The electric vehicle transition and the economics of banning gasoline vehicles

Stephen P. Holland* Erin T. Mansur† Nicholas Z. Muller‡
Andrew J. Yates§

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Abstract

We analyze the transition to electric vehicles and recent policy proposals to ban gasoline vehicles. Our model captures declining electric vehicle damages; declining electric vehicle production costs due to exogenous changes or to learning by doing; stock effects; and the introduction of complementary infrastructure such as charging stations. We derive conditions under which it is socially optimal to ban gasoline vehicle production in the long run. We derive two classes of solutions. In one, it is optimal to ban gasoline vehicle production before beginning production of electric vehicles. This solution obtains if electric vehicles are perfect substitutes for gasoline vehicles. In the other solution, it is only optimal to ban gasoline vehicle production after beginning the production of electric vehicles. Simulation results show that the optimal time to ban gasoline vehicles depends critically on the parameters that describe preferences for the two types of vehicles.

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*Department of Economics, University of North Carolina at Greensboro and National Bureau of Economic Research. Mailing Address: Bryan School of Business and Economics, Department of Economics, Bryan 462, PO Box 26170, Greensboro, NC 27402-6170. Phone: 336- 334-4925. Fax: 336-334-5580. Email: `sphollan@uncg.edu`

†Tuck School of Business at Dartmouth and National Bureau of Economic Research. Email: `erin.mansur@dartmouth.edu`

‡Department of Engineering and Public Policy, Tepper School of Business, Carnegie Mellon University and National Bureau of Economic Research. Email: `nzm@andrew.cmu.edu`

§Department of Economics and Curriculum for the Environment and Ecology, University of North Carolina at Chapel Hill. Email: `ajyates@email.unc.edu`.

1 Introduction

Two changes have dramatically altered markets in personal transportation. First, the introduction of the Tesla Roadster in 2006 marked a modern resurgence in sales of electric vehicles. In Norway, electric vehicles have surpassed 60% of new vehicle sales and are expected to exceed 90% by 2024 (Accenture). In the US, sales are currently less than 10%, but are projected to exceed 90% in California by 2040. This growth has been fueled by technological advances, for example in batteries, and by substantial public monetary and non-monetary support.[1] Second, the electricity grid has become substantially cleaner. Holland et al. (2018) document dramatic declines in emissions from U.S. power plants and show that the decline in total emissions has led to a decline in marginal damages in the East of about 5% per year. Public support for electric vehicles is justified at least in part by their environmental benefits, which should increase as electricity becomes cleaner.

These two changes raise questions about policies regarding the optimal transition to electric vehicles. Should electric vehicle subsidies be increasing or decreasing over time? Should the transition be abrupt or gradual? How should electric vehicle adoption be sequenced with complementary technologies such as charging infrastructure? Perhaps most importantly, under what conditions would a ban on gasoline vehicles be justified? This last question arises because a number of countries are considering bans on the sale of new gasoline vehicles within the next few decades: Norway (proposed date of ban is 2025), India (2030), Britain (2040), and France (2040).[2]

In this paper, we address these questions with a theoretical model of the transition from gasoline to electric vehicles. We construct an optimal control problem for a planner who determines the production levels for gasoline and electric vehicles over time to maximize welfare. Welfare includes the utility from using the vehicles, the costs of producing them, and the pollution damages and other costs associated with their use. We assume that gasoline vehicles and electric vehicles are not necessarily perfect substitutes, which is a distinguishing characteristic of our work relative to other papers that have examined the transition from

[1] See Holland et al (2016) and Yuksel et al (2016) for analyses of the economic implications of electric vehicles.

[2] http://money.cnn.com/2017/07/26/autos/countries-that-are-banning-gas-cars-for-electric/index.html. Accessed 9/1/2017.

dirty to clean vehicles or from dirty to clean fuels more generally (Chakravorty et al 2012, Amigues et al 2016, Bahel and Chakravorty 2016, Creti et al 2018).

Our model accounts for a number of important dynamic aspects to the problem. First, the model allows for declining damages from electric vehicles. Holland et al (2018) document an extraordinary decline in air pollution from electricity generation in recent years and determine the corresponding decline in damages from electric vehicles. Our model assumes that damages decline exogenously over time. Second, we allow for declining production costs of electric vehicles. The resurgence of electric vehicles was led by high performance but high cost Teslas. Prices of other electric vehicles have typically been \$20-\$30,000 higher than comparable gasoline vehicles primarily due to the high initial costs of batteries. However, production costs of electric vehicles have declined dramatically due to improvements e.g., in battery technology or electric motors. We allow for production costs to decline either exogenously or endogenously due to learning by doing in the manufacturing of electric vehicles. Third, both gasoline and electric vehicles are durable goods. Stocks of durable goods decay naturally over time but can be increased through the production of new goods. We model the stock effects of production and depreciation for both electric and gasoline vehicles. The final dynamic consideration is the production of complementary infrastructure for electric vehicles such as charging stations. Complementary infrastructure increases the utility of electric vehicles, and thus changes the degree of substitutability between the vehicles. In the most complete version of the model, the planner selects the both the production of vehicles and the roll out of infrastructure.

We first consider the basic question of whether or not a ban is justified by the economic fundamentals in the model. We provide a simple condition under which the planner optimally bans production of gasoline vehicles in the long run. The condition is a function of the marginal rate of substitution, the long run production costs of the vehicles, and the long run pollution damages. A numerical calulation based on plausible values for the parameters of the model for the United States shows that the ban is much more strongly influenced by percentage changes in production costs than by equal percentage changes in pollution damages.

Next, we turn to the question of when electric vehicle production should be started and when gasoline vehicle production should be halted (i.e. the point in time in which the ban takes effect.) We show that there are two types of solutions to the planner's problem, depending on the degree of substitutability. In one solution, which we refer to as the *gap solution*, the planner optimally bans gasoline vehicle production before production of electric vehicles starts. The gap solution arises, for example, if electric vehicles are perfect substitutes for gasoline vehicles. A second solution occurs when electric vehicles are not perfect substitutes for gasoline vehicles. We call this solution the *simultaneous solution* because there is a period of time in which both vehicles are produced. In the simultaneous solution, it is only optimal to ban gasoline vehicle production after starting production of electric vehicles.

We calibrate the model for the United States and simulate the solution numerically. These simulations show that the optimal time to implement a ban on gasoline vehicle production in the Unites States is quite sensitive to the parameters that characterize the substitutability of electric vehicles for gasoline vehicles. For many values of the parameters, the ban on gasoline vehicle production occurs well after 2030, if it occurs at all. We also conduct simulations of the business as usual (BAU) case in which the planner ignores the externalities from air pollution from both gasoline and electric vehicles. This mimics the market outcome. There are parameter values for which the market stops producing gasoline vehicles, but the stop times are generally at least a decade later than the stop times in the planner's problem. Finally, we analyze the welfare benefits of several types of subsidies on the purchase of electric vehicles.

2 Model

Consider a continuous time model in which society benefits from the stock of gasoline and/or electric vehicles. The benefit per unit of time in dollars is given by $U(G, X)$ where $G(t)$ denotes the stock of gasoline vehicles and $X(t)$ the stock of electric vehicles at time t.[3] Letting U_G and U_X denote the partial derivatives, we assume U is concave with $U_G > 0$ and $U_X > 0$.

The stocks of gasoline and electric vehicles evolve over time due to production of new

[3]For notational convenience, we often suppress writing variables as explicit functions of time.

vehicles and retirement of vehicles from events such as accidents and mechanical failure. Let $g(t)$ denote the production of gasoline vehicles at time t and $a > 0$ denote the retirement rate. The state equation for the stock of gasoline vehicles is $\dot{G} = -aG + g$ where $\dot{G}$ is the time derivative of G. Likewise, let $x(t)$ denote the production of electric vehicles. The state equation for the stock of electric vehicles is $\dot{X} = -aX + x$. Note that the expected lifetime of a vehicle is given by $\frac{1}{a}$.

Each vehicle has production costs and usage costs. These costs may include both private costs and externalities from, for example, emissions of air pollution. Let c_g denote the one time production cost of a gasoline vehicle, and let δ_g denote the usage costs of driving a gasoline vehicle per unit of time. In our simulation, gasoline usage costs are equal to the sum of the operating costs and damages from air pollution from tailpipes. We assume that both c_g and δ_g are constant over time. Also we assume that $U_G(0,0) > (a+r)c_g + \delta_g$ so that it is optimal to have some gasoline cars if there are no electric cars.

We assume electric vehicles initially have greater production costs and/or greater usage costs than gasoline vehicles, but that these costs are falling over time. Let production costs of an electric vehicle at time t be $c_x(t)$ with $\dot{c}_x < 0$ and $\ddot{c}_x \geq 0$.[4] Decreases in c_x over time are due to, for example, exogenous improvements in battery technology.[5] Let $\delta_x(t)$ denote the usage cost of driving an electric vehicle per unit of time at time t. In our simulation, the electric vehicle usage costs equals the sum of operating costs and damages from air pollution from electric power plants. We assume $\dot{\delta}_x \leq 0$. Decreases in δ_x over time are due to, for example, decreases in damages as the electricity grid gets cleaner.[6] Define the limits of production and usage costs for electric vehicles as $\lim_{t\to\infty} c_x(t) = \hat{c}_x$ and $\lim_{t\to\infty} \delta_x(t) = \hat{\delta}_x$.

Several elements of the model deserve additional explanation. First, the stocks of gasoline and electric vehicles G and X are only differentiated by fuel type. Many other attributes matter to drivers including the age of the vehicle. Second the benefit function U represents the benefits to society from optimally allocating the stocks of vehicles to consumers. Adding an electric vehicle may require the existing stock of vehicles to be reallocated among

[4] We assume that $U_x(G^{ss}, 0) < (a+r)c_x(0) + \delta_x(0)$ so that no electric vehicles are produced at $t = 0$.

[5] Kittner et al (2017). Below we extend the model to include endogenous reductions in costs, for example, from learning by doing.

[6] Holland et al 2018.

consumers. This assumption is consistent with well functioning markets for new and used vehicles. Third, the usage cost is per vehicle, so the total usage cost is simply the product of the stock of vehicles and the usage cost per vehicle. We assume the usage cost for a gasoline vehicle, δ_g is constant over time. If certain components of the usage cost, e.g., the social cost of carbon, are increasing over time, then we are implicitly assuming that other components of the usage cost (e.g., driving or fuel economy) are changing to offset these increases. On the electric side, we assume usage costs are decreasing due to declines in pollution damages over time. Note that this does not require us to model the vintages of the electric vehicles if declines in pollution damages per mile arise due to improvements in the electricity grid, rather than efficiency improvements to the electric drivetrains. An improvement in the grid leads to a contemporaneous decrease in damages from the entire fleet of electric vehicles regardless of the age of the vehicle.

The planner determines the production of gasoline and electric vehicles to maximize discounted net benefits. If r is the interest rate and the planner starts in an initial steady state with no electric vehicles, the planner's problem is

$$\begin{array}{rlcl} \max\limits_{g,x} & \int_0^\infty e^{-rt} \left(U(G,X) - c_g g - c_x x - \delta_g G - \delta_x X \right) dt & & \\ \text{s.t.} & \dot{G} & = & -aG + g \ ; \ G(0) = G^{ss} \\ & \dot{X} & = & -aX + x \ ; \ X(0) = 0 \\ & & & x \geq 0 \ ; \ g \geq 0, \end{array}$$

where G^{ss}, defined by $U_G(G^{ss}, 0) = (a+r)c_g + \delta_g$, is the initial steady state stock of gasoline vehicles. In this optimal control problem, the control variables are the production levels g and x, and the state variables are G and X, which are assumed to be continuous.[7]

The derivation of necessary conditions for the planner's problem are given in Online Appendix A. These conditions include the state equations for G and X and the corresponding adjoint equations

$$\dot{\alpha} = (a+r)\alpha + \delta_g - U_G, \tag{1}$$

eq:adjG

[7] This rules out using impulse functions for the controls x and g.

and

$$\dot{\beta} = (a+r)\beta + \delta_x - U_X,$$

where α is the adjoint variable for G and β is the adjoint variable for X. Because the objective and state equations are linear in the controls, and the controls must be non-negative, we have the following Kuhn-Tucker first order conditions for the controls:

$$\begin{aligned} g \geq 0 \quad \alpha - c_g \leq 0 \quad g(\alpha - c_g) = 0 \\ x \geq 0 \quad \beta - c_x \leq 0 \quad x(\beta - c_x) = 0. \end{aligned}$$

These equations show that the adjoint variables are bounded above by the production costs and equal production costs when g or x is interior.

Consider interior production of gasoline vehicles. When $g > 0$, the first order condition implies that $\alpha = c_g$ and hence because c_g is constant over time we have $\dot{\alpha} = 0$. The adjoint equation for G then implies

$$U_G = (a+r)c_g + \delta_g. \tag{2}$$

eq:g_int

To interpret this equation, we first define the *full marginal cost* of the gasoline vehicle as $(a+r)c_g + \delta_g$, which is the sum of annualized depreciation, investment, and operating costs. It follows that the marginal benefit of a gasoline vehicle per unit of time equals the full marginal cost.

