Interest Rates Under Falling Stars

Michael D. Bauer and Glenn D. Rudebusch

Federal Reserve Bank of San Francisco

July 10, 2019
NBER Summer Institute
Forecasting & Empirical Methods

Long-run trends in macroeconomics

- Long-run steady state of the economy changes.
  - Example: productivity and growth slowdown in 1970s
- Stochastic trends are widely used in empirical work.

Trend inflation: π_t^*

- Perceived inflation target of central bank
- Important for inflation forecasting

Equilibrium real interest rate: r_t^*

- Real interest rate consistent with output at potential and inflation at target, neutral rate for monetary policy
- Determined by fundamentals: productivity growth, demographics, price of capital goods, etc.

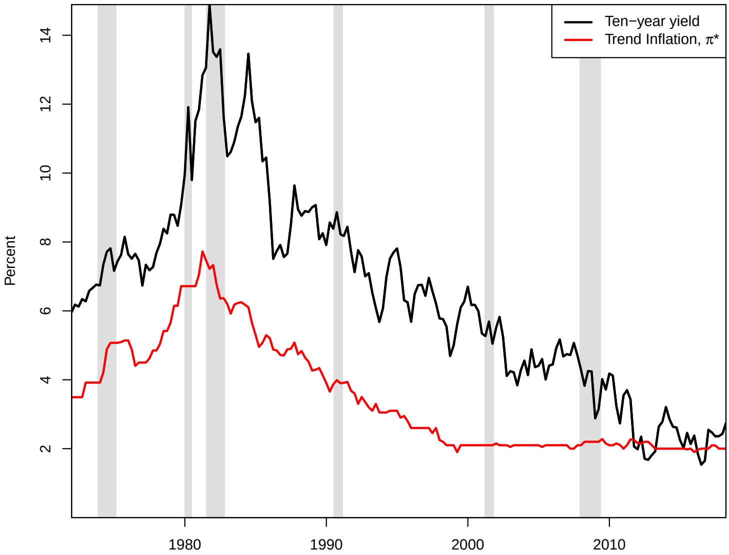
Macro trends and interest rates

- Interest rates are extremely persistent.
  - Longstanding challenge for financial econometrics/yield curve literature
- Theory says that interest rate trend driven by macro trends:

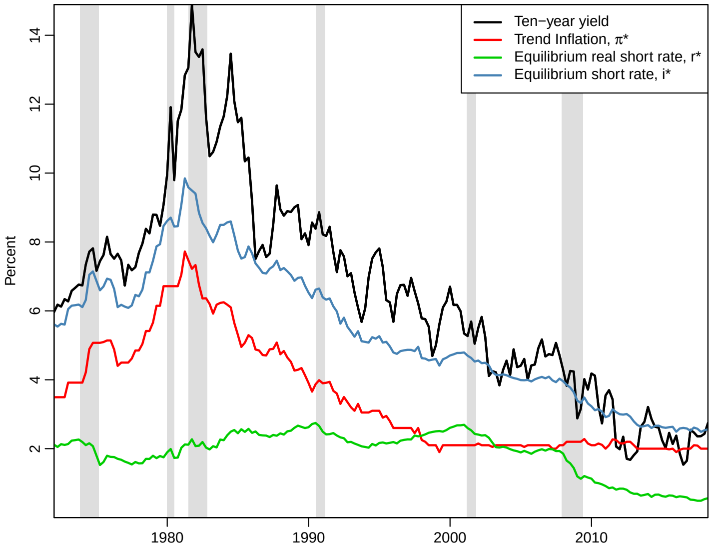
$$i_t^* = r_t^* + \pi_t^*$$

- Do we see this in the data?

Secular decline in U.S. long-term interest rates



Secular decline in U.S. long-term interest rates



Long-run trends in finance models

- No-arbitrage yield curve models assume **stationary** interest rates and don't allow for (macro) trends.

  "The level of nominal interest rates is surely a stationary variable in a fundamental sense: we have observations near 6% as far back as ancient Babylon, and it is about 6% again today." (Cochrane, 2005)

- Long-run expectations are stable because of mean reversion.
- Swings in long-term interest rates are attributed mainly to term premium.
- Models do poorly in forecasting (don't beat random walk).

