The Nature of Firm Growth*

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*Any opinions and conclusions expressed herein are those of the author(s) and do not necessarily represent the views of the U.S. Census Bureau. All results have been reviewed to ensure that no confidential information is disclosed.

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- What is the “nature” of these shocks?
- Typically disciplined by matching long-run size distribution and sometimes a *stationary* autocorrelation
- Could we find more direct evidence?

Research question

Can we use administrative panel data now available to better understand "nature" of firm life-cycle dynamics/growth in macro models with firm dynamics ...

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... and their macro implications, i.e., should we care?

What we do

1. Use large administrative panel data to estimate contributions of **ex-post shocks** and **ex-ante heterogeneity** to dispersion in firm size
   - New moments: autocovariance matrix of log employment
   - Statistical approach: estimate an employment process to match moments
   - Structural approach: estimate parameters in a macro firm dynamics model

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3. Changes over time: decline in "dynamism"; disappearance of gazelles

   $\Longrightarrow$ Evidence for shift in distribution of ex-ante profiles

Estimating the "nature" of firm growth

A statistical model

Estimating a new set of moments: employment autocovariance matrix

- Autocovariance matrix, $[c_{aa'}]$, of firm-level (residual) log employment

$$c_{aa'} = \text{Cov}\left[\widetilde{\log n_{ia}}, \widetilde{\log n_{ia'}}\right] \qquad a = 0, 1, \ldots, 19$$

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  1. Employment determined only by permanent **ex-ante heterogeneity**:
     $\implies$ perfect correlation of size over firm's life-cycle

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– (Borrowing from income dynamics literature) can estimate parameters of employment processes by matching autocovariance patterns

Generalized employment process

$$\log n_{ia} = \sum_{k=0}^{a} \rho_u^k \theta_i + \rho_u^{a+1} \tilde{u}_i + \rho_v^{a+1} \tilde{v}_i + \sum_{k=0}^{a} \rho_v^k \varepsilon_{ia-k} + z_{ia}$$

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Ex-ante profile $(a < 0)$:

$$\begin{aligned} \theta_i &\sim \quad IID\left(\mu_\theta, \sigma_\theta\right) \\ \tilde{u}_i &\sim \quad IID\left(\mu_{\tilde{u}}, \sigma_{\tilde{u}}\right) \\ \tilde{v}_i &\sim \quad IID\left(\mu_{\tilde{v}}, \sigma_{\tilde{v}}\right) \end{aligned}$$

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Ex-ante profile $(a < 0)$:

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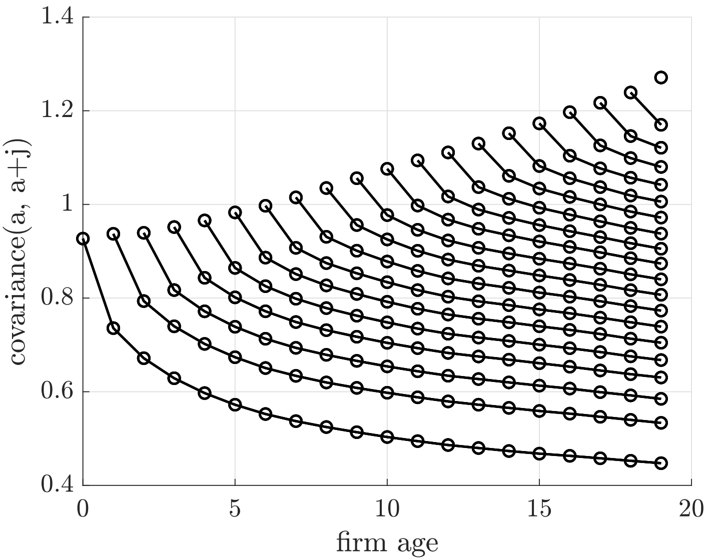
- Nests models of Hopenhayn and Rogerson (1993) and Melitz (2003) and allows for richer ex-ante profiles as in e.g. Luttmer (2011)
- Closed-form autocovariance function in model parameters
- Estimate parameters by matching data and process autocovariance (EWMD)

Autocovariance Function

Data and model fit



RW | Ex-ante+RW | Ex-anter+RW + AR(1) | Age dep. | AR(1) | Panel AR(1) | Panel AR(2) | Panel ARMA(1,1)

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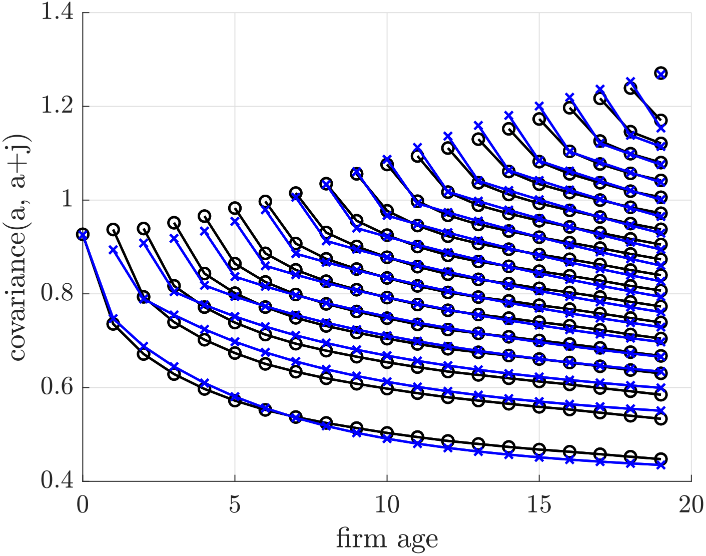

