# Global Sourcing and Domestic Value-added in Exports*

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**Abstract**

This paper employs data from the World Input-Output Database (WIOD) to document the evolution of the domestic content in exports, as measured by the domestic value added to gross exports ratio (DVAR), across countries and sectors over the period 1995-2008. We develop a multiple-sector general equilibrium model of Eaton and Kortum (2002) with domestic and global input-output linkages (a la Caliendo and Parro (2015)) to provide structural interpretations of individual countries' DVAR. We use the calibrated version of the model to fully decompose the time-series changes of the global DVAR and selected countries' DVAR into separate parts that are due to changes in technology, bilateral trade frictions, unilateral export fixed costs, and other exogenous factors such as changes in factor endowments and trade balances. We find that while the partial effects of both technology and trade costs are negative, there is a positive and significant interactive effect from the two. Taking into account the interactive effects, we find that the total effect of technology, which has been either overlooked or misinterpreted in the existing analyses of the evolution of global value chains, is significantly positive, while the total effect of trade frictions is far from capable of explaining the changes in DVAR over the sample period. The contributions of other determinants are quantitatively very small.

JEL Classification codes: F10, F11, F14, F17

Keyword: fragmentation, gravity, global value chain, domestic value-added ratio

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# 1 Introduction

Advances in information and communication technologies and declining trade barriers have encouraged firms to source inputs more globally. Global fragmentation of production tasks implies that a country's exports may contain a significant amount of foreign content, making gross trade flow statistics increasingly less accurate for describing the actual value-added of a country in its exports. Consider consumer electronics (which includes iPhones) as an example. According to Kee and Tang (2016), only 53% of the exports of electronics from China in 2007 was value-added attributed to China. As documented by Koopman, Wang, and Wei (2014) and Johnson and Noguera (2017), the share of domestic value-added in gross exports is significantly lower than 1 for most countries and sectors, and has been decreasing for decades.

Analyzing the determinants of the DVAR of a country's exports can help us understand the drivers of global production fragmentation. In this paper, we employ data from the multi-country multi-sector input-output tables from World Input-Output Database (WIOT) to document the evolution of the domestic content in exports, as measured by the domestic value added to gross exports ratio (DVAR), across countries and sectors over the period 1995-2008. To guide our quantitative analysis of the evolution of the DVAR in exports across countries and years, we develop a multi-country, multi-sector, quantitative trade model with inter-sectoral input-output linkages, based on Eaton and Kortum (2002) and Caliendo and Parro (2015). The model offers structural interpretations of the DVAR at the country-sector level for each year.

After documenting the patterns of DVAR across time and countries, we study the determinants of a country's DVAR. Specifically, we use the calibrated version of the model to fully decompose the time-series changes of the global and selected countries' DVAR into separate parts that are due to (exogenous) changes in three sets of factors, namely (i) technology, (ii) trade frictions, and (iii) other exogenous factors such as factor endowments and trade balances. We find that while the stand-alone effects of both technology and trade costs are negative, there is a positive and significant interactive effect from the two. Taking into account the interactive effect, we find that the total effect of a country's technology on its DVAR, an overlooked or misinterpreted aspect in existing analyses of the determinants of DVAR, is significantly positive, and even larger in absolute value than the negative total effect of trade frictions. The contributions of other determinants (i.e., factor endowments and trade balances) are quantitatively tiny.

Table 1 summarizes the calibrated decomposition of the decline in the global, developed, and developing countries' DVAR during the sample period. It reports the total effect (pure effect plus interactive effects) of each set of factor(s). Adding the stand-alone effect and the interactive effects together, we find that for the world as a whole, as well as the developing and developed country samples respectively, changes in technology have a significant and positive impact on its DVAR, while trade liberalization has a comparably significant negative effect. In fact, certain fast-growing countries like China, can have their DVAR increasing over time, despite falling trade frictions, due to a larger technology effect.

Table 1: Percentage-point Changes in DVAR (1995-2008)

| | Global | Developed | Developing |
|---|---|---|---|
| Total | -4.36 | -4.20 | -4.58 |
| total effect of | | | |
| Technology | 2.83 | 1.51 | 4.40 |
| Trade Costs | -3.11 | -1.66 | -4.78 |
| Other Factors | 0.36 | 0.01 | 0.77 |

Notice that the sum of the three total effects is far away from the data, as each total effect embeds interactive effects with other factors. The potentially large interactive effect is an outcome of the non-linearity in the structural gravity equations derived from a large class of quantitative trade models, to which Eaton and Kortum (2002) belongs, together with a negative correlation between changes in technology and changes in import barriers across country-sector pairs (e.g., China's estimated sectoral TFP was rising faster in sectors that experienced larger declines in import barriers). However, the total effect of the change in technology on a country's DVAR cannot be isolated from that of trade frictions, or vice versa. The total effect of each factor depends on the underlying empirical distribution of the changes in technology and trade frictions across time and sectors within a country. Since trade frictions act in conjunction with technology to shape a country's trade patterns, ignoring such interactive effects may result in biased estimates of the contribution of any single determinant of the DVAR and other global value chain (GVC) measures.

We also use our calibrated model to conduct a series of counterfactual exercises. As a first pass, we study quantitatively how shutting down China's technological growth or trade liberalization (i.e., turning the clock for both estimated parameters back to the 1995 levels) will affect the DVAR of China's, US's, and world exports. We find that the effect of China's technological growth on its own exports' DVAR is significantly positive, while the effect of China's trade liberalization on its own DVAR is significantly negative. We also find that while shutting down China's technological growth or trade liberalization have little impact on the DVAR of global exports, both have non-negligible positive impacts on the US's DVAR.

This paper relates to various strands of literature on GVC. First, it contributes to the studies that develop models of fragmentation (Baldwin, 2012, Baldwin and Venables, 2013; Eaton and Kortum,, 2002; Alvarez and Lucas, 2007; Yi, 2003; 2010; Antras and Chor, 2017). See Feenstra (1998) for a review of the early literature on foreign outsourcing.

Second, our paper contributes to the literature that provides methods to measure of various aspects of GVC (e.g., domestic value added, upstreamness, length of production chains). This literature starts with Hummels, Ishii and Yi (2001) to use industry input-output (IO) tables to calculate the value added to exports ratios for many countries. Recent related work includes Antràs, Chor, Fally, and Hillberry (2012), Johnson and Noguera (2012 and 2014), Koopman, Wang and Wei (2012, 2014), Antràs and Chor (2013), De la Cruz, Koopman, Wang and Wei (2013), and Johnson (2014).

The third strand of studies bridges the first largely theoretical literature and the second literature on measurement by calibrating quantifiable models of GVC (Antras and Chor, 2017; Antras and de Gortari, 2017; Johnson and Noguera, 2017; Fally and Hillberry, 2018; de Gortari, 2019). Our paper belongs to this frontier of research, by linking the literature that documents the domestic content in countries' exports (e.g. Hummels, Ishii and Yi, 2001 and Koopman, Wang, and Wei, 2014, among others) and the one that develops quantitative trade models to answer the "welfare gains from trade" and other macroeconomic questions (e.g., Arkolakis, Costinot, and Rodriguez-clare, 2012).

The paper is organized as follows. Section 2 introduces the theoretical model we use to quantify the determinants of countries' and sectors' DVAR. Section 3 describes how to bring our model to the data. Section 4 presents the quantitative results using our calibrated model. Section 5 attempts to establish a relation between the welfare gains from trade and DVAR. The final section concludes.

# 2 A Model of production fragmentation

## 2.1 Setup

Our model is built on Eaton and Kortum (2002) and thus adopts the same set of model assumptions: (1) All countries have the capability to produce all intermediates and final goods; (2) international trade (but not domestic trade) is costly; and (3) all markets are perfectly competitive.

There are $N$ countries in the world, indexed by $n = 1, ..., N$. Each country has (time-varying) labor ($L_n$) and capital ($K_n$) endowments. Labor and capital are fully mobile across sectors within a country, but not mobile across countries. There are $J$ final goods available for consumption, indexed by $i = 1, ..., J$.

In each country there is a representative household, who uses labor and capital income to purchase an optimal consumption bundle of final goods to maximize utility:

$$U = \prod_{i=1}^{J} \left\{ \left[ \int_0^1 \left( q^i \left( \omega \right) \right)^{\frac{\sigma^i - 1}{\sigma^i}} d\omega \right]^{\frac{\sigma^i}{\sigma^i - 1}} \right\}^{\alpha^i}, \text{ with } \sum_{i=1}^{J} \alpha^i = 1. \tag{1}$$

where $q^i(\omega)$ is the consumption of variety $\omega$ in sector $i$; $\sigma^i > 1$ is the elasticity of substitution between any pair of varieties within sector $i$; $\alpha^i$ is the expenditure share of final good $i$.

The production function of a variety is given by

$$y^i(\omega) = z^i(\omega)\left(M^i(\omega)\right)^{1-\beta^i}(l(\omega))^{\beta^i\mu^i}(k(\omega))^{\beta^i\left(1-\mu^i\right)} \tag{2}$$

where $y^i(\omega)$ is the quantity produced by firm $\omega$ in sector $i$; $z^i(\omega)$ is the total factor productivity (TFP) of the firm; $M^i$ is the sector-$i$ specific intermediate composite (to be defined later); $l(\omega)$ and $k(\omega)$ are labor and capital inputs, respectively.

The production function of an intermediate composite $M^i$ in the same country takes the same functional form:

$$M^i = \prod_{k=1}^{J}\left\{\left[\int_0^1\left(q^k(\omega)\right)^{\frac{\sigma^k-1}{\sigma^k}}d\omega\right]^{\frac{\sigma^k}{\sigma^k-1}}\right\}^{\gamma^{ik}}, \text{ with } \sum_{k=1}^{J}\gamma^{ik} = 1. \tag{3}$$

where $q^k(\omega)$ is the quantity of input variety $\omega$ in sector $k$; $\sigma^k > 1$ is the elasticity of substitution between any pair of varieties within sector $k$; $\gamma^{ik}$ is the sector-pair specific cost share of (upstream) input $k$ in the total cost of producing (downstream) input composite $i$.

International trade is costly. Whenever an intermediate or final good variety from sector $i$ is shipped from country $n$ to country $m$ to be used as input in sector $j$, an iceberg trade cost $\tau_{mn}^{ji}$ is incurred ($j = F$ if it is used as final good). That is, $\tau_{mn}^{ji} > 1$ units of good are shipped from the origin for one unit to arrive the destination. As usual, we normalize $\tau_{nn}^{ji} = 1$ for all $n$, $j$ and $i$, implying frictionless domestic trade.[1] We also assume that the triangle inequality $\tau_{mn}^{ji}\tau_{nl}^{ji} \geq \tau_{ml}^{ji}$ holds for $\forall l, m, n, i$ and $j$.

As such, the competitive price of a variety in sector $i$ shipped from country $l$ to country $n$ to be used as input in sector $j$ takes the following form.

$$p_{nl}^{ji}(\omega) = \frac{\tau_{nl}^{ji}c_l^i}{z_l^i(\omega)} \qquad \text{for all } \omega \in [0,1],$$

where

$$c_l^i = \left(\frac{P_l^i}{1-\beta_l^i}\right)^{1-\beta_l^i}\left(\frac{w_l}{\beta_l^i\mu_l^i}\right)^{\beta_l^i\mu_l^i}\left(\frac{r_l}{\beta_l^i\left(1-\mu_l^i\right)}\right)^{\beta_l^i\left(1-\mu_l^i\right)}$$

where $P_l^i$ is the price index of the intermediate composite used in sector $i$ and (source) country $l$, while $w_l$ and $r_l$ are the equilibrium wage rate and rental cost of capital in country $l$, respectively.

The final remark about the supply side is about firm-specific productivity. We assume that country $l$ possesses a technology stock of $T_l^i$ in producing sector-$i$ varieties. The technology stock

[1] Alternatively, one can interpret the international trade costs as relative to the domestic trade costs.

$T_l^i$ reflects country $l$'s absolute advantage in producing sector-$i$ goods. Following Eaton and Kortum (2002), we assume that firms in country $l$ draw efficiency $z_l^i$ for each variety $\omega \in [0,1]$ from the Fréchet distribution:

$$F\left(z_l^i < z\right) = e^{-T_l^i z^{-\theta}},$$

where $\theta$ is a parameter governing the (inverse) dispersion of productivity draw $z$ from the distribution. For simplicity, we assume the same constant $\theta$ for all countries and sectors.

### 2.2 Price indices and trade shares

Perfect competition implies that firms of each variety of sector j in country $n$ will purchase the intermediates from the firm that offers the lowest cost across all possible source countries. Thanks to Fréchet distribution of $z$, the price index of intermediates in country $n$ and sector $j$ is given by

$$P_n^j = \Upsilon_n^j \prod_{i=1}^J \left(p_n^{ji}\right)^{\gamma_n^{ji}} = \Upsilon_n^j \prod_{i=1}^J \left(\Phi_n^{ji}\right)^{-\frac{\gamma_n^{ji}}{\theta}},$$

where $\Upsilon_n^j = \prod_{i=1}^J \left(\gamma_n^{ii}\right)^{-\gamma_n^{ii}}$ is a constant and

$$\Phi_n^{ji} = \sum_l T_l^i \left(c_l^i \tau_{nl}^{ji}\right)^{-\theta}.$$

For sector-$j$ in country $n$, the cost share of intermediates $i$ from source country $l$ in total costs spent on intermediates $i$ is

$$\pi_{nl}^{ji} = \frac{T_l^i \left(c_l^i \tau_{nl}^{ji}\right)^{-\theta}}{\Phi_n^{ji}}. \tag{4}$$

### 2.3 Expressions of DVAR

Now let us derive the accounting expressions of DVAR in sales (domestic or exports) at the country-sector level. Let us denote the DVAR of country $n$ embodied in country $m$'s production of sector-$i$ goods by $r_{mn}^i$.

A complete accounting of a country-sector's DVAR should incorporate (1) domestic value added (DVA) from foreign countries embodied in imported intermediates; (2) DVA embodied in domestically produced intermediates; (3) Primary factors (i.e. capital and labor) employed directly (direct DVA). Formally, domestic country $n$'s value added in its own output from sector $i$:

$$r_{nn}^i = \beta_n^i + (1 - \beta_n^i) \sum_{h=1}^N \sum_{k=1}^J \pi_{nh}^{ik} \gamma_n^{ik} r_{hn}^k, \tag{5}$$

and for foreign country $n$'s value added in sales of sector $i$ from producing country $m$:

$$r_{mn}^i = (1 - \beta_m^i) \sum_{h=1}^N \sum_{k=1}^J \pi_{mh}^{ik} \gamma_m^{ik} r_{hn}^k \quad \text{for } m \neq n. \tag{6}$$

Two remarks are in order First, the main difference between $r^i_{nn}$ and $r^i_{mn}$ is that $\beta^i_n$ appears in the former, as domestic content in a country's exports obviously includes direct value added generated by domestic primary factors, including labor and physical capital.

Second, both (5) and (6) feature recursive nature of a country's own sector-specific DVAR, as the domestic content of a country's sectoral exports will be used as intermediates by other countries' production, which can be exported back to the source country of the domestic content. To more systematically analyze such recursivity of DVAR, we express the DVAR in matrix $\mathbf{r}$ for all country-sector pairs as follows:

$$\underbrace{\mathbf{r}}_{NJ\times N} = \underbrace{\beta}_{NJ\times N} + \underbrace{(\mathbf{I}-\mathbf{B})}_{NJ\times NJ}\underbrace{\mathbf{G}}_{NJ\times NJ}\underbrace{\mathbf{r}}_{NJ\times N}$$

where $\mathbf{r}$ is a $NJ \times N$ matrix whose $((m-1)\times N+i,n)$-th element is $r^i_{mn}$. The matrix $\mathbf{B}$ is the $NJ \times NJ$ square matrix with all off-diagonal elements equal to 0 and the $((n-1)\times N+i)$-th diagonal element equal to $\beta^i_n$. The matrix $\mathbf{G}$ is the $NJ \times NJ$ global intermediate goods cost share matrix, whose $((m-1)\times N+i,(n-1)\times N+k)$-th element is $\pi^{ik}_{mn}\gamma^{ik}_m$. Finally, $\beta$ is a $NJ \times N$ matrix, formed by stacking up $J$ $N \times N$ matrixes, each containing 0 off-diagonal elements and its $((j-1)\times N+n)$-th element equal to $\beta^i_n$.

