the dependent variable time series, and the correlation between the time aggregated shocks ($\sum_{t=1}^{T} \theta_t$) and X is 0.3. We first focus on the results in Panel A of Table 3 with 1000 cross-sectional and 15 time-series observations. In this setting where units respond differently to shocks, TFE does not capture the heterogeneity and results in around 60% bias for β_1 (Bias=0.595 where β_1=1) and 30% bias for β_2 (Bias=0.599 for β_2=2). On the other hand, GFE successfully captures the heterogeneous response, and it correctly classifies more than 85% of units. The GFE coefficient estimate has a small bias (around 0.01).

IFE$_{gt}$ is the best among all estimators as it correctly *specifies* (not estimates) all the groups and the shocks. IFE$_{gt}$ is more accurate than GFE because there is no estimation of groups involved. However, it is very difficult, if not impossible, to always exactly specify all groups and shocks in practice. If there is any misspecification, IFE may perform differently. To assess how misspecification influences the IFE estimator performance, we compare IFE$_g$, IFE$_t$, and IFE$_{null}$. Results in Panel A of Table 3 show that incorrectly specifying the shocks (IFE$_g$) leads to 50% bias for β_1 and 25% bias for β_2, and misspecification in grouping (IFE$_t$) causes 25% bias for β_1 and 12% bias for β_2. Naturally, misspecifying both shocks and groups (IFE$_{null}$) leads to an even larger bias.[15]

Comparing the standard deviation of different estimators, we find that GFE produces a slightly larger standard deviation than IFE$_{gt}$, suggesting that estimating the groups introduces little extra uncertainty. The standard deviation of TFE is almost twice as large as GFE. The misspecified IFE also has much larger standard deviation than GFE. Turning to the tradeoff between bias and variance, the RMSE of GFE is only slightly higher than that of the infeasible estimator IFE$_{gt}$. However, the RMSE of TFE is almost 50 times larger than that of GFE. Incorrectly specifying the groups and shocks also dramatically decreases the performance of IFE (with RMSE 45 times larger than that of GFE), and the performance depends on the degree of misspecification.

Panels B and C of Table 3 examine how the sample size affects the performance of different models. Increasing the sample size (N and/or T) improves the accuracy of GFE and IFE$_{gt}$, but not the accuracy of TFE and other versions of IFE. For GFE, increasing the time dimension remarkably reduces the misclassification frequency, but increasing the unit dimension does not improve the classification, because only the time series variation contributes to the accurate classification of units. In fact, the convergence rate of group estimation is $1/T^{\delta}$, where δ is any arbitrary positive number, and N does not affect the convergence rate. Both the bias and the standard deviation of

[15]The impact of misspecification obviously depends on the degree of misspecification. Unreported results (available upon demand from the authors) confirm that if we increase the proportion of individuals that are misclassified or decrease the number of specified shocks, the bias of IFE increases.

GFE estimates decrease by increasing N and T. The accuracy of IFE$_{\mathsf{gt}}$ also improves by increasing either dimension of the sample, since it is asymptotically equivalent to GFE. On the contrary, the RMSEs of TFE and other versions of IFE do not decrease by increasing the sample size. The improvement in GFE and IFE$_{\mathsf{gt}}$ performance confirms the consistency of these two estimators.

Sparse shocks

Next, we study the case where shocks occur infrequently, as presented in Table 4. In this DGP, we add another IFE estimator, which correctly specifies groups but over-specifies the shocks by interacting groups with time dummies of each time period, denoted as IFE$_{\mathsf{got}}$. Since the shocks remain of the same size but appear less frequently, the aggregate heterogeneous response of the dependent variable to the shocks is of a smaller magnitude. Hence, although the estimators that omit these heterogeneous responses, i.e. TFE, IFE$_{\mathsf{g}}$, IFE$_{\mathsf{t}}$, and IFE$_{\mathsf{null}}$, continue to be biased, the biases are all smaller than in the case of frequent shocks.

GFE continues to perform well in grouping with misclassification frequency less than 10% in all cases and its coefficient estimates remain highly accurate, although the standard deviations are slightly larger than in DGP1. This is because GFE unnecessarily models the shock at each time period while there are actually fewer shocks. This also explains the difference between GFE and IFE$_{\mathsf{gt}}$, which exactly identifies both grouping and shocks. Similarly, we find that imposing more shocks (IFE$_{\mathsf{got}}$) does not affect the consistency but only weakens the efficiency of IFE. Given sufficient observations in our case, the efficiency loss in $\mathsf{IFE}_{\mathsf{got}}$ is ignorable (compared to IFE$_{\mathsf{gt}}$).[16]

Homogeneous shocks

The previous results have shown that when individuals respond differently to shocks, ignoring such heterogeneity can lead to severely misleading results for TFE and IFE estimations with misspecified groups. A natural question is, if the data generating process has homogeneous shocks, i.e. the impact of shocks is homogeneous across individuals, then how much inefficiency will be caused by assuming a relatively complicated model of grouped fixed effects?

To answer this question, we generate the data by model (11) but set $\mathsf{G} = 1$. Table 5 compares various estimators in the homogeneous-shock case. We see that TFE and IFE$_{\mathsf{gt}}$ now perform equivalently well, because both models coincide with the DGP. As for GFE, we consider two cases

[16]This result suggests that one could simply include time dummies for all periods, and not necessarily specify the time of shocks precisely.

with estimated number of groups $\hat{G} = 1$ and $\hat{G} = 4$.

When $\hat{G} = 1$, GFE boils down to TFE and IFE_{gt}, and it performs identically to the other two models. When $\hat{G} = 4$, we over-specify the number of groups, which only marginally increases the RMSE for GFE. This confirms the theoretical result of Bonhomme and Manresa (2015a) that over-specifying the number of groups does not affect the consistency of the slope coefficient estimators. In such large sample sizes, the potential efficiency loss is ignorable.[17] We also observe that when the sample size increases, the RMSE decreases as expected.

Larger and more correlated shocks

Next we change the parametrization of the data to investigate how the size of the shocks and their correlation with regressors affect the performance of different models. We first consider larger shocks ($c_\theta = 5$ and other parameters remaining the same) that is roughly 60% of the magnitude of the dependent variable on average. The results in Panel A of Table 6 show that the biases of TFE and misspecified IFE (IFE_g, IFE_t, and IFE_{null}) all increase, as expected. However, GFE is not affected by the size of shocks, and it remains as accurate as the infeasible estimator IFE_{gt}. The classification of GFE is even more accurate with less than 1% misclassification frequency, because larger shocks allow for clearer group separation.

Panel B of Table 6 examines the case when the correlation between shocks and regressors is high. To mimic this situation, we set $c_\tau = 1$ while other parameters remain the same. This setting has a correlation of 0.5 between shocks and regressors, and the size of shocks remains 35% of the dependent variable on average. We find that a high correlation increases the bias of all estimators (compared to Panel B of Table 3). However, GFE remains the most accurate among all feasible estimators, with 1% bias. GFE also produces the second smallest RMSE after the infeasible estimator IFE_{gt}. The RMSEs produced by TFE and IFE_g are both 100 times larger than those of IFE_{gt}.

Incorrect $\widehat{G}$

The estimated number of groups $\widehat{G}$ may be misspecified in two ways: underestimated or overestimated, see Section 2.3.4. The experiment in DGP3 shows that overestimating the number of groups does not affect the consistency of GFE slope coefficient estimates. However, underestimation of G theoretically affects consistency. If the number of groups is under-specified, then some groups will

[17] The net effect of overspecifying G on standard error estimates depends on the degree of improvement in model fitness and efficiency loss caused by increasing the number of parameters.

contain heterogeneous individuals. Since units within a group are treated homogeneously, this leads to ignoring heterogeneity and a biased GFE estimator.

Table 7 presents the case where the true number of groups $\mathsf{G} = 4$ is incorrectly specified as $\widehat{\mathsf{G}} = 2$ in the GFE estimation. It shows that GFE is biased when G is under-specified, and this bias does not decrease as the sample size increases. However, we find that even in this case the degree of bias and the RMSE of GFE are still much smaller than those of TFE. The direction of the bias of GFE estimates depends on the correlation between regressors and heterogenous responses.

Based on these results, a recommendation for best practice is to choose a larger G, if the number of groups is uncertain and the slope coefficients are the main variable of interest. In general, the cost of over-specification seems much smaller than under-specification, especially for large sample sizes. Therefore, one can use GFE also for TFE and IFE DGPs.

Large number of groups

Finally, we consider the situation where the data are generated by a large number of groups, i.e. $\mathsf{G} = 10$ and $\mathsf{G} = 20$, respectively. Enlarging the number of groups results in an even larger degree of heterogeneity. Therefore, we should expect the bias of TFE and misspecified IFE to increase, and GFE to remain consistent.[18] Table 8 shows the performance under $\mathsf{G} = 10$ and $\mathsf{G} = 20$. The RMSEs of TFE and misspecified IFE increase at least 7 times compared to those in DGP1, while the accuracy of GFE is unaffected.

3.2 Accuracy of the estimated group membership

The previous sections have shown that GFE can correctly estimate group membership even when membership is unknown, theoretically and in simulations. In practice, we would also like to know how accurately the group memberships of individual units are estimated by GFE. In this section, we first evaluate the accuracy of the group estimator through an empirical application and supplement the first exercise with a simulation based on parameters that closely match the empirical data.

The design of a perfect setting for an empirical application to evaluate the group estimator is challenging, as it is nearly impossible to know ex-ante the correct endogenous groupings that are driven by both observable and unobservable determinants. As a result, we can never fully verify the accuracy of the grouping because of the unobservable determinants. However, if the grouping

[18] Bonhomme and Manresa (2015b) propose an alternative algorithm (variable neighborhood search) that potentially works more efficiently than the iterative algorithm for high-dimensional heterogeneity.

is predominantly driven by observable determinants and we know these determinants ex-ante, we can evaluate the GFE group estimator based on those observables.

We study the performance of the GFE group estimator using the empirical application of the effect of the severity of major natural disasters on sales growth. The use of natural disasters as an observable determinant of firm grouping is ideal, as we can compare the GFE estimated group membership to the firms affected by the natural disasters. Moreover, the timing and geographical severity of the disasters is exogenous but correlated to the explanatory variables. The shortcoming of the exercise is that we ignore the role of unobservable characteristics in grouping. As a result, the GFE estimated grouping, which is based on observables and unobservables, will be positively but imperfectly related to groups based on only the location of natural disasters.

3.2.1 Sales growth and natural disasters

Section 2.3 discusses the estimation of the latent group membership and the "oracle" property of the GFE estimator, as both dimensions of the panel data increase. In this application, we study the determinants of firms' sales growth in the presence of major natural disasters. Barrot and Sauvagnat (2016) finds that firms' sales growth decreases by 3.3 to 4.5 percent for three consecutive quarters following a natural disaster. From Barrot and Sauvagnat (2016), we expect $\theta_{g_i,t}$ in the sales growth regression to be negative and to pick up some of these sizable geographical effects for the disaster-affected firms in each year, even in the presence of other unobservable determinants of heterogeneity (firm and managerial characteristics). This application provides some degree of validity check on the group membership estimator and focuses on how the estimated time effect corresponds to the severity of the disaster impact on the work force in the affected firms.

