The Sandwich Effect: Challenges for Middle-Income Countries *

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May 31, 2019

Abstract

We develop a tractable growth model to show how a middle-income country (M) can be sandwiched by an innovating north country (N) and an imitating south country (S) through international trade. An increase in labor productivity of existing varieties (intensive margin) or in the number of varieties (extensive margin) produced in S may result in non-convergence of M to N, but this chasing effect from S disappears when S is sufficiently unproductive. Meanwhile, an increase in innovation in N not only enlarges the income gap between N and M (pressing effect from N) but also makes M more vulnerable to the chasing effect from S. We characterize how M should optimally allocate its resources between production and R&D to respond to the chasing effect from S and pressing effect from N.

1 Introduction

Since the World War II, only thirteen economies have successfully graduated from the middle-income status and become high-income economies, whereas most of middle-income economies have failed to converge to rich countries quickly enough. This phenomenon is generally referred to as the middle-income trap. How to avoid the middle-income trap is a key challenge facing all the emerging markets. For instance, South Korea has managed to escape the middle-income trap, but large emerging markets such as Brazil and South Africa have failed so far. Now China is a middle-income country; will it be able to escape the middle-income trap? These questions have attracted increasing attentions from both the academia and policy circles in recent years.

Unfortunately and surprisingly, however, despite the popularity of the term "middle-income trap" in public media and government policy reports, there

*preliminary and incomplete, please do not circulate.

exist very few formal models that are specifically trying to explain the mechanisms that cause an economy to fall into the middle-income trap, let alone specific policy implications for how to escape middle-income trap based on rigorous academic analyses. The closest pertinent academic research is mostly still preliminary attempts to sort out possible empirical patterns of the middle-income trap or to debate on whether middle-income trap is a real phenomenon (see, Aiya et al (2013), Eichengreen et al (2012, 2013), Quah (2013), and Im and Rosenblatt (2013), Han and Wei (2018)). Such delays could be partly because we are lack of sufficient understanding of key structural differences in the growth challenges between low-income and middle-income countries, while exisiting theories have largely assumed that their differences are only quantitative.

The primary objective of this paper is to fill this void. Certainly, unhappy families are unhappy for different reasons, and we have no reason to believe ex ante that there is only one specific mechanism that causes middle-income trap. In this paper, we propose one specific channel through which an economy at the middle-income status may fail to converge to high-income countries, that is, how well this economy makes adjustment in response to the changes in the growth behaviors of its trade partners. We observe that in the modern globalized world, middle-income countries on the one hand are losing comparative advantage in those industries whose technologies are relatively easy to imitate and adopt by low-income countries, where labor costs are lower, and at the same time, middle-income countries must upgrade their industries and overtake at least some industries that are currently dominated by high-income countries at a sufficiently rapid way in order to catch up as high-income countries have comparative advantages in innovation. As a result, middle-income countries are potentially "sandwiched" by low-income and high-income countries.

South Korea has successfully escaped the middle-income trap. In the 1990s, Korea found that Indonesia took advantage of its cheaper labor and was quickly chasing up from behind in the electronic industry, one of Korea's pillar industries. Moreover, Korea also faces fierce competition from more advanced economies such as the US and Germany. In response to the sandwiching pressures, Korea's government formed a national innovation committee chaired by the president of Korea to boost innovation and swiftly upgraded its industries. By contrast, Mexico is caught in the middle-income trap. Whereas it enjoys the advantage of exporting labor-intensive and relatively low value added products to the US and Canada thanks to the NAFTA for some period, Mexico quickly finds itself losing the market share of those industries in the US, overtaken by lower-income countries such as China. Unfortunately, Mexico fails to upgrade its industries quickly enough to compensate the loss of existing industries while the US continues to innovate and grow, so the Mexican economy is sandwiched to stagnate. In retrospection, the four Asian tigers all managed to escape the middle-income countries from 1960s to early 1990s,during which period mainland China was not competitive enough to impose a chasing effect. [[it seems useful to make a motivating plot to show a negative correlation between GDP growth rate of a middle-income country and the export or GDP growth performance of its close chasers]]]

To formalize the idea of the potential sandwiching forces, we develop a general equilibrium three-country trade model to show how a middle-income country (M) can be "sandwiched" by an innovating north country (N) and an imitating south country (S) through international trade. In the static part of the model, we establish two main theoretical results. First, the chasing effect from S, defined as the behaviors of S which tends to result in non-convergence of M to N, works either through the intensive margin (i.e., an increase in labor productivity of existing varieties in S) or through the extensive margin (i.e., an increase in the number of varieties produced in S) , but never both simultaneously. Moreover, the chasing effect is absent when the labor productivity in S is sufficiently low. Second, an increase in the number of new varieties due to innovation in N not only enlarges the income gap between N and M, which is referred to as the pressing effect from N, but also makes M more vulnerable to the chasing effect from S. In other words, chasing effect and pressing effect may intensify each other endogenously. In the dynamic part of the model, we show that the sandwiching forces are still effective on the balanced growth path, but the chasing effect now takes the form of immitation speed of varieties or the diffusion speed of productivity in S, whereas the pressing effect takes the form of innovation speed in N.

In particular, we also characterize how country M should optimally allocate its resources between production and innovation to balance its productivity growth of existing varieties and the expansion speed of the number of varieties to best respond to the behaviors of S and N.

We characterize how M should optimally allocate its resources between production and R&D to respond to the chasing effect from S and depressing effect from N.

The paper is organized as follows. Section II examines the sandwiching forces in a static environment. Section III extends the static model into a dynamic framework. Section IV concludes.

2 Static Model

2.1 Simple Setting

This model extends Krugman (1979) by adding a middle-income country as a third country into his North-South two-country setting. There are three countries in the world: North (N), Middle (M), and South (S). The populations are L_N, L_M, and L_S, respectively. Each household is endowed with one unit of labor, which is inelastically supplied. All households in the world share the same utility function as follows:

$$U = \left[\int_0^n c(i)^\theta di\right]^{1/\theta}, \theta \in (0,1), \tag{1}$$

where $c(i)$ denotes consumption of variety i and n is the measure of the whole set of horizontally differentiated varieties.

Firms in country N have free access to technologies for all the varieties $[0, n]$. Firms in country M only know how to produce a subset of varieties $[0, n_M]$, where $n_M < n$. Firms in country S have access to technologies for each $i \in [0, n_s]$ with $n_s < n_M$. The following figure shows how the knowledge space is partitioned.

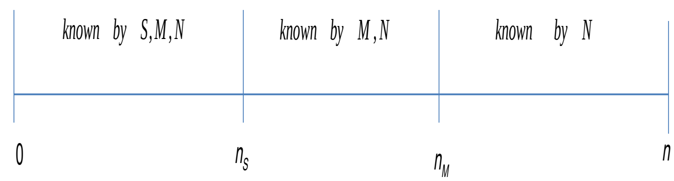

Figure 1: Technology Partition

Labor is the only production factor. For each variety that a firm knows how to produce, one unit of labor produces one unit of that variety. All the markets are perfectly competitive. The wage rates, equivalent to GDP per capita, in the three countries are denoted by w_N, w_M and w_S, respectively. Trade is free across countries.

In this simple Ricardian trade model, we easily obtain the following Lemma.

Lemma 1 *In the static trade equilibrium, $w_N > w_M > w_S$ if and only if*

$$\frac{L_N}{n - n_M} < \frac{L_M}{n_M - n_S} < \frac{L_S}{n_S}, \tag{2}$$

in which case all varieties $[0, n_s]$ are produced in country S, all varieties $(n_s, n_M]$ are produced in country M, and the rest varieties $(n_M, n]$ are all produced in country N, in addition, the following is true:

$$\frac{w_N}{w_M} = \left(\frac{n - n_M}{n_M - n_S} \frac{L_M}{L_N} \right)^{1-\theta}, \tag{3}$$

and

$$\frac{w_M}{w_S} = \left(\frac{n_M - n_S}{n_S} \frac{L_S}{L_M} \right)^{1-\theta}.$$

Proof. Refer to Appendix.

The trade specialization pattern characterized in the above Lemma can be intuitively illustrated in the following figure:

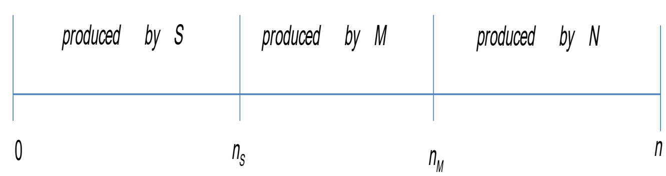

Figure 2: Free Trade Equilibrium Specialization

Our main focus is on what determines $\frac{w_N}{w_M}$, the GDP per capita gap between country M and country N. By (3), $\frac{w_N}{w_M}$ becomes larger when n_S increases or when n_M decreases (or when n increases). In other words, country M can be "sandwiched" by country S and country N from both directions when the total number of varieties produce by M (equal to $n_M - n_S$) decreases, either by an increase in n_S (from behind) or a decrease in n_M (from front). The intuition is simple. When n_S increases, the total number of varieties produced by M decreases, *ceteris paribus*, so the induced world demand for the labor of country M decreases, resulting in a lower w_M and hence a larger $\frac{w_N}{w_M}$. We refer to this force (impact of n_S on $\frac{w_N}{w_M}$) as the *chasing effect* that country S has on the "convergence" performance of country M relative to country N. Such an effect cannot be discussed in the standard North-South two-country setting, but we will argue that the chasing effect could be crucial for us to understand the non-convergence behavior of middle-income countries. On the other hand, when n_M decreases, the world demand for the labor of country M decreases whereas the demand for the labor of country N increases, so $\frac{w_N}{w_M}$ becomes larger. Likewise, when n becomes larger, the wage gap is also widened because the world demand for labor in country N increases. We refer to these forces (the impact of n_M and n on $\frac{w_N}{w_M}$) as the *pressing effect* that country N imposes on country M.

The lemma states that $w_N > w_M > w_S$ holds if and only if labor per variety is highest in country S and lowest in country N, as specified in (2). The cross-country difference in wage rates and variety prices reflects their difference in the technology accessibility. Ricardian comparative advantages fully dictate the trade pattern and absolute advantages translate into the wage (price) premium even though all varieties enter the utility function symmetrically. A closer scrutiny suggests that $w_N > w_M$ is ensured by $\frac{L_N}{n-n_M} < \frac{L_M}{n_M-n_S}$ and $w_M > w_S$ is ensured by $\frac{L_M}{n_M-n_S} < \frac{L_S}{n_S}$. When $\frac{L_N}{n-n_M} \geq \frac{L_M}{n_M-n_S}$, we would have $w_N = w_M$. When $\frac{L_M}{n_M-n_S} \geq \frac{L_S}{n_S}$, we would have $w_M = w_S$.

More generally, what happens when labor productivities for the same accessible varieties are heterogeneous across countries? What happens when (2) is violated? To address these questions, we will now explore a more general model setting.

2.2 General Setting

Suppose that everything is identical to the previous simple setting except that labor productivities in the three countries are now A_N, A_M, and A_S, respectively. That is, in country $j \in \{N, M, S\}$, one unit of labor produces A_j units of goods for each variety that country j knows how to produce, so the domestic unit cost for such a variety is $\frac{w_j}{A_j}$. When $A_j = 1$ for all $j \in \{N, M, S\}$, we are back to the simple setting examined earlier.

Lemma 2 *In the static trade equilibrium, we have*

$$\frac{w_N}{w_M} = \left(\frac{n - n_M}{n_M - n_S}\frac{L_M}{L_N}\right)^{1-\theta}\frac{A_N^\theta}{A_M^\theta}, \tag{4}$$

when $\frac{w_S}{A_S} < \frac{w_M}{A_M} < \frac{w_N}{A_N}$ *holds, which is true if and only if*

$$\frac{A_S L_S}{n_S} > \frac{A_M L_M}{n_M - n_S}, \tag{5}$$

$$\text{and } \frac{A_M L_M}{n_M - n_S} > \frac{A_N L_N}{n - n_M}. \tag{6}$$

Proof. Refer to Appendix.

Obviously, Lemma 1 is a special case of Lemma 2 when $A_j = 1$ for all $j \in \{N, M, S\}$. Observe that (4) can be rewritten as

$$\frac{w_N/A_N}{w_M/A_M} = \left[\left(\frac{A_M L_M}{n_M - n_S}\right) / \left(\frac{A_N L_N}{n - n_M}\right)\right]^{1-\theta},$$

which means that the price ratio of a variety produced in N and a variety produced in M is now determined by the ratio of their effective labor, instead of raw labor, per variety in country M and country N. After considering the cases when (5) and (6) are not simultaneously satisfied, we obtain the following results.[1]

Theorem 3 *Suppose (6) is true (i.e.* $\frac{A_M L_M}{n_M - n_S} > \frac{A_N L_N}{n - n_M}$*), we have*

$$\frac{w_N}{w_M} = \begin{cases} \frac{A_N}{A_M} & \text{if } A_S \in (0, A_0] \\ \left[\frac{A_N L_N}{A_S L_S + A_M L_M} \frac{n_M}{n - n_M}\right]^{\theta - 1} \frac{A_N}{A_M} & \text{if } A_S \in (A_0, A_1] \\ \left(\frac{n - n_M}{n_M - n_S} \frac{L_M}{L_N}\right)^{1-\theta} \frac{A_N^\theta}{A_M^\theta} & \text{if } A_S \in (A_1, \infty) \end{cases}, \tag{7}$$

where

$$A_0 \equiv \frac{n_M A_N L_N - (n - n_M) A_M L_M}{(n - n_M) L_S}; A_1 \equiv \frac{n_S A_M L_M}{(n_M - n_S) L_S}. \tag{8}$$

Proof. Refer to Appendix. Q.E.D

This theorem states that there are three different scenarios depending on A_S, the productivity level of country S. The result is intuitively seen in Figure 3.

