The Two-Pillar Policy for the RMB*

Urban J. Jermann, Bin Wei, and Vivian Z. Yue†

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Abstract

We document stylized facts about China's recent exchange rate policy for its currency, the Renminbi (RMB). Our empirical findings suggest that a "*two-pillar* policy" is in place, aiming to balance RMB index *stability* and exchange rate *flexibility.* We then develop a tractable no-arbitrage model of the RMB under the two-pillar policy. Using derivatives data on the RMB and the US dollar index, we estimate the model to assess financial markets' views about the fundamental exchange rate and sustainability of the policy. Our model is able to predict the modification of the two-pillar policy in May 2017 when a discretion-based "countercyclical factor" was introduced for the first time. We also examine the model's ability to forecast RMB movements.

Keywords: Exchange Rate Policy, Two-Pillar Policy, Managed Float, Chinese currency, Renminbi, RMB, Central Parity, RMB Index.

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†Jermann is at the Wharton School of University of Pennsylvania and NBER, E-mail: jermann@ wharton.upenn.edu. Wei is at the Federal Reserve Bank of Atlanta, E-mail: bin.wei@atl.frb.org. Yue is at Emory University, the Federal Reserve Bank of Atlanta, and NBER, E-mail: vyue@emory.edu.

1 Introduction

How China manages its currency, the Renminbi or RMB, is one of the most consequential decisions in global financial markets. China has been the world's largest exporter since 2009. The value of the RMB is of paramount importance in determining the prices of China's exports. Many people have argued that China's undervalued currency contributes to its trade surplus that China runs consistently since 1993.[1] Hence it is important to understand how China's monetary authority conducts its exchange rate policy.

Since July 21, 2005 when the RMB was depegged from the U.S. dollar, China has adopted a *managed floating* regime. In the current regime, the People's Bank of China (PBOC) announces the *central parity* (or *fixing*) rate of the RMB against the U.S. dollar before the opening of the market each business day. The central parity rate serves as the midpoint of the daily trading range such that the intraday spot rate is allowed to fluctuate only within a narrow band around it. For a long time, little was revealed about how the central parity was determined. Since August 2015, the PBOC has implemented several reforms to improve the mechanism of setting the central parity, and to make it more transparent and more market-oriented. However, it still remains largely opaque and inscrutable to investors how the policy is implemented.

In this paper we first document stylized facts about China's recent exchange rate policy. Our empirical findings suggest that a "*two-pillar* policy" is in place, aiming to balance RMB index *stability* and exchange rate *flexibility.* According to the PBOC's Monetary Policy Report in the first quarter of 2016, the formation mechanism of the central parity depends on two key factors, or two pillars: the first pillar refers to "*the closing rates of the previous business day to reflect changes in market demand and supply conditions*", while the second pillar is related to changes in the currency basket, "*as a means to maintain the overall stability of the RMB to the currency basket.*" Empirically, we construct these two pillars and find that they explain as much as 80% of the variations in the central parity. Furthermore, we also present empirical evidence that both pillars receive roughly equal weights in setting the central parity.

Based on the stylized facts, we develop a tractable no-arbitrage model of the RMB under the two-pillar policy. Using derivatives data on the RMB and the US dollar index, we estimate the model to assess financial markets' views about the fundamental exchange rate and sustainability of the policy. Based on the daily estimation results for the sample

[1] For example, the United States has since 2003 been pressuring China to allow the RMB to appreciate and be more flexible (see Frankel and Wei (2007) for more analysis on this issue). On the other hand, the RMB was assessed in 2015 by the International Monetary Fund (IMF) to be no longer undervalued given its recent appreciation (see the press release at https://www.imf.org/en/News/Articles/2015/09/14/01/49/pr15237).

period between December 11, 2015 and December 31, 2018, we find that the RMB is valued on average about 1.7% higher than its fundamental value, which is consistent with recent expectations of further depreciation in the RMB. The gap between the RMB spot rate and its estimated fundamental value averages much higher at 2.8% in the first half the sample period that witnessed the first interest rate hike by the U.S. Federal Reserve. By contrast, the average deviation declines to 0.7% in the second half of the sample period, implying that the observed spot rate is very close to its fundamental value lately.

In addition, our model allows us to estimate how credible the two-pillar policy is as viewed by financial market participants. We find that during the whole sample period financial markets attached an average probability of 66% to the policy still being in place three months later. Furthermore, the estimated probability of policy continuation has fluctuated mostly between 60% and 90% until the PBOC unexpectedly changed the central parity rule in May 2017 by introducing a new "*countercyclical factor*." Despite the fact that the policy change was largely a surprise to financial markets, our model is able to predict the change, in the sense that our model-implied probability of policy continuation drops to the lowest level around 15% in the week preceding the PBOC's confirmation of the change on May 26, 2017.[2]

Our model outperforms a random walk model in predicting both the central parity and spot rates during the period when the two-pillar policy was in place (i.e., between December 11, 2015 and May 26, 2017). It is well known that it is very difficult to outperform a random walk model in predicting exchange rates (Meese and Rogoff (1983)). In the case of China, predicting its currency is an even more daunting task, given the opaqueness in how the RMB is managed. The superior forecasting performance of our model stems from two important sources. One is our formulation of the two-pillar policy that seems to closely capture the policy in practice as evidenced in our empirical analysis. This explains why our model can better forecast the central parity relative to the random walk model. Second, the key input for our model estimation is data on RMB options. The forward looking nature of the options makes our model estimation results informative about future exchange rate movements. It thus explains why our model can outperform a random walk model in predicting the spot rate. Furthermore, we show that after May 2017 when the discretion-based countercyclical factor was introduced, the RMB has become less predictable and it is increasingly more difficult to beat the random walk model in RMB forecast.

This paper is related to the literature on exchange rate target-zones initially developed for Europe's path to monetary union. Krugman (1991) presents a model of exchange rate behavior under a target zone regime. Bertola and Caballero (1992) and Bertola and Svensson

[2] See the statement on the CFETS website: http://www.chinamoney.com.cn/fe/Info/38244066.

(1993) extend the target-zone model to allow for realignment risk that the target cannot be credibly maintained. Dumas et al. (1995), Campa and Chang (1996), Malz (1996), Söderlind (2000), Hui and Lo (2009) utilize options data to estimate the realignment risk.

In a recent study, Jermann (2017) develops a no-arbitrage model in a spirit similar to the target-zone models to study the Euro-Swiss franc exchange rate floor policy. He derives the exchange rate endogenously based on a no-arbitrage condition and uses options data to estimate the survival probability of the floor policy as well as the exchange rate without the policy. Our no-arbitrage modeling approach is built on Jermann (2017). Our focus on China's exchange rate policy distinguishes this paper from Jermann (2017) and requires a substantially different modeling approach. Specifically, unlike Switzerland with a free floating regime, China maintains a managed floating regime and has an opaque exchange rate policy. Therefore, we first empirically analyze the formation mechanism of the central parity for the RMB. Based on our formulation of the two-pillar policy, we develop a model that characterizes the behavior of the RMB endogenously.

Our paper is also related to the literature on the Chinese exchange rate.[3] Many earlier research papers study the RMB undervaluation or misalignment, such as Frankel (2006), Cheung, Chinn, Fujii (2007), Yu (2007). After China depegged the RMB from the dollar, some papers attempted to characterize how China managed its exchange rate, such as Frankel (2009), Frankel and Wei (2009), Sun (2010). The recent exchange rate policy since 2015 is however underresearched. Cheung, Hui and Tsang (2018), Clark (2018), and McCauley and Shu (2018) empirically investigated the determinants of the central parity. To our best knowledge, our paper is the first one that explicitly characterizes and model the two-pillar policy. Furthermore, we incorporate the two-pillar policy into a no-arbitrage model of the RMB. Our model allows for the deviation from the uncovered interest rate parity as well as the endogenous determination of the exchange rate.[4] The estimates of the fundamental exchange rate and the probability of policy continuation are novel in the literature.

In the rest of the paper, we review the recent reforms on China's exchange rate policy and present empirical facts in Section 2. Section 3 presents our model. Section 4 contains the estimation results and examines the model's forecasting performance. Section 5 concludes.

[3] A related literature is on China's monetary policy and capital control, e.g., Prasad et al. (2005), Chang, Liu, and Spiegel (2015).

[4] The model is distinct from other equilibrium exchange rate models such as Verdelhan (2010), Gabaix and Maggiori (2015), Lustig and Verdelhan (2018).

2 Stylized Facts

We start by describing official policies for the RMB in the recent years. We argue that China's exchange rate policy since 2015 can be formulated by a two–pillar approach. Based on the data publicly available from the PBOC and Bloomberg, we provide empirical evidence for our formulation and document some novel empirical facts that provide more clarity about recent exchange rate policy in China.

2.1 Managed Floating RMB Regime

During the last three decades, China's transition into a market-based economy has been remarkable. However, the Chinese government continues to keep a firm grip on the RMB. Table 1 lists recent major events for the RMB.[5]

[Insert Table 1 Here.]

Since July 21, 2005 when the RMB was depegged from the dollar, China has implemented a managed floating regime for its currency. In the current regime, the PBOC announces the central parity (or fixing) rate of the RMB against the dollar at 9:15AM before the opening of the market each business day. The central parity rate serves as the midpoint of daily trading range in the sense that the intraday spot rate is allowed to fluctuate within a narrow band around it.

We obtain daily central parity data from the website of China Foreign Exchange Trade System (CFETS), a subsidiary of the PBOC.[6] The daily closing rate of the RMB against the dollar is obtained from Bloomberg.[7] Figure 1's Panel A displays the RMB central parity and closing rates since 2004. It is evident from the panel that the deviation of the closing rate from the central parity rate is typically very small and falls within the official trading band.

To strengthen the role of demand and supply force, China has gradually widened the trading band from an initial width of 0.3% to the current width of 2%.[8] Figure 1's Panel B

[5] See Goldstein and Lardy (2009) for more background information on China's exchange rate policy.

[6] The CFETS's website address is http://www.chinamoney.com.cn.

[7] To be more precise, the closing rate of the RMB against the dollar is the rate at the close (i.e., 5PM) in New York time. The central parity and closing rates in Figure 1 are plotted in New York time. Specifically, for each day, the closing rate is plotted together with the central parity rate known around 8:15PM or 9:15PM the previous day. As we show in Online Appendix, the figure is very similar if it is plotted in China time. To be coherent with the rest of the paper, we use the closing rate at 5PM in New York time, unless otherwise stated.

[8] Starting from the initial 0.3%, the bandwidth has been widened to 0.5% on May 21, 2007, to 1% on April 16, 2012, and 2% on March 17, 2014.

plots the deviation between the central parity and the close since 2004. The largest deviation occurred on July 21, 2005 when China depegged its currency from the dollar and the RMB appreciated by about 2 percent that day. It shows that as the trading band widened, the deviations have become more volatile, reflecting more flexibility of the RMB.

However, the PBOC can intervene in the foreign exchange market and control the extent to which the spot rate can deviate from the central parity. As a result, the effective width of the trading band can be much smaller than the officially announced width. For example, during the recent financial crisis, the RMB was essentially re-pegged to the dollar. As another example, since August 11, 2015, the band around the central parity has been effectively limited to 0.5%, with an exception of a few dates.

[Insert Figure 1 Here.]

2.2 The Two-Pillar Policy

Over the past decade, China has stepped up its efforts to internationalize the RMB (e.g., its inclusion in the IMF's SDR basket of reserve currencies in October 2016).[9] The internationalization of the RMB requires a more market-driven exchange rate policy. Accordingly, on August 11, 2015, China reformed its procedure of setting the daily central parity of the RMB against the dollar. The reformed formation mechanism is meant to be more transparent and more market driven. In particular, "quotes of the central parity of the RMB to the USD should refer to the closing rates of the previous business day to reflect changes in market demand and supply conditions", according to the PBOC's Monetary Policy Report in the first quarter of 2016. Following the reform the central parity of the RMB against the dollar depreciated 1.9%, 1.6%, and 1.1%, respectively, in the first three trading days under the reformed formation mechanism until the PBOC intervened to halt further depreciation.

Against the backdrop of the slowing economy and the first possible interest rate liftoff by the Federal Reserve on December 16, 2015, there was tremendous depreciation pressure on the RMB. In order to mitigate depreciation expectations, the PBOC introduced three RMB indices and reformed the formation mechanism of the central parity on December 11, 2015. The PBOC's Monetary Policy Report in the first quarter of 2016 provides more details about the new formation mechanism of the central parity. It states that

> "*a formation mechanism for the RMB to the USD central parity rate [consisting] of 'the previous closing rate plus changes in the currency basket' has been*

[9] Some recent studies on RMB internationalization include Chen and Cheung (2011), Cheung, Ma and McCauley (2011), Frankel (2012), Eichengreen and Kawai (2015), and Prasad (2016), among others.

preliminarily in place. The 'previous closing rate plus changes in the currency basket' formation mechanism means that market makers must consider both factors when quoting the central parity of the RMB to the USD, namely the 'previous closing rate' and the 'changes in the currency basket'."

As such, the formation mechanism of the central parity can be characterized by the following *two-pillar* policy whereby the central parity is a weighted average of the basket target and previous day's close:

$$S_{t+1}^{CP} = \left(\bar{S}_{t+1}\right)^{w} \left(S_t^{CL}\right)^{1-w}, \tag{1}$$

where $\overline{S}_{t+1}$ denotes the hypothetical rate that achieves stability of the basket, and S_t^{CL} the spot exchange rate of the RMB against the dollar at the close of day t. These two components are the two pillars of the central parity. As explained in the same report, the former is referred to as "the amount of the adjustment in the exchange rate of the RMB to the dollar, as a means to maintain the overall stability of the RMB to the currency basket", while the latter reflects the "market demand and supply situation."

