Simon Says?
(Interpersonal) Authority in Organizations

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Abstract

This paper contrasts the efficiency of two coordination mechanisms, decentralized coordination and authority, in resolving coordination problems in organizations. Under decentralized coordination, the agents (subordinates) responsible for different tasks are also responsible for acquiring and sharing information about their tasks and then executing a plan of action. In contrast, under authority, a principal (superior) processes all relevant information and then instructs the subordinates as to what actions to take, and the subordinates choose to follow these instructions without further evaluation. The analysis thus revisits the classic notion of authority as the ability of an individual to instruct others on what to do and to expect obedience, and formalizes it as an endogenous equilibrium information structure of a game. Both types of equilibria can co-exist, with the advantage of authority lying in the strategic management of information made available to the subordinates, which facilitates coordinated adaptation to opportunities without exposing the organization to excessive oportunism. Authority dominates decentralized coordination whenever the parties exhibit moderate patience and cost disadvantage of authority in processing information is not too large.

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1 Introduction

The existence of authority relationships has long been recognized as one of the defining characteristics of organizations. While there are different notions of authority, a particularly common and much-discussed notion is that of interpersonal authority – the ability of one party ("superior") to instruct the behavior of another ("subordinate") and to expect obedience.[1] This notion of authority dates back to at least Chester Barnard and Herbert Simon, who define authority "as the power to make decisions which guide the actions of another... The superior frames and transmits decisions with the expectation that they will be accepted by the subordinate. The subordinate expects such decisions, and his conduct is determined by them." (Simon, 1947/1997:179)

The purpose of this paper is to formalize this notion of authority, and examine the conditions under which it can be sustained and when its use can be value-enhancing to an organization. This notion of authority as the right to instruct (and to expect obedience) is quite different from the notion of authority as the "right to decide," which has been at the heart of a large and growing literature in economics (following Aghion and Tirole, 1997, and Dessein, 2002, among others). Most importantly, while the right to decide may be moved around contractually, the key feature of (interpersonal) authority is that such authority is always granted by the subordinates who are expected to carry out the decision, and without such authority the right to decide may have limited value. As noted by Barnard (1938/1968:163), "[t]he decision as to whether an order has authority or not lies with persons to whom it is addressed, and does not reside in "persons of authority" or those who issue these orders." If such conditions are not met, "there is no authority, whatever may be the "paper" theory of organization." (Simon, 1947/1997:179)

To sustain such authority relationships, the literature identifies two broad channels. The first is the *authority of sanctions*. Simply put, when the superior has available sufficient sanctions that she can impose on the subordinate in case of disobedience, it will be in the subordinate's best interest to follow instructions. However, the sanctions generally available to organizations are limited, and, as noted by Arrow (1974:72), "the point is clear. It is not that authority is not in fact usually exercised; it is that the control mechanisms, the sanctions we usually think of as enforcing authority, cannot be the sole or even the major basis for the acceptance of authority. Employees follow instructions, and citizens obey law to a much greater extent than can be explained on the basis of control mechanisms."[2] Instead, he proposes that "[a]n individual obeys authority because he expects that others will obey it." (Arrow, 1974:72). In other words, authority is a mutually beneficial illusion to those who submit themselves to it. The extent of obedience, however, is likely to depend on the characteristics of the superior. In particular, the second source of authority is *authority of expertise*, whereby, as long as the subordinate is sufficiently confident in the decision-making abilities and

[1] As noted by Arrow (1974:63), "Among the most widespread characteristics of organizations is the prevalence of authoritative allocations. Virtually universally, in organizations of any size, decisions are made by some individuals and carried out by others."

[2] To make the observation about the limits of sanctions even stronger, Barnard notes that "[m]any men have destroyed all authority as to themselves by dying rather than yield." (1938/1968:184), although in the context of organizations, the destruction of authority generally takes less extreme forms, with "[c]ases of voluntary resignation from all sorts of organizations are common for this sole reason. Malingering and intentional lack of dependability are the more usual methods." (1938/1968:166).

motives of the superior, he is willing to hold "in abeyance his own critical faculties for choosing between alternatives and uses the formal criterion of the receipt of a command or signal as his basis for choice." (Simon 1947/1997:179) It is this latter source of authority that is the focus of the present work.[3]

Separately, the basic premise of a large management literature is that organizations exist to provide conscious and intentional coordination among its various activities, applying a variety of different methods of coordinating embedded in the formal and informal structure of the organization, with authority being one of them.[4] For example, Mintzberg (1979) makes a distinction between *mutual adjustment*, which "achieves coordination of work by the simple process of informal communication " (p.3), and *direct supervision*, which "achieves coordination by having one individual take responsibility for the work of others, issuing instructions to them and monitoring their actions." (p.4) In the first mechanism, the agents responsible for different tasks both evaluate and execute plans, and no authority relationships exist. There is communication, but it is only to share information and coordinate, with no expectation of obedience. The second mechanism, however, separates the formulation of the plan from the execution of the plan. In this case, the organization needs to ensure that the instructions will actually be followed - the decision-maker needs to have *authority* over the executor. The question is then under what conditions can such an authority relationship exist and when does it dominate coordination based on mutual adjustment.

To formalize the above discussion, I consider a repeated game of coordinated adaptation played among three players. There are two tasks, where the appropriate action for each task is dependent on the local conditions for both tasks. Two of the players (agents/subordinates) are responsible for the execution of their respective tasks, while the third player (principal/superior) plays no directly productive role. To minimize the role of formal structures, I assume that nothing in the game is contractible. The subordinates are always responsible for executing the tasks they are responsible for, and any player can acquire information and share that information or their plans with any other player. They are also free to make any monetary transfers among each other, but nothing is verifiable to outside parties. The only question is who will acquire information and when, with the constraint that processing information about the tasks is costly, and that information is costlier to the superior than the subordinates responsible for a given task. If the superior does process information and issue instructions, it is always up to the subordinates whether they follow the instructions or engage in their own evaluation or direct disobedience, and the superior has no formal sanctions available: all rewards and punishments are informal, sustained as a part of the ongoing relationship.[5] For example, Barnard notes that "since the efficiency of organization is affected by the

[3] "The superior who possesses such advantages of information will have much less occasion to invoke the formal sanctions of authority than the superior whose subordinates are in a better situation than he, from the standpoint of information, to make the decision." (Simon, 1947/1997:190)

[4] For example, Arrow (1974:19) writes, "interpersonal organization is needed to secure the gains that can accrue from cooperation."

[5] This sentiment is echoed by Alchian and Demsetz (1972:777), who note that while "[i]t is common to see the firm characterized by the power to settle issues by fiat, by authority, or by disciplinary action superior to that available in the conventional market. This is delusion. The firm does not own all its inputs. It has no power of fiat, no authority, no disciplinary action any different in the slightest degree from ordinary market contracting between any two people. I can "punish" you only by withholding future business or by seeking redress in the courts for any failure to honor our exchange agreement. That is eactly all that any employer can do. He can fire or sue, just as I can fire my grocer by stopping purchases from him or sue him for delivering faulty products."

degree to which individuals assent to orders, denying the authority of an organization communication is a threat to the interests of all individuals who derive a net advantage from their connection with the organization." (1938/1968:169) In other words, one of the main consequences of disobedience is the breakdown of authority, which is the focus here.

In this framework, I compare the performance of the two modes of coordination: decentralized coordination (mutual adjustment) and authority (direct supervision). Under decentralized coordination, the subordinates acquire information about their local conditions, share that information with each other and then execute a plan. Not surprisingly, when the subordinates are sufficiently patient, they are able to implement the first-best solution, but as they become increasingly impatient, collaboration slowly erodes, eventually unraveling to a situation where the subordinates act purely selfishly. More interestingly, the setting highlights why successful mutual adjustment is difficult to achieve. The reason is that information available to the subordinates plays a dual role. On one hand, information about both tasks is necessary for formulating the right course of action. This necessitates both the acquisition and sharing of information by both subordinates. On the other hand, as the subordinates acquire and share information, they will simultaneously become aware of opportunities for opportunistic behavior, where they can take advantage of the situation to their own benefit. And it is the knowledge of such opportunities that is the eventual downfall of mutual adjustment.

The authority structure differs from decentralized coordination in two key dimensions. First, it is now the superior who processes the information regarding both tasks, comes up with a plan on how the tasks should be executed and instructs the subordinates. Second, the subordinates accept this instruction and follow it, despite having the freedom to acquire further information to re-evaluate the optimality of the plan, exactly as under the decentralized structure. In other words, the subordinate "permits his behavior to be guided by the decision of a superior, without independently examining the merits of that decision." (Simon 1947/1997:9).

Such an authority structure, with appropriately structured communication, provides three core benefits over the decentralized structure, all resulting from the ability of the superior to control what information is actually communicated to the subordinates. First, by first processing information and then only issuing an instruction to the subordinate, the superior is able to convey the socially valuable information regarding what action should be taken while hiding information about the possibly individually valuable deviations. In other words, instructions are able to decouple the social and opportunistic values of information and thus ensure compliance for wider range of parameters. Second, because of the social value of information embedded in the instruction, the value of further information acquisition by the subordinate is decreased and thus we can sustain an equilibrium where the subordinate does *not* acquire information and instead accepts instruction as the basis of choice.[6] Third, by hiding information about the complete plan of action and only disclosing the parts that the subordinate needs to know to execute his task, the superior is able to transfer surplus across different states to further motivate obedience.

But as with decentralized coordination, as the subordinates become sufficiently impatient, the

[6] As noted by Barnard, "The practical difficulties in organization seldom lie in the excessive desire of individuals to assume responsibility for the organization action of themselves or others." (1938/1968:170)

superior can no longer implement the first-best decision rule and must, instead, allow increasingly selfish behavior by the subordinates. This observation reflects the important idea that wise superiors realize the limitations of their authority and behave in a way that the authority relationship is sustained by not requesting actions that would lead to disobedience. As noted by Simon (1947/1997:186), "[r]estraint of the superior is as important as obedience of the subordinate in maintaining the relationship." And as noted by Barnard (1938/1968:168), "[i]t is generally recognized that those who least understand this fact - newly appointed minor or "first line" executives - are often guilty of "disorganizing" their groups for this reason, as do experienced executives who lose self-control or become unbalanced by a delusion of power for some other reason."

The remaining question is then which coordination mechanism performs relatively better. When the subordinates are sufficiently patient, decentralized coordination dominates. The subordinates are able to sustain the first-best decision rule and the costs of information are minimized by leaving the processing in the hands of the parties in the best position to do that. For intermediate patience levels, on the other hand, authority dominates. By being able to manage the information available to the agents, the organization is able to maintain collaborative behavior without exposing itself to excessive opportunism, a benefit that can outweigh even considerable cost disadvantage by the superior in terms of processing information. This dominance remains even after the superior needs to start making concessions in terms of the equilibrium decisions requested.

But as the players grow increasingly impatient, the superior's ability to manage the information available to the subordinates decreases as the first-best decision rule becomes increasingly unresponsive to the joint information regarding the two tasks. As a result, decentralized coordination becomes again preferred to take advantage of the lower cost of information, until collaboration breaks down completely. The analysis thus reveals a non-monotone relationship between decentralization and patience, with both high- and low-patience environments characterized by decentralized coordination, but with highly different surplus levels, while the authority structure is preferred only in the middle range.

The rest of the manuscript is organized as follows. Next section reviews the related literature and Section 3 outlines the model. Section 4 discusses the first-best decision rules and conditions under which both decentralization and authority are able to sustain the second-best decision rule. Section 5 derives the efficient equilibria under the two structures when the first-best decision rule is not sustainable, Section 6 performs a comparison between the two and Section 7 concludes.

2 Related literature

To relate this framework to the broader literature on authority and decision-making in organizations, it is instructive to consider the different stages of the decision process. Mintzberg (1979), for example, breaks the decision process into five stages, flowing from information to advice to choice to authorization and, finally, execution (as illustrated in Figure 1), and where a crucial component of organizational design is how the responsibility for these five tasks is allocated among the organizational members. A large and growing literature, building on Aghion and Tirole (1997) and Dessein

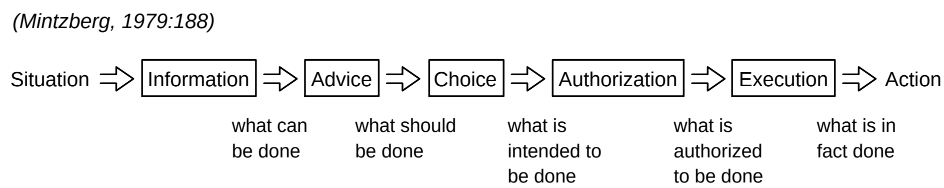

Figure 1: Decision process

(2002), among others, has examined the decision process from information to choice (and authorization), with valuable insights into how incentive conflicts affect the transmission of information, when is it better to co-locate choice with information, and how the identity of the decision-maker affects the incentives of different parties to acquire information in the first place, among many other results. However, much less attention has been given to the completion of the decision process, that is, moving from authorization to execution, which determines what is in fact done, as opposed to what is authorized to be done.

The papers most closely related to the present work are Van den Steen (2007, 2010a, 2010b), which also consider how interpersonal authority (defined by the obedience of the subordinate) arises, but which focus on differing priors as the source of disagreements and on settings where information is readily available to the organizational participants. In contrast, the present work derives the authority relationship in the presence of common priors and focuses on the implications of costly information and the benefits that the authority structure offers in terms of structuring the information dissemination throughout the organization. The mechanisms that create the value of the authority relationship (and so sustain it) are thus very different. In terms of sources of authority discussed earlier, Van den Steen largely builds on the authority of sanctions (the ability of the person with authority to sanction an individual in the case of disobedience, and how the value of disobedience can be managed through organizational design), while the present framework builds on authority of expertise.[7]

In settings with common priors, there exists a few recent papers on implementation problems and dissent. Zabojnik (2002) considers when it is optimal to delegate the action choice to the agent responsible for implementation when the agent has additional private information regarding the profitability of the task. Marino et al. (2010) consider how an agent's knowledge over preferred actions leads to potential disobedience of instructions, limits the principal's incentives to become informed and how the extent of disobedience is influenced by the costs of separation and monetary incentives. Bester and Krahmer (2008) consider how the principal's need to rely on an agent's effort for implementation restricts the principal's ability to choose her preferred project. Relatedly, Landier et al. (2009) illustrate the optimality of dissent by implementers to regulate the potentially selfish behavior of the decision-maker. In addition to considering very different organizational problems, these papers also assume informational structures with exogenously informed agents and its implications.[8] In contrast, the current paper derives the authority relationship endogenously as

[7] A partial exception is Van den Steen (2009), where authority may be supplemented by persuasion, i.e. with provision of hard information, to convince the agent.

[8] The exception is the second half of Landier et al. (2009), which considers a privately informed principal, but

an equilibrium information structure of a game, where the key feature is that the agents choose to remain *un*informed.

The second related literature is the small literature on leadership, such as Hermalin (1998) and Komai et al. (2007), who consider how the existence of a leader with priviledged access to information can improve effort provision in team production problem. Beyond analyzing a different setting, the key difference is that in their setting, the leader takes costly actions to signal his priviledged information to convince the followers, while the similarity lies in the potential benefits of limiting the agents' access to information.[9] In another related paper, Bolton et al. (2013) consider how leadership helps to coordinate the behavior of the subordinates, but in a setting where the leader and the followers both have direct access to their own private signals and the followers have an intrinsic motive to coordinate both with each other and with the leader. Again, while there is transmission of information from the leader to the followers through the leader's actions, there is no obedience in the sense of the present paper, nor are many of the benefits of authority identified here present.

Technically, the paper builds on the large repeated games literature, in particular Levin (2003). In terms of focus, this paper provides a complemented to Baker et al. (1999), who argue that in organizations, formal decision rights always reside at the top and can only be informally delegated down the organizational hierarchy through self-enforcing relational contracts. In this paper, interpersonal authority is allocated up the organizational hierarchy through the same. In addition, a few other recent papers use repeated games to consider elements of authority and information. Bolton and Rajan (2003) consider a repeated game where a buyer and a seller interact repeatedly, with the buyer having private information on both the value of the item to herself and the cost of supplying it. A repeated interaction allows for an equilibrium where the buyer simply instructs the seller which action to perform and then compensates the seller appropriately after the fact, which the authors interpret as an employment relationship. The key feature of repetition is the ability of the seller to walk if taken advantage of, disciplining the buyer not to take advantage of her informational advantage. The present setting, in contrast, deals with the obedience of the agents and highlights the important role of information control embedded in instructions. Rayo (2007) considers a repeated team production problem with no private information, and derives the result that when the effort levels of the individuals are sufficiently hard to observe (while total output is fully contractible), the optimal arrangement allocates all formal incentives in the hands of a single agent, who then manages relational contracts with subjective bonuses with the rest of the team members. He thus derives an endogenous principal analogous to the principal proposed by Alchian and Demsetz (1972), but where, instead of monitoring work (since the information is freely available to all), the agent manages the relational rewards based on that work, while here, the principal arises endogenously through the choice of who chooses to become informed. Finally, Kolotilin and Li (2018) consider a repeated game between a sender and a receiver in a setting analogous to Crawford and Sobel (1982). Their key insight is that voluntary transfers can be used to eliminate the strategic communication

where the benefit of dissent continues to arise from regulating the choice of the principal, resulting from the inference about project quality made by the implementer following the choice.

[9] See also Blanes i Vidal and Moller (2007) on how the motivational effects can bias equilibrium decisions when some information can be disclosed to the agents.

problem, analogous to the results here, and the main constraint is to manage the choice of the decision-maker, who is tempted to deviate from the socially efficient decision rule. In their setting, this is achieved under some parameter values by coarsened communication at the extremes, which limits the deviation temptation of the decision maker. This logic is analogous to the present paper, with information control used to manage deviation temptations.