Interior production of electric vehicles has a similar interpretation. When $x > 0$, we have $\beta = c_x$. Taking the time derivative gives $\dot{\beta} = \dot{c}_x$, so the adjoint equation for X implies

$$U_X = (a+r)c_x + \delta_x - \dot{c}_x. \tag{3}$$

eq:x_int

This equation is analogous to (2) except it has an additional cost $-\dot{c}_x > 0$, which is the opportunity cost of producing the electric vehicle at time t instead of waiting until it is cheaper to produce in the future. In this case the full marginal cost, $(a+r)c_x + \delta_x - \dot{c}_x$ includes the opportunity cost $-\dot{c}_x$. The full marginal cost of the electric vehicle is decreasing over time.[8]

[8] The time derivative of the full marginal cost is $(a+r)\dot{c}_x + \dot{\delta}_x - \ddot{c}_x$ which is negative by assumption.

In the initial steady state, we have $x(0) = 0$ and $X(0) = 0$, so the steady state stock of gasoline vehicles, G^{ss}, is determined by $U_G(G^{ss}, 0) = (a + r)c_g + \delta_g$ and initial production of gasoline vehicles is $g = aG^{ss}$. Note that, if gasoline production was simply set to zero, then the gasoline vehicle stock would asymptote to zero according to $G(t) = G^{ss}e^{-at}$.

2.1 Banning gasoline vehicle production

To explore whether it is optimal to ban gasoline vehicle production, we first explore the terminal steady state in which all improvements to electric vehicles and the electricity grid have been completed, so usage costs have converged to $\hat{\delta}_x$ and production costs have converged to $\hat{c}_x$. Let g^∞ be gasoline vehicle production in the terminal steady state. We say that gasoline vehicle production is optimally banned in the terminal steady state if $g^\infty = 0$, or more precisely if there exists some T such that $g(t) = 0$ for all $t > T$. Our first proposition describes parameter combinations that determine whether or not gasoline vehicle production is banned in the terminal steady state. All proofs are in Online Appendix B.

lrprop **Proposition 1.** *Let X^* be defined by*

$$U_X(0, X^*) = (a + r)\hat{c}_x + \hat{\delta}_x.$$

Gasoline vehicle production is banned in the terminal steady state, i.e., $g^\infty = 0$ if

$$U_G(0, X^*) < (a + r)c_g + \delta_g.$$

Conversely, we have $g^\infty > 0$ if $U_G(0, X^*) > (a + r)c_g + \delta_g$

In the proposition, X^* is the number of electric vehicles which would be optimal if there were no gasoline vehicles in the terminal steady state. The proposition states that gasoline vehicle production should be banned if the marginal benefit of a gasoline vehicle, when there are no gasoline vehicles but lots of electric vehicles, is less than the full marginal cost of a gasoline vehicle, i.e., is less than the sum of the depreciation, interest, and usage costs of the vehicle. If U is derived from an underlying discrete choice model, the marginal benefit $U_G(0, X^*)$ would be interpreted as the utility gain to the individual with the highest gain in valuation from a gasoline vehicle relative to optimally having either no vehicle or an electric vehicle. The planner would ban gasoline vehicle production if this gain is small, but would not ban gasoline vehicle production if this gain is large.

Dividing the first two equations in Proposition 1 allows us to derive a condition on society's marginal rate of substitution (MRS) between gasoline and electric vehicles. The proposition implies that gasoline vehicle production should be banned if this MRS is smaller than the ratio of full marginal costs. If vehicles are perfect substitutes such that the MRS is one, the result implies that gasoline vehicles should be banned if the full marginal cost of an electric vehicles is cheaper. Alternatively, if vehicles are not good substitutes, such that the MRS is not one, then the full marginal cost of electric vehicles would need to be substantially cheaper in order for a ban to be optimal.

We could also define a gasoline vehicle ban as $g(t) = 0$ for some interval of time. In particular, it may be optimal to stop producing gasoline vehicles for a time and draw down the existing stock of gasoline vehicles even if gasoline vehicles must eventually be produced in the terminal steady state. We refer to this is a temporary ban. We will explore conditions under which gasoline vehicles are subject to a temporary ban in future work.

2.2 Transition From Gasoline to Electric Vehicles

We now turn our attention to the transition from gasoline to electric vehicles. By assumption, $g(0) = aG^{ss} > 0$ but $x(0) = 0$, i.e., gasoline vehicles are initially produced but electric vehicles are not. This leads to two key transition times: t^g, which is the time when gasoline vehicle production stops, and t^e which is when electric vehicle production starts. More precisely t^g is defined such that $g(t) > 0$ for $t \in [0, t^g]$ but $g(t) = 0$ for $t > t^g$.[9] If $t_g = \infty$ then gasoline vehicle production is not banned. Similarly, t^e is defined such that $x(t) = 0$ for all $t \in [0, t^e)$ but $x(t) > 0$ for all $t \in [t^e, \infty)$.

For these transition times, there are two possible solutions to the planner's problem. If $t^e < t^g$, then there is a period of time in which gasoline and electric vehicles are both produced. We can this the *simultaneous solution*. If $t^g < t^e$, then there is a period of time in which neither gasoline nor electric vehicles are produced. We call this the *gap solution*

[9]In the case of a temporary ban, $g(t)$ need not equal zero for all $t > t^g$.

due to the gap in vehicle production. Surprisingly, this solution obtains for reasonable parameterizations of the model. We first characterize the transition times in the simultaneous solution and then in the gap solution.

2.2.1 The simultaneous solution

The simultaneous solution is characterized first by production of gasoline vehicles only, then by production of both gasoline and electric vehicles, and finally by production of electric vehicles only. Before t^e, the solution has $g = aG^{ss} > 0$ but no electric vehicle production. Electric vehicle production begins at t^e. Over the interval $[t^e, t^g]$, both gasoline and electric vehicles are produced so both (2) and (3) must hold and the vehicle stocks (and hence production) are determined by these equations. Note that the costs of electric vehicles are falling so more vehicles are produced and $\dot{X} > 0$. Because the right-hand-side of (2) is constant over time, it follows that $\dot{G} < 0$ over this interval. If $t^g < \infty$, then gasoline vehicle production ceases at t^g and the stock of gasoline vehicles simply depreciates thereafter, i.e., $G(t) = G(t^g)e^{-a(t-t^g)}$ for every $t > t^g$. As the gasoline vehicle stock is depreciating, electric vehicle production is determined by (3), generally increasing toward a terminal steady state.

Characterizing t^e in the simultaneous solution is relatively simple because $G(t^e) = G^{ss}$ and $X(t^e) = 0$. Substituting these values for vehicles stocks into (3) yields $U_X(G^{ss}, 0) = (a + r)c_x(t^e) + \delta_x(t^e) - \dot{c}_x(t^e)$ which can then be solved for t^e. The characterization of t^g is trickier since it involves solving a differential equation. Details are in the proof of the following proposition which characterizes the transition times:

prop:simul **Proposition 2.** *In the simultaneous solution, the transition time t^e is the solution to*

$$U_X(G^{ss}, 0) = (a + r)c_x(t^e) + \delta_x(t^e) - \dot{c}_x(t^e). \tag{4}$$

eq:simul_t

If $t^g < \infty$, the transition time t^g is the solution to

$$c_g = \int_{t^g}^{\infty} e^{-(a+r)(\tau-t^g)} \left[U_G(G^{no}(\tau), X^{no}(\tau)) - \delta_g\right] d\tau \tag{5}$$

eq:simul_t

where $G^{no}(t) = G(t^g)e^{-a(t-t^g)}$ and $X^{no}(t)$ satisfies $U_X(G^{no}(t), X^{no}(t)) = (a+r)c_x + \delta_x - \dot{c}_x$ for *all* $t > t^g$.

The characterization of t^e in (4) shows that electric vehicle production should begin when the full marginal cost of the electric vehicle falls such that it exactly equals the marginal benefit of an electric vehicle given a zero stock of electric vehicles. Because gasoline vehicles are in steady state at t^e, equation (4) can also be written in terms of the MRS:

$$\frac{U_X(G^{ss},0)}{U_G(G^{ss},0)} = \frac{(a+r)c_x(t^e)+\delta_x(t^e)-\dot{c}_x(t^e)}{(a+r)c_g+\delta_g}.$$

If gasoline and electric vehicles are close substitutes, this MRS should be close to one, and electric vehicle production should begin when full marginal costs of the electric and gasoline vehicles are approximately equal. If, however, there are individuals who highly value electric vehicles, the MRS when $X = 0$ could be quite large. In this case it might be optimal to produce electric vehicles even if their full marginal costs substantially exceed the full marginal costs of a gasoline vehicle. Conversely, if electric vehicles are seen as inferior even by the individuals with the highest relative valuations (perhaps due to range anxiety), then the full marginal costs would need to fall below the full marginal costs of a gasoline vehicle before electric vehicles should be produced.

The characterization of t^g in (5) shows that gasoline vehicle production should stop when the cost of producing a gasoline vehicle equals the present value of the lifetime benefit of driving a gasoline vehicle net of usage costs from that time on. Two points are worth noting about (5). First, the discount factor $e^{-(a+r)(\tau-t^g)}$ reflects both the time cost of money, r, and the depreciation cost, a, of the vehicle. Second, the path of the gasoline vehicle stock after t^g, given by $G^{no}(t) = G(t^g)e^{-a(t-t^g)}$, is simply the stock decaying because no new gasoline vehicles are produced after t^g. However, $G(t^g)$ is not equal to G^{ss} because during the interval $[t^e, t^g]$ both gasoline and electric vehicles are produced so the stocks evolve to satisfy both (2) and (3).

2.2.2 Gap Solution

The gap solution is characterized first by gasoline vehicle production, then by no vehicle production (the gap) and finally by electric vehicle production. During $[0, t^g]$, gasoline vehicles are produced in steady state with gasoline stock equal to G^{ss} and interior gasoline production $g = aG^{ss} > 0$. At time t^g, production of gasoline vehicles stops and from then on, the stock of gasoline vehicles decays exponentially so $G(t) = G^{ss}e^{-a(t-t^g)}$ for all $t > t^g$. At time t^e, production of electric vehicles starts with interior x determined so that X satisfies (3) with $G(t) = G^{ss}e^{-a(t-t^g)}$. Let $X^{gap}(t)$ denote this stock of electric vehicles during the period $[t^e, \infty]$.

We can now characterize the transition times t^e and t^g in the gap solution. In this case, the transition times must be solved for jointly.

gapprop **Proposition 3.** *In the gap solution, the transition times t^e and t^g are the solutions to the system of equations described by*

$$U_X(G^{ss}e^{-a(t^e-t^g)}, 0) = (a+r)c_x(t^e) + \delta_x(t^e) - \dot{c}_x(t^e) \tag{6}$$

gapone

and

$$c_g = \int_{t^g}^{t^e} e^{-(a+r)(\tau-t^g)}(U_G(G^{ss}e^{-a(\tau-t^g)}, 0) - \delta_g)d\tau + \int_{t^e}^{\infty} e^{-(a+r)(\tau-t^g)}\left[U_G(G^{ss}e^{-a(\tau-t^g)}, X^{gap}(\tau)) - \delta_g\right]d\tau. \tag{7}$$

gaptwo

Equation (6) shows that electric vehicle production should begin when the full marginal cost of the electric vehicle falls to the marginal benefit of an electric vehicle. However because of the gap in production, the stock of gasoline vehicles has depreciated and the marginal benefit of an electric vehicle is higher than in the initial steady state value. Thus the gap in production increases the marginal benefit of an electric vehicle and causes their production to begin earlier. Notice that (6) is a function of both t^e and t^g so the marginal benefit depends on the stock of gasoline vehicles, which in turn depends on how long it has been since production of these vehicles was stopped. Equation (7) shows that at time t^g, the production cost of the vehicle should equal the discounted net benefits of an additional gasoline vehicle at each time in the future. This equation is similar to (5) except that it depends on t^e because the production paths and hence the marginal benefit of a gasoline vehicle change when electric vehicle production begins. Thus equations (6) and (7) each depend on both t^g and t^e.

The gap solution has a period of time in which no vehicles are produced. Because this is counterintuitive, it is useful to point out that the gap solution can occur for reasonable parameterizations of the model. The next proposition shows that we obtain the gap solution in the rather important special case in which gasoline and electric vehicles are perfect substitutes.

nobothg **Proposition 4.** *If the benefit function is $U(G,X) = u(G + \eta X)$ with u concave, then the solution to the planner's problem has $t^g < t^e$.*

The perfect substitutes case thus provides a useful benchmark for the analysis of the transition from gasoline to electric vehicles. The planner accounts for the decreasing damages and production costs of electric vehicles when determining the optimal time to introduce them. If electric vehicles are perfect substitutes for gasoline vehicles, there is no loss in benefit from banning gasoline vehicle production if they are replaced by electric vehicles. In this case, the planner bans production of gasoline vehicles before beginning production of electric vehicles because gasoline vehicles produced today will remain in the fleet for some time, and they will cause more damages than the increasingly clean electric vehicles. In addition, banning production of gasoline vehicles increases the marginal benefit of an electric vehicle, thus leading to an earlier introduction of electric vehicles.