$\Rightarrow$ **Disconnect between macroeconomics and finance**

This paper

Question

How much do macro trends matter for the yield curve?

What we do

- Quantify importance of macro trends for yield curve dynamics
- Model-free stylized facts: use various proxies for π_t^*, r_t^* and i_t^*, link to yield curve, cointegration, predict excess bond returns
- Dynamic term structure model with time-varying i_t^*

What we find

- Quantitatively important role for *both* macro trends
- Yields revert to i_t^*, not to constant mean
- Accounting for trends changes estimated bond risk premia and improves out-of-sample yield forecasts

Literature

Inflation trend and interest rates

Kozicki and Tinsley (2001), Dewachter and Lyrio (2006), Rudebusch and Wu (2008), Cieslak and Povala (2015)

Equilibrium real interest rate

Laubach and Williams (2003), Kiley (2015), Rachel and Smith (2015), Hamilton et al. (2016), Johannsen and Mertens (2016), Christensen and Rudebusch (2017), Holston et al. (2017)

Interest-rate forecasting

Diebold and Li (2006), Christensen et al. (2011), Dijk et al. (2014)

Time series models for interest rates with both π_t^* and r_t^*

Del Negro et al. (2017), Johannsen and Mertens (2018), Crump et al. (2018)

Outline

Outline

The equilibrium nominal interest rate: i_t^*

- Decomposition of long-term interest rates:

$$y_t^{(n)} = \frac{1}{n}\sum_{j=0}^{n-1} E_t i_{t+j} + TP_t^{(n)} = i_t^* + \frac{1}{n}\sum_{j=0}^{n-1} E_t i_{t+j}^c + TP_t^{(n)},$$

- Equilibrium/long-run mean/Beveridge-Nelson trend in nominal short rate:

$$i_t^* \equiv \lim_{j\to\infty} E_t i_{t+j}$$

- Fisher equation: $i_t = r_t + E_t \pi_{t+1} \quad \Rightarrow \quad i_t^* = r_t^* + \pi_t^*$

- Macro trends shift the level of expectations and yields, but are changes in i_t^* quantitatively important for yields and risk premia?

Trend inflation: π_t^*

- Inflation target of the central bank (in policy rule/obj.fn.)
  - Most recent DSGE models allow for time-varying π_t^*
- Trend component of inflation (e.g., UCSV model)
  - To forecast π_t need nowcast and π_t^* (Faust and Wright, 2013)
- Our preferred estimate of π_t^*: Perceived Target Rate (PTR) from FRB/US model
  - Long-run expectations of PCE inflation
  - Survey of Professional Forecasters since 1979
  - Model-based (Kozicki and Tinsley, 2001) before 1979
  - Stable at two percent since 2000
  - Widely used in research studies
  - Generally in line with other estimates of π_t^*

Equilibrium real interest rate: r_t^*

- Several different definitions of r_t^*: equilibrium/neutral/natural rate
  - For example, neutral rate: real short rate at which monetary policy is neither expansionary nor contractionary
- We focus on long-run trend in real rate:

$$r_t^* = \lim_{h \to \infty} E_t r_{t+h}$$

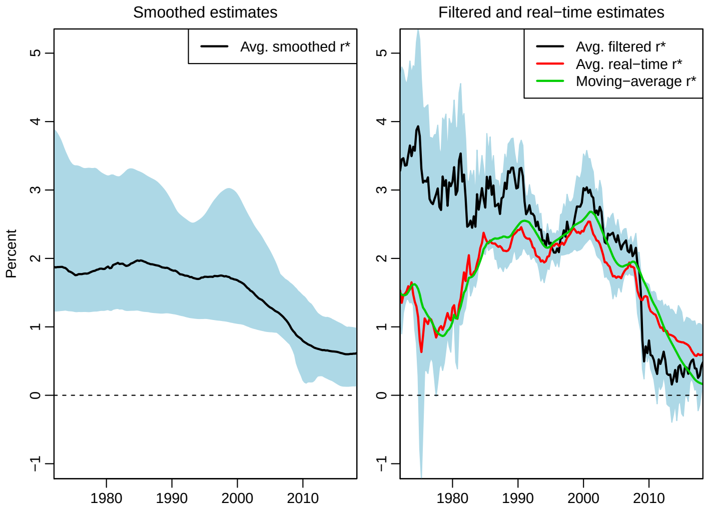
- Growing literature estimates and analyzes r_t^*
- Difficult to pin down the level of r_t^*
- But estimates point to a substantial decline over past 20 years
  - Slower productivity growth, aging population, secular stagnation, ...