The recursive relationship in $DVAR$ through global IO linkages can be solved in matrix form by collecting all $\mathbf{r}$ on the left hand side:

$$\mathbf{r} = [\mathbf{I} - (\mathbf{I}-\mathbf{B})\,\mathbf{G}]^{-1}\,\beta. \tag{7}$$

Totally differentiating $\mathbf{r}$ gives us the following expression

$$d\mathbf{r} = \underbrace{[\mathbf{I} - (\mathbf{I}-\mathbf{B})\,\mathbf{G}]^{-1}\,[d\beta - (d\mathbf{B})\,\mathbf{G}\mathbf{r}]}_{\text{pure effects of changing } \beta^i_n} + \underbrace{[\mathbf{I} - (\mathbf{I}-\mathbf{B})\,\mathbf{G}]^{-1}\,(\mathbf{I}-\mathbf{B})\,(d\mathbf{G})\,\mathbf{r}}_{\text{pure effects of changing } \pi^{ik}_{nm} \text{ and/or } \gamma^{ik}_n} \tag{8}$$

The first term of the right hand side captures the pure effect of changing $\beta^i_n$. The second term captures the effect of the changes in intermediate goods shares $\pi^{ik}_{nm}$ and input-output coefficients $\gamma^{ik}_n$. In the structural estimation exercises below, we will quantify the magnitude of each of the channels.

## 2.4 A simplified model for illustration

Let us develop a simple two-country, one-factor, one-sector model with round-about IO linkages to obtain some insights about both the stand-alone and interactive effects of changes in technology and trade frictions on country's DVAR. Let us denote $t = T_1/T_2$; $c = c_1/c_2$. For simplicity, let us also assume that $\tau_1 \equiv \tau_{12}$ and $\tau_2 \equiv \tau_{21}$.

Using the trade share equation (4) in the general model, we can express the trade share from country $n$ in country $m$ ($\pi_{mn}$) as

$$\begin{aligned}\pi_{11} &= \frac{tc^{-\theta}}{tc^{-\theta}+\tau_1^{-\theta}}, \pi_{12} = \frac{\tau_1^{-\theta}}{tc^{-\theta}+\tau_1^{-\theta}}, \\ \pi_{22} &= \frac{1}{1+tc^{-\theta}\tau_2^{-\theta}}, \pi_{21} = \frac{tc^{-\theta}\tau_2^{-\theta}}{1+tc^{-\theta}\tau_2^{-\theta}}.\end{aligned}$$

Using the accounting identities of DVAR (4) and (4), we can express the DVAR of country 1's firms in country 1's exports and country 2's exports respectively as

$$\begin{aligned}r_{11} &= \beta + (1-\beta)\left(\pi_{11}r_{11} + \pi_{12}r_{21}\right); \\ r_{21} &= (1-\beta)\left(\pi_{21}r_{11} + \pi_{22}r_{21}\right).\end{aligned}$$

Totally differentiating this system of the two equations yields

$$\begin{aligned}dr_{11} &= (1-\beta)\left(\pi_{11}dr_{11} + \pi_{12}dr_{21}\right) + (1-\beta)\left(r_{11}-r_{21}\right)d\pi_{11} \\ dr_{21} &= (1-\beta)\left(\pi_{21}dr_{11} + \pi_{22}dr_{21}\right) - (1-\beta)\left(r_{11}-r_{21}\right)d\pi_{22}\end{aligned}$$

which leads to

$$dr_{11} = Ad\pi_{11} - Bd\pi_{22}$$

where $A$ and $B$ are some constants, with $A > B > 0$.[2]

Taylor series expansion of $d\pi_{11}$ and $d\pi_{22}$ to the second order derivative gives the decomposition of effects on DVAR due to different forces (See the appendix for details). Rearranging the terms and ignoring the second order effects on $c$, the effect can be decomposed into

(i) The pure effect of technology

$$(C+D)\frac{dt}{t} - \left[C\pi_{11} + D\pi_{21}\right]\left(\frac{dt}{t}\right)^2 \tag{9}$$

(ii) The pure effect of trade frictions

$$-C\left[\frac{d\tau_1^{-\theta}}{\tau_1^{-\theta}} - \pi_{12}\left(\frac{d\tau_1^{-\theta}}{\tau_1^{-\theta}}\right)^2\right] + D\left[\frac{d\tau_2^{-\theta}}{\tau_2^{-\theta}} - \pi_{21}\left(\frac{d\tau_2^{-\theta}}{\tau_2^{-\theta}}\right)^2\right] \tag{10}$$

(iii) The interactive effect from technology and trade frictions

$$C\left(\pi_{11}-\pi_{12}\right)\left(\frac{dt}{t}\right)\left(\frac{d\tau_1^{-\theta}}{\tau_1^{-\theta}}\right) + D\left(\pi_{22}-\pi_{21}\right)\left(\frac{dt}{t}\right)\left(\frac{d\tau_2^{-\theta}}{\tau_2^{-\theta}}\right) \tag{11}$$

[2] $A = \frac{(1-\beta)(r_{11}-r_{21})(1-(1-\beta)\pi_{22})}{(2\beta-\beta^2)-\beta(1-\beta)(\pi_{11}+\pi_{22})}$; $B = \frac{(1-\beta)^2(r_{11}-r_{21})\pi_{12}}{(2\beta-\beta^2)-\beta(1-\beta)(\pi_{11}+\pi_{22})}$.

where $C = A\pi_{11}(1-\pi_{11})$ and $D = B\pi_{22}(1-\pi_{22})$.

Given $\pi_{11} < 1$ and $\pi_{21} < 1$, the pure effect of technology will be positive if $\frac{dt}{t}$ is sufficiently larger than $\left(\frac{dt}{t}\right)^2$, which is likely to be the case. The intuition is that when a country's productivity increases, the prices of its output will decline, raising the competitiveness of domestic sectors relative to foreign sectors, and thus increasing the domestic content in exports.

On the other hand, the pure effect of trade frictions will be negative, if $\frac{d\tau_1^{-\theta}}{\tau_1^{-\theta}} - \pi_{12}\left(\frac{d\tau_1^{-\theta}}{\tau_1^{-\theta}}\right)^2 > 0$ and $\frac{d\tau_2^{-\theta}}{\tau_2^{-\theta}} - \pi_{21}\left(\frac{d\tau_2^{-\theta}}{\tau_2^{-\theta}}\right)^2$ (i.e., if the first-order derivative is larger than the second-order derivative with respect to increases in trade frictions) and $C > D$. This would be the case if the country 1 is more closed and/ or cuts trade costs more than country 2. If only country 1 reduce trade frictions unilaterally (i.e., $\frac{d\tau_1^{-\theta}}{\tau_1^{-\theta}} > 0$ and $\frac{d\tau_2^{-\theta}}{\tau_2^{-\theta}} = 0$), then for sure the pure effect of unilateral trade reduction by country 1 will surely be negative if the first-order effect is larger.

Finally and most importantly, the effect which we refer to as the interactive effect from technology and trade frictions, as a result of the cross-partial comparative static exercise. The sign of the effect depends on the sign of $\frac{dt}{t}$ and $\frac{d\tau_1^{-\theta}}{\tau_1^{-\theta}}$, respectively. The existing empirical literature overall finds the same signs of the two effects, that is, countries that unilaterally cuts import costs ($\frac{d\tau_1^{-\theta}}{\tau_1^{-\theta}} > 0$) tend to experience more positive technological change ($\frac{dt}{t} > 0$) (see Bernard et al., 2012 for a review). If the sign of $\left(\frac{dt}{t}\right)\left(\frac{d\tau_1^{-\theta}}{\tau_1^{-\theta}}\right)$ is positive, and that domestic total trade shares tend to be larger than foreign trade shares (i.e., $\pi_{ii} > \pi_{ij}$) in both countries, the interactive effect will be positive.

# 3 Taking the model to data

## 3.1 Main data sources

We use the 2013 edition of the World Input-Output Database (WIOD), which contains trade data between any sector pairs and country pairs for 40 countries plus the rest of the world (RoW) (indexed by $j$) and 35 sectors (indexed by $s$), over 14 years (indexed by $t$) from 1995 and 2008.[3] In particular, we use yearly changes in the $NJ \times NJ$ (2,059,225) trade shares (i.e., $\pi_{nm}^{ik}$) as targets for our calibration of the general-equilibrium model. Another data set we use is the Social Economic Accounts (SEA) (2013 version) of the WIOD, from which we obtain the factor endowment data for all 40 countries every year in our sample.

[3] There is a 2016 version that covers more industries and more recent years but we chose to use the 2013 version to avoid dealing with the trade collapse during the 2008-2009 global financial crises.

### 3.2 Estimating trade frictions and productivity

We calibrate the following set of parameters in the model: (i) moments of the productivity distributions $T_n^i$ and $\theta$; (ii) trade costs $\tau_{nl}^{ji}$; (iii) production function parameters $\beta_n^i$ and $\gamma_n^{ik}$; (iv) preference parameters $\alpha^i$; and (v) country factor endowments $L_n$ and $K_n$. We will discuss the calibration of each in turn, in particular which parameter we estimate and take directly from existing studies.

The first step of our quantitative exercise is to estimate the change in competitiveness (relative to the US) at the exporter-sector level. To this end, we estimate the following structural gravity equation year by year, derived from our model:

$$\ln\left(\frac{\pi_{nlt}^{ji}}{\pi_{nnt}^{ji}}\right) = \ln\left(\frac{X_{nlt}^{ji}}{X_{nnt}^{ji}}\right) = \ln\left(T_{lt}^i\left(c_{lt}^i\right)^{-\theta}\right) - \theta ex_{lt}^i - \ln\left(T_{nt}^i\left(c_{nt}^i\right)^{-\theta}\right) - \theta v_{nlt}^{ji}, \qquad (12)$$

where $X_{nlt}^{ji}$ is the country-pair-sector-pair export value, obtained from the WIOT;[4] $\ln\left(T_{lt}^i\left(c_{lt}^i\right)^{-\theta}\right) - \theta ex_{lt}^i$ is estimated as the exporter-sector fixed effect, $-\ln\left(T_{nt}^i\left(c_{nt}^i\right)^{-\theta}\right)$ is estimated as the importer-sector fixed effect, and $-\theta v_{nlt}^{ji}$ is the residual of the estimation.

We follow Waugh (2010) to interpret $ex_{lt}^i$ as the exporter-sector fixed effects in year $t$, which captures the additional costs facing sector $i$'s exports exporting country $l$, compared to the US. The estimated asymmetric bilateral trade costs $\{\tau_{nl}^{ji}\}$ comprises two parts, the exporter fixed costs in Waugh (2010) and the actual "bilateral" trade costs:

$$\ln\tau_{nlt}^{ji} = ex_{lt}^i + v_{nlt}^{ji}. \qquad (13)$$

### 3.3 Solving the general-equilibrium model computationally

To reduce the burden of notation, let us suppress the time subscript and express variables associated with the following period by a superscript $'$. Following Dekle, Eaton, and Kortum (2008), we use exact hat algebra to characterize the equilibrium changes: $\widehat{x} = x'/x$. For each year, using $\ln\left(T_{lt}^i\left(c_{lt}^i\right)^{-\theta}\right)$ and $\ln\tau_{nlt}^{ji}$ estimated from the gravity equation (12), we calculate $\{\widehat{T}_l^i\left(\widehat{c}_l^i\right)^{-\theta}\}$ and $\{\widehat{\tau}_{nl}^{ji}\}$ as initial values. Starting with some guesses of $\{\widehat{w}_l\}$ and $\{\widehat{r}_l\}$, we solve for $\{\widehat{c}_l^i\}$, $\{\widehat{p}_l^{ji}\}$ and $\{\widehat{P}_l^i\}$ in the following system of three equations derived directly from our model:

$$\widehat{c}_l^i = \left(\widehat{P}_l^i\right)^{1-\beta_l^i}\left(\widehat{w}_l\right)^{\beta_l^i\mu_l^i}\left(\widehat{r}_l\right)^{\beta_l^i\left(1-\mu_l^i\right)} \qquad (14)$$

$$\widehat{p}_n^{ji} = \left[\sum\nolimits_{l=1}^{N}\pi_{nl}^{ji}\widehat{T}_l^i\left(\widehat{c}_l^i\widehat{\tau}_{nl}^{ji}\right)^{-\theta}\right]^{-\frac{1}{\theta}} \qquad (15)$$

[4] In order to deal with the trearment of inventories in the WIOD table, which causes some negative export volumes in the sample, we follow Antras et al. (2012) to apply a "net inventory" adjustment, which apportions the reported net inventory of each destination-sector across purchasing countries and sectors, according to the corresponding proportions computed using data on intermediate uses.

$$\widehat{P}_n^j = \prod_{i=1}^{J} \left(\widehat{p}_n^{ji}\right)^{\gamma_n^{ji}} \tag{16}$$

The above system, which comprises $2NJ + NJ^2$ equations and the same number of unknowns, forms the inner loop of our general-equilibrium model. Given estimated $\{\widehat{T}_l^i \left(\widehat{c}_l^i\right)^{-\theta}\}$ and $\{\widehat{\tau}_{nl}^{ji}\}$, the values of unknowns can be directly obtained from solving the system iteratively, given some initial guesses of $\{\widehat{w}_l\}$ and $\{\widehat{r}_l\}$ and the following adding-up constraints (our outer loop):

1. The total expenditure on final goods must be equal to payments to factors of production and trade deficit ($D_n'$):
$$E_n' = w_n L_n \widehat{w}_n \widehat{L}_n + r_n K_n \widehat{r}_n \widehat{K}_n + D_n'. \tag{17}$$

2. Output of sector $i$ in country $n$, $\{X_n^{i\prime}\}$, should be equal to the sum across all possible usages across the globe:
$$X_n^{i\prime} = \underbrace{\sum_{k=1}^{J} \sum_{l=1}^{N} \left(1 - \beta_l^k\right) \gamma_l^{ki} \pi_l^{ki\prime} X_l^{k\prime}}_{\text{intermediate input uses}} + \underbrace{\sum_{l=1}^{N} \pi_l^{Fi\prime} \alpha_l^i E_l'}_{\text{final good consumption}} . \tag{18}$$

3. Capital and labor market clearing conditions for each country $n$:
$$\begin{aligned} r_n K_n \widehat{r}_n \widehat{K}_n &= \sum_{i=1}^{J} \beta_n^i \left(1 - \mu_n^i\right) X_n^{i\prime}; \\ w_n L_n \widehat{w}_n \widehat{L}_n &= \sum_{i=1}^{J} \beta_n^i \mu_n^i X_n^{i\prime}. \end{aligned} \tag{19}$$

We use the inner loop (eq. (14) to (15)) and the estimates of the gravity equation (12) to obtain the changes in trade shares at the country-pair sector-pair level $\{\widehat{\pi}_{nl}^{ji}\}$ as

$$\widehat{\pi}_{nl}^{ji} = \widehat{T}_l^i \left(\frac{\widehat{c}_l^i \widehat{\tau}_{nl}^{ji}}{\widehat{p}_n^{ji}}\right)^{-\theta}.$$

This change in the trade shares together with the current period trade shares obtained directly from the WIOT gives us predicted trade shares of the following year as

$$\pi_{nl}^{ji\prime} = \pi_{nl}^{ji} \widehat{\pi}_{nl}^{ji}.$$

For each year, given (i) predicted $\pi_{nl}^{ji\prime}$ $\forall i, k, n, l$, (ii) production and preference parameters ($\alpha_n^i$, $\beta_n^k$, $\gamma_n^{ki}$ $\forall i, k, n$) computed using data from the first sample year of WIOT (i.e., 1995) and (iii) changes in factor endowment $\widehat{K}_n$ and $\widehat{L}_n$, constructed using data from the SEA of WIOD, we solve for country $n$'s production in sector $i$ the following year $\{X_n^{i\prime}\}$ and consumption $\{E_n'\}$ based on constraint (18). The country-sector level gross production, in turn, offers a new set of values of $\{\widehat{w}_l\}$, $\{\widehat{r}_l\}$, according to (19), given exogenous $\{K_n\}$, $\{L_n\}$ and trade deficit $\{D_n'\}$. We then repeat the entire process using the inner loop (eq. (14) to (16)) until the values of $\{\widehat{w}_l\}$ and $\{\widehat{r}_l\}$ converge.

