# Can Global Uncertainty Promote International Trade?

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ABSTRACT

Common wisdom holds that uncertainty impedes trade—yet we show that uncertainty can fuel more trade in a simple general equilibrium trade model with information frictions. In equilibrium, increases in uncertainty increase both the mean and variance in returns to exporting. This implies that trade can increase or decrease with uncertainty, depending on preferences. Under general conditions on preferences, we characterize the importance of these forces using a sufficient statistics approach. Higher uncertainty leads to increases in trade because agents receive improved terms of trade, particularly in states of nature in which consumption is most valuable. Trade creates value, in part, by offering a mechanism for risk sharing, and risk sharing is most effective when both parties are uninformed.

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In any discussion of frictions in cross-border trade, inevitably one that arises concerns information and uncertainty. Portes and Rey (2005) show that the volume of phone calls between two countries predicts how much they trade. Gould (1994) and Rauch and Trindade (2002) argue that immigrants trade more with their home countries. The argument is so simple that it needs no formalization: information frictions create uncertainty, and this uncertainty deters risk-averse exporters. In this paper, we show that uncertainty can fuel more trade in a simple general equilibrium trade model.

Anecdotal evidence suggests that the effects of uncertainty on trade are far from clear. There has been increased uncertainty about the future international trading environment, with the United States government adopting a hostile stance to existing trade agreements and others threatening retaliation. In particular, measures of policy uncertainty have increased dramatically since late 2016; see, e.g., Figure 1. Despite this uncertain environment, U.S. exports relative to GDP have grown by 17 percent since early 2016.

In a simple general equilibrium trade model with information frictions, we show how outcomes of this nature—uncertainty-fueled booms in trade—are possible. We deliver two insights about the relationship between uncertainty and international trade. The first concerns mechanics: Uncertainty increases both the mean and the variance in the returns to exporting. The implication is that trade can increase or decrease with uncertainty, depending on preferences over these different forces. Under general conditions on preferences, we characterize the importance of these forces using a sufficient statistics approach. Once one understands certain risk, prudence, and temperance properties of preferences, changes in mean and variance are sufficient to characterize the change in trade flows to aggregate uncertainty. In the commonly used CES case, these sufficient statistics simply boil down to functions of the elasticity of substitution across home and foreign varieties, or "trade elasticity."

The second insight regards interpretation: Uncertainty facilitates cross-country risk sharing and, hence, more trade. When uncertainty is high, other countries do not realize that bad states of nature are prevailing domestically. Their exports provide the home country with lots of goods in exactly the states in which consumption is most needed. In contrast, when uncertainty is low, this risk-sharing mechanism is muted; informed countries substitute away from trade in states in which they would prefer to not insure their trading partner. Thus, one interpretation of our results is that uncertainty-fueled increases in trade occur because risk sharing is most effective when both parties are uninformed.

We demonstrate these results in a standard, simple general equilibrium trade model—a two-good, two-country Armington model. We introduce cross-country uncertainty in the most obvious way: Each country experiences a random shock that affects its export choice. Home firms observe home shocks perfectly. Foreigners observe foreign shocks perfectly. But each group observes the other's shocks imperfectly, with a noisy signal. Then every firm chooses how much to export to an international market. The international relative price clears that market, goods are immediately shipped to their destination country, and agents consume.

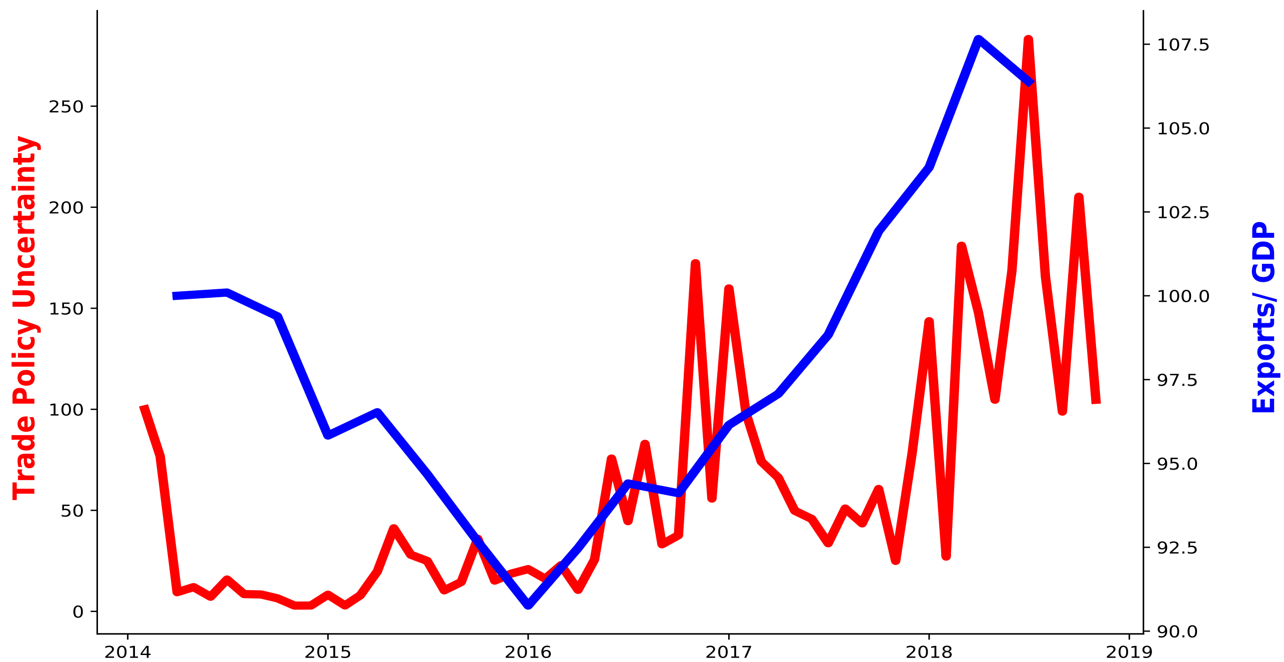
In our model, uncertainty is about another country's endowment. But endowment risk is a simple economic primitive that is a stand-in for many other sorts of uncertainty. For example, uncertainty about the quality of foreign exports or preference shocks is isomorphic to the model we wrote down. Similarly, the endowment could represent production, net of deadweight "iceberg" losses, created by trade policy. Trade policy uncertainty creates randomness in the residual supply. What matters to an exporting firm is not the source of uncertainty or the quantity or quality of foreign exports; it cares about the distribution of the relative price of their good. Section 3 establishes that the key to our results is that this relative price has a distribution that is right-skewed. Such a distribution arises naturally in this context because terms of trade are never negative, but can be arbitrarily large. If some uncertain outcome makes the right-skewed terms of trade more variable, it raises risk—but, under some plausible conditions, also raises the average relative price. These competing mean and variance forces are central to our analysis, and arise in settings in which aggregate uncertainty affects trade.

None of this proves that uncertainty causes an increase in trade. It is possible that it does not. But if not, these results teach us that many standard parameterized trade models are logically inconsistent with uncertainty's being a barrier to trade. Either our intuition about uncertainty and trade or our models should change.

Our analysis proceeds in several steps. First, we consider the effects of uncertainty on the terms of trade. The key insight is that—in general equilibrium—uncertainty affects not only the volatility, but also the expected terms of trade. Mathematically, the mechanism is that uncertainty impairs home agents' ability to condition their exporting behavior on the foreign country's state (and vice versa). As a result, home and foreign exports covary less. In equilibrium, the terms of trade depend on the ratio of home and foreign exports. If home and foreign exports are always proportional, the terms of trade are constant. Less coordination creates more volatile terms of trade. The mechanism encodes the conventional wisdom that uncertainty deters risk-averse exporters from exporting.

This conventional wisdom, however, is incomplete. A fall in export covariance causes the numerator and denominator of the terms of trade to covary less, while always remaining positive. This results in terms of trade that occasionally reach a very high level, but never fall below zero. When such a positive ratio varies more, its mean increases. Thus, high uncertainty, which results in more volatile terms of trade, also increases the expected level of the terms of trade, making exporting more lucrative on average.

The effect of information on the risk and the expected return from exporting permeates a broad




Source: Exports and GDP from NIPA. Trade policy uncertainty comes from the Categorical Economic Policy Uncertainty Index for the U.S. constructed by Baker, Bloom, and Davis and downloaded from http://www.policyuncertainty.com.

**Figure 1: Trade Policy Uncertainty and Exports**

class of general equilibrium trade models. However, how risk and return affect the incentives— to export and which effect dominates—depends on preferences and their parameters. While our analysis ultimately identifies fundamental features of preferences that cause information to affect trade volumes one way or another we begin with a specific but commonly used form of preferences to identify these forces in a well-understood setting and build intuition for how and why they arise. With constant elasticity of substitution (CES) preferences, these comparative statics boil down to functions of the elasticity of substitution across home and foreign varieties. If goods are highly substitutable, the risk effect dominates and information frictions decrease trade. When goods are less substitutable, the rise in the expected terms of trade more than offsets the increase in risk, and firms choose to export more when information is less precise. In other words, uncertainty facilitates trade.

With other preferences outside the CES class, the same forces are at play, but may result in different net effects on trade volume. One possibility is that the increase in the terms of trade can reduce exports. The logic is that if I'm expecting to get lots of the foreign good back in return for my exports, and I like a balanced consumption bundle, then I should export less when the relative price of my good rises. Otherwise, I'll have too much of the foreign good to consume. Another possibility is that when the terms of trade become more uncertain, an agent chooses to export more for purely precautionary reasons. Our general results characterize preferences where substitution or precautionary effects dominate. We can distinguish these well-understood substitution and precaution effects from our equilibrium terms of trade effect.

These arguments have a tight link to risk-sharing and insurance motives. We point out how the change in covariance, which governs risk sharing, affects the mean return to trade. How uncertainty actually affects trade volume depends on which of these forces—the increase in risk or increase in return—dominate. One obvious reason that uncertainty might encourage more trade is that agents have precautionary motives to trade. Agents who export, not knowing how much of the foreign good they will get in return, might export more to make sure they get enough of the foreign good back. For preferences with the right type of curvature, precautionary exporting emerges. But even when preferences do not normally induce precautionary behavior, we show that equilibrium movements in the terms of trade can induce countries to export more in the face of more mutual uncertainty. Just as borrowing constraints can change interest rate dynamics to induce precautionary behavior in a savings problem, equilibrium movements in the terms of trade can induce precautionary exporting in trade models with a wide range of non-precautionary preferences.

In other words, the terms of trade vary, in such a way as to share risk between countries (Cole and Obstfeld, 1991). When uncertainty is low, the terms of trade vary less and pose less risk to the exporter. But terms of trade that are not variable cannot hedge risk effectively. As uncertainty rises, and the terms of trade are less predictable, they also covary more negatively with endowments, so as to hedge each country's risk. This is what makes trade more attractive.

Our results rely on the assumption that there are no financial instruments or contracts that formally share risk. Just as in Newbery and Stiglitz (1984), we eliminate such instruments because we cannot logically study uncertainty if all uncertainty can be hedged and thereby effectively eliminated. We relax this restriction and describe the average amount of trade in settings in which some agents can write fully state-contingent contracts and others cannot. We find that allowing more risk-sharing works just like reducing uncertainty. If you can condition exports on the realized price, then it is just like knowing the price. Both reduce the average amount of trade.

Our results are most applicable to existing trading relationships. Our argument does not apply when two countries are new trading partners and many new trading relationships are potentially being formed. The reason is that new trading relationships surely involve fixed costs to set up. Uncertainty affects the willingness to bear those fixed costs in a way that is not captured by this model. However, much of the world's trade takes place between trading partners that are already established, like the U.S. and Mexican car manufacturers. The question there is not whether to start exporting, but how much to trade within an existing relationship. This question is a natural starting point because the setting is simpler, but also because the answer is more surprising.

**Related Literature**

A handful of classic and more recent papers explore how openness to trade affects utility and economic volatility, which is close to the reverse of our question. Newbery and Stiglitz (1984) and Caselli, Korenz, Lisicky, and Tenreyro (2017) debate whether, in incomplete markets, trade can reduce welfare or increase economic volatility. While our focus is on trade volume rather than volatility, some of the mechanisms are similar. Specifically, their focus on the relative price of goods as a risk-sharing mechanism is present in our results. In their setting, opening to trade affects terms of trade volatility; in our setting, information about trading partners increases the coordination of exports, which reduces terms of trade volatility.

Closer to our main point are papers that measure the negative effect of uncertainty on trade. Some focus on firm-specific or product-specific uncertainty.[1] Our model complements these findings by focusing on an element these theories abstract from, the role of uncertainty about a foreign economy. Other authors use the term "uncertainty" to mean volatility and show that volatility reduces trade (de Sousa, Disdier, and Gaigne, 2018). Our work does not dispute or contradict these facts. We also find that volatility reduces trade, but uncertainty does not. The difference between the two is information. A process can be volatile but predictable, like brightness over night and day. Our analysis leaves the volatility of shocks unchanged and simply varies the quality of the information about those shocks. This clarifies the distinction between aggregate and idiosyncratic risk and between volatility and uncertainty. In doing so, it informs measurement.

Several recent papers have measured the impact of the alleviation of information frictions on international trade. Steinwender (2014) showed that transatlantic connectivity through the introduction of the telegraph lowered average prices, lowered volatility, and increased imports of U.S. cotton in the mid 19th century. The fact that decreases in price uncertainty led to a reduction in prices and a reduction in price volatility supports our main mechanism. But our model changes the interpretation of their increase in trade volume. One interpretation, as suggested by Juhasz and Steinwender (2019) is that the uncertainty concerns product characteristics, not aggregate conditions. However, if the reduction in uncertainty were about aggregate shocks and did cause a surge in trade, then it would imply that agents' preferences or market mechanics should be modeled differently.

In financial markets, lower uncertainty also frequently inhibits risk-sharing. The Hirshleifer (1971) effect arises when information precludes trade in assets whose payoffs are contingent on an outcome revealed by the information. Our effect is distinct because (1) our signals are

[1] E.g., Allen (2013), Petropoulou (2011), Rauch and Watson (2004), and Eaton, Eslava, Krizan, Kugler, and Tybout (2011).

not public, (2) the existence of two distinct consumption goods matters, and (3) our mechanism works through changes in the international relative price. We discuss the importance of each of these differences when we explore risk sharing in Section 2.3.

## 1. A Benchmark Equilibrium Model of Trade Under Uncertainty

This section develops a simple model with two countries, stochastic nationally differentiated endowments, and a cross-border information friction. The first two ingredients are standard ingredients of trade and international business cycle models, as in Armington (1969) and Backus, Kehoe, and Kydland (1995). The cross-border information friction is that agents in each country know their own country's aggregate endowment, but have imperfect information about the other country's endowment. This information friction gives rise to aggregate uncertainty about the terms of trade. Below we discuss the economic environment and then discuss our modeling choices at the end of the section.

### 1.1. The Economic Environment

The economic environment is a repeated static model with the following features.

**Preferences.** There are two countries ($x$ and $y$) and a continuum of agents within each country. We denote individual variables with lower case and aggregates with upper case. Agents like to consume two goods, $x$ and $y$ (which are nationally differentiated), and their utility flow each period is

$$U(c_x, c_y). \tag{1}$$

where for now we only restrict $U$ to be increasing and concave in both goods. Section 2 solves the model for the constant elasticity of substitution case; Section 3 characterizes the general case.

**Endowments.** Each agent in the domestic country has an idiosyncratic endowment of $z_x$ units of good $x$, where $\ln z_x \sim \mathcal{N}(\mu_x, \sigma_x^2)$. Agents in the foreign country have an idiosyncratic endowment of $z_y$ units of good $y$, where $\ln z_y \sim \mathcal{N}(\mu_y, \sigma_y^2)$. Thus, production of each good is nationally differentiated, as in Armington (1969). Most important is that we let the mean of these distributions be independent random variables: $\mu_x \sim \mathcal{N}(m_x, s_x^2)$ and $\mu_y \sim \mathcal{N}(m_y, s_y^2)$. Because they represent the average endowment realization for each country, $\mu_x$ and $\mu_y$ are aggregate shocks.

Endowment shocks are the source of uncertainty in the model. They could equivalently be quality or preference shocks. See Appendix B for an isomorphic model. Since trade policy is often modeled as a preference change, this could represent policy as well. What is important is that the shocks are aggregate, not firm-specific.

**Information**: At the beginning of the period, agents in country $x$ observe their own endowment $z_x$ and the mean of their country's endowment $\mu_x$. Likewise, agents in country $y$ observe $z_y$ and $\mu_y$. Furthermore, agents know the distribution from which mean productivity is drawn and the cross-sectional distribution of firm outcomes. In other words, $m_x, m_y, s_x, s_y, \sigma_x$ and $\sigma_y$ are common knowledge.

Agents in each country receive signals about the other countries' aggregate endowment realization. Specifically, agents in country $x$ observe a signal about the $y$-endowment

$$\tilde{m}_y = \mu_y + \eta_y \tag{2}$$

where $\eta_y \sim N(0, \tilde{s}_y^2)$. Similarly, agents in country $y$ observe a signal about the $x$-endowment

$$\tilde{m}_x = \mu_x + \eta_x \tag{3}$$

where $\eta_x \sim N(0, \tilde{s}_x^2)$. Thus agents in each country receive an imprecise but unbiased signal about fundamentals in the foreign country. How precise or imprecise the signal is will depend on the variance of the noise, $\tilde{s}_y^2$ and $\tilde{s}_y^2$. Changing these variances allows us to vary fundamental uncertainty, in a continuous way, and study the response of the economy.

Let $\mathcal{I}_x$ denote the information set of an agent in the home country and $\mathcal{I}_y$ denote the information set of a foreign agent. All country $x$ choices will be a function of the three random variables in the home agents' information set: $\mathcal{I}_x = \{z_x, \mu_x, \tilde{m}_y\}$. Likewise, country $y$ choices depend on $\mathcal{I}_y = \{z_y, \mu_y, \tilde{m}_x\}$.

