Valuing Private Equity Investments Strip by Strip

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Figure 4  Firm size, industry, and listing propensity

Source: The Center for Research in Security Prices (CRSP), Compustat, and the U.S. Census Bureau’s Longitudinal Business Database.

Notes: Listed firms include U.S. firms in CRSP and Compustat on the NYSE, AMEX, and Nasdaq that we can assign to an employee size group. Investment companies, mutual funds, REITs, and other collective investment vehicles are excluded. The percentage of firms that are listed in each employee size group equals listed firms/total firms, where total firms includes public and private firms. The sample period is from 1977 to 2015.
Which Complicates the Assessment of Factor Exposure
1. The shift in capital towards private markets, especially for small growth firms, means that we need to know how risk is priced in private markets.
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2. Existing approaches to private equity valuation have not taken into account the multivariate nature of risk nor the temporal composition of risk.
   - Standard approaches:
     - TVPI (no discounting, no risk)
     - IRR (no risk)
     - PME (beta = 1)
     - GPME (beta constant)
   - Limitations to all approaches: only one aggregate source of risk
     - If a bad assumption in equities (CAPM) likely also the case in PE
     - Especially in “alternative” categories like Real Estate Funds
1. Shift to private markets
2. Literature has struggled with cross-section and term structure of risk

3. **To address these limitations, we draw from asset pricing literature emphasizing rich cross-section of factors and term structure of risk**


     - Term structure of risk potentially upward or downward sloping, depending on factor
     - Term structure of strips on other factors not known or traded
Measurement Problems in Assessment of Risk and Return in Private Equity

1. Shift to private markets
2. Literature has struggled with cross-section and term structure of risk
3. We draw from other asset pricing literature emphasizing multifactor models
4. **Problem:** Observe cashflows, not returns
1. Shift to private markets
2. Literature has struggled with cross-section and term structure of risk
3. We draw from other asset pricing literature emphasizing multifactor models
4. Problem: Observe cashflows, not returns

5. **Our Solution:**
   - Estimate exposures of PE fund cash flows to cash flows on bond and cross-section of stock strips
   - Use asset pricing model to price these strips

Delivers PE factor exposure, expected return, risk-adjusted profit, NAV
1. Alternative PE Categories have sector-specific factor loadings in the cross-section
Key Takeaways

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2. Risk exposure is more equity-like early in fund life
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3. Decreasing realized profits and expected returns in more recent vintages
Key Takeaways

1. Alternative PE Categories have sector-specific factor loadings in the cross-section

2. Risk exposure is more equity-like early in fund life

3. Decreasing realized profits and expected returns in more recent vintages

4. Substantial small and growth factor exposure, suggesting these factor loadings are prominent in PE
Want to Understand Cash-Flow Profiles of Private Equity Funds

Cash Flows by Age
Call or Distribution Amount Relative to $1 Commitment

Future Cash Flow
Actual Cash Flow

0 5 10 15

Fund of Funds
Restructuring
Debt Fund

Buyout
Venture Capital
Real Estate
Infrastructure

0.00
0.05
0.10
0.00
0.05
0.10

Cash Flows by Age
Cash-Flow Variation Across Horizon and Vintage — Buyout
Break out Factor Exposure Strip by Strip to Match PE Cash Distributions to LPs

- Construct $F_{t+h}$ cash flows on replicating portfolio:
  - Zero coupon bond, pays out $1$ in horizon $t+h$
  - Dividend strips: pay one risky cash flow at $t+h$

- $h$-period stochastic discount factor chains one-period SDFs:

$$M_{t+h}^h = \prod_{k=1}^{h} M_{t+k}$$

- Defining prices of these strips:

$$P_{t,h} = \mathbb{E}_t[M_{t+h}^h F_{t+h}]$$
Estimate Factor Exposure Strip-by-Strip

- Three-factor model fitting PE fund cash flows, in quarterly strips, against factor cash flows:

\[ X_{t+h}^i = \beta_{t,h}^i F_{t+h} + e_{t+h}^i \]

Factors are:

- Buyout: bond, stock, small
- VC: bond, growth, small
- Real Estate: bond, stock, REIT
- Infrastructure: bond, stock, infra
Shrinkage Estimators Measure Factor Exposure

