Skilled Immigration, Firms, and Policy*

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Abstract

This paper studies the macroeconomic general equilibrium effects of skilled immigration policy changes by explicitly taking into account the role of firm demand for foreign skilled labor. To this end, I develop a two-sector dynamic stochastic general equilibrium model with monopolistically competitive firms and heterogeneous workers. Unlike most previous studies that view immigration as a labor supply shock, the paper models skilled labor immigration as an endogenous response to an increase in firm labor demand in the receiving economy. The model is calibrated to mimic the U.S. economy with its current immigration policy: Firms face hiring costs and there is an occasionally binding cap on the foreign skilled workers that can be hired each period. The results indicate that a less restrictive skilled immigration policy via an immigration cap increase leads to heterogeneous effects on skilled and unskilled workers — unskilled domestic workers gain but skilled domestic workers lose. However, the magnitude of the welfare impacts depend on the structure of the labor market (presence of search frictions). This paper also evaluates the welfare gain from moving toward an alternate skilled immigration policy with a market-driven allocation of permits for hiring skilled foreign workers. Such a policy increases welfare and brings the economy’s allocation closer to the social planner’s first-best allocation.

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1 Introduction

There has been a rapid increase in the number of foreign skilled workers in the U.S. labor force. Among all foreign-born individuals, those with at least a bachelor’s degree witnessed the sharpest increase (42 percent) during the 2004 - 2015 period (Figure A.1). The corresponding increase for the native born in the same skill group was 26 percent. This led to an increase in the proportion of college-educated foreign born in the U.S. labor force from 14 percent to 16 percent (Figure A.2).  

Firm demand for foreign skilled workers has played an important role in generating this increase. Since its inception in 1990, the H1-B visa program remains the dominant entry route of foreign skilled workers into the U.S. labor force (Figure A.3). Firms play a crucial role in hiring, sponsoring, and incurring costs at each stage of the H1-B application process for a foreign worker. The first step requires a firm that wants to hire a foreign worker to file a Labor Condition Application (LCA) with the Department of Labor in which one of the items that they need to specify is the number of foreign workers they would like to hire for a particular occupation. These LCAs signal vacancies or firm demand for foreign skilled labor. However, the actual number of visas issued to foreign workers is determined by a policy-imposed cap.  

The gap between firm demand, measured by the number of workers requested in the LCAs filed, and visas issued tends to grow during expansionary periods (Figure 1). Moreover, the visa cap was met in each year since 2004, in less than a week in seven of those years (Figure 2), and visas for foreign skilled workers were allocated according to a lottery process. These facts indicate that there is a strong demand for foreign skilled workers that is not accommodated by the current immigration policy.

The role of firm demand of foreign skilled workers and the current allocation mechanism of foreign workers across them have implications for how an increase in skilled immigration and changes in immigration policies impact the aggregate economy. However, these implications are not fully understood in the current literature since most studies view immigration as a labor supply-induced shock.

\footnote{Over a longer horizon of two and a half decades, the foreign-born share of the total population with a bachelor’s degree in the U.S. labor force increased from 10 percent in 1990 to 17 percent in 2015 (U.S. Census Bureau’s 1990 and 2000 Decennial Census). Data from 2004 - 2015 is compiled from the Current Population Survey (CPS). Foreign born in this survey include legally-admitted immigrants, refugees, temporary residents and temporary workers, and undocumented immigrants. However, the number of undocumented unskilled immigrants is likely to be underreported. In this study, I do not distinguish between foreign born and immigrants even though the legal definitions are different.}

\footnote{The Appendix B discusses details on the H1-B visa program.}

\footnote{Firms have from April 1st until the beginning of the next fiscal year to file petitions for H1-B visa applications.}
Motivated by these facts, the main goal of this paper is to address the following question within a macroeconomic general equilibrium framework: What are the determinants of firm demand for foreign skilled workers and what are the impacts of skilled immigration policy changes on labor market outcomes, output, and welfare of domestic households? I study how the presence of labor market search frictions influences the quantitative impact of an immigration cap change. Moreover, unlike much of the previous literature, I begin to analyze the welfare impact of moving towards alternate skilled immigration policies - in particular, a market-driven allocation of permits for hiring skilled foreign workers.
To this end, I develop a two-sector dynamic stochastic general equilibrium (DSGE) model. The baseline model features perfectly competitive labor markets with no labor market search frictions. Monopolistically competitive firms in the skill-intensive sector produce output by employing skilled domestic and foreign labor. Skilled labor immigration is modeled as an endogenous response to an increase in firm labor demand in the domestic economy, subject to immigration policy restrictions that mimic current U.S. policy: Firms face hiring costs and there is a cap on the number of foreign workers that can be hired each period. This cap binds when economic conditions are such that the aggregate demand for foreign labor exceeds the policy-imposed quota. If the cap is met, the endogenously determined probability of being able to hire each foreign worker is less than one. Firms take into account these immigration policy restrictions and optimally choose to hire foreign labor until the expected discounted benefit from hiring foreign skilled workers is equal to the expected cost. Since a significant proportion of foreign skilled workers on an H1-B visa are temporary workers, the model allows for an exogenous probability of return to the country of origin. Perfectly competitive firms in the second sector employ unskilled labor to produce a homogeneous product.\footnote{This two-sector model is qualitatively similar to a one-sector version of the model in which firms hire both skilled and unskilled workers. In the two-sector version, complementarities between skilled and unskilled workers exist through the consumption basket, while in the one-sector version, these complementarities exist through the production technology. Moreover, in the context of skilled foreign-born labor, around 73 percent of the Labor Condition Applications are requested by the relatively skill-intensive NAICS Sector 54 — Professional, Scientific, Technical services Sector (United States Department of Labor). The Professional and Business sector as a whole contributes around 12.4 percent of value added as a percentage of U.S. Gross Domestic Product (Bureau of Economic Analysis).}

The baseline model is tractable and the analytical solution produces insights regarding factors that influence firm demand for foreign skilled workers and how immigration policy distorts firm hiring. The model is consistent with evidence regarding U.S. labor markets. More productive firms demand more skilled labor. Moreover, the model is consistent with the empirical evidence that firms’ response to an immigration cap change depends on the realized state $Z_t$ of the economy. Therefore, during recessionary periods, foreign workers hired may be less than the increase in the cap, as was witnessed in 2001 (Figure 2).

I calibrate the main parameters of the baseline model that pertain to immigration to match the U.S. economy during the 2004 - 2017 period. I then employ the calibrated model to study dynamics of economic variables in response to productivity shocks and changes in immigration policy. I calculate the welfare effect on domestic households of an increase in the policy-imposed cap. Under the baseline model, unskilled domestic households gain (due to complementarities that increase unskilled wages) but skilled domestic households lose (due to substitutabilities that reduce skilled wages) from a ten percent immigration cap increase. The welfare gain (including transitional dynamics) amount to 0.0102 percent of...
quarterly steady-state consumption for unskilled workers. For skilled domestic workers, the welfare loss amounts to 0.0086 percent of quarterly steady-state consumption. An increase in the stock of foreign skilled labor increases firm output and profits over time. I show that the profit distribution across households is important for the quantitative impact of the cap change. Moreover, the magnitude and impact of an increase in the cap may depend on the presence of labor market search frictions. Extending the model to include search frictions highlight some key insights that emphasize the importance of focusing on the response of firms when studying skilled immigration policy changes. In this case, even though domestic skilled wages fall as in the baseline case, an increase in domestic employment offsets part of the negative impact on domestic skilled workers. The reduction in welfare loss for skilled domestic workers ranges from 54.2% to 79%, depending on the profit distribution. The welfare gain for unskilled domestic households is 33% higher compared to the baseline model due to higher demand for their output stemming from a larger pool of employed workers.

The main intuition behind the above result is that the increase in the cap encourages firms to post more vacancies, and this increases both domestic and foreign matches. In the search and matching framework, firms can be matched with either domestic or foreign workers (depending on the relative proportion of job searchers). However, under the current immigration policy, if the cap binds, firms are able to hire only a fraction of their foreign matches (given by the probability of an application being selected as in the baseline model). When the cap increases, this probability increases, and firms are able to retain more foreign matches. This increases their expected discounted benefit from posting a vacancy. More vacancies posted lead to an increase in the number of matched domestic skilled workers, which mitigates their welfare losses.

The main result from analyzing the alternate skilled immigration policy with a market-driven allocation of permits is that such a policy increases welfare of both skilled and unskilled domestic workers, compared to the baseline model. The main channel is through higher government revenues. This indicates that an alternate allocation of the same quota of foreign workers can potentially increase welfare and reduce inefficiency between the decentralized economy’s allocation and the first-best allocation chosen by a social planner. However, closing much of the gap between the market economy’s allocation and the socially planner’s allocation requires an increase in the cap (or equivalently an increase in the number of permits allocated.)

This paper has three main contributions. First, the model focuses on the role of firms and includes a more realistic skilled immigration policy. The results indicate that this is relevant for evaluating the impact of a cap change in the U.S. economy. Second, apart from studying the impact of skilled immigration policy reform via changes in the cap alone, this paper
also begins to evaluate the impact of an alternate skilled immigration policy setup through a market-driven allocation of permits, which is related to the skilled immigration policy reform proposed in Peri (2012). Third, by incorporating a more realistic skilled immigration policy setup within the search and matching framework, the paper shows that even when U.S. and foreign workers are perfectly substitutable, an increase in the immigration cap can generate positive employment effects for domestic skilled workers.

2 Related Literature

This research adds to the emerging literature that examines the implications of high skilled migration. This includes Borjas and Doran (2012), Ottaviano, Shih, and Sparber (2015), and Kerr and Lincoln (2010). The paper is also related to studies that measure the welfare gains from lowering barriers to labor mobility (Urrutia (1998); Klein and Ventura (2007, 2009); Iranzo and Peri (2009); Levchenko et. al. (2015); Ehrlich and Kim (2015)). In the context of DSGE models of international business cycles, the paper is related to Mandelman and Zlate (2012), who develop a two-country business cycle model with unskilled labor migration.

This paper is also related to empirical studies that highlight the role of firms in the context of skilled immigration (Kerr et al. (2013) and Ottaviano, Peri, and Wright (2015)). Kerr et al. (2014) stress the “need to increasingly develop a better understanding of the general equilibrium effects of skilled immigration with firms as a central element.” Some recent studies have explicitly focused on the role of firms in a macroeconomic general equilibrium framework while discussing impacts of skilled immigration. Waugh (2017) studies the impact of a larger labor force (through an expansion of the H1-B visa program) on dynamics of firm entry and exit, and the effect on wages, aggregate output, and welfare. Bound et al. (2016) also use a general equilibrium model to study the effect of an increase in high-skill foreign born on domestic workers, consumers and firms, during the 1990s. My baseline model is consistent with their results — skilled immigration reduces wages of domestic skilled households, while redistributing gains to complements in production. Immigration lowers prices and raises output and profits of firms in the relevant sectors. However, skilled immigration is modeled as a labor supply shock within a competitive labor market setting in these studies. The explicit focus on the role of firm hiring of foreign skilled labor leads to some new insights that are relevant for evaluating skilled immigration policy changes — for instance, that the welfare impacts of immigration policy changes may depend on firms’ response to the policy change, which may be affected by the realized state of the economy or the presence of search frictions in the economy.

The extended model with search and matching frictions is related to recent literature
that studies the effects of immigration on the welfare of native individuals in a general equilibrium model featuring search frictions and wage bargaining (Chassamboulli and Palivos (2014), Battisti et al. (2014), Kingi (2015)). In all these, as long as immigrants have inferior outside options compared to natives, an increase in immigration raises firms’ incentives to create vacancies which benefits all workers, including native skilled workers. My results are consistent with these papers — unskilled native workers gain unambiguously. Skilled native workers, on the other hand, gain in terms of employment despite wage losses. However, by focusing on a more realistic immigration policy that is relevant to the U.S., the extended version of my model with search and matching frictions is able to capture an additional channel by which immigration impacts vacancy postings and hence employment of domestic households — an increase in the immigration cap increases the probability of being able to retain a foreign match, and therefore the overall surplus from posting a vacancy.

3 Baseline Model

The baseline model features a two-sector economy that is populated by skilled and unskilled households and households with the same skill level are identical. Heterogeneous monopolistically competitive firms (as in Melitz (2003)) in sector 1 (the skill-intensive sector) produce differentiated goods using domestic and foreign skilled labor[5] In the background, there is a foreign country that is assumed to have a large elastic supply of skilled workers that can be hired by domestic firms, subject to domestic firm demand and migration policy restrictions that mimic U.S. immigration policies - costs of hiring and a cap that occasionally binds, depending on the state of the economy[6] Therefore, the constraint that firms face for hiring skilled labor is an outcome of immigration policy, rather than the supply of foreign skilled labor[7]

Foreign and domestic skilled workers are substitutes in the baseline model and earn the same wage under competitive labor markets[8] This is consistent with the overall evidence on

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[5] Kerr et al. (2014) highlight heterogeneity in demand of foreign skilled workers across firms (Figure A.4).

[6] For the H1-B visa program, it does not matter whether the foreign-born worker is employed directly from the foreign country or from the domestic economy (for instance, after studying in the U.S.), as firms need to go through the same procedures in both cases. Although there is an additional quota for workers who obtain a master’s degree or higher from a U.S. institution, I ignore this distinction in the model.

[7] This assumption is realistic due to the significant wage differences between OECD and developing countries. If hired, there is a strong incentive for a foreign skilled worker to migrate. Empirically, Clemson (2012) estimates that there is a six-fold increase in salary for skilled workers who migrate to the US.

[8] The estimated degree of imperfect substitutability between native and immigrants within the same education and experience group is around 20 (Ottaviano and Peri (2012)). In the model with complementaries (Appendix C), this corresponds to $\gamma = 0.9477$, and therefore results are similar to the baseline model where $\gamma = 1$. 
relative wage earnings of domestic born as a percent of foreign-born workers with a bachelors degree or higher (Figure 3)\textsuperscript{9} Moreover, when filing a Labor Condition Application, firms attest that they will pay the worker the prevailing compensation for that occupation.

![Figure 3: Ratio of average quarterly domestic and foreign wages](image)

I ignore emigration from the domestic economy as I treat the domestic economy as OECD countries like the U.S. and the foreign economy as developing countries (China, India, and the Philippines) and there is a very small share of migration from OECD to developing countries (OECD, 2013)\textsuperscript{10} Here, I focus on the domestic economy and do not model the foreign country explicitly. For immigrants, there is an exogenous probability of return to the country of origin, to account for the fact that a bulk of foreign skilled workers are on a temporary work visa and a fraction returns every period. Moreover, the exogenous return to the country of origin helps ensure that even in the absence of shocks, there is some demand for foreign skilled labor in every period, as is evident in the data.

\textsuperscript{9}Average quarterly wages are computed using weekly wage data of skilled foreign and native-born workers in private non-farm employment from the CPS monthly survey data. The wage data is deflated using the CPI deflator to convert it into real units. Each data series is seasonally adjusted using the Census ARIMA method.

\textsuperscript{10}Theoretically, this can be justified in terms of technology differences between the domestic economy and the foreign economy that ensure that wages in the home economy will always be higher than wages at foreign, which would removes the incentive to migrate to the foreign economy.
Representative perfectly competitive firms in sector 2 (the unskilled sector) produce output using unskilled domestic labor. All contracts and prices are written in nominal terms, and prices and wages are flexible. Thus, the model solution will focus only on real variables.

3.1 Domestic Households

The economy consists of a continuum of two types of infinitely-lived domestic households that supply units of skilled and unskilled labor inelastically. The labor supply of the representative skilled household is normalized to 1, and that of the representative unskilled household is \( \bar{l}_u \).