Estimates of r_t^*

- Existing estimates of long-run trend in real rate
  - Johannsen and Mertens (2016, 2018), JM
  - Del Negro et al. (2017), DGGT
- Existing estimates of neutral real rate from macro models
  - Laubach and Williams (2003, 2016), LW
  - Holston, Laubach and Williams (2017), HLW
  - Kiley (2015)

  $\Rightarrow$ Consistent with long-run trend because r_t^* is martingale
- Our own estimates of long-run trend
  - Univariate unobserved components model
  - State space model with nominal rates and inflation (and PTR)
  - SSM with macro trend proxies – long moving averages of GDP growth and labor hours growth (Lunsford and West, 2017)
  - Simple exponentially-weighted moving average ($\alpha = 0.98$)

Estimates of r_t^*



Smoothed: LW, Kiley, JM, DGGT, our own three models. **Filtered**: LW, HLW, Kiley
Real-time: JM, DGGT, our own three models, moving average

Outline

Data

- Sample period: 1971:Q4 to 2018:Q1
- Interest rates: 3m, 6m T-bill rates and 1-15y zero-coupon Treasury yields from Gürkaynak, Sack and Wright (2007)
- PTR estimate of π_t^*
- filtered, real-time and moving-average estimates of r_t^*

Macro-finance trends and persistence in yields

Theoretical prediction

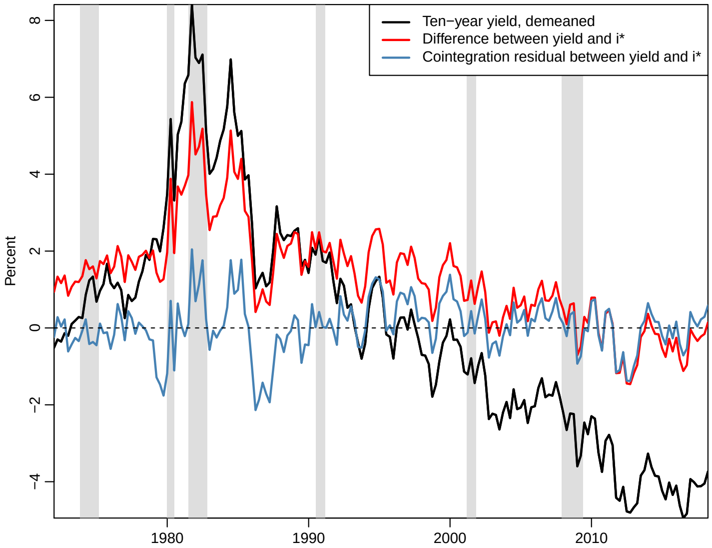
Persistence in yields driven by underlying macro trend $i_t^* = \pi_t^* + r_t^*$

- $y_t^{(n)}$ and i_t^* cointegrated
- $y_t^{(n)}$ and π_t^* not cointegrated because of r_t^*

Empirical investigation

Is the variation in macro trends large enough to matter?