Intuitively, the two-pillar policy allows the PBOC to make the RMB flexible and more market-driven through the second pillar, S_t^{CL}, and at the same time keep it stable relative to the RMB index through the first pillar, $\bar{S}_{t+1}$. At one extreme, when weight w is fixed at 100 percent, the central parity is fully determined by the first pillar; that is, the exchange rate policy is essentially basket pegging and the RMB index does not change over time. At the other extreme, when weight w is fixed at zero, the central parity is fully determined by the second pillar and is thus market driven to the extent that the spot exchange rate is permitted to fluctuate within a band around the central parity rate under possible interventions by the PBOC.

To explicitly represent the pillar associated with the currency basket $\bar{S}_{t+1}$, we turn to discussion of the RMB indices in the next subsection.

2.3 RMB Indices

On December 11, 2015, China Foreign Exchange Trade System (CFETS), a subsidiary of the PBOC, introduced the CFETS index as another measure of the RMB's performance against a basket of 13 currencies with weighting based mainly on international trade. The basket was unveiled as a way of shifting focus away from the RMB's moves against the dollar following China's unexpected devaluation in August that year. Besides the CFETS index, the PBOC also started publishing two other trade-weighted RMB indices based on the IMF's

Special Drawing Rights (SDR) and Bank for International Settlement (BIS) baskets since December 2015. We refer to these two RMB indices as the SDR and BIS indices throughout the paper. All three indices have the same base level of 100 in the end of 2014 and are published regularly.

Table 2 reports the composition of these three RMB indices. Note that the composition for the CFETS and SDR indices has changed since 2017. Take the CFETS index as an example. Initially in this index the dollar had the largest weight of 26.4 percent, followed by the euro and the yen with 21.4 percent and 14.7 percent, respectively. On December 29, 2016, the PBOC decided to expand the CFETS basket by adding 11 new currencies and at the same time reduced the dollar's weight to 22.4 percent, effective in 2017.

[Insert Table 2 Here.]

In essence, an RMB index (e.g., CFETS) is a geometric average of a basket of currencies:

$$B_t = C_B \left(S_t^{CP,USD/CNY}\right)^{w_{USD}} \left(S_t^{CP,EUR/CNY}\right)^{w_{EUR}} \left(S_t^{CP,JPY/CNY}\right)^{w_{JPY}} \cdots \quad (2)$$

where C_B is a scaling constant used to normalize the index level to 100 in the end of 2014, $S_t^{CP,i/CNY}$ denotes the central parity rate in terms of the RMB for the currency i in the basket, and w_i the corresponding weight for $i = USD$, EUR, JPY, etc. When the RMB strengthens (or weakens) relative to the currency basket, the RMB index goes up (or down).

The key central parity rate is the one of the RMB against the dollar, denoted as $S_t^{CP} \equiv 1/S_t^{CP,USD/CNY}$. According to the website of CFETS, once S_t^{CP} is determined, the central parity rates for other non-dollar currencies are determined as the cross rates between S_t^{CP} and the spot exchange rates of the dollar against those currencies. Therefore, we focus on the formation mechanism of the central parity rate S_t^{CP}. For this reason, we refer to it simply as the central parity wherever there is no confusion.

The RMB index can be rewritten in terms of the central parity rate of the RMB against the dollar, S_t^{CP}, and a US dollar index of all the non-RMB currencies, X_t:

$$B_t = \chi \frac{X_t^{1-w_{USD}}}{S_t^{CP}}, \quad (3)$$

where X_t denotes the index-implied dollar index, defined by

$$X_t \equiv C_X \left(\frac{S_t^{CP,EUR/CNY}}{S_t^{CP,USD/CNY}}\right)^{\frac{w_{EUR}}{1-w_{USD}}} \left(\frac{S_t^{CP,JPY/CNY}}{S_t^{CP,USD/CNY}}\right)^{\frac{w_{JPY}}{1-w_{USD}}} \cdots \quad (4)$$

with a scaling constant C_X, and $\chi \equiv C_B/C_X^{1-w_{USD}}$. In the next subsection we show that the index-implied dollar index is highly correlated with the well known U.S. Dollar Index that is actively traded on the Intercontinental Exchange under the ticker "DXY". The scaling constant C_X is chosen such that X_t coincides with the U.S. dollar index DXY in the end of 2014.

The pillar associated with the currency basket $\bar{S}_{t+1}$ is determined so as to achieve "stability" of the basket.[10] Put differently, $\bar{S}_{t+1}$ is the value that would keep the RMB index unchanged if the central parity were set at such value. Therefore it is straightforward to show[11]

$$\overline{S}_{t+1} = S_t^{CP} \left(\frac{X_{t+1}}{X_t} \right)^{1-w_{USD}} . \tag{5}$$

The expression of $\overline{S}_{t+1}$ in equation (5) is intuitive. The key idea is that movements in the RMB index are attributable to movements in either the value of the RMB relative to the dollar, or the value of the dollar relative to the basket of non-dollar currencies in the RMB index, or both. The relative contributions of these two types of the movements are determined by w_{USD} and $(1 - w_{USD})$, respectively. As a result, in order for the RMB index to remain unchanged in response to movement in the dollar index, hypothetically, the value of the RMB relative to the dollar should be at a level that exactly offsets such movement.

Substituting the above equation into equation (1), the two-pillar policy can be described by the following equation:

$$S_{t+1}^{CP} = \left[S_t^{CP} \left(\frac{X_{t+1}}{X_t} \right)^{1-w_{USD}} \right]^w \left(S_t^{CL} \right)^{(1-w)} . \tag{6}$$

Before we present in the following subsection empirical evidence for the two-pillar policy based on the above equation, we also need to discuss two modifications of the policy.

- First, on February 20, 2017, the PBOC adjusted the formation mechanism of the central parity. Specifically, the PBOC reduced the reference period for the central parity against the RMB index to 15 hours (i.e., between 4:30PM on the previous day and 7:30AM) from 24 hours under the previous mechanism. According to Monetary

[10] For expositional purpose, the weight of the dollar w_{USD} in the RMB index is assumed to be fixed here. However, the dollar's weights in both CFETS and SDR indices have been adjusted in the end of 2016. In Appendix A, we provide detailed explanation about how to account for the change in w_{USD} in our analysis.

[11] Specifically, the expression of $\overline{S}_{t+1}$ can be derived as follows. At time t, the RMB index is given by $B_t = \chi \frac{X_t^{1-w_{USD}}}{S_t^{CP}}$. At time $t+1$, if the index-implied dollar baset changes its value to X_{t+1}, the RMB index would become $B_{t+1} = \chi \frac{X_{t+1}^{1-w_{USD}}}{S_{t+1}^{CP}}$ if the central parity were set as S_{t+1}^{CP}. Equalizing B_t and B_{t+1} (i.e., $B_t = \chi \frac{X_t^{1-w_{USD}}}{S_t^{CP}} = \chi \frac{X_{t+1}^{1-w_{USD}}}{S_{t+1}^{CP}}$) determines the hypothetical value of S_{t+1}^{CP}, or $\overline{S}_{t+1}$, which would keep the RMB index unchanged.

Policy Report in the second quarter of 2017, the rational for the adjustment is to avoid "*repeated references to the daily movements of the USD exchange rate in the central parity of the following day*" since the previous close has already incorporated such information to a large extent. This adjustment, however, is widely believed to have limited impact on the RMB exchange rate.

- Second, on May 26, 2017, the PBOC confirmed that it had modified the formation mechanism of the central parity by introducing a new "countercyclical factor." The modification is believed to "give authorities more control over the fixing and restrain the influence of market pricing."[12] The policy change is perceived by many market participants as a tool to address depreciation pressure without draining foreign reserves. However, it undermines earlier efforts to make the RMB more market driven. The countercyclical factor was then subsequently removed as reported by Bloomberg on January 9, 2018. It signals the return to the previous two-pillar policy. The removal of the countercyclical factor in January 2018 reflects the RMB's strength over the past year as well as the dollar's protracted decline.

2.4 Empirical Evidence

We document here that the RMB central parity has closely tracked our equation (1) summarizing the official policy statements. In addition, we find strong empirical support for a central parity rule that gives equal weights to each of the two pillars (i.e., $w = 1/2$).

We use daily central parity rates to reconstruct the three RMB indices on a daily basis as stated in Appendix A. Figure 2 plots the reconstructed indices (blue lines) as well as the official indices (red circles). The figure shows almost perfect fit of our reconstructed indices with the official ones.

[Insert Figure 2 Here.]

Next, we construct the index-implied basket X_t that is directly comparable to the US dollar Index DXY_t. Based on equation (4) and the estimated coefficient C_X,[13] we construct and plot the index-implied dollar basket X_t as well as the DXY index (blue solid line) in

[12] See the article "China Considers Changing Yuan Fixing Formula to Curb Swings" on Bloomberg News on May 25, 2017.

[13] Recall that C_X is set such that the resulting X_t coincides with the dollar index on December 31, 2014; that is, $C_X = \frac{DXY_{12/31/2014}}{X_{1/1/2015}}$. For each of the three indices, the coefficient C_X is determined below: $C_X^{CFETS} = 47.1452$, $C_X^{BIS} = 14.1169$, and $C_X^{SDR} = 108.6492$.

Figure 3 below. The figure shows high correlation between the index-implied dollar basket X_t and the US dollar index DXY_t.

[Insert Figure 3 Here.]

Based on the constructed index-implied dollar basket X_t, we can empirically examine our formulation of the two-pillar policy, particularly, the value of w. Specifically, we run the following regression for the whole sample period between December 11, 2015 and December 31, 2018:

$$\log\left(\frac{S_{t+1}^{CP}}{S_t^{CP}}\right) = \alpha \cdot (1 - w_{USD}) \log\left(\frac{X_{t+1}}{X_t}\right) + \beta \cdot \log\left(\frac{S_t^{CL}}{S_t^{CP}}\right) + \epsilon_{t+1}. \tag{7}$$

In addition, we repeat the analysis for two subsample periods: one is "*Subsample I*" or the period between December 11, 2015 and February 19, 2017, and the other is "*Subsample II*" or the period between February 20, 2017 and December 31, 2018. As mentioned before, these two subsample periods differ on whether a 24-hour or 15-hour reference period is used.

Under the two-pillar policy formulated in equation (1), the coefficients α and β correspond to w and $1-w$, respectively. The regression results are reported in Column "Whole Sample" of Panel A in Table 3. Interestingly, even though we do not impose the restriction that the coefficients should sum up to one (i.e., $\alpha + \beta = 1$), the regression results suggest it roughly holds in the data. Most importantly, the regression results support that $w = 1/2$ as both of the coefficients α and β are roughly equal to one half. The PBOC's Monetary Policy Report in the first quarter of 2016 has an example that seems to suggest equal weights for both pillars. Consistent with the report, our empirical analysis provides supportive empirical evidence for $w = 1/2$ for the period following December 11, 2015 when the RMB indices were announced for the first time. Moreover, the regression has a very high R-squared at around 80%, which suggests that our formulation of the two-pillar policy has a large explanatory power in describing the formation mechanism of the central parity in practice.

The regression in equation (7) is unconstrained in the sense that we do not impose the restriction that the coefficients should sum up to one. We also run the regression with the restriction imposed. The results are reported in Column "Whole Sample" of Panel A in Table 4. The constrained regression results confirm $w = 1/2$, especially for the subsample period I.

It is important to point out that the index-implied dollar basket X_t used in the regression (7) is constructed using the daily central parity rates for all constituent currencies in the RMB index. As we show in the online appendix, our analysis suggests that the central parity rates for non-dollar currencies are highly dependent on the spot exchange rates around 9AM in China time. Furthermore, as mentioned before, the closing rate S_t^{CL} used in the regression (7) is obtained from the daily closing rate in New York time from Bloomberg. So roughly speaking, the first regressor in (7) incorporates market information within the 24-hour reference period from 9AM to 9AM on the previous day, while the second regressor uses the spot rate information around 5AM of the same day.

To more closely follow the formation mechanism of the central parity in practice, we obtain intraday exchange rate data from the Bloomberg BFIX data, which are available every 30 minutes on the hour and half-hour throughout the day. For each week, the BFIX data begin Sunday 5:30 PM New York time and end Friday 5 PM New York time. We then use the BFIX data to construct intraday values for the US dollar index (DXY), CFETS, and SDR indices. Because Bloomberg has stopped producing the BFIX data for Venezuela currency, we do not construct intraday spot fixings for the BIS index.

For a given index, we collect the BFIX data for all constituent currencies and then convert the data in China local time, taking into account time-zone difference and daylight saving period. Based on the BFIX data, we can thus construct the index-implied dollar basket X_t and the spot rate S_t^{CL} for all 48 half-hour intervals throughout the day. As a result, we can now run the following regression that utilizes the intraday information:

$$\log \frac{S_{t+1}^{CP}}{S_t^{CP}} = \alpha \cdot (1 - w_{USD}) \log \frac{X_{t+1}^o}{X_t^o} + \beta \cdot \log \frac{S_t^{CL}}{S_t^{CP}} + \epsilon_{t+1}, \tag{8}$$

where X_t^o denotes the index-implied dollar basket X_t at a certain time t^o on date t before the open, and, similarly, with a slight abuse of notation S_t^{CL} denotes the spot rate at a certain time t^c before the open on date t. Based on the BFIX data, we run the above regression for all $2304\,(= 48 \times 48)$ possible values of (t^o, t^c). According to the results reported in the online appendix, we find that the regression that has the highest R-squared for both CFETS and SDR indices is the one where t^o and t^c are associated with 7:30AM and 4:30PM, respectively. Interestingly, our finding seems consistent with the formation mechanism in practice. For example, both times coincide with the 15-hour reference period when the PBOC adjusted the formation mechanism on February 20, 2017 by reducing the 24-hour reference period to the 15-hour reference period between 7:30AM and 4:30PM on the previous day. Therefore, we focus on the regression where X_t^o represents the index-implied dollar basket at 7:30AM and S_t^{CL} represents the close at 4:30PM in China time on date t.