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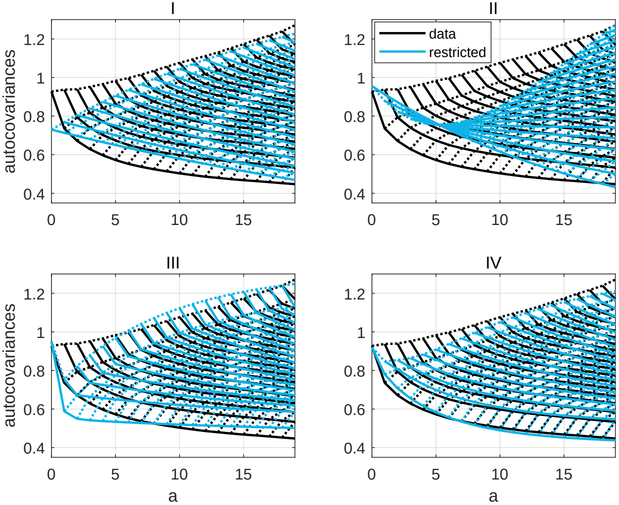
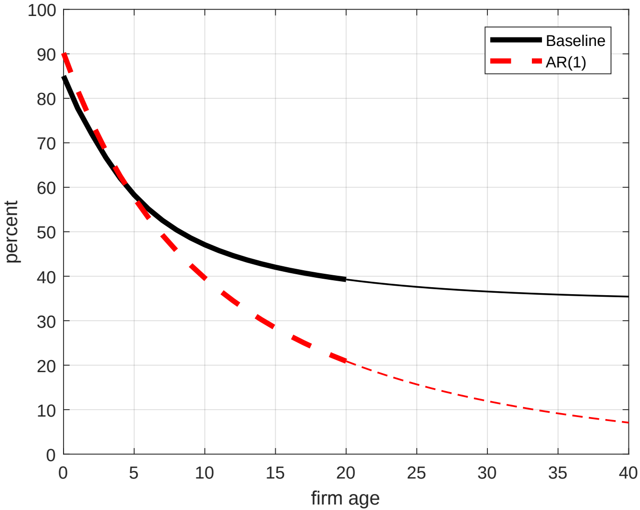
Data and restricted models



Note: (I) only persistent shocks and initial condition, (II) only cumulated permanent effect and initial condition, (III) baseline with only one initial condition, (IV) baseline without transitory ex-post shocks.

Ex-ante factors and the dispersion of firm sizes by age



Alternatives

Estimating the “nature” of firm growth

A general equilibrium model with firm dynamics

Important advantages of structural model

- accounts for endogenous firm selection
- enables us to address aggregate questions
- can address various frictions and their effect on observed patterns
- show why getting source of heterogeneity right is important

Sketch of model

Environment à la Hopenhayn (1992), Melitz (2003) and Luttmer (2011)

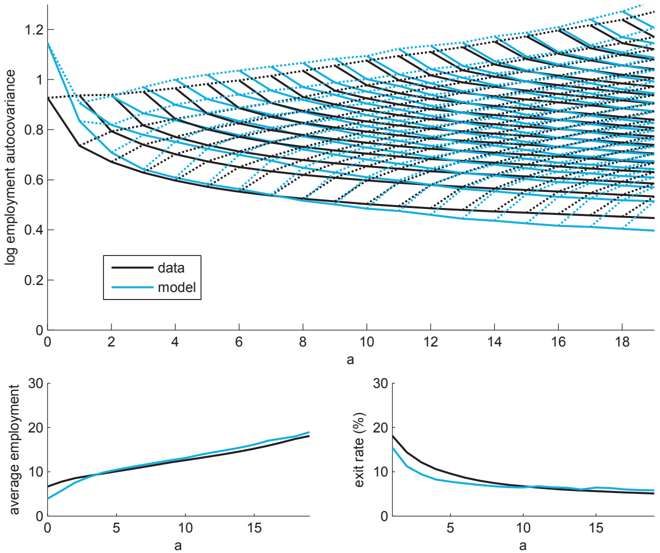
- stationary, no aggregate uncertainty
- general equilibrium closed economy
- endogenous entry and exit
- frictionless factor adjustment
- inelastic labor supply
- demand heterogeneity (Foster et al., 2015; Hottman et al., 2016)
- idiosyncratic shock process as in reduced-form analysis

  ex-ante profiles + ex-post shocks

Estimate by matching autocovariance and life-cycle profiles of exit and size

Autocovariance matrix, average size and exit rates

size distr



Variance decomposition in estimated model with selection

$$\begin{aligned}\mathsf{Var}\left[\ln n\right] &= \mathsf{Var}\left[\ln n^{EXA}\right] + \mathsf{Var}\left[\ln n^{EXP}\right] + 2\mathsf{Cov}\left[\ln n^{EXA}, \ln n^{EXP}\right] \\ &= \mathsf{Cov}\left[\ln n^{EXA}, \ln n\right] + \mathsf{Cov}\left[\ln n^{EXP}, \ln n\right]\end{aligned}$$

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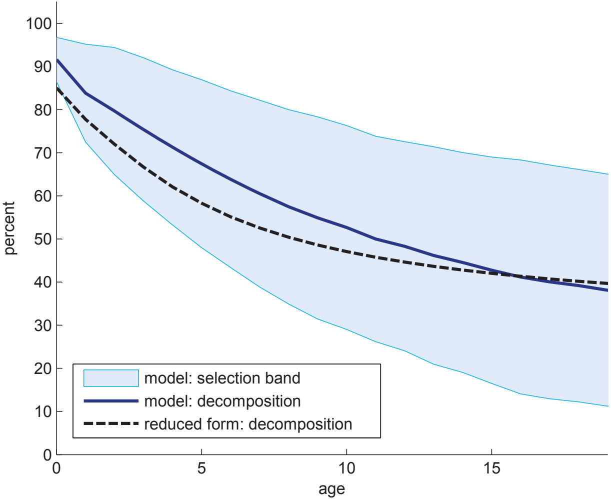
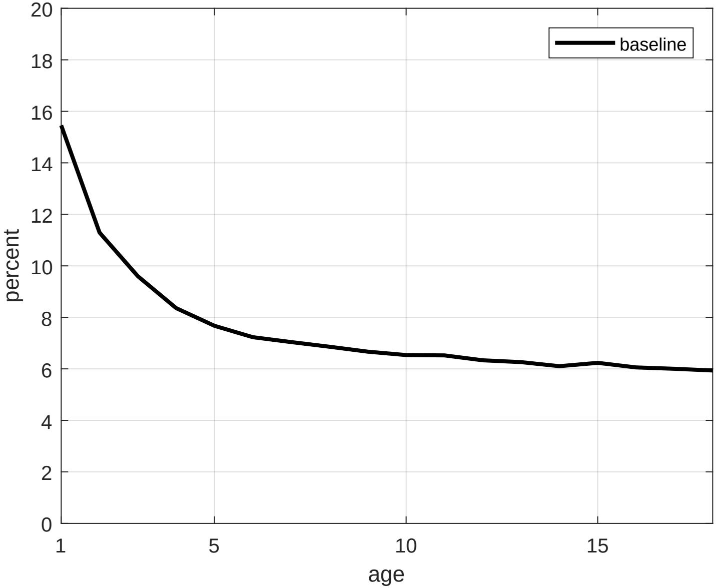


Quantifying the importance of ex-ante heterogeneity

How important is ex-ante heterogeneity for firm selection and growth?