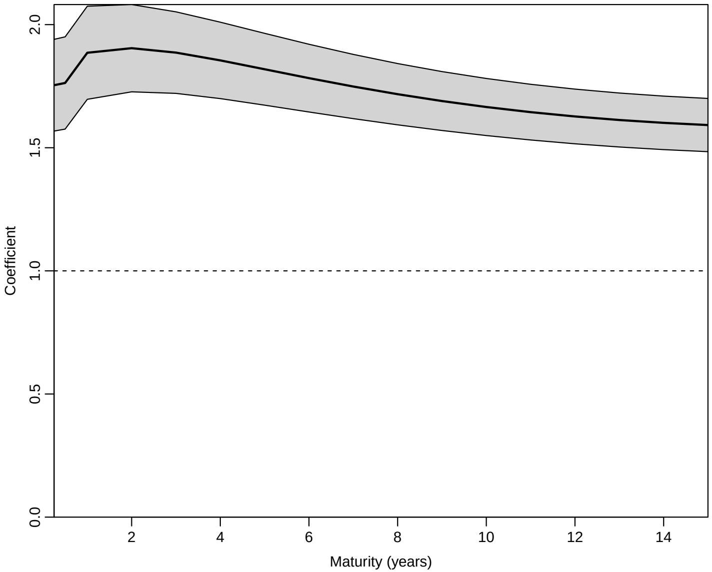
Interest rate cycles



Cointegration (DOLS) regressions for ten-year yield

	Yield	(1)	(2)	(3)
π_t^*		1.65	1.53	
		(0.11)	(0.07)	
r_t^*			1.76	
			(0.16)	
i_t^*				1.67
				(0.06)
R^2		0.85	0.96	0.95
SD	2.94	1.31	0.70	0.70
$\hat{\rho}$	0.97	0.88	0.65	0.64
Half-life	26.4	5.6	1.6	1.5
ADF	-1.13	-2.60	-5.32***	-5.37***
LFST	0.00	0.03	0.72	0.71
Johansen $r = 0$		13.34	46.83***	30.69***
Johansen $r = 1$		1.29	11.57	0.73
ECM $\hat{\alpha}$		-0.11	-0.44	-0.45
		(0.03)	(0.08)	(0.08)

Cointegration (DOLS) regressions of yields on i_t^*



Predictive regressions for bond returns

- ECM suggests that trend i_t^* determines future evolution of bond yields.
- What matters for investors are excess bond returns:

$$rx_{t,t+h}^{(n)} = p_{t+h}^{(n-h)} - p_t^{(n)} - y_t^{(h)}$$

- We use one-quarter holding period ($h = 1$) and predict average $\overline{rx}_{t,t+1}$
- Do π_t^*, r_t^*, i_t^* have incremental predictive power beyond what's in the yield curve at time t?
  1. Test spanning hypothesis using bootstrap (Bauer and Hamilton, 2017)
  2. Compare predictive power of principal components (PCs) of yields and PCs of *detrended* yields

Predictive regressions for excess bond returns

	(1)	(2)	(3)	(4)	(5)	(6)
PC1	0.08	0.98	1.39	2.38	2.04	2.47
	(0.17)	(0.26)	(0.39)	(0.67)	(0.56)	(0.61)
PC2	0.43	0.47	0.43	0.67	0.68	0.70
	(0.17)	(0.17)	(0.17)	(0.15)	(0.15)	(0.15)
PC3	-2.37	-1.79	-1.92	-0.92	-0.90	-0.86
	(1.34)	(1.27)	(1.22)	(1.39)	(1.43)	(1.35)
π_t^*		-1.95	-2.21	-4.40	-3.88	
		(0.44)	(0.47)	(1.10)	(0.92)	
		[0.00]	[0.00]	[0.00]	[0.00]	
r_t^*			-1.19	-3.89	-2.70	
			(0.59)	(1.47)	(1.04)	
			[0.14]	[0.07]	[0.04]	
i_t^*						-4.50
						(1.05)
						[0.00]
R^2	0.09	0.16	0.18	0.21	0.20	0.21
Memo: r^*			filtered	real-time	mov. avg.	real-time

Predictive regressions with detrended yields (residuals)

	(1)	(2)	(3)	(4)
PC1	0.08	0.98	1.25	1.36
	(0.17)	(0.25)	(0.51)	(0.50)
PC2	0.43	0.48	0.76	0.78
	(0.17)	(0.17)	(0.16)	(0.16)
PC3	-2.37	-1.77	-0.79	-0.73
	(1.34)	(1.26)	(1.38)	(1.33)
R^2	0.09	0.15	0.18	0.20

(1) PCs of yields

(2) PCs of yields detrended by π_t^*

(3) PCs of yields detrended by π_t^* and r_t^* (real-time)

(4) PCs of yields detrended by i_t^*

Outline

Specification of no-arbitrage model

- Four state variables: i_t^* and $P_t = WY_t$ (3 linear combinations of yields)
- Short rate: $i_t = \delta_0 + \delta_1' P_t$
- Risk-neutral dynamics (stationary; Joslin-Singleton-Zhu normalization):

$$P_t = \mu^{\mathbb{Q}} + \Phi^{\mathbb{Q}} P_{t-1} + u_t^{\mathbb{Q}}$$

- Yields are affine: $Y_t = A + BP_t$
- Real-world dynamics (our main innovation):

$$\begin{aligned} P_t &= \bar{P} + \gamma i_t^* + \tilde{P}_t && \text{one common trend} \\ i_t^* &= i_{t-1}^* + \xi_t && \text{trend} \\ \tilde{P}_t &= \Phi \tilde{P}_{t-1} + w_t && \text{cycles} \end{aligned}$$