[1] From now on, for the sake of expositional convenience, we make the following slight adjustment in model assumptions:

$$\frac{dA_J(i)}{di} = \varepsilon \text{ for all varieties } i \text{ acccessible to country } J \text{ for all } J \in \{N, M, S\},$$

where ε is an infinitesimally small but strictly negative number. This ensures that lower-indexed varieties are cheaper to produce in any country, so all countries in equilibrium will produce the full amount of lower-indexed varieties before producing higher-indexed varieties.

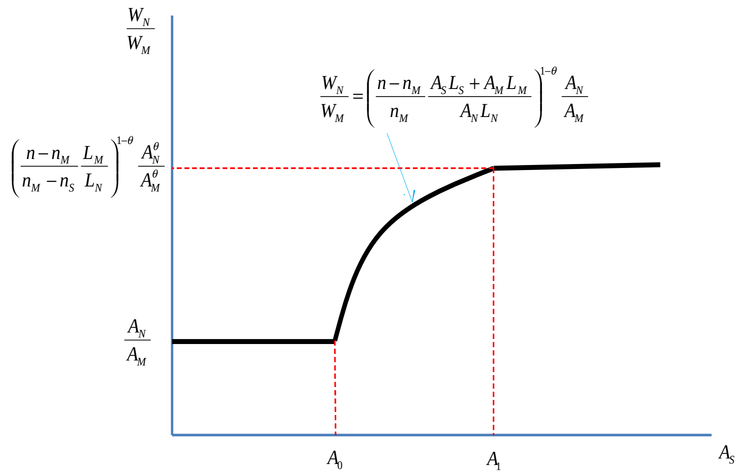

Figure 3. How w_N/w_M Changes with A_S under (6)

When A_S is sufficiently small ($A_S \in (0, A_0]$), country S only produces varieties on $[0, \widetilde{n_S}]$, where $\widetilde{n_s} < n_s$. So country M has to serve the world demand for some varieties accessible to S. It then drives up the labor demand and hence the wage rate in M, which in turn allows N to produce some varieties accessible to M as well. That is, country M produces varieties on some interval $(\widetilde{n_S}, \widetilde{n_M}]$, where $\widetilde{n_M} < n_M$, and country N not only produces varieties on $(n_M, n]$ but also produces varieties on $(\widetilde{n_M}, n_M]$, which are accessible to country M. Thus prices of these overlapping varieties must be equal across the two countries: $\frac{w_N}{A_N} = \frac{w_M}{A_M}$, or equivalently, $\frac{w_N}{w_M} = \frac{A_N}{A_M}$. So the wage gap is solely determined by their relative labor productivity, independent of A_S, n_s or L_s. In other words, there exists no chasing effect from country S, either on the intensive margin (A_S) or the extensive margin (n_S). Here, the binding constraint for productions in country M is not technology diffusion barrier n_M (because the highest-indexed variety produced in M is $\widetilde{n_M} < n_M$). In this case, $\frac{w_M}{A_M} = \frac{w_S}{A_S}$ because $\widetilde{n_s} < n_s$.

When A_S is on a medium range ($A_S \in (A_0, A_1]$), country S produces and exports more varieties than in the previous case (now $\widetilde{n_S}$ becomes larger, but $\widetilde{n_s} < n_s$ still holds), which pushes up $\widetilde{n_M}$ till $\widetilde{n_M} = n_M$. That is, country N only produces varieties on $(n_M, n]$. Now there is a chasing effect from country S in the sense that $\frac{w_N}{w_M}$ increases with A_S, because more varieties are taken over from M by country S when A_S increases, reducing the world demand for the labor in country M and hence resulting in an increase in $\frac{w_N}{w_M}$. The induced demand for labor in country M decreaes not only because it produces less varieties, but also because demand for each variety it produces also decreases due to the substitution effect as prices of those varieties taken over by S become lower than before. However, $\frac{w_N}{w_M}$ is still independent of n_S because the knowledge frontier constraint is not binding for S (*i.e.*, $\widetilde{n_s} < n_s$), so $\frac{w_M}{A_M} = \frac{w_S}{A_S}$, same as before. In other words, chasing effect only takes place in the intensive margin (through A_S), but not the extensive margin (through n_S). Moreover, different from the previous case, country size now also affects $\frac{w_N}{w_M}$. More specifically, when L_s becomes larger, w_S becomes lower due to a larger labor supply, so country S takes over some varieties produced by M, imposing a chasing effect on M. Similarly, when L_m becomes larger, w_M becomes lower

When A_S is sufficiently large ($A_S \in (A_1, \infty)$), country S serves the world demand for all the varieties it has access to, pushing M to only produce what S cannot produce ($\widetilde{n_S} = n_S$). In this case, an increase in A_S can no longer affect the set of the varieties produced by M or the set produced by country N in equilibrium because country S faces the binding constraint on its knowledge frontier n_S and country M also faces the binding constraint on its knowledge frontier n_M, so A_S has no impact on $\frac{w_N}{w_M}$. In this scenario, we have $\frac{w_N}{A_N} > \frac{w_M}{A_M} > \frac{w_S}{A_S}$, and the price differentials across the three countries simply reflect their differences in knowledge frontiers. A higher knowledge frontier of a country translates into a premium in the market price of each good it exclusively produces, even though all varieties are symmetric in terms of marginal utilities, as implied by (1). In fact, this is the case shown in Lemma 2.

In summary, no chasing effect exists when A_S is sufficiently small. In addition, when chasing effect exists, it is either through intensive margin (via A_S) or through extensive margin (via n_S), but never both margins simultaneously.

The theorem also suggests that an increase in A_N always enlarges the GDP gap between N and M, because it enhances the labor productivity and thus the wage rate in country N. Whereas the pressing effect through the intensive margin from country N exists unconditionally, the pressing effect through the extensive margin exists conditionally. More specifically, an increase in n or a decrease in n_M amplifies the gap between N and M only when A_S is sufficiently large ($A_S \in (A_0, \infty)$), because $\widetilde{n_M} < n_M$ when $A_S < A_0$, in which case country N produces some varieties that country M also knows how to produce, so their prices must be equal: $\frac{w_N}{A_N} = \frac{w_M}{A_M}$, so n and n_M do not affect $\frac{w_N}{w_M}$.

To summerize, we learn the following:

Remark 4 *Suppose (6) is true. Country M is sandwiched in both directions if and only if A_S is sufficiently large ($A_S \in (A_0, \infty)$), in which case the pressing effect exists both at intensive and extensive margins whereas the chasing effect exists only at either intensive or extensive margin, but not both. When A_S is sufficiently small ($A_S \in (0, A_0]$), there is no chasing effect but pressing effect exists at the intensive margin only.*

To see the sandwiching forces more intuitively, we now conduct more comparative static analyses.

From (7) and (8), we see that a forward push in the knowledge frontier of country S (that is, a larger n_S) enlarges the range for A_S on which $\frac{w_N}{w_M}$ strictly increases with A_S. More precisely, this range changes from $[A_0, A_1]$ to $[A_0, A_1']$ when n_S increases to n_S', where $A_1' \equiv \frac{n_S' A_M L_M}{(n_M - n_S') L_S}$ (see Figure 4).

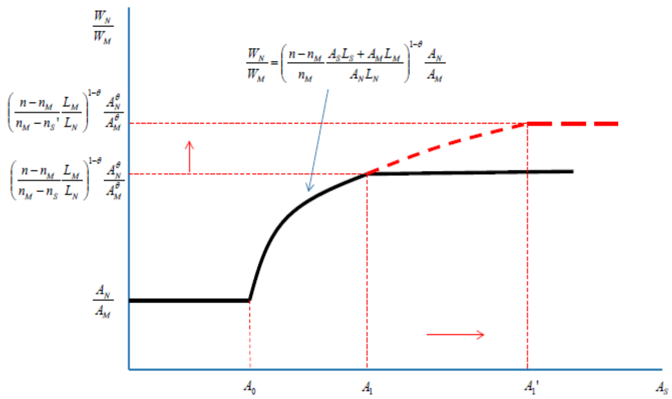

Figure 4. How w_N/w_M Changes with A_S when n_s increases under (6)

Moreover, $\frac{w_N}{w_M}$ strictly increases with n_S if and only if $A_S > A_1$. The economic intuition has been already explained earlier. It shows that an increase in n_S not only renders country M more vulnerable to the chasing effect through the intensive margin A_S but also through the extensive margin n_S directly.

Figure 5 shows that when the global knowledge frontier n increases to n' (due to, for example, more innovation arises in country N), $\frac{w_N}{w_M}$ remains unaffected only when $A_S < A_0'$, where $A_0' \equiv \frac{n_M A_N L_N - (n' - n_M) A_M L_M}{(n' - n_M) L_S}$. Consider any given $A_S \in [A_0', A_0)$, a marginal increase in A_S has no effect on $\frac{w_N}{w_M}$ before the increase in n. But after n increases to n', a marginal increase in that specific A_S would raise $\frac{w_N}{w_M}$, meaning that when the global knowledge frontier advances, country M becomes more vulnable to the chasing effect from country S. In other words, the pressing effect from N via the extensive margin endogenously augments the chasing effect from S, which intensifies the sandwiching forces. A real life example is that Mexico as a middle-income country became more likely to suffer the chasing effect from low-income countries including China during the IT technology boom in the late 1990s and early 2000s in the US.

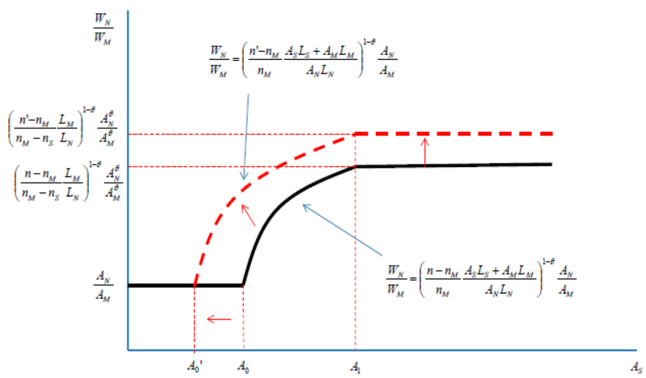

Figure 5. How w_N/w_M Changes with A_S when n increases under (6)

Figure 6 below shows that, so long as $A_M > A_0$, an increase in n_M to n'_M not only strictly reduces the GDP gap between N and M but also shortens the interval for A_S on which $\frac{w_N}{w_M}$ strictly increases with A_S (that is, $A'_0 \equiv \frac{n_M A_N L_N - (n-n'_M) A_M L_M}{(n-n'_M) L_S} > A_0$ and $A'_1 \equiv \frac{n_S A_M L_M}{(n'_M - n_S) L_S} < A_1$), and *vice versa.* So if n_M decreases due to the "push" from country N (for example, if N strengthens the international intellectual property rights protection, so that knowledge frontier of country M is dampened), country M not only becomes more likely to suffer the chasing effect from country S, but also "diverges" further from country N when the chasing effect exists. Again, the pressing effect from N and the chasing effect from S work jointly to sandwich country M.

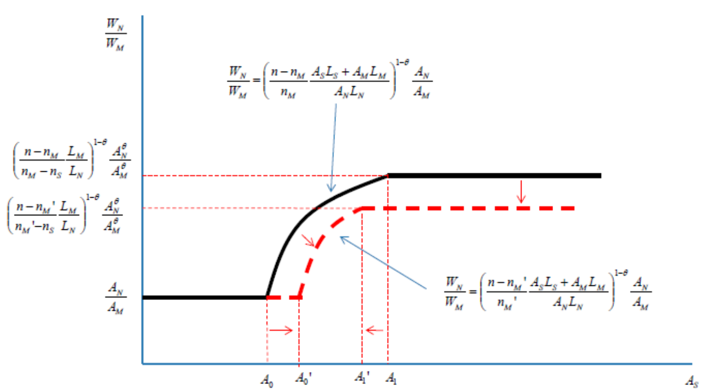

Figure 6. How w_N/w_M Changes with A_S when n_M increases under (6)

Figure 7 shows that when A_M increases to A'_M, $\frac{w_N}{w_M}$ decreases for any given A_s, but country M becomes more vulnerable to the chasing effect from country S on the intensive margin, namely, $(A_0, A_1]$ becomes larger on both sides.