The regression results are reported in Column "Whole Sample" of Panel B in Table 3. The results suggest that the coefficients α and β are roughly equal, although they do not add up to one as in Panel A. The constrained regression results in Panel B of Table 4 lend further support to the finding of equal weights.

To account for the modification announced on February 20, 2017 that the reference period for the central parity against the RMB index is shortened from 24 to 15 hours (i.e., between 4:30PM on the previous day and 7:30AM), we also run the following regression that uses the 15-hour overnight reference period:

$$\log \frac{S_{t+1}^{CP}}{S_t^{CP}} = \alpha \cdot (1 - w_{USD}) \log \frac{X_{t+1}^o}{X_t^c} + \beta \cdot \log \frac{S_t^{CL}}{S_t^{CP}} + \epsilon_{t+1}, \tag{9}$$

where X_t^c denotes the index-implied dollar basket X_t at 4:30PM on date t, and X_t^o and S_t^{CL} are defined in the same way as before.

We refer to the regression in (8) using the 24-hour window as 24-hour "*Regression I*", and the one in (9) as overnight "*Regression II*". Arguably, regression I (or regression II) is more relevant for subsample period 1 (or subsample period 2). The results of overnight regression II are reported in Panel C of Tables 3 and 4. The results are similar as before.

[Insert Tables 3 and 4 Here.]

The above results from running the regression in equation (7) shed light on the *average* weights on the two pillars during a fixed period, but are silent about possible time variation in the weights. To further investigate how the weights may possibly vary over time, we run the above regression by using 60-day rolling windows, starting from 60 business days after August 11, 2015. Figure 4 plots the estimate of weight w implied by the rolling-window regressions.

[Insert Figure 4 Here.]

Figure 4 shows that the weight w is initially around 0.1 in the period prior to the introduction of the RMB indices, and steadily increases and is then stabilized around 0.5 until May 2017 when a countercyclical factor was introduced. The finding is consistent with the regression results for the period between August 11, 2015 and December 10, 2015, reported in the online appendix, that the regression-based estimate of w is close to zero and the R-squared is very high (around 0.95) for this period. The findings suggest that since the reform announced on August 11, 2015 but before the introduction of the RMB indices on December 11, 2015, the formation mechanism of the central parity indeed has started incorporating information from the previous day's closing rate. To be more precise, the formation mechanism during this period seems to be described well by a "one-pillar" policy in the sense that it is almost completely determined by the previous day's close as the dollar basket implied in the RMB index carries very little weight.

From Figure 4, we can also see that the two-pillar policy with equal weighting is in place between 2016 and mid 2017. Since mid 2017, the estimate of weight w exhibits more variability under the modified two-pillar policy with the new countercyclical factor.

2.5 How Is the Two-Pillar Policy Implemented?

Piecing together all the findings, we conjecture that the implementation of the two-pillar policy may look like something below. For each day t, before the market open the PBOC conducts the following steps:

1. Collect the previous close S_{t-1}^{CL} and the index-implied basket X_{t-1}^{c} at 4:30PM on the previous day $t-1$, as well as X_{t-1}^{o} at 7:30AM of day $t-1$;

2. Collect 7:30AM foreign exchange rate data for all constituent currencies against the dollar and compute the index-implied basket X_t^o based on the information on 7:30AM of day t;

3. Compute the central parity of the RMB against the dollar S_t^{CP} under the two-pillar policy, using S_{t-1}^{CP}, X_t^o, X_{t-1}^o (or X_{t-1}^c), S_{t-1}^{CL};[14]

4. Compute the central parity of the RMB against other currencies based on S_t^{CP} and 7:30AM (or 9AM) foreign exchange rates. For example, $S_t^{CP,CNY/EUR} = S_t^{CP}/S_t^{EUR/USD}$, etc.

5. Compute the RMB index B_t using the central parity rates for all constituent currencies;

6. At 9:15AM, announce day t's central parity rates S_t^{CP}, $S_t^{CP,CNY/EUR}$, etc., as well as the RMB index B_t.

3 A Model for the RMB

In this section we build a no-arbitrage model for the RMB based on the previous stylized facts about the two-pillar policy.

[14]During the period under the modified two-pillar policy with the countercyclical factor, in step 3 the PBOC also uses discretion or so-called the counterycyclical factor to determine S_t^{CP}, besides the two pillars.

3.1 Setup

Building on Jermann (2017), we assume that there is a probability p that the two-pillar policy regime continues, and a probability $(1-p)$ that the policy ends tomorrow. If it ends, the exchange rate equals the fundamental exchange rate V_t that is arbitrage-free itself, satisfying

$$\frac{1+r^{\$}}{1+r^{C}} E_t^Q \left[V_{t+1}\right] = V_t. \tag{10}$$

The interpretation of V_t can be quite broad; for example, we can interpret it as the exchange rate that prevails when the RMB becomes freely floating.

The equilibrium exchange rate $\widetilde{S}_t$ is given by

$$\begin{aligned} \tilde{S}_t &= \frac{1+r^{\$}}{1+r^{C}} \left[p E_t^Q H\left(\tilde{S}_{t+1}, S_{t+1}^{CP}, b\right) + (1-p)\, E_t^Q V_{t+1}\right] \\ &= \frac{1+r^{\$}}{1+r^{C}} p E_t^Q H\left(\tilde{S}_{t+1}, S_{t+1}^{CP}, b\right) + (1-p)\, V_t, \end{aligned} \tag{11}$$

where b denotes the width of the trading band around the central parity,

$$H\left(\tilde{S}_{t+1}, S_{t+1}^{CP}; b\right) = \max\left(\min\left(\tilde{S}_{t+1}, S_{t+1}^{CP}\left(1+b\right)\right), S_t^{CP}\left(1-b\right)\right), \tag{12}$$

and $E_t^Q[\cdot]$ refers to expectations under the RMB risk-neutral measure, and $r^{\$}$ and r^C are per-period interest rates in the U.S. and China, respectively. Intuitively, the current spot rate is the expected value of the exchange rate in the two regimes, appropriately adjusted for the yields. This follows the same approach as in Jermann (2017), in particular, the second line is derived under the assumption in equation (10). Note that in the case where the bandwidth b is so large that the band is never binding, $H\left(\tilde{S}_{t+1}, S_{t+1}^{CP}; b\right)$ is always equal to $\tilde{S}_{t+1}$ and the solution to equation (11) is $\tilde{S}_t = V_t$. In practice, the band with $b = 2\%$ is still occasionally binding.[15]

If the equilibrium exchange rate $\widetilde{S}_t$ falls within the band around the central parity, the observed spot exchange rate at the close is equal to $\widetilde{S}_t$. Otherwise, the spot exchange rate is equal to $\widetilde{S}_t$ truncated at the (lower or upper) boundary of the band. Therefore, the model-implied spot exchange rate at the close then equals

$$S_t^{CL} = H\left(\tilde{S}_t, S_t^{CP}; b\right). \tag{13}$$

Next, we turn to the formation mechanism of the central parity S_t^{CP}. Interpreted in the

[15] By simulation we show that under some realistic parameter values the band with $b = 2\%$ is binding 1.1% of the time during a 3-month period. The results are unreported here, but available upon request.

narrow sense, the mechanism described in the PBOC's Monetary Policy Report in the first quarter of 2016 implies the following "*two-pillar rule*":

$$S_{t+1}^{CP} = \left[S_t^{CP} \left(\frac{X_{t+1}}{X_t} \right)^{(1-w_{USD})} \right]^w H\left(\tilde{S}_t, S_t^{CP}; b \right)^{1-w}. \tag{14}$$

Recall that X_t denotes the dollar basket implied in the RMB index. In the above rule, the term in the first square brackets, $S_t^{CP} \left(\frac{X_{t+1}}{X_t} \right)^{(1-w_{USD})}$, is the first pillar to maintain the stability of the RMB index. On the other hand, the term in the second square brackets, $H\left(\tilde{S}_t, S_t^{CP}; b \right)$, is the previous close or the second pillar to account for the "market demand and supply situation".

To make the model tractable enough for estimation, we consider the following two-pillar rule, which represents a good approximation of the two-pillar policy empirically and is also consistent with the formation mechanism of the central parity in the PBOC's report in the broad sense. Specifically, we keep the first pillar unchanged, but model the second pillar as $S_t^{CP} \left(\frac{V_{t+1}}{V_t} \right)^\gamma$ where $0 \leq \gamma \leq 1$. That is, the two-pillar rule modeled as the following:

$$\begin{aligned} S_{t+1}^{CP} &= \left[S_t^{CP} \left(\frac{X_{t+1}}{X_t} \right)^{(1-w_{USD})} \right]^w \left[S_t^{CP} \left(\frac{V_{t+1}}{V_t} \right)^\gamma \right]^{1-w} \\ &\equiv S_t^{CP} \left(\frac{X_{t+1}}{X_t} \right)^\alpha \left(\frac{V_{t+1}}{V_t} \right)^\beta, \end{aligned} \tag{15}$$

where $\alpha \equiv (1 - w_{USD})\, w$ and $\beta \equiv \gamma\,(1 - w)$ are some constants bounded between 0 and 1.

The modified two-pillar approach is broadly consistent with the PBOC's report for the following reasons. First, the second pillar in the modified rule takes into account the "market demand and supply situation" captured by the term $(V_{t+1}/V_t)^\gamma$. Second, in the case with a non-binding band (i.e., b is sufficiently large), the equilibrium exchange rate $\tilde{S}$ and the close S_t^{CL} are always equal to V_t. If the PBOC had exclusively focused on the second pillar and ignored the first one, it would set $S_{t+1}^{CP} = S_t^{CL} = V_t$ or $S_{t+1}^{CP} = S_t^{CP} V_t/V_{t-1}$, which is almost identical to the modified second pillar $S_t^{CP} \left(\frac{V_{t+1}}{V_t} \right)^\gamma$ if we set $\gamma = 1$ and ignore the one-period lag.[16] In addition, it is symmetric to use V_{t+1}/V_t in modeling the second pillar as we use X_{t+1}/X_t in modeling the first pillar. The symmetry makes the model very tractable, especially in continuous time. Third, with a possibly binding band (i.e., b is not too large), the second pillar in the original rule in equation (14) is bounded between $(1 - b)$ and $(1 + b)$. That is, not all market-driven forces are considered in forming the central parity in equation

[16] Because in our estimation one period is taken as one day, the lag of one period makes essentially no difference.

(14). To capture this feature, we set $\gamma < 1$. Later on, we calibrate this parameter γ such that the term $\left(\frac{V_{t+1}}{V_t}\right)^{\gamma}$ is roughly bounded within the limits $(1-b)$ and $(1+b)$ for reasonable fluctuations in the fundamental rate V_t. In summary, the two-pillar approach in equation (15) is a very realistic way to model the formation mechanism of the central parity.

Equation (11) implies that the equilibrium exchange rate $\tilde{S}_t = \tilde{S}\left(S_t^{CP}, V_t\right)$ is a function of S_t^{CP} and V_t. Due to homogeneity of function $H(x, y; b)$, we can scale the equilibrium exchange rate $\tilde{S}$ and the fundamental rate V_t by the central parity S_t^{CP} and obtain $\widehat{S} \equiv \tilde{S}/S_t^{CP}$ and $\widehat{V} \equiv V_t/S_t^{CP}$. As a result, determination of the equilibrium exchange rate $\tilde{S}_t$ boils down to determination of the univariate function $\widehat{S}(\widehat{V}_t)$. Based on this, we can simplify equation (11) to

$$\begin{aligned} \widehat{S}(\widehat{V}_t) &= \frac{\tilde{S}\left(V_t, S_t^{CP}\right)}{S_t^{CP}} \\ &= \frac{1+r^{\$}}{1+r^{C}} p E_t^Q \left[H\left(\frac{\tilde{S}\left(V_{t+1}, S_{t+1}^{CP}\right)}{S_t^{CP}}, \frac{S_{t+1}^{CP}}{S_t^{CP}}; b \right) \right] + (1-p)\widehat{V}_t \\ &= \frac{1+r^{\$}}{1+r^{C}} p E_t^Q \left[\frac{S_{t+1}^{CP}}{S_t^{CP}} H\left(\widehat{S}\left(\widehat{V}_{t+1}\right), 1; b \right) \right] + (1-p)\widehat{V}_t. \end{aligned} \tag{16}$$

For ease of notation from now on we simply write $H(\widehat{S}(\widehat{V}_{t+1}), 1; b)$ as $H(\widehat{S}(\widehat{V}_{t+1}))$.

Substituting the two-pillar rule into the above equation, we can show that both $\widehat{S}(\widehat{V}_t)$ and $\widehat{V}_t$ are determined as follows:

$$\widehat{S}\left(\widehat{V}_t\right) = \frac{1+r^{\$}}{1+r^{C}} p E^Q \left[\left(\frac{X_{t+1}}{X_t}\right)^{\alpha} \left(\frac{V_{t+1}}{V_t}\right)^{\beta} H\left(\widehat{S}\left(\widehat{V}_{t+1}\right)\right) \right] + (1-p)\widehat{V}_t, \tag{17}$$

$$\frac{\widehat{V}_{t+1}}{\widehat{V}_t} = \left(\frac{X_{t+1}}{X_t}\right)^{-\alpha} \left(\frac{V_{t+1}}{V_t}\right)^{1-\beta}. \tag{18}$$

We estimate the model to match the close and four RMB options. Under this model, the price of a call option with maturity τ and strike K is given by

$$C(K;\tau) = e^{-r_{CNY}\tau} \left(p^{\tau} E^Q \left[\max\left(H\left(\widetilde{S}_{t+\tau}, S_{t+\tau}^{CP}, b \right), K \right) \right] + (1-p^{\tau}) E^Q \left[\max\left(V_{t+\tau}, K\right) \right] \right). \tag{19}$$

The price of a put option can be represented in a similar way.