The converged values of $\{\widehat{w}_l\}$ and $\{\widehat{r}_l\}$ allow us to express all general equilibrium solutions to the inner loop. Of note, we can use the solutions of $\{\widehat{c}_l^i\}$ to separate out the estimated $\{\widehat{T}_l^i\}$ from the importer-sector fixed effects $\{\widehat{T}_l^i \left(\widehat{c}_l^i\right)^{-\theta}\}$ that are estimated from the gravity equation (12).

We start the aforementioned calibration using data on trade shares ($\pi_{nl}^{ji}$), production and preference parameters ($\alpha_n^i$, $\beta_n^k$, $\gamma_n^{ki}$ $\forall i, k, n$), factor endowment ($\{K_n\}$, $\{L_n\}$) and trade deficit ($\{D_n\}$) to solve for the general equilibrium of the model for the first sample year (i.e., 1995). We then use the model-predicted trade shares for the following years $\{\pi_{nl}^{ji'}\}$, together with exogenous $\{K_n\}$, $\{L_n\}$, $\{D_n\}$ and some guesses of $\{\widehat{w}_l\}$, $\{\widehat{r}_l\}$, as the initial conditions of the calibration the following year (1996). The process repeats itself every year until we have the full dynamic path of $\{\pi_{nl}^{ji}\}$, $\{w_l\}$, $\{r_l\}$, $\{X_n^i\}$ and prices ($\{P_l^i\}$ and $\{p_l^{ji}\}$) every year between 1995 and 2008. Using these endogenous variables and parameters, we can compute the dynamic path of DVARs at the country-sector level based on equation (7) for each year.

Notice that during the calibration process, all production and preference parameters are kept constant at the 1995 value, while we take values of endowments and trade deficit directly from the data.

For all the decomposition and counterfactual exercises conducted below, we repeat the calibration exercise by starting with the same set of exogenous variables $\{K_n\}$, $\{L_n\}$, $\{D_n'\}$ and production and preference parameters ($\alpha_n^i$, $\beta_n^k$, $\gamma_n^{ki}$), along with exercise-specific values of $\{\widehat{T}_l^i\}$ and $\{\widehat{\tau}_{nl}^{ji}\}$. In each exercise, we start with some initial guesses of $\{\widehat{w}_l\}$ and $\{\widehat{r}_l\}$, solve for $\{\widehat{c}_l^i\}$, $\{\widehat{p}_l^{ji}\}$ and $\{\widehat{P}_l^i\}$, and thus new trade shares $\pi_{nl}^{ji\prime}$. A new general equilibrium is computationally solved until $\{\widehat{w}_l\}$ and $\{\widehat{r}_l\}$ converge.

# 4 Quantitative results

Before reporting the results of our calibration exercises, let us present the trends of countries' DVAR in gross exports computed using data from the WIOT (1995-2008).

## 4.1 DVAR trends

Figure 1 shows the change in the DVAR of exports from developed and developing countries, respectively, based on eq. (7). As the figure shows, the DVAR of exports from both samples of countries have been declining continuously, with the cumulative decline for developing countries equal to 5% (solid line), compared to 4% for developed countries (dash line).

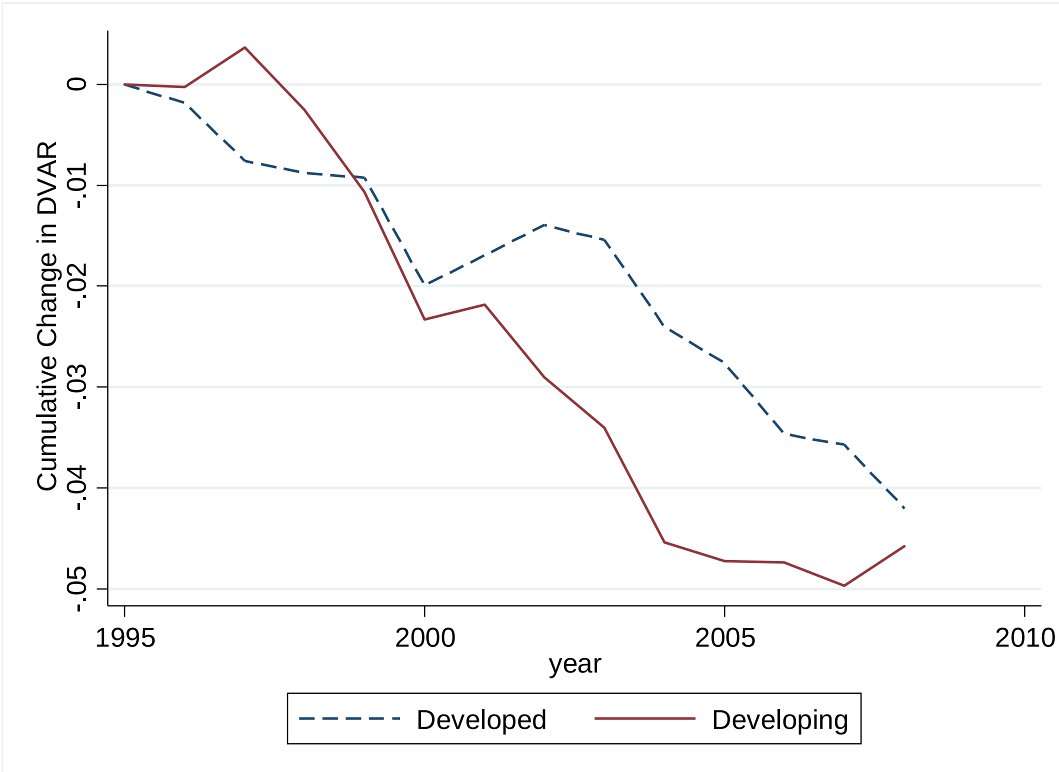


Fig 1: Developed and Developing Countries' DVAR

The following two graphs show the individual countries' DVAR for the fast-growing countries and the developed (OECD) countries respectively. Some countries, like Canada, China, Indonesia, Ireland, Luxembourg, Russia, had their DVAR increasing over time, while the trend is decreasing for all other 34 (out of 40 countries) in the sample.


Fig 2: Fast-Growing Countries' DVAR

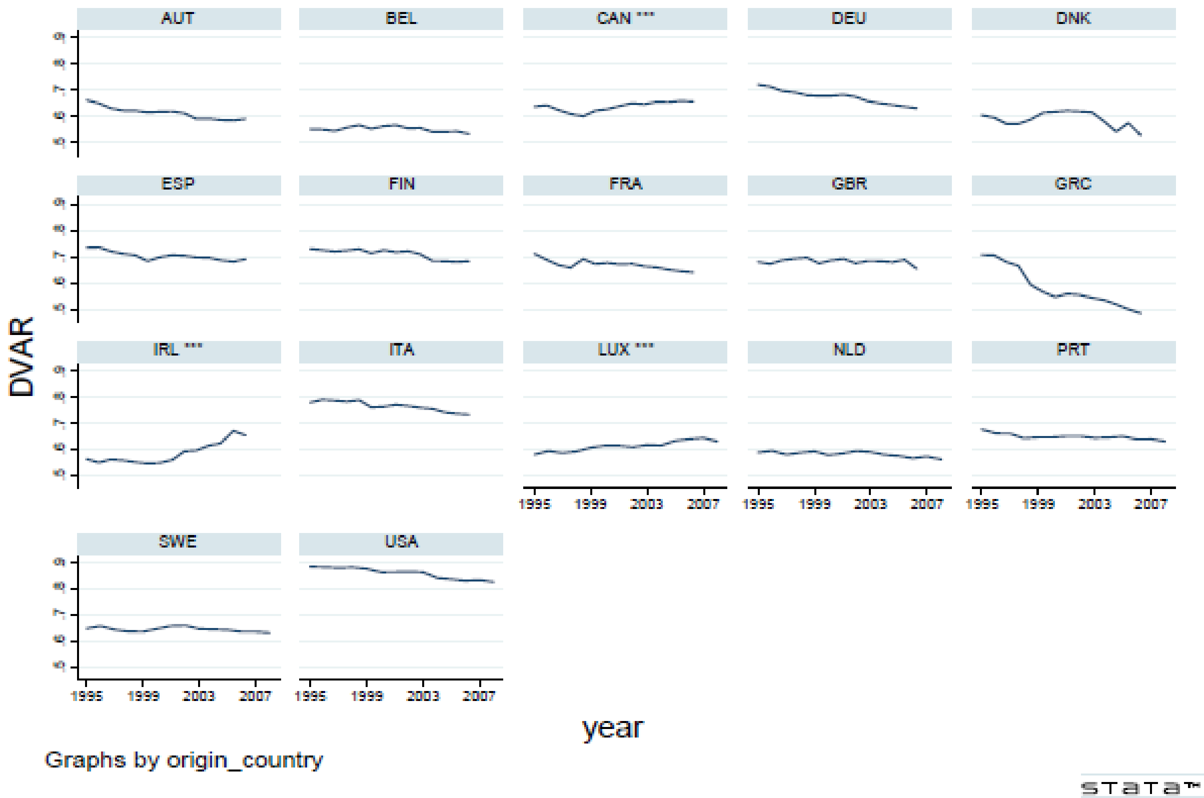


Fig 3: Developed Countries' (OECD) DVAR

### 4.2 Calibration results

Now let us discuss the results of our calibration exercises. First, we examine the fit of our calibration. Figure 4 plots the simulated cumulative change in DVAR against data for each country from 1995 to 2008. As is shown, the simulated cumulative changes in DVAR are very close to the 45-degree line, implying that our calibrated model, which focuses on exogenous technology, trade costs, factor endowments, and trade imbalances performs very well. Notice that even we target each country-pair-sector-pair trade shares in the WIOT, the fit is not perfect as we assume that the production and preference parameters in the Cobb-Douglas functions ($\alpha's$, $\beta's$ and $\gamma's$) are constants (specifically, equal to the 1995 computed values) across years within countries. In the data, however, they are changing, though our model has nothing to say about those changes. Another reason for the imperfect fit is that we replace zero trade with $1 in our sample (see the appendix for details).

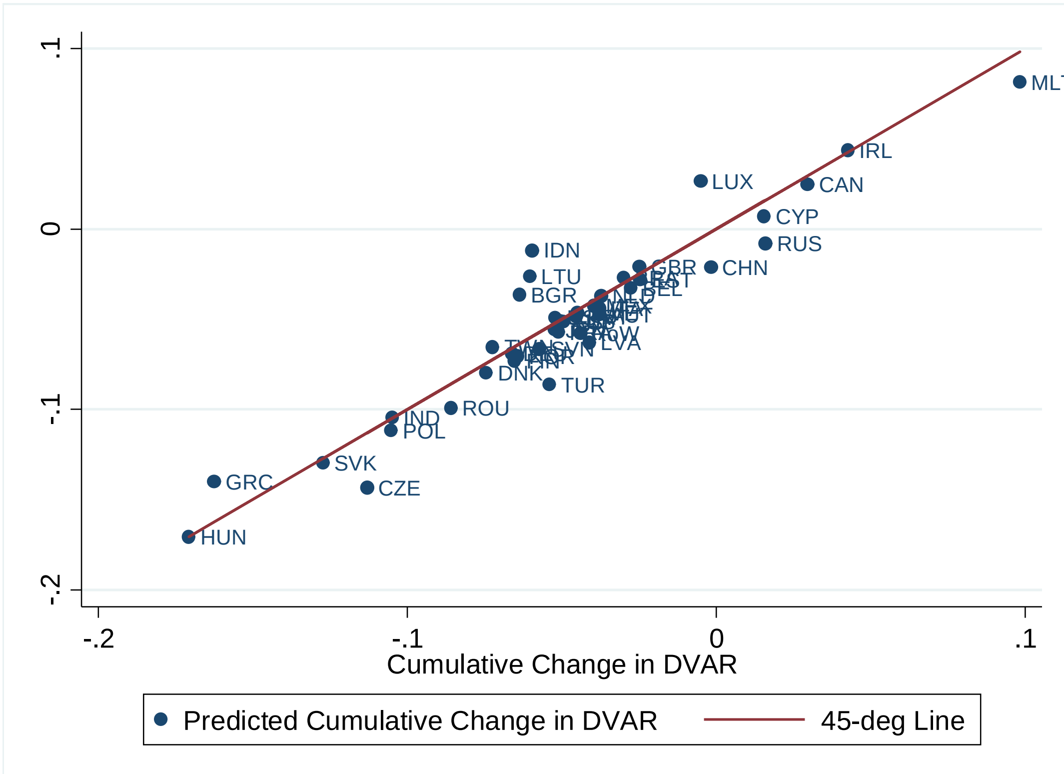


Fig 4: Fit

Figure 5 shows the pure effect of changes in trade costs ($\tau^{ji}_{nlt} = ex^{i}_{lt} + v^{ji}_{nlt}$) on cumulative changes in individual country's DVAR, the focus of the existing literature (e.g., Johnson and Noguera, 2017). The pure effect of trade costs are obtained by shutting down any change in technology ($T$) and other factors (factor endowment and trade imbalances), that is, assuming that the values of all $T$'s and other factors take the same values as those in the first sample year (i.e., 1995). Not surprisingly, changes in trade costs alone cannot explain the data well, and in general predict lower DVAR's for most countries (as revealed by many predicted values scattered below the 45-degree line). Not surprisingly, as our two-country simple model in Section 2.4 illustrates, when the trade cost are declining, the effect on the DVAR of a country's exports is negative under reasonable assumptions. When we shut down changes in technology and other factors in this exercise, the counter-factual world with only trade costs declining over time will likely imply more imports of intermediates in most countries and thus lower DVAR's.

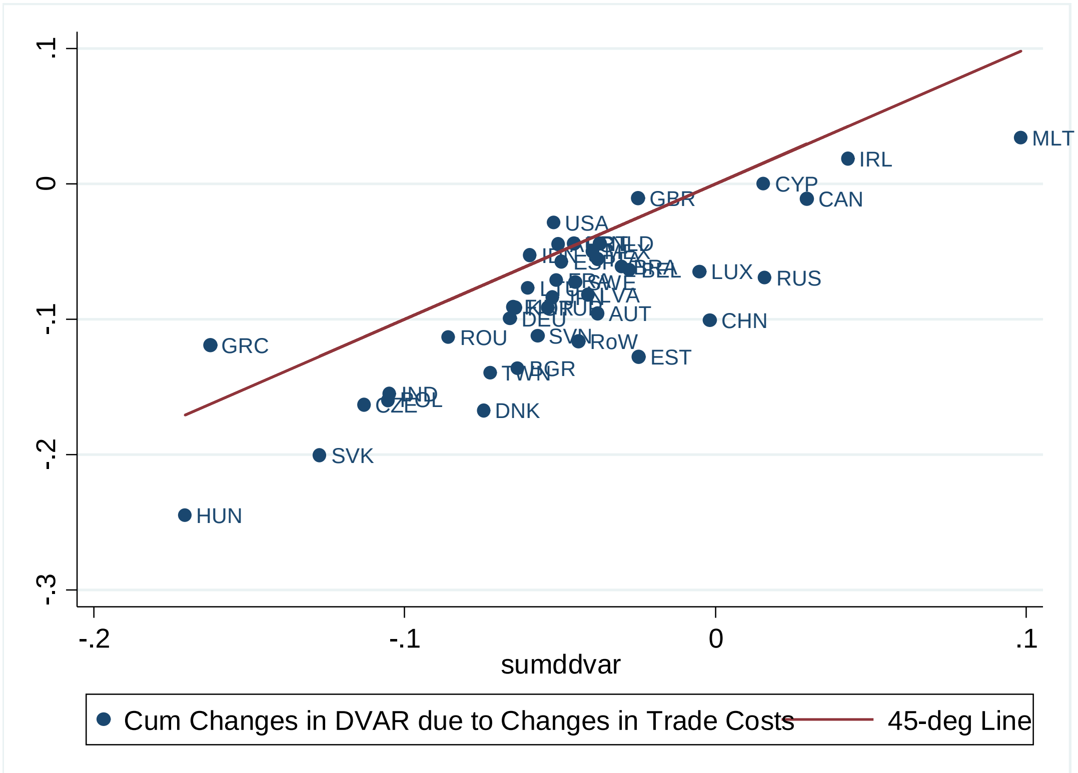


Fig 5: Pure Effects of $\triangle\tau$

Figure 6 shows the pure effect of changes in technology on cumulative changes in individual country's DVAR instead, an aspect that has been downplayed in the existing literature. To gauge the pure effects of technology, we shut down all changes in trade costs and other factors (factor endowment and trade imbalances) in our calibration exercises. Not surprisingly, technology alone cannot explain the data well, and in general over-predicts the level of DVAR for many countries, with some notable exceptions that lie significantly below the 45-degree line, like Bulgaria, Canada, Russia, and Malta. As our simple model predicts, under reasonable assumptions, when a productivity of a country improved on average, the output prices will decline, encouraging more sectors to use domestic intermediates rather than foreign intermediates. The domestic content in the country's exports will increase a result. Of course, the figure considers estimated productivity growth in all countries, thus the opposite can happen if the productivity growth of certain open economies leads to a reduction in the DVAR of other countries' exports. That could be the reason for why with only technological growth allowed in the counterfactual exercises, some countries' DVAR were predicted to be lower than the data