We regress log sales growth on several firms characteristics and different fixed effects. We estimate the following regression:

$$\ln(\text{Sales growth})_{i,t} = \alpha_i + \theta_{g_{i,t}} + X'_{i,t-1}\beta + \epsilon_{i,t},\ \ g_i \in \{1, ..., G\} \tag{12}$$

where $\ln(\text{Sales growth})_{i,t}$ is the natural log of sales growth for firm i in year t. $X_{i,t-1}$ is a 6×1 vector of lagged variables that affect sales growth. We include the following commonly used explanatory variables in our regressions: natural log total assets (TA), Tobin's q (q), return on assets (ROA), capital expenditure (Capex), leverage (Leverage), and cash flow (CF). α_i is the firm fixed effect and $\theta_{g_{i,t}}$ is a time-varying group effect. For TFE, $\theta_{g_{i,t}} = \theta_t$ and for IFE, $\theta_{g_{i,t}}$ is industry interacted with year fixed effects.[19] We use Compustat data for firm characteristics and natural disaster information from Barrot and Sauvagnat (2016). The sample period is 1987 to 2010.

We expect the estimates of θ_t from TFE, $\theta_{industry_i,t}$ from IFE, and $\theta_{g_i,t}$ from GFE to be negative for affected firms in disaster states on the year of the disasters, as firms sales growth decrease due to disasters. Furthermore, we also expect the cross-sectional heterogeneity to be negatively correlated with the share of total U.S. employment affected by the disasters, according to Barrot and Sauvagnat (2016). In other words, firms based in disaster states which have a higher percentage of employment affected by disasters should experience a larger drop in sales.

For exposition purposes, we first focus on 2004 as a year of very large natural disasters. Figure 1 depicts the estimates of the state aggregated effects for 2004. We average the estimates of θ across firms according to their headquarter states, as in Barrot and Sauvagnat (2016). Panel A of Figure 1 shows the share of total U.S. employment affected by four major hurricanes (Charles, Fracis, Ivan and Jeanne), as computed from the County Business Pattern data, provided by the U.S. Census Bureau. Panel B of Figure 1 shows the GFE estimates, where $\theta_{g_i,t}$ is negative and is negatively correlated with states with larger disaster-affected work force. IFE estimates $\theta_{industry_i,t}$ in Panel C are also negative, but do not coincide with the state dispersion of workforce affected in Panel A. Panel D shows that while the TFE estimates are negative, they are uncorrelated with the share of total U.S. employment affected by disasters at the state level.

Using the sample from 1987-2010, Panel A of Table 9 reports the univariate regression estimates of θ for TFE, IFE, and GFE on disaster-affected employment across all natural disasters for the sample period. Consistent with the above findings, disaster-affected employment is negatively related to GFE $\theta_{g_{i,t}}$. Barrot and Sauvagnat (2016) report an average drop in employment of 22 percent in major disaster-affected states, which leads to an estimated drop of three to five percent in sales based on the GFE coefficient of -0.25, close to Barrot and Sauvagnat's reported estimate of 3.3 to 4.5 percent. On the other hand, TFE and IFE estimates suggest a positive but economically small increase in sales with an increase in affected employment, contrary to what we expect from theory and empirical observations. These results demonstrate the reliability of the GFE group estimator.

To quantify the economic significance of misspecifying the econometric model for typical sales growth regressions, Panel B of Table 9 presents the coefficient estimates of the determinants of sales growth using the TFE, IFE based on industry grouping, and GFE based on $G = 10, 15, 30$ models.[20]

[19] For IFE we include individual fixed effects to capture individual heterogeneity. The interacted fixed effect is based on industry, because the assumption is that firms respond to shocks heterogeneously based on industry characteristics.

[20] To implement the GFE estimation, we first need to determine the number of groups G. We consider two evaluation

In general, there are substantial differences between GFE, TFE and IFE estimates, confirmed by the Hausman test statistics. The test statistics suggest that TFE and IFE estimates are incorrect. Particularly, the coefficient estimates for total assets, capex, and cash flow are significant for all GFE specifications, but are much larger for TFE and insignificant for IFE. The leverage effect is insignificant for TFE and negative for GFE. We also see economically significant differences across estimates for Tobin's q, where TFE yields a coefficient 5 times larger than GFE. Overall, the results in this section show that GFE accurately classifies firms into groups when a group structure exists. If the group structure is ignored, coefficient estimates are biased and inference is incorrect. Later, we demonstrate the economic importance of using GFE and how economic and statistical inferences change by replicating a recently published paper.

3.2.2 Simulation based on empirical parameters

The shortcoming of the above exercise is that we are never sure about the true grouping in the data, due to the unobserved determinants of group membership. To more critically evaluate the GFE group estimator, we perform Monte Carlo simulations based on empirical parameters from the observed data. For simulation convenience, we drop firms if any of the six explanatory variables are missing during the sample period 1987–2001 to construct a balanced panel.[21] This leads to a sample containing 207 firms with 15 years of observations. All the data is from Compustat.

For a given number of groups, we generate the dependent variable by

$$y_{it} = \theta_{g_i,t} + X'_{i,t-1}\beta + \epsilon_{it}, \quad i = 1, \dots, N; \quad t = 1, \dots, T; \quad \epsilon_{it}\ N(0, \sigma^2_\epsilon) \tag{13}$$

where X_{it} contains the six explanatory variables in regression (12). To specify the slope coefficients and group fixed effects, we first estimate model (13) on the empirical dataset using the GFE approach. Then, we fix the GFE estimates of β and $\theta_{g_i,t}$ as their true values, respectively. The true group memberships g_i for all i are also fixed to their corresponding GFE estimated memberships. Finally, we generate the error terms from i.i.d. normal distributions with zero mean and variance equal to the variance of the GFE residuals.

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criteria, the Bayesian Information Criterion (BIC) given by equation (10) and the convergence of slope coefficient estimates. The BIC serves as an effective criterion to tradeoff between the model fitness and the number of parameters (degree of complication of the model), and we select the number of groups that minimizes this criterion. Figure 2(a) depicts how the BIC varies for different numbers of groups in the sales growth regression. The value of BIC decreases dramatically when we increase the number of groups from 1 to 10, and it reaches its minimum when G = 10.

[21] We use a shorter sample for this simulation to maximize the number of firms (cross-section) of the balanced sample and to have similar T to the baseline simulations in Section 3.1.2.

We evaluate the performance of GFE based on the accuracy of grouping and coefficient estimates. We use the average misclassification frequency to measure grouping accuracy. For each coefficient estimator $\widehat{\beta}_l$, we employ relative bias to facilitate comparison, different from the absolute bias used in Section 3.1, because the magnitude of the true coefficients varies significantly across variables. Specifically, we compute the absolute value of the relative bias for each $\widehat{\beta}_l$ as

$$|\text{Relative bias}| = \left| \frac{\mathrm{Bias}(\hat{\beta}_l)}{\beta_l^0} \right|,$$

where β_l^0 is the true value of β_l, and $l = 1, \dots, 6$.

Table 10 presents the misclassification frequency, absolute relative bias, and overall RMSE. We find that when $T = 15$ and $G = 5$, GFE can correctly classify more than 90% of the units. The relative bias of GFE is much smaller than TFE and IFE for all variables, and the RMSE of GFE is almost half of that produced by TFE and IFE. Increasing the number of groups marginally affects the performance of GFE as expected, but the bias and RMSE of TFE and IFE increase sharply.

In summary, we have demonstrated the effectiveness of the GFE group membership estimator through an empirical application where we recover the grouping based on ex-ante known and observable determinants. We further show the reliability of the group estimator through a simulation based on parameters drawn from the empirical distributions of the variables of interests.

4 Economic importance of GFE

The previous section (3.2.1) examined the reliability of the group membership estimation and showed that economic inference can vary, when using different econometric approaches. In this section, we demonstrate the importance of GFE for economic and statistical inference through revisiting the study of pilot CEO's influence on corporate innovation, Sunder et al. (2017).

Sunder et al. (2017) argues that "CEOs who combine risk tolerance with a desire for new experiences achieve greater innovation success." The paper collects information about CEO's pilot credentials from Federal Aviation Administration (FAA) records and studies the differences in innovation outcomes across firms with pilot and non-pilot CEOs. The paper finds that CEOs' hobby of flying airplanes is associated with significantly better innovation outcomes, measured by patents and citations. The paper argues that "sensation seeking combines risk taking with a desire to pursue novel experiences and has been associated with creativity." The econometric model in the paper controls for year and industry fixed effects, CEO and firm characteristics, including age, human capital proxied by CEO tenure and academic achievement, explicit risk-taking incentives, military experience, and overconfidence. However, Sunder et al. (2017) cautions about the interpretation of their findings and acknowledges the important role of time-varying corporate risk preferences and the issue of endogenous matching between CEOs and firms.

For example, a group of firms with preferences for risk will select corporate leaders with similar risk preferences. As a result, studying the association between the risk taking behavior of a firm and an observable variable that measures manager's taste for risk might lead to misguided economic inference without accounting for firms' time-varying unobserved heterogeneity of such groups, i.e. their changing strategy on risk taking. In this section, we revisit and replicate the study of pilot CEO and the success of innovation investment by estimating various econometric models using TFE, IFE, and GFE and compare the resulting estimates.[22]

4.1 Pilot CEO and corporate innovation

We model corporate innovation success as follows:

$$\ln(1+\text{citation})_{it} = \alpha_{ind} + \theta_{g_{i,t}} + X_{i,t}'\beta + \epsilon_{i,t}, \;\; g_i \in \{1, ..., G\}, \tag{14}$$

where $\ln(1+\text{citation})_{it}$ is the natural log of the number of raw citations multiplied by the weighting index of Hall et al. (2001) to all the patents applied for by firm i in year t. α_{ind} is the industry fixed effect and $\theta_{g_{i,t}}$ is a time-varying group effect. As in Sunder et al. (2017), X_{it} is a 17×1 vector of variables affecting patent citations: pilot CEO dummy, log total assets (TA), ratio of net property, plant, and equipment over the number of employees (PPE/EMP), stock returns, Tobin's q, institutional holdings, CEO tenure, CEO stock and option delta and vega, CEO age, CEO overconfidence, and dummy variables for: top university, PhD, no school information, finance education, technical education, CEO military background. Table A.2 in the appendix provides all variable definitions. The data is from ExecuComp, BoardEx, Compustat, NBER patent database, and the U.S. Federal Aviation Administration airmen certification records. We exclude financial firms and regulated utilities and apply similar filters as in Sunder et al. (2017). The sample period is 1993 to 2003.

The TFE estimator includes industry fixed effects and year dummies as in Sunder et al. (2017),

[22] For brevity, we will focus on the results for patent citation, see Column 6 of Table 4 in Sunder et al. (2017). We also study other outcome variables in their paper and reach similar conclusions (results are available from the authors upon demand).

$\theta_{g_{i,t}} = \theta_t$. IFE includes an industry fixed effect and $\theta_{g_{i,t}}$ is the industry fixed effect interacted with year fixed effects. To implement the GFE estimation, we first need to determine the number of groups G. We consider two evaluation criteria, the Bayesian Information Criterion (BIC) given by (10) and the convergence of slope coefficient estimates. The BIC serves as an effective criterion to tradeoff between the model fitness and the number of parameters (degree of complication of the model), and we select the number of groups that minimizes this criterion. Figure 2(b) depicts how the BIC varies for different numbers of groups in regression (14). The value of BIC decreases dramatically when we increase the number of groups from 1 to 10, and reaches its minimum when $G = 10$. From $G = 10$ onwards, increasing the number of groups does not significantly improve the model fitness, while introducing a much larger set of parameters to estimate. For robustness, we examine the behavior of slope coefficient estimates for G=15 and find little difference with G=10.