Each skilled and unskilled representative household has the same preferences over a basket of goods produced at Home. The lifetime utility of skilled and unskilled households is given by:

\[
\max_{C_{j,t}} E_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left( \ln C_{j,\tau} \right) \quad \forall j \in \{s, u\}
\]

where the consumption basket of each household is given by

\[
C_{j,t} = \left( \frac{c_{1,t}}{\alpha} \right)^\alpha \left( \frac{c_{2,t}}{1 - \alpha} \right)^{1 - \alpha}
\]

\( c_{1,t} \) is the basket of sector 1 goods consumed, and \( c_{2,t} \) is the sector 2 good consumed by each household. The weight of sector 1 goods in consumption is \( \alpha \in (0, 1] \).

The consumption-based price index is \( P_t = (p_{1,t})^\alpha (p_{2,t})^{1-\alpha} \), where \( p_{1,t} \) and \( p_{2,t} \) are the price indices of sector 1 and sector 2 goods, respectively. The price indices in units of the consumption basket are \( \rho_{1,t} = p_{1,t}/P_t \) and \( \rho_{2,t} = p_{2,t}/P_t \). Therefore, the consumption-based price index can also be expressed as \( 1 = (p_{1,t})^\alpha (p_{2,t})^{1-\alpha} \) in units of the consumption basket.

The basket of sector 1 goods is given by

\[
c_{1,t} = \int_{\omega \in \Omega} \left( p_{1,t}(\omega)^{1-\theta} d\omega \right)^{\theta/\delta} \quad \text{where} \quad \theta > 1 \text{ is households’ symmetric elasticity of substitution across sector 1 goods.}
\]

Thus, the price index of sector 1 output is \( p_{1,t} = \int_{\omega \in \Omega} (p_{1,t}(\omega))^{1-\theta} d\omega \), where \( p_{1,t}(\omega) \) is the price of the good \( \omega \). The demand for each good in sector 1 by household type \( j \in \{s, u\} \) is given by

\[
\alpha \left( \frac{p_{1,t}(\omega)}{p_{1,t}} \right)^{-\theta} \rho_{1,t} C_{j,t}
\]

or

\[
\alpha \left( \frac{p_{1,t}(\omega)}{p_{1,t}} \right)^{-\theta} \frac{1}{p_{1,t}} C_{j,t}.
\]

The demand for the sector 2 good by household \( j \) is given by

\[
(1 - \alpha) \frac{\rho_{2,t}}{p_{2,t}} C_{j,t} = (1 - \alpha) \frac{1}{p_{2,t}} C_{j,t},
\]

where \( \rho_{2,t} \) is the price of sector 2 output in units of the consumption basket. The budget constraint for the domestic skilled household is \( w_{s,t} + d_t = C_{s,t} \) where \( d_t \) is the profit income of sector 1 firms, in units of the consumption basket. Domestic skilled households are the firm owners in the baseline model. However, I consider alternate scenarios of profit distribution for the welfare calculations. \( w_{s,t} \) is the real wage paid to skilled labor, which
will be determined in the competitive labor market for skilled workers. Unskilled households consume their labor income \( C_{u,t} = w_u t u_{u,t} \), where \( w_{u,t} \) is the real wage paid to unskilled labor, and is also determined competitively in a separate labor market for unskilled labor.

3.2 Production

3.2.1 Skill-Intensive Sector (Sector 1)

There are a continuum of heterogeneous monopolistically competitive firms, each producing a differentiated variety \( \omega \in \Omega \). There is no endogenous firm entry or exit. The constant mass of firms is normalized to 1. Production requires skilled (domestic or foreign) labor. Aggregate labor productivity is \( Z_t \) which is exogenous and follows an AR[1] process in logs. Firms are heterogeneous as they produce with different technologies indexed by relative productivity \( z \). Firm specific productivity \( z \) follows a Pareto distribution \( G(z) \), with shape parameter \( k \), and support on \((z_{min}, \infty)\). Output supplied by firm \( z \) in sector 1 is \( y_{1,t}(z) = Z_t z l_{s,t}(z) \), where the total mass of skilled labor employed is

\[
l_{s,t}(z) = l_{d,t}(z) + a l_{f,t}(z)
\]

where \( d \) and \( f \) denote domestic and foreign skilled labor respectively. The coefficient \( a \) denotes the productivity of a foreign skilled worker relative to a domestic skilled worker. Given a competitive labor market, additional costs of hiring foreign workers (described below), and flexible real skilled wages, firms would have an incentive to hire foreign workers only if there is additional benefit of hiring foreign workers. Either, foreign workers are paid lower wages or they generate an additional productivity increase for firms. The data on wage earnings of foreign vs domestic born does not indicate that foreign-born workers are paid lower wages on average. The relative productively of foreign-born workers, \( a \), creates a wedge between wages paid to foreign born (determined in the competitive labor market for all skilled workers), and the marginal revenue product from an additional foreign worker hired. I do not impose that \( a > 1 \). However, this turns out to be an outcome of the calibration described in Section 5.

Skilled Immigration Policy

Domestic firms face certain immigration policy restrictions when hiring foreign workers: Firms have to pay hiring costs, and there is a government-imposed cap on the number of foreign workers that can be hired each period. The sunk hiring costs can be decomposed into two components — cost due to immigration policy, and technological costs of hiring foreign
workers. Firms have to incur a sunk cost, $f_{R,t}$, for all foreign workers they apply for, which reflects the regulatory component of the immigration policy cost — legal fees and other administrative costs involved in the various processes for hiring foreign skilled workers.\[1\]

To facilitate comparison with the Social Planner’s allocation, an additional cost that firms face is the technologically imposed cost of hiring skilled foreign workers, $f_{T,t}$, which is the same cost that a social planner would face for hiring a foreign worker. One way to interpret this cost would be to think of this as the cost incurred by firms on airfare or relocation of foreign workers. All costs are in units of the consumption basket.

If firm $z$ optimally chooses to submit applications for $N_{e,t}(z)$ workers, then the total cost that the firm will incur is $(f_{R,t} + f_{T,t})N_{e,t}(z)$, where $q_t$ turns out to be the endogenous probability or fraction of workers that firms are allocated if the immigration cap binds, and is described below. Higher immigration policy costs imply a more restrictive immigration policy.

The entry cap for foreign skilled workers is exogenously set at $\bar{N}_{e,t}$. Suppose firm $z$ files $N_{e,t}(z)$ applications for foreign skilled workers. Then, the probability of each application being selected is given by:

\[
q_t = \min \left( \frac{\bar{N}_{e,t}}{\left( \int_{z_{\min}}^{\infty} N_{e,t}(z) dG(z) \right)}, 1 \right)
\]

where $q_t < 1$ if the aggregate demand of foreign skilled workers, $\int_{z_{\min}}^{\infty} N_{e,t}(z) dG(z)$, exceeds the cap, and the cap endogenously binds. Each firm knows that if it submits $N_{e,t}(z)$ applications, it will get $q_t N_{e,t}(z)$ workers. Each firm is of measure 0 and takes $q_t$ as given in its hiring decision.\[12\]

The timing is as follows. A fraction $\delta$ of the foreign skilled workers currently employed by domestic firms (including newly hired workers from the previous period) are separated from firms at the beginning of each period $t$. The state $Z_t$ of the economy is realized, wages are determined, and firms produce period-$t$ output. Firms then maximize expected discounted profits and optimally choose the number of foreign skilled workers to hire (or submit appli-

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\[1\] Firms in multiple surveys (for instance, by the Government Accountability Office (GAO)), document a range of direct and indirect costs associated with the H-1B program, including legal and administrative costs. Firms note that apart from the filing fees paid to the Department of Homeland Security, the main cost incurred is due to the opportunity cost of the time and effort spent in the process, which is captured by the regulatory component of the sunk cost, $f_{R,t}$, in the model. The various filing fees that firms have to pay when submitting the H1-B petition for each worker itself on average amount to $3,000.

\[12\] While no actual risk in the realization of the share of foreign workers allocated to firms may seem restrictive, it is consistent with how U.S. firms in the economy behave. To mitigate risk, U.S. firms subcontract a large part of the H1-B hiring process to large IT management firms. These firms substantially reduce the risk of procuring H-1B visas by applying in bulk. These outsourcing firms were awarded almost 20 percent of total H-1Bs in 2016, and the workers were then allocated to U.S. employers through subcontracting.
cations for), after taking into account the immigration policy restrictions. The realized state of the economy and the corresponding firm demand for foreign workers determine whether the cap binds, and an endogenously-determined fraction $q_t$ of the applications are approved. These are the workers that firms are able to ‘bring’ to the firm for production. There is a time to build lag and those workers that survive the separation shock are added to the stock of next period’s skilled labor stock.

Thus, the stock of foreign skilled labor at firm $z$ in period $t + 1$ is given by:

$$ l_{f,t+1}(z) = (1 - \delta)(l_{f,t}(z) + q_t N_{e,t}(z)) \quad (1) $$

Expressed in units of the consumption basket, the inter-temporal profit function of firm $z$ is given by:

$$ E_t \sum_{\tau=t}^{\infty} \beta_{\tau,t} \left[ \rho_{1,\tau}(z) Z_{\tau} z l_{s,\tau}(z) - w_{s,\tau} l_{s,\tau}(z) - (f_{R,\tau} + f_{T,\tau}) N_{e,\tau}(z) \right] $$

(2)

The inter-temporal discount factor that the firm applies to its profits is $\beta_{\tau,t} \equiv \beta(u'(C_{s,\tau})/u'(C_{s,t}))$, which is the inter-temporal discount factor of domestic skilled households, who are assumed to be domestic firm owners.

**Optimal Hiring of Skilled Foreign Workers**

Firms maximize the inter-temporal profit function (2) subject to the law of motion of foreign skilled workers, the production technology, and subject to the demand curve faced by them i.e. $y_{1,t}(z) = \alpha(\rho_{1,t}(z))^{-\theta} Y^c_t / \tilde{\rho}_{1,t}$. Profit maximization implies that the price $\rho_{1,t}(z)$ set by the firm is a proportional markup over the marginal cost: $\rho_{1,t}(z) = \frac{\theta}{\theta - 1} \psi_{1,t}(z)$. The marginal cost $\psi_{1,t}(z)$ is the Lagrange multiplier on the production technology in the optimization problem.

The average sector 1 price, $\tilde{\rho}_{1,t}$, and aggregate demand, $Y^c_t$, are described in section 3.2.3.

Firms take the wages paid to skilled workers as given and this wage $w_{s,t}$ is determined in the competitive labor market for skilled workers.

Each period, firms submit applications for skilled foreign workers such that the expected discounted profit generated from an additional skilled foreign worker, $v_t$, is equal to the expected sunk hiring cost:

$$ v_t = (f_{R,t} + f_{T,t})/q_t $$

where

$$ v_t = \sum_{\tau=t+1}^{\infty} E_t \{ \beta_{\tau,t}(1 - \delta)^{\tau-t}[a \psi_{1,t}(z) Z_{t+1} z - w_{s,t}] \} $$

(3)
$u_t$ can be expressed as:

$$\frac{f_{R,t} + f_{T,t}}{q_t} = (1 - \delta) E_t \left\{ \beta(C_{s,t+1}/C_{s,t})^{-1} \left[ \alpha \psi_{1,t+1}(z)Z_{t+1}z - w_{s,t+1} + \frac{f_{R,t+1} + f_{T,t+1}}{q_{t+1}} \right] \right\}$$  \hspace{1cm} (4)

The expected cost of hiring immigrant workers is $(f_{R,t} + f_{T,t})/q_t$. Since firms have to pay $f_{R,t} + f_{T,t}$ for each immigrant worker that they apply for, and $q_t$ is the probability of the application being selected, a binding cap implies a higher cost of hiring foreign skilled workers. The right hand side of (4) gives the expected benefit of hiring immigrant workers — the marginal revenue product from an additional worker net of wage paid, plus the future cost saving of hiring workers today. Hiring immigrant labor is like an investment decision for firms and the stock of immigrant workers is governed by an Euler equation, the forward iteration of which gives (3).

Firms serve only the domestic market. Market clearing for each firm $z$ implies that the total supply equals to the total demand for the product i.e.

$$Z_t z l_{s,t}(z) = \alpha \left( \frac{\rho_{1,t}(z)}{\tilde{\rho}_{1,t}} \right)^{-\theta} Y_t^c / \tilde{\rho}_{1,t}$$

As is standard in the Melitz (2003) model, more productive firms face a higher demand for their output due to lower prices, and hence employ more skilled labor, including skilled foreign labor. We can see this by expressing the aggregate stock of foreign labor as a function of firm specific productivity i.e. $l_{s,t}(z) = f(z^{\theta-1})$.

Firm profits in period $t$ are given by $d_t(z) = \rho_{1,t}(z) y_{1,t}(z) - w_{s,t} l_{s,t}(z) - (f_{R,t} + f_{T,t}) N_{e,t}(z)$.

### 3.2.2 Unskilled Sector (Sector 2)

Sector 2 output is produced by competitive firms that have an identical technology:

$$Y_{2,t} = Z_t l_{u,t}$$

where $l_{u,t}$ is the unskilled labor employed by the representative firm. The marginal cost of production for the firm is $w_{u,t}/Z_t$. Therefore, the price of the representative sector 2 good in units of the consumption basket is given by $\rho_{2,t} = w_{u,t}/Z_t$.

### 3.2.3 Aggregate Accounting and Equilibrium

The distribution of firm productivities is given by a Pareto distribution $G(z) = 1 - \left( \frac{z_{\min}}{z} \right)^k$, with lower bound $z_{\min}$ and shape parameter $k > \theta - 1$. As in Melitz (2003), aggregate productivity is defined as $\tilde{z} = [\int_{z_{\min}}^{\infty} z^{\theta-1} dG(z)]^{\frac{1}{\theta-1}}$. Since, $l_{s,t}(z) = f(z^{\theta-1})$, we can aggregate
skilled labor as $\tilde{l}_{s,t} = l_{s,t}(\tilde{z}_t) = 1 + \tilde{l}_{f,t}$. The aggregate sector 1 output is $Y_{1,t} = Z_t \tilde{z}(1 + a \tilde{l}_{f,t})$. The aggregate sector 1 price index is given by $	ilde{\rho}_{1,t} = \int_{z_{\text{min}}}^{\infty} \rho_{1,t}(z)^{1-\theta} dG(z))^{\frac{1}{1-\theta}} = \rho_{1,t}(\tilde{z}_t)$. The corresponding aggregate marginal cost of production is $\tilde{\psi}_{1,t}$.

Aggregate consumption by households in the economy is given by $C_{s,t} + C_{u,t} + C_{f,t}$ i.e. the sum of consumption by domestic skilled, unskilled, and foreign workers residing in the domestic economy. Immigrants consume their labor income, $C_{f,t} = w_{s,t} \tilde{l}_{f,t}$. Domestic labor market clearing requires that the aggregate domestic labor employed is equal to the inelastic supply i.e. $\int_{z_{\text{min}}}^{\infty} l_{d,t}(z) dG(z) = 1$ and $l_{u,t} = \bar{l}_{u}$.

Aggregate accounting in the economy requires that the aggregate supply in the economy is equal to the aggregate demand in the economy.