Structure of exposure (Buyout):

\[ X_{t+h}^{i \in c} = \beta_{t,h}^b + \beta_{t,h}^{mkt} F_{t+h}^{m} + \beta_{t,h}^{stock} F_{t+h}^{stock} + e_{t+h}^i \]

\[ = a_t^1 b_h^1 + a_t^2 b_h^2 F_{t+h}^{m} + a_t^3 b_h^3 F_{t+h}^{stock} + e_{t+h}^i \]

Allow \( b_h \) to vary for each horizon (quarter)

\( a_t \) varies for each tercile of the P/D distribution of vintage year

Two estimation techniques:

1. OLS
2. Lasso:

\[ \hat{\beta}_{lasso} = \arg \min_{\beta \in \mathbb{R}^{189}} \| X_{t+h}^i - \beta_{t,h}^i F_{t+h} \|^2 + \lambda 1\{\beta > 0\}, \quad \lambda = \infty \]
We connect Strip Prices + Fund Exposures = Replicating Portfolio

- Define scaled long-positions in each factor that are budget feasible, where

$$q_{t,h}^i = \frac{\beta_{t,h}^i}{\sum_{h=1}^H \beta_{t,h}^i P_{t,h}} \Rightarrow \sum_{h=1}^H q_{t,h}^i P_{t,h} = 1.$$  

where $P_{t,h}$ comes from an asset pricing model

- Null: present discounted value of fund cash distributions is 1:

$$\mathbb{E}_t \left[ \sum_{h=1}^H M_{t+h}^h X_{t+h}^i \right] = \mathbb{E}_t \left[ \sum_{h=1}^H M_{t+h}^h q_{t,h}^i F_{t+h} \right] = \sum_{h=1}^H q_{t,h}^i P_{t,h} = 1$$
Estimation Enables Novel Understanding of PE Asset Pricing

- Use model to understand expected returns:

\[
\mathbb{E}_t \left[ R^i \right] = \sum_{h=1}^{H} \sum_{k=1}^{K} w^i_{t+h}(k) \mathbb{E}_t \left[ R_{t+h}(k) \right]
\]

- Profit corrects for risk, but may include premium for illiquidity:

\[
v^i_{t+h} = X^i_{t+h} - q^i_{t+h} F_{t+h}
\]

\[
RAP^i_t = \sum_{h=1}^{H} P^s_{t+h} v^i_{t+h}
\]
Asset Pricing Model

- State variables follow Gaussian first-order VAR:

\[ z_t = \Psi z_{t-1} + \Sigma^{\frac{1}{2}} \varepsilon_t, \quad \varepsilon_t \sim i.i.d. \mathcal{N}(0, I) \]

- Bond variables: nominal short rate, realized inflation, 5-year - 1-month Treasury spread

- Stock variables: log price-dividend, log real dividend growth for: CRSP, NAREIT real estate, listed infra, small, growth

- SDF:

\[ m_{t+1}^\$ = -y_t^\$(1) - \frac{1}{2} \Lambda'_t \Lambda_t - \Lambda'_t \varepsilon_{t+1} \]
Model Matches Time-Series of Bond Yields
Also Matches underlying Components of Bond Yield: Real + Nominal

![Graphs showing average nominal yield, average real yield, risk premium, and decomposing 5-year nominal bond yield over time.](chart.png)
Fits Equity Risk Premia as well as Stock Price Levels
Rich Patterns in Temporal Pricing of Risk

Average zero-coupon bond risk premium

Average REIT div strip risk premium

Average Infra div strip risk premium

Average Small div strip risk premium

Average Growth div strip risk premium

Average Market div strip risk premium

Average Small div strip risk premium
Imputed Dividend Strip Model Matches Data when Available
Outcome of Model: Bond + Dividend Strip Prices

Zero-coupon Bond Prices

Dividend Strip Prices - Stock Market

Dividend Strip Prices - REITS

Dividend Strip Prices - Infrastructure

Dividend Strip Prices - Small

Dividend Strip Prices - growth
Factor Exposure in PE Funds by Horizon — Buyout

Factor Exposure (q) by Horizon

Panel R^2: 0.17 | Collapsed R^2: 0.93

- Bond
- Small
- Stock

Years from Fund Inception

b Coefficient

-0.1
0.0
0.1
0.2
0.3

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Factor Exposure in PE Funds by P/D Ratio – Buyout

- Venture Capital
- Real Estate
- Infrastructure

Factor Exposure (q) by P/D Tercile

Legend:
- Bond
- Small
- Stock

PD Tercile

Coefficient

1 2 3
Factor Exposure in PE Funds by Horizon — Buyout, Lasso

Venture Capital  Real Estate  Infrastructure
Factor Exposure in PE Funds by Horizon – VC

