Resolving Braess's Paradox through Information Design: Routing for Heterogeneous Autonomous Vehicles

Abstract

In the era of the Internet of Things (IoT), devices are all connected, which makes it possible to gather and share a vast amount of real-time data. Besides technology advancements, the way to distribute information is also critical to increase social benefits. In this paper, we look specifically into the information-based routing problem for heterogeneous autonomous vehicles. To address the issue, we synthesize theoretical economics and artificial intelligence (AI). In a fully autonomous transportation system, based on the idea of Bayesian persuasion introduced by Kamenica and Gentzkow (2011), its extension by Kolotilin et al. (2016) and the unified information design framework by Bergemann and Morris (2017), we propose a method to design the information structure conveyed to a vehicle based on its private preference, strategic interactions with other vehicles, expressway tolling, and other traffic conditions. In a simple road system, we present both a multi-vehicle model and a multi-period model. We also build up a framework to deal with urban traffic. To implement the method we propose, we introduce the idea of creating a hardware engine to accelerate the calculation and demonstrate it by a flow chart.

Keywords: Internet of Things; autonomous vehicles; information design; Bayesian persuasion

1 Introduction

In the era of the Internet of Things (IoT), we see remarkable advancements in robotics. The IoT concept has been integrated into robot systems that every robot can collect and communicate data to other connected devices and the hub. As real-time data processing becomes possible, robots are getting "smarter" and capable of performing tasks with a high degree of autonomy. Among all kinds of robots, autonomous vehicles have attracted much attention as the market keeps growing and a mass adoption can be expected in a decade. People are looking forward to a fully autonomous transportation system shortly. The host of 2024 Olympic Games, Paris, is likely to be the first post-car metropolis. It will transform into a city with no individual-driving vehicles. In the future Paris, driverless shuttles/taxis will be going up and down all day (Kuper, 2017). However, even with self-driving vehicles, traffic congestion can still be a huge problem if there no control or direction over the vehicles. One possible solution is to build up a tolling system. To deal with the severely congested traffic in New York City, a plan of reducing traffic through a redesigned system of tolls is under discussed (NYT Editorials, 2017). The idea is to use a pricing mechanism to discourage the entrance of cars and trucks to certain streets during peak business hours and decongest the main streets. Congestion pricing seems to be a viable tool to alleviate traffic jams. We believe a well-designed directions for autonomous vehicles in a tolling traffic system will bring enormous benefits to people.

One thing that is vital to support self-driving is the map. The map which is necessary for autonomous vehicles is totally different from the traditional static one telling us how roads are connected in a region. Self-driving requires a highly accurate 3D map. On top of it, a live map that knows the route conditions, traffic flows, and even the precise location and destination of each vehicle is necessary. It makes Waze, a crowdsourcing mapping app, critical since it is pervasive and equipped on each car. Waze collects the accurate location data of every vehicle, and as vehicles move, Waze obtains the real-time traffic updates and the traffic speeds. Besides, Waze users also actively contribute accidents and construction data to Waze. In the world of IoT, all devices are connected, which makes it possible for Waze to cluster and exchange comprehensive real-time traffic data. In addition to Waze, various firms are exploring the advantages of IoT. For instance, WeatherCloud provides weather data from the ground level which is powered by connected sensors attached to vehicles. This real-time data transmission is faster and more accurate compared to the traditional satellite data.

Given the large scale data benefitted from IoT, then the question is how to properly distribute information to robots leading to a more beneficial outcome. We see this problem being related to an old traffic paradox – Braess's Paradox. Braess's Paradox refers to the counterintuitive phenomenon that increasing the capacity of a road system can worsen the overall traffic congestions, or in other words, decreasing road capacity can improve the overall traffic performance. The idea behind the paradox is that, because of each player's selfish decision that does not take into account the strategies of others, by adding extra resources, although the best scenario can be enhanced, the equilibrium of the game can be worse. People may think Braess's Paradox will not frequently occur in the real world. However, studies have shown that, in a natural random network model, Braess's Paradox occurs with high probability. Furthermore, Roughgarden and Tardos (2002) and Roughgarden (2005) provide a quantitative measure of the inefficiency resulting from drivers selfish routing behavior, which is proved to be extremely bad under certain conditions.

The paradox was proposed in the 1960s and had been forgotten for a while. However, it happens in many aspects of our lives, not restricted to road systems. Especially nowadays, with technology advancement in the era of IoT, it can be found even more frequent when people are careless about how to distribute information. The popularization of traffic navigation apps, e.g., Waze, witnesses that the IoT technology can significantly boost the mobility of people in cities. However, as reported in [News], navigation apps use real-time traffic information to re-route vehicles to quiet residential areas, resulting in congestions in local roads and a disturbance to the community. That is to say, providing information recklessly to all drivers can hurt not only the drivers but also the residents, and thus, decreases the social welfare. Inspired by the new observations in today's world, we would like to provide a new information-based interpretation of Braess's Paradox. That is, providing additional information (which is equivalent to increasing road capacity) can hurt the benefits of parties involved. In other words, blocking information can improve the efficiency of a system.

Autonomous vehicles, based on the traffic information, choose the route with least travel time or achieve other goals defined by travelers. More generally speaking, robots all make their decisions on actions to take based on the information conveyed to them. In fact, if provided with different information sets, robots may make different choices even though the objectives are the same. For traffic congestion problems, Acemoglu et al. (2016) consider the informational Braess's paradox and address the question of whether additional information could harm the system efficiency. They find that under certain conditions, vehicles receiving additional information could choose a route with longer delay. In many other setting, these kind of informational paradox could occur. Especially in the era of IoT, the problem could be even more complicated with higher volume of real-time information. Thus, besides technology advance, information distribution design is also an important question in IoT, which will be the focus of this paper.

The existence of Braess's Paradox in the information-provision process calls for an information-based approach to resolve it. We believe one the most natural method is the information design framework stemmed from Bayesian persuasion by Kamenica and Gentzkow (2011) and its extension by Kolotilin et al. (2016), and proposed by Bergemann and Morris (2017). Intuitively, if distributing information inattentively may cause a problem, then we should research on how to design the information allocation mechanisms. We believe the information design framework gives some hints and there is an inherent link between this framework and Braess's Paradox. Our approach to address the question of information distribution, we synthesize theoretical economics and artificial intelligence (AI) by building up an information-based framework to direct autonomous vehicles. Though neoclassical economics are sometime criticized for its ideal assumption that decision makers are perfectly rational, we believe that economic theories are relevant as basis for robotic design. In the robotic field, AI researchers strive to build up autonomous entities that perceive the environment and act to achieve certain goals, which are nothing but the rational agents in economics. Thus, in this paper, we propose and apply an economic framework to direct autonomous vehicles, which exploits the traffic data crowd-sourced by Waze. Our approach of information design under tolling assumes that each vehicle makes rational road choices based on the (adjusted) information it receives. Thus, our model optimizes on the information structure delivered to autonomous vehicles in order to improve the efficiency of the transportation system.

To define the rational choices and better manage autonomous vehicles, or more broadly, robots, we introduce utility functions for robots as a high-level specification to regulate and direct their behaviors. We believe it is a proper approach for two reasons. Firstly, introducing utility functions for robots is helpful to build up a framework for regulations and resolve the concerns about robotic moral and legal issues. There have been heated debates on robots' moral and legal responsibilities. People in both the industry and the academia are calling for a code for robots (Thornhill, 2017). In early 2017, members of the European Parliament urged for a regulation draft including giving robots a form of "electronic personhood," which would make it easier to ensure robots' rights and responsibilities (Hern, 2017). Among all types of robots, autonomous vehicles are of particular interest since they have attracted tremendous R&D investments and will fundamentally change the transportation industry and people's everyday life soon. Thus, it is even more urgent to establish a regulation code to ensure that their development and implementation is in for the best interest of human beings.

Besides the moral and legal issues, in practice, introducing utility functions is also necessary to direct robots' optimal behavior from an economic point of view, which is the focus of our work. In the transportation area, congestion is an inevitable problem nowadays. For autonomous vehicles, passengers on mobility service vehicles may have various attitudes towards congestion. In particular, business travelers may have tight schedules whereas leisure travelers are fine with traffic jams. Similarly, for autonomous freight vehicles, supply chain managers may also have different attitudes since some vehicles may be required to arrive at their destinations within a short period, while others may not. To formally model these disparities, we assign a utility function for each driverless vehicle to characterize its tolerance towards delay as incorporated by its passengers or managers. Specifically, for the disutility of random travel latency, we assume a mean-variance form which is frequently used in finance (Luenberger et al., 1997). Since we can specify the distribution of parameters associated with the mean and the variance, this formulation can be adjusted to actual needs and cover various scenarios.