Drivers of firm selection



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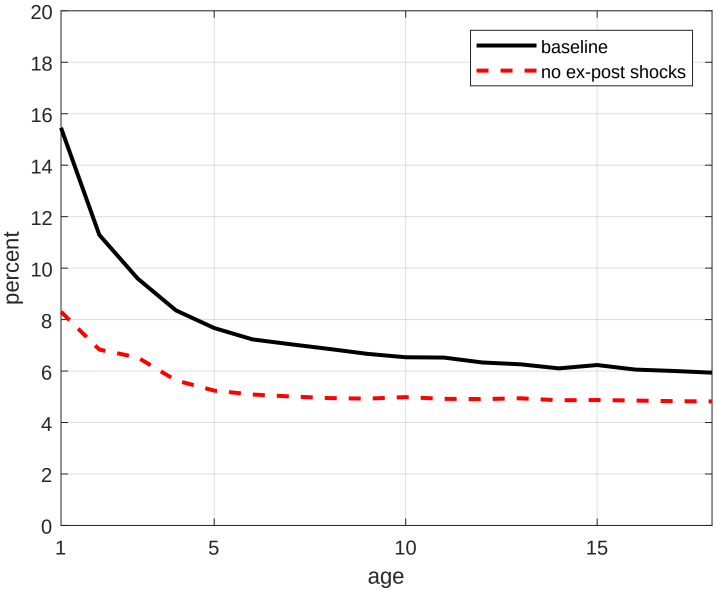
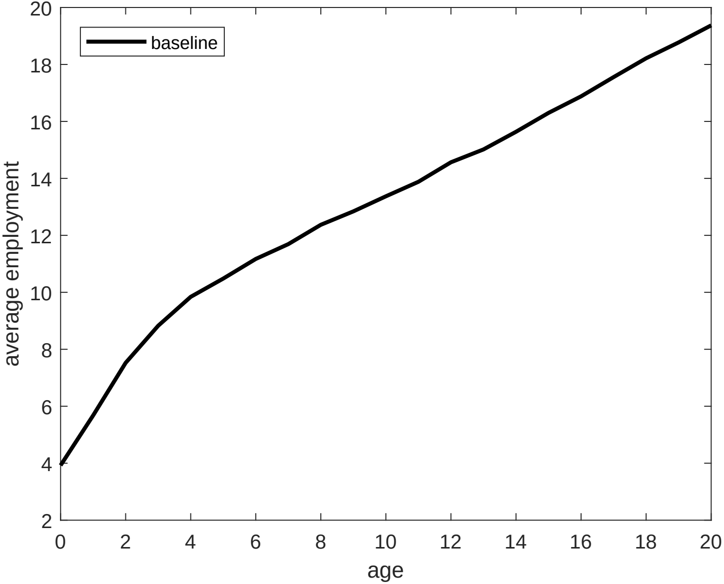


Quantify impact of firm selection: firm growth



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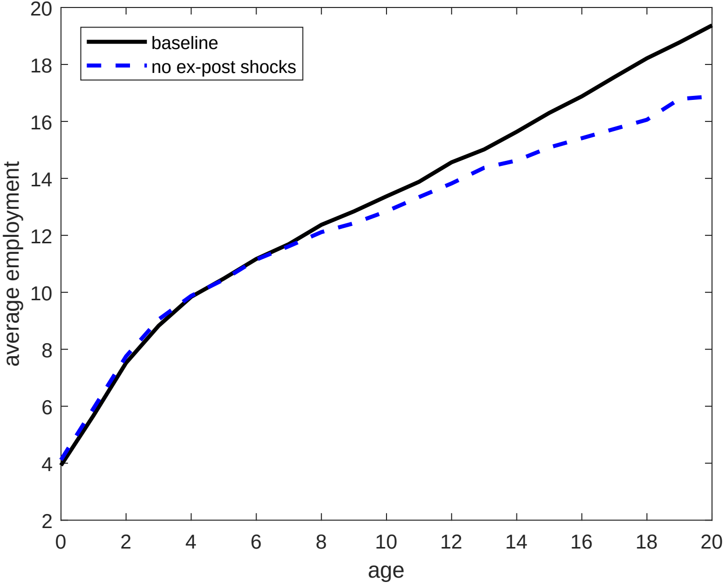

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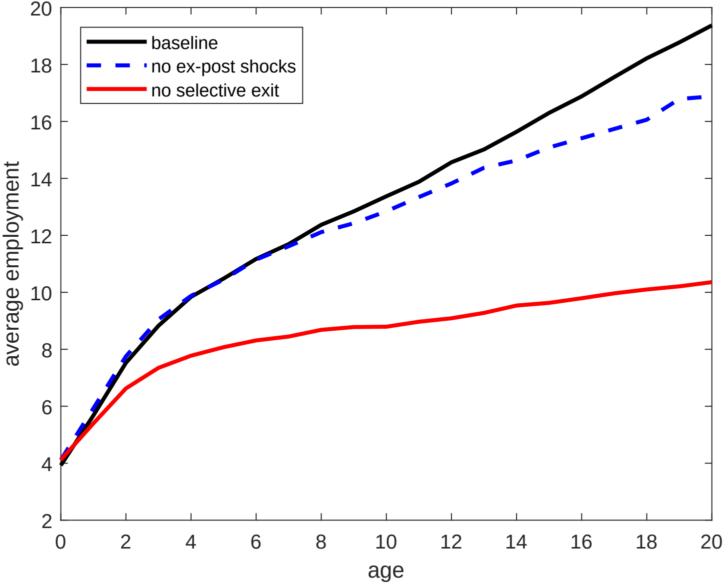

Quantify impact of firm selection: aggregate output

- Aggregate output can be written

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  - **38** percent lower without effects of selection altogether
  - **4** percent lower without effects of selection on ex-post shocks

Quantifying the importance of ex-ante heterogeneity

How does source of heterogeneity matter for macro outcomes?

The macro impact of micro-level frictions

Consider two economies:

1. baseline

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Introduce following adjustment cost into both versions of the model

- Now, must pay cost κ to accept next period's demand shock
- "Active" accumulation of demand as in Foster, Haltiwanger and Syverson (2016)

The macro impact of micro-level frictions

Table: Aggregate impact of adjustment costs (percent change)

	output	wage	size	exit	firms
restricted model	-3.2	-0.6	-26.2	$+7.2$	$+31.9$
baseline model	-0.1	-0.1	$+2.5$	-0.2	-2.4

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- Reason: key decisions made by firms depend on firm **values**, which are forward looking and hence depend on the process
- Dispersion of firm values much greater in baseline model
  - $\Rightarrow$ fewer "marginal" firms
  - $\Rightarrow$ smaller impact of frictions

Quantifying the importance of ex-ante heterogeneity

Apply to estimating changing nature of firm dynamics.