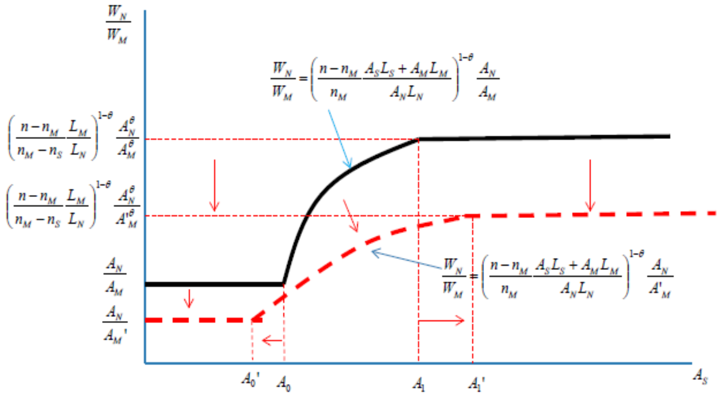

Figure 7. How w_N/w_M Changes with A_S when A_M increases under (6)

Theorem 5 *When (6) is violated, we already have* $\frac{w_N}{w_M} = \frac{A_N}{A_M}$*. Moreover,* $\frac{w_M}{w_S} = \frac{A_M}{A_S}$ *when* $A_S \in (0, \frac{n_S[L_M A_M + L_N A_N]}{(n-n_S)L_S}]$*, and* $\frac{w_M}{w_S} > \frac{A_M}{A_S}$ *when* $A_S \in (\frac{n_S[L_M A_M + L_N A_N]}{(n-n_S)L_S}, \infty)$*.*

Proof. Refer to Appendix.

The intuition is the following. When A_N is sufficiently large such that (6) no longer holds, country N is so productive that it also produces some varieties that are accessible to country M, that is, $\widetilde{n_M} < n_M$. So equal prices across the two countries for these varieties imply $\frac{w_N}{w_M} = \frac{A_N}{A_M}$. An alternative way to interpret the invalidity of condition (6) is that n_M is sufficiently large, so the induced world demand for the labor of country M becomes so high that its wage is driven up to a level that the commodity price is equal to that produced by country N, that is $\frac{A_M}{w_M} = \frac{A_N}{w_N}$, so country N also produces some varieties with indexes lower than n_M. Graphically, this case can be seen in Figure 6 when n_M becomes large enough such that A_0 becomes smaller than A_1, so $\frac{w_N}{w_M} = \frac{A_N}{A_M}$ holds for any A_S. In fact, (6) is the necessary and sufficient condition for $A_0 < A_1$.

Observe that an increase in A_N now has two effects on the sandwiching forces. One is to strengthen the pressing effect from country N, and the other is to help eliminate the chasing effect from country S because the set of varieties produced by country M is pushed leftward in equilibrium so that n_s becomes binding for country S (see Figure 1).

The rest of the theorem can be understood as follows: When A_S is sufficiently small ($A_S \in (0, \frac{n_S[L_M A_M + L_N A_N]}{(n-n_S)L_S}]$), country M also produces some varieties accessible to country S, that is, $\widetilde{n_s} < n_s$, so $\frac{w_M}{A_M} = \frac{w_S}{A_S}$. When A_S is sufficiently large, country S is constrained by the binding technology diffusion barrier ($\widetilde{n_s} = n_s$), so $\frac{w_M}{A_M} > \frac{w_S}{A_S}$ and A_S has no impact on $\frac{w_N}{w_M}$ as $\widetilde{n_M} < n_M$.

So far, all analyses are static and take the three important threshold values n_s, n_M, n and labor productivities A_N, A_M, and A_S as exogenous. Next we will make the model dynamic and all of these cutoff values and productivies endogenous.

3 Dynamic Model

Consider a continuous-time world, where all households are infinitely lived. To make the analysis simple, we assume all goods are non-storable and trade is free and balanced at any time point. So a representative household in country $J \in \{N, M, S\}$ maximizes her following utility function

$$\max_{c_J(t,i)} \int_0^{\infty} \left[\int_0^{n(t)} c_J(t,i)^{\theta} di \right]^{1/\theta} e^{-\rho t} dt \tag{9}$$

subject to

$$\int_0^{n(t)} c_J(t,i) p(t,i) di = w_J(t), \forall t, \tag{10}$$

where $c_J(t,i)$ denotes consumption of variety i at time point t for a household in country J, ρ denotes the time discount rate, and $p(t,i)$ denotes the price of variety i at time point t, which is the same across countries due to free trade:

$$p(t,i) = \min_{I \in \{M,N,S\}} \{\frac{w_I(t)}{A_I(t)}\} \text{ for any } i \text{ produced at time } t. \tag{11}$$

Note that the total number of varieties $n(t)$ and the set of varieties produced by each country may change over time. Suppose country N keeps innovating at an exogenous and positive speed α:

$$\dot{n} = \alpha n. \tag{12}$$

Country M adapts technologies (varieties with measure n_M) learned from country N at an exogenous positive speed β :

$$\dot{n}_M = \beta(n - n_M), \tag{13}$$

where $n - n_M$ is the measure of varieties that only country N knows how to produce. Country S knows how to produce varieties with measure n_S and it imitates from country M. The positive imitation speed is γ, so

$$\dot{n}_S = \gamma(n_M - n_S). \tag{14}$$

Please refer to Figure 1, where now all the three technology threshold values n_s, n_M, n change dynamically.

Without loss of generality, a variety $i \in [0, n]$ is indexed following a strictly increasing order in terms of difficulty of technology diffusion. So a variety with a lower index i is easier to adapt or imitate and, therefore, diffuses abroad earlier. The technology diffusion of each variety involves an infinitesimally small ε labor cost, so M and S will produce the full amount of lower-indexed varieties before producing newly learned higher-indexed varieties.

Let us start by considering the following simplest case:

$$A_S(t) = A_M(t) = A_N(t) = 1, \text{ for } \forall t. \tag{15}$$

Theorem 6 *Under conditions (12)-(15), there exists a unique balanced growth path, on which the following is true*

$$\begin{aligned} \frac{\dot{n}_M}{n_M} &= \frac{\dot{n}_S}{n_S} = \frac{\dot{n}}{n} = \alpha, \\ \frac{w_N}{w_M} &= \left(\frac{\alpha+\gamma}{\beta}\frac{L_M}{L_N}\right)^{1-\theta}, \frac{w_M}{w_S} = \left(\frac{\alpha}{\gamma}\frac{L_S}{L_M}\right)^{1-\theta}, \end{aligned} \tag{16}$$

if and only if the following condition is satisfied

$$\frac{\beta}{\alpha+\gamma} L_N < L_M < \frac{\alpha}{\gamma} L_S. \tag{17}$$

Proof. Refer to Appendix.

Note that the engine for growth in the model is innovation in country N, accompanied by sequential technology diffusion to country M and then to country S. Under condition (17), in the long run, the world reaches a unique balanced growth path (BGP), on which everything characterized in Lemma 1 remains true for any time t. That is, the pattern of complete trade specialization is endogenously maintained over time (same as Figure 2), wage ratio (3) and condition (2) are also satisfied. Different from the static model, now the measure of the set of varieties produced by each country grows over time at the same rate α, that is,

$$\frac{d\log(n - n_M)}{dt} = \frac{d\log(n_M - n_S)}{dt} = \frac{d\log(n_S)}{dt} = \alpha.$$

This theorem shows that an increase in γ, the imitation speed of S, enlarges the per capita income gap between N and M on the BGP, which is a new type of chasing effect in terms of the technological diffusion speed.

Note that $\frac{w_M}{w_S}$ is independent of β under (17), because $\frac{\dot{n}_M}{n_M} = \alpha$ holds on the BGP and only the interaction between S and M determines their per capita income gap. However, as implied by (16), the per capita GDP gap between N and S is still affected by β:

$$\frac{w_N}{w_S} = \left(\frac{\alpha+\gamma}{\beta}\frac{\alpha}{\gamma}\frac{L_S}{L_N}\right)^{1-\theta}.$$

(17) is satisfied when α, the innovation rate in N, is sufficiently large, *ceteris paribus*, in which case permanent gaps in GDP per capita will always exist in the long run (that is, $w_N > w_M > w_S$).

When (17) is not satisfied, in particular, if $\frac{\beta}{\alpha+\gamma}L_N \geq L_M$, then $\frac{w_N}{w_M} = 1$ on the BGP, this is because the technology adaption speed of country M is high enough to dominate the joint forces of chasing by country S and pressing by country N, so eventually $\frac{L_N}{n-n_M} \geq \frac{L_M}{n_M-n_S}$. Similar analyses apply when $L_M \geq \frac{\alpha}{\gamma}L_S$ as it would result in $\frac{L_M}{n_M-n_S} \geq \frac{L_S}{n_S}$, so $\frac{w_M}{w_S} = 1$ on the BGP. The intuitions are similar to the static model.

Next, we extend assumption (15) to allow for both heterogeneity and dynamic changes of labor productivities across countries. For future references, define

$$\begin{aligned} g_J(t) &\equiv \frac{\dot{A}_J(t)}{A_J(t)} \text{ for } J \in \{N, M, S\}; \\ a_0(t) &\equiv \frac{A_0(t)}{A_M(t)}; a_1(t) \equiv \frac{A_1(t)}{A_M(t)}, \end{aligned}$$

where A_0 and A_1 are defined in (8). Suppose labor productivities grow and diffuse sequentially across countries in the following fashion:

$$\dot{A}_N = \phi_N A_N, \tag{18}$$

$$\dot{A}_M = \phi_M(A_N - A_M), \tag{19}$$

$$\dot{A}_S = \phi_S(A_M - A_S), \tag{20}$$

where ϕ_N is the exogenous growth rate of labor productivity in country N, ϕ_M is productivity adaption rate of country M, and ϕ_S denotes the productivity imitation rate of country S. Let $\phi_J \geq 0$ for all $J \in \{N, M, S\}$.

Theorem 7 *When varieties diffuse sequentially across countries as characterized by (12)-(14) and productivities also diffuse sequentially across countries as characterized by (18)-(**??**), the following is true in the long run equilibrium:*

$$\frac{w_N}{w_M} = \begin{cases} \frac{\phi_N}{\phi_M}+1 & \text{if } \begin{array}{c} \frac{\alpha+\gamma}{\beta} \leq \left(\frac{\phi_N}{\phi_M}+1\right)\frac{L_N}{L_M}, \text{ or} \\ \frac{\phi_S}{\phi_N+\phi_S} \in (0, a_0] \text{ and } \frac{\alpha+\gamma}{\beta} > \left(\frac{\phi_N}{\phi_M}+1\right)\frac{L_N}{L_M} \end{array} \\ \left[\frac{\left(\frac{\phi_S}{\phi_N+\phi_S}\right)L_S+L_M}{L_N}\frac{\alpha}{\beta}\right]^{1-\theta}\left(\frac{\phi_N}{\phi_M}+1\right)^{\theta} & \text{if } \frac{\phi_S}{\phi_N+\phi_S} \in (a_0, a_1] \text{ and } \frac{\alpha+\gamma}{\beta} > \left(\frac{\phi_N}{\phi_M}+1\right)\frac{L_N}{L_M} \\ \left(\frac{\alpha+\gamma}{\beta}\frac{L_M}{L_N}\right)^{1-\theta}\left(\frac{\phi_N}{\phi_M}+1\right)^{\theta} & \text{if } \frac{\phi_S}{\phi_N+\phi_S} \in (a_1, \infty) \text{ and } \frac{\alpha+\gamma}{\beta} > \left(\frac{\phi_N}{\phi_M}+1\right)\frac{L_N}{L_M} \end{cases},$$

where

$$a_0 = \frac{\beta}{\alpha}\left(\frac{\phi_N}{\phi_M}+1\right)\frac{L_N}{L_S} - \frac{L_M}{L_S}; a_1 = \frac{\gamma L_M}{\alpha L_S}.$$

Proof. See the appendix.

This theorem shows that no chasing effect exists when variety imitation speed γ is sufficiently small (that is, $\frac{\alpha+\gamma}{\beta} \leq \left(\frac{\phi_N}{\phi_M}+1\right)\frac{L_N}{L_M}$), in which case permanent gap between N and M exists if and only if $\phi_N > 0$. When γ is sufficiently large, there are three different cases. The first case is when productivity imitation speed of S ϕ_S is sufficiently small (that is, $\frac{\phi_S}{\phi_N+\phi_S} \in (0, a_0]$), chasing effect still does not exist. The second case is when ϕ_S is in an intermediate range (that is, $\frac{\phi_S}{\phi_N+\phi_S} \in (a_0, a_1]$), chasing effect exists through the productivity imitation speed in the sense that $\frac{\partial}{\partial \phi_S}\left(\frac{w_N}{w_M}\right) > 0$. Observe that in this case, the country sizes also matter. More specifically, the larger the country size of M, the stronger the sandwich effect in the following sense: $\frac{w_N}{w_M}$ increases with L_M and the interval $(a_0, a_1]$ becomes "wider" (that is, $\frac{\partial a_0}{\partial L_M} < 0$ and $\frac{\partial a_1}{\partial L_M} > 0$). Moreover, $\frac{w_N}{w_M}$ increases with L_S, meaning that the larger the size of the chasing South, the stronger the chasing effect. However, the variety imitation speed γ does not affect the gap of N and M. The third case is when ϕ_S is sufficiently large (that is, $\frac{\phi_S}{\phi_N+\phi_S} \in (a_1, \infty)]$), chasing effect exists through the variety imitation speed in the sense that $\frac{\partial}{\partial \gamma}\left(\frac{w_N}{w_M}\right) > 0$, but country size of L_S or productivity imitation speed of country S no longer exerts any effect on the GDP gap between N and M. The intuitions are largely analogous to those in Theorem 2.