Table 11 presents the coefficient estimates of the determinants of patent citations obtained from two-way fixed effect, interacted fixed effect based on industry grouping, and grouped fixed effect regressions. Hausman tests strongly reject the null hypothesis that TFE, IFE and GFE produce the same coefficient estimates. In general, we observe substantial differences in the estimated coefficients, which leads to different economic inferences between GFE and other estimators.

Consistent with Sunder et al. (2017), we find that pilot CEO is positively correlated with firms' total patent citations, when we account for industry and year fixed effect (TFE).[23] In particular, firms with pilot CEOs have an estimated 0.46 (exp$^{0.38} - 1$) more citations that firms with non-pilot CEOs. We also find qualitatively similar statistical relations between the covariates and patent citations. These relations remain similar and the pilot CEO coefficient remains positive and significant when we control for interacted fixed effects with industry-year interaction fixed effects.

Column 3 of Table 11 reports the estimated coefficients from GFE. Contrary to the TFE and IFE results, we find that there is statistically **no difference** in citations per year for firms with pilot CEOs compared to firms with non-pilot CEOs. This is consistent with the idea that a firm seeking to change its corporate culture or risk taking strategy will select a CEO with similar risk preferences. The inclusion of θ_{gt}, which might potentially capture the endogenous grouping of this changing culture, allows the model to compare pilot versus non-pilot CEOs across firms with similar time-varying risk culture or management strategy. If firm heterogeneous responses are only driven by observables that are correlated with pilot CEOs, controlling for these observables may reduce the

[23]Our point estimate of 0.38 for TFE is slightly different from the Sunder et al. (2017) reported estimate of 0.61, see Table 4 Column 6.

heterogeneity bias. The bias of pilot CEOs exists because there is an unobserved variable, such as corporate risk preferences, that is correlated with the appointment of pilot CEOs and heterogeneous responses to shocks. Although it would be ideal to be able to identify the economic mechanism that gives rise to the bias, the unobserved source of the bias prevents us from further investigation.

GFE's estimates for other CEO characteristics and compensation variables are mainly consistent with the literature's findings. We find that larger firms with higher capital intensity, higher Tobin's q and lower institutional holdings have more cited patents. We find a positive relation between innovation and delta indicating that a strong CEO incentive alignment with shareholders generates better innovation outcomes. We also find that CEOs at the beginning of their tenure engage with riskier and more successful innovations. Education-related coefficients also suggest that education especially those related to finance and technical degrees play an important part in firms' innovation outcomes. We find that firms with military CEOs have slightly lower patent citations consistent with the literature's finding that military CEOs allocate less resources to research and development, see Benmelech and Frydman (2015), which is different from Sunder et al. (2017).

Overall, we demonstrate how the failure of TFE and IFE to appropriately account for time-varying group unobserved heterogeneity, possibly related to business strategy and corporate risk culture, can lead to incorrect economic inference and biased estimates of treatment and control variables.

5 Endogeneity and standard errors

5.1 Endogeneity

So far, we have focused on the model with exogenous regressors. However, often some regressors of interest may be endogenous. In this case, least squares estimation (9) leads to inconsistent estimates of slope coefficients, and instrument variable estimation is required. We extend the GFE approach to incorporate endogenous variables in a two-stage least squares (TSLS) framework.

Consider the structural equation of interest

$$y_{it} = \theta_{g_i,t} + X_{it}\beta + \epsilon_{it}, \tag{15}$$

where X_{it} is a $K \times 1$ vector of variables, (some of) which may be correlated with ϵ_{it} and thus endogeneous. The conditional expectation of X_{it} can be written as the following reduced form model given a $P \times 1$ vector of variables Z_{it}:

$$X_{it} = f(Z_{it}, \pi) + u_{it}, \tag{16}$$

where π is an $M \times 1$ vector of parameters, $f(\cdot, \cdot)$ is a presumed known function that maps $\mathcal{R}^P \times \mathcal{R}^M$ to $\mathcal{R}^K$, and u_{it} is $K \times 1$ satisfying $E(u_{it}|Z_{it}) = 0$. This suggests that Z_{it} can serve as valid instruments for $K \times 1$ endogenous variables X_{it}, if $\text{rank}(Z_{it}) \geq \text{rank}(X_{it})$. The reduced form (16) can be applied to widely used econometric models, such as linear or threshold regressions, etc. In the following analysis, we use linear pooled regressions as an example, and in this case we have:

$$X_{it} = \alpha + Z_{it}'\gamma + u_{it}. \tag{17}$$

Note that we do not allow for endogeneity in the group membership determination, namely that the group membership is not determined by endogenous variables. Neither do we allow for a group pattern of heterogeneity in the reduced form model (16). Moreover, the standard rank condition must be satisfied to identify the structure parameters.

For this system, we can estimate the group membership, group fixed effects, and slope coefficients jointly by minimizing the following objective function as

$$(\tilde{\theta}, \tilde{\beta}, \tilde{g}) = \arg\min_{\theta,\beta,g} \sum_{i=1}^{N} \sum_{t=1}^{T} \left(y_{it} - \hat{X}_{it}'\beta - \theta_{g_i,t}\right)^2, \tag{18}$$

where $\hat{X}_{it}$ is the predicted value of X_{it} based on the first-stage regression model (17). To solve this optimization problem, one can again employ the iterative algorithm that iteratively estimates the group membership parameter g and slope and intercept parameters. In particular, we can modify Algorithm 1 as follows:

Algorithm 2

1. Compute: $\hat{X}_{it} = \hat{\alpha} + Z_{it}'\hat{\gamma}$, where $\hat{\alpha}$ and $\hat{\gamma}$ are the least squares estimates from (17).

2. Let $g^{(0)}$ be an initial value of grouping. Set $s = 0$.

3. For the given $g^{(s)}$ and $\hat{X}_{it}$, compute:

$$(\theta^{(s+1)}, \beta^{(s+1)}) = \arg\min_{\beta,\theta} \sum_{i=1}^{N} \sum_{t=1}^{T} \left(y_{it} - \hat{X}_{it}'\beta - \theta_{g_i,t}\right)^2,$$

where $\hat{X}_{it}$ is the predicted value of X_{it} from the first stage regression of X_{it} on Z_{it}.

4. Compute for all $i \in \{1, \ldots, N\}$:

$$g_i^{(s+1)} = \arg \min_{g \in \{1,\ldots,G\}} \sum_{t=1}^{T} \left(y_{it} - \hat{X}_{it}' \beta^{(s+1)} - \theta_{g_i,t}^{(s+1)} \right)^2 .$$

5. Set $s = s + 1$ and go to Step 3 (until numerical convergence).

Our TSLS approach complements the IV estimation method for GFE briefly discussed in Bonhomme and Manresa (2015b), because we address endogeneity in both group membership and coefficient estimation. Bonhomme and Manresa (2015b) first estimate group memberships in a GFE regression with endogenous variables using least squares as in equation (9), and then use this group membership in the IV regression. In some situations, the endogeneous variables can influence the estimation of group memberships either directly or indirectly via slope coefficients, leading to inconsistent membership estimates. Using inconsistent group membership estimates further leads to inconsistent coefficient estimates. In contrast, we explicitly address the endogeneity issue in estimating both group memberships and slope coefficients in a TSLS framework, and thus obtain consistent estimates.

We show the effectiveness of this procedure through simulations. We generate the data from the linear panel data model with time-varying heterogeneous shocks as in our benchmark simulation design (11), except that now $X_{1,it}$ is endogenous. We generate $X_{1,it} = Z_{it} + \varepsilon_{it}/5$, where Z_{it} is independent of ε_{it} and generated by $Z_{it} = c_\tau \tau_{1,g_i,t} + \nu_{it}$ with the same parameterization as in Section 3. The remaining parameters are also set in the same way as in Section 3, and we focus on DGP1, the case of frequent but moderate shocks. Coefficients are estimated using TSLS.

We evaluate the performance of GFE estimators using the misclassification frequency, bias, and RMSE, and compare with the TSLS with two-way fixed effect estimator. Table 12 shows that the misclassification frequency is slightly higher than in the cases of exogenous regressors (see Table 3). This could be due to an efficiency loss caused by the two-stage estimation. Nevertheless, GFE still correctly classifies more than 80% of units even in the smallest sample with $T = 15$. The misclassification rate decreases as T increases, confirming the consistency of group membership estimates, but it does not decrease as N increases, as expected. The accuracy of GFE coefficient estimates improves as either N or T increases. On the contrary, the estimates produced by TFE are severely biased. Increasing the sample size does not seem to help to reduce the bias and RMSE of the TFE estimates.

In principle, we can replace the two-stage least squares estimation by (optimal) GMM. In that case, the estimation of the optimal weight matrix requires a careful treatment since it depends on the group membership estimates.

5.2 Standard errors

Petersen (2009) highlights the importance of using the correct methods to estimate standard errors. Here we discuss the standard error estimation in the presence of group fixed effects. Our discussion is twofold. First, we investigate the properties of standard error estimates produced by GFE. Second, we discuss how other standard error estimates perform, if the data generating process is characterized by group-specific heterogeneous responses to shocks.

There are two versions of standard errors for the GFE method, an asymptotic standard error and a finite sample standard error. The asymptotic standard error estimate is identical to White's (1984) robust standard error for OLS estimators under true group membership (see also Corollary 1 of Bonhomme and Manresa (2015a)). This asymptotic estimate is often a good proxy because the group membership is estimated super-consistently, and the estimation errors caused by unknown grouping can be ignored. When the time dimension is particularly short, the finite sample standard error estimates need to be used to account for the group estimation error.[24] Since most financial data have moderate time series, our discussion here focuses on asymptotic standard error estimates of GFE.

The GFE asymptotic standard error is unbiased in the presence of fixed firm and homogeneous time effects, because α_i and $\theta_{g,t}$ control the firm and time effects, leaving an idiosyncratic error term. The GFE asymptotic standard error is also unbiased if the time effect is group-specifically heterogeneous, i.e., if the firm effect changes over time but in a group pattern. Such time-varying group effect is captured by $\theta_{g,t}$, and thus leaves only the idiosyncratic component in the error term.