Declines in business dynamism and its macro effects

Recently documented decline in dispersion/skewness of firm growth rates

- i.e. firms are not growing as fast as they used to don't trust me?

Decker et al. (2016) link declining dynamism to

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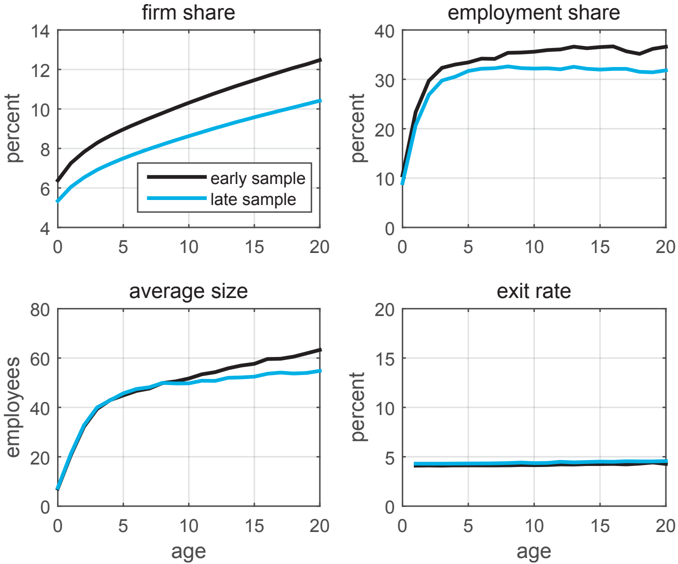
We use our framework to answer whether

- dynamism decline is caused by a change in ex-post shocks
- or ex-ante factors, i.e. are high-growth firms ("gazelles") dying out?

Estimating model on split sample: "gazelles"



Conclusions

- Estimate (and release) the autocovariance structure of firm-level employment over first 20 years of lifecycle for U.S. firms

- Using both statistical and structural approaches, **ex-ante profile heterogeneity** explains *almost half* of within-industry size dispersion, even at long horizons

- Important for firm-level and macro outcomes
  - Selection and age-profile of firm size
  - Aggregate productivity gains from selection
  - Macro effects of micro frictions
  - Decline in dynamism is in part explained by change in ex-ante "gazelles"

- Even more applications in the paper

Thank you!

Two popular processes in firm dynamics literature

1. Hopenhayn and Rogerson (1993)

$$\ln n_{i,a} = \mu + \rho \ln n_{i,a-1} + \varepsilon_{i,a}$$

$$\begin{aligned} \widetilde{n}_{i,-1} &\sim IID\left(\mu_{\widetilde{v}}, \sigma_{\widetilde{v}}\right), \\ \varepsilon_{i,a} &\sim IID\left(0, \sigma_{\varepsilon}\right) \end{aligned}$$

2. Melitz (2003)

$$\ln n_{i,a} = \theta_i$$

$$\theta_i \sim IID\left(\mu_\theta, \sigma_\theta\right)$$

Generalized reduced-form process

$$\ln n_{i,a} = \underbrace{u_{i,a} + v_{i,a}}_{\text{Ex-ante components}} + \underbrace{w_{i,a} + z_{i,a}}_{\text{Ex-post components}} \qquad a = 0, 1, 2, \ldots$$

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Ex-ante components:

$$\begin{aligned} u_{i,a} &= \theta_i + \rho_u u_{i,a-1} \\ v_{i,a} &= \rho_v v_{i,a-1} \end{aligned}$$

"pre startup" draws $a < 0$

$$\begin{aligned} \theta_i &\sim IID\left(\mu_\theta, \sigma_\theta\right) \\ u_{i,-1} &\sim IID\left(\mu_{\widetilde{u}}, \sigma_{\widetilde{u}}\right) \\ v_{i,-1} &\sim IID\left(\mu_{\widetilde{v}}, \sigma_{\widetilde{v}}\right) \end{aligned}$$

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Ex-post components:

$$w_{i,a} = \rho_w w_{i,a-1} + \varepsilon_{i,a}$$

with $w_{i,-1} = 0$ and for $a \geq 0$

$$\begin{aligned} \varepsilon_{i,a} &\sim IID\left(0, \sigma_\varepsilon\right) \\ z_{i,a} &\sim IID\left(0, \sigma_z\right) \end{aligned}$$

Autocovariance function

$$\text{Cov}\left[\log n_{ia}, \log n_{a-j}\right] = \left(\sum_{k=0}^{a} \rho_u^k\right)\left(\sum_{k=0}^{a-j} \rho_u^k\right)\sigma_\theta^2 + \rho_u^{2(a+1)-j}\sigma_{\tilde{u}}^2 + \rho_v^{2(a+1)-j}\sigma_{\tilde{v}}^2$$
$$+ \sigma_\varepsilon^2 \rho^j \sum_{k=0}^{a-j} \rho_w^{2k} + \sigma_z^2 \mathbf{1}_{j=0}.$$

- autocovariance matrix (over-)identifies persistence and variance parameters
  - $\rho_u, \rho_v, \rho_w, \sigma_\theta^2, \sigma_{\tilde{u}}^2, \sigma_{\tilde{v}}^2, \sigma_\varepsilon^2, \sigma_z^2$
- does not identify levels
  - $\mu_\theta,\ \mu_{\widetilde{u}},\ \mu_{\widetilde{v}}$

Back

Estimation

- minimum distance estimation using empirical autocovariance
- 210 moments, 8 parameters, identity weighting matrix
- include 4-digit NAICS and cohort fixed effects
- estimate version restricted to AR(1) and other processes

Related literature

Firm dynamics models

- e.g. Jovanovic (1982), Hopenhayn (1992), Hopenhayn and Rogerson (1993), Melitz (2003), Abbring and Campbell (2005), Luttmer (2007, 2011), Arkolakis (2016), Arkolakis, Papageorgiou, Timoshenko (2017), Sedláček and Sterk (2017)

Empirical evidence:

- e.g. Davis and Haltiwanger (1992), Haltiwanger, Jarmin, Kulick, Miranda (2016), Hurst and Pugsley (2011), Guzman and Stern (2015), Sedláček and Sterk (2017)