3.1 Endogenous Productivity

We extend the model by making both the variety adaption speed in country M and the productivity growth rate g_M endogenous. More specifically, let $\mu(t)$ denote the employment share in the R&D sector in country M at time t, and the rest of labor is employed in the manufacturing sector. Different from (13), now how fast M adapts new varieties from N not only depends on the existing gap $n - n_M$, but also on the size of R&D employment:

$$\dot{n}_M = \beta(n - n_M)\left[\mu L_M + 1\right]^{\xi}, \tag{21}$$

where $\xi \geq 0$ captures the impact of R&D on the adaptation speed. A higher employment in the R&D sector leads to a faster technology adaptation. When $\xi = 0$ or $\mu = 0$, the adaptation speed is still positive, back to the previous case (13). Different from (19), now the labor productivity in M evolves as follows

$$\dot{A}_M = \phi(A_N - A_M)\left[(1-\mu)L_M + 1\right]^{\eta}, \tag{22}$$

which means that the speed of labor productivity diffusion from N to M not only depends on the gap $A_N - A_M$ but also on the size of manufacturing employment $(1-\mu)L_M$. ϕ and η are both positive parameters. A larger η implies a stronger learning-by-doing effect in the manufacturing sector. Diffusion does not stop even when $\mu = 1$. Everything else is same as before. In particular, (12), (14), (18) and (20) are still valid for simplicity.

An artificial social planner for country M chooses $c_M(t,i)$, the time path of consumption bundle, and $\mu(t)$, the time path for R&D employment share, to maximize the welfare of a representative household in country M. The optimal choice of $\mu(t)$ concerns the trade-off between whether to facilitate variety expansion or to boost productivity growth of existing varieties.

The Ramsey government's problem in M is

$$\max_{c_M(t,i),\mu(t)} \int_0^\infty \left[\int_0^{n(t)} c_M(t,i)^\theta di\right]^{1/\theta} e^{-\rho t} dt$$

subject to

$$\int_0^{n(t)} c_M(t,i)p(t,i)di = w_M(t), \forall t,$$

where $p(t,i)$ is governed by (11), whereas a representative household in country $J \in \{N,S\}$ still solves (9) subject to (10), and all decision makers are subject to (12), (14), (18), (20), (21) and (22).

$$\widetilde{A_0} \equiv \frac{n_M A_N L_N - (n-n_M)(1-\mu)A_M L_M}{(n-n_M)L_S}; \widetilde{A_1} \equiv \frac{n_S(1-\mu)A_M L_M}{(n_M - n_S)\, L_S}$$

$$\begin{aligned} \widetilde{A_0} &< \widetilde{A_1} \Leftrightarrow \mu < 1 - \frac{(n_M - n_S)\, A_N L_N}{(n-n_M)A_M L_M} \\ \widetilde{A_0} &> 0 \Leftrightarrow \mu > 1 - \frac{n_M A_N L_N}{(n-n_M)A_M L_M} \end{aligned}$$

Consider a given time point τ, country M chooses $\mu(\tau)$, and suppose $A_S(\tau) \in (\widetilde{A_0}(\tau), \widetilde{A_1}(\tau)]$. Thus country N produces varieties $[n_M(\tau), n(\tau)]$, country M produces $(\widetilde{n}_S(\tau), n_M(\tau))$, and country S produces $[0, \widetilde{n}_S(\tau)]$, where $\widetilde{n}_S(\tau) < n_S(\tau)$.

Let $c_{IJ}(\tau)$ denote consumption of a variety produced in country J by a household in country I at time τ for $I, J \in \{M,N,S\}$.

The labor market clearing condition for country M is implied by

$$(1-\mu(\tau))L_M A_M(t) = (n_M(\tau) - \widetilde{n}_S(\tau)) \sum_{I\in\{M,N,S\}} L_I c_{IM}(\tau), \tag{23}$$

where the left hand side is the total amount of goods produced in country M and the right hand side is the world demand for goods produced in country M. Similarly, the labor market clearing for country N is implied by

$$L_N A_N(t) = (n(\tau) - n_M(\tau)) \sum_{I \in \{M,N,S\}} L_I c_{IN}(\tau) \tag{24}$$

and the labor market clearing for country S is implied by

$$L_S A_S(t) = \widetilde{n}_S(\tau) \sum_{I \in \{M,N,S\}} L_I c_{IS}(\tau). \tag{25}$$

Observe that

$$\frac{c_{SM}(\tau)}{c_{SN}(\tau)} = \frac{c_{MM}(\tau)}{c_{MN}(\tau)} = \frac{c_{NM}(\tau)}{c_{NN}(\tau)} = \left(\frac{\frac{w_M(t)}{A_M(t)}}{\frac{w_N(t)}{A_N(t)}} \right)^{\frac{1}{\theta-1}},$$

together with (23) and (24), we obtain

$$\frac{w_M(t)}{w_N(t)} = \left[\frac{(1-\mu(\tau))L_M A_M(t)}{L_N A_N(t)} \frac{(n(\tau) - n_M(\tau))}{(n_M(\tau) - \widetilde{n}_S(\tau))} \right]^{\theta-1} \frac{A_M(t)}{A_N(t)}. \tag{26}$$

Similarly, by (23) and (25), we obtain

$$\frac{(1-\mu(\tau))L_M A_M(t) + L_S A_S(t)}{L_S A_S(t)} = \frac{n_M(\tau)}{\widetilde{n}_S(\tau)}.$$

Solving out $\widetilde{n}_S(\tau)$ and substituting it back into (26) yields

$$\frac{w_M(\tau)}{w_N(\tau)} = \left[\frac{(1-\mu(\tau))L_M A_M(\tau) + L_S A_S(\tau)}{L_N A_N(\tau)} \frac{(n(\tau) - n_M(\tau))}{n_M(\tau)} \right]^{\theta-1} \frac{A_M(\tau)}{A_N(\tau)}, \tag{27}$$

which is smaller than one if and only if

$$\frac{A_M(\tau)}{A_N(\tau)} < \left[\frac{(1-\mu(\tau))L_M A_M(\tau) + L_S A_S(\tau)}{L_N A_N(\tau)} \frac{(n(\tau) - n_M(\tau))}{n_M(\tau)} \right]^{1-\theta}.$$

Suppose $g_M^* = g_N = g_S$,

$$g_N = \frac{\dot{A}_M}{A_M} = \phi(\frac{A_N}{A_M} - 1) \left[(1-\mu)L_M + 1\right]^{\eta}, \tag{28}$$

which implies

$$\frac{g_N + \phi\left[(1-\mu)L_M + 1\right]^{\eta}}{\phi\left[(1-\mu)L_M + 1\right]^{\eta}} = \frac{A_N}{A_M}.$$

$$\alpha = \frac{\dot{n}_M}{n_M} = \beta(\frac{n-n_M}{n_M})\left[\mu L_M+1\right]^{\xi}, \tag{29}$$

$$\frac{n-n_M}{n_M} = \frac{\alpha}{\beta\left[\mu L_M+1\right]^{\xi}} \tag{30}$$

(27) becomes

$$\frac{w_M(\tau)}{w_N(\tau)} = \left[\left[\frac{(1-\mu(\tau))L_M}{L_N}\frac{g_N+\phi\left[(1-\mu)L_M+1\right]^{\eta}}{\phi\left[(1-\mu)L_M+1\right]^{\eta}}+\frac{L_S A_S(\tau)}{L_N A_N(\tau)}\right]\frac{\alpha}{\beta\left[\mu L_M+1\right]^{\xi}}\right]^{\theta-1}\frac{g_N+\phi\left[(1-\mu)L_M+1\right]^{\eta}}{\phi\left[(1-\mu)L_M+1\right]^{\eta}}$$

Obviously, when $\eta=0$, the above equation is reduced to

$$\frac{w_M(\tau)}{w_N(\tau)} = \left[\left[\frac{(1-\mu(\tau))L_M}{L_N}\frac{g_N+\phi}{\phi}+\frac{L_S A_S(\tau)}{L_N A_N(\tau)}\right]\frac{\alpha}{\beta\left[\mu L_M+1\right]^{\xi}}\right]^{\theta-1}\frac{g_N+\phi}{\phi},$$

the RHS of which is maximized when $\mu(\tau)=1$, unreasonable.

On the other hand, when $\xi=0$,

$$\begin{aligned}\frac{w_M(\tau)}{w_N(\tau)} &= \left[\left[\frac{(1-\mu(\tau))L_M}{L_N}\frac{g_N+\phi\left[(1-\mu)L_M+1\right]^{\eta}}{\phi\left[(1-\mu)L_M+1\right]^{\eta}}+\frac{L_S A_S(\tau)}{L_N A_N(\tau)}\right]\frac{\alpha}{\beta}\right]^{\theta-1}\frac{g_N+\phi\left[(1-\mu)L_M+1\right]^{\eta}}{\phi\left[(1-\mu)L_M+1\right]^{\eta}} \\ &= \left[\left[\frac{(1-\mu(\tau))L_M}{L_N}\frac{g_N+\phi\left[(1-\mu)L_M+1\right]^{\eta}}{\phi\left[(1-\mu)L_M+1\right]^{\eta}}+\frac{L_S A_S(\tau)}{L_N A_N(\tau)}\right]\frac{\alpha}{\beta}\right]^{\theta} \\ &\quad \frac{1}{\left[\frac{(1-\mu(\tau))L_M}{L_N}+\frac{L_S A_S(\tau)}{L_N A_N(\tau)}\frac{\phi\left[(1-\mu)L_M+1\right]^{\eta}}{g_N+\phi\left[(1-\mu)L_M+1\right]^{\eta}}\right]\frac{\alpha}{\beta}}\end{aligned}$$

$$\begin{aligned}A_0 &\equiv \frac{n_M A_N L_N-(n-n_M)A_M L_M}{(n-n_M)L_S} \\ &= \frac{n_M A_N L_N}{(n-n_M)L_S}-\frac{A_M L_M}{L_S} \\ &= \frac{\beta}{\alpha}\left[\mu L_M+1\right]^{\xi}\frac{A_N L_N}{L_S}-\frac{A_M L_M}{L_S} \\ \frac{n_M}{n-n_M} &= \frac{\beta}{\alpha}\left[\mu L_M+1\right]^{\xi} \\ A_1 &\equiv \frac{n_S A_M L_M}{(n_M-n_S)L_S}. \\ &= \frac{\gamma}{\alpha}\frac{A_M L_M}{L_S} \\ \frac{n_M}{n_S} &= \frac{\alpha+\gamma}{\gamma}\end{aligned}$$

we require

$$\begin{aligned} A_1 &> A_0 \\ &\Leftrightarrow \frac{\gamma}{\alpha}\frac{A_M L_M}{L_S} > \frac{\beta}{\alpha}\left[\mu L_M+1\right]^{\xi}\frac{A_N L_N}{L_S} - \frac{A_M L_M}{L_S} \\ &\Leftrightarrow (\gamma+\alpha)\frac{L_M}{L_N} > \beta\left[\mu L_M+1\right]^{\xi}\frac{g_N+\phi\left[(1-\mu)L_M+1\right]^{\eta}}{\phi\left[(1-\mu)L_M+1\right]^{\eta}}. \end{aligned}$$

It turns out that $A_1 > A_0 \Leftrightarrow \frac{A_M L_M}{A_N L_N} > \frac{n_M - n_S}{n - n_M}$, the latter of which holds if and only if the following is true

$$\frac{L_M}{L_N}\frac{\alpha+\gamma}{\beta\left[\mu L_M+1\right]^{\xi}} > \frac{g_N+\phi\left[(1-\mu)L_M+1\right]^{\eta}}{\phi\left[(1-\mu)L_M+1\right]^{\eta}} \tag{31}$$

[Proof. Because we require

$$\frac{A_M L_M}{A_N L_N} > \frac{n_M - n_S}{n - n_M},$$

which is eqivalent to (31) after utilizing the following three conditions:

$$\begin{aligned} \frac{g_N+\phi\left[(1-\mu)L_M+1\right]^{\eta}}{\phi\left[(1-\mu)L_M+1\right]^{\eta}} &= \frac{A_N}{A_M} \\ \frac{n_M}{n_S} &= \frac{\alpha+\gamma}{\gamma} \\ \frac{n_M}{n-n_M} &= \frac{\beta}{\alpha}\left[\mu L_M+1\right]^{\xi} \end{aligned}$$

Q.E.D.]

Obviously, the LHS decreases with μ while the RHS increases with μ.

Suppose the following two are true: (When $\mu = 0$),

$$\frac{L_M}{L_N}\frac{\alpha+\gamma}{\beta} > \frac{g_N+\phi\left[L_M+1\right]^{\eta}}{\phi\left[L_M+1\right]^{\eta}}.$$

and (When $\mu = 1,$)

$$\frac{L_M}{L_N}\frac{\alpha+\gamma}{\beta\left[L_M+1\right]^{\xi}} < \frac{g_N+\phi}{\phi}.$$

Then there exists a unique $\widehat{\mu} \in (0,1)$ such that $A_1 > A_0 \Leftrightarrow \mu \in [0, \widehat{\mu})$.