We have seen that perfect substitutes leads to the gap solution. With more general preferences, either the gap or simultaneous solution may occur. Loosely speaking, if electric cars are good substitutes for gasoline vehicles, then the gap solution occurs. If, however, the vehicles are not good substitutes, then the planner accounts for this by extending the production lifetime of gasoline vehicles past the point at which electric vehicles are introduced.

2.3 Extensions

In this section we extend the model to investment in charging infrastructure and learning by doing. Investment in charging infrastructure can increase the benefit of an electric vehicle by facilitating intercity driving and reducing range anxiety. To model charging infrastructure, let the state variable $W(t)$ be the stock of charging infrastructure and the control variable $w(t)$ be the additional charging infrastructure (e.g., number of stations) produced at time

t which costs c_w per unit. Assuming charging infrastructure does not depreciate, the state equation for the stock of charging infrastructure is $\dot{W} = w$. The benefit of driving an electric vehicle depends on the charging infrastructure. Charging infrastructure increases the utility of electric vehicles because, for example, it is easier to take long trips. We can write the aggregate benefit function as $U(G, X, W)$ with $U_W > 0$ and $U_{XW} > 0$. We also assume that $U_W(G, 0, W) = 0$, i.e., there is no benefit to charging stations when there are no electric vehicles.

Charging infrastructure investment is subject to the "chicken and egg" problem of two-sided externalities. The planner in our framework avoids this externality, and the following proposition delineates conditions under which investment in charging infrastructure should begin before or after the production of electric vehicles.

op-chargef **Proposition 5.** *Let t^{w1} be the time at which production of charging infrastructure begins. If w at t^{w1} is interior and $r > 0$, then $t^{w1} > t^e$. Conversely, if $r = 0$, then $t^{w1} \leq t^e$.*

The condition that w is interior essentially means that any upper bound on the production of charging infrastructure must be large enough so that it is not binding. If the interest rate is positive, the planner does not invest in charging infrastructure until there is a sufficient stock of electric vehicles to raise its marginal benefit above its marginal cost. Alternatively, if the interest rate is zero, then the planner wants to build out charging stations in advance because they do not depreciate and interest payments are zero.

The second extension introduces learning by doing in the production of electric vehicles. With learning by doing, the marginal cost of producing electric vehicles decreases in cumulative electric vehicle production which is denoted by the state variable $Z(t)$ with state equation $\dot{Z} = x$. The marginal cost of producing electric vehicles is given by a function $c_x = f(Z, t)$ with $f_Z < 0$ and $\dot{f} < 0$.

These extensions allow for a more general analysis of policy in the simulation section. A detailed discussion of the solution to the planner's problem with the extensions is given in Online Appendix C. The qualitative features of the solutions described above are generally robust to the extensions. For example, with learning by doing, Proposition 4 still holds and the equations for the transition times t_e and t_g have a similar structure as in Proposition 3.

2.4 Second Best Subsidies

In a market economy, of course, a planner does not make decisions to produce gasoline and electric vehicles. Rather these decisions are decentralized to firms and consumers through prices. Government policy can nudge the consumer toward a lower polluting technology by subsidizing its purchase. Without pricing all externalities, the outcome is not efficient but can be second best. Langer and Lemoine (2017) consider a problem that is similar in spirit but analyzes adoption of residential solar units in California.

In theory, the optimal second best subsidy can be determined in the following manner. Given an arbitrary price path for electric vehicles, consumers determine the purchases of vehicles in each instant of time to maximize utility net of operating costs but ignoring any externalities. The solution to this consumer's problem describes the choices of g and x, and thus X and G, as a function of the price path for electric vehicles. Using this solution, the planner can select the price path of electric vehicles (i.e., uses a tax or subsidy to manipulate the market price as desired) to maximize welfare.

In practice, it difficult to determine an analytical solution to the general second best problem. See Holland et al (2019) for a discussion and solution in a special case in which electric vehicles are perfect substitutes for gasoline vehicles. Simpler subsidies, for example, a constant subsidy or a subsidy which declines exponentially in time, can be readily analyzed in our framework. For a given functional form for the subsidies, we can select the parameters that maximize welfare. The resulting subsidies need not be second best but may provide a guide to policy nonetheless.

3 Model Calibration

In this section we describe the components of the simulation exercise and the calibration of parameters for these components. The starting year is 2005. We value any terminal stocks using production costs and assume a sufficiently distant terminal time T such that the results are not affected by the terminal conditions. Baseline values are shown in Table 1. We now describe how we determine these values.

ab-baseline

Table 1: Baseline parameter values

Parameter	Value	Description
T	70	Terminal time
a	0.067	Stock decay
δ_g	295.9 + 2726	Usage costs of gasoline vehicles: (externality) + operating
δ_x	$632e^{(-0.05*t)} + 1535$	Usage costs of electric vehicles: (externality) + operating
c_g	36113	Production costs of gasoline vehicles
c_x	$c_g + 21961e^{-0.06t}$	Production costs of electric vehicles: 60 kWh battery
G^{ss}	110 million	Initial steady state stock of gas passenger vehicles
r	0.05	Interest rate

3.1 Usage costs for gasoline vehicles: δ_g

In the theoretical model, we assume that δ_g is constant over time. To put this assumption in context, consider the externality component of usage costs. To determine how air pollution damages from gasoline vehicles evolved over time, we calculate a historical time series of tail pipe emission regulations, fleet MPG, and sulphur content in gasoline. Using data from Argonne National Laboratory's GREET model,[10] we can calculate emissions in grams per mile for 5 pollutants (VOC, CO_2, SO_2, $PM_{2.5}$, and NO_X) during the years 1975-2015 (see Online Appendix D for details.) To determine damages from these emissions, we follow the methodology of Holland et al (2016). We combine the emissions in grams per mile with the estimates of damage valuations in dollars per gram by county using the AP2 integrate assessment model, the EPA social cost of carbon, and VMT by county.

To illustrate this data, we conduct an experiment in which we assume the 2015 fleet of vehicles has the polluting characteristics of, say, 1975 vintage passenger vehicles. Repeating this for each year in our historical time series gives the results shown in Figure 1. In this figure, the damage valuations of pollutants are kept constant. The only thing that is changing is the emissions rates of the vehicles. The major improvements in emission reductions occur in the late 1970's and the 1990's. Although there has been less improvement in the last decade, the compound annual growth rate over this time period is negative two percent per year. To calibrate the model, we us 2015 vintage vehicles as baseline. This gives the externality component of usage costs to be 1.97 cents per mile and hence \$295.9 dollars

[10] https://greet.es.anl.gov/

per year per vehicle (assuming 15,000 miles per year).[11] The operating costs of $2726 are determined by data from the American Automobile Association (see Online Appendix F).

We assume that usage costs are constant over time for gasoline vehicles. However, usage costs depend on the social cost of carbon, which increases over time. We are implicitly assuming that that reductions in emissions from gas cars over time just balance out increasing damage valuations for carbon and other pollutants. In particular, each new production cohort must improve enough such that the externality from the entire stock of vehicles stays constant. This is perhaps overly optimistic about the rate of improvements from gasoline technology.

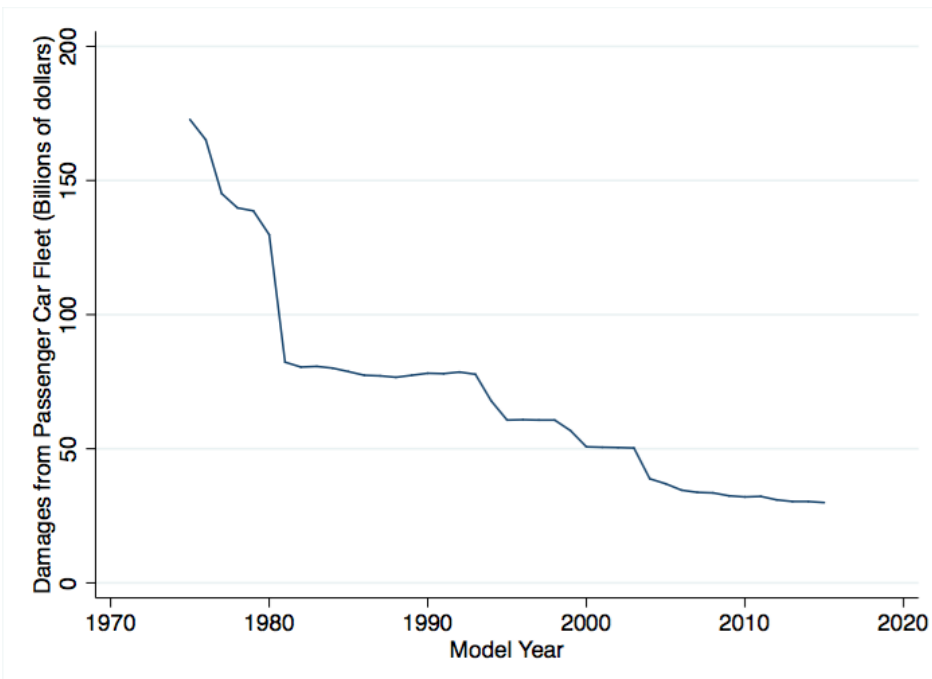

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Figure 1: Damages from Emissions of Gasoline Vehicles over Time

3.2 Usage costs for electric vehicles: δ_x

For the externality component of electric vehicle usage costs, Holland et al (2018) determine the marginal damages from a unit of electricity for the three electricity interconnections in the United States from 2010-2017. Marginal damages are decreasing at approximately 5 percent per year in the Eastern interconnection and we use this result in our baseline

[11] All dollar amounts are delineated in year 2017 dollars.

specification. The operating costs for electric vehicles are determined by data from the American Automobile Association (see Online Appendix F) and are assumed to stay constant over time at a value of \$1535.

The estimates in Holland et al (2018) do account for increasing damage valuations over time. But similar to what we discussed for gasoline vehicles, it is likely that our usage cost assumptions are overly optimistic about the rate of improvements from electric technology. First, Holland et al (2018) point out that the time period in their analysis is characterized by unusually rapid decreases in emissions. Second, our equation for usage costs assumes the externality from electric vehicles decreases to zero in the terminal steady state.

3.3 Benefit Function and Other parameters

Calibration of the model requires a functional form for the benefit function $U(G, X)$. The functional form must satisfy our concavity assumption and should ideally be parsimonious with parameters which we can identify from estimates in the literature. In addition, our focus on the banning of gasoline vehicle production requires the functional form to admit corner solutions with only gasoline or only electric vehicles.[12] For tractability, we assume a benefit function given by

$$U(G, X) = A \ln(G + \eta X + \gamma \eta G X), \tag{8}$$

eq-utility

where A, γ, and η are parameters. There are several things to note about this functional form assumption. First, the functional form nests linear indifference curves (if $\gamma = 0$) and one-to-one perfect substitutes (if $\gamma = 0$ and $\eta = 1$). We can interpret η as the relative preference for electric vehicles, and γ as the degree of substitutability. When the parameter γ is zero, we have perfect substitutes and η describes the slope of the linear indifference curves. When γ is greater than zero we have convex indifference curves, reflecting a social preference for balanced consumption of the two types of vehicles. Second, the ln function implies a unitary demand elasticity if $G = 0$ or $X = 0$. Third, the cross derivative U_{GX} can be either positive

[12] This requirement rules out the simplest versions of widely used functional forms such as Cobb-Douglas or constant elastiticity of substitution (CES).

or negative.[13] Finally, the parameter A is a demand shift parameter and can be determined from the initial steady state.[14]

Data from the Department of Transportation and the Bureau of Economic Analysis enables us to determine $a = 0.067$ (see Online Appendix G). For the production cost of gasoline vehicles, c_g, we use average sales price of light duty vehicles. This is consistent with perfect competition in which vehicles are priced at marginal cost. For the production cost of electric vehicles, we assume that most of the changes in cost will be due to decreased cost for the batteries. We assume an electric car has a 60 kWh battery. Online Appendix E contains time series data on costs of lithium ion batteries. Using this data, we determine an exponential model to predict the decrease in electric vehicle production costs over time. The resulting function approaches the gasoline vehicle production cost over time.

Our calibration shows that the initial annual usage costs of electric vehicles, \$2167, is less than the usage cost for the gasoline vehicle, \$3022. However, the production cost of the electric vehicle is higher. Full marginal costs are initially higher for the electric vehicle (about \$9000) but fall to about \$6000 in the limit. Because full marginal costs for the gasoline vehicle are about \$7000, it is optimal to initially produced gasoline vehicles and transition to electric vehicles as they become cheaper.