**Bayesian updating.** Agents in each country combine their signal (i.e., equations (2) and (3)) with their prior knowledge of the endowment distribution to form posterior beliefs. Agents must form posterior beliefs over two outcomes: First, they must form a belief about the endowment realization in the foreign country; we will call these first-order beliefs. Second, those in the home country must form beliefs about the foreign country's belief about themselves; we will call these second-order beliefs. Characterizing first- and second-order beliefs are sufficient to characterize optimal actions.[2]

To compute country $x$'s first-order beliefs about country $y$'s endowment distribution, note that by Bayes' law, the posterior probability distribution is normal with mean $\hat{m}_y$ and variance $\hat{s}_y^2$ given by

$$F(\mu_y | \, \mathcal{I}_x) = \Phi\left(\frac{\mu_y - \hat{m}_y}{\hat{s}_y}\right) \quad \text{where} \quad \hat{m}_y = \frac{s_y^{-2} m_y + \tilde{s}_y^{-2}\tilde{m}_y}{s_y^{-2} + \tilde{s}_y^{-2}}, \quad \hat{s}_y^2 = \frac{1}{s_y^{-2} + \tilde{s}_y^{-2}}. \tag{4}$$

[2]In fact, all higher orders of beliefs can matter for export choices. But because there are only two shocks observed by each country, the first two orders of beliefs are sufficient to characterize the entire hierarchy.

where the posterior mean is a precision weighted average of the signal and unconditional mean; $\Phi$ is the standard normal distribution. Similarly country $y$'s first-order belief about county $x$'s endowment distribution is:

$$F(\mu_x|\,\mathcal{I}_y) = \Phi\left(\frac{\mu_x - \hat{m}_x}{\hat{s}_x}\right) \quad \text{where} \quad \hat{m}_x = \frac{s_x^{-2} m_x + \tilde{s}_x^{-2} \tilde{m}_x}{s_x^{-2} + \tilde{s}_x^{-2}}, \quad \hat{s}_x^2 = \frac{1}{s_x^{-2} + \tilde{s}_x^{-2}} \tag{5}$$

Then to compute country $x$'s second-order belief—its belief about country $y$'s belief about itself—these second-order beliefs are

$$F(\hat{m}_x|\mathcal{I}_x) = \Phi\left(\frac{\hat{m}_x - \hat{\hat{m}}_x}{\hat{\hat{s}}_x}\right) \quad \text{where} \quad \hat{\hat{m}}_x = \frac{s_x^{-2} m_x + \tilde{s}_x^{-2} \mu_x}{s_x^{-2} + \tilde{s}_x^{-2}}, \quad \hat{\hat{s}}_x^2 = \frac{\tilde{s}_x^{-2}}{(s_x^{-2} + \tilde{s}_x^{-2})^2} \tag{6}$$

$$F(\hat{m}_y|\mathcal{I}_y) = \Phi\left(\frac{\hat{m}_y - \hat{\hat{m}}_y}{\hat{\hat{s}}_y}\right) \quad \text{where} \quad \hat{\hat{m}}_y = \frac{s_y^{-2} m_y + \tilde{s}_y^{-2} \mu_y}{s_y^{-2} + \tilde{s}_y^{-2}}, \quad \hat{\hat{s}}_y^2 = \frac{\tilde{s}_y^{-2}}{(s_y^{-2} + \tilde{s}_y^{-2})^2} \tag{7}$$

Here the second-order beliefs posterior mean ($\hat{m}_x$ , $\hat{m}_y$) is a precision weighted average of a country's own realization and the unconditional mean.

A final note regarding notation. Since there is a one-to-one mapping between signals $\tilde{m}$ and posterior beliefs $\hat{m}$, we will use posterior beliefs as a state variable rather than using signals. This simplifies the notational burden. Thus, we write $\mathcal{I}_x = \{z_x, \mu_x, \hat{m}_y\}$ and $\mathcal{I}_y = \{z_y, \mu_y, \hat{m}_x\}$.

**Price and budget set.** Given their information sets, agents chose how much to export, $t_x$ or $t_y$. In return, they receive the other country's goods at relative price $p$, which is denominated in units of $y$ good. For example, an agent who exports $t_x$ units of the $x$ goods receives $pt_x$ units of $y$ for immediate consumption. Finally, we assume that there is no secondary resale market or storage and we restrict exports and consumption to be nonnegative. This implies that country $x$'s budget set is:

$$c_x \;\; \epsilon \;\; [0, z_x - t_x], \tag{8}$$

$$c_y \;\; \epsilon \;\; [0, pt_x]\,, \tag{9}$$

and country $y$'s budget set is:

$$c_x \;\; \epsilon \;\; \left[0, \frac{t_y}{p}\right], \tag{10}$$

$$c_y \;\; \epsilon \;\; [0, z_y - t_y]. \tag{11}$$

**Timing.** The timing protocol is as follows: First, agents see their endowments and receive signals about the foreign county's endowments. Agents then make export decisions. Thus, they are exporting prior to knowing the actual price $p$. This timing protocol allows information frictions to matter: Uncertainty about the foreign country's endowment gives rise to aggregate uncertainty about the terms of trade and this uncertainty, in turn, feeds back into the decision to export.

**Equilibrium.** An equilibrium is given by export policy functions for domestic $t_x(z_x, \mu_x, \hat{m}_y)$ and foreign $t_y(z_y, \mu_y, \hat{m}_x)$ countries; aggregate exports $T_x(\mu_x, \hat{m}_y), T_y(\mu_y, \hat{m}_x)$; a perceived price function $\tilde{p}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)$ for each country; and an actual price function $p(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)$ such that:

1. Given perceived price functions $\tilde{p}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)$, export policies maximize expected consumption of every firm in each country. Substituting budget sets (8) to (11) into utility $\mathbb{E}[U(c_x, c_y)]$, we can write this problem as

$$t_x(z_x, \mu_x, \hat{m}_y) = \arg\max \mathbb{E}\left[U\left(z_x - t_x, \tilde{p}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)t_x\right)\middle|\, \mathcal{I}_x\right] \tag{12}$$

$$t_y(z_y, \mu_y, \hat{m}_x) = \arg\max \mathbb{E}\left[U\left(\frac{t_y}{\tilde{p}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)}, z_y - t_y\right)\middle|\, \mathcal{I}_y\right] \tag{13}$$

   Using the conditional densities (4), (5), (7), and (6), we can compute expectations as

$$t_x(z_x, \mu_x, \hat{m}_y) = \arg\max \int\int U\left(z_x - t_x, \tilde{p}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)t_x\right) dF(\mu_y|\mathcal{I}_x)dF(\hat{m}_x|\mathcal{I}_x) \tag{14}$$

$$t_y(z_y, \mu_y, \hat{m}_x) = \arg\max \int\int U\left(\frac{t_y}{\tilde{p}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)}, z_y - t_y\right) dF(\mu_x|\mathcal{I}_y)dF(\hat{m}_y|\mathcal{I}_y). \tag{15}$$

   To understand the expectations in (14) and (15), note how expected utility is computed by integrating over my beliefs about the foreign country's endowment (the inside integral), and then my beliefs about their beliefs about me (the outside integral).

2. The relative price $p$ clears the international market. Since every unit of $x$-good exported must be sold and paid for with $y$ exports, and conversely, every unit of $y$ exports must be sold and paid for with $x$ exports, the only price that clears the international market is the ratio of aggregate exports:

$$p(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) = \frac{T_y(\mu_y, \hat{m}_x)}{T_x(\mu_x, \hat{m}_y)} \tag{16}$$

   where aggregate exports in each country are

$$T_x(\mu_x, \hat{m}_y) = \int t_x(z_x, \mu_x, \hat{m}_y)dF(z_x|\mu_x) \tag{17}$$

$$T_y(\mu_y, \hat{m}_x) = \int t_y(z_y, \mu_y, \hat{m}_x) dF(z_y|\mu_y), \tag{18}$$

which simply integrate over the individual heterogeneity within each country.

3. The perceived and actual price functions coincide:

$$\tilde{p}(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) = p(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) \qquad \forall(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y). \tag{19}$$

This definition is relatively straightforward. Agents maximize utility, markets clear, and then the mapping from endowments and signals to prices is consistent with agents' expectations about prices.

**Aggregation.** Given the model assumptions, the individual trade and consumption policies are multiplicative in the idiosyncratic shock

$$t_x(z_x, \mu_x, \hat{m}_y) = z_x \Psi(\mu_x, \hat{m}_y), \qquad c_x(z_x, \mu_x, \hat{m}_y) = z_x(1 - \Psi(\mu_x, \hat{m}_y)), \tag{20}$$

where $\Psi(\cdot)$ is a function that depends only on the aggregate domestic endowment and the beliefs about aggregate foreign endowment. With this decomposition, aggregate exports become $T_x(\mu_x, \hat{m}_y) = f_x \Psi(\mu_x, \hat{m}_y)$, where $f_x \equiv \int z_x dF(z_x|\mu_x) = e^{\mu_x - \sigma_x^2/2}$ represents the aggregate fundamental. This allows us to aggregate each economy and consider two representative agents, with utility $\mathbb{E}\left[U(C_x, C_y)|\mathcal{I}\right]$ over aggregate consumption, computed as

$$\begin{aligned} C_x(\mu_x, \hat{m}_y) &= f_x - T_x(\mu_x, \hat{m}_y), & (21) \\ C_y(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) &= \frac{T_x(\mu_x, \hat{m}_y)}{p(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)}. & (22) \end{aligned}$$

for the domestic country (and analogously for the foreign country, i.e., $T_y(\mu_y, \hat{m}_x) = f_y \Psi^*(\mu_y, \hat{m}_x)$, where $f_y \equiv \int z_y dF(z_y|\mu_y) = e^{\mu_y - \sigma_y^2/2}$).

The rest of the analysis focuses on how the trade policies of the representative agents change with uncertainty.

### 1.2. The Economic Environment

Several comments regarding our modeling choices are in order. While our model makes a clear connection between fundamental frictions and aggregate uncertainty, we abstract from two aspects of uncertainty discussed in the literature. First, uncertainty could arise from firm-specific conditions and second, it could arise from product characteristics. While these issues are interesting, in this case uncertainty would be idiosyncratic rather than aggregate, and hence its effect on aggregate trade would only concern aggregation properties (e.g., distortions to the extensive margin of trade) rather than the uncertainty itself. Moreover, uncertainty does not mean volatility. The shocks in both countries have a fixed variance. Only the uncertainty—what is not known about those shocks—is changing.

Timing is a form of friction here, which allows the information friction to generate aggregate uncertainty. Forcing firms to export before knowing shocks or prices causes uncertainty about the foreign country's endowment to matter; it is the aggregate uncertainty about the terms of trade that feeds back into the decision to export. Absent any timing friction, information frictions and uncertainty would play no role. Realistically, this modeling choice captures the idea that shipping lags and certain payment arrangements make exporting risky, because the terms of trade might not be known with certainty. For example, Hummels and Schaur (2010) and Hummels and Schaur (2013) demonstrate the time-intensive nature of trade and show how it shapes the cross-sectional pattern of trade. Similarly, Antras and Foley (2015), International Monetary Fund (2009, 2011), and Asmundson, Dorsey, Khachatryan, Niculcea, and Saito (2011) present evidence on the cash-in-advance-like arrangements under which import transactions are often carried out.

The financial contracts agents have access to also matters. If a complete set of risk-sharing instruments exists, then agents can contract on outcomes of all unknown variables and the effects of information asymmetry would disappear. In other words, some market incompleteness is necessary for informational frictions to matter. If we want to explore the effects of information frictions, complete markets render that investigation impossible. When markets are partially incomplete, end results show that information frictions facilitate more risk sharing through movements in international prices than would otherwise be insured with financial instruments.

## 2. Trade and Uncertainty with Constant Elasticity (CES) Preferences

This section illustrates the main argument of the paper in the context of a specific utility function. We start with a form of CES preferences that are standard in the trade and international macro literature. Section 3 generalizes the results to cases outside of CES.

We argue the following. First, we show how uncertainty—in general equilibrium—affects both the mean and variance of the terms of trade. Second, we characterize how changes in the volume of trade depend on both changes in the mean and variance of the terms of trade and trade elasticity.

As discussed above, we work with the following CES utility function:

$$\mathbb{E}\left[C_x^\theta + C_y^\theta \middle| \mathcal{I}_x\right],\ \theta < 1. \tag{23}$$

The restriction $\theta < 1$ is required for the function to be concave in both goods.[3] Expectations are

[3]How to interpret this utility function? Consider a consumer with CRRA utility $\mathbb{E}\left[\frac{C^{1-\sigma}-1}{1-\sigma}\right]$ and a consumption

conditional on the information set of the home country, which contains its own realization of the aggregate shock and the signal about the realization abroad.

### 2.1. Information and the Terms of Trade

How does uncertainty affect the stochastic properties of the terms of trade? We proceed in three steps: how uncertainty affects the covariance of exports, how the covariance affects the average terms of trade, and how the covariance affects volatility in the terms of trade. This set of results holds for all elasticity parameters $\theta$ and, as we see later, holds for a much broader class of preferences as well.

Result 1 states that more uncertainty (less precise information about the other country's endowment) decreases the covariance between aggregate exports.

**Result 1** ***Uncertainty reduces the covariance of aggregate exports.*** *In a neighborhood around complete certainty ($\tilde{s}_x^2$ and $\tilde{s}_y^2$ equal zero), more uncertainty moves the covariance between aggregate exports toward zero.*

The intuition behind this result is easy to understand: Agents cannot condition their action on a variable that is not known to them. In our context, this implies that home agents cannot export conditional on foreign outcomes if the foreign state is unknown. Thus, when signal precision approaches zero, the covariance of exports must be zero.

In contrast, as each country becomes less uncertain about the other, they are able to trade in a more sophisticated way. By "sophisticated", we mean that the home country's export decision is better informed about the foreign country's endowment and, in turn, the resulting terms of trade. This leads to more coordinated actions and a stronger covariance in export behavior.

As an example, in the substitute case (i.e., $\theta > 0$), accurate information about a high endowment realization in the foreign country suggests that foreigners will export a lot. Foreign goods will be abundant and cheap; home goods will be relatively expensive. The expectation of high returns to exports incentivizes agents at home to export more. Both countries export more together, i.e., actions are positively covarying.

Figure 2 illustrates this point. The top panel illustrates the case in which there is perfect information; one sees that exports are highly correlated across countries. When one country exports more, the other country has a strong desire to export more as well. The bottom panel illustrates the opposite extreme. Here neither country has any information about the other country, and thus their exports are independent.

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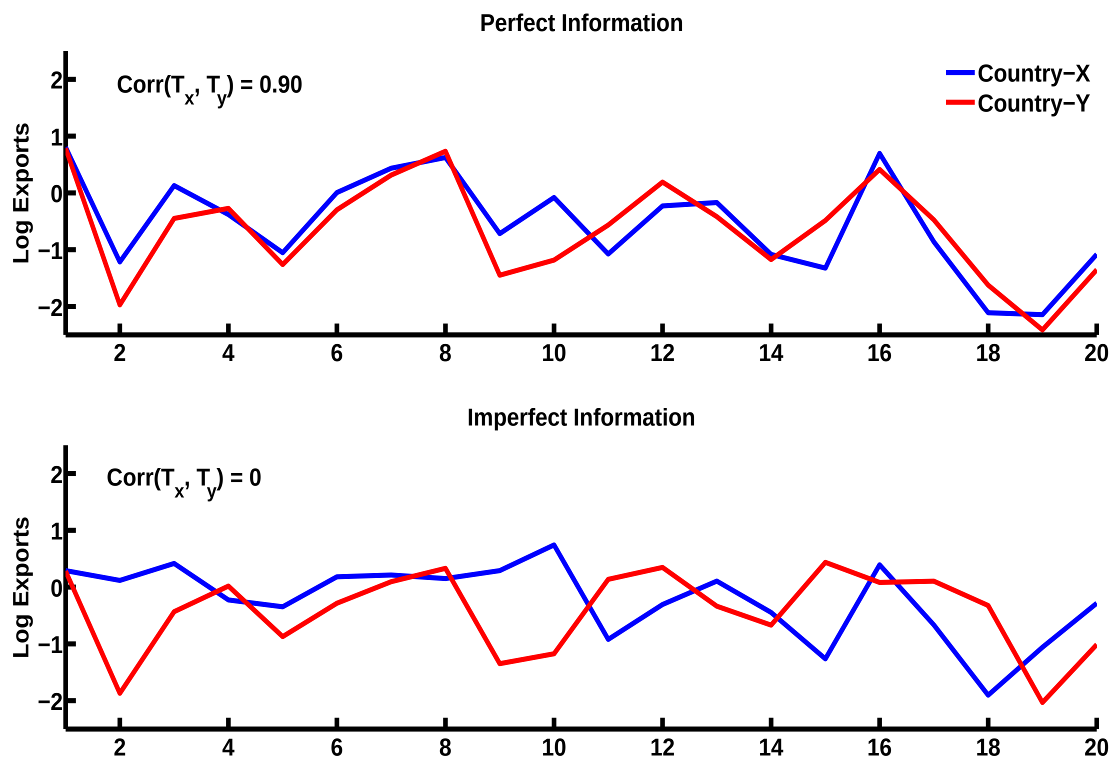
bundle given by an aggregator $C = \left(C_x^\theta + C_y^\theta\right)^{1/\theta}$. Then the CES-like case is a special case in which $\sigma + \theta = 1$. If $0 < \theta < 1$, then $\sigma < 1$ and it is a risk-averse agent. If $\theta < 0$, then $\sigma$ could be above one and it is a risk-loving agent. Recall that the elasticity of intertemporal substitution is $1/\sigma$.




Notes: Simulation of $T = 20$ periods of the model with CES preferences. Parameter values are $\theta = 0.75$, $m_x = m_y = 0$, $s_x = s_y = 1$, and $\sigma_x = \sigma_y = \sqrt{2}$. Perfect information sets signal noise $\tilde{s}^2$ in both countries to zero and imperfect information takes its limit to infinity. See Appendix C for details on computation.

**Figure 2: Export Coordination under Perfect and Imperfect Information**

The fact that information allows exports to covary underpins the following result: In equilibrium, higher uncertainty increases both the mean and the variance of the terms of trade.