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Consider at a given time point τ, country M chooses $\mu(\tau)$, and suppose $A_S(\tau) \in (A_1(\tau), \infty)$. Thus country N produces varieties $[n_M(\tau), n(\tau)]$, country

M produces $(n_S(\tau), n_M(\tau))$, and country S produces $[0, n_S(\tau)]$. The market clearing conditions are:

Labor market clearing for country M:

$$\begin{aligned}(1-\mu(\tau))L_M A_M(t) &= L_S(n_M(\tau)-n_S(\tau))c_{SM}(\tau) \\ &\quad +L_M(n_M(\tau)-n_S(\tau))c_{MM}(\tau) \\ &\quad +L_N(n_M(\tau)-n_S(\tau))c_{NM}(\tau)\end{aligned} \tag{32}$$

Labor market clearing for country N:

$$\begin{aligned}L_N A_N(t) &= L_S(n(\tau)-n_M(\tau))c_{SN}(\tau) \\ &\quad +L_M(n(\tau)-n_M(\tau))c_{MN}(\tau) \\ &\quad +L_N(n(\tau)-n_M(\tau))c_{NN}(\tau)\end{aligned} \tag{33}$$

Labor market clearing for country S:

$$\begin{aligned}L_S A_S(t) = L_S n_S(\tau)c_{SS}(\tau) &\quad +L_M n_S(\tau)c_{MS}(\tau) \\ &\quad +L_N n_S(\tau)c_{NS}(\tau)\end{aligned} \tag{34}$$

so when taking ratio for (32) and (33) on both sides, we obtain

$$\frac{w_M(t)}{w_N(t)} = \left[\frac{(1-\mu(\tau))L_M}{L_N}\frac{(n(\tau)-n_M(\tau))}{(n_M(\tau)-n_S(\tau))}\right]^{\theta-1}\left[\frac{A_M(t)}{A_N(t)}\right]^{\theta} \tag{35}$$

$$= \left[\frac{(1-\mu)L_M}{L_N}\frac{\alpha+\gamma}{\beta\left[\mu L_M+1\right]^{\xi}}\right]^{\theta-1}\left[\frac{\phi\left[(1-\mu)L_M+1\right]^{\eta}}{g_N+\phi\left[(1-\mu)L_M+1\right]^{\eta}}\right]^{\theta} \tag{36}$$

$$\frac{w_N}{w_M} = \begin{cases} \frac{A_N}{A_M} & \text{if} \quad A_S \in (0, A_0] \\ \left[\frac{A_N L_N}{A_S L_S + A_M L_M}\frac{n_M}{n-n_M}\right]^{\theta-1}\frac{A_N}{A_M} & \text{if} \quad A_S \in (A_0, A_1] \\ \left(\frac{n-n_M}{n_M-n_S}\frac{L_M}{L_N}\right)^{1-\theta}\frac{A_N^{\theta}}{A_M^{\theta}} & \text{if} \quad A_S \in (A_1, \infty) \end{cases}, \tag{37}$$

$$\max\frac{w_M(t)}{w_N(t)} \Leftrightarrow \max_{\mu}\Psi(\mu) \equiv \frac{\left[1-\mu\right]^{\theta-1}\left[(1-\mu)L_M+1\right]^{\eta\theta}}{\left[\mu L_M+1\right]^{\xi(\theta-1)}\left[g_N+\phi\left[(1-\mu)L_M+1\right]^{\eta}\right]^{\theta}}$$

$$\begin{aligned}
\Psi'(\mu) &> 0 \Leftrightarrow \\
\frac{\frac{\partial\left\{\left[1-\mu\right]^{\theta-1}\left[(1-\mu)L_M+1\right]^{\eta\theta}\right\}}{\partial\mu}}{\left[1-\mu\right]^{\theta-1}\left[(1-\mu)L_M+1\right]^{\eta\theta}} &> \frac{\frac{\partial\left\{\left[\mu L_M+1\right]^{\xi(\theta-1)}\left[g_N+\phi\left[(1-\mu)L_M+1\right]^{\eta}\right]^{\theta}\right\}}{\partial\mu}}{\left[\mu L_M+1\right]^{\xi(\theta-1)}\left[g_N+\phi\left[(1-\mu)L_M+1\right]^{\eta}\right]^{\theta}} \Leftrightarrow \\
\frac{\partial\log\left[1-\mu\right]^{\theta-1}\left[(1-\mu)L_M+1\right]^{\eta\theta}}{\partial\mu} &> \frac{\partial\log\left[\mu L_M+1\right]^{\xi(\theta-1)}\left[g_N+\phi\left[(1-\mu)L_M+1\right]^{\eta}\right]^{\theta}}{\partial\mu} \Leftrightarrow \\
\frac{(\theta-1)\,\partial\log\left[1-\mu\right]+\eta\theta\partial\log\left[(1-\mu)L_M+1\right]}{\partial\mu} &> \frac{\xi\,(\theta-1)\,\partial\log\left[\mu L_M+1\right]+\theta\partial\log\left[g_N+\phi\left[(1-\mu)L_M+1\right]^{\eta}\right]}{\partial\mu} \\
\frac{\left[(1-\theta)-\eta\theta\right](1-\mu)\,L_M+(1-\theta)}{(1-\mu)\left[(1-\mu)L_M+1\right]} &> L_M\left[\frac{\xi\,(\theta-1)}{\mu L_M+1}+\frac{-\theta\phi\eta\left[(1-\mu)L_M+1\right]^{\eta-1}}{g_N+\phi\left[(1-\mu)L_M+1\right]^{\eta}}\right]
\end{aligned}$$

when $\mu = 1$, LHS>RHS; when $\mu = 0$, LHS>RHS,

$$\frac{[(1-\theta) - \eta\theta]\, L_M + (1-\theta)}{[L_M + 1]} > L_M \left[\xi (\theta - 1) + \frac{-\theta\phi\eta\, [L_M + 1]^{\eta-1}}{g_N + \phi\, [L_M + 1]^{\eta}} \right]$$

$$\begin{aligned} \frac{g_N + \phi\, [(1-\mu)L_M + 1]^{\eta}}{\phi\, [(1-\mu)L_M + 1]^{\eta}} &= \frac{A_N}{A_M} \\ \frac{n_M}{n_S} &= \frac{\alpha + \gamma}{\gamma} \\ \frac{n_M}{n - n_M} &= \frac{\beta\, [\mu L_M + 1]^{\xi}}{\alpha} \end{aligned}$$

when taking ratio for (32) and (34) on both sides, we obtain

$$\begin{aligned} \frac{(1-\mu(\tau))L_M A_M(t) n_S(\tau)}{L_S A_S(t)(n_M(\tau) - n_S(\tau))} &= \frac{c_{SM}(\tau)}{c_{SS}(\tau)} = \left(\frac{\frac{w_M(t)}{A_M(t)}}{\frac{w_S(t)}{A_s(t)}} \right)^{\frac{1}{\theta-1}} \\ \left[\frac{(1-\mu(\tau))L_M n_S(\tau)}{L_S(n_M(\tau) - n_S(\tau))} \right]^{\theta-1} \left(\frac{A_M(t)}{A_s(t)} \right)^{\theta} &= \frac{w_M(t)}{w_S(t)} \end{aligned}$$

$$\frac{c_{SM}(\tau)}{c_{SS}(\tau)} = \left(\frac{\frac{w_M(t)}{A_M(t)}}{\frac{w_S(t)}{A_s(t)}} \right)^{\frac{1}{\theta-1}}$$

Proposition 8 *When $g_N > g_S \geq 0$, there exists a Balanced Growth Path (BGP), on which the following is true*

$$\begin{aligned} \mu^* &= 0; \\ g_M^* &= g_N; \\ \frac{w_N}{w_M} &= \frac{A_N}{A_M} = \frac{g_N}{\phi\, [L_M + 1]^{\eta}} + 1; \end{aligned}$$

Proof. Refer to the Appendix.

This proposition states that, when the labor productivity growth rate of N exceeds that of S, all workers in country M should eb employed in the manufacturing sector in the long run equilibrium, in which per capita GDP ratio between North and Middle is equal to their labor productivity ratio. The intuition behind this result is that in the long run, A_s becomes sufficiently small so there will be no chasing effect, the logic of which has been explained in the static model. To maximize welfare of country M is equivalent to maximizing its income w_M, or equivalently, to minimize the per capita gap between N and M. Since $\frac{w_N}{w_M} = \frac{A_N}{A_M}$ with the absence of chasing effect, the best way to boost A_M is to choose $\mu^* = 0$ so that all labor works in the manufacturing sector to maximize the learning by doing effect on A_M.

Proposition 9 *When $g_N = g_S > 0$ and the initial conditions are such that $A_S \in (A_0, A_1]$ in the long run, there exists a BGP, on which the following is true:*

$$
\begin{aligned}
g_M^* &= g_N = g_S; \\
\mu^* &\in \min_{\mu \in [0,1]} F(\mu) \text{ where} \\
F(\mu) &\equiv \left[\frac{g_N}{\phi\left[(1-\mu)L_M+1\right]^{\eta}}+1\right]\left[\frac{\frac{A_S}{A_N}\left(\frac{g_N}{\phi[(1-\mu)L_M+1]^{\eta}}+1\right)L_S+(1-\mu)L_M}{L_N}\frac{\alpha}{\beta\left[\mu L_M+1\right]^{\xi}}\right]^{1-\theta} \\
\frac{w_N}{w_M} &= F(\mu^*);
\end{aligned}
$$

Proof. Refer to the Appendix.

This proposition demonstrates how labor in country M should be optimally allocated between the R&D sector and the manufacturing sector in the long run when South labor productivity exhibits a chasing effect. Fraction μ^* of the labor should be allocated to the R&D sector to enhance the technology adaptation from N in order to partly offset the chasing effect from S, and fraction $1-\mu^*$ of the labor should be allocated to the manufacturing sector to produce goods and boost labor productivity on existing varieties. The above proposition implies that in equilibrium, $\frac{w_N}{w_M}$ increases in α and A_S but decreases in β, ϕ A_N and L_N. However, μ^* is independent of α, β and L_N. Moreover,

$$
\frac{\partial \mu^*}{\partial A_S} < 0; \frac{\partial \mu^*}{\partial A_N} > 0; \frac{\partial \mu^*}{\partial L_S} < 0; \frac{\partial \mu^*}{\partial g_N} < 0.
$$

4 Conclusion

In this paper, we develops a tractable growth model to show how a middle-income country (M) can be sandwiched by an innovating north country (N) and an imitating south country (S) through international trade. An increase in labor productivity of existing varieties (intensive margin) or in the number of varieties (extensive margin) produced in S may result in non-convergence of M to N, but this chasing effect from S disappears when S is sufficiently unproductive. Meanwhile, an increase in innovation in N not only enlarges the income gap between N and M (pressing effect from N) but also makes M more vulnerable to the chasing effect from S. We characterize how M should optimally allocate its resources between production and R&D to respond to the chasing effect from S and depressing effect from N.

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Appendix

Let $\widetilde{n_S}$ denote the highest-indexed variety produced in S in equilibrium ($\widetilde{n_S} \leq n_S$). All the varieties $[0, \widetilde{n_S})$ are only produced in S, while variety $\widetilde{n_S}$ is generically produced in both S and M. Obviously, $\widetilde{n_S}'(A_S) \geq 0$ for any $\widetilde{n_S} < n_S$. Obviously, $\underset{A_S \to 0}{lim} \widetilde{n_S}(A_S) = 0$ and $\underset{A_S \to \infty}{lim} \widetilde{n_S}(A_S) = n_S$.

Pattern A: $\frac{w_S}{A_S} < \frac{w_M}{A_M} < \frac{w_N}{A_N}$.

The necessary and sufficient conditions are (5) and (6).

Pattern B: $\frac{w_N}{A_N} = \frac{w_M}{A_M} = \frac{w_S}{A_S}$.

Then we must have

$$c_J(i) = \frac{A_J}{n}.$$

$$\begin{aligned} \sum_{J \in \{N,M,S\}} \frac{L_J A_J}{n} \widetilde{n_S} &= L_S A_S \\ \sum_{J \in \{N,M,S\}} \frac{L_J A_J}{n} (n - \widetilde{n_M}) &= L_N A_N \\ \sum_{J \in \{N,M,S\}} \frac{L_J A_J}{n} (\widetilde{n_M} - \widetilde{n_S}) &= L_M A_M \end{aligned}$$

So

$$\widetilde{n_S} = \frac{n L_S A_S}{\sum_{J \in \{N,M,S\}} L_J A_J} \leq n_S \tag{38}$$

$$0 < \widetilde{n_M} = n - \frac{n L_N A_N}{\sum_{J \in \{N,M,S\}} L_J A_J} \leq n_M \tag{39}$$

(38) holds if and only if

$$A_S \leq \frac{n_S \left[L_M A_M + L_N A_N \right]}{(n - n_S) L_S}. \tag{40}$$

(39) holds if and only if

$$A_S \leq \frac{(n_M - n) L_M A_M + n_M L_N A_N}{(n - n_M) L_S}. \tag{41}$$

(38) and (39) are mutually compatible iff

$$A_S \leq \begin{cases} \frac{n_S[L_M A_M + L_N A_N]}{(n-n_S)L_S}, & \text{when} \quad \frac{(n_M - n_S) L_N A_N}{(n - n_M)} \geq L_M A_M \\ \frac{(n_M - n) L_M A_M + n_M L_N A_N}{(n - n_M) L_S}, & \text{when} \quad \frac{(n_M - n_S) L_N A_N}{(n - n_M)} < L_M A_M \end{cases}.$$

Pattern C: $\frac{w_N}{A_N} > \frac{w_M}{A_M} = \frac{w_S}{A_S}$.