By suggesting a principal advantage of authority relationships to be the ability of the superior to manage the information that is revealed to the subordinates and thus associating authority with an equilibrium information structure, the model bears a link to the literature on information control and Bayesian persuasion that has followed Kamenica and Gentzkow (2011), while by modeling all communication as non-verifiable messages, the analysis bears a link to the large literature on cheap talk that has followed Crawford and Sobel (1982). Finally, the analysis also links to the view of laws and authority as cheap talk coordination devices, as considered in Mailath et al (2001,2007). In particular, sustaining authority bears tensions similar to Mailath et al (2007).

3 Model

The game consists of two tasks and three players. Two of the players are productive agents (subordinates), each responsible for one of the tasks, and accruing the payoff to that task. The third player has no active interest in the game. For each task, the player responsible for that task will choose between two different ways of performing the task: collaborative/cooperative and selfish/adaptive. The payoffs to the game are given by the following table:

		Player 2	
		(C)ooperative	(A)daptive
Player 1	(C)ooperative	$1, 1$	$-\theta_1, (1+\alpha)+\theta_2$
	(A)daptive	$(1+\alpha)+\theta_1, -\theta_2$	β, β

,

where $\theta_i \in \left\{-\overline{\theta}, \overline{\theta}\right\}$ with equal probabilities (and independent across the players and over time). The game is thus a slightly modified prisoner's dilemma, intended to capture the following intuition. The players can either operate selfishly (A), pursuing individual opportunities but foregoing the gains from collaboration, or cooperate with the other player. Mutual cooperation yields 1 while mutual selfishness yields $\beta < 1$. While collaboration is thus socially preferred over selfish behavior, the problem is that collaboration exposes the player to opportunistic behavior by the other player. In particular, $(1+\alpha)+\theta_i > 1$, so that if the other player cooperates, each player has an immediate temptation to take advantage of the other player.

The key addition to the model are the shocks to the benefits of adaptative behavior and the costs of accommodating that. As a result, instead of simply playing (C,C), it may be optimal to play (C,A) or (A,C), depending on the costs and benefits of such coordinated adaptation. This captures the idea that, in addition to simply routinized cooperation, value can be created by a coordinated response to market opportunities that may arise. However, to realize that value requires a concerted move by all members of the organization. In the absence of that, those benefits cannot be realized.

To restrict our attention to games that capture the above logic, I assume the following:

Assumption 1:
(i) $(1+\alpha)+2\overline{\theta} > 2 > (1+\alpha) > 2\beta > (1+\alpha) - 2\overline{\theta}$
(ii) $(1+\alpha) - \overline{\theta} > 1 > \beta > \overline{\theta}$

The first line determines the socially optimal actions. In short, the optimal policy is to cooperate unless one player has a high benefit to adapting and the other player faces a low cost of cooperating. Further, such coordinated adaptation dominates mutually selfish behavior (in other words, coordinated adaptation is better than uncoordinated adaptation), except if the player adapting has a low benefit to adaptation and the other player has a high cost of accommodation. The second line looks at the game from an individual player's perspective and simply states that the selfish/adaptive action is a dominant action for the player, independent of the realized costs of accommodation. These assumptions restrict the feasible parameters to $\alpha \in (1/3, 1), \beta \in (1/3, \frac{1+\alpha}{2})$ and $\overline{\theta} \in (\max\left(\frac{1-\alpha}{2}, \frac{(1+\alpha)-2\beta}{2}\right), \min(\alpha, \beta))$.

Before the choice of action, the players may engage in information acquisition about the states of the two actions and share that information or make recommendations for the action plan. Acquiring information is costly and given by $c_L \leq c_M \leq c_H$, where the first is the cost of learning the state to the specific agent, the second is the cost of learning either state to the third player (superior) who is not directly involved in the productive process itself, and the third is the cost for a subordinate to learn the state of the other task. Regarding the cost, I assume the following:

Assumption 2:
(i) $c_L \leq \overline{\theta}/2$
(ii) $c_H >> c_M$

The first assumption simply restricts the set of cost parameters that I consider in a way that all situations where information acquisition is actually desirable are covered while also allowing us to highglight the problem that sometimes the agents may engage in information acquisition even when it is socially undesirable. The second assumption restricts the set of feasible solutions by assuming that it is never part of an equilibrium for a subordinate to directly learn the information regarding both tasks. This assumption is relaxed in parallel work, which leads to additional possible coordination mechanisms.

In addition to learning the value of adaptation for a given task, the player who invested c_i learns also the realization of a random variable $\omega \in [0, 1]$, which can be considered as payoff-irrelevant facts about the state of the world. If a player does not make the investment, he/she remains uninformed about both θ_i and ω.

Remark: While information acquisition is costly to all players, I am agnostic as to the source of that information. In other words, a lot of the relevant data for decision-making may reside at the local task level, but the problem is that analyzing that data and converting it into information is

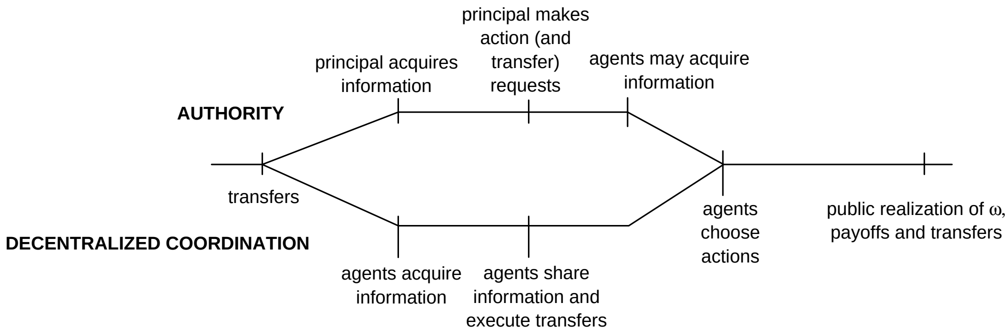

Figure 2: Timeline of the stage game.

costly. Therefore, it is possible that a subordinate may be tasked to pass on any raw data from his operations to someone else for processing, without the subordinate actually gaining an understanding of the content of that information. In other words, even if data is increasingly available, a "new scarcity has been created: the scarcity of human time for attending to the information that flows in on us." (Simon 1947/1997:23).

Following the information acquisition (or data processing) stage, the players are free to communicate with each other and make any transfers with each other at any time before and after the decision-making stage, with the limitation that nothing is formally contractible. In addition, players who initially chose not to acquire information may acquire that information later should they want to do so. This general game form leads to two particular equilibrium structures for the process of information acquisition and communication, as illustrated in Figure 2.

Decentralized coordination: Under decentralized coordination, the two subordinates acquire information about their tasks, share that information with each other and then execute a plan based on the information communicated. There is no role for the superior since the subordinates will be in possession of all the relevant information.

Authority: Under authority, the superior expends effort to acquire information, after which she communicates action plans to the subordinates. At this stage, the subordinates may acquire information themselves, after which they choose their actions. For an authority relationship to exist, it must be that the subordinates choose not to acquire information and instead follow the instruction of the superior. That is, the "roles played by two persons over a period of time ... involve an *expectation of obedience* by the one and a *willingness to obey* by the other." (Simon 1947/1997:180)

Independent of the exact coordination mechanism, the decisions are followed by the realization of the subordinates' payoffs and a public realization of the "facts," ω. The purpose of these "facts" is to simplify the analysis by eliminating the possibility of private histories. In particular, while deviations in the actions can be detected naturally, deviations from information acquisition may not be detectable in this simple setting because of the ability to simply guess the state. Efficient strategies may then be more complicated and history-dependent. But by introducing this state of "facts," and expecting its disclosure as a part of the communication stage, even if deviation may still take place, it will be caught at the end of the period because the disclosed facts do not match the real facts that have become apparent.

Because of these assumptions, we can apply Levin (2003) and note that for any efficient equilibrium of the game, there is exists a stationary equilibrium that achieves the same payoff. Thus, I will refrain from introducing any additional notation regarding histories and note simply that we will be looking for the best stationary equilibrium that can be sustained under either coordination mechanism. The game is infinitely repeated and all the players discount the future at a common rate, $\delta < 1$. The off-path punishment is provided by the threat of Nash reversion (breakdown of collaboration), which in this game matches the min-max payoff of the stage game.

4 Attainability of the first-best decision rule

From the payoff structure, we can immediately derive that the first-best decision rule has the players coordinating unless one player has a high benefit to adapting and the other has a low cost of coordinating. In this case, the expected payoff (for each player) can be written as $u_i^{\text{info}} = \frac{3+\alpha}{4} + \frac{1}{2}\overline{\theta} - c_i$. But if information is too costly, it may be optimal not to acquire information in the first place and just focus on playing the cooperative action, as in a classic prisoner's dilemma. In this case, the stage-game payoff would be simply $u_i^{\text{no-info}} = 1$. Then, information acquisition is desirable as long as $u_i^{\text{info}} \geq u_i^{\text{no-info}}$, or $\frac{1}{2}\overline{\theta} - \frac{1-\alpha}{4} \geq c_i$.[10] These optimal decision rules as a function of the states are summarized in the following matrices (using notation H for $\overline{\theta}$ and L for $-\overline{\theta}$ to indicate high and low gain to adapting, respectively):

		Player 2	
		H	L
Player 1	H	(C,C)	(A,C)
	L	(C,A)	(C,C)

first-best decision rule with information - $u_i^{\text{info}} = \frac{3+\alpha}{4} + \frac{1}{2}\overline{\theta} - c_i$

		Player 2	
		H	L
Player 1	H	(C,C)	(C,C)
	L	(C,C)	(C,C)

first-best decision rule without information - $u_i^{\text{no-info}} = 1$

The next question is then under what conditions either of the coordination mechanisms is able to sustain the first-best decision rule. But before considering that, I will summarize some terminology that will facilitate the discussion that follows. First, the game itself can, each period, be in one of four states. In state (H,H), both players have a high gain to adapting, in state (L,L), both players have a low gain to adapting, and in the (H,L)/(L,H) states, one player has a high gain to adapting while the other player has a low gain to adapting (and so low cost to accommodating). For each state, the action profile can take the form of (C,C), which I will call mutual coordination, (C,A)/(A,C), which I will call coordinated adaptation (with one agent adapting and the other accommodating that adaptation), or (A,A), which I will call uncoordinated adaptation. At parts, I

[10] As an aside, due to the payoff structure assumed, acquiring a single piece of information is always suboptimal.

will refer to both (C,C) and (A,C) as collaboration, since they create value above (A,A), and (A,A) as fully selfish behavior.

4.1 Attainability of the first-best decision rule under decentralized coordination

Consider first the decentralized coordination mechanism, and assume that information acquisition by the agents is desired. There are five different stages in the game where we must make sure that incentive-compatibility holds. Working backwards, these constraints are:

(i) ex post transfer constraint: once the decisions have been made and payoffs realized, it needs to be in the agents' interest to complete any promised transfers. Given the stationary nature of the game, this transfer constraint can be written as $w_{i,j} \leq \delta \Delta V$, where ΔV is the reduction in the continuation payoff following a deviation.

(ii) decision constraint: at the decision stage, the choice of the agent needs to be incentive-compatible, conditional on expected outcome and resulting transfers. Given that A is the dominant action in the stage game, this constraint may bind whenever a player is called to play C, but where the gains to deviating will depend both on the agent's state and his expectations about the other agent's play.

(iii) communication constraint: at the communication stage, if communication is desired, the agents need to be willing to disclose their information truthfully, given the expected associated transfers τ_i and the impact of the message on the continuation of the stage game.

(iv) information acquisition constraint: at the information acquisition stage, it must be optimal for the agents to acquire the signal instead of communicating and choosing without information.

(v) ex ante transfer constraint: before any actions taken by the players, it must be incentive-compatible to make any transfers agreed upon.

Given that the efficient equilibrium actions will be symmetric, no ex ante transfers are needed in any equilibrium, and we can focus on only the first four constraints. Further, the role of ex post transfers is simply to relax any of the earlier constraints so that can also be subsumed in the analysis, and we are left with the three constraints for information acquisition, communication and decisions. The attainability of the first-best decision rule is summarized in the following proposition:

Proposition 1 ***Attainability of the first-best decision rule under decentralized coordination:***
Under decentralized coordination and positive information acquisition, the binding constraint is inducing mutual coordination when the returns to adaptation are high. The equilibrium is sustainable as long as

$$\delta \Delta V^{FB\text{-}info} \geq \alpha + \overline{\theta} \Leftrightarrow \delta \geq \delta^{fb\text{-}dec} = \frac{(\alpha+\overline{\theta})}{\Delta u_i + (\alpha+\overline{\theta})},$$

where $\Delta u_i = \frac{3+\alpha-4\beta}{4} + \frac{\overline{\theta}}{2} - c_L$, *the net surplus created by the relationship.*

Proof. See Appendix A.1 ■

Intuitively, the largest deviation temptation for the agents arises when they are supposed to coordinate but could gain a lot by choosing an adaptive action due to high returns. All the other constraints can be satisfied by appropriate use of incentive-compatible transfers. As to the sustainability of the equilibrium itself, it is immediate that the threshold patience level is increasing in both β and c_L, which reduce the surplus created by the relationship. With respect to α and $\overline{\theta}$, their effect is ambiguous because they both increase the deviation gain and the surplus created by the relationship.

The above, however, applies only to the case where information acquisition is actually desired. The downside of information is that it exposes the organization to opportunism, and even when used appropriately, the value created needs to be weighted against its cost. If information is sufficiently costly, then it will be optimal to not acquire information and simply choose to play (C,C). But in this case, the players also face two potential temptations. First, they may simply choose to deviate to (A), independent of the underlying state. Second, contrary to what is expected, they may choose to acquire information and then deviate if the gains to deviating turn out to be high. The attainability of the uninformed coordination equilibrium is summarized below:

Proposition 2 ***Attainability of the first-best decision rule under decentralized coordination (2):***
Under decentralized coordination and no information acquisition, the binding constraint is having the agents remain uninformed. The equilibrium is sustainable as long as

$$\frac{\delta}{2}\Delta V^{FB\text{-}noinfo} \geq \frac{\alpha+\overline{\theta}}{2} - c_L \Leftrightarrow \delta \geq \delta^{fb\text{-}dec\text{-}noinfo} = \frac{\alpha+\overline{\theta}-2c_L}{\Delta u_i+\alpha+\overline{\theta}-2c_L},$$

where $\Delta u_i = 1-\beta$, *the net surplus created by the relationship*

Proof. See Appendix A.1 ■

In this case, sustaining the equilibrium continues to become harder in β, which influences the surplus created by the relationship, but now the sustainability is also unambiguously decreasing in α and $\overline{\theta}$, which only affect the gains to reneging. Further, the relationship becomes more sustainable in c_L, because, instead of reducing the surplus created when information acquisition is expected, it is now reducing the gains from deviating (since information acquisition is not expected). Comparing the two, we obtain the following corollary:

Corollary 3 ***Sustainability of the first-best decision rules:***
When no information acquisition is preferred, sustaining no information acquisition is easier than sustaining the first-best decision rule under information acquisition. That is, for $\frac{1}{2}\overline{\theta} - \frac{1-\alpha}{4} \leq c_i$, $\delta^{fb\text{-}dec\text{-}noinfo} < \delta^{fb\text{-}dec}$.

This observation simply highlights that when the uninformed equilibrium is both desired and becomes unsustainable, it cannot be replaced by the informed equilibrium under a first-best decision rule. But conversely, when the first-best decision rule is not attainable, the second-best solution may be to forego information acquisition altogether to limit the deviation temptation.

4.2 Attainability of the first-best decision rule under authority

Having derived the conditions under which the first-best decision rule can be attained under decentralized coordination, the next step is to consider under what conditions the authority structure can obtain the same. But before that, it is important to consider how the authority structure can add value despite the subordinates being responsible for task execution and being free to acquire additional information in any stage they would like.

The key behind the value created comes from the method of communication. To obtain the first-best decision rule, the organization needs to have information about both tasks and to aggregate that into an appropriate plan of action. But when the agents are responsible for information acquisition, this social value of information gets confounded by the additional knowledge the agents gain about profitable opportunities for deviating from the plan. Thus, the authority structure is beneficial only when (i) the instructions delivered to the agents are able to convey the social value of information by providing the appropriate plan of action, while limiting knowledge about profitable deviations, and (ii) the information provided is sufficient so that the agents do not want to engage in further investigation of their tasks and thus discover profitable deviations themselves.

To this end, note that the superior has effectively three ways of delivering the instructions. The first involves disclosing a plan of action, together with the underlying state. But this would replicate the information structure under decentralization as the agents, and so that cannot be optimal. The second option is an "open" communication protocol, where the superior shares the joint action plan with both subordinates, i.e. discloses to both subordinates (C,C), (C,A)/(A,C) or (A,A), and the third option is a "closed" communication protocol, where the superior shares only the part of the plan relevant to a given subordinate, i.e. instructs each subordinate to perform either (C) or (A). Comparison of the latter two protocols gives us the following lemma:

Lemma 4 *The closed communication protocol, where each subordinate is only disclosed their part of the plan, dominates the open communication protocol.*

The proof of the lemma is immediate, by noting that the closed communication protocol pools the incentive constraints of the plans (C,C) and (C,A), which makes the relationship weakly more sustainable. But the lemma contains a valuable simple insight: the efficiency of an authority structure is maximized when the subordinates are disclosed the smallest amount of information necessary for the subordinates to perform their tasks, because that minimizes the subordinates' temptation for strategic opportunism.

