Monetary Policy in an Era of Global Supply Chains*

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Abstract

We study the implications of global supply chains for the design of monetary policy, using a small-open economy New Keynesian model with multiple stages of production. First, within the family of simple monetary policy rules, a rule that targets separate producer price inflation at different production stages, in addition to output gap and real exchange rate, is found to outperform alternative policy rules. Second, as an economy becomes more open, the optimal weight on the upstream inflation also rises relative to that on the final stage inflation. Third, if we have to choose among aggregate price indicators, targeting PPI inflation is significantly better than targeting CPI inflation alone. Fourth, as the production chain becomes longer, the optimal weight on PPI inflation should also rise. Fifth, a trade cost shock such as a rise in the import tariff can alter the optimal weights on different inflation variables.

Keywords: Taylor rule; CPI; PPI; global supply chains; monetary policy; multi-stage production

JEL classification: E31; E52; E58; F11; F12; F41; F62

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1 Introduction

We live in a world of supply chains. From the data of World Input-output Tables, in 2000, the world gross output was 1.97 times that of the world value added, suggesting a large role of intermediate inputs in production and supply chains in the modern economy. Many supply chains are global. Trade in intermediate goods has been growing faster than trade in final goods. The importance of supply chains has also grown over time; the ratio of gross output to value added has increased to 2.18 by the end of 2014. In this paper, we study the implications of global supply chains for the design of optimal monetary policy.

There is an active research on outsourcing and offshoring in the field of international trade (e.g., Feenstra, 1999; Hummels, Ishii, and Yi, 2001; Koopman, Wang, and Wei, 2014; Johnson and Nuguero, 2016; Antràs, 2016), where firms purchasing intermediate inputs from other firms, sometimes foreign firms, for further processing. Global supply chains are rising in importance as an increased fraction of output is produced as intermediate inputs rather than final consumption. As important, it is accompanied by an increase in the number of production stages in many sectors (e.g., Wang et al., 2017).

A voluminous but separate literature in monetary economics studies optimal monetary policy. Woodford (2010) provides an excellent survey of the subject in an closed-economy setting, whereas Corsetti, Dedola, and Leduc (2010) supply an excellent survey of issues in the new open-economy macroeconomics. While central banks typically target only CPI inflation, the literature has studied whether CPI or PPI is more appropriate for monetary policy goal (e.g., De Paoli, 2009; De Gregorio, 2012). Two pioneering papers are especially worth noting. In an open-economy model featuring a single stage of production (i.e., no supply chains), Gali and Monacelli (2005) suggest that PPI is a better target, where PPI in their context is the price index for domestically produced final products. In a closed-economy featuring two stages of production (i.e., there are simple national supply chains but not global supply chains), Huang and Liu (2005) demonstrate that the optimal simple rule should include PPI inflation as well as CPI, where PPI is the sales weighted average of producer prices of domestically produced products. Strum (2009) expands on this line of inquiry. The intuition is that, in a New Keynesian model, a PPI inflation causes distortions in the allocation of productive resources, including among domestic producers of intermediate goods. Since all firms are owned by the households, the distortions associated with a PPI inflation reduce household welfare too.

Interactions beween multi-stage production and economic openness and their implications for the design of monetary policy have not been much explored. For example, when an economy becomes more open, should the optimal weights on the upstream sector inflation rise or fall relative to those on the final stage inflation? Should trade frictions such as a rise in the tariff rate affect the design of monetary policy?

We build a New Keynesian model that features simultaneously multi-stage production and openness. A noteworthy feature of the equilibrium is that there are separate Philips curve relationships for each production stage that link the producer price inflation of a given stage to both the expected next-period inflation and log deviation of that stage's real marginal cost from the steady state. The real marginal cost term for each production stage, in turn, is a function of change in the real exchange rate (due to the openness of the economy) and a relative price gap between the production stages (due to multiple stages of production).

Following Rotemberg and Wooford (1999), Gali and Monacelli (2005), and Huang and Liu (2005), we assume that the central bank maximizes the welfare of the household which is approximated by a second order expansion of the utility function. By making use of equilibrium conditions, we can see that the welfare loss function contains not only output gap and change in the real exchange rate, but also separate producer price inflation in each production stage, separate terms for employment fluctuations in each production stage, and the relative price gap between the production stages. Parameters describing the openness of the economy (shares each sector's output that are sold abroad and shares of inputs imported from abroad) appear in the welfare loss function as well.

We consider a family of simple monetary rules, including (a) the classic Taylor (2003) rule that features output gap, CPI inflation, and change in the real exchange rate, (b) the Gali-Monacelli (2005) in which PPI inflation takes the place of CPI inflaiton in the Taylor rule, (c) a rule that includes separate producer price inflation in each stage of production as well as output gap and change in the real exchange rate, and (d) some variations of the previous rules that omit either output gap, change in the real exchange rate, or both. For each rule, we compute optimal weights on each term in the monetary policy rule.

Within the family of simple monetary policy rules, a rule that targets separate producer price inflation at each stage of production (as well as output gap and change in the real exchange rate) outperforms all other rules. As an economy becomes more open, the optimal weight on the upstream sector inflation rises relative to that on the final stage inflation.

Greater trade frictions reduce the openness of an economy. This, in the model, would dampen the optimal weight on the upstream sector inflation. However, we document a direct welfare loss associated with greater trade frictions even if the monetary policy rule adjusts optimally. In other words, the central bank cannot completely offset the negative effects of greater trade frictions. (Of course, the welfare loss would be even greater without the re-optimization by the central bank.)

In general, because the optimal weights on the inflation at different production stages are not proportional to the sales weights, the PPI inflation would not be sufficient to replace these production-stage-specific inflation. At the same time, if we restrict ourselves to only consider aggregate inflation measurs (PPI and CPI), targeting PPI inflation is better than targeting CPI inflation (in addition to output gap and change in the real exchange rate). That is because PPI inflation puts at least some weight on the upstream sector inflation whereas CPI inflation puts none.

We also consider a general version of the model that can feature an arbitrary number of production stages (but in a closed economy). In this case, as the number of production stages increases, the optimal weights on the upstream sector inflation, or the optimal weight on the PPI inflation (if we only consider aggregate price index), should increase as well. This discussion is collected in an appendix.

Is it possible for countries to obtain separate producer price index for upstream and downstream sectors? In turns out that the United States, Japan, Australia, Korea and Canada already collect and report such data. For instance, the US Bureau of Labor Statistics (BLS) has considered a four-stage production process and accordingly constructed a stage-of-processing price indices in the PPI Final Demand-Intermediate Demand indice.[1] Figure 1 presents separate inflation paths for producer price indexes at different production stages as well as core CPI for the United States (on the left) and Australia (on the right). It can be seen clearly that the producer price inflation in the upstream sectors and the final stage do not move together during the sample years. This means that the monetary policy implied by a rule that includes separate producer price inflation would look different from the classic Taylor rule.

Figure 1: Stage-of-processing producer price index and core CPI, US and Australia

This paper builds on the literature on monetary policy by introducing global supply chains. Corsetti, Dedola and Leduc (2010) provide a comprehensive survey on early literature. Gali and Monacelli (2005) build a small-open economy New Keynesian model that features a single stage

[1] Details about PPI Final Demand-Intermediate Demand indices can be found at https://www.bls.gov/ppi/fdidsummary.htm.

of production, and compare three alternative simple policy rules: CPI-based Taylor rule, PPI-based rule, and an exchange rate peg. De Paoli (2009) demonstrates in a model with more general parameterization but also a single stage of production and focuses on terms of trade externality in driving the optimal monetary policy. Matsumura (2018) studies monetary policy in an economy with multiple sectors but still with only one stage of production. Our point of departure is a simultaneous introduction of multi-stage production and economic openness. We pay special attention to an interaction between openness and multi-stage production and its implication for the monetary policy rule. We also investigate effects of a lengthening of the supply chains.

Gong, Wang, and Zou (2016) study optimal simple monetary policy rules in a two-country New Keynesian model with two stages of production. However, since labor is assumed to be only used in the first stage of production, there is no misallocation of labor across production stages. In comparison, we do allow for potential misallocation across production stages. This qualitatively changes the results of the analysis.

This paper is also related to a literature that explores the effects of globalization on national inflation. Auer, Levchenko, and Saurè (2016), Auer, Borio, and Filardo (2017), and Forbes (2018) study how national inflation dyanmics are altered by inter-country connections through supply chains, and how the trade-offs in inflation targeting policies may be changed for central banks. Wei and Xie (2019) demonstrate how an increase in the number of production stages can lead to a weakening in the correlation between PPI and CPI inflation. However, that literature does not explore how an interaction between multi-stage production and globalization affects the design of the monetary policy.

The rest of the paper proceeds as follows: Section 2 introduces the basic model with global supply chains; Section 3 characterizes the steady-state, flexible-price, and sticky-price equilibrium in the special case of two stages of production, derives an expression for the welfare loss function, and discusses the comparative statics of changes in import tariff. Section 4 compares several monetary policy rules via calibrations. Finally, Section 5 concludes the paper. An appendix discusses a more general case with an arbitrary number of production stages.

2 The model setting

Consider a small-open economy New Keynesian model with an infinitely lived representative household. The household maximizes its utility through consumption and leisure. The household owns all domestic firms and receives dividends from them.

The production of a final good requires N-stages of production, which constitutes a vertical production chain. In each stage of production, a large number of domestic firms produce a unit continuum of differentiated outputs, i.e., $u \in [0, 1]$. In the first stage of production, firms use domestic labor as the only input. In each of the subsequent stages of production, intermediary inputs purchased from the previous stage (from both domestic and foreign sources) together with labor are used together for production. All production features constant returns to scale.

The firms and households take international prices of inputs and foreign demand of outputs as given (the small open economy assumption). While the firms are price-takers in the factor markets, they are assumed to be monopolistic competitive in their outputs and set the output prices in their own currency (the producer currency pricing assumption, or PCP).

2.1 Household

The representative household has the following utility function and budget constraint:

$$E\Sigma_{t=0}^{\infty}\beta^{t}[U(C_t) - V(L_t)]$$

$$s.t.\, P_tC_t + E_t\{D_{t,t+1}B_{t+1}\} \leq W_tL_t + \Pi_t + B_t$$

where C_t is the final consumption good, L_t is the supply of labor, $D_{t,t+1}$ is the price of a one-period nominal bond paying off in domestic currency, W_t is the nominal wage, Π_t includes both firm profits and a lump-sum transfer of any government tax revenue, and B_t denotes the holding of a riskless one-period bond.

The consumption good is a composite of both domestically produced and imported final goods, i.e.,

$$C_t = \Theta\bar{Y}_{NH,t}^{\gamma}\bar{Y}_{NF,t}^{1-\gamma}$$

where $\bar{Y}_{NH,t} = [\int_0^1 Y_{NH,t}(u)^{\frac{\theta-1}{\theta}}du]^{\frac{\theta}{\theta-1}}$ is a bundle of domestically produced differentiated final goods, and $\bar{Y}_{NF,t}$ is a bundle of foreign produced differentiated final goods. γ is the share of the household expenditure on domestically produced final goods, $1-\gamma$ is the share of the expenditure on imports, and θ is the elasticity of substitution among the differentiated final goods. $\Theta = [\gamma^{\gamma}(1-\gamma)^{1-\gamma}]^{-1}$ is a constant for normalization.

By the household's expenditure minimization problem, the demand function for the final goods are:

$$Y_{NH,t}^{d}(u) = [\frac{P_{NH,t}(u)}{\bar{P}_{NH,t}}]^{-\theta}\frac{\gamma P_t}{\bar{P}_{NH,t}}C_t$$

$$Y_{NF,t}^{d} = \frac{(1-\gamma)P_t}{\bar{P}_{NF,t}}C_t$$

where the aggregate price index for the final consumption is $P_t = \bar{P}_{NH,t}^{\gamma}\bar{P}_{NF,t}^{1-\gamma}$, the aggregate price index for the domestic produced final goods is $\bar{P}_{NH,t} = (\int_0^1 P_{NH,t}(\mu)^{1-\theta}du)^{\frac{1}{1-\theta}}$, and the aggregate price index for the foreign produced final goods is $\bar{P}_{NF,t} = T_t\mathcal{E}_tP_{NF,t}^{*}$. The term $\mathcal{E}_t$ is the price of foreign currency in units of domestic currency, $P_{NF,t}^{*}$ is the exogenous foreign price in foreign currency, and T_t is a uniform tariff on imports. An upper star $*$ is used to denote variables in the foreign country (denominated in the foreign currency).

By the household's utility maximization problem, we obtain the labor supply and Euler Equation, respectively, as

$$\frac{W_t}{P_t} = \frac{V'_{N,t}}{U'_{c,t}} \tag{1}$$

and

$$U'_{c,t} = \beta R_t E_t[U'_{c,t+1}\frac{P_t}{P_{t+1}}] \tag{2}$$

where $R_t = \frac{1}{E_t D_{t,t+1}}$ is the gross return on a one-period risk-free nominal bond in domestic currency.

Assuming a CRRA utility function, i.e., $U(C_t) = \frac{C_t^{1-\sigma}-1}{1-\sigma}$ and $V(N_t) = \frac{L_t^{1+\psi}}{1+\psi}$, the labor supply (1) and Euler equation (2) can be written in log-linearized form as

$$w_t - p_t = \sigma c_t + \psi n_t \tag{3}$$

$$c_t = E_t(c_{t+1}) - \frac{1}{\sigma}[i_t - E_t(\pi_{t+1}) - \rho] \tag{4}$$

where lower-case letters denote the logarithm of the respective variables, $i_t = R_t - 1$ is the nominal interest rate in domestic currency, $\pi_{t+1} = p_{t+1} - p_t$ is the CPI inflation, and $\rho = \beta^{-1} - 1$.[2]

The household is assumed to have access to a complete set of state-contingent securities (both domestic and international).[3] Then, the intertemporal marginal rates of substitution must be equalized across countries, i.e.,

$$(\frac{C^*_{t+1}}{C^*_t})^{-\sigma}\frac{\mathcal{E}_t P^*_t}{\mathcal{E}_{t+1}P^*_{t+1}} = (\frac{C_{t+1}}{C_t})^{-\sigma}\frac{P_t}{P_{t+1}}$$

which implies

$$C_t = \theta^* C^*_t Q_t^{1/\sigma} \tag{5}$$

i.e.,

$$c_t = c^*_t + \sigma^{-1} q_t \tag{6}$$

where $Q_t = \frac{\mathcal{E}_t P^*_t}{P_t}$ is the real exchange rate, θ^* is a constant whose value depends on initial asset position. Without loss of generality, we set $\theta^* = 1$. Furthermore, we assume that the foreign consumption follows an $AR(1)$ process, i.e., $c^*_t = \rho_{c^*} c^*_{t-1} + \epsilon_{n,t}$ with $\rho_{c^*} \in (0,1)$ and $\epsilon_{c^*} \sim N(0, \sigma^2_{c^*})$.

From the risk sharing condition (6), given exogenous foreign consumption, there is an increase in consumption if and only if the real exchange rate depreciates. Under the assumption of complete international financial markets, it also implies uncovered interest parity, i.e., $i_t - i^*_t = E_t(\Delta e_{t+1})$, where $e_t = ln\mathcal{E}_t$.

[2] It's equivalent to write $i_t = -logE_t D_{t,t+1}$ and $\rho = -log\beta$.

[3] The same assumption of complete asset market is also imposed in Gali and Monacelli (2005) and De Paoli (2009).

2.2 Firms

Each final good requires N-stages of production, with a large number of domestic firms producing a unit continuum of differentiated outputs and featuring constant returns to scale at each stage. In the first stage, the production function for good $u \in [0,1]$ is given by

$$Y_{1H}(u) + Y_{1H}^{X}(u) = A_1 L_1(u)$$

where A_1 is the productivity in stage 1 and $L_1(u)$ is the quantity of labor employed in the production of good u. The output is either sold at home $Y_{1H}(u)$ or exported abroad $Y_{1H}^{X}(u)$.

The stage-1 output sold at home and the corresponding price are given, respectively, by

$$Y_{1H} = [\int_0^1 Y_{1H}(u)^{\frac{\theta-1}{\theta}} du]^{\frac{\theta}{1-\theta}}$$

$$P_{1H} = [\int_0^1 P_{1H}(u)^{1-\theta} du]^{\frac{1}{1-\theta}}$$

In each subsequent stage, the production needs to use intermediate inputs. The production in stage n (for $n = 2, \ldots, N$) can be viewed as a two-step process. In the first step, a firm purchases all differentiated outputs produced in the previous stage $n-1$ from all global sources and form a bundle of intermediate inputs. Specifically, the intermediate input bundle to be used in the production stage n, $\bar{Y}_n$, is a bundle of two composites of stage $n-1$ outputs:

$$\bar{Y}_n = \Theta \bar{Y}_{(n-1)H}^{\gamma} \bar{Y}_{(n-1)F}^{1-\gamma}$$

$$\bar{Y}_{(n-1)H} = [\int_0^1 Y_{(n-1)H}(j)^{\frac{\theta-1}{\theta}} dj]^{\frac{\theta}{\theta-1}}$$

where $Y_{n-1,H}(j)$ is the amount of good j that is domestically produced in stage $n-1$ and purchased by the firm in stage n, and $\bar{Y}_{(n-1)F}$ is the amounts of composite good that foreign firms produced in stage $n-1$. In the factor market, domestic firms are price takers in purchasing foreign composite goods $\bar{Y}_{(n-1)F}$, and because of the small-open economy setup, the supply of foreign composite goods is perfectly elastic in price.