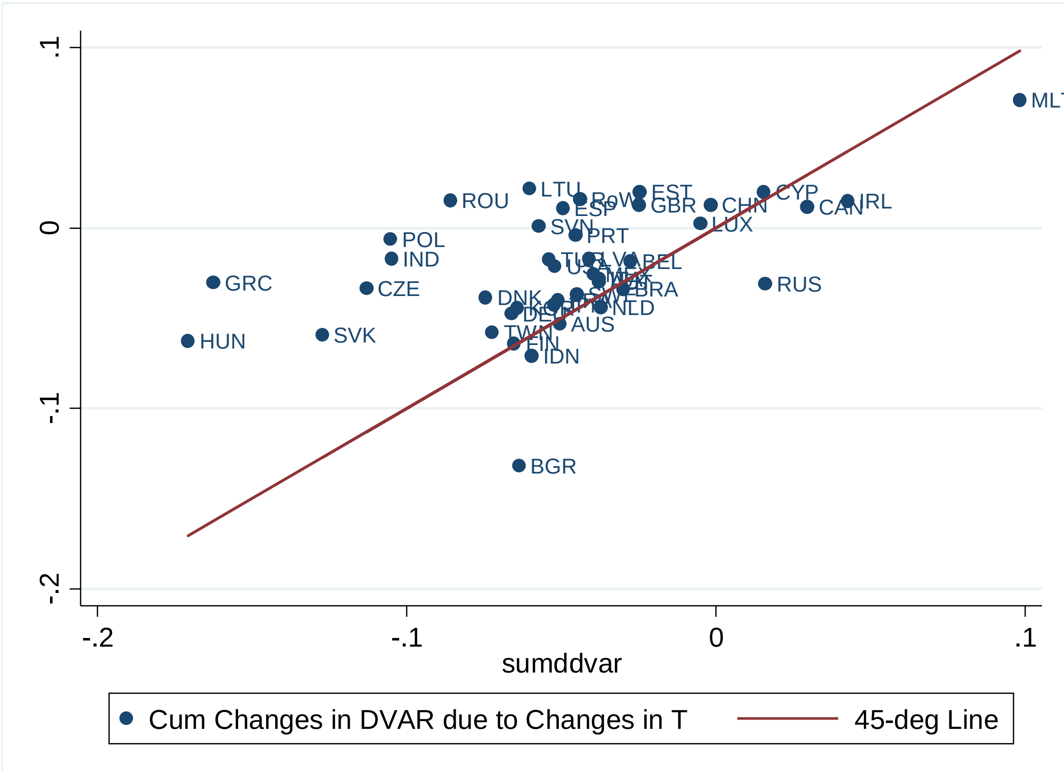


Fig 6: Pure Effect of $\triangle T$

Figure 7 shows the pure effect of changes in other factors (factor endowment and trade imbalances) on cumulative changes in individual country's DVAR, with changes in technology and trade costs assumed to be zero. Not surprisingly, changes in other factors alone cannot explain the data well, and in general over-predicts the level of DVAR for most countries. Moreover, with the exception of a few countries (e.g, Cyprus and Turkey), considering only (working) population growth and capital accumulation implies higher DVAR, with the joint effect of the other two determinants, especially the decline in trade costs, largely pull the predicted DVAR's down, as suggested by Figure 5.

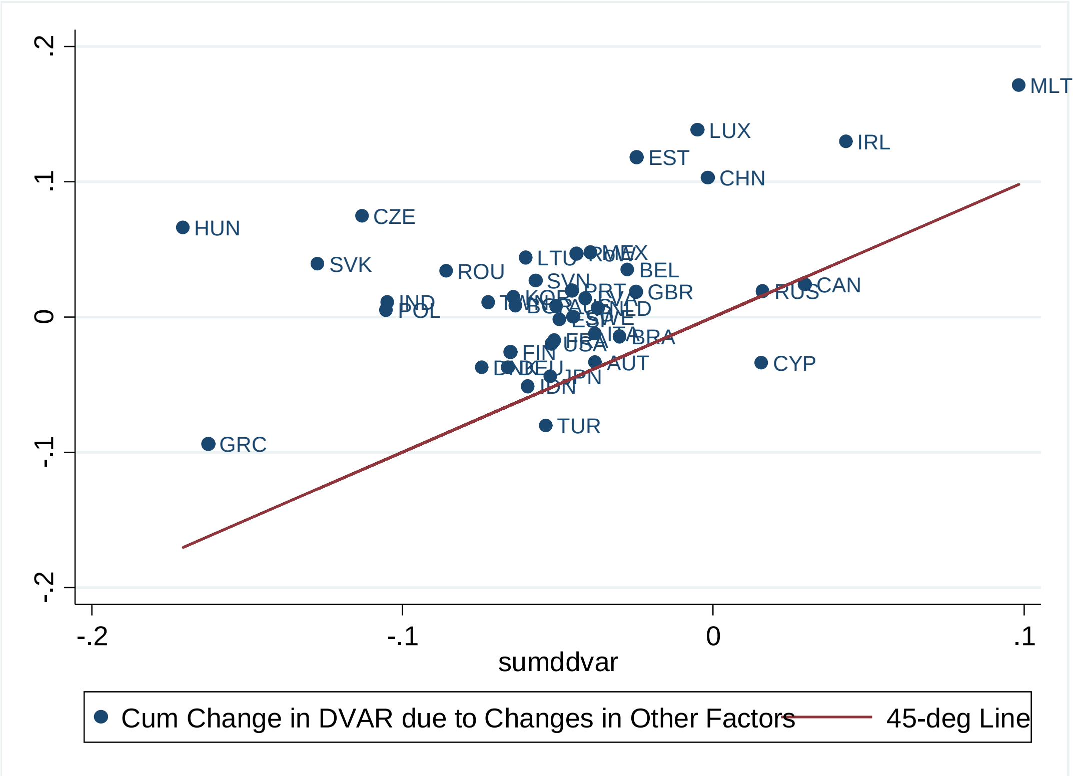


Fig 7: Pure Effects of Changes in Other Factors (i.e. $K$, $L$ and trade balance)

Finally, before moving to the next section about the decomposition of countries' DVAR, we examine the joint effects of the changes in technology and trade costs on the cumulative change in countries' DVAR's. The joint effect includes not only the sum of the pure effects of technology and trade costs, as illustrated Figure 5 and 6, but also the interactive effect between the two that cannot be separated out either theoretically or quantitatively. Figure 8 shows the combined effect of technology and trade costs on cumulative changes in countries' DVAR, with changes in other factors assumed to be zero. The joint effect can explain quite a lot of the cross-country variation observed in the data, with the simulated DVAR being highly correlated with the data. However, the model with only the two determinants considered systematically under-predict countries' DVAR's, as illustrated by the simulated changes in countries DVAR generally scattered below the 45-degree line. It suggests that population growth and capital accumulation on average raise a country's DVAR in exports.

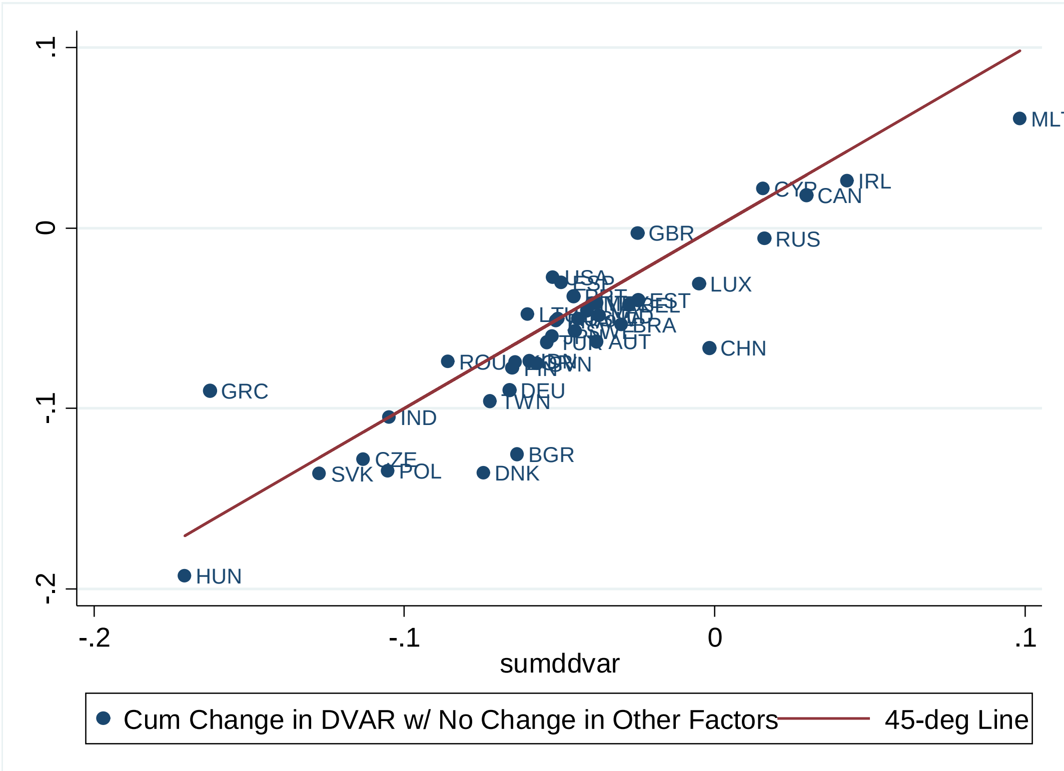


Fig 8: Counterfactuals of No Change in Other Factors

### 4.3 Decomposition exercises

We now discuss how to use our quantitative model as an accounting framework to decompose the changes in countries' DVAR in exports over the sample period due to the various estimated exogenous changes in the data. We first quantitatively assess the change in global DVAR due to changes in only one of the determinants, by shutting down all other determinants in each counterfactual exercise.

Figure 9 shows the pure effect of each determinant. The blue solid line shows the data. The green dash line shows that by shutting down the changes in all determinants but trade costs, there is a significantly larger decline in the predicted DVAR compared to the data. Specifically, the predicted DVAR with only changes in trade costs will be 8% lower than its 1995 level, compared to slightly over 4% decline in the data. Again, this is not surprising, as we have discussed both theoretically and quantitatively the likelihood of a strong negative effect of declining trade costs during the sample period on countries' DVAR.

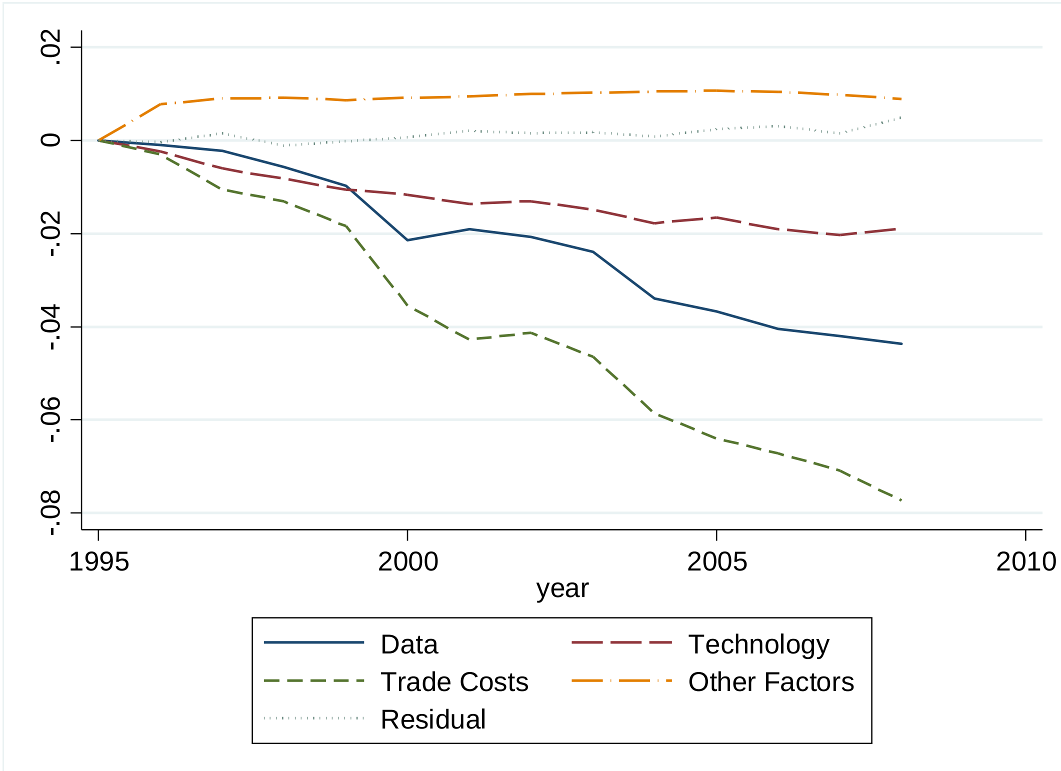


Fig 9: Different Pure Effects on Global DVAR

When we shut down all other changes but keep changes in technology, we find that the pure effect of technology is negative! Specifically, as the red long-dash line shows, the predicted DVAR with only changes in technology will be 2% lower than its 1995 level, compared to over 4% decline in the data. We also examine the effect of shutting down all changes but keep changes in other factors, including factor endowments and trade imbalances. As the orange dot-dash line shows, the pure effect of other factors is positive, implying a 1% increase in the DVAR of world exports, compared to the value in 1995. The remaining plot is for the residuals, which are insignificant. This is not surprising as we allow the trade friction to vary at the country-pair sector-pair level, the most granular level in our data.

Next we examine the interaction effect. To gauge each interactive effect, we need to conduct three counterfactuals. For instance, to quantitatively assess the interactive effect from changes in technology and trade costs, we first conduct the counterfactual calibration with only changes in technology, and obtain predicted DVAR's (call them $DVAR^T$). We then conduct another counterfactual calibration due to changes in trade costs, and obtain another set of DVARs (call them $DVAR^\tau$). Finally, we conduct the counterfactual calibration with changes in both technology and trade costs, from which we obtain predicted DVAR's that we refer to as $DVAR^{T\tau}$. The interactive effect of $T$ and $\tau$ is obtained by computing $DVAR^{T\tau} - DVAR^{T} - DVAR^{\tau}$. We repeat the same exercises to gauge the interactive effects for other combinations of the changes in other exogenous determinants, namely, technology and other factors, trade costs and other factors, as well as the interactive effects across all three determinants.

As Figure 10 shows, the interactive effect from changes in technology and trade costs, as represented by the red dash line, is significantly positive. The impact on the cumulative change up to 2008 is about 4.2%. On the other hand, the interactive effect from the changes in technology and other factors, as represented by the green dotted line, is negative but tiny (less than 1%). The interactive effect from the changes in trade costs and other factors, is also negative and tiny (also less than 1%). Finally, the triple interaction effect is positive and remained constant at around positive 1%. In sum, among the interactive effects, the most important one is the one that involves changes in technology and trade costs.