Next, we discuss the properties of other standard error estimates in the presence of group-specific heterogeneous responses. The standard errors of OLS, fixed effects (FE), and TFE are all biased to different extents. The bias of FE standard errors has been demonstrated by equation (22) in Appendix A.1. Given the bias of FE standard errors, we can expect that the OLS standard errors are even more biased, because OLS fails to take into account both firm and group fixed effects. We

[24]There are two ways to compute the finite sample standard error estimates. One can use the analytical formula proposed in Section 2.2.2 of Bonhomme and Manresa (2015a) or bootstrap the standard error estimate by resampling unit specific blocks of observations from the original sample.

show the bias of TFE standard errors using the case with only one explanatory variable. The within individual-time transformed variables are denoted by $\widetilde{X}_{it}$ and $\widetilde{\theta}_{g_i,t}$. Then the variance of the TFE estimator is

$$
\begin{aligned}
var(\hat{\beta}_{TFE}) &= \left[\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\widetilde{X}_{it}(\widetilde{\theta}_{g_i,t}+\widetilde{\epsilon}_{it})\right)^2\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\widetilde{X}_{it}^2\right)^{-2}\right] \\
&= \left(\sum_{t=1}^{T}\sum_{i=1}^{N}\widetilde{X}_{it}^2(\widetilde{\theta}_{g_i,t}+\widetilde{\epsilon}_{it})^2+\sum_{t=1}^{T}\sum_{i\neq j}\widetilde{X}_{it}\widetilde{X}_{jt}(\widetilde{\theta}_{g_i,t}+\widetilde{\epsilon}_{it})(\widetilde{\theta}_{g_j,t}+\widetilde{\epsilon}_{jt})\right)\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\widetilde{X}_{it}^2\right)^{-2} \\
&= \left[\sum_{t=1}^{T}\sum_{i=1}^{N}\widetilde{X}_{it}^2\widetilde{\epsilon}_{it}^2+C\right]\left(NT\sigma_X^2\right)^{-2} \\
&= \frac{\sigma_\epsilon^2}{NT\sigma_X^2}\left[1+NT\sigma_X^2 C\right], \qquad (19)
\end{aligned}
$$

where $C = \sum_{t=1}^{T}\sum_{i=1}^{N}\widetilde{X}_{it}^2\widetilde{\theta}_{g_i,t}^2 + 2\widetilde{X}_{it}\widetilde{\theta}_{g_i,t}\widetilde{\epsilon}_{it} + \sum_{t=1}^{T}\sum_{i\neq j}\widetilde{X}_{it}\widetilde{X}_{jt}(\widetilde{\theta}_{g_i,t}+\widetilde{\epsilon}_{it})(\widetilde{\theta}_{g_j,t}+\widetilde{\epsilon}_{jt})$, which is nonzero as long as $\widetilde{X}_{it}$ and $\widetilde{\theta}_{g_i,t}$ are correlated.

Table 13 presents the standard errors structure in the presence of group fixed effects. This structure resembles that of two-way fixed effects except that the firms in distinct groups are not correlated even at the same time point, because the time shocks impose heterogeneous effects across groups (assuming groups are independent). In the presence of group fixed effects, standard errors only clustered by firm are clearly biased because it imposes that residuals between firms are completely uncorrelated. In contrast, standard errors clustered by both firm and time are unbiased, but can be inefficient because they allow all firms (across groups) to be correlated at the same time. In Appendix A.1, we also discuss the issue of standard errors for event studies, which are widely used in empirical studies, in the presence of group heterogeneous time effects.

6 Conclusions

Firms are often exposed to market-wide shocks such as regulatory changes and financial crises. Failing to appropriately incorporate the effects of these market-wide shocks in regression analysis can lead to inconsistent estimates of treatment effects and can affect statistical inference. The most widely used approach to model market-wide shocks is the two-way fixed effects model, which assumes that all firms or individuals respond homogeneously to these common shocks. However, it is well-documented that regulatory changes like the Sarbanes-Oxley Act and financial crises have heterogenous impact on firms outcomes. In this paper, we demonstrate how ignoring this heterogeneity results in biased slope coefficient estimates, when heterogeneous shocks are correlated with explanatory variables. More specifically, we formally quantify the bias of the estimates of existing models in using two-way fixed effects and interacted industry-year fixed effects models.

We propose the use of the "grouped fixed effects" estimator to account for heterogenous responses to market-wide shocks. The group fixed effect estimators assume that there is an underlying latent group pattern for the population of firms. Within the group, firms respond similarly to each shock, while they respond differently across groups. One important advantage of this approach is that the group structure can be consistently estimated from the data, jointly with the other regression parameters. Therefore, no a priori knowledge of grouping is required. This avoids possible estimate inconsistency caused by the misspecification of the group structure.

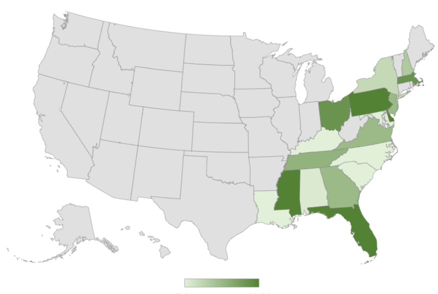
We theoretically compare the GFE estimators with popular alternatives, such as two-way fixed effects and interacted fixed effects with artificially imposed group structure (e.g. industry-based grouping). We show that two-way fixed effect estimates are inconsistent if firms respond differently to the shocks. On the contrary, grouped fixed effects models always produce consistent slope coefficient estimates, when the number of groups is not under-specified. GFE is asymptotically equivalent to the interacted fixed effects model, if the imposed group structure in IFE happens to coincide with the data generating process. However, if the group structure of IFE is misspecified, IFE estimates are no longer consistent. We provide guidelines on how to appropriately correct for standard errors based on the group structure. We propose new methods to select between GFE and TFE/IFE model specifications and to estimate group membership and coefficient parameters in the presence of endogenous explanatory variables.

We conduct a large number of Monte Carlo simulation experiments, controlling for the size and the frequency of shocks, the degree of heterogeneity among units, the correlation between responses and regressors, the number of groups, etc. Simulation results confirm our theoretical argument that conventional models are biased to different extents, depending on the feature of shocks, the correlation, and specification of groups, etc. On the contrary, GFE remains a consistent approach even when all firms respond homogeneously.

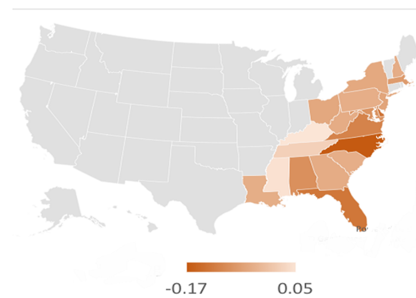
We demonstrate the economic importance of accounting for heterogeneity through an empirical model of CEO attributes and corporate innovation. We find that accounting for time-varying heterogeneity plays an important role in practice. Our findings and the proposed GFE estimator are likely to be of increasing importance and of practical use to empirical researchers.

Figure 1
Firms' sales growth responses to natural disasters in 2004

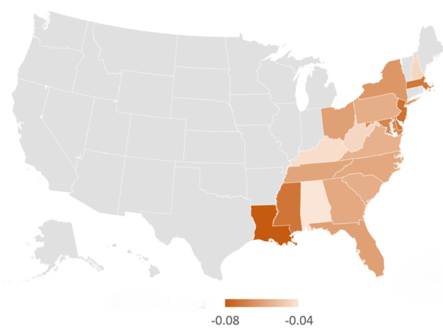
The figure presents the effect of natural disasters on employment and the estimated effect on sales growth in 2004. Panel A shows the share of total U.S. employment affected by 4 natural disasters with damages exceeding \$1 billion in 2004: Hurricane Charley, Hurricane Frances, Hurricane Ivan, and Hurricane Jeanne. Employment numbers are computed from the County Business Pattern data, provided by the U.S. Census Bureau. Panels B, C, and D present the time effect estimates of the sales growth regression using GFE, TFE and IFE respectively for firms in the affected states in 2004, estimated for the sample period from 1987-2010 using data from Compustat. Figure B shows the estimate $\theta_{g_i,t}$ for GFE: $y_{it} = \alpha_i + \theta_{g_i,t} + X_{it}'\beta + \epsilon_{it}, \quad g_i \in \{1, ..., G\}$. Figure C shows the estimate θ_t for TFE $y_{it} = \alpha_i + \theta_t + X_{it}'\beta + \epsilon_{it}$. Figure D shows the estimate $\theta_{industry,i,t}$ for IFE $y_{it} = \alpha_i + \theta_{industry,i,t} + X_{it}'\beta + \epsilon_{it}$. y_{it} is log annual sale changes and X_{it} is a 6×1 vector of variables: natural log total assets, Tobin's q, return on assets, capital expenditure, leverage, and cash flow. Table A.2 in the Appendix provides all variable descriptions. Darker colors indicate stronger effects.



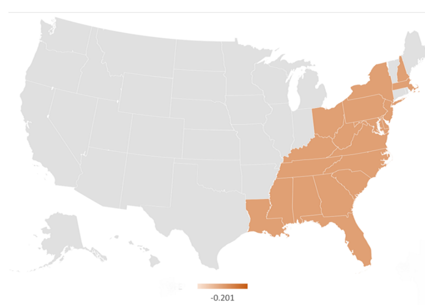
Panel A: Percentage of affected employment



Panel B: GFE $\theta_{g_i,t}$



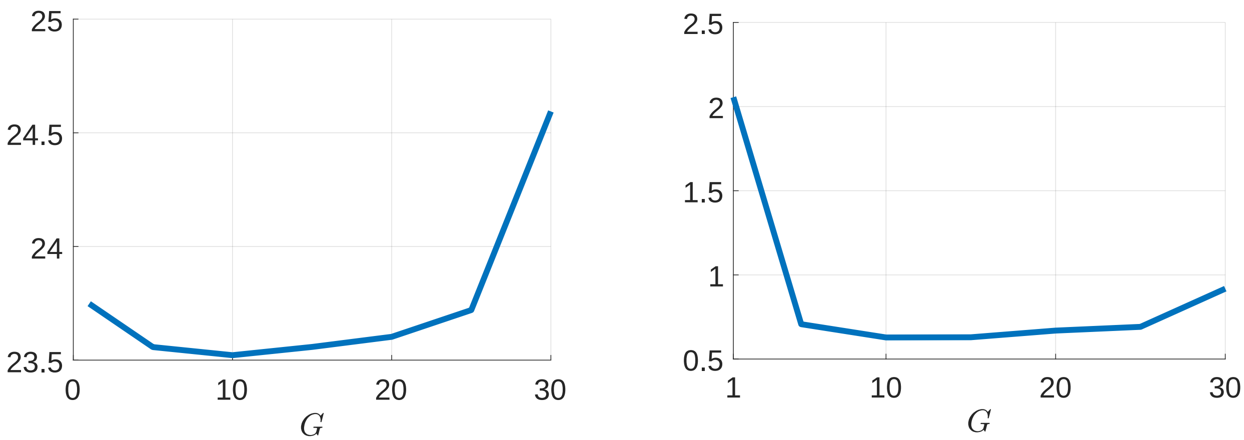
Panel C: TFE θ_t



Panel D: IFE $\theta_{industry,i,t}$

Figure 2
Bayesian information criterion and number of groups

The figure presents the Bayesian information criterion (BIC) for different numbers of groups for the empirical examples. Figure A presents the BIC for the sales growth regression: $\ln(\text{Sales growth})_{i,t} = \alpha_i + \theta_{g_{i,t}} + X'_{i,t-1}\beta + \epsilon_{i,t},\ g_i \in \{1, ..., G\}$, where X_{it} is a 6×1 vector of variables: natural log total assets, Tobin's q, return on assets, capital expenditure, leverage, and cash flow. Figure B presents the BIC for the pilot CEO regression: $\ln(1+\text{citation})_{it} = \alpha_{ind} + \theta_{g_{i,t}} + X'_{i,t}\beta + \epsilon_{i,t},\ g_i \in \{1, ..., G\}$. $\ln(1+\text{citation})_{it}$ is the natural log of the number of raw citations adjusted by the weighting index of Hall et al. (2001) to all the patents applied for by firm i in year t, α_{ind} is the industry fixed effect, $\theta_{g_{i,t}}$ is the time-varying group effect, and X_{it} is a 17×1 vector of variables: pilot CEO dummy, natural log total assets, ratio of net property, plant, and equipment over the number of employees, stock returns, Tobin's q, institutional holdings, CEO tenure, CEO stock and option delta and vega, CEO age, CEO overconfidence, and dummy variables for: top university, PhD, no school information, finance education, technical education, CEO military background. The x-axis is the number of groups, and the y-axis is the BIC. Table A.2 in the Appendix provides all variable descriptions.