The aggregate price index for the inputs in stage n is then given by

$$\bar{P}_n = \bar{P}_{(n-1)H}^{\gamma} \bar{P}_{(n-1)F}^{1-\gamma}$$

$$\bar{P}_{(n-1)H} = [\int_0^1 P_{(n-1)H}(u)^{1-\theta} du]^{\frac{1}{1-\theta}}$$

$$\bar{P}_{(n-1)F} = T_t \mathcal{E}_t P^*_{(n-1)F}$$

where $P^*_{(n-1)F}$ is the price of composite goods in foreign currency produced in stage $n-1$ by foreign firms. Note that the output price in stage n satisfies $P_{nH} = \bar{P}_{nH}$ for $\forall n = 1, 2, \ldots, N$.

In the second step, the firm combines the composite intermediate good with labor input to produce an output. The production function for good u in stage n is given by

$$Y_{nH}(u) + Y^X_{nH}(u) = \Theta^* A_n \bar{Y}_n(u)^{\phi} L_n(u)^{1-\phi}$$

where $\Theta^* = [(1-\phi)^{1-\phi}\phi^{\phi}]^{-1}$ is a constant for normalization. We assume the technology in each stage following the $AR(1)$ process $a_{n,t} = \rho_n a_{n,t-1} + \epsilon_{n,t}$ with $a_{n,t} = lnA_{n,t}$ and $\rho_n \in (0,1)$ for $n = 1, 2, \ldots, N$. Note that $\{\epsilon_n\}_{n=1}^N$ are i.i.d. shocks with the same normal distribution, i.e., $\epsilon_n \sim N(0, \sigma_a^2)$.

Since the production of any good in stage n needs a bundle of output from the previous stage as inputs, it captures a feature of a typical input-output table in which the output from all sectors may be used as inputs into the production. In the language of Baldwin and Venables (2013), the entire manufacturing production process follows a combination of a snake and a spider patterns. At a given stage, outputs from the previous stage from all over the world are purchased to form a composite intermediate input, resembling a spider pattern. Going from one stage of production to the next, the process resembles a snake pattern.

By the small-open economy set-up, the foreign demand for domestic output in stage $n = 1, 2, \ldots, N$ is given by

$$Y^{Xd}_{nH}(u) = (\frac{P_{nH}(u)}{P_{nH}})^{-\theta} \frac{Y^*_{nH} P^*_{nH} \mathcal{E}_t}{P_{nH}}$$

where Y^*_{nH} is exogenous foreign demand and P^*_{nH} is the price for domestic produced composite goods in foreign currency. This foreign demand function can be derived from the cost minimization problem of a foreign buyer who aggregates the composite of domestic produced goods.

Similarly, the domestic demand function in stage $n = 1, 2, \ldots, N$ is given by

$$Y_{nH}(u) = (\frac{P_{nH}(u)}{P_{nH}})^{-\theta} \frac{\bar{Y}_{nH} \bar{P}_{nH}}{P_{nH}}$$

It is worth noting that nominal exchange rate is not a sufficient statistics for import tariffs in a world of multi-stage production. An increase in the nominal exchange rate (i.e., a depreciation of the domestic currency) raises both the input costs and foreign demand for domestic goods simultaneously. In comparison, an increase in import tariffs only affects production cost through higher costs of imported inputs.

2.3 The firms' pricing problem

Firms in each stage of production are price-takers in factor markets, but are monopolistic competitors in their outputs. We assume firms following Calvo pricing, and the probability that firms in stage n can adjust prices freely is $1-\alpha_n$, $n=1,\ldots,N$. Then, by the law of large numbers, in each period, a fraction $1-\alpha_n$ of firms in stage n can adjust prices while the rest of firms have to stay unchanged. For a firm producing good u in stage n, which can set a new price in period t, it chooses price $P_{nH}(u)$ in domestic currency for its product sold both at home and in the foreign market. Its maximization problem becomes

$$max_{P_{nH,t}(u)} E_t \Sigma_{k=t}^{\infty} \alpha_n^{k-t} D_{t,k}[(1+\tau)P_{nH,t}(u) - \Psi_{n,k}(u)][Y_{nH,k}^d(u) + Y_{nH,k}^{Xd}(u)]$$

where τ is the subsidy to firms that corrects the distortion from monopolistic competition, $\Psi_{n,k}(u) = \bar{P}_{n,k}^{\phi} W_k^{1-\phi}/A_{n,k}$ is the nominal unit production cost for $n=2,\ldots,N$ and $\Psi_{1,k}(u) = W_k/A_{1,k}$, $\bar{P}_{n,k}$ is the price for the composite of intermediate input goods at stage n, and $Y_{nH,k}^d(u)$ and $Y_{nH,k}^{Xd}(u)$ denote the output demand from both domestic and foreign market respectively.

The optimal pricing decision is given by

$$P_{nH,t}^o(u) = \frac{\mu}{1+\tau} \frac{E_t \Sigma_{\tau=t}^{\infty} \alpha_n^{\tau-t} D_{t,\tau} \Psi_{n,\tau}(u)[Y_{nH,\tau}^d(u) + Y_{nH,k}^{Xd}(u)]}{E_t \Sigma_{\tau=t}^{\infty} \alpha_n^{\tau-t} D_{t,\tau}[Y_{nH,\tau}^d(u) + Y_{nH,k}^{Xd}(u)]}$$

where $\mu = \frac{\theta}{\theta-1}$ is the markup in the market for producing outputs in each stage.[4] To be abstract from the distortion generated by monopolistic competition, a subsidy is imposed such that $1+\tau=\mu$.

Taking input prices as given, the cost minimization problem for the firms at stage n for $n = 2,\ldots N$ in period t yields a factor demand function as

$$\bar{Y}_{n,t}^d = \phi \frac{\Psi_{n,t}}{\bar{P}_{n,t}} \int_0^1 [Y_{nH,t}^d(u) + Y_{nH,t}^{Xd}(u)]du \tag{7}$$

$$L_{n,t}^d = (1-\phi) \frac{\Psi_{n,t}}{W_t} \int_0^1 [Y_{nH,t}^d(u) + Y_{nH,t}^{Xd}(u)]du \tag{8}$$

$$\bar{Y}_{(n-1)H,t}^d = \frac{\gamma \bar{P}_{n,t}}{\bar{P}_{(n-1)H,t}} \bar{Y}_{n,t}^d \tag{9}$$

$$\bar{Y}_{(n-1)F,t}^d = \frac{(1-\gamma)\bar{P}_{n,t}}{\bar{P}_{(n-1)F,t}} \bar{Y}_{n,t}^d \tag{10}$$

[4] Since we assume the elasticity of substitution among differentiated goods to be the same across stages, it implies the same markup across different stages.

and

$$Y^d_{(n-1)H,t}(u) = (\frac{P_{(n-1)H,t}(u)}{\bar{P}_{(n-1)H,t}})^{-\theta}\bar{Y}^d_{(n-1)H,t} \tag{11}$$

In the first stage of production, the firm problem is simpler since labor is the only input. Specifically, the optimal pricing decision for a firm in stage 1 is

$$P^o_{1H,t}(u) = \frac{E_t\Sigma_{\tau=t}^{\infty}\alpha_1^{\tau-t}D_{t,\tau}\Psi_{1,\tau}(u)[Y^d_{1,\tau}(u) + Y^{Xd}_{1H,k}(u)]}{E_t\Sigma_{\tau=t}^{\infty}\alpha_1^{\tau-t}D_{t,\tau}[Y^d_{1,\tau}(u) + Y^{Xd}_{1H,k}(u)]}$$

where $\Psi_{1,\tau}(u) = W_\tau/A_{1,\tau}$ is the unit production cost in stage 1, and the subsidy has been imposed to offset the markup.

Since labor is the only input in the first stage, the labor demand is

$$L^d_{1,t} = \frac{\Psi_{1,t}}{W_t}\int_0^1 [Y^d_{1H,t}(u) + Y^{Xd}_{1H,t}(u)]du$$

As the goods are symmetric, we drop good index u in the price variable. The aggregate price index for the outputs in stage n, $n = 1, 2, \ldots, N$, is thus given by

$$P_{nH,t} = [\alpha_n P^{1-\theta}_{nH,t-1} + (1-\alpha_n)(P^o_{nH,t})^{1-\theta}]^{\frac{1}{1-\theta}} \tag{12}$$

2.4 The market clearing conditions and equilibrium definition

Equilibrium definition: given exogenous monetary policy (the rule of nominal interest rate or nominal exchange rate $\{i_t, \mathcal{E}_t\}$) and tariffs $\{T_t\}$, as well as exogenous foreign demand and foreign prices $\{C^*_t, P^*_{nH,t}, P^*_{nF,t}, Y^*_{nH,t}\}_{n=1}^N$, the market equilibrium consists of a set of stochastic processes – $\{C_t, L_t\}$ for domestic households, $\{L^d_{n,t}(u), Y_{nH,t}(u), Y^X_{nH,t}(u), P_{nH,t}(u)\}_{n=1}^N$ for firms $u \in [0,1]$ and price indices $\{P_{nH,t}\}_{n=1}^N$, and wages and real exchange rate $\{W_t, Q_t\}$, satisfying the following conditions:

1. Taking prices and wages as given, the representative household maximizes its utility.
2. Taking intermediate input goods prices, wages, and all output prices except their own's as given, firms in each stage maximize their profits.
3. The intertemporal trade balance condition holds.
4. The labor market clears, and the goods markets clear in all production stages, i.e.,

$$L_t = \sum_{n=1}^N L^d_t, \quad Y_{nH} = Y^d_{nH}, \quad Y^X_{nH} = Y^{Xd}_{nH}$$

It is worth noting that the intertemporal trade condition is derived from no-Ponzi game in the household's debt, which does not necessarily require trade balance in each period.

3 The case of two-stage production

For a small-open economy with two stages of production, we can obtain more analytical expressions. We now characterize the equilibrium of this economy. (For the general case of N-stage of production, we obtain some interesting results and present them in Appendix A.)

We characterize the steady-state, flexible-price, and sticky-price equilibria, in turn, where the steady-state equilibrium is efficient. We derive the second-order approximation of the welfare loss function for the sticky price case. With sticky prices, there is misallocation of labor across production stages. Because the terms of trade externality and the labor allocation distortions interact with each other, the real exchange rate and the relative price gaps between the production stages enter the welfare loss function.

The model lends itself well to thought experiments on how a change in openness affects the optimal monetary policy rule. This also facilitates a discussion on how a change in the import tariff affects monetary policy. While we only consider domestic productivity shocks in this section, a broader set of stochastic shocks are considered in the numerical analysis in Section 4.

3.1 The steady-state equilibrium

In the steady state, $A_1 = A_2 = 1$, and foreign variables are kept constant. The price index satisfies $P_{1H} = \bar{P}_{1H} = P_{1H}(u)$ and $P_{2H} = \bar{P}_{2H} = P_{2H}(u)$. By $\bar{P}_2 = \bar{P}_{1H}^{\gamma}\bar{P}_{1F}^{1-\gamma}$, from Section 2.2, the price indexes of domestically produced goods across stages are given by

$$P_{1H} = W$$

and

$$\begin{aligned} P_{2H} &= W^{1-\phi}[\bar{P}_{1H}^{\gamma}\bar{P}_{1F}^{1-\gamma}]^{\phi} \\ &= W^{1-\phi+\gamma\phi}(T\mathcal{E})^{(1-\gamma)\phi}(P_{1F}^{*})^{(1-\gamma)\phi} \end{aligned}$$

Since $P_{1H}(u) = P_{1H}$ and $P_{2H}(u) = P_{2H}$, it follows that $Y_{1H}^d(u) = \bar{Y}_{1H}^d$ and $Y_{2H}^d(u) = \bar{Y}_{2H}^d$. The goods market clearing condition requires $Y_{1H} = \bar{Y}_{1H}^d$ and $Y_{2H} = \bar{Y}_{2H}^d$. Therefore, the factor demand functions are given by

$$\bar{Y}_2^d = \phi W^{1-\phi}(\bar{P}_2)^{1-\phi}(Y_{2H}^d + Y_{2H}^{Xd})$$

$$L_2^d = (1-\phi)W^{-\phi}\bar{P}_2^{-\phi}(Y_{2H}^d + Y_{2H}^{Xd})$$

$$Y_{1H}^d = \frac{\gamma \bar{P}_2 \bar{Y}_2^d}{\bar{P}_{1H}}$$

and

$$L_1^d = Y_{1H}^d + Y_{1H}^X$$

where $Y_{2H}^{Xd} = \frac{Y_{2H}^* P_{2H}^* \mathcal{E}}{P_{2H}}$, $Y_{1H}^{Xd} = \frac{Y_{1H}^* P_{1H}^* \mathcal{E}}{P_{1H}}$, $Y_{2H}^d = \gamma C \bar{P}_{2H}^{-(1-\gamma)} \bar{P}_{2F}^{1-\gamma}$, and $\bar{P}_{2F} = T P_{2F}^* \mathcal{E}$. By backward induction, we can obtain the labor demand function in each stage of production.

The equilibrium in the steady state $\{C, L\}$ is then fully characterized by the labor supply equation (1), the risk sharing condition (5), and the labor demand function $L_1^d + L_2^d$ as derived above, where all price indices are a function of W and $\mathcal{E}$. Following Huang and Liu (2005), we set $\psi = 0$ to simplify expressions, which can be justified by indivisible labor (e.g., Hansen, 1985).[5] Then, equations (1) and (5) give

$$w = \sigma c^* + e + p^*$$

$$c = c^* + \frac{1}{\sigma}[e + p^* - p]$$

By substituting w into $p = \gamma \bar{p}_{2H} + (1-\gamma)\bar{p}_{2F}$, together with $\sigma c = \sigma c^* + e + p^* - p$, we obtain an expression of c, which includes neither the domestic price index nor nominal exchange rate. Similarly, by substituting w into price index, together with $L = L_1^d + L_2^d$, we obtain an expression of state-state labor l.

3.2 The flexible-price equilibrium

In the flexible price equilibrium, $\alpha_n = 0$ for $n = 1, 2$. The optimal pricing decision for firms at stage n becomes $P_{nH,t}^o = \Psi_{n,t}$ and thus $P_{nH,t} = \bar{P}_{nH,t} = P_{nH,t}^o$.

With $\bar{P}_{2,t} = \bar{P}_{1H,t}^{\gamma} \bar{P}_{1F,t}^{1-\gamma}$, stage-specific price indexes are given as below

$$P_{1H,t} = \frac{W_t}{A_{1,t}}$$

and

$$P_{2H,t} = W^{1-\phi+\gamma\phi}(T_t \mathcal{E}_t)^{(1-\gamma)\phi}(P_{1F}^*)^{(1-\gamma)\phi} A_{1,t}^{-\gamma\phi} A_{2,t}^{-1}$$

$$P_t = (P_{2H,t})^{\gamma}(T_t P_{2F,t}^* \mathcal{E}_t)^{1-\gamma}$$

where we have plugged in the expression of $\bar{P}_{2F,t}$ in the last equation.

Similar to the analysis in the steady-state equilibrium, we have $Y_{1H,t}^d(u) = \bar{Y}_{1H,t}^d$, $Y_{2H,t}^d(u) =$

[5] In the numerical analysis in Section 4, we do not require $\psi = 0$ but take a more general calibrated value.

$\bar{Y}^d_{2H,t}$, $Y_{1H,t} = \bar{Y}^d_{1H,t}$ and $Y_{2H,t} = \bar{Y}^d_{2H,t}$. The factor demand functions are given by

$$\bar{Y}^d_{2,t} = \phi \frac{W_t^{1-\phi}(\bar{P}_{2,t})^{1-\phi}}{A_{2,t}}(Y^d_{2H,t} + Y^{Xd}_{2H,t})$$

$$L^d_{2,t} = (1-\phi)\frac{W_t^{1-\phi}(\bar{P}_{2,t})^{1-\phi}}{A_{2,t}}(Y^d_{2H,t} + Y^{Xd}_{2H,t})$$

$$Y^d_{1H,t} = \frac{\gamma \bar{P}_{2,t}\bar{Y}^d_{2,t}}{\bar{P}_{1H,t}}$$

and

$$L^d_{1,t} = Y^d_{1H,t} + Y^X_{1H,t}$$

where $Y^X_{2H,t} = \frac{Y^*_{2H,t}P^*_{2H,t}\mathcal{E}_t}{P_{2H,t}}$, $Y^X_{1H,t} = \frac{Y^*_{1H,t}P^*_{1H,t}\mathcal{E}_t}{P_{1H,t}}$, $Y^d_{2H,t} = \gamma C \bar{P}_{2H,t}^{-(1-\gamma)}\bar{P}_{2F,t}^{1-\gamma}$, and $\bar{P}_{2F,t} = T_t P^*_{2F,t}\mathcal{E}_t$.