Household earnings processes:

- e.g.Lillard and Weiss (1979), MaCurdy (1982), Abowd and Card (1989), Guvenen (2009)

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Firm employment data

U.S. Census Bureau Longitudinal Business Database (LBD)

- administrative data
- nearly universal coverage of U.S. employers
- annual data from 1976 until 2012
- observe firms and establishments
- observe employment, age and 4-digit industry

Use only within-industry and within-cohort *residual* employment:

$$\widetilde{\log n_{i,a,t,j}} = \log n_{i,a,t,j} - \mu_j - \lambda_{t-a}$$

ALTERNATIVE STATISTICAL MODELS

Table: EWMD estimation of alternative firm employment processes

	(1)	(2)	(3)	(4)	(5)		(6)	(7)	(8)
							Dynamic Panel Data Models		
	Base	RW	RW+Base	Age Dep.	AR(1)	AR(1) + FE	AR(1)	AR(2)	ARMA(1,1)
ρ_u	0.2184	0.5853	0.2199	0.0002	—	—	—	—	—
ρ_v	0.8323	0.9608	0.8246	0.8123	0.9771	0.9716	—	—	—
ρ_w/ρ	0.9625	1	0.9492	0.9693	0.9771	0.9716	0.9749	0.684	0.9756
ρ_2	—	—	—	—	—	0	—	0.2817	—
σ_θ	0.5545	0.2142	0.5572	0.6669	—	0.3782	0.0179	0.0316	0.0270
σ_u	1.7425	0.7402	1.7305	—	—	0	—	—	—
σ_v	0.6951	0.7709	0.6992	0.7605	0.8304	0.7308	0.8420	1.1019	0.8617
σ_ε	0.2548	0.2020	0.2408	0.2476	0.2676	0.2732	0.2641	0.3313	0.4371
σ_x	—	—	0.0945	0.276	—	—	—	—	—
σ_z	0.2716	0.3313	0.2660	—	—	—	—	—	—
γ	—	—	—	—	—	—	—	—	- 0.4478
RMSE	0.0120	0.0191	0.0119	0.0083	0.0368	0.0367	0.0367	0.032	0.0272
# Params	8	7	9	27	3	4	4	5	5

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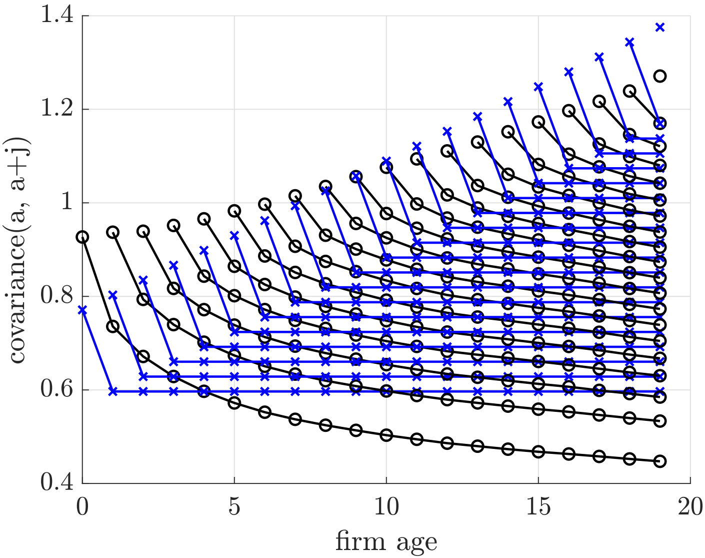
ALTERNATIVES: MODEL FIT

Model fit: Random walk with heterogeneous drift



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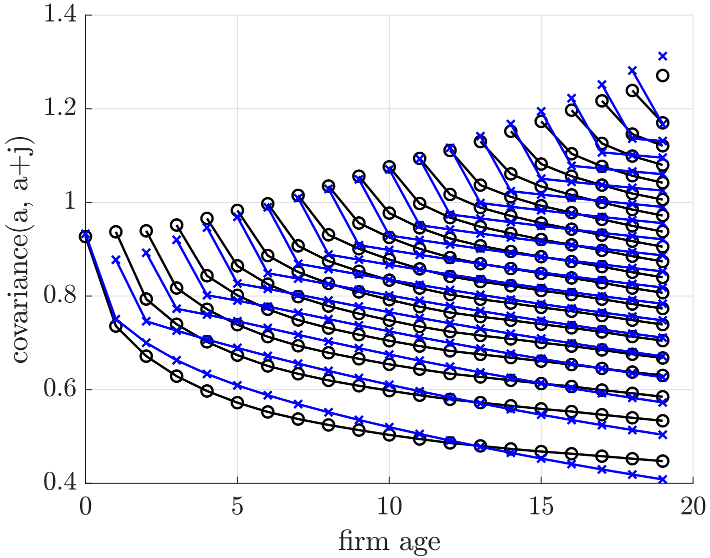
Model fit: Ex-ante + Ex-post growth rate shocks (random-walk)



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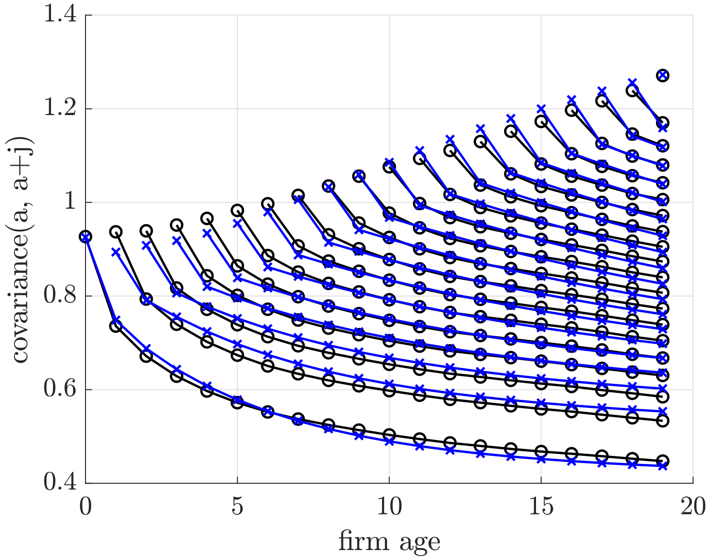
Model fit: Ex-ante + Ex-post persistent *and* permanent shocks



