Knowledge Diffusion, Trade and Innovation across Countries and Sectors

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Abstract

We provide a unified framework to quantify the cross-country and cross-sector interactions between trade, innovation and knowledge diffusion. We study the effect of trade liberalization in a multi-country and multi-sector endogenous growth model in which comparative advantage and the stock of knowledge are endogenously determined by innovation and diffusion. A reduction in trade costs induces a reallocation of innovation and production comparative advantage across sectors, which translates into higher growth in the counterfactual balanced growth path (BGP). Welfare gains from trade are significantly larger than in static multi-sector models of trade. Heterogeneous knowledge diffusion across country-sectors further amplifies the specialization effects of trade-induced R&D reallocation and increases dispersion in comparative advantage, becoming an additional source of growth and welfare.

Keywords: Technology Diffusion; Knowledge Network; R&D; International Trade; Sectoral Linkages

JEL Classification: F12, O33, O41, O47

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1 Introduction

The world has increasingly become a highly interconnected network of countries and sectors that not only trade goods and services but also exchange ideas. Much has been understood about how the benefits of trade liberalization may spread across sectors through production input-output linkages, thanks to the growing literature on sectoral linkages (Eaton, Kortum, Neiman, and Romalis (2016); Caliendo and Parro (2015); Costinot, Donaldson, and Komunjer (2012); Arkolakis, Costinot, and Rodríguez-Clare (2012)). At the same time, countries and sectors are linked through another dimension that has been studied much less so far—innovation and knowledge spillovers and how they interact with trade to affect productivity and growth. Knowledge spillovers are pervasive and highly heterogeneous—knowledge in one sector of a country can be used to enhance innovation in another country-sector, and much like production input-output linkages, knowledge linkages across countries and sectors are far from uniform (Cai and Li (2018) and Acemoglu, Akcigit, and Kerr (2016)). In a world with multiple sectors, while trade affects countries' knowledge composition and diffusion, the reverse link is also prominent—productivity differences induced by innovation and diffusion also condition the patterns of trade and aggregate growth.[1]

This paper provides a novel unified framework to quantify the interactions between trade, innovation and knowledge diffusion in a multi-sector environment in which country-sectors are interconnected both in the product and knowledge spaces. Our framework is a multi-country and multi-sector endogenous growth model in which productivity evolves endogenously through innovation and knowledge diffusion. The model advances existing multi-sector models of trade with input-output linkages (see Caliendo and Parro (2015), and Costinot, Donaldson, and Komunjer (2012)) by adding dynamics through innovation and diffusion across countries and sectors.

Countries and sectors are heterogeneous in production, in the efficiency of innovation and in the strength of knowledge spillovers. The model is composed of two blocks. A 'trade block' that determines the static equilibrium, taking as given country-sector productivity. A 'growth block' in which productivity evolves endogenously through innovation and knowledge diffusion. The trade block is built upon the Ricardian model of trade with Bertrand competition (Bernard, Eaton, Jensen, and Kortum (2003)). The growth block is modeled in the spirit of Eaton and Kortum (1996, 1999), who analyze an endogenous growth model without trade or sectoral linkages.

[1] A vast empirical literature has documented the significant impact of trade on innovation (e.g. Aghion, Bergeaud, Lequien, and Melitz (2018); Bloom, Draca, and Van Reenen (2016); Autor, Dorn, Hanson, Pisano, and Shu (2016); Bustos (2011); Lileeva and Trefler (2010)) and its effect on knowledge diffusion (e.g. Keller (2010); MacGarvie (2006)). Regarding the reverse relationship, Santacreu and Zhu (2018) and Cameron, Proudman, and Redding (2000) find that innovation and knowledge diffusion help determine trade patterns. Hausmann, Hwang, and Rodrik (2007) and Hidalgo, Klinger, Barabási, and Hausmann (2007) argue that a country's knowledge composition conditions its income.

Firms choose their research effort to create new ideas, building on existing stock of knowledge. Ideas diffuse across *all* sectors and countries, although the speed of diffusion may differ.[2] In our model, knowledge diffusion increases the stock of knowledge in two ways. First, it increases the stock of knowledge in the receiving country-sector. Second, the increase in the stock of knowledge in turn enhances the innovation efficiency there, further fostering creation of new knowledge. While all new ideas contribute to the stock of knowledge, only ideas with the highest quality would be adopted for production. Different from recent papers in the literature (e.g. Buera and Oberfield (2016), Grossman and Helpman (1991)), diffusion in our model takes place independently from trade, reflecting that in practice there are other channels such as foreign direct investment, migration or direct communications, among others, that help diffuse ideas across countries and sectors (see Fons-Rosen, Kalemli-Ozcan, Sorensen, Villegas-Sanchez, and Volosovych (2017), Ramondo and Rodríguez-Clare (2013), Keller (1998)). Innovation and diffusion determine the distribution of knowledge stock across countries and sectors and economic growth. We solve for the BGP of the model in which all countries and sectors grow at a common and constant rate.

In our multi-sector model, trade liberalization induces an endogenous reallocation of research effort across countries and sectors, increasing aggregate innovation and long-run growth. This contrasts with standard one-sector models, in which trade has a negligible effect on innovation and growth as the market size effect exactly offsets the competition effect (Eaton and Kortum (2006); Atkeson and Burstein (2010); Buera and Oberfield (2016)). In addition, comparative advantage of production is endogenous, and it results in welfare gains from trade beyond the specialization effects present in static multi-sector models.[3] Heterogeneous knowledge spillovers across country-sectors could further amplify or dampen the specialization effect of trade-induced R&D reallocation on growth and welfare, depending on the exact pattern of diffusion. If diffusion forces are stronger for already innovative countries (i.e. innovative countries also diffuse ideas fast among themselves), the dispersion of comparative advantage resulting from R&D reallocation could be even higher, *amplifying* the specialization effect. On the other hand, if less innovative country-sectors are better at absorbing knowledge, knowledge diffusion could *dampen* the aforementioned specialization effect by enabling faster productivity convergence and by making the stock of knowledge more similar across country-sectors. Our estimation of the diffusion pattern suggests that it is the former that dominates in the data.

[2] Although in contrast with the technology adoption literature, which assumes only the best knowledge is adopted or diffused from the technologically frontier economies (Comin and Hobijn (2010)), this assumption is supported by observations that a non-negligible amount of novel inventions are initiated outside the traditional frontier economies.

[3] In Levchenko and Zhang (2016) and Hanson, Lind, and Muendler (2015), comparative advantage is also endogenous. They use gravity equations to estimate comparative advantage and they characterize its the evolution over time.

We calibrate the model to data on production, bilateral trade, R&D intensity, and patent citations for 19 countries and 19 sectors (including a nontradable sector). The main empirical contribution is to use patent citations and R&D expenditure data to estimate key parameters related to knowledge spillovers and innovation. In particular, we calibrate the speed of cross-country and cross-sector knowledge spillovers by fitting a citation function that depends on the diffusion lag among other factors.[4] Specific advantages of this approach are that (i) it allows for patents in different country-sectors to vary in terms of their obsolescence rates and their ability to generate spillovers, in addition to their diffusion speed; (ii) it does not need to impose the assumption that citations are mapped into knowledge spillovers one-to-one. Since the diffusion speed parameters are estimated jointly with other parameters that also govern the citation process, this procedure helps obtain a more accurate estimate of our parameters of interest—diffusion speed across country and sectors. The innovation parameters are calibrated by jointly solving the two blocks of the model. We use an algorithm based on excess demand iterations to solve for the static trade equilibrium given a distribution of productivity, and a fixed-point algorithm to solves for the endogenous growth rate and average productivity using the growth block of the model. This algorithm helps us determine the exogenous cross-country and cross-sector efficiency of innovation as well as the stock of knowledge of the economy on the BGP.

We conduct a counterfactual exercise to study the effect of a 25% reduction in trade costs on innovation, comparative advantage and growth along the BGP. Changes in trade costs have a non-negligible effect on innovation in our model, as there is a reallocation of R&D towards sectors in which the country has comparative advantageAs our estimated speed of diffusion implies that innovative countries are also faster at absorbing and diffusing knowledge from and to each other, the presence of knowledge spillovers coupled with research effort reallocation leads to higher dispersion in endogenous comparative advantage, both across countries for a given sector, and across sectors within a country. As a result, we find that productivity grows at a higher rate in the counterfactual BGP, increasing from 2.8% to 3.4%. These results have implications for welfare. We find that all countries experience positive welfare gains from lowering trade costs, although the size of the gains differ, ranging from 8% to 34%.