- **In a nutshell**: model with *unspanned* shifting endpoint (in the Joslin-Priebsch-Singleton sense); unit root under $\mathbb{P}$-measure but not under $\mathbb{Q}$

Estimation

Problem

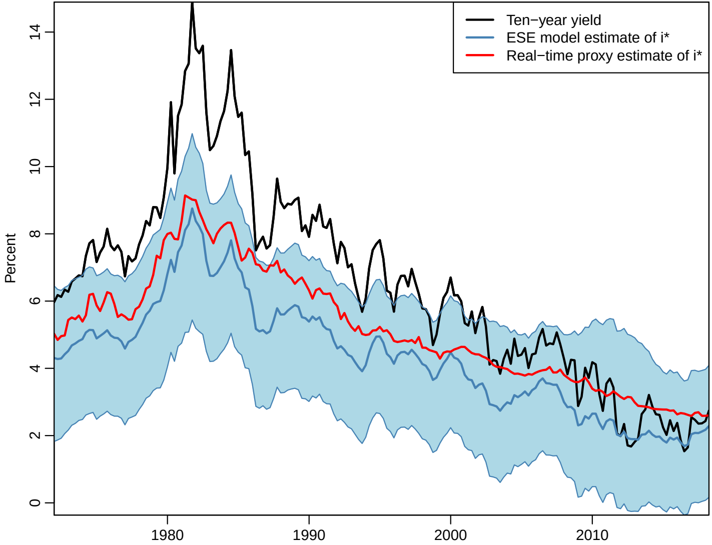
- Estimation simple in principle (Kalman filter, MLE), but estimation of long-run trends with limited samples is difficult (Watson, 1986)
- Estimates of i_t^* not sufficiently stable and robust, as others have found for π_t^*, r_t^*

Solution: Two ways to add more information

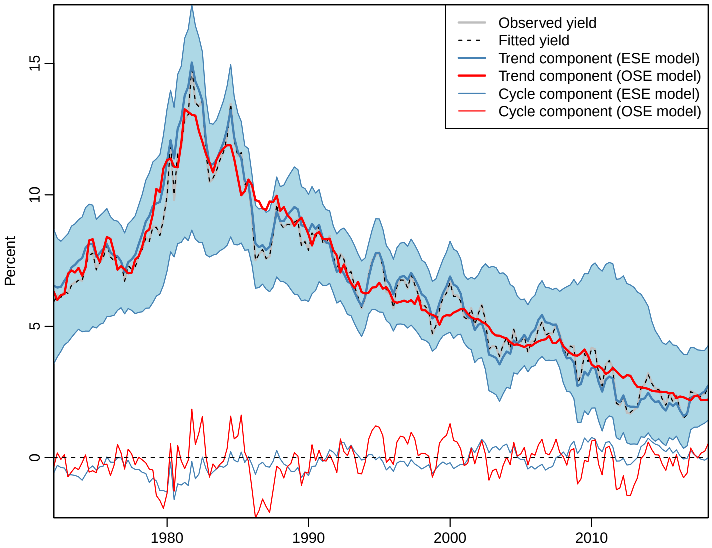
(1) Data: pin down i_t^* with external proxy
$\rightarrow$ Observed Shifting Endpoint (*OSE*) Model

(2) Prior: impose tight prior on variance of Δi_t^* to have smooth path of i_t^*, estimate with MCMC
$\rightarrow$ Estimated Shifting Endpoint (*ESE*) Model