The objective of the study is to propose a framework to obtain the optimal information structure in real-time for heterogeneous autonomous vehicles given the traffic condition and prediction. It requires to build up a multi-period model based on traffic network that accounts for dynamic information updates and personalization based vehicle's private type to maximize the efficiency of the transportation system over time. Note that, it is critical to consider both the strategic interactions between vehicles that are close to each other and the future impact of routing decisions for current vehicles. Firstly, it has long been known as Braess' Paradox (Braess, Nagurney and Wakolbinger, 2005) that, without coordination, vehicles choosing the best route from their perspectives may end up with a socially suboptimal outcome. Also, as we deal with vehicles with various tolerances towards delay, a myopic optimal decision may be disadvantageous for later vehicles. For example, with the toll road having less traffic, directing some vehicles to the toll road may seem to be beneficial. However, when considering its impact on vehicles with tight schedules possibly coming in the future, it may be more advantageous to "sacrifice" current vehicles (especially if they are not urgent) and route them to the free road with longer waiting time.

We start to build up our theoretical framework by introducing the idea of information design based on a single vehicle model. We consider an open road structure with one toll road and one free road connecting a starting point and a destination. Based on the traffic condition and the

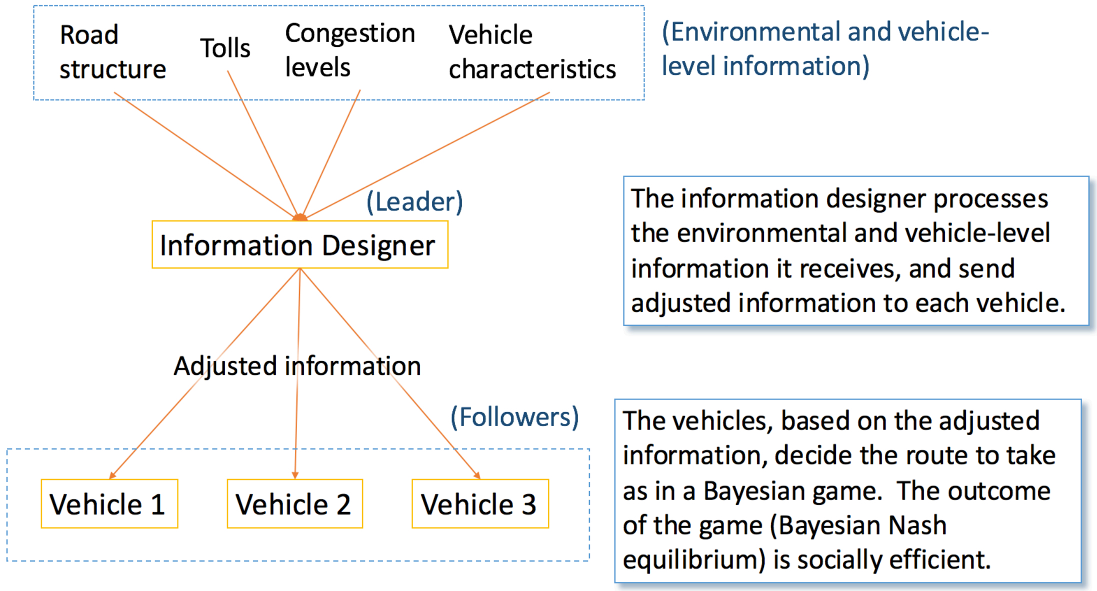

Figure 1: The model diagram

vehicle's reported preference (type), the information designer solves for the optimal information structure in the sense that, by receiving information conveyed to it and updating the prediction on traffic condition, the vehicle chooses the best road from its own perspective, which will also be socially optimal. Then, we extend our study to a multiple-vehicle model and consider the strategic interactions between vehicles. We assume several vehicles depart at almost the same time, and when selecting the road, will consider other vehicles' strategies. To coordinate vehicles' choices, the information designer finds the optimal information conveyed to each vehicle. The transformed information can be divergent even among the same-type vehicles since it may be necessary to differentiate the choices to achieve the socially optimal outcome. An illustration diagram is shown in Figure 1. Next, we formulate a multi-period model to study the dynamics of the information structure. A more practical model for urban traffic is built up, which considers the dynamic navigation update and the different levels of forecast accuracy. We also consider the extension to risk-averse vehicles. Instead of maximizing the expected utility, the vehicle also worries about the variance, and thus maximize a mean-variance utility function. To demonstrate the implementation of our framework in practice, we propose a hardware engine that could accelerate the calculation. As the accuracy of traffic prediction is the key to guarantee the optimality of the information structures, we also discuss the possibility of incorporating data from other sources (e.g., crowdsourced weather data) to better the forecast.

To realize the method we propose and achieve the efficiency improvement, specific institutional arrangements are necessary. Ideally, a government authority should be built up to manage the transportation-related affairs. The authority contracts with companies like Waze to obtain real-time traffic data. Autonomous vehicles are required to be equipped with the hardware engines which receive information exclusively from the authority. Then, as vehicles move on roads, hardware engines work as information designers based on the data transmitted by the authority to direct the vehicles. As discussed in Financial Times (2018), there is a rising concern that autonomous vehicles can be turned into "weapons" and governments should pay more attention to the autonomous vehicle security issues. There are various ways to improve the security. One possible way, as proposed in our paper, is to have a more centralized operation of the information distribution process. In addition, application-specific integrated circuits (ASICs) that are suggested in our paper are evaluated to be less in danger of attacks compared with field programmable gate arrays (FPGAs) and other integrated chips. A detailed discussion of security issues is in §8.5.

The remainder of the paper is organized as follows. §2 reviews the related literature. After that, we first present our single period models that capture the specification of heterogeneous autonomous vehicles and the key mechanisms of information design in §3. The advantages of the proposed methods are demonstrated in §4. Then, in §5, we extend our discussion to multi-period models. A more realistic urban traffic model is built in §6. A risk-averse version of the model is presented in §7. Various implementation-related topics are considered in §8. In §9, we conclude our paper with a list of topics that are worth further research.

2 Literature review

The real-time routing problem for autonomous vehicles through information design has been studied in Liu and Whinston (2017). By applying the idea of Bayesian persuasion (Kamenica and Gentzkow, 2011) in the basic model of a single vehicle and implement the unified information design framework (Bergemann and Morris, 2017) in the general model, we formulate a model to obtain the optimal information structures to eliminate "the price of anarchy" (Roughgarden and Tardos, 2002; Roughgarden, 2005)[1]. Kamenica and Gentzkow (2011) investigate a symmetric information model where a sender sends a signal to a receiver to persuade her to change her actions. This work, together with some other papers, is unified by Bergemann and Morris (2017) from an information

[1]It is a quantitative measure of the inefficiency resulting from drivers selfish routing behavior.

design perspective. A work that is closely related is Das, Kamenica and Mirka (2017). They study the problem of reducing congestion through information design based on a two-path example and the Wheatstone network. The idea of strategic information transmission has been studied in early economic papers (e.g., Crawford and Sobel, 1982). Except for our previous work, to the best of our knowledge, there is no application of strategic information design/distribution on routing autonomous vehicles or in cyber-physics area. In this paper, we extend Liu and Whinston (2017) by allowing vehicles to have private types associated with their utilities. Theoretically, the utility of each vehicle is a function of its own decision and type, decisions of other vehicles (if any) and the state of the world. We further assume that the type of a vehicle is unobservable to the information designer, and thus, a vehicle could report a type that differs from its true type. In fact, we allow vehicles to be "strategic" to some degree since the owners of vehicles who design the high-level specification of the robots could strategically ask their robots to "lie" when competing for public road resources with other (owners of) vehicles. From an economic perspective, we build up an information design model with elicitation in a traffic game when receivers have private information. The framework we build up stems from Kolotilin et al. (2016) who consider the persuasion mechanism for a sender who wants to influence the action of a single privately informed receiver. Their paper assumes a linear environment, and thus boundary conditions imply the obedience constraints. The framework deviates from Bayesian persuasion (Kamenica and Gentzkow, 2011) in the sense that, under elicitation, additional constraints to ensure truth-telling are necessary. It also deviates from Myerson (1981) as the notion of incentive compatibility guarantees no "double deviations" when receivers both disobey the recommendation and report untruthful private type. A perspective unifies existing works concerning information design in various settings can be found in Bergemann and Morris (2017). When applying the information design framework in the vehicle routing setting we are interested in, in addition to the single receiver (vehicle) model, we further formulate multiple-vehicle and multiple-period models to address the practical issues of traffic problems. For the multiple-vehicle model, an incentive compatibility constraint (truth-telling obedient constraint) is required for each vehicle, given it has taken into account the strategies of other vehicles. As for the multi-period model, the optimal information structure will be shaped to incorporate long-run considerations.