Similar to the steady-state equilibrium, the flexible-price equilibrium $\{C_t, L_t\}$ is then fully characterized by the labor supply equation (1), the risk sharing condition (5), and the labor demand function $L^d_1 + L^d_2$ as derived above. With the assumption of $\psi = 0$, the equations (1) and (5) again give

$$w_t = \sigma c^*_t + e_t + p^*_t$$

$$c_t = c^*_t + \frac{1}{\sigma}[e_t + p^*_t - p_t]$$

By substituting w_t into price index, we obtain the expressions of c_t and l_t, which does not include domestic price index or nominal exchange rate. Note that, by denoting $t_t = lnT_t$, we have the expression of CPI index p_t as

$$p_t = \gamma[(1-\phi+\gamma\phi)w_t + (1-\gamma)\phi(e_t + t_t) + (1-\gamma)\phi p^*_{1F,t} - \gamma\phi a_{1,t} - a_{2,t}]$$

$$+(1-\gamma)(e_t + t_t + p^*_{2F,t})$$

By substituting p_t into the risk sharing condition, we obtain the natural rate of interest rate as

$$\bar{rr} = \sigma E(c_{t+1} - c_t)$$

$$= \gamma[\gamma\phi\rho_1\Delta a_{1,t} + \rho_2\Delta a_{2,t}]$$

where we treat exogenous foreign variables and import tariff as constant.

3.3 The sticky-price equilibrium

We now derive the New Keynesian Phillips curves for each stage as a function of the relative price gap and output gap, and characterize the equilibrium with sticky prices. Similar to the derivation in Gali (2015), the Phillips curve for each stage is given by

$$\pi_{1H,t} = \beta E_t \pi_{1H,t+1} + \lambda_1 \hat{\gamma}_{1,t}$$

$$\pi_{2H,t} = \beta E_t \pi_{2H,t+1} + \lambda_2 \hat{\gamma}_{2,t}$$

where $\lambda_n = \frac{(1-\beta\alpha_n)(1-\alpha_n)}{\alpha_n}$ for $n = 1, 2$ and $\hat{\gamma}_n$ is the log-derivation of real marginal cost from steady-state equilibrium, i.e.,

$$\hat{\gamma}_{n,t} = ln(\Psi_{n,t}/P_{nH,t}) - ln(\Psi_n/P_{nH})$$

Since Ψ_n and P_{nH} are the marginal cost and aggregate price in stage n under steady state equilibrium, we have

$$\hat{\gamma}_{1,t} = \sigma\hat{c}_t + \frac{1-\gamma}{\gamma}\hat{q}_t - \hat{g}_{2H,t} - a_{1,t}$$

$$\hat{\gamma}_{2,t} = \gamma\phi\hat{g}_{2H,t} + \frac{1-\gamma}{\gamma}\hat{q}_t + (1-\phi)\sigma\hat{c}_t - a_{2,t}$$

where $\hat{g}_{2H,t}$ is the log-deviation of relative price gap between stage-2 output price with respect to stage-1 output price from the steady-state equilibrium, i.e., $\hat{g}_{2H,t} = ln(P_{1H,t}/P_{2H,t}) - ln(P_{1H}/P_{2H})$.[6] In terms of notation, we use the variable with hat to denote deviations from the steady-state equilibrium, and use tilde to denote the deviation from the flexible price equilibrium.

After log-linearizing the Euler equation of the household around the steady state and subtracting the steady-state IS curve, we obtain the IS curve with sticky prices as

$$\hat{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\sigma}[\hat{\imath}_t - E_t(\pi_{t+1})]$$

The aggregate inflation π_t (CPI index) can be written as

$$\pi_t = \pi_{2H,t} + \frac{1-\gamma}{\gamma}\Delta\hat{q}_t$$

The derivation of the aggregate inflation can be found in Appendix B.

The law of motion for relative price gap between stage 1 and stage 2 is characterized by

$$\hat{g}_{2H,t} = \hat{g}_{2H,t-1} + \pi_{1H,t} - \pi_{2H,t}$$

The above equations together with the risk-sharing condition (6) fully characterize the sticky-price equilibrium.

[6] The proofs for deriving the expression of $\hat{\gamma}_1$and $\hat{\gamma}_2$ are in Appendix A.3, where Appendix A.3 characterizes the sticky-price equilibrium with N stages of production.

3.4 A utility-based welfare loss function for optimal monetary policy

We assume that the central bank aims to maximize the household's utility, and represent its objective function by a second-order approxmiation. This follows the approach of Rotemberg and Woodford (1999), Benigno and Woodford (2006), and Gali I2015). Due to simultaneous presence of openness and multiple stages of production, the first order terms do not cancel each others out, unlike in the standard literature. This means that the welfare loss function in our setting includes inflation for each stage of production, the relative price gap across production stages, as well as the real exchange rate and output gap. The flexible-price equilibrium is, in general, not Pareto-optimal. Only in the limit case of a closed economy would the first order terms cancel out and the welfare loss function is left only with the second order terms. In such a case, both the steady-state equilibrium and the flexible-price equilibrium are Pareto-efficient.

Since labor is present in all stages of production and prices are sticky, there is labor misallocation across production stages, which reduces the utility of the household. There is also a terms-of-trade externality since some of the domestically produced intermediate goods are exported and some imported intermediate inputs are used in domestic production.

The household's utility function is given by

$$E\Sigma_{t=0}^{\infty}\beta^t[U(C_t) - V(L_t)]$$

where $U(C_t) = \frac{C_t^{1-\sigma}-1}{1-\sigma}$ and $V(L_t) = \frac{L_t^{1+\psi}}{1+\psi}$.

A second-order Taylor expansion around the steady state (C, L) for the period utility of consumption gives

$$U(C_t) - U = U_c C(\hat{c}_t + \frac{1-\sigma}{2}\hat{c}_t^2) + t.i.p.$$

where $\hat{c}_t$ denotes the log-deviation of consumption from steady state and $t.i.p.$ stands for "terms independent of policy" following Woodford (2003).

By the labor market clearing condition, we obtain the second order Taylor expansion around the steady state for the period utility of employment, i.e., $V(L_t)$, as

$$V(L_t) - V = V_L L\{\sum_{n=1}^{2}\frac{L_n}{L}[\hat{l}_{n,t} + \frac{1}{2}\sum_{n=1}^{N}\hat{l}_{n,t}^2]\} + t.i.p.$$

where L_n/L is the share of labor in stage n in total labor in the steady state, as described in Section 3.1, with the assumption of $\psi = 0$.[7]

It is useful to rewrite the employment gap in the two production stages in terms of the output

[7] As specified for the steady-state variables in Section 3.1, we have $\frac{L_1}{L} = \frac{\gamma\phi}{\gamma\phi+(1-\phi)\bar{a}_1}$ and $\frac{L_2}{L} = 1 - \frac{L_1}{L}$, where $\bar{a}_1 = \frac{Y_{1H}^d}{Y_{1H}^d+Y_{1H}^X}$ and $\bar{a}_2 = \frac{Y_{2H}^d}{Y_{2H}^d+Y_{2H}^X}$ are the share of goods sold, respectively, to the domestic market in stages 1 and 2.

gap and the relative price gap:

$$\hat{l}_{1,t} = [-\bar{a}_1\sigma\phi + \bar{a}_1\bar{a}_2 + \bar{a}_1\sigma]\hat{c}_t + [\bar{a}_1\phi\gamma - 1]\hat{g}_{2H,t}$$

$$+\frac{1+\bar{a}_1 - \bar{a}_1\gamma - \bar{a}_1\bar{a}_2\gamma}{\gamma}\hat{q}_t + \bar{a}_1 d_{2,t} + d_{1,t} - a_{1,t} - \bar{a}_1 a_{2,t}$$

$$\hat{l}_{2,t} = (\bar{a}_2 - \phi\sigma)\hat{c}_t + \phi\gamma\hat{g}_{2H,t} + \frac{1-\bar{a}_2\gamma}{\gamma}\hat{q}_t + d_{2,t} - a_{2,t}$$

where $d_{n,t} = ln(\int_0^1 (\frac{P_{nH,t}(u)}{P_{nH,t}})^{-\theta} du)$ for $n = 1, 2$ measures the price dispersion in stage n, $\bar{a}_1 = \frac{Y_{1H}^d}{Y_{1H}^d + Y_{1H}^X}$ and $\bar{a}_2 = \frac{Y_{2H}^d}{Y_{2H}^d + Y_{2H}^X}$ are the shares of goods sold, respectively, to the domestic market in stages 1 and 2 in the steady state. Details can be found in Appendix C.

Following Gali (2015), up to a second-order approximation around the steady state, the price dispersion term $d_{n,t}$ for $n = 1, 2$ can be written as

$$d_{n,t} = \frac{\theta}{2}\int_0^1 [p_{nH,t}(i) - p_{nH,t}]^2 di \equiv \frac{\theta}{2} var\{p_{nH,t}(i)\}$$

By Woodford (2003), the price dispersion can be re-written as a function of inflation in each stage of production, i.e.,

$$\sum_{t=0}^{\infty}\beta^t var\{p_{nH,t}(i)\} = \lambda_n^{-1}\sum_{t=0}^{\infty}\beta^t \pi_{nH,t}^2 + t.i.p.$$

We then substitute $\hat{l}_{n,t}$ and $d_{n,t}$ into the period utility of employment. Since the total labor income of households are given by $WL = \frac{PC\gamma}{\bar{a}_2}(1-\phi) + \phi\frac{PC\gamma}{\bar{a}_2}\frac{\gamma}{\bar{a}_1}$, the steady state equilibrium implies $WL = PC(1-\Phi)$, where $1 - \Phi = \frac{\gamma}{\bar{a}_2}(1-\phi) + \phi\frac{\gamma^2}{\bar{a}_2\bar{a}_1}$, and thus $U_c C = V_L L(1-\Phi)$.[8] In addition, in the steady state, the labor shares in the two stages are given, respectively, by $L_1/L = \frac{\gamma\phi}{\gamma\phi + (1-\phi)\bar{a}_1}$ and $L_2/L = \frac{(1-\phi)\bar{a}_1}{\gamma\phi+(1-\phi)\bar{a}_1}$.

By summing up $U(C_t) - U$ and $V(L_t) - V$, the household's welfare loss as a fraction of the steady state consumption is given by

$$W = E_0\sum_{t=0}^{\infty}\beta^t \frac{U(C_t) - V(L_t) - (U - V)}{U_c C}$$

$$= E_0\sum_{t=0}^{\infty}\beta^t\{\hat{c}_t - \sum_{n=1}^{2}(1-\Phi)\frac{L_n}{L}\hat{h}_{n,t} - \frac{1}{2}\{-(1-\sigma)\hat{c}_t^2 + (1-\Phi)[\frac{L_1}{L}(\hat{h}_{1,t} - a_{1,t} - \bar{a}_1 a_{2,t})^2$$

[8] In the small-open economy New Keynesian literature, Φ is normally assumed to be zero due to the symmetry assumption across countries in a two-country structure model, e.g., Faia and Monacelli (2008), De Paoli (2009), or in a model of continuum of small countries, e.g., Gali and Monacelli (2005).

$$+\frac{L_2}{L}(\hat{h}_{2,t}-a_{2,t})^2+\frac{L_1}{L}\theta\lambda_1^{-1}\pi_{1H,t}^2+(\frac{L_1}{L}\bar{a}_1+\frac{L_2}{L})\theta\lambda_2^{-1}\pi_{2H,t}^2]\}\}+t.i.p. \tag{13}$$

where

$$\hat{h}_{1,t}=(-\bar{a}_1\sigma\phi+\bar{a}_1\bar{a}_2+\bar{a}_1\sigma)\hat{c}_t+(\bar{a}_1\phi\gamma-1)\hat{g}_{2H,t}+\frac{1+\bar{a}_1-\bar{a}_1\gamma-\bar{a}_1\bar{a}_2\gamma}{\gamma}\hat{q}_t$$

$$\hat{h}_{2,t}=(\bar{a}_2-\phi\sigma)\hat{c}_t+\phi\gamma\hat{g}_{2H,t}+\frac{1-\bar{a}_2\gamma}{\gamma}\hat{q}_t$$

The first-order terms can be eliminated by approximating the equilibrium conditions specified in Section 3.3 to a second-order expansion using the approach developed by Sutherland (2002) and Benigno and Woodford (2006). Though we do not present an explicit expression of the welfare loss function purely in second-order terms due to the complexity arising from the multi-stage production, the numerical analysis in Section 4 approximates the full equilibrium in the second-order expansion.

It is worth noting that, in the special case of a closed economy with $N=2$, $\gamma=1$, and $\bar{a}_1=\bar{a}_2=1$, the expression (13) reproduces the welfare loss function in Huang and Liu (2005). In a special case of a small-open economy with one stage of production ($N=1$), $\bar{a}_1=\gamma$, and the additional assumption of symmetry in the foreign country, the expression (13) reproduces the welfare loss function in De Paoli (2009).

The central bank is to choose a monetary policy to minimize welfare loss with endogenous variables $\{\hat{c}_t,\pi_{1H,t},\pi_{2H,t},\pi_t,\hat{q}_t,\hat{g}_{2H,t},\hat{\imath}_t\}$ subject to

(1). IS curve: $\hat{c}_t=E_t\hat{c}_{t+1}-\frac{1}{\sigma}[\hat{\imath}_t-E_t(\pi_{t+1})]$

(2). Aggregate CPI inflation: $\pi_t=\pi_{2H,t}+\frac{1-\gamma}{\gamma}\Delta\hat{q}_t$

(3). Risk sharing: $\hat{c}_t=\frac{1}{\sigma}\hat{q}_t$

(4). The Phillips curve for production stage 1: $\pi_{1H,t}=\beta E_t\pi_{1H,t+1}+\lambda_1\hat{\gamma}_{1,t}$, where $\hat{\gamma}_{1,t}=\sigma\hat{c}_t+\frac{1-\gamma}{\gamma}\hat{q}_t-\hat{g}_{2H,t}-a_{1,t}$

(5). The Phillips curve for production stage 2: $\pi_{2H,t}=\beta E_t\pi_{2H,t+1}+\lambda_2\hat{\gamma}_{2,t}$, where $\hat{\gamma}_{2,t}=\gamma\phi\hat{g}_{2H,t}+\frac{1-\gamma}{\gamma}\hat{q}_t+(1-\phi)\sigma\hat{c}_t-a_{2,t}$

(6). The evolution of the relative price gap between the production stages: $\hat{g}_{2H,t}=\hat{g}_{2H,t-1}+\pi_{1H,t}-\pi_{2H,t}$

3.5 Comments on the welfare loss function

There are two distortions in the model, i.e., the labor allocation distortion (caused by sticky prices along the production chain) and the terms of trade externality. Those terms measuring stage-specific unemployment gaps $\hat{h}_{1,t}$ and $\hat{h}_{2,t}$ show up in welfare loss function because of sticky prices and misallocation of labor across production stage. The real exchange rate $\hat{q}_t$ appears due to the terms-of-trade externality.

In an open economy with a finite elasticity of foreign demand for export, the social planner wishes to exploit a domestic monopoly power in trade. This gives rise to a terms of trade effect. As the real exchange rate $\hat{q}_t$ and the relative price gap between production stage, $\hat{g}_{2H,t}$, jointly enter $\hat{h}_{1,t}$ and $\hat{h}_{2,t}$, we see an interaction between the labor allocation distortion and the terms of trade distortion. This interaction suggests that the monetary policy discussion is not a simple sum of the results from an open-economy with one stage of production and a closed economy with two stages of production.

The second-order terms in the welfare loss function consist of three parts: (a) a consumption gap, and stage-specific unemployment gaps which can be written in terms of the consumption gap, (b) separate inflation terms for each production stage, and the relative price gap between production stages, and (c) the real exchange rate. The consumption gap is connected with the output gap and real exchange rate via $\hat{y} = \gamma\hat{c} + \frac{1-\gamma^2}{\gamma}\hat{q}$.

The welfare loss function indicates that targeting CPI and PPI are not satisfactory. Instead, the central bank needs to pay attention to stage-specific inflation terms along the production process as well as the price gap across the production stages. These terms will become more important as the economy becomes more open or when the number of production stages increase. The last point is elaborated in Appendix E.2 when we consider the case of N-stage production.

3.6 Effects of import tariff

Motivated by a recent rise in international trade tensions, we study how a change in trade policy, which alters the cost of supply chain trade, may affect the design of the monetary policy. We consider two counter-factual cases: a high tariff and a low tariff. In each case, the import tariff affects the welfare loss function through its impact on the steady-state shares of domestic demand in total demand for domestically produced goods in the two stages of production, i.e., $\bar{a}_1$ and $\bar{a}_2$.

It can be shown that

$$\frac{\bar{a}_2}{1-\bar{a}_2} = f_2(*) \cdot T^{(1-\gamma)(1+\phi\gamma)(1-\frac{1}{\sigma})}$$

and

$$\frac{\bar{a}_1}{1-\bar{a}_1} = f_2(*) \cdot \frac{1}{1-\bar{a}_2}$$

where $f_1(*)$ and $f_2(*)$ are functions of exogenous foreign variables. The explicit expression of $f_1(*)$ and $f_2(*)$ can be found in Appendix D. We then proceed with the following proposition.[9]

Proposition 1: If the relative risk aversion $\sigma = 1$, a higher import tariff does not affect the steady-state allocation, i.e., $\frac{\partial \bar{a}_1}{\partial T} = \frac{\partial \bar{a}_2}{\partial T} = 0$ and $\frac{\partial L_1/L}{\partial T} = \frac{\partial L_2/L}{\partial T} = 0$; if $\sigma > 1$, a higher import tariff will lead to a higher share of domestic demand for domestically produced goods, i.e., $\frac{\partial \bar{a}_1}{\partial T}, \frac{\partial \bar{a}_2}{\partial T} > 0$, and the labor share in the upstream production relative to the downstream decreases, i.e., $\frac{\partial L_1/L}{\partial T} < 0$ and $\frac{\partial L_2/L}{\partial T} > 0$.