**Result 2** ***Uncertainty increases the mean and the variance of terms of trade*** *If the unconditional expectations and variances of aggregate exports are kept constant, an increase in uncertainty:*

1. *Increases the expected terms of trade for both countries. Furthermore, if countries are symmetric, the average terms of trade can be expressed as*

$$\mathbb{E}\left[p\right] = 1 + \mathbb{CV}^2[T_x]\left(1 - corr[T_x, T_y]\right) \tag{24}$$

   *where $\mathbb{CV}^2$ is the squared coefficient of variation and $corr$ is the correlation.*

2. *Increases the volatility of the terms of trade for both countries.*

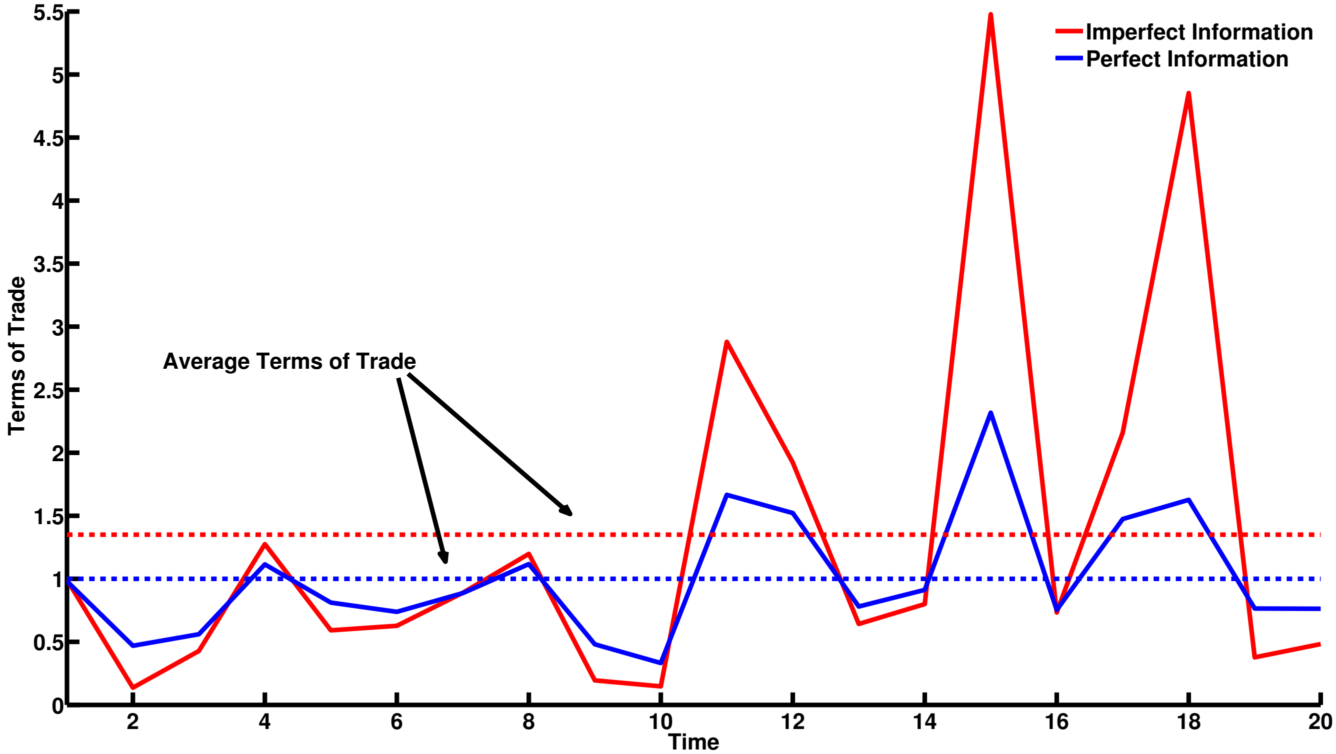
The terms of trade are the price that clears the international export market. The only price that clears that market is the ratio of exports. When home and foreign exports covary, the numerator and denominator of the terms of trade covary. Imagine an extreme case in which home and foreign exports had perfect correlation. Home exports were exactly proportional to foreign exports, in every state. Then the ratio of home and foreign exports would be constant. That would imply constant terms of trade, with zero variance. When home and foreign exports covary less, the terms of trade become more volatile. This is the logic formalized in the previous result.

Figure 3 illustrates the link between the reduction in export covariance and the volatility of the terms of trade. As the figure shows, more uncertainty means less covariance in exports, and thus the terms of trade are much more volatile (compare the red to the blue line).

Figure 3 also shows the link between uncertainty and the average terms of trade. Greater uncertainty reduces export covariance, which causes the numerator and denominator of the terms of trade to covary less, while always remaining positive. This high-uncertainty case is depicted by the red line. Notice that the high-uncertainty terms of trade occasionally spike. These are states in which home exports are quite low, and therefore scarce and valuable. With a sufficiently low productivity state, exports can become arbitrarily low, which makes the terms of trade arbitrarily high. Yet the terms of trade never fall below zero. By its nature, the process for the terms of trade is asymmetric.

This is not to say that this is an asymmetry that systematically favors one country over the other. When information is scarce, *both countries simultaneously have high expected terms of trade*. Indeed, high terms of trade for one country imply low terms of trade for the other. But high expected terms of trade do not imply that expectation of inverse terms of trade is low. In short, information frictions increase the expected terms of trade for both countries.

Importantly, these results are proven, without reference to the preference specification. The relationship between export covariance and the properties of the terms of trade is a statement




Notes: Simulation of $T = 20$ periods of the model with CES preferences. Parameters as in Figure 2.

**Figure 3: Export Coordination under Perfect and Imperfect Information**

about the statistical properties of the ratio of two lognormal random variables. These statistical properties are independent of preferences. Therefore, these two results will carry over when we discuss the model with general preferences at the end. The first result about information that enables correlation is not general because of its sign. In some cases, preferences will make agents want to coordinate their exports negatively. It is always true that the only feasible level of coordination with no information is zero covariance. But less uncertainty might enable either positive or negative export covariance strategies, depending on preferences.

The previous results showed how uncertainty affected mean and variance in the terms of trade through a coordination motive. The next section connects these forces to firms' decisions on how much to export.

### 2.2. How the Terms of Trade Distribution Affects the Volume of Trade

The second part of our argument links the expected terms of trade and its variance to the volume of trade. There are no surprises here. Our main point is not that agents react to changes in the mean and variance of the terms of trade in some strange way. With CES preferences and sufficient substitutability, firms export more when the return to exporting is higher and export less when exporting is risky. So conventional wisdom about uncertainty deterring trade is correct in the substitutable-good, CES model. The unexpected part of the relationship between uncertainty and trade comes from the previous section, in which uncertainty about others' exports raises the expected returns to exporting one's own good.

The next result shows that, as one would expect, an increase in the expected terms of trade—the return to exporting—increases the average level of exports. It also shows that higher variance in the terms of trade deters exports, because agents are averse to the risky return of exporting.

**Proposition 1** *Suppose the terms of trade mean and variance change in* $\frac{d\mathbb{E}_x[p]}{\mathbb{E}_x[p]}$ *and* $\frac{d\mathbb{V}ar_x[p]}{\mathbb{V}ar_x[p]}$*, respectively. Then the sign of the change in exports is equal to the sign of the following expression:*

$$\theta \frac{d\mathbb{E}_x[p]}{\mathbb{E}_x[p]} + \frac{\mathbb{CV}_x^2[p]}{2}\left(\theta(1-\theta)(2-\theta)\frac{d\mathbb{E}_x[p]}{\mathbb{E}_x[p]} - \theta(1-\theta)\frac{d\mathbb{V}ar_x[p]}{\mathbb{V}ar_x[p]}\right) \tag{25}$$

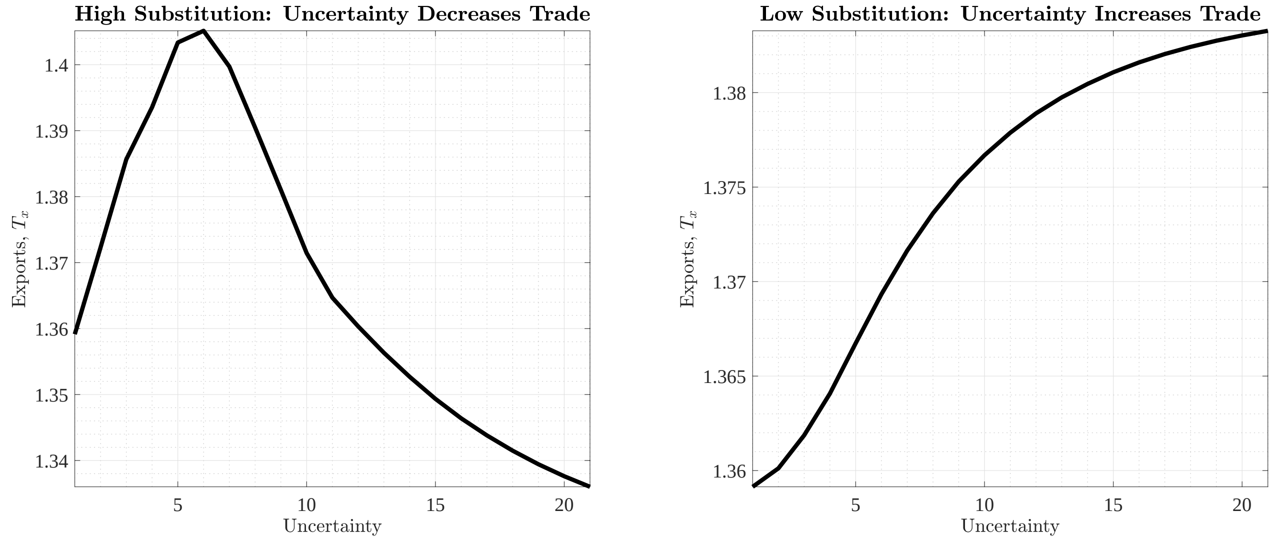
Proposition 1 tell us how much exports will rise or fall from a given percentage change in the expected mean or variance of the terms of trade. It reveals many facets of the relationship between the terms of trade $p$ and trade volume. First, it tells us that an improvement in the expected terms of trade, holding other moments equal, causes firms to export more. This is true for elasticity of substitution $0 < \theta \leq 1$ or for $\theta \geq 2$. The elasticity $\theta$ governs the size of the trade volume effect.

Variance in the terms of trade also changes with uncertainty. More variance—or risk—in the terms of trade deters trade. And this is true for any level of elasticity of substitution. Combined with (25), this result means that uncertainty deters trade through risk. This is the conventional wisdom—that is, noisier signals increase uncertainty, and this force deters exports.

We have identified two competing forces. Consistent with conventional wisdom, uncertainty creates risk and this deters trade. However, uncertainty also raises the return on trade, encouraging more trade. Which force wins?

The relative strengths of the mean and variance forces depend on the degree of uncertainty, as well as the elasticity of substitution. Figure 4 illustrates this by plotting average exports of the home country as uncertainty increases in two alternative economies: The left panel considers a high substitution economy with $\theta = 0.8$, which features a nonmonotonic effect of uncertainty on trade with a large decreasing segment. The right panel considers a low substitution economy with $\theta = 0.3$ that generates an increasing relationship between uncertainty and trade.

The mean effect is always positive, meaning that increases in the mean terms of trade always increase average exports. The variance effect is always negative. More volatile terms of trade alone always deter trade. When there is a low degree of substitutability between home and foreign goods, the trade-increasing effect is stronger. This reflects the idea that as goods become




Notes: Equilibrium exports for domestic country $T_x$ for different levels of uncertainty (signal noise $\tilde{s}^2$). Common parameters values are $m_x = m_y = 0$, $s_x = s_y = 1$, and $\sigma_x = \sigma_y = \sqrt{2}$. The left panel uses a high elasticity of substitution $\theta = 0.8$, and the right panel uses a low elasticity of substitution $\theta = 0.3$. See Appendix C for details on computation.

**Figure 4: Effect of Uncertainty in Trade Depends on the Elasticity of Substitution**

more complementary, uncertainty matters more because agents want to ensure balanced consumption bundles across goods. As a consequence, agents export more to reach that balance in expectation. Since the trade-increasing mean effect is stronger and the trade-reducing variance effect is weaker for low substitutable goods, these low-$\theta$ economies are ones in which greater uncertainty promotes trade.

### 2.3. A Risk-sharing Interpretation

One way of understanding why uncertainty can facilitate trade is to explore why uncertainty enables better risk-sharing. In our trade model, countries would achieve full risk sharing if each country consistently exported half its endowment. In such a world, both countries would consume the same bundle: half of the home goods produced and half of the foreign goods produced in that period. Consumption in both countries would be perfectly correlated. This full risk-sharing world also achieves the maximum level of average trade. Exporting more than half of one's endowment, on average, never makes sense. So if full risk sharing implies maximum trade, the question of why uncertainty promotes trade amounts to asking why uncertainty brings the world economy closer to full risk sharing.

In finance, the argument for why uncertainty facilitates risk sharing is well understood and is often called the "Hirschleifer effect." Hirschleifer considers the example of two bettors, each with a ticket on an identical but independent lottery. The bettors can diversify their risk by splitting the two claims, so that if either lottery pays off, both get half the winnings. Now, suppose that both bettors observe noisy signals about the outcome of each lottery. The bettor whose claim is on the lottery with the more favorable signal would want to keep a larger claim on his own lottery. The signal reduces uncertainty about both lottery outcomes, but at the same time undermines risk-sharing. The only way both bettors will consistently share all their risk is if they know nothing about the lottery outcomes.

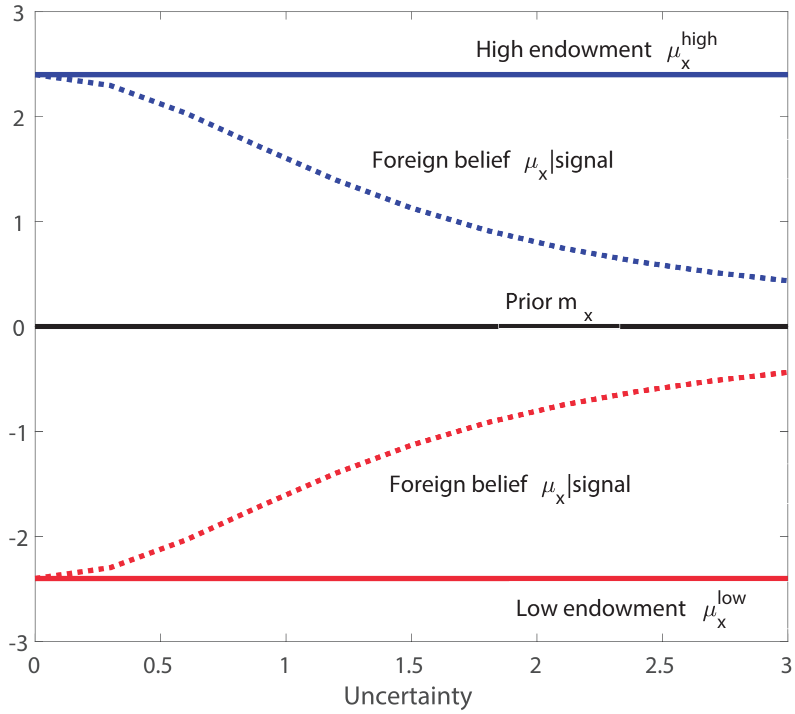
The analogy between financial markets and trade is not perfect. The fact that trade involves two or more goods, rather than two lottery tickets that pay identical currency units, matters. Risk-sharing in trade takes a different form. There are no ex ante agreements to share output. Instead, the mechanism for international risk-sharing is movement in the terms of trade. When agents in one country get a low endowment, their good is scarce; therefore their good is valuable, and they get lots of foreign goods in return for their exports. The abundance of foreign goods helps to insure the risk of a low endowment.

This insurance mechanism through terms of trade is already present in worlds with perfect information, as in Cole and Obstfeld (1991). What is new here is that uncertainty strengthens the terms of trade as a risk-sharing mechanism, because it prevents countries from backing away from trade in states when they would prefer not insure their trading partner. To understand the logic of this argument, let us focus on how foreign beliefs are affected by changes in uncertainty, as illustrated by Figure 5.

Suppose there is a low realization of domestic endowment $\mu_x$ (solid red line) and uncertainty is very low (toward the left side of the figure). Then foreign firms expect domestic firms to export little, and those are states in which they would prefer to walk away from full insurance. The full insurance action would be for foreign to export lots, home to export little, and both to consume the same amount. But foreign agents do not want to export much at those terms. Just like the bettor who no longer wants to share his half ticket in return for one with lower odds, the foreign country who knows that the home endowment is low no longer wants to send lots away for little in return.

The previous logic breaks when uncertainty is high (toward the right side of the figure), as the foreigners then do not realize that the home endowment is low as their belief moves closer to the prior and away from the true realization. This is pure Bayesian updating. In this case, foreign will export more in a low-endowment $(\mu_x^{low})$ state, providing the home country with better insurance in exactly the states in which it is more needed.

Clearly, the arguments above about uncertainty increasing insurance in bad states applies in reverse for good states. In other words, if the foreigners know that the domestic endowment is high, they would exports lots. But as uncertainty increases, foreigners' beliefs again move toward the prior and away from the true realization, decreasing exports. However, the lack of



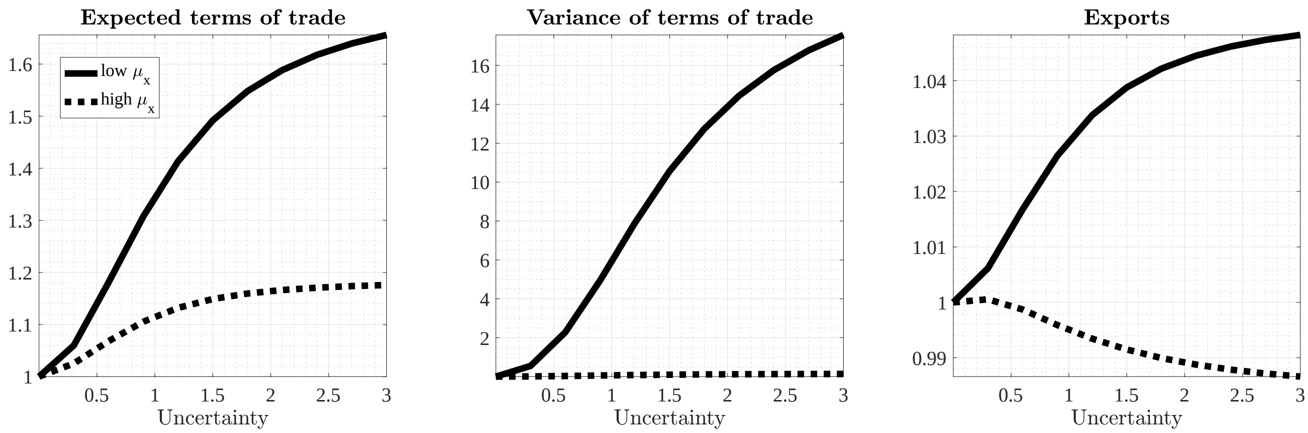

Notes: Average foreign beliefs for different realizations of the domestic endowment, for various levels of uncertainty (signal noise $\tilde{s}^2$).