We want to emphasize the difference between our paper and the vast body of literature in the transportation area that studies the traffic routing problems. Firstly, most transportation papers investigate the techniques for an individual to solve for the optimal path, the departing time, or

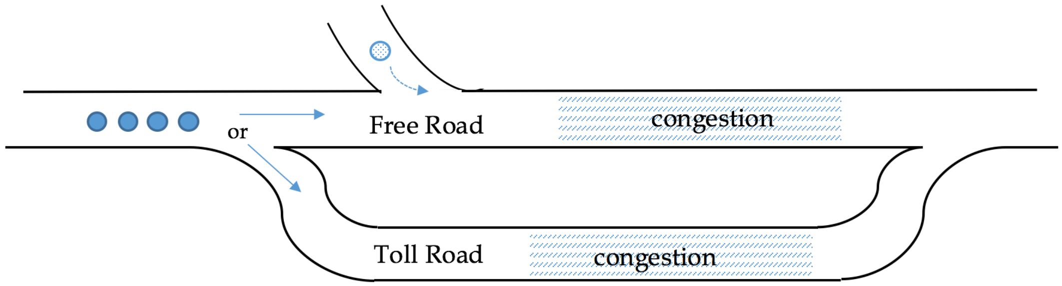

Figure 2: The model

other issues with the objective of minimizing the travel time or other goals (e.g., Franceschetti et al. (2018), Savelsbergh and Van Woensel (2016)). Contrastingly, we are interested in the perspective of a social planner who wants to maximize the social welfare by coordinating multiple parties interacting with each other. Secondly, most transportation papers do not study the mechanism of information distribution but assume a given fixed information environment. The fundamental difference between the information-based approach and others is that the information designer cannot force the individuals to take specific actions, but can manipulate how the individuals view the state of the worlds and indirectly persuade them to take the actions that meet the designer's goal. Given what the individuals believe, the techniques studies by transportation paper can be helpful to decide which action to choose.

3 Single period models

3.1 Single vehicle

Consider an example that two roads connect a given starting point and a given destination (Figure 2). One road is free and the other is a toll road with fee τ. Congestion may occur on either road. The capacities of the free road and the toll road are s_0 and s_1, respectively. Some vehicles may merge onto the free road, causing or aggravating congestion.

An autonomous vehicle departs from the starting point and heads for the destination by either the free road or the toll road. Let $a \in A = \{0, 1\}$ denote the vehicle's road choice between the toll road ($a = 1$) and the free road ($a = 0$). Before departing, the vehicle has access to the real-time traffic information on both roads.[2] However, as the journey starts, the vehicle is not clear about

[2]We make this assumption since the owner of the vehicle or the passenger on the vehicle may schedule the departure

vehicles merging onto the free road in front of it. Specifically, we assume that, if there is congestion, upon departure, the vehicle is informed about the number of vehicles D_0 on the free road and D_1 on the toll road. Let $\theta \in \Theta$ denote the number of merging vehicles. We assume Θ is a finite set and the random variable θ has a PMF $f(\theta)$, which characterizes the vehicle's belief. Let t_1 and t_0 denote the vehicle's waiting time if it travels on the toll road and the free road, respectively. Then,

$$t_1 = \frac{D_1}{s_1},$$

and

$$t_0(\theta) = \frac{D_0 + \theta}{s_0}.$$

Vehicles may have different attitudes towards congestion and thus have different risk profiles. We define a vehicle's private type $c \in \mathcal{C}$ which is a finite set and the utility function of the vehicle $u : \Theta \times \mathcal{C} \times A \to \mathbb{R}$,

$$u(\theta, c, a) \quad = \quad \begin{cases} b - ct_0(\theta), & \text{if } a = 0, \\ b - ct_1 - \tau, & \text{if } a = 1, \end{cases} \tag{1}$$

where b is the utility of arriving at the destination from the starting point. The vehicle's type c represents the disutility from per unit time of waiting (delay). The higher the c, the tighter schedule the vehicle may have. The PMF of c is given by $g(c)$.

The information designer informs the vehicle of τ. Given the toll, the vehicle reports its type c' (which can be different from its true type c). In fact, the vehicle can indicate whether it is in a rush. The information designer considers the possibility that a vehicle may "cheat" by reporting a type different from its true type, e.g., a vehicle not in a rush may report that it in a rush. Then, the information designer receives the updated traffic information θ and makes a stochastic recommendation to the vehicle: conditional on the traffic state θ and vehicle's reported type c', the information designer recommends the toll road with probability $\pi(\theta, c')$ and the free road with probability $1 - \pi(\theta, c')$. Receiving the recommendation, the vehicle takes actions $a_0 \in \{0, 1\}$ and $a_1 \in \{0, 1\}$ if recommended $\hat{a} = 0$ and $\hat{a} = 1$, respectively.

Thus, the vehicle's ex post probability of taking the toll road is $a_0(1 - \pi(\theta, c')) + a_1\pi(\theta, c')$.

based on the traffic information. We do not consider the schedule problem in the current paper. However, it will be an interesting extension to consider the route selection together with the schedule problems.

Then, its expected utility is

$$U_\pi(c, c', a_0, a_1) = \sum_{\theta \in \Theta} u(\theta, c, a_0(1 - \pi(\theta, c')) + a_1 \pi(\theta, c')) f(\theta).$$

If the vehicle reports its true type and obeys the recommendation, i.e., $c' = c$, $a_0 = 0$ and $a_1 = 1$, then its utility is

$$U_\pi(c) \triangleq U_\pi(c, c, 0, 1).$$

A mechanism is *incentive compatible* if the vehicle finds it optimal to report its true type and obey the recommendation. Equivalently, the mechanism satisfies

$$U_\pi(c) \geq U_\pi(c, c', a_0, a_1) \text{ for all } c, c' \in \mathcal{C} \text{ and } a_0, a_1 \in A.$$

That is, comparing to disobey and/or misreport the type, the vehicle gains the highest utility if it obeys the recommendation and truthfully report the type.

The objective of the information designer is to minimize the expected disutility from delay. Given the mechanism π, the objective of the information designer is

$$V_\pi = \sum_{c \in \mathcal{C},\ \theta \in \Theta} c[t_0(\theta) + \pi(\theta, c)(t_1 - t_0(\theta))] f(\theta) g(c).$$

Thus, we summarize the information designer optimization problem (P1) as follows.

$$\begin{aligned} \text{(P1)} \quad \min \quad & \textstyle\sum_{c \in \mathcal{C},\ \theta \in \Theta} c[t_0(\theta) + \pi(\theta, c)(t_1 - t_0(\theta))] f(\theta) g(c) \\ \text{s.t.} \quad & U_\pi(c) \geq U_\pi(c, c', a_0, a_1)\ \forall c, c' \in \mathcal{C},\ \forall a_0, a_1 \in A, \\ & \pi(\theta, c) \in [0, 1]. \end{aligned}$$

Remark 1. Note that, though we have incentive compatibility conditions in the formulation as in mechanism design problems, they are different as explained below.

In the classical mechanism design problem, the mechanism designer commits to a mechanism for the players who are not able to control over the outcomes. In addition, the mechanism designer has no additional information to provide. Differently, in the information design problem, the information designer commits to giving information to the players. To propose pricing mechanisms to indirectly route vehicles and reduce the congestions, the central planner may assign various prices to different roads, and vehicles of different types will choose the roads such that some objective of the mechanism designer is achieved. Note that, the mechanism designer does not require the players (vehicles) to directly elicit their types. In contrast, in the routing problem we study, the pricing scheme is assumed to be given while what information to provide to players is optimized. We also assume that the vehicles can elicit their types (e.g., indicating whether the passengers are in a hurry). Paralleling the truth-telling constraint in the mechanism design problem, that vehicles report their true types is also required in the information design problem we study. Furthermore, like the information design problem with homogeneous vehicles, we require obedience conditions such that vehicles follow the directions provided by the designer.