(a) Natural disaster

(b) Pilot CEO

Table 3
Frequent moderate shocks

The table presents the results for simulations under DGP1 for different econometric specifications. DGP1 is generated using the procedure described in Section 3, where shocks occur at every time t and heterogenous responses to shocks are allowed. The parameters in the DGP are set as $c_\tau = 0.5$, $c_\theta = 15$, and $G = 5$. c_τ is a constant which determines the degree of correlation between X and θ, c_θ is a constant which controls the size of shocks, G is the number of groups. We present the bias: $\text{Bias}(\hat{\beta}) = \frac{1}{R}\sum_{r=1}^{R}\hat{\beta}^r - \beta^0$, the estimated standard deviation (Std.): $\text{Std}(\hat{\beta}) = \sqrt{\frac{1}{R-1}\sum_{r=1}^{R}(\hat{\beta}^r - \bar{\hat{\beta}})^2}$, and the root mean square error (RMSE): $\text{RMSE}(\hat{\beta}) = \sqrt{\frac{1}{R}\sum_{r=1}^{R}\|\hat{\beta}^r - \beta^0\|^2}$ of estimated coefficients, where $\hat{\beta}^r$ is the estimator in the r-th replication, β^0 is the true value, and $\bar{\hat{\beta}} = 1/R\sum_{r=1}^{R}\hat{\beta}^r$ is the sample average across replications. Misclassification frequency (Misclass. Freq.) is the average proportion of units that are misclassified by GFE across replications: $AMF = 1/R\sum_{r=1}^{R}\text{MF}_r$, where R is the number of replications and $\text{MF}_r = \frac{1}{N}\sum_{i=1}^{N}\text{I}(\hat{g}_i \neq g_i^0)$. The table reports the results using the following methods: grouped fixed effects (GFE), two-way fixed effects (TFE), and group-time interacted fixed effects (IFE). The IFE method includes 4 different specifications: IFE_{gt} exactly specifies the group memberships of all individuals and the timing of all shocks (perfect hindsight), IFE_g exactly specifies the groups but correctly specifies the shocks every 1/3 of the time, IFE_t correctly specifies the shocks but not the groups, and IFE_{null} misspecifies both groups and shocks. When the groups are misspecified, we assign 30% of individuals to the closest neighboring groups. The details of these methods are presented in Section 3.1.2. N is the number of units, and T is the number of time series observations. Panels A to C report results for three different sample sizes N and T. Each simulation consists of 1000 replications.

		GFE	TFE	IFE_{gt}	IFE_g	IFE_t	IFE_{null}
Panel A. $N = 1000$, $T = 15$							
Misclass. Freq.		0.1491					
β_1	Bias	0.0109	0.5954	0.0002	0.5125	0.2548	0.5693
	Std.	0.0119	0.2057	0.0108	0.2203	0.1061	0.2148
β_2	Bias	0.0102	0.5991	−0.0006	0.5145	0.2590	0.5720
	Std.	0.0124	0.2102	0.0109	0.2206	0.1089	0.2133
β	RMSE	0.0183	0.8858	0.0144	0.9898	1.0035	0.8500
Panel B. $N = 1000$, $T = 30$							
Misclass. Freq.		0.0739					
β_1	Bias	0.0054	0.6107	0.0004	0.6336	0.2625	0.6577
	Std.	0.0075	0.1383	0.0072	0.1465	0.0733	0.1449
β_2	Bias	0.0051	0.6110	0.0002	0.6347	0.2591	0.6588
	Std.	0.0077	0.1381	0.0072	0.1453	0.0719	0.1433
β	RMSE	0.0120	0.8869	0.0102	0.9235	0.3831	0.9558
Panel C. $N = 2000$, $T = 15$							
Misclass. Freq.		0.1460					
β_1	Bias	0.0096	0.6045	0.0005	0.5208	0.2571	0.5778
	Std.	0.0093	0.2006	0.0077	0.1978	0.1055	0.1950
β_2	Bias	0.0092	0.6072	−0.0002	0.5217	0.2666	0.5806
	Std.	0.0089	0.2064	0.0072	0.2102	0.1088	0.2052
β	RMSE	0.0147	0.9008	0.0105	0.7929	0.3969	0.8664

Table 4
Sparse moderate shocks

The table presents the results for simulations under DGP2 for different econometric specifications. DGP2 is generated using the procedure described in Section 3, where shocks occur 25% of the time and heterogenous responses to shocks are allowed. The parameters in the DGP are set as $c_\tau = 0.5$, $c_\theta = 15$, and $G = 5$. c_τ is a constant which determines the degree of correlation between X and θ, c_θ is a constant which controls the size of shocks, G is the number of groups. We present the bias: $\text{Bias}(\hat{\beta}) = \frac{1}{R}\sum_{r=1}^{R}\hat{\beta}^r - \beta^0$, the estimated standard deviation (Std.): $\text{Std}(\hat{\beta}) = \sqrt{\frac{1}{R-1}\sum_{r=1}^{R}(\hat{\beta}^r - \bar{\hat{\beta}})^2}$, and the root mean square error (RMSE): $\text{RMSE}(\hat{\beta}) = \sqrt{\frac{1}{R}\sum_{r=1}^{R}\|\hat{\beta}^r - \beta^0\|^2}$ of estimated coefficients, where $\hat{\beta}^r$ is the estimator in the r-th replication, β^0 is the true value, and $\bar{\hat{\beta}} = 1/R\sum_{r=1}^{R}\hat{\beta}^r$ is the sample average across replications. Misclassification frequency (Misclass. Freq.) is the average proportion of units that are misclassified by GFE across replications: $AMF = 1/R\sum_{r=1}^{R}\text{MF}_r$, where R is the number of replications and $\text{MF}_r = \frac{1}{N}\sum_{i=1}^{N}I(\hat{g}_i \neq g_i^0)$. The table reports the results using the following methods: grouped fixed effects (GFE), two-way fixed effects (TFE), and group-time interacted fixed effects (IFE). The IFE method includes 5 different specifications: IFE_{gt} exactly specifies the group memberships of all individuals and the timing of all shocks, IFE_g exactly specifies the groups but correctly specifies the shocks every 1/3 of the time, IFE_{got} exactly specifies the groups but over-specifies the shocks at each time period, IFE_t correctly specifies the shocks but not the groups, and IFE_{null} misspecifies both groups and shocks. When the groups are misspecified, we assign 30% of individuals to the closest neighboring groups. The details of these methods are presented in Section 3.1.2. N is the number of units, and T is the number of time series observations. Panels A to C report results for different N and T. Each simulation consists of 1000 replications.

		GFE	TFE	IFE_{gt}	IFE_g	IFE_{got}	IFE_t	IFE_{null}
				Panel A. $N = 1000$, $T = 15$				
Misclass. Freq.		0.0953						
β_1	Bias	0.0032	0.3584	−0.0002	0.2712	−0.0001	0.1340	0.3178
	Std.	0.0130	0.3537	0.0093	0.3835	0.0105	0.1328	0.3609
β_2	Bias	0.0035	0.3386	−0.0008	0.2410	−0.0005	0.1268	0.2897
	Std.	0.0161	0.3191	0.0086	0.3345	0.0100	0.1257	0.3159
β	RMSE	0.0212	0.6852	0.0127	0.6246	0.0144	0.2596	0.6438
				Panel B. $N = 1000$, $T = 30$				
Misclass. Freq.		0.0482						
β_1	Bias	0.0018	0.4515	0.0003	0.4489	0.0001	0.1657	0.4744
	Std.	0.0067	0.2559	0.0062	0.3171	0.0068	0.0978	0.3068
β_2	Bias	0.0016	0.4450	0.0001	0.4425	0.0003	0.1618	0.4672
	Std.	0.0081	0.2639	0.0062	0.3214	0.0071	0.0973	0.3069
β	RMSE	0.0114	0.7326	0.0087	0.7751	0.0098	0.2695	0.7945
				Panel C. $N = 2000$, $T = 15$				
Misclass. Freq.		0.0973						
β_1	Bias	0.0027	0.3636	0.0004	0.2754	0.0004	0.1280	0.3186
	Std.	0.0097	0.3438	0.0063	0.3622	0.0072	0.1240	0.3456
β_2	Bias	0.0023	0.3682	−0.0005	0.2866	−0.0005	0.1262	0.3316
	Std.	0.0125	0.3216	0.0063	0.3655	0.0071	0.1178	0.3381
β	RMSE	0.0192	0.6993	0.0089	0.6498	0.0101	0.2480	0.6669

Table 5
Homogeneous shocks

The table presents the results for simulations under DGP3 for different econometric specifications. DGP3 is generated using the procedure described in Section 3, where shocks occur at every time t and there is a homogenous response to the shock. The parameters in the DGP are set as $c_\tau = 0.5$, $c_\theta = 15$, and $G = 1$. c_τ is a constant which determines the degree of correlation between X and θ, c_θ is a constant which controls the size of shocks, G is the number of groups.We present the bias: Bias$(\hat{\beta}) = \frac{1}{R}\sum_{r=1}^{R}\hat{\beta}^r - \beta^0$, the estimated standard deviation (Std.): Std$(\hat{\beta}) = \sqrt{\frac{1}{R-1}\sum_{r=1}^{R}(\hat{\beta}^r - \bar{\hat{\beta}})^2}$, and the root mean square error (RMSE): RMSE$(\hat{\beta}) = \sqrt{\frac{1}{R}\sum_{r=1}^{R}\|\hat{\beta}^r - \beta^0\|^2}$ of estimated coefficients, where $\hat{\beta}^r$ is the estimator in the r-th replication, β^0 is the true value, and $\bar{\hat{\beta}} = 1/R\sum_{r=1}^{R}\hat{\beta}^r$ is the sample average across replications. The table reports the results using the following methods: grouped fixed effects (GFE), two-way fixed effects (TFE), and group-time interacted fixed effects (IFE). GFE is estimated under two specifications of groups, $\hat{G} = 1$ and $\hat{G} = 4$. The IFE method includes 4 different specifications: IFE$_{gt}$ exactly specifies the group memberships of all individuals and the timing of all shocks, IFE$_g$ exactly specifies the groups but correctly specifies the shocks every 1/3 of the time, IFE$_t$ correctly specifies the shocks but not the groups, and IFE$_{null}$ misspecifies both groups and shocks. The details of these methods are presented in Section 3.1.2. N is the number of units, and T is the number of time series observations. Panels A to C report results for different N and T. Each simulation consists of 1000 replications.