Finally, we study the role of the different channels by considering three alternative versions of our model: (i) a static model in which productivity is taken as exogenously given, (ii) homogeneous knowledge spillovers across counties and sectors, and (iii) almost negligible knowledge spillovers across countries and sectors. We recalibrate each version to match the same moments of the data. We find that welfare gains from trade are lower and less disperse in those alternative models, which

[4]Our method extends the approach proposed in Caballero and Jaffe (1993) into a multi-country multi-sector environment.

exposes the importance of considering multi-sector models with heterogeneous knowledge spillovers in quantifying the effect of trade liberalization. In particular, the average gain from trade in the dynamic model is 2 times larger than those in the static model, 1.3 times larger than in a model with homogeneous cross-country cross-sector knowledge spillovers and 1.2 times larger than in the model with (almost) negligible cross-country cross-sector knowledge spillovers. Finally, we find that the presence of heterogeneous knowledge spillovers plays an important role on the impact of trade on growth and welfare. If cross-country-sector knowledge spillovers are uniform, then there is zero dispersion in the stock of knowledge across-country-sectors and hence no dispersion in productivity. In addition, the presence of knowledge diffusion greatly enhances the growth effect of trade liberalization, as countries and sectors now have access to innovations developed elsewhere.

A few points merit mention regarding our calibration strategy for knowledge diffusion. Naturally, direct measures of technology spillovers do not exist. Patent citation data have been used extensively in a growing body of economic research as a way of tracking technological diffusion across time and geographic boundaries.[5] One patent application citing an earlier patent generally indicates that the applicant has benefited from the earlier patent. Although patent citations provide valuable rare insight into the knowledge spillovers, we first note as a caveat that they are subject to certain limitations. For example, they do not capture technology transfer or any types of learning that do not result in a patent, such as reverse-engineer, imitation or replication. Moreover, a substantial amount of inventions are not patented but are protected through trade secrets and other informal mechanisms. Although there are several considerations, all difficult to quantify, there is no pervasive evidence suggesting that we should expect nonpatented knowledge to diffuse at a systematically and significantly different speed than patented knowledge. Second and more importantly, our estimation procedure builds on the approach proposed by Caballero and Jaffe (1993) by extending it into a multi-country multi-sector environment. This approach is designed to incorporate, in addition to heterogeneous cross-country-sector diffusion speed, how variations in obsolescence rates, quality of patents and citations in different country-sectors affect citations. Controlling for these additional variations with a fairly rich structure of citation process helps to obtain a more accurate estimation of the diffusion speed. Third, consider the alternative regression approach in the literature which estimates how related domestic TFP in a certain sector is with foreign R&D capital stock in another sector and uses the estimated elasticity to proxy spillovers. Apparently such estimation requires data that are either not available (such as sectoral capital stock and R&D stock) or hard to measure (such as sectoral TFP) for most countries. In addition, using outcome-based measures may confound technology spillovers with other factors that lead to

[5] For example, see Li (2014); Jaffe, Trajtenberg, and Henderson (1993); Thompson and Fox-Kean (2005); Peri (2005); Griffith, Lee, and Van Reenen (2011).

comovement between country-sectors.

Literature Review Our paper connects and extends existing theoretical literature on the relationship between trade and innovation (e.g. Somale (2014); Atkeson and Burstein (2010); Rivera-Batiz and Romer (1991); Grossman and Helpman (1991)), between trade and diffusion (e.g. Perla, Tonetti, and Waugh (2015); Sampson (2016)), and between innovation and diffusion (e.g. Eaton and Kortum (1999); Eaton and Kortum (1996)). Yet, rarely are trade, innovation and diffusion analyzed in one unified framework. Notable exceptions are Buera and Oberfield (2016), Santacreu (2015), Eaton and Kortum (2006) and Lind and Ramondo (2018). In both Buera and Oberfield (2016) and Santacreu (2015), trade is the only channel for cross-border exchange of ideas. We allow knowledge linkages and trade linkages to operate separately, even though trade liberalization would impact knowledge accumulation and the strength of diffusion as an endogenous outcome. In their survey paper, Lind and Ramondo (2018) consider multinational production as the channel for diffusion (as in Ramondo and Rodríguez-Clare (2013)). Eaton and Kortum (2006)'s theoretical investigation analyzes the effect of faster diffusion and lower trade barriers on the incentive to innovate. In general, our main departure from all these papers is that we consider a *multi-sector environment* in which sectors are interconnected both through the input-output linkages and knowledge linkages. As thoroughly discussed in Eaton and Kortum (2006), in the absence of diffusion, the one-sector model predicts the same share of resources towards research regardless of trade barriers. Our multi-sector model, however, generates changes in aggregate innovation and growth via the additional mechanism of research reallocation across sectors and its interactions between knowledge spillovers.

This paper also joins forces on the growing literature quantifying dynamic gains from trade (Perla, Tonetti, and Waugh (2015), Buera and Oberfield (2016), Akcigit, Ates, and Impullitti (2018), Ramondo and Rodríguez-Clare (2013), Ravikumar, Santacreu, and Sposi (2017)). In a Melitz type of model, Perla, Tonetti, and Waugh (2015) find that lowering trade barrier induces faster technology adoption and growth as the relative profit gains between the average and marginal adopting firms become larger. However, they obtain lower gains from trade owing to a decrease in the number of varieties due to entry. Akcigit, Ates, and Impullitti (2018) focuses on the role of strategic interaction between firms in shaping their innovation responses to policy changes (such as tariffs and R&D subsidies) and the dynamic gains from trade. Ramondo and Rodríguez-Clare (2013) study the interaction between trade and multinational production. Ravikumar, Santacreu, and Sposi (2017) analyze the role of capital accumulation on welfare gains from trade. Although each has a different focus, these studies also show the gains from trade increase substantially compared to the static counterparts of those models, a result also found in our paper.

In analyzing multi-sector trade models of innovation with endogenous comparative advantage, our paper relates to two recent works by Somale (2014) and Sampson (2016). Somale (2014) studies the two-way relationship of trade and innovation in a multi-sector semi-endogenous model with only level effects of research on the BGP, while our model allows for growth effect as well. More importantly we analyze the three-way interactions between trade, innovation and knowledge spillovers, and allow for sectors to be interconnected. Our analysis shows that both considerations of knowledge spillovers and interconnections between country-sectors are important in understanding the endogenous evolution of comparative advantage and quantitatively contributes significantly to the welfare gains. Sampson (2016) develops a theoretical Armington framework of innovation and learning as sources of endogenous comparative advantage. Our emphasis is on the quantification of the model, which allows for useful counterfactual analyses.

The paper is also related to Arkolakis, Ramondo, Rodríguez-Clare, and Yeaple (2018), who develop a quantitative one-sector model of multinational production and trade in which comparative advantage in production and in innovation are modeled separately. They study the effect of openness on specialization in innovation and production. They focus on the role of multinational production to determine specialization in innovation. We focus on the role of trade to determine specialization in innovation and production, and the role of knowledge spillovers to spread innovations around the world.

The paper also contributes to a burgeoning strand of research that analyzes the implications of interconnections between different sectors (e.g. Carvalho (2014), Carvalho and Gabaix (2013), Acemoglu et al. (2012) and Gabaix (2011) in close economies, and (Eaton et al. (2016); Caliendo and Parro (2015); Costinot, Donaldson, and Komunjer (2012) in open-economy setup). Most of these papers focus on factor-demand linkages of production. In addition to the input-output linkages, this paper also simultaneously considers the intrinsic interconnections of technologies embodied in different sectors, which turns out to be significant and relevant when studying innovation and diffusion (Cai and Li (2016, 2018), Acemoglu, Akcigit, and Kerr (2016)). Most related, Cai and Li (2016) study knowledge spillovers across sectors within a country and how trade costs affect the distribution of endogenous knowledge accumulation across sectors in a theoretical model. Different from our paper, however, cross-country knowledge diffusion is not considered and intermediate input-demand linkages across sectors are absent.

2 The Model

We develop a general equilibrium model of trade in intermediate goods, with sector heterogeneity and input-output linkages, in which technology evolves endogenously through innovation and knowledge diffusion. The model can be decomposed into two blocks: (i) a *trade block* which, given a distribution of technology and trade barriers, determines the static equilibrium, and (ii) a *growth block*, which determines the dynamics of technology through innovation and knowledge spillovers.

There are M countries and J sectors. Countries are denoted by i and n and sectors are denoted by j and k. Labor is the only factor of production, which is assumed to be mobile across sectors within a country but immobile across countries. Ricardian trade takes place in the form of intermediate goods.

2.1 Consumers

In each country there is a representative household with life-time utility

$$U_{nt} = \int_{t=0}^{\infty} \rho^t \log(C_{nt}) dt, \tag{1}$$

where $\rho \in (0,1)$ is the discount factor and C_{nt} represents consumption of country n at time t.

The household consumes and finances R&D activities of the entrepreneurs and owns all the firms. In return, she receives labor income and the profits of the entrepreneurs.

The budget constraint of the household is given by

$$P_{nt}C_{nt} + \dot{a}_{nt} = r_{nt}a_{nt} + \Pi_{nt} + W_{nt}L_{nt} + TD_{nt},$$

where P_{nt} is the price of the final good, to be defined later, a_{nt} is the household's holding of firms shares, r_{nt} is the return on assets, Π_{nt} is the profit of firms that the household obtains from financing firm's R&D activities, $W_{nt}L_{nt}$ is the labor income and TD_{nt} denotes trade deficit.