3.1.1 An example

Now, we consider a simple example. Vehicles can belong to one of two types. Specifically, $c_i \in \mathcal{C} = \{c_L,\ c_H\},\ c_H > c_L$. In addition, the traffic state of free road follows the following distribution,

$$\theta = \begin{cases} \lambda, & \text{with probability } \psi, \\ 0, & \text{with probability } 1-\psi. \end{cases}$$

Then,

$$\begin{aligned} & U_\pi(c, c', a_0, a_1) \\ =& b - c\frac{D_0 + \lambda\psi}{s_0} + a_0\left(c\frac{D_0+\lambda}{s_0} - c\frac{D_1}{s_1} - \tau\right) \\ & + (a_1 - a_0)\left(c\frac{D_0}{s_0} - c\frac{D_1}{s_1} - \tau\right)(1-\psi)\pi(0, c') + (a_1 - a_0)\left(c\frac{D_0+\lambda}{s_0} - c\frac{D_1}{s_1} - \tau\right)\psi\pi(\lambda, c'), \end{aligned}$$

and

$$U_\pi(c) = b - c\frac{D_0 + \lambda\psi}{s_0} + \left(c\frac{D_0}{s_0} - c\frac{D_1}{s_1} - \tau\right)(1-\psi)\pi(0, c) + \left(c\frac{D_0+\lambda}{s_0} - c\frac{D_1}{s_1} - \tau\right)\psi\pi(\lambda, c),$$

We conduct several numerical experiments to illustrate the idea we propose. We limit our focus on the effects of vehicles' type and congestion levels.

First, consider a special case of $c_L = 0$. The optimal recommendations under different congestion levels are presented in Table 1.

The first observation is that $\pi(\cdot, c_L) = 0$. In fact, for a low type ($c_L = 0$) vehicle, the incentive

	$D_0 = 0$	$D_0 = 1$	$D_0 = 1.5$
$\pi(0, c_H)$	0	0	1
$\pi(\lambda, c_H)$	0	1	1
$\pi(0, c_L)$	0	0	0
$\pi(\lambda, c_L)$	0	0	0

Table 1: Optimal recommendations under different congestion levels on the free road. Settings: $D_1 = 1, c_H = 1, g(c_H) = 0.5, \lambda = 1, f(\lambda) = 0.5, s_0 = s_1 = 1, b = 20, \tau = 0.2$.

compatibility conditions are

$$b - a_0\tau - (a_1 - a_0)\tau[(1-\psi)\pi(0, c') + \psi\pi(\lambda, c')] \geq b - \tau[(1-\psi)\pi(0, c') + \psi\pi(\lambda, c')], \ \forall a_0, a_1 \in A.$$

It is implied that the recommendation for low-type vehicles is always to take the free road no matter the traffic conditions. It is intuitive since these vehicles experience no disutility from delay.

Next, we increase c_L to see how the recommendations change, which is summarized in Table 2. When the low type vehicles experience more disutility from congestion, they will be more likely to be routed to the toll road, especially when there is merging traffic on the free road. As for high type vehicles, under this set of experiments, the recommendations are not affected by the change in c_L. When there is no merging traffic on the free road, they will take it; otherwise, they are routed to the toll road.

	$c_L = 0$	$c_L = 0.2$	$c_L = 0.3$	$c_L = 0.4$
$\pi(0, c_H)$	0	0	0	0
$\pi(\lambda, c_H)$	1	1	1	1
$\pi(0, c_L)$	0	0	0.5	1
$\pi(\lambda, c_L)$	0	1	1	1

Table 2: Optimal recommendations with different levels of c_L. Settings: $D_0 = 1$ and the rest is the same with Table 1.

Then, we fix c_L to a strictly positive level and see how optimal recommendations change under different congestion levels on free road. The results are summarized in Table 3.

	$D_0 = 0$	$D_0 = 1$	$D_0 = 1.5$
$\pi(0, c_H)$	0	1	1
$\pi(\lambda, c_H)$	1	1	1
$\pi(0, c_L)$	0.5	0.76	1
$\pi(\lambda, c_L)$	1	1	1

Table 3: Optimal recommendations under different congestion levels on free road. Settings: $c_L = 0.3$ and the rest is the same with Table 1.

For a high-type vehicle, the recommendation varies with the traffic conditions. Specifically, when there is little congestion on the free road and the expected merging traffic on it is small, then, intuitively, the vehicle will be recommended to take the free road. As the congestion level (or the expected merging traffic volume) on the free road increases, it becomes more likely to route the high-type vehicle to the toll road when there is indeed some merging traffic ($\theta = \lambda$) but it is not necessary to do so when there is no merging traffic ($\theta = 0$).

3.2 Multiple vehicles

In this subsection, we assume n vehicles depart at the same time from the starting point to the destination. Let $a_i \in A = \{0, 1\}$ denote the vehicle i's road choice between the toll road ($a_i = 1$) and the free road ($a_i = 0$). The private type of vehicle i is $c_i \in \mathcal{C}$. We assume the probability of vehicles' types to be $(c_1, \cdots, c_n)$ is $g(c_1, \cdots, c_n)$. Let t_{i1} and t_{i0} denote the vehicle i's waiting time if it travels on the toll road and the free road, respectively. Then,

$$t_{i1}(a_i, a_{-i}) = \frac{D_1 + \frac{1}{2}(\sum_{i=1}^{n} a_i - 1)}{s_1},$$

and

$$t_{i0}((a_i, a_{-i}), \theta) = \frac{D_0 + \theta + \frac{1}{2}(\sum_{i=1}^{n}(1 - a_i) - 1)}{s_0}.$$

In sum,

$$t_i((a_i, a_{-i}), \theta) = a_i t_{i1}(a_i, a_{-i}) + (1 - a_i) t_{i0}((a_i, a_{-i}), \theta).$$

We define the utility function of vehicle i $u_i : \Theta \times C \times A \to \mathbb{R}$,

$$u_i((a_i, a_{-i}), c_i, \theta) = \begin{cases} b - c_i t_{i0}((a_i, a_{-i}), \theta), & \text{if } a_i = 0, \\ b - c_i t_{i1}(a_i, a_{-i}) - \tau, & \text{if } a_i = 1, \end{cases}$$

where b is the utility of arriving at the destination from the starting point, which is assumed to be the same for all vehicles.

The information designer informs the vehicles of τ, and makes a stochastic recommendation to each vehicle. Conditional on the traffic state θ and vehicle i's reported type c_i, the information designer recommends action a_i to vehicle i with probability $\pi(a_1, \cdots, a_n | (c_1, \cdots, c_n), \theta)$. In fact, the decision rule of recommendation specifies a mapping from $\mathcal{C}^n \times \Theta$ to $\Delta(A^n)$.

A vehicle chooses to report a type c_i' which can differs from its true type c_i and an action deviation $\delta_i : A \to A$ with $\delta_i(a_i)$ being the action chosen by the vehicle if recommended a_i. Then, the utility function of vehicle i which chooses a reported type c_i' and a deviation δ_i is defined as

$$U_{i,\pi}(c_i, c_i', \delta_i) = \sum_{\substack{(a_i,a_{-i})\in A^n,\\ c_{-i}\in\mathcal{C}^{n-1},\theta\in\Theta}} u_i((\delta_i(a_i), a_{-i}), c_i, \theta)\pi(a_1, \cdots, a_n|(c_i', c_{-i}), \theta)g(c_i, c_{-i})f(\theta).$$

If the vehicle reports its true type and obeys the recommendation, then its utility is

$$U_{i,\pi}(c_i) \triangleq U_{i,\pi}(c_i, c_i, \mathbf{1}),$$

where $\mathbf{1}(a) = a, \forall a$.

The designer's problem is to find a mechanism π such that all vehicles find it optimal to report their true types and obey the recommendation in order to minimize the total disutility from waiting time. Thus, we have the following optimization problem.

$$\begin{aligned}
\min \quad & \sum_{\substack{(a_1,\cdots,a_n)\in A^n,\\ (c_1,\cdots,c_n)\in\mathcal{C}^n,\theta\in\Theta}} \sum_{i=1}^n c_i t_i((a_i, a_{-i}), \theta)\pi(a_1, \cdots, a_n|(c_1, \cdots, c_n), \theta)g(c_1, \cdots, c_n)f(\theta) \\
\text{s.t.} \quad & U_{i,\pi}(c_i) \geq U_{i,\pi}(c_i, c_i', \delta_i) \ \forall c_i, c_i' \in \mathcal{C}, \ \forall \delta_i, \ \forall i \\
& \pi(a_1, \cdots, a_n|(c_1, \cdots, c_n), \theta) \in [0, 1].
\end{aligned}$$

4 Comparison with other methods

4.1 Comparison with human-operated vehicles

Without being directed by the information designer proposed in the previous section, human drivers make uncooperative (selfish) decisions. Specifically, human drivers choose the route giving a higher expected utility among two route.