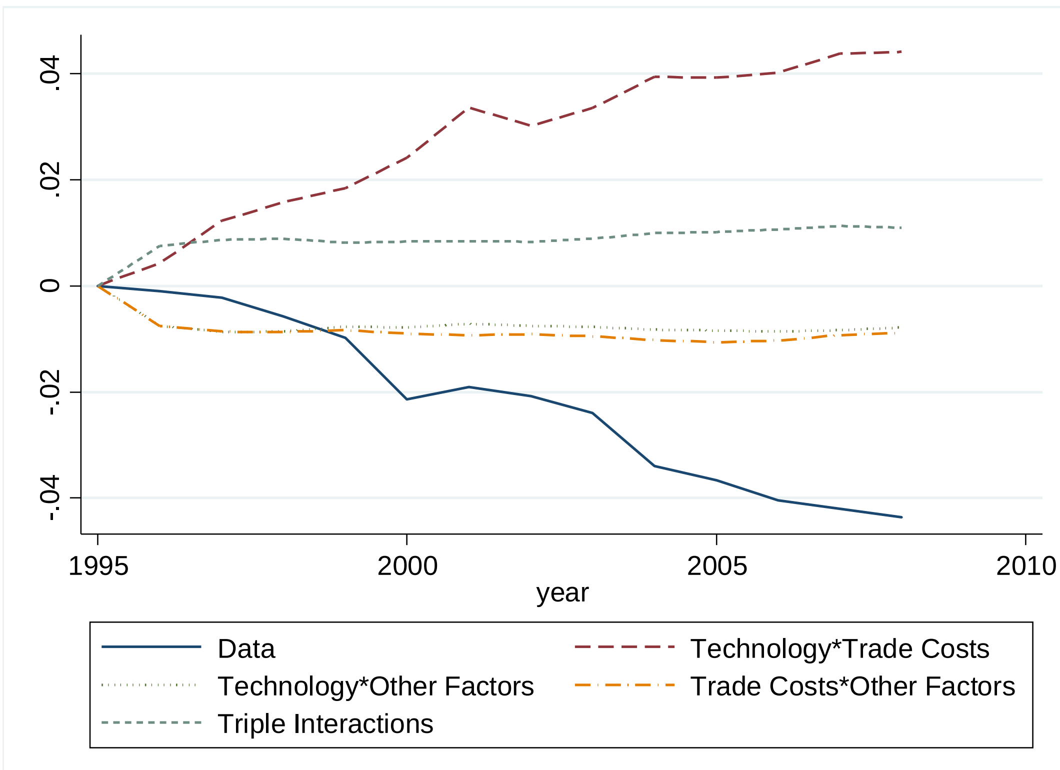


Fig 10: Effects of Interaction Terms on Global DVAR

The findings of the interactive effects encourage us to examine the total effects, which can be obtained by studying the difference between the calibration with all changes allowed and the counterfactual with one of the forces shut down. For instance, to examine the total effect of technology (T), we subtract the predicted DVAR with all changes but T shut down (call it $DVAR^{-T}$) from the predicted DVAR with all changes calibrated.

Figure 11 shows the results. The total effect of technology, as illustrated by the blue solid line, is significantly positive, accounting for a close to 3% increase in the world's DVAR. The total effect of trade costs is significantly negative (around -3%), consistent with previous findings (e.g., Johnson and Noguera, 2017). The total effect is about negative 3%. The total effect of other factors is marginally positive. Notice that the sum of the three total effects is not supposed to be equal to the data, as there will be double accounting exactly due to the interactive effects we highlighted in Figure 10.

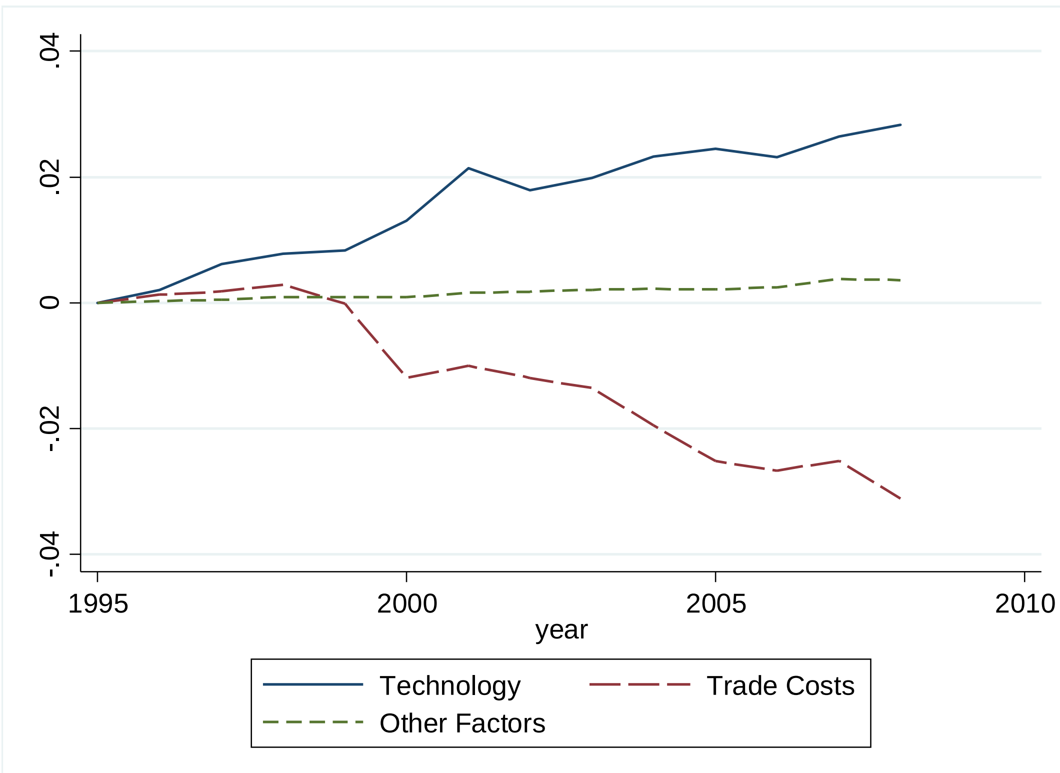


Fig 11: Total Effects of $T$, $\tau$, and Other Factors on Global DVAR

In Figures 12 and 13, we repeat the same exercises to gauge the total effects of the three exogenous determinants for the developed and developing country samples, respectively. The results look quite similar to the one we showed for the whole global sample in Figure 11. If anything, the size of the total technology and trade cost effects are larger for the developing countries as a whole.

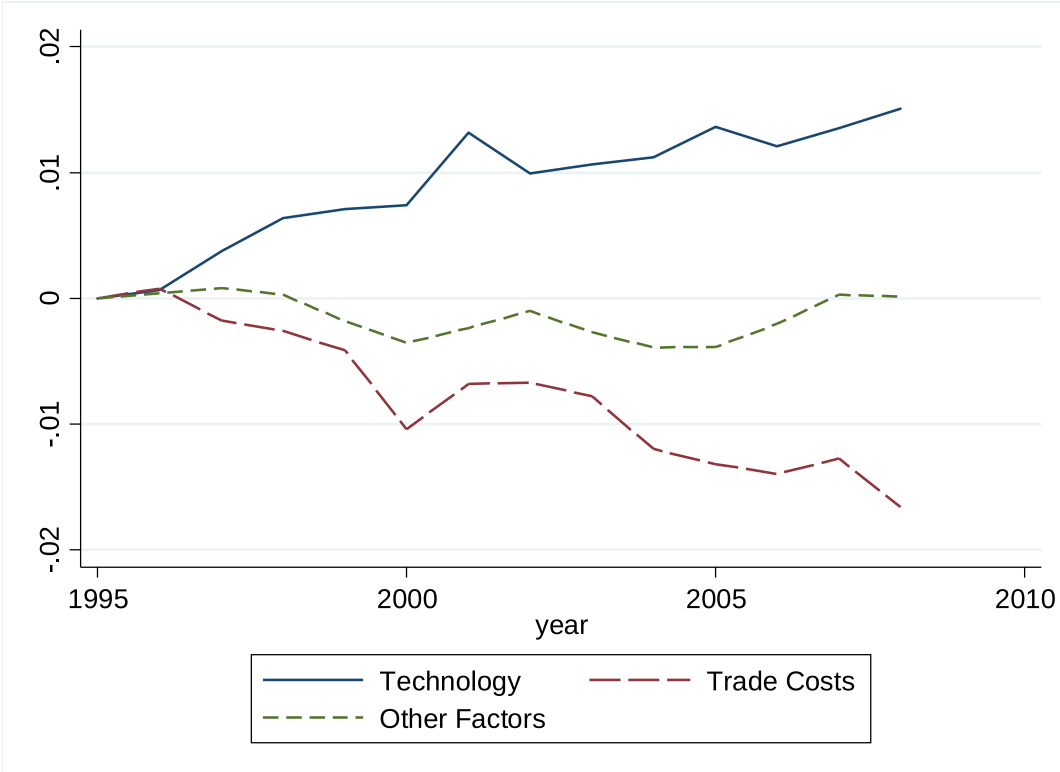


Fig 13: Developed Countries

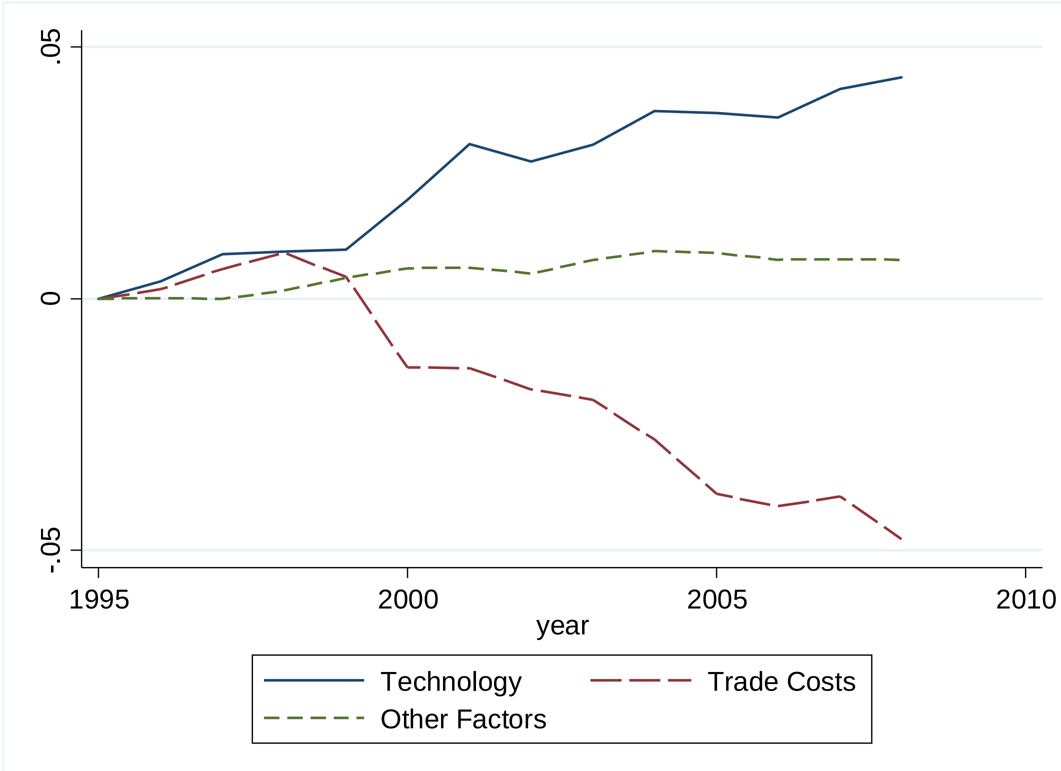


Fig 14: Developing Countries

Table 2 summarizes the pure (i.e. stand-alone) effects of each set of factor(s), as well as their interactive effects. The stand-alone effects of technology and trade costs are -1.9 and -7.7, respectively, while the stand-alone effect of other factors (i.e., changes in trade balances and factor endowments) is only 0.9. The three stand-alone effects add up to a number that is much more negative than the data, because there are positive interactive effects, in particular, the one that involves the interaction between changes in technology and change in trade costs. It contributes about 4.4 percentage-point *increase* in the global DVAR. The large positive interactive term is an outcome of the non-linearity in the structural gravity equations derived from a large class of quantitative trade models, to which Eaton and Kortum (2002) belongs, together with a negative correlation between changes in technology and changes in import barriers across country-sector pairs (e.g., China's estimated sectoral TFP was rising faster in sectors that experienced larger declines in import barriers).

| Table 2: Percentage-point Changes in DVAR (1995-2008) | | | |
|---|---|---|---|
| | Global | Developed | Developing |
| Total | -4.36 | -4.20 | -4.58 |
| due to changes in | | | |
| Technology | -1.89 | -2.40 | -1.35 |
| Trade Costs | -7.74 | -5.55 | -10.32 |
| Other Factors | 0.90 | -0.70 | 2.68 |
| Tech * Trade Costs | 4.41 | 3.60 | 5.42 |
| Tech * Other Factors | -0.75 | 0.43 | -2.03 |
| Trade Costs * Other Factors | -0.85 | 0.40 | -2.24 |
| All Three Forces | 1.06 | -0.12 | 2.36 |
| Residual | 0.49 | 0.13 | 0.90 |

Before discussing further counterfactual exercises, let us show the results of our calibration to assess the total effects of the three determinants for China's exports. As Figure 15 shows, the total effect of technology, as represented by the blue solid line, is significantly positive, reaching over 5% by 2008. The total effect of trade costs is first positive and reaches the peak of over 7% in 1998, before declining continuously to a -5% by 2008. China's accession to the WTO is obviously an important reason behind this trend. To understand the overall trend of China's DVAR, one would need to consider both the total effects and the interactive effects discussed, which will be our first counterfactual exercise in the next section.

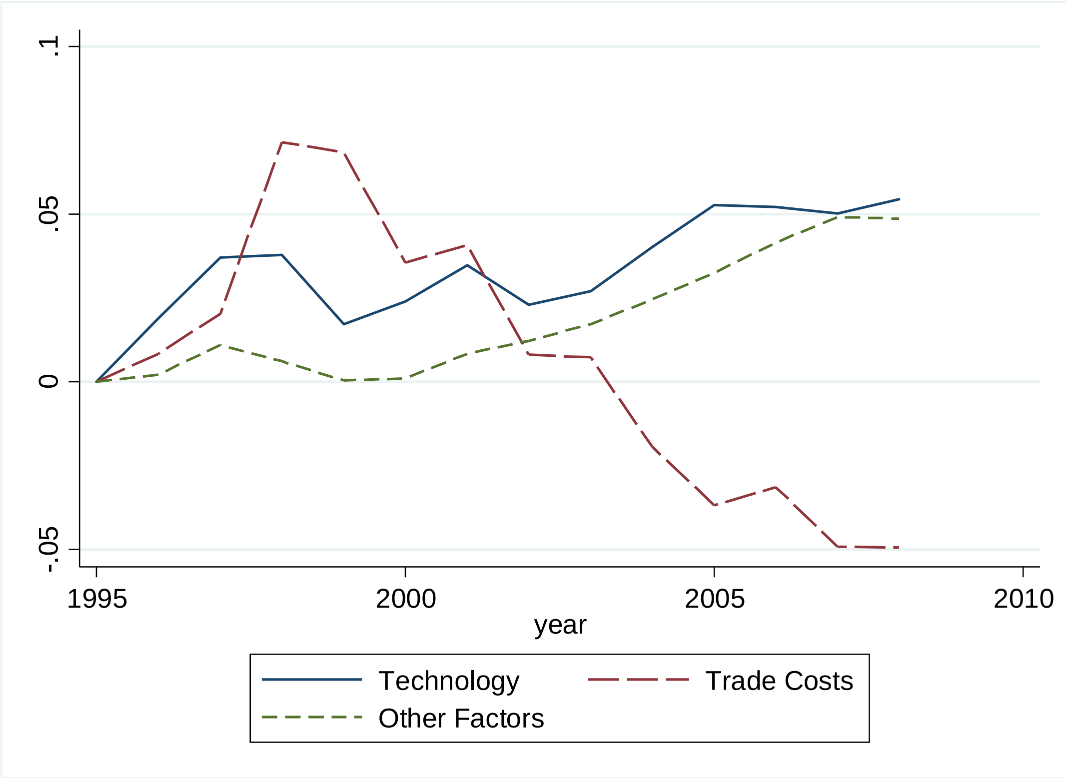


Fig 15: China

### 4.4 Counterfactuals

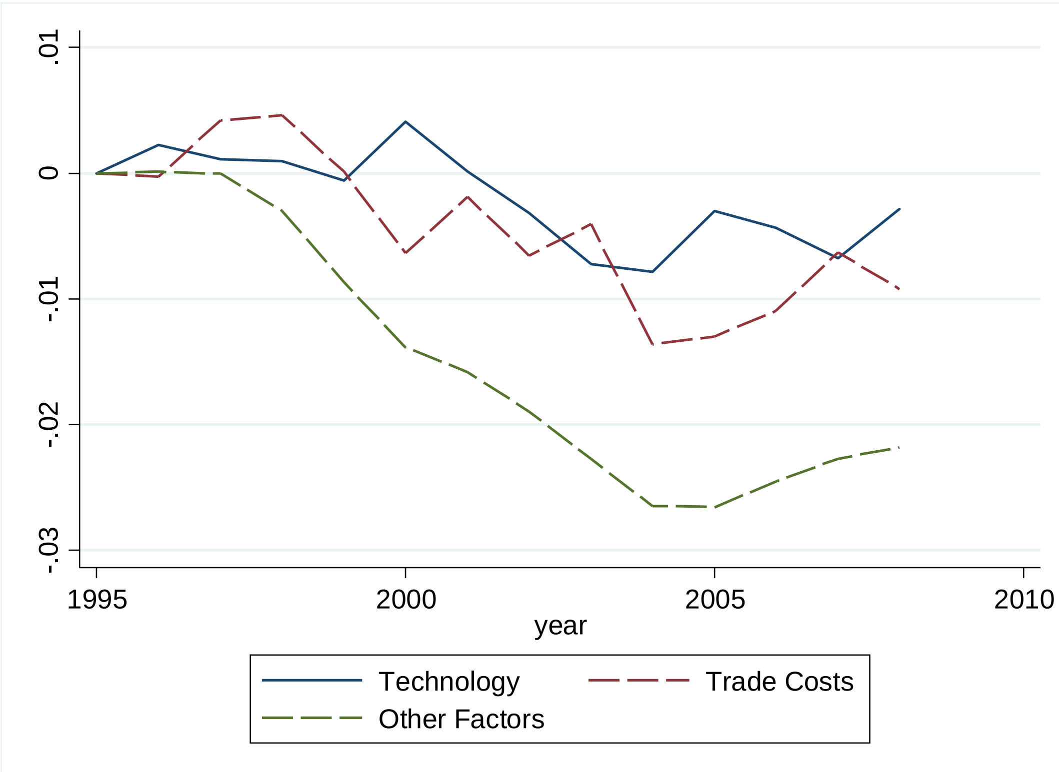


Fig 16: US

We first conduct the counterfactual exercise of shutting down China's technological growth (i.e., assuming that all China's T of each sector across year is equal to its initial level in 1995). As Figure 17 shows, the predicted DVAR of China's exports in the absence of technological growth will be significantly lower than the data (dash line).