4 Simulation Results

4.1 Terminal Steady State: Banning gasoline vehicle production

We first analyze the conditions under which gasoline vehicle production is optimally banned as implied by Proposition 1. Our calibration assumes that the terminal steady state production cost of electric vehicles is equal to the production cost of gasoline vehicles (so that $\hat{c}_x = c_g$.) We also assume that in the terminal steady state, the emissions from electric power generation are equal to zero, so that $\hat{\delta}_x$ consists of operating costs only. Under these assumptions, the critical value for the marginal rate of substitution, $[(a+r)\hat{c}_x + \hat{\delta}_x]/[(a+r)c_g + \delta_g]$,

[13] U_{GX} can be positive when $X = 0$ or $G = 0$ and can be negative when both G and X are large.

[14] The equation for the initial steady state is $U_G(G^{ss}, 0) = (a+r)c_g + \delta_g$. Using the values in Table 1 gives $A = 797,184$.

that determines whether or not gasoline vehicles are banned in the terminal steady state depends on four underlying parameters: gasoline global damages (due to CO_2, evaluated at the social cost of carbon), gasoline local damages (due to VOC, SO_2, $PM_{2.5}$, NO_X), gasoline operating costs, and electric operating costs.

Figure 2 shows the sensitivity of the critical value of the marginal rate of substitution to changes in these four parameters. Consider the red line. Holding the other three parameters at their baseline value, the red line shows the relationship between the critical value and different values of the electric operating costs, expressed as a percentage of its baseline value. As electric operating costs increase, the critical value that leads to a ban on gasoline vehicle production in the long run increases. An increase in the costs of operating electric vehicles makes electric vehicles less desirable, hence the marginal rate of substitution must increase if the ban on gasoline vehicle production is to be justified. The other three variables have a negative relationship with the critical value. For example, the blue line shows that as the social cost of carbon increases, gasoline vehicles impose greater external costs, and hence the marginal rate of substitution required to justify a ban decreases. The general message from Figure 2 is that the critical value is much more sensitive to operating expenses than it is to externalities.

For the specific functional form of the utility function (8), we have

$$MRS(0, X^*) = \frac{U_X(0, X^*)}{U_G(0, X^*)} = \frac{\eta}{1 + X^*\gamma\eta}.$$

It follows that in the special case in which $\gamma = 0$, the MRS is equal to the relative preference for electric vehicles η. This gives a simpler interpretation of the lines in Figure 2. An increase in the costs of operating electric vehicles must be offset by an increase in the relative preference for electric vehicles if the ban on gasoline vehicle production is to be justified.

4.2 Transition from gasoline to electric vehicles

Next we turn to analyzing the transition from gasoline to electric vehicles, focusing in particular on the benefit function in (8). We begin by illustrating the production and stocks for $\eta = 0.85$ and three different values of the parameter γ. Panels (a) and (b) in Figure 3 have

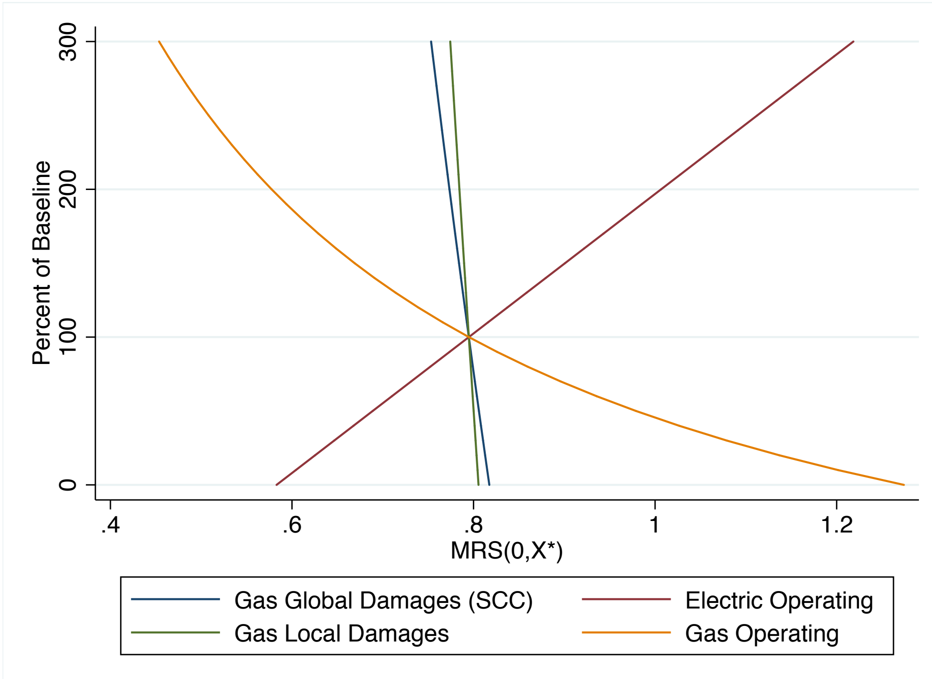
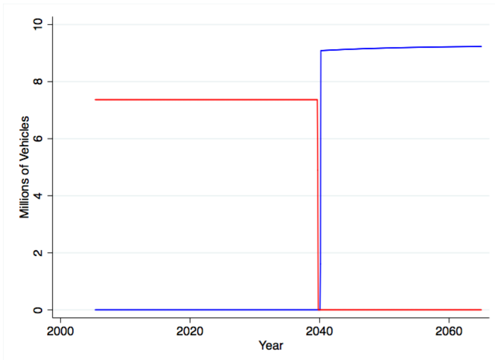


fig-prop1

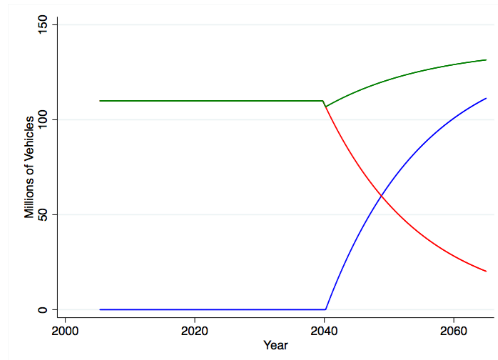
Figure 2: Parameter Combinations Implying a Ban on Gasoline Vehicle Production

$\gamma = 0$, indicating that electric vehicles are perfect substitutes for gasoline vehicles. Panel (a) shows that, as expected from Proposition 4, there is a gap in production of vehicles around the year 2040, although it is quite short. Panel (b) shows that during the gap, the total stock of vehicles (green line) decreases, but then rebounds when electric production (blue line) begins. Panels (c) and (d) increase γ to 0.0005. This parameter combination implies a simultaneous solution in which gasoline vehicle production is still banned eventually. Electric vehicle production begins around 2035, and there is a period of about five years in which both electric and gasoline vehicles are produced. Gasoline vehicle production ends about five years later, despite the fact that electric vehicles are not perfect substitutes. When electric vehicle production begins and when gasoline production ends, there are jumps in vehicle production, but the overall stock of vehicles increases smoothly. Panels (e) and (f) increase the value of γ to 0.0006. This case illustrates the simultaneous solution in which gasoline vehicle production is never banned. There is a large jump in gasoline vehicle production when electric vehicle production begins, but the overall stock of vehicles increases smoothly.

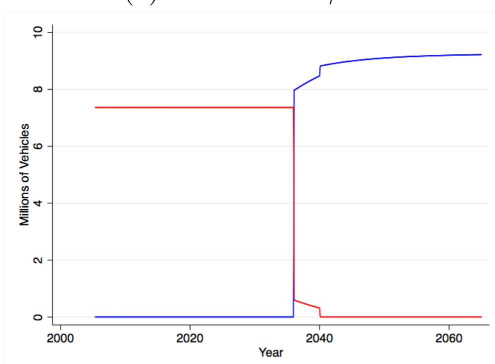
We next analyze the transition times for a broader range of values for γ and η. We focus on parameter values such that $g(\infty) = 0$ in the terminal steady state. Essentially, we repeat



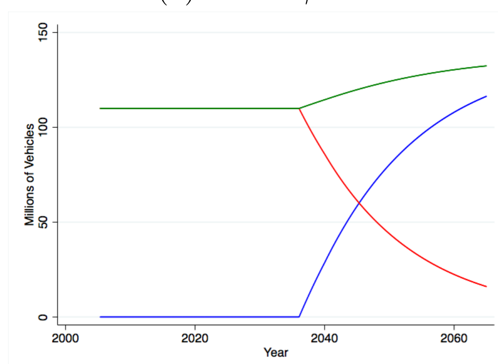
(a) Production $\gamma = 0$



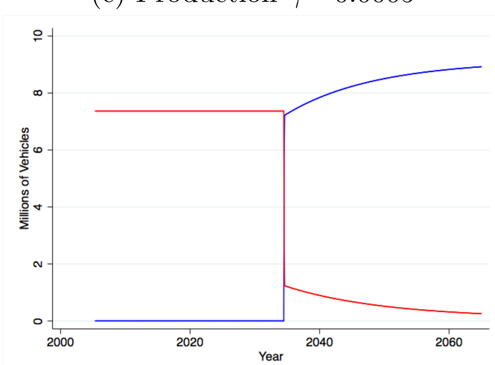
(b) Stocks $\gamma = 0$



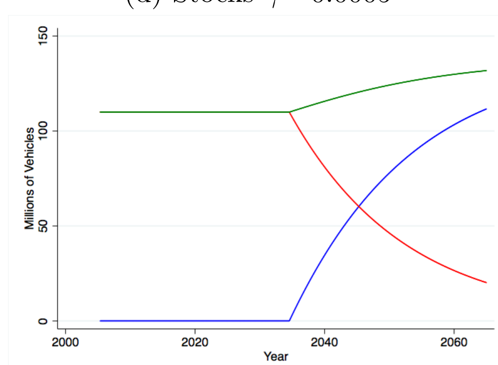
(c) Production $\gamma = 0.0005$



(d) Stocks $\gamma = 0.0005$



(e) Production $\gamma = 0.0006$



(f) Stocks $\gamma = 0.0006$

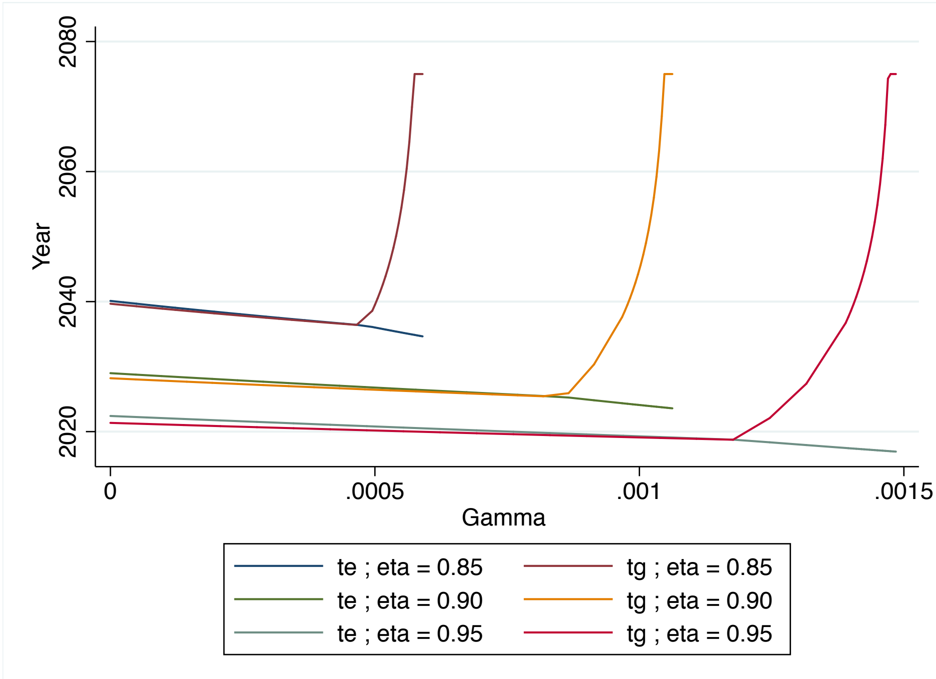
fig-gamma0

Figure 3: Production and Stocks for various values of γ holding $\eta = 0.85$
Notes: Red is gasoline vehicles, blue is electric vehicles, and green is total vehicles.

the analyses shown in Figure 3 for different parameter values and then catalog the resulting transition times. These results are shown in Figure 4. The red and blue curves show the values for t^g and t^e corresponding to the baseline value of $\eta = 0.85$. There are three different regions represented as a function γ. For small values of γ we have the gap solution. Here $t^g < t^e$, although for practical purposes the period of time in which there is no production is essentially zero. As γ increases, this at first leads to slightly earlier transition times within the gap solution. But eventually we cross a threshold in which we move from the gap solution to the simultaneous solution (the red curve moves above the blue curve.) Here increases in γ lead to much later transition times for gasoline vehicles, and in fact only a small increase in γ is needed for the the transition time for gasoline vehicles t^g to increase dramatically. Finally, we reach the final region in which gasoline vehicles are never banned. The threshold of this region is indicated by the point at which the blue and red curves stop. For all values of γ, the point at which electric vehicle production begins is around 2040 (approximately 35 years from the starting year). Also shown in Figure 4 is an alternative set of results for two other values of η. As η increases, the intensity of preferences for electric vehicles increases, and hence the value of t^e decreases and electric vehicles are produced earlier, even before 2020 in some cases. The sensitivity of the transition times to γ is quite similar for the different values of η.