**Figure 5: Higher uncertainty brings average foreign beliefs closer to the prior**

foreign exports under this scenario is not very costly for the home country, as it is enjoying a high endowment anyway (this is especially true with high substitutability across foreign and domestic goods). This asymmetry in the demands for insurance is at the core of our results, as the average response of trade to uncertainty is mainly driven by its response at low states, when the insurance premium is larger.

To further investigate how the relationship between uncertainty and trade varies across states, Figure 6 plots moments of the terms of trade and the volume of exports conditional on the realization of the domestic state $\mu_x$, which is either high or low. Without loss of generality, we fix the belief about foreign endowment to its prior value $\hat{m}_y = m_y$. We show results for a parametrization with the preferences in (23) with a low level of substitutability $\theta = 0.3$, for which the average response of trade is increasing in uncertainty, as shown above. As a normalization, we express results for expected terms of trade and trade volume as multiples of the value under perfect information (zero uncertainty).

We observe that when the domestic country has a low endowment (solid line), the expectations and volatility of the terms of trade, conditional on the domestic country's information set, dramatically increase with uncertainty and in a monotonic way. In this case, given the preferences and low degree of substitutability, domestic exports increase with uncertainty as well:




Notes: Equilibrium terms of trade and exports for the domestic country for different levels of uncertainty (signal noise $\tilde{s}^2$) and a low elasticity of substitution $\theta = 0.3$. Other parameters are $m_x = m_y = 0$, $s_x = s_y = 1$, and $\sigma_x = \sigma_y = \sqrt{2}$. Expected terms of trade and trade volume are shown as multiples of the value under perfect information (zero uncertainty). See Appendix C for details on computation.

**Figure 6: State-dependent responses to uncertainty**

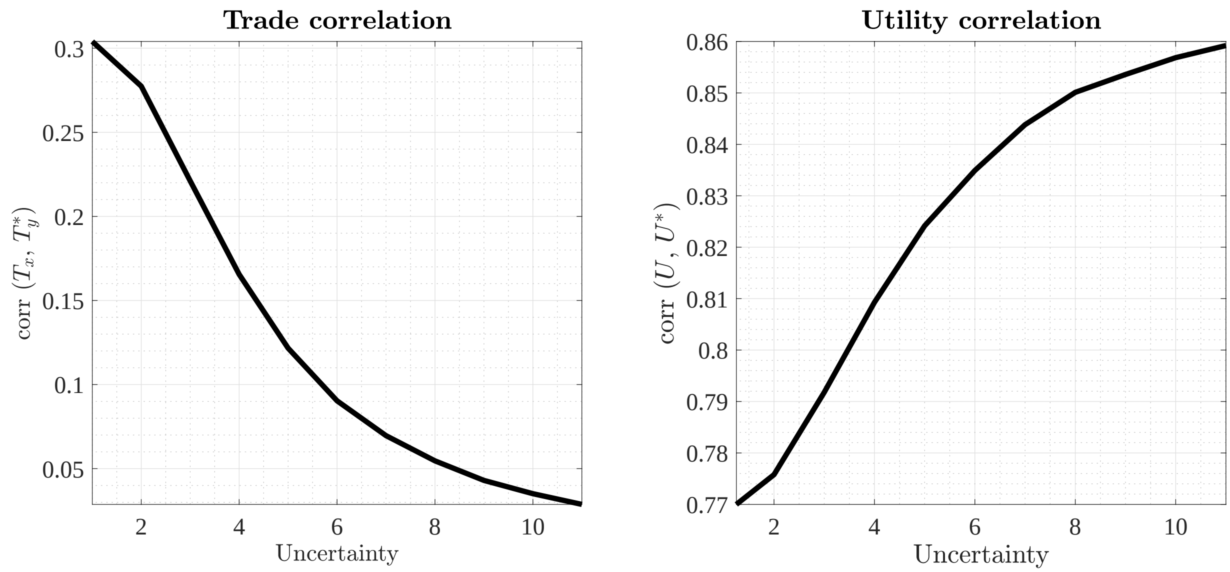
The trade-increasing average effect dominates the trade-reducing volatility effect.

Now let us consider the opposite case with a high domestic endowment (dashed line). Moments of the terms of trade are still monotonically increasing (the increase in the volatility is not observable in the figure due to the scale), but in this case, the volatility effect dominates the average effect and exports decrease with uncertainty. When we average across all states, the strong positive response of exports to uncertainty in bad states dominates the weak negative response in good states, and we recover the result that for low substitutability, uncertainty increases trade on average.

By increasing returns to trade $p$ when the endowment is low and reducing returns when the endowment is high, uncertainty smooths out the utility of each country's residents. That is, uncertainty improves risk-sharing. The left panel in Figure 7 shows that cross-country correlation in exports decreases toward zero with higher mutual uncertainty, which illustrates the coordination failure stated in Result 1 above. The right panel shows how cross-country correlation in utility—a commonly used measure of risk-sharing—increases with uncertainty.

## 3. The General Case

Information reduces uncertainty. Conditioning on that information makes random variables more predictable, and thus less risky. That is the nature of information. Depending on agents' desire to undertake precautionary savings, the reduction in risk could prompt them to export less or export more. In an equilibrium trade model, the risk effect is not the only effect of information. The other effect is to reduce the expected terms of trade. This shift in the terms of trade




Notes: Trade and utility correlation across countries for different levels of uncertainty (signal noise $\tilde{s}^2$) and a low elasticity of substitution $\theta = 0.3$. Other parameters are $m_x = m_y = 0$, $s_x = s_y = 1$, and $\sigma_x = \sigma_y = \sqrt{2}$. Simulation of $T = 100,000$ periods. See Appendix C for details on computation.

**Figure 7: Uncertainty Decreases Trade Coordination and Increases Utility Correlation**

distribution can also move the desire to export in either direction. However, the combination of the mean effect and the variance effect reveals the total impact of information on trade. We now examine these forces more generally.

While CES preferences are commonly used and useful for illustrating our results and the mechanisms behind them, focusing on only one type of preferences raises questions: Is CES a special, knife-edge case that generates unusual results? It turns out that CES is not special or knife-edge. A broad class of preferences also has the property by which uncertainty promotes trade. But not all preferences have this property. This section delineates which preferences are like CES, in the sense that uncertainty promotes trade. We also offer practical guidance for those who wish to pursue aggregate uncertainty as a trade barrier. Our results reveal what sorts of preferences are required for mutual uncertainty to deter trade. They also clarify to what extent the CES results reflect risk aversion or good substitutability. With CES preferences, both are tied to one parameter. When we work with a general utility function, we can expose what effects come from each force. The general characterization of the forces at work also uncovers a mathematical foundation for our risk-sharing interpretation: Uncertainty about trade raises the returns to trade because it facilitates better international risk-sharing.

### 3.1. How Export Uncertainty Affects Terms of Trade

The results from Section 2.1, which describe the relationship between trade uncertainty and the mean and variance of the terms of trade, do not depend on any preference specification, i.e., CES preferences. They use the fact that the terms of trade are the ratio of the two countries' exports. The result by which countries whose exports have lower covariance have higher expected terms of trade does use the fact that countries' endowments are lognormally distributed. Is this result specific to lognormal variables?

The key to the relationship between trade uncertainty and the expected terms of trade lies in the distributional assumptions. What is essential is that exports can be arbitrarily close to zero but can never be negative. If neither country can ever exports a negative amount, then the ratio of exports, which is the terms of trade, are bounded below by zero. But if there exist states of the world in which either country would choose an export amount arbitrarily close to zero, then the ratio of exports can be arbitrarily large. If the terms of trade are $T_y/T_x$ and $T_x$ can be arbitrarily close to zero, then $p = T_y/T_x$ will occasionally be huge. The point is that the economics of exporting skew the distribution of the terms of trade. There is no way this distribution can be symmetric if it is bounded below and unbounded above. Exactly how skewed and what form the skewness takes depend on the distributions and preferences. But the histogram of the terms of trade, for either country, will always have a bigger right tail.

Once we understand that the terms of trade are a skewed distribution, we can see why signals about exports reduce the mean. Think of a skewed distribution as a function of a normal distribution. Left-skewed distributions would be a concave function of a normal; the concavity accentuates the left tail (bad events). In this case, we have a right-skewed distribution. This can be constructed as a convex function of a normal. Lemma 1 in the Appendix proves that the distribution of the terms of trade must be a somewhere-convex function of a normal probability density. Now, recall that by the definition of convexity, lotteries of convex functions have higher expected values than the median lottery realization. The more uncertain the lottery, the higher the expected value. Firms face terms of trade that are like this convex lottery. The more uncertain the lottery, the higher the expected terms of trade. The higher expected terms of trade are what make exporting more attractive.

For the CES case, Figure 3 illustrates the same effect in a time-series plot. Recall that when information was scarce, the terms of trade were very volatile; this resulted in occasional spikes in the terms of trade that raised the average. The average terms of trade effect originates in the asymmetry of the terms of trade distribution. This asymmetry arises naturally, whenever exports are required to be nonnegative.

What does this convex lottery look like economically? Consider trade policy uncertainty of the following form: When you export goods, you may get very little in return; the least you can get is $\epsilon > 0$. But there is also a possibility that your good gets through, is relatively scarce, and earns an enormous rate of return. A firm that exports more in the face of trade uncertainty is gambling on the possibility that it is one of the few units that get into the foreign country. If it is, it earns enormous rents on its scarce good. This is a risky lottery, and firms dislike risk. But they also understand that the odds are stacked in their favor: The more uncertain the trade policy, the greater the possibility of winning an enormous rate of return.

### 3.2. How Terms of Trade Moments Affect Exports: Sufficient Statistics for Preferences

With more general preferences, the uncertainty and trade relationship can work either way: Firms may export more or less when the expected terms of trade rise; they may export more or less when the variance increases, or mean and variance effects can trade off differently. We classify preferences, according to a few sufficient statistics, that allow us to say how firms with these preferences will react to trade uncertainty and why.

In a multi-good setting, risk aversion and its related higher-order risk preferences must reflect the interaction of preferences for the two goods. We encode risk attitudes with the following coefficients, which turn out to be the sufficient statistics for determining whether our export paradox holds:

- $\tilde{\rho}_y^{(1)} = \rho_y^{(1)} \left(1 - \frac{U_{xy}}{p\,U_{yy}}\right)$ where $\rho_y^{(1)} \equiv -\frac{C_y U_{yy}}{U_y}$ relative risk aversion;
- $\tilde{\rho}_y^{(2)} = \rho_y^{(2)} \left(1 - \frac{U_{xyy}}{p\,U_{yyy}}\right)$ where $\rho_y^{(2)} \equiv -\frac{C_y U_{yyy}}{U_{yy}}$ relative prudence; and
- $\tilde{\rho}_y^{(3)} = \rho_y^{(3)} \left(1 - \frac{U_{xyyy}}{p\,U_{yyyy}}\right)$ where $\rho_y^{(3)} \equiv -\frac{C_y U_{yyyy}}{U_{yyy}}$ relative temperance.

Coefficients without tildes are the standard single-good expressions for risk aversion, prudence, and temperance. Coefficients with tildes are adjusted by cross-good derivatives and the terms of trade to reflect the fact that there are two goods.

Before we proceed it is useful to interpret each of these statistics, so that we understand what economic forces we are discussing. Start with risk aversion. Note that $\rho_y \equiv -\frac{C_y U_{yy}}{U_y}$ is the standard coefficient of relative risk aversion (RRA) for risky good $y$. Why adjust relative risk aversion for the two goods? If agents do not have any preference for correlated consumption of the two goods ($U_{xy} \leq 0$), then consumption of $y$ is a hedge for the risk of low consumption of $x$. If agents prefer correlated consumption of both goods, then utility is very high when both goods are abundant and very low when both are scarce. This increases utility risk. Following Kihlstrom and Mirman (1974), when $U_{xy} > 0$ $\tilde{\rho}_y > \rho_y$ the adjusting factor in the RRA amplifies risk aversion compared to a one-good case.

Prudence is a third derivative of preferences and is related to the desire for precautionary savings. It governs whether agents want to export more to insure a modicum of foreign consumption or export less when the return to exporting is riskier. Following Eeckhoudt, Rey, and Schlesinger (2007), we adjust prudence for cross-prudence in $x$. Given a zero-mean $\delta$ random variable, an individual is cross-prudent in $x$ if the lottery $[(x, y + \delta); (x - k, y)]$ is preferred to the lottery $[(x, y); (x - k, y + \delta)]$; that is, higher $x$ consumption dampens the detrimental effects of risk in $y$. Eeckhoudt, Rey, and Schlesinger (2007) show that cross-prudent preference for $x$ implies that $U_{xyy} > 0$.

Temperance is a negative fourth derivative of utility (see Eeckhoudt and Schlesinger (2006)), which can be interpreted as a preference for risk disaggregation. Consider two zero mean random variables $\varepsilon_1, \varepsilon_2$. An individual is said to be temperate if the lottery $[\varepsilon_1; \varepsilon_2]$ is preferred to the lottery $[0; \varepsilon_1 + \varepsilon_2]$, where all outcomes of the lotteries have equal probability. A temperate individual prefers that risks to be spread across states. With multiple goods, relative temperance also implies that some risk in each good is preferred to concentrating all risk on one good.

Now, we use these sufficient statistics to characterize the relationship between the terms of trade moments and export volume. The first general proposition comes from the firm's first-order condition. It says that the marginal rate of substitution of a unit of home good for a unit of foreign good should be equal to the risk-adjusted rate of exchange of the two goods. If this were not true, a firm could improve its utility by exporting less or more.

**Proposition 2** *Optimal exports can be approximated as a function of the conditional mean and variance of the terms of trade distribution, $T(f_x, \mathbb{E}_x[p], \mathbb{V}ar_x[p])$, and are determined by equating the marginal rate of substitution, evaluated at the expected terms of trade, with the risk-adjusted expected terms of trade.*

$$\frac{U_x(f_x - T, \mathbb{E}_x[p]T)}{U_y(f_x - T, \mathbb{E}_x[p]T)} = \underbrace{\mathbb{E}_x[p]}_{\textit{expectation}} \left[ 1 - \underbrace{\rho_y^{(1)}(\mathbb{E}_x[p])}_{\textit{relative risk aversion}} \underbrace{\left(2 - \tilde{\rho}_y^{(2)}(\mathbb{E}_x[p])\right)}_{\textit{adjusted relative prudence}} \underbrace{\frac{\mathbb{CV}_x^2(p)}{2}}_{\textit{variance/expectation}^2} \right] \quad (26)$$

What is useful about this way of expressing the first-order condition is that it expresses the risk-adjusted terms of trade in terms of our first two sufficient statistics and the mean and variance of the terms of trade. Once we know the mean and variance of the terms of trade, and we know these two features of preferences, we can describe the firms' optimal export condition.

Higher expected terms of trade make exporting more desirable, unless the term in square brackets is negative. If risk aversion and prudence are sufficiently high, then when a firm believes that it will get more foreign goods back in return for each unit of home exports, it reasons that it can send fewer exports and still have plenty of foreign goods to eat. So it exports less.

More variable terms of trade raise the coefficient of variation, $\mathbb{CV}_x^2(p)$. This deters exporting, unless the adjusted relative prudence term $2 - \tilde{\rho}_y^{(2)}(\mathbb{E}_x[p])$ is negative. When this prudence term is negative, the firm that faces more uncertain terms of trade exports more to ensure that they will have enough foreign good to consume, even if the rate of exchange turns out to be low.

This effect is similar to the increase in precautionary savings observed when earnings are more volatile in consumption/savings problems.

The next result simply differentiates (26) with respect to the mean and variance of the terms of trade. The resulting expression clarifies how changes in the terms of trade distribution change the volume of exports.

**Proposition 3** *Suppose the terms of trade mean and variance change in* $\frac{d\mathbb{E}_x[p]}{\mathbb{E}_x[p]}$ *and* $\frac{d\mathbb{V}ar_x[p]}{\mathbb{V}ar_x[p]}$*, respectively. Then the sign of the change in exports is equal to the sign of the following expression:*

$$\underbrace{\left(1-\tilde{\rho}_y^{(1)}\right)}_{\textit{risk aversion}}\frac{d\mathbb{E}_x[p]}{\mathbb{E}_x[p]}+\frac{\mathbb{CV}_x^2[p]}{2}\left(\rho_y^{(1)}\rho_y^{(2)}\underbrace{\left(3-\tilde{\rho}_y^{(3)}\right)}_{\textit{temperance}}\frac{d\mathbb{E}_x[p]}{\mathbb{E}_x[p]}-\rho_y^{(1)}\underbrace{\left(2-\tilde{\rho}_y^{(2)}\right)}_{\textit{prudence}}\frac{d\mathbb{V}ar_x[p]}{\mathbb{V}ar_x[p]}\right) \tag{27}$$

What we learn from this is that it clarifies many reasons why exports might rise or fall in response to clearer mutual information. Information can change risk, it can change whether risk is highest when consumption is low or high, and it can change whether risk in one consumption good is high when the other is high or low. How a country responds to each of these changes depends on their preferences. Specifically, it depends on their risk aversion, temperance, and prudence, as described above.

**How can information frictions boost trade?** The main question we aim to answer is how cross-border information frictions affect trade volume. We've described some competing effects: Endowment uncertainty raises the mean terms of trade, but also raises variance. This leaves the question of why these competing effects facilitate trade on average.

The mean effect of the terms of trade dominates the variance effect because of preferences. The combination of risk aversion, prudence, and temperance is not strong enough to overcome the higher mean returns to exporting. Under some preferences, higher risk and higher return correspond to less trade. But under commonly used preferences—like CES, with elasticities of substitution consistent with other trade facts—the net effect is more trade. This is not an oddity of CES preferences; a broad class of preferences produces the same effect. So, might President Trump's, or anyone else's, trade threats promote trade? Yes—if these threats create mutual uncertainty about the quantity of foreigners' exports and if preferences are not too risk-averse, too prudent, or too temperate, this surge in trade is a logical equilibrium outcome.