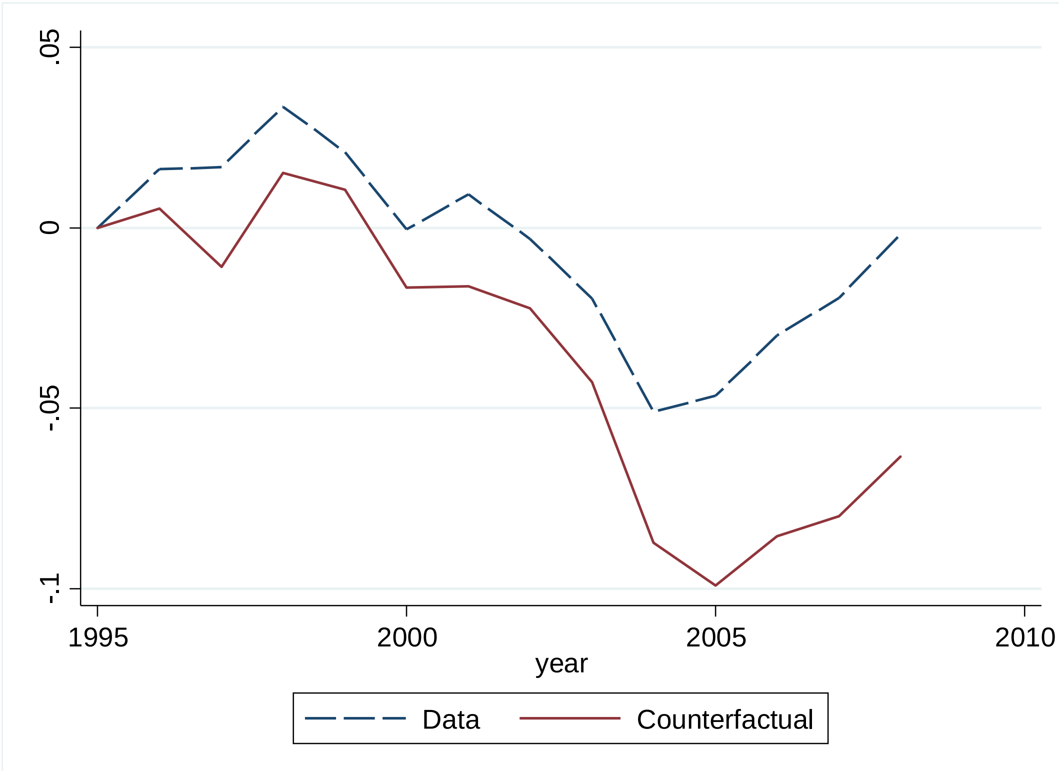


Fig 17: Effects of Shutting Down Changes in China's $T$ on China's DVAR

We then conduct the counterfactual exercise of shutting down China's trade liberalization (i.e., assuming that all China's estimated $\tau$'s at each country-sector -pair level across year is equal to its initial level in 1995). As Figure 18 shows, the predicted DVAR of China's exports in the absence of trade liberalization will be significantly higher than the data (dash line), as expected.

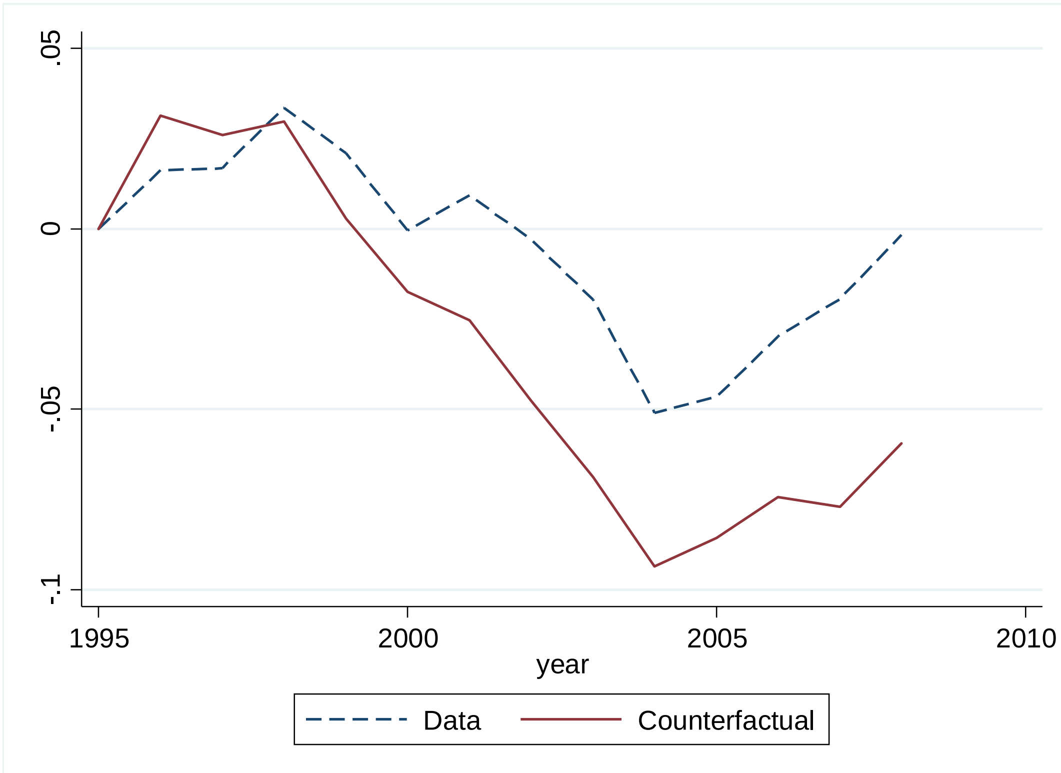


Fig 18: Effects of Shutting Down Changes in China's $\tau$ on China's DVAR

Next, we conduct the counterfactual exercise of shutting down any change in China's factor endowment and trade imbalances (i.e., assuming that all China's estimated $K$, $L$, and $D$ in all year take their initial value in 1995). As Figure 19 shows, the predicted DVAR of China's exports in the absence of changes in other factors will be lower than the data (dash line), as expected.

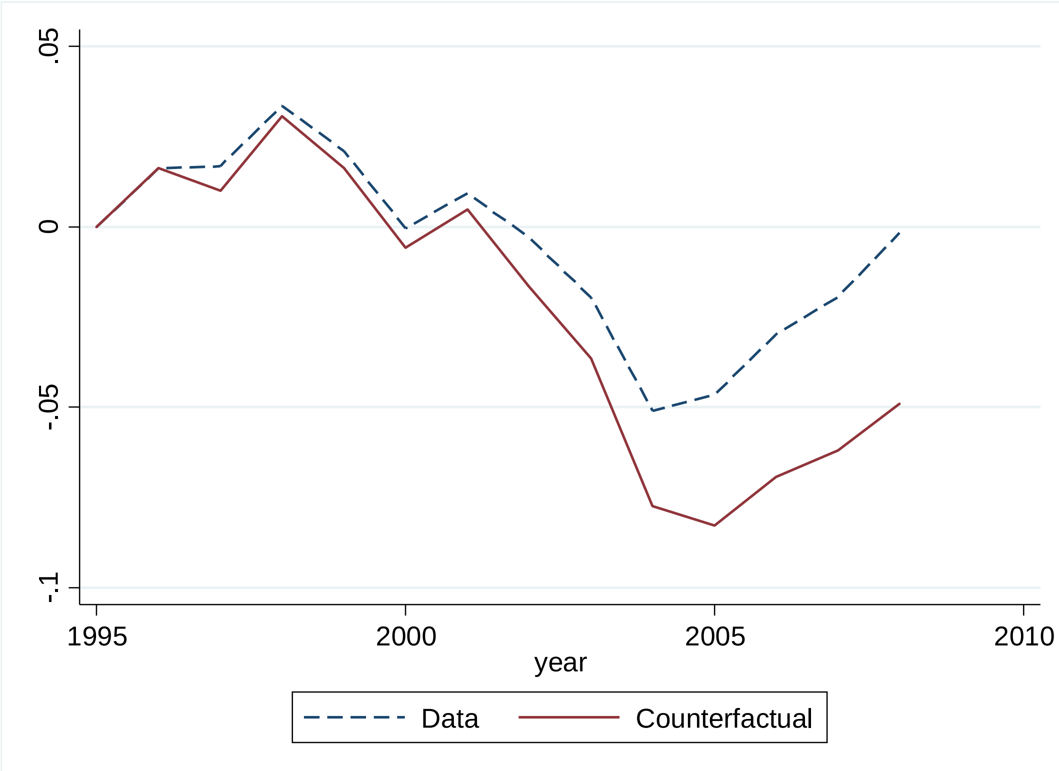


Fig 19: Effects of Shutting Down Changes in China's Capital on China's DVAR

The next set of counterfactual exercises is to examine how shutting down changes in the three exogenous factors in China will affect the DVAR of global exports and the US exports, respectively. Figures 20-21 shows that shutting down changes in China's technology or trade costs have minimal effects on the DVAR of global exports.

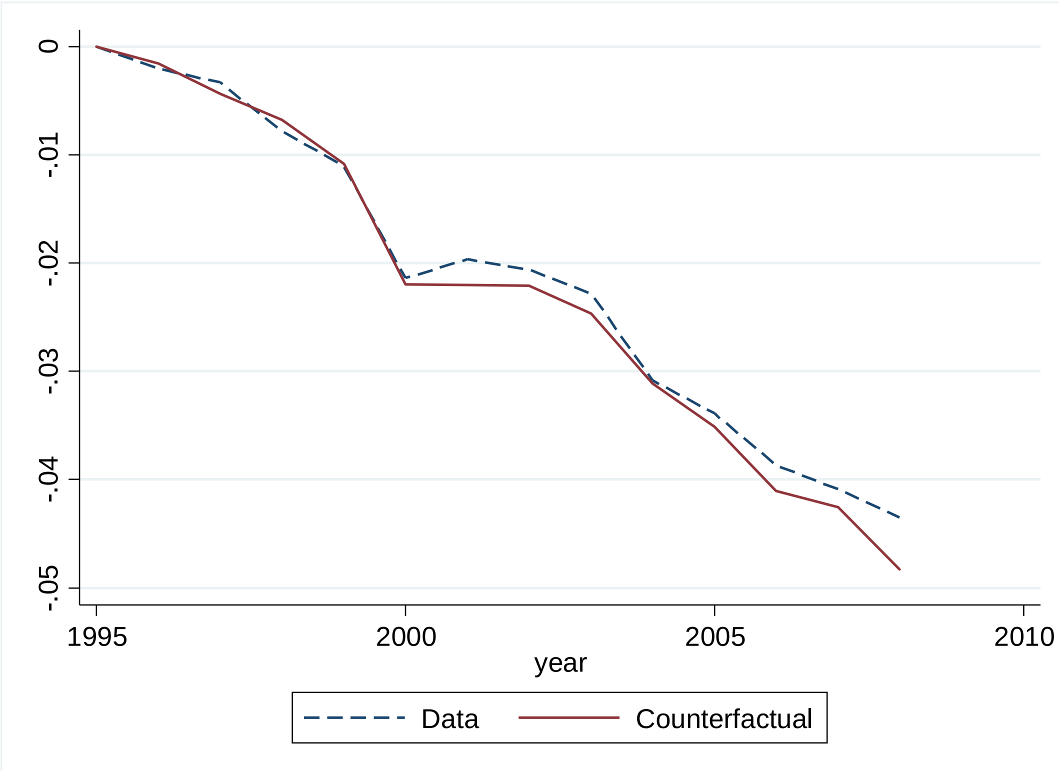


Fig 20: Effects of Shutting Down Changes in China's $T$ on ROW's DVAR

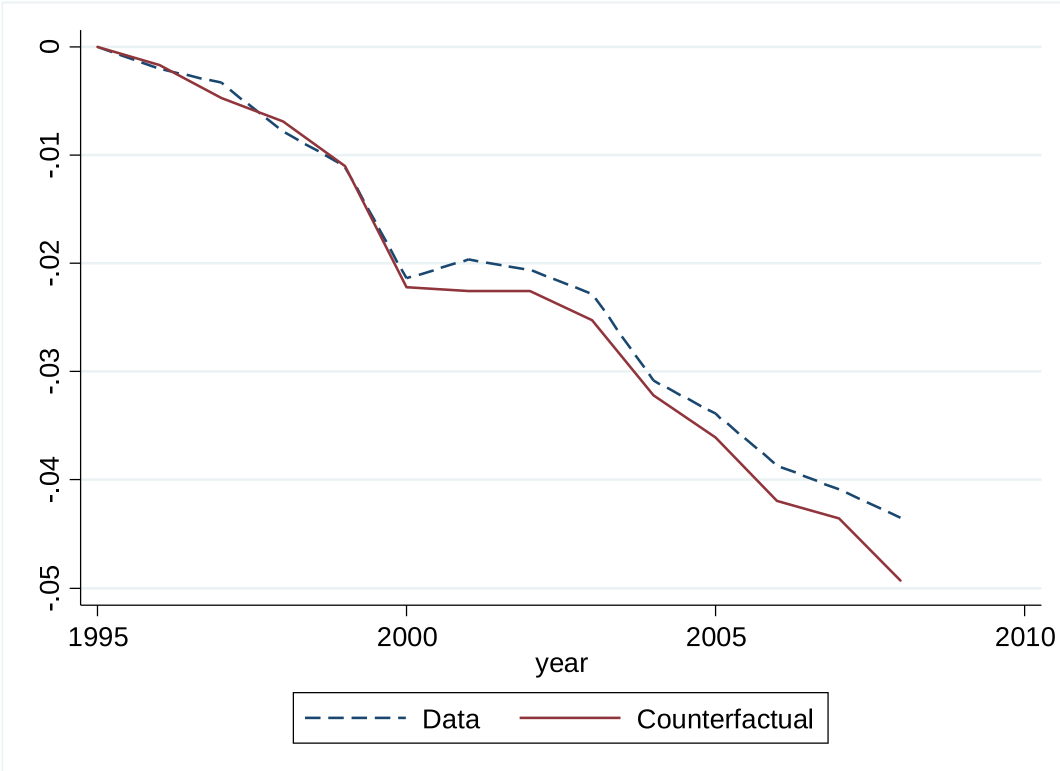


Fig 21: Effects of Shutting Down Changes in China's $\tau$ on ROW's DVAR

That said, the effects of shutting down China's technological growth or trade liberalization on the DVAR of US exports are more noticeable. Specifically, Figure 22 shows that the DVAR of US exports will be higher than the data (dash line), in the absence of China's productivity growth during the sample period; while Figure 23 reveals that the DVAR of US exports will also be higher than the data (dash line) in the counterfactual world when China's trade costs were at their high levels in 1995.