In the single-vehicle model, for a driver with type c, he chooses the toll road if $b - ct_1 - \tau \geq b - c\mathbb{E}_\theta[t_0(\theta)]$; otherwise, the driver chooses the free road. Thus, under human drivers' selfish choices, the expected social welfare is

$$V_0 = \sum_{c\in\mathcal{C},\ \theta\in\Theta} c[t_0(\theta) + \pi_0(c)(t_1 - t_0(\theta))]f(\theta)g(c)$$

$$\text{s.t.} \quad \pi_0(c) = \begin{cases} 1, & \text{if } b - ct_1 - \tau \geq b - c\mathbb{E}_\theta[t_0(\theta)], \\ 0, & \text{otherwise,} \end{cases}$$

where $\pi_0(c)$ denotes the type-c driver's choice.

Note that, V_0 can be written as the objective function of the following programming.

$$
\begin{aligned}
V_0 = \min \quad & \textstyle\sum_{c\in\mathcal{C},\ \theta\in\Theta} c[t_0(\theta)+\pi_0(c)(t_1-t_0(\theta))]f(\theta)g(c) \\
\text{s.t.} \quad & \textstyle\sum_{\theta\in\Theta} u(\theta,c,\pi_0(c))f(\theta) \geq \sum_{\theta\in\Theta} u(\theta,c,1-\pi_0(c))f(\theta), \\
& \pi_0(c)\in[0,1],
\end{aligned}
$$

which is equivalent to

$$
\begin{aligned}
V_0 = \min \quad & \textstyle\sum_{c\in\mathcal{C},\ \theta\in\Theta} c[t_0(\theta)+\pi(\theta,c)(t_1-t_0(\theta))]f(\theta)g(c) \\
\text{s.t.} \quad & U_\pi(c) \geq U_\pi(c,c',a_0,a_1)\ \forall c,c'\in\mathcal{C},\ \forall a_0,a_1\in A, \\
& \pi(\theta,c)=\pi(\theta',c),\ \forall\theta,\theta'\in\Theta, \\
& \pi(\theta,c)\in[0,1].
\end{aligned}
$$

Comparing the above LP with (P1), the above LP requires more constraints. Therefore, through information design, routing autonomous vehicles will give a lower socially aggregated waiting time, and thus, a higher social welfare. We summarize the above finding in the following proposition.

Proposition 1. *In the single vehicle model, comparing with human-operated vehicles, an autonomous vehicle under routing based on the information design approach gives a shorter delay (higher utility).*

Similarly, for multiple-vehicle model, if the vehicles do not consider each other's strategy, then all of them will make the same decision. Specifically, if $b-ct_1-\tau \geq b-c\mathbb{E}_\theta[t_0(\theta)]$, all type-$c$ vehicles choose the toll road; otherwise, they choose the free road. Thus, the total utilities of n vehicles are

$$
\begin{aligned}
V_{n0} = \min \quad & \textstyle\sum_{i=1}^n \sum_{(c_1,\cdots,c_n)\in\mathcal{C}^n,\ \theta\in\Theta} c_i[t_0(\theta)+\pi(\theta,c_i)(t_1-t_0(\theta))]g(c_1,\cdots,c_n)f(\theta) \\
\text{s.t.} \quad & U_\pi(c_i) \geq U_\pi(c_i,c_i',a_0,a_1)\ \forall c_i,c_i'\in\mathcal{C},\ \forall a_0,a_1\in A, \\
& \pi(\theta,c_i)=\pi(\theta',c_i),\ \forall\theta,\theta'\in\Theta, \\
& \pi(\theta,c_i)\in[0,1].
\end{aligned}
$$

For the multiple-vehicle model, if the vehicles are strategic and consider each other's strategy, we will show that information design routing approach always gives a shorter total waiting time (delay), compared with the human-operated vehicles. If we disallow the vehicles to misrepresent its true type, then each vehicle plays the best response given other vehicles' strategies. That is, given a_{-i}, vehicle i chooses the free road ($a_i = 0$) if $b - c_i t_{i0}((a_i, a_{-i}), \theta) > b - c_i t_{i1}(a_i, a_{-i}) - \tau$; otherwise, vehicle i chooses the toll road ($a_i = 1$). Thus, we can write the total utility of all vehicles in the following format:

$$\sum_{i=1}^{n} \sum_{(c_1,\cdots,c_n)\in\mathcal{C}^n,\ \theta\in\Theta} b - c_i[t_0(\theta) + \pi((a_i, a_{-i})|(c_1, \cdots, c_n), \theta)(t_1 - t_0(\theta))]g(c_1, \cdots, c_n)f(\theta)$$

$$\text{s.t.} \quad \pi((a_i, a_{-i})|(c_1, \cdots, c_n), \theta) = \begin{cases} 1, & \text{if } b - c_i t_{i1}(a_i, a_{-i}) - \tau \geq b - c_i t_{i0}((a_i, a_{-i}), \theta), \\ 0, & \text{otherwise,} \end{cases}$$

where $\pi((a_i, a_{-i})|(c_1, \cdots, c_n), \theta)$ denotes vehicle i's best response.

Note that, the best response of each vehicle forms a feasible solution to the information design problem in the multiple vehicle model. Therefore, we have the following proposition.

Proposition 2. *In the multiple-vehicle model, comparing with human-operated vehicles that are required to report their true types, autonomous vehicles under routing based on the information design approach give a shorter total delay (higher total utility).*

5 Multi period models

5.1 Dynamic model

We first build up a dynamic model for routing autonomous vehicles. We assume that each time interval is sufficiently small such that only one vehicle departs from the starting point. Given all the notations being the same with §2, the following optimization problem gives the optimal route selection for the current vehicle:

$$\begin{aligned} V(D_0, D_1) = \min \quad & \textstyle\sum_{c\in\mathcal{C},\ \theta\in\Theta}(1 - \pi(\theta, c))[t_0(\theta) + V((D_0 + \theta + 1 - s_0)^+, (D_1 - s_1)^+)] \\ & \quad + \pi(\theta, c)[t_1 + V((D_0 + \theta - s_0)^+, (D_1 + 1 - s_1)^+)]f(\theta)g(c) \\ \text{s.t.} \quad & U_\pi(c) \geq U_\pi(c, c', a_0, a_1)\ \forall c, c' \in \mathcal{C},\ \forall a_0, a_1 \in A \\ & \pi(\theta, c) \in [0, 1]. \end{aligned}$$

The terminal function is given by $V(0, 0) = 0$.

5.2 Implementable model

The dynamic model in the previous subsection assumes that one vehicle departs at the beginning of each period. However, in practice, the departure patterns vary over time. Also, to accelerate the computation and achieve a near-optimal solution, we can only consider a short future periods.

Assume that the length of each period is h, and from now (time point 0) to the following T periods (time point Th), the departure pattern is stable. With a little abuse of notation, we assume that $\{\theta, f(\theta)\}$ gives the distribution of number of merging vehicles in a time interval of h. Then, we have the following optimization problem.

$$
\begin{aligned}
&\text{For } t = 0, \cdots, T-1, \\
&V_t(D_0, D_1) = \min \quad \textstyle\sum_{c\in\mathcal{C},\ \theta\in\Theta}(1-\pi(\theta,c))[\frac{D_0+\theta}{s_0 h} + V_{t+1}((D_0+\theta+1-s_0h)^+, (D_1-s_1h)^+)] \\
&\qquad\qquad\qquad +\pi(\theta,c)[\tfrac{D_1}{s_1 h} + V_{t+1}((D_0+\theta-s_0h)^+, (D_1+1-s_1h)^+)]f(\theta)g(c) \\
&\qquad\qquad \text{s.t.} \qquad U_\pi(c) \geq U_\pi(c, c', a_0, a_1)\ \forall c, c' \in \mathcal{C},\ \forall a_0, a_1 \in A \\
&\qquad\qquad\qquad\qquad \pi(\theta, c) \in [0,1].
\end{aligned}
$$

The terminal function is $V_T(D_0, D_1) = \min\{\frac{D_0+\mathbb{E}\theta}{s_0 h}, \frac{D_1}{s_1 h}\}$.

Furthermore, h and T can be adjusted gradually for each vehicle with respect to the departure pattern.

6 Urban traffic

In this section, we consider the information design problem for urban traffic (Figure 3). Instead of selecting from two routes, as discussed in previous sections, we consider the routing problem in a urban road network denoted by a directed graph (V, E). Here, V is the set of nodes (intersections) and two special nodes, the source (starting point) s and the sink (destination) t, and $E = (e)$ is the set of edges. Each vehicle, upon departure, chooses a path to the sink. Let $P(s,t)$ be the set of all paths from the source (s) to the sink (t).