**When is uncertainty a barrier to trade?** So far, we have focused on cases in which uncertainty increases trade because these are the most surprising. In many cases, however, a researcher might want to build a model in which uncertainty is a barrier to trade. What preferences make that possible?

We can use the previous proposition to precisely define this set of preferences. Suppose information increases both the mean and the variance in equal proportions. Then applying Proposition 3 tells us that average exports will fall if

$$\underbrace{\left[\frac{1-\tilde{\rho}_y{}^{(1)}}{\rho_y^{(1)}}\right]}_{\text{risk aversion}} + \frac{\mathbb{CV}[p]^2}{2}\rho_y^{(2)}\left\{\underbrace{\left[\frac{\tilde{\rho}_y{}^{(2)}-2}{\rho_y^{(2)}}\right]}_{\text{prudence}} + \rho_y^{(3)}\underbrace{\left[\frac{3-\tilde{\rho}_y{}^{(3)}}{\rho_y^{(3)}}\right]}_{\text{temperance}}\right\} < 0. \tag{28}$$

The inequality requires that preferences exhibit high risk aversion ($\tilde{\rho}_y{}^{(1)} > 1$), low prudence ($\tilde{\rho}_y{}^{(2)} < 2$), high temperance ($\tilde{\rho}_y{}^{(3)} > 3$), or a combination of these. This condition describes a test that can be applied to any preferences to determine whether the mean and variance effect combine to deliver a decrease in trade from a rise in uncertainty.

Conceptually, the test is this: Preferences must have the feature that uncertain terms of trade deter, rather than promote, trade, and that this force is strong enough to overcome the increase in the expected terms of trade. High risk aversion helps; it tempers the reaction to changes in expected terms of trade and amplifies the effect of risk. For positive adjusted risk aversion ($\rho_y^{(1)} > 0$), low prudence implies a lower precautionary motive. Agents do not want to export more in the face of risk to ensure they have some foreign goods to consume. Instead, they export less to expose themselves less to the unknown rate of return. If the urge to step away from risk is strong, then resolving uncertainty about trade will reduce the terms of trade risk and promote more trade. Low prudence (low $\tilde{\rho}_y^{(2)}$) also helps to render trade volumes more sensitive to changes in terms of trade variance. Finally, high temperance helps because temperate agents who face more risk in their consumption of one good want to shift some of that risk to another good. In this case, exporting less is a way of shifting the composition of consumption risk.

**Relating the general case and the CES case** The CES results we presented earlier for change in trade volume were a special case of this more general result. Differentiating our CES preferences (23) reveals that the risk aversion term is $1-\tilde{\rho}_y^{(1)} = \theta$; the prudence term is $2-\tilde{\rho}_y^{(2)} = \theta$; and the temperance term is $3-\tilde{\rho}_y^{(3)} = \theta$ (because cross-derivatives are equal to zero, and thus the adjusted preference and standard preference parameters are equal). Substituting these $\theta$ terms into (28), we find the following.

**Corollary** *For CES-like preferences with $\theta < 1$, an equal percent increase in the average and the volatility of terms of trade inhibits exports if*

$$\theta\left[1 + \frac{\mathbb{CV}[p]^2}{2}(1-\theta)^2\right] < 0.$$

Since the second term is squared and thus always positive, the only way this expression can be negative is if $\theta < 0$ (goods are complementary). The general preference results now shed light on why the numerical CES results reverse at $\theta = 0$. This is the threshold at which risk aversion, prudence, and temperance combine to make the terms of trade mean effect smaller than the risk affect.

**Sufficient statistics in broader classes of models.** These results give us conceptual guidance about how to assess the effect of information in models outside this class. They point to two sets of statistics that could be computed for any model. The first pair of statistics maps uncertainty into a mean and variance of the terms of trade. In our model, where the terms of trade is the ratio of exports, uncertainty raises both. It other models with frictions or different market clearing mechanisms, these are the two statistics we need to know from the equilibrium side of the model. The second pair of statistics we need governs how a firm's export decision reacts to raising the expected return and the variance of the return to exporting. We derive such conditions in a frictionless model. In a richer model, similar conditions, depending on the derivatives of the frictions-adjusted marginal utility of exports, would emerge.

### 3.3. Completing the Contracting Space

So far, we have assumed away all instruments that agents might use to share international risk. Exchange rate futures, international equity holdings, profit-sharing contracts, and secondary markets could all help to share international risk. If we included a complete set of risk-sharing instruments, then agents could hedge the outcomes of all unknown variables and the effects of information asymmetry would disappear. In fact, just allowing firms to write price-contingent export contracts (where the export fractions in (20) are $\Psi_C(p)$ and $\Psi^*_C(p)$ for the domestic and foreign country, respectively, and the subscript indicates that these are contingent contracts) would yield outcomes identical to the full-information setting. At every price $p$, firms would decide how much they would optimally send at that price. Thus, at realized price $p$, every firm sends a quantity that is equal to what they would send if they had full information and knew that price in advance. If we want to have some meaningful role for trade uncertainty, we need to step away from complete contingent export contracts.

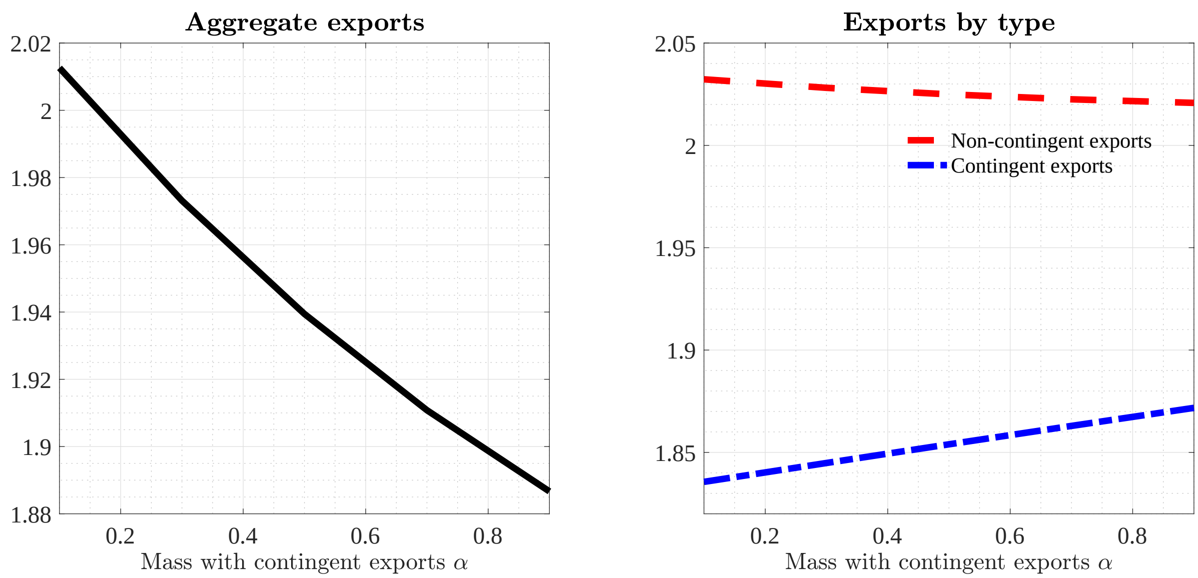
To explore an economy with some contingent claims and some meaningful uncertainty, we consider an environment in which a fraction $\alpha$ of firms submit price-contingent export plans

$(\Psi_C(p), \Psi_C^*(p))$ and a fraction $(1-\alpha)$ of firms choose noncontingent exports $(\Psi_N(\mu_x), \Psi_N^*(\mu_y))$ that depend only on their home productivity. This captures the idea that in reality, firms use a variety of contracting arrangements for international transactions that allocate terms of trade risk to different parties.

We eliminate any signals about the other country's productivity (the complete uncertainty environment) and study what happens as we change the fraction $\alpha$ of price-contingent exporters. The equilibrium relative price is still the ratio of foreign exports to home exports:

$$p = e^{\mu_y - \mu_x + \frac{1}{2}(\sigma_y^2 - \sigma_x^2)} \times \left( \frac{\alpha \Psi_C^*(p) + (1-\alpha)\Psi_N^*(\mu_y)}{\alpha \Psi_C(p) + (1-\alpha)\Psi_N(\mu_x)} \right)$$

However, the total export of each country is now $\alpha$ times the export amount of price-contingent firms plus $1-\alpha$ times the export amount chosen by noncontingent firms.




Notes: Equilibrium exports in the domestic country for different fractions of agents with price-contingent contracts $\alpha \in [0,1]$. Figure assumes a low elasticity of substitution $\theta = 0.3$ and perfect information (signal noise $\tilde{s}^2 = 0$). Other parameters are $m_x = m_y = 0$, $s_x = s_y = 1$, and $\sigma_x = \sigma_y = \sqrt{2}$. The left panel plots aggregate export volume and the right panel plots exports by type of contract.

**Figure 8: Completing the market reduces exports**

Having a more complete contracting space and having more information are similar: Both cause the average export volume to fall. When we solve the model with contracts numerically (for a low elasticity of substitution of $\theta = 0.3$), we see that as the number of price-contingent exporters rises, non-price-contingent firms export less and price-contingent firms export more. However, the responses are not very large quantitatively. Still, as the number of price-contingent exporters rises, each noncontingent firm is trading against an average foreign firm that is more likely to have chosen a price-contingent quantity. This is like trading against a foreign country that is better informed. The price-contingent export share rises, but is relatively flat on a per-firm basis. The noncontingent export share falls in aggregate and each firm exports less. The net effect is a decline in exports.

This model extension demonstrates that introducing complete contingent contracts undermines the effect of asymmetric information. At the same time, however, completing the market and reducing information asymmetry work almost identically to reduce trade, for the same reasons. Conditioning exports on the outcome of a random variable and knowing that random variable before exports are chosen are functionally equivalent.

## 4. Conclusions

Information frictions are often invoked as reasons for low levels of international trade. But in an equilibrium model, the link between information friction and trade volume is not simple. Our model shows how information also changes the expected terms of trade. It also highlights that in the face of risk, some types of agents may prefer to export more to ensure that they have a sufficient amount of the foreign good to consume. This depends on agents' preferences.

With constant elasticity of substitution (CES) preferences, information frictions impede trade when goods are very substitutable. The decline in trade occurs because the increase in risk from lower-precision information deters trade, and that risk effect is stronger than the effect on the mean terms of trade, which encourages exporting. But with empirically plausible elasticity parameters, the opposite is true: Information frictions encourage trade. CES preference is not a special or anomalous case. We derive a a broad class of preferences for which similar effects arise.

Our results demonstrate that, if we believe that information frictions are truly an important barriers to international trade, we need to amend standard trade models to be consistent with this belief. The could mean changing the elasticities or types of preferences used, adding new frictions that interact with information, or finding some way to change the relationship between uncertainty and the expected terms of trade.

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# A. Proofs and Solution Details

## 1.1. Preliminaries for CES model

The maximization problem for country $x$ is given by

$$V(z_x, \mu_x, \hat{m}_y) = \max_{t_x} \mathbb{E}\left[\frac{1}{1-\sigma}\left(c_x^\theta + c_y^\theta\right)^{\frac{1-\sigma}{\theta}}\right] = \max_{t_x} \mathbb{E}\left[\frac{1}{1-\sigma}\left((z_x - t_x)^\theta + \left(\frac{t_x}{(1+\tau)q}\right)^\theta\right)^{\frac{1-\sigma}{\theta}}\right].$$

The FOC of the maximization problem is given by

$$(z_x - t_x)^{\theta-1}\mathbb{E}\left[\left((z_x - t_x)^\theta + \left(\frac{t_x}{(1+\tau)q}\right)^\theta\right)^{\frac{1-\sigma-\theta}{\theta}}\right] = t_x^{\theta-1}\mathbb{E}\left[\frac{1}{(1+\tau)^\theta q^\theta}\left((z_x - t_x)^\theta + \left(\frac{t_x}{(1+\tau)q}\right)^\theta\right)^{\frac{1-\sigma-\theta}{\theta}}\right].$$

Rearranging yields

$$t_x = z_x\left(\frac{Q(z_x, \mu_x, \hat{m}_y)}{\bar{\lambda}(z_x, \mu_x, \hat{m}_y)}\right)^{\frac{\theta}{1-\theta}},$$

where we define the following objects:

$$\begin{aligned}
Q(z_x, \mu_x, \hat{m}_y) &\equiv \left(\mathbb{E}\left[\lambda(z_x, \mu_x, \hat{m}_y)^\theta\right]^{\frac{1}{\theta-1}} + \mathbb{E}\left[\left(\frac{\lambda(z_x, \mu_x, \hat{m}_y)}{(1+\tau)q}\right)^\theta\right]^{\frac{1}{\theta-1}}\right)^{\frac{\theta-1}{\theta}}; \\
\bar{\lambda}(z_x, \mu_x, \hat{m}_y) &\equiv \mathbb{E}\left[\lambda(z_x, \mu_x, \hat{m}_y)^\theta\right]^{\frac{1}{\theta}}; \\
\lambda(z_x, \mu_x, \hat{m}_y)^\theta &\equiv c(z_x, \mu_x, \hat{m}_y)^{1-\sigma-\theta} = \left((z_x - t_x)^\theta + \left(\frac{t_x}{(1+\tau)q}\right)^\theta\right)^{\frac{1-\sigma-\theta}{\theta}}.
\end{aligned}$$

We guess a multiplicative solution $t(z_x, \mu_x, \hat{m}_y) = z_x\Psi(\mu_x, \hat{m}_y)$. Substituting, we get:

$$c(z_x, \mu_x, \hat{m}_y) = \frac{1}{1-\sigma}\left((z_x - t_x)^\theta + \left(\frac{t_x}{(1+\tau)q}\right)^\theta\right)^{\frac{1-\sigma}{\theta}} = z_x^{1-\sigma}\Psi_2(\mu_x, \hat{m}_y; q^{-1})$$

where

$$\Psi_2(\mu_x, \hat{m}_y; q^{-1}) \equiv \frac{1}{1-\sigma}\left((1 - \Psi(\mu_x, \hat{m}_y))^\theta + \left(\frac{\Psi(\mu_x, \hat{m}_y)}{(1+\tau)q}\right)^\theta\right)^{\frac{1-\sigma}{\theta}}.$$

Now we have that

$$\begin{aligned}
\bar{\lambda}(z_x, \mu_x, \hat{m}_y) &= \mathbb{E}\left[\lambda(z_x, \mu_x, \hat{m}_y)^\theta\right]^{\frac{1}{\theta}} = z_x^{\frac{1-\sigma}{\theta}(1-\sigma-\theta)}\mathbb{E}\left[\Psi_2(\mu_x, \hat{m}_y; q^{-1})^{(1-\sigma)(1-\sigma-\theta)}\right]^{\frac{1}{\theta}}, \\
Q(z_x, \mu_x, \mu_y) &= z_x^{\frac{1-\sigma}{\theta}(1-\sigma-\theta)}\left(\mathbb{E}\left[\Psi_2(\mu_x, \hat{m}_y; q^{-1})^{(1-\sigma)(1-\sigma-\theta)}\right]^{\frac{1}{\theta-1}} + \mathbb{E}\left[\left(\frac{1}{(1+\tau)q}\right)^\theta\Psi_2(\mu_x, \hat{m}_y; q^{-1})^{(1-\sigma)(1-\sigma-\theta)}\right]^{\frac{1}{\theta-1}}\right)^{\frac{\theta-1}{\theta}}.
\end{aligned}$$

Therefore we get the export policy for a domestic firm:

$$t_x = z_x \left( \frac{Q(z_x, \mu_x, \hat{m}_y)}{\bar{\lambda}(z_x, \mu_x, \hat{m}_y)} \right)^{\frac{\theta}{1-\theta}} = z_x \Psi(\mu_x, \hat{m}_y),$$

where we define the aggregate export share of income as

$$\Psi(\mu_x, \hat{m}_y) = \left( \frac{\left( \mathbb{E}\left[ \Psi_2(\mu_x, \hat{m}_y; q^{-1})^{(1-\sigma)(1-\sigma-\theta)} \right]^{\frac{1}{\theta-1}} + \mathbb{E}\left[ \left( \frac{1}{(1+\tau)q} \right)^{\theta} \Psi_2(\mu_x, \hat{m}_y; q^{-1})^{(1-\sigma)(1-\sigma-\theta)} \right]^{\frac{1}{\theta-1}} \right)^{\frac{\theta-1}{\theta}}}{\mathbb{E}\left[ \Psi_2(\mu_x, \hat{m}_y; q^{-1})^{(1-\sigma)(1-\sigma-\theta)} \right]^{\frac{1}{\theta}}} \right)^{\frac{\theta}{1-\theta}}.$$

Notice that for country $y$, we get the export policy for a foreign firm as

$$\begin{aligned}
t_y &= z_y \Gamma(\mu_y, \hat{m}_x), \\
\Gamma(\mu_y, \hat{m}_x) &= \left( \frac{\left( \mathbb{E}\left[ \Gamma_2(\mu_y, \hat{m}_x; q)^{(1-\sigma)(1-\sigma-\theta)} \right]^{\frac{1}{\theta-1}} + \mathbb{E}\left[ \left( \frac{q}{(1+\tau)} \right)^{\theta} \Gamma_2(\mu_y, \hat{m}_x; q)^{(1-\sigma)(1-\sigma-\theta)} \right]^{\frac{1}{\theta-1}} \right)^{\frac{\theta-1}{\theta}}}{\mathbb{E}\left[ \Gamma_2(\mu_y, \hat{m}_x; q)^{(1-\sigma)(1-\sigma-\theta)} \right]^{\frac{1}{\theta}}} \right)^{\frac{\theta}{1-\theta}}, \\
\Gamma_2(\mu_y, \hat{m}_x; q) &\equiv \frac{1}{1-\sigma} \left( (1 - \Gamma(\mu_y, \hat{m}_x))^{\theta} + \left( \frac{\Gamma(\mu_y, \hat{m}_x) q}{(1+\tau)} \right) \theta \right)^{\frac{1-\sigma}{\theta}}.
\end{aligned}$$