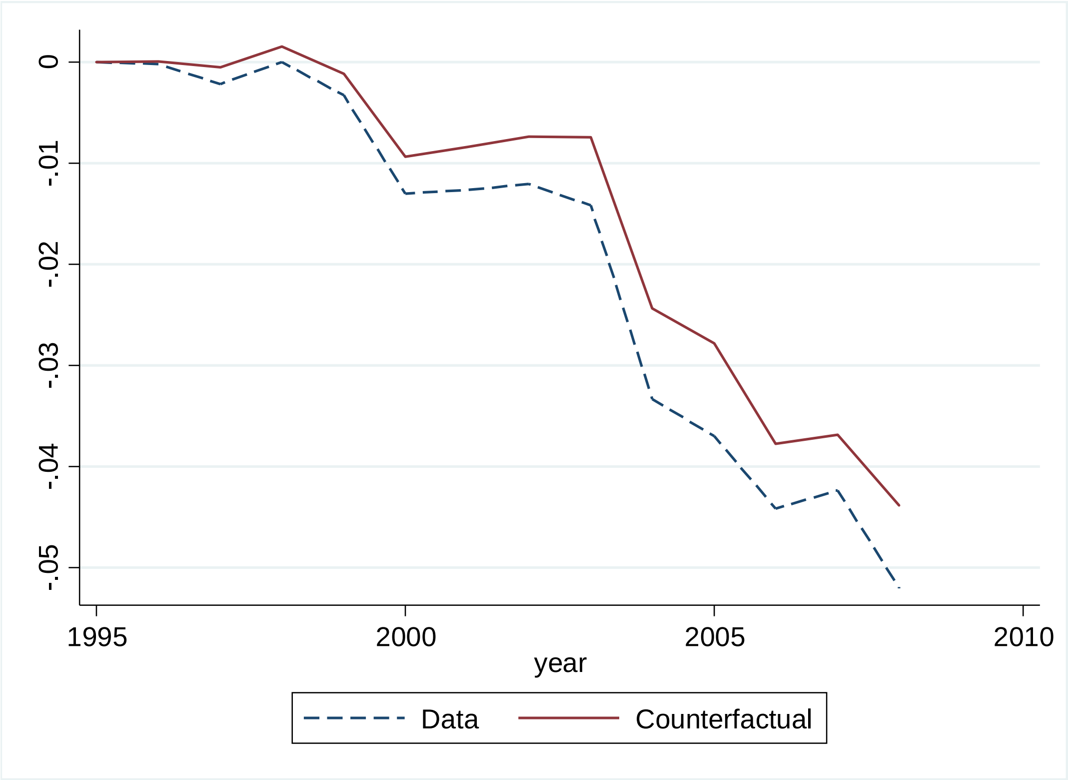


Fig 22: Effects of Shutting Down Changes in China's $T$ on US's DVAR

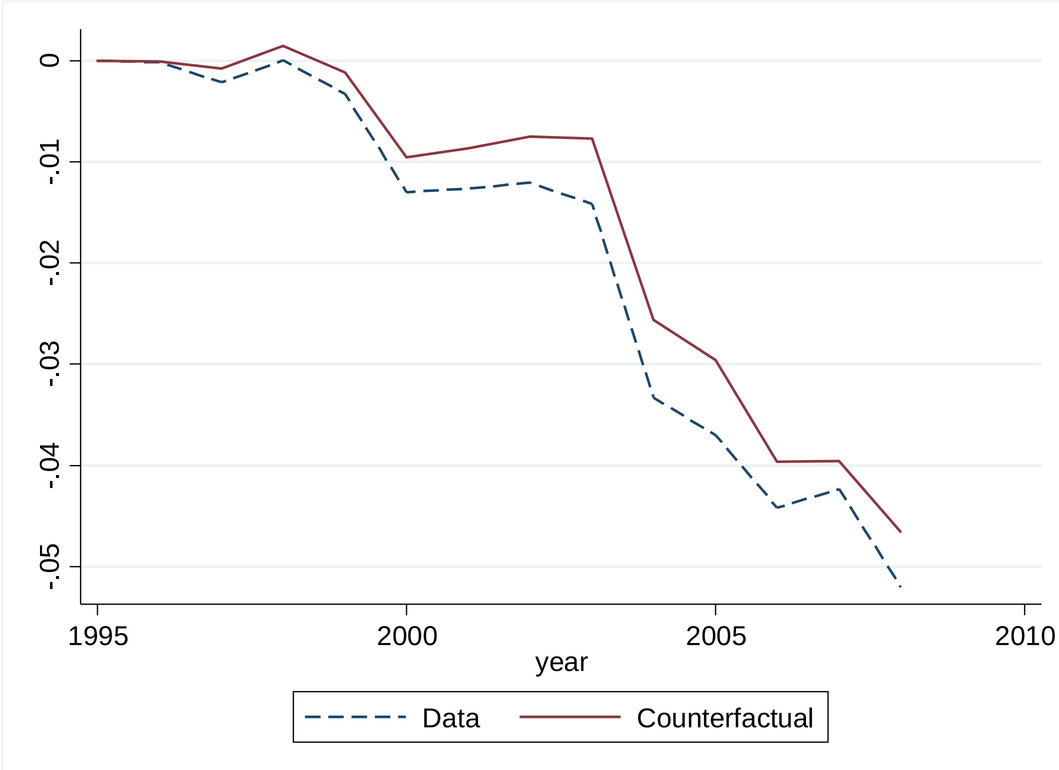


Fig 23: Effects of Shutting Down Changes in China's $\tau$ on US's DVAR

## 5 Is a country' DVAR related to the gains from trade? (PRELIMINARY)

A question one may ask is whether a country' DVAR related to the welfare gains from trade; and if so, whether the determinants of the changes in a country's DVAR matter for such a welfare calculation.

Let us define the consumer welfare of country $n$, $W_n$, as its real income:

$$W_n = \frac{w_n}{P_n^F}$$

where $P_n^F$ is the price index of the final consumption. Denote the price index of sector $j$ final goods in country $n$ to be $P_n^{jF} = \left(\Phi_n^{jF}\right)^{-\frac{1}{\theta}}$, where

$$\Phi_n^{jF} = \sum_{l=1}^{N} T_l^j (c_l^j \tau_{nl}^{jF})^{-\theta}$$

Thus the percentage change in real income in country $n$ is given by (see the appendix for details):

$$d\ln W_n = \sum_{j=1}^{J} \alpha_n^j \left( \frac{d\ln T_n^j}{\theta} - \frac{d\ln s_{nn}^j}{\theta} + d\ln w_n - d\ln c_n^j \right)$$

where the last equality follows from $d\ln s_{nn}^j = d\ln T_n^j - \theta d\ln c_n^j - d\ln \Phi_n^{jF}$.

Detailed derivation as shown in the appendix yields the following welfare change equation

$$d\ln W_n = \sum_{j=1}^{J} \alpha_n^j \left[ \frac{1}{\theta} \sum_{i=1}^{J} \delta_n^{ji} d\ln T_n^i - \frac{1}{\theta} \left( d\ln s_{nn}^j + \sum_{i=1}^{J} \delta_n^{ji} \left(1-\beta_n^i\right) \sum_{k=1}^{J} \gamma_n^{ik} d\ln \pi_{nn}^{ik} \right) \right] \tag{20}$$

Notice that the welfare gains can come from direct imports of final goods, and imports of intermediate goods for the final goods production through the IO linkage, as well as technological progress in either final goods or intermediate goods. In the tradeable intermediate goods setting of Eaton and Kortum (2002), $s_{nn}^i = \pi_{nn}^{ji} \equiv \pi_{nn}^i$, the term $d\ln s_{nn}^j + \sum_{i=1}^{J} \delta_n^{ji} \left(1-\beta_n^i\right) \sum_{k=1}^{J} \gamma_n^{ik} d\ln \pi_{nn}^{ik}$ will degenerate to a simple term $\sum_{i=1}^{J} \delta_n^{ji} d\ln \pi_{nn}^i$, similar to the effect of changes in technology. When there is no roundabout production and IO linkage, $\mathbf{B}_n = \mathbf{\Gamma}_n = \mathbf{I}$, $\left[\mathbf{I} - \left(\mathbf{I} - \mathbf{B}_n\right) \mathbf{\Gamma}_n\right]^{-1} = \mathbf{I}$, equation (20) will degenerate to equation (11) in Donaldson (2018).

Equation (20) implies that

$$\widehat{W}_n = \prod_{j=1}^{J} \prod_{i=1}^{J} \left(\widehat{T}_n^i\right)^{\frac{\alpha_n^j \delta_n^{ji}}{\theta}} \cdot \prod_{j=1}^{J} \left(\widehat{s}_{nn}^j\right)^{-\frac{\alpha_n^j}{\theta}} \cdot \prod_{j=1}^{J} \prod_{i=1}^{J} \prod_{k=1}^{J} \left(\widehat{\pi}_{nn}^{ik}\right)^{-\frac{\alpha_n^j \delta_n^{ji} \left(1-\beta_n^i\right) \gamma_n^{ik}}{\theta}} \tag{21}$$

For the yearly change in welfare, $\widehat{s}_{nn}^j$ and $\widehat{\pi}_{nn}^{ji}$ can be obtained from data, $\widehat{T}_n^j$ are solved from the general equilibrium of the model, and $\theta$, $\alpha_n^j$, $\beta_n^j$, $\gamma_n^{ji}$, $\mu_n^j$ as well as $\delta_n^{ji}$ are exogenous parameters. Thus the value $\widehat{W}_n$ of each year can be directly calculated.

We can also compute the compute the **gains from trade** compared to autarky. When country $n$ moves from autarky, where $s_{nn}^j = \pi_{nn}^{ji} = 1$, to the observed equilibrium, the gains from trade is

$$\widehat{W}_{n,A} - 1 = \prod_{j=1}^{J} \left(s_{nn}^j\right)^{-\frac{\alpha_n^j}{\theta}} \cdot \prod_{j=1}^{J} \prod_{i=1}^{J} \prod_{k=1}^{J} \left(\pi_{nn}^{ik}\right)^{-\frac{\alpha_n^j \delta_n^{ji} \left(1-\beta_n^i\right) \gamma_n^{ik}}{\theta}} - 1 \tag{22}$$

In the one-sector setting, $\mathbf{B}_n = \beta_n$ and $\mathbf{\Gamma}_n = 1$, which leads to $\left[\mathbf{I} - \left(\mathbf{I} - \mathbf{B}_n\right) \mathbf{\Gamma}_n\right]^{-1} = 1/\beta_n$, the RHS of equation (22) will degenerate to

$$\left(s_{nn}\right)^{-\frac{1}{\theta}} \left(\pi_{nn}\right)^{-\frac{1-\beta_n}{\theta\beta_n}} - 1$$

which is the gains from trade expression in Section 7 of Antras and de Gortari (2017) when $N = 1$.

[to be completed]

## 6 Conclusion

Based on a multi-sector Eaton-Kortum (2002) model with domestic and global input-output linkages, we quantify the contributions of different sets of factors to the changes in individual countries' DVARs and global DVAR during 1995-2008. In addition to trade frictions, we emphasize the importance of the positive effect of technology on individual countries' DVARs and global DVAR. The contribution of other exogenous factors (factor endowment, trade imbalance) are small. Last but not least, fast-growing countries, like China, which experienced a substantial improvement in technology, despite falling trade frictions, could have DVAR increasing over time.

Counterfactual exercises show that the effect of China's technological growth on its own exports' DVAR is significantly positive, while the effect of its trade liberalization on its DVAR is significantly negative. While shutting down China's technological growth or trade liberalization have little impact on the DVAR of global exports, both have non-negligible positive impact on the US's DVAR.

In research in progress, we will relate the DVAR of a country's exports with its welfare gains from trade. Theoretical results show that depending on the ultimate drivers of a country's DVAR, the relationship between a country's DVAR and its welfare gains from trade can be positive or negative.

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## 6.2 Appendix

### 6.2.1 Proofs of the two-country one-sector simple model

The DVAR of country 1 $r_{11}$ is a function of $\pi_{11}$ and $\pi_{22}$, thus a function of $t$, $\tau_1$ and $\tau_2$. The second-order Taylor expansion gives

$$\begin{aligned}
dr_{11} &= A d\pi_{11} - B d\pi_{22} \\
&= \left(A\frac{\partial \pi_{11}}{\partial t} - B\frac{\partial \pi_{22}}{\partial t}\right) dt + \frac{1}{2}\left(A\frac{\partial^2 \pi_{11}}{\partial t^2} - B\frac{\partial^2 \pi_{22}}{\partial t^2}\right)(dt)^2 \\
&\quad + \left(A\frac{\partial \pi_{11}}{\partial \tau_1^{-\theta}} - B\frac{\partial \pi_{22}}{\partial \tau_1^{-\theta}}\right) d\tau_1^{-\theta} + \frac{1}{2}\left(A\frac{\partial^2 \pi_{11}}{\left(\partial \tau_1^{-\theta}\right)^2} - B\frac{\partial^2 \pi_{22}}{\left(\partial \tau_1^{-\theta}\right)^2}\right)\left(d\tau_1^{-\theta}\right)^2 \\
&\quad + \left(A\frac{\partial \pi_{11}}{\partial \tau_2^{-\theta}} - B\frac{\partial \pi_{22}}{\partial \tau_2^{-\theta}}\right) d\tau_2^{-\theta} + \frac{1}{2}\left(A\frac{\partial^2 \pi_{11}}{\left(\partial \tau_2^{-\theta}\right)^2} - B\frac{\partial^2 \pi_{22}}{\left(\partial \tau_2^{-\theta}\right)^2}\right)\left(d\tau_2^{-\theta}\right)^2 \\
&\quad + \left(A\frac{\partial^2 \pi_{11}}{\partial t \partial \tau_1^{-\theta}} - B\frac{\partial^2 \pi_{22}}{\partial t \partial \tau_1^{-\theta}}\right) dt d\tau_1^{-\theta} + \left(A\frac{\partial^2 \pi_{11}}{\partial t \partial \tau_2^{-\theta}} - B\frac{\partial^2 \pi_{22}}{\partial t \partial \tau_2^{-\theta}}\right) dt d\tau_2^{-\theta} \\
&\quad + \left(A\frac{\partial^2 \pi_{11}}{\partial \tau_1^{-\theta} \partial \tau_2^{-\theta}} - B\frac{\partial^2 \pi_{22}}{\partial \tau_1^{-\theta} \partial \tau_2^{-\theta}}\right) d\tau_1^{-\theta} d\tau_2^{-\theta} + \textit{higher order terms}
\end{aligned}$$

$$\begin{aligned}
&= (C + D)\frac{dt}{t} - \left[C\pi_{11} + D(1 - \pi_{22})\right]\left(\frac{dt}{t}\right)^2 \\
&\quad - C\left[\frac{d\tau_1^{-\theta}}{\tau_1^{-\theta}} - (1 - \pi_{11})\left(\frac{d\tau_1^{-\theta}}{\tau_1^{-\theta}}\right)^2\right] + D\left[\frac{d\tau_2^{-\theta}}{\tau_2^{-\theta}} - (1 - \pi_{22})\left(\frac{d\tau_2^{-\theta}}{\tau_2^{-\theta}}\right)^2\right] \\
&\quad + C(2\pi_{11} - 1)\left(\frac{dt}{t}\right)\left(\frac{d\tau_1^{-\theta}}{\tau_1^{-\theta}}\right) + D(2\pi_{22} - 1)\left(\frac{dt}{t}\right)\left(\frac{d\tau_2^{-\theta}}{\tau_2^{-\theta}}\right) \\
&\quad + \textit{higher order terms}
\end{aligned}$$

where $C = A\pi_{11}(1-\pi_{11}) > 0$ and $D = B\pi_{22}(1-\pi_{22}) > 0$. The last equality follows from

$$\begin{aligned}
\frac{\partial \pi_{11}}{\partial t} &= \frac{\pi_{11}(1-\pi_{11})}{t}, \frac{\partial \pi_{11}}{\partial \tau_1^{-\theta}} = -\frac{\pi_{11}(1-\pi_{11})}{\tau_1^{-\theta}}, \frac{\partial \pi_{11}}{\partial \tau_2^{-\theta}} = 0, \\
\frac{\partial \pi_{22}}{\partial t} &= -\frac{\pi_{22}(1-\pi_{22})}{t}, \frac{\partial \pi_{22}}{\partial \tau_1^{-\theta}} = 0, \frac{\partial \pi_{22}}{\partial \tau_2^{-\theta}} = -\frac{\pi_{22}(1-\pi_{22})}{\tau_2^{-\theta}}, \\
\frac{\partial^2 \pi_{11}}{\partial t^2} &= -\frac{2(\pi_{11})^2(1-\pi_{11})}{t^2}, \frac{\partial^2 \pi_{11}}{\left(\partial \tau_1^{-\theta}\right)^2} = \frac{2\pi_{11}(1-\pi_{11})^2}{\left(\tau_1^{-\theta}\right)^2}, \frac{\partial^2 \pi_{11}}{\left(\partial \tau_2^{-\theta}\right)^2} = 0, \\
\frac{\partial^2 \pi_{22}}{\partial t^2} &= \frac{2\pi_{22}(1-\pi_{22})^2}{t^2}, \frac{\partial^2 \pi_{22}}{\left(\partial \tau_1^{-\theta}\right)^2} = 0, \frac{\partial^2 \pi_{22}}{\left(\partial \tau_2^{-\theta}\right)^2} = \frac{2\pi_{22}(1-\pi_{22})^2}{\left(\tau_2^{-\theta}\right)^2}, \\
\frac{\partial^2 \pi_{11}}{\partial t \partial \tau_1^{-\theta}} &= \frac{\pi_{11}(1-\pi_{11})(2\pi_{11}-1)}{t\tau_1^{-\theta}}, \frac{\partial^2 \pi_{11}}{\partial t \partial \tau_2^{-\theta}} = 0, \frac{\partial^2 \pi_{11}}{\partial \tau_1^{-\theta}\partial \tau_2^{-\theta}} = 0, \\
\frac{\partial^2 \pi_{22}}{\partial t \partial \tau_1^{-\theta}} &= 0, \frac{\partial^2 \pi_{22}}{\partial t \partial \tau_2^{-\theta}} = -\frac{\pi_{22}(1-\pi_{22})(2\pi_{22}-1)}{t\tau_2^{-\theta}}, \frac{\partial^2 \pi_{22}}{\partial \tau_1^{-\theta}\partial \tau_2^{-\theta}} = 0.
\end{aligned}$$