		GFE ($\hat{G} = 1$)	GFE ($\hat{G} = 4$)	TFE	IFE$_{gt}$	IFE$_g$	IFE$_t$	IFE$_{null}$
		Panel A. $N = 1000$, $T = 15$						
β_1	Bias	0.0003	0.0007	0.0003	0.0003	0.0557	0.0003	0.0531
	Std.	0.0106	0.0095	0.0106	0.0106	0.0632	0.0106	0.0384
β_2	Bias	−0.0002	0.0000	−0.0002	−0.0002	0.0542	−0.0001	0.0494
	Std.	0.0110	0.0095	0.0110	0.0110	0.0633	0.0110	0.0381
β	RMSE	0.0152	0.0167	0.0152	0.0152	0.1184	0.0153	0.0905
		Panel B. $N = 1000$, $T = 30$						
β_1	Bias	0.0002	0.0003	0.0002	0.0002	0.0666	0.0002	0.0658
	Std.	0.0073	0.0069	0.0073	0.0073	0.0385	0.0073	0.0289
β_2	Bias	0.0001	0.0000	0.0001	0.0001	0.0667	0.0001	0.0669
	Std.	0.0071	0.0069	0.0071	0.0071	0.0402	0.0071	0.0275
β	RMSE	0.0102	0.0106	0.0102	0.0102	0.1089	0.0102	0.1006
		Panel C. $N = 2000$, $T = 15$						
β_1	Bias	0.0000	−0.0001	0.0000	0.0000	0.0566	0.0000	0.0527
	Std.	0.0071	0.0068	0.0071	0.0071	0.0627	0.0071	0.0362
β_2	Bias	0.0000	−0.0001	0.0000	0.0000	0.0535	0.0000	0.0490
	Std.	0.0074	0.0068	0.0074	0.0074	0.0623	0.0074	0.0344
β	RMSE	0.0102	0.0113	0.0102	0.0102	0.1177	0.0103	0.0875

Table 6
Large and highly correlated shocks

The table presents the results for simulations under DGP1 for different shock and correlation sizes across different econometric specifications. DGP1 is generated using the procedure described in Section 3, where shocks occur at every time t and heterogenous responses to shocks are allowed. We present the bias: Bias($\hat{\beta}$) = $\frac{1}{R}\sum_{r=1}^{R}\hat{\beta}^r - \beta^0$, the estimated standard deviation (Std.): Std($\hat{\beta}$) = $\sqrt{\frac{1}{R-1}\sum_{r=1}^{R}(\hat{\beta}^r - \bar{\hat{\beta}})^2}$, and the root mean square error (RMSE): RMSE($\hat{\beta}$) = $\sqrt{\frac{1}{R}\sum_{r=1}^{R}\|\hat{\beta}^r - \beta^0\|^2}$ of estimated coefficients, where $\hat{\beta}^r$ is the estimator in the r-th replication, β^0 is the true value, and $\bar{\hat{\beta}} = 1/R\sum_{r=1}^{R}\hat{\beta}^r$ is the sample average across replications. Misclassification frequency (Misclass. Freq.) is the average proportion of units that are misclassified by GFE across replications: $AMF = 1/R\sum_{r=1}^{R}\mathrm{MF}_r$, where R is the number of replications and $\mathrm{MF}_r = \frac{1}{N}\sum_{i=1}^{N}\mathrm{I}(\hat{g}_i \neq g_i^0)$. The table reports the results using the following methods: grouped fixed effects (GFE), two-way fixed effects (TFE), and group-time interacted fixed effects (IFE). The IFE method includes 4 different specifications: IFE_{gt} exactly specifies the group memberships of all individuals and the timing of all shocks, IFE_g exactly specifies the groups but correctly specifies the shocks every 1/3 of the time, IFE_t correctly specifies the shocks but not the groups, and IFE_{null} misspecifies both groups and shocks. When the groups are misspecified, we assign 30% of individuals to the closest neighboring groups. The details of these methods are presented in Section 3.1.2. Panel A presents the results for $c_\tau = 0.5$, a larger time series shock $c_\theta = 5$, and $G = 5$, and Panel B presents the results for a larger correlation c_τ=1. c_τ is a constant which determines the degree of correlation between X and θ, c_θ is a constant which controls the size of shocks, G is the number of groups. We report the results for 1000 cross-sectional units (N), 30 time periods (T), and 5 groups (G). Each simulation consists of 1000 replications.

($N = 1000$, $T = 30$)		GFE	TFE	IFE_{gt}	IFE_g	IFE_t	IFE_{null}
Panel A. Large shocks: $c_\tau = 0.5$, $c_\theta = 5$, and $G = 5$							
Misclass. Freq.		0.0061					
β_1	Bias	0.0012	1.8553	−0.0002	1.9234	0.8076	1.9969
	Std.	0.0073	0.4343	0.0068	0.4613	0.2225	0.4565
β_2	Bias	0.0013	1.8431	0.0000	1.9104	0.7990	1.9844
	Std.	0.0076	0.4295	0.0073	0.4588	0.2189	0.4542
β	RMSE	0.0107	2.6855	0.0100	2.7878	1.1781	2.8878
Panel B. Highly correlated shocks: $c_\tau = 1$, $c_\theta = 15$, and $G = 5$							
Misclass. Freq.		0.1385					
β_1	Bias	0.0148	0.6809	−0.0004	0.6930	0.3723	0.7051
	Std.	0.0091	0.1079	0.0071	0.1174	0.0828	0.1119
β_2	Bias	0.0144	0.6833	−0.0004	0.6961	0.3745	0.7085
	Std.	0.0087	0.1101	0.0069	0.1161	0.0826	0.1125
β	RMSE	0.0242	0.9769	0.0099	0.9960	0.5409	1.0120

Table 7
Under-specify G

The table presents the results for simulations under DGP1 for an underspecified number of groups G for GFE and TFE. DGP1 is generated using the procedure described in Section 3, where shocks occur at every time t and heterogenous responses to shocks are allowed. The parameters in the DGP are set as $c_\tau = 0.5$, $c_\theta = 15$, and $G = 4$. c_τ is a constant which determines the degree of correlation between X and θ, c_θ is a constant which controls the size of shocks, G is the number of groups. We present the bias: Bias($\hat{\beta}$) $= \frac{1}{R}\sum_{r=1}^{R}\hat{\beta}^r - \beta^0$, the estimated standard deviation (Std.): Std($\hat{\beta}$) $= \sqrt{\frac{1}{R-1}\sum_{r=1}^{R}(\hat{\beta}^r - \bar{\hat{\beta}})^2}$, and the root mean square error (RMSE): RMSE($\hat{\beta}$) $= \sqrt{\frac{1}{R}\sum_{r=1}^{R}\|\hat{\beta}^r - \beta^0\|^2}$ of estimated coefficients, where $\hat{\beta}^r$ is the estimator in the r-th replication, β^0 is the true value, and $\bar{\hat{\beta}} = 1/R\sum_{r=1}^{R}\hat{\beta}^r$ is the sample average across replications. The table reports the results using grouped fixed effects (GFE) and two-way fixed effects (TFE). The estimates are generated using two groups, $\widehat{G} = 2$, where the true DGP has G=4. The details of these methods are presented in Section 3.1.2. N is the number of units, and T is the number of time series observations. Each simulation consists of 1000 replications.

		$N = 1000, T = 15$		$N = 1000, T = 30$		$N = 2000, T = 15$	
		GFE	TFE	GFE	TFE	GFE	TFE
β_1	Bias	0.0799	0.2772	0.0808	0.2848	0.0781	0.2833
	Std.	0.0397	0.1082	0.0277	0.0793	0.0365	0.1027
β_2	Bias	0.0778	0.2762	0.0819	0.2826	0.0777	0.2782
	Std.	0.0416	0.1061	0.0266	0.0764	0.0391	0.1117
β	RMSE	0.1255	0.4196	0.1213	0.4160	0.1224	0.4250

Table 8
Large number of groups

The table presents the results for simulations under DGP1 for two different number of groups G and different econometric specifications. DGP1 is generated using the procedure described in Section 3, where shocks occur at every time t and heterogenous responses to shocks are allowed. The parameters in the DGP are set as $c_\tau = 0.5$, $c_\theta = 15$. c_τ is a constant which determines the degree of correlation between X and θ, c_θ is a constant which controls the size of shocks. We consider the number of groups $G = 10$ and $G = 20$ in DGP. We present the bias: $\text{Bias}(\hat{\beta}) = \frac{1}{R}\sum_{r=1}^{R}\hat{\beta}^r - \beta^0$, the estimated standard deviation (Std.): $\text{Std}(\hat{\beta}) = \sqrt{\frac{1}{R-1}\sum_{r=1}^{R}(\hat{\beta}^r - \bar{\hat{\beta}})^2}$, and the root mean square error (RMSE): $\text{RMSE}(\hat{\beta}) = \sqrt{\frac{1}{R}\sum_{r=1}^{R}\|\hat{\beta}^r - \beta^0\|^2}$ of estimated coefficients, where $\hat{\beta}^r$ is the estimator in the r-th replication, β^0 is the true value, and $\bar{\hat{\beta}} = 1/R\sum_{r=1}^{R}\hat{\beta}^r$ is the sample average across replications. Misclassification frequency (Misclass. Freq.) is the average proportion of units that are misclassified by GFE across replications: $AMF = 1/R\sum_{r=1}^{R}\text{MF}_r$, where R is the number of replications and $\text{MF}_r = \frac{1}{N}\sum_{i=1}^{N} I(\hat{g}_i \neq g_i^0)$. The table reports the results using the following methods: grouped fixed effects (GFE), two-way fixed effects (TFE), and group-time interacted fixed effects (IFE). The IFE method includes 4 different specifications: IFE_{gt} exactly specifies the group memberships of all individuals and the timing of all shocks, IFE_g exactly specifies the groups but correctly specifies the shocks every 1/3 of the time, IFE_t correctly specifies the shocks but not the groups, and IFE_{null} misspecifies both groups and shocks. When the groups are misspecified, we assign 30% of individuals to the closest neighboring groups. The details of these methods are presented in Section 3.1.2. We report the results for 1000 cross-sectional units (N) and 30 time periods (T). Each simulation consists of 1000 replications.

		GFE	TFE	IFE_{gt}	IFE_g	IFE_t	IFE_{null}
			$G = 10$ $(N = 1000,\ T = 30)$				
Misclass. Freq.		0.0734					
β_1	Bias	0.0040	5.0593	−0.0002	4.9302	2.8958	5.0519
	Std.	0.0073	0.6131	0.0071	0.6419	0.4988	0.6227
β_2	Bias	0.0048	5.0731	0.0006	4.9358	2.8899	5.0528
	Std.	0.0079	0.5423	0.0077	0.5978	0.4745	0.5704
β	RMSE	0.0124	7.2112	0.0105	7.0312	4.1485	7.1947
			$G = 20$ $(N = 1000,\ T = 30)$				
Misclass. Freq.		0.0613					
β_1	Bias	0.0051	5.0402	0.0011	4.9088	2.8528	5.0238
	Std.	0.0078	0.6268	0.0075	0.6510	0.4893	0.6313
β_2	Bias	0.0039	5.0259	−0.0002	4.8785	2.8488	4.9996
	Std.	0.0076	0.5917	0.0075	0.6173	0.4883	0.6025
β	RMSE	0.0126	7.1697	0.0107	6.9785	4.0904	7.1411

Table 9
Sales growth and major natural disasters

The table presents the relation between sales growth and natural disasters. Panel A reports results from univariate regressions of TFE, IFE and GFE estimated time effects on the percentage of employment affected by U.S. major disasters (in %), $\theta = \alpha + \beta A.Employment_{it} + \epsilon_{i,t}$, where i is the state and θ is the estimated year/state effect from the following regressions regressions: θ_t is the TFE estimate from $y_{it} = \alpha_i + \theta_t + X'_{it}\beta + \epsilon_{it}$, $\theta_{industry,i,t}$ are the IFE estimates from $y_{it} = \alpha_i + \theta_{industry,i,t} + X'_{it}\beta + \epsilon_{it}$, and $\theta_{g_i,t}$ are the GFE estimates from $y_{it} = \alpha_i + \theta_{g_i,t} + X'_{it}\beta + \epsilon_{it}, \quad g_i = 1, 2, ..10$. y_{it} is log annual sales changes and X_{it} is a 6×1 vector of variables: natural log total assets (TA), Tobin's q, return on assets (ROA), capital expenditure (Capex), leverage, and cash flow (CF). Disaster affected employment numbers are computed from County Business Pattern data, publicly provided by the U.S. Census Bureau. Panel B presents regressions of log annual sales changes on explanatory variables using different estimation models. TFE is the firm and year fixed effect estimator, IFE is the firm fixed effect with industry and year interaction estimator. GFE estimates are obtained based on $G = 10, 15, 30$, selected from a range of G based on various information criteria. For IFE, preliminary within-transformation is taken to integrate out the individual specific effect as in GFE. The industry IFE is based on 2-digit SIC code. Clustered standard errors are reported in parentheses for TFE and IFE. For GFE, we present the asymptotic standard errors. ***, **, * denote significance at 1%, 5%, and 10% level, respectively. p-values of Hausman tests for TFE and IFE against GFE ($G = 10$) are presented. All variables are from Compustat and the sample period is 1987-2010. All variables are described in Table A.2 in the Appendix.