2.2 Final Production

In each country n, a domestic final producer uses the composite output from each domestic sector j at time t, Y_{nt}^j, to produce a non-traded final output, Y_{nt}, according to the following Cobb-Douglas production function:

$$Y_{nt} = \prod_{j=1}^{J} \left(Y_{nt}^j\right)^{\alpha^j}, \tag{2}$$

with $\alpha^j \in (0,1)$ being the share of sector production in total final output, and $\sum_{j=1}^{J} \alpha^j = 1$.

Final producers operate under perfect competition, with profits given by

$$\Pi_{nt} = P_{nt}Y_{nt} - \sum_{j=1}^{J} P_{nt}^{j} Y_{nt}^{j},$$

where P_{nt} is the price of the final product and P_{nt}^{j} is the price of the composite good produced in sector j from country n.

Under perfect competition, the price charged by the final producer to the consumers is equal to the marginal cost; that is

$$P_{nt} = \prod_{j=1}^{J} \left(\frac{P_{nt}^{j}}{\alpha^{j}} \right)^{\alpha^{j}}.$$

The demand by final producers for the sectoral composite good is given by

$$Y_{nt}^{j} = \alpha^{j} \frac{P_{nt}}{P_{nt}^{j}} Y_{nt}.$$

2.3 Intermediate Producers

In each sector j there is a continuum of intermediate producers indexed by $\omega \in [0,1]$ that use labor, $l_{nt}^{j}(\omega)$, and a composite intermediate good from every other sector k in the country, $m_{nt}^{jk}(\omega)$, to produce a variety ω according to the following constant returns to scale technology[6]:

$$y_{nt}^{j}(\omega) = z_{n}^{j}(\omega)[l_{nt}^{j}(\omega)]^{\gamma^{j}} \prod_{k=1}^{J} [m_{nt}^{jk}(\omega)]^{\gamma^{jk}}, \tag{3}$$

with $\gamma^{j} + \sum_{k=1}^{J} \gamma^{jk} = 1$. Here γ^{jk} is the share of materials from sector k used in the production of intermediate ω is sector j, and γ^{j} is the share of value added. Firms are heterogeneous in their productivity $z_{n}^{j}(\omega)$.

The cost of producing each intermediate good ω is

$$c_{nt}^{j}(\omega) = \frac{c_{nt}^{j}}{z_{nt}^{j}(\omega)},$$

where c_{n}^{j} denotes the cost of the input bundle. With constant returns to scale in production, we have

$$c_{nt}^{j} = \Upsilon^{j} W_{nt}^{\gamma^{j}} \prod_{k=1}^{J} (P_{nt}^{k})^{\gamma^{jk}}, \tag{4}$$

[6]The notation in the paper is such that every time there are two subscripts or two superscripts, the one on the right corresponds to the source country and the one on the left corresponds to the destination country.

with $\Upsilon^j = \prod_{k=1}^{J}(\gamma^{jk})^{-\gamma^{jk}}(\gamma^j)^{-\gamma^j}$ and W_{nt} the nominal wage rate.

Intermediate producers operate with Bertrand competition. We characterize their equilibrium prices later in the paper.

2.4 Composite Intermediate Goods (Materials)

Each sector j produces a composite good at minimum cost by combining domestic and foreign varieties from that sector. Composite producers operate under perfect competition and buy intermediate products ω from the lowest cost supplier.

The production for a composite good in sector j and country n is given by the Ethier (1982) CES function,

$$Q^j_{nt} = \left(\int e^j_{nt}(\omega)^{1-1/\sigma} d\omega\right)^{\sigma/(\sigma-1)}, \tag{5}$$

where $\sigma > 0$ is the elasticity of substitution across intermediate goods and $e^j_{nt}(\omega)$ is the demand of intermediate goods from the lowest cost supplier in sector j.

The demand for each intermediate good ω is given by

$$e^j_{nt}(\omega) = \left(\frac{p^j_{nt}(\omega)}{P^j_{nt}}\right)^{-\sigma} Q^j_{nt},$$

where P^j_{nt} is the unit price of the composite intermediate good

$$P^j_{nt} = \left(\int p^j_{nt}(\omega)^{1-\sigma} d\omega\right)^{\frac{1}{1-\sigma}}. \tag{6}$$

and $p^j_{nt}(w)$ is the lowest price of intermediate good ω across all countries n.

Composite intermediate goods are used as final goods in the final production and as materials for the production of the intermediate goods:

$$Q^j_{nt} = Y^j_{nt} + \sum_{k=1}^{J} \int m^{kj}_{nt}(\omega) d\omega.$$

2.5 International Trade

Trade in goods is costly. In particular, there are iceberg transport costs so that shipping a good that is produced in sector j from country i to country n requires producing $d^j_{ni} > 1$ units of good in sector j and country i. We assume that the triangular inequality, $d^j_{ih}d^j_{hn} > d^j_{in}$. We follow Bernard et al. (2003) and assume Bertrand competition. With Bertrand competition, as with perfect competition, composite producers in each sector buy from the lowest cost supplier and the price charged by the producer relates to the production cost of the second-lowest producer.

Ricardian motives for trade are introduced as in Eaton and Kortum (2002), since productivity is allowed to vary by country-sector. The productivity of producing intermediate good ω in country i and sector j is drawn from a Frechet distribution described by T_{it}^{j} (which is endogenous and will be explained later) and the shape parameter θ: $F(z_i^j) = Pr\left[Z \leq z_i^j\right] = e^{-T_{it}^{j} z^{-\theta}}$. A higher T_{it}^{j} implies a higher average fundamental productivity of that country-sector, while a lower θ implies more dispersion of productivity across varieties. Here , T_{it}^{j} determines cross-sector comparative advantage, and θ determines intra-industry comparative advantage (see Costinot, Donaldson, and Komunjer (2012)).

Given these assumptions, Bernard et al. (2003) show that the price index of goods in sector j in country n is

$$P_{nt}^{j} = B\left(\Phi_{nt}^{j}\right)^{-1/\theta}, \tag{7}$$

with $B = \left[\frac{1+\theta-\sigma+(\sigma-1)(\bar{m})^{-\theta}}{1+\theta-\sigma}\Gamma\left(\frac{2\theta+1-\sigma}{\theta}\right)\right]^{1/(1-\sigma)}$ and

$$\Phi_{nt}^{j} = \sum_{i=1}^{M} T_{it}^{j}(d_{ni}^{j}c_{it}^{j})^{-\theta}. \tag{8}$$

For prices to be well defined, we assume $\sigma < (1+\theta)$.[7]

Consumers buy the final output at the price

$$P_{nt} = \prod_{j=1}^{J}\left(\frac{P_{nt}^{j}}{\alpha_n^j}\right)^{\alpha_n^j} \tag{9}$$

Expenditure shares Given the distributional assumptions of productivity, the probability that country i is the lowest cost supplier of a good in sector j to be exported to country n is

$$\pi_{nit}^{j} = \frac{T_{it}^{j}\left(c_{it}^{j}d_{ni}^{j}\right)^{-\theta}}{\Phi_{nt}^{j}}, \tag{10}$$

where c_{it}^{j} is defined in equation (4). Because there is a continuum of intermediate goods, π_{nit}^{j} is also the fraction of goods that sector j in country i sells to any sector in country n. In particular,

[7]Details of these derivations can be found in Bernard et al. (2003).

the share that country n spends on sector-j products from country i is

$$\pi^j_{nit} = \frac{X^j_{nit}}{X^j_{nt}}. \tag{11}$$

with $X^j_{nt} = P^j_{nt} Q^j_{nt}$ being total expenditures on goods from sector j and country n and X^j_{nit} the value of intermediate products from sector j that country n buys from country i.

2.6 Total Expenditures and Balanced Trade

Total expenditures on goods from sector j and country n are given by the sum of what the composite producers from each sector k and country i buys and the spending from final producers. Thus, X^j_n is given by

$$X^j_{nt} = \sum_{k=1}^{J} \gamma^{kj} \sum_{i=1}^{M} X^k_{it} \pi^k_{int} + \alpha^j P_{nt} Y_{nt}. \tag{12}$$

We assume trade is balanced period by period. Total imports in country n are given by

$$IM_{nt} = \sum_{i=1 i \neq n}^{M} \sum_{k=1}^{J} X^k_{nit} = \sum_{k=1}^{J} X^k_{nt} \sum_{i=1 i \neq n}^{M} \pi^k_{nit}. \tag{13}$$

Total exports in country n are given by

$$EX_{nt} = \sum_{i=1 i \neq n}^{M} \sum_{k=1}^{J} X^k_{int} = \sum_{i=1 i \neq n}^{M} \sum_{k=1}^{J} \pi^k_{int} X^k_{it}.$$

Balanced trade implies

$$EX_{nt} = IM_{nt}.$$

2.7 Endogenous Growth: Innovation and Knowledge Spillovers

So far, we have described the *trade block* of the model, which, given a distribution of technology, T^j_{it} and trade barriers, d^j_{ni}, determines the static equilibrium. Note that, different from static models of trade, T^j_{it} depends on t. Next we describe the *growth block* of the model, which determines the endogenous evolution of T^j_{it}. In this model, T^j_{nt} which also represents the stock of knowledge of sector j and country n.