[9] We assume that $\gamma < 1$, i.e., the share of import is not zero.

4 Comparing monetary policy rules

We consider a family of simple monetary policy rules. As discussed in Section 3, the first order approximation for the equilibrium conditions is not enough for welfare analysis, and thus we estimate the general nonlinear model specified in Section 2 with $N = 2$ and approximate the equilibrium to the second order expansion. We relax the assumption of $\psi = 0$ and include a broader set of stochastic shocks, i.e., stage-specific productivity shocks and shocks on foreign consumption (which are the two types of shocks most commonly studied in the literature).

We consider the following set of policy rules: (a) a classic Taylor (1993) rule that is based on CPI inflation (and output gap),[10] (b) a Gali-Monacelli rule (2005) that replaces CPI inflation with PPI inflation, (c) a rule that targets separate inflation terms for each production stage (i.e., stage-specific producer price indices), (d) combinations of the above with the real exchange rate, and (e) an exchange rate peg. For each rule, we examine both the case with imposed coefficients as specified in the literature (such as 1.5 and 0.5 on CPI inflation and output gap in the classic Taylor rule) and optimally estimated coefficients.

Since global supply chains have been gaining importance over the last two decades but face disruptions by recent tariff wars, we conduct comparative statics exercies on how the optimal weight on upstream inflation relative to the final stage inflation changes in response to changes in an economy's openness. Specifically, we consider a range of openness parameters. For each scenario, we estimate the optimal weights on the production-stage-specific inflation terms as well as on other variables. We then look at how the relative weights evolve as the degree of openness changes.

Asymmetric price stickiness along the production chain appears to be empirically relevant. Cornille and Dossche (2008) and Nakamura and Steinsson (2008) both suggest that the price contracts in more upstream production stages tend to have a shorter duration than those in the finished product sectors. Gong, Wang, and Zou (2016) argue that different degrees of price stickiness in different stages would affect which price index (i.e., CPI, final-goods-based PPI, or intermediate-goods-based PPI) should be included in a simple monetary policy rule.[11]

4.1 Model Parameters

We begin with the calibration of parameters for the baseline model. Each period in the model corresponds to a quarter. Following Gali and Monacelli (2005) and De Paoli (2009), the model economy is meant to resemble Canada in some key dimensions. The calibrated parameters are summarized in Table 1.

[10] Henderson and McKibbon (1993) have discussed similar rules.

[11] Instead of including all stage-specific price indices in a simple monetary rule, Gong, Wang, and Zou (2016) consider CPI-based Taylor rule, final-goods PPI-based Taylor rule, and intermediate-goods PPI-based Taylor rule, where the Taylor-type rule (only) includes one specific inflation index and output gap.

The subjective discount factor is set to be $\beta = 0.99$, which implies a 4% annual real interest rate in the steady state. Following Arellano (2008) and De Paoli (2009), the inverse of intertemporal elasticity of substitution is set to be $\sigma = 2$. The parameter in Calvo pricing in both production stages is set to be $\alpha_1 = \alpha_2 = 0.66$, implying an average contract duration of 3 quarters. Following Benigno and Woodford (2005), the elasticity of substitution in the consumption bundle is set to be $\theta = 10$. Consistent with Huang and Liu (2005), we set the share of intermediate goods in production to be $\phi = 0.6$.

We set the shares of goods sold to the domestic markets in both stages to be $\bar{a}_1 = \bar{a}_2 = 0.7$, implying a 30% export share of GDP (approximately the level observed for Canada). The parameters $\bar{a}_1, \bar{a}_2$ are the sufficient statistics for the (exogenous) foreign demand for output in the two production stages. Following Gali and Monacelli (2005), the process of productivity shock is set to follow an AR(1) process with persistence parameter $\rho_a = 0.66$ and standard deviation $\sigma_a = 0.0071$, which is calibrated from Canada data. Following De Paoli (2009), the foreign consumption shock is set to an AR(1) process with persistence $\rho_{C^*} = 0.66$ and standard deviation $\sigma_{C^*} = 0.0129$. We normalize the import tariff in the baseline numerical exercise to be $T = 1$ (implying a zero tariff).

Table 1: Parameter calibration

Parameter	Name	Value	Notes
β	Subjective discount factor	0.99	4% annual interest rate with a quarterly model
σ	Inverse of intertemporal elasticity of substitution	2	Standard value in literature, e.g., Arellano (2008)
α_1,α_2	Parameter in Calvo pricing	0.66	An average length of price contract of 3 quarters
γ	Share of goods purchased in domestic market	0.6	Implying 40% import share of GDP
θ	Elasticity of substitution in consumption bundle	10	Following Benigno and Woodford (2005)
ϕ	Share of intermediate goods in production	0.6	Following Huang and Liu (2005)
$\bar{a}_1, \bar{a}_2$	Share of goods selling to domestic market	0.7	Implying 30% export share of GDP
ρ_a	Persistency of productivity shock	0.66	Following Gali and Maonacelli (2005), De Paoli (2009)
σ_a	Standard deviation of productivity shock	0.0071	Following Gali and Monacelli (2005), De Paoli (2009)
ρ_{C^*}	Persistency of foreign consumption shock	0.66	Following De Paoli (2009)
σ_{C^*}	Standard deviation of foreign consumption shock	0.0129	Following De Paoli (2009)

4.2 Welfare losses

The numerical estimation is conducted based on the general nonlinear model specified in Section 2 with $N = 2$. The equilibrium is estimated up to second order approximation (for both the constraints and the welfare loss function). We define the welfare loss χ in percentage term relative to the steady-state consumption , i.e.,

$$E\Sigma_{t=0}^{\infty}\beta^t[\frac{[C(1-\chi)]^{1-\sigma}-1}{1-\sigma} - \frac{L^{1+\psi}}{1+\psi}] = V^a$$

where C and L are steady-state consumption and employment, and V^a is the welfare estimated from a given policy rule.

The aggregated PPI index is a sales-weighted average of producer prices index:

$$\pi_{PPI} = (1-\omega)\pi_{1H} + \omega\pi_{2H}$$

where $\omega = \frac{P_{1h}(Y_{1h}+Y_{1h}^X)}{P_{1h}(Y_{1h}+Y_{1h}^X)+P_{2h}(Y_{2h}+Y_{2h}^X)}$ is the relative sales-weight in the upstream production stage.

We assume that neither the real marginal cost by production stage nor the relative price gap across production stages can be observed by the central bank. So they do not enter any monetary policy rule. Within the family of simple rules, the best that the central bank can do is to make the interest rate a function of the upstream producer price inflation, the final stage producer price inflation, change in the real exchange rate, the output gap, and one-period lagged interest rate. Since the PPI inflation is a sales-weighted average of the first two terms, and the CPI inflation is a linear combination of the second and the third terms, there is no need to include these terms separately. We estimate the optimal coefficients on these variables, label this best possible rule as Policy Rule 1, normalize its welfare loss to one. Table 2 reports the optimally estimated coefficients for ten different monetary policy rules. The welfare loss for each rule is expressed as relative to that under Policy Rule 1[12] "Peg" in the table indicates a nominal exchange rate peg.

A classic Taylor rule that targets only CPI inflation and output gap (Policy Rule 2) does terribly in this economy. The welfare loss is 80% higher than Policy Rule 1. The Gali-Monacelli (2005) rule that replaces the CPI inflation with PPI inflation (Policy Rule 3) represents a significant improvement over the classic Taylor Rule in terms of a much smaller welfare loss. Still, the Gali-Monacelli rule is not as good as Policy Rule 1. That is because, with the input-output linkages across production stages, the optimal weights on the upstream sector and final stage inflation terms in Policy Rule 1 are not proportional to the relative sales of the two sectors. Including both PPI and CPI inflation (Policy Rule 4) yields a small improvement over Policy Rule 3 (but a larger improvement over Policy Rule 2. Adding the (change in the) real exchange rate to Rule 4 (Policy 5) produces more noticeable improvement over Rules 2, 3, or 4. Still, Policy Rule 1 dominates Policy Rule 5.

Policy Rules 6-9 suggest that inflation measures in both the upstream stage and the final stage contain important information. Dropping either one of them from a monetary policy rule could lead to a significant increase in the welfare loss. A nominal exchange rate peg (Policy Rule 10) yields a welfare loss that is 173% higher than Policy Rule 1. It is the worst rule among the ten rules considered.

To summarize, the best simple rule would target separate producer price inflation in different stages of production and the real exchange rate (as well as the output gap). If we have to choose

[12] The welfare loss for P1 in Table 2 in term of consumption is $0.319 \times 0.01\%$.

among aggregate price indicators, PPI targetting is superior to CPI targetting.

Table 2: Optimal alternative simple rules of monetary policy

	π_{1H}	π_{2H}	π_{PPI}	π_{CPI}	$\hat{c}$	$\hat{q}$	$\hat{\imath}_{t-1}$	Welfare loss
P1	6.3861	9.8675			0.7570	-1.5549	0.2048	1
P2				5.1882	0.0006		0.0215	1.809
P3			9.9999		0.1000		1.0441	1.031
P4			9.9888	0.0009	0.0004		0.8085	1.022
P5			9.1138	0.0747	0.1697	-0.6933	0.1432	1.003
P6	5.2870	9.9965			0.0001		0.6948	1.009
P7	4.2548				0.0000		0.8757	1.793
P8		2.6975			0.0003		0.8327	1.257
P9	5.2741	9.9825					0.7004	1.009
Peg								2.730

Notes: PPI index (sales-weighted): $\pi_{PPI} = (1-\omega)\pi_{1H} + \omega\pi_{2H}$ with $\omega = \frac{P_{1h}(Y_{1h}+Y_{1h}^X)}{P_{1h}(Y_{1h}+Y_{1h}^X)+P_{2h}(Y_{2h}+Y_{2h}^X)}$.
CPI index: $\pi_{CPI,t} = \pi_t$.

For each type of policy rule, besides optimally estimated coefficients, we also evaluate a version where the coefficients are imposed using the values suggested in the literature. Table 3 reports the welfare performance proposed by the original Taylor calibration (i.e., Taylor, 1993), alternative rules adopted in Gali and Monacelli (2005), and in Huang and Liu (2005). The welfare losses in the table are still reported as relative to that under Policy Rule P1 in Table 2. Evidently, simple monetary policy rules that target aggregate PPI, or stage-specific producer indices, outperform those targeting just the CPI index.

4.3 Comparative statics: Effects of openness and intermediate goods share

A country's openness and share of intermediate goods in production are the two most important features of global supply chains. In order to study the role of these two factors in optimal simple rules, we conduct comparative statics on how the relative optimal weight on upstream sector inflation changes with respect to these two parameters.

Table 3: Alternative simple rules of monetary policy in literature

	π_{1H}	π_{2H}	π_{CPI}	$\hat{c}$	$\hat{\imath}_{t-1}$	Welfare loss	Notes
P1			1.5	0.5		5.862	Taylor (1993)
P2	1.42	1.68		0.04	1.12	1.166	Huang and Liu (2005)
P3			1.5			2.661	Gali and Monacelli (2005) – CPI based
P4		1.5				3.843	Gali and Monacelli (2005) – PPI based

We calibrate foreign demand in both production stages such that the shares of exporting in the steady state are the same in both stages ($1 - \bar{a}_1 = 1 - \bar{a}_2 \equiv 1 - \bar{a}$.), and both vary from 10% to 90%.

Figure 2 shows the estimated optimal weight on the upstream inflation relative to that on the final stage inflation, as a function of the openness (the export share, assumed to be common in both stages of production). The y-axis in the figure is defined as the estimated optimal coefficient on the upstream inflation index divided by the sum of the estimated coefficients for the two inflation measures in the upstream and final stages.The calibrations show that the optimal relative weight on the upstream sector inflation rises as an economy becomes more open. This is especially true when the economy evolves from median open to very open (an increase in openness from, say, 0.6 to 0.9). It also implies that the welfare loss under the classific Taylor rule becomes larger as the degree of openness increases.

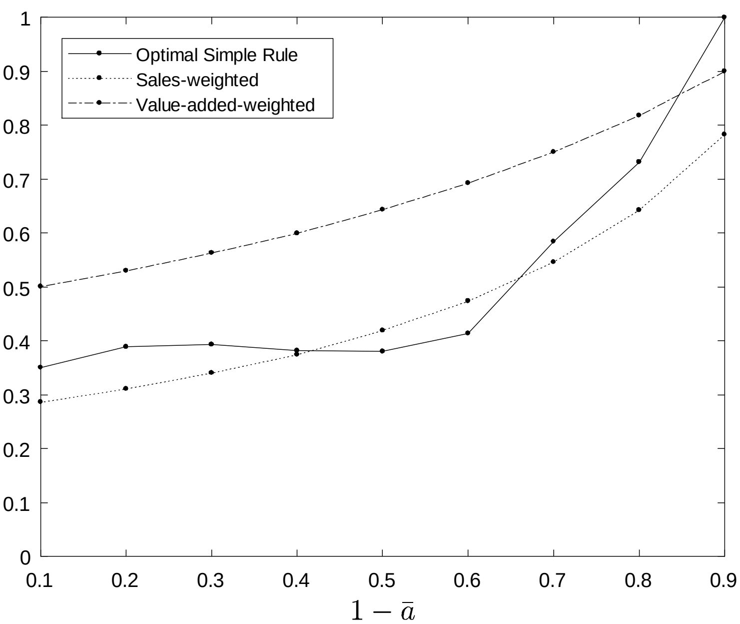

Figure 2: Relative weight of upstream inflation index in optimal simple rule with respect to country openness

In the same graph, we also plot the reative weight of the two sectors based on their relative sales (the thin dotted line) and relative value added (the dash dotted line), respectively. They serve to show that the optimal weights on the stage specific producer inflation are not proportional to either sales or value added of the sectors. Hence, targetting aggregate PPI inflation cannot achieve the same level of welfare as targetting production state specific producer price inflation.

We also vary the intermediate goods share (ϕ) and compute the optimal weights under Policy

Rule 1. Figure 3 traces out the estimated optimal relative weight on the upstream sector inflation as a function of the share of intermediate goods in production. The calibrations show that, as the share of intermediate goods increases, the relative optimal weight on the upstream sector inflation also needs to go up.

To contrast the optimal rule that targets separate producer price inflation in different production stages with those that targets CPI inflation, we construct an implied coefficient on CPI inflation as the weighted average of the estimated coefficients on the final stage inflation and on the change in the real exchange rate and check it changes as the share of intermediate goods increases.[13] Figure 4 reports the ratio of the estimated coefficient for upstream inflation index (in optimal simple rule) divided by the sum of the coefficients on the upstream inflation and the implied coefficient for CPI. There is a clear upward trend in Figure 4. This means that targeting CPI alone becomes increasingly sub-optimal as the share of intermediate goods goes up.

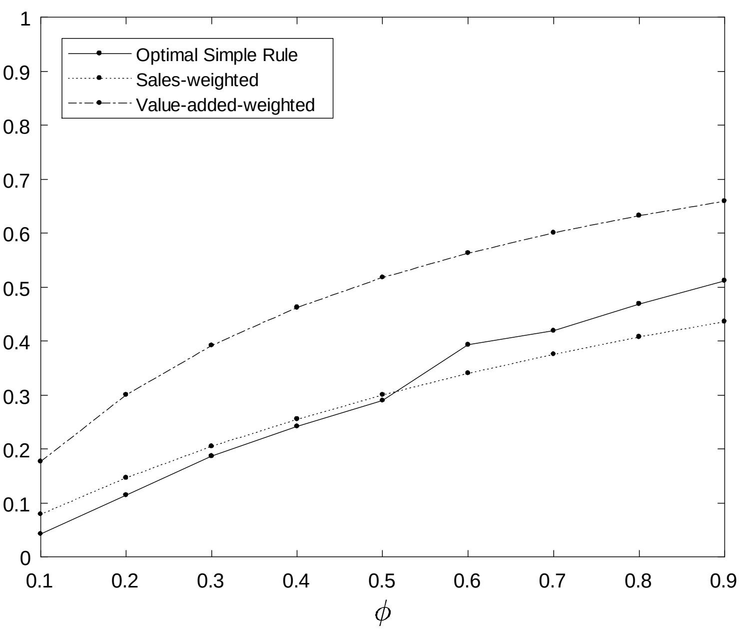

Figure 3: Relative weight of upstream inflation index in optimal simple rule with respect to intermediate goods share

4.4 Effects of a higher import tariff

In this setup, trade frictions can be thought of as a reduction in an economy's openness. Let us consider a case of doubling the import tariff (a change to $T = 2$). In such a scenario, the shares

[13] As shown in the expression for CPI in Section 3.3, the weight on final stage estimated coefficient is set to be γ, while the weight on the estimated coefficient of exchange rate is set to be $1 - \gamma$.