The prices for all the varieties $i \in [0, n_M]$ are identical and the consumed quantity is also equal across these varieties in each country. N only produces varieties $(n_M, n]$. The world total output for all the varieties $[0, n_M]$ is equal to the total demand:

$$A_S L_S + A_M L_M = n_M \sum_{J \in \{N,M,S\}} L_J c_J(i), \tag{42}$$

where $c_J(i)$ denotes the consumption quantity of some variety $i \in [0, n_M]$ by a representative household in country $J \in \{N, M, S\}$. Household utility maximization implies

$$c_J(i) \equiv \frac{W_J}{\frac{w_M}{A_M} n_M + (n - n_M)\left(\frac{w_N}{w_M}\frac{A_M}{A_N}\right)^{\frac{1}{\theta-1}} \frac{w_N}{A_N}}. \tag{43}$$

Substituting (43) and $\frac{w_M}{A_M} = \frac{w_S}{A_S}$ into (42), we obtain

$$\frac{w_N}{w_M} = \left[\frac{A_N L_N}{A_S L_S + A_M L_M} \frac{n_M}{(n - n_M)}\right]^{\theta-1} \frac{A_N}{A_M}, \tag{44}$$

the right hand side of which is a strictly increasing function of A_S when $A_S \in (0, A_1)$, where A_1 is given by (18).

This pattern requires that

$$\frac{A_N L_N}{A_S L_S + A_M L_M} \frac{n_M}{(n - n_M)} < 1, \tag{45}$$

which comes from (44) and $\frac{w_N}{A_N} > \frac{w_M}{A_M}$. We also require

$$n_M \sum_{J \in \{N,M,S\}} L_J c_J(i) > A_M L_M \geq (n_M - n_S) \sum_{J \in \{N,M,S\}} L_J c_J(i),$$

which comes from $0 < \widetilde{n_M} \leq n_M$. This is reduce to

$$n_S A_M L_M \geq (n_M - n_S) A_S L_S, \tag{46}$$

which is exactly the opposite for (5). That is, (46) is true iff $A_S \leq A_1$.

In summary, $\frac{w_N}{A_N} > \frac{w_M}{A_M} = \frac{w_S}{A_S}$ iff (45) and (46) are both satisfied, or equivalently,

$$\frac{n_S A_M L_M}{(n_M - n_S)} \geq A_S L_S > \frac{n_M A_N L_N - (n - n_M) A_M L_M}{(n - n_M)},$$

which is feasible iff (6) holds. For future reference, define

$$A_0 \equiv \frac{n_M A_N L_N - (n - n_M) A_M L_M}{(n - n_M) L_S}. \tag{47}$$

Pattern D: $\frac{w_N}{A_N} = \frac{w_M}{A_M} > \frac{w_S}{A_S} > 0$

The prices for all the varieties $(n_S, n]$ are identical and the consumed quantity is also equal across these varieties in each country. S only produces varieties $[0, n_S]$. The world total output for all the varieties $i \in (n_S, n]$ is equal to the total demand:

$$A_N L_N + A_M L_M = (n - n_S) \sum_{J \in \{N,M,S\}} L_J c_J(i),$$

and market clearing condition for the goods produced by S :

$$A_s L_s = n_S \sum_{J \in \{N,M,S\}} L_J c_J(j) \text{ for } j \in [0, n_S].$$

Household utility maximization implies

$$c_J(j) = \left(\frac{A_M w_S}{A_S w_M} \right)^{\frac{1}{\theta - 1}} c_J(i).$$

For country J,

$$n_S c_J(j) + (n - n_S) c_J(i) = W_J$$

Thus

$$n_S \left(\frac{A_M w_S}{A_S w_M} \right)^{\frac{1}{\theta - 1}} c_J(i) + (n - n_S) c_J(i) = W_J$$

$$c_J(i) = \frac{W_J}{n_S \left(\frac{A_M w_S}{A_S w_M} \right)^{\frac{1}{\theta - 1}} + (n - n_S)} \tag{48}$$

$$c_J(j) = \frac{\left(\frac{A_M w_S}{A_S w_M} \right)^{\frac{1}{\theta - 1}}}{n_S \left(\frac{A_M w_S}{A_S w_M} \right)^{\frac{1}{\theta - 1}} + (n - n_S)} W_J$$

Thus

$$A_N L_N + A_M L_M = (n - n_S) \sum_{J \in \{N,M,S\}} \frac{W_J L_J}{n_S \left(\frac{A_M w_S}{A_S w_M} \right)^{\frac{1}{\theta - 1}} + (n - n_S)},$$

and

$$A_s L_s = n_S \sum_{J \in \{N,M,S\}} \frac{\left(\frac{A_M w_S}{A_S w_M} \right)^{\frac{1}{\theta - 1}}}{n_S \left(\frac{A_M w_S}{A_S w_M} \right)^{\frac{1}{\theta - 1}} + (n - n_S)} W_J L_J.$$

The above two equations imply

$$\left[\frac{A_N L_N + A_M L_M}{A_s L_s} \frac{n_S}{(n - n_S)} \right]^{1 - \theta} = \frac{A_M w_S}{A_S w_M}. \tag{49}$$

Thus

$$\begin{aligned}
\sum_{J\in\{N,M,S\}} W_J L_J &= A_N L_N + A_M L_M + A_s L_s \\
w_M &= \frac{A_M\left(A_N L_N + A_M L_M + A_s L_s\right)}{\left[\frac{A_N L_N + A_M L_M}{A_s L_s}\frac{n_S}{(n-n_S)}\right]^{1-\theta} A_S L_S + A_M L_M + A_N L_N} \\
W_N &= \frac{A_N\left(A_N L_N + A_M L_M + A_s L_s\right)}{\left[\frac{A_N L_N + A_M L_M}{A_s L_s}\frac{n_S}{(n-n_S)}\right]^{1-\theta} A_S L_S + A_M L_M + A_N L_N} \\
W_S &= \frac{\left[\frac{A_N L_N + A_M L_M}{A_s L_s}\frac{n_S}{(n-n_S)}\right]^{1-\theta} A_S\left(A_N L_N + A_M L_M + A_s L_s\right)}{\left[\frac{A_N L_N + A_M L_M}{A_s L_s}\frac{n_S}{(n-n_S)}\right]^{1-\theta} A_S L_S + A_M L_M + A_N L_N}
\end{aligned}$$

Since we require $\frac{w_M}{A_M} > \frac{w_S}{A_S}$, which, by (49), is equivalent to

$$\frac{A_N L_N + A_M L_M}{A_S L_S}\frac{n_S}{(n-n_S)} < 1. \tag{50}$$

We require N to produce at least $(n-n_M)$ varieties and at most $(n-n_S)$ varieties:

$$\frac{(A_N L_N + A_M L_M)}{(A_s L_s + A_N L_N + A_M L_M)}\sum_{J\in\{N,M,S\}} L_J W_J > A_N L_N \geq (n-n_M)\sum_{J\in\{N,M,S\}}\frac{L_J W_J}{n_S\left(\frac{A_M w_S}{A_S w_M}\right)^{\frac{1}{\theta-1}} + (n-n_S)},$$

which is true iff

$$(n_M - n_S) A_N L_N \geq (n-n_M) A_M L_M. \tag{51}$$

Observe that (6) is diametrically opposite to (51).

We also must ensure that M produces no more than $(n_M - n_S)$ varieties:

$$A_M L_M \leq (n_M - n_S)\sum_{J\in\{N,M,S\}} L_J c_J(i),$$

for all the varieties $i \in (n_S, n_M]$. This is equivalent to

$$\begin{aligned}
A_M L_M &\leq (n_M - n_S)\sum_{J\in\{N,M,S\}} L_J \frac{W_J}{n_S\left(\frac{A_M w_S}{A_S w_M}\right)^{\frac{1}{\theta-1}} + (n-n_S)} \\
&= \frac{(n_M - n_S)}{n_S\left(\frac{A_M w_S}{A_S w_M}\right)^{\frac{1}{\theta-1}} + (n-n_S)}\sum_{J\in\{N,M,S\}} L_J W_J \\
&= \frac{(n_M - n_S)\left(A_N L_N + A_M L_M\right)}{(n-n_S)},
\end{aligned}$$

which is further reduced to

$$(n - n_M)A_M L_M \leq (n_M - n_S)A_N L_N,$$

same as (51). In summary, $\frac{w_N}{A_N} = \frac{w_M}{A_M} > \frac{w_S}{A_S} > 0$ is true iff (50) and (51) are both satisfied.

Proposition 1. Suppose (6) is true. $\frac{w_N}{w_M} = \frac{A_N}{A_M}$ (Pattern B) holds for any $A_S \in (0, A_0]$; (44) (Pattern C) holds for any $A_S \in (A_0, A_1]$;(4) (Pattern A) holds for any $A_S \in (A_1, \infty)$, where A_0 and A_1 are defined in (47) and (18), respectively.

Proof. Trivial. **Q.E.D.**

This implies that, when (6) is true, the following is true: as A_S increases, $\frac{w_N}{w_M}$ is first independent of A_S, and then strictly increases with A_S, and finally becomes independent of A_S again. See Figure 3.

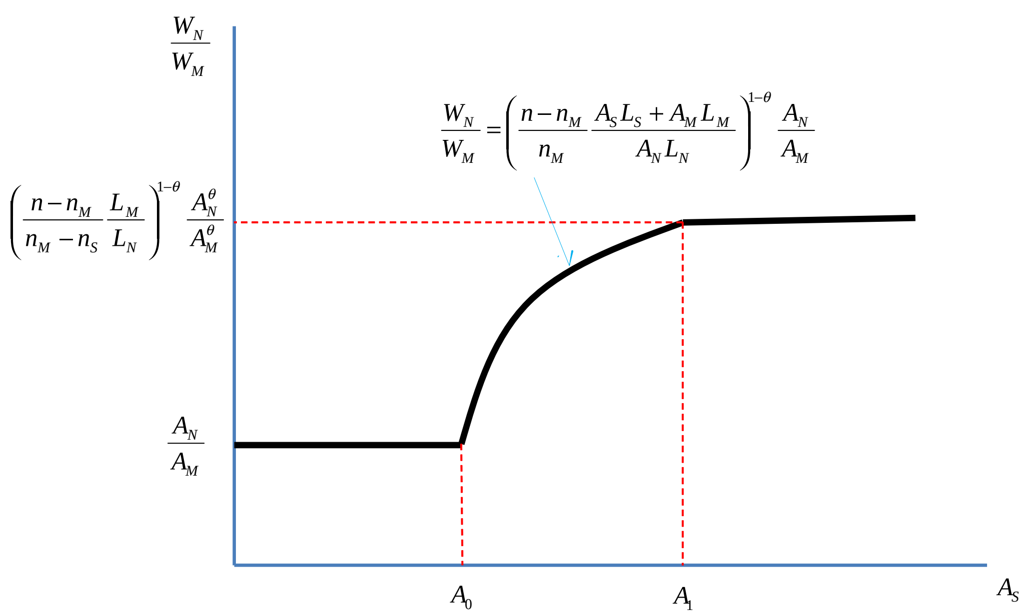

Figure 3. How $\frac{w_N}{w_M}$ Changes with A_S under (6)

The intuition is the following. When A_N is sufficiently small ((6) is true), the trade specialization pattern shall depend on A_S.

When A_S is sufficiently small ($A_S \in (0, A_0]$), M has to serve the world demand for some varieties accessible to S. This drives up the wage rate in M, which in turn allows N to produce some varieties accessible to M as well. So $\frac{w_N}{w_M}$

is determined by their productivity ratio, independent of A_S. Here, the binding constraint for M is not technology diffusion barrier (because the highest-indexed variety produced in M is $\widetilde{n_M} < n_M$).

When S productivity is in a medium range ($A_S \in (A_0,\, A_1]$), S produces and exports more varieties, which allows M to serve the world total demand for all the varieties accessible to M but not accessible to S. Now $\frac{w_N}{w_M}$ increases with A_S because more business is stolen away from M to S when S productivity A_S increases. This is a negative pecuniary externality S imposes on M.

When S productivity is sufficiently large ($A_S \in (\, A_1, \infty)$), S serves the world total demand for all the goods accessible to it, pushing M to only produce what S cannot produce. However, S has encountered technology diffusion barrier ($\widetilde{n_S} = n_S$), so it cannot effectively steal business from country M at the extensive margin, implying no impact of A_S on $\frac{w_N}{w_M}$.