Innovation and knowledge spillovers determine the endogenous evolution of the distribution of productivity. Innovation is conducted in a particular country and sector and requires R&D investment. Knowledge spillovers across countries and sectors are costless. Firms in a country and sector learn about technologies that have been developed elsewhere, despite with different lags. Both innovation and knowledge spillovers increase the stock of knowledge of a particular country and sector.

Innovation In each sector j and country n, there is a continuum of entrepreneurs that invest final output, R^j_{nt}, to come up with a new idea. Ideas are blueprints used to produce an intermediate good with higher efficiency.[8] Research efforts are targeted at any of the continuum of intermediate goods in that sector. In each country n and sector j, ideas are drawn at a Poisson rate given by

$$\lambda^j_n T^j_{nt} \left(s^j_{nt}\right)^{\beta_r}, \tag{14}$$

where λ^j_n is a country- and sector-specific parameter that determines the efficiency of innovation, T^j_{nt} the stock of knowledge in sector j, $s^j_{nt} = R^j_{nt}/\bar{Y}_t$, with $\bar{Y}_t$ the world output in the BGP, and $\beta_r \in (0,1)$ is a parameter governing diminishing returns to R&D investment. This process ensures that there is a balanced-growth path without scale effects (see Eaton and Kortum (1996, 1999) and Santacreu (2015)). In our specification for the Poisson arrival of new ideas, $\lambda^j_n T^j_{nt}$ determines comparative advantage in innovation, which depends on an exogenous component, λ^j_n, as well as an endogenous component, T^j_{nt}. All else being equal, countries that have accumulated more knowledge stock over time (i.e., a higher T^j_{nt}) become more productive at conducting innovation.

As it is standard in the quality-ladders literature, an idea is the realization of two random variables. One is the good ω to which the idea applies. An idea applies to only one good in the continuum. The good ω with which it is associated is drawn from the uniform distribution $[0,1]$. The other is the quality of the idea q, which is drawn from a Pareto distribution of qualities, $H(q) = 1 - q^{-\theta}$. In equilibrium, only the best idea for each input is actually used to produce an intermediate good in any sector and country. In that case, the idea would be used to produce an intermediate product ω in sector j and country n with efficiency $z^j_n(\omega)$. Therefore, the efficient technology $z^j_n(\omega)$ for producing good ω in country n is the best idea for producing it yet discovered (see Eaton and Kortum (2006)).

The stock of ideas at time t in each sector j and country n is T^j_{nt}. Because there is a unit interval of intermediate goods, the number of ideas for producing a specific good is Poisson with parameter T^j_{nt}. This Poisson arrival implies that the quality distribution of best ideas is $F(q) = e^{-T^j_{nt} q^{-\theta}}$.[9] Therefore, the quality distribution of successful ideas inherits the distribution of productivity of

[8] We model the innovation process within each industry in a country as in Kortum (1997).

[9] Under poisson arrival rate of new ideas, the probability of k ideas for producing a good by period t in sector j and country n is $\left(T^j_{nt}\right)^k e^{-T^j_{nt}}/k!$. If there are k ideas, the probability that the best one is below the best quality q is $[H(q)]^k$. Summing over all possible k, we have $F(q) = e^{-T^j_{nt} q^{-\theta}}$.

the intermediate goods produced in a country. Our probabilistic distribution assumption for the quality of an idea implies that the probability of an idea being successful (i.e. being the best idea) is $1/T^j_{nt}$.

Entrepreneurs finance R&D activities by issuing equity claims to households. These claims pay nothing if the entrepreneur is not successful in introducing a new technology in the market, and it pays the stream of future profits if the innovation succeeds. Because of the probabilistic distribution of productivity, entrepreneurs are indifferent to what product ω to devote their efforts, all products within a sector deliver the same expected profit. Innovators choose the amount of R&D investment, in terms of final output, R^j_{nt}, to maximize

$$\lambda^j_n T^j_{nt} \left(s^j_{nt}\right)^{\beta_r} V^j_{nt} - P_{nt} R^j_{nt}$$

Here, V^j_{nt} is the value of an innovation created in sector j and country n, which is the expected flow of profits that will last until a new producer is able to produce the good at a lower cost. It is given by

$$V^j_{nt} = \int\limits_t^\infty e^{-\int_t^s r_{iu} du} \frac{\Pi^j_{ns}}{T^j_{ns}} ds, \tag{15}$$

where as mentioned earlier $1/T^j_{nt}$ governs the probability of an idea being successful, and Π^j_{nt} denotes total profits generated from selling the goods in all countries, which can be expressed as

$$\Pi^j_{nt} = \frac{\sum_{i=1}^M \pi^j_{int} X^j_{it}}{1+\theta}.$$

The expression for V^j_{nt} introduces a competitive effect, by which the larger the stock of knowledge in a sector-country, the lower the probability that the new idea lowers the cost there. Furthermore, conditional on the idea being successful, expected profits of the innovator are determined by the probability that the intermediate good produced with her idea is produced at the lowest cost, which is determined by π^j_{int}. In this specification, the probability that an idea in sector j and country n is successful in country i is given by $\frac{\pi^j_{int}}{T^j_{nt}}$.

The first-order condition for optimal R&D investment is

$$s^j_{nt} = \left(\beta_r \lambda^j_n T^j_{nt} \frac{V^j_{nt}}{P_{nt} Y_{nt}} \frac{Y_{nt}}{\bar{Y}_t}\right)^{\frac{1}{1-\beta_r}}. \tag{16}$$

Knowledge Spillovers New ideas created in each sector j and country n increase its own stock of knowledge, T^j_{nt}. Furthermore, ideas can diffuse across sectors and countries, contributing to the stock of knowledge elsewhere.

Diffusion takes time. An idea discovered at time t in country i and sector k diffuses to country n and sector j at time $t + \tau_{ni}^{jk}$. We assume that the diffusion lag, τ_{ni}^{jk}, follows an exponential distribution with parameter ε_{ni}^{jk}: $Pr[\tau_{ni}^{jk} \leq x] = 1 - e^{-\varepsilon_{ni}^{jk} x}$. Thus, ε_{ni}^{jk} is the speed of diffusion from country i sector k to country n sector j and its inverse is the mean diffusion lag.

An idea, therefore, may be the outcome of domestic research in the same sector, or may arrive from other countries or from other sectors. It could be the result of recent innovation or previous ones and diffused here. Summing over the past research conducted in all countries and sectors, the flow of ideas diffusing to country n and sector j is given by

$$\dot{T}_{nt}^{j} = \sum_{i=1}^{M} \sum_{k=1}^{J} \varepsilon_{ni}^{jk} \int_{-\infty}^{t} e^{-\varepsilon_{ni}^{jk}(t-s)} \lambda_i^k T_{is}^k \left(s_{is}^k\right)^{\beta_r} ds. \tag{17}$$

The evolution of the stock of knowledge in sector j and country n at time t depends on the past research outcome by each other country-sector up to time t and diffused at rate ε_{ni}^{jk}. We assume every idea eventually diffuses to every other country-sector (i.e. the ε_{ni}^{jk} values are strictly positive).

2.8 Resource Constraint

Final output in a given country is used for either consumption or R&D investment. The resource constraint equation is

$$Y_{nt} = C_{nt} + \sum_{j=1}^{J} R_{nt}^{j}. \tag{18}$$

3 Balanced Growth Path

We define the balanced growth (BGP) as an equilibrium in which all variables grow at a constant rate. In our model, growth along the BGP is endogenous and it depends on policy parameters. Changes in trade costs have both growth and level effects. We stationarize all the endogenous variables so that they are constant on the BGP: We denote the normalized variables with a hat, and remove all time subscripts in our derivation. Here we characterize the BGP growth rate of the economy and provide details about the normalization of the endogenous variables in Appendix B.

Cross-country and cross-sector knowledge spillovers guarantee that the stock of knowledge T_n^j grows at a constant rate, g_T, which is common across all countries and sectors.