$\tilde{\rho}_{1,t} Z_t \tilde{z}_{s,t} + \rho_{2,t} Z_t \bar{l}_u = C_{s,t} + C_{u,t} + C_{f,t} + (f_{R,t} + f_{T,t}) \tilde{N}_{e,t}$

The aggregate demand in the economy is expressed as $Y^c_t = C_{s,t} + C_{u,t} + C_{f,t} + (f_{R,t} + f_{T,t}) \tilde{N}_{e,t}$

Table 1 summarizes the key equilibrium conditions in the model. There are 15 equations in 15 endogenous variables of interest: $Y_{1,t}, Y_{2,t}, \tilde{l}_{f,t}, \tilde{N}_{e,t}, q_t, w_{u,t}, w_{s,t}, \tilde{d}_t, \tilde{\rho}_{1,t}, \rho_{2,t}, \tilde{\psi}_{1,t}, C_{u,t}, C_{s,t}, Y^c_t, C_{f,t}, Z_t$ follows an exogenous AR[1] process in logs. The immigration policy costs and cap are exogenous and calibrated in Section 5.

4 Steady State and Intuition

I next turn to the consequences of skilled immigration and skilled immigration policy changes by studying how skilled immigration responds to a temporary productivity shock, as well as the transitional dynamics and the long-run effects of a permanent increase in the immigration cap. Before presenting these results, I discuss implications of some key steady-state relationships that highlight some of the main model mechanisms. The baseline model is tractable enough to generate analytical solutions. The analytical solution for the steady state of the model is given in Appendix D.

Steady-State Stock of Foreign Born

Since the model features an occasionally binding constraint, the model is equivalent to one with two regimes. The constraint is binding under one regime and slack under the other and each regime has a separate non-stochastic steady state. In the appendix, I derive the steady-state stock of foreign-born labor in the regime when the cap does not bind ($q = 1$)

\footnote{As in Cacciatore (2012), aggregate demand ($Y^c_t$) includes a component other than household consumption. However, it is in the same units as the consumption basket.}

\footnote{Guerrieri and Iacoviello (2015)}
as follows:

\[
\tilde{l}_f = \frac{1}{a} \left\{ \alpha \tilde{z} \tilde{l}_f \left[ (1 - \delta) \beta Z (a - 1)(\theta - 1) \right] \right\}^{\frac{1}{1 - \alpha}} - 1
\]  

Equation (5) helps identify the factors that influence a larger firm demand for foreign skilled labor. The model predicts that firm demand for foreign skilled labor is increasing in:

- The aggregate labor productivity, \( Z \)
- The aggregate firm specific productivity, \( \tilde{z} \)
- The weight of skill-intensive sector output, \( \alpha \), in the consumption basket
- The relative productivity of foreign skilled workers, \( a \)
- The stock of domestic unskilled workers, \( \bar{l}_u \)
- Lower hiring costs of foreign skilled workers, \( f_R \) and \( f_T \)
- A lower probability of return, \( \delta \), to the country of origin
- A lower elasticity of substitution across goods, \( \theta \), (which increases firm profits from each unit produced)

However, the actual level of foreign skilled workers depends on the level of the cap and whether the cap binds or not. Let (5) be \( \tilde{l}_m^f \) which denotes the profit maximizing level of steady state foreign skilled workers. Then, depending on whether the cap binds or not, the actual level of foreign skilled workers (\( \tilde{l}_f^{actual} \)) is given by

\[
\tilde{l}_f^{actual} = \min \left[ \tilde{l}_m^f, \frac{(1 - \delta) \bar{N}_e}{\delta} \right]
\]

The demand for foreign workers in turn determines the probability \( q \). When the cap binds, the probability is given by

\[
q^{bind} = \frac{(f_R + f_T)(1 - (1 - \delta) \beta) \theta}{(1 - \delta) \beta (a - 1)(\theta - 1) Z \tilde{z}} \left( \frac{(1 - \alpha) \tilde{z} (1 + a \tilde{l}_f)}{\alpha \bar{l}_u} \right)^{1 - \alpha}
\]

Any of the factors that increases the demand for foreign skilled workers, lowers the probability of hiring foreign workers. Depending on whether the cap binds or not, the probability of hiring foreign workers is given by \( q = \min[q^{bind}, 1] \).
5 Calibration

In order to study the dynamics numerically, I calibrate the parameters of the model under the assumption that the steady state in the domestic economy mimics the U.S. economy during the 2004-2017 period. The immigration policy cap was binding during this period. I interpret each period as a quarter. I calibrate the parameters that pertain to immigration to match data from the Current Population Survey (CPS) and data from the United States Citizenship and Immigration Service (USCIS), between 2004 to 2017.\footnote{Plots of some data series used in calibration is plotted in the appendix.}

I rely on existing literature for some parameters. The discount rate $\beta$ is set to 0.99, as is standard in the literature. I set the elasticity of substitution across product varieties to $\theta = 3.8$ (Bernard, Eaton, Jensen, and Kortum (2003)). Following Ghironi and Melitz (2004), I set the dispersion of firm productivity draws, $k = 3.4$, and normalize $z_{min}$ to 1. The share of sector 1 goods in consumption is set at $\alpha = 0.4$ to match the average share of unskilled income in total aggregate income over the period 2004 to 2017.\footnote{This follows from sector two’s good market clearing: $Z\tilde{l}_u = (1 - \alpha)Y^c/\rho_2$, which implies that $w_u\tilde{l}_u/Y^c = (1 - \alpha)$.}

The domestic skilled labor force $l_d$ is normalized to 1 and given this normalization, the domestic unskilled labor supply is calibrated to $\tilde{l}_u = 1.92$ to match the average share of domestic workers over this time period with less than a bachelor’s degree of 52 percent. The immigration cap $\tilde{N}_e$ is set to 0.0002 in order to match the average cap (converted to a quarterly value) imposed by skilled immigration policy over the 2004 to 2017 period as a proportion of the normalized average domestic skilled labor in the economy. The separation rates $\delta_d$ of foreign workers is set to 0.07 to match the average quarterly separation rate of foreign workers, computed using monthly CPS data as in Shimer (2005).\footnote{In reality, H1-B visas are allocated for a period of three years, and can be extended for another three years. However a sizeable proportion of H1-B workers stay for longer as firms sponsor their green card application. There is no concrete estimate of this proportion. Moreover some workers may end up leaving before the visa expires. Therefore, I compute average separation rates.} I set the technological part of the hiring cost at 0.0833 to target one month’s real skilled wage in the data.\footnote{While there is no direct estimate of this cost in the data, a month’s wage is a reasonable estimate of the relocation and other expenses that are meant to capture this cost.}

The sunk regulatory cost $f_R$ and productivity parameter $a$ are jointly calibrated to target a skill premium of 1.85 (computed from the average wage data of skilled workers and unskilled workers in the U.S.) and the average number of petitions for foreign skilled workers filed in the U.S that generates a steady-state application selection probability of about 0.4 (USCIS).

Average quarterly wages used in the calibration are computed using weekly wage data of skilled foreign and native-born workers in private non-farm employment from the CPS.
wage data is deflated using the CPI deflator to convert it in real units, and is seasonally adjusted using the X-12 ARIMA method. The parameters are calibrated to minimize the squared residuals between model moments and targets in Table 3. The resulting parameter values are highlighted in Table 4.

6 Transition Dynamics and Welfare Results in the Baseline Model

In this section, I solve the calibrated model numerically to study dynamics in response to a temporary positive productivity shock and changes in immigration policy. I then calculate the welfare effects of a perfect foresight increase in the immigration policy cap. These are the benchmark welfare results. In section 9, I analyze how these welfare impacts vary in the presence of search frictions. All the welfare experiments in the paper are summarized in Table 9.

6.1 Dynamic Response to a Temporary Productivity Increase

I study the responses (percent deviation from the steady state) to a temporary 1 percent increase in aggregate productivity ($Z_t$) in the baseline model. In order to analyze the impact of the cap on the economy’s dynamic responses to a productivity shock, I compare two cases. In case 1 (the benchmark case), there is a policy-imposed entry cap on foreign skilled workers in the economy. In case 2, there is no entry cap.

Figure 4 shows that in the presence of an entry cap, the increase in firm demand for foreign skilled workers in response to the productivity increase is less than half, when compared to case without the cap. This is because firms have to pay costs $f_{R,t}$ and $f_{T,t}$ for hiring workers that may not eventually be able to join the firm due to the binding cap, and thus would not contribute to firms’ output and profit. Therefore, costs associated with the current immigration policy may distort firms’ incentives for hiring foreign skilled workers. I analyze these distortions in greater detail in section 7.1. An implication of this is that the costs incurred due to the burdensome current immigration policy may lead to an inaccurate signal of firm demand for foreign workers.

Figure 4 also shows that the stock of skilled labor is inelastic in the short run and rises only slowly due to the time-to-build lag. In the presence of the cap, the stock of foreign skilled labor rises by much less. As a result, the increase in output, profits, and real wages of unskilled workers (and thus their consumption) is smaller in the presence of the cap. Without the entry cap, firm profits initially fall more as more resources are spent on hiring. However,
this quickly recovers as the stock of foreign skilled workers increases over time and firms are able to produce more output. Unskilled wages are higher without the cap as a larger stock of foreign workers increases demand for goods produced in sector 2, which increases demand for unskilled labor and puts an upward pressure on their wages.\footnote{19} However, the real wage of skilled labor falls by less in the presence of the entry cap due to the smaller increase in the stock of skilled labor in the domestic economy. Overall, the presence of the constraint dampens the economy’s response to aggregate shocks.

**Response to a Cap Change Under Different States**

An important insight from explicitly taking into account the role of firm demand is that we can study how firms’ response to an immigration cap change depends on the realized state $Z_t$ of the economy. To illustrate how the state of the economy matters, Figure 5 plots how firm demand for foreign labor and how the probability of hiring foreign workers, responds after a perfect foresight increase in the cap, under two different states of the economy. In the first state, there is a 10% cap increase in period 1 but there is no change in aggregate productivity. In the second state, there is a 10% cap increase in period one but there is also a temporary negative productivity shock in period one. In the second case, the demand for foreign workers does not respond to the cap change immediately and it actually falls because labor is less productive. It is only when the effect of the negative productivity shock wanes out, that we see the demand (petitions filed) for foreign labor rising in response to the cap increase. In contrast when there is no negative productivity shock, the demand for foreign skilled labor increases immediately. These effects are also reflected in the probability of hiring foreign skilled labor. A lower demand for foreign labor combined with a higher cap leads to a huge jump in $q$ in case 2 compared to case 1.

This is similar to the episode in 2001. In 2001, the cap was raised from 115,000 to 195,000, a 69.5 percent increase. At that point, the economy entered a recessionary phase, and firms did not increase their hiring of foreign workers by 69.5 percent. In fact, the unused cap in 2001, 2002, and 2003, was 31,400, 115,900, and 117,000, respectively\footnote{20} The average additional cap used over the entire period was about 20 percent. The model can be used to explain this evidence. The key point is that if the increase in foreign workers hired is less than the increase in the policy-imposed cap, then evaluating the impact of the policy change as a labor supply increase would be misleading.

\footnote{19}{This is consistent with empirical evidence in Hong and McLaren (2015).}  
\footnote{20}{Using USCIS historical data.}
6.2 Welfare Analysis

The above dynamic responses indicate that an increase in skilled foreign labor has different impacts on heterogeneous workers. In order to draw inferences about the impact of current migration policy changes, it is important to quantify the welfare changes across different sets of workers. I calculate welfare impacts after a 10 percent perfect foresight increase in the entry cap. The long-run welfare gain of each type of native worker from the immigration policy easing is computed as the percentage increase ($\Delta$) in consumption that would leave the households indifferent between the initial policy and the new policy with the higher cap, when the new policy is implemented at time $t = 0$. Transitional dynamics have been included in the welfare computations. Thus, $\Delta$ solves:

$$u\left[C_j(1 + \frac{\Delta}{100})\right] = (1 - \beta) \sum_{t=0}^{\infty} \beta^t u(C_{jt}) \quad \forall\{j \in s, u\}$$

Suppose the cap increases by 10 percent. Since the baseline model is calibrated to a period when the cap is 'very' binding in the sense that the gap between firm demand for foreign labor and the cap is very large, firms increase hiring by the full 10 percent. The dynamics following this cap change are given in Figure 6. Firms increase hiring of foreign skilled labor following the cap increase and as the stock of skilled workers builds up, sector 1 output, unskilled wages, and consumption rise. As before, the increase in unskilled wages and therefore consumption is due to the higher demand for sector 2 output by the larger stock of skilled foreign labor. Average firm profits in sector 1 fall initially as they have to bear costs of hiring more foreign workers (the immigration policy cost of hiring foreign skilled workers remains unchanged). However, profits recover over time as output increases. Skilled wages fall due to the larger inflow of foreign workers. The net effect on real consumption of domestic skilled workers is negative, despite their profit income increasing. The probability of hiring a foreign worker is higher in the new steady state due to the higher cap.

Table 5 shows that the welfare gain (including transitional dynamics) amount to 0.0102 percent of quarterly steady-state consumption for unskilled workers. For skilled domestic workers, the welfare loss amounts to 0.0086 percent of quarterly steady-state consumption. Thus, there are different effects of a skilled immigration policy change on heterogeneous workers — workers most complimentary to skilled immigrants gain, while those most substitutable lose. Part of the negative impact on consumption of skilled workers is mitigated as firm profits increase. The transitional dynamics in Figure 6 show that most of the welfare changes are realized slowly over the longer horizon.

An important consideration is that the welfare impacts will depend on the profit distri-
bution across households. Therefore, in order to carry out accurate welfare analysis of immigration policy changes, it is important to empirically estimate how profits are distributed across households. To analyze the role of profit distribution in the model, I introduce a new type of household, the entrepreneurs. The entrepreneurs own the firms and thus their budget constraint is $C_{e,t} = \tilde{d}_t$. Therefore, the new stochastic discount factor for the firms is $\beta_{k,t} \equiv \beta(u'(C_{e,k})/u'(C_{e,t}))$. In this scenario, domestic skilled households consume only their wage income. Table 5 compares the welfare results in this scenario with the benchmark case. The welfare loss for domestic skilled workers is much larger (0.0153 percent of quarterly steady-state consumption). The welfare gain to domestic entrepreneurs from a 10% cap increase amounts to 0.0102 percent of quarterly steady-state consumption.

7 Social Planner’s Solution

The discussion in Section 6.1 indicates that the presence of an immigration cap distorts firm demand for foreign workers. To analyze these distortions further, I solve for the social planner’s solution in the baseline setup. The solution to the social planner’s problem gives us the optimal skilled foreign entry into the domestic economy. Understanding some of the potential distortions also gives us a framework for evaluating the impact of alternate policies. Therefore, in this section, I discuss the first-best, efficient allocation chosen by a social planner. In order to identify distortions in the model economy, I compare the equilibrium conditions in the baseline decentralized economy (Table 1) to those implied by the planner’s solution (Table 6).

Consider the problem of a Social Planner that maximizes welfare of domestic households and chooses the optimal entry of foreign skilled workers in the domestic labor force, taking the firm size distribution, preferences, technology, and resources available in the economy as given. Let $f_T$ be the technologically imposed cost of hiring skilled immigrants in the economy. The planner’s problem is given by:

$$
\max_{\{C_{u,t}, C_{s,t}, N_{e,t}(z), l_{f,t+1}(z), y_{1,t}(z), y_{2,t}(z)\}} E_t \left[ \mu \sum_{t=0}^{\infty} \beta^t \ln(C_{u,t}) + (1 - \mu) \sum_{t=0}^{\infty} \beta^t \ln(C_{s,t}) \right]
$$
\[ C_{u,t} + C_{s,t} + f_T \int_{z_{\text{min}}}^{\infty} N_{e,t}(z) dG(z) = \left( \int_{z_{\text{min}}}^{\infty} y_{1,t}(z) \frac{\theta - 1}{\alpha} dG(z) \right) \alpha \left( \frac{y_{2,t}}{1 - \alpha} \right)^{1 - \alpha} \]

\[ \int_{z_{\text{min}}}^{\infty} y_{1,t}(z) dG(z) = \int_{z_{\text{min}}}^{\infty} Z_t(z(l_{d,t}(z) + a l_{f,t}(z))) dG(z) \]

\[ Y_{2,t} = Z_t \bar{l}_u \]

\[ \int_{z_{\text{min}}}^{\infty} l_{f,t+1}(z) dG(z) = (1 - \delta) \int_{z_{\text{min}}}^{\infty} (l_{f,t}(z) dG(z) + \int_{z_{\text{min}}}^{\infty} N_{e,t}(z) dG(z)) \]

\[ \int_{z_{\text{min}}}^{\infty} l_{d,t}(z) dG(z) = 1 \]

where \( \mu \in [0, 1] \) is the planner’s weight on domestic unskilled households’ welfare. Appendix E describes the constraints in the planners problem and gives the solution to the social planner’s problem. Table 6 summarizes the equilibrium conditions in the planner’s environment.