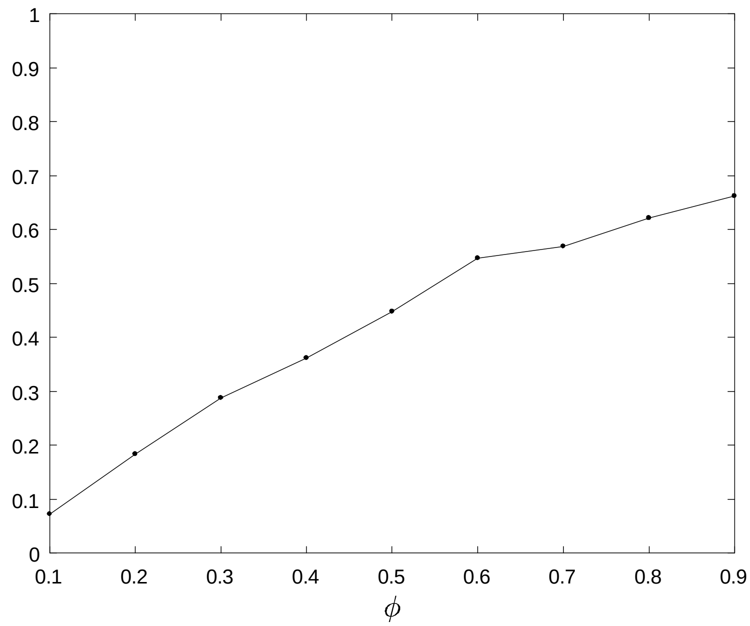

Figure 4: Ratio of the weight on upstream inflation index versus CPI with respect to intermediate goods share

of the demand for domestic goods in the two production stages become $\bar{a}_1 = 0.73$ and $\bar{a}_2 = 0.74$, which are larger than those in the original calibration. The direction of the change is exactly as predicted by Proposition 1.

While the central bank cannot undo the increase in tariff directly, it will re-optimize by choosing a different set of coefficients on the variables in a monetary policy rule in response to a higher import tariff. We compute the new optimal weights and new welfare losses for the best simple rule, the classic Taylor rule, the Gali-Monacelli rule, a rule that includes both PPI and CPI inflation as well as the output gap and real exchange rate but not stage-specific producer price inflation, and finally an exchange rate peg.

The results are reported in Table 4. Note that the welfare losses are relative to the case of Policy Rule P1 before the tariff increase (i.e., relative to the welfare for Policy Rule 1 in Table 2). Comparing across different policy rules, it is still the case that Policy Rule 1 that includes stage-specific producer price inflation is the best monetary policy rule. Afterwards, including PPI inflation and the real exchange rate would beat the classific Taylor rule. The exchange rate peg and the classic Taylor yield the biggest and the second biggest welfare losses, respectively.

Recall that Proposition 1 suggests that a higher import tariff would reduce the optimal weight on the upstream sector inflation in the monetary policy rule relative to that on the final stage inflation. This can be confirmed in our calibrations. The ratio of the optimal relative weight on the upstream producer price inflation under Policy Rule 1 has changed from 0.647 (=6.3861/9/8675) in the case of T=1 in Table 2) to 0.564 (=5.0113/8.8786) in the case of T=2 in Table 4).

It is important to note that a higher tariff reduces welfare directly, as we can see from the greater welfare losses in Table 4 relative to their counterparts in Table 2, in spite of the best adjustments made by the central bank. If the central bank does not re-optimize, the welfare loss would be even greater.

Table 4: Optimal alternative simple rules of monetary policy with higher imported tariff

	π_{1H}	π_{2H}	π_{PPI}	π_{CPI}	$\hat{c}$	$\hat{q}$	$\hat{\imath}_{t-1}$	Welfare loss
P1	5.0113	8.8786			0.2298	-0.7860	0.1629	1.182
P2				5.2378	0.0000		0.0007	2.144
P3			9.9988		0.1000		1.0509	1.223
P4			9.3569	0.0487	0.0766	-0.5579	0.1445	1.188
Peg								3.310

PPI index (sales-weighted): $\pi_{PPI} = (1-\omega)\pi_{1H} + \omega\pi_{2H}$ with $\omega = \frac{P_{1h}(Y_{1h}+Y_{1h}^X)}{P_{1h}(Y_{1h}+Y_{1h}^X)+P_{2h}(Y_{2h}+Y_{2h}^X)}$.

CPI index: $\pi_{CPI,t} = \pi_t$.

Since Policy Rule 1 already includes the real exchange rate, it implies that the central bank cannot offset the effects of a higher import tariff by simply changing the exchange rate. Appreciation in the domestic currency reduces the cost of imported intermediate inputs or imported final consumption goods, but also increases the price of domestically produced outputs of both intermediate goods and final goods. Since the foreign demand is price-elastic, firms obtain a smaller revenue from exporting.

4.5 Asymmetric price stickiness

We now consider the implications of uneven price stickiness in different stages of production. Cornille and Dossche (2008) and Nakamura and Steinsson (2008) argue that the duration of price contracts in the upstream production stages is shorter than the downstream stages. For instance, Nakamura and Steinsson (2008) document that the median price contract for finished producer goods in 1998-2005 lasts for 8.7 months, while the median duration of price contracts for intermediate goods is about 7.0 months.

To investigate the implications of such difference, we reduce the Calvo pricing parameter in the first stage of production to be $\alpha_1 = 0.5$, indicating an average length of price contracts of 2 quarters.

The estimated results are showed in Table 5, where the welfare loss of Policy Rule P1 in the table has been normalized to be one. The loss becomes smaller as compared to the baseline case in Table 2 since the prices are less sticky overall. Furthermore, the optimal relative weight on the upstream producer price inflation also becomes smaller.[14]

[14] This is consistent with the findings in Gong, Wang, and Zou (2016). They argue that, when the degree of price

Table 5: Optimal alternative simple rules of monetary policy with lower price stickiness in upstream production

	π_{1H}	π_{2H}	π_{PPI}	π_{CPI}	$\hat{c}$	$\hat{q}$	$\hat{\imath}_{t-1}$	Welfare loss
P1	3.1846	9.8760			0.0100	-0.5776	0.0328	1
P2				5.0515	0.0001		0.0038	1.889
P3			9.9981		0.1001		1.0715	1.083
P4			9.8058	0.0240	0.0110	-0.6126	0.0174	1.038
Peg								3.101

PPI index (sales-weighted): $\pi_{PPI} = (1-\omega)\pi_{1H} + \omega\pi_{2H}$ with $\omega = \frac{P_{1h}(Y_{1h}+Y_{1h}^X)}{P_{1h}(Y_{1h}+Y_{1h}^X)+P_{2h}(Y_{2h}+Y_{2h}^X)}$.

CPI index: $\pi_{CPI,t} = \pi_t$.

5 Concluding remarks

Supply chains are everywhere and supply chains are often global. This paper studies the implications of global supply chains on the design of optimal monetary policy using a small-open economy New Keynesian model with multiple stages of production. The optimal simple policy rule should target separate producer price inflation in each production stage (in addition to output gap and real exchange rate).

Importantly, the optimal weights on the upstream sector inflation versus the final stage inflation are not proportional to the sectors' sales or value added. As an economy becomes more open, the optimal relative weight on the upstream sector inflation should also rise. Separately, as the share of intermediate goods in total production increases, the optimal relative weight on the upstream sector inflation should also rise. In both cases, the classic Taylor rule that targets only CPI inflation would become more and more inferior (in the sense of an ever greater relative welfare loss).

Trade frictions can be thought of as a shock to an economy's openness. With a higher tariff, the optimal weights on various terms in the monetary policy rule would have to change. Importantly, a higher tariff reduces the welfare directly even if the central re-optimizes. In particular, the negative effect of a higher tariff cannot be offset completely by a change in the real exchange rate.

If we only consider aggregate price indices in the simple monetary policy rule, then targeting aggregate PPI inflation (as well as the output gap) is superior to just targeting CPI inflation. Adding real exchange rate is even better. Still, no simple rule is better than the one that includes separate producer price inflation in each production stage on top of the output gap, the real exchange rate, and the lagged interest rate.

As the production chain becomes longer, the optimal weights on the upstream sector inflation or the PPI inflation should also increase.[15]

stickiness for intermediate-goods production is high, the central bank should follow intermediate-goods PPI-based Taylor rule. However, in their paper, there is no labor allocation distortion between production stages since labor is assumed to be used only in the production of intermediate goods.

[15] A rigorous theoretical discussion regarding an increase in the length of production chain can be found in Appendix

Is it feasible to obtain separate producer price inflation for different production stages? Official statistics agencies in the United States and Australia actually already collect producer prices by production stages. For example, the US Bureau of Labor Statistics has a system of producer price indices featuring a four-stage vertical production chain, which is named under the PPI Final Demand-Intermediate Demand indice. However, central banks do not typically take advantage of these data.

Ironically, central banks use PPI inflation information only to the extent that it helps to forecast CPI inflation. When PPI and CPI diverge, as they often do in recent periods, central banks would ignore PPI. However, our theory suggests that these would be the times when the central bank should pay more attention to producer price inflation.

There are several other directions to further the discussion in this paper. The model adopted in our analysis assumes producer currency pricing, and as mentioned in the literature (e.g., Smets and Wouters, 2002; Corsetti, Dedola, and Leduc, 2011; Engel, 2011), local currency pricing may have a different implications on the trade-offs for monetary policy. Another direction could be to consider a more broad set of exogenous shocks, especially shocks on foreign input prices or foreign demand shocks along the production chain. One challenge in line with this extension would be how to appropriately calibrate the shocks to empirical data.

E.

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A Equilibrium characterization with N-stage of production in a small-open economy

A.1 The steady state equilibrium

We first characterize the steady state with perfect foresight. The steady state is defined as the equilibrium under non-stochastic and constant exogenous variables. Since the whole economy does not change with timing, we can ignore the timing index t in all variables, and $A_n = 1$ for $n = 1, 2, \ldots, N$. The optimal pricing decision for firms at stage n, $n = 1, 2, \ldots, N$, becomes

$$P^o_{nH} = \Psi_n = P_{nH} = \bar{P}_{nH}$$

and for $n = 2, \ldots, N$, we have

$$\bar{P}_n = \bar{P}^{\gamma}_{(n-1)H} \bar{P}^{1-\gamma}_{(n-1)F}$$

where $\bar{P}_{(n-1)F} = T\mathcal{E}P^*_{(n-1)F}$.

Now, we solve for the price indices in terms of wages and derive the labor demand function. Note that $\Psi_n = \bar{P}^{\phi}_n W^{1-\phi}$ for $n = 2, 3, \ldots, N$ with $\Psi_1 = W$. By substituting $\bar{P}_n$, the relationship of output price index across adjacent stages is given by

$$P_{nH} = W^{1-\phi}(\bar{P}_n)^{\phi}$$

$$= W^{1-\phi} P^{\gamma\phi}_{(n-1)H} P^{(1-\gamma)\phi}_{(n-1)F}$$

for $n = 2, \ldots, N$ and $P_{1H} = W$.

By writing all price index in terms of wage and exogenous variables through forward induction, we get

$$P_{nH} = W^{(1-\phi)\frac{1-(\gamma\phi)^{n-1}}{1-\gamma\phi}+(\gamma\phi)^{n-1}} (T\mathcal{E})^{\phi(1-\gamma)\frac{1-[\phi(1-\gamma)]^{n-1}}{1-\phi(1-\gamma)}} \Pi_{i=1}^{n-1} (P^*_{nF})^{[\phi(1-\gamma)]^{n-i}} \tag{14}$$

for $n = 2, \ldots, N$ with $P_{1H} = W$.

Since $P_{nH}(u) = P_{nH}$ for $u \in [0, 1]$ in steady state, by Equation (11), for $n = 1, \ldots, N$, we have

$$Y^d_{nH,t}(u) = \bar{Y}^d_{nH,t}$$

Together with goods market clear condition $Y_{nH} = \bar{Y}^d_{nH}$, and factor market demand function (8) and (7), for $n = 2, \ldots, N$, we get

$$\bar{Y}^d_n = \phi \frac{\Psi_n}{\bar{P}_n} [\bar{Y}^d_{nH} + Y^X_{nH}]$$

$$L_n^d = (1-\phi)\frac{\Psi_n}{W}[\bar{Y}_{nH}^d + Y_{nH}^X]$$

$$\bar{Y}_{(n-1)H}^d = \frac{\gamma \bar{P}_n \bar{Y}_n^d}{\bar{P}_{(n-1)H}}$$

where $\bar{Y}_{NH}^d = \gamma C P_{nH}^{-(1-\gamma)} \bar{P}_{NF}^{1-\gamma}$, and $L_1^d = \frac{\Psi_1}{W}\bar{Y}_{1H}^d$. By substituting the price index and unit cost function in each stage, for $n = 1, \ldots, N$, and through backward induction, the factor demand functions for labor can be written in an implicit form as

$$L_n^d = f(W, C, \mathcal{E}, T, P_{1F}^*, \cdots, P_{NF}^*, P_{nH}^*, \cdots, P_{NH}^*, Y_{nH}^*, \cdots, Y_{NH}^*) \tag{15}$$

Therefore, by summing up labor demand across all stages, the total labor demand function becomes

$$L^d = \sum_{n=1}^{N} L_n^d \tag{16}$$

Therefore, the three equations (1), (5), and (16) fully characterize the steady-state wage, consumption and employment.

A.2 The flexible price equilibrium

In this subsection, we solve the flexible price equilibrium similarly as in the steady state equilibrium. In flexible price equilibrium, $\alpha_n = 0$ for $\forall n$, and then the optimal pricing decision for firms at stage n, $n = 1, 2, \ldots, N$, becomes

$$P_{nH,t}^o = \Psi_{n,t} = P_{nH,t} = \bar{P}_{nH,t}$$

and for $n = 2, \ldots, N$, we have

$$\bar{P}_{n,t} = \bar{P}_{(n-1)H,t}^{\gamma} \bar{P}_{(n-1)F,t}^{1-\gamma}$$

Similar to the steady state case, we solve for the price indices in terms of wages and productivity. Note that $\Psi_{n,t} = \bar{P}_{n,t}^{\phi} W_t^{1-\phi}/A_{n,t}$ for $n = 2, 3, \ldots, N$ with $\Psi_1 = W_t/A_{1,t}$. By substituting $\bar{P}_{n,t}$, the relationship of price index across adjacent stages is given by

$$P_{nH,t} = W^{1-\phi} P_{(n-1)H,t}^{\gamma\phi} P_{(n-1)F,t}^{(1-\gamma)\phi}$$

for $n = 2, \ldots, N$ and $P_{1H,t} = W_t/A_{1,t}$.

By writing all price index in terms of wage through forward induction, we similarly get

$$P_{nH,t} = W^{(1-\phi)\frac{1-(\gamma\phi)^{n-1}}{1-\gamma\phi} + (\gamma\phi)^{n-1}} (\mathcal{E}_t T_t)^{\phi(1-\gamma)\frac{1-[\phi(1-\gamma)]^{n-1}}{1-\phi(1-\gamma)}}$$

$$\cdot\Pi_{i=1}^{n-1}(P_{nF}^*)^{[\phi(1-\gamma)]^{n-i}}\cdot\Pi_{i=1}^{n}(A_{i,t})^{-(\gamma\phi)^{n-i}}$$

for $n = 2, \ldots, N$ with $P_{1H,t} = W_t/A_{1,t}$.

Due to flexible price, the expressions for factor market in each stage of production are exactly the same as in the steady state case. Therefore, we can derive labor demand function in each stage, i.e., $n = 1, \ldots, N$, as

$$L_{n,t}^{fd} = f(W_t, C_t, \mathcal{E}_t, T_t, P_{1F,t}^*, \cdots, P_{NF,t}^*, P_{nH,t}^*, \cdots, P_{NH,t}^*, Y_{nH,t}^*, \cdots, Y_{NH,t}^*, A_{1,t}, \cdots, A_{N,t})$$

where we denote the labor demand under flexible price with an upper symbol f.

The total labor demand function becomes

$$L_t^{fd} = \sum_{n=1}^{N} L_{n,t}^{fd} \tag{17}$$

The three equations (1), (5), and (17) fully characterize the wage, consumption and employment in flexible price equilibrium, where the consumption can be written in

$$C_t^f = f(T_t, P_{1F,t}^*, \cdots, P_{NF,t}^*, P_{nH,t}^*, \cdots, P_{NH,t}^*, Y_{nH,t}^*, \cdots, Y_{NH,t}^*, A_{1,t}, \cdots, A_{N,t})$$

which can be re-written in log-linearized form, i.e.,

$$c_t^f = f(t_t, p_{1F,t}^*, \cdots, p_{NF,t}^*, p_{nH,t}^*, \cdots, p_{NH,t}^*, y_{nH,t}^*, \cdots, y_{NH,t}^*, a_{1,t}, \cdots, a_{N,t})$$

By the Euler equation (4), the IS curve is characterized by

$$c_t^f = E_t(c_{t+1}^f) - \frac{1}{\sigma}[i_t - E_t(\pi_{t+1}) - \rho]$$

which implies the natural rate of interest as

$$r\bar{r}_t = \sigma E_t\{c_{t+1}^f - c_t^f\} \tag{18}$$

A.3 The sticky-price equilibrium

We now derive New Keynesian Phillips curves for each stage as a function of relative price gap and output gap, and characterize the equilibrium with sticky prices. Similar to the derivation in Gali (2015), in each stage of production $n = 1, 2, \ldots, N$, firms' optimal pricing decision gives

$$\pi_{n,t} = \beta E_t \pi_{n,t+1} + \lambda_n \hat{\gamma}_{n,t}$$

where $\lambda_n = \frac{(1-\beta\alpha_n)(1-\alpha_n)}{\alpha_n}$ and $\hat{\gamma}_n$ is the log-derivation of real marginal cost from steady-state equilibrium, i.e.,

$$\hat{\gamma}_{n,t} = ln(\Psi_{n,t}/P_{nH,t}) - ln(\Psi_n/P_{nH})$$

where Ψ_n and P_{nH} is the marginal cost and aggregate price in stage n under steady state equilibrium.