The second preliminary observation, which again impacts the set of potential solutions that we need to examine relates to the timing of transfers. To this end, we can observe the following:

Lemma 5 *Under the authority structure, all transfers (both between the subordinates and between a subordinate and a superior) take place ex post, after the realization of the payoffs.*

The reason for this result is that it is only at the last stage that the behavior of the players can be confirmed and so rewarded. Importantly, the use of ex ante or interim transfers to expand the amount of value available for redistribution in the ex post stage is not possible. The reason is that any such ex ante or interim 'deposit' to the system simply creates a new incentive-compatibility constraint for the party who is supposed to pay that money back, the satisfaction of which requires reallocation of rents that negate the value of the transfer. For example, suppose the subordinates deposited ex ante some money with the superior to be used as rewards in the ex post stage. But then the superior has always the option of walking away with the money, requiring a reallocation of surplus from the subordinates to the superior, which lowers the continuation value of the subordinates by an amount exactly equal to the original deposit, nullifying its effect as a compensation tool for later.

Now, following these two simple observations, the authority structure needs to satisfy three constraints to ensure that the proposed actions constitute an equilibrium

(i) ex post transfer constraint: As under decentralized coordination, the transfers from the subordinates are bounded above by the continuation value, $\delta \Delta V$. The additional constraint is that the transfers to the principal must equal, in expectation, the cost of effort: $2c_M = E\sum_{i=1,2} c_i$, where c_i are the transfers from the subordinates to the superior.

(ii) Obedience constraint: Having received the instruction, the subordinate prefers to execute the instruction instead of choosing directly against it. In the present setting, this amounts to simply an agent receiving an instruction (C) and obeying it instead of directly choosing (A).

(iii) Second-guessing constraint: Having received the instruction, the subordinate must also prefer to execute the instruction instead of choosing to further investigate his own state and disobeying the instruction if found to be beneficial. In the present setting, this amounts to the agent receiving an instruction (C) and obeying instead of investing in information acquisition and, if he finds the benefits to adaptation to be high, disobeying the instruction.

Given these constraints, we can summarize the attainability of the first-best decision rule under authority as follows:

Proposition 6 ***Attainability of the first-best decision rule under authority:***
Let $\Delta u_i = \left(\frac{3+\alpha-4\beta}{4} + \frac{\overline{\theta}}{2} - c_M\right)$*, the per-period surplus created by the relationship.*
(i) When c_L *and* c_M *are low-enough (*$3c_L + c_M \leq \frac{2\alpha-\beta+5\overline{\theta}}{4}$*), the binding constraint is the second guessing constraint and the decision rule is sustainable as long as*

$$\delta\Delta V \geq \left(\alpha + \overline{\theta}\right) - 3c_L \Leftrightarrow \delta \geq \delta^{fb\text{-}2ndguess} = \frac{\left(\alpha+\overline{\theta}-3c_L\right)}{\Delta u_i + \left(\alpha+\overline{\theta}-3c_L\right)}.$$

(ii) When c_L *and* c_M *are high-enough (*$3c_L + c_M > \frac{2\alpha-\beta+5\overline{\theta}}{4}$*), the binding constraint is the obedience constraint and the decision rule is sustainable as long as*

$$\frac{4}{3}\left(\delta\Delta V - c_M\right) \geq \frac{2\alpha}{3} + \frac{1}{3}\left(\beta - \overline{\theta}\right) \Leftrightarrow \delta \geq \delta^{fb\text{-}obedience} = \frac{\frac{\left(2\alpha+\beta-\overline{\theta}\right)}{4} + c_M}{\Delta u_i + \frac{\left(2\alpha+\beta-\overline{\theta}\right)}{4} + c_M}.$$

Proof. See Appendix A.? ■

The proposition then also leads to the following immediate corollary:

Corollary 7 ***Attainability of the first-best under decentralized coordination and authority:***
As long as $(c_M - c_L)$ *is not too large,* $\delta^{fb\text{-}dec} \geq \max\left(\delta^{fb\text{-}2ndguess}, \delta^{fb\text{-}obedience}\right)$*. That is, there is a range of patience levels under which the first-best decision rule can be attained under authority but not under decentralized coordination (with informed decision-making).*

In other words, authority is able to achieve the first-best decision rule for a wider range of discount rates than decentralized coordination, as long as the cost-disadvantage of authority is not too large. To understand the sources of this advantage, they all follow from changing in the informational environment that the subordinates are facing when making the decision. Recall that under decentralized coordination, the specialist agent knows that when called to play (C,C) when his gains from adaptation are high, he could gain $\alpha+\overline{\theta}$ by deviating from the agreement. In contrast, under authority, he is only told to play (C). When told to play (C) under the first-best decision rule, he attributes only a probability $\Pr(H|C) = \frac{1}{3}$ to having a high benefit to adapting. Further, he attributes a probability $\frac{2}{3}$ to the other player also coordinating while a probability $\frac{1}{3}$ to the situation where he is actually accommodating adaptation by the other agent. These changes in the beliefs that the agent has have three interrelated benefits.

First, for the obedience constraint, the added uncertainty over the exact state lowers the deviation payoff to $\frac{2\alpha}{3} + \frac{1}{3}\left(\beta - \overline{\theta}\right)$ instead of $\alpha+\overline{\theta}$ because of the uncertainty over both the gain from adapting and the play of the other agent. Second, while the agent could go and acquire further information to see if deviation is actually worthwhile, the fact that the recommendation of C lowered the probability of high gains of adapting down to $\frac{1}{3}$, the expected value of that information acquisition is lowered (relaxing the no-second guessing constraint). Third, and related to the first, when the realized states call for the play of (A,C), then it is efficient for the principal to instruct the adapting agent

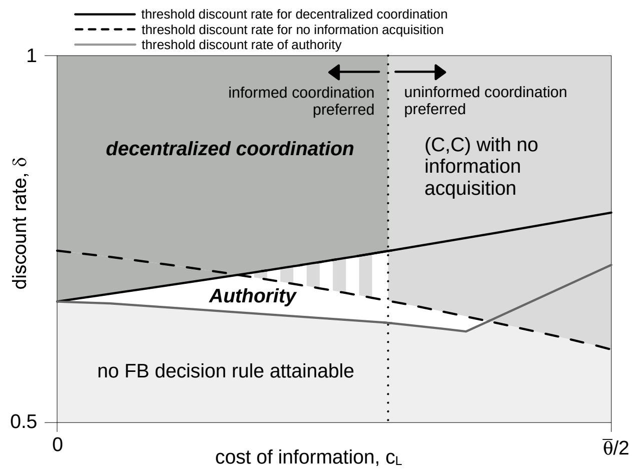

Figure 3: Attainability of a first-best decision rule: $\alpha = \beta = 0.6, \overline{\theta} = 0.5$ and $c_M = 1.2c_L$.

to compensate the accommodating agent for the compromise. So when the player is called to play (C), there is $\frac{1}{3}$ chance that there is such an additional reward waiting as long as he chooses to play according to the realized strategies, which further relaxes the reneging temptation. Indeed, this is the reason why the left-hand side of the obedience constraint has a coefficient $\frac{4}{3}$ instead of 1 in the proposition - we are leveraging the surplus created in one state to relax the overall deviation temptation.

Incindentally, going back to the optimality of the closed communication protocol, suppose that the superior used an open communication protocol which reveals (C,C) or (C,A). Then, the binding constraint would be the state (C,C), where now (i) the agent has no expectation of additional compensation going forward (elimininating the third channel) and (ii) the agent attributes probability $\frac{1}{2}$ to actually having high gains to adapting, increasing the value of further information.

Two final observations are as follows. First, if information becomes sufficiently cheap, then it becomes increasingly tempting for the agents to still check what their actual state is because even if that information is likely to be wasted, it doesn't cost so much and can always be ignored. Second, the above discussion relates to the comparison across the two informed equilibria. Sometimes, it is optimal to keep the agents uninformed and simply coordinate their actions. For that case, there is no clear comparison in the sense that when no information acquisition is desired but is no longer sustainable, the authority structure may or may not be sustainable as a potential second-best solution. An illustration of the attainability of the first-best decision rule is given in Figure 3.

5 Second-best equilibria when the first-best decision rule is not attainable

The discussion so far has only considered the attainability of the first-best decision rule. But what this illustrates is only that when decentralized coordination is able to attain the first-best decision rule, that is the globally efficient solution and thus the preferred outcome. But we don't yet know if there exists a second-best outcome under decentralized coordination that is able to dominate authority (because of the cost disadvantage of authority) and, overall, what is the best sustainable equilibrium when the first-best decision rule cannot be sustained under any structure. The derivation of the efficienct equilibria is undertaken in this section for the two coordination mechanisms and they are compared in the next section. The basic logic behind the construction is that as cooperation becomes unsustainable, the organization needs to accept inefficient decision-making in some states of the world, and the situation becomes increasingly worse as the agents become increasingly impatient until no collaboration can be sustained.

5.1 Decentralized coordination

To construct the optimal equilibrium when the first-best is not sustainable, we can build the general intuition by making a few broader observations regarding the game. The first two observations are preliminary and simplify the discussion that follows. First, given the assumptions regarding information and transfers and the payoff structure of the game, we can focus on symmetric, stationary equilibria. Second, given the access to transfers, the communication constraint is not binding in any of the potentially optimal equilibria. In other words, transfers can be used to signal private information without loss of efficiency, as in Kolotilin and Li (2018).

Having been left with the decision constraints and the information acquisition constraint, the main point is then simply how to manage them. While the analysis is somewhat arduous, the logic is straightforward and the flow chart for the sustainability of equilibria is illustrated in Figure 4, where H and L continue to denote high and low value to adapting.

To understand the logic behind the solution, suppose first that information is sufficiently cheap so that the only constraint that ever binds is the decision constraint, which the subordinate faces whenever he is called to play (C). However, as is immediate, the gain to deviating from (C) depends on both whether the subordinate faces a high or low gain to adapting and whether the other subordinate is coordinating or adapting himself. In particular, above we established that the first constraint to bind is having the subordinates to play (C,C) in the state (H,H), where both subordinates have a high value to adapting. But by implication, the decision constraint remains slack in all the other states.

Focusing then first just on the state (H,H), the fact that (C,C) can no longer be sustained leads the organization to switch to using (C,A) or (A,C) instead, while maintaining the first-best decision rule for the other three states. By assumption 1, such coordinated adaptation is still better than

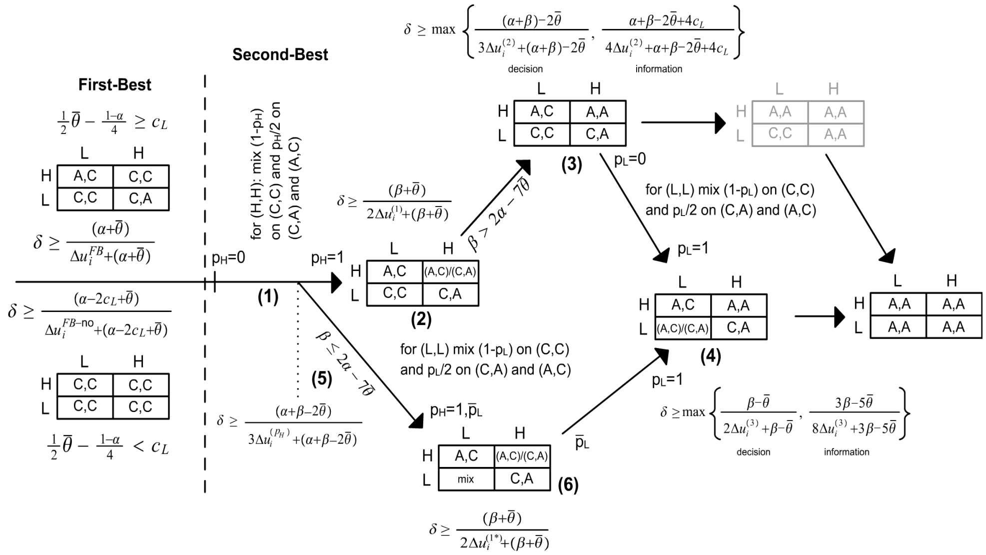

Figure 4: Equilibria sustainable under decentralized coordination

mutually selfish behavior and thus generates more value. And while the deviation temptation is still there, it is now muted because the subordinate who accommodates the adapting subordinate can now expect a compensating transfer after the actions, and so the action profile can be sustained for a wider range of discount factors than the first-best decision rule. However, when the subordinates become even more impatient, such coordinated adaptation profile can also no longer be supported and the organization needs to give up on any value creation in (H,H), allowing for profile (A,A) instead. Exactly the same logic applies for the state (L,L). While the fact that the gains to adapting are lower than in the case of (H,H), eventually the subordinates are impatient enough that even this temptation becomes too strong. Then, the decision rule again must allow the agents to switch, first, to (C,A) or (A,C) and, eventually, to (A,A). And once a subordinate is unwilling to take the accommodating action even when his cost of accommodating is low and he can expect the compensating transfer from the adapting subordinate, by implication collaboration also collapses in the states (H,L) and (L,H) and the repeated game is unable to generate any additional surplus. We will thus observe a gradual destruction of collaboration as the subordinates become increasingly impatient.

Now, we can refine this general logic by making four additional observations. First, whether in the state (H,H) or (L,L), the transition from mutual coordination to coordinated adaptation causes a discrete drop in the deviation temptation and thus a slack decision constraint. Thus, the subordinates can do better by using, instead of a pure strategy, a mixed strategy where each subordinate is uncertain whether the other subordinate will actually coordinate or adapt. While attaining the right information structure can be challenging in face-to-face communication, it is straightforward to achieve through mediation, which creates an additional role for the superior: each subordinate communicates his private information to the superior, who then issues instructions based on the agreed-upon mixed strategy, which slowly lowers the likelihood of recommending (C,C) in a way

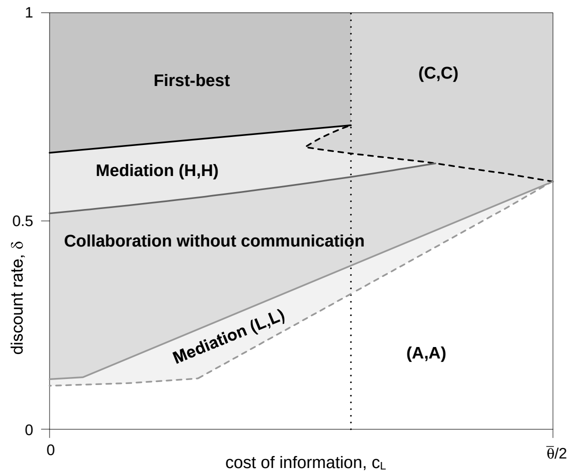

Figure 5: Second-best equilibrium under decentralized coordination, $\alpha = \beta = 0.6, \overline{\theta} = 0.5$.

that keeps the decision constraint just satisfied. The reason is that by facing some probability that the other subordinate plays (A) and the associated compensatory transfer, the decision constraint can be appropriately relaxed. Second, similar beneficial uncertainty cannot be generated when transitioning from (C,A) to (A,A). The reason is that since the subordinate now knows that the other subordinate always adapts, the only way to restore incentive-compatibility is to allow the first subordinate to adapt as well.

Third, while the first binding constraint is always the cooperation constraint in (H,H), the order in which rest of the decision constraints come to bind depends on the exact parameters. This is illustrated with the two branches of the Figure 4. In the upper branch, it is the accommodation constraint of (C,A) in state (H,H) that comes to bind before the coordination constraint in (L,L), and in the lower branch the order is reversed. But both paths lead to the eventual collapse of collaboration, which occurs when the collaboration constraint for low cost of accommodation becomes binding (stage (4) in Figure), after which any collaboration becomes impossible.

Fourth, note that as the agents become increasingly impatient, the remaining decision constraints involve lower and lower absolute gains to deviating, while the fact that the equilibrium decision rule becomes increasingly inefficient, the value of information decreases. Thus, if information is sufficiently costly, the information acquisition constraint may become the binding constraint.

A typical solution of the optimal equilibrium is illustrated in Figure 5, which plots the optimal equilibrium decision rule as a function of both the cost of information and the patience of the subordinates, in this case for parameters that follow the upper branch of Figure 4. When the players are sufficiently patient, they can achieve the first-best decision rule, either under information acquisition or without it. Just below that, the subordinates will use a mediation mechanism for the (H,H) state to introduce appropriate degree of coordinated adaptation to sustain the decision constraint, while right around the threshold where uninformed coordination becomes preferred, the organization prefers that equilibrium. Further decrease in patience leads to the collapse of any collaboration in (H,H), and leads to the region of collaboration without communication (step (3) of

Figure 4) In this region, the subordinates simply follow their private signals: high value to adapting leads to adaptation and low value leads to coordination. Some value is still created, but without any need for communication. Finally, even the coordination constraint in the (L,L) state becomes binding, after which similar mediation is used to introduce coordinated adaptation in the (L,L) state to sustain some collaboration, until even that becomes unsustainable and no further collaboration is possible: both subordinate simply chooses the selfish action with no information acquired. The kinks, in turn, represent the transition from a binding decision constraint to a binding information acquisition constraint.

5.2 Authority

The second-best solution under authority follows a logic similar to the case of decentralized coordination. In particular, once the superior cannot trust the obedience of the subordinates under the first-best decision rule, she will need to start adjusting the equilibrium decision rule in a way that restores the subordinates' incentive-compatibility to obey and not to engage in second-guessing of the issued instructions. The only (and simplifying) difference to the case of decentralized coordination is that since the information of the subordinate is compressed simply to a recommendation to either (A)dapt or (C)oordinate, there are only two incentive-compatibility constraints that we need to keep track of (the obedience constraint and the second guessing constraint), as opposed to the multiple decision constraints that the subordinate can face under decentralized coordination.