7.1 Distortions in the Baseline Model’s Decentralized Economy

I compare the equilibrium conditions in the decentralized economy to under the planned economy. Consider the entry conditions in the planned vs decentralized economy:

Entry condition under the planned economy:

\[ f_T = E_t \left[ \beta_{t,t+1}(1 - \delta) \left( \frac{\nu_{t+1}}{\chi_{t+1}} a Z_{t+1} \bar{z} + f_T \right) \right] \]

Entry condition under the decentralized economy:

\[ \frac{f_{R,t} + f_T}{q_t} = E_t \left[ \beta_{t,t+1}(1 - \delta) \left( a \bar{\psi}_{1,t+1} Z_{t+1} \bar{z} - w_{s,t+1} + \frac{f_{R,t+1} + f_T}{q_{t+1}} \right) \right] \]

The major distortions in the decentralized economy due to immigration policy are listed below. Note that in this model, there is no distortion in the relative allocation of workers across firms. This is because a social planner would choose an allocation across firms such that the marginal rate of substitution is equal to the marginal rate of transformation i.e. \( z_1/z_2 = (\frac{y_1(z_1)}{y_1(z_2)})^{1/\theta} \). This is preserved under the decentralized equilibrium due to market clearing in each firm \( z \): \( y_1(z) = \alpha \bar{\psi}_{1,t+1} Z_{t+1} \bar{z} - w_{s,t+1} + \frac{f_{R,t+1} + f_T}{q_{t+1}} \). Therefore, as long as market clearing

\[ \text{there is also an additional distortion due to monopoly power. Monopoly power distorts the job creation decision by inducing a lower return from hiring foreign skilled workers. This would be captured by } Y_\theta = \frac{y_1(z_1)}{y_1(z_2)} \]
holds under the decentralized economy, the relative allocation across firms is efficient\(^{22}\).

- **Distortion because of the cap:** When the cap binds, the probability that a firm will be able to hire a worker is \(q_t\), which is less than 1. Hence the distortion (relative to the socially planned economy) because of the cap is \(\Upsilon_{q,t} = 1 - q_t\).

- **Distortion because of the immigration policy cost:** Let \(\Upsilon_{x,t} = f_{R,t}\). This is the additional cost that a firm faces in the decentralized economy due to immigration policy when the cap is non-binding. If the cap doesn’t bind (\(q_t = 1\)), then \(\Upsilon_{x,t}\) would be the only distortion w.r.t. immigration policy in this framework. However, when the cap binds and \(q_t < 1\) in the decentralized economy, firms face additional costs. To hire one worker, they need to submit \(1/q_t\) applications and hence incur \((f_R + f_T)/q_t\) as costs. Define \(\Upsilon_{R,t} = (f_R + f_T)/q_t - f_T\) as the additional costs that firms have to pay in the decentralized economy, relative to the socially planned economy. Then, we can get

\[
\Upsilon_{R,t} = \frac{\Upsilon_{x,t} + f_T \Upsilon_{q,t}}{1 - \Upsilon_{q,t}}
\]

Thus, if \(\Upsilon_{q,t} = 0\) (no distortion because of the cap), then \(\Upsilon_{R,t} = \Upsilon_{x,t}\). However, a binding cap amplifies the distortion due to the immigration policy cost and \(\Upsilon_{R,t} > \Upsilon_{x,t}\) is the relevant distortion due to skilled immigration policy costs.

These distortions lead to inefficiency in the market economy’s equilibrium allocation relative to the social planner’s equilibrium allocation. This is because, the additional costs and cap in the decentralized economy distort firm hiring. There is also inefficiency w.r.t diversion of resources from consumption due to sunk immigration costs.

### 7.2 Market-Driven Allocation of Permits

Focusing on the role of firms allows us to evaluate the impact of moving towards alternate immigration policies that go beyond analyzing the impact of an immigration cap change alone. I begin to analyze one such policy and study the impact of moving toward a market-driven allocation of visas (via auction of a fixed number of permits) for skilled foreign workers. In particular, I analyze whether such a policy would increase welfare of domestic workers. If it does, then the alternate policy would bring the economy’s equilibrium closer to the social planner’s first-best allocation\(^{23}\).

\(^{22}\)Market clearing across firms facilitates aggregation as \(\xi = \sum_{i} \theta_i z_i = (\bar{z})^\theta\) is the condition that allows us to interpret \(\bar{z}\) as the weighted average of productivity of firms, where weights reflect relative output share of firms.

\(^{23}\)In the first-best allocation, the welfare of both types of domestic workers is higher.
The motivation for the market-driven allocation of permits stems from the advantages of this alternate skilled immigration policy highlighted in Peri (2012)\textsuperscript{24} The main idea is that introducing a market-driven system of allocating permits to firms who hire immigrants would introduce a price mechanism to allocate visas and would quantify the value attributed by the U.S. market to a foreign skilled worker. According to the proposal, the market-driven price of permits would also provide potential flexibility across the business cycle via price feedback and these prices could be a potential signal for raising/lowering total number of permits. For instance, if the price of the permit rises during expansionary times, it would signal a true shortage in the number of permits relative to firm demand for foreign skilled labor. This may be relevant for policy makers as the costs under the current immigration policy setup may distort firm demand of foreign skilled labor, and thus do not always give a true indication of firm demand. Moreover, such a mechanism may generate additional revenue for the government, which could help compensate domestic households. The key idea in Peri’s proposal is that “a simpler, more flexible, and more market-driven system of labor-sponsored permits for immigrants would enhance the economic benefits of employment-based visas”.

**Market-Clearing Price of Permits**

The preferences, technology, and the economic setup are exactly the same as in the baseline model (Section 3). The main difference is that the immigration policy-imposed cost of hiring skilled foreign workers will vary with economic conditions via the optimally varying price of permits. Also, there is no lottery and hence firms get allocated their optimal demand of permits at the market clearing price of permits.

I evaluate the impact of moving from the baseline skilled immigration policy toward an alternate market-driven policy, in a simple framework with no informational asymmetries, and one in which firms bid for permits according to their demand schedule. I set the number of permits to be allocated to be the same as the cap imposed under the baseline policy\textsuperscript{25} I evaluate the implications of this policy under the assumption that the demand for foreign workers is high enough, so that the immigration cap would otherwise be binding. I do not consider the implications of this alternate policy when the cap is non binding.

The timing is as follows: the state of the economy is realized and a fraction $\delta$ of foreign skilled workers separate from the domestic labor market. Wages are determined competitively, and firms produce period-$t$ output. The Government announces the period $t$ auction

\textsuperscript{24} Peri (2012) focuses on reform of all immigration policies rather than only on skilled immigration policy.
\textsuperscript{25} According to Peri (2012), permits for the H-1B category (as well as the L-1A, L-1B (intra-company transfers) and TN visas (professionals from NAFTA), all included as one category) should be sold in a quarterly electronic auction and the number of permits could initially be set equal to the number of annual average temporary visas issued over the past ten years.
of a fixed mass of permits — each permit would allow firms in the skilled sector to hire a foreign skilled worker that will become productive in period, $t+1$. Each firm decides its price-quantity schedule for permits in the manner described below. Firms then hire labor subject to the constraint that the foreign labor hired can’t exceed the number of permits they hold. This labor becomes productive in the next period $t+1$. A brief time line is given below:

<table>
<thead>
<tr>
<th>Period</th>
<th>δ shock</th>
<th>Period-$t$ output</th>
<th>Labor hiring</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t$</td>
<td>$Z_t$ realized</td>
<td>Permit auction</td>
<td>$t+1$</td>
</tr>
</tbody>
</table>

To determine its bidding strategy, each firm must anticipate how much foreign skilled labor it would want to hire following the auction. Firms first solve their optimization problem under the constraint that when they make the hiring decision, they would be constraint by the number of permits they hold. Then, the value placed on a permit in the auction by the firm is determined endogenously by the marginal value associated with the permit in the hiring stage.

The optimization problem of firm $z$ for hiring new foreign skilled labor ($N_e(z)$) subject to the constraint on the permits it holds is given below. Suppose that the outcome of the auction in period $t$ was that firm $z$ bought $N^p_e(z)$ permits at a market clearing price for permits $\zeta^p$. Note that the cost $N^p_e(z)\zeta^p$ has already been incurred before making the hiring decision, and therefore it does not affect the profit maximization problem for hiring foreign skilled labor, which is given by:

$$\max_{N_e,t(z),l_{f,t+1}(z)} E_t \sum_{\tau=t}^{\infty} \beta_{\tau,t} \left[ \rho_{\tau}(z)y_{\tau}(z) - w_{s,\tau}(l_{d,\tau}(z) + l_{f,\tau}(z)) - f_T N_{e,\tau}(z) \right]$$

subject to

1. $y_{1,t}(z) = Z_t z(l_{d,t}(z) + a l_{f,t}(z))$
2. $l_{f,t+1}(z) = (1 - \delta)(l_{f,t}(z) + N_{e,t}(z))$
3. $N_{e,t}(z) \leq N^p_e(z)$

The equilibrium condition for hiring foreign skilled workers is given by:

$$\lambda_t(z) + f_T = (1 - \delta) E_t \left\{ \beta(C_{s,t+1}/C_{s,t})^{-1} [a \psi_{1,t+1}(z) Z_{t+1} z - w_{s,t+1} + \lambda_{t+1}(z) + f_T] \right\} \quad (7)$$
\(\psi_{1,t+1}\) is the marginal cost of production and is the Lagrange multiplier on the first constraint. \(\lambda_t(z)\) is the Lagrange multiplier on the third constraint. This is the shadow price corresponding to the permit constraint. Note that the inequality constraint implies that 
\[\lambda_t(z)[N_{e,t}(z) - N_{pe,t}] = 0.\] I assume that there is no additional uncertainty and the permit constraint binds i.e. firms end up using all their permits. Therefore, \(\lambda_t(z)\) is the positive marginal value associated with a permit.

In this paper, I analyze the aggregate market clearing price of permits that firms end up paying.\(^{26}\) Since wages, and therefore prices in period \(t+1\) are a function of the immigration cap in period \(t\) (which determines the foreign labor stock in \(t+1\)), (7) determines the demand schedule for permits.\(^{27}\) This is because firms can infer the price they are willing to pay for each additional permit from the marginal value of a permit. Since there are no informational asymmetries across firms, they have no incentive to submit any other price and quantity bid other than according to their demand schedule. The equilibrium market-clearing permit price that firms pay is such that the aggregate demand for permits is equal to the aggregate supply i.e. \(\tilde{N}^p_{e,t} = \tilde{N}_e\), where \(\tilde{N}_e\) is the exogenous number of permits allocated by the government. To get the market clearing permit price, the hiring condition of all firms can be aggregated to obtain the following.\(^{28}\)

\[
\zeta^p_t + f_T = (1 - \delta)E_t\left\{\beta(C_{s,t+1}/C_{s,t})^{-1}\left[a\tilde{\psi}_{1,t+1}Z_{t+1}\tilde{z} - w_{s,t+1} + \zeta^p_{t+1} + f_T\right]\right\}
\] (8)

In Appendix F, I derive the steady-state equilibrium price of permits as a function of number of permits (cap). The expression for the steady-state equilibrium price of permits indicates that it is an increasing function of aggregate productivity \(Z\) and aggregate firm specific productivity \(\tilde{z}\). Intuitively, an increase in aggregate productivity \(Z\) raises demand for permits by all firms and therefore shifts the aggregate demand schedule to the right. Given an exogenously fixed number of permits, this raises the equilibrium permit price paid by each firm to the government. The equilibrium price of permits is also an increasing function of the factors that increase the demand for skilled workers in the economy i.e. a larger stock of unskilled labor, a larger relative productivity of foreign labor, a lower return rate of skilled foreign labor, a larger share of the skilled sector in consumption. The key message is that because price of permits is allowed to endogenously vary with economic conditions, the market-clearing price of permits would reflect firms’ value of foreign skilled labor.

Firms pay the government the market clearing price of permits. The government collects

\(^{26}\) I leave the implications of heterogeneous prices paid by heterogeneous firms to future work.

\(^{27}\) As mentioned above, this alternate policy is evaluated under a binding cap and therefore the total number of permits sold (and the total foreign skill labor hired) will be constrained to be equal to the cap.

\(^{28}\) The tilde variables denote aggregates across firms. Aggregation in this model is similar to the baseline model because all firms pay the same wages and pay the same market-clearing price for permits \(\zeta^p_t\).
the revenue and rebates it back to skilled and unskilled domestic households in a lump-sum and symmetric manner. Firms then hire foreign skilled workers according to the number of permits they hold and also incur the technological hiring costs for each worker hired. There are no secondary market sales for permits.

The rest of the equilibrium conditions are similar to the baseline model. Aggregate accounting requires $\rho_1, t Z_t \tilde{z}_s, t + \rho_2, t Z_t \tilde{z}_u = C_{s, t} + C_{t, t} + C_{u, t} + f_T \tilde{N}_e$. Consumpion of skilled domestic households is given by $C_{s, t} = w_{s, t} + \tilde{d}_t + T_t / 2$, where $T_t$ is the government transfer, which is equal to $\tilde{N}_e \zeta_p$ (the revenue from immigration policy). Unskilled domestic households are assumed to receive the rest of the transfers, and immigrants consume their labor income, as before. Average firm profits are given by $\tilde{d}_t = \tilde{p}_t Y_1, t - w_{s, t} (1 + \tilde{l}_f, t) - \zeta_p \tilde{N}_e, t - f_T \tilde{N}_e, t$.

Welfare Implications of Moving Toward the Market-Driven Allocation of Permits

I calculate the welfare implications for each domestic household of a perfect foresight change in policy from the current immigration policy setup, toward the market-driven allocation of permits. In order to measure the welfare change, I calibrate the model with the alternate immigration policy by setting the sunk regulatory cost $f_R$ to zero. The rest of the calibration is the same as in the baseline case. The equilibrium permit price is determined endogenously. The welfare change is measured as the percentage change ($\Delta$) in consumption that would leave the household indifferent between the baseline skilled immigration policy and the policy with the market-driven allocation of permits for hiring skilled foreign workers.

The main impact of the policy change is that government revenue increases. If the revenue from permit sales ($\zeta_p \tilde{N}_e$) is rebated back to domestic households in a symmetric manner, both households witness a welfare gain as a result of the policy implementation. The welfare gain amounts to 0.0055 percent of quarterly steady-state consumption for unskilled workers. For skilled workers, the welfare gain amounts to 0.0113 percent of quarterly steady-state consumption. Therefore, both households can be made potentially better off.