A vehicle departing at time t is informed about the current traffic volume (queue length) on each edge e, $D_e(t)$. However, it is not sure about the queue length at time $t' > t$. Thus, for a path $p = (e_1, \cdots, e_n) \in P$, the vehicle assumes that the queue length $\{\tilde{D}_{e_i}(t'), t' \geq t\}$ is a stochastic process with $\tilde{D}_{e_i}(t) = D_e(t)$. $\tilde{D}_E = (\tilde{D}_e)_{e\in E}$ follows a distribution with CDF $\tilde{F}$. From the vehicle's

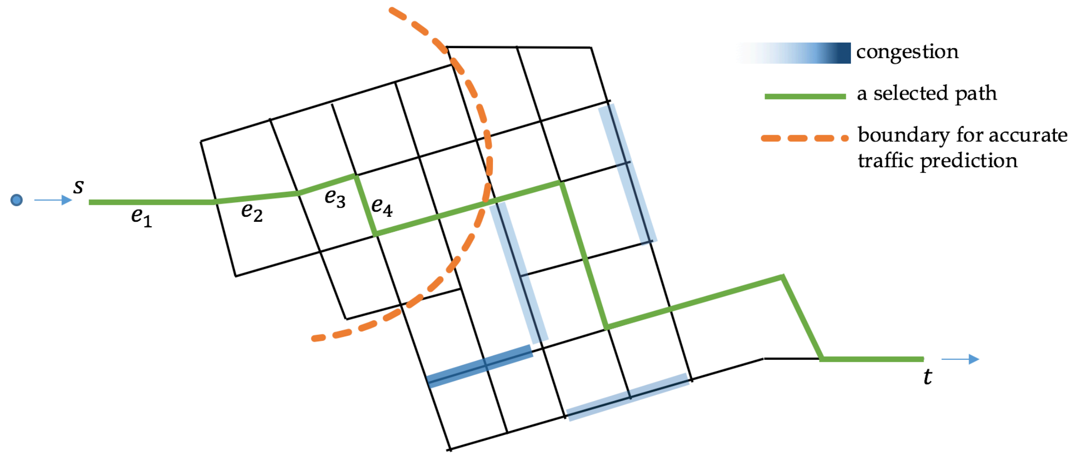

Figure 3: Urban traffic model

perspective, the travel time along e_1 is given by

$$\delta_{e_1} = \frac{D_{e_1}(t) + 1}{s_{e_1}}.$$

At time $t + \delta_{e_1}$, the vehicle arrives at the head of edge e_2. Its travel time on e_2 (which is a random variable) is

$$\delta_{e_2} = \frac{\tilde{D}_{e_2}(t + \delta_{e_1}) + 1}{s_{e_2}}.$$

Thus, we have the following equations

$$\delta_{e_i} = \frac{\tilde{D}_{e_i}(t + \sum_{k=1}^{i-1} \delta_{e_k}) + 1}{s_{e_i}}, \quad i = 2, \cdots, n.$$

The total travel time along path p is

$$\delta_p = \sum_{i=1}^{n} \delta_{e_i}.$$

We allow the toll to be dynamic with with the congestion level on each road based on a *known* function. Specifically, we assume

$$\tau_e(t') = h_e(\tilde{D}_e(t')),$$

where $h_e(\cdot)$ is an increasing function. Thus, if the vehicle travels along p, then the total toll is

$$\tau_p = \sum_{i=1}^{n} \tau_{e_i}(t + \sum_{k=1}^{i-1} \delta_{e_k}).$$

The utility function of the vehicle is defined as

$$u(\tilde{D}_E, c, \{a_p\}_{p\in P}) = b - \sum_{p\in P} a_p(c\delta_p + \tau_p),$$

where $a_p = 1$ if route p is chosen; otherwise $a_p = 0$.

Now, we define the recommendation mechanism. We assume that the hardware engine has access to accurate traffic flow data of roads nearby. Specifically, we assume that for any route starting from s, it has accurate forecast of traffic of the first $\hat{J}$ roads. Let $\hat{D}_{e_j}$ be the realization of $\tilde{D}_{e_j}$. Note that, since we assume for a single route, the number of roads with known traffic is up to $\hat{J}$, to simplify the analysis, we allow the hardware engine to know the realization of traffic on these roads for any future time.

The recommendation mechanism is represented by $\pi_p(\{\hat{D}_{e_j}\}_{1\leq j\leq \hat{J}}, c)$ which denotes the probability that route p is recommended given the accurate traffic flow forecast on $\{e_j\}_{1\leq j\leq \hat{J}}$ and the vehicle's reported type c. $\sum_{p\in P} \pi_p(\{\hat{D}_{e_j}\}_{1\leq j\leq \hat{J}}, c) = 1$.

Given the vehicle's reported type c' (which can be different from its true type c), the chosen action $p'(p)$ if recommended p, its expected utility is

$$U_\pi(c, c', \{p'(p)\}_{p\in P}) = \mathbb{E}_{\tilde{D}_E}\left[u(\tilde{D}_E, c, \sum_{p\in P} \pi_p(\{\hat{D}_{e_j}\}_{1\leq j\leq \hat{J}}, c')a_{p'(p)})\right]$$

If the vehicle reports its true type and obeys the recommendation, then its utility is

$$U_\pi(c) \triangleq U_\pi(c, c, \{p\}_{p\in P}).$$

We define the hardware engine's problem as

$$\begin{aligned}
&\min \mathbb{E}_c \left[\mathbb{E}_{\tilde{D}_E} \left[\pi_p(\{\hat{D}_{e_j}\}_{1\le j\le \hat{j}}, c)\delta_p \right] \right] \\
&\text{s.t. } U_\pi(c) \ge U_\pi(c, c', \{p'(p)\}_{p\in P}) \ \forall c, c' \in C, \ \forall p', \\
&\quad \sum_{p\in P} \pi_p(\{\hat{D}_{e_j}\}_{1\le j\le \hat{j}}, c) = 1, \\
&\quad \pi_p(\{\hat{D}_{e_j}\}_{1\le j\le \hat{j}}, c) \ge 0.
\end{aligned}$$

Budget constraints

Since a journey may pass several toll roads, the owner of an autonomous vehicle or the traveler on the vehicle may want to specify at the beginning their budgets for the total toll paid during the journey. Thus, we reformulate the model by replacing the individual rationality constrain with a budget constraint.

By an abuse of notation, we use b to denote the budget set by the owner/traveler before the journey starts. Then, the budget constraint is

$$b - \mathbb{E}_{\tilde{D}_E} \left[\sum_{p\in P} \pi_p(\{\hat{D}_{e_j}\}_{1\le j\le \hat{j}}, c) a_p \tau_p \right] \ge 0.$$

Thus, the hardware engine's problem with budget constraints is

$$\begin{aligned}
\text{(Pb)} \quad &\min \mathbb{E}_c \left[\mathbb{E}_{\tilde{D}_E} \left[\pi_p(\{\hat{D}_{e_j}\}_{1\le j\le \hat{j}}, c)\delta_p \right] \right] \\
&\text{s.t. } U_\pi(c) \ge U_\pi(c, c', \{p'(p)\}_{p\in P}) \ \forall c, c' \in C, \ \forall p', \\
&\quad b - \mathbb{E}_{\tilde{D}_E} \left[\sum_{p\in P} \pi_p(\{\hat{D}_{e_j}\}_{1\le j\le \hat{j}}, c) a_p \tau_p \right] \ge 0 \ \forall c \in C.
\end{aligned}$$

Dynamic update

To better the traffic outcome of routing, dynamic update should be implemented to refine route selection decisions as more traffic prediction data becomes available. Thus, after passing through the first road on the optimal path, the hardware engine re-optimizes the problem by setting the end point of the first road as the new starting point, and updating the traffic prediction and the budget constraint.

7 Risk-averse vehicles

In previous sections, we assume that vehicles are risk neutral and maximize their expected utilities. In this section, we extend our model and allow vehicles to have private risk attitude. Specifically, for a single vehicle, if it reports its type c', λ' and takes actions $a_0 \in \{0,1\}$ and $a_1 \in \{0,1\}$ if recommended $\hat{a} = 0$ and $\hat{a} = 1$, we assume its utility is

$$\begin{aligned} U'_\pi(c, c', \lambda, \lambda', a_0, a_1) \quad &= \quad \mathbb{E}_\theta[u(\theta, c, a_0(1 - \pi(\theta, c', \lambda')) + a_1\pi(\theta, c', \lambda'))] \\ &\quad -\lambda \mathbb{V}_\theta[u(\theta, c, a_0(1 - \pi(\theta, c', \lambda')) + a_1\pi(\theta, c', \lambda'))], \end{aligned}$$

where $\lambda \in \Lambda$ is the parameter associated with delay variance and $\pi(\theta, c', \lambda')$ is the recommendation given by Waze given the vehicle's reported c' and λ'.