In the next subsection we show that if we cast the model in continuous time, we are able to derive the equilibrium exchange rate $\widehat{S}(\widehat{V}_t)$ in closed form. Furthermore in the special case with $b = 0$, we are also able to derive option prices in closed form.

3.2 Model Solution in Continuous Time

We describe the model setup in discrete time in the last subsection where each period lasts for Δt in years. The model is particularly tractable in continuous time once we let Δt tend to zero. Under the RMB risk-neutral measure Q,

$$
\begin{aligned}
\frac{dV_t}{V_t} &= (r_{CNY} - r_{USD})\,dt + \sigma_V dW_{V,t} && (20)\\
&\equiv \mu_V dt + \sigma_V dW_{V,t}, \\
\frac{dX_t}{X_t} &= \left(r_{DXY} - r_{USD} - \rho\sigma_X\sigma_V + \sigma_X^2\right) dt + \sigma_X \left(\rho dW_{V,t} + \sqrt{1-\rho^2} dW_{X,t}\right) && (21)\\
&\equiv \mu_X dt + \sigma_X \left(\rho dW_{V,t} + \sqrt{1-\rho^2} dW_{X,t}\right),
\end{aligned}
$$

where $W_{V,t}$ and $W_{X,t}$ are independent Brownian motions under the measure Q, and r_{CNY}, r_{USD}, r_{DXY} are instantaneous interest rates for the RMB, the dollar, and the currency basket in the dollar index DXY, respectively. That is, the per-period interest rates in the preceding discrete-time setup satisfy: $r^{\$} = \exp(r_{USD}\Delta t)$ and $r^C = \exp(r_{CNY}\Delta t)$. Similarly, the per-period probability $p = 1 - \lambda\Delta t$ while we assume that the current managed floating regime will be abandoned upon arrival of a Poisson process with intensity λ. The processes $\{X_t\}$ and $\{V_t\}$ are assumed to have a correlation ρ. Their drifts are specified in the above equations so as to exclude any arbitrage opportunities.

We summarize the above result in the proposition below.

Proposition 1 *In the continuous-time model, the scaled equilibrium exchange rate $\widehat{S}(\widehat{V}_t)$ is determined as follows:*

$$
\widehat{S}\left(\widehat{V}_t\right) = \begin{cases} 1-b, & \text{if } \widehat{V} \le \widehat{V}_*; \\ C_0\widehat{V} + C_1\widehat{V}^{\eta_1} + C_2\widehat{V}^{\eta_2}, & \text{if } \widehat{V}_* < \widehat{V} < \widehat{V}^*; \\ 1+b, & \text{if } \widehat{V} \ge \widehat{V}^*, \end{cases} \tag{22}
$$

where the thresholds $\widehat{V}_*$ and $\widehat{V}^*$ are endogenously determined and the expressions of η_1, η_2, and C_0 through C_2 are given in the proof in the appendix.

Proof. See Appendix. ■

Based on our estimation results, we find that $\widehat{S}(\widehat{V}_t)$ has the S shape in the middle range when $\widehat{V}$ falls within the interval between $\widehat{V}_*$ and $\widehat{V}^*$. Figure 5 plots the equilibrium exchange rate $\widehat{S}(\widehat{V}_t)$. Mathematically, $\eta_1 > 0 > \eta_2$ and $C_1 < 0 < C_2$. Intuitively, as the fundamental rate deviates from the central parity in the positive direction, $\widehat{V}_t$ increases, so does the term

$\widehat{V}^{\eta_1}$. When $\widehat{V}$ gets closer to $\widehat{V}^*$, the shape of $\widehat{S}$ gets flatter because of expected interventions in the near future. This is reminiscent of the exchange rate behavior in a target-zone model (see Krugman (1991)) that the expectation of possible interventions affects exchange rate behavior even when the exchange rate lies inside the zone.

[Insert Figure 5 Here.]

Even though we are able to determine $\widehat{S}(\widehat{V}_t)$ in closed form, we rely on simulation-based method to compute options prices because they do not have closed form solutions in most cases. However, in a special case with $b = 0$, we are able to derive analytical expressions for option prices. This case is also interesting because it provides some insights into how we can identify certain parameters, particularly p. We turn to this special case next before we present our estimation results in the next section.

3.3 Special case: $b = 0$

In the special case with $b = 0$, by construction, the model-implied spot rate always coincides with the central parity rate S_t^{CP}. We can also show that the scaled equilibrium exchange rate $\widehat{S}(\widehat{V}_t)$ is always equal to one. That is, the actual and equilibrium exchange rate in this special case are identical and both equal to S_t^{CP}.

In the proposition below we derive option prices in closed form.

Proposition 2 *In the special case where $b = 0$, RMB call and put option prices with maturity τ and strike K are given by:*

$$\begin{aligned} C(K) &= p^\tau C^S(K) + (1 - p^\tau) C^V(K), \\ P(K) &= p^\tau P^S(K) + (1 - p^\tau) P^V(K), \end{aligned}$$

where

$$\begin{aligned} C^S(K) &= e^{-r_{CNY}\tau} E_t^Q \max\left[S_{t+\tau}^{CP} - K, 0\right] = e^{-r_{CNY}\tau}\left(S_t^{CP} e^{\mu_{CP}\tau}\Phi(d_{1,X}) - K\Phi(d_{2,X})\right) \\ P^S(K) &= e^{-r_{CNY}\tau} E_t^Q \max\left[K - S_{t+\tau}^{CP}, 0\right] = e^{-r_{CNY}\tau}\left(K\Phi(-d_{2,X}) - S_t^{CP} e^{\mu_{CP}\tau}\Phi(-d_{1,X})\right) \end{aligned}$$

and

$$\begin{aligned} C^V(K) &= e^{-r_{CNY}\tau} E_t^Q \max\left[V_{t+\tau} - K, 0\right] = e^{-r_{USD}\tau} V_t \Phi(d_{1,V}) - e^{-r_{CNY}\tau} K\Phi(d_{2,V}) \\ P^V(K) &= e^{-r_{CNY}\tau} E_t^Q \max\left[K - V_{t+\tau}, 0\right] = -V_t e^{-r_{USD}\tau} N(-d_{1,V}) + K e^{-r_{CNY}\tau} N(-d_{2,V}) \end{aligned}$$

where the coefficients $d_{1,X}$, $d_{2,X}$, $d_{1,V}$, and $d_{2,V}$ are given in the proof.

Proof. See Appendix. ■

Proposition 2 shows that in the special case with $b = 0$, the option prices can be expressed as the weighted average of the prices that would prevail if the current policy would last or be abandoned for ever. The weights are the corresponding probabilities, p^{τ} and $(1 - p^{\tau})$, respectively. Therefore, fitting the model-implied option prices to the data pins down the probability p. In fact, the estimated model in this special case can fit the option prices in the data pretty well.[17] However, the model-implied close in this special case is always tied to the central parity due to the assumption of $b = 0$. As a result, it cannot be used to fit the close in the data. In the next section we report the estimation results for the model with a non-zero b that is used to fit the close as well as option prices.

4 Estimation Results

Based on the model, we estimate (V, p, σ_V) for each day t in the sample period between December 11, 2015 and December 31, 2018 to fit the close and four option prices in the data. During this period, the limit on the trading band is officially $\pm 2\%$. However, the effective width is much smaller, around 0.5%. As a result, we choose $b = 0.5\%$ in estimating the model.[18] To simplify the estimation, we fix $\rho = 0$.

The parameter γ determines how sensitive the central parity is to the changing market conditions. We choose $\gamma = 1/4$ so that the frequency of the second pillar, $S_t^{CP}(V_{t+1}/V_t)^{\gamma}$, staying within the band $b = 0.5\%$ around S_t^{CP} is roughly similar to that of the close S_t^{CL}.

No closed-form option pricing formula exist in the model with a nonzero b. Therefore, we simulate the model 20,000 times and for each simulation we simulate the paths of the fundamental rate V_t, the central parity S_t^{CP}, the resulting scaled rate $\widehat{V}_t$, and the implied equilibrium exchange rate $\widehat{S}_t$ based on equation (22). In the end we can obtain 20,000 simulated option payoffs at maturity and by properly discounting and averaging we compute the model-implied option prices based on these simulations.

In the rest of this section, we first describe the data and then describe our estimation results.

[17]The estimation results of this special case are not reported, but available upon request.

[18]The results for the case of $b = 2\%$, unreported here, are similar and available upon request.

4.1 Data

Our sample period is chosen from December 11, 2015 to December 31, 2018. The beginning date of the sample period is chosen as December 11, 2015 because the RMB indices were introduced on that date for the first time.[19]

Main sources of our data are the CFETS and Bloomberg. The former source is used to retrieve the historical data of the central parity rates and the RMB indices. The latter source is used to obtain the rest of the data, for example, daily 3-month SHIBOR and LIBOR interest rates.

Besides the exchange rate data and the interest rate data, we also collect data from the derivatives markets, particularly the Bloomberg data of the forwards/futures and options written on the RMB and the dollar index.

The RMB option data consist of implied volatility quotes for at-the-money options ("ATM"), risk reversals ("RR"), and butterfly spreads ("BY"), with a maturity of 3 months. The RR and BY quotes are available for strike prices corresponding to 25% and 10% delta (labeled as "25Δ" and "10Δ" respectively). These quotes can then be used to infer implied volatilities of 25Δ or 10Δ options by the standard approach (e.g., see Jermann (2017) or Bisesti et al. (2015) for more details). For 25Δ calls and puts, for instance, implied volatilities are computed as

$$\begin{aligned}\sigma_{25C} &= \sigma_{ATM} + \sigma_{25BY} + \frac{1}{2}\sigma_{25RR}, \\ \sigma_{25P} &= \sigma_{ATM} + \sigma_{25BY} - \frac{1}{2}\sigma_{25RR}.\end{aligned}$$

Following the quoting convention of the RMB options markets, the Black-Scholes formula is then used to compute the prices and strike prices corresponding to 25Δ and 10Δ options. In the end, we obtain four option price series: two for puts and two for calls. Table 5 below reports average strike prices, option prices, and implied volatility quotes for these four options during the sample period.

[Insert Table 5 Here.]

We obtain the data on the implied volatility σ_X directly from Bloomberg.[20] We then use the futures of the dollar index (DXY) to infer the drift term of the $\{X_t\}$ process. Note that

[19] Note that starting from January 1, 2017 the PBOC has changed the weight of the USD in the CFETS index from 0.2640 to 0.2240. We adjust the value of the parameter w_{USD} accordingly when estimating the model for the period after January 1, 2017.

[20] Alternatively, we have estimated σ_X using the options on the US dollar index (DXY). The dollar index options are written on the dollar index futures and traded on the ICE electronic platform. Furthermore they are quoted in dollar, rather than in terms of implied volatility as in the case of the RMB options. To measure the implied volatility, we obtain the data of the dollar index options with strike prices ranging from 50 to

based on equation (21) and Girsanov's theorem we can price the dollar index futures with maturity τ under the dollar risk-neutral measure as $X_t \exp\left[\left(r_{DXY} - r_{USD} + \sigma_X^2\right)\tau\right]$. Given the estimate of σ_X, the futures data are used to back out the drift term $(r_{DXY} - r_{USD} - \rho\sigma_X\sigma_V + \sigma_X^2)$ of the process $\{X_t\}$ based on equation (21).

4.2 Results

Figure 6 displays the main estimation results. As shown in the first panel, the fundamental rate V_t is estimated to be always greater than both the central parity and the spot rates, which is consistent with the expectations of further depreciation in the RMB. Implied from the estimate of V_t, the RMB is valued on average about 1.7% higher than its fundamental value during the whole sample period. The gap between the RMB spot rate and its estimated fundamental value averages much higher at 2.8% in the first half the sample period that witnessed the first interest rate hike by the U.S. Federal Reserve. For example, the gap is particularly elevated in the first several months following the liftoff by the Federal Reserve in December 2015, ranging between 2% and 9%. By contrast, the average deviation becomes significantly smaller at the level of 0.7% in the second half of the sample period, implying that the observed spot rate is very close to its fundamental value. Another interesting observation is that our estimate of the fundamental value peaks at 7.18 on June 24, 2016 immediately after the Brexit referendum.

The second panel in Figure 6 plots the probability that the current policy would still be in place three months later. It suggests that during the whole sample period markets attached an average probability of 66% to the policy still being in place three months later. Following the interest rate hikes by the Federal Reserve, the probabilities dramatically decreased. Although the credibility of the policy increased in the beginning of 2016, it gradually decreased until August 2016. Since then, the probability remained at a relatively low level within a narrow range between 70% and 80% until May 2017.

A particularly interesting finding is that the model-implied probability of policy continuation dropped precipitately to as low as 38% around May 15, 2017 and afterwards it continued to decrease and dropped to the lowest level around 15% on May 23, 2017. An article on the Wall Street Journal published on May 25, 2017 quoted a study by Brooks and Ma (2017) who argue that "the central bank has moved away from a rule-based method of fixing the yuan and is exerting more 'discretion' over the exchange rate."[21] Then on May 26, 2017,

115 from Bloomberg. We then construct the implied volatility measure by following a model-free method similarly as how the CBOE constructs the VIX index. Our model-free estimate of σ_X is very close to that obtained directly from Bloomberg.