The important corollary of the above is that the authority of the superior is always limited by the willingness of the subordinates to follow instructions. Such restrictions are recognized by effective superiors and they keep themselves from issuing instructions that would lead to the destruction of the authority relationship. In other words, not only do the subordinates delegate authority to the superior by their choice to obey, but the superior reciprocates that by not issuing orders that would lead to the dissolution of the mutually beneficial illusion of authority.

How the superior manages the decision rule is similar to decentralized authority, in that she will slowly give up on achieving mutual coordination in (H,H) by starting to introduce some coordinated adaptation in the form of (C,A)/(A,C) and, when even that is not enough, introducing purely selfish behavior (A,A). In addition, the superior may further start to give up on mutual coordination in (L,L) as well. The only key qualitative difference to decentralized coordination is that, since the subordinate will not be aware of his own state, the key through which the principal manages the obedience and second-guessing constraints is through the belief $\Pr(H|C)$ – how likely it is that the gains to adapting are actually high when called to coordinate. By first introducing mixing between (C,C) and (C,A) and later between (C,A) and (A,A), the superior pushes this belief downward and relaxes the two constraints. A key threshold to this process arises when the superior's decision rule places a weight 1 on playing (A,A) when the state is (H,H). At this point, the recommendation of the superior becomes perfectly revealing of the underlying state, in the sense that the subordinate is never called to play (C) when his gain to adapting is actually high. Conversely, mixing on the low state (L,L) actually pushes the subordinates' beliefs about his own state up, and is useful only in

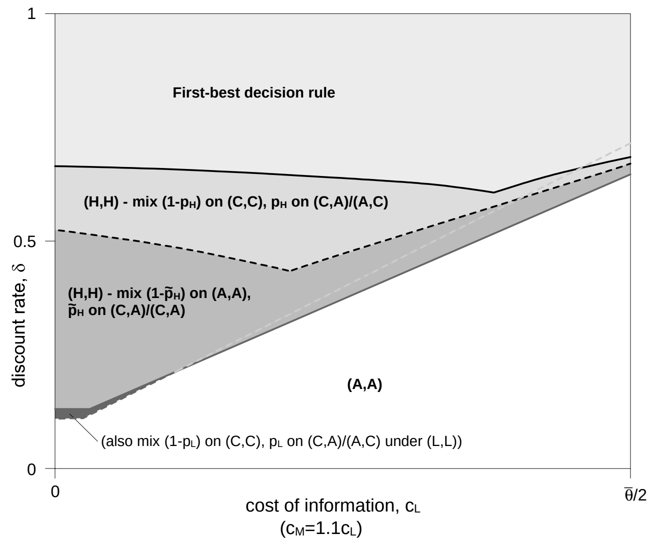

Figure 6: Second-best equilibrium under authority, $\alpha = \beta = 0.6, \overline{\theta} = 0.5$ and $c_M = 1.1c_L$

the same way as under decentralized coordination, by creating uncertainty over the behavior of the other subordinate.

An illustration of the solution is provided in Figure 6, which plots the second-best decision rule under authority for the same parameter values as the solution for decentralized coordination above. When the subordinates are patient enough, the first-best decision rule can be sustained. When that is no longer enough, the principal will begin to mix between recommending (C,C) and (C,A)/(C,A) in the state (H,H), to push the subordinates' beliefs about their gains to adapting down and to maintain incentive-compatibility. At the dashed line, placing any probability on playing (C,C) becomes suboptimal and, to continue to push the beliefs down, the superior switches to mixing between (C,A)/(A,C) and (A,A) to continue to push the beliefs down. As we will see below, this is a crucial difference to the decentralized coordination mechanism, where the subordinate's knowledge of his own state prevents such a smooth transition. For both regions, the superior is able to maintain the first-best decision rule for the other three states. Finally, when the subordinates are really impatient and information is not too expensive, the superior introduces some mixing even in the (L,L) state until any collaboration becomes impossible.

6 Equilibrium Organization

Having derived the basic features of the optimal second-best decision rule under both decentralized coordination and authority, we can finally compare the two in terms of their expected performance. The earlier discussion has already highlighted the basic advantages and disadvantages of the two. The main advantage of decentralized coordination is its ability to minimize the costs of information processing by relying on the local subordinates that specialize in the task at hand while its main downside is the fact that when the information is processed by the subordinates, the subordinates also learn information about the existence of profitable opportunities for opportunism, which makes maintaining collaboration difficult. In contrast, the main advantage of authority is its ability to control the information that is available to the subordinates through the use of instruction that is able to convey the social value of information embedded in the recommended action while hiding information about profitable opportunities for selfish behavior, thus facilitating collaboration while limiting the organization's exposure to opportunism, while the disadvantage lies in the additional costs of information processing that are faced by the subordinate less versed in the local conditions.

The resulting equilibrium choice is illustrated in Figure 7 for various cost differences of the form $c_M = (1+x)c_L$, continuing the example from the previous sections. The first panel illustrates the choice when the superior is nearly as efficient as the subordinate in processing the information regarding a particular task. In this case, following the discussion from above, decentralized coordination is preferred as long as it is able to achieve the first-best decision rule (whether informed or uninformed), but the moment the first-best decision rule is no longer sustainable, the authority structure becomes preferred, either because it is still able to achieve the first-best decision rule, or, because while deviating from the first-best decision rule, the ability of the superior to manage the information available to the subordinates helps to sustain a decision rule that is more efficient than that under decentralized coordination. But, returning to the above, this informational advantage is slowly eroding as the subordinates are becoming increasingly impatient and the instruction issued to the subordinate becomes increasingly revealing of his underlying state. The crucial boundary occurs at the boundary where the no-communication equilibrium is just sustainable under decentralized coordination and the recommendations under authority converge to the same decision rule, where (A,A) is instructed with probability one in the state (H,H). At that point, the information advantage of authority is eliminated, and the remaining collaboration, which involves partial cooperation in state (L,L), is best achieved under the decentralized coordination mechanism: since the information advantage of authority is eliminated, the only component that matters is the cost paid for information acquisition, which is minimized under decentralized coordination.

As we increase the cost advantage of decentralized coordination, the preference for authority starts to naturally shrink, beginning at the boundaries where the decision rules are otherwise identical. But authority retains a role for even quite high cost disadvantages. The reason is the particular disadvantage of decentralized coordination, whereby there is a discrete drop in the performance between the boundary of mediating the (H,H) state and the collapse to the equilibrium with collaboration without communication. Recall that this drop occurred because the knowledge of high costs to accommodation made it impossible to transition smoothly from the coordinated adaptation boundary of (A,C)/(C,A) to the selfish behavior (A,A). Once (C,A) became unsustainable in (H,H), the decision rule discretely collapsed to (A,A). In contrast, authority, by keeping the subordinate uninformed of the state, is able to achieve this transition and smoothly push the belief $\Pr(H|C)$ to zero by slowly increasing the frequency at which (A,A) is played. This is the reason why the preference for authority shrinks towards the mediation threshold.

The basic outcome of the game is then a non-monotone relationship between the choice of coordination mechanism and the patience of the players. Decentralized coordination is preferred for both high and low patience levels, but for very different reasons and with very different performance

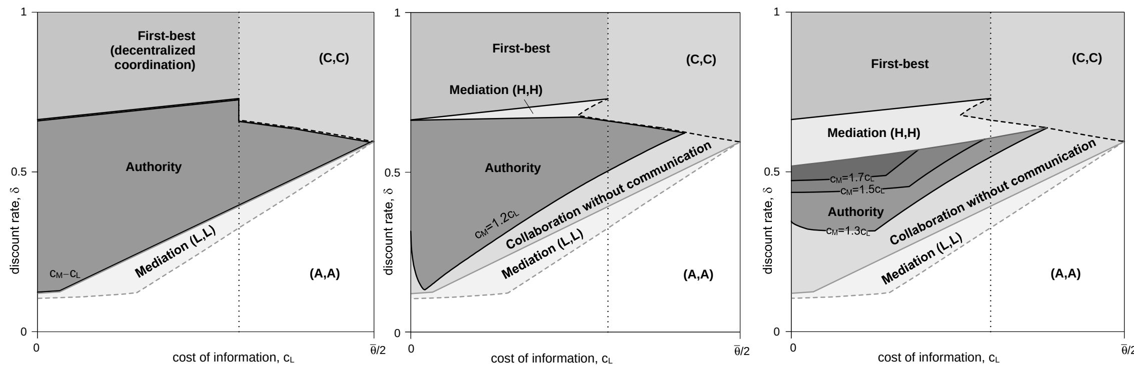

Figure 7: Preferred coordination mechanism and the cost advantage of decentralized coordination, $\alpha = \beta = 0.6, \overline{\theta} = 0.5$.

implications. For high patience levels, decentralized coordination is preferred because it is able to achieve the first-best decision rule, while for low patience levels, the subordinates are so impatient that an authority relationship can no longer effectively manage the information available to the subordinates and so the only thing that matters is minimizing the costs of information processing, even if the behavior is quite disfunctional. Authority is preferred only for intermediate levels of patience, where the ability to strategically manage the information available to the subordinates creates enough value, with the preference for authority naturally decreasing in the cost disadvantage of authority.

7 Conclusion

The main propositions of this manuscript are two-fold. First, it proposed a formalization of the classic notion of authority (as the right to instruct others and to expect obedience) as an equilibrium information structure of a game that satisfies the two basic characteristics associated with it in the literature: (i) a superior instructs subordinates as to what to do and (ii) subordinates take those instructions and execute them, without further evaluation. In particular, by allowing the subordinates to access information at any part of the game, should they choose to, the analysis highlighted that a crucial part of a sustainable authority relationship is the choice by the subordinates *not* to become informed themselves, which may be equally if not more difficult to satisfy than to have the subordinates to become informed. These obedience and second-guessing constraints limit the scope of authority and smart superiors understand that, so that while the subordinates delegate authority to them by the voluntary act of submission, the superiors reciprocate that by issuing instructions that will not threaten the sustainability of the relationship.

Second, it applied this notion of authority to analyze how it can help the organization to solve coordination problems by contrasting the relative performance of decentralized coordination and authority in solving a simple repeated coordination problem. The key observation was the dual role of information when available to the subordinates: while creating the information was necessary to achieve appropriate decisions, direct access to such information by the subordinates also informed them of opportunities for opportunistic behavior, which led to its eventual downfall. The main advantage of the authority structure was the ability to decouple these two roles of information. By issuing instructions, the superior could convey the socially valuable information embedded in the instruction itself while hiding the information about possible opportunities for selfish gain. By conveying the socially valuable part of the information, the instruction then could also keep the subordinate from acquiring information and instead simply obey the instruction, thus sustaining collaborative behavior without exposing the organization to opportunism. However, the benefits of authority were still bound by the incentive-compatibility of the subordinates' behavior, so that when the subordinates were sufficiently impatient, authority relationship could not create value over the decentralized coordination mechanism. Conversely, when the subordinates were sufficiently patient and so could restrain themselves from opportunism even under decentralized coordination, no authority relationship was needed in the first place.

While the basic message of the analysis is quite straightforward, the framework was intentionally highly simplified and a number of additional questions remain that may shed further light on the value and sustainability of authority relationships. First, the analysis assumed that the cost for a subordinate to learn the state of the other subordinate was too high to be ever optimal in equilibrium. This precluded the possibility that one of the subordinates may arise endogenously as the "superior." Such solution would be a blend of the two arrangements considered here, with one subordinate being in possession of both pieces of information but being still able to control the information of the uninformed subordinate through his instructions. Second, the analysis assumed that no formal contracting is possible, which left open the potentially important question of how the authority of expertise, as considered here, may interact with the authority of sanctions. Third, and related, because of the assumption that the productive task and its payoffs were inseparable from the subordinates, the superior in the present setting faced no conflict of interest with the subordinates, which is unlikely to be the case in many practical situations, potentially complicating the sustainability of the authority relationship. Finally, to allow for the derivation of the optimal solution when the first-best could not be achieved, the analysis made very strong informational assumptions by assuming that everything could be perfectly observed at the end of every period. An interesting question for future research is how informational frictions may affect the relative performance of the two coordination mechanisms considered. Relatedly, the analysis assumed access to voluntary transfers, which in practice may be limited, and the implications of which are currently unknown. In short, the present work has taken some first steps towards understanding the value and the nature of the classic notion of authority, but much work remains to be done to deepen our understanding of its benefits and potential downfalls.

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A Proofs and derivations

A.1 First-best decisions under decentralized coordination

A.1.1 First-best under information acquisition

We work through the potential IC constraints backwards. First, under the first-best decision rule, the continuation value is given by

$$\delta \Delta V = \tfrac{\delta}{1-\delta}\left(u^{cont} - u^{dev}\right) = \tfrac{\delta}{1-\delta}\left(\tfrac{3+\alpha-4\beta}{4} + \tfrac{1}{2}\overline{\theta} - c_L\right)$$

(i) incentive-compatibilit of ex post transfers: Once all actions are taken, any outgoing transfers are bounded by $w_{i,j} \leq \delta \Delta V$. This will hold for all configurations below and not repeated.

(ii) incentive-compatibility of equilibrium decisions: At the decision stage, there are two possible constraints that may bind. First, the players need to be willing to coordinate. Since any transfers would net out, the cooperation constraint under value of adapting is

$$1 + \delta V^{cont} \geq (1+\alpha) + \overline{\theta} + \delta V^{dev} \Leftrightarrow \delta \Delta V \geq \alpha + \overline{\theta}.$$

Correspondingly, the constraint to accommodate when the cost is low is

$$\overline{\theta} + w_{C,A} + \delta V^{cont} \geq \beta + \delta V^{dev} \Leftrightarrow \delta \Delta V \geq \beta - \overline{\theta} - w_{C,A}.$$

Now, from (i) the maximal transfer is $w_{C,A} = \delta \Delta V$, which allows us to write this constraint as

$$\delta \Delta V \geq \tfrac{\beta - \overline{\theta}}{2},$$

and comparing, we have $\alpha + \overline{\theta} > \frac{\beta-\overline{\theta}}{2} \Leftrightarrow 2\alpha + 3\overline{\theta} > \beta$. Now, smallest $\overline{\theta} = \frac{1-\alpha}{2}$, and so the inequality is always satisfied and the first binding constraint is the coordination constraint.

(iii) incentive-compatibility of equilibrium messages: In the communication stage, the agents can send payments τ_L, τ_H with the messages as an additional signal. The agent may deviate simply by not making the required payment or make a payment associated with a different message. Since a lie is always detected at the end of the period, the agent will always choose an adaptive action following a lie.

To ensure honesty, we have for the high value agent that

$$\tfrac{1}{2} + \tfrac{1}{2}\left((1+\alpha) + \overline{\theta} - w_{C,A}\right) - \tau_H + E(\tau) + \delta V^{cont} \geq \max\left\{\beta + E(\tau), \tfrac{1}{2}\left((1+\alpha) + \overline{\theta}\right) + \tfrac{1}{2}\beta - \tau_L + E(\tau)\right\} + \delta V^{dev},$$

where the deviations are to either not making a payment at all or sending the low message but adapting, and, of course, not making any further payments. As established below, it is without loss of generality to set $\tau_L = 0$, which allows us to write the constraint as

$$\frac{1}{2}(1-\beta) + \delta\Delta V \geq \tau_H + \frac{1}{2}(w_{C,A})$$

as the upper bound on the transfer. For the low value agent, we have

$$\frac{1}{2} + \frac{1}{2}\left(\overline{\theta} + w_{C,A}\right) + E(\tau) + \delta V^{cont} \geq \max\left\{\beta + E(\tau), (1+\alpha) - \overline{\theta} - \tau_H + E(\tau)\right\} + \delta V^{dev},$$

which gives the requirement that

$$\frac{1}{2}(w_{C,A}) + \tau_H \geq \frac{(1+2\alpha)}{2} - \frac{3}{2}\overline{\theta} - \delta\Delta V.$$

Now, suppose that $w_{C,A}$ is maximized and $\tau_H = 0$. We have

$$\begin{aligned}&0 \geq \frac{(1+2\alpha)}{2} - \frac{3}{2}\overline{\theta} - \frac{3}{2}\delta\Delta V\\ &6\overline{\theta} \geq (1-2\alpha),\end{aligned}$$

which is satisfied since $\overline{\theta} \geq \frac{1-\alpha}{2}$. So as long as the constraint (ii) is satisfied, truth-telling can be sustained just with ex post transfers.

(iv) incentive-compatibility of information acquisition: The last step is to make sure the agents want to acquire information as required. To this end, we can write the IC constraint as

$$\frac{3+\alpha}{4} + \frac{1}{2}\overline{\theta} - c_L + \delta V^{cont} \geq \max\{\beta + E(\tau), \frac{1}{2}(1+\alpha) + \frac{1}{2}\beta - \tau_L + E(\tau), (1+\alpha) - \tau_H + E(\tau)\} + \delta V^{dev}.$$

Now, since (ii) could be satisfied by just ex post transfers, let $\tau_i = 0$. Then the most attractive deviation is to a high message and we have

$$\frac{-1+\alpha}{4} + \frac{1}{2}\overline{\theta} + \alpha + \overline{\theta} \geq c_L$$

but information acquisition is efficient only if $\left(\frac{1}{2}\overline{\theta} - \frac{1-\alpha}{4}\right) \geq c_L$, so the inequality simplifies to

$$\alpha + \overline{\theta} \geq 0,$$

which is true and so the information acquisition constraint is also immediately satisfied. Thus, if we can attain the first-best, we can attain the first-best simply by using ex post transfers. Finally, to verify that this is the only range over which we need to ensure cooperation, we can show that when it is better to not acquire information, the not-acquiring constraint is more slack than the full information constraint for the first-best, done in the next subsection.