Factor Exposure (q) by Horizon

Panel R^2: 0.05 | Collapsed R^2: 0.89

Years from Fund Inception

b Coefficient

Bond
Growth
Small
Factor Exposure in PE Funds by Horizon — Real Estate

Factor Exposure (q) by Horizon

Panel $R^2$: 0.2 | Collapsed $R^2$: 0.97
Histogram of Fund–Level Profit Relative to Replicating Portfolio

Avg Profit is: 0.261  Fraction above 10% is: 0.633

Fund Count

Profit Relative to $1 Committed to Replicating Portfolio

-1 0 1 2 3

Losses
Excess Profits
Average Fund-level profit by Vintage

RAP Profit, OLS

Venture Capital
Real Estate
Infrastructure
Burgiss - Venture Capital
Buyout - Lasso
Venture Capital - Lasso
Real Estate - Lasso
Infrastructure - Lasso


Vintage

RAP Profit, OLS

0.75
0.50
0.25
0.00
-0.25
-0.50


Vintage

Average Fund-level profit by Vintage
Replicating Portfolio and Private Equity Returns (IRR)

- Actual Fund
- Replicating Portfolio

Expected Return by Horizon and Risk Exposure

- Bond
- Small
- Stock

Fund Age (Years)
PE Expected Return — Buyout

- Venture Capital
- Real Estate
- Infrastructure
- Duration
- Betas

Buyout - Lasso  Venture Capital - Lasso  Real Estate - Lasso  Infrastructure - Lasso

Expected Return by Vintage

Expected Return

Vintage
PE Comparison with PME – Buyout

- Venture Capital
- Real Estate
- Infrastructure

- Buyout, Lasso
- Venture Capital, Lasso
- Real Estate, Lasso
- Infrastructure, Lasso

RAP from OLS Model against K–S PME. Correlation: 0.76

Correlation: 0.76
1. Develop methodology to value and understand risk/return characteristics when only cash flows, not returns, are available

2. Find PE funds take asset-specific specific exposure. Small, growth, real estate, infra exposure has migrated to PE

3. Risk-adjusted profit (and compensation for illiquidity), as well as expected return on replicating portfolios, declining over time
Cash-Flow Variation Across Horizon and Vintage — Venture Capital

Capital Distribution Relative to $1 Invested vs. Year

Vintage
- 1990
- 1991
- 1992
- 1993
- 1994
- 1995
- 1996
- 1997
- 1998
- 1999
- 2000
- 2001
- 2002
- 2003
- 2004
- 2005
- 2006
- 2007
- 2008
- 2009
- 2010
- 2011
- 2012
- 2013
- 2014
- 2015

Capital Distribution Relative to $1 Invested

Year
Cash-Flow Variation Across Horizon and Vintage — Real Estate
Cash-Flow Variation Across Horizon and Vintage — Infrastructure

![Graph showing capital distribution relative to $1 invested for different vintages over years. The x-axis represents years from 2000 to 2015, and the y-axis represents capital distribution relative to $1 invested. Multiple lines represent different vintages, each with distinct colors.](image-url)
Factor Exposure in PE Funds by P/D Ratio – VC

Factor Exposure (q) by P/D Tercile

- Bond
- Growth
- Small

PD Tercile

a Coefficient

Factor Exposure (q) by P/D Tercile chart with lines indicating exposure for Bond, Growth, and Small categories across different P/D terciles.
Factor Exposure (q) by P/D Tercile

- Bond
- REIT
- Stock

PD Tercile
Factor Exposure in PE Funds by P/D Ratio – Infrastructure

Factor Exposure (q) by P/D Tercile

- Bond
- Infra
- Stock
Factor Exposure in PE Funds by Horizon — VC, Lasso

Factor Exposure (q) by Horizon: $R^2: 0.02$

- Bond
- Growth
- Small
Factor Exposure in PE Funds by Horizon — Real Estate, Lasso

Factor Exposure (q) by Horizon: $R^2: 0.06$
Factor Exposure in PE Funds by Horizon — Infrastructure, Lasso

Factor Exposure (q) by Horizon: \(R^2: 0.06\)
Factor Exposure in PE Funds by Horizon — Buyout, Lasso
Factor Exposure in PE Funds by Horizon – VC, Lasso
Factor Exposure in PE Funds by Horizon — Real Estate, Lasso
Factor Exposure in PE Funds by Horizon — Infrastructure, Lasso