Now we have all we need to find the aggregate variables. Aggregate exports are given by

$$\begin{aligned}
T_x(\mu_x, \hat{m}_y) &= \int z_x \Psi(\mu_x, \hat{m}_y) dF(z_x|\mu_x) = e^{\mu_x + \frac{1}{2}\sigma_x^2} \Psi(\mu_x, \hat{m}_y), \\
T_y(\mu_y, \hat{m}_x) &= \int z_y \Gamma(\mu_y, \hat{m}_x) dF(z_y|\mu_y) = e^{\mu_y + \frac{1}{2}\sigma_y^2} \Gamma(\mu_y, \hat{m}_x).
\end{aligned}$$

Therefore, the terms of trade are given by

$$q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) = \frac{e^{\mu_x + \frac{1}{2}\sigma_x^2} \Psi(\mu_x, \hat{m}_y)}{e^{\mu_y + \frac{1}{2}\sigma_y^2} \Gamma(\mu_y, \hat{m}_x)} = f \frac{\Psi(\mu_x, \hat{m}_y)}{\Gamma(\mu_y, \hat{m}_x)} \quad ,$$

where aggregate fundamentals are

$$f \equiv e^{\mu_x - \mu_y} e^{\frac{1}{2}(\sigma_x^2 - \sigma_y^2)}.$$

The equilibrium conditions of the model under **imperfect information** are given by

$$\Psi(\mu_x, \hat{m}_y) = \frac{1}{1 + (1+\tau)^{\frac{\theta}{1-\theta}} \left( \frac{\mathbb{E}\left[ \Psi_2(\mu_x, \hat{m}_y; q^{-1})^{(1-\sigma)(1-\sigma-\theta)} \right]}{\mathbb{E}\left[ \Psi_2(\mu_x, \hat{m}_y; q^{-1})^{(1-\sigma)(1-\sigma-\theta)} q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)^{-\theta} \right]} \right)^{\frac{1}{1-\theta}}};$$

$$\Gamma(\mu_y, \hat{m}_x) = \frac{1}{1 + (1+\tau)^{\frac{\theta}{1-\theta}} \left( \frac{\mathbb{E}\left[ \Gamma_2(\mu_y, \hat{m}_x; q)^{(1-\sigma)(1-\sigma-\theta)} \right]}{\mathbb{E}\left[ \Gamma_2(\mu_y, \hat{m}_x; q)^{(1-\sigma)(1-\sigma-\theta)} q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)^{\theta} \right]} \right)^{\frac{1}{1-\theta}}};$$

$$q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) = f \frac{\Psi(\mu_x, \hat{m}_y)}{\Gamma(\mu_y, \hat{m}_x)}.$$

where

$$\Psi_2(\mu_x, \hat{m}_y; q^{-1}) \equiv \frac{1}{1-\sigma} \left( (1 - \Psi(\mu_x, \hat{m}_y))^\theta + \left( \frac{\Psi(\mu_x, \hat{m}_y)}{(1+\tau) q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)} \right)^\theta \right)^{\frac{1-\sigma}{\theta}}$$

$$\Gamma_2(\mu_y, \hat{m}_x; q) \equiv \frac{1}{1-\sigma} \left( (1 - \Gamma(\mu_y, \hat{m}_x))^\theta + \left( \frac{\Gamma(\mu_y, \hat{m}_x) q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y)}{(1+\tau)} \right)^\theta \right)^{\frac{1-\sigma}{\theta}}.$$

Whereas, the equilibrium conditions of the model under **perfect information** are given by

$$\Psi^{PI}(\mu_x, \mu_y) = \frac{1}{1 + [(1+\tau) q^{PI}(\mu_x, \mu_y)]^{\frac{\theta}{1-\theta}}};$$

$$\Gamma^{PI}(\mu_x, \mu_y) = \frac{1}{1 + \left[ \frac{(1+\tau)}{q^{PI}(\mu_x, \mu_y)} \right]^{\frac{\theta}{1-\theta}}};$$

$$q^{PI}(\mu_x, \mu_y) = f \frac{\Psi^{PI}(\mu_x, \mu_y)}{\Gamma^{PI}(\mu_x, \mu_y)}.$$

## 1.2. Proofs

**Proof of Result 1: Uncertainty Reduces the Covariance of Aggregate Exports** **Proof.** *Part 1: From derivative to covariance.* The first step is to connect the derivative $\frac{dT_x}{dT_y}$ with the covariance $\mathbb{C}ov[T_x, T_y | \mathcal{I}_x]$. Note that $T_y$ and $\mu_x$ are the only random variables for the agent in country $x$. A first-order approximation of the policy function $T_x(T_y, \mu_x)$ yields

$$T_x(\mu_x, T_y) \approx T_x(\mu_x, \mathbb{E}[T_y | \mathcal{I}_x]) + \beta \left( T_y - \mathbb{E}[T_y | \mathcal{I}_x] \right) + \gamma \left( \mu_x - m_x \right) = \alpha + \beta T_y + \gamma \mu_x$$

where $\alpha$ gathers all constants. From an ex ante perspective, $T_x$ is a random variable. With this approximation, the covariance with $T_y$ is given by

$$\mathbb{C}ov[T_x, T_y] \approx \mathbb{C}ov(\alpha + \beta T_y + \gamma \mu_x, T_y) = \beta \mathbb{V}ar(T_y)$$

i.e., the own aggregate shock does not induce covariance with other countries' exports. Therefore, the slope is

$$\beta = \frac{\mathbb{C}ov[T_x, T_y]}{\mathbb{V}ar(T_y)} = \frac{dT_x}{dT_y}\Big|_{T_y = \mathbb{E}[T_y]}.$$

With no information $\beta = 0$. With perfect information, $\frac{dT_x}{dT_y} \neq 0 \; \forall T_y$, and therefore, $\beta > 0$. We have established that

$$sign\left(\frac{dT_x}{dT_y}\right) = sign\left(\mathbb{C}ov(T_x, T_y)\right) = sign(\beta).$$

*Part 2: Continuity of the covariance in the amount of information.* If the conditional distribution of terms of trade $p$ is a continuous function of the signal and its precision, then the continuity of Bayesian updating, together with the continuity of the integral operator, ensures that any conditional expectation is also continuous. Since the covariance is an expectation, it is also a continuous function of the signal precision. By (i) for no information (zero precision) the covariance is zero, and for perfect information (infinity precision) the covariance is positive. By the continuity established in (ii), there exists an interval for precision between 0 and infinity for which the covariance is increasing in precision. Therefore, more information increases the covariance of aggregate exports. ■

### Proof of Result 2: Uncertainty Increases Mean and Variance of Terms of Trade **Proof.**

A second-order approximation of $p = \frac{T_y}{T_x}$ around the unconditional expectation of exports $(\mathbb{E}[T_x], \mathbb{E}[T_y])$ yields

$$\frac{T_y}{T_x} \approx \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]} + \frac{1}{\mathbb{E}[T_x]}(T_y - \mathbb{E}[T_y]) - \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]^2}(T_x - \mathbb{E}[T_x]) + \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]^3}(T_x - \mathbb{E}[T_x])^2 - \frac{1}{\mathbb{E}[T_x]^2}(T_x - \mathbb{E}[T_x])(T_y - \mathbb{E}[T_y]).$$

Taking expectations on both sides, which makes the first-order terms equal to zero, yields

$$\mathbb{E}\left[\frac{T_y}{T_x}\right] \approx \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]} + \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]^3}\mathbb{V}ar[T_x] - \frac{1}{\mathbb{E}[T_x]^2}\mathbb{C}ov[T_x, T_y].$$

By symmetry, $\mathbb{E}[T_y] = \mathbb{E}[T_x]$ and $\mathbb{V}ar[T_y] = \mathbb{V}ar[T_x]$, we can simplify to

$$\mathbb{E}\left[\frac{T_y}{T_x}\right] = 1 + \frac{\mathbb{V}ar[T_x]}{\mathbb{E}[T_x]^2} - \frac{\mathbb{C}ov[T_x, T_y]}{\mathbb{E}[T_x]^2}.$$

Furthermore, using the definition of coefficient of variation $\mathbb{CV}^2[z] = \frac{\mathbb{V}ar[z]}{\mathbb{E}[z]^2}$ and the correlation coefficient, together with symmetry across countries, we obtain:

$$\mathbb{E}[p] = 1 + \mathbb{CV}^2[T_x]\left(1 - corr([T_x, T_y])\right).$$

The proof is analogous from the foreign country's perspective, using an approximation of $1/p$.

Now for the variance, a first-order approximation of $p = \frac{T_y}{T_x}$ around the expectation of aggregate exports $(\mathbb{E}[T_x], \mathbb{E}[T_y])$ yields

$$\frac{T_y}{T_x} \approx \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]} + \frac{1}{\mathbb{E}[T_x]}(T_y - \mathbb{E}[T_y]) - \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]^2}(T_x - \mathbb{E}[T_x]).$$

Now take expectation on both sides and cancel the first-order terms:

$$\mathbb{E}\left[\frac{T_y}{T_x}\right] \approx \frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]}.$$

Subtract the two previous expressions to compute the variance:

$$\begin{aligned}\mathbb{V}ar\left[\frac{T_y}{T_x}\right] &= \mathbb{E}\left[\left(\frac{T_y}{T_x}-\mathbb{E}\left[\frac{T_y}{T_x}\right]\right)^2\right] = \mathbb{E}\left[\left(\frac{1}{\mathbb{E}[T_x]}(T_y-\mathbb{E}[T_y])-\frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]^2}(T_x-\mathbb{E}[T_x])\right)^2\right]\\ &= \frac{1}{\mathbb{E}[T_x]^2}\left[\mathbb{V}ar[T_y]+\frac{\mathbb{E}[T_y]^2}{\mathbb{E}[T_x]^2}\mathbb{V}ar[T_x]-2\frac{\mathbb{E}[T_y]}{\mathbb{E}[T_x]}\mathbb{C}ov[T_x,T_y]\right].\end{aligned}$$

Keeping the variance of exports constant, the larger covariance decreases the variance of the terms of trade. By symmetry, $\mathbb{E}[T_y]=\mathbb{E}[T_x]$ and $\mathbb{V}ar[T_y]=\mathbb{V}ar[T_x]$, we can simplify to

$$\mathbb{V}ar\left[\frac{T_y}{T_x}\right]=\frac{2}{\mathbb{E}[T_x]^2}\left(\mathbb{V}ar[T_x]-\mathbb{C}ov[T_x,T_y]\right).$$

Again, using the definition of coefficient of variation and the correlation coefficient and symmetry:

$$\frac{\mathbb{V}ar\,[p]}{2}=CV^2[T_x]\left(1-corr([T_x,T_y])\right).$$

The proof is analogous for the foreign country. ∎

## Proof of Proposition 1

This is a special case of Proposition 3, proven below.

## Proof of Lemma 1

**Lemma 1** *Let $g(\cdot)$ be the probability density function of the terms of trade and $\phi(\cdot)$ be a normal probability density. Suppose that $g(\cdot)=h(\cdot)*\phi(\cdot)$, where $h$ is continuous. Then the function $h$ must be somewhere convex.*

**Proof.** The terms of trade are a ratio of two nonnegative stochastic variables: $T_x/T_y$. Both $T_x$ and $T_y$ are proportional to a log-normal variable. As such, they can take any positive value with strictly positive probability density. Thus, the ratio of the two variables has positive density over the positive real line. Thus, the function $g(p)$ takes on value zero for all $p<0$.

We can thus deduce three properties on the function $h$: (1) If $g(p)$ takes on value zero for all $p<0$ and the normal density is positive-valued over the whole real line, then $h(p)$ must be zero for all $p<0$. (2) For $g$ to be a probability density, it must be that $h(p)$ does not fall below zero. (3) $h$ cannot be a constant function. Since we know it takes value zero, it would then be zero everywhere. If that were true, $g$ would be zero everywhere, which is not a probability density because it does not integrate to one.

From these three properties we can deduce that $h$ must be somewhere convex. Suppose not. If the function $h$ is nowhere convex, then it is globally, weakly concave. Since it is not a constant function, there exists some $x^*$ such that $h'(x^*)\neq 0$. Let $m$ be a linear function with slope $h'(x^*)$ that passes $x^*$. Then for all $x\in\mathbb{R}$, $h(x)\leq m(x)$. Since $g$ is a linear function with nonzero slope, there exists $p^*$ such that $m(p^*)<0$. This means that $h(p^*)<0$, which violates the assumption that $h(p)$ does not fall below zero. This contradiction proves that under the assumptions stipulated, $h$ must be somewhere convex. ∎

**Proof of Proposition 2: Optimal exports as a function of terms of trade moments.** **Proof.** Given the domestic country state—endowment $\mu_x$ and signal about foreign endowment $\tilde{m}_y$—the FOC of the maximization problem yields

$$\mathbb{E}_x[w(p)] = 0 \qquad \text{with} \qquad w(p) = pU_y(f_x - T_x(p), pT_x(p)) - U_x(f_x - T_x(p), pT_x(p))$$

where the expectation operator is conditional on its information set $\mathbb{E}_x[\cdot] = \mathbb{E}[\cdot|\mu_x, \tilde{m}_y]$ (equal to its state), $w(p)$ is the marginal utility of exports, and $f_x = e^{\mu_x}$ is the country's aggregate endowment.

A second-order approximation of $w(p)$ around the terms of trade conditional expectation $\mathbb{E}_x[p]$ gives:

$$\begin{aligned}
0 = \mathbb{E}_x[w(p)] &\approx w(\mathbb{E}_x[p]) + w'(\mathbb{E}_x[p])\mathbb{E}_x[p - \mathbb{E}_x[p]] + \frac{1}{2}w''(\mathbb{E}_x[p])\mathbb{E}_x[p - \mathbb{E}_x[p])^2] \\
0 &= w(\mathbb{E}_x[p]) + \frac{\mathbb{V}ar_x[p]}{2}w''(\mathbb{E}_x[p]) \\
0 &= U_y\left(\mathbb{E}_x[p]\right)\left\{\mathbb{E}_x[p] - \rho_y^{(1)}(\mathbb{E}_x[p])\left(\frac{2 - \tilde{\rho}_y^{(2)}(\mathbb{E}_x[p])}{2}\right)\frac{\mathbb{V}ar_x[p]}{\mathbb{E}_x[p]}\right\} - U_x\left(\mathbb{E}_x[p]\right) \\
0 &= \varphi(T, \mathbb{E}_x[p], \mathbb{V}ar_x[p]).
\end{aligned}$$

The expression $\varphi(\cdot) = 0$ determines optimal exports as a function of conditional moments of the terms of trade. Rearranging the expression in terms of the marginal rate, we obtain the result:

$$\frac{U_x\left(\mathbb{E}_x[p]\right)}{U_y\left(\mathbb{E}_x[p]\right)} = \mathbb{E}_x[p]\left\{1 - \rho_y^{(1)}(\mathbb{E}_x[p])\left(2 - \tilde{\rho}_y^{(2)}(\mathbb{E}_x[p])\right)\frac{\mathbb{CV}[p]^2}{2}\right\}.$$

■

**Proof of Proposition 3 Sign of change in exports for general case** **Proof.** By Implicit Function Theorem applied to $\varphi(T, \mathbb{E}_x[p], \mathbb{V}ar_x[p]) = 0$, we have that

$$\begin{gathered}
\frac{\partial \varphi}{\partial T_x}\left(\frac{\partial T_x}{\partial \mathbb{E}_x[p]}d\mathbb{E}_x[p] + \frac{\partial T_x}{\partial \mathbb{V}ar_x[p]}d\mathbb{V}ar_x[p]\right) + \frac{\partial \varphi}{\partial \mathbb{E}_x[p]}d\mathbb{E}_x[p] + \frac{\partial \varphi}{\partial \mathbb{V}ar_x[p]}d\mathbb{V}ar_x[p] = 0 \\
\frac{\partial T_x}{\partial \mathbb{E}_x[p]}d\mathbb{E}_x[p] + \frac{\partial T_x}{\partial \mathbb{V}ar_x[p]}d\mathbb{V}ar_x[p] = -\left(\frac{\partial \varphi}{\partial \mathbb{E}_x[p]}d\mathbb{E}_x[p] + \frac{\partial \varphi}{\partial \mathbb{V}ar_x[p]}d\mathbb{V}ar_x[p]\right)\Big/\frac{\partial \varphi}{\partial T_x}.
\end{gathered}$$

Since the denominator is negative (utility is concave in exports), the sign of the derivative is given by the numerator.

$$\begin{aligned}
num &= \frac{\partial \varphi}{\partial \mathbb{E}_x[p]}d\mathbb{E}_x[p] + \frac{\partial \varphi}{\partial \mathbb{V}ar_x[p]}d\mathbb{V}ar_x[p] \\
&= \left(w'(\mathbb{E}_x[p]) + \frac{\mathbb{V}ar_x(p)}{2}w'''(\mathbb{E}_x[p])\right)d\mathbb{E}_x[p] + \frac{1}{2}w''(\mathbb{E}_x[p])d\mathbb{V}ar_x[p] \\
&= U_y(\mathbb{E}_x[p])\left[\left(\left(1 - \tilde{\rho}_y^{(1)}\right) + \frac{\rho_y^{(1)}\rho_y^{(2)}}{\mathbb{E}_x[p]^2}\frac{\mathbb{V}ar_x[p]}{2}(3 - \tilde{\rho}_y^{(3)})\right)d\mathbb{E}_x[p] - \frac{\rho_y^{(1)}}{\mathbb{E}_x[p]}(2 - \tilde{\rho}_y^{(2)})\frac{d\mathbb{V}ar_x[p]}{2}\right]
\end{aligned}$$

where the new term $w'''(p)$ is equal to $w'''(p) = U_y\frac{\rho_y^{(1)}\rho_y^{(2)}}{p^2}(\tilde{\rho}_y^{(3)} - 3)$, where $\tilde{\rho}_y^{(3)} = \rho_y^{(3)}\left(1 - \frac{U_{xyyy}/p}{U_{yyyy}}\right)$ and $\rho_y^{(3)} \equiv -\frac{C_y U_{yyyy}}{U_{yyy}}$ is the coefficient of relative temperance. If this expression is positive, then an increase in the conditional mean and variance of the terms of trade increases exports. ■

**Proof of Corollary: Sign of change in exports for CES** Recall that for CES-like cases, all cross-derivatives are equal to zero and thus adjusted risk attitudes (denoted with tildes) are equal to the standard ones, i.e., $\tilde{\rho}_y^{(k)} = \rho_y^{(k)}$. With this preference, we have that $\rho_y^{(k)} = k - \theta$ for all $k$. Substituting this result into the required condition yields

$$\underbrace{\left[\frac{\theta}{1-\theta}\right]}_{\text{risk aversion}} + \frac{\mathbb{CV}[p]^2}{2}(2-\theta)\left\{\underbrace{\left[\frac{-\theta}{(2-\theta)}\right]}_{\text{prudence}} + (3-\theta)\underbrace{\left[\frac{\theta}{(3-\theta)}\right]}_{\text{temperance}}\right\} < 0$$

Finally, we simplify the expression to obtain the condition for the CES-like case:

$$\theta\left[1 + \frac{\mathbb{CV}[p]^2}{2}(1-\theta)^2\right] < 0$$

## B. A Model with Preference Shocks

In this section, we consider a related model in which there are preference shocks instead of aggregate endowment shocks. We want to show that this related model is equivalent to the original model, once we redefine variables.