Our framework also allows us to analyze the business as usual (BAU) time path of electric vehicle adoption. This time path corresponds to how market forces ignoring the externality would respond to changes in vehicle costs. To determine this, we simply eliminate the externalities from the planner's problem and solve the resulting problem for the optimal production over time. As before, we first look at the production and vehicle stocks for a given set of parameters and then look at a figure that catalogs the transition times for many parameter values.

Figure 5 compares the first-best and BAU vehicle production and stocks for $\eta = 0.85$ and $\gamma = 0.0005$. In panel (a), the blue and red curves simply reproduce the first-best results from panel (c) in Figure 3. Relative to first best, the BAU production of electric vehicles (purple curve) starts about five years later and has lower production. Also notice that the BAU solution does not lead to zero production of gasoline vehicles (black curve) in the terminal

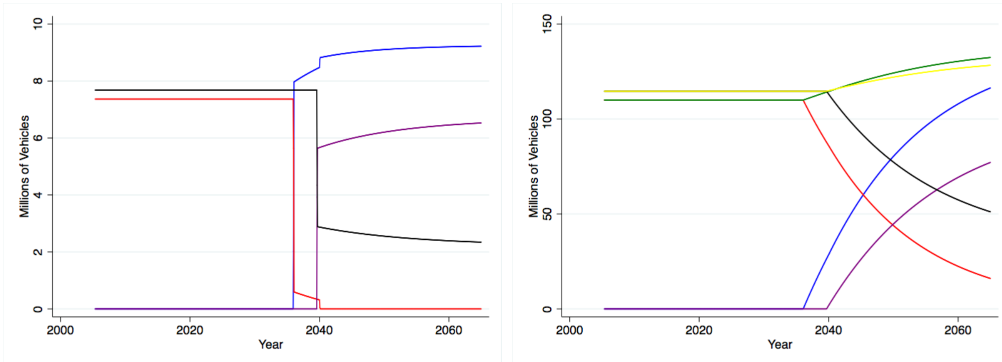


-aggregate

Figure 4: Transition times as a function of utility parameters

steady state. Even with these differences in production, the stocks of vehicles are similar, as shown in Panel (b). Notice that the BAU solution has a greater initial steady state stock of gasoline vehicles than first best. This is because the BAU solution ignores the externalities from gasoline vehicles. In contrast, the BAU solution has a smaller stock of total vehicles the terminal steady state. This is because the BAU solution has a mixture of gasoline and electric vehicles in the terminal steady state. Therefore the usage cost of the fleet is a mixture of the usage cost of gasoline and electric vehicles. In the first best solution, the usage cost of the fleet is simply the usage cost of the electric vehicles. Thus the fleet usage cost is lower than in BAU, so there are more vehicles in the first best terminal steady state than in the BAU steady state.

First best and BAU transition times are compared in Figure 6 for a variety of values for γ and with $\eta = 0.85$. The BAU curves are in the upper left of the figure, and the first best curves (reproduced from Figure 4) are on the right below. We see that for small values of γ (less than 0.0002), the market would eventually adopt electric vehicles and reduce production of gasoline vehicles to zero, although about 15 years later than in the first best solution. Next, there are parameter combinations (around $\gamma = 0.0002$) in which the market



(a) Production (b) Stocks

fig-bau

Figure 5: First Best vs. BAU: Production and Stocks

Notes: Red (gasoline vehicles), blue (electric vehicles), and green (total vehicles) are identical to the curves in Figure 3 panels (c) and (d). The other colors are BAU.

has a simultaneous solution but the first best has a gap solution. For these parameter combinations, the market would keep gasoline vehicles in production for several decades more than the first best solution. Next, there are parameter combinations (γ from about 0.0002 to 0.0005) in which the market would never eliminate gasoline production but the first best solution would. And finally there are parameter combinations (γ above 0.0005) in which neither solution eliminates gasoline vehicle production.

4.3 Extensions: Learning by doing and charging infrastructure

Next we consider the effects of learning by doing in the production of electric vehicles. The specification of the learning by doing function is described in Online Appendix E. [15] Once again we consider the case in which $\eta = 0.85$ and $\gamma = 0.0005$. The blue and red curves in panel (a) are the original solution without learning by doing. Adding learning by doing leads to the production of electric vehicles about a decade earlier. The production of electric vehicles with learning by doing (purple line) has a large spike at t^e. This spike helps drive down costs on subsequent production. Also notice that the learning by doing solution features a gap in production, whereas the original solution has simultaneous production of both vehicles.

The next set of results analyzes investment in charging infrastructure. We assume that charging infrastructure makes electric vehicles better substitutes for gasoline vehicles by

[15] The function is $c_x = c_g + 60 * 351.95e^{-0.0641t - 0.160 \ln Z}$ where Z is cumulative production of electric vehicles.

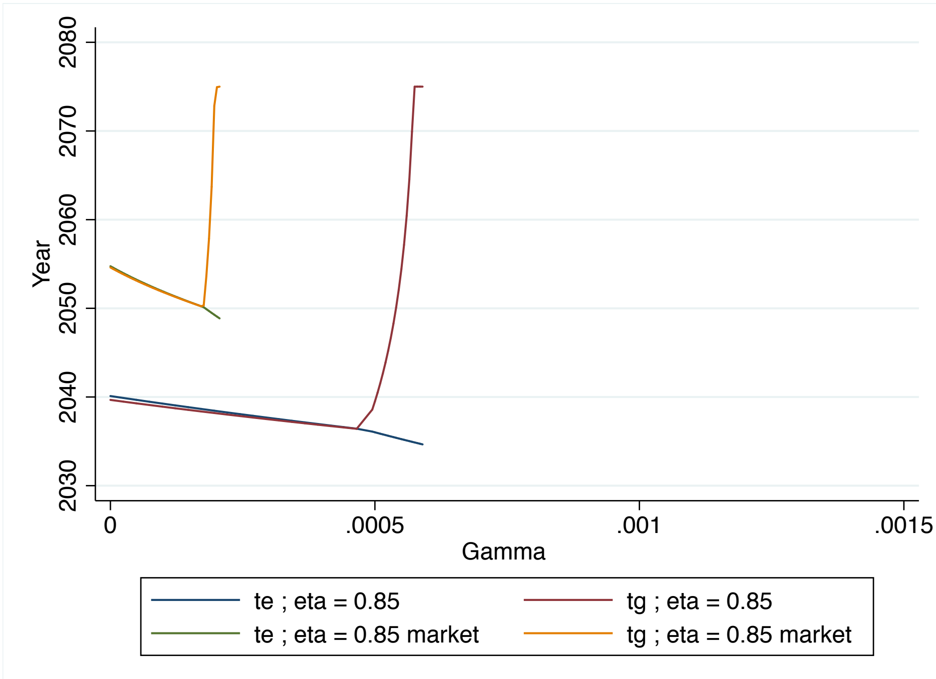


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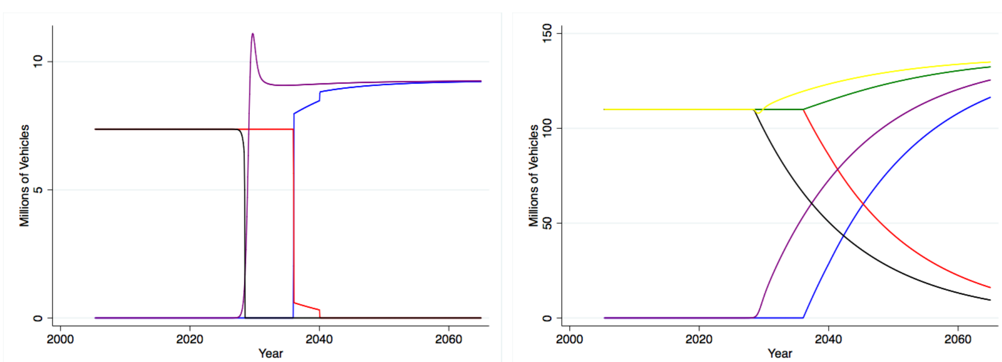
Figure 6: First best versus BAU: Transition times



(a) Production (b) Stocks

fig-lbd

Figure 7: Effect of adding Learning by Doing

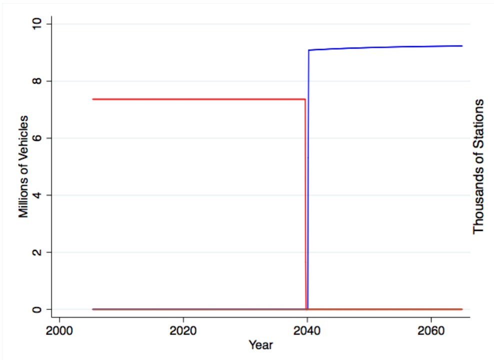
Notes: Red (gasoline vehicles), blue (electric vehicles), and green (total vehicles) are identical to the curves in Figure 3 panels (c) and (d). The other colors add learning by doing.

increasing η from 0.85 to 1 while holding $\gamma = 0$. Thus the effect of charging stations is to increase the relative preference for electric vehicles. Details of this specification are given in Online Appendix H. In this calibration, the interest rate is positive and the upper bound on production of charging stations is not binding, so Proposition 5 implies that charging stations should only be built after production of electric vehicles has started. The results are shown in Figure 8 for high, medium, and low costs of charging infrastructure. When charging infrastructure is very expensive, as shown in panels (a) and (b), its cost outweighs the increased benefits of electric vehicles and hence no stations are built. As the costs decrease, more are built. In Panel (c) and (d), the transition time for electric vehicles t^e occurs well before the first charging station is built. And, when charging stations do start to be constructed, there is a spike in the production of electric vehicles. As charging stations become even more inexpensive, as shown in panel (e) and (f), charging stations are introduced closer to t^e and the electric vehicle production spike becomes more pronounced.

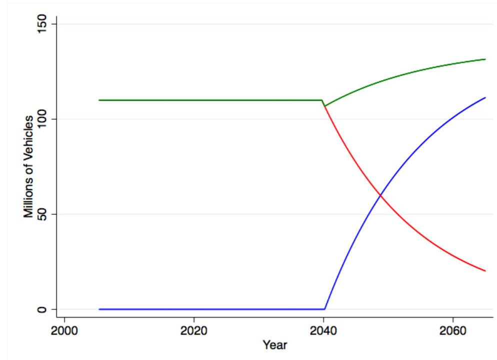
4.4 Subsidies

We conclude the results section by analyzing electric vehicle purchase subsidies. In the absence of Pigouvian taxation of all externalities, subsidizing electric vehicle purchases can improve welfare. In our base calibration the production cost (and hence price path) of an electric vehicle is $c_x = c_g + 21961e^{-0.06t}$. If consumers ignore externalities in their driving, a subsidy could modify this price path to potentially improve welfare. For example, a constant subsidy ψ would make the new price path $\tilde{c}_x = c_g - \psi + 21961e^{-0.06t}$. Whether this subsidy would increase or decrease welfare depends on the level of the subsidy ψ. However, we can choose the subsidy ψ optimally to maximize the welfare gain from this simple subsidy. In fact, for any price path, $\tilde{c}_x$, which depends on a few parameters we can follow a similar procedure to optimize the price path conditional on the functional form. The implied subsidy would then be the difference between c_x and $\tilde{c}_x$. In the limit, with sufficient parameters and functional form flexibility, we could approximate the second-best subsidy with this procedure.

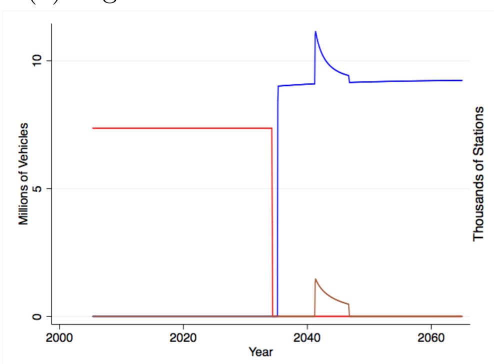
Table 2 shows the optimized parameters, deadweight loss, and transition times for five optimized price paths with different implied subsidies. The first row shows the BAU market outcome, i.e., zero subsidy. Relative to first best, BAU has deadweight loss of \$29.3 billion



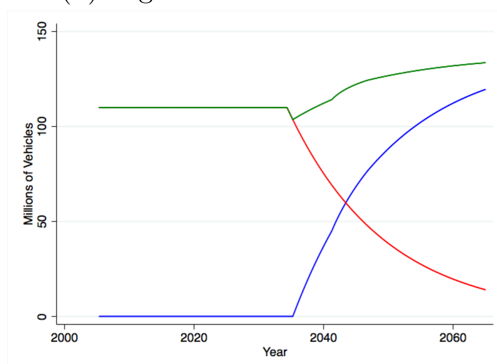
(a) High Cost Stations: Production



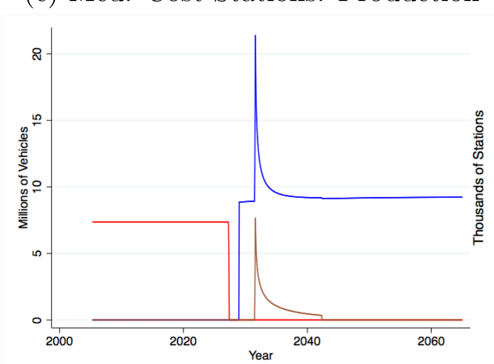
(b) High Cost Stations: Stocks



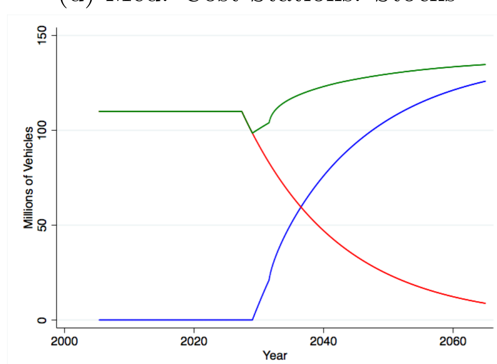
(c) Med. Cost Stations: Production



(d) Med. Cost Stations: Stocks



(e) Low Cost Stations: Production



(f) Low Cost Stations: Stocks

fig-charge

Figure 8: Production and Stocks with Charging Infrastructure Investments
Notes: Red is gasoline vehicles, blue is electric vehicles, green is total vehicles, and brown is charging infrastructure.