The pure effect of technology on $r_{11}$ will be

$$(C+D)\frac{dt}{t} - \left[C\pi_{11} + D(1-\pi_{22})\right]\left(\frac{dt}{t}\right)^2$$

which is positive if $dt > 0$ and negative if $dt < 0$.

The pure effect of trade friction on $r_{11}$ will be

$$-C\left[\frac{d\tau_1^{-\theta}}{\tau_1^{-\theta}} - (1-\pi_{11})\left(\frac{d\tau_1^{-\theta}}{\tau_1^{-\theta}}\right)^2\right] + D\left[\frac{d\tau_2^{-\theta}}{\tau_2^{-\theta}} - (1-\pi_{22})\left(\frac{d\tau_2^{-\theta}}{\tau_2^{-\theta}}\right)^2\right]$$

which is positive if $d\tau_1^{-\theta} < 0$ and $d\tau_2^{-\theta} > 0$, and negative if $d\tau_1^{-\theta} > 0$ and $d\tau_2^{-\theta} < 0$. (In the symmetric case, $\tau_1 = \tau_2$ and $d\tau_1 = d\tau_2$, the term is negative as $C > D$).

The interaction effect of technology and trade frictions will be

$$C(2\pi_{11}-1)\left(\frac{dt}{t}\right)\left(\frac{d\tau_1^{-\theta}}{\tau_1^{-\theta}}\right) + D(2\pi_{22}-1)\left(\frac{dt}{t}\right)\left(\frac{d\tau_2^{-\theta}}{\tau_2^{-\theta}}\right)$$

which is positve when $dt > 0$, $d\tau_1^{-\theta} \geq 0$, $d\tau_2^{-\theta} \geq 0$ or $dt < 0$, $d\tau_1^{-\theta} \leq 0$, $d\tau_2^{-\theta} \leq 0$,and negative when $dt > 0$, $d\tau_1^{-\theta} \leq 0$, $d\tau_2^{-\theta} \leq 0$ or $dt < 0$, $d\tau_1^{-\theta} \geq 0$, $d\tau_2^{-\theta} \geq 0$.

### 6.2.2 Details about the calibration exercises

This section contains some additional technical details about our estimation and calibration process, which have been omitted in the main text to save space.

- We have combined the last two sectors of the World Input-Output Tables from the WIOD, namely, the "Other community, social and personal services" and "Private households with employed persons", into one. The main reason is that most countries do not have statistics for the last sector "Private households with employed persons" and it contributes about 2/3 of zeros in the WIOT.

- In estimating the structural gravity equations, we follows Antras and Chor (2017) to set zero trade flows to $1.

- To smooth the yearly fluctuations in the trade volumes, we winsorize the estimated competitiveness and the changes in competitiveness by setting the bottom and top 2.5% values to the 2.5% and 97.5% percentile respectively. Similarly, we winsorize the estimated changes in trade costs by setting the bottom and top 0.5% values to the 0.5% and 99.5% percentile respectively. We have tried different cutoffs for the winsorizing and the results are not sensitive to the cutoffs used.

- All the data comes from the 2013 version of the WIOD Table, or the corresponding Socio Economic Accounts (SEA) dataset of the WIOD database for consistency purpose. All the variables and parameters are either directly obtained from the data, or calculated from values in the data.

#### 6.2.3 Detailed derivation about the relationship between country' DVAR and the gains from trade?

Let us define the consumer welfare of country $n$, $W_n$, as its real income:

$$W_n = \frac{w_n}{P_n^F}$$

where $P_n^F$ is the price index of the final consumption. Denote the price index of sector $j$ final goods in country $n$ to be $P_n^{jF} = \left(\Phi_n^{jF}\right)^{-\frac{1}{\theta}}$, where

$$\Phi_n^{jF} = \textstyle\sum_{l=1}^{N} T_l^j (c_l^j \tau_{nl}^{jF})^{-\theta}$$

Thus the percentage change in real income in country $n$ is given by

$$\begin{aligned} d\ln W_n &= d\ln w_n - \textstyle\sum_{j=1}^{J} \alpha_n^j d\ln P_n^{jF} \\ &= \textstyle\sum_{j=1}^{J} \alpha_n^j \left(\frac{1}{\theta} d\ln \Phi_n^{jF} + d\ln w_n\right) \\ &= \textstyle\sum_{j=1}^{J} \alpha_n^j \left(\frac{d\ln T_n^j}{\theta} - \frac{d\ln s_{nn}^j}{\theta} + d\ln w_n - d\ln c_n^j\right) \end{aligned}$$

where the last equality follows from $d\ln s_{nn}^j = d\ln T_n^j - \theta d\ln c_n^j - d\ln \Phi_n^{jF}$.

The percentage change in the unit cost of the input bundle is given by

$$\begin{aligned} d\ln c_n^j &= \beta_n^j d\ln w_n + \left(1-\beta_n^j\right) d\ln P_n^j \\ &= \beta_n^j d\ln w_n - \left(1-\beta_n^j\right) \textstyle\sum_{i=1}^J \gamma_n^{ji} \frac{d\ln \Phi_n^{ji}}{\theta} \\ \Rightarrow d\ln w_n - d\ln c_n^j &= \left(1-\beta_n^j\right) \textstyle\sum_{i=1}^J \gamma_n^{ji} \left( d\ln w_n + \frac{d\ln \Phi_n^{ji}}{\theta} \right) \end{aligned}$$

which leads to

$$d\ln w_n - d\ln c_n^j = \left(1-\beta_n^j\right) \textstyle\sum_{i=1}^J \gamma_n^{ji} \left[ \left( d\ln w_n - d\ln c_n^i \right) + \left( \frac{d\ln T_n^i}{\theta} - \frac{d\ln \pi_{nn}^{ji}}{\theta} \right) \right] \qquad (23)$$

where the last equality follows $d\ln \pi_{nn}^{ji} = d\ln T_n^i - \theta d\ln c_n^i - d\ln \Phi_n^{ji}$.

For country $n$, define $\mathbf{c}_n \equiv \{d\ln w_n - d\ln c_n^i\}$, which is a $J*1$ vector, $\mathbf{B}_n$ is defined as a diagonal matrix with the $j$th diagonal element being $\beta_n^j$, $\mathbf{\Gamma}_n \equiv \{\gamma_n^{ji}\}$ is the $J*J$ input-output matrix of country $n$, $\mathbf{\Pi}_n \equiv \{\left(1-\beta_n^j\right) \sum_{i=1}^J \gamma_n^{ji} \left( \frac{d\ln T_n^i}{\theta} - \frac{d\ln \pi_{nn}^{ji}}{\theta} \right)\}$, which is a $J*1$ vector. Thus, equation (23) can be rewritten as

$$\begin{aligned} \mathbf{c}_n &= \left(\mathbf{I}-\mathbf{B}_n\right) \mathbf{\Gamma}_n \mathbf{c}_n + \mathbf{\Pi}_n \\ \Rightarrow \quad \mathbf{c}_n &= \left[\mathbf{I}- \left(\mathbf{I}-\mathbf{B}_n\right) \mathbf{\Gamma}_n\right]^{-1} \mathbf{\Pi}_n \end{aligned}$$

where $\left[\mathbf{I}- \left(\mathbf{I}-\mathbf{B}_n\right) \mathbf{\Gamma}_n\right]^{-1}$ is a typical Leontief inverse matrix. Define $\delta_n^{ji}$ as the row $j$ column $i$ element of $\left[\mathbf{I}- \left(\mathbf{I}-\mathbf{B}_n\right) \mathbf{\Gamma}_n\right]^{-1}$, then

$$d\ln w_n - d\ln c_n^j = \sum_{i=1}^J \delta_n^{ji} \left(1-\beta_n^i\right) \sum_{k=1}^J \gamma_n^{ik} \left( \frac{d\ln T_n^k}{\theta} - \frac{d\ln \pi_{nn}^{ik}}{\theta} \right)$$

Thus

$$\begin{aligned} d\ln W_n &= \sum_{j=1}^J \alpha_n^j \left[ \frac{d\ln T_n^j}{\theta} - \frac{d\ln s_{nn}^j}{\theta} + \sum_{i=1}^J \delta_n^{ji} \left(1-\beta_n^i\right) \sum_{k=1}^J \gamma_n^{ik} \left( \frac{d\ln T_n^k}{\theta} - \frac{d\ln \pi_{nn}^{ik}}{\theta} \right) \right] \\ &= \sum_{j=1}^J \alpha_n^j [\frac{1}{\theta} \left( d\ln T_n^j + \sum_{i=1}^J \delta_n^{ji} \left(1-\beta_n^i\right) \sum_{k=1}^J \gamma_n^{ik} d\ln T_n^k \right) - \frac{1}{\theta} \left( d\ln s_{nn}^j + \sum_{i=1}^J \delta_n^{ji} \left(1-\beta_n^i\right) \sum_{k=1}^J \gamma_n^{ik} d\ln \pi_{nn}^{ik} \right) \end{aligned}$$

For the term $d\ln T_n^j + \sum_{i=1}^J \delta_n^{ji} \left(1-\beta_n^i\right) \sum_{k=1}^J \gamma_n^{ik} d\ln T_n^k$, it can be rewritten as

$$\begin{aligned} & \mathbf{T}_n + \left[\mathbf{I}- \left(\mathbf{I}-\mathbf{B}_n\right) \mathbf{\Gamma}_n\right]^{-1} \left(\mathbf{I}-\mathbf{B}_n\right) \mathbf{\Gamma}_n \mathbf{T}_n \\ = \; & \left[\mathbf{I}- \left(\mathbf{I}-\mathbf{B}_n\right) \mathbf{\Gamma}_n\right]^{-1} \mathbf{T}_n \end{aligned}$$

where $\mathbf{T}_n \equiv \{d\ln T_n^i\}$. Thus $d\ln T_n^j + \sum_{i=1}^J \delta_n^{ji} \left(1-\beta_n^i\right) \sum_{k=1}^J \gamma_n^{ik} d\ln T_n^k = \sum_{i=1}^J \delta_n^{ji} d\ln T_n^i$, which leads to

$$d\ln W_n = \sum_{j=1}^J \alpha_n^j \left[ \frac{1}{\theta} \sum_{i=1}^J \delta_n^{ji} d\ln T_n^i - \frac{1}{\theta} \left( d\ln s_{nn}^j + \sum_{i=1}^J \delta_n^{ji} \left(1-\beta_n^i\right) \sum_{k=1}^J \gamma_n^{ik} d\ln \pi_{nn}^{ik} \right) \right]$$

The welfare gains can come from direct imports of final goods, and imports of intermediate goods for the final goods production through the IO linkage, as well as technological progress in either final goods or intermediate goods. In the tradeable intermediate goods setting of Eaton and Kortum (2002), $s^{i}_{nn} = \pi^{ji}_{nn} \equiv \pi^{i}_{nn}$, the term $d\ln s^{j}_{nn} + \sum_{i=1}^{J} \delta^{ji}_{n}\left(1-\beta^{i}_{n}\right)\sum_{k=1}^{J}\gamma^{ik}_{n} d\ln \pi^{ik}_{nn}$ will degenerate to a simple term $\sum_{i=1}^{J}\delta^{ji}_{n} d\ln \pi^{i}_{nn}$, similar to the effect of changes in technology. When there is no roundabout production and IO linkage, $\mathbf{B}_n = \mathbf{\Gamma}_n = \mathbf{I}$, $[\mathbf{I}-(\mathbf{I}-\mathbf{B}_n)\mathbf{\Gamma}_n]^{-1} = \mathbf{I}$, equation (20) will degenerate to equation (11) in Donaldson (2018).

Equation (20) implies that

$$\widehat{W}_n = \prod_{j=1}^{J}\prod_{i=1}^{J}\left(\widehat{T}^{i}_{n}\right)^{\frac{\alpha^{j}_{n}\delta^{ji}_{n}}{\theta}} \cdot \prod_{j=1}^{J}\left(\widehat{s}^{j}_{nn}\right)^{-\frac{\alpha^{j}_{n}}{\theta}} \cdot \prod_{j=1}^{J}\prod_{i=1}^{J}\prod_{k=1}^{J}\left(\widehat{\pi}^{ik}_{nn}\right)^{-\frac{\alpha^{j}_{n}\delta^{ji}_{n}\left(1-\beta^{i}_{n}\right)\gamma^{ik}_{n}}{\theta}}$$

For the yearly change in welfare, $\widehat{s}^{j}_{nn}$ and $\widehat{\pi}^{ji}_{nn}$ can be obtained from data, $\widehat{T}^{j}_{n}$ are solved from the general equilibrium of the model, and $\theta$, $\alpha^{j}_{n}$, $\beta^{j}_{n}$, $\gamma^{ji}_{n}$, $\mu^{j}_{n}$ as well as $\delta^{ji}_{n}$ are exogenous parameters. Thus the value $\widehat{W}_n$ of each year can be directly calculated.

**Gains from Trade**

For the gains from trade compared to autarky, when we move from the autarky, where $s^{j}_{nn} = \pi^{ji}_{nn} = 1$, to the current equilibrium, the gains from trade is

$$\widehat{W}_{n,A} - 1 = \prod_{j=1}^{J}\left(s^{j}_{nn}\right)^{-\frac{\alpha^{j}_{n}}{\theta}} \cdot \prod_{j=1}^{J}\prod_{i=1}^{J}\prod_{k=1}^{J}\left(\pi^{ik}_{nn}\right)^{-\frac{\alpha^{j}_{n}\delta^{ji}_{n}\left(1-\beta^{i}_{n}\right)\gamma^{ik}_{n}}{\theta}} - 1$$

In the one-sector setting, $\mathbf{B}_n = \beta_n$ and $\mathbf{\Gamma}_n = 1$, which leads to $[\mathbf{I}-(\mathbf{I}-\mathbf{B}_n)\mathbf{\Gamma}_n]^{-1} = 1/\beta_n$, the RHS of equation (22) will degenerate to

$$\left(s_{nn}\right)^{-\frac{1}{\theta}}\left(\pi_{nn}\right)^{-\frac{1-\beta_n}{\theta\beta_n}} - 1$$

which is the gains from trade expression in Section 7 of Antras and de Gortari (2017) when $N = 1$.