[21] See the Wall Street Journal article on May 25, 2017 titled "China hitches Yuan to the Dollar, Buying Rare Calm" by Linglin Wei and Saumya Vaishampayan.

the PBOC confirmed the change in the central parity's formation mechanism and stated in a report that a "countercyclical factor" has been introduced to the formation mechanism, although no detailed information has been disclosed about how the countercyclical factor is constructed. The introduction of the countercyclical factor signals the end of the two-pillar policy. It is very interesting that our model is able to predict it.

Since the introduction of the countercyclical factor in May 2017, the average probability of policy continuation had stayed low around 36% in the last quarter of 2017. On January 9, 2018, Bloomberg reports that the PBOC has decided to effectively remove the countercyclical factor, resulting in a return to the two-pillar policy.[22] Such policy change is also predicted by our model to some extent as the model-implied probability of policy continuation has increased to about 50% on average in January 2018.

The implied volatility of the fundamental exchange rate shown in the third panel displays a relatively stable pattern in that the implied volatility fluctuates around its average value 8.6% during the whole sample period. In the end of 2016 following the rate hike by the Federal Reserve, the implied volatility has picked up, hovering around 14%. It is worthwhile to point out that since the U.S. election, the implied volatility has steadily decreased from around 14% in mid-December of 2016 to only 4% in the end of May, 2017. It suggests "damping currency volatility against the dollar [is] now a bigger priority" as in the aforementioned Wall Street Journal article.

[Insert Figure 6 Here.]

Next, let us examine the goodness of fit of our model. Figure 7 plots model-implied and actual spot exchange rates as well as the four option prices. The results suggest that the model in general does a good job at fitting the data. In fact, the median fitting error is only about 4.5%.

[Insert Figure 7 Here.]

4.3 Forecast of the central parity and the spot exchange rate

It is well known that it is very difficult to outperform a random walk model in predicting exchange rates (Meese and Rogoff (1983) and Messe and Singleton (1982)). In the case of China, it is a more daunting task to predict its currency, given the opaqueness in how the

[22] See the article "China Changes the Way It Manages Yuan After Currency's Jump" on Bloomberg News on January 9, 2018.

RMB is managed. In this subsection we investigate our model's predictive power relative to a random walk model.

Specifically, consider a subsample period $t \in \{1, \cdots, T_0\}$ that falls within our whole sample period. Fixing the forecasting horizon τ, for each $t \in \{1, \cdots, T_0 - \tau\}$, we can forecast the central parity and spot rates at date $t + \tau$ using the model estimates $(V_t, p_t, \sigma_{V,t})$ at date t. In particular, recall that from equations (26) and (27) in Appendix B both S_t^{CP} and $\widehat{V}_t$ follow geometric Brownian motion processes with drifts given by μ_{CP} and $\mu_{\widehat{V}}$, respectively. As a result, our model-implied forecasts of $S_{t+\tau}^{CP}$ and $\widehat{V}_{t+\tau}$ are given by:

$$
\begin{aligned}
S_{t \to t+\tau}^{CP} &\equiv S_t^{CP} \exp\left(\mu_{CP}\tau\right), \\
\widehat{V}_{t \to t+\tau} &\equiv \widehat{V}_t \exp\left(\mu_{\widehat{V}}\tau\right).
\end{aligned}
$$

Given our model-implied $\widehat{S}(\widehat{V})$ in equation (16), we can then forecast the spot rate $S_{t+\tau}^{CL}$ at date $t + \tau$:

$$
S_{t \to t+\tau}^{CL} = S_{t \to t+\tau}^{CP} H\left(\widehat{S}(\widehat{V}_{t \to t+\tau}), 1; b\right),
$$

where function $H(\cdot, 1; b)$ is given in equation (12). We can then calculate our model's forecasting root mean squared errors (RMSE):

$$
\begin{aligned}
RMSE_{CP} &= \sqrt{\frac{1}{T_0 - \tau} \sum_{t=1}^{T_0-\tau} \left(S_{t+\tau}^{CP} - S_{t \to t+\tau}^{CP}\right)^2}, \\
RMSE_{CL} &= \sqrt{\frac{1}{T_0 - \tau} \sum_{t=1}^{T_0-\tau} \left(S_{t+\tau}^{CL} - S_{t \to t+\tau}^{CL}\right)^2}.
\end{aligned}
$$

Similarly, we can compute the RMSEs for a random walk model in which the central parity and spot rates at date t are used in the forecast:

$$
\begin{aligned}
RMSE_{CP}^{RW} &= \sqrt{\frac{1}{T_0 - \tau} \sum_{t=1}^{T_0-\tau} \left(S_{t+\tau}^{CP} - S_t^{CP}\right)^2}, \\
RMSE_{CL}^{RW} &= \sqrt{\frac{1}{T_0 - \tau} \sum_{t=1}^{T_0-\tau} \left(S_{t+\tau}^{CL} - S_t^{CL}\right)^2}.
\end{aligned}
$$

The ratios $RMSE_{CP}/RMSE_{CP}^{RW}$ and $RMSE_{CL}/RMSE_{CL}^{RW}$ measures the model's forecasting performance relative to the random walk model in forecasting the central parity and spot rates, respectively. Ratios less than one indicate that our model outperforms the random walk model. Similarly, ratios greater than one suggest that the random walk model outperforms our model.

In Figure 8, we plot ratios of the out-of-sample forecasting RMSEs for both the central parity (Panel A) and the spot rate (Panel B) at the 90-day forecasting horizon. Specifically, we use the expanding windows that always start at December 11, 2015, but expand the last date month by month from July 2016 to December 2018.

[Insert Figure 8 Here.]

Figure 8 shows that at the 90-day forecasting horizon our model outperforms a random walk model in predicting both the central parity and the spot rate during the two-pillar policy period between December 11, 2015 and May 31, 2017.[23] We think that the superior forecasting performance of our model stems from two important sources. One is our formulation of the two-pillar policy that seems to closely capture the policy in practice as evidenced in our empirical analysis. This explains why our model can better forecast the central parity relative to a random walk model. Second, the key input for our model estimation is the data on RMB options. The forward looking nature of the options makes our model estimation results very informative about future exchange rate movements. It thus explains why our model can outperform a random walk model in predicting the spot rate.

Furthermore, we show that only after May 2017 when the countercyclical factor was introduced does a random walk model start to outperform our model. Intuitively, by introducing the countercyclical factor the PBOC essentially deviates away from the rule-based formation mechanism, exerting more discretion in determining the central parity and tighter control on the spot rate. Consequently, the RMB becomes less predictable.

4.4 Robustness Checks

The model estimation results are based on the data at the 5PM closing time in New York time, except that the central parity data that is announced at 9:15AM in China local time. Not all input data are available at the close in China local time. Therefore, we cannot estimate the model and obtain the estimates in China local time. However, as a robustness check, we can use certain data series in China local time and estimate the model to see how robust the model estimation results are. We find that if we use the BFIX-based spot rate at 4:30PM in China local time and keep all other input data the same, the estimation results, reported in the online appendix, are little changed.

[23] The results, unreported here, are similar for horizons of 60 days or longer.

As another robustness check, we use the interest rate implied from the covered interest rate parity (CIP) to estimate the model.[24] It would be very interesting to see how robust the estimation results are if the CIP-implied interest rate is used instead. Specifically, we can back out the CIP-implied interest rate at date t as $\widetilde{r}^C = \left(1 + r^{\$}\right) F_t / S_t^{CL} - 1$, where F_t denotes RMB non-deliverable forward rate. If the CIP had held exactly, then the CIP-implied rate $\widetilde{r}^C$ would coincide with the interest rate r^C. Using the CIP-implied rate $\widetilde{r}^C$ and keep the rest of the data unchanged, we re-estimate the model and find that the estimation results are largely unchanged from before. More details and results are reported in the online appendix.

5 Conclusions

Understanding China's exchange rate policy is a key global monetary issue. China's exchange rate policy not only affects the Chinese economy but also impacts the global financial markets as seen in August 2015. Our paper is the first academic paper that provides an in-depth analysis of China's recent two-pillar policy for the RMB. We provide empirical evidence for the implementation of the two pillar policy that aims to achieve balance between exchange rate flexibility and stability against a RMB index. More importantly, we develop a no-arbitrage model that incorporates the two-pillar exchange rate policy for the RMB. The model allows us to extract information from derivatives data (particularly RMB options) to assess financial markets' views about the fundamental value of the RMB as well as the sustainability of the current policy. The estimation results show that the RMB is valued on average about 2.8% higher than its fundamental value until mid 2017, and 0.7% since then. In addition, financial markets attached an average probability of 66% to the policy still being in place three months later. Our model is able to predict the modification of the two-pillar policy in May 2017 when the new countercyclical factor was introduced. Finally, we show that our model can forecast RMB movements better than a random walk model.

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Appendix A: Reconstructing RMB Indices

We use daily central parity rates to reconstruct the indices on a daily basis as follows. Let t_0 denote December 31, 2014, the date on which the RMB indices are set to the benchmark level of 100. Furthermore, the composition of both CFETS and SDR indices has been revised and the new composition became effective starting January 1, 2017. Below we let t_1 denote December 31, 2016 and let w_{USD}, w_{EUR}, $\cdots (w'_{USD}, w'_{EUR}, \cdots)$ denote the weight of the dollar, the Euro, $\cdots$, prior to and including (after) t_1.

1. For each non-dollar currency i (i.e., $i \neq USD$) in a given RMB index, we construct the implied central parity rate of this currency against the dollar:[25]

$$S_t^{CP,i/USD} = \frac{S_t^{CP,i/CNY}}{S_t^{CP,USD/CNY}} \tag{23}$$

2. We then collect all non-dollar currencies in the index into a basket for the dollar without the RMB. Specifically, for $t \leq t_1$, the index-implied dollar basket $\widehat{X}_t$ is constructed as follows:

$$\begin{aligned} B_t &= C_B \left(S_t^{CP,USD/CNY}\right) \left[\left(S_t^{CP,EUR/USD}\right)^{\frac{w_{EUR}}{1-w_{USD}}} \left(S_t^{CP,JPY/USD}\right)^{\frac{w_{JPY}}{1-w_{USD}}} \ldots\right]^{1-w_{USD}} \\ &\equiv C_B \left(S_t^{CP,USD/CNY}\right) \widehat{X}_t^{1-w_{USD}} \end{aligned} \tag{24}$$

where $\widehat{X}_t \equiv \left(S_t^{CP,EUR/USD}\right)^{\frac{w_{EUR}}{1-w_{USD}}} \left(S_t^{CP,JPY/USD}\right)^{\frac{w_{JPY}}{1-w_{USD}}} \cdots$ and the weights in the basket $\widehat{X}_t$ are determined by the weights from the index. For $t > t_1$, new weights are used to calculate the basket, denoted by $\widehat{X}'_t$.

3. We estimate the scaling constant C_B by matching the index level of 100 on December 31, 2014 (i.e., date t_0). That is,

$$C_B = \frac{100 \cdot S_{t_0}^{CP,CNY/USD}}{\widehat{X}_{t_0}^{1-w_{USD}}}. \tag{25}$$

[25] There are a couple of currencies in which the CNY central parity rates only become available in the recent years. For example, $S_t^{CP,CNY/CHF}$ is available only since 11/10/2015 and $S_t^{CP,THB/CNY}$ is not available. For these currencies, say CHF, in the period when their CNY central parity rates are not available, we use the market cross rates in the previous day between these currencies and the USD to approximate $S_t^{CP,CHF/USD}$.

For each of the three indices, the coefficient C_B is determined as

$$
\begin{aligned}
C_{B,CFETS} &= \frac{100 \cdot 6.1190}{1.9147^{1-0.2640}} = 379.3657, \\
C_{B,BIS} &= \frac{100 \cdot 6.1190}{6.3944^{1-0.1780}} = 133.1417, \\
C_{B,SDR} &= \frac{100 \cdot 6.1190}{0.8308^{1-0.4190}} = 681.4643.
\end{aligned}
$$

4. We then adjust the dollar basket $\widehat{X}_t$ by a scaling factor C_X to make it comparable to the US dollar index (DXY):

$$X_t = C_X \cdot \widehat{X}_t.$$

To be precise, the scaling factor C_X is chosen such that X_t coincides with DXY_t at date $t_0 = 12/31/2014$. That is,

$$C_X = \frac{DXY_{t_0}}{\widehat{X}_{t_0}}.$$

For each of the three indices, the coefficient C_X is determined as

$$
\begin{aligned}
C_{X,CFETS} &= \frac{90.269}{1.91502} = 47.1374, \\
C_{X,BIS} &= \frac{90.269}{6.38983} = 14.1270, \\
C_{X,SDR} &= \frac{90.269}{0.83083} = 108.6492.
\end{aligned}
$$

5. For $t > t_1$ when the new weighting scheme applies, the RMB index is similarly constructed:

$$B_t = C'_B \cdot S_t^{CP,USD/CNY} \cdot \left(\widehat{X}'_t\right)^{1-w'_{USD}},$$

where the scaling factor C'_B is chosen such that the level of the index is the same at date t_1 no matter whether the index is under the old or new weighting scheme. That is,

$$C'_B = C_B \frac{\widehat{X}_{t_1}^{1-w_{USD}}}{\left(\widehat{X}'_{t_1}\right)^{1-w'_{USD}}}.$$