From equation (15), the value of an innovation on the BGP can be expressed as

$$\hat{V}_n^j = \left(\frac{1}{r - g_T/\theta + g_T}\right) \frac{\sum_{i=1}^{M} \pi_{in}^{j} \hat{X}_i^j}{(1+\theta)}, \tag{19}$$

where $\hat{V}_n^j = \frac{V_n^j T_n^j}{W_M}$ and $\hat{X}_i^j = \frac{X_i^j}{W_M}$. We impose $r - g_T/\theta + g_T > 0$. Substituting this expression into equation (16), the optimal R&D intensity can be expressed as

$$s_n^j = \left(\beta_r \lambda_n^j \frac{1}{(1+\theta)} \frac{1}{r - g_T/\theta + g_T} \frac{\sum_{i=1}^M \pi_{in}^j \hat{X}_i^j}{\hat{Y}_n} \frac{\hat{Y}_n}{\hat{\bar{Y}}} \right)^{\frac{1}{1-\beta_r}}. \tag{20}$$

with $\hat{Y}_n = \frac{Y_n}{W_M}$ and $\hat{\bar{Y}} = \frac{\bar{Y}}{W_M}$. Combining the above equation with equation (17), the growth rate of the stock of knowledge on the BGP is

$$g_T = \sum_{i=1}^M \sum_{k=1}^J \frac{\varepsilon_{ni}^{jk}}{g_T + \varepsilon_{ni}^{jk}} \lambda_i^k \frac{\hat{T}_i^k}{\hat{T}_n^j} \left(s_i^k \right)^{\beta_r}, \tag{21}$$

where $\hat{T}_i^k = \frac{T_i^k}{T_M^J}$. Finally, substituting equation (20) into the above equation, we obtain

$$g_T = \sum_{i=1}^M \sum_{k=1}^J \frac{\varepsilon_{ni}^{jk}}{g_T + \varepsilon_{ni}^{jk}} \lambda_i^k \frac{\hat{T}_i^k}{\hat{T}_n^j} \left(\frac{\beta_r \lambda_i^k}{r - g_T/\theta + g_T} \frac{\sum_{n=1}^M \pi_{ni}^k \hat{X}_n^k}{(1+\theta)\hat{Y}_i} \frac{\hat{Y}_i}{\hat{\bar{Y}}} \right)^{\frac{\beta_r}{1-\beta_r}}. \tag{22}$$

The growth rate of the stock of knowledge on the BGP depends positively on the speed of diffusion, expected future profits, and the efficiency of innovation, and it depends negatively on the dispersion parameter. Following Eaton and Kortum (1999), the Frobenius theorem guarantees that there is a unique growth rate on the BGP in which all countries and sectors grow at the same rate g_T. The expression for the growth rate can be expressed in matrix form as

$$g_T T = \Delta(g_T) T.$$

If the matrix $\Delta(g_T)$ is definite positive, then there exists a unique positive BGP rate of technology $g_T > 0$, given research intensities and diffusion parameters. Associated with that growth rate is a vector T (defined up to a scalar multiple), with every element positive, which reflects each country-sector's relative level of knowledge along that BGP.

In Appendix C, we summarize the equations of the equilibrium conditions of the model after normalizing all endogenous variables. In Appendix D we analyze the existence and uniqueness of the equilibrium in our model. We follow the methodology developed in Allen, Arkolakis, and Li (2015) to study uniqueness of the trade block of the model, given T_n^j. We then prove uniqueness of R&D intensity, s_n^j, given g_T and the trade equilibrium. Finally, we use the Frobenius theorem and the growth block of the model to prove uniqueness of g_T and T_n^j given the static trade equilibrium.[10]

[10] Kucheryavyy, Lyn, and Rodríguez-Clare (2016) also study the existence and uniqueness of equilibrium in a

4 The Mechanism

In this section we describe the mechanism through which a reduction of trade costs, d_{in}^{j}, has an impact on innovation, growth and comparative advantage. In multi-sector *static* models of trade, there is the well-known specialization effect: A decrease in d_{in}^{j} induces a reallocation of production towards those sectors in which the country has comparative advantage (Caliendo and Parro (2015)). The larger the dispersion in relative productivity, the stronger is comparative advantage, and hence the specialization effect.

In a multi-sector *dynamic* model, there are additional effects of trade liberalization that can potentially generate welfare gains. The first is the R&D reallocation effect. Through the specialization effect just described, profits increase in sectors with stronger comparative advantage due to the market size effect following the decline in trade costs. As a result, R&D resources reallocate towards sectors that experience a higher increase in production. Consider two sectors j and j' in country n. From Equation (20), we can obtain an expression of the relative R&D expenditure between these two sectors as:

$$\left(\frac{s_n^j}{s_n^{j'}}\right)^{1-\beta_r} = \frac{\lambda_n^j}{\lambda_n^{j'}} \frac{\sum_{i=1}^{M} \pi_{in}^{j} X_i^j}{\sum_{i=1}^{M} \pi_{in}^{j} X_i^{j'}}. \tag{23}$$

Everything else constant, lowering trade costs affects the production patterns in the economy and shifts R&D towards sectors that experience larger increase in profits (higher $\sum_{i=1}^{M} \pi_{in}^{j} X_i^j$). This reallocation effect changes the aggregate R&D intensity at the country level. The exact magnitude is a quantitative question and we will provide more details in the quantitative analysis.

The following two extreme cases further illustrate the R&D reallocation effect after a change in trade costs.

Case 1 (Autarky): Suppose all countries are closed from international trade. That is, $d_{in}^{j} \to \infty$, $\forall i, n, j$. Equation (23) can be rewritten as:

$$\left(\frac{s_n^j}{s_n^{j'}}\right)^{1-\beta_r} = \frac{\lambda_n^j X_{nn}^j}{\lambda_n^{j'} X_{nn}^{j'}}, \tag{24}$$

where X_{nn}^{j} is total domestic expenditure on sector j product. Compared to another country-sector,

multi-sector model with external economies and without knowledge spillovers.

we have

$$\left(\frac{s_n^j/s_n^{j'}}{s_{n'}^j/s_{n'}^{j'}}\right)^{1-\beta_r} = \underbrace{\frac{\lambda_n^j/\lambda_n^{j'}}{\lambda_{n'}^j/\lambda_{n'}^{j'}}}_{\substack{\text{exogenous innovation}\\ \text{comparative advantage}}} \times \underbrace{\frac{X_{nn}^j/X_{nn}^{j'}}{X_{n'n'}^j/X_{n'n'}^{j'}}}_{\substack{\text{relative domestic}\\ \text{market size}}}, \tag{25}$$

Thus, innovation efforts are distributed across country-sectors according to the exogenous component of innovation efficiency and the relative domestic market share. The exogenous component of innovation efficiency determines comparative advantage in innovation in our model.

Case 2 (Free Trade): In the case of free trade, $d_{in}^j = 1$. Equation (23) then becomes

$$\left(\frac{s_n^j}{s_n^{j'}}\right)^{1-\beta_r} = \frac{\lambda_n^j}{\lambda_n^{j'}} \frac{T_n^j(c_n^j)^{-\theta}/\sum_n T_n^j(c_n^j)^{-\theta}}{T_n^{j'}(c_n^{j'})^{-\theta}/\sum_n T_n^{j'}(c_n^{j'})^{-\theta}} \frac{X^j}{X^{j'}}, \tag{26}$$

where $X^j = \sum_n X_n^j$ denotes the world demand for sector-j good. This equation shows that under free trade, in addition to the sector-specific relative innovation efficiency, a country's R&D resources would be distributed according to production comparative advantage[11], and the world expenditure share $(\frac{X^j}{X^{j'}})$. The latter captures the traditional market size effect of opening trade.

Since the last two terms are common across countries for given sectors, we can further express the relative R&D allocation across country and sectors as

$$\left(\frac{s_n^j/s_n^{j'}}{s_{n'}^j/s_{n'}^{j'}}\right)^{1-\beta_r} = \underbrace{\frac{\lambda_n^j/\lambda_n^{j'}}{\lambda_{n'}^j/\lambda_{n'}^{j'}}}_{\substack{\text{exogenous innovation}\\ \text{comparative advantage}}} \times \underbrace{\frac{T_n^j(c_n^j)^{-\theta}/T_n^{j'}(c_n^{j'})^{-\theta}}{T_{n'}^{j'}(c_{n'}^{j'})^{-\theta}/T_{n'}^{j'}(c_{n'}^{j'})^{-\theta}}}_{\substack{\text{production}\\ \text{comparative advantage}}}. \tag{27}$$

A comparison between these two extreme cases shows that when a country opens up to trade, research efforts are directed more into country-sectors with production comparative advantage. Furthermore, all else equal, a higher share of R&D investment in a sector translate into higher relative knowledge stock. Production comparative advantage thus evolves with the distribution of innovation efforts over time, which in turn affects the R&D allocation as shown in Equation (26).[12]

The second effect is driven by cross-country cross-sector knowledge spillovers, which further add to the complexity of the interactions between innovation and production. Equation (21) implies that without cross-country cross-sector spillovers (i.e. $\varepsilon_{ni}^{jk} = 0$ for $nj \neq ik$), the evolution of

[11]Note that $T_n^j(c_n^j)^{-\theta}/(\frac{1}{N}\sum_n T_n^j(c_n^j)^{-\theta})$ signals the absolute comparative advantage based on cost adjusted technology. When expressing as a ratio to another sector j' in the same country as in $\frac{T_n^j(c_n^j)^{-\theta}/\sum_n T_n^j(c_n^j)^{-\theta}}{T_n^{j'}(c_n^{j'})^{-\theta}/\sum_n T_n^{j'}(c_n^j)^{-\theta}}$, it provides an indicator for comparative advantage

[12]This result is similar to what Somale (2014) obtains in a semi-endogenous growth model without knowledge spillovers.

the technology distribution ($T_n^j/T_n^{j'}$) eventually reflects the underlying specialization in innovation $\left(\lambda_n^j/\lambda_n^{j'}\right)$. In the presence of spillovers, the technology level is also determined by the amount of ideas diffused from elsewhere.