Similarly, we define the utility of a vehicle truthfully reports its type and obey the recommendation,

$$U'_\pi(c, \lambda) \triangleq U_\pi(c, c, \lambda, \lambda, 0, 1).$$

We have Waze's optimization problem for a risk-averse vehicle as follows.

$$\begin{aligned} \min \quad & \mathbb{E}_{\theta,c}[t_0(\theta) + \pi(\theta, c)(t_1 - t_0(\theta))] \\ \text{s.t.} \quad & U'_\pi(c, \lambda) \geq U'_\pi(c, c', \lambda, \lambda', a_0, a_1) \;\forall c, c' \in C,\; \forall \lambda, \lambda' \in \Lambda,\; \forall a_0, a_1 \in A \\ & \pi(\theta, c) \in [0, 1]. \end{aligned}$$

8 Implementation

8.1 Localization

In the model we discussed in §2-6, we mainly deal with a group of vehicles that are close to each other and treat the decisions of other vehicles exogenously. In fact, we make an underlying assumption that vehicles are able to identify other vehicles that are close and localize the routing decisions to this group of vehicles. V2V (Vehicle-to-Vehicle) communication technology is available to realize the wireless information exchange between vehicles. As illustrated in Figure 4, vehicles are segmented into several groups. One possible way of segmentation is to segment the road map into regions depending on the road structures. For example, the areas before intersections and

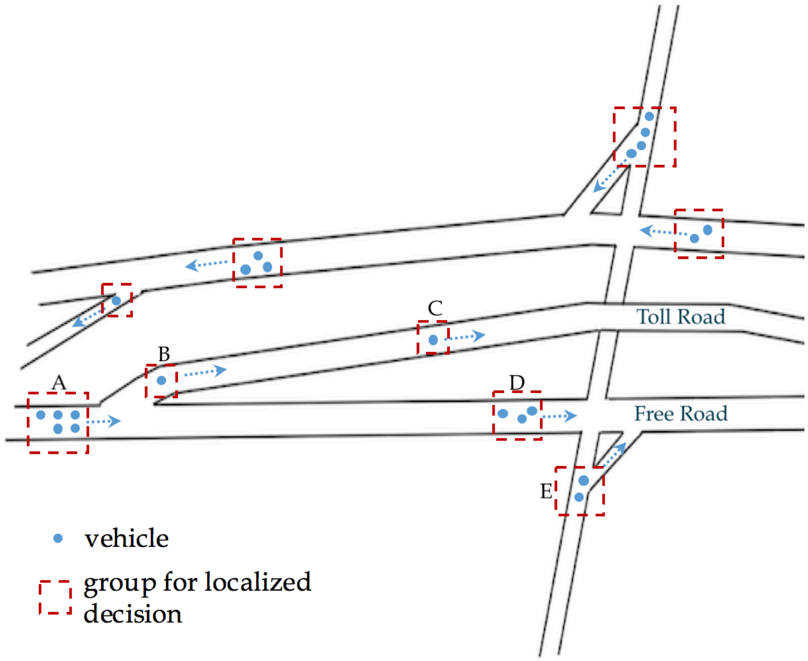

Figure 4: Illustration for localized decision

entrances/exists to highways/frontage road could be separate regions. For a sufficiently short time interval for decision making, vehicles in the same region are allocated to one group. Simulations, experiments and tests are necessary to define a proper mechanism of segmentation of maps. After segmentation and grouping, for a group of vehicles, if the behavior of vehicles in adjacent groups may affect the utility of this group of vehicles, then the behavior of adjacent groups are treated as an exogenous input to the information design model. For instance, consider the information design problem for vehicles in group A as shown in Figure 4. For these vehicles, they can either choose the toll road or the free road. To convert the situation into the model we proposed, we need to first identify other groups that may impact the utility of group A. Note that, group B and C are on the toll road, which could cause congestion for group A. Thus, group B and C are related to D_1 in the model. Similarly, group D is on the free road and is related to D_0 in the model. However, for group E, some of vehicles in this group may enter the free road and the number of vehicles that possibly enter the free road before group A is model as the random variable θ.

8.2 Institutional arrangement

Figure 5 illustrates the institutional arrangement. Authority is founded that contracts with companies like Waze to obtain all traffic-related raw data. The data are the input to a prediction model (which will be discussed in §8.3) for traffic forecast. Each autonomous vehicle is required to be equipped with a hardware engine (which will be discussed in §8.4). Hardware engines are extensions of the authority that connect locally with each other. Each hardware engine obtains nearby traffic predictions exclusively from the authority. Also, it also acquires the preference parameters of the vehicle it is equipped. External data including tolls and road capacities will also be fed into the hardware engine.

8.3 Prediction

The movement of vehicles has both endogenous portion and exogenous portion. As the information conveyed to each vehicle is optimized and controlled, Waze is clear about the route decision of each vehicle. Thus, ideally, since the locations of vehicles are tracked, and the speeds are calculable, the near future traffic condition is entirely predictable. However, in practice, other exogenous factors may affect the exact time that a particular vehicle arrives at a particular location. For example, weather conditions may contribute to an additional variance of vehicles' movement. Thus, the prediction model should also incorporate the features of weather, climate or other natural conditions. In industry, there are companies providing support to improve road weather-related decisions through "Ground Truth®" data (WeatherCloud). In addition, historical accidents data and other traffic-related data can also be incorporated to enhance the accuracy of prediction.

8.4 Hardware engine

As demonstrated in Figure 5, a hardware engine loads traffic predictions from the authority together with other external and internal information. It solves for the optimal road choice for the vehicle based on the information design approach proposed in §2-5 and outputs the directions to steer the vehicle.

There are many algorithms that can solve linear programming. Simplex algorithm is one of the most widely used algorithms. In the following, we introduce the idea of creating a hardware engine to solve the linear routing problem by implementing Simplex algorithm. Within each Simplex iteration, numerous matrix multiplications and comparisons are needed, especially for large-scale problems. Thus, parallelism can be exploited in several ways (Schutz and Klindworth, 1992; Bayliss et al., 2006). As autonomous vehicles have to road selection decisions in real time, it is critical to complete each step of calculation very fast. Therefore, we believe it is a proper to accelerate the calculation through hardware implementation.

We propose to create an application-specific integrated circuit (ASIC) to rapidly solve the road