A.1.2 First-best: no information acquisition

If information is too costly, it is better not to induce any information acquisition. In this case, we can write the sustainability of the equilibrium as

$$1 + \frac{\delta}{1-\delta} \geq \max\left\{(1+\alpha) + \frac{\delta}{1-\delta}\beta, \frac{1}{2}\left((1+\alpha) + \overline{\theta} + \frac{\delta}{1-\delta}\beta\right) + \frac{1}{2}\left((1+\alpha) + \frac{\delta}{1-\delta}(1+\alpha)\right) - c_L\right\},$$

where the two deviations are choosing A without any further information and choosing to acquire information and deviating only if the gains to deviation turn out to be high. We can write these two constraints as

$$\frac{\delta}{1-\delta}(1-\beta) \geq \alpha \qquad \textit{and} \qquad \frac{\delta}{1-\delta}(1-\beta) \geq \left(\alpha + \overline{\theta}\right) - 2c_L,$$

with the first giving $\delta \geq \frac{\alpha}{1+\alpha-\beta}$ and the second giving $\delta \geq \frac{\alpha - 2c_L + \overline{\theta}}{1+\alpha-\beta-2c_L+\overline{\theta}}$. The second constraint is more binding as long as $\overline{\theta} \geq 2c_L$, which is the region considered. Thus, it is the acquisition constraint that will be binding in equilibrium. In relation to the above, to show that the no-information constraint is more slack than the information acquisition constraint when no acquisition is desired, note that the no-information constraint is relaxed in c_L while the other constraint becomes more binding. Thus, it is sufficient to show the ordering at the boundary at which no acquisition becomes preferred. At the boundary, we can solve the patience needed to sustain the equilibrium as

$$\frac{\delta}{1-\delta}(1-\beta) \geq \frac{1+\alpha}{2},$$

giving $\delta = \frac{1+\alpha}{3+\alpha-2\beta}$. While for the sustainability of the informed first-best equilibrium, we have

$$\frac{\delta}{1-\delta}\left(\frac{3+\alpha-4\beta}{4} + \frac{1}{2}\overline{\theta} - c_L\right) \geq \alpha + \overline{\theta},$$

and again evaluating at the boundary, we get

$$\frac{\delta}{1-\delta}(1-\beta) \geq \alpha + \overline{\theta} \rightarrow \delta = \frac{\alpha+\overline{\theta}}{(1+\alpha-\beta)+\overline{\theta}},$$

so that the result holds as long as $\frac{\alpha+\overline{\theta}}{(1+\alpha-\beta)+\overline{\theta}} > \frac{1+\alpha}{3+\alpha-2\beta}$. To minimize LHS to see if information acquisition can be easier, set $\overline{\theta} = \frac{1-\alpha}{2}$ as the minimum value allowed by Assumption (1). Then, the two sides converge, which implies that for any larger $\overline{\theta}$ the inequality holds.

A.2 First-best decisions under authority

As discussed in the body of the paper, the optimal communication protocol involves a closed-door policy, where each agent only receives a recommendation to either adapt or coordinate. Let $w_{A,C}$ denote a compensating ex post transfer between the agents from the adapting agent to the accommodating agent, and let c_i^k denote ex post transfers from the subordinates to the superior to compensate for the information acquired. Then we have for the three constraints that

(i) transfer constraints:

$\max\left\{c_i^L, c_i^H, c_i^A + w_{A,C}, c_i^C\right\} \leq \delta\Delta V,$

which summarize how much the subordinates will be willing to transfer at the ex post stage. Finally, the superior needs to be compensated sufficiently to reward information acquisition, which gives us

$$2c_M = \tfrac{1}{4}\left(2c_i^H\right) + \tfrac{1}{4}\left(2c_i^L\right) + \tfrac{1}{2}\left(c_i^A + c_i^C\right)$$

(ii) obedience constraint: Given a recommendation (C), the agent's beliefs are $\Pr(H|C) = \frac{1}{3}$ and so the condition for the agent not to deviate to adapting is given by

$$\tfrac{1}{3}\left(1 - c_i^H\right) + \tfrac{1}{3}\left(1 - c_i^L\right) + \tfrac{1}{3}\left(\overline{\theta} + w_{A,C} - c_i^C\right) + \delta V^{cont} \geq \tfrac{1}{3}\left((1+\alpha) + \overline{\theta}\right) + \tfrac{1}{3}\left((1+\alpha) - \overline{\theta}\right) + \tfrac{1}{3}(\beta) + \delta V^{dev}.$$

To relax the constraint the most, we will set $w_{A,C} = \delta\Delta V - c_j^A$, which then allows us to write the constraint as

$$-\tfrac{1}{3}\left[c_i^H + c_i^L + \left(c_i^A + c_i^C\right)\right] + \tfrac{4}{3}\delta\Delta V \geq \tfrac{2\alpha}{3} + \tfrac{1}{3}\left(\beta - \overline{\theta}\right).$$

Importantly, note that how the compensation of the superior is distributed across the states is irrelevant to the satisfaction of the obedience constraint, and so we can further simplify the expression to

$$\tfrac{4}{3}\left(\delta\Delta V - c_M\right) \geq \tfrac{2\alpha}{3} + \tfrac{1}{3}\left(\beta - \overline{\theta}\right)$$

(iii) second-guessing constraint: The second requirement is that the subordinate prefers to follow the instruction instead of examining his own state further to find out the state and responding optimally to that information, which we can write as

$$\begin{aligned}
&\tfrac{1}{3}\left(1 - c_i^H\right) + \tfrac{1}{3}\left(1 - c_i^L\right) + \tfrac{1}{3}\left(\overline{\theta} + w_{A,C} - c_i^C\right) + \delta V^{cont} \\
&\geq \tfrac{1}{3}\max\left\{\left((1+\alpha) + \overline{\theta}\right) + \delta V^{dev}, \tfrac{1}{3}\left(1 - c_i^H\right) + \delta V^{cont}\right\} \\
&+ \tfrac{2}{3}\max\left\{\tfrac{1}{2}\left((1+\alpha) - \overline{\theta}\right) + \tfrac{1}{2}(\beta) + \delta V^{dev}, \tfrac{1}{2}\left(1 - c_i^L\right) + \tfrac{1}{2}\left(\overline{\theta} + w_{A,C} - c_i^C\right) + \delta V^{cont}\right\} - c_L.
\end{aligned}$$

But now, note that for the last line, following the instruction must be optimal, because otherwise deviation would always be optimal and thus it is better to deviate without any information acquisition. Thus, the constraint becomes

$$\tfrac{1}{3}\delta\Delta V \geq \tfrac{1}{3}\left((\alpha + \overline{\theta} + c_i^H\right) - c_L.$$

Thus, we want to minimize c_i^H needed. If cost of information is sufficiently low, the superior can be fully compensated by transfers in the other states and we can set $c_i^H = 0$, in which case the solution is

$$\delta\Delta V \geq \left(\alpha + \overline{\theta}\right) - 3c_L.$$

If not, then we can use $4c_M = c_i^H + c_i^L + c_j^A + c_j^C$ and the fact that to satisfy the order of deviations, we cannot try to charge too much compensation in the case of low value of adaptation so that it would be actually deviating in that case that is attractive. To that end, we have

$$\tfrac{1}{2}\left((1+\alpha) - \overline{\theta}\right) + \tfrac{1}{2}(\beta) + \delta V^{dev} \leq \tfrac{1}{2}\left(1 - c_i^L\right) + \tfrac{1}{2}\left(\overline{\theta} + w_{A,C} - c_j^C\right) + \delta V^{cont},$$

which we can rearrange to yield

$$3\delta\Delta V + 2\overline{\theta} - \alpha - \beta = \left(c_i^L + c_j^A + c_j^C\right),$$

which leaves

$$4c_M - \left(3\delta\Delta V + 2\overline{\theta} - \alpha - \beta\right) = c_i^H$$

as the minimum necessary payment in this state, giving us

$$\delta\Delta V \geq \alpha + \overline{\theta} + 4c_M - \left(3\delta\Delta V + 2\overline{\theta} - \alpha - \beta\right) - 3c_L$$

$$4\left(\delta\Delta V - c_M\right) \geq 2\alpha + \left(\beta - \overline{\theta}\right) - 3c_L.$$

But note that this constraint is subsumed by the obedience constraint, which was

$$\tfrac{4}{3}\left(\delta\Delta V - c_M\right) \geq \tfrac{2\alpha}{3} + \tfrac{1}{3}\left(\beta - \overline{\theta}\right).$$

Thus, the binding constraint is either the obedience constraint, which we can solve to yield

$$\delta \geq \frac{\frac{\left(2\alpha+\beta-\overline{\theta}\right)}{4} + c_M}{\left(\frac{3+\alpha-4\beta}{4} + \frac{\overline{\theta}}{2} - c_M\right) + \frac{\left(2\alpha+\beta-\overline{\theta}\right)}{4} + c_M},$$

or the first second-guessing constraint, which we can solve to yield

$$\delta \geq \frac{\left(\alpha+\overline{\theta}-3c_L\right)}{\left(\frac{3+\alpha-4\beta}{4} + \frac{\overline{\theta}}{2} - c_M\right) + \left(\alpha+\overline{\theta}-3c_L\right)}.$$

In addition, the second-guessing constraint is the binding constraint whenever

$$\left(\alpha + \overline{\theta} - 3c_L\right) \geq \frac{\left(2\alpha+\beta-\overline{\theta}\right)}{4} + c_M \Leftrightarrow \frac{\left(2\alpha-\beta+5\overline{\theta}\right)}{4} \geq 3c_L + c_M,$$

and vice versa.

A.3 Second-best equilibria under decentralization

In this section, I derive the binding constraints for the various equilibrium decision rules, following the flow from Figure 4. Assume first that $\beta + 7\overline{\theta} \geq 2\alpha$, with the converse considered later for reasons that will be apparent. The logic is to first consider the boundaries at which the second-best equilibrium arises at particular strategy profiles, and then consider how we can potentially smooth out the transitions with the help of mediation.

A.3.1 Second-best (2): (H,H)→(A,C)/(C,A), (H,L)→(A,C), (L,L)→(C,C)

Given the assumed parameter configuration, the first step is to give up on mutual cooperation in (H,H) and replace it with coordinated adaptation. In this case, the value created in this equilibrium as

$$\tfrac{1}{4}\left((1+\alpha)+\overline{\theta}\right)+\tfrac{1}{4}\left(\overline{\theta}\right)+\tfrac{1}{8}\left((1+\alpha)+\overline{\theta}\right)+\tfrac{1}{8}\left(-\overline{\theta}\right)+\tfrac{1}{4}=\tfrac{5+3\alpha}{8}+\tfrac{1}{2}\overline{\theta},$$

so that the net continuation value is $\frac{5+3\alpha-8\beta}{8}+\frac{1}{2}\overline{\theta}-c_L$. Then we can consider the constraints.

(i) Decision constraints: From above, we know that the coordination constraint is violated, so that $\delta\Delta V < \alpha+\overline{\theta}$. There are two possible constraints that may now be binding First, since both high and low value require accommodation in equilibrium, the high accommodation cost is the binding one, which gives us

$$-\overline{\theta}+w_{A,C}+\delta V^{cont}\geq\beta+\delta V^{dev}\Leftrightarrow w_{A,C}+\delta\Delta V\geq\beta+\overline{\theta}.$$

Alternatively, for a low-type agent, the optimal mediation protocol reveals no information since he is always expected to cooperate. Thus, his decision constraint boils down to

$$\tfrac{1}{2}+\tfrac{1}{2}\left(\overline{\theta}+w_{A,C}\right)+\delta V^{cont}\geq\tfrac{1}{2}\left(1+\alpha-\overline{\theta}\right)+\tfrac{1}{2}\beta+\delta V^{dev},$$

which we can rearrange to

$$\tfrac{1}{2}\left(w_{A,C}\right)+\delta\Delta V\geq\tfrac{1}{2}\left(\alpha+\beta\right)-\overline{\theta}.$$

Under maximal transfers, $w_{A,C}=\delta\Delta V$, which simplifies the two constraints to

$$\delta\Delta V\geq\tfrac{\beta+\overline{\theta}}{2}\qquad and\qquad\delta\Delta V\geq\tfrac{1}{3}\left(\alpha+\beta\right)-\tfrac{2}{3}\overline{\theta}.$$

The accommodation constraint for H is thus more binding than the coordination constraint of L as long as

$$\frac{\beta+\overline{\theta}}{2} \geq \frac{1}{3}(\alpha+\beta) - \frac{2}{3}\overline{\theta} \Leftrightarrow \beta + 7\overline{\theta} \geq 2\alpha,$$

which is the case that we are considering first. For the remainder of the constraints, begin with the information acquisition constraint:

(ii) Information acquisition constraint: We can write down the information acquisition constraint as

$$\frac{5+3\alpha}{8} + \frac{1}{2}\overline{\theta} - c_L + \delta V^{cont} \geq \max\left\{\beta + E(\tau), \frac{1}{2}\beta + \frac{1}{2}(1+\alpha) - \tau_L + E(\tau), \frac{3}{4}(1+\alpha) + \frac{1}{4}\beta - \tau_H + E(\tau)\right\} + \delta V^{dev},$$

where τ_i are the transfers required with the equilibrium messages to be believed, with the resulting strategies by the other player and the payoffs. Thus, we have that information acquisition is preferred over the false low signal as long as

$$\frac{5+3\alpha}{8} + \frac{1}{2}\overline{\theta} - c_L + \delta V^{cont} \geq \frac{1}{2}\beta + \frac{1}{2}(1+\alpha) - \tau_L + E(\tau) + \delta V^{dev} \Leftrightarrow$$
$$\frac{1-\alpha-4\beta}{8} + \frac{1}{2}\overline{\theta} - c_L + \delta\Delta V \geq \frac{1}{2}(\tau_H - \tau_L)$$

And correspondingly, deviating to the high signal is not optimal as long as

$$\frac{1}{2}(\tau_H - \tau_L) \geq c_L + \frac{1+3\alpha+2\beta}{8} - \frac{1}{2}\overline{\theta} - \delta\Delta V.$$

So a transfer that is able to satisfy this constraint exists as long as

$$\frac{1-\alpha-4\beta}{8} + \frac{1}{2}\overline{\theta} - c_L + \delta\Delta V \geq c_L + \frac{1+3\alpha+2\beta}{8} - \frac{1}{2}\overline{\theta} - \delta\Delta V$$
$$\overline{\theta} - \frac{2\alpha+3\beta}{4} + 2\delta\Delta V \geq 2c_L \Leftrightarrow \delta\Delta V \geq \frac{2\alpha+3\beta}{8} - \frac{\left(\overline{\theta}-2c_L\right)}{2}.$$

(iii) communication constraints:

With the low transfer set at zero, the constraint for the low-value agent to be truthful is

$$\frac{1}{2}\left(\overline{\theta}\right) + \frac{1}{2} + \frac{1}{2}\widetilde{w}_{A,C} + \frac{1}{2}\tau_H + \delta V^{cont} \geq \frac{1}{2}\left(\frac{1}{2}\beta + \frac{1}{2}\left((1+\alpha) - \overline{\theta}\right)\right) + \frac{1}{2}\left((1+\alpha) - \overline{\theta}\right) - \frac{1}{2}\tau_H + \delta V^{dev}$$
$$\tau_H + \frac{1}{2}\widetilde{w}_{A,C} \geq \left(\frac{1+3\alpha+\beta}{4} - \frac{5}{4}\overline{\theta}\right) - \delta\Delta V,$$

and similarly for the high-value agent to remain truthful, we have that

$$\left(\frac{1}{4}(1+\alpha)\right) + \frac{1}{2}\left((1+\alpha) + \overline{\theta} - \widetilde{w}_{A,C}\right) - \tau_H + \delta\Delta V \geq \frac{1}{2}\left((1+\alpha) + \overline{\theta}\right) + \frac{1}{2}\beta$$
$$\left(\frac{1+\alpha-2\beta}{4}\right) + \delta\Delta V \geq \frac{1}{2}\widetilde{w}_{A,C} + \tau_H.,$$

So we need to have that

$$\left(\frac{1+\alpha-2\beta}{4}\right) + \delta\Delta V \geq \frac{1}{2}\widetilde{w}_{A,C} + \tau_H > \left(\frac{1+3\alpha+\beta}{4} - \frac{5}{4}\overline{\theta}\right) - \delta\Delta V$$

$$\delta \Delta V > \left(\frac{2\alpha + 3\beta}{8} - \frac{5}{8}\overline{\theta} \right),$$

but from the acquisition constraint, $\frac{2\alpha+3\beta}{8} - \frac{\left(\overline{\theta}-2c_L\right)}{2} > \frac{2\alpha+3\beta}{8} - \frac{5}{8}\overline{\theta}$, so that it is the acquisition constraint that will bind first, and also because of the access to $\widetilde{w}_{A,C}$, the level differences between the two constraints can be adjusted for. The remaining comparison is between the acquisition constraint and the decision constraint. Here, on the upper path we have

$$\frac{\beta+\overline{\theta}}{2} \geq \frac{2\alpha+3\beta}{8} - \frac{\left(\overline{\theta}-2c_L\right)}{2}$$
$$\beta + 8\left(\overline{\theta} - c_L\right) \geq 2\alpha,$$

where LHS is minimized when $\overline{\theta} = 2c_L$, which simplifies the expression to $\beta + 4\overline{\theta} \geq 2\alpha$. Thus, there is a small region for some parameter values, including very high cost of information for which the acquisition constraint can become binding before the decision constraint. A sufficient condition for this not to arise is $\overline{\theta} - \frac{2\alpha-\beta}{8} \geq c_L$.