Whether knowledge diffusion would dampen or amplify the specialization effect induced by trade liberalization depends on the exact pattern of diffusion. To see this, rearranging terms in Equation (21), we have

$$g_T T_n^j = \sum_{i=1}^{M} \sum_{k=1}^{J} \frac{\varepsilon_{ni}^{jk}}{g_T + \varepsilon_{ni}^{jk}} \lambda_i^k T_i^k \left(s_i^k\right)^{\beta_r}. \tag{28}$$

First of all, if for a given source country-sector, the diffusion speed is common across all destination countries ($\varepsilon_{ni}^{jk} = \varepsilon_i^k$, $\forall n, j$), then according to the above equation, the stock of knowledge (T_n^j) will be the same everywhere. When the diffusion speed is heterogeneous, however, the effect of R&D reallocation on the stock of knowledge will be stronger in those country-sectors (nj) that receive faster diffusion from the country-sectors (ik) that experience larger increases in innovation ($\lambda_i^k T_i^k (s_i^k)^{\beta_r}$). Therefore, if less innovative country-sectors are better at absorbing knowledge, knowledge diffusion can *dampen* the aforementioned specialization effect by enabling faster productivity convergence and by making the stock of knowledge more similar across country-sectors. On the other hand, if diffusion forces are stronger for already innovative countries (i.e. innovative countries also diffuse ideas fast among themselves), diffusion would *amplify* the specialization effects of trade-induced R&D reallocation, as stock of knowledge and productivity becomes even more dispersed. In our calibration section 5.1.2, we find that it is the latter case that holds true in the data. Thus, heterogeneous knowledge spillovers propagate the effect of trade liberalization, and introduce another source of dispersion to the distribution of stock of knowledge and innovation specialization.

The last effect of trade liberalization on welfare is a growth effect. From Equation (21), the change in R&D spending across country-sectors induces changes in the growth rate along the BGP. More R&D investment overall increases the world growth rate. Furthermore, if R&D reallocates towards the sectors that are better at doing R&D (i.e. higher $\lambda_i^k \hat{T}_i^k$) and have faster diffusion speed, the growth rate of the world also increases. The existence of knowledge spillovers reinforces this channel, as changes in R&D across sectors will have a larger impact on BGP growth as countries and sectors can benefit from R&D in other countries and sectors.

5 Quantitative Analysis

We calibrate the model to quantify the effect of a trade liberalization on innovation, comparative advantage and growth. To explore the role of knowledge spillovers we simulate three versions of the model: (i) our baseline model with innovation and cross-sector and cross-country knowledge linkages; (ii) a model with (almost) negligible knowledge spillovers; and (iii) a model with knowledge spillovers that are homogeneous across countries and sectors.We then study effect of each of the channels proposed in Section 4 on welfare gains from trade. We isolate the effect of our dynamic model by comparing the results to those of a a static version of the model in which productivity is exogenously determined, In all cases, we recalibrate the parameters of the model to match the same moments of the data.

5.1 Calibration

We use data on bilateral trade flows, production, R&D intensity and data on patent citations to calibrate the main parameters of the model. In this section, we focus on the calibration of the trade costs, d_{in}^{j}, the diffusion parameters ε_{in}^{jk}, and the parameters governing the innovation process—the elasticity of innovation, β_r, the efficiency of innovation, λ_i^j, and the stock of knowledge, T_{it}^{j}. Details on the calibration of the production parameters and the data used in the calibration are relegated to Appendix E.

5.1.1 Estimation of d_{ni}^{j}: Gravity Equation at the Sector Level

To estimate the trade costs for tradable sectors, $j \leq J-1$, we estimate the model-consistent gravity equations for each sector in 2005 using bilateral trade flow data. We start from the trade shares in equation (11):

$$\pi_{ni}^{j} = \frac{X_{ni}^{j}}{X_n^{j}} = \frac{T_i^{j}\left(c_i^{j} d_{ni}^{j}\right)^{-\theta}}{\Phi_n^{j}}. \tag{29}$$

Dividing the trade shares by their domestic counterpart as in Eaton and Kortum (2002) and assuming $d_{nn}^{j} = 1$, we have

$$\frac{\pi_{ni}^{j}}{\pi_{nn}^{j}} = \frac{X_{ni}^{j}}{X_{nn}^{j}} = \frac{T_i^{j}\left(c_i^{j} d_{ni}^{j}\right)^{-\theta}}{T_n^{j}\left(c_n^{j}\right)^{-\theta}}. \tag{30}$$

Taking logs of both sides, we have

$$\log\left(\frac{X_{ni}^{j}}{X_{nn}^{j}}\right) = \log\left(T_i^{j}\left(c_i^{j}\right)^{-\theta}\right) - \log\left(T_n^{j}\left(c_n^{j}\right)^{-\theta}\right) - \theta\log(d_{ni}^{j}), \tag{31}$$

where the log of the trade costs can be expressed as

$$\log(d_{ni}^{j}) = D_{ni,k}^{j} + B_{ni}^{j} + CU_{ni}^{j} + RTA_{ni}^{j} + ex_i^{j} + \nu_{ni}^{j}. \tag{32}$$

Following Eaton and Kortum (2002), $D^j_{ni,k}$ is the contribution to trade costs of the distance between country n and i falling into the k^{th} interval (in miles), defined as [0,350], [350, 750], [750, 1500], [1500, 3000], [3000, 6000], [6000, maximum). The other control variables include common border effect, B_{ni}, common currency effect, CU_{ni}, and regional trade agreement RTA_{ni}, between country n and country i. We include an exporter fixed effect, ex^j_i, which has been shown to fit better the patterns in both country incomes and observed price levels (see Waugh (2010)). ν^j_{ni} is the error term.

Substituting (32) back into (31) results in the following gravity equation at the sector level:

$$\log\left(\frac{X^j_{ni}}{X^j_{nn}}\right) = \log\left(T^j_i\left(c^j_i\right)^{-\theta}\right) - \theta ex^j_i - \log\left(T^j_n\left(c^j_n\right)^{-\theta}\right) - \theta(D^j_{ni,k} + B^j_{ni} + CU^j_{ni} + RTA^j_{ni} + \nu^j_{ni}). \quad (33)$$

Define $\hat{F}^j_i = \log\left(T^j_i\left(c^j_i\right)^{-\theta}\right) - \theta ex^j_i$ and $F^j_n = \log\left(T^j_n\left(c^j_n\right)^{-\theta}\right)$. We then estimate the following equation using fixed effects and observables related to symmetric trade barriers, taking θ as known:

$$\log\left(\frac{X^j_{ni}}{X^j_{nn}}\right) = \hat{F}^j_i - F^j_n - \theta(D^j_{ni,k} + B^j_{ni} + CU^j_{ni} + RTA^j_{ni} + \nu^j_{ni}). \quad (34)$$

Using the coefficient estimates of equation (34), we can back out $\log(d^j_{ni})$ based on equation (32). The exporter fixed effect in trade cost, ex^j_i, can then be estimated using importer and exporter fixed effects estimates from the Gravity equation (34): $ex^j_i = (F^j_i - \hat{F}^j_i)/\theta$. Figure 1 plots the distance parameters that we obtain from the sectoral gravity equations, d^j_{in}, against the trade share from the data that we use to estimate the gravity equations at the sector level, assuming $\theta = 4$.

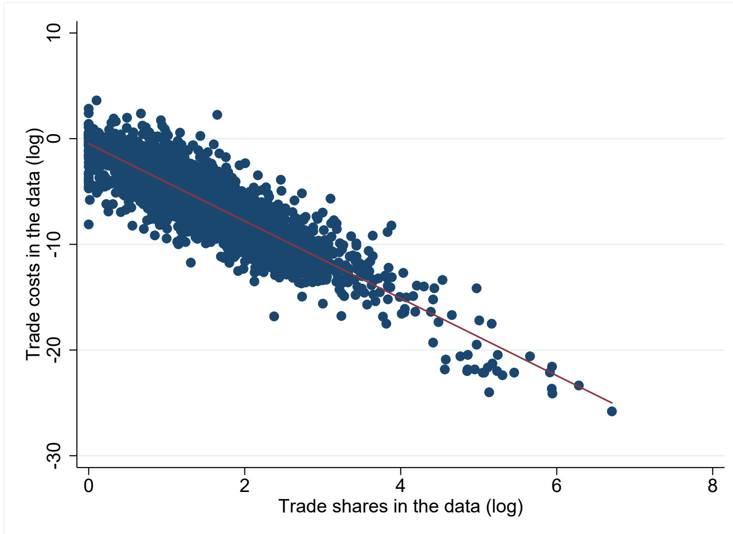


Figure 1: Trade shares and distance

Notes: This figure shows the negative relationship between trade flows, π_{in}^{j}, and trade costs, d_{in}^{j}. Trade flows are observed in the data. Trade costs are estimated running gravity regressions at the sector level.

5.1.2 The Speed of Knowledge Diffusion

Estimating the speed of knowledge diffusion is not a trivial task, as diffusion is conceptual and difficult to measure. The diffusion literature has typically found patent citations to represent a reasonable indicator of diffusion albeit with some degree of noise (Jaffe, Trajtenberg, and Fogarty (2000); Bottazzi and Peri (2003))[13] When a patent is granted, its document identifies a list of citations made to previous patents upon which the current one builds. Thus, citations are informative of links between innovations. If a single technology is cited in numerous patents, it is apparently involved in many developmental efforts.