Given $P_{nH,t} = \bar{P}_{nH,t}$ for $n = 1, 2, \ldots, N$ and the expression of production cost function, we have for stages $n = 2, \ldots, N$

$$\hat{\gamma}_{n,t} = \gamma\phi\hat{g}_{nH,t} + (1-\gamma)\phi\hat{g}_{nF,t} + (1-\phi)(\hat{w}_t - \hat{p}_{nH,t}) - a_{n,t}$$

where $\hat{g}_{nH,t}$ and $\hat{g}_{nF,t}$ are the log-deviation of relative output price gap with respect to input price from steady state equilibrium, i.e., $\hat{g}_{nH,t} = ln(\frac{\bar{P}_{(n-1)H,t}}{P_{nH,t}}) - ln(\frac{\bar{P}_{(n-1)H}}{P_{nH}})$, and $\hat{g}_{nF,t} = ln(\frac{\bar{P}_{(n-1)F,t}}{P_{nH,t}}) - ln(\frac{\bar{P}_{(n-1)F}}{P_{nH,t}})$. Since $\bar{P}_{(n-1)H,t} = P_{(n-1)H,t}$, $\hat{g}_{nH,t}$ also indicates the log-deviation of relative output price gap between adjacent stages n and $n-1$ from steady state equilibrium. By the definition of $\hat{g}_{nH,t}$ and $\hat{g}_{nF,t}$, we also have $\hat{p}_{nH,t} = \hat{p}_{NH,t} + \Sigma_{i=n+1}^{N}\hat{g}_{iH,t}$ for $n = 1, \ldots, N-1$.

Following Huang and Liu (2005), without losing generality, we assume $\psi = 0$. Then, from 3, we have $w_t - p_t = \sigma c_t$. By substituting $\hat{w}$ and $\hat{p}_{nH,t}$ into the real marginal cost function, for $n = 2, \ldots, N-1$, we get

$$\hat{\gamma}_{n,t} = \gamma\phi\hat{g}_{nH,t} + (1-\gamma)\phi\hat{g}_{nF,t}$$
$$+(1-\phi)[\sigma\tilde{c}_t - (1-\gamma)\hat{p}_{NH,t} + (1-\gamma)\hat{p}_{NF,t} - \Sigma_{i=n+1}^{N}\hat{g}_{iH,t}] - a_{n,t}$$

and

$$\hat{\gamma}_{N,t} = \gamma\phi\hat{q}_{NH,t} + (1-\gamma)\phi\hat{q}_{NF,t} + (1-\phi)[\sigma\hat{c}_t - (1-\gamma)\hat{p}_{NH,t} + (1-\gamma)\hat{p}_{NF,t}] - a_{N,t}$$

$$\hat{\gamma}_{1,t} = \sigma\hat{c}_t - (1-\gamma)\hat{p}_{NH,t} + (1-\gamma)\hat{p}_{NF,t} - \Sigma_{i=2}^{N}\hat{q}_{iH,t} - a_{1,t}$$

Note that $\hat{p}_{nF,t} = \hat{e}_t$ and $\hat{g}_{nF,t} = \hat{e}_t - \hat{p}_{nH,t}$ for $n = 1, \ldots, N$, and $\hat{q}_t = \gamma(\hat{e}_t - \hat{p}_{NH,t})$.

After log-linearizing the Euler equation of households around steady state and subtracting the natural rate IS curve, we get the IS curve with stick price as

$$\hat{c}_t = E_t\hat{c}_{t+1} - \frac{1}{\sigma}[i_t - E_t(\pi_{t+1}) - \bar{rr}_t]$$

where $\bar{rr}_t$ is the natural rate of interest.

The law of motion for relative price gap between stage n and stage $n-1$, for $n = 2, 3, \ldots, N$, is characterized by

$$\hat{q}_{nH,t} = \hat{q}_{nH,t-1} + \pi_{(n-1)H,t} - \pi_{nH,t}$$

Give the policy rule $\{i_t, \mathcal{E}_t\}$, risk-sharing condition 6, IS curve, the stage-specific Phillips curves, and the law of motion for relative price gap fully pin down the dynamic equilibrium under sticky prices.

A.4 Stage-specific employment in small open-economy with N-stage production

We derive the stage-specific employment gap in terms of output gap and relative price gap. By the factor demand function (7), (8), and (11) in each stage, and substituting with the unit cost, for $n = 2, 3, \ldots, N$, we have

$$lnL_{n,t} = ln(1-\phi) + \phi[ln\bar{P}_{n,t} - lnW_t] - lnA_{n,t} + ln[Y^d_{nH,t} + Y^X_{nH,t}] + d_{n,t} \tag{19}$$

where $d_{n,t} = ln(\int_0^1 (\frac{P_{n,t}(u)}{P_{n,t}})^{-\theta} du)$ and $lnL_{1,t} = -lnA_{1,t} + ln[Y^d_{1H,t} + Y^X_{1H,t}] + d_{1,t}$.

By the factor demand function for intermediate goods and labor in each stage, i.e., Expression (7) and (8), for $n = 2, 3, \ldots, N$, we get

$$lnL_{n,t} = ln(\frac{1-\phi}{\phi}) + ln\bar{P}_{n-1,t} - lnW_t + ln\bar{Y}^d_{n,t}$$

Also, not that

$$Y^d_{(n-1)H,t} = \frac{\bar{Y}^d_{n,t}\bar{P}_{n,t}\gamma}{\bar{P}_{(n-1)H,t}}$$

Then, by substituting $lnY^d_{nH,t}$ using $lnL_{n,t}$ and $ln\bar{Y}^d_{(n+1),t}$ into (19), we get backward induction relationship for the stage-specific employment, i.e., for $n = 2, 3, \ldots, N$,

$$l_{n,t} = l_{n+1,t} + d_{n,t} + g_n(\hat{c}_t, e_t, t_t, a_{1,t}, \ldots, a_{N,t}, p^*_{1F,t}, \ldots, p^*_{NF,t}, p^*_{nH,t}, \ldots, p^*_{NH,t}, y^*_{nH,t}, \ldots, y^*_{NH,t}, \hat{q}_{1,t}, \ldots, \hat{q}_{N,t})$$

where $l_{n,t} = lnL_{n,t}$.

B Aggregate price expression with two-stage production

Given exogenous foreign variables and import tariff constant, the aggregate inflation index (CPI index) can be written as

$$\pi_{t+1} = \gamma\pi_{2H,t+1} + (1-\gamma)\Delta e_{t+1}$$

Since $\hat{q}_t = \gamma(\hat{e}_t - \hat{p}_{2H,t})$, we have

$$\Delta e_{t+1} = \frac{\Delta\hat{q}}{\gamma} + \pi_{2H,t}$$

Then, the aggregate inflation index can be re-written as

$$\pi_{t+1} = \gamma\pi_{2H,t+1} + (1-\gamma)\Delta e_{t+1}$$

$$= \pi_{2H,t} + \frac{1-\gamma}{\gamma}\Delta\hat{q}_t$$

C Stage-specific employment with two-stage production

In this section, we derive explicit expression of the employment gap with two-stage of production, i.e, $N = 2$. As specified in Section A.4, and also note that $\hat{w}_t = \gamma\hat{p}_{2H,t} + (1-\gamma)\hat{p}_{2f,t} + \sigma\hat{c}_t$, $\hat{p}_{1H,t} = \hat{g}_{2,t} + \hat{p}_{2H,t}$, and $\hat{p}_{1F,t} = \hat{p}_{2F,t} = \hat{e}_t$,[16] we have for the second stage

$$\hat{l}_{2,t} = \phi[\gamma\hat{p}_{1H,t} + (1-\gamma)\hat{p}_{1F,t} - \hat{w}_t] + \bar{a}_2[\gamma\hat{p}_{2H,t} + (1-\gamma)\hat{p}_{2F,t} - \hat{p}_{2H,t} + \hat{c}_t]$$

$$+(1-\bar{a}_2)(\hat{e}_t - \hat{p}_{2H}) - a_{2,t} + d_{2,t}$$

$$= \phi[\gamma\hat{g}_{2,t} - \sigma\hat{c}] + \bar{a}_2[(1-\gamma)(\hat{e}_t - \hat{p}_{2H,t})] + (1-\bar{a}_2)(\hat{e}_t - \hat{p}_{2H,t}) - a_{2,t} + d_{2,t}$$

$$= (\bar{a}_2 - \phi\sigma)\hat{c}_t + \phi\gamma\hat{g}_{2,t} + \frac{1-\bar{a}_2\gamma}{\gamma}\hat{q}_t - a_{2,t} + d_{2,t}$$

where, in the first equality, we have used $Y^d_{1H,t} = \frac{\gamma\bar{P}_{2,t}\bar{Y}^d_{2,t}}{\bar{P}_{1H,t}}$, $Y^{Xd}_{1H,t} = \frac{Y^*_{1H}P^*_{1H}\mathcal{E}_t}{P_{1H,t}}$, $Y^d_{2H,t} = \frac{\gamma C_t P_t}{\bar{P}_{2H,t}}$, and $Y^d_{2H,t} = \frac{Y^*_{2H}P^*_{2H}\mathcal{E}_t}{P_{2H,t}}$; in the last equality uses the condition that $\hat{q}_t = \gamma(\hat{e}_t - \hat{p}_{2H,t})$.

For the first stage, the employment is given by

$$\hat{l}_{1,t} = \bar{a}_1(\hat{\bar{p}}_{2,t} + \hat{\bar{y}}^d_{2,t} - \hat{p}_{1H,t}) + (1-\bar{a}_1)(\hat{e}_t - \hat{p}_{1H,t}) + d_{1,t} - a_{1,t}$$

$$= \bar{a}_1(\hat{\bar{p}}_{2,t} + \hat{w}_t + \hat{l}_{2,t} - \hat{\bar{p}}_{2,t} - \hat{p}_{1H,t}) + (1-\bar{a}_1)(\hat{e}_t - \hat{p}_{1H,t}) + d_{1,t} - a_{1,t}$$

$$= \bar{a}_1\hat{l}_{2,t} + \bar{a}_1[\gamma\hat{p}_{2H,t} + (1-\gamma)\hat{e}_t + \sigma\hat{c}_t - \hat{g}_{2,t} - \hat{p}_{2H,t}] + (1-\bar{a}_1)[\hat{e}_t - \hat{p}_{2H,t} - \hat{g}_{2,t}]$$

$$= [-\bar{a}_1\sigma\phi + \bar{a}_1\bar{a}_2 + \bar{a}_1\sigma]\hat{c}_t + [\bar{a}_1\phi\gamma - 1]\hat{g}_{2H,t}$$

$$+\frac{1+\bar{a}_1 - \bar{a}_1\gamma - \bar{a}_1\bar{a}_2\gamma}{\gamma}\hat{q}_t + \bar{a}_1 d_{2,t} + d_{1,t} - a_{1,t} - \bar{a}_1 a_{2,t}$$

where the second equality uses the condition that $\hat{l}_{2,t} = \hat{\bar{y}}^d_{2,t} - \hat{w}_t + \hat{\bar{p}}_{2,t}$.

[16] We have used the assumption of $\psi = 0$.

D Steady-state share of domestic demand in total demand with respect to import tariff

In this section, we characterize how the import tariff affects the steady-state share of domestic demand in total demand in both of two production stages, i.e., $\bar{a}_1$ and $\bar{a}_2$, as specified in Section 3.

By the definition of $\bar{a}_2$, we have

$$\frac{\bar{a}_2}{1-\bar{a}_2} = \frac{\gamma CP}{Y_{2H}^* P_{2H}^* \mathcal{E}}$$

$$= \frac{\gamma C^* (P_t^*)^{1/\sigma}}{Y_{2H}^* P_{2H}^*} P_t^{1-\frac{1}{\sigma}} \mathcal{E}_t^{1/\sigma}$$

$$= \frac{\gamma C^* (P_t^*)^{1/\sigma}}{Y_{2H}^* P_{2H}^*} \{[W^{1-\phi+\gamma\phi} (\mathcal{E}T)^{(1-\gamma)\phi} (P_{1F}^*)^{(1-\gamma)\phi}]^{\gamma} [\mathcal{E}TP_{2F}^*]^{1-\gamma}\}^{1-1/\sigma} \mathcal{E}^{1/\sigma}$$

where the second equality uses $C = C^*(\frac{\mathcal{E}P_t^*}{P_t})^{1/\sigma}$ and the third equality uses the condition specified in Section 3.1. Since $W = \mathcal{E}P^*(C^*)^{\sigma}$ under the assumption of $\psi = 0$, by plugging W into the expression of $\bar{a}_2/(1-\bar{a}_2)$, domestic price index all cancel out (including nominal exchange rate) and thus we have

$$\frac{\bar{a}_2}{1-\bar{a}_2} = f_2(*) \cdot T^{(1-\gamma)(1+\phi\gamma)(1-\frac{1}{\sigma})}$$

where $f_2(*)$ are functions of exogenous foreign variables.

For the first stage, we have

$$\frac{\bar{a}_1}{1-\bar{a}_1} = \frac{\gamma \bar{P}_2 \bar{Y}_2^d}{\mathcal{E} P_{1H}^* Y_{1H}^*}$$

$$= \frac{\gamma\phi P_{2H} (Y_{2H}^d + Y_{2H}^{Xd})}{\mathcal{E} P_{1H}^* Y_{1H}^*}$$

$$= \frac{\gamma\phi}{P_{1H}^* Y_{1H}^*} \frac{P_{2H} Y_{2H}^d}{\mathcal{E}\bar{a}_2}$$

$$= \frac{\gamma\phi}{P_{1H}^* Y_{1H}^*} \frac{\gamma CP}{\mathcal{E}\bar{a}_2}$$

$$= \frac{\gamma\phi}{P_{1H}^* Y_{1H}^*} \frac{Y_{2H}^* P_{2H}^*}{1-\bar{a}_2}$$

where we have used $\frac{\bar{a}_2}{1-\bar{a}_2} = \frac{\gamma CP}{Y_{2H}^* P_{2H}^* \mathcal{E}}$ in the last equality. Therefore, for $\bar{a}_1$, we have

$$\frac{\bar{a}_1}{1-\bar{a}_1} = f_2(*) \cdot \frac{1}{1-\bar{a}_2}$$

where $f_1(*)$ are functions of exogenous foreign variables.

E N-stage production in a closed economy

In this section, we consider the case of N-stage production in a closed-economy, and focus on the effects of lengthening of production chain on welfare loss function. We can similarly characterize equilibrium as the case of the open-economy model with two stages of production, which is shown in Appendix F. In the closed-economy model, since the distortion from monopolistic competition is assumed to be corrected by a subsidy tax, the only distortion in the economy comes from sticky price. Thus, the flexible price equilibrium is Pareto optimal and we can write each variable in the deviation from flexible price equilibrium. The shocks considered in this section are stage-specific productivity shock.

E.1 A utility-based objective welfare loss function for optimal monetary policy

Similarly to the derivation in Section 3.4, the households utility function is given by

$$E\Sigma_{t=0}^{\infty}\beta^t[U(C_t) - V(L_t)]$$

where $U(C_t) = \frac{C_t^{1-\sigma}-1}{1-\sigma}$ and $V(L_t) = \frac{L_t^{1+\psi}}{1+\psi}$.

A second-order Taylor expansion around steady state (C, L) for the period utility of consumption gives

$$U(C_t) - U = U_c C(\hat{c}_t + \frac{1-\sigma}{2}\hat{c}_t^2) + t.i.p.$$

where $\hat{c}_t$ denotes the log-deviation of consumption from steady state. To write the output gap in terms of the gap between the output with sticky-price and natural output (flexible-price equilibrium), the period utility of consumption can be re-written as

$$U(C_t) - U = U_c C(\tilde{c}_t + \frac{1-\sigma}{2}\tilde{c}_t^2 + (1-\sigma)c_t^f\tilde{c}_t) + t.i.p.$$

where $\tilde{c}_t = c_t - c_t^f$ and c_t^f is the log-deviation of consumption in flexible-price equilibrium from steady state equilibrium.