For each of the three indices, the coefficient C'_B is determined as

$$\begin{aligned} C'_{B,CFETS} &= \frac{379.3158 \times 2.11099^{(1-0.2640)}}{5.82815^{(1-0.2240)}} = 167.4060, \\ C'_{B,BIS} &= C_{B,BIS}, \\ C'_{B,SDR} &= \frac{681.4655 \times 0.95258^{(1-0.4190)}}{0.95878^{(1-0.4685)}} = 677.4886. \end{aligned}$$

6. For $t > t_1$, we adjust the dollar basket $\widehat{X}'_t$ by a scaling factor C'_X to make it consistent with X_t and comparable with the DXY index. To be precise, the scaling factor C'_X is chosen such that the adjusted dollar basket coincides on December 31, 2016 (i.e., t_1). That is,

$$\begin{aligned} X'_t &= C'_X \cdot \widehat{X}'_t. \\ C_X \widehat{X}_{t_1} &= C'_X \widehat{X}'_{t_1}. \end{aligned}$$

For each of the three indices, the coefficient C'_X is determined as

$$\begin{aligned} C'_{X,CFETS} &= 47.1374 \frac{2.11099}{5.82815} = 17.0734, \\ C'_{X,BIS} &= C_{X,BIS}, \\ C'_{X,SDR} &= 108.6492 \frac{0.95258}{0.95878} = 107.9466. \end{aligned}$$

After some tedious algebra, we can show that the first pillar $\overline{S}_{t+1}$ at date $(t_1 + 1)$ is given by

$$\begin{aligned} \overline{S}_{t_1+1} &= S_t^{CP} \frac{\chi'}{\chi} \frac{\left(X'_{t_1+1}\right)^{1-w'_{USD}}}{\left(X_{t_1}\right)^{1-w_{USD}}} \\ &= S_t^{CP} \frac{X_{t_1}^{1-w_{USD}}}{X_{t_1}^{1-w'_{USD}}} \frac{\left(X'_{t_1+1}\right)^{1-w'_{USD}}}{\left(X_{t_1}\right)^{1-w_{USD}}} \\ &= S_t^{CP} \left(\frac{X'_{t_1+1}}{X_{t_1}}\right)^{1-w'_{USD}}, \end{aligned}$$

where $\chi \equiv \frac{C_B}{C_X^{1-w_{USD}}}$ and $\chi' \equiv \frac{C'_B}{\left(C'_X\right)^{1-w'_{USD}}}$. Because $X_{t_1} = X'_{t_1}$, it implies that $\overline{S}_{t_1+1} = S_t^{CP}\left(\frac{X'_{t_1+1}}{X_{t_1}}\right)^{1-w'_{USD}}$. As a result,

$$\overline{S}_{t+1} = \begin{cases} S_t^{CP}\left(X_{t+1}/X_t\right)^{1-w_{USD}}, \text{ if } t < t_1; \\ S_t^{CP}\left(X'_{t+1}/X'_t\right)^{1-w'_{USD}}, \text{ if } t \geq t_1. \end{cases}$$

Appendix B: Proofs

Proof of Proposition 1. Below we use lowercase variables to denote the logarithm of the corresponding uppercase variables. For example, $v_t \equiv \log V_t$, $s_t^{CP} \equiv \log S_t^{CP}$, $\widehat{v}_t \equiv \log \widehat{V}_t$, etc. Under the two-pillar policy in equation (15), by Ito's lemma the dynamics of S_t^{CP} is given by:

$$\frac{dS_t^{CP}}{S_t^{CP}} \equiv \mu_{CP}dt + \left(\alpha\rho\sigma_X + \beta\sigma_V\right)dW_{V,t} + \alpha\sqrt{1-\rho^2}\sigma_X dW_{X,t}, \tag{26}$$

where $\mu_{CP} \equiv \alpha\left(\mu_X - 1/2\sigma_X^2\right) + \beta\left(\mu_V - 1/2\sigma_V^2\right) + \frac{1}{2}\left(\alpha\rho\sigma_X + \beta\sigma_V\right)^2 + \frac{1}{2}\alpha^2\left(1-\rho^2\right)\sigma_X^2$ denotes the expected growth rate of the central parity; that is, $E_t^Q\left[S_{t+\tau}^{CP}\right] = S_t^{CP}\exp\left(\mu_{CP}\tau\right)$.

Similarly, we derive the dynamics of $\widehat{V}_t$ as follows:

$$\frac{d\widehat{V}_t}{\widehat{V}_t} = \mu_{\widehat{V}}dt + \left(\left(1-\beta\right)\sigma_V - \alpha\sigma_X\rho\right)dW_{V,t} - \alpha\sigma_X\sqrt{1-\rho^2}dW_{X,t}, \tag{27}$$

where $\mu_{\widehat{V}} \equiv -\alpha\left(\mu_X - 1/2\sigma_X^2\right) + \left(1-\beta\right)\left(\mu_V - 1/2\sigma_V^2\right) + \frac{1}{2}\sigma_{\widehat{V}}^2$ denotes the expected growth rate of the scaled fundamental rate and $\sigma_{\widehat{V}} \equiv \sqrt{\left(\left(1-\beta\right)\sigma_V - \alpha\sigma_X\rho\right)^2 + (\alpha\sigma_X\sqrt{1-\rho^2})^2}$.

We are now ready to solve the (scaled) equilibrium exchange rate $\widehat{S}(\widehat{V}_t)$. It is straightforward to prove that $\widehat{S}(\widehat{V}_t)$ is monotonically increasing. Define $\widehat{V}_*$ and $\widehat{V}^*$ such that $\widehat{S}(\widehat{V}_*) = 1-b$ and $\widehat{S}(\widehat{V}^*) = 1+b$. As the length of the period Δt converges to zero, with probability one $\widehat{V}_{t+\Delta t} > \widehat{V}^*$ (or $\widehat{V}_{t+\Delta t} < \widehat{V}_*$) if $\widehat{V}_t > \widehat{V}^*$ (or $\widehat{V}_t < \widehat{V}_*$). Therefore, from equation (17), it must be true that: $\widehat{S}(\widehat{V}_t) = 1-b$ if $\widehat{V} < \widehat{V}_*$, and $1+b$ if $\widehat{V} > \widehat{V}^*$.

If $\widehat{V} \in (\widehat{V}_*, \widehat{V}^*)$, it is straightforward to show that $\widehat{S}(\widehat{V}_t)$ must satisfy the following equation based on equation (17):

$$\widehat{S}'\left(\widehat{V}_t\right)\widehat{V}_t\mu_{\widehat{V}} + \frac{1}{2}\widehat{S}''\left(\widehat{V}_t\right)\widehat{V}_t^2\sigma_{\widehat{V}}^2 + \left(\mu_{CP} - \mu_V - \lambda\right)\widehat{S}\left(\widehat{V}_t\right) + \lambda\widehat{V}_t = 0. \tag{28}$$

The solution to this ordinary differential equation is: $\widehat{S}(\widehat{V}_t) = C_0\widehat{V} + C_1\widehat{V}^{\eta_1} + C_2\widehat{V}^{\eta_2}$, where

η_1 and η_2 are the two roots of the quadratic equation:

$$\frac{1}{2}\sigma_{\widehat{V}}^2\eta^2 + \left(\mu_{\widehat{V}} - \frac{1}{2}\sigma_{\widehat{V}}^2\right)\eta + (\mu_{CP} - \mu_V - \lambda) = 0, \tag{29}$$

and the coefficient C_0 is given by

$$C_0 = \frac{\lambda}{\lambda + \mu_V - \mu_{\widehat{V}} - \mu_{CP}}, \tag{30}$$

and the coefficients C_1, C_2 and the thresholds $\widehat{V}_*$, $\widehat{V}^*$ are determined from the value-matching and smooth-pasting conditions:

$$C_0\widehat{V}_* + C_1(\widehat{V}_*)^{\eta_1} + C_2(\widehat{V}_*)^{\eta_2} = 1 - b, \tag{31}$$

$$C_0\widehat{V}^* + C_1(\widehat{V}^*)^{\eta_1} + C_2(\widehat{V}^*)^{\eta_2} = 1 + b, \tag{32}$$

$$C_0 + \eta_1 C_1(\widehat{V}_*)^{\eta_1 - 1} + \eta_2 C_2(\widehat{V}_*)^{\eta_2 - 1} = 0, \tag{33}$$

$$C_0 + \eta_1 C_1(\widehat{V}^*)^{\eta_1 - 1} + \eta_2 C_2(\widehat{V}^*)^{\eta_2 - 1} = 0. \tag{34}$$

■

Proof of Proposition 2. In the special case with $b = 0$, the function $H(\widehat{S}(\widehat{V}_t))$ is always equal to one, implying:

$$\frac{S_{t+1}^{CP}}{S_t^{CP}} = \left(\frac{X_{t+1}}{X_t}\right)^{w(1-w_{USD})}, \tag{35}$$

and

$$\widehat{S}(\widehat{V}_t) = p\kappa + (1-p)\,\widehat{V}_t, \tag{36}$$

where $\kappa \equiv \frac{1+r^{\$}}{1+r^{C}} E^Q\left[\left(\frac{X_{t+1}}{X_t}\right)^{w(1-w_{USD})}\right]$ is a constant. That is, the equilibrium exchange rate is a weighted average of the scaled central parity rate (κS_t^{CP}) and the fundamental value V_t, with the weights being the probabilities p and $(1-p)$. By construction, the model-implied spot rate in this special case always coincides with the central parity rate S_t^{CP} because $b = 0$.

The option prices can be pinned down based on a Black-Scholes type of formula. Note that when $b = 0$, we have $H\left(\widetilde{S}_{t+\tau}, S_{t+\tau}^{CP}, 0\right) = S_{t+\tau}^{CP}$. Below we derive $C^S(K) \equiv e^{-r_{CNY}\tau}E^Q\left[\max\left(S_{t+\tau}^{CP} - K, 0\right)\right]$. Recall that from equation (26) we can rewrite the dynamics of S_t^{CP} below:

$$\frac{dS_t^{CP}}{S_t^{CP}} = \mu_{CP}dt + \sigma_{CP}dW_{CP,t},$$

where

$$W_{CP,t} \equiv \frac{1}{\sigma_{CP}} \left(\left(\alpha\rho\sigma_X + \beta\sigma_V \right) dW_{V,t} + \alpha\sqrt{1-\rho^2}\sigma_X dW_{X,t} \right),$$

and

$$\sigma_{CP} \equiv \sqrt{\left(\alpha\rho\sigma_X + \beta\sigma_V \right)^2 + \alpha^2 \left(1-\rho^2\right) \sigma_X^2}.$$

Therefore,

$$\begin{aligned} S_{t+\tau}^{CP} &= S_t^{CP} \exp\left\{ \left(\mu_{CP} - \sigma_{CP}^2/2 \right) \tau + \sigma_{CP} \left(W_{CP,t+\tau} - W_{CP,t} \right) \right\} \\ &\equiv S_t^{CP} \exp\left\{ \left(\mu_{CP} - \sigma_{CP}^2/2 \right) \tau + \sigma_{CP}\sqrt{\tau} z_{CP} \right\}, \end{aligned}$$

where we define $z_{CP} \equiv \tau^{-1/2} \left(W_{CP,t+\tau} - W_{CP,t} \right)$ and by definition z_{CP} follows the standard normal distribution with mean zero and variance one.

As a result, $S_{t+\tau}^{CP} > K$ is equivalent to

$$z_{CP} > -\frac{\log\left(S_t^{CP}/K \right) + \left(\mu_{CP} - \sigma_{CP}^2/2 \right)\tau}{\sigma_{CP}\sqrt{\tau}} \equiv -d_{2,X}.$$

It follows that:

$$\begin{aligned} C_I(K;\tau) &\equiv e^{-r_{CNY}\tau} E^Q \left[\max\left(S_{t+\tau}^{CP} - K, 0 \right) \right] \\ &= e^{-r_{CNY}\tau} \int_{-d_{2,X}}^{\infty} \left[S_t^{CP} \exp\left\{ \left(\mu_{CP} - \sigma_{CP}^2/2 \right)\tau + \sigma_{CP}\sqrt{\tau} z_{CP} \right\} - K \right] \phi\left(z_{CP} \right) dz_{CP} \\ &= e^{-r_{CNY}\tau} S_t^{CP} e^{\left(\mu_{CP} - \sigma_{CP}^2/2 \right)\tau} \int_{-d_{2,X}}^{\infty} e^{\sigma_{CP}\sqrt{\tau} z_{CP}} \phi\left(z_{CP} \right) dz_{CP} - e^{-r_{CNY}\tau} K \Phi\left(d_{2,X} \right) \\ &= S_t^{CP} e^{\left(\mu_{CP} - r_{CNY} \right)\tau} \left(1 - \Phi\left(-d_{2,X} - \sigma_{CP}\sqrt{\tau} \right) \right) - e^{-r_{CNY}\tau} K \Phi\left(d_{2,X} \right) \\ &= S_t^{CP} e^{\left(\mu_{CP} - r_{CNY} \right)\tau} \Phi\left(d_{1,X} \right) - e^{-r_{CNY}\tau} K \Phi\left(d_{2,X} \right) \end{aligned}$$

where $d_{1,X} \equiv d_{2,X} + \sigma_{CP}\sqrt{\tau}$. That is,

$$\begin{aligned} d_{1,X} &= d_{2,X} + \sigma_{CP}\sqrt{\tau} = \frac{\log\left(S_t^{CP}/K \right) + \left(\mu_{CP} + \sigma_{CP}^2/2 \right)\tau}{\sigma_{CP}\sqrt{\tau}} \\ d_{2,X} &= \frac{\log\left(S_t^{CP}/K \right) + \left(\mu_{CP} - \sigma_{CP}^2/2 \right)\tau}{\sigma_{CP}\sqrt{\tau}} \end{aligned}$$

The expression for the put option $P^S(K)$ can be similarly derived.