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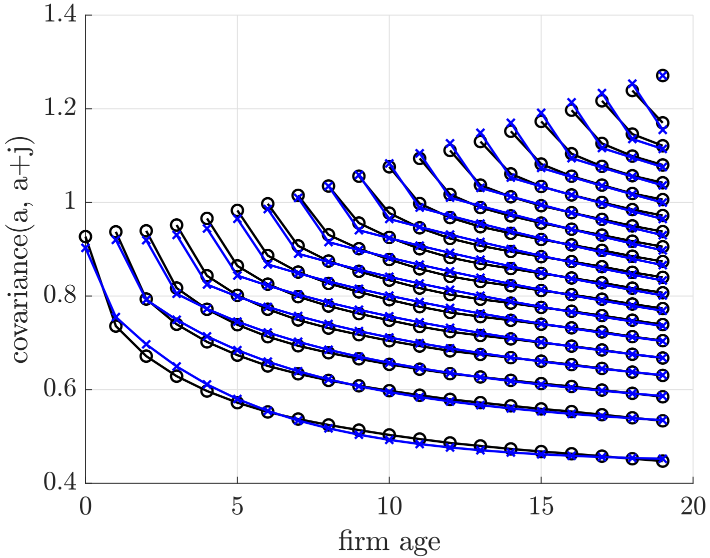
Model fit: Age dependent ex-post shocks



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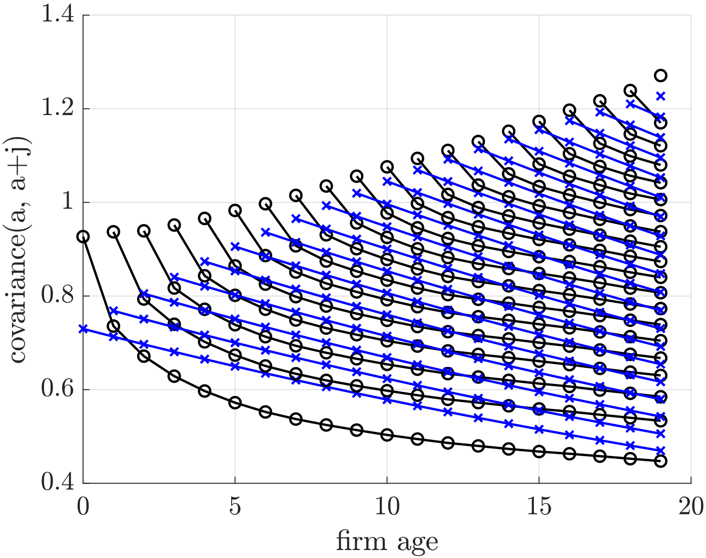
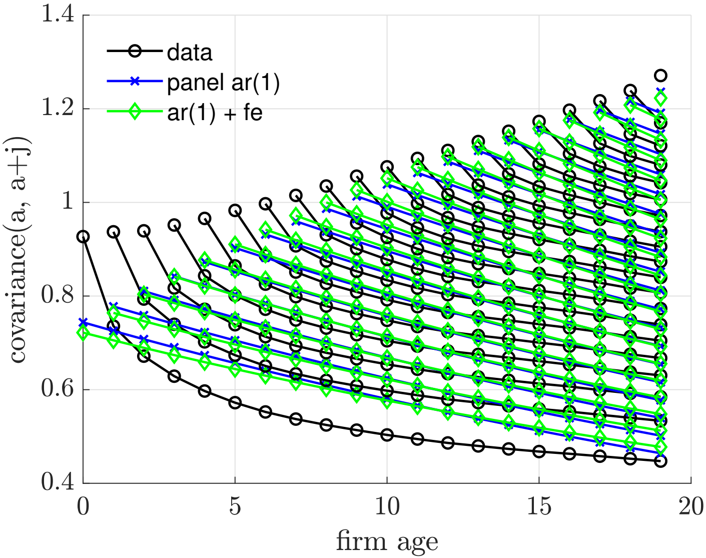
Model fit: AR(1)



Model fit: Panel AR(1) vs. separate AR(1) and FE terms



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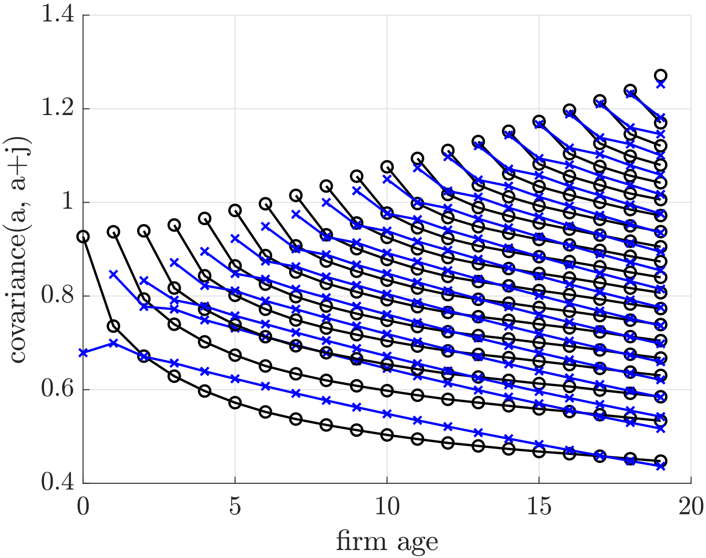
Model fit: Panel AR(2)