By labor market clearing condition, we obtain the second order Taylor expansion around steady state for the period utility of employment, i.e., $V(L_t)$, as

$$V(L_t) - V = V_L L\{\sum_{n=1}^{N}\frac{L_n}{L}[\hat{l}_{n,t} + \frac{1}{2}\sum_{n=1}^{N}\hat{l}_{n,t}^2]\} + t.i.p.$$

where L_n/L is the share of labor demand in stage n in total labor demand under steady state, given by equations (17) and (23) in Appendix F, and we have used the assumption of $\psi = 0$. More specifically, the stage-specific labor share under steady state is given by

$$\frac{L_n}{L} = (1-\phi)\phi^{N-n},\ n = 2, 3, \dots, N$$

$$\frac{L_1}{L} = \phi^{N-1}$$

The period utility of employment can then be re-written as the gap between labor demand with sticky-price and natural rate of labor demand in each stage, i.e.,

$$V(L_t) - V = V_L L\{\sum_{n=1}^{N} \frac{L_n}{L}[\tilde{l}_{n,t} + \frac{1}{2}\tilde{l}_{n,t}^2 + l_{n,t}^f \tilde{l}_{n,t}]\} + t.i.p \tag{20}$$

where $\tilde{l}_{n,t} = l_{n,t} - l_{n,t}^f$.

As showed in Appendix I, the stage-specific employment gap in terms of output gap and relative price gap for $n = 2, 3, \dots, N-1$ are given by

$$\tilde{l}_{n,t} = \phi[\sum_{i=n}^{N} \tilde{g}_{i,t} - \sigma\tilde{c}_t] + \tilde{l}_{n+1,t} - [\sum_{i=n+1}^{N} \tilde{g}_{i,t} - \sigma\tilde{c}_t] + d_{n,t}$$

with

$$\tilde{l}_{N,t} = \phi[\tilde{g}_{N,t} - \sigma\tilde{c}_t] + \tilde{c}_t + d_{N,t}$$

$$\tilde{l}_{1,t} = \tilde{l}_{2,t} - [\sum_{i=2}^{N} \tilde{g}_{i,t} - \sigma\tilde{c}_t] + d_{1,t}$$

where $d_{n,t} = ln(\int_0^1 (\frac{P_{n,t}(u)}{P_{n,t}})^{-\theta} du)$ measures the price dispersion in stage n. Details can be found in Appendix I.

For simplicity, we denote

$$\tilde{l}_{n,t} = f_n(\tilde{g}_{n,t}, \dots, \tilde{g}_{N,t}) + g(n)\tilde{c}_t + \sum_{g=n}^{N} d_{g,t}$$

where $g(n) = (N-n)(1-\phi)\sigma + 1 - \phi\sigma$ for $n = 2, 3, \dots, N$, and $g(1) = (N-1)(1-\phi)\sigma + 1$.

Then, by summing up $U(C_t) - U$ and $V(L_t) - V$ and also noting that efficiency of steady state implies $U_c C = V_L L$ in closed-economy, the first order terms all cancel out, and only the second order terms are left. The welfare loss function as a fraction of steady state consumption is thus given by

$$W = E_0 \sum_{t=0}^{\infty} \beta^t \frac{U(C_t) - V(L_t) - (U - V)}{U_c C}$$

$$= -\frac{1}{2}E_0 \sum_{t=0}^{\infty} \beta^t \{-(1-\sigma)\tilde{c}_t^2 + \sum_{n=1}^{N} \frac{L_n}{L}[g(n)\tilde{c}_t + f_n(\tilde{g}_{n,t}, \dots, \tilde{g}_{N,t})]^2 + \sum_{n=1}^{N} \theta\phi^{N-g}\lambda_n^{-1}\pi_{n,t}^2\} \qquad (21)$$

where $\frac{L_n}{L} = (1-\phi)\phi^{N-n}$ for $n = 2, 3, \dots, N$ and $\frac{L_1}{L} = \phi^{N-1}$.

Appendix (J) shows the welfare loss function for the case of $N = 2$ and $N = 3$ without abbreviation (i.e., expanding $\frac{L_n}{L}$, $g(n)$, and $f_n(\cdot)$ for n).

In general, monetary policy cannot attain Pareto optimal allocation except in special cases with restrictions on productivity shocks. We proceed with the following proposition.

Proposition 2: in the closed-economy model with N-stage of production and labor being used in each production stage (i.e., $0 < \phi < 1$), there is no monetary policy that can replicate flexible price equilibrium (Pareto-optimal allocation) unless the stage-specific productivity shocks satisfy $\sum_{i=1}^{n-1} \phi^{n-i-1}(\phi - 1)\Delta a_{i,t} + \Delta a_{n,t} = 0$ for $n = 2, \dots, N$ and for all t.

E.2 Discussion about the terms and coefficients in welfare loss function

There are three main parts in the welfare loss function: (a) output gap, and terms measuring stage-specific unemployment gap written in output gap, (b) the relative price gap, and (c) terms measuring stage-specific inflation. More specifically, as showed in the expression of welfare loss function (21), the coefficients before output gap $\tilde{c}_t^2$ and the stage-specific inflation, i.e., $\pi_{n,t}^2$ for $\forall n$, are all positive.[17] Therefore, similar to the standard welfare loss function (e.g., Rotemberg and Woodford, 1999; Woodford, 2003), the objective for a benevolent central bank still includes stabilizing output gap and inflation (i.e., the final stage inflation corresponding to typical "inflation" in literature).

Besides the output gap and final-stage inflation, there are many more terms included in the welfare loss function, classified by those measuring stage-specific unemployment gaps and stage-specific inflation. It suggests that the central bank should not only care about the output gap and CPI, but also need to pay attention to the variations in PPI inflation and the gaps of the real marginal cost in the production of intermediate goods.

Importantly, as showed in the expression of welfare loss function (21), by aggregating the terms of output gap $\tilde{c}_t^2$, the coefficient before output gap is $\sum_{n=1}^{N} \frac{L_n}{L} g(n)^2 - (1-\sigma)$, which is a function of the production structure, and changes with the number of total production stage N. In contrast, the coefficient before CPI (i.e., the final stage inflation $\pi_{N,t}^2$) is a constant $\theta\lambda_N^{-1}$. That is to say, even the central bank follows the Taylor Rule, or the monetary rule suggested by Huang and Liu (2005), i.e., targeting both CPI and PPI, the optimal weights before output gap and CPI (or PPI) are changing with the production structure of the economy.

The welfare loss function with multi-stage production indicates that targeting both CPI and PPI are not satisfactory. Instead, the central bank needs to pay attention to all stage-specific

[17] Details about the proof can be found in Appendix K.

inflation during production process, especially in the case of lengthening production chain. Those terms measuring stage-specific inflation, i.e., $\sum_{n=1}^{N} \theta\phi^{N-n}\lambda_n^{-1}\pi_{n,t}^2$, in welfare loss function have two important implications. On the one hand, given the total number of production stages N and the price stickiness being the same across different stages, the coefficients before inflation in downstream stages are larger compared with those in upstream stages. On the other hand, as the number of total stages N increases, there are more terms of upstream inflation included in the welfare loss function, while the terms for downstream inflation do not change. In the latter case, the relative importance of final stage inflation (i.e., CPI) in welfare loss function becomes smaller; however, the inflation in upstream stages becomes more relatively important. That is to say, as the production length becomes longer, the central bank needs to care more about inflation in intermediate stages but less on final stage inflation (i.e, CPI).

From the perspective of practice, if the stage-specific inflation cannot be attained, PPI, as a sales-weighted price index for intermediate goods across all stages, can be a approximation. But, in general, if PPI index is available, the information used to construct PPI index is enough to construct the stage-specific inflation. For instance, in the PPI program of US Bureau of Labor Statistics, it not only constructs the aggregate PPI index, but also constructs the stage-specific inflation indices in a four-stage vertical production framework with the same original data.[18] Their idea in constructing this system of indices is to choose the total number of stages and assign industries to stages of production in such a manner that maximizes forward goods flow along vertical chain while minimizes backward flow and internal goods flow within the system.

F N-stage of production in a closed economy

F.1 The steady state equilibrium

We first characterize the steady state with perfect foresight. We drop the time subscript t for all variables, and set $A_n = 1$ for $n = 1, 2, \ldots, N$. The optimal pricing decision for firms at stage n, $n = 1, 2, \ldots, N$, becomes

$$P_n^* = \mu\Psi_n$$

By the aggregate price expression (12), in the steady state,

$$P_n = P_n^* = \mu\Psi_n$$

Now, we solve for the price indices in terms of wages and derive the labor demand function. Note that $P_n = \bar{P}_{n+1}$ for $n = 1, 2, \ldots, N-1$, and $\Gamma_n = \bar{P}_n^{\phi} W^{1-\phi}$ for $n = 2, 3, \ldots, N$ with $\Gamma_1 = W$.

[18] Details for the stage-specific inflation indices constructed by US Bureau of Labor Statistics can be found at https://www.bls.gov/ppi/fdidsummary.htm, or Weinhagen (2011).

By substituting $\bar{P}_n$, the relationship of price index between adjacent stages is given by

$$P_n = \mu W^{1-\phi}(P_{n-1})^{\phi}$$

for $n = 2, \ldots, N$ and $P_1 = \mu W$.

By rewriting all price indexes in terms of wages, it comes

$$P_n = (\mu)^{1+\phi+\cdots+\phi^{n-2}} W^{1-\phi^{n-1}} (P_1)^{\phi^{n-1}}$$
$$= \mu^{\frac{1-\phi^n}{1-\phi}} W$$

for $n = 2, \ldots, N$ with $P_1 = \mu W$.

Since $P_n(u) = P_n$ for $u \in [0,1]$ in the steady state, we have

$$Y^d_{n-1,t}(u) = \bar{Y}^d_{n,t}$$

Together with the goods markets clearing condition $Y_{n,t} = \bar{Y}^d_{n+1,t}$, and factor market demand function (8) and (7), for $n = 2, \ldots, N-1$, we obtain

$$\bar{Y}^d_n = \phi \frac{\Gamma_n}{\bar{P}_n} \bar{Y}^d_{n+1}$$

$$L^d_n = (1-\phi) \frac{\Gamma_n}{W} \bar{Y}^d_{n+1}$$

where $Y_N = C$, $\bar{Y}_N = \phi \frac{\Gamma_N}{\bar{P}_N} C$, $L^d_N = (1-\phi)\frac{\Gamma_N}{W} C$, and $L^d_1 = \frac{\Gamma_1}{W}\bar{Y}^d_2$. By substituting the price index and unit cost function in each stage, for $n = 2, \ldots, N$, the factor demand functions for both labor and composite intermediate goods are given by

$$\bar{Y}^d_n = \phi^{N+1-n}(\mu)^{-(N+1-n)+\frac{\phi^{n-1}-\phi^N}{1-\phi}} C$$

$$L^d_n = (1-\phi)\phi^{N-n}(\mu)^{-(N+1-n)+\frac{1-\phi^N}{1-\phi}} C \tag{22}$$

with $L^d_1 = \bar{Y}^d_2$.[19]

[19] A simple way of thinking labor demand in each stage can be viewed as backward deduction (which is helpful when taking log-linearization), i.e.,

$$L^d_n = L^d_{n+1} \cdot \phi \cdot \mu^{-1},\ n = 2, \ldots, N$$
$$L^d_1 = (\frac{\phi}{1-\phi})\mu^{-1} L^d_2$$

By summing up labor demand across all stages, the total labor demand function becomes

$$L^d = \sum_{n=2}^{N} [(1-\phi)\phi^{N-n}\mu^{-(N+1-n)+\frac{1-\phi^N}{1-\phi}}C] + \phi^{N-1}\mu^{-(N-1)+\frac{\phi-\phi^N}{1-\phi}}C \tag{23}$$

By the labor supply function (1) together with the price index, the labor supply in steady state becomes

$$L^{\psi}C^{\sigma} = (\mu)^{-\frac{1-\phi^N}{1-\phi}} \tag{24}$$

Given $L^d = L$, the two equations (23) and (24) fully characterize the steady state total consumption and total employment.

F.2 The flexible price equilibrium

In order to obtain efficient allocation in the model economy, i.e., the natural rate of the output, we solve for the flexible price equilibrium in a similar way as in the steady state. In the flexible price equilibrium, $\alpha_n = 0$ for $\forall n$, and the optimal pricing decision for firms at stage n, $n = 1, 2, \ldots, N$, becomes

$$P^*_{n,t} = \mu\Gamma_{n,t}$$

By the aggregate price expression (12), we have

$$P_{n,t} = P^*_{n,t} = \mu\Gamma_{n,t}$$

Similar to the steady state case, we solve for the price indices in terms of wages and productivity. Note that $P_{n,t} = \bar{P}_{n+1,t}$ for $n = 1, 2, \ldots, N-1$, and $\Gamma_{n,t} = \bar{P}^{\phi}_{n,t}W^{1-\phi}_t/A_{n,t}$ for $n = 2, 3, \ldots, N$ with $\Gamma_1 = W_t/A_{1,t}$. By substituting $\bar{P}_{n,t}$, the relationship of price index across adjacent stages is given by

$$P_{n,t} = \mu W^{1-\phi}_t (P_{n-1,t})^{\phi}/A_{n,t}$$

for $n = 2, \ldots, N$ and $P_1 = \mu W/A_{1,t}$.

By writing all price index in terms of wage, we obtain

$$\begin{aligned} P_{n,t} &= (\mu)^{1+\phi+\cdots+\phi^{n-2}} W^{1-\phi^{n-1}} (P_1)^{\phi^{n-1}} \Pi^{n}_{g=2} A^{-\phi^{n-g}}_{g,t} \\ &= \mu^{\frac{1-\phi^n}{1-\phi}} W \cdot \Pi^{n}_{g=1} A^{-\phi^{n-g}}_{g,t} \end{aligned} \tag{25}$$

for $n = 2, \ldots, N$.

Similar to the derivation for the steady state case, the labor demand function in each stage in flexible price equilibrium is given by

$$L^d_{n,t} = (1-\phi)\phi^{N-n}\mu^{-(N+1-n)+\frac{1-\phi^N}{1-\phi}}\Pi^N_{g=1}A^{-\phi^{N-g}}_{g,t}C_t \tag{26}$$

for $n = 2, 3, \ldots, N$ with $L^d_{1,t} = (\frac{\phi}{1-\phi})\mu^{-1}L^d_{2,t}$. Details can be found in Appendix G.

Therefore, the total labor demand is given by

$$L^d_t = \eta\Pi^N_{n=1}A^{-\phi^{N-n}}_{n,t}C_t \tag{27}$$

where η is a constant, and it is given by

$$\eta = \sum_{n=2}^{N}[(1-\phi)\phi^{N-n}\mu^{-(N+1-n)+\frac{1-\phi^N}{1-\phi}}C] + \phi^{N-1}\mu^{-(N-1)+\frac{\phi-\phi^N}{1-\phi}}$$

By the labor supply function (1) together with the price index, we know

$$L^\psi C^\sigma = \mu^{-\frac{1-\phi^N}{1-\phi}}\Pi^N_{n=1}A^{\phi^{N-n}}_{n,t} \tag{28}$$

After taking log-deviation from the steady state for both the total labor demand (27) and the labor supply (28), we get

$$l^f_t = c^f_t - [\sum_{n=1}^{N}\phi^{N-n}a_{n,t}] \tag{29}$$

and

$$\psi l^f_t + \sigma c^f_t = \sum_{n=1}^{N}\phi^{N-n}a_{n,t} \tag{30}$$

Therefore, the log-deviation of output from the steady state under flexible prices c^f_t is given by

$$c^f_t = \frac{1+\psi}{\psi+\sigma}[\sum_{n=1}^{N}\phi^{N-n}a_{n,t}]$$

By the Euler equation (4), the IS curve is characterized by

$$c^f_t = E_t(c^f_{t+1}) - \frac{1}{\sigma}[i_t - E_t(\pi_{N,t+1}) - \rho]$$

which yields the natural rate of interest as

$$\bar{r}r_t = i_t - E_t(\pi_{N,t+1})$$

$$= \rho + \sigma E_t\{c^f_{t+1} - c^f_t\}$$

Given the expression of output and the process of productivity shocks, we have

$$\bar{rr}_t = \rho + \frac{\sigma(1+\psi)}{\psi+\sigma} E_t[\sum_{n=1}^{N} \phi^{N-n} \Delta a_{n,t+1}] \tag{31}$$

where $\Delta a_{n,t} = a_{n,t} - a_{n,t-1}$ is the growth rate of productivity in stage n.

F.3 The sticky-price equilibrium

We now derive New Keynesian Phillips curves for each stage as a function of relative price gap and output gap, and characterize the equilibrium with sticky prices. Similar to the derivation in Gali (2015), in each stage of production $n = 1, 2, \ldots, N$, firms' optimal pricing decision gives

$$\pi_{n,t} = \beta E_t \pi_{n,t+1} + \lambda_n \tilde{\gamma}_{n,t}$$

where $\lambda_n = \frac{(1-\beta\alpha_n)(1-\alpha_n)}{\alpha_n}$ and $\tilde{\gamma}_n$ is the log-derivation of the real marginal cost from the flexible price equilibrium, i.e.,

$$\tilde{\gamma}_{n,t} = ln(\Gamma_{n,t}/P_{n,t}) - ln(\Gamma^f_{n,t}/P^f_{n,t})$$

where $\Gamma^*_{n,t}$ and $P^*_{n,t}$ are, respectively, the marginal cost and aggregate price in stage n in the flexible price equilibrium.