A.3.2 Mixed equilibrium

Having derived the first threshold equilibrium, we can bridge the gap from the first-best equilibrium to this equilibrium. To construct this equilibrium, continue to assume that we undertake maximal transfers off diagonal to provide rewards for compliance and otherwise the first-best decision rule is followed, except that in the state (H,H) the mediator mixes so that with probability $(1-p)$ he recommends (C,C) and with probability p he randomizes 50/50 between recommendations (A,C) and (C,A), with associated compensatory transfers in the latter case.

Then, knowing that it is the decision constraint that binds across the states (with the caveat from above), we need to make sure that that constraint is satisfied. Note that conditional on receiving the recommendation (C) and the mixing probabilities, the agent attaches a probability $\frac{2(1-p)}{2-p}$ to the other agent also coordinating and probability $\frac{p}{2-p}$ to the other agent adapting. Then, we can write the constraint for cooperation as

$$\frac{2(1-p)}{2-p} + \frac{p}{2-p}\left(-\overline{\theta} + w_{A,C}\right) + \delta\Delta V \geq \frac{2(1-p)}{2-p}\left((1+\alpha) + \overline{\theta}\right) + \frac{p}{2-p}\beta,$$

which simplifies, using $w_{A,C} = \delta\Delta V$, to

$$2\delta\Delta V \geq 2(1-p)\alpha + (2-p)\overline{\theta} + p\beta.$$

Finally, the relevant continuation value is now

$$\frac{1}{4} + \frac{1}{4}\left((1+\alpha) + \overline{\theta}\right) + \frac{1}{4}\overline{\theta} + \frac{1}{4}\left((1-p) + \frac{p}{2}((1+\alpha) + \overline{\theta}) + \frac{p}{2}(1-p)(-\overline{\theta})\right) = \frac{3+\alpha}{4} + \frac{1}{2}\overline{\theta} - p\left(\frac{1-\alpha}{8}\right).$$

With this, we can then solve for the probability p that can be sustained as

$$p \geq \frac{(\alpha+\overline{\theta})-\frac{\delta}{1-\delta}\left(\frac{3+\alpha-4\beta}{4}+\frac{1}{2}\overline{\theta}-c_L\right)}{(\alpha+\overline{\theta})-\frac{1}{2}(\beta+\overline{\theta})-\frac{\delta}{1-\delta}\left(\frac{1-\alpha}{8}\right)},$$

which provides the transition between the two equilibria.

A.3.3 Second-best: (H,H)→(A,A), (H,L)→(A,C), (L,L)→(C,C)

Continue to assume that $\beta > 2\alpha - 7\overline{\theta}$, so that the binding constraint the the equilibrium (2) is the accommodation constraint by the high-type agent. In this case, we must give up on any collaboration in the (H,H) state and the surplus created is equal to

$$\frac{1}{4}\beta + \frac{1}{4}\left((1+\alpha)+\overline{\theta}\right) + \frac{1}{4}\overline{\theta} + \frac{1}{4} = \frac{2+\alpha+\beta}{4} + \frac{1}{2}\overline{\theta},$$

creating net value $\frac{2+\alpha-3\beta}{4} + \frac{1}{2}\overline{\theta} - c_L$. Importantly, note that this equilibrium does not require any communication as the action is determined solely by the signal that the agent receives. Notationally, let

$$\Delta u_i^{(3)} = \frac{2+\alpha-3\beta}{4} + \frac{1}{2}\overline{\theta} - c_L = \Delta u_i^{FB} - \frac{1-\beta}{4}$$

(i) decision constraint: Given the lack of communication, the only question is whether the low-type agent will actually coordinate as required. We can write this constraint (where we continue to use transfers in the (H,L) state to relax the IC constraint) as:

$$\frac{1}{2}\left(\overline{\theta} + \widetilde{w}_{A,C}\right) + \frac{1}{2} + \delta V^{cont} \geq \frac{1}{2}\left((1+\alpha)-\overline{\theta}\right) + \frac{1}{2}\beta + \delta V^{dev}$$
$$\overline{\theta} - \frac{(\alpha+\beta)}{2} + \frac{3}{2}\delta\Delta V \geq 0,$$

which we can solve to give $\delta^{decision} \geq \frac{(\alpha+\beta)-2\overline{\theta}}{3\Delta u_i^{(3)}+(\alpha+\beta)-2\overline{\theta}}$.

(ii) information acquisition: To solve for the information acquisition constraint, note that without acquiring information the payoff is $\frac{1}{2}\beta + \frac{1}{2}(1+\alpha)$. Then, the acquisition constraint becomes

$$\frac{2+\alpha+\beta}{4} + \frac{1}{2}\overline{\theta} - c_L + \delta\Delta V \geq \frac{1}{2}\beta + \frac{1}{2}(1+\alpha)$$

$$\delta\Delta V \geq \left(\frac{\alpha+\beta}{4} - \frac{1}{2}\overline{\theta} + c_L\right) \Leftrightarrow \delta^{\text{information}} \geq \frac{\left(\alpha+\beta-2\overline{\theta}+4c_L\right)}{4\Delta u_i^{(3)}+\left(\alpha+\beta-2\overline{\theta}+4c_L\right)}.$$

Looking at the difference between the two constraints, the information constraint is binding as long as $\left(\frac{\alpha+\beta}{4} - \frac{1}{2}\overline{\theta}\right) \leq 3c_L$, which is true unless c_L is sufficiently low $\left(\frac{(\alpha+\beta)-2\overline{\theta}}{12} \geq c_L\right)$.

A.3.4 Mixed equilibrium

Unlike the first bridge that we could build from (C,C) to (C,A)/(A,C), we cannot do the same for the transition from (C,A)/(A,C) to (A,A). The reason is that when the agent knows his state, if he is called to play C, he knows the other agent would adapt and there is no way of relaxing this constraint further. There is thus no mixed equilibrium between these two.

A.3.5 Second-best: (H,H)→(A,A), (H,L)→(A,A), (L,L)→(C,C) and (H,H)→(A,A), (H,L)→(A,C), (L,L)→(A,C)/(C,A)

From the above, there are then only two paths left, depending on whether it is the accommodate-low decision or the coordinate-low decision constraint that is binding. From above, we had that the decision constraint is given by

$$\delta^{decision} \geq \frac{(\alpha+\beta)-2\overline{\theta}}{3\Delta u_i^{(3)}+(\alpha+\beta)-2\overline{\theta}}.$$

Now, there are two possible outcomes once this constraint is violated. First, even if the two constraints are pooled together, suppose that the accommodate-low constraint is more binding, which arises when $\frac{\beta-\overline{\theta}}{2} > \alpha - \overline{\theta}$. Then, we cannot use the same trick of mixing the (C,C) outcome with (C,A)/(A,C) because that would increase the likelihood that the other agent is adapting and thus increases the deviation temptation. In this case, the only feasible solution is to give up on any collaboration in any state except (L,L). Below, I will show that because such an equilibrium creates so little value, if it is sustainable, a more attractive decision rule can be sustained at the same time, so it is never the preferred arrangement. Second, if it is the case that the coordination constraint in (L,L) is more binding, then we can follow a logic similar to the (H,H) state – mix in (A,C)/(C,A) to relax the decision constraint, until we must give up on collaboration altogether. But following the above, I will first establish the limit equilibrium where (A,C)/(C,A) is played for sure in (L,L) and then use mixing to bridge the gap.

(H,H)→(A,A), (H,L)→(A,A), (L,L)→(C,C) Suppose the acommodate constraint comes to bind first. Then, the only possibility for creating value is to achieve coordination when both agents have a lost value of adapting. In this case, the stage-game value becomes $\frac{3}{4}\beta + \frac{1}{4} = \frac{1+3\beta}{4}$, with the net payoff (value of the relationship) in each period equal to $\frac{1-\beta}{4} - c_L$.

(i) decisions: Cooperation is sustainable if and only if $1 + \delta V^{cont} \geq (1+\alpha) - \overline{\theta} + \delta V^{cont} \Leftrightarrow \delta \Delta V \geq \alpha - \overline{\theta}$.

(ii) communication: The only way to induce cooperation is now to send the low message, which becomes the potential deviation. To sustain truth-telling by the high type, we need

$$\beta + E(\tau) + \delta V^{cont} \geq \frac{1}{2}\left((1+\alpha)+\overline{\theta}\right) + \frac{1}{2}\beta + E(\tau) - \tau_L + \delta V^{dev},$$

which we can rearrange to

$$\tau_L \geq \frac{1}{2}\left((1+\alpha)+\overline{\theta}-\beta\right) - \delta\Delta V,$$

while for the low type to be willing to make the payment, we have

$$\frac{1}{2}\beta + \frac{1}{2} - \tau_L + \delta V^{cont} \geq \beta + \delta V^{dev} \Leftrightarrow \frac{1}{2}(1-\beta) + \delta\Delta V \geq \tau_L.$$

Thus, a transfer exists as long as

$$\frac{1}{2}(1-\beta) + \delta\Delta V \geq \frac{1}{2}\left((1+\alpha)+\overline{\theta}-\beta\right) - \delta\Delta V$$
$$\delta\Delta V \geq \frac{1}{4}\left(\alpha+\overline{\theta}\right).$$

(iii) information acquisition: Finally, inducing information acquisition requires that

$$\frac{3}{4}(\beta) + \frac{1}{4} - c_L + \delta V^{cont} \geq \max\left(\frac{1}{2}\beta + \frac{1}{2}(1+\alpha) - \frac{1}{2}\tau_L, \beta + \frac{1}{2}\tau_L\right) + \delta V^{dev},$$

where we can rearrange the two constraints to give

$$\frac{3}{4}(\beta) + \frac{1}{4} - c_L + \delta\Delta V \geq \frac{1}{2}\beta + \frac{1}{2}(1+\alpha) - \frac{1}{2}\tau_L \Leftrightarrow \frac{1}{2}\tau_L \geq \frac{1+2\alpha-\beta}{4} + c_L - \delta\Delta V$$

$$\frac{3}{4}(\beta) + \frac{1}{4} - c_L + \delta\Delta V \geq \beta + \frac{1}{2}\tau_L \Leftrightarrow \frac{1}{4}(1-\beta) - c_L + \delta\Delta V \geq \frac{1}{2}\tau_L,$$

which gives us the sustainable range of transfers as

$$\frac{1}{4}(1-\beta) - c_L + \delta\Delta V \geq \frac{1+2\alpha-\beta}{4} + c_L - \delta\Delta V$$
$$\delta\Delta V \geq \frac{\alpha}{4} + c_L.$$

Thus, the communication constraint now always dominates the acquisition constraint. Further, we can solve the communication constraint to yield $\delta^{comm} \geq \frac{a+\overline{\theta}}{\left(1+\alpha-\beta-4c_L+\overline{\theta}\right)}$, whereas from earlier, we have that $\delta \geq \frac{\alpha-2c_L+\overline{\theta}}{\left(1+\alpha-\beta-2c_L+\overline{\theta}\right)}$ sustains no information acquisition (which would be preferred since coordinating is better than coordinating only sometimes, and at a cost). And taking the difference between the two, we get

$$\delta^{comm} - \delta = \frac{2c_L\left(2\alpha-\beta+2\overline{\theta}-4c_L+1\right)}{\left(1+\alpha-\beta-4c_L+\overline{\theta}\right)\left(1+\alpha-\beta-2c_L+\overline{\theta}\right)} > 0,$$

since $2\alpha - \beta + 2\overline{\theta} - 4c_L + 1 > 1 + 2\alpha - \beta > 0$. Thus, this equilibrium is never preferred as a no-information equilibrium dominates and is easier to sustain than this partial collaboration equilibrium.

(H,H)→(A,A), (H,L)→(A,C), (L,L)→(A,C)/(C,A) The last remaining arrangement for us to look is the case where the cooperation constraint binds first, so that we are left with the adaptation constraint for the low agent (given by $\beta < 2\alpha - \overline{\theta}$). In this equilibrium, the high type always adapts, and the low mixes between (A,C) and (C,A) when matched with another low type while accommodating a high type. The expected payoff is

$$\frac{1}{4}\beta B + \frac{1}{4}\left(A + \overline{\theta}\right) + \frac{1}{4}\overline{\theta} + \frac{1}{8}\left(\left(A - \overline{\theta}\right)\right) + \frac{1}{8}\left(\left(\overline{\theta}\right)\right) = \frac{3+3\alpha-6\beta}{8}B + \frac{1}{2}\overline{\theta} - c_L.$$

(i) decision constraint: For the low agent, the decision constraint becomes $\widetilde{w}_{A,C} + \delta\Delta V \geq \beta - \overline{\theta} \Leftrightarrow \delta\Delta V \geq \frac{\beta-\overline{\theta}}{2}$ for both the high and low opponents.

(ii) communication constraint: We can next write the communication constraint for the high type as

$$\frac{1}{2}\beta+\frac{1}{2}\left((1+\alpha)+\overline{\theta}-\widetilde{w}_{A,C}\right)-\frac{1}{2}\left(\tau_H-\tau_L\right)+\delta V^{cont} \geq \frac{1}{2}\beta+\frac{1}{2}\left(\frac{1}{2}\beta+\frac{1}{2}\left((1+\alpha)+\overline{\theta}\right)\right)+\frac{1}{2}\left(\tau_H-\tau_L\right)+\delta V^{dev},$$
$$\delta\Delta V + \frac{1}{4}\left(1+\alpha-\beta+\overline{\theta}\right) \geq \left(\tau_H-\tau_L\right) + \frac{1}{2}\widetilde{w}_{A,C}.$$

Similarly, for the low type, we have

$$\frac{1}{2}\left(\overline{\theta}+\widetilde{w}_{A,C}\right) + \frac{1}{2}\left(\frac{1}{2}\overline{\theta}+\frac{1}{2}\left((1+\alpha)-\overline{\theta}\right)\right) + \frac{1}{2}\left(\tau_H-\tau_L\right) + \delta V^{cont} \geq \frac{1}{2}\left((1+\alpha)-\overline{\theta}\right) + \frac{1}{2}\left(\beta\right) - \frac{1}{2}\left(\tau_H-\tau_L\right) + \delta V^{dev}$$
$$\left(\tau_H-\tau_L\right) + \frac{1}{2}\widetilde{w}_{A,C} \geq \frac{1}{4}(1+\alpha) - \overline{\theta} + \frac{1}{2}\left(\beta\right) - \delta\Delta V$$

together, for a solution to exist, we have that

$$\delta\Delta V + \frac{1}{4}\left(1+\alpha-\beta+\overline{\theta}\right) \geq \frac{1}{4}(1+\alpha) - \overline{\theta} + \frac{1}{2}\left(\beta\right) - \delta\Delta V$$
$$2\delta\Delta V \geq \frac{3\beta-5\overline{\theta}}{4}$$

(iii) information acquisition constraint: The final constraint that needs to be satisfied is to induce information acquisition. We can write this constraint as

$$\frac{3+3\alpha+2\beta}{8} + \frac{1}{2}\overline{\theta} - c_L + \delta V^{cont} \geq \max\left\{\frac{3}{4}\beta + \frac{1}{4}(1+\alpha) + \frac{1}{2}\tau_H, \frac{1}{2}\beta + \frac{1}{2}(1+\alpha) - \frac{1}{2}\tau_H\right\} + \delta V^{dev},$$

where the two bounds are then

$$\frac{3+3\alpha+2\beta}{8} + \frac{1}{2}\overline{\theta} - c_L + \delta V^{cont} \geq \frac{6}{8}\beta + \frac{2+2\alpha}{8} + \frac{1}{2}\tau_H + \delta V^{dev}$$
$$\frac{1+\alpha-4\beta}{8}B + \frac{1}{2}\overline{\theta} - c_L + \delta\Delta V \geq \frac{1}{2}\tau_H$$

$$\frac{3+3\alpha+2\beta}{8}B + \frac{1}{2}\overline{\theta} - c_L + \delta V^{cont} \geq \frac{1}{2}\beta B + \frac{1}{2}A - \frac{1}{2}\tau_H + \delta V^{dev}$$
$$\frac{1}{2}\tau_H \geq c_L + \frac{1+\alpha+2\beta}{8}B - \frac{1}{2}\overline{\theta} - \delta\Delta V,$$

which gives

$$\frac{1+\alpha-4\beta}{8}B + \frac{1}{2}\overline{\theta} - c_L + \delta\Delta V \geq c_L + \frac{1+\alpha+2\beta}{8}B - \frac{1}{2}\overline{\theta} - \delta\Delta V$$
$$2\delta\Delta V \geq 2c_L + \frac{3\beta-4\overline{\theta}}{4}.$$

Bringing the components together, we have

information: $\delta\Delta V \geq c_L + \frac{3\beta-4\overline{\theta}}{8}$
communication: $\delta\Delta V \geq \frac{3\beta-5\overline{\theta}}{8}$
decision: $\delta\Delta V \geq \frac{\beta-\overline{\theta}}{2}$,

so that the information constraint always dominates the communication constraint, while for the other two constraints, either one may be binding.

A.3.6 Mixed equilibrium

To bridge the gap to this equilibrium, we can follow the same logic as under (H,H) – we introducing mixing to the (L,L) state with some probability of (C,C) and some (A,C)/(C,A). However, we do need to be careful because now the binding constraint may no longer be only the decision constraint as the information acquisition constraint may bind as well. The decision constraint in this segment is given by

$$6\delta\Delta V \geq 2(1-p_L)\left(\alpha - \overline{\theta}\right) + (2+p_L)\left(\beta - \overline{\theta}\right),$$

where $\Delta u_i^{FB} - \frac{p_L}{8}(1-\alpha) - \frac{(1-\beta)}{4}$.