Panel A. Relation of TFE, IFE, GFE, and disaster-affected employment

	TFE	IFE	GFE
	(θ_t)	($\theta_{industry,i,t}$)	($\theta_{g_i,t}$)
Disaster-affected Employment	0.01	0.03	-0.25
t-stats	2.13	3.86	-3.85
# Obs		66,185	

Panel B. Sales growth regression

	TFE	IFE	GFE	GFE	GFE
			$G = 10$	$G = 15$	$G = 30$
TA	0.0232***	0.0045	0.0074***	0.0094***	0.0085***
	(4.47)	(1.05)	(2.30)	(3.24)	(3.27)
ROA	-0.0001	0.0113	0.0101	0.0066	0.0149
	(-0.01)	(0.89)	(1.15)	(0.75)	(1.18)
Capex	5.60e-06***	3.08e-06	2.87e-06*	3.16e-06**	3.46e-06**
	(2.52)	(1.61)	(1.70)	(1.98)	(1.93)
Leverage	-0.0146	-0.0215**	-0.0141*	-0.0165**	-0.0148*
	(-1.19)	(-2.05)	(-1.82)	(-2.26)	(-1.76)
Tobin's q	0.0003***	0.0001*	0.0001***	0.0001***	0.0001***
	(3.44)	(1.72)	(4.09)	(5.06)	(4.12)
CF	0.0010**	0.0000	0.0002***	0.0001***	0.0002***
	(1.92)	(1.64)	(7.37)	(4.64)	(3.02)
p-val of Hausman test	0.000	0.000	–	–	–
Obs	66,185	66,185	66,185	66,185	66,185

Table 10
Compustat data based simulation

The table presents the results for simulations using empirical parameters from Compustat data for different econometric specifications, as specified in Section 3.2.2. The simulation is based on the Compustat data for sales growth, major natural disasters, and six control variables: natural log total assets (TA), Tobin's q, return on assets (ROA), capital expenditure (Capex), leverage, and cash flow (CF). See Table A.2 in the Appendix for all variable definitions. We present the absolute relative bias: |Relative bias| = $\left|\frac{Bias(\hat{\beta}_l)}{\beta_l^0}\right|$, where β_l^0 is the true value of β_l, the root mean square error (RMSE): RMSE($\hat{\beta}$) = $\sqrt{\frac{1}{R}\sum_{r=1}^{R}||\hat{\beta}^r - \beta^0||^2}$ of estimated coefficients, where $\hat{\beta}^r$ is the estimator in the r-th replication, β^0 is the true value, and $\bar{\hat{\beta}} = 1/R\sum_{r=1}^{R}\hat{\beta}^r$ is the sample average across replications. Misclassification frequency (Misclass. Freq.) is the average proportion of units that are misclassified by GFE across replications: $AMF = 1/R\sum_{r=1}^{R}$ MF$_r$, where R is the number of replications and MF$_r$ = $\frac{1}{N}\sum_{i=1}^{N}$ I($\hat{g}_i \neq g_i^0$). The table reports the results using the following methods: grouped fixed effects (GFE), two-way fixed effects (TFE), and group-time interacted fixed effects (IFE). Each simulation consists of 1000 replications.

		$T = 15, G = 5$			$T = 15, G = 10$		
		GFE	TFE	IFE	GFE	TFE	IFE
Misclass. Freq.		0.0565			0.1439		
|Relative bias|	TA	-0.0154	0.2015	0.1999	-0.0340	0.6930	0.6316
	ROA	0.0122	0.1009	0.8163	-0.1235	1.0103	3.9146
	Capex	-0.0227	-0.1423	-0.0882	0.3648	2.0938	1.4327
	Leverage	-0.0030	0.6245	0.8416	-0.5154	-9.4913	-16.5195
	Tobin's q	-0.0264	-8.2434	-1.5863	-0.0592	-0.6009	-1.0451
	CF	-0.0086	0.5552	0.5819	-0.1762	-3.7295	-3.4110
Overall RMSE		0.0378	0.0775	0.0765	0.0528	0.1862	0.1791

Table 11
Pilot CEOs and patent citations

The table presents the results of the effect of pilot CEOs on patent citation counts. TFE is the industry and year fixed effect estimator, IFE is a firm fixed effect with industry and year interaction estimator. GFE estimates are obtained based on $G = 10, 15$, selected from a range of G from 1 to 30 based on various information criteria, and includes industry fixed effects. For IFE, preliminary within-transformation is taken to integrate out the industry specific effect as in GFE. The industry IFE is based on two-digit SIC codes. Following Sunder et al. (2017), the dependent variable is Log(1 + citation). Citation is the number of raw citations multiplied by the weighting index of Hall et al. (2001) to all the patents applied for during the year. Pilot CEO is an indicator variable equal to one if the CEO has been a pilot and zero otherwise. All regressors are lagged by one year. All variable definitions are provided in Table A.2 in the Appendix. Following Sunder et al. (2017), the sample period is from 1993 to 2003. Standard errors are clustered at firm level for TFE and IFE, and the asymptotic standard errors for GFE. t-statistics are reported in parentheses. ***, **, * denote significance at 1%, 5%, and 10% level, respectively. p-val are the p-values of Hausman tests for TFE and IFE against GFE ($G = 10$).

	TFE	IFE	GFE $G = 10$	GFE $G = 15$
Pilot CEO	0.376*	0.203*	-0.027	-0.043
	(1.67)	(1.66)	(-0.44)	(-0.65)
TA	0.589***	0.611***	0.264***	0.335***
	(9.06)	(9.15)	(13.73)	(14.64)
PPE/EMP	0.000	-0.0002***	0.000	0.000
	(2.70)	(-3.64)	(1.07)	(-0.40)
Stock return	-0.098	-0.168***	0.001	-0.020
	(-1.33)	(-4.31)	(-0.01)	(-0.62)
Tobin's q	0.156***	0.114***	0.020**	0.045***
	(3.04)	(8.38)	(2.01)	(3.48)
Institutional holdings	0.148	-0.061	-0.331***	-0.136
	(0.53)	(-0.43)	(-5.92)	(-1.43)
Log(1 + tenure)	-0.149***	0.0002	-0.079**	-0.104***
	(-2.22)	(0.00)	(-3.34)	(-3.45)
Log(1 + delta)	-0.001	0.035	0.029**	0.031***
	(-0.01)	(1.11)	(2.18)	(3.30)
Log(1 + vega)	0.145**	0.066	0.022	0.002
	(2.18)	(1.30)	(1.28)	(0.14)
Log(CEO age)	0.296	-0.562	0.213	0.400
	(0.45)	(-1.34)	(0.87)	(1.63)
Top university	-0.002	-0.037	-0.240***	-0.065
	(-0.01)	(-0.21)	(-3.96)	(-0.90)
Finance education	0.520***	0.243	1.448***	1.438***
	(3.02)	(0.40)	(3.21)	(3.05)
Technical education	1.253	0.960	1.795***	1.513***
	(1.00)	(1.61)	(10.02)	(8.67)
PhD	0.154	-0.042	0.090	0.201***
	(0.56)	(-0.26)	(1.08)	(2.72)
No school information	0.299*	0.044	0.028	0.086
	(1.84)	(0.60)	(0.54)	(1.59)
Military	0.004	-0.043***	-0.017***	-0.014***
	(0.62)	(-7.11)	(-7.51)	(-4.72)
Overconfidence	0.109	0.034	0.051	0.002
	(0.92)	(0.31)	(1.15)	(0.05)
p-val of Hausman test	0.000	0.000	–	–
Obs	2450	2450	2450	2450

Table 12
Simulation with endogenous regressors

The table presents the results for simulations under DGP1 with endogenous regressors for grouped fixed effects (GFE) and two-way fixed effects (TFE). DGP1 is generated using the procedure described in Section 5.1, where shocks occur at every time t and heterogenous responses to shocks are allowed. X_{it} is correlated with the error term. The parameters in the DGP are set as $c_\tau = 0.5$, $c_\theta = 15$, and $G = 5$. c_τ is a constant which determines the degree of correlation between X and θ, c_θ is a constant which controls the size of shocks, G is the number of groups. We present the bias: Bias($\hat{\beta}$) $= \frac{1}{R}\sum_{r=1}^{R}\hat{\beta}^r - \beta^0$, the estimated standard deviation (Std.): Std($\hat{\beta}$) $= \sqrt{\frac{1}{R-1}\sum_{r=1}^{R}(\hat{\beta}^r - \bar{\hat{\beta}})^2}$, and the root mean square error (RMSE): RMSE($\hat{\beta}$) $= \sqrt{\frac{1}{R}\sum_{r=1}^{R}\|\hat{\beta}^r - \beta^0\|^2}$ of estimated coefficients, where $\hat{\beta}^r$ is the estimator in the r-th replication, β^0 is the true value, and $\bar{\hat{\beta}} = 1/R\sum_{r=1}^{R}\hat{\beta}^r$ is the sample average across replications. Misclassification frequency (Misclass. Freq.) is the average proportion of units that are misclassified by GFE across replications: $AMF = 1/R\sum_{r=1}^{R}$ MF$_r$, where R is the number of replications and MF$_r = \frac{1}{N}\sum_{i=1}^{N} I(\hat{g}_i \neq g_i^0)$. The table reports the results using grouped fixed effects and two-way fixed effects. Both GFE and TFE employ TSLS estimation with X_{it} instrumented by Z_{it}. Each simulation consists of 1000 replications.

		$N = 1000, T = 15$		$N = 1000, T = 30$		$N = 2000, T = 30$	
		GFE	TFE	GFE	TFE	GFE	TFE
Misclass. Freq.		0.1877		0.1252		0.1232	
β_1	Bias	0.0215	0.6010	0.0093	0.6112	0.0101	0.6255
β_2	Bias	0.0232	0.6087	0.0105	0.6156	0.0099	0.6323
β	RMSE	0.0378	0.9028	0.0172	0.8561	0.0221	0.9063

Table 13
Residual cross product matrix: Group fixed effects

The table presents a sample covariance matrix of the residuals for a group fixed effect DGP. In the presence of group fixed effecs, residuals of the same firm across different years as well as residuals of the same year and the same group, may be correlated. However, residuals of the same year but different groups are assumed to be uncorrelated, and thus reported as zero in the matrix.