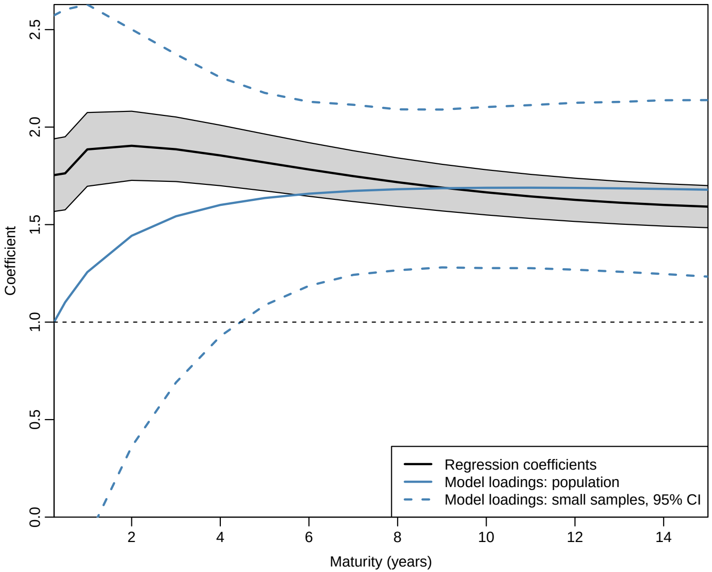
Model-based estimate of i_t^*



Ten-year yield – trend and cycle



Loadings/regressions of yields on i_t^*



Model matches stylized facts: predicting excess returns

	R^2 PCs only	R^2 with i_t^*	ΔR^2
Data	0.09	0.21	0.12
FE model	0.09	0.10	0.01
	[0.04, 0.17]	[0.05, 0.17]	[0.00, 0.04]
OSE model	0.10	0.19	0.09
	[0.04, 0.18]	[0.13, 0.26]	[0.02, 0.18]
ESE model	0.07	0.15	0.08
	[0.02, 0.13]	[0.05, 0.27]	[0.00, 0.20]

- We simulate 10,000 short samples and run excess-return regressions with and without i_t^*, report mean and 95% intervals of R^2 in sim. data
- Our shifting-endpoint models are able to capture substantial predictive gains.

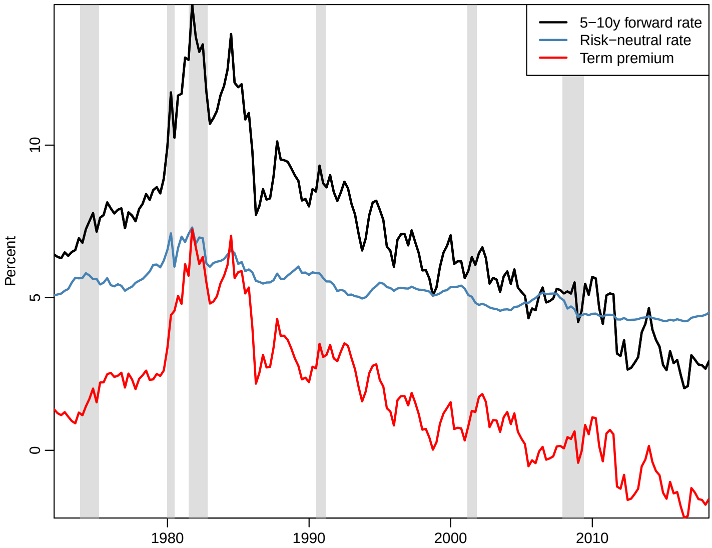
Term premium in long-term yields

- Crucial question in macroeconomics and finance: What are the underlying drivers of changes in long-term interest rates?
- Two components: expectations and term premium

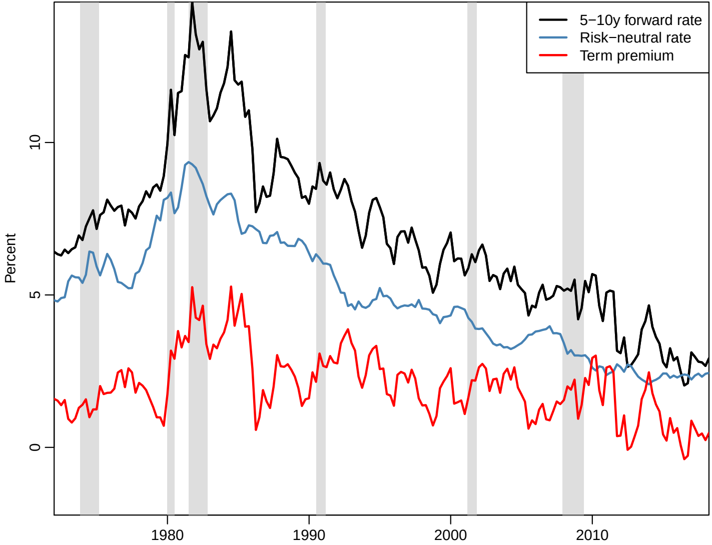
$$y_t^{(n)} = \frac{1}{n} \sum_{j=0}^{n-1} E_t i_{t+j} + TP_t^{(n)}$$