Lastly, the derivation is almost the same for $C^V(K)$ and $P^V(K)$, if we replace S_t^{CP} by

V_t and its drift and diffusion terms accordingly. In particular,

$$\begin{aligned} d_{1,V} &= d_{2,V} + \sigma_V \sqrt{\tau} = \frac{\log\left(V_t/K\right) + \left(\mu_V + \frac{1}{2}\sigma_V^2\right)\tau}{\sigma_V\sqrt{\tau}} \\ d_{2,V} &= \frac{\log\left(V_t/K\right) + \left(\mu_V - \frac{1}{2}\sigma_V^2\right)\tau}{\sigma_V\sqrt{\tau}} \end{aligned}$$

■

Table 1: Recent Major Events for the RMB

Date	Major RMB Event
July 21, 2005	The RMB was depegged from the U.S. dollar.
August 11, 2015	The PBOC reformed on the formation mechanism of the central parity to make it more market driven.
December 11, 2015	The PBOC introduced three RMB indices, signalling the beginning of the two-pillar policy.
October 1, 2016	The RMB was included in the IMF's SDR basket of reserve currencies, along with the dollar, the euro, the yen, and the British pound.
May 26, 2017	The PBOC confirmed the recent addition of a countercyclical factor to the formation mechanism of the central parity.
February 20, 2017	The reference period of the central parity against the RMB index was shortened to 15 hours.
January 9, 2018	Bloomberg reported the removal of the countercyclical factor by the PBOC.
August 24, 2018	The countercyclical factor was reapplied.

Notes: This table reports recent major RMB events or reforms since 2005.

Table 2: RMB Indices

Curr.	CFETS		SDR		BIS
	old	new	old	new	
USD	0.2640	0.2240	0.4685	0.4190	0.178
EUR	0.2139	0.1634	0.3472	0.3740	0.187
JPY	0.1468	0.1153	0.0935	0.0940	0.141
GBP	0.0386	0.0316	0.0908	0.1130	0.029
HKD	0.0655	0.0428	–	–	0.008
AUD	0.0627	0.044	–	–	0.015
NZD	0.0065	0.0044	–	–	0.002
SGD	0.0382	0.0321	–	–	0.027
CHF	0.0151	0.0171	–	–	0.014
CAD	0.0253	0.0215	–	–	0.021
MYR	0.0467	0.0375	–	–	0.022
RUB	0.0436	0.0263	–	–	0.018
THB	0.0333	0.0291	–	–	0.021
ZAR	–	0.0178	–	–	0.006
KRW	–	0.1077	–	–	0.085
AED	–	0.0187	–	–	0.007
SAR	–	0.0199	–	–	0.010
HUF	–	0.0031	–	–	0.004
PLN	–	0.0066	–	–	0.009
DKK	–	0.0040	–	–	0.004
SEK	–	0.0052	–	–	0.008
NOK	–	0.0027	–	–	0.004
TRY	–	0.0083	–	–	0.008
MXN	–	0.0169	–	–	0.023
other	–	–	–	–	0.149

Notes: This table reports the composition of three RMB indices: CFETS, SDR, and BIS. The composition of the CFETS and SDR indices has been changed by the PBOC since 2017. The columns "old" and "new" display the weights before and after 2017, respectively.

Table 3: Empirical Evidence for the Two-Pillar Policy: Unconstrained Regressions

Panel A: Central Parity Data and Bloomberg Close									
	Whole Sample			Subsample I			Subsample II		
	α	β	R^2	α	β	R^2	α	β	R^2
CFETS	0.47	0.53	0.81	0.51	0.49	0.83	0.42	0.55	0.80
SDR	0.45	0.55	0.80	0.48	0.50	0.79	0.42	0.57	0.80
BIS	0.42	0.55	0.78	0.48	0.51	0.79	0.36	0.57	0.78
Panel B: BFIX & 24-hour Regression I									
	Whole Sample			Subsample I			Subsample II		
	α	β	R^2	α	β	R^2	α	β	R^2
CFETS	0.62	0.60	0.83	0.64	0.56	0.82	0.58	0.63	0.83
SDR	0.57	0.64	0.81	0.58	0.57	0.76	0.55	0.66	0.84
BIS	–	–	–	–	–	–	–	–	–
Panel C: BFIX & Overnight Regression II									
	Whole Sample			Subsample I			Subsample II		
	α	β	R^2	α	β	R^2	α	β	R^2
CFETS	0.67	0.73	0.78	0.64	0.71	0.66	0.71	0.74	0.86
SDR	0.59	0.72	0.76	0.58	0.68	0.63	0.61	0.74	0.85
BIS	–	–	–	–	–	–	–	–	–

Notes: This table reports the results of regressing the daily change in the log central parity (i.e., $\log\left(S_{t+1}^{CP}/S_t^{CP}\right)$ on the two pillars scaled by the previous central parity (i.e., $(1-w_{USD})\log\left(X_{t+1}/X_t\right)$ and $\log\left(S_t^{CL}/S_t^{CP}\right)$). The regression is conducted in three different periods: whole sample period, subsample period 1, and subsample period 2. Whole sample period is the period between December 11, 2015 and December 31, 2018. Subsample period 1 is the period between December 11, 2015 and February 19, 2017. Subsample period 2 is the period between February 20, 2017 and December 31, 2018.

Table 4: Empirical Evidence for the Two-Pillar Policy: Constrained Regressions

	Whole Sample		Subsample I		Subsample II	
Panel A: Central Parity Data and Bloomberg Close						
	α	β	α	β	α	β
CFETS	0.44	0.56	0.51	0.49	0.44	0.56
SDR	0.43	0.57	0.49	0.51	0.43	0.57
BIS	0.40	0.60	0.48	0.52	0.40	0.60
Panel B: BFIX & 24-hour Regression I						
	α	β	α	β	α	β
CFETS	0.52	0.48	0.59	0.41	0.46	0.54
SDR	0.48	0.52	0.54	0.46	0.43	0.57
BIS	–	–	–	–	–	–
Panel C: BFIX & Overnight Regression II						
	α	β	α	β	α	β
CFETS	0.45	0.55	0.50	0.50	0.41	0.59
SDR	0.43	0.57	0.49	0.51	0.40	0.60
BIS	–	–	–	–	–	–

Notes: This table reports the results of regressing the daily change in the log central parity (i.e., $\log\left(S_{t+1}^{CP}/S_t^{CP}\right)$ on the two pillars scaled by the previous central parity (i.e., $(1-w_{USD})\log\left(X_{t+1}/X_t\right)$ and $\log\left(S_t^{CL}/S_t^{CP}\right)$). The regression is conducted in three different periods: whole sample period, subsample period 1, and subsample period 2. Whole sample period is the period between December 11, 2015 and December 31, 2018. Subsample period 1 is the period between December 11, 2015 and February 19, 2017. Subsample period 2 is the period between February 20, 2017 and December 31, 2018.

Table 5: **Summary Statistics for 3-month RMB Options**

Option	avg. strike price	avg. option price	avg. imp. vol.
Put 10Δ	6.492	0.008	5.21
Put 25Δ	6.601	0.025	4.99
Call 10Δ	7.038	0.011	7.34
Call 25Δ	6.851	0.030	6.07

Notes: This table reports summary statistics for 3-month RMB options during the sample period between December 11, 2015 and December 31, 2018. In particular, there are four (call and put) options with 25% and 10% delta (labeled as "25Δ" and "10Δ"). For each option, the table reports the sample average of the strike price, the option price, and the implied volatility in percent during the sample period.

Figure 1: RMB Central Parity and Closing Rates between 2004 and 2018

Panel A of this figure plots historical central parity rate (blue solid line) and closing rate (red dashed line) between 2004 and 2018. Panel B of this figure plots in blue solid line the difference between the logarithms of the central parity and closing rates, and in red solid lines the bounds imposed by the PBOC.

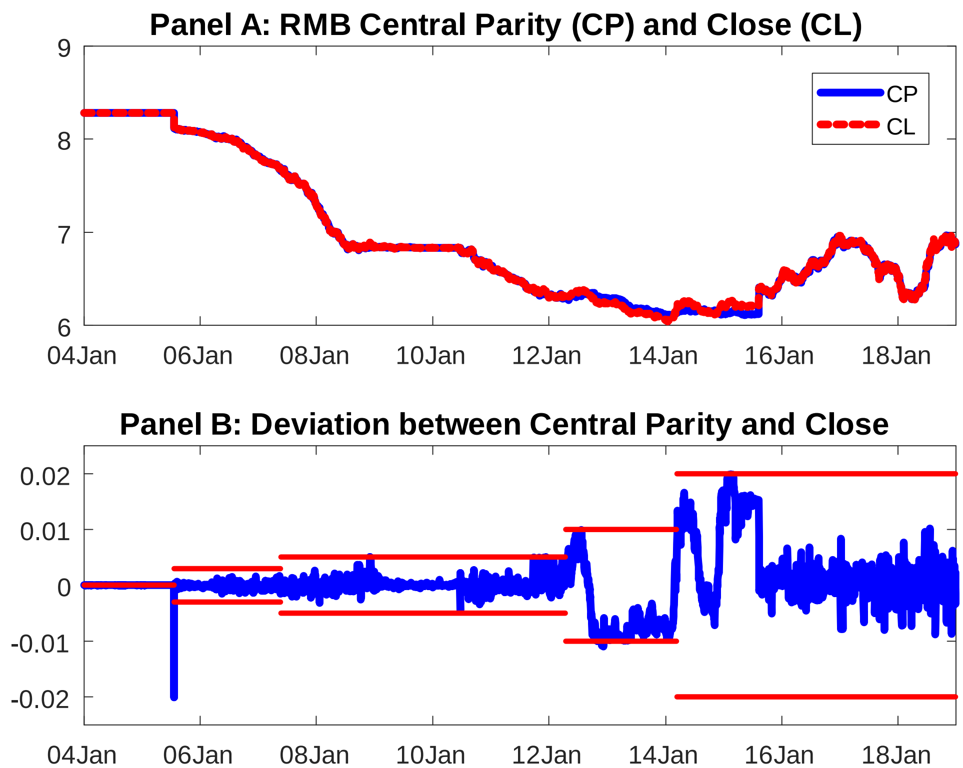

Figure 2: RMB Indices: reconstructed vs. actual

This figure plots the actual periodic RMB indices (red circle) vs. the daily indices we reconstruct (blue solid lines). The three panels correspond to the three RMB indices: CFETS, BIS, and SDR.

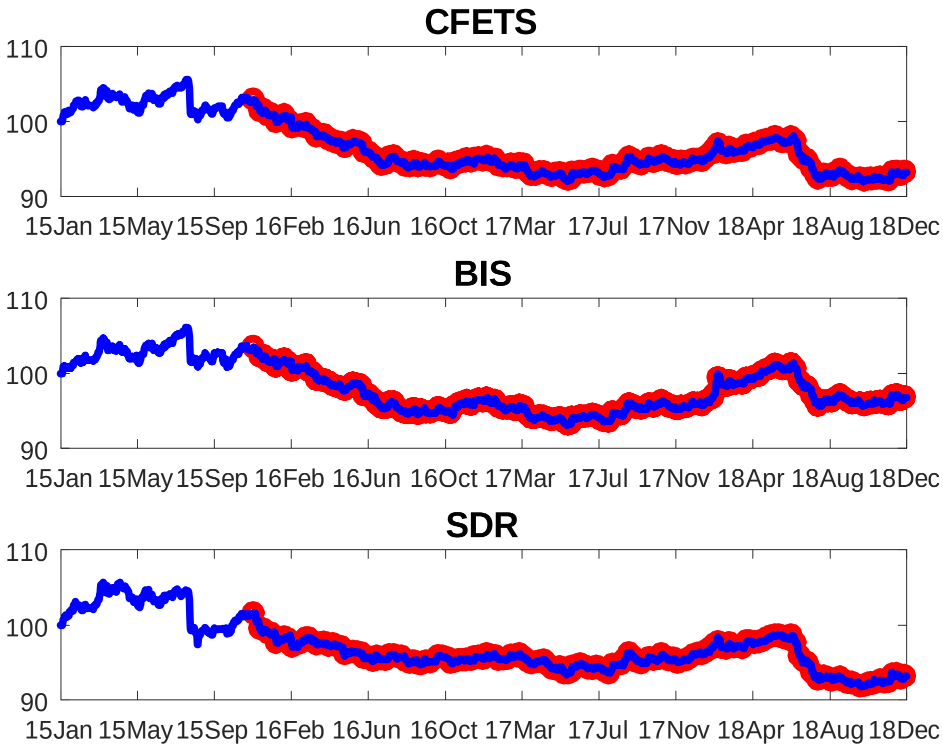

Figure 3: Index-implied Dollar Basket vs. DXY

In this figure we plot the historical dollar index (blue solid line) together with the dollar baskets implied in three indices. The dollar basket implied in the CFETS (respectively, BIS or SDR) index is plotted using red dashed line (respectively, pink dotted or black dash-dotted lines).