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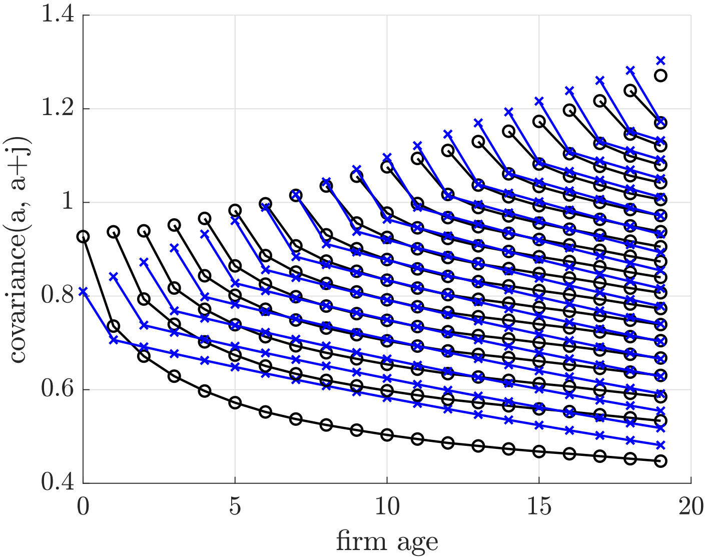
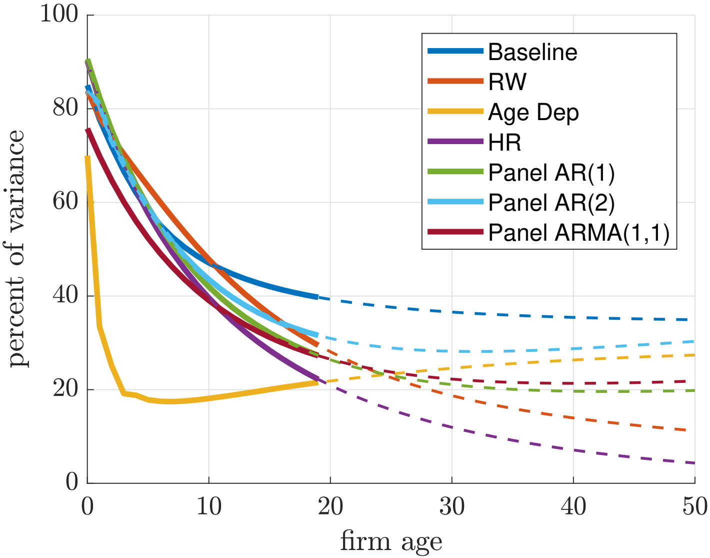
Model fit: Panel ARMA(1,1)



ALTERNATIVES: VARIANCE DECOMPOSITION

“Ex-ante” share of total variance by firm age



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EQUILIBRIUM MODEL DETAILS

Idiosyncratic demand shocks

$$\ln n_{i,a} = \underbrace{u_{i,a} + v_{i,a}}_{\text{Ex-ante components}} + \underbrace{w_{i,a} + z_{i,a}}_{\text{Ex-post components}} \qquad a = 0, 1, 2, \ldots$$

Ex-ante components:

$$u_{i,a} = \theta_i + \rho_u u_{i,a-1}$$
$$v_{i,a} = \rho_{\mathbf{w}} v_{i,a-1}$$

"pre startup" draws $a < 0$

$$\theta_i \sim IID\left(\mu_\theta, \sigma_\theta\right)$$
$$u_{i,-1} \sim IID\left(\mu_{\widetilde{u}}, \sigma_{\widetilde{u}}\right)$$
$$v_{i,-1} \sim IID\left(\mu_{\widetilde{v}}, \sigma_{\widetilde{v}}\right)$$

Ex-post components:

$$w_{i,a} = \rho_{\mathbf{w}} w_{i,a-1} + \varepsilon_{i,a}$$

with $w_{i,-1} = 0$ and for $a \geq 0$

$$\varepsilon_{i,a} \sim IID\left(0, \sigma_\varepsilon\right)$$
$$z_{i,a} \sim IID\left(0, \sigma_z\right)$$

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Parametrization

- demand process as in reduced-form part look
- all shocks drawn from normal distribution
- $\beta = 0.96$, $\eta = 6$
- $\frac{f^e}{f} = 0.82$ (Barseghyan and DiCecio, 2011)
- remaining parameters target firm-level values (0-19 years) look
  - autocovariance matrix of log employment (persistence and variance)
  - average size and exit rates by age (level μ_θ and fixed cost f)
  - normalize $\mu_{\widetilde{u}} = \mu_{\widetilde{v}} = 0$

Parameters of structural model

parameter		value
		set a priori
β	discount factor	0.96
η	elasticity of substitution	6.00
f^e	entry cost	0.44
		used to target moments
f	fixed cost of operation	0.539
δ	exogenous exit rate	0.041
μ_θ	permanent component θ, mean	-1.762
σ_θ	permanent component θ, st. dev.	1.304
$\sigma_{\widetilde{u}}$	initial condition u_{-1}, st. dev.	1.572
$\sigma_{\widetilde{v}}$	initial condition v_{-1}, st. dev.	1.208
σ_ϵ	transitory shock ϵ, st. dev.	0.307
σ_z	noise shock z, st. dev.	0.203
ρ_u	permanent component, persistence	0.393
ρ_v	transitory component, persistence	0.988

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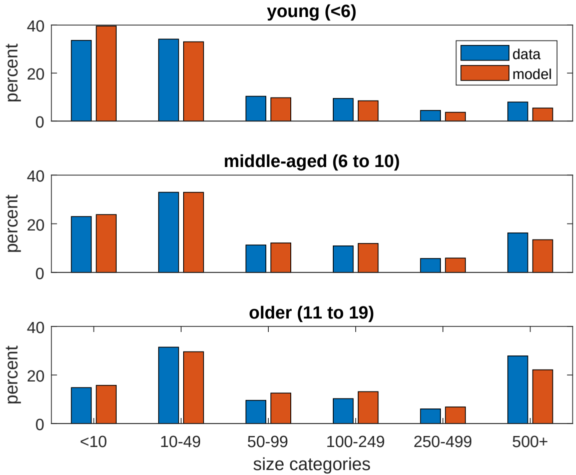
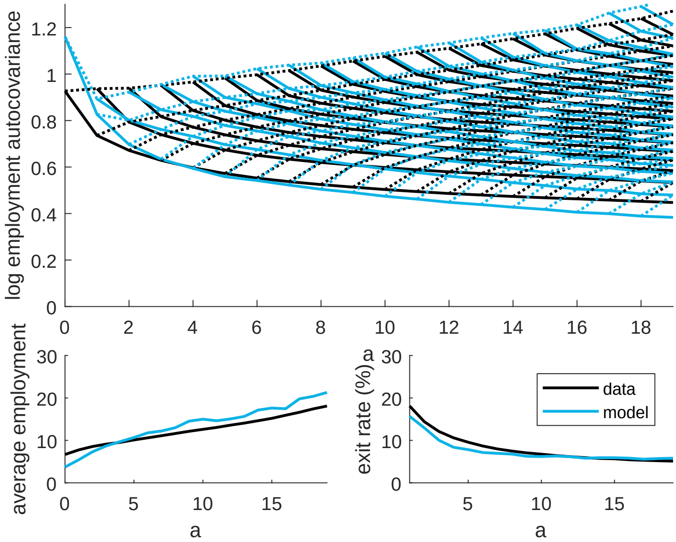
Model fit: untargeted