Under the current framework, the major distortion in the decentralized economy is due to the presence of the immigration cap. Therefore, reducing much of the inefficiency with respect to the socially optimal allocation requires an increase in the cap. However, even in the absence of a cap increase, easing some of the burdensome immigration policy procedures, and moving towards a simpler market-driven mechanism could potentially increase welfare. An implication of this is that the recent political discussions on tightening the procedures related to skilled immigration policy in the U.S. would have the opposite impact of widening

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Note that now firms pay technological costs only for the workers they hire based on the number of permits they own.
the gap between the market economy’s allocation and the social planner’s optimal allocation and therefore would increase inefficiency in the economy.

8 Model Extension: Search and Matching Framework

In this section, I extend the benchmark model in order to analyze how firms respond to an immigration cap change in the presence of search frictions, and how this in turn effects the welfare impacts of an immigration cap change. Such a framework allows us to study potential unemployment effects on skilled workers.

In the framework with search frictions, firms in the skill-intensive sector post vacancies and they can be matched with either a skilled foreign worker or to a skilled domestic worker. While posting vacancies, firms cannot differentiate between domestic and foreign workers. This is realistic as firms cannot legally differentiate between the two worker types in their job postings. This also follows Chassamboulli and Palivos (2014), Battisti et al. (2014), Kingi (2015), in which domestic and immigrant workers are ex ante identical from a firm’s perspective but may have different outcomes depending on their bargaining power, outside options, and separation rates.

The probability of getting matched to a domestic or foreign worker depends on the relative fraction of each type of worker searching for jobs. However, there is still a policy-imposed cap and additional costs of hiring foreign workers. For each foreign worker that is matched, firms have to pay immigration policy and technological hiring costs as in the baseline model. Also, if the aggregate number of matches with foreign workers (which is the aggregate demand for foreign workers) exceeds the cap, there is a probability that each application will be selected, similar to before.

8.1 Search and Matching in the Skilled Labor Market

Suppose firm $z$ posts $v_t(z)$ vacancies for skilled workers in period $t$. The cost of posting a vacancy is $\kappa$. The matching function is given by $m(V_t, U_t) = \chi_t U_t V_t^{1-\epsilon}$ with unemployment elasticity $\epsilon$ and matching efficiency $\chi$. The aggregate vacancies posted is given by $V_t$ and $U_t = u_{d,t} + u_{f,t}$ is the aggregate mass of domestic and foreign skilled workers searching for a job. $\frac{V_t}{U_t}$ is the market tightness. Since the H1-B policy in the U.S. does not allow foreign workers to remain in the U.S. for an extended period if they are unemployed, I interpret $u_{f,t}$ not necessarily as the unemployment of foreign skilled workers in the domestic economy, but instead as the aggregate number of foreign skilled workers who are seeking a job in the domestic labor market, and can be located either in the domestic economy, or abroad. The
probability that a firm will be matched with a skilled worker (domestic or foreign) is given by \( \mu_t = \chi(V_t)^{-\epsilon} \).

The probability that a firm is matched to worker type \( j \in \{d, f\} \) is given by

\[
q_{j,t} = \frac{u_{j,t}}{u_{d,t} + u_{f,t}} \chi(V_t)^{-\epsilon}
\]

where \( d \) and \( f \) denote domestic and foreign skilled respectively, and \( \frac{u_{j,t}}{u_{d,t} + u_{f,t}} \) is the relative share of job searchers of each type. Note that \( q_{d,t} + q_{f,t} = \mu_t \). However, if a firm is matched with a foreign worker, it faces additional costs due to immigration policy (sunk costs \( f_R \) and \( f_T \) as in the baseline model).

The second immigration policy restriction, as before, is the cap on the total number of foreign workers that can be hired each period, \( \bar{N}_{e,t} \). In this framework, the total number of matches determines the demand by each firm \( z \):

\[
N_{e,t}(z) = q_{f,t}v_t(z)
\]

Then, firm \( z \) will apply for \( N_{e,t}(z) \) workers to join the firm and the probability that each foreign worker that was matched would eventually be able to join the firm is \( q_t = \min\left[\frac{N_{e,t}}{\int_{z=0}^{\infty} N_{e,t}(z)dG(z)}, 1\right] \), which is endogenously determined, as before. Both the match probability and the probability of application being selected is determined by aggregate labor market conditions and are taken as given by each firm, as each firm is of measure zero. Therefore, if the flow of matches for foreign workers is \( q_{f,t}v_t(z) \), the mass of foreign workers that eventually join firm \( z \) is \( q_tq_{f,t}v_t(z) \).

The exogenous separation rate for domestic workers is \( \delta_d \), and that of foreign workers is \( \delta_f \). As a significant proportion of foreign workers are likely to be temporary workers due to the nature of immigration policy, one can postulate that \( \delta_f > \delta_d \) (Battisti et al., 2014) and this turns out to be the case in the calibration. Workers hired this period join the firm in the next period and the separation shock is realized at the beginning of every period. Thus the stock of employed domestic and foreign skilled workers at firm \( z \) is given by

\[
l_{d,t}(z) = (1 - \delta_d)(l_{d,t-1}(z) + q_{d,t-1}v_{t-1}(z))
\]

\[
l_{f,t}(z) = (1 - \delta_f)(l_{f,t-1}(z) + q_{t-1}q_{f,t-1}v_{t-1}(z))
\]

The timing is as follows — in each period, first the separations are realized, the aggregate productivity shock is realized, wages are negotiated by a surplus-sharing rule, and firms produce period-\( t \) output. Firms then post vacancies and workers are matched. For foreign
matches, firms pay the immigration policy costs and submit applications for the workers to join the firm. If the cap binds, only a fraction $q_t$ of the foreign matches are allocated to the firms.

There are three households - skilled domestic, unskilled domestic, and skilled foreign. Each household consists of a continuum of workers and the measure of workers that are employed in Sector 1 is determined by the matching process. Let the total measure of domestic skilled workers in the labor force be $\bar{L}_d$ and that of foreign be $\bar{L}_f$ (both fixed to begin with). Then $u_{d,t} = \bar{L}_d - \bar{d}_d$ and $u_{f,t} = \bar{L}_f - \bar{d}_f$ are the domestic and foreign unemployed/job searchers in each period. $\bar{d}_d$ and $\bar{d}_f$ are the aggregate domestic and foreign workers that are employed, respectively. Employed and unemployed households of each type pool labor income, as is standard. Thus, the budget constraints are similar to the simple model except that now labor income is earned only by the measure of employed households of each type. Household preferences and optimal consumption choices are exactly the same as in the baseline model.

**Profit Maximization**

Firm $z$'s profit maximization problem (taking wages paid to domestic and foreign workers as given) is given by:

$$
\max_{\{\rho_{1,t}(z),v_{1,t}(z),l_{f,t+1}(z),l_{d,t+1}(z)\}} \sum_{t=0}^{\infty} \beta_{t,t} \left[ \rho_{1,t}(z)y_{1,t}(z) - w_{d,t}(z)l_{d,t}(z) - w_{f,t}(z)l_{f,t}(z) - \kappa v_{1,t}(z) - q_{f,t}v_{1,t}(f_R + f_T) \right]
$$

subject to following constraints:

1. Production Technology:
   $$y_{1,t}(z) = Z_t z(l_{f,t}(z) + l_{d,t}(z))$$

2. Stock of domestic workers:
   $$l_{d,t+1}(z) = (1 - \delta_d)l_{d,t}(z) + v_t(z)q_{d,t}$$

3. Stock of foreign workers:
   $$l_{f,t+1}(z) = (1 - \delta_f)l_{f,t}(z) + v_t(z)q_{f,t}$$

I follow Cacciatore (2014) in assuming this.
4. Demand: \( y_{1,t}(z) = \left( \frac{\theta_t(z)}{\bar{\rho}_{t,t}} \right)^{-\theta} Y_t^c / \bar{\rho}_{t,t} \)

Note that in the production technology, there are no relative productivity differences between domestic and foreign workers. This is because under a non-competitive labor market setting, firms don’t need any additional incentives to hire foreign workers.

The first order condition w.r.t \( v_t(z) \) gives the optimal condition for posting vacancies. The hiring condition is given by:

\[
\kappa = (1 - \delta_d) \Gamma_{d,t}(z) q_{d,t} + (1 - \delta_f) \Gamma_{f,t}(z) q_{f,t} q_t - (f_R + f_T) q_{f,t}
\]

(12)

where \( \Gamma_{d,t} \) is the Lagrange Multiplier (LM) on constraint (11) and \( \Gamma_{f,t} \) is the LM on (12). These represent the surplus from the respective matches. The hiring condition implies that in equilibrium, the cost of posting a vacancy is equal to the expected surplus from a domestic match and the expected surplus from a foreign match, both weighed by the probability of each match, net of the sunk hiring cost for foreign matches. Since there is a time to build lag, the surplus in period \( t \) is realized in period \( t + 1 \), and therefore the surplus from each worker is realized only with probability \( 1 - \delta_d \) and \( 1 - \delta_f \), respectively.

The first order conditions w.r.t \( l_{d,t+1} \) and \( l_{f,t+1} \) give:

\[
\Gamma_{d,t}(z) = E_t[\beta_{t,t+1} \{ \Xi_{t+1}(z) Z_{t+1} z - w_{d,t+1}(z) + (1 - \delta_d) \Gamma_{d,t+1}(z) \}]
\]

(13)

\[
\Gamma_{f,t}(z) = E_t[\beta_{t,t+1} \{ \Xi_{t+1}(z) Z_{t+1} z - w_{f,t+1}(z) + (1 - \delta_f) \Gamma_{f,t+1}(z) \}]
\]

(14)

where \( \Xi_t \) is the LM on production technology and represents the real marginal cost of production. The above equations show that the surplus from each match is the additional value generated from a skilled labor (when the match becomes productive in period \( t + 1 \)) net of the real wage in that period, plus the continuation value of the match.

**Wages: Nash Bargaining**

There is no match specific productivity so the surplus to the firm from every domestic match and from every foreign match is the same. However, the surplus from a foreign match may differ from the surplus from a domestic match. Wages are determined by the following surplus sharing rule:

\[
\eta_i S_{i,t+1}^F(z) = (1 - \eta_i) S_{i,t+1}^W(z) \quad \forall i \in \{ d, f \}
\]

\( \eta_i \) is bargaining power of worker \( i \in \{ \text{domestic, foreign} \} \). \( S^F \) is the firm’s surplus and \( S^W \) is the worker’s surplus from the match. These are described below.

Each workers’ surplus is given by the wage received minus the outside option plus the
continuation value of match. In the model, the only outside option of the worker is to search for another job, weighed by the probability that the job match (with any potential firm $z$) would survive. The worker surplus equations below reflect the fact that the surplus from a match in period $t$ is realized from wages received in the next period (due to the time to build). Given the timing, firms post vacancies only once in each period and therefore the outside option of a worker (if he loses the match or doesn’t get matched in period $t$) would depend on the probability of getting matched next period ($i_{t+1}$), weighed by the probability that the match would survive.

$$S_{d,t}^W(z) = E_t \beta_{t,t+1} [w_{d,t+1}(z) - \varpi_{d,t+1} + (1 - \delta_d)S_{d,t+1}^W(z)]$$

$$S_{f,t}^W(z) = E_t \beta_{t,t+1} [w_{f,t+1}(z) - \varpi_{f,t+1} + (1 - \delta_f)S_{f,t+1}^W(z)]$$

The outside options are given by:

$$\varpi_{d,t} = i_t (1 - \delta_d) \int_{z_{min}}^\infty \frac{v_t(z)}{V_t} S_{d,t}^W(z) dG(z)$$

$$\varpi_{f,t} = i_t (1 - \delta_f) \int_{z_{min}}^\infty \frac{v_t(z)}{V_t} S_{f,t}^W(z) dG(z)$$

where $i_t$ is the average job finding probability of a skilled worker in the domestic economy. The average job finding probability of a skilled worker is given by

$$i_t = \chi_t \left( \frac{V_t}{u_{d,t} + u_{f,t}} \right)^{1-\epsilon}$$

Appendix G describes the solution of the model and summarizes the main equations.\textsuperscript{31}

The appendix also describes the solution of the steady state of the model.

8.2 Calibration: Search and Matching Framework

In order to study the dynamics numerically, I calibrate the parameters of the model under the assumption that the steady state in the domestic economy mimics the U.S. during the 2004-2017 period. The immigration policy cap was binding during this period. I interpret each period as a quarter.

I rely on existing literature for some parameters. The discount rate $\beta$ is set to 0.99, as is standard in the literature. I set the elasticity of substitution across product varieties to $\theta = 3.8$ (Bernard, Eaton, Jensen, and Kortum (2003)). I choose the matching elasticity $\epsilon$ as

\textsuperscript{31}The unskilled sector is the same as the baseline model.
0.5, which is the commonly used value in the literature (Petrongolo and Pissarides (2001)), and the bargaining power of both workers, \( \eta_d = \eta_f = 0.5 \), as the same as \( \epsilon \) so that the Hosios condition holds. The share of sector 1 goods in consumption is set at \( \alpha = 0.4 \) to match the average share of unskilled income in total aggregate income over the period 2004 to 2017.

Monthly Current Population Survey (CPS) data is used to compute quarterly data from 2004 to 2017 for employment, unemployment, and wages of domestic and foreign workers. Average quarterly domestic and foreign wages are computed using weekly wage data of skilled foreign and native-born workers in private non-farm employment from the CPS. The wage data is deflated using the CPI deflater to convert it in real units. The job finding probability for skilled workers and the separation rate for domestic and foreign skilled workers is computed using de-seasonalized monthly CPS data as in Shimer (2005). All seasonally adjusted data used in the calibration is plotted in Appendix A.

The separation rates \( \delta_d \) and \( \delta_f \) are set to 0.046 and 0.07 to match the average quarterly separation rates computed using monthly CPS data as in Shimer (2005). The domestic skilled labor force \( \bar{L}_d \) is normalized to 1 and given this normalization, the domestic unskilled labor supply is calibrated to \( \bar{l}_u = 1.92 \), as in the baseline model. The total foreign skilled workers in the U.S. labor force is set to \( \bar{L}_f = 0.2 \) to match the average skilled foreign labor in the U.S as a proportion of the average skilled domestic labor.

The immigration policy costs \( (f_R \text{ and } f_T) \) and cap \( (\bar{N}_{e,t}) \) are taken to be same as the baseline model, in order to compare results with the baseline model. The vacancy posting cost is normalized to 1. The efficiency of the matching function is calibrated to 0.68 in order to match the average job finding probability for skilled workers of 0.75 and an average domestic unemployment rate of 0.03%. The calibrated model yields a domestic to foreign skilled wage ratio of 1.04, which is close to the average value seen in the data (0.99).

Under this calibration, the steady-state outside option of foreign skilled workers in units of consumption turns out to be larger than that of the domestic worker (0.86 vs 0.76). This is in line with evidence that immigrants and natives differ according to their outside options (Chassamboulli and Palivos, 2014)[32]

### 8.3 Welfare Impact of an Immigration Cap Increase in the Presence of Search Frictions

Figure 7 shows that following an increase in the immigration cap, firms post more vacancies, which increases not only the foreign skilled labor employed, but also the domestic skilled labor force. The efficiency of the matching function is calibrated to 0.68 in order to match the average job finding probability for skilled workers of 0.75 and an average domestic unemployment rate of 0.03%. The calibrated model yields a domestic to foreign skilled wage ratio of 1.04, which is close to the average value seen in the data (0.99). Under this calibration, the steady-state outside option of foreign skilled workers in units of consumption turns out to be larger than that of the domestic worker (0.86 vs 0.76). This is in line with evidence that immigrants and natives differ according to their outside options (Chassamboulli and Palivos, 2014)[32]

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32 Since I do not model the foreign economy explicitly in this paper, I do not calibrate the outside options that would exist for foreign workers in the foreign economy.
labor employed. Therefore, even though domestic skilled wages fall as in the baseline model, an increase in domestic employment offsets part of the negative impact on domestic skilled workers and the negative welfare impact on domestic skilled workers is much smaller. However, under the current calibration, the increase in domestic labor employment is not enough to completely offset the wage losses. Tables 7 and 8 indicates that the reduction in welfare losses compared to the benchmark case ranges from 54.2% to 79%. This is because the impact depends on the profit distribution across households.