Table 2: Deadweight Loss and Transition Times for Optimized Purchase Subsidies

ab-subsidy

Subsidy Type	Price Path Formula for $\tilde{c}_x$	Optimal Parameter	Deadweight Loss ($ billions)	Transition Time t^e
BAU	$c_g + 21961e^{-0.06t}$	n.a.	29.256	49.75
Constant	$c_g - \psi + 21961e^{-0.06t}$	$\psi^* = 1482$	14.387	35.51
Initial Price	$c_g + \psi e^{-0.06t}$	$\psi^* = 9600$	14.118	35.05
Decay Rate	$c_g + 21961e^{-\psi t}$	$\psi^* = 0.084$	14.140	35.14
Two Parameter	$c_g + \psi_1 e^{-\psi_2 t}$	$\psi_1^* = 8027$ $\psi_2^* = 0.055$	14.114	35.04

Notes: Benefit function assumes $\gamma = 0$ and $\eta = 0.85$.

(about $400 million per year) and $t^e = 49.75$, which is 2055. The second row shows the optimized constant subsidy. The optimal constant subsidy of $1482 reduces the deadweight loss to about $14.4 billion (cuts the deadweight loss in half) and decreases the transition time to 35.51. This transition time is only slightly later than the first best transition time of 35.10. The third through fifth rows have time varying subsidies. In the third row, "Initial Price", the subsidy starts at $21961 - \psi$ and exponentially decays to zero over time. The optimal value of ψ is $9600, which implies that the initial subsidy is $12,361, much higher than the constant subsidy of $1482. This falling subsidy reduces deadweight loss further to $14.1 billion, again about half the BAU deadweight loss, and has a transition time close to the first best. The fourth row considers a subsidy that declines at the different rate than the baseline decline in c_x, and the last row optimizes over both the initial price and the decay rate. All of the optimized subsidies reduce the deadweight loss by about 50% relative to BAU and lead to electric vehicle adoption times very close to the first best time of 2040. Interestingly, adding a second degree of freedom leads to only a trivial decrease in the deadweight loss suggesting that there may not be substantial efficiency gains from introducing additional flexibility into the subsidy.

5 Conclusion

The small parameter ranges for which the simultaneous solution exists illustrates a fundamental tension in the decision to ban gasoline vehicle production. If such a ban is ever justified in the long run, then it must be the case that electric vehicles are fairly good substitutes for gasoline vehicles. But, if they are indeed good substitutes, then this suggests that the gap solution is likely to occur. Hence gasoline vehicles are banned quite early, before production of electric vehicles commences! There is only a narrow range for parameters that yields the much more intuitively appealing simultaneous solution. In this solution, gasoline and electric vehicles are both produced for a period of time before gasoline vehicles are banned.

Within the simultaneous solution, the optimal time to ban gasoline vehicles is quite sensitive to the consumer preferences for the two types of vehicles. A small decrease in the substitutability of electric vehicles can increase the optimal time to ban gasoline vehicles by a decade or more. A similar dynamic occurs in the BAU outcome, although the BAU outcome generally leads to the termination of gasoline production much later than in the first best solution. This suggests that optimal policy will depend critically on accurate assessment of consumer preferences for electric vehicles.

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Online Appendices

A Planners Problem

planprob

Consider a finite horizon version of the planner's problem with terminal time T. We will take the limit as T goes to infinity as needed to determine the features of the infinite horizon problem. The salvage value of a gasoline or electric vehicle at terminal time is c_g or $\hat{c}_x$ (in current value.) The planner's problem is

$$\begin{aligned}
\max_{g,x} \quad & c_g e^{-rT} G(T) + \hat{c}_x e^{-rT} X(T) + \int_0^T e^{-rt} \left(U(G,X) - c_g g - c_x x - \delta_g G - \delta_x X \right) dt \\
\text{s.t.} \quad \dot{G} &= -aG + g \ ; \ G(0) = G^{ss} \\
\dot{X} &= -aX + x \ ; \ X(0) = 0 \\
g &\geq 0, \\
x &\geq 0
\end{aligned}$$

where G^{ss} is the initial steady state stock of gasoline vehicles.

Let $\tilde{\alpha}$ and $\tilde{\beta}$ be the adjoint variables corresponding to the system equations for G and X. The Hamiltonian is

$$H = \tilde{\alpha}(-aG + g) + \tilde{\beta}(-aX + x) + e^{-rt} \left(U(G,X) - c_g g - c_x x - \delta_g G - \delta_x X \right).$$

From the Maximum Principle[16] the necessary conditions for the optimal control are the state equations, the initial conditions, and

$$\begin{aligned}
-\dot{\tilde{\alpha}} + \tilde{\alpha} a - e^{-rt} \left(U_G - \delta_g \right) &= 0 \qquad \text{(adjoint equations)} \\
-\dot{\tilde{\beta}} + \tilde{\beta} a - e^{-rt} \left(U_X - \delta_x \right) &= 0 \\
\tilde{\alpha}(T) &= c_g e^{-rT} \qquad \text{(adjoint final conditions)} \\
\tilde{\beta}(T) &= \hat{c}_x e^{-rT},
\end{aligned}$$

In addition, the controls g and x must maximize the Hamiltonian subject to the nonnegativity constraints. Because the Hamiltonian is linear in the controls, we use the Kuhn-Tucker

[16] See for example Kamien and Schwartz (2012).

necessary conditions for the controls:

$$g \geq 0 \quad \tilde{\alpha} - e^{-rt}c_g \leq 0 \quad (\tilde{\alpha} - e^{-rt}c_g)g = 0 \quad \text{(necessary condition for } g\text{)}$$
$$x \geq 0 \quad \tilde{\beta} - e^{-rt}c_x \leq 0 \quad (\tilde{\beta} - e^{-rt}c_x)x = 0 \quad \text{(necessary condition for } x\text{)}$$

If the control g satisfies $g > 0$, then it must be the case that $\tilde{\alpha} - e^{-rt}c_g = 0$. Conversely, if $\tilde{\alpha} - e^{-rt}c_g < 0$, it must be that $g = 0$. If $\tilde{\alpha} - e^{-rt}c_g = 0$ then any g such that $g \geq 0$ maximizes the Hamiltonian.

Using the change of variables $\alpha = e^{rt}\tilde{\alpha}$ and $\beta = e^{rt}\tilde{\beta}$ (i.e. current values instead of present values) the adjoint equations become

$$\dot{\alpha} = (a + r)\alpha - U_G + \delta_g \ ; \alpha(T) = c_g.$$

$$\dot{\beta} = (a + r)\beta - U_X + \delta_x \ ; \beta(T) = c_x$$

The necessary conditions for interior g and x become

$$g \geq 0 \quad \alpha - c_g \leq 0 \quad (\alpha - c_g)g = 0 \quad \text{(necessary condition for } g\text{)}$$
$$x \geq 0 \quad \beta - c_x \leq 0 \quad (\beta - c_x)x = 0 \quad \text{(necessary condition for } x\text{)}$$

B Proofs

proofss

Proof of Lemma 1

Before proving the propositions, we state the following lemma, which allows us to solve for the adjoint variable for gasoline vehicles from the adjoint equation:

lemmaDEq **Lemma 1.** *The adjoint equation (1) for gasoline vehicles is solved by the function*

$$\alpha(t) = e^{(a+r)t}\left[\int_{t_0}^{t} e^{-(a+r)\tau}[\delta_g - U_G(G(\tau), X(\tau)]d\tau + K\right] \tag{A-1}$$

eq:DEqSol

for an arbitrary constant K.

With an initial condition $\alpha(t_0) = \alpha_0$, the adjoint equation is solved by

$$\alpha(t) = e^{(a+r)t}\left[\int_{t_0}^{t} e^{-(a+r)\tau}[\delta_g - U_G(G(\tau), X(\tau))]d\tau + \alpha_0 e^{-(a+r)t_0}\right] \tag{A-2}$$

eq:alphaIn

With a terminal condition, the adjoint equation is solved by

$$\alpha(t) = e^{(a+r)t}\int_{t}^{\infty} e^{-(a+r)\tau}[U_G(G(\tau), X(\tau)) - \delta_g]d\tau. \tag{A-3}$$

eq:alphaTe

Proof. The adjoint equation (1) is a first-order differential equation of the form

$$\dot{\alpha} - (a + r)\alpha = f(t)$$

where $f(t) = \delta_g - U_G(G(t), X(t))$ is a function of t. Using the integrating factor method, the solution to a differential equation of this form, which can be easily verified, is

$$\alpha(t) = e^{(a+r)t}\left[\int_{t_0}^{t} e^{-(a+r)\tau} f(\tau)d\tau + K\right],$$

where K is an arbitrary constant.

If $\alpha(t_0) = \alpha_0$, we set (A-1) equal to α_0 and solve for K, which implies $K = \alpha_0 e^{-(a+r)t_0}$. Substitution in (A-1) yields (A-2).

For the terminal condition, instead of using the transversality conditions, we use the terminal condition $\alpha(T) = \alpha_1$ with an arbitrary end period T. We then determine the constant K_T with this arbitrary end period and take the limit of K_T as $T \to \infty$. With this terminal condition in (A-1) we can solve for K_T which yields

$$K_T = \alpha_1 e^{-(a+r)T} - \int_{t_0}^{T} e^{-(a+r)\tau}[\delta_g - U_G(G(\tau), X(\tau))]d\tau.$$

We then take the limit as $T \to \infty$ of K_T and substitute the result into (A-1) which gives

$$\alpha(t) = e^{(a+r)t}\left[\int_{t_0}^{t} e^{-(a+r)\tau}[\delta_g - U_G(G(\tau), X(\tau)]d\tau - \int_{t_0}^{\infty} e^{-(a+r)\tau}[\delta_g - U_G(G(\tau), X(\tau))]d\tau\right]$$

which simplifies to (A-3). Note that the solution does not depend on the arbitrary constant α_1. □

Proof of Proposition 1

Proof. To prove the condition for banning gasoline vehicles, we show that the solution satisfies the first order conditions with $g^\infty = 0$. As $t \to \infty$, if $g = 0$ and x is such that (3) is satisfied, then for some T_1 we have $(G, X) \approx (0, X^*)$. We can then evaluate the adjoint variable using (A-3) from Lemma 1, which shows that for $t > T_1$

$$\alpha(t) = e^{(a+r)t} \int_t^\infty e^{-(a+r)\tau} [U_G(G(\tau), X(\tau)) - \delta_g] d\tau.$$

$$\approx e^{(a+r)t} \int_t^\infty e^{-(a+r)\tau} [U_G(0, X^*) - \delta_g] d\tau.$$

$$= (U_G(0, X^*) - \delta_g)/(a + r) < c_g$$

which implies $\alpha(t) < c_g$. Together with $g = 0$, the remaining first order condition is satisfied.

A proof by contradiction demonstrates the condition under which gasoline vehicles are not banned. Suppose $g^\infty = 0$ which implies that $g(t) = 0$ for all $t > T_1$ for some T_1. But this implies that for some T_2 with $t > T_2 > T_1$ we have $G \approx 0$ and $X \approx X^*$ so that $U_G(G, X) \approx U_G(0, X^*)$ where $\approx$ means "arbitrarily close to", i.e., within an ϵ-ball. Again using (A-3) from Lemma 1, we have that for $t > T_2$

$$\alpha(t) = e^{(a+r)t} \int_t^\infty e^{-(a+r)\tau} [U_G(G(\tau), X(\tau)) - \delta_g] d\tau.$$

$$\approx e^{(a+r)t} \int_t^\infty e^{-(a+r)\tau} [U_G(0, X^*) - \delta_g] d\tau.$$

$$= (U_G(0, X^*) - \delta_g)/(a + r) > c_g$$

which contradicts the first order condition $\alpha \le c_g$.