Size distribution: alternative calibration

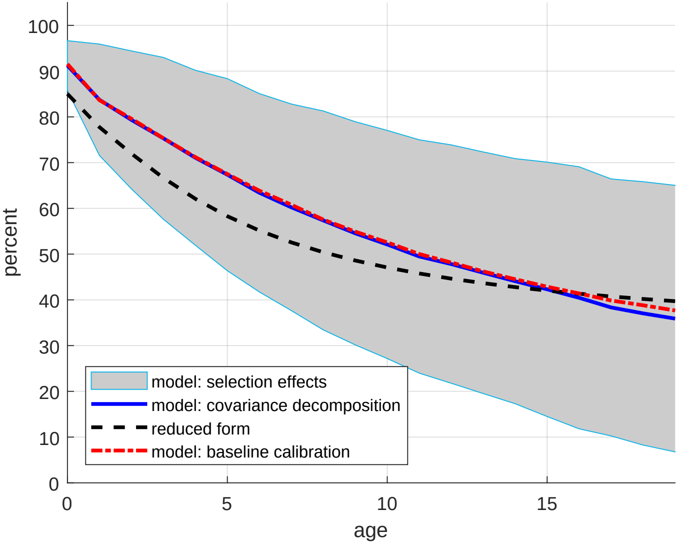


Note: Employment shares by firm age and size (employment). Values are expressed as percentages of total employment in firms between 0 to 19 year old firms, both in the data and the model. Data from the Business Dynamics Statistics.

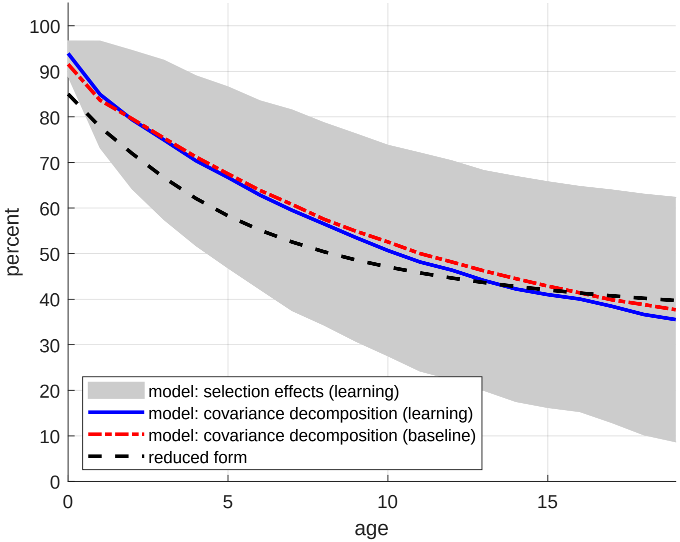
Remaining targets: alternative calibration



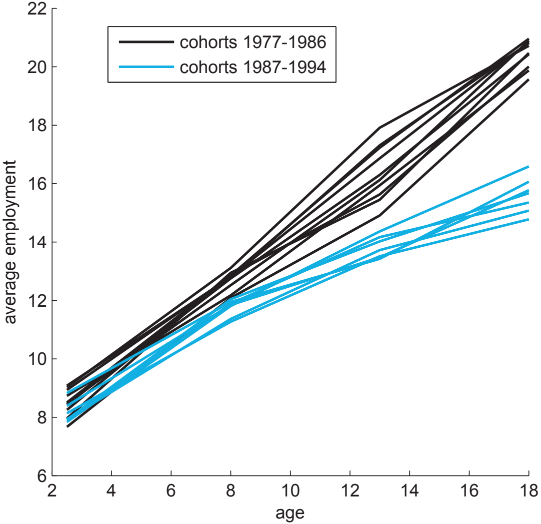
Variance decomposition: alternative calibration



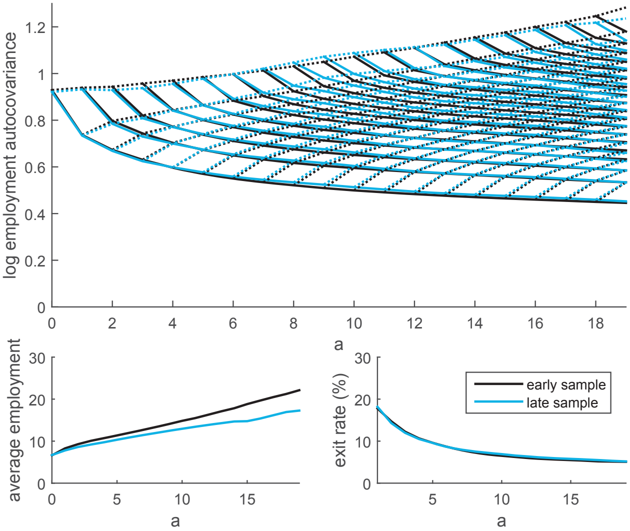
Variance decomposition: model with learning



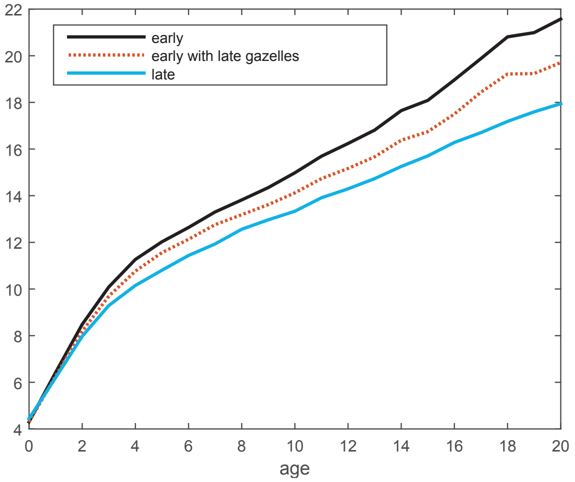
Supportive evidence: firm cohorts



How has the “nature” of firm growth changed?



"Gazelles" and average life-cycle profiles



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Extensions: learning

Limited information

- what if entrepreneurs do not know ex-ante profiles?
- using estimated parameters and optimal Bayesian learning
  - the majority of information uncovered in first years
  - survival is very informative about large ex-ante heterogeneity!
- extend model to include learning in early years
  - decisions based on (updated) beliefs about demand fundamental
  - main results very similar to baseline model!

Extensions: adjustment costs

Active accumulation of demand

- in addition to "passive accumulation of demand" (of θ_i via ρ_u)
- include option to invest into demand accumulation
- as in e.g. Foster, Haltiwanger, Syverson (2016)
- parameter estimates affected:
  - dispersion of θ lower
- main results very similar to baseline model!