As the stock of skilled labor employed in sector 1 builds up, output and profit of the skilled sector firms increase. Since a larger skilled labor stock employed (both domestic and foreign) implies a higher demand for sector 2 output, unskilled wages increase. Therefore, the impact of an immigration cap increase on the welfare of unskilled natives is still positive. In fact, the welfare gain for unskilled domestic households is 33% higher compared to the welfare gain in the baseline model. This is due to larger employment gains for skilled workers.

**Intuition**

In order to get some intuition regarding the impact of an immigration cap change on the employment of domestic skilled workers in this framework, in the appendix, I derive the steady-state relationship between aggregate domestic skilled labor employed by firms, $\tilde{l}_d$, and the immigration policy cap, $\bar{N}_e$ as:

$$\tilde{l}_d = \bar{L}_d \left( \frac{\bar{L}_f \delta_d}{(1 - \delta_d) \bar{N}_e - \delta_d(1 - \delta_f)} + 1 \right)^{-1}$$

(15)

The equation above shows that as the immigration cap on foreign skilled workers increases, the aggregate domestic workers employed, $\tilde{l}_d$, increases, for a given mass of aggregate domestic and foreign labor in the labor force (i.e. for a given $\bar{L}_d$ and $\bar{L}_f$ ). Intuitively, an increase in the entry cap increases firms’ incentive to post more vacancies. This happens because there is a higher probability that a foreign worker that was matched would eventually be able to join the firm, which increases firms’ overall surplus from posting vacancies. This is also evident from the vacancy posting condition (12). Part of the expected benefit from posting a vacancy for each firm is the surplus from a foreign match, as with probability $q_{f,t}$, a firm may be matched with a foreign worker. However, because of the cap on foreign workers, the firm will only be able to retain a fraction $q_t$ of its foreign matches. When the immigration cap increases, the probability of being able to bring the worker to the firm increases as $q_t$ is a positive function of the cap. This increases firms’ expected discounted benefit from posting.

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33In a technical appendix that will be available soon, I log linearize this version of the model to study the intuition behind the dynamic responses of the economy to a cap change.
a vacancy and firms end up posting more vacancies. More vacancies posted increases firm matches with domestic skilled workers, and therefore, their employment.

This result confirms the fact that firms’ endogenous response to the immigration cap change depends on the structure of the labor market. Therefore, it is important to take this into account when determining the welfare impact of an increase in skilled immigration on domestic workers.

9 Conclusion

This paper takes a step in the direction of studying the impact of skilled immigration and skilled immigration policy changes in a macroeconomic general equilibrium framework by explicitly focusing on the role of firm demand for foreign skilled labor. The framework I propose — featuring monopolistically competitive firms and a realistic modeling of the current skilled immigration policy setup — facilitates a better understanding of the determinants of firm demand for foreign skilled labor and the aggregate impacts of changes in current skilled immigration policies. This framework leads to some new insights — the realized state of the economy and the structure of the labor market potentially matter, for evaluating the impact of current immigration policy changes. These insights are lost if we evaluate the impact of a cap increase as a pure labor supply change within a competitive labor market setting. Also, even if the government does not want to change the cap on foreign workers, an allocation of the same quota of foreign workers via an alternate mechanism — a market-driven allocation of permits for hiring skilled foreign workers, can be welfare improving for domestic households. However, an increase in the immigration cap is required to close much of the gap between the decentralized economy’s allocation and the social planner’s equilibrium allocation.

Extensive work remains. The framework does not account for heterogeneity within skill groups or the endogenous response of domestic households to skilled immigration. Moreover, it is important to study the implications of firm heterogeneity and the misallocation effect of the current immigration policy across firms in greater detail. I leave these for future extensions. One direction in which I currently start extending this work is to study the impact of skilled immigration policy changes on the offshoring decision of firms in a two-country framework.
Tables and Figures

Table 1: Baseline Model Summary

<table>
<thead>
<tr>
<th>Economic Variable</th>
<th>Equilibrium Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sector 1 Output</td>
<td>$Y_{1,t} = Z_l z (1 + l_{f,t})$</td>
</tr>
<tr>
<td>Sector 2 Output</td>
<td>$Y_{2,t} = Z_d l_u$</td>
</tr>
<tr>
<td>Price Index</td>
<td>$1 = (\hat{\rho}_1 t) (\rho_2 t)^{1-\alpha}$</td>
</tr>
<tr>
<td>Hiring Condition</td>
<td>$(f_{R,t} + f_{T,t})/q_t = (1 - \delta) \beta_t {(C_{s,t+1} - C_{s,s})^{-1} [\tilde{\psi}<em>1 t + 1 Z_t \tilde{z} - w</em>{s,t+1} + \tilde{f}<em>{R,t+1} \tilde{f}</em>{T,t+1}]}$</td>
</tr>
<tr>
<td>Hiring Probability</td>
<td>$q_t = \min[\tilde{N}_e t, 1]$</td>
</tr>
<tr>
<td>Foreign Labor Stock</td>
<td>$\tilde{l}<em>{f,t+1} = (1 - \delta) (\tilde{l}</em>{f,t} + q_t \tilde{N}_e t)$</td>
</tr>
<tr>
<td>Sector 1 Profits</td>
<td>$\tilde{d}<em>t = \hat{\rho}<em>1 t y</em>{1,t} - w</em>{s,t}(1 + \tilde{l}<em>{f,t}) - (f</em>{R,t} + f_{T,t}) \tilde{N}_e t$</td>
</tr>
<tr>
<td>Skilled Wages</td>
<td>$w_{s,t} = \tilde{\psi}_1 z_t \tilde{z}$</td>
</tr>
<tr>
<td>Sector 1 Prices</td>
<td>$\tilde{\rho}_1 t = \theta \tilde{\psi}_1 t \tilde{z}$</td>
</tr>
<tr>
<td>Sector 2 Prices</td>
<td>$\tilde{\rho}<em>2 t = w</em>{u,t} / Z_t$</td>
</tr>
<tr>
<td>Aggregate Demand</td>
<td>$Y^c_t = C_{s,t} + C_{u,t} + C_{f,t} + (f_{R,t} + f_{T,t}) \tilde{N}_e t$</td>
</tr>
<tr>
<td>Market Clearing (Both Sectors)</td>
<td>$\tilde{\rho}_1 t Y^c_t / \alpha = \tilde{\rho}_2 t Y^c_t / (1 - \alpha)$</td>
</tr>
<tr>
<td>Consumption by Unskilled</td>
<td>$C_{u,t} = w_{u,t} \tilde{l}_u$</td>
</tr>
<tr>
<td>Consumption by Domestic Skilled</td>
<td>$C_{s,t} = w_{s,t} \tilde{d}_t$</td>
</tr>
<tr>
<td>Consumption by Immigrants</td>
<td>$C_{f,t} = w_{s,t} \tilde{l}_{f,t}$</td>
</tr>
</tbody>
</table>

Table 2: A Priori Parameters: Baseline Model

<table>
<thead>
<tr>
<th>A Priori Parameters</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Factor</td>
<td>$\beta = 0.99$</td>
<td>Literature</td>
</tr>
<tr>
<td>Literature Elasticity of Substitution</td>
<td>$\theta = 3.8$</td>
<td>Literature</td>
</tr>
<tr>
<td>Separation of foreign skilled</td>
<td>$\delta_f = 0.07$</td>
<td>Computed using Shimer (2005) and CPS data</td>
</tr>
<tr>
<td>Cap</td>
<td>$N_e = 0.0002$</td>
<td>Cap/Domestic skilled labor (USCIS, CPS)</td>
</tr>
<tr>
<td>Domestic unskilled labor</td>
<td>$l_u = 1.92$</td>
<td>Proportion of unskilled labor (CPS)</td>
</tr>
<tr>
<td>Foreign skilled in U.S. labor force</td>
<td>$l_f = 0.2$</td>
<td>Skilled foreign labor/skilled domestic labor (CPS)</td>
</tr>
<tr>
<td>Technological Hiring Cost</td>
<td>$f_t = 0.0833$</td>
<td>One month real skilled wage (CPS)</td>
</tr>
</tbody>
</table>

Table 3: Targeted Moments

<table>
<thead>
<tr>
<th>Target</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average fraction of immigrant applications approved</td>
<td>0.40</td>
<td>0.39</td>
</tr>
<tr>
<td>Skill Premium</td>
<td>1.85</td>
<td>1.85</td>
</tr>
</tbody>
</table>

Table 4: Calibrated Parameters

<table>
<thead>
<tr>
<th>Calibrated Parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Foreign relative productivity</td>
<td>$a = 1.1035$</td>
</tr>
<tr>
<td>Sunk immigration cost</td>
<td>$f_R = 0.6699$</td>
</tr>
</tbody>
</table>
Table 5: Welfare Impact of 10 Percent Immigration Cap Increase: Baseline

<table>
<thead>
<tr>
<th>Profit Earners (Scenario)</th>
<th>Domestic Skilled</th>
<th>Domestic Unskilled</th>
<th>Domestic Entrepreneurs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home Skilled</td>
<td>-0.0086</td>
<td>0.0102</td>
<td></td>
</tr>
<tr>
<td>Entrepreneurs</td>
<td>-0.0153</td>
<td>0.0102</td>
<td>0.0102</td>
</tr>
</tbody>
</table>

Note: Values reported above are in percent of initial steady-state consumption.

Table 6: Efficient Allocation

<table>
<thead>
<tr>
<th>Economic Variable</th>
<th>Equilibrium Condition</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td>$C_{u,t} = C_{s,t}$</td>
</tr>
<tr>
<td>Demand for sector 1 output</td>
<td>$Y_{1,t} = \alpha \frac{\varsigma}{\eta_c} Y_t^c$</td>
</tr>
<tr>
<td>Demand for sector 2 output</td>
<td>$Y_{2,t} = (1 - \alpha) \frac{\varsigma}{\eta_c} Y_t^c$</td>
</tr>
<tr>
<td>Sector 1 output</td>
<td>$Y_{1,t} = Z_t \hat{z}(1 + a L_{f,t})$</td>
</tr>
<tr>
<td>Sector 2 output</td>
<td>$Y_{2,t} = Z_t \tilde{l}$</td>
</tr>
<tr>
<td>Aggregate accounting</td>
<td>$C_{u,t} + C_{s,t} + f_{T,t} N_{e,t} = Y_t^c$</td>
</tr>
<tr>
<td>Price index</td>
<td>$1 = \left( \frac{\varsigma}{\eta_c} \right)^{\alpha \left( \frac{\varsigma}{\eta_c} \right)^{1-\alpha}}$</td>
</tr>
<tr>
<td>Entry condition of foreign workers</td>
<td>$f_{T,t} = E_t \left[ \beta (1 - \delta) \frac{C_{s,t+1}}{\varsigma \eta_c} \left( \frac{\varsigma}{\eta_c} a Z_{t+1} \hat{z} + f_{T,t+1} \right) \right]$</td>
</tr>
<tr>
<td>Stock of foreign workers</td>
<td>$L_{f,t+1} = (1 - \delta)(L_{f,t} + N_{e,t})$</td>
</tr>
</tbody>
</table>

Note: $\frac{\varsigma}{\eta_c} = \vartheta_{1,t}$ and $\frac{\varsigma}{\eta_c} = \vartheta_{2,t}$ are the relative prices of sector 1 and 2 output in the planner’s equilibrium.

Table 7: A Priori Parameters: Search and Matching Model

<table>
<thead>
<tr>
<th>A Priori Parameters</th>
<th>Value</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Discount Factor</td>
<td>$\beta = 0.99$</td>
<td></td>
</tr>
<tr>
<td>Literature Elasticity of Substitution</td>
<td>$\theta = 3.8$</td>
<td>Literature</td>
</tr>
<tr>
<td>Matching Function Elasticity</td>
<td>$\epsilon = 0.5$</td>
<td>Literature</td>
</tr>
<tr>
<td>Bargaining power: foreign skilled</td>
<td>$\eta_f = 0.5$</td>
<td>Literature</td>
</tr>
<tr>
<td>Bargaining power: domestic skilled</td>
<td>$\eta_d = 0.45$</td>
<td>Literature</td>
</tr>
<tr>
<td>Separation of domestic skilled</td>
<td>$\delta_d = 0.046$</td>
<td>Computed using Shimer (2005) and CPS data</td>
</tr>
<tr>
<td>Separation of foreign skilled</td>
<td>$\delta_f = 0.07$</td>
<td>Computed using Shimer (2005) and CPS data</td>
</tr>
<tr>
<td>Domestic unskilled labor</td>
<td>$\hat{l}_u = 1.92$</td>
<td>Proportion of unskilled labor (CPS)</td>
</tr>
</tbody>
</table>

Table 8: Welfare Impact of 10 Percent Immigration Cap Increase: Search Frictions

<table>
<thead>
<tr>
<th>Profit Earners (Scenario)</th>
<th>Domestic Skilled</th>
<th>Domestic Unskilled</th>
<th>Domestic Entrepreneurs</th>
</tr>
</thead>
<tbody>
<tr>
<td>Home Skilled</td>
<td>-0.0018</td>
<td>0.0136</td>
<td></td>
</tr>
<tr>
<td>Entrepreneurs</td>
<td>-0.007</td>
<td>0.0136</td>
<td>0.0115</td>
</tr>
</tbody>
</table>

Note: Values reported above are in percent of initial steady-state consumption.

Table 9: Summary of Welfare Impacts: With Entrepreneurs Separated

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Domestic Skilled</th>
<th>Domestic Unskilled</th>
<th>Domestic Entrepreneurs</th>
</tr>
</thead>
<tbody>
<tr>
<td>10% cap increase: Baseline model</td>
<td>-0.0153</td>
<td>0.0102</td>
<td>0.0102</td>
</tr>
<tr>
<td>Policy change to alternate policy</td>
<td>0.0113</td>
<td>0.0055</td>
<td>0</td>
</tr>
<tr>
<td>10% cap increase: Search frictions</td>
<td>-0.007</td>
<td>0.0136</td>
<td>0.0115</td>
</tr>
</tbody>
</table>

Note: Values reported above are in percent of initial steady-state consumption.
Figure 4: Response to a temporary productivity shock in the presence of an occasionally binding constraint (solid blue line) (baseline case) vs the case without an entry cap (dotted red line).
Figure 5: Response to a 10% cap change under two states of the economy. In the first state, the economy is not transitioning from a negative productivity shock. In the second state, the economy is transitioning from a negative productivity shock.
Figure 6: Transitional dynamics after a 10 percent skilled immigration cap increase in the baseline model. All variables (except probability) are in percent deviation from the steady state.
Figure 7: Transition dynamics after a 10 percent skilled immigration cap change in the search and matching model. All variables are in percent deviation from the steady state.
Appendix A  Figures

Figure A.1: Source: Current Population Survey (CPS) on foreign-born population in the U.S.
Figure A.2: Proportion of foreign born in the U.S. skilled labor force
Figure A.3: Entry of foreign skilled workers by visa category

Source: U.S. Department of State.
Figure A.4: Firm sales and Labor Condition Applications (LCA)

Source: Kerr et. al. (2014)
A.1 Plots of Data used in Calibration

Figure A.5: Seasonally Adjusted data computed from monthly Current Population Survey (CPS) Surveys.
Figure A.6: Seasonally Adjusted data computed from monthly Current Population Survey (CPS) Surveys.
Figure A.7: Seasonally Adjusted data computed from monthly Current Population Survey (CPS) Surveys.
Appendix B  H1-B Program: Institutional Framework and Background

Since the implementation of the H1-B visa program in 1990, it has been the main method of entry into the U.S. workforce for foreign college educated professionals. Table 10 shows that H1-B visa holders constituted 66 percent of all skilled foreign entrants in 2014. A significant proportion of H1-B recipients (over 70 percent) are from emerging economies — India and China. The other major visa categories for foreign skilled workers are L-1 (for transfer of employees across multinational firms) and TN (visas for Canadian and Mexican NAFTA professional workers). The proportion of entrants from the latter two visa categories has been increasing since 2001, but the H1-B visa program still remains the dominant entry mode. Thus, most studies that analyze the impact of skilled foreign workers in the U.S. focus on the H1-B visa program. Though the H-1B visa is a temporary visa as it is issued for three years (and can be renewed for another three years), it is a dual intent visa as it
can lead to permanent residency if the employer is willing to sponsor the worker for a green card.