Following Huang and Liu (2005), without a loss of generality, we assume that $\psi = 0$. Together with labor supply function (28), for $n = 2, 3, \ldots, N-1$, the log-derivation of the real marginal cost can be written as a function of relative price gap and output gap, i.e.,

$$\tilde{\gamma}_{n,t} = \phi \tilde{q}_{n,t} + (1-\phi)[\sigma \tilde{c}_t - \sum_{i=n+1}^{N} \tilde{g}_{i,t}] \tag{32}$$

$$\tilde{\gamma}_{1,t} = \sigma \tilde{c}_t - \sum_{i=2}^{N} \tilde{g}_{i,t}$$

$$\tilde{\gamma}_{N,t} = \phi \tilde{g}_{N,t} + (1-\phi)\sigma \tilde{c}_t$$

where $\tilde{g}_{n,t}$ is the relative price gap between stage n and stage $n-1$, i.e., $\tilde{g}_{n,t} = ln(\frac{P_{n-1,t}}{P_{n,t}}) - ln(\frac{P^f_{n-1,t}}{P^f_{n,t}})$. Details for Expression (32) can be found in Appendix H.

After log-linearizing the Euler equation around the steady state and subtracting the natural rate IS curve, we obtain the IS curve with stick prices as

$$\tilde{c}_t = E_t \tilde{c}_{t+1} - \frac{1}{\sigma}[i_t - E_t(\pi_{N,t+1}) - \bar{rr}_t]$$

where $\bar{rr}_t$ is the natural rate of interest.

The law of motion for the relative price gap between stage n and stage $n-1$, for $n = 2, 3, \ldots, N$, is characterized by

$$\tilde{g}_{n,t} = \tilde{g}_{n,t-1} + \pi_{n-1,t} - \pi_{n,t} - \Delta g^f_{n,t}$$

where $\Delta g^f_{n,t} = g^f_{n,t} - g^f_{n,t-1}$. By Equation (25), we have

$$\Delta g^f_{n,t} = \sum_{i=1}^{n-1} \phi^{n-i-1}(\phi - 1)\Delta a_{i,t} + \Delta a_{n,t}$$

Give the monetary policy rule, the Phillips curve, IS curve, and the law of motion for the relative price gap fully pin down the dynamic equilibrium under sticky prices.

G Labor demand function in the flexible price equilibrium

Similar to the steady state equilibrium, we derive the labor demand function in the flexible price equilibrium. Note that, with flexible prices, $P_n(u) = P_n$ for $u \in [0, 1]$. We then obtain

$$Y^d_{n-1,t}(u) = \bar{Y}^d_{n,t}$$

Together with goods markets clearing condition $Y_{n,t} = \bar{Y}^d_{n+1,t}$, and factor market demand function (8) and (7), for $n = 2, \ldots, N-1$, we obtain

$$\bar{Y}^d_{n,t} = \phi \frac{\Gamma_{n,t}}{\bar{P}_{n,t}} \bar{Y}^d_{n+1,t}$$

$$L^d_{n,t} = (1 - \phi) \frac{\Gamma_{n,t}}{W_t} \bar{Y}^d_{n+1,t}$$

where $Y_{N,t} = C_t$, $\bar{Y}_{N,t} = \phi \frac{\Gamma_{N,t}}{\bar{P}_N} C_t$, $L^d_{N,t} = (1-\phi)\frac{\Gamma_{N,t}}{W_t} C_t$, and $L^d_{1,t} = \frac{\Gamma_{1,t}}{W_t} \bar{Y}^d_{2,t}$.

Note that $P_{n,t} = \bar{P}_{n+1,t}$ for $n = 1, 2, \ldots, N-1$, and $\Gamma_{n,t} = \bar{P}^{\phi}_{n,t} W^{1-\phi}_t / A_{n,t}$ for $n = 2, 3, \ldots, N$ with $\Gamma_1 = W_t / A_{1,t}$. By substituting the unit cost function in each stage, for $n = 2, \ldots, N$, we obtain the labor demand in each stage as follows:

$$\bar{Y}^d_{n,t} = \frac{\phi}{1-\phi} \frac{W_t}{P_{n-1,t}} L^d_{n,t}$$

and thus

$$L^d_{n,t} = \phi \frac{P_{n-1,t}}{P_{n,t}} \left(\frac{W_t}{P_{n-1,t}}\right)^{1-\phi} A^{-1}_{n,t} L^d_{n+1,t}$$

$$L^d_{1,t} = \frac{1}{A_{1,t}}\frac{\phi}{1-\phi}\frac{W_t}{P_{1,t}}L^d_{2,t}$$

One can view the labor demand in each stage as backward induction (which is helpful when taking log-linearization), i.e.,

$$L^d_{n,t} = L^d_{n+1,t} \cdot \phi \cdot \mu^{-1},\ n = 2, \ldots, N$$

$$L^d_{1,t} = (\frac{\phi}{1-\phi})\mu^{-1}L^d_{2,t}$$

Note that $L^d_{N,t} = (1-\phi)\frac{\Gamma_{N,t}}{W_t}C_t$, which indicates

$$L^d_{N,t} = (1-\phi)\mu^{\frac{\phi-\phi^N}{1-\phi}}\Pi^N_{g=1}A^{-\phi^{N-g}}_{g,t}C_t$$

Therefore, for $n = 2, 3, \ldots, N$, we obtain the labor demand function in each stage as

$$L^d_{n,t} = (1-\phi)\phi^{N-n}\mu^{-(N+1-n)+\frac{1-\phi^N}{1-\phi}}\Pi^N_{g=1}A^{-\phi^{N-g}}_{g,t}C_t$$

with $L^d_{1,t} = (\frac{\phi}{1-\phi})\mu^{-1}L^d_{2,t}$.

H The log-derivation of the real marginal cost from the flexible price equilibrium

Note that, for $n = 2, 3, \ldots, N$, $\Gamma_{n,t} = \bar{P}^{\phi}_{n,t}W^{1-\phi}_t/A_{n,t}$ and $\bar{P}_{n,t} = P_{n-1,t}$. The log-derivation of the real marginal cost is given by

$$\tilde{\gamma}_{n,t} = ln(\Gamma_{n,t}/P_{n,t}) - ln(\Gamma^*_{n,t}/P^*_{n,t})$$

$$= \phi[ln(P_{n-1,t}/P_{n,t}) - ln(P^*_{n-1,t}/P^*_{n,t})] + (1-\phi)[ln(W_t/P_{n,t}) - ln(W^f_t/P^f_{n,t})]$$

Denote $g_{n,t} = ln(P_{n-1,t}/P_{n,t})$ and $\tilde{g}_{n,t} = ln(P_{n-1,t}/P_{n,t}) - g^f_{n,t}$. For $n = 1, 2, \ldots, N-1$, we have

$$lnP_{n,t} = \sum^N_{i=n+1} g_{i,t} + lnP_{N,t}$$

$$\Longleftrightarrow p_{n,t} = \sum^N_{i=n+1} g_{i,t} + p_{N,t}$$

Also, by the labor supply equation (28), by assuming $\psi = 0$, we have

$$w_t - p_{N,t} = \sigma c_t$$

Therefore, for $n = 2, 3, \ldots N-1$, the log-derivation of real marginal cost can be written as

$$\tilde{\gamma}_{n,t} = \phi\tilde{g}_{n,t} + (1-\phi)[\tilde{w}_t - \tilde{p}_{n,t}]$$

$$= \phi\tilde{g}_{n,t} + (1-\phi)[\sigma\tilde{c}_t + \tilde{p}_{N,t} - \tilde{p}_{n,t}]$$

$$= \phi\tilde{g}_{n,t} + (1-\phi)[\sigma\tilde{c}_t - \sum_{i=n+1}^{N}\tilde{g}_{i,t}]$$

with $\tilde{\gamma}_{N,t} = \phi\tilde{g}_{N,t} + (1-\phi)\sigma\tilde{c}_t$.

Similarly, for the first stage $n = 1$, since $\Gamma_1 = W_t/A_{1,t}$, we have

$$\tilde{\gamma}_{1,t} = \tilde{w}_t - \tilde{p}_{1,t}$$

$$= \sigma\tilde{c}_t - \sum_{i=2}^{N}\tilde{g}_{i,t}$$

I Stage-specific employment gaps in a closed economy with N-stage production

We derive the stage-specific employment gap in terms of output gap and relative price gap. By the factor demand function (7), (8), and (11) in each stage, and substituting with the unit cost, for $n = 2, 3, \ldots, N$, we have

$$lnL_{n,t} = ln(1-\phi) + \phi[lnP_{n-1,t} - lnW_t] - lnA_{n,t} + ln\bar{Y}^d_{n+1,t} + d_{n,t}$$

$$\Longleftrightarrow l_{n,t} = ln(1-\phi) + \phi[p_{n-1,t} - w_t] - a_{n,t} + ln\bar{Y}^d_{n+1,t} + d_{n,t}$$

where $d_{n,t} = ln(\int_0^1 (\frac{P_{n,t}(u)}{P_{n,t}})^{-\theta} du)$ and $l_{1,t} = -a_{1,t} + ln\bar{Y}^d_{2,t} + d_{1,t}$.

By the factor demand function for intermediate goods and labor in each stage, i.e., Expression (7) and (8), for $n = 2, 3, \ldots, N$, we get

$$l_{n,t} = ln(\frac{1-\phi}{\phi}) + p_{n-1,t} - w_t + ln\bar{Y}^d_{n,t}$$

Note that $\bar{Y}^d_{N,t} = C_t$. Then, by substituting $ln\bar{Y}^d_{n+1,t}$, we obtain via backward induction the relationship for the stage-specific employment, i.e., for the stage of $n = N$,

$$l_{N,t} = ln(1-\phi) + \phi[p_{N-1,t} - w_t] - a_{N,t} + c_t + d_{N,t}$$

for $n = 2, 3, \dots, N-1$,

$$l_{n,t} = ln(\phi) + \phi[p_{n-1,t} - w_t] - a_{n,t} + l_{n+1,t} - [p_{n,t} - w_t] + d_{n,t}$$

for $n = 1$,

$$l_{1,t} = -a_{1,t} + l_{2,t} - [p_{1,t} - w_t] + d_{1,t}$$

As shown in Appendix H, for $n = 1, 2, \dots, N$, we have $p_{n,t} = \sum_{i=n+1}^{N} g_{i,t} + p_{N,t}$, and, by assuming $\psi = 0$, $w_t - p_{N,t} = \sigma c_t$. The stage specific employment can be written in terms of relative price and output as, for $n = N$,

$$l_{N,t} = ln(1-\phi) + \phi[g_{N,t} - \sigma c_t] - a_{N,t} + c_t + d_{N,t}$$

for $n = 2, 3, \dots, N-1$,

$$l_{n,t} = ln(\phi) + \phi[\sum_{i=n}^{N} g_{i,t} - \sigma c_t] - a_{n,t} + l_{n+1,t} - [\sum_{i=n+1}^{N} g_{i,t} - \sigma c_t] + d_{n,t}$$

for $n = 1$,

$$l_{1,t} = -a_{1,t} + l_{2,t} - [\sum_{i=2}^{N} g_{i,t} - \sigma c_t] + d_{1,t}$$

By subtracting the corresponding equations for the flexible price equilibrium, the stage-specific employment gap in terms of output gap and the relative price gap is given by, for $n = 2, 3, \dots, N-1$,

$$\tilde{l}_{n,t} = \phi[\sum_{i=n}^{N} \tilde{g}_{i,t} - \sigma \tilde{c}_t] + \tilde{l}_{n+1,t} - [\sum_{i=n+1}^{N} \tilde{g}_{i,t} - \sigma \tilde{c}_t] + d_{n,t}$$

with

$$\tilde{l}_{N,t} = \phi[\tilde{g}_{N,t} - \sigma \tilde{c}_t] + \tilde{c}_t + d_{N,t}$$

$$\tilde{l}_{1,t} = \tilde{l}_{2,t} - [\sum_{i=2}^{N} \tilde{g}_{i,t} - \sigma \tilde{c}_t] + d_{1,t}$$

Therefore, by solving forward induction, the stage-specific employment gap is given by, for

$n = 2, 3, \dots, N-1$,

$$\tilde{l}_{n,t} = \phi[\sum_{i=n}^{N} \tilde{g}_{i,t} - \sigma \tilde{c}_t] + \tilde{l}_{n+1,t} - [\sum_{i=n+1}^{N} \tilde{g}_{i,t} - \sigma \tilde{c}_t] + d_{n,t}$$

with

$$\tilde{l}_{N,t} = \phi[\tilde{g}_{N,t} - \sigma \tilde{c}_t] + \tilde{c}_t + d_{N,t}$$

$$\tilde{l}_{1,t} = \tilde{l}_{2,t} - [\sum_{i=2}^{N} \tilde{g}_{i,t} - \sigma \tilde{c}_t] + d_{1,t}$$

J The closed-form welfare loss function for the case of $N = 2$ and $N = 3$ in closed-economy

To illustrate the welfare loss function, we show the analytical welfare loss function for the cases of $N = 2$ and $N = 3$ without abbreviation. For the case of $N = 2$, by Appendix I, the stage-specific employment gap in terms of output gap and relative price gap is given by

$$\tilde{l}_{1,t} = (1 + \sigma - \sigma\phi)\tilde{c}_t + (\phi - 1)\tilde{g}_{2,t} + d_{1,t} + d_{2,t}$$

$$\tilde{l}_{2,t} = (1 - \sigma\phi)\tilde{c}_t + \phi\tilde{g}_{2,t} + d_{2,t}$$

Since $\frac{L_1}{L} = \phi$ and $\frac{L_2}{L} = 1 - \phi$, by plugging into Equation (21), the welfare loss function with $N = 2$ is given by

$$W = -\frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \{\sigma \tilde{c}_t^2 + \phi(1-\phi)[\sigma\tilde{c}_t - \tilde{g}_{2,t}]^2 + \theta\lambda_2^{-1}\pi_{2,t}^2 + \theta\phi\lambda_1^{-1}\pi_{1,t}^2\}$$

which is exactly the same as in Huang and Liu (2005).

Similarly, for the case of $N = 3$, the stage-specific employment gap in terms of output gap and relative price gap is given by

$$\tilde{l}_{1,t} = (1 + 2\sigma - 2\sigma\phi)\tilde{c}_t + 2(\phi - 1)\tilde{g}_{3,t} + (\phi - 1)\tilde{g}_{2,t} + d_{1,t} + d_{2,t} + d_{3,t}$$

$$\tilde{l}_{2,t} = (1 + \sigma - 2\sigma\phi)\tilde{c}_t + (2\phi - 1)\tilde{g}_{3,t} + \phi\tilde{g}_{2,t} + d_{2,t} + d_{3,t}$$

$$\tilde{l}_{1,t} = (1 - \sigma\phi)\tilde{c}_t + \phi\tilde{g}_{3,t} + d_{3,t}$$

Since $\frac{L_1}{L} = \phi^2$, $\frac{L_2}{L} = \phi(1-\phi)$ and $\frac{L_3}{L} = 1 - \phi$, by plugging into Equation (21), the welfare loss function with $N = 3$ is given by

$$W = -\frac{1}{2}E_0 \sum_{t=0}^{\infty} \beta^t \{-(1-\sigma)\tilde{c}_t^2 + \phi^2[(1+2\sigma-2\sigma\phi)\tilde{c}_t + 2(\phi-1)\tilde{g}_{3,t} + (\phi-1)\tilde{g}_{2,t}]^2$$

$$+(1-\phi)\phi[(1+\sigma-2\sigma\phi)\tilde{c}_t + (2\phi-1)\tilde{g}_{3,t} + \phi\tilde{g}_{2,t}]^2$$

$$+(1-\phi)[(1-\sigma\phi)\tilde{c}_t + \phi\tilde{g}_{3,t}]^2$$

$$+\theta\lambda_3^{-1}\pi_{3,t}^2 + \theta\phi\lambda_2^{-1}\pi_{2,t}^2 + \theta\phi^2\lambda_3^{-1}\pi_{3,t}^2\}$$

K The proof for positive coefficient of output gap in welfare loss function

The coefficient of output gap $\tilde{c}_t^2$ in the welfare loss function (21) is given by

$$-(1-\sigma) + \sum_{n=1}^{N} \frac{L_n}{L} g(n)^2 \equiv f$$

Note that the stage-specific labor share under efficient steady state yields

$$\frac{L_n}{L} = (1-\phi)\phi^{N-n},\ n = 2, 3, \ldots, N$$

$$\frac{L_1}{L} = \phi^{N-1}$$

and $g(n) = (N-n)(1-\phi)\sigma + 1 - \phi\sigma$ for $n = 2, 3, \ldots, N$, and $g(1) = (N-1)(1-\phi)\sigma + 1$. Also, since $\phi < 1$ and $\sigma \leq 1$, it is obvious that $g(n) \geq 1 - \sigma\phi > 0$ for $\forall n$.

Therefore,

$$f = -(1-\sigma) + \sum_{n=1}^{N} \frac{L_n}{L} g(n)^2$$

$$\geq -(1-\sigma) + \sum_{n=1}^{N} \frac{L_n}{L} g(n) \cdot (1-\sigma\phi)$$

$$= (1-\sigma\phi) - (1-\sigma)$$

$$= \sigma(1-\phi) > 0$$

In other words, the coefficient on the output gap in the welfare loss function is positive.