**Preferences:** Suppose that the utility of all agents depends on preference shocks $\rho_x$ and $\rho_y$:

$$\mathbb{E}\left[(\rho_x c_x^\theta + \rho_y c_y^\theta)^{1/\theta}\right]$$

where $\ln \rho_x \sim N(0, s_x^2)$ and $\ln \rho_y \sim N(0, s_y^2)$.

**Endowments:** Each agent in the domestic country has an idiosyncratic endowment of $\tilde{z}_x$ units of good $x$, where $\ln \tilde{z}_x \sim \mathcal{N}(m_x, \sigma_x^2)$. Each agent in the foreign country has an idiosyncratic endowment $\tilde{z}_y$ units of good $y$, where $\ln \tilde{z}_y \sim \mathcal{N}(m_y, \sigma_y^2)$. The means of these distributions are constants (in contrast to the previous model, in which they were random variables). These constants are common knowledge.

**Information:** All agents in country $x$ know $\rho_x$, but not $\rho_y$. All agents in country $y$ know $\rho_y$ but not $\rho_x$. Each set of agents can receive normally distributed signals about the unknown preference shock. They form posterior beliefs by Bayes' Law.

**Solution:** Under these new preferences, the new optimization problem becomes

$$\tilde{t}_x = \arg\max \mathbb{E}\left[\left(\rho_x(\tilde{z}_x - \tilde{t}_x)^\theta + \rho_y\left(\frac{\tilde{t}_x}{\tilde{q}}\right)^\theta\right)^{1/\theta}\middle| \mathcal{I}_x\right] \tag{29}$$

$$\tilde{t}_y = \arg\max \mathbb{E}\left[\left(\rho_x\left(\tilde{t}_y\tilde{q}\right)^\theta + \rho_y(\tilde{z}_y - \tilde{t}_y)^\theta\right)^{1/\theta}\middle| \mathcal{I}_y\right]. \tag{30}$$

Market clearing has the same form as (16).

We can rewrite the country-$x$ problem as

$$\tilde{t}_x = \arg\max \mathbb{E}\left[\left((\rho_x^{1/\theta}\tilde{z}_x - \rho_x^{1/\theta}\tilde{t}_x)^\theta + \left(\rho_y^{1/\theta}\tilde{t}_x\right)^\theta \tilde{q}^{-\theta}\right)^{1/\theta}\middle| \mathcal{I}_x\right]. \tag{31}$$

Next, the market-clearing condition becomes

$$\tilde{q} = \frac{\tilde{T}_x(m_x, \hat{m}_y)}{\tilde{T}_y(m_y, \hat{m}_x)} = \frac{\int \tilde{t}_x(\tilde{z}_x, m_x, \hat{m}_y)dF(\tilde{z}_x|m_x)}{\int \tilde{t}_y(\tilde{z}_y, m_y, \hat{m}_x)dF(\tilde{z}_y|m_y)}. \tag{32}$$

Since the agent in the $x$ country knows $x$, we can redefine the choice variable to be $t_x \equiv \rho_x^{1/\theta}\tilde{t}_x$ and do a simple change of variable in the optimization problem and the constraint:

$$t_x = \arg\max \mathbb{E}\left[\left((\rho_x^{1/\theta}\tilde{z}_x - t_x)^\theta + t_x^\theta\left(\frac{\rho_x^{1/\theta}}{\rho_y^{1/\theta}}\tilde{q}\right)^{-\theta}\right)^{1/\theta}\middle| \mathcal{I}_x\right] \tag{33}$$

$$\tilde{q} = \frac{\rho_y^{1/\theta}}{\rho_x^{1/\theta}}\frac{\int t_x(\tilde{z}_x, m_x, \hat{m}_y)dF(\tilde{z}_x|m_x)}{\int t_y(\tilde{z}_y, m_y, \hat{m}_x)dF(\tilde{z}_y|m_y)}.$$

Comparing the expression above to (16), we see that

$$\tilde{q} = \frac{\rho_y^{1/\theta}}{\rho_x^{1/\theta}} q.$$

Substituting in for $\tilde{q}$ in (35), we get

$$t_x = \arg\max \mathbb{E}\left[ \left( (\rho_x^{1/\theta} \tilde{z}_x - t_x)^\theta + t_x^\theta q^{-\theta} \right)^{1/\theta} \middle| \mathcal{I}_x \right]. \tag{34}$$

The last step is to define a random variable $z_x \equiv \rho_x^{1/\theta} \tilde{z}_x$ and substitute it into our problem:

$$t_x = \arg\max \mathbb{E}\left[ \left( (z_x - t_x)^\theta + t_x^\theta q^{-\theta} \right)^{1/\theta} \middle| \mathcal{I}_x \right]. \tag{35}$$

Notice that this optimization problem is identical to the benchmark problem. The expectation measures are also identical, because the distribution of $\rho_x^{1/\theta} \tilde{z}_x$ is the same as the distribution of $z_x$. For $\ln(\rho_x^{1/\theta} \tilde{z}_x) \sim N(m_x, s_x^2 + \sigma_x^2)$. Similarly, $\ln z_x \sim N(m_x, s_x^2 + \sigma_x^2)$. Thus, the two problems are equivalent; one is just a change of variable of the other.

**Interpretation** The one difference between these two models is the interpretation of what trade volume consists of. In the second problem, $T_x$ is $\rho_x^{1/\theta}$ times trade volume. However, the trade share is identical in both models. To see this, note that in the preference shock model, the country-$x$ trade share is

$$\frac{\tilde{t}_x}{\tilde{c}_x} = \frac{\tilde{t}_x}{\tilde{z}_x - \tilde{t}_x}.$$

Substituting in $t_x = \rho_x^{1/\theta} \tilde{t}_x$ and $z_x = \rho_x^{1/\theta} \tilde{z}_x$, we get

$$\frac{\tilde{t}_x}{\tilde{c}_x} = \frac{\rho_x^{-1/\theta} t_x}{\rho_x^{-1/\theta} z_x - \rho_x^{-1/\theta} t_x} = \frac{t_x}{z_x - t_x} = \frac{t_x}{c_x}.$$

Thus, the trade share in the preference shock model is equal to the trade share in the original model. Following the same steps reveals that the country-$y$ trade share is also equal in both models. Since the aggregate supply shock model and the preference shock model are equivalent problems, it follows that providing both countries more information about preference shocks must reduce the trade share, for the same reason that more information about aggregate supply does.

# C. Details of Computational Algorithm

This section describes the algorithm to compute the equilibrium, the simulation strategy, and parameter choices.

## 3.1. Polynomial approximation to policy functions

**Functional Basis** Let $\{\Phi_k\}_{k=1}^{M}$ be a basis of polynomials with support $x \in [a,b]$. We use linear splines and uniform nodes for the two states of each country.

1. **Grid for state 1: Own productivity**:

   - In $x$ country it is distributed $\mu_x \sim \mathcal{N}(m_x, s_x^2)$, where $m_x$, $s_x$ are parameters. We construct uniform nodes $\{\mu_x^i\}_{i=1}^{N}$ in the support $[m_x - 4s_x, m_x + 4s_x]$.

   - In $y$ country it is distributed $\mu_y \sim \mathcal{N}(m_y, s_y^2)$, where $m_y$, $s_y$ are parameters. We construct uniform nodes $\{\mu_y^i\}_{i=1}^{N}$ in the support $[m_y - 4s_y, m_y + 4s_y]$.

2. **Grid for state 2: Posterior mean of foreign productivity**:

   - In $x$ country, the posterior mean of foreign productivity is $\hat{m}_y \sim \mathcal{N}(m_y, \bar{s}_y^2)$ where $\bar{s}_y^2 = \frac{s_y^4}{s_y^2+\bar{s}_y^2}$. However, to use a fixed grid that does not change with the precision of information, we construct the nodes $\{\hat{\mu}_y^j\}_{j=1}^{N}$ over the support $[m_y - 4s_y, m_y + 4s_y]$.

   - In $y$ country, the posterior mean of foreign productivity is $\hat{m}_x \sim \mathcal{N}(m_x, \bar{s}_x^2)$ where $\bar{s}_x^2 = \frac{s_x^4}{s_x^2+\bar{s}_x^2}$. Analogously, we construct the nodes $\{\hat{m}_x^j\}_{j=1}^{N}$ over the support $[m_x - 4s_x, m_x + 4s_x]$.

**Approximating functions** We approximate four conditional expectations with polynomials:

$$
\begin{aligned}
\mathbb{E}_{\mu_y,\hat{m}_x}\left[\Psi_2(\mu_x,\hat{m}_y,\mu_y,\hat{m}_x)^{1-\theta}\middle|\mathcal{I}_x\right] &\approx g^1(\mu_x,\hat{m}_y)\\
\mathbb{E}_{\mu_y,\hat{m}_x}\left[\Psi_2(\mu_x,\hat{m}_y,\mu_y,\hat{m}_x)^{1-\theta}q(\mu_x,\hat{m}_y,\mu_y,\hat{m}_x)^{-\theta}\middle|\mathcal{I}_x\right] &\approx g^2(\mu_x,\hat{m}_y)\\
\mathbb{E}_{\mu_x,\hat{m}_y}\left[\Gamma_2(\mu_y,\hat{m}_x,\mu_x,\hat{m}_y)^{1-\theta}\middle|\mathcal{I}_y\right] &\approx h^1(\mu_y,\hat{m}_x)\\
\mathbb{E}_{\mu_x,\hat{m}_y}\left[\Gamma_2(\mu_y,\hat{m}_x,\mu_x,\hat{m}_y)^{1-\theta}q(\mu_x,\hat{m}_y,\mu_y,\hat{m}_x)^{\theta}\middle|\mathcal{I}_y\right] &\approx h^2(\mu_y,\hat{m}_x)
\end{aligned}
$$

where the polynomials are constructed using the basis for each dimension evaluated at the nodes described above:

$$
\begin{aligned}
g^1(\mu_x^i,\hat{m}_y^j) &\equiv \sum_{k,k'\in K\times K'} g^1_{k,k',i,j}\Phi_k(\mu_x^i)\Phi_{k'}(\hat{m}_y^j)\\
g^2(\mu_x^i,\hat{m}_y^j) &\equiv \sum_{k,k'\in K\times K'} g^2_{k,k',i,j}\Phi_k(\mu_x^i)\Phi_{k'}(\hat{m}_y^j)\\
h^1(\mu_y^i,\hat{m}_x^j) &\equiv \sum_{k,k'\in K\times K'} h^1_{k,k',i,j}\Phi_k(\mu_y^i)\Phi_{k'}(\hat{m}_x^j),\\
h^2(\mu_y^i,\hat{m}_x^j) &\equiv \sum_{k,k'\in K\times K'} h^2_{k,k',i,j}\Phi_k(\mu_y^i)\Phi_{k'}(\hat{m}_x^j).
\end{aligned}
$$

## 3.2. Computing expectations

For each country, we have two random variables—foreign productivity and second-order beliefs—for which we will evaluate expectations using Gaussian quadrature. For this, we must define a set of nodes $\{x_a\}_{a=1}^{N_q}$ and weights

$\{w_a\}_{a=1}^{N_q}$ such that

$$\mathbb{E}[f(X)] = \sum_{a=1}^{N_q} w_a f(x_a).$$

and further moments conditions are satisfied.

- **Grid for random variable 1: Foreign productivity**: The distribution of foreign aggregate productivity depends on the second state, the posterior mean $\hat{m}$.

  - In $x$ country, for each value of the second state (the posterior mean) we have that foreign productivity is normal with mean equal to the posterior mean $\hat{m}_y^j$ and variance equal to the posterior variance $\hat{s}_y^2 = \left(s_y^{-2} + \tilde{s}_y^{-2}\right)^{-1} = \frac{1}{\frac{1}{s_y^2}+\frac{1}{\tilde{s}_y^2}}$

  $$\mu_y^j \sim \mathcal{N}(\hat{m}_y^j, \hat{s}_y^2).$$

  Then for each $j = 1, ..., N$, Gaussian quadrature constructs nodes of foreign productivity $\{\mu_y^{j,b}\}_b = 1, ..., N_q$ and corresponding weights $\{\omega^b\}_b = 1, ..., N_q$. Note that the weights do not depend on $j$.

  - In $y$ country, for each value of the second state (the posterior mean $\hat{m}_x^j$) we have that foreign productivity is normal with mean equal to the posterior mean $\hat{m}_x^j$ and variance equal to the posterior variance $\hat{s}_x^2 = \left(s_x^{-2} + \tilde{s}_x^{-2}\right)^{-1} = \frac{1}{\frac{1}{s_x^2}+\frac{1}{\tilde{s}_x^2}}$

  $$\mu_x^j \sim \mathcal{N}(\hat{m}_x^j, \hat{s}_x^2).$$

  Then for each $j = 1, ..., N$, Gaussian quadrature constructs nodes of foreign productivity $\{\mu_x^{j,b}\}_b = 1, ..., N_q$ and corresponding weights $\{\omega^b\}_b = 1, ..., N_q$.

## Extreme cases

  – Perfect Info: As $\tilde{s}_y \to 0$, $\mu_y^j \sim \mathcal{N}(\hat{m}_y^j, 0) = \mathcal{N}(\mu_y^j, 0)$. The grid degenerates to a single point for each $j$: $\mu_y^{j,b} = \mu_y^j$.

  – No Info: As $\tilde{s}_y \to \infty$, $\mu_y^j \sim \mathcal{N}(\hat{m}_y^j, s_y^2) = \mathcal{N}(m_y, \bar{s}_y^2)$ which is equal to the distribution of the posterior mean (the second state). Clearly, $\bar{s}_y^2 = \frac{s_y^4}{s_y^2+\tilde{s}_y^2} \to s_y^2$ as well, which makes the distribution of foreign productivity equal to the prior. However, in the code we have fixed grids for the states so that they do not depend on signal precision. Therefore, as we reduce signal precision, the grid will not converge to the prior. However, the simulations take care of this.

- **Grid for random variable 2: Second-order beliefs**: From the perspective of the domestic country, second-order beliefs about the posterior mean (that is, what the domestic country thinks the posterior mean of the foreign country is) is a normal random variable that depends on the first state, the domestic aggregate productivity $\mu$.

  - In the $x$ country, for each value of the first state (aggregate productivity $\mu_x^i$), we have that the second-order belief is normal with mean and variance as follows:

  $$\hat{m}_x^i \sim \mathcal{N}(\hat{\hat{m}}_x^i, \hat{\hat{s}}_x^2) \qquad \text{with} \qquad \hat{\hat{m}}_x^i \equiv \frac{s_x^{-2} m_x + \tilde{s}_{p_x}^{-2}\mu_x^i}{s_x^{-2} + \tilde{s}_{p_x}^{-2}}, \qquad \hat{\hat{s}}_x^2 \equiv \tilde{s}_{p_x}^{-2}(s_x^{-2} + \tilde{s}_{p_x}^{-2})^{-2} = \frac{1}{(\frac{\tilde{s}_{px}}{s_x^2} + \frac{1}{\tilde{s}_{px}})^2}$$

  where $\tilde{s}_{p_x}$ is the foreign signal noise as perceived by the domestic country. With known information structures $\tilde{s}_{p_x} = \tilde{s}_x$, but with unknown information structures $\tilde{s}_{p_x} \neq \tilde{s}_x$.

  Then for each $i = 1, ..., N$, Gaussian quadrature constructs nodes for second-order beliefs $\{\mu_x^{i,a}\}_a = 1, ..., N_q$ and corresponding weights $\{\gamma^a\}_a = 1, ..., N_q$.

- In the $y$ country, we have that for each value of the first state $\mu_y^i$ the second-order belief is distributed:

$$\hat{m}_y^i \sim \mathcal{N}(\hat{\hat{m}}_y^i, \hat{\hat{s}}_y^2) \qquad \text{with} \qquad \hat{\hat{m}}_y^i \equiv \frac{s_y^{-2} m_y + \tilde{s}_{p_y}^{-2} \mu_y^i}{s_y^{-2} + \tilde{s}_{p_y}^{-2}}, \qquad \hat{\hat{s}}_y^2 \equiv \tilde{s}_{p_y}^{-2} (s_y^{-2} + \tilde{s}_{p_y}^{-2})^{-2} = \frac{1}{(\frac{\tilde{s}_{p_y}}{s_y^2} + \frac{1}{\tilde{s}_{p_y}})^2}.$$

### Extreme cases

- Perfect Info: As $\tilde{s}_{p_x} \to 0$, then the distribution becomes degenerate at the true realizations: $\hat{m}_x^i \sim \mathcal{N}(\mu_x^i, 0) \quad \forall i$ and the grid becomes: $\hat{m}_x^{i,a} = \mu_x^i, \ a = 1, ..., N_q$.
- Imperfect Info: As $\tilde{s}_{p_x} \to \infty$, then the distribution becomes degenerate at the prior means $\hat{m}_x^i \sim \mathcal{N}(m_x, 0) \quad \forall i$ and the grid becomes $\hat{m}_x^{i,a} = m_x, \ a = 1, ..., N$.