Table 10: Major Entry Routes for Foreign Skilled Workers (2014)

<table>
<thead>
<tr>
<th>Visa Category</th>
<th>Proportion of Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>H1-B</td>
<td>66.1 %</td>
</tr>
<tr>
<td>L-1</td>
<td>29.3 %</td>
</tr>
<tr>
<td>TN</td>
<td>4.6 %</td>
</tr>
</tbody>
</table>

Source: U.S. Department of State

The H1-B program has been subject to an annual quota on new visa issuances. The initial visa cap was 65,000 which was subsequently increased to 115,000 in 1999 and 2000, after the cap was met in 1997. The cap was further increased to 195,000 for 2001 through 2003. In 2001, cap exemptions were introduced for employees of higher education, non-profit, and government research organizations. In 2004, the cap was reduced back to 65,000, but 20,000 additional visas were allocated for workers who had obtained a master’s degree or higher from a U.S. institution. The cap applies only to new H1-B visa issuances for for-profit firms.

In order to obtain an H1-B visa, there are several steps to be followed and firms are central to this process. The first step requires the firm that wants to hire a foreign worker to file a Labor Condition Application (LCA) with the department of labor. In the application, the firm specifies the nature of occupation and attests that the firm will pay the worker the greater of the actual compensation paid to other employees in the same job or the prevailing compensation for that occupation. The rationale given for this attestation is to help protect domestic worker wages.

LCA forms can request for one or more foreign workers for a particular occupation and thus they signal firm vacancies in specific occupations for foreign workers. However, there are some limitations of using the LCA database. The LCA database contains records for every request submitted, but this is only an intermediate step in the process towards the final visa approval. An LCA is submitted for every H-1B request, whether new or a renewal, and each LCA can contain multiple H-1B workers. A more conservative estimate of the demand for foreign skilled workers would be to count each LCA filed as a request for one employee. Plotting the total number of LCAs filed as compared to Figure 1 that plots the total number of employees requested in LCAs filed each year does not change the main motivation regarding business
cycle correlation and rising excess demand during expansionary periods.

Once the LCA has been approved by the Department of Labor, it is sent to the United States Citizenship and Immigration Services (USCIS), along with the I-129 form and the required visa fees. The crucial fact is that employees can apply for an H1-B visa only if they have a job offer from an employer with an LCA approval. The employer cannot file more than one I-129 for the same prospective employee. Most of the filing and legal fees are borne by the employer. If the number of H1-B visa petitions (I-129 forms) that fall within the non-exempt category exceed the cap, the USCIS randomly selects visas for processing via a lottery system, until the 65,000 cap is reached. The filing fees paid for the unsuccessful visa applications is returned (unless it is discovered that multiple H1-B petitions are submitted for the same employee).

Appendix C  Model with Complementarities between Domestic and Foreign Skilled Labor

The production technology is now given by: \( y_{1,t}(z) = z l_{s,t}(z) \) where:

\[
l_{s,t}(z) = \left( \lambda^{(1-\gamma)} l_{d,t}(z)^\gamma + (1 - \lambda)(1-\gamma)(l_{f,t}(z))^{\gamma-1} \right)^{1/\gamma}.
\]

\( \gamma = 1 \rightarrow \) gives the baseline model. When \( \gamma < 1 \), foreign and domestic skilled labor are imperfect substitutes. As \( \gamma \) falls, foreign and domestic skilled labor increasingly become complementary. Now wages of domestic and foreign skilled households are given by:

\[
w_{d,t} = \psi_{1,t}(z)Z_t z(l_{s,t}(z))^{(1-\gamma)}(l_{d,t}(z))^{\gamma-1}\lambda^{(1-\gamma)}
\]

\[
w_{f,t} = \psi_{1,t}(z)Z_t z(l_{s,t}(z))^{(1-\gamma)}(l_{f,t}(z))^{\gamma}(1 - \lambda)(1-\gamma)
\]

Appendix D  Steady-State Solution in the Baseline Model

Since the model features an occasionally binding constraint, the model is equivalent to one with two regimes. The constraint is binding under one regime and slack under the other and each regime has a separate non-stochastic steady state. I first solve for the steady state when the cap is not binding.

\[^{34}\text{This proves the worker's qualifications}\]
When the cap does not bind \((q = 1)\):

Here, I derive expressions for the average real prices in sector 1, \(\tilde{\rho}_1\), and the average stock of foreign skilled workers, \(\tilde{l}_f\), and all other endogenous steady-state variables can be expressed as a function of these.

From the market clearing condition in sector 1 we can get: \(Z\tilde{z}(1 + a\tilde{l}_f) = \alpha Y_c / \tilde{\rho}_1\), where the aggregate demand is given by \(Y_c = C_s + C_i + C_u + (f_R + f_T)\tilde{N}_e\). Substituting the households’ budget constraint, we can write aggregate demand as \(Y_c = w_s(1 + \tilde{l}_f) + \tilde{d} + w_u\tilde{l}_u + (f_R + f_T)\tilde{N}_e\).

Consider the average firm profit in sector 1, \(\tilde{d} = \tilde{\rho}_1\tilde{y}_1 - w_s(1 + \tilde{l}_f) - (f_R + f_T)\tilde{N}_e\). Substituting this in the aggregate demand, we can write the skilled sector market clearing as:

\[
\tilde{\rho}_1Z\tilde{z}(1 + a\tilde{l}_f) = \alpha(\tilde{\rho}_1Z\tilde{z}(1 + \tilde{l}_f) + w_u\tilde{l}_u).
\]

Using expression for real wages in sector 2 \(w_u = \rho_2Z\), and also the price index equation \(\tilde{\rho}^\alpha_1\rho_2^{1-\alpha} = 1\), we can obtain:

\[
(1 - \alpha)\tilde{z}(1 + a\tilde{l}_f) = \alpha(\tilde{\rho}_1)^{\frac{1}{\alpha}}\tilde{l}_u \quad (D.1)
\]

Combining the steady-state hiring condition for foreign skilled workers with the equilibrium condition for wages and prices, we can obtain:

\[
(f_R + f_T) = \frac{(1 - \delta)\beta(a - 1)(\theta - 1)\tilde{\rho}_1Z\tilde{z}}{\theta(1 - (1 - \delta)\beta)} \quad (D.2)
\]

Using D.2, we can get the average price of sector 1 output as \(\tilde{\rho}_1 = \frac{\theta(f_R + f_T)(a - 1)(\theta - 1)(1 - (1 - \delta)\beta)}{(1 - \delta)\beta Z\tilde{z}}\).

Finally, substituting the expression obtained for average skilled sector prices into D.1, we can get:

\[
\tilde{l}_f = \frac{1}{a}\left\{ \frac{\alpha Z_{\tilde{z}}(a - 1)(\theta - 1)}{(1 - \alpha)} \left[ \theta(f_R + f_T)(1 - (1 - \delta)\beta) \right]^{\frac{1}{\alpha - 1}} - 1 \right\}
\]

Once we have \(\tilde{\rho}_1\) and \(\tilde{l}_f\), we can obtain the other steady state expressions for \(w_s, \tilde{N}_e, Y_1, \rho_2, w_u,\)
\( C_u, C_s, C_f, Y^c \), since these variables can be expressed as a function of \( \tilde{l}_f \), and \( \tilde{\rho}_1 \):

\[
\begin{align*}
    w_s &= (\theta - 1)\tilde{\rho}_1 Z \tilde{z}/\theta \\
    \rho_2 &= \tilde{\rho}_1^{\frac{\alpha}{\alpha - 1}} \\
    Y_1 &= Z \tilde{z}(1 + a \tilde{l}_f) \\
    \tilde{N}_e &= \delta \tilde{l}_f / (1 - \delta)
\end{align*}
\]

All other variables can be expressed as a function of these variables, as shown below.

**Steady State When the Cap Binds**

When the cap binds, \( q\tilde{N}_e = \bar{N}_e \) and therefore from the law of motion for foreign skilled workers, we have \( \tilde{l}_f = (1 - \delta)\bar{N}_e/\delta \). Since the market clearing in the skilled sector still needs to hold, we have \( \tilde{\rho}_1 = \left( \frac{(1-\alpha)\tilde{z}(1+a\tilde{l}_f)}{\alpha \tilde{u}} \right)^{\alpha - 1} \). Then, using the optimal hiring condition, we can get:

\[
q = \frac{(f_R + f_T)(1 - (1 - \delta)\beta)\theta}{(1 - \delta)\beta(a - 1)(\theta - 1)\tilde{\rho}_1 Z \tilde{z}} \tag{D.3}
\]

Substituting the steady-state value of prices into D.3, we get:

\[
q = \frac{(f_R + f_T)(1 - (1 - \delta)\beta)\theta}{(1 - \delta)\beta(a - 1)(\theta - 1)Z \tilde{z}} \left( \frac{(1 - \alpha)\tilde{z}(1 + a \tilde{l}_f)}{\alpha \tilde{u}} \right)^{1-\alpha}
\]

**Appendix E  Social Planner Allocation**

Consider the problem of a Social Planner who maximizes welfare of domestic households and chooses the optimal entry of foreign skilled workers in the domestic labor force, taking the firm size distribution, preferences, technology, and resources available in the economy as given. Let \( f_T \) be the technologically imposed cost of hiring skilled immigrants in the economy. The planner’s problem is given by:

\[
\max_{\{C_{u,t}, C_{s,t}, N_{e,t}(z), l_{f,t+1}(z), y_{1,t}(z), y_{2,t}\}} \mathbb{E}_t \left[ \mu \sum_{t=0}^{\infty} \beta^t \ln(C_{u,t}) + (1 - \mu) \sum_{t=0}^{\infty} \beta^t \ln(C_{s,t}) \right]
\]
s.t.

\[ C_{u,t} + C_{s,t} + f_T \int_{z_{\text{min}}}^{\infty} N_{e,t}(z)dG(z) = \left( \int_{z_{\text{min}}}^{\infty} \frac{y_{1,t}(z)^{\theta-1}}{\alpha} dG(z) \right)^{\frac{\theta}{\theta-1}} \left( \frac{y_{2,t}}{1 - \alpha} \right)^{1-\alpha} \]  \hspace{1cm} (E.1)

\[ \int_{z_{\text{min}}}^{\infty} y_{1,t}(z)dG(z) = \int_{z_{\text{min}}}^{\infty} Z_t z(l_{d,t}(z) + a l_{f,t}(z))dG(z) \]  \hspace{1cm} (E.2)

\[ Y_{2,t} = Z_t \bar{L}_u \]  \hspace{1cm} (E.3)

\[ \int_{z_{\text{min}}}^{\infty} l_{f,t+1}(z)dG(z) = (1 - \delta) \int_{z_{\text{min}}}^{\infty} (l_{f,t}(z)dG(z) + \int_{z_{\text{min}}}^{\infty} N_{e,t}(z)dG(z)) \]  \hspace{1cm} (E.4)

\[ \int_{z_{\text{min}}}^{\infty} l_{d,t}(z)dG(z) = 1 \]  \hspace{1cm} (E.5)

where \( \mu \in [0, 1] \) is the planner’s weight on domestic unskilled households’ welfare.

The first constraint is the aggregate resource constraint — the total output can be used for aggregate consumption and incurring the technological component of the sunk cost for hiring foreign skilled workers. The Lagrange multiplier on this constraint, \( \varsigma_t \), represents the social marginal utility of consumption resources. In the Planner’s environment, \( Y_{t}^c = C_{u,t} + C_{s,t} + f_T N_{e,t} \).

The second constraint defines the production technology for firms in sector 1. The Lagrange multiplier on this, \( \nu_t \), denotes the social marginal cost of producing one more unit of sector 1 output. Similarly, the Lagrange multiplier on constraint (E.3), \( \eta_t \) gives the social marginal cost of producing one more unit of sector 2 output.

Constraint (E.4) gives the law of motion of foreign skilled workers. Note that the social planner’s constraint does not include the probability of an application being selected, \( q_t \), because the Social Planner does not face a cap. The Lagrange multiplier associated to this constraint, \( \xi_t \), denotes the real marginal value of a foreign skilled worker to society.

The first order conditions are given by:
\[
\frac{a}{C_{u,t}} = \varsigma_t \quad \forall t
\]
\[
\frac{(1 - a)}{C_{s,t}} = \varsigma_t \quad \forall t
\]
\[
E_t[\beta(\alpha \nu_{t+1} Z_{t+1} \tilde{z} + (1 - \delta) \xi_{t+1})] = \xi_t \quad \forall t
\]
\[
-\varsigma_t f_T + (1 - \delta) \xi_t = 0 \quad \forall t
\]
\[
\varsigma_t \left( \frac{Y_{1,t}}{\alpha} \right)^{\alpha - 1} \left( \frac{Y_{2,t}}{1 - \alpha} \right)^{1 - \alpha} \left( \frac{y_{1,t}(z)}{Y_{1,t}} \right)^{\frac{1}{\theta}} = \nu_t \quad \forall t
\]
\[
\varsigma_t \left( \frac{Y_{1,t}}{\alpha} \right)^{\alpha} \left( \frac{Y_{2,t}}{1 - \alpha} \right)^{-\alpha} = \eta_t \quad \forall t
\]
\[
C_{u,t} + C_{s,t} + f_T N_{e,t} = \left( \frac{Y_{1,t}}{\alpha} \right)^{\alpha} \left( \frac{Y_{2,t}}{1 - \alpha} \right)^{1 - \alpha} \quad \forall t
\]
\[
Y_{1,t} = Z_t \tilde{z} (1 + aL_{f,t}) \quad \forall t
\]
\[
Y_{2,t} = Z_t \bar{L}_u \quad \forall t
\]
\[
L_{f,t+1} = (1 - \delta)(L_{f,t} + N_{e,t}) \quad \forall t
\]

where \( Y_{1,t} = \int_{z_{min}}^{\infty} y_{1,t}(z) \frac{\theta - 1}{\theta} dG(z) \). Also, \( L_{f,t} = \int_{z_{min}}^{\infty} l_{f,t}(z)dG(z) \). \( L_{d,t} = 1 \) and \( N_{e,t} \) are similarly aggregate values across all firms.