Factor Exposure (q) by P/D Tercile

- Bond
- Infra
- Stock

PD Tercile

Factor Exposure (q) by P/D Tercile
Histogram of Fund–Level Profit Relative to Replicating Portfolio

Avg Profit is: 0.08  Fraction above 10% is: 0.311
histogram of fund-level profit relative to replicating portfolio.

average profit is: 0.089  fraction above 10% is: 0.475

profit relative to $1 committed to replicating portfolio.

losses
excess profits

fund count

profit relative to $1 committed to replicating portfolio

47
Histogram of Fund–Level Profit Relative to Replicating Portfolio

Avg Profit is: 0.252  Fraction above 10% is: 0.579
Average Fund-level profit by Vintage

RAP Profit, OLS

Vintage
Average Fund–level profit by Vintage

RAP Profit, OLS

Vintage

1995 2000 2005 2010
Average Fund-level profit by Vintage

RAP Profit, OLS

Vintage

Histogram of Fund–Level Profit Relative to Replicating Portfolio

Avg Profit is: 0.216   Fraction above 10% is: 0.577
Histogram of Fund-Level Profit Relative to Replicating Portfolio

Avg Profit is: 0.039  Fraction above 10% is: 0.281

Fund Count

Profit Relative to $1 Committed to Replicating Portfolio

Losses
Excess Profits
Histogram of Fund–Level Profit Relative to Replicating Portfolio

Avg Profit is: 0.08  Fraction above 10% is: 0.456

Profit Relative to $1 Committed to Replicating Portfolio

Fund Count

Losses
Excess Profits
Histogram of Fund–Level Profit Relative to Replicating Portfolio

Avg Profit is: 0.155   Fraction above 10% is: 0.474
Average Fund–level profit by Vintage

RAP Profit, Lasso

Vintage

1995 2000 2005 2010
PE Fund Risk-Adjusted Profits — Infrastructure, Lasso

Average Fund–level profit by Vintage

RAP Profit, Lasso

Vintage

1996 2000 2004

0.0

0.4

-0.4

-0.8
Replicating Portfolio and Private Equity Returns (IRR)
Aggregated Replicating Portfolios Match Fund Cash Flows Each Vintage — VC, Lasso
Aggregated Replicating Portfolios Match Fund Cash Flows Each Vintage — Real Estate, Lasso
Replicating Portfolio and Private Equity Returns (IRR)
Private Equity Fund Expected Return — Buyout, Lasso

Expected Return by Horizon and Risk Exposure

- Bond
- Small
- Stock

Fund Age (Years) vs. Expected Return
Expected Return by Horizon and Risk Exposure

- Bond
- Growth
- Small

Fund Age (Years)

Expected Return
Private Equity Fund Expected Return — Infrastructure, Lasso

Expected Return by Horizon and Risk Exposure

Bond
Infra
Stock

Fund Age (Years)

Expected Return
Expected Return by Vintage

- Expected Return
- Vintage

- 1980
- 1990
- 2000
- 2010
Expected Return by Vintage

Vintage

Expected Return

0.00 0.05 0.10 0.15 0.20

1980 1990 2000 2010
Expected Return by Vintage

Expected Return

Vintage
Expected Return by Vintage

Expected Return

Vintage
RAP from OLS Model against K−S PME. Correlation: 0.87
RAP from OLS Model against K–S PME. Correlation: 0.6
RAP from OLS Model against K–S PME. Correlation: 0.38
RAP from Lasso Model against PME. Correlation: 0.76
PE Comparison with PME — VC, Lasso

RAP from Lasso Model against PME. Correlation: 0.89
RAP from Lasso Model against PME. Correlation: 0.6
RAP from Lasso Model against PME. Correlation: 0.46
Cash-flow weighted average maturity

- Buyout
- VC
- RE
- Infra

Implied Stock and Bond Betas

Stock beta of PE funds

Bond beta of PE funds

- Buyout
- VC
- RE
- Infra

1980 1990 2000 2010

-1 0 1 2 3

1980 1990 2000 2010

-1 0 1 2 3
Average Fund–level profit by Vintage


RAP Profit, OLS

Profit trend over vintage years.
Replicating Portfolio and Private Equity Returns (IRR)

- Actual Fund
- Replicating Portfolio