**Computational algorithm** We solve the fixed-point problem by iterating on the export policy functions $\Psi$ and $\Gamma$, which are approximated using linear splines. For each country we define grids for their two states: aggregate productivity and posterior mean of foreign productivity. We also define grids for foreign productivity and the second-order beliefs countries use to evaluate their perceived price function. Expectations with respect to foreign productivity and second-order beliefs are computed using Gaussian quadrature. Once we have solved the fixed-point problem, we simulate the repeated economy for T=100,000 periods and compute average statistics across simulations.

## 3.3. Finding the fixed point

1. For reference, we organize the states as follows. For x - country: $(\mu_x, \hat{m}_y)$ and for y - country: $(\mu_y, \hat{m}_x)$. For the price and other economy-wide variables, we make the following convention: $q(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)$.
2. Guess an initial set of coefficients for polynomials $\{g^1_{k,k',i,j}, g^2_{k,k',i,j}, h^1_{k,k',i,j}, h^2_{k,k',i,j}\}$.
   - We start by solving the perfect information case and approximate the policies with the polynomials to get the first set of coefficients. Since with perfect information $\hat{m}_y = \mu_y$ and $\hat{m}_x = \mu_x$, we have the following system of equations:

$$\begin{aligned} \Psi^{PI}(\mu_x, \mu_y) &= \frac{1}{1 + [(1+\tau) q^{PI}(\mu_x, \mu_y)]^{\frac{\theta}{1-\theta}}} \\ \Gamma^{PI}(\mu_y, \mu_x) &= \frac{1}{1 + \left[\frac{(1+\tau)}{q^{PI}(\mu_x, \mu_y)}\right]^{\frac{\theta}{1-\theta}}} \\ q^{PI}(\mu_x, \mu_y) &= f \frac{\Psi^{PI}(\mu_x, \mu_y)}{\Gamma(\mu_y, \mu_x)}. \end{aligned}$$

Thus $q^{PI}$ solves the following equation[4]:

$$q^{PI} - f \frac{\frac{1}{1 + [(1+\tau) q^{PI}(\mu_x, \mu_y)]^{\frac{\theta}{1-\theta}}}}{\frac{1}{1 + \left[\frac{(1+\tau)}{q^{PI}(\mu_x, \mu_y)}\right]^{\frac{\theta}{1-\theta}}}} = 0.$$

Once we have the price, we recover the policies and construct the first guess of coefficients and approximating functions.

[4]Notice that without the trade cost $\tau$, the price is: $q^{PI}(\mu_x, \mu_y) = f^{1-\theta} \quad \forall (\mu_y, \mu_x)$.

3. For the X - country:

   - For each state $(\mu_x^i, \hat{m}_y^j)$, approximate $\Psi$ using the polynomials $g^1$ and $g^2$ evaluated at the state:

$$\Psi(\mu_x^i, \hat{m}_y^j) \approx \frac{1}{1+(1+\tau)^{\frac{\theta}{1-\theta}}\left(\frac{g^1(\mu_x^i,\hat{m}_y^j)}{g^2(\mu_x^i,\hat{m}_y^j)}\right)^{\frac{1}{1-\theta}}}.$$

   - For each quadrature node $(\mu_y^a, \hat{m}_x^b)$ approximate $\Gamma$ using the polynomials $h^1$ and $h^2$ evaluate at the nodes $\{\mu_y^a\}_{a=1}^{N_q}, \{\hat{m}_x^b\}_{b=1}^{N_q}$

$$\Gamma(\mu_y^a, \hat{m}_x^b) \approx \frac{1}{1+(1+\tau)^{\frac{\theta}{1-\theta}}\left(\frac{h^1(\mu_y^a,\hat{m}_x^b)}{h^2(\mu_y^a,\hat{m}_x^b)}\right)^{\frac{1}{1-\theta}}}.$$

   - Construct $q$ and $\Psi_2$ in 4 dimensions using $\Psi(\mu_x^i, \hat{m}_y^j)$ and $\Gamma(\mu_y^a, \hat{m}_x^b)$:

$$q(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b) \approx e^{(\mu_x^i-\mu_y^a)} e^{\frac{1}{2}(\sigma_x^2-\sigma_y^2)} \frac{\Psi(\mu_x^i, \hat{m}_y^j)}{\Gamma(\mu_y^a, \hat{m}_x^b)}$$

$$\Psi_2(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b) = \left((1-\Psi(\mu_x^i, \hat{m}_y^j))^\theta + \left(\frac{\Psi(\mu_x^i, \hat{m}_y^j)}{(1+\tau)q(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b)}\right)^\theta\right)^{\frac{1}{\theta}}.$$

   - Compute the conditional expectations of $\Psi_2^{1-\theta}$ and $\Psi_2^{1-\theta}q^{-\theta}$ that integrate out the two random variables $(\mu_y, \hat{m}_x)$ as the weighted sum of the functions evaluated at the quadrature nodes, using the quadrature weights $\{\omega^a\}_{a=1}^{N_q}$ and $\{\gamma^b\}_{b=1}^{N_q}$:

$$\begin{aligned}
&\mathbb{E}_{\mu_y,\hat{m}_x}\left[\Psi_2^{1-\theta}(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)\middle|\mathcal{I}_x\right] \\
=& \int_{\mu_y}\int_{\hat{m}_x} \Psi_2^{1-\theta}(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x) \quad \phi\left(\frac{\mu_y-\hat{m}_y}{\hat{s}_y}\right)\phi\left(\frac{\hat{m}_x-\hat{\hat{m}}_x}{\hat{\hat{s}}_x}\right)d\mu_y d\hat{m}_x \\
\approx& \sum_{a=1}^{N_q}\sum_{b=1}^{N_q}\omega^a\gamma^b\Psi_2^{1-\theta}(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b)
\end{aligned}$$

$$\begin{aligned}
&\mathbb{E}_{\mu_y,\hat{m}_x}\left[\Psi_2^{1-\theta}(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)q^{-\theta}(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)\middle|\mathcal{I}_x\right] \\
=& \int_{\mu_y}\int_{\hat{m}_x} \Psi_2^{1-\theta}(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)q^{-\theta}(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x)\phi\left(\frac{\mu_y-\hat{m}_y}{\hat{s}_y}\right)\phi\left(\frac{\hat{m}_x-\hat{\hat{m}}_x}{\hat{\hat{s}}_x}\right)d\mu_y d\hat{m}_x \\
\approx& \sum_{a=1}^{N_q}\sum_{b=1}^{N_q}\omega^a\gamma^b\Psi_2^{1-\theta}(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b)q^{-\theta}(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b).
\end{aligned}$$

4. For the Y- country, we do analogous calculations.

   - For each state $(\mu_y^i, \hat{m}_x^j)$, approximate $\Gamma$ using the polynomials $h^1$ and $h^2$ evaluated at the state:

$$\Gamma(\mu_y^i, \hat{m}_x^j) \approx \frac{1}{1+(1+\tau)^{\frac{\theta}{1-\theta}}\left(\frac{h^1(\mu_y^i,\hat{m}_x^j)}{h^2(\mu_y^i,\hat{m}_x^j)}\right)^{\frac{1}{1-\theta}}}.$$

- For each quadrature node $(\mu_x^a, \hat{m}_y^b)$ approximate $\Psi$ using the polynomials $g^1$ and $g^2$ evaluated at the nodes $\{\mu_x^a\}_{a=1}^{N_q}, \{\hat{m}_y^b\}_{b=1}^{N_q}$

$$\Psi(\mu_x^a, \hat{m}_y^b) \approx \frac{1}{1 + (1+\tau)^{\frac{\theta}{1-\theta}} \left(\frac{g^1(\mu_x^a, \hat{m}_y^b)}{g^2(\mu_x^a, \hat{m}_y^b)}\right)^{\frac{1}{1-\theta}}}.$$

- Construct $q$ and $\Gamma_2$ in 4 dimensions using $\Gamma(\mu_y^i, \hat{m}_x^j)$ and $\Psi(\mu_x^a, \hat{m}_y^b)$ (note that the state for the price is in the same order as for the X-country):

$$q(\mu_x^a, \hat{m}_y^b, \mu_y^i, \hat{m}_x^j) \approx e^{(\mu_x^a - \mu_y^i)} e^{\frac{1}{2}(\sigma_x^2 - \sigma_y^2)} \frac{\Psi(\mu_x^a, \hat{m}_y^b)}{\Gamma(\mu_y^i, \hat{m}_x^j)}$$

$$\Gamma_2(\mu_y^i, \hat{m}_x^j, \mu_x^a, \hat{m}_y^b) = \left( (1 - \Gamma(\mu_y^i, \hat{m}_x^j))^\theta + \left( \frac{\Gamma(\mu_y^i, \hat{m}_x^j) q(\mu_x^a, \hat{m}_y^b, \mu_y^i, \hat{m}_x^j)}{(1+\tau)} \right)^\theta \right)^{\frac{1}{\theta}}.$$

- Compute the conditional expectations of $\Gamma_2^{1-\theta}$ and $\Gamma_2^{1-\theta} q^\theta$ that integrate out the two random variables $(\mu_x, \hat{m}_y)$. This is just the weighted sum of the functions evaluated at the quadrature nodes, using the quadrature weights $\{\omega^a\}_{a=1}^{N_q}$ and $\{\gamma^b\}_{b=1}^{N_q}$:

$$\begin{aligned}
&\mathbb{E}_{\mu_x, \hat{m}_y} \left[ \Gamma_2^{1-\theta}(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y) \middle| \mathcal{I}_y \right] \\
= & \int_{\mu_y} \int_{\hat{m}_x} \Gamma_2^{1-\theta}(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y) \quad \phi\left(\frac{\mu_x - \hat{m}_x}{\hat{s}_x}\right) \phi\left(\frac{\hat{m}_y - \hat{\hat{m}}_y}{\hat{\hat{s}}_y}\right) d\mu_x d\hat{m}_y \\
\approx & \sum_{a=1}^{N_q} \sum_{b=1}^{N_q} \omega^a \gamma^b \Gamma_2^{1-\theta}(\mu_y^i, \hat{m}_x^j, \mu_x^a, \hat{m}_y^b)
\end{aligned}$$

$$\begin{aligned}
&\mathbb{E}_{\mu_x, \hat{m}_y} \left[ \Gamma_2^{1-\theta}(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y) q^\theta(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x) \middle| \mathcal{I}_y \right] \\
= & \int_{\mu_y} \int_{\hat{m}_x} \Gamma_2^{1-\theta}(\mu_y, \hat{m}_x, \mu_x, \hat{m}_y) q^\theta(\mu_x, \hat{m}_y, \mu_y, \hat{m}_x) \phi\left(\frac{\mu_x - \hat{m}_x}{\hat{s}_x}\right) \phi\left(\frac{\hat{m}_y - \hat{\hat{m}}_y}{\hat{\hat{s}}_y}\right) d\mu_x d\hat{m}_y \\
\approx & \sum_{a=1}^{N_q} \sum_{b=1}^{N_q} \omega^a \gamma^b \Gamma_2^{1-\theta}(\mu_x^i, \hat{m}_y^j, \mu_y^a, \hat{m}_x^b) q^\theta(\mu_x^a, \hat{m}_y^b, \mu_y^i, \hat{m}_x^j).
\end{aligned}$$

5. Update coefficients by (i) fitting polynomials to approximate the conditional expectations and (ii) using a linear combination of the new coefficients with the previous guess.
6. Repeat steps until convergence of coefficients.
7. Once convergence is achieved, recover all variables at the firm level and at the aggregate level.

   Recall the definitions of domestic, foreign, and relative fundamentals:

$$f_x \equiv e^{\mu_x + \frac{1}{2}\sigma_x^2}, \qquad f_y \equiv e^{\mu_y + \frac{1}{2}\sigma_y^2} \qquad f \equiv e^{(\mu_x - \mu_y)} e^{\frac{1}{2}(\sigma_x^2 - \sigma_y^2)}.$$

   (a) Price function:

$$q(\mu_x, \mu_y, \hat{m}_x, \hat{m}_y) = f \frac{\Psi(\mu_x, \hat{m}_y)}{\Gamma(\mu_y, \hat{m}_x)}.$$

(b) Firms' export policy and consumptions in $x$ country:

$$\begin{aligned} t_x(z_x,\mu_x,\hat{m}_y) &= z_x\Psi(\mu_x,\hat{m}_y) \\ c_x(z_x,\mu_x,\hat{m}_y) &= z_x(1-\Psi(\mu_x,\hat{m}_y)) \\ c_y(z_x,\mu_x,\hat{m}_y,\mu_y,\hat{m}_x) &= \frac{t_x(z_x,\mu_x,\hat{m}_y)}{(1+\tau)q(\mu_x,\mu_y,\hat{m}_x,\hat{m}_y)} \\ c(z_x,\mu_x,\hat{m}_y,\mu_y,\hat{m}_x) &= z_x\Psi_2(\mu_x,\hat{m}_y,\mu_y,\hat{m}_x). \end{aligned}$$

(c) Firms' export policy and consumptions in $y$ country:

$$\begin{aligned} t_y(z_y,\mu_y,\hat{m}_x) &= z_y\Gamma(\mu_y,\hat{m}_x) \\ c_y^*(z_y,\mu_y,\hat{m}_x) &= z_y(1-\Gamma(\mu_y,\hat{m}_x)) \\ c_x^*(z_y,\mu_y,\hat{m}_x,\mu_x,\hat{m}_y) &= \frac{t_y(z_y,\mu_y,\hat{m}_x)q(\mu_x,\mu_y,\hat{m}_x,\hat{m}_y)}{(1+\tau)} \\ c^*(z_x,\mu_x,\hat{m}_y,\mu_y,\hat{m}_x) &= z_y\Gamma_2(\mu_y,\hat{m}_x,\mu_x,\hat{m}_y). \end{aligned}$$

(d) Aggregate variables in $x$ country:

$$\begin{aligned} T_x(\mu_x,\hat{m}_y) &= f_x\Psi(\mu_x,\hat{m}_y) \\ C_x(\mu_x,\hat{m}_y) &= f_x(1-\Psi(\mu_x,\hat{m}_y)) \\ C_y(\mu_x,\hat{m}_y,\mu_y,\hat{m}_x) &= \frac{T_x(\mu_x,\hat{m}_y)}{(1+\tau)q(\mu_x,\mu_y,\hat{m}_x,\hat{m}_y)} \\ C(\mu_x,\hat{m}_y,\mu_y,\hat{m}_x) &= f_x\Psi_2(\mu_x,\mu_y,\hat{m}_x,\hat{m}_y). \end{aligned}$$

(e) Aggregate variables in $y$ country:

$$\begin{aligned} T_y(\mu_y,\hat{m}_x) &= f_y\Gamma(\mu_y,\hat{m}_x) \\ C_y^*(\mu_y,\hat{m}_x) &= f_y(1-\Gamma(\mu_y,\hat{m}_x)) \\ C_x^*(\mu_y,\hat{m}_x,\mu_x,\hat{m}_y) &= \frac{T_y(\mu_y,\hat{m}_x)q(\mu_x,\mu_y,\hat{m}_x,\hat{m}_y)}{(1+\tau)} \\ C^*(\mu_y,\hat{m}_x,\mu_x,\hat{m}_y) &= f_y\Gamma_2(\mu_y,\hat{m}_x,\mu_x,\hat{m}_y). \end{aligned}$$

### 3.4. Simulation strategy and parameter choice

We solve and simulate different versions of the model: (i) full information, where both countries know their own productivity and the other country's productivity exactly ($\hat{m}_x = \mu_x$ and $\hat{m}_y = \mu_y$); (ii) no information, where each country knows its own productivity, but since neither country gets any signal, their beliefs about the other country's productivity are given by the unconditional distribution ($\hat{m}_x = m_x$ and $\hat{m}_y = m_y$); and (iii) noisy signals, where each country knows its own productivity and receives signals about the other country's productivity. In this last case we will solve and simulate the model for various levels of signal precision, always keeping the precision symmetric across countries.

For each information model, we solve the fixed-point problem by iterating on export policy functions $\Psi$ and $\Gamma$, which are approximated using linear splines. For each country we define grids for their two states: aggregate productivity and posterior mean of foreign productivity. We also define grids for foreign productivity and second-order beliefs that countries use to evaluate their perceived price function. Expectations with respect to foreign productivity and second-order beliefs are computed using Gaussian quadrature.

Once we have solved the fixed-point problem, we simulate the repeated economy for T=100,000 periods and compute average statistics across simulations. We first draw a series of aggregate productivities and fix it across information models. Then for each information model we generate posterior means of foreign productivity by drawing signals centered on the realized aggregate productivities and with precision determined by the model at hand.

**Parameters** Table 1 describes the parameters we use to simulate the models and how they are chosen or calibrated. The unconditional mean of aggregate productivity $m$ and the dispersion of firm productivity $\sigma$ only appear in the solution as differences between home and foreign values. Only the differences $m_x - m_y$ or $\sigma_x - \sigma_y$ affect the export shares or the relative prices. The absolute quantities produced and traded do depend on these parameters, but not any relative quantities or prices. Since the question of fundamental asymmetry between two countries and its effect on trade is not the focus of this paper, we assume that there is no difference in the distribution of fundamentals by imposing symmetry: $m_x = m_y$ and $\sigma_x = \sigma_y$. We normalize $m = 0$ and $\sigma = \sqrt{2}$ so that the mean of aggregate (log) endowment is unity: $\mathbb{E}[\log f_x] = m_x + \frac{1}{2}\sigma_x^2 = 1$. To set volatility of the aggregate productivity shocks, we note that it is the ratio of signal to aggregate dispersion that matters for outcomes $\frac{\tilde{s}}{s}$. Since we will vary signal precision across simulations, we assume symmetry and normalize the volatility of aggregate shocks to unity ($s_x = s_y = 1$). We consider two values for $\theta \in \{0.3, 0.8\}$ that imply an elasticity of substitution across the two consumption goods of $\nu = \frac{1}{1-\theta} \in \{1.42, 5\}$, respectively. Finally, the precision of the signals each country observes about the other's productivity $\tilde{s}^{-2}$ are varied across simulations in the range $[0, \infty]$.

Table 1: **Summary of Model Parameters**

| Parameter | $\theta$ | $\sigma$ | $m_x = m_y$ | $s_x = s_y$ | $\sigma_x = \sigma_y$ | $\tilde{s}_x = \tilde{s}_y$ |
|---|---|---|---|---|---|---|
| Value | $0.3, 0.8$ | $1 - \theta$ | $0$ | $1$ | $\sqrt{2}$ | $[0, \infty]$ |