Now, for the information acquisition constraint, we need to have that

$$\Delta u_i^{FB} - \frac{p_L}{8}(1-\alpha) - \frac{(1-\beta)}{4} + \delta\Delta V \geq \max\left\{\beta + E(\tau), \frac{1}{2}\left(1+\frac{p_L}{2}\right)\beta + \frac{1}{2}\left(1-\frac{p_L}{2}\right)(1+\alpha) + E(\tau) - \tau_L, \frac{1}{2}\beta + \frac{1}{2}(1+\alpha) +$$

and note that the deviation is to claiming the state is high, so $\tau_H > 0$ (at the no communication equilibrium, the two are equivalent, but once mixing is introduced, the reduction in cooperation under (L,L) makes the high message strictly preferred). Thus, it needs to be that

$$\Delta u_i^{FB} - \frac{p_L}{8}(1-\alpha) - \frac{(1-\beta)}{4} + \delta\Delta V \geq \frac{1}{2}\left(1+\frac{p_L}{2}\right)\beta + \frac{1}{2}\left(1-\frac{p_L}{2}\right)(1+\alpha) + E(\tau)$$
$$\left(\Delta u_i^{FB} - \frac{p_L}{8}(1-\alpha) - \frac{(1-\beta)}{4}\right) + \delta\Delta V - \frac{1}{2}\left(1+\frac{p_L}{2}\right)\beta - \frac{1}{2}\left(1-\frac{p_L}{2}\right)(1+\alpha) \geq \frac{1}{2}\tau_H$$

and

$$\left(\Delta u_i^{FB} - \frac{p_L}{8}(1-\alpha) - \frac{(1-\beta)}{4}\right) + \delta\Delta V \geq \frac{1}{2}\beta + \frac{1}{2}(1+\alpha) - \frac{1}{2}\tau_H$$
$$\frac{1}{2}\tau_H \geq \frac{1}{2}\beta + \frac{1}{2}(1+\alpha) - \left(\Delta u_i^{FB} - \frac{p_L}{8}(1-\alpha) - \frac{(1-\beta)}{4}\right) - \delta\Delta V,$$

which then gives

$2\Delta u_i^{FB} - \frac{p_L}{4}(1-\alpha) - 2\frac{(1-\beta)}{4} + 2\delta\Delta V \geq \beta + (1+\alpha) - \frac{p_L}{4}(1+\alpha-\beta)$
$2\delta\Delta V \geq \frac{\alpha+\beta-2\overline{\theta}}{2} + 2c_L - \frac{p_L}{4}(2\alpha-\beta)$

Finally, for the communication constraint, we have that, for the high type,

$\frac{1}{2}\beta + \frac{1}{2}\left(1+\alpha+\overline{\theta}-w_{A,C}\right) - \tau_H + \delta\Delta V \geq \frac{1}{2}\left(1+\frac{p_L}{2}\right)\beta + \frac{1}{2}\left(1-\frac{pL}{2}\right)\left(1+\alpha+\overline{\theta}\right)$
$\frac{pL}{4}\left[1+\alpha+\overline{\theta}-\beta\right] + \delta\Delta V \geq \frac{1}{2}w_{A,C} + \tau_H$

and for the low type to be honest, we need

$\frac{1}{2}\left(\overline{\theta}+w_{A,C}\right) + \frac{1}{2}\left((1-p_L) + \frac{p_L}{2}(1+\alpha-\overline{\theta}-\widetilde{w}_{A,C}) + \frac{p_L}{2}\left(\overline{\theta}+\widetilde{w}_{A,C}\right)\right) + \delta\Delta V \geq \frac{1}{2}\beta + \frac{1}{2}\left(1+\alpha-\overline{\theta}\right) - \tau_H$
$\frac{1}{2}w_{A,C} + \tau_H \geq \frac{\alpha+\beta-2\overline{\theta}}{2} + \frac{p_L}{4}(1-\alpha) - \delta\Delta V$

and the sustainable range then becomes

$\frac{pL}{4}\left[1+\alpha+\overline{\theta}-\beta\right] + \delta\Delta V \geq \frac{\alpha+\beta-2\overline{\theta}}{2} + \frac{p_L}{4}(1-\alpha) - \delta\Delta V$
$2\delta\Delta V \geq \frac{\alpha+\beta-2\overline{\theta}}{2} - \frac{pL}{4}\left[2\alpha+\overline{\theta}-\beta\right].$

Thus, it is again that the information acquisition constraint trumps the communication constraint, while the tradeoff between the decision and information constraints is ambiguous, and cannot be ranked in terms of p_L. Thus, we just need to make sure that both are satisfied. A quick manipulation of the expressions gives us

information: $p_L \geq \frac{2\left(\alpha+\beta-2\overline{\theta}\right)+8c_L-8\frac{\delta}{1-\delta}\left(\Delta u_i^{FB}-\frac{(1-\beta)}{4}\right)}{\left((2\alpha-\beta)-\frac{\delta}{1-\delta}(1-\alpha)\right)}$
communication: $p_L \geq \frac{2\left(\alpha+\beta-2\overline{\theta}\right)-6\frac{\delta}{1-\delta}\left(\Delta u_i^{FB}-\frac{(1-\beta)}{4}\right)}{\left[2\alpha-\beta-\overline{\theta}-\frac{\delta}{1-\delta}\frac{3}{4}(1-\alpha)\right]}.$

As $p_L \to 1$, the solution converges to the above and after that, no further collaboration is sustainable and the play becomes fully selfish.

Now, the only remaining part of the analysis is to follow the lower branch of the figure. The only difference is that in that branch, the gains to adapating are so high even in the low state that the constraint for playing (C) in L and not knowing whether the other agent plays (C) or (A) becomes binding before the constraint for playing (C) in H when knowing the opponent plays (A).

A.3.7 Second-best path, $\beta \leq 2\alpha - 7\overline{\theta}$

From above, we know that along this path, the constraint for playing (C) comes to bind for L-agent before the mixing from (C,C) to (C,A)/(A,C) is complete for the H-agent. But the logic follows already-familiar steps. Once the mediator is instructing (C,C) with probability $(1-p_H)$ and (A,C) and (C,A) with probability $p_H/2$, the coordination constraint becomes in (L,L) becomes binding for some p_H^*, after which the mediator will need to start instructing (C,C) with probability

$(1 - p_L)$ and (A,C) and(C,A) with probability $p_L/2$. Then, we can write the two IC constraints for decision-making as

$$\begin{aligned} \delta\Delta V &\geq \tfrac{(1-p_L)}{3}\left(\alpha - \overline{\theta}\right) + \tfrac{(2+p_L)}{6}\left(\beta - \overline{\theta}\right) \\ \delta\Delta V &\geq (1-p_H)\left(\alpha + \overline{\theta}\right) + \tfrac{p_H}{2}\left(\beta + \overline{\theta}\right). \end{aligned}$$

The lower decision constraint thus becomes binding at p_H^* for which

$$(1-p_H^*)\left(\alpha + \overline{\theta}\right) + \tfrac{p_H^*}{2}\left(\beta + \overline{\theta}\right) = \tfrac{1}{3}\left(\alpha - \overline{\theta}\right) + \tfrac{1}{3}\left(\beta - \overline{\theta}\right),$$

giving $p_H^* = \frac{2\left(2\alpha+5\overline{\theta}-\beta\right)}{3\left(2\alpha+\overline{\theta}-\beta\right)}$, which allows us then to write the critical threshold at which p_L must become positive as

$$\delta \geq \tfrac{\left(\alpha+\beta-2\overline{\theta}\right)}{\Delta u_i^{(p_H)}+\left(\alpha+\beta-2\overline{\theta}\right)},$$

where $\Delta u_i^{(p_H)} = \Delta u_i^{FB} - \frac{2\left(2\alpha+5\overline{\theta}-\beta\right)}{3\left(2\alpha+\overline{\theta}-\beta\right)}\left(\frac{1-\alpha}{8}\right)$. Then, once mixing at (L,L) becomes necessary, the two constraints need to be satisfied simultaneously, with

$$\begin{aligned} \delta\Delta V &= \tfrac{(1-p_L)}{3}\left(\alpha - \overline{\theta}\right) + \tfrac{(2+p_L)}{6}\left(\beta - \overline{\theta}\right) \\ \delta\Delta V &= (1-p_H)\left(\alpha + \overline{\theta}\right) + \tfrac{p_H}{2}\left(\beta + \overline{\theta}\right), \end{aligned}$$

where $\delta\Delta V = \frac{\delta}{1-\delta}\Delta u_i^{FB} - \frac{\delta}{1-\delta}\left(p_L + p_H\right)\left(\frac{1-\alpha}{8}\right)$. Solving the two equations with two unknowns then gives us the solution as

$$p_H = \tfrac{4\left(2\alpha-\beta-\overline{\theta}\right)\left(\alpha+\overline{\theta}-\frac{\delta}{1-\delta}\Delta u_i^{FB}\right)-\frac{\delta}{1-\delta}(1-\alpha)\left(2\alpha-\beta+5\overline{\theta}\right)}{2(2\alpha+\beta+3\overline{\theta})(2\alpha-\beta-\overline{\theta})-y(1-\alpha)\left[4\alpha+4\overline{\theta}+\beta\right]}$$

$$p_L = \tfrac{4\left(2\alpha+\beta+3\overline{\theta}\right)\left((\alpha+\beta-2\overline{\theta})-3\frac{\delta}{1-\delta}\Delta u_i^{FB}\right)+\frac{\delta}{1-\delta}(1-\alpha)\left(2\alpha-\beta+5\overline{\theta}\right)}{2(2\alpha+\beta+3\overline{\theta})(2\alpha-\beta-\overline{\theta})-y(1-\alpha)\left[4\alpha+4\overline{\theta}+\beta\right]}.$$

The next limit arises when $p_H = 1$, after which the accommodation constraint in (H,H) becomes binding and any collaboration becomes unsustainable. This limit is then given by conditions

$$\begin{aligned} \delta\Delta V &= \tfrac{(1-p_L)}{3}\left(\alpha - \overline{\theta}\right) + \tfrac{(2+p_L)}{6}\left(\beta - \overline{\theta}\right) \\ \delta\Delta V &= \tfrac{1}{2}\left(\beta + \overline{\theta}\right), \end{aligned}$$

which then gives the equilibrium mixing probability at (L,L) as

$$\tfrac{1}{2}\left(\beta + \overline{\theta}\right) = \tfrac{(1-p_L)}{3}\left(\alpha - \overline{\theta}\right) + \tfrac{(2+p_L)}{6}\left(\beta - \overline{\theta}\right) \Leftrightarrow p_L = \tfrac{2\alpha-\beta-7\overline{\theta}}{2\alpha-\beta-\overline{\theta}}.$$

At this point, the continuation value is then

$$\text{where } \delta\Delta V = \tfrac{\delta}{1-\delta}\Delta u_i^{FB} - \tfrac{\delta}{1-\delta}\left(\tfrac{2\alpha-\beta-4\overline{\theta}}{2\alpha-\beta-\overline{\theta}}\right)\left(\tfrac{1-\alpha}{4}\right),$$

which allows us to solve the critical threshold as $\delta \geq \frac{(\beta+\overline{\theta})}{\Delta u_i^{(1*)}+(\beta+\overline{\theta})}$. At this point, the solution then converges back to the same path as the upper branch of the Figure. The information acquisition constraint follows similarly [to be completed].

A.4 Second-best equilibria under authority

The optimal equilibrium under authority follows a logic similar to the case under decentralized coordination. When the first-best decision rule can no longer be sustained, the superior will first relax the constraints by introducing a positive probability to playing (C,A)/(A,C) in (H,H), followed by (A,A) in (H,H) and the introduction of (A,C)/(C,A) in (L,L), until no collaboration can be sustained.

As the first step, we need to construct the conditional beliefs of the subordinate following a recommendation (C). To this end, let first p_H be the probability that the superior recommends (A,C)/(C,A) when the gains to adapting are high and p_L for the low state (with then 50/50 coin flip on who gets to adapt). Then, we have

$$\Pr(L|C) = \frac{4-p_L}{6-(p_L+p_H)} \qquad \textit{and} \qquad \Pr(H|C) = \frac{2-p_H}{6-(p_L+p_H)},$$

and for the conditional probability of the other player's behavior, we have

$$\begin{aligned} &\Pr(d_2 = C|C,L) = \frac{2(1-p_L)}{4-p_L}, \qquad \Pr(d_2 = A|C,L) = \frac{2+p_L}{4-p_L} \\ &\Pr(d_2 = C|C,H) = \left(\frac{2(1-p_H)}{2-p_H}\right), \qquad \Pr(d_2 = A|C,H) = \left(\frac{p_H}{2-p_H}\right). \end{aligned}$$

Then, we can construct the obedience constraint as follows. First, the transfer constraints are as before, with

$$\max\left\{c_i^L, c_i^H, c_i^A + w_{A,C}, c_i^C, \widetilde{c}_i^A + \widetilde{w}_{A,C}, \widetilde{c}_i^C\right\} \leq \delta \Delta V,$$

allowing the compensatory transfers differ whether performed by a low- or high-cost accommodation agent. For the superior, the compensation needs to satisfy

$$\begin{aligned} 2c_M &= \frac{1}{2}\left(c_j^A + c_j^C\right) + \frac{1}{4}\left((1-p_L)\,2c_i^L + p_L\left(c_j^A + c_j^C\right)\right) + \frac{1}{4}\left((1-p_H)\,2c_i^H + p_H\left(\widetilde{c}_j^A + \widetilde{c}_j^C\right)\right) \\ 2c_M &= \frac{2+p_L}{4}\left(c_j^A + c_j^C\right) + \frac{2(1-p_L)c_i^L}{4} + \frac{2c_i^H(1-p_H)}{4} + \frac{p_H\left(\widetilde{c}_j^A+\widetilde{c}_j^C\right)}{4} \end{aligned}$$

For the obedience constraint, we then get

$$\begin{aligned} &\Pr(L|C)\left(\Pr(d_2 = C|C,L)\,(1) + \Pr(d_2 = A|C,L)\left(\overline{\theta} + w_{A,C}\right)\right) \\ &+\Pr(H|C)\left(\Pr(d_2 = C|C,H)(1) + \Pr(d_2 = A|C,H)\left(\overline{\theta} + w_{A,C}\right)\right) - E(c) + \delta V^{cont} \geq \\ &\Pr(L|C)\left(\Pr(d_2 = C|C,L)\left(1+\alpha-\overline{\theta}\right) + \Pr(d_2 = A|C,L)\beta\right) \end{aligned}$$

$$+\Pr(H|C)\left(\Pr(d_2=C|C,H)\left(1+\alpha+\overline{\theta}\right)+\Pr(d_2=A|C,H)\beta\right)+\delta V^{dev},$$

which simplifies to

$$\left(\tfrac{4-p_L}{6-(p_L+p_H)}\right)\left(\left(\tfrac{2(1-p_L)}{4-p_L}\right)(1-c_i^L)+\left(\tfrac{2+p_L}{4-p_L}\right)\left(\overline{\theta}+w_{A,C}-c_j^C\right)\right)$$
$$+\left(\tfrac{2-p_H}{6-(p_L+p_H)}\right)\left(\left(\tfrac{2(1-p_H)}{2-p_H}\right)(1-c_i^H)+\left(\tfrac{p_H}{2-p_H}\right)\left(-\overline{\theta}+\widetilde{w}_{A,C}-\widetilde{c}_j^C\right)\right)+\delta V^{cont}\geq$$
$$\left(\tfrac{4-p_L}{6-(p_L+p_H)}\right)\left(\left(\tfrac{2(1-p_L)}{4-p_L}\right)\left(1+\alpha-\overline{\theta}\right)+\left(\tfrac{2+p_L}{4-p_L}\right)\beta\right)$$
$$+\left(\tfrac{2-p_H}{6-(p_L+p_H)}\right)\left(\left(\tfrac{2(1-p_H)}{2-p_H}\right)\left(1+\alpha+\overline{\theta}\right)+\left(\tfrac{p_H}{2-p_H}\right)\beta\right)+\delta V^{dev}$$

$$\left((2(1-p_L))\left(-c_i^L\right)+(2+p_L)\left(\overline{\theta}+w_{A,C}-c_j^C\right)\right)$$
$$+\left((2(1-p_H))\left(-c_i^H\right)+(p_H)\left(-\overline{\theta}+\widetilde{w}_{A,C}-\widetilde{c}_j^C\right)\right)+(6-(p_L+p_H))\,\delta\Delta V\geq$$
$$\left((2(1-p_L))\left(\alpha-\overline{\theta}\right)+(2+p_L)\,\beta\right)+\left((2(1-p_H))\left(\alpha+\overline{\theta}\right)+(p_H)\,\beta\right)$$

$$8\delta\Delta V-\left[2(1-p_L)c_i^L+(2+p_L)\left(c_j^A+c_j^C\right)+(2(1-p_H))\,c_i^H+p_H\left(\widetilde{c}_j^A+\widetilde{c}_j^C\right)\right]\geq$$
$$\left((2(1-p_L))\left(\alpha-\overline{\theta}\right)+(2+p_L)\left(\beta-\overline{\theta}\right)\right)+\left((2(1-p_H))\left(\alpha+\overline{\theta}\right)+(p_H)\left(\beta+\overline{\theta}\right)\right)$$

so that the structure of compensation continues to be irrelevant here, and we have

$$8\delta\Delta V-8c_M\geq 2(1-p_L)\left(\alpha-\overline{\theta}\right)+(2+p_L)\left(\beta-\overline{\theta}\right)+2(1-p_H)\left(\alpha+\overline{\theta}\right)+p_H\left(\beta+\overline{\theta}\right),$$

where $\delta\Delta V=\frac{\delta}{1-\delta}\Delta u_i^{FB}-\frac{\delta}{1-\delta}\left(p_L+p_H\right)\left(\frac{1-\alpha}{8}\right).$

Since the two probabilities are perfect substitutes on LHS, we can look at RHS and see which is more effective in relaxing the constraint. We have, for p_H :

$$-2\left(\alpha+\overline{\theta}\right)+\left(\beta+\overline{\theta}\right)=-2\alpha-\overline{\theta}+\beta,$$

while for p_L gives us

$$-2\left(\alpha-\overline{\theta}\right)+\left(\beta-\overline{\theta}\right)=-2\alpha+\overline{\theta}+\beta,$$

so that we will use p_H alone first, until reaching (A,C)/(C,A) at the top, after which we will start switching to (A,A) for (H,H) and, potentially, mixing at the bottom as well. However, note that the last expression shows that mixing at the bottom can help only when $\beta<2\alpha-\overline{\theta}$. Otherwise, we are increasing the reneging temptation which in this case is stronger for (C,A) than (C,C).