□

Proof of Proposition 2

Proof. To characterize t^e note that $x > 0$ over the interval $[t^e, t^g]$ so (3) must hold including at t^e. But at t^e we have $G(t^e) = G^{ss}$ and $X(t^e) = 0$. Substituting these into (3) $U_X(G^{ss}, 0) = (a+r)c_x(t^e) + \delta_x(t^e) + \dot{c}_x(t^e)$ which can then be solved for t^e and characterizes t^e. (4) follows directly.

To characterize t^g, we focus on the adjoint variable α. During $[t^e, t^g]$, we have g interior, so that $\alpha = c_g$. After t_g, α evolves according to the adjoint equation (1) which we can solve using (A-3) from Lemma 1 to have

$$\alpha(t) = e^{(a+r)t} \int_t^\infty e^{-(a+r)\tau}[U_G(G^{no}(\tau), X^{no}(\tau)) - \delta_g]d\tau. \qquad \text{(A-4)}$$ eq:alpha

But at t^g, we have $\alpha(t^g) = c_g$ which implies the result

$$c_g = \int_{t^g}^\infty e^{-(a+r)(\tau - t^g)}(U_G(G^{no}(\tau), X^{no}(\tau)) - \delta_g)d\tau.$$

□

Proof of Proposition 3

Proof. After t^g, the gasoline vehicle stock is simply $G(t) = G^{ss}e^{-a(t-t^g)}$. So at t^e we have

$$U_X(G^{ss}e^{-a(t^e - t^g)}, 0) = (a+r)c_x(t^e) + \delta_x(t^e) - \dot{c}_x(t^e)$$

This equation is a function of both t^e and t^g, so we need another equation to pin down the transition times.

The other equation comes from the continuity of $\alpha(t)$. For $t \in [0, t^g]$, we have $\alpha(t) = c_g$. For $t \in [t^g, t^e]$ and for $t \in [t^e, \infty]$, $\alpha(t)$ evolves according to two different differential equations, which we can solve using Lemma 1. Continuity at t^e gives the additional equation.

For the interval $[t^g, t^e]$, the adjoint equation can be solved using (A-2) from Lemma 1 with the initial condition $\alpha(t^g) = c_g$ as

$$\alpha(t) = e^{(a+r)t}\left[\int_{t^g}^{t} e^{-(a+r)\tau}[\delta_g - U_G(G^{ss}e^{-a(\tau-t^g)}, 0)]d\tau + c_g e^{-(a+r)t^g}\right] \tag{A-5}$$ eq:Gap1

for the interval $[t^g, t^e]$

For the interval $[t^e, \infty)$, the adjoint equation can be solved using (A-3) from Lemma 1 as

$$\alpha(t) = e^{(a+r)t}\left[\int_{t}^{\infty} e^{-(a+r)\tau}\left[U_G(G^{ss}e^{-a(\tau-t^g)}, X^{gap}(\tau)) - \delta_g\right]d\tau\right] \tag{A-6}$$ eq:Gap2

for $t \geq t^e$. Since (A-5) and (A-6) must both hold at t^e, we have the result

$$c_g = \int_{t^g}^{t^e} e^{-(a+r)(\tau-t^g)}(U_G(G^{ss}e^{-a(\tau-t^g)}, 0) - \delta_g)d\tau + \int_{t^e}^{\infty} e^{-(a+r)(\tau-t^g)}\left[U_G(G^{ss}e^{-a(\tau-t^g)}, X^{gap}(\tau)) - \delta_g\right]d\tau$$

□

Proof of Proposition 4.

Proof. First we show that both g and x cannot be interior during the same time interval. For $U(G, X) = u(G + \eta X)$, we have $U_G = u'$ and $U_X = u'\eta$ If g is interior, (2) can be written $u' = (a + r)c_g + \delta_g$. Similarly, if x is interior, (3) can be written $u'\eta = (a + r)c_x + \delta_x - \dot{c}_x$. Combining these implies

$$\eta[(a + r)c_g + \delta_g] = (a + r)c_x(t) + \delta_x(t) - \dot{c}_x(t).$$

Since the left side of this equation is constant but the right side is decreasing, both g and x cannot be interior over an open interval, which implies $t^g \leq t^e$.

We now show that $t^g < t^e$. Suppose that $t^g = t^e$. Because g is interior, $\alpha = c_g$ on the interval $[0, t^g]$. In particular, $\alpha(t^g) = c_g$. However, using (A-3) from Lemma 1, we have

$$\alpha(t) = e^{(a+r)t}\int_{t}^{\infty} e^{-(a+r)\tau}[u'(G(\tau) + \eta X(\tau)) - \delta_g]d\tau$$

when we substitute in $U_G = u'$. Because $u'\eta = (a + r)c_x + \delta_x - \dot{c}_x$ after t^e, we have that u' is decreasing after t^e. This implies that

$$
\begin{aligned}
\alpha(t^g) &= e^{(a+r)t^g} \int_{t^g}^{\infty} e^{-(a+r)\tau} [u'(G(\tau) + \eta X(\tau)) - \delta_g] d\tau \\
&< e^{(a+r)t^g} \int_{t^g}^{\infty} e^{-(a+r)\tau} [u'(G(t^g)) - \delta_g] d\tau \\
&= e^{(a+r)t^g} [u'(G(t^g)) - \delta_g] \int_{t^g}^{\infty} e^{-(a+r)\tau} d\tau \\
&= e^{(a+r)t^g} [(a+r)c_g] e^{-(a+r)t^g} / (a+r) = c_g
\end{aligned}
$$

which is a contradiction.

$\square$

C Extensions

extend

Here we introduce learning by doing and investment in charging infrastructure. To account for charging infrastructure, the aggregate utility function becomes $U(G, X, W)$ where W is the stock of charging infrastructure and $U_W > 0$. Charging infrastructure grows based on investment, w, which costs c_w per unit and increases the stock according to the state equation $\dot{W} = w$. To account for learning by doing, we assume the cost for producing an electric vehicle depends on the cumulative number of electric vehicles produced, Z, which follows the state equation $\dot{Z} = x$. The cost per electric vehicle then becomes

$$c_x = f(Z),$$

where $f'(Z) \leq 0$ and $f(\infty) = \hat{c}_x$.

The planner's problem with terminal time T is

$$\begin{array}{rccl}
\max\limits_{g,x,w} & \multicolumn{3}{l}{c_g e^{-rT} G(T) + f(Z(T)) e^{-rT} X(T) + \int_0^T e^{-rt} \left(U(G,X,W) - c_g g - f(Z)x - c_w w - \delta_g G - \delta_x X \right) dt} \\
\text{s.t.} & \dot{G} & = & -aG + g \ ; \ G(0) = G^{ss} \\
& \dot{X} & = & -aX + x \ ; \ X(0) = 0 \\
& \dot{W} & = & w \ ; \ W(0) = 0 \\
& \dot{Z} & = & x \ ; \ Z(0) = 0 \\
& \bar{g} & \geq g \geq & 0 \\
& \bar{x} & \geq x \geq & 0 \\
& \bar{w} & \geq w \geq & 0.
\end{array}$$

Let $\tilde{\alpha}$, $\tilde{\beta}$, and $\tilde{\phi}$, and $\tilde{\lambda}$, be the adjoint variables corresponding to the system equations for G, X, W and Z. The Hamiltonian is

$$H = \tilde{\alpha}(-aG + g) + \tilde{\beta}(-aX + x) + \tilde{\phi} w + \tilde{\lambda} x + e^{-rt} \left(U(G,X,W) - c_g g - f(Z)x - c_w w - \delta_g G - \delta_x X \right).$$

From the Maximum Principle the necessary conditions for the optimal control are the state equations, the initial conditions, and

$$\begin{array}{rclr}
-\dot{\tilde{\alpha}} + \tilde{\alpha} a - e^{-rt} \left(U_G - \delta_g \right) & = & 0 & \text{(adjoint equations)} \\
-\dot{\tilde{\beta}} + \tilde{\beta} a - e^{-rt} \left(U_X - \delta_x \right) & = & 0 & \\
-\dot{\tilde{\phi}} - e^{-rt} U_W & = & 0 & \\
-\dot{\tilde{\lambda}} - e^{-rt} \left(-f'(Z)x \right) & = & 0 & \\
\tilde{\alpha}(T) & = & c_g e^{-rT} & \text{(adjoint final conditions)} \\
\tilde{\beta}(T) & = & f(Z(T)) e^{-rT} & \\
\tilde{\phi}(T) & = & 0, & \\
\tilde{\lambda}(T) & = & f'(Z(T)) X(T) e^{-rT} &
\end{array}$$

and, in addition, the controls g, x, and w maximize the Hamiltonian subject to the feasibility constraints. Because the Hamiltonian is linear in the controls, we use the Kuhn-Tucker necessary conditions:

$$
\begin{array}{llll}
g \geq 0 & \tilde{\alpha} - e^{-rt}c_g \leq 0 & (\tilde{\alpha} - e^{-rt}c_g)g = 0 & \text{(necessary condition for } g) \\
x \geq 0 & \tilde{\lambda} + \tilde{\beta} - e^{-rt}f(Z) \leq 0 & (\tilde{\lambda} + \tilde{\beta} - e^{-rt}f(Z))x = 0 & \text{(necessary condition for } x) \\
w \geq 0 & \tilde{\phi} - e^{-rt}c_w \leq 0 & (\tilde{\phi} - e^{-rt}c_w)w = 0 & \text{(necessary condition for } w)
\end{array}
$$

In other words, if the control g satisfies $\bar{g} > g > 0$, then it must be the case that $\tilde{\alpha} - e^{-rt}(c_g) = 0$. We also have the boundary conditions. For example, if $\tilde{\alpha} - e^{-rt}c_g < 0$ then $g = 0$ and if $\tilde{\alpha} - e^{-rt}c_g > 0$ then $g = \bar{g}$. Finally, we note that if $\tilde{\alpha} - e^{-rt}c_g = 0$ then any g such that $\bar{g} \geq g \geq 0$ maximizes the Hamiltonian.

Assume in what follows that T is long enough (and/or $Z(T)$ is big enough) so that $f(Z(T)) = \hat{c}_x$ and $f'(Z(T)) = 0$. Using the change of variables $\alpha = e^{rt}\tilde{\alpha}, \beta = e^{rt}\tilde{\beta}, \phi = e^{rt}\tilde{\phi}$, and $\lambda = e^{rt}\tilde{\lambda}$ (i.e. current values instead of present values) the adjoint equations become

$$\dot{\alpha} = (a + r)\alpha - U_G + \delta_g \ ; \alpha(T) = c_g$$

$$\dot{\beta} = (a + r)\beta - U_X + \delta_x \ ; \beta(T) = \hat{c}_x$$

$$\dot{\phi} = r\phi - U_W \ ; \ \phi(T) = 0.$$

$$\dot{\lambda} = r\lambda + f'(Z)x \ ; \lambda(T) = 0$$

The necessary conditions for g, x, and w become

$$
\begin{array}{llll}
g \geq 0 & \alpha - c_g \leq 0 & (\alpha - c_g)g = 0 & \text{(necessary condition for } g) \\
x \geq 0 & \lambda + \beta - f(Z) \leq 0 & (\lambda + \beta - f(Z))x = 0 & \text{(necessary condition for } x) \\
w \geq 0 & \phi - c_w \leq 0 & (\phi - c_w)w = 0 & \text{(necessary condition for } w)
\end{array}
$$

Next we show a few facts about the adjoint variables.

lem-pos **Lemma 2.** *1. For all $t \in [0, T]$ we have $\lambda \geq 0$ and $\phi \geq 0$.*

2. If x is interior, then $\dot{\beta} = -r\lambda$.

3. If x is zero for $t \in [0, t^s)$ and interior for $t \in [t^s, T]$ for some t^s, then $\beta(t^s) < f(0)$ and

$\beta \geq c_g$ for $t \in [t^s, T]$.

Proof. Suppose that at some point in time $\lambda < 0$. Because $f'(Z)x \leq 0$, it follows from the adjoint equation for λ that $\dot{\lambda} < 0$. Thus λ must continue to fall for the rest of the time period. But this is a contradiction with $\lambda(T) = 0$. The proof for ϕ is similar.

Next suppose that x is interior. Take the time derivative of (C). This gives

$$\dot{\lambda} + \dot{\beta} = f'(Z)x.$$

Using the adjoint equation for λ this implies

$$r\lambda + f'(Z)x + \dot{\beta} = f'(Z)x.$$

Simplifying gives the desired result that

$$\dot{\beta} = -r\lambda.$$

The fact that $\beta(t^s) < f(0)$ follows directly from (C) and the fact that $\lambda \geq 0$. To prove that $\beta \geq c_g$, we combine

$$\dot{\beta} = -r\lambda \leq 0.$$

with $\beta(T) = c_g$. $\square$

Learning By Doing

In this section, we focus learning by doing and ignore charging infrastructure. Furthermore, we assume that we have the representative consumer model so that aggregate utility is given by $U(G + \eta X)$. In the main text, we showed that such a utility function leads to the gap solution. The next proposition shows this result is robust to the having the cost of electric vehicles be determined by learning by doing.