Proposition 2. Suppose (6) is not true. $\frac{w_N}{w_M} = \frac{A_N}{A_M}$ (Pattern B) holds for any $A_S \in (0, \frac{n_S[L_M A_M + L_N A_N]}{(n-n_S)L_S}]$; $\frac{w_N}{w_M} = \frac{A_N}{A_M}$ (Pattern D) holds for any $A_S \in (\frac{n_S[L_M A_M + L_N A_N]}{(n-n_S)L_S}, \infty)$.

Proof. Trivial. **Q.E.D.**

This implies that, when (6) is violated, the following is true: $\frac{w_N}{w_M}$ is always independent of A_S. The intuition is the following. When A_N is sufficiently large ((6) is violated), country N not only produces those varieties that are only accessible to it but also produces some varieties accessible to country M, that is, $\widetilde{n_M} < n_M$. So $\frac{w_N}{w_M} = \frac{A_N}{A_M}$, which is independent of A_S. In particular, when A_S is sufficiently small ($A_S \in (0, \frac{n_S[L_M A_M + L_N A_N]}{(n-n_S)L_S}]$), country M also produces some varieties accessible to country S, that is, $\widetilde{n_s} < n_s$. When A_S is sufficiently large, country S is contrained by technology diffusion barrier and it supplies all the varieties accessible to it, however, it encounters the technology diffusion barrier, so its productivity has no impact on $\frac{w_N}{w_M}$; the unit cost for any varieties on $(n_s, n_M]$ is still identical for country M and country N, so $\frac{w_N}{w_M} = \frac{A_N}{A_M}$.

Appendix for the dynamic model part:

Assumption B: Labor productivities grow and diffuse across countries in the following fashion:

$$\begin{aligned}
\dot{A_N} &= \phi_N A_N, \\
\dot{A_M} &= \phi_M A_N^{\chi_M} A_M^{1-\chi_M}, \\
\dot{A_S} &= \phi_S A_N^{\chi_{SN}} A_M^{\chi_{SM}} A_S^{1-\chi_{SN}-\chi_{SM}},
\end{aligned}$$

which implies the following is true on the BGP:

$$
\begin{aligned}
\frac{A_N}{A_M} &= \left(\frac{\phi_N}{\phi_M}\right)^{\frac{1}{\chi_M}} \\
\frac{A_S}{A_M} &= \Phi(\phi_N, \phi_M, \phi_S) \equiv \phi_N^{-\frac{\chi_M - \chi_{SN}}{\chi_M}\frac{1}{\chi_{SN}+\chi_{SM}}} \phi_S^{\frac{1}{\chi_{SN}+\chi_{SM}}} \phi_M^{-\frac{\chi_{SN}}{\chi_M}\frac{1}{\chi_{SN}+\chi_{SM}}}
\end{aligned}
$$

Theorem 10 *When Assumption B is satisfied, the following is true in the long run equilibrium:*

$$
\frac{w_N}{w_M} = \begin{cases}
\left(\frac{\phi_N}{\phi_M}\right)^{\frac{1}{\chi_M}} & \text{if} \quad \frac{\alpha+\gamma}{\beta} \le \left(\frac{\phi_N}{\phi_M}\right)^{\frac{1}{\chi_M}} \frac{L_N}{L_M}\ , \textit{ or} \\
 & \quad \Phi(\phi_N, \phi_M, \phi_S) \in (0, a_0] \textit{ and } \frac{\alpha+\gamma}{\beta} > \left(\frac{\phi_N}{\phi_M}\right)^{\frac{1}{\chi_M}} \frac{L_N}{L_M} \\
\left[\frac{L_N}{\Phi(\phi_N,\phi_M,\phi_S)L_S + L_M}\frac{\beta}{\alpha}\right]^{\theta-1} \left(\frac{\phi_N}{\phi_M}\right)^{\frac{\theta}{\chi_M}} & \text{if} \quad \Phi(\phi_N, \phi_M, \phi_S) \in (a_0, a_1] \textit{ and } \frac{\alpha+\gamma}{\beta} > \left(\frac{\phi_N}{\phi_M}\right)^{\frac{1}{\chi_M}} \frac{L_N}{L_M} \\
\left(\frac{\alpha+\gamma}{\beta}\frac{L_M}{L_N}\right)^{1-\theta} \left(\frac{\phi_N}{\phi_M}\right)^{\frac{\theta}{\chi_M}} & \text{if} \quad \Phi(\phi_N, \phi_M, \phi_S) \in (a_1, \infty) \textit{ and } \frac{\alpha+\gamma}{\beta} > \left(\frac{\phi_N}{\phi_M}\right)^{\frac{1}{\chi_M}} \frac{L_N}{L_M}
\end{cases}, \tag{52}
$$

where

$$
a_0 = \frac{\beta}{\alpha}\left(\frac{\phi_N}{\phi_M}\right)^{\frac{1}{\chi_M}} \frac{L_N}{L_S} - \frac{L_M}{L_S}; a_1 = \frac{\gamma L_M}{\alpha L_S}. \tag{53}
$$

The opposite is true when For example, this must be true in the long run if labor productivity growth rate $g_J(t)$ is constant over time at $\overline{g_J}$ for all countries and $\overline{g_S} > \overline{g_M} > \overline{g_N}$. Notice that in this case, neither A_S nor $\overline{g_S}$ can affect $\frac{w_N}{w_M}$ in the long run, so the chasing effect only works at the speed margin (via γ). If, instead, $\overline{g_S} = \overline{g_M} > \overline{g_N}$ such that $\frac{A_S}{A_M} \le a_1$ in the long run, we would have $\frac{w_N}{w_M} = \left[\frac{A_N L_N}{A_S L_S + A_M L_M}\frac{\beta}{\alpha}\right]^{\theta-1} \frac{A_N}{A_M}$, which means that chasing effect only works through level A_S with the speed margin absent. If $\overline{g_M} < \overline{g_N}$,

Theorem 11 *In the long run, the following is true:*

$$
\frac{w_N}{w_M} = \begin{cases}
\frac{A_N}{A_M} & \text{if} \quad \frac{\alpha+\gamma}{\beta} \le \frac{A_N L_N}{A_M L_M}\ , \textit{ or} \\
 & \quad \frac{A_S}{A_M} \in (0, a_0] \textit{ and } \frac{\alpha+\gamma}{\beta} > \frac{A_N L_N}{A_M L_M} \\
\left[\frac{A_N L_N}{A_S L_S + A_M L_M}\frac{\beta}{\alpha}\right]^{\theta-1} \frac{A_N}{A_M} & \text{if} \quad \frac{A_S}{A_M} \in (a_0, a_1] \textit{ and } \frac{\alpha+\gamma}{\beta} > \frac{A_N L_N}{A_M L_M} \\
\left(\frac{\alpha+\gamma}{\beta}\frac{L_M}{L_N}\right)^{1-\theta} \frac{A_N^{\theta}}{A_M^{\theta}} & \text{if} \quad \frac{A_S}{A_M} \in (a_1, \infty) \textit{ and } \frac{\alpha+\gamma}{\beta} > \frac{A_N L_N}{A_M L_M}
\end{cases}, \tag{54}
$$

where

$$
a_0 = \frac{\beta}{\alpha}\frac{A_N}{A_M}\frac{L_N}{L_S} - \frac{L_M}{L_S}; a_1 = \frac{\gamma L_M}{\alpha L_S}. \tag{55}
$$

First observe that $a_0 < a_1$ if and only if $\frac{\alpha+\gamma}{\beta} > \frac{A_N L_N}{A_M L_M}$. Second, this theorem shows that the chasing effect through the imitation speed γ exists if and only if $\frac{A_S}{A_M}$ is sufficiently large whereas $\frac{A_N}{A_M}$ is sufficiently small. For example, this must be true in the long run if labor productivity growth rate $g_J(t)$ is constant over time at $\overline{g_J}$ for all countries and $\overline{g_S} > \overline{g_M} > \overline{g_N}$. Notice that in this case, neither A_S nor $\overline{g_S}$ can affect $\frac{w_N}{w_M}$ in the long run, so the chasing effect only works at the speed margin (via γ). If, instead, $\overline{g_S} = \overline{g_M} > \overline{g_N}$ such that $\frac{A_S}{A_M} \leq a_1$ in the long run, we would have $\frac{w_N}{w_M} = \left[\frac{A_N L_N}{A_S L_S + A_M L_M} \frac{\beta}{\alpha}\right]^{\theta-1} \frac{A_N}{A_M}$, which means that chasing effect only works through level A_S with the speed margin absent. If $\overline{g_M} < \overline{g_N}$,

$$\frac{\alpha}{\beta} = \frac{n - n_M}{n_M}; \frac{\alpha}{\alpha+\gamma} = \frac{n_M - n_S}{n_M}$$

5 III. Frictional Trade

Now consider frictional trade in the static economy with technology accessibility depicted by Figure 1. Assume $A_j = 1$ for any $j \in \{M, N, S\}$. Suppose iceberg trade cost d is symmetric between the trade partners. For an importing country to receive one unit, the exporting country needs to export d units because part of it will be "melted" away like an iceberg. So $d \geq 1$. Let d_{SM} denote the iceberg cost between S and M, d_{SN} between S and N, and d_{MN} between M and N. Observe that the two-country model in Krugman (1979) is a special case of this general setting when $d_{SN} = d_{SM} = +\infty$ and $d_{MN} = 1$).

Questions to be addressed:

- Please derive $\frac{w_N}{w_M}$ in the static equilibrium when $d_{SN} = d_{SM} = +\infty$ and $d_{MN} = d$. Plot how it changes with d for $d \in [1, \infty)$. Explain the economic intuitions. Also derive welfare for the **three** countries using (1).

First of all, observe that country S is isolated from the world trade system, so the consumers there only consume the goods produced domestically, the measure of which is n_s according to Figure 1. Here it is assumed that trade barrier cannot prevent the idea flows (imitation by S from M).

Let $c_J(i)$ denote the amount of consumption of variety $i \in [0, n]$ by a representative household in country $J \in \{S, M, N\}$. In equilibrium, we must have

$$c_s(i) = \begin{cases} \frac{1}{n_s}, & \text{when} \quad i \in [0, n_S] \\ 0, & \text{when} \quad i \in (n_S, n] \end{cases} .$$

It remains to analyze the two-country world consisting of M and N. Observe that the utility function satisfies the Inada condition for each variety (because $\theta < 1$), so the demand for each variety must be strictly positive so long as its price is less than infinity. So all the varieties will be consumed by all the consumers in both countries whenever $d < \infty$.

There are three possible cases, depending on whether $d \cdot w_M$ is larger than, or equal to, or smaller than w_N.

Case 1. Suppose $d \cdot w_M > w_N$, then N would rather produce everything by itself than import from M, in which case M cannot export anything and therefore cannot import anything due to trade balance condition. However, this contradicts the fact that consumers in M must import from N due to Inada condition because $dw_N < \infty$. Therefore, this case is never possible.

Case 2. Suppose $d \cdot w_M = w_N$, then M will export some variety $i \in [0, n_M]$ to N because M must import each variety $j \in (n_M, n]$ from N.

Case 3. $d \cdot w_M < w_N$, so M exports each variety $i \in [0, n_M]$ to N and imports each variety $j \in (n_M, n]$ from N.

Then from the Household Problem in M, we have

$$\left[\frac{c_M(i)}{c_M(j)}\right]^{\theta-1} = \frac{w_M}{w_N d},$$

for $\forall i \in [0, n_M]$ and $\forall j \in (n_M, n]$. By symmetry, we can write the budget constraint for each household in M as follows

$$n_M c_M(i) w_M + (n - n_M) c_M(j) w_N d = w_M.$$

Similarly, for N we have

$$\left[\frac{c_N(i)}{c_N(j)}\right]^{\theta-1} = \frac{w_M d}{w_N},$$

and

$$n_M c_N(i) w_M d + (n - n_M) c_N(j) w_N = w_N,$$

for $\forall i \in [0, n_M]$ and $\forall j \in (n_M, n]$.

Markets clear for labor in M and N, respectively:

$$\begin{aligned} L_M c_M(i) + L_N c_N(i) d &= \frac{L_M}{n_M}, \\ L_M c_M(j) d + L_N c_N(j) &= \frac{L_N}{n - n_M}, \end{aligned}$$

for $\forall i \in [0, n_M]$ and $\forall j \in (n_M, n]$. Define $x \equiv \frac{w_N}{w_M}$. Solving these equations, we obtain

$$G(x) \equiv \frac{L_M}{\left[n_M x^{\frac{1}{1-\theta}} d^{\frac{\theta}{1-\theta}} + (n - n_M) x\right]} + \frac{L_N}{\left[n_M \left(\frac{x}{d}\right)^{\frac{\theta}{1-\theta}} + (n - n_M)\right]} = \frac{L_N}{n - n_M}, \tag{56}$$

Since $G'(x) < 0$, there exists a unique solution for x if the solution exists. Observe that $\lim_{x\to\infty} G(x) = 0$.