Using \( C_{u,t} + C_{s,t} + f_T N_{e,t} = Y^c_t \), and \( \mu = 1/2 \) (Assuming that the social planner puts equal weights on skilled and unskilled domestic workers), the first order conditions can be
expressed as:

\[ C_{u,t} = C_{s,t} \] (E.6)

\[ Y_{1,t} = \frac{\alpha S_t}{\nu_t} Y_t^c \] (E.7)

\[ Y_{2,t} = (1 - \alpha)\frac{S_t}{\eta_t} Y_t^c \] (E.8)

\[ 1 = \left( \frac{\nu_t}{\varsigma_t} \right)^\alpha \left( \frac{\eta_t}{\varsigma_t} \right)^{1-\alpha} \] (E.9)

\[ f_{T,t} = E_t \left[ \beta(1 - \delta) \frac{C_{s,t}}{C_{s,t+1}} \left( \frac{\nu_{t+1}}{\varsigma_{t+1}} aZ_{t+1} \tilde{z} + f_T \right) \right] \] (E.10)

\[ Y_{1,t} = Z_t \tilde{z}(1 + aL_{f,t}) \] (E.11)

\[ Y_{2,t} = Z_t \bar{L}_u \] (E.12)

\[ L_{f,t+1} = (1 - \delta)(L_{f,t} + N_{e,t}) \] (E.13)

\[ C_{u,t} + C_{s,t} + f_{T,t}N_{e,t} = Y_t^c \] (E.14)

(E.7) and (E.8) give the demand schedules for sector 1 and sector 2 output. To facilitate the comparison between the planned and decentralized economy, define the following relative prices in sector 1 and 2 for the planner’s equilibrium: \( \frac{\nu_t}{\varsigma_t} = \varrho_{1,t} \) and \( \frac{\eta_t}{\varsigma_t} = \varrho_{2,t} \). Then (E.9) is the price index in the planner’s economy.

Equation (E.10), obtained by combining the first order condition w.r.t \( N_{e,t} \) with \( L_{f,t+1}^* \) is the social planner’s entry condition for foreign skilled workers. It shows that the social planner will allow entry of foreign skilled workers till the technological cost of hiring foreign workers is equal to the expected social benefit — the expected discounted value of output produced by an additional foreign skilled worker.

The social planner’s equilibrium allocation can be solved using the 9 equations (E.6 - E.14) and 9 variables — \( \varrho_{1,t}, \varrho_{2,t}, Y_{1,t}, Y_{2,t}, C_{s,t}, C_{u,t}, Y_t^c, L_{f,t}, N_{e,t} \).

### Appendix F Alternate Policy: Market Driven Allocation of Permits

**Steady-State Aggregate Demand Schedule and Equilibrium for Permits**

Because the alternate policy is evaluated under the case of a binding cap, from the law of motion for foreign skilled workers, we have \( \bar{I}_f = (1 - \delta)\bar{N}_e/\delta \). From the market clearing in
the skilled sector, we have \( \tilde{\rho}_1 = \left( \frac{(1-\alpha)\tilde{z}(1+a_{lf})}{\alpha l_u} \right)^{\alpha-1} \). Then, using the optimal hiring condition (8) in the main text, we can get:

\[
\zeta^p = \frac{(1-\delta)\beta(a-1)(\theta-1)\tilde{\rho}_1 Z \tilde{z}}{(1-(1-\delta)\beta)\theta} - f_T
\] (F.1)

Substituting the steady-state value of aggregate prices and the steady state stock of foreign skilled labor into F.1, we get:

\[
\zeta^p = \frac{(1-\delta)(a-1)(\theta-1)\tilde{\rho}_1 Z \tilde{z}}{(1-(1-\delta)\beta)\theta} \left( \frac{\alpha \delta l_u}{(1-\alpha)\tilde{z}(\delta + a(1-\delta)N_e)} \right)^{1-\alpha} - f_T
\] (F.2)

This gives the market-clearing permit price as a function of the immigration policy cap (or the number of permits being allocated).

**Appendix G  Extension to the Baseline Model: Search and Matching Frictions**

Since there are no firing costs, firm’s surplus from each skilled worker is given by \( S_{i,t}^F(z) = \Gamma_{i,t}(z) \). Using \( S_{i,t}^W(z) = \frac{\eta_i}{1-\eta_i} S_{i,t}^F(z) \) (from the surplus sharing rule) and using \( S_{i,t}(z)^F = \Gamma_{i,t}(z) \), we can write the outside options of each worker as:

\[
\varpi_{d,t} = i_t(1-\delta_d)\frac{\eta_d}{(1-\eta_d)} \Gamma_{d,t}(z)
\] (G.1)

\[
\varpi_{f,t} = i_t(1-\delta_f)\frac{\eta_f}{(1-\eta_f)} \Gamma_{f,t}(z)
\] (G.2)

Using expressions for \( \Gamma_{i,t}, S_{i,t}^W \), and the surplus sharing rule, we can solve for wages in period \( t+1 \) as given in the equations below. Since period \( t+1 \) is arbitrary, we can get period \( t \) wages:

\[
w_{d,t+1} = \eta_d(\Xi_{t+1}Z_{t+1}z) + (1-\eta_d)\varpi_{d,t+1}
\] (G.3)

\[
w_{f,t+1} = \eta_f(\Xi_{t+1}Z_{t+1}z) + (1-\eta_f)\varpi_{f,t+1} + (1-\delta_f)E_t[\Gamma_{f,t+1}(\beta_{t,t+1} - \beta_{t+1}^f)]
\] (G.4)

\footnote{From the wage setting equations, we see that since domestic and foreign workers are perfect substitutes and contribute equally to production, any differences in wages between the two skilled workers has to be because of potential differences in the bargaining power or outside options of the two workers. Evidence shows that immigrants and domestic workers differ according to their outside options (Chassamboulli and Palivos, 2014), and in their separation rates (Battisti et al., 2014).}
where the last term in G.4 comes from different stochastic discount factors of firms and foreign households.

Profit maximization also implies that prices are a markup over marginal cost:

\[ \rho_{1,t}(z) = \frac{\theta}{\theta - 1} \Xi_t(z) \]  \hspace{1cm} (G.5)

Period \( t \) profits are given by:

\[ d_t(z) = \rho_{1,t}(z)y_{1,t}(z) - w_{d,t}d_t(z) - w_{f,t}l_{f,t}(z) - \kappa v_t(z) - (f_R + f_T)q_{f,t}v_t(z) \]  \hspace{1cm} (G.6)

### Aggregate Accounting in the Search and Matching Framework

Since firms face the same costs and the same probabilities of being matched with workers, it can be shown as in Cacciatore (2014) that \( \Xi_t(z) = \Xi_t/z \) (the real marginal cost is symmetric across producers up to firm-specific productivity differentials). This facilitates aggregation as in the standard Melitz (2003) model. The ‘tilde’ variables and uppercase variables denote aggregates.

Aggregate accounting implies that aggregate output is equal to the aggregate demand in units of the consumption basket: \( \tilde{\rho}_{1,t}Y_{1,t} + \rho_{2,t}Y_{2,t} = Y_{t}^c \), where \( Y_{t}^c = C_{u,t} + C_{s,t} + C_{f,t} + \kappa V_t + (f_R + f_T)q_{f,t}V_t \) (as vacancy posting costs and immigration costs are sunk). As in the baseline model, the goods market clearing for the aggregate skilled sector output is given by \( Y_{1,t} = \alpha Y_{t}^c/\tilde{\rho}_{1,t} \) and for unskilled sector output is given by \( Y_{2,t} = (1 - \alpha)Y_{t}^c/\rho_{2,t} \).

The decentralized equilibrium allocation is determined as a solution of 26 equations in 26 endogenous variables: \( \tilde{\rho}_{1,t}, \rho_{2,t}, C_{f,t}, C_{d,t}, C_{u,t}, w_{d,t}, w_{f,t}, w_{u,t}, \tilde{d}_t, \tilde{l}_{f,t}, \tilde{l}_{d,t}, u_{d,t}, u_{f,t}, q_t, q_{d,t}, q_{f,t}, Y_{1,t}, V_t, \Gamma_{d,t}, \Gamma_{f,t}, \tilde{\Xi}_t, \tilde{\omega}_d, \tilde{\omega}_f, Y_{2,t}, \tilde{i}_t, Z_t \)

\(^3\)These together imply that \( \rho_{1,t}y_{1,t}/\alpha = \rho_{2,t}y_{2,t}/(1 - \alpha) \)
Equation Summary
The summary of conditions can be written as:

\[ 1 = (\tilde{\rho}_{1,t})^{\alpha}(\rho_{2,t})^{1-\alpha} \] (G.7)
\[ C_{f,t} = w_{f,t}\tilde{l}_{f,t} \] (G.8)
\[ C_{d,t} = w_{d,t}\tilde{l}_{d,t} + d_{t} \] (G.9)
\[ C_{u,t} = w_{u,t}\tilde{L}_{u} \] (G.10)
\[ u_{d,t} = \tilde{L}_{d} - \tilde{l}_{d,t} \] (G.11)
\[ u_{f,t} = \tilde{L}_{f} - \tilde{l}_{f,t} \] (G.12)
\[ q_{d,t} = \frac{u_{d,t}}{u_{d,t} + u_{f,t}}\chi\left(\frac{V_{t}}{u_{d,t} + u_{f,t}}\right)^{-\epsilon} \] (G.13)
\[ q_{f,t} = \frac{u_{f,t}}{u_{d,t} + u_{f,t}}\chi\left(\frac{V_{t}}{u_{d,t} + u_{f,t}}\right)^{-\epsilon} \] (G.14)
\[ q_{t} = \frac{\tilde{N}_{v,t}}{q_{f,t}v_{t}} \] (G.15)
\[ Y_{1,t} = Z_{t}\tilde{z}(\tilde{l}_{f,t} + \tilde{l}_{d,t}) \] (G.16)
\[ \tilde{l}_{d,t+1} = (1 - \delta_{d})(\tilde{l}_{d,t} + \tilde{V}_{t}q_{d,t}) \] (G.17)
\[ \tilde{l}_{f,t+1} = (1 - \delta_{f})(\tilde{l}_{f,t} + \tilde{V}_{t}q_{f,t}q_{t}) \] (G.18)
\[ \kappa = (1 - \delta_{d})\Gamma_{d,t}q_{d,t} + (1 - \delta_{f})\Gamma_{f,t}q_{f,t}q_{t} - (f_{R} + f_{T})q_{f,t} \] (G.19)
\[ \Gamma_{d,t} = E_{t}[\beta_{t,t+1}\{\tilde{Z}_{t+1}Z_{t+1}\tilde{z} - w_{d,t+1} + (1 - \delta_{d})\Gamma_{d,t+1}\}] \] (G.20)
\[ \Gamma_{f,t} = E_{t}[\beta_{t,t+1}\{\tilde{Z}_{t+1}Z_{t+1}\tilde{z} - w_{f,t+1} + (1 - \delta_{f})\Gamma_{f,t+1}\}] \] (G.21)
\[ w_{d,t} = \eta_{d}(\tilde{Z}_{t+1}Z_{t+1}\tilde{z}) + (1 - \eta_{d})\varpi_{d,t} \] (G.22)
\[ w_{f,t} = \eta_{f}(\tilde{Z}_{t+1}Z_{t+1}\tilde{z}) + (1 - \eta_{f})\varpi_{f,t} + (1 - \delta_{f})E_{t}[\Gamma_{f,t+1}(\beta_{t,t+1} - \beta_{f,t,t+1}^{f})] \] (G.23)
\[ \varpi_{d,t} = i_{t}(1 - \delta_{d})\frac{\eta_{d}}{1 - \eta_{d}}\Gamma_{d,t} \] (G.24)
\[ \varpi_{f,t} = i_{t}(1 - \delta_{f})\frac{\eta_{f}}{1 - \eta_{f}}\Gamma_{f,t} \] (G.25)
\[ i_{t} = \chi\left(\frac{V_{t}}{u_{d,t} + u_{f,t}}\right)^{1-\epsilon} \] (G.26)
\[ \tilde{\rho}_{t} = \frac{\theta}{\theta - 1}\tilde{Z}_{t} \] (G.27)
\[ d_{t} = \tilde{\rho}_{1,t}Y_{1,t} - w_{d,t}\tilde{l}_{d,t} - w_{f,t}\tilde{l}_{f,t} - \kappa v_{t} - f_{R}q_{f,t}v_{t} \] (G.28)
\[ Y_{2,t} = Z_{t}\tilde{l}_{u} \] (G.29)
\[ \rho_{2,t} = \frac{w_{u,t}}{Z_{t}} \] (G.30)
\[ \tilde{\rho}_{1,t}Y_{1,t}/\alpha = \rho_{2,t}Y_{2,t}/(1 - \alpha) \] (G.31)
\[ \log(Z_{t}) = \phi\log(Z_{t-1}) + e \] (G.32)
Steady State for the Search and Matching Model

The steady state for the search and matching model boils down to a system of 9 equations in 9 endogenous variables — $\tilde{l}_f$, $\tilde{l}_d$, $V$, $\tilde{\rho}_1$, $q_d$, $q_f$, $\varpi_d$, $\varpi_f$, and $i$. I show this for the steady state under the non-binding regime. Using (G.20), (G.21), (G.22), and (G.23), we can write the steady-state surplus obtained from domestic and foreign workers as

$$\Gamma_d = (1 - \eta_d) \beta (\tilde{\Xi} Z \tilde{z} - \varpi_d) (1 - (1 - \delta_d) \beta) q_d q - (f_R + f_T) q_f$$

and

$$\Gamma_f = (1 - \eta_f) \beta (\tilde{\Xi} Z \tilde{z} - \varpi_f) (1 - (1 - \delta_f) \beta) q_f q - (f_R + f_T) q_f$$

Using (G.37), (G.38), and (G.39) can be written as:

$$L_d = (1 - \delta_d) V q_d / \delta_d$$

and

$$L_f = (1 - \delta_f) V q_f / \delta_f$$

The steady-state outside option of domestic skilled workers is given by:

$$\varpi_d = (1 - \delta_d) i \eta_d / (1 - \eta_d) \Gamma_d$$

The steady-state outside option of foreign skilled workers is given by:

$$\varpi_f = (1 - \delta_f) i \eta_f / (1 - \eta_f) \Gamma_f$$

The steady-state job finding probability of skilled workers is given by:

$$\text{For the binding case, there will be 10 equations in 10 variables with } q = \tilde{N}_e / (q_f V) \text{ as an additional equation for determining } q.
\[ i = \chi \left( \frac{V}{\bar{L}_d + \bar{L}_f - \bar{L}_d - \bar{L}_f} \right)^{1-\epsilon} \]  

(G.41)

(G.33) - (G.41) constitute a system of 9 equations in 9 variables.

**Relationship Between Domestic Skilled Employment and the Immigration Cap**

When the cap binds in steady state, the probability of hiring a foreign skilled worker is given by \( q = \frac{N_e}{q_f V} \), and therefore from the law of motion of foreign labor, we get that the steady-state foreign labor employed is \( \bar{L}_f = (1 - \delta_f) \bar{N}_e / \delta_f \). To see relationship between \( \bar{L}_d \) and \( \bar{N}_e \), take the ratio of (G.37) and (G.38), and use \( \bar{L}_f = (1 - \delta_f) \bar{N}_e / \delta_f \), and after rearranging terms, we get:

\[
\bar{L}_d = \bar{L}_d \left( \frac{\bar{L}_f \delta_d}{(1 - \delta_d) \bar{N}_e} - \frac{\delta_d (1 - \delta_f)}{\delta_f (1 - \delta_d)} \right) + 1 \]  

(G.42)

i.e. as the cap on foreign skilled workers increases, for a given pool of domestic and foreign workers in the labor force, the aggregate domestic skilled workers employed increases. Intuitively, an increase in the entry cap increases firms’ incentive to post more vacancies as there is a higher probability that a foreign worker that was matched would eventually be able to join the firm.
References