Next, we need to look at the no-second guessing constraint, which becomes

$$0\geq\Pr(L|C)\max\left\{\left(\Pr(d_2=C|C,L)\left(\alpha-\overline{\theta}+c_i^L\right)+\Pr(d_2=A|C,L)\left(\beta-\overline{\theta}-\delta\Delta V+\left(c_j^A+c_j^C\right)\right)\right)-\delta\Delta V,0\right\}$$
$$+\Pr(H|C)\max\left\{\left(\Pr(d_2=C|C,H)\left(\alpha+\overline{\theta}+c_i^H\right)+\Pr(d_2=A|C,H)\left(\beta+\overline{\theta}-\delta\Delta V+\left(\widetilde{c}_j^A+\widetilde{c}_j^C\right)\right)\right)-\delta\Delta V,0\right\}-$$
c_L

Now, for the constraint to be relevant, we need to have

$$\left(\Pr(d_2 = C|C, L)\left(\alpha - \overline{\theta} + c_i^L\right) + \Pr(d_2 = A|C, L)\left(\beta - \overline{\theta} - \delta\Delta V + \left(c_j^A + c_j^C\right)\right)\right) - \delta\Delta V \leq 0,$$

after which we can write the remaining constraint as

$$2\delta\Delta V \geq 2(1 - p_H)\left(\alpha + \overline{\theta}\right) + p_H\left(\beta + \overline{\theta}\right) + 2(1 - p_H)c_i^H + p_H\left(\widetilde{c}_j^A + \widetilde{c}_j^C\right) - \left(6 - (p_L + p_H)\right)c_L.$$

As before, $p_L > 0$ only makes this constraint tighter, and so this constraint also warrants us to first use up all of p_H. So our first pass involves solving for the ranges for which only mixing at the top is sustainable. First, to derive the lower bound for both constraints, we have, for the obedience constraint:

$$\begin{aligned} 8\delta\Delta V - 8c_M &\geq 2\left(\alpha - \overline{\theta}\right) + 2\left(\beta - \overline{\theta}\right) + 2(1 - p_H)\left(\alpha + \overline{\theta}\right) + p_H\left(\beta + \overline{\theta}\right) \\ 8\delta\Delta V - 8c_M &\geq 4\alpha + 2\left(\beta - \overline{\theta}\right) - p_H\left(2\alpha + \overline{\theta} - \beta\right) \end{aligned}$$

where $\delta\Delta V = \frac{\delta}{1-\delta}\Delta u_i^{FB} - \frac{\delta}{1-\delta}(p_H)\left(\frac{1-\alpha}{8}\right)$,

with limit $8\delta\Delta V \geq 2\alpha + 3\left(\beta - \overline{\theta}\right) + 8c_M$, so we have

$$\delta \geq \frac{2\alpha + 3\left(\beta - \overline{\theta}\right) + 8c_M}{8\Delta u_i^{(1)} + \left(2\alpha + 3\left(\beta - \overline{\theta}\right) + 8c_M\right)},$$

while the interior mixing probability needs to solve

$$\begin{aligned} &8\left(\frac{\delta}{1-\delta}\Delta u_i^{FB} - \frac{\delta}{1-\delta}(p_H)\left(\frac{1-\alpha}{8}\right)\right) - 8c_M \geq 4\alpha + 2\left(\beta - \overline{\theta}\right) - p_H\left(2\alpha + \overline{\theta} - \beta\right) \\ &p_H \geq \frac{\left(4\alpha + 2\left(\beta - \overline{\theta}\right) + 8c_M - 8\frac{\delta}{1-\delta}\Delta u_i^{FB}\right)}{\left[\left(2\alpha + \overline{\theta} - \beta\right) - \frac{\delta}{1-\delta}(1 - \alpha)\right]}. \end{aligned}$$

Similarly, for the second-guessing constraint, we have

$$2\delta\Delta V \geq 2(1 - p_H)\left(\alpha + \overline{\theta}\right) + p_H\left(\beta + \overline{\theta}\right) + 2(1 - p_H)c_i^H + p_H\left(\widetilde{c}_j^A + \widetilde{c}_j^C\right) - (6 - p_H)c_L.$$

Now, as before, we can ideally set $2(1 - p_H)c_i^H + p_H\left(\widetilde{c}_j^A + \widetilde{c}_j^C\right) = 0$ and use the low states to provide the full compensation for the superior, which would simplify this constraint to

$$2\delta\Delta V \geq 2\left(\alpha + \overline{\theta}\right) - p_H\left(2\alpha + \overline{\theta} - \beta\right) - (6 - p_H)c_L,$$

which obtains the limit of

$$2\delta\Delta V \geq \overline{\theta} + \beta - 5c_L \rightarrow \delta \geq \frac{\overline{\theta} + \beta - 5c_L}{2\Delta u_i^{(1)} + \overline{\theta} + \beta - 5c_L}$$

and the interior probability needs to satisfy

$$2\left(\frac{\delta}{1-\delta}\Delta u_i^{FB} - \frac{\delta}{1-\delta}(p_H)\left(\frac{1-\alpha}{8}\right)\right) \geq 2\left(\alpha + \overline{\theta}\right) - p_H\left(2\alpha + \overline{\theta} - \beta - c_L\right) - 6c_L$$

$$p_H \geq \frac{2\left(\alpha+\overline{\theta}\right)-6c_L-2\frac{\delta}{1-\delta}\Delta u_i^{FB}}{\left[\left(2\alpha+\overline{\theta}-\beta-c_L\right)-\frac{\delta}{1-\delta}\left(\frac{1-\alpha}{4}\right)\right]}.$$

Finally, it is possible that the cost constraint is binding. Then, to minimize the payments we need to revert to the budget constraint, which was

$$8c_M = 2\left(c_j^A + c_j^C\right) + 2c_i^L + 2c_i^H\left(1-p_H\right) + p_H\left(\widetilde{c}_j^A + \widetilde{c}_j^C\right),$$

and preventing the deviation in the other state as well, we must have

$$\left(2c_i^L + 2\left(c_j^A + c_j^C\right)\right) = 6\delta\Delta V - 2(1-p_L)\left(\alpha-\overline{\theta}\right) - (2+p_L)\left(\beta-\overline{\theta}\right),$$

which then gives us

$$\begin{aligned}
&8c_M = 6\delta\Delta V - 2\left(\alpha-\overline{\theta}\right) - 2\left(\beta-\overline{\theta}\right) + 2c_i^H\left(1-p_H\right) + p_H\left(\widetilde{c}_j^A + \widetilde{c}_j^C\right)\\
&8c_M + 2\left(\alpha-\overline{\theta}\right) + 2\left(\beta-\overline{\theta}\right) - 6\delta\Delta V = 2c_i^H\left(1-p_H\right) + p_H\left(\widetilde{c}_j^A + \widetilde{c}_j^C\right),
\end{aligned}$$

and going back to the constraint, we have

$$\begin{aligned}
&2\delta\Delta V \geq 2(1-p_H)\left(\alpha+\overline{\theta}\right) + p_H\left(\beta+\overline{\theta}\right) + 8c_M + 2\left(\alpha-\overline{\theta}\right) + 2\left(\beta-\overline{\theta}\right) - 6\delta\Delta V - (6-p_H)\,c_L\\
&8\delta\Delta V - 8c_M \geq 2\left(2\alpha+\beta-\overline{\theta}\right) - p_H\left[2\alpha+\overline{\theta}-\beta\right] - (6-p_H)\,c_L.
\end{aligned}$$

But going back to the obedience constraint, which was

$$8\delta\Delta V - 8c_M \geq 4\alpha + 2\left(\beta-\overline{\theta}\right) - p_H\left(2\alpha+\overline{\theta}-\beta\right),$$

it is clear that the obedience constraint subsumes the second-guessing constraint. Indeed, this is a general idea because if we push the agent to be just indifferent to violating the second-guessing constraint, and he needs to pay a price for it, he would just violate the obedience constraint instead.

Once $p_H = 1$, the superior needs to start mixing in (A,A), which allows the superior to push the belief of the subordinate regarding his state even lower, and now potentially also start mixing at the bottom. In this case, the probabilities become (letting $(1-\widetilde{p}_H)$ denote the probability of playing (A,A))

$$\Pr(L|C) = \frac{4-p_L}{4-p_L+\widetilde{p}_H} \qquad \Pr(H|C) = \left(\frac{\widetilde{p}_H}{4-p_L+\widetilde{p}_H}\right)$$

$$\begin{aligned}
&\Pr(d_2 = C|C,L) = \left(\frac{2(1-p_L)}{4-p_L}\right), \qquad \Pr(d_2 = A|C,L) = \left(\frac{2+p_L}{4-p_L}\right)\\
&\Pr(d_2 = C|C,H) = 0, \qquad \Pr(d_2 = A|C,H) = 1.
\end{aligned}$$

With this, we can then write the obedience constraint as

$$\begin{aligned}
&\Pr(L|C)\left(\Pr(d_2 = C|C,L)\left(1-c_i^L\right) + \Pr(d_2 = A|C,L)\left(\overline{\theta} + w_{A,C} - c_j^C\right)\right)\\
&+\Pr(H|C)\left(\Pr(d_2 = C|C,H)(1-c_i^H) + \Pr(d_2 = A|C,H)\left(-\overline{\theta} + \widetilde{w}_{A,C} - \widetilde{c}_j^C\right)\right) + \delta V^{cont} \geq
\end{aligned}$$

$$\Pr(L|C)\left(\Pr(d_2=C|C,L)\left(1+\alpha-\overline{\theta}\right)+\Pr(d_2=A|C,L)\beta\right)$$
$$+\Pr(H|C)\left(\Pr(d_2=C|C,H)\left(1+\alpha+\overline{\theta}\right)+\Pr(d_2=A|C,H)\beta\right)+\delta V^{dev}$$

$$\frac{4-p_L}{4-p_L+\widetilde{p}_H}\left(\left(\frac{2(1-p_L)}{4-p_L}\right)\left(1-c_i^L\right)+\left(\frac{2+p_L}{4-p_L}\right)\left(\overline{\theta}+\delta V^{cont}-c_j^A-c_j^C\right)\right)$$
$$+\left(\frac{\widetilde{p}_H}{4-p_L+\widetilde{p}_H}\right)\left(\left(-\overline{\theta}+\delta V^{cont}-\widetilde{c}_j^A-\widetilde{c}_j^C\right)\right)+\delta V^{cont}\geq$$
$$\frac{4-p_L}{4-p_L+\widetilde{p}_H}\left(\left(\frac{2(1-p_L)}{4-p_L}\right)\left(1+\alpha-\overline{\theta}\right)+\left(\frac{2+p_L}{4-p_L}\right)\beta\right)$$
$$+\left(\frac{\widetilde{p}_H}{4-p_L+\widetilde{p}_H}\right)\left(\Pr(d_2=A|C,H)\beta\right)+\delta V^{dev}$$

$$\left((2(1-p_L))\left(1-c_i^L\right)+(2+p_L)\left(\overline{\theta}-c_j^A-c_j^C\right)\right)+6\delta\Delta V\geq$$
$$\left((2(1-p_L))\left(1+\alpha-\overline{\theta}\right)+(2+p_L)\beta\right),$$

while the superior's information constraint is

$$2c_M=\tfrac{1}{4}\left(c_j^A+c_j^C\right)+\tfrac{1}{4}\left(c_j^A+c_j^C\right)+\tfrac{1}{4}\left(\widetilde{p}_H\left(\widetilde{c}_j^A+\widetilde{c}_j^C\right)+(1-\widetilde{p}_H)2c_i^H\right)+\tfrac{1}{4}\left((1-p_L)\,2c_i^L+p_L\left(c_j^A+c_j^C\right)\right)$$
$$8c_M=(2+p_L)\left(c_j^A+c_j^C\right)+2\left(1-p_L\right)c_i^L+\widetilde{p}_H\left(\widetilde{c}_j^A+\widetilde{c}_j^C\right)+(1-\widetilde{p}_H)2c_i^H.$$

Now, since the (A,A) state has no constraints, the superior can charge the maximum rent in that state and set $c_i^H=\delta\Delta V$. with that, we can finally write the information constraint as

$$8c_M-(1-\widetilde{p}_H)2\delta\Delta V=(2+p_L)\left(c_j^A+c_j^C\right)+2\left(1-p_L\right)c_i^L+\widetilde{p}_H\left(\widetilde{c}_j^A+\widetilde{c}_j^C\right),$$

which we can substitute in the obedience constraint to give us

$$8\delta\Delta V-8c_M\geq(2(1-p_L))\left(\alpha-\overline{\theta}\right)+(2+p_L)\left(\beta-\overline{\theta}\right)+(\widetilde{p}_H)\left(\beta+\overline{\theta}\right),$$

where the continuation value is now

$$\tfrac{1}{4}\left(1+\alpha+\overline{\theta}\right)+\tfrac{1}{4}\left(\overline{\theta}\right)+\tfrac{1}{4}\left((1-p_L)+\tfrac{p_L}{2}\left(\overline{\theta}+w_{A,C}\right)+\tfrac{p_L}{2}\left(1+\alpha-\overline{\theta}-w_{A,C}\right)\right)$$
$$+\tfrac{1}{4}\left((1-\widetilde{p}_H)\,\beta+\tfrac{\widetilde{p}_H}{2}\left(-\overline{\theta}+\widetilde{w}_{A,C}\right)+\tfrac{\widetilde{p}_H}{2}\left(1+\alpha+\overline{\theta}-\widetilde{w}_{A,C}\right)\right)=$$
$$\tfrac{1}{4}\left(1+\alpha+\overline{\theta}\right)+\tfrac{1}{4}\left(\overline{\theta}\right)+\tfrac{1}{4}\left(1-\tfrac{p_L}{2}\left(1-\alpha\right)\right)+\tfrac{1}{4}\left((1-\widetilde{p}_H)\,\beta+\tfrac{\widetilde{p}_H}{2}\left(1+\alpha\right)\right)$$
$$=\tfrac{(3+\alpha)}{4}+\tfrac{1}{2}\overline{\theta}-\tfrac{p_L}{8}\left(1-\alpha\right)-\left(\tfrac{1-\beta}{4}\right)+\tfrac{\widetilde{p}_H}{8}\left(1+\alpha-2\beta\right)$$

But note that this computation is valid only when the transfer is positive - so if $8c_M\leq(1-\widetilde{p}_H)2\delta\Delta V$, then the expression becomes simply

$$(6+2\widetilde{p}_H)\,\delta\Delta V\geq\left((2(1-p_L))\left(\alpha-\overline{\theta}\right)+(2+p_L)\left(\beta-\overline{\theta}\right)\right)+(\widetilde{p}_H)\left(\beta+\overline{\theta}\right),$$

so we have for the obedience constraint that

$$8\delta\Delta V-8c_M\geq(2(1-p_L))\left(\alpha-\overline{\theta}\right)+(2+p_L)\left(\beta-\overline{\theta}\right)+(\widetilde{p}_H)\left(\beta+\overline{\theta}\right)\qquad 8c_M>(1-\widetilde{p}_H)2\delta\Delta V$$

$$(6+2\widetilde{p}_H)\,\delta\Delta V \geq \left((2(1-p_L))\left(\alpha-\overline{\theta}\right)+(2+p_L)\left(\beta-\overline{\theta}\right)\right)+(\widetilde{p}_H)\left(\beta+\overline{\theta}\right) \qquad 8c_M \leq (1-\widetilde{p}_H)2\delta\Delta V$$

The second-guessing constraint is given by

$$2\widetilde{p}_H\delta\Delta V \geq (\widetilde{p}_H)\left(\beta+\overline{\theta}\right)-c_L\left(4-p_L+\widetilde{p}_H\right).$$

Now, we know that $\widetilde{p}_H$ will always be used, so the second-guessing constraint will be slack at the limit. For the other constraint, we do better either using only $\widetilde{p}_H$ or both. The relevant limits are then, first if only $\widetilde{p}_H$ is used:

$$4\delta\Delta V-4c_M \geq \left(\alpha+\beta-2\overline{\theta}\right) \qquad 4c_M > \delta\Delta V$$
$$3\delta\Delta V \geq \left(\alpha+\beta-2\overline{\theta}\right) \qquad 4c_M \leq \delta\Delta V,$$

with value created as $\frac{(3+\alpha)}{4}+\frac{1}{2}\overline{\theta}-\left(\frac{1-\beta}{4}\right)$. Conversely, if it is optimal to use both, the limit is

$$8\delta\Delta V-8c_M \geq 3\left(\beta-\overline{\theta}\right) \qquad 4c_M > \delta\Delta V$$
$$6\delta\Delta V \geq 3\left(\beta-\overline{\theta}\right) \qquad 4c_M \leq \delta\Delta V,$$

with the value created as $\frac{(3+\alpha)}{4}+\frac{1}{2}\overline{\theta}-\frac{1}{8}(1-\alpha)-\left(\frac{1-\beta}{4}\right)$. These provide the lower bounds for the randomization. The interior probabilities, in turn, are given by the earlier inequalities. [to be completed and cleaned]