Thus the solution for x, which must satisfy $x > d$, exists iff $G(d) > \frac{L_N}{n-n_M}$, or equivalently,

$$\frac{(n-n_M)\,nL_M}{n_M L_N} > n_M d^{\frac{1+\theta}{1-\theta}} + (n-n_M)d,$$

from which we obtain a unique cutoff value $\overline{d}$ so that x exists iff $d \in [1, \overline{d})$. To ensure $\overline{d} > 1$, we must have

$$\frac{(n-n_M)}{n_M} > \frac{L_N}{L_M}. \tag{57}$$

[Suppose (57) is violated, then for any $d > 1$, so solution exists for (56). Thus Case 3 is impossible, and we must have $d \cdot w_M = w_N$.]

When $d \in [\overline{d}, \infty)$, we must have $d \cdot w_M = w_N$ (Case 2). Now consider the case when $d \in [1, \overline{d})$. (56) is equivalent to

$$\left[\frac{L_M}{L_N} - x\right] d^{\frac{-\theta}{1-\theta}} = \frac{n_M}{(n-n_M)} x^{\frac{1}{1-\theta}} - \frac{L_M(n-n_M)}{L_N n_M x^{\frac{\theta}{1-\theta}}}, \tag{58}$$

which implies that:

(1)When $\frac{L_M}{L_N} - x > 0$, we need

$$\frac{L_M}{L_N} > x > \left[\frac{L_M(n-n_M)^2}{L_N n_M^2}\right]^{\frac{1-\theta}{1+\theta}}, \tag{59}$$

the second inequality of which is due to that the right hand side of (58) must be postive. In this case, we have $\frac{\partial x}{\partial d} < 0$. (59) holds iff

$$G(\frac{L_M}{L_N}) < \frac{L_N}{n-n_M} < G(\left[\frac{L_M(n-n_M)^2}{L_N n_M^2}\right]^{\frac{1-\theta}{1+\theta}}),$$

which is equivalent to

$$\frac{(n-n_M)}{n_M} < \left(\frac{L_M}{L_N}\right)^{\frac{\theta}{1-\theta}}.$$

Together with (57), we also require

$$\frac{L_N}{L_M} < 1. \tag{60}$$

In summary, we require

$$\frac{L_N}{L_M} < \min\{\frac{(n-n_M)}{n_M}, \left[\frac{(n-n_M)}{n_M}\right]^{-\frac{1-\theta}{\theta}}\}.$$

(2)When $\frac{L_M}{L_N} - x < 0$, we require $\frac{L_M}{L_N} < x < \left[\frac{L_M(n-n_M)^2}{L_N n_M^2}\right]^{\frac{1-\theta}{1+\theta}}$. Then $\frac{\partial x}{\partial d} > 0$, which holds iff

$$\left[\frac{(n-n_M)}{n_M}\right]^{\frac{1-\theta}{\theta}} > \left(\frac{L_M}{L_N}\right).$$

Together with (57), we also require

$$\frac{L_M}{L_N} > \frac{n_M}{(n-n_M)}.$$

In summary, we require

$$\left[\frac{(n-n_M)}{n_M}\right]^{\frac{\theta-1}{\theta}} < \frac{L_N}{L_M} < \frac{(n-n_M)}{n_M}$$

which is feasible only if

$$n > 2n_M.$$

(3)When $\frac{L_M}{L_N} - x = 0$, we have $\frac{\partial x}{\partial d} = 0$, which holds iff

$$\frac{L_N}{L_M} = \left[\frac{(n-n_M)}{n_M}\right]^{\frac{\theta-1}{\theta}}.$$

Together with (57), we also require

$$\frac{L_N}{L_M} < 1,$$

which is possible only if

$$n > 2n_M.$$

These findings can be summarized as follows.

Proposition 3. [a]. $x = d$ for any $d \in [1, \infty)$ whenever $\frac{L_N}{L_M} \geq \frac{(n-n_M)}{n_M}$. [b] Suppose $\frac{L_N}{L_M} < \frac{(n-n_M)}{n_M}$ and $n \leq 2n_M$. For any $d \in [1, \overline{d})$, $\frac{\partial x}{\partial d} < 0$. $\frac{\partial x}{\partial d} = 1$ for $d \in [\overline{d}, \infty)$, where $\overline{d}$ is the uniquely determined by $G(\overline{d}) = \frac{L_N}{n-n_M}$. [c]Suppose $\frac{L_N}{L_M} < \frac{(n-n_M)}{n_M}$ and $n > 2n_M$. For any $d \in [1, \overline{d})$, $\frac{\partial x}{\partial d} < 0$ iff $\frac{L_N}{L_M} < \left[\frac{(n-n_M)}{n_M}\right]^{\frac{\theta-1}{\theta}}$; $\frac{\partial x}{\partial d} = 0$ iff $\frac{L_N}{L_M} = \left[\frac{(n-n_M)}{n_M}\right]^{\frac{\theta-1}{\theta}}$; $\frac{\partial x}{\partial d} > 0$ iff $\left[\frac{(n-n_M)}{n_M}\right]^{\frac{\theta-1}{\theta}} < \frac{L_N}{L_M} < \frac{(n-n_M)}{n_M}$. $\frac{\partial x}{\partial d} = 1$ for $d \in [\overline{d}, \infty)$.

The above proposition is illustrated in Figure 4. Panels a, b, and c capture, respectively, cases [a], [b] and [c] in Proposition 3. In Panel a, $\frac{w_N}{w_M}$ always strictly increases in d for any $d \geq 1$. In Panel b. $\frac{w_N}{w_M}$ first strictly decreases with d when $d \in [1, \overline{d})$ following the curve CD, and then $\frac{w_N}{w_M} = d$, so $\frac{w_N}{w_M}$ strictly increases with d when $d \in [\overline{d}, \infty)$, following the straight line DE. Panel c is more complicated. When $\frac{L_N}{L_M} < \left[\frac{(n-n_M)}{n_M}\right]^{\frac{\theta-1}{\theta}}$, $\frac{w_N}{w_M}$ first strictly decreases with d when

$d \in [1, \overline{d})$ following the curve CD, and then $\frac{w_N}{w_M} = d$, so $\frac{w_N}{w_M}$ strictly increases with d when $d \in [\overline{d}, \infty)$, following the 45 degree line. When $\frac{L_N}{L_M} = \left[\frac{(n-n_M)}{n_M}\right]^{\frac{\theta-1}{\theta}}$, $\frac{w_N}{w_M} = \frac{L_N}{L_M} = \overline{d}$, so $\frac{w_N}{w_M}$ is independent of d when $d \in [1, \overline{d})$, following horizontal line BD, and then $\frac{w_N}{w_M} = d$, so $\frac{w_N}{w_M}$ strictly increases with d when $d \in [\overline{d}, \infty)$, following the 45 degree line. When $\left[\frac{(n-n_M)}{n_M}\right]^{\frac{\theta-1}{\theta}} < \frac{L_N}{L_M} < \frac{(n-n_M)}{n_M}$, $\frac{w_N}{w_M}$ is strictly increasing in d when $d \in [1, \overline{d})$, following the curve AD, and then $\frac{w_N}{w_M} = d$, so $\frac{w_N}{w_M}$ strictly increases with d when $d \in [\overline{d}, \infty)$, following the 45 degree line.

- (multilateral trade liberalization) Please derive $\frac{w_N}{w_M}$ and $\frac{w_M}{w_S}$ in the static equilibrium when $d_{SM} = d_{SN} = d_{MN} = d$. Please also plot in two separate graphs how these two ratios change with d for $d \in [1, \infty)$. Explain the economic intuitions for the two graphs. [Observe that Section II treats a special case when $d = 1$].

- (How bilateral S-N trade liberalization affects M) Please derive $\frac{w_N}{w_M}$ in the static equilibrium when $d_{SM} = d_{MN} = 1, d_{SN} = d$. Please also plot in two separate graphs how these two ratios change with d for $d \in [1, \infty)$. Explain the economic intuitions for the two graphs.

First consider the simplest case in which the following holds,

$$w_S d < w_M < w_N,$$

so the production patterns are still given by Figure 2. Let c_{ij} denote the consumption of a representative household in country j for a variety produced in country i.

For the household problem in S,

$$\begin{aligned}
\left(\frac{c_{ss}}{c_{ms}}\right)^{\theta-1} &= \frac{w_s}{w_M} \\
\left(\frac{c_{ss}}{c_{Ns}}\right)^{\theta-1} &= \frac{w_s}{dw_N}
\end{aligned}$$

Budget constraint

$$w_s c_{ss} n_s + w_M(n_M - n_S)c_{MS} + dw_N(n_N - n_M)c_{NS} = w_S.$$

For the household problem in M,

$$\begin{aligned}
\left(\frac{c_{SM}}{c_{mM}}\right)^{\theta-1} &= \frac{w_s}{w_M} \\
\left(\frac{c_{NM}}{c_{MM}}\right)^{\theta-1} &= \frac{w_N}{w_M}
\end{aligned}$$

Budget constraint

$$w_s c_{sM} n_s + w_M(n_M - n_S)c_{MM} + w_N(n_N - n_M)c_{NM} = w_M.$$

For the household problem in N,

$$\begin{aligned}
\left(\frac{c_{SN}}{c_{MN}}\right)^{\theta-1} &= \frac{dw_S}{w_M} \\
\left(\frac{c_{NN}}{c_{MN}}\right)^{\theta-1} &= \frac{w_N}{w_M}
\end{aligned}$$

Budget constraint

$$dw_S c_{SN} n_S + w_M(n_M - n_S)c_{MN} + w_N(n_N - n_M)c_{NN} = w_N.$$

Labor market clearing conditions

$$\begin{aligned}
\frac{L_S}{n_S} &= L_S c_{SS} + L_M c_{SM} + dL_N c_{SN} && (61) \\
\frac{L_M}{n_M - n_S} &= L_S c_{MS} + L_M c_{MM} + L_N c_{MN} && (62) \\
\frac{L_N}{n_N - n_M} &= dL_S c_{NS} + L_M c_{NM} + L_N c_{NN} && (63)
\end{aligned}$$

(61) and (62) imply

$$\begin{aligned}
\frac{\frac{L_S}{n_S} - dL_N c_{SN}}{\frac{L_M}{n_M - n_S} - L_N c_{MN}} &= \frac{w_S}{w_M} \\
c_{SN} &= c_{MN}\left(\frac{dw_S}{w_M}\right)^{\frac{1}{\theta-1}} \\
c_{MN} &= \frac{\frac{w_S}{w_M}\frac{L_M}{n_M - n_S} - \frac{L_S}{n_S}}{L_N\left[\frac{w_S}{w_M} - d\left(\frac{dw_S}{w_M}\right)^{\frac{1}{\theta-1}}\right]} \\
c_{SN} &= \left(\frac{dw_S}{w_M}\right)^{\frac{1}{\theta-1}} \frac{\frac{w_S}{w_M}\frac{L_M}{n_M - n_S} - \frac{L_S}{n_S}}{L_N\left[\frac{w_S}{w_M} - d\left(\frac{dw_S}{w_M}\right)^{\frac{1}{\theta-1}}\right]}
\end{aligned}$$

(62) and (63) imply

$$\begin{aligned}
\frac{\frac{L_M}{n_M - n_S} - L_S c_{MS}}{\frac{L_N}{n_N - n_M} - dL_S c_{NS}} &= \frac{w_M}{w_N} \\
\left(\frac{c_{MS}}{c_{NS}}\right)^{\theta-1} &= \frac{w_M}{dw_N}
\end{aligned}$$

so we have

$$\begin{aligned}
\frac{L_M}{n_M - n_S} &= \frac{w_M}{w_N}\frac{L_N}{n_N - n_M} + c_{NS}\left[L_S\left(\frac{w_M}{dw_N}\right)^{\frac{1}{\theta-1}} - \frac{w_M}{w_N}dL_S\right] \\
c_{NS} &= \frac{\frac{L_M}{n_M-n_S} - \frac{w_M}{w_N}\frac{L_N}{n_N-n_M}}{L_S\left[\left(\frac{w_M}{dw_N}\right)^{\frac{1}{\theta-1}} - \frac{w_M}{w_N}d\right]} \\
c_{MS} &= \left[\frac{w_M}{dw_N}\right]^{\frac{1}{\theta-1}}\frac{\frac{L_M}{n_M-n_S} - \frac{w_M}{w_N}\frac{L_N}{n_N-n_M}}{L_S\left[\left(\frac{w_M}{dw_N}\right)^{\frac{1}{\theta-1}} - \frac{w_M}{w_N}d\right]}
\end{aligned}$$

Recall S budget is

$$\frac{w_M}{w_N}(n_M - n_S)c_{MS} + d(n - n_M)c_{NS} = \frac{w_S}{w_N}\left[1 - c_{ss}n_s\right].$$

Trade balance between S and N:

$$L_s(n - n_M)c_{NS}dw_N = L_N n_S c_{SN} dw_S$$

which is

$$\frac{L_s(n - n_M)c_{NS}}{L_N n_S c_{SN}} = \frac{w_S}{w_N}$$

- (North trade liberalization without S-M trade) Please derive $\frac{w_N}{w_M}$ and $\frac{w_M}{w_S}$ in the static equilibrium when $d_{SM} = +\infty, d_{SN} = d_{MN} = d$. Please also plot in two separate graphs how these two ratios change with d for $d \in [1, \infty)$. Explain the economic intuitions for the two graphs.