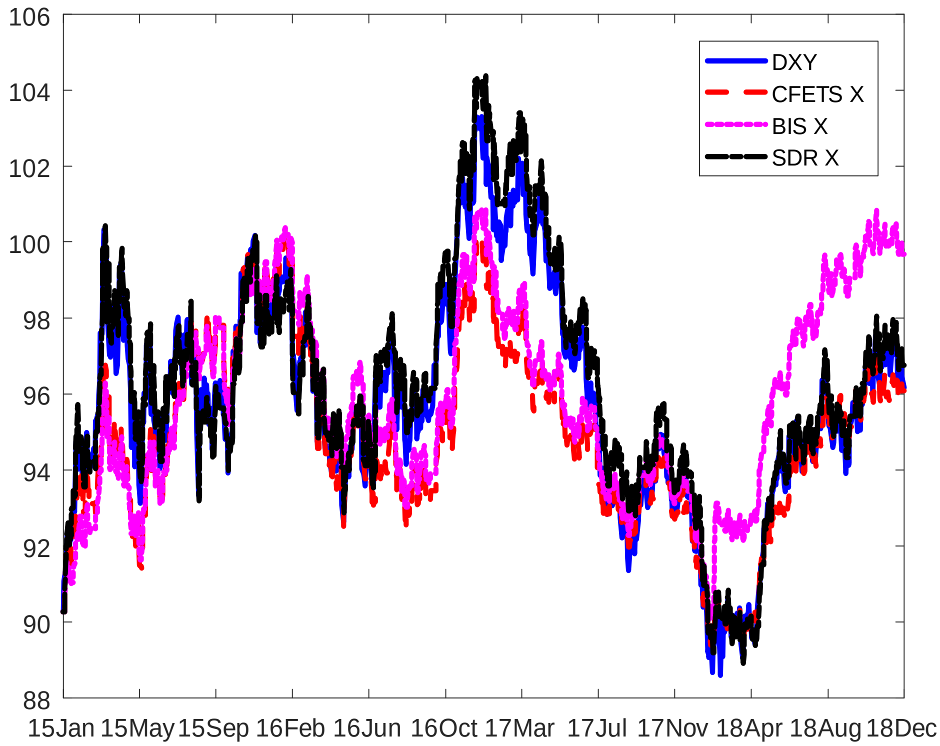

Figure 4: Empirical Evidence for the Two-Pillar Approach

In this figure we report results from running 60-day rolling-window regressions: $\log\left(\frac{S_{t+1}^{CP}}{S_t^{CP}}\right) = \alpha \cdot \left(1 - w_{USD}^{ind}\right) \log\left(\frac{X_{t+1}^{ind}}{X_t^{ind}}\right) + \beta \cdot \log\left(\frac{S_t^{CL}}{S_t^{CP}}\right) + \epsilon_{t+1}$, where superscript "*ind*" indicates one of the three indices CFETS, BIS, and SDR. For each index, we regress the logged growth rate of the central parity on $(1 - w_{USD}) \log(X_{t+1}/X_t)$ and the logged ratio between the close and the central parity. The regression coefficient α, which corresponds to the weight w, is plotted in the figure (blue solid line for CFETS, red solid line for BIS, and yellow solid line for SDR indices).

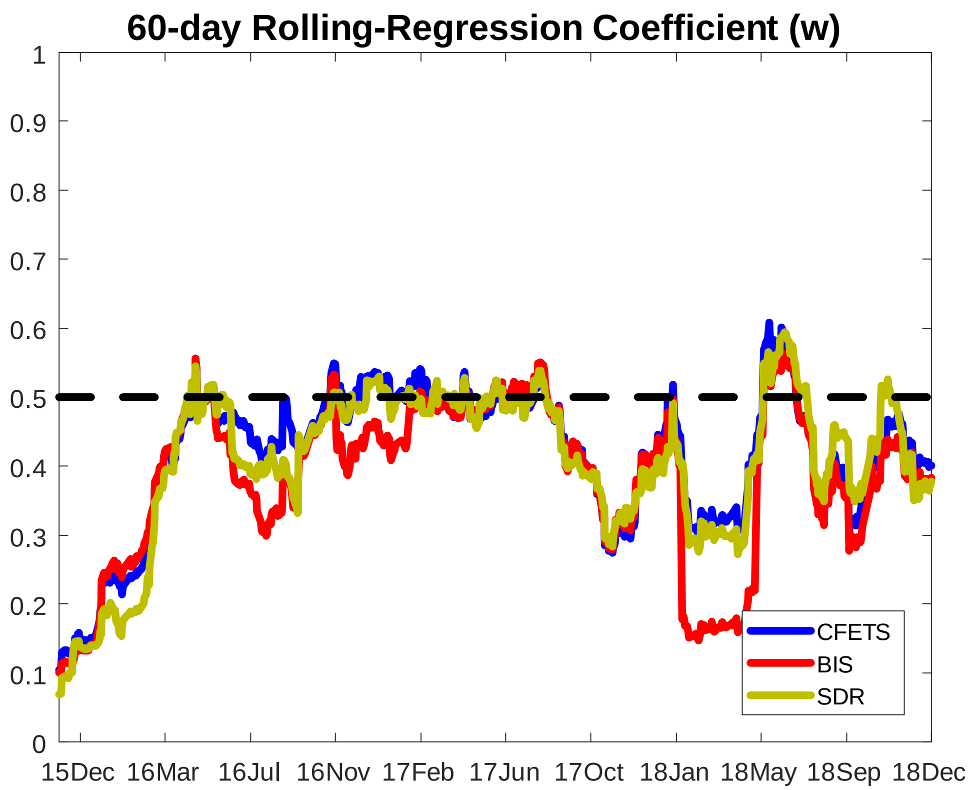

Figure 5: Equilibrium Exchange Rate $\widehat{S}(\widehat{V}_t)$

This figure plots in blue solid line the scaled equilibrium exchange rate $\widehat{S}(\widehat{V})$ as a function of $\widehat{V}$.

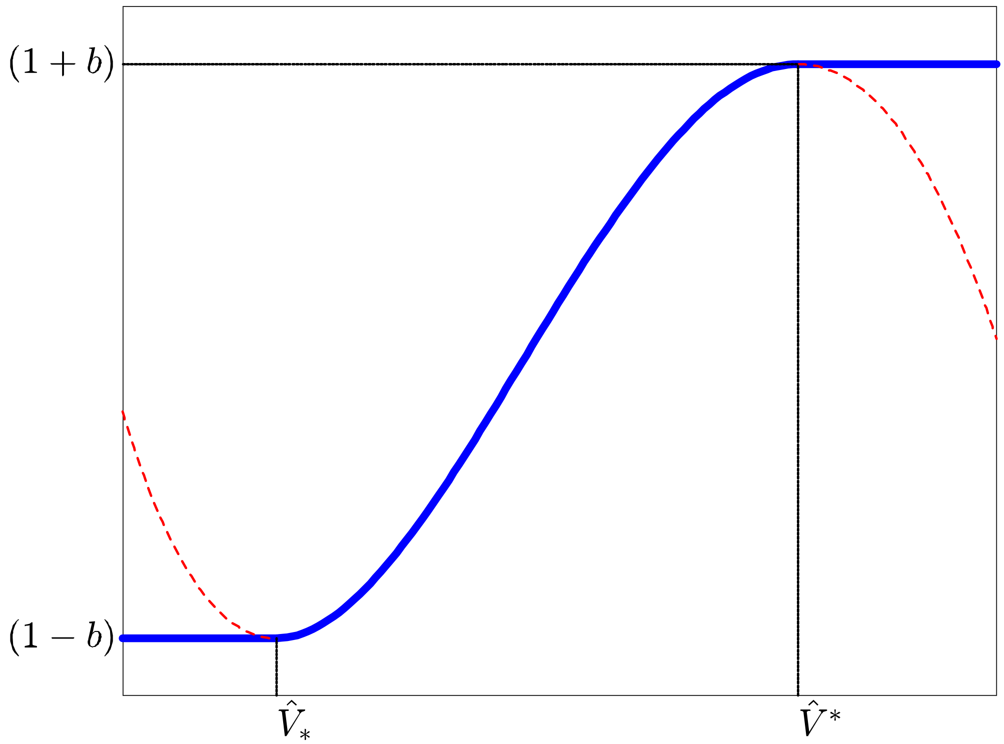

Figure 6: Baseline Parameter Estimates

This figure reports the results of the baseline estimation where we fix w to 0.5. In the top panel, we plot V_t, the estimated fundamental exchange rate, in blue solid line, the historical central parity rate in red dashed line, as well as the close in black dashed line. In the middle panel, we plot p_t, the probability of the two-pillar approach still being in place three months late. In the bottom panel, we plot σ_V, the estimated annualized volatility of the fundamental rate process, in blue solid line, and the average implied volatilities of 10-delta options (red dashed line) and 25-delta options (black dashed line) in the data.

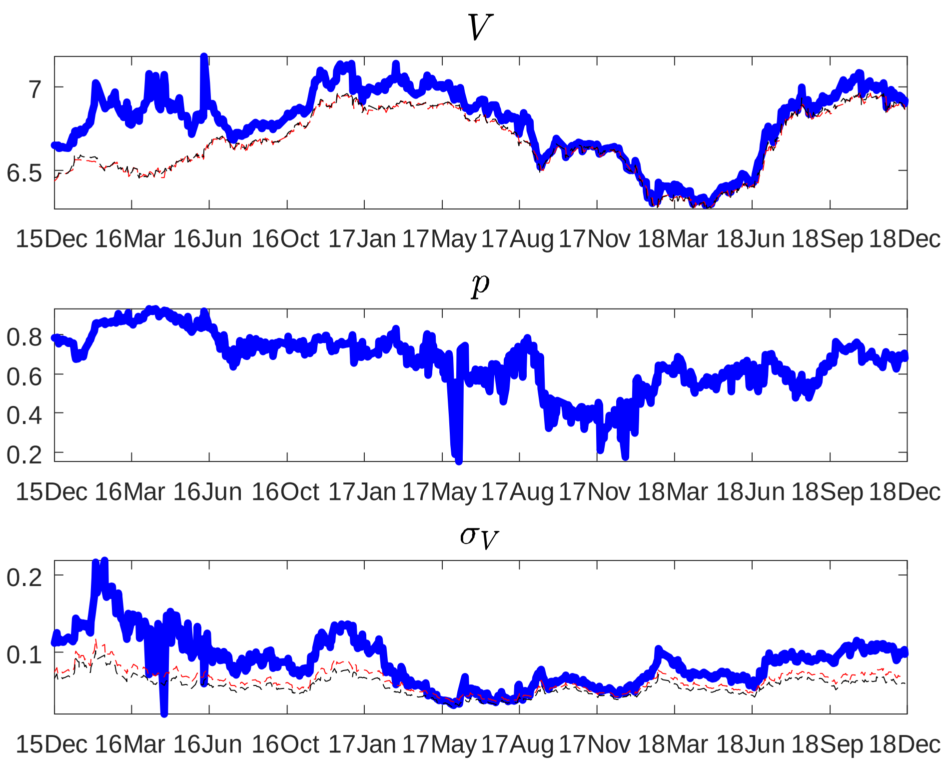

Figure 7: Goodness-of-fit of the Model

The top four panels plot the model-implied option prices (blue cross) against the actual option prices (red circle). For example, the left upper panel plots the model-implied 10-delta option prices $P_t^{10\Delta,model}$ using blue crosses and the actual 10-delta option prices in the data $P_t^{10\Delta,data}$ using red circles. The bottom panel plots the model-implied close $S_t^{CL,model}$ (blue cross) against the actual close $S_t^{CL,data}$ in the data (red circle).

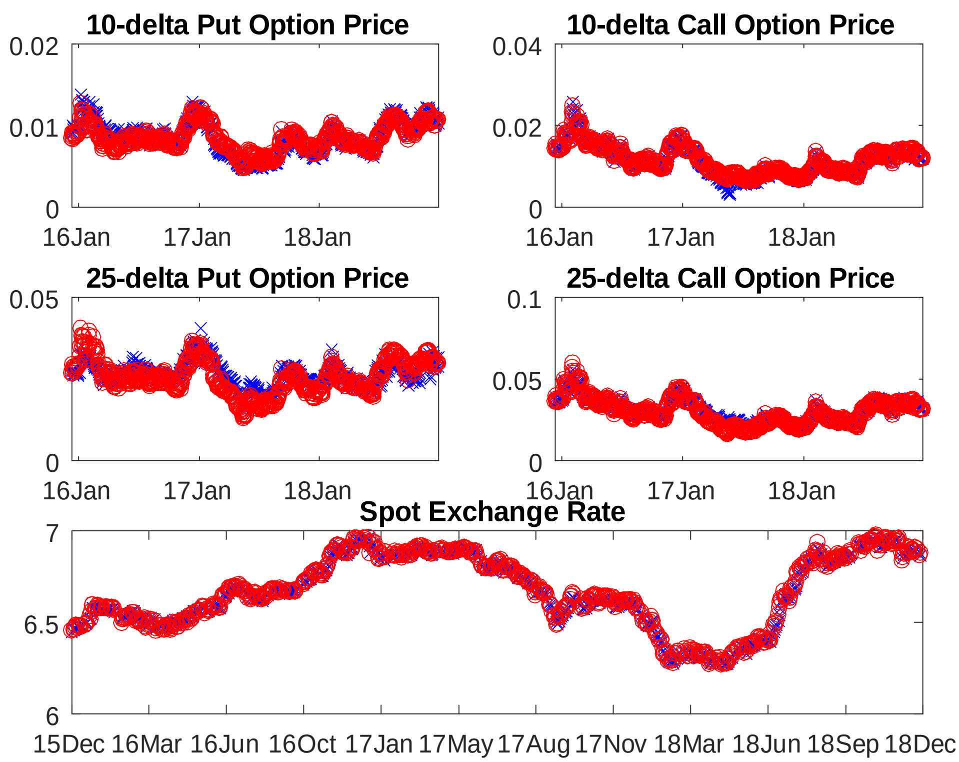

Figure 8: Forecasting Errors

This figure plots the ratio of forecasting root mean squared errors (RMSEs) of our model relative to a random walk model, for both the central parity forecast (Panel A) and the spot rate forecast (Panel B). The forecasting RMSE ratio is calculated using the 90-day forecasting horizon and is tracked throughout the sample period.