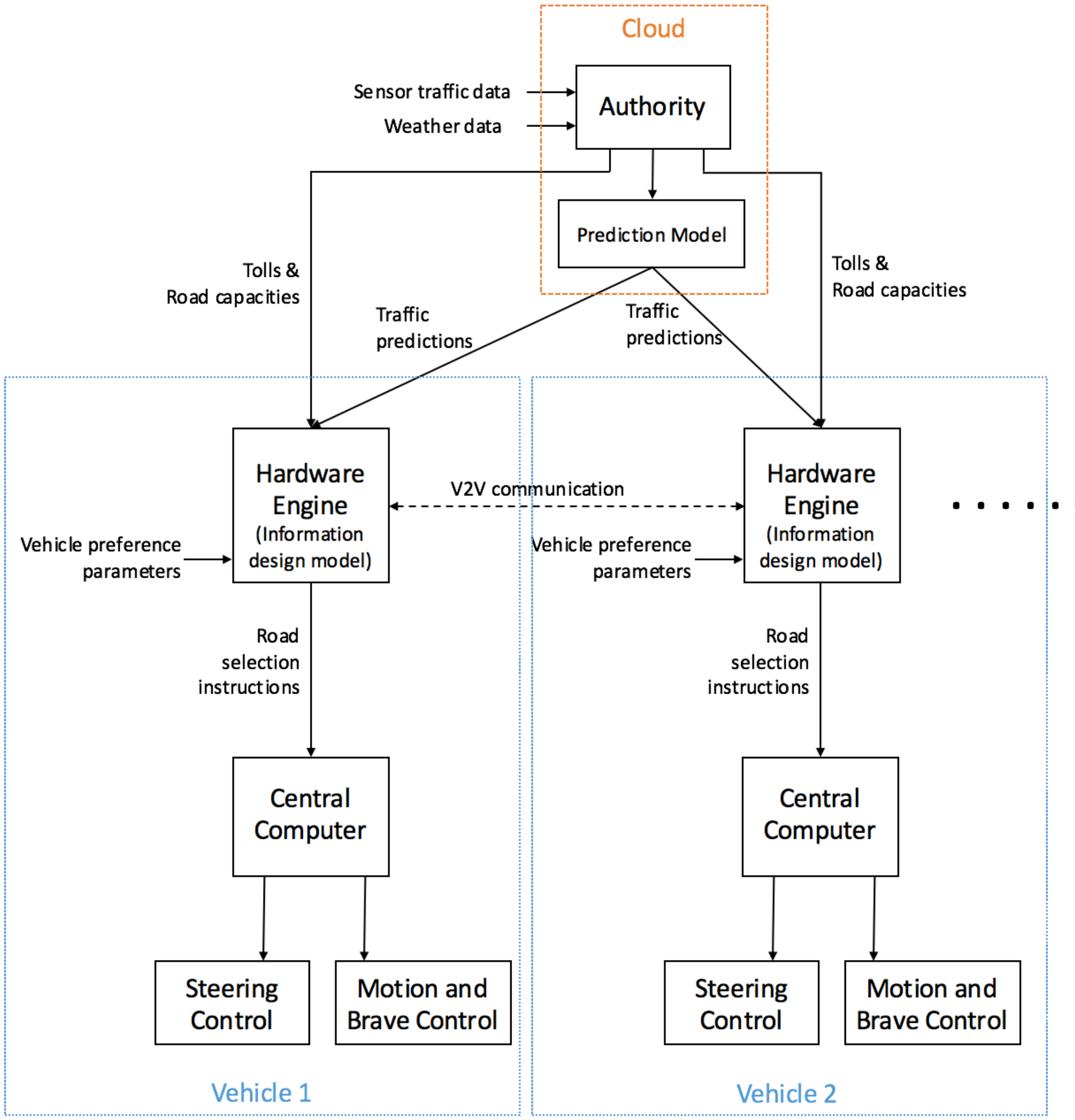

Figure 5: Institutional arrangement

selection problems continuously for autonomous vehicles. We first use a field-programmable gate array (FPGA) to prototype based on the specification using hardware description language (e.g., Verilog). After testing the performance on FPGA, we then convert to an ASIC, which would give significant speed improvement.

8.5 Security

Compared with various general-purpose solutions, ASICs are less in danger of security-related attacks. However, attackers can still use a variety of methods to compromise ASICs.

ASICs are usually designed and produced to complete specific tasks which could also be realized by running software on general-purpose solutions. For example, to solve the linear programming problems proposed in this paper, CPLEX, GAMS (General Algebraic Modeling System) and much other software are available. A large portion of designs in the hardware elements of ASICs are pre-wired to perform only the required tasks, and thus, their configuration cannot be changed after manufacture. Therefore, since no software is running on ASICs, they are immune to many types of attacks that target security problems in software and operating systems. Slightly different from ASICs, FPGAs have reprogrammable logic and could be vulnerable to more types of attacks. Like ASICs, FPGAs also have no software running on them and cannot be compromised by attacks aiming at computer software.

As shown in the diagram (Figure 5), the hardware engines we design are connected to the cloud to transmit information. Thus, they are susceptible to attacks that direct at the communication between vehicles and the rest of the world. In general, when ASICs or FPGAs are connected to the internet, engineers must prevent attackers from seeing into the connection between the devices and the cloud, and from modifying the data transmitted over the Internet. To satisfy security requirements, hardware engines need to have security settings that are similar to what general computers need for secure Internet connections. The first line of defense is encryption (Van Tilborg, 2012). In addition, verification of data integrity that the data is not altered while transferred is another important step of preventing attackers from seeing into the communication. Also, it is critical to have authentication functionality that verifying the identity of parties that send and receive information. Last but not least, non-repudiation is necessary to make sure that senders cannot deny that they have sent the data that has been received. Since the building of the Internet, protocols and standards (e.g. HTTPS, SSL, etc.) has been developed, which can provide and support encryption, data integrity, authentication, and non-repudiation. Hardware implementations of these standards have existed and are available on the market. For further details on the topic of Internet security, the interested readers are referred to Andress (2014)

Generally, integrated chips, and by extension ASICs and FPGAs, are vulnerable to manufacture-related attacks (Goertzel and Hamilton, 2013). Individual or companies with malicious intent may produce counterfeit versions of chips that do not meet all the original design but can function in the same way. If attackers have physical access to the integrated chips, then they could potentially reverse engineer the chips. For other related security issues, readers are referred to Abramovici and Bradley (2009) and Majzoobi, Koushanfar and Potkonjak (2011).

9 Discussion

We list a few possible extensions below.

Utility Function design. In the paper, we do not go deep into the utility function design problem. In fact, in practice, it is a critical question to formulate a function form of utility based on the needs of the firms that own the vehicles or the passengers on the vehicles. This problem will be more related the decision analysis field.

Insurance. Following the previous point of designing utility functions, one factor that could affect this problem is the insurance policy. It will be interesting to study how to design insurance policies for autonomous vehicles under full autonomy or when driverless vehicles and vehicles with drivers coexist on roads.

Intersections. In the urban model we build up, we do not model how vehicles go through intersections in detail. In effect, the information design framework can also be applied to coordinate vehicles from various directions to pass through road intersections efficiently without the control of traffic lights. This implementation further decreases the travel time and reduce congestions, which will be advantageous to the social welfare.

Tolling. In this paper, we assume that the toll is given. A possible next step is to study the optimal toll. The toll can be a constant, or vary with the congestion level.

Accidents and vehicles with priority. Traffic accidents usually cause severe congestion and require emergency vehicles such as ambulances and police cars to arrive at the scene as soon as possible to save the injured person, handle the accident, and direct the traffic. Thus, it is important for Waze to react quickly to accidents and reroute cars to alleviate the impact of accidents.

Scheduling. As mentioned at the beginning of the paper, the transportation in a future city may primarily rely on autonomous taxis and buses. Then, a fundamental problem is to schedule the departure of these vehicles. The study on vehicle departure scheduling problems has a long history (e.g., Angel et al., 1972). However, in a fully autonomous world with extensive information, it is possible to simulate various departure times to find a proper one based on the historical data. In addition to city transportation planning problem, scheduling is also important in logistics and supply chain management, which is worth further research.

Unique information. When vehicles belonging to a certain firm run on the roads, they could collect various information. In fact, in addition to transporting products or bringing travelers to all kinds of destinations, autonomous vehicles are also data collectors that moving and gathering information all over the place. The unique experience of a firm's vehicles provides the firm a unique dataset could be beneficial to the firm in one way or another.

Repositioning. Following our previous point of unique information, we believe companies like Uber and Lyft can combine private sensor data with public traffic data to reposition their vehicles through information design. Given a firm's (private) dataset on demand surges in peak times, it can design a method to distribute information strategically such that vehicles, following the information provided to them, move to proper places, and in overall, spatially distributed demands can be met by supplies appropriately.

Information purchase. As shown in the urban traffic model, the level of prediction accuracy has a direct impact on the routing decisions. In the current paper, we assume the only source of prediction information is the crowdsourced data provided by Waze. However, many firms are in this business and collect all kinds of related data. Thus, it will be interesting to study the routing problem with the option of purchasing more accurate traffic prediction data. From a theoretical point of view, it will be related to the selling information literature (Bergemann, Bonatti and Smolin, 2017).

Warehouse robots. Robots currently are used in warehouses, for example, those owned by Amazon, to carry out logistic tasks. Route design for robots is a critical step that directly influences the efficiency of goods flows. The design problem has to take into account the priorities of robots performing various tasks. We believe our framework is also applicable in this setting to optimize routes for robots with different priorities.

Worst-case scenario. Roughgarden and Tardos (2002) and Roughgarden (2005) study the worst-case scenario for human-operated vehicles. The worst-case scenario for autonomous vehicles has not been clearly defined and fully studied so far. Given the concerns of "weaponisation" of autonomous vehicles (Financial Times, 2018), it is worth investigating from a security perspective.

Long-distance travel. In the current paper, we mainly deal with short-distance travel. It would be interesting to study the long-distance travel problem where the vehicle has to decide how to allocate its budget throughout the journey. The problem can be studied in a similar fashion as discussed in §5.

Public vs. private information. In the models discussed in the current paper, we assume that the congestion levels when vehicles departing are public information (known to each vehicle), but the additional information about the merging vehicles is private (specific to each vehicle). It will be interesting to extend the settings and allow the information designer to choose the public available and private signals. The problem can be related to Mathevet, Perego and Taneva (2017).

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