- Does accounting for movements in i_t^* make a difference for estimates of the term premium?
- Compare our models to restricted special case with Fixed Endpoint (FE), $i_t^* = i^*$
  - This is the stationary DTSM of Joslin, Singleton, Zhu (2011)

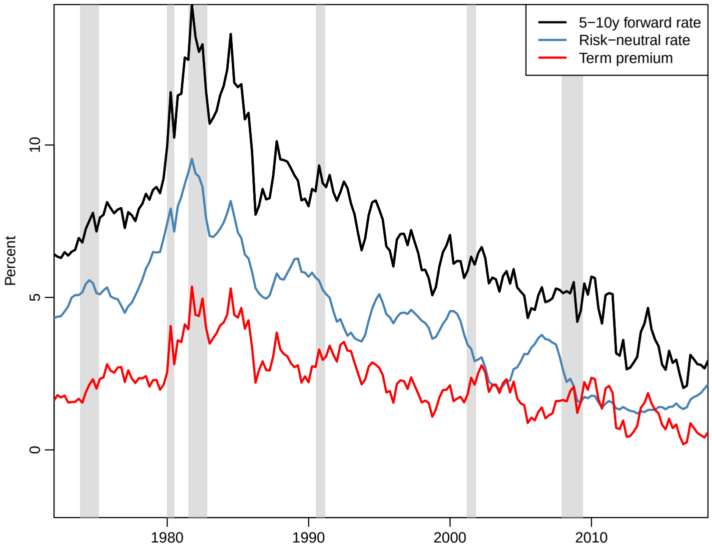
Term premium: stationary FE model



Term premium: OSE model



Term premium: ESE model



Out-of-sample forecasts for 10-year yield

Root-mean-squared forecast errors and Diebold-Mariano p-values

Horizon h (quarters):	4	10	20	30	40
Random walk (RW)	1.33	1.85	2.52	2.60	2.88
Fixed endpoint (FE)	1.42	2.25	3.28	3.72	4.19
Observed shifting endpoint (OSE)	1.17	1.76	2.37	2.39	2.60
p-value: $OSE \geq RW$	0.05	0.00	0.00	0.03	0.04
p-value: $OSE \geq FE$	0.00	0.00	0.01	0.03	0.05

First OOS forecast in 1976:Q3 (five years of data)

Out-of-sample forecasts: models vs. Blue Chip survey

Root-mean-squared forecast errors and Diebold-Mariano p-values

Horizon in years	1	2	3	4	5
Blue Chip (BC)	1.06	1.39	1.59	1.79	1.99
Random walk (RW)	0.85	1.08	1.21	1.37	1.56
Fixed endpoint (FE)	1.53	2.08	2.52	2.96	3.34
Observed shifting endpoint (OSE)	0.87	0.95	1.04	1.18	1.37
p-value: $OSE \geq BC$	0.10	0.08	0.15	0.18	0.20
p-value: $OSE \geq RW$	0.58	0.05	0.01	0.04	0.08
p-value: $OSE \geq FE$	0.00	0.00	0.00	0.00	0.00

- Forecasts are for 10-year yield
- Forecast dates: 1988:Q1 to 2011:Q4 (48 Blue Chip surveys)

Conclusion

- Long-run trends important in macroeconomics but largely ignored in finance
- Theory predicts that macro trends π_t^* and r_t^* are reflected in nominal yield curve
- Macro trends indeed quantitatively important for interest rates
  - Strong evidence for cointegration: trends account for persistence in interest rates
  - Trends change dynamics of estimated bond risk premia (expected excess returns)
- Novel term structure model with time-varying i_t^*
  - Consistent with stylized facts
  - New and different estimates of the term premium
  - More accurate out-of-sample yield forecasts
- Trends are a crucial macro-finance link