Proposition 6. *Consider the model with learning by doing. Suppose that aggregate utility is given by $U(G + \eta X)$. Then the solution to the planner's problem has $t^g < t^e$.*

Proof. First we show that both g and x cannot be interior during the same time interval. Suppose both g and x are interior during some time interval. Because g is interior we have $\alpha = c_g$. Thus $\dot{\alpha} = 0$ and from the adjoint equation for α we have

$$\delta_g + (a + r)c_g = U'.$$

Because x is interior we have $\lambda + \beta = f(Z)$. Taking the time derivative gives

$$\dot{\lambda} + \dot{\beta} = f'(Z)x.$$

Substituting in from the adjoint equations for λ and β gives

$$r\lambda + f'(Z)x + (a + r)\beta - U'\eta + \delta_x = f'(Z)x.$$

Simplifying gives

$$r(\lambda + \beta) + a\beta + \delta_x = U'\eta. \tag{A-7}$$

keyforx

Substituting the value for U' from above and using (C) gives

$$rf(Z) + a\beta + \delta_x = (\delta_g + (a + r)c_g)\eta.$$

Taking the time derivative gives

$$rf'(Z)x + a\dot{\beta} + \dot{\delta}_x = 0 \tag{A-8}$$

fstep

From Lemma 2 we know that

$$\dot{\beta} = -r\lambda.$$

So we have

$$rf'(Z)x + \dot{\delta}_x = ar\lambda.$$

The first expression on the left-hand-side is non-positive and the second expression is negative. This implies that λ is negative, which contradicts Lemma 2.

So far we have shown that $t^g \leq t^e$. So we must rule out the case that $t^g = t^e$. Suppose that t^g does indeed equal t^e. In steady state with g interior, we have $\alpha = c_g$, thus $\dot{\alpha} = 0$, and hence $U' - (a+r)c_g - \delta_g = 0$. These equations hold at $t = t^g$. Because $t^g = t^e$, we are also producing electric vehicles with interior x. From (A-7) and (C) we have

$$r(f(Z)) + a\beta + \delta_x = U'\eta. \tag{A-9}$$

interxu

Taking the time derivative gives

$$\eta\dot{U}' = rf'(Z)x + a\dot{\beta} + \dot{\delta}_x.$$

Every term on the right hand side is negative, hence $\dot{U}'$ is negative. Thus marginal utility is decreasing over time.

Now consider some point in time $\tilde{t} = t^g + \varepsilon$. Because $\dot{\alpha} = 0$ at t^g, we have $\alpha = c_g$ at $\tilde{t}$. So at $\tilde{t}$, the adjoint equation for α is

$$\dot{\alpha} = -(U' - (a+r)c_g - \delta_g).$$

Because marginal utility is decreasing over time, U' is less at $\tilde{t}$ than it is at t^g. At t^g we have $U' - (a+r)c_g - \delta_g = 0$. So at $\tilde{t}$ we have $U' - (a+r)c_g - \delta_g < 0$. Hence at $\tilde{t}$ we have $\dot{\alpha} > 0$. So this implies α will become greater than c_g in the next time instant. But, from the necessary conditions, if $\alpha > c_g$ then it is optimal for g to be positive (equal to the maximum production level.) This contradicts the definition of t^g. □

Charging Infrastructure

In this section, we focus on charging infrastructure and ignore learning by doing. Aggregate utility $U(G, X, W)$ is a function of the stock of electric vehicles, gas vehicles, and charging stations. As W increases, electric vehicles become better substitutes for gasoline vehicles. We assume that $U_W(G, X, W) = 0$, which implies that the marginal utility of charging stations is zero when there are no electric vehicles. Let t^{w1} be the time at which the planner begins production of charging stations.

We now prove Proposition 5 in the main text.

Proof. We prove both statements by contradiction. To prove the first statement, suppose $t^{w1} \leq t^e$. By assumption at t^{w1}, we have w interior which implies that $\phi = c_w$. It follows that $\dot{\phi} = 0$. The adjoint equation then implies that $U_W(G, X, W) = rc_w$ at t^{w1}. But since $t^{w1} \leq t^e$, we have $X(t^{w1}) = 0$, so $rc_w = U_W(G(t^{w1}), X(t^{w1}), W(t^{w1})) = U_W(G(t^{w1}), 0, W(t^{w1})) = 0$ which is a contradiction because $r > 0$.

To prove the second statement, suppose $t^{w1} > t^e$. At t^e, we have $w = 0$. Thus $\phi \leq c_w$. Because $r = 0$, the adjoint equation for ϕ implies that $\dot{\phi} < 0$ for all t. But this contradicts the fact that at t^{w1} we must have $\phi \geq c_w$. $\square$

The next proposition shows that if the planner starts producing charging stations before electric vehicles, then there is a period of time in which charging stations are produced at the maximum rate.

lem-char **Proposition 7.** *If $t^{w1} < t^e$, then there is a period of time starting defined by $[t^{w1}, t^{w2}]$, with $t^{w2} > t^e$, such that charging infrastructure is at the maximum rate ($w = \bar{w}$) during this period.*

Proof. Production of charging infrastructure implies that $\phi \geq c_w$. At t^{w1}, we have $X = 0$, and thus $U_W = 0$. It follows from the adjoint equation for ϕ that $\dot{\phi} > 0$. Thus there exists a period of time $[t^{w1}, t^{w2}]$ for some t_{w2} such that $\phi > c_w$ and hence $w = \bar{w}$ in this period.

Next we argue that $t^{w2} > t^e$. If $t^{w2} = T$, then this is trivially true. If $t^{w2} < T$, then at t^{w2} production of charging infrastructure is less than $\bar{w}$. Thus $\phi(t^{w2}) \leq c_w$. But this implies that $\dot{\phi}$ has to become negative at some point in $[t^{w1}, t^{w2}]$, and the only way that can happen is for U_W to be positive, and this requires $X > 0$. Hence t^e must be in the interior of this period. □

D Emissions from gasoline vehicles

hisgas

The historical emissions from gasoline vehicles are given in Table C. Many of these values simply reflect the emissions regulations in place at various points in time. For 1975-2003, the NO_X standard comes from data in Mondt (2000). After 2003 the NO_X standard is average for Tier 2 bins phased in from 2004 to 2009. VOC emissions include tailpipe and evaporation. For 1975-2003, the tailpipe VOC emissions come from Lee et al (2010) and after 2003 tailpipe VOC emissions come from Tier 2 Bin 5. Evaporation VOC is fixed at the value specified in GREET 2013. $PM_{2.5}$ includes tailpipe emissions (Tier 2 bin 5 standard) and tire and break wear from GREET 2013. SO_2 emissions calculated from GREET 2013 and EPA (1999). Emissions of CO_2 are derived from fleet average MPG figures (EPA 2015, Table 9.1).

E Lithium ion battery prices over time

lithium

Data on prices and production of Lithium ion batteries comes from Kittner, Lill, and Kammen (2017) and is shown in Table D. All numbers in this section are expressed in 2015 dollars. (We convert to 2017 dollars to get the numbers used in the main text.)We assume that the cost premium of electric vehicles relative to gasoline vehicles is largely driven by battery prices. The general procedure for specifying c_x is to determine an initial cost premium in conjunction with a decay function that is a function of time and cumulative production. We assume that electric vehicles have a 60 KWh battery and convert the units of cumulative production into millions of vehicles (divide MWh by 60,0000) to be consistent with the rest of the parameters. We then estimate the following model for battery prices

$$\ln(\text{Price}) = constant + \alpha\text{Year} + \beta\ln(\text{Cumulative Production}) + \varepsilon.$$

Table C: Historical Emissions from Gasoline Vehicles (g/mile)

hisgastab

Model Year	CO_2	SO_2	$PM_{2.5}$	VOC	NO_X
1975	658	0.20881	0.0173	1.557	3.1
1976	596	0.18919	0.0173	1.557	3.1
1977	570	0.18071	0.0173	1.557	2
1978	526	0.16680	0.0173	1.557	2
1979	517	0.16390	0.0173	1.557	2
1980	444	0.14095	0.0173	1.557	2
1981	415	0.13173	0.0173	0.467	1
1982	400	0.12698	0.0173	0.467	1
1983	402	0.12756	0.0173	0.467	1
1984	397	0.12585	0.0173	0.467	1
1985	386	0.12257	0.0173	0.467	1
1986	375	0.11895	0.0173	0.467	1
1987	373	0.11845	0.0173	0.467	1
1988	369	0.11697	0.0173	0.467	1
1989	375	0.11895	0.0173	0.467	1
1990	381	0.12099	0.0173	0.467	1
1991	380	0.12047	0.0173	0.467	1
1992	385	0.12203	0.0173	0.467	1
1993	378	0.11996	0.0173	0.467	1
1994	381	0.12099	0.0173	0.307	0.6
1995	380	0.08192	0.0173	0.307	0.6
1996	381	0.08227	0.0173	0.307	0.6
1997	380	0.08192	0.0173	0.307	0.6
1998	380	0.08192	0.0173	0.307	0.6
1999	386	0.08334	0.0173	0.307	0.3
2000	388	0.04924	0.0173	0.307	0.3
2001	386	0.04903	0.0173	0.307	0.3
2002	385	0.04881	0.0173	0.307	0.3
2003	383	0.04860	0.0173	0.307	0.3
2004	385	0.02929	0.0173	0.132	0.07
2005	378	0.02159	0.0173	0.132	0.07
2006	381	0.00726	0.0173	0.132	0.07
2007	369	0.00702	0.0173	0.132	0.07
2008	366	0.00696	0.0173	0.132	0.07
2009	350	0.00666	0.0173	0.132	0.07
2010	344	0.00656	0.0173	0.132	0.07
2011	347	0.00661	0.0173	0.132	0.07
2012	328	0.00624	0.0173	0.132	0.07
2013	319	0.00606	0.0173	0.132	0.07
2014	319	0.00606	0.0173	0.132	0.07
2015	313	0.00596	0.0173	0.132	0.07

Table D: Price and Cumulative Production of Lithium Ion Batteries

lithprices

Year	Price (2015 Dollars per KWh)	Cumulative Production (MWh)
1991	5394.66	0.13
1992	4392.33	1.68
1993	3444.54	14.55
1994	2718.82	50.08
1995	2566.13	121.13
1996	1888.28	547.42
1997	1329.07	1257.9
1998	872.46	2288.09
1999	711.34	3815.63
2000	619.38	5982.59
2001	508.85	8504.79
2002	487.47	12092.71
2003	437.97	17350.26
2004	401.51	24526.11
2005	351.95	33371.58
2006	317.43	44916.87
2007	320.07	58806.74
2008	319.25	75616.08
2009	298.29	94954.08
2010	260.87	119308.1
2011	231.81	149029.1
2012	185.82	183816.1
2013	183.14	226555.1
2014	170.2	276384.1
2015	150	337871.1

Using OLS, we obtain $\alpha = -0.0641$ (Std. Err. 0.01) and $\beta = -0.160$ (Std. Err. 0.02). For the simulation, we start in year 2005. Under the assumption that costs are a function of time alone we get

$$c_x = cg + 60 * 351.95e^{-0.0641t}.$$

Adding learning by doing gives

$$c_x = cg + 60 * 351.95e^{-0.0641t-0.160\ln Z},$$

where Z is the cumulative production of electric vehicles.

F Operating costs for gasoline and electric vehicles

opcosts

The American Automobile Association (AAA) issues an annual report on the cost of driving. For 2017, the operating costs for the average gasoline vehicle are 18.18 cents per mile, or \$2726 dollars per year (assuming 15000 miles driven per year). For the first time in 2017, AAA determined operating costs for electric vehicles. The values are 10.23 cents per mile or

$1535 dollars per year (assuming 15000 mlles driven per year.)

G Stock decay

stockdecay

Over the period from 2012 to 2016, the average production of light duty vehicles is 16.3 million units and the average production of passenger cars (a subset of light duty vehicles) is 7.4 million.[17] The 2015 stock of light duty vehicles is 190 million.[18] Approximately 58 percent of all light duty production from 1976 to 2016 is passenger cars. Assuming that passenger cars and light trucks retired at same rate suggests that the stock of passenger cars is 110 million. In steady state the decay rate is the annual production divided by the stock. So we have

$$a = 7.4/110 = 0.067.$$

H Charging Infrastructure

charge

We model η and γ as a function of the stock of charging stations W. We have

$$\eta = 1 - (1 - \eta_o)e^{-k_1 W}$$

and

$$\gamma = \gamma_o e^{-k_2 W}.$$

In this formulation, η is equal to η_o when $W = 0$ and converges to 1 as W approaches infinity. Likewise, γ starts at γ_0 and approaches 0.

[17] See "light vehicle production.xlsx", produced by Bureau of Economics Analysis, 2017.

[18] https://www.rita.dot.gov/bts/sites/rita.dot.gov.bts/files/publications/national transportation statistics/html/table 01 11.html. Accessed Dec 18, 2017.

References for Online Appendices

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