Using patent data, this section adapts the approach proposed in Caballero and Jaffe (1993) to estimate the diffusion speed parameters. To be consistent with our model, we extend their approach to a multi-sector multi-country environment. Similar to their paper, we use patents as an indicator of the creation of new ideas, and the citations as an indicator of use of existing ideas in the creation of new ideas.

A vast literature discusses the potential issues of using patent data to proxy ideas and spillovers.[14] First, a considerable number of inventions or ideas are never patented but are protected by secrecy

[13] Although patent statistics have been widely used in studies of firm innovations, not all innovations are patented, especially process innovations, which are often protected in other ways such as copyright, trademarks and secrecy (see Levin et al. (1987)). Our measure implicitly assumes that for any sector, the nonpatented and patented knowledge utilizes knowledge (patented or nonpatented) from other sectors in the same manner, particularly with the same speed.

[14] See, for example, the survey by Griliches (1990).

or other informal mechanism. Second, sectors differ in their propensity to patent and propensity to cite. Therefore, a relative abundant stock of patents in one sector may not necessarily imply a large accumulation of ideas. Third, individual patent varies in terms of its quality (the number of ideas embodied or the ability to generate spillovers). Lastly, not all citations necessarily represent spillovers as the decision to cite another patent sometimes rests with the patent examiner, who is supposed to be an expert in the area and able to identify relevant prior art that the applicant misses or conceals. This implies that the inventor may not be aware of the earlier work and the citation may not represent the true knowledge transmission.

A particular virtue of Caballero and Jaffe (1993) approach is that it is designed to deal with some of these issues by estimating these sector-specific factors—such as propensity to patent and to cite, the ability to generate spillovers and knowledge obsolescence rate, and the discrepancy between citations and spillovers—jointly with the diffusion speed parameters. Controlling for these additional sectoral variations with a fairly rich structure of citation process helps to obtain a more accurate estimation of the parameter of interest, the cross-country cross-sector speed of diffusion parameters, $\{\varepsilon_{ni}^{jk}\}_{MJ\times MJ}$.

In particular, we first specify a "gravity-type" citations function which describes citations made to patents developed at time s in country-sector ik by patents in country-sector nj at time t ($t \geq s$), given the size of both citing and cited patent applications. Let $C_{ni}^{jk}(t,s)$ be the citations from patents applied by country n sector j in year t to patents by country i sector k in year s. $P_{i,s}^{k}$ represent the number of patent applications by country i sector k in period s. The expected citation function is written as below:

$$C_{ni}^{jk}(t,s) = \phi_{n,t}^{j}\delta_{i,s}^{k}(\psi_{i,s}^{k}P_{i,s}^{k})^{\beta_g}(\psi_{n,t}^{j}P_{n,t}^{j})^{\beta_l}e^{-\sum_{\tau=s}^{t}O_{i,\tau}^{k}\tilde{P}_{i,\tau}^{k}}(1-e^{-\varepsilon_{ni}^{jk}(t-s)}). \tag{35}$$

Here, $\phi_{n,t}^{j}$ gives the ratio between the number of citations and actual idea used by nj at time t, which also captures the sector-specific variation in propensity to cite. $\delta_{i,s}^{k}$, represents the ability to generate spillovers from or quality of ideas in country-sector ik dated in period s. It is also assumed that the number of ideas embodied in each patent in country-sector nj at time t is given by $\psi_{n,t}^{j}$. Note that this function is a generalization of Caballero and Jaffe (1993)'s approach, in which β_l and β_g are restricted to be 1.

The discount term, $e^{-\sum_{\tau=s}^{t}O_{i,\tau}^{k}\tilde{P}_{i,\tau}^{k}}$, can be interpreted as an index of knowledge obsolescence, in which $\tilde{P}_{i,t}^{k} = \psi_{i,t}^{k}P_{i,t}^{k}/\sum_{\tau=1}^{T}\psi_{i,\tau}^{k}P_{i,\tau}^{k}$ denotes the relative size of ideas in ik in period τ (relative to the total stock of ideas in ik over the sample period). It decreases with the (normalized) size of inventions that take place between the recipient cohorts t and the source cohorts $\tau(\in[s,t])$, with a time-varying obsolent rate $O^k_{i,\tau}$. This term captures the notation that old knowledge eventually is made obsolete by the emergence of superior new knowledge. Thus, the accumulation of new inventions (rather than simply the passage of time) that occur after the source cohorts increases the rate of which the source knowledge becomes obsolete.

The last term represents the probability of ideas in s having been seen by period t. Given the model assumption of the exponential distribution of diffusion lags, it follows that the probability of seeing an idea $(t-s)$ years old is given by $(1-e^{-\varepsilon^{jk}_{ni}(t-s)})$, where ε^{jk}_{ni} is the constant diffusion speed from ik to nj. $\varepsilon^{jk}_{ni} \rightarrow \infty$ indicates instantaneous diffusion, whereas $\varepsilon^{jk}_{ni} = 0$ implies no diffusion. This is the parameter we are particularly interested in estimating.

We obtain patent and patent citation data across countries and sectors from the U.S. Patent and Trade Office (USPTO) for the period 2000-2010.[15] In the dataset, each patent is assigned to one of the IPC (International patent classification) categories. We use the probability mapping between IPC and ISIC Rev.3 provided by the World Intellectual Property Organization (WIPO) to assign patents into our 19 sectors. Our sample contains 1.15 million patents and over 13 million citations between the 28 countries and 19 sectors.

Equation (35) is estimated using Generalized Method of Moments (GMM) based on observations about citation count from nj, t to ik, s, $\hat{C}^{jk}_{ni}(t,s)$, with $t \in [2001, 2010]$, $s \in [2001, t]$, $j, k \in [1, J]$ and $n, i \in [1, M]$. Define $\Theta_{nj,ik}(t,s) = \{\varepsilon^{jk}_{ni}, \phi^j_{n,t}, \psi^j_{n,t}, \delta^j_{n,t}\}$ as the set of parameters to be estimated and $\Gamma(\Theta)$ the difference between the model-generated moments and data moments:

$$\Gamma[\Theta_{nj,ik}(t,s)] = C^{jk}_{nt,is} - \hat{C}^{jk}_{nt,is}.$$

Our GMM estimators solve:

$$\Theta^* = \mathrm{arg}min_{\Theta} \sum_{n,i=1}^{M} \sum_{j,k=1}^{J} \sum_{t=2001}^{2010} \sum_{s=2001}^{t} \Gamma[\Theta_{nj,ik}(t,s)]^2. \tag{36}$$

The GMM estimates $\phi^j_{n,t}, \psi^j_{n,t}, \delta^j_{n,t}$ and ε^{jk}_{ni} separately through two iterating steps. First, given

[15]Note that patents applied in U.S. are not necessarily created by U.S. inventors. According to the territorial principle in U.S. patent laws, anyone intending to claim exclusive rights for inventions is required to file U.S. patents. In fact, about 50 percent of patents applied in the United States in the early 2000s were from foreign inventors. Given that the United States has been the largest technology consumption market in the world over the past few decades, it is reasonable to assume that most important innovations from other countries have been patented in the U.S. Therefore, the knowledge linkages uncovered in the U.S. patent data are reasonably representative of the deep fundamental relationship of technologies. All we really need is that statements of the following sort hold: If a patent that belongs to a German inventor in electronic components sector cites a Japanese patent in radio and television receiving equipment in the U.S. patent data, similar relationship also holds for German inventors filing a patent in Europe.

initial level of $\phi^j_{n,t}, \psi^j_{n,t}, \delta^j_{n,t}$, we find the $\hat{\varepsilon}^{jk}_{ni}$ that minimizes.

$$\Theta^* = \text{arg}min_{\Theta} \sum_{t=2001}^{2010} \sum_{s=2001}^{t} \Gamma[\Theta_{nj,ik}(t,s)]^2.$$