		Firm 1			Firm 2			Firm 3		
Group 1	Firm 1	ϵ_{11}^2	$\epsilon_{11}\epsilon_{12}$	$\epsilon_{11}\epsilon_{13}$	$\epsilon_{11}\epsilon_{21}$	0	0	0	0	0
		$\epsilon_{21}\epsilon_{11}$	ϵ_{12}^2	$\epsilon_{12}\epsilon_{13}$	0	$\epsilon_{12}\epsilon_{22}$	0	0	0	0
		$\epsilon_{13}\epsilon_{11}$	$\epsilon_{13}\epsilon_{12}$	ϵ_{13}^2	0	0	$\epsilon_{13}\epsilon_{23}$	0	0	0
	Firm 2	$\epsilon_{21}\epsilon_{11}$	0	0	ϵ_{21}^2	$\epsilon_{21}\epsilon_{22}$	$\epsilon_{21}\epsilon_{23}$	0	0	0
		0	$\epsilon_{22}\epsilon_{12}$	0	$\epsilon_{22}\epsilon_{21}$	ϵ_{22}^2	$\epsilon_{22}\epsilon_{23}$	0	0	0
		0	0	$\epsilon_{23}\epsilon_{13}$	$\epsilon_{23}\epsilon_{21}$	$\epsilon_{23}\epsilon_{22}$	ϵ_{23}^2	0	0	0
Group 2	Firm 3	0	0	0	0	0	0	ϵ_{31}^2	$\epsilon_{31}\epsilon_{32}$	$\epsilon_{31}\epsilon_{33}$
		0	0	0	0	0	0	$\epsilon_{32}\epsilon_{31}$	ϵ_{32}^2	$\epsilon_{32}\epsilon_{33}$
		0	0	0	0	0	0	$\epsilon_{33}\epsilon_{31}$	$\epsilon_{33}\epsilon_{32}$	ϵ_{33}^2

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A Appendix

A.1 Event study standard errors

Another popular method to control for heterogeneous responses in empirical finance is event studies. There are two potential problems with event studies. First, event studies require that there are relatively few shocks over the entire sample period, and the timing of the shocks needs to be precisely specified. However, in some cases, there may be many shocks over the whole sample period, or different time effects in each period. Then event studies are less reliable, since the time period within each subsample is short. More importantly, if the timing of shocks is incorrectly identified or several shocks are ignored, event studies generally result in inconsistent estimates.

Second, the conventional standard errors provided by event studies are often inappropriate, because they do not take into account cross-sectional correlation. The group specific responses suggest that an unobserved group pattern of features drives heterogeneity in responses, e.g. managerial and institutional characteristics, and it is likely that these characteristics will also affect firm's responses to any kind of corporate related events. Therefore, we can expect unobserved group characteristics in both the independent variables and the residuals to possibly change over time. These time-varying group effects cannot be canceled out by the fixed effects transformation.

To obtain appropriate standard error estimates in event studies, one should explicitly take into account the latent group pattern, and calculate the group-clustered standard error. Our GFE estimates of the group membership thus provide a natural and reliable estimate of the latent group pattern, which can be used to construct clustered standard errors.

To see how the correlation causes bias in standard errors, suppose we apply FE estimators to each regime segmented by the pre-specified event. In each regime, the within-transformed residuals still contain two components, a group-specific time-varying component $\gamma_{g_i,t}$ and an idiosyncratic component η_{it} for each individual unit, namely

$$\dot{\epsilon}_{it} = \gamma_{g_i,t} + \eta_{it}. \tag{20}$$

Also, the transformed independent variable $\dot{X}$ is driven by a group-specific time-varying component and an idiosyncratic component, and it can specified as

$$\dot{X}_{it} = \delta_{g_i,t} + \nu_{it}. \tag{21}$$

For notation simplicity, we assume that all four components $\gamma_{g_i,t}$, η_{it}, $\delta_{g_i,t}$, and ν_{it} are independent from each other and across time, and they all have a zero mean and finite variance, σ^2_γ, σ^2_η, σ^2_δ, and σ^2_ν, respectively. We consider an example of a single regressor, thus X_{it} is a scalar. Due to the group-specific components, the residual and regressor of individual units (firms) are correlated with each other with the following correlation structure

$$\begin{aligned}
\mathrm{corr}(\dot{X}_{it}, \dot{X}_{js}) &= 1 \quad \text{for } i = j \text{ and } t = s \\
&= \rho_x = \sigma_\delta^2/\sigma_X^2 \quad \text{for } g_i = g_j \text{ and } t = s \\
&= 0 \quad \text{for all } g_i \neq g_j \text{ or } t \neq s,
\end{aligned}$$

and

$$\begin{aligned}
\mathrm{corr}(\dot{\epsilon}_{it}, \dot{\epsilon}_{js}) &= 1 \quad \text{for } i = j \text{ and } t = s \\
&= \rho_\epsilon = \sigma_\gamma^2/\sigma_\epsilon^2 \quad \text{for } g_i = g_j \text{ and } t = s \\
&= 0 \quad \text{for all } g_i \neq g_j \text{ or } t \neq s.
\end{aligned}$$

Under such residual and regressor structure, the fixed effects coefficient estimate in a given regime is still consistent, but its standard error estimate is downward biased due to disregarding the group correlation. In particular, we obtain the asymptotic variance of the fixed effects coefficient estimate as

$$\begin{aligned}
var(\hat{\beta}_{FE}) &= \left[\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\dot{X}_{it}\dot{\epsilon}_{it}\right)^2 \left(\sum_{i=1}^{N}\sum_{t=1}^{T}\dot{X}_{it}^2\right)^{-2}\right] \\
&= \left[\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\dot{X}_{it}^2\dot{\epsilon}_{it}^2 + \sum_{i,j\in g}\sum_{t=1}^{T}\dot{X}_{it}\dot{X}_{jt}\dot{\epsilon}_{it}\dot{\epsilon}_{jt}\right)\left(\sum_{i=1}^{N}\sum_{t=1}^{T}\dot{X}_{it}^2\right)^{-2}\right] \\
&= \left[NT\sigma_X^2\sigma_\epsilon^2 + \sum_{g=1}^{G}N_g(N_g-1)T\rho_X\rho_\epsilon\sigma_X^2\sigma_\epsilon^2\right]\left[NT\sigma_X^2\right]^{-2} \\
&= \frac{\sigma_\epsilon^2}{NT\sigma_X^2}\left[1 + 1/N\sum_{g=1}^{G}N_g(N_g-1)\rho_X\rho_\epsilon\right], \qquad (22)
\end{aligned}$$

where N_g is the number of individuals in the g-th group. Under a group structure, individuals within a group share the same component in both the independent variable and residuals, leading to a positive correlation. Therefore, the fixed effects standard error estimates underestimate the true standard errors.

Table A.1
Demonstrating example: Time-invariant heterogeneity

The table presents a simple numerical example of the two-way fixed effect absorbing the time invariant unobserved heterogeneity. Columns (1)-(3) present the basic data structure: (1) is the individual unique identification, (2) is the time period denomination, and (3) is the group identification. Columns (4)-(6) present the data: (4) is the individual fixed effect, (5) is the time effect, and (6) is the total value of fixed effects, i.e. $(6) = (4) + (5)$. Column (7) shows the demeaned cross-sectional fixed effect, $\widetilde{\alpha}_i = \alpha_i - \sum_{t=1}^{T} \alpha_i - \sum_{i=1}^{N} \alpha_i + \sum_{t=1}^{T} \sum_{i=1}^{N} \alpha_i = 0$, column (8) shows the demeaned time fixed effect $\widetilde{\theta}_{g,t} = \theta_{g,t} - \sum_{t=1}^{T} \theta_{g,t} - \sum_{i=1}^{N} \theta_{g,t} + \sum_{t=1}^{T} \sum_{i=1}^{N} \theta_{g,t}$, and column (9) shows the demeaned total FE, i.e. $(12) = (10) + (11)$.

Unit & Time			Data			Transformed data		
Indiv. ID (1)	Time (2)	Group ID (3)	α_i (4)	$\theta_{g,t}$ (5)	Total FE (6)	$\widetilde{\alpha}_i$ (7)	$\widetilde{\theta}_{g,t}$ (8)	Total FE (9)
1	1	1	2	3	5	0	0	0
1	2	1	2	4	6	0	0	0
1	3	1	2	5	7	0	0	0
2	1	1	4	3	7	0	0	0
2	2	1	4	4	8	0	0	0
2	3	1	4	5	9	0	0	0
3	1	2	6	4	10	0	0	0
3	2	2	6	5	11	0	0	0
3	3	2	6	6	12	0	0	0
4	1	2	8	4	12	0	0	0
4	2	2	8	5	13	0	0	0
4	3	2	8	6	14	0	0	0

Table A.2
Variable description

Variable	Description
Citation	The number of citations per firm summed across all patents applied for during the year. Each patent's number of citations is multiplied by the weighting index from Hall et al. (2001).
Employment affected by major disasters	The number of employees affected by natural disasters by U.S. state as computed by County Business Pattern data, publicly provided by the U.S. Census Bureau.
Pilot CEO	An indicator variable equal to one for CEOs with a pilot license and zero otherwise.
TA	Natural log of total assets in millions.
ROA	Return on assets.
Leverage	The long-term debt plus short-term debt divided by total assets.
Capex	The capital expenditure divided by total assets.
Cash flow	Lagged Operating Income before depreciation minus total income taxes, minus change in deferred taxes from the previous year to the current year minus gross interest expense minus preferred dividend requirement on cumulative preferred stock and dividends paid on non-cumulative preferred stock minus total dollar amount of dividends declared on common stock. Cash flow is divided by TA.
Tobin's q	The market value of assets divided by the book value of assets where the market value of assets equals the book value of assets plus the market value of common equity less the sum of the book value of common equity and balance sheet deferred taxes.
PPE/EMP	The ratio of net property, plant, and equipment over the number of employees.
Stock return	Firm buy-and-hold return over the fiscal year.
Institutional holdings	Percentage of shares outstanding held by financial institutions.
Tenure	CEO tenure in months.
Delta	Dollar change in CEO stock and option portfolio for a 1% change in stock price.
Vega	Dollar change in CEO option holdings for a 1% change in stock return volatility.
CEO age	CEO age in years.
Top university	An indicator variable equal to one if the CEO's undergraduate institution is listed as one of the top 50 schools ranked by U.S. News & World Report in any year during the period 1983 through 2007 and zero otherwise.
Finance education	An indicator variable equal to one if the CEO received a degree in accounting, finance, business (including MBA), or economics and zero otherwise.
Technical education	An indicator variable equal to one for CEOs with undergraduate or graduate degrees in engineering, physics, operations research, chemistry, mathematics, biology, pharmacy, or other applied science and zero otherwise.
PhD	An indicator variable equal to one for CEOs with a PhD and zero otherwise.
No school information	An indicator variable equal to one if we cannot identify the CEO's undergraduate school and zero otherwise.
Military	An indicator variable equal to one for CEOs with military background and zero otherwise.
Overconfidence	An indicator variable equal to one for all years after the CEO's options exceed 67% moneyness and zero otherwise.