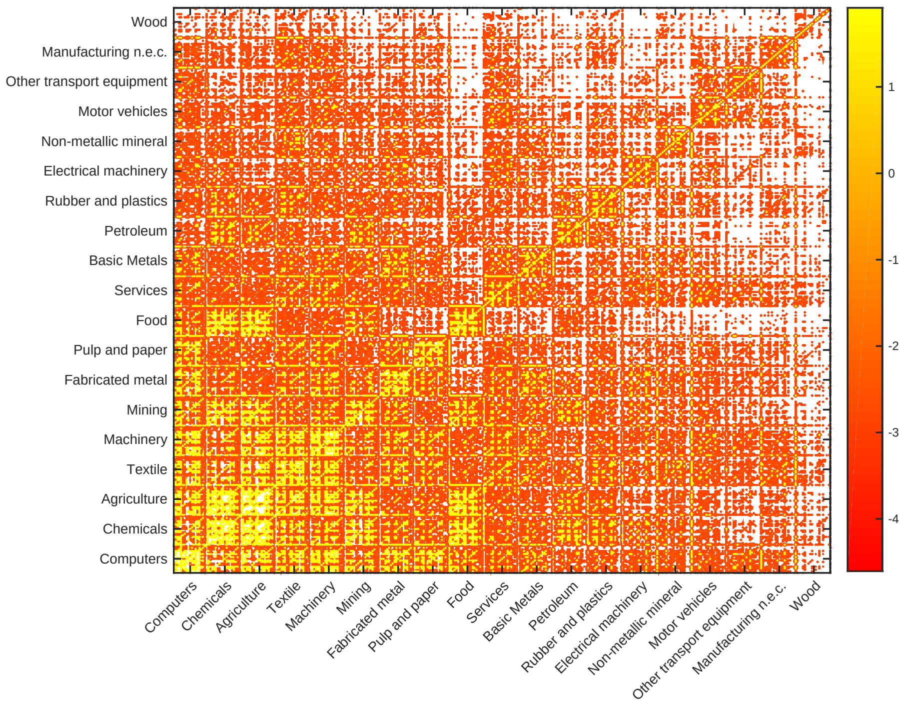
Second, we take the estimated $\hat{\varepsilon}^{jk}_{ni}$ from the first step as given and estimate $\hat{\phi}^j_{n,t}, \hat{\psi}^j_{n,t}, \hat{\delta}^j_{n,t}$ to minimizes (36). Then we set $\hat{\phi}^j_{n,t}, \hat{\psi}^j_{n,t}, \hat{\delta}^j_{n,t}$ as initial value again and iterate the above two steps until the two sets of estimated parameters converge.

The estimated ε^{jk}_{ni}s are then normalized following Eaton and Kortum (1999). Specifically, we fix the within-sector adoption speed in the U.S. to 2 years (taking the mid-point of the evidence reported by Pakes and Schankerman (1984)). The adoption lag in the model is given by $1/(\bar{\varepsilon}_{USUS} + g) = 2$, which implies $\bar{\varepsilon}_{USUS} = 0.38$ with $g = 0.12$ in our calibration. We then use this restriction to normalize all ε^{jk}_{ni}.

Several interesting findings emerge from estimating the citations function. First, there is a large heterogeneity in the diffusion speed across countries and sectors (i.e. between $(nj,\ ik)$ cells), with a large number of country-sector pairs that diffuse knowledge very slowly to each other. The mean diffusion lag (i.e. $1/\varepsilon$) is found to be about 12 years for cross-country-sector diffusion ($nj \neq ik$) and the within-country-sector averages slightly over 1 year. Second, although not reported here, we find that the estimated diffusion speeds vary across country-sector pairs in intuitively sensible patterns: they significantly decrease with geographic distances, linguistic distances and landlocked status, and increase when the two countries are in the same trade union, continent, or share a common border or have colonial ties.

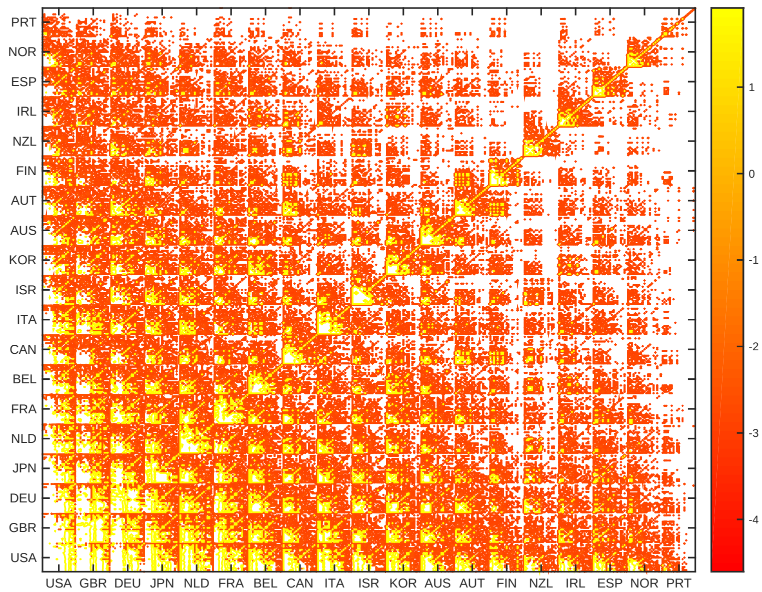
We present the contour maps of the estimated ε^{jk}_{ni} in Figures 2 and 3. Figure 2, organizes the sub-blocks by sector and Figure 3 by country. A brighter color means higher value. It is evident that there is large heterogeneity across country-sector-pairs. In general, patents in the computer, electronic and medical instruments, chemicals and chemical products sectors have the highest diffusion speed, while patents in the wood products sector have the lowest diffusion speed. In addition, sectors being cited with a faster speed also tend to cite others with a higher speed.

Figure 2: Contour map of ϵ_{ni}^{jk}, by sectors



Notes: Cited sectors are on the x-axis, and citing sectors are listed on the y-axis. Sectors are ranked by their average cited speed.

Figure 3: Contour map of ϵ_{ni}^{jk}, by countries



Notes: Cited countries are on the x-axis, and citing countries are listed on the y-axis. Countries are ranked by their average cited speed

Among countries, unsurprisingly, new knowledge created in the United States, Japan, Germany, and the United Kingdom diffuse more rapidly on average, while those in Portugal, Norway and Spain diffuse the slowest. Countries that diffuse knowledge (get cited) rapidly also tend to acquire new knowledge from other countries (citing others) fast. Since the advanced innovative countries (e.g. US, Japan, Germany and UK) also share knowledge faster among themselves than they do with less developed less innovative countries, as discussed in Section 4, knowledge diffusion further magnifies the specialization effect of R&D reallocation across country-sectors.

5.1.3 Parameters of Innovation

Given the calibrated values for trade costs d_{in}^{j}–estimated with gravity regressions- production input-output linkages parameters $\{\alpha^j, \gamma^j, \gamma^{jk}\}$–estimated using the U.S. input-output table for 2005—and knowledge spillovers—estimated with patent citation data— we use the trade and growth blocks of the model to calibrate the parameters of innovation $\{\beta_r, \lambda_n^j, \hat{T}_n^j\}$. In this section, we describe the algorithm used to calibrate the innovation parameters and show the main results.

The algorithm The calibration of the parameters of innovation, $\{\lambda_n^j, \beta_r, \hat{T}_n^j\}$ follows a recursive algorithm. We proceed in four steps. First, we guess a value of $\hat{T}_n^j$. Second we apply the standard procedure to solve for the static equilibrium of the model. Third, we solve for the growth equilibrium, and fourth, we iterate to find a fixed point solution in $\hat{T}_n^j$. Next, we describe in detail the steps of the algorithm:

1. We guess a value for $\hat{T}_n^j$

2. We use the calibrated values for $\{\gamma^j, \gamma^{jk}, \alpha^j, d_{in}^j, \theta\}$ together with the equations that define the static trade equilibrium to obtain wages, prices, expenditures, trade shares, and output.[16].

3. We use the growth structure of the model to calibrate the innovation parameters:

   (a) We start by setting a value for the growth rate of the economy in the BGP, g_y. This corresponds to a growth rate for the stock of knowledge along the BGP of $g_T = \theta\left(1+\sum_{j=1}^{J}\alpha_j\Lambda_j\right)^{-1} g_y$. See Appendix B for details of this derivation. Because all countries and sectors' stock of knowledge grows at the same rate, all countries have the same productivity growth on the BGP.

   (b) Given the calibrated values for ε_{in}^{jk}, data on R&D intensity, s_n^j, and the value for g_T, we iterate over equations (19, 20, and 21)) and use the Frobenious theorem to obtain $\{\beta_r, \hat{T}_n^j\}$. The Frobenius theorem guarantees that there is a unique balanced-growth path in which all countries and sectors grow at the same rate g_T.

   (c) We update β_r so that the growth rate of the stock of knowledge is g_T and obtain $\hat{T}_n^j$ from the eigenvector associated to $\Delta(g_T)$.

4. With β_r and $\hat{T}_n^j$, we go back to solving for the new static trade equilibrium and keep iterating until we find a fixed point solution for $\hat{T}_n^j$.

5. Finally, we compute λ_n^j so that R&D intensity s_n^j matches the data.

Calibration results We follow the steps of the algorithm that we just described to calibrate the innovation parameters $\lambda_n^j, \beta_r, T_n^j$. We proceed as follows: First, we solve for the static trade equilibrium. Then, we assume a growth of income per capita (productivity) on the BGP of $g_y =$ 2.8%. This corresponds to a growth rate for the stock of knowledge on the BGP of $g_T = 12\%$. Given data for s_n^j, together with the estimated values for ε_{ni}^{jk}, and g_T, and the results from the static equilibrium, we can use the Frobenius theorem and iterate on equation (21) to obtain β_r and λ_n^j. We repeat the procedure until $\hat{T}_n^j$ converges to a fixed point solution.

[16] See equations (65), (66), (67), (68), (69), (71), (72), (73), and (74) in Appendix C

Calibration results Table 1 reports the calibrated values of the parameters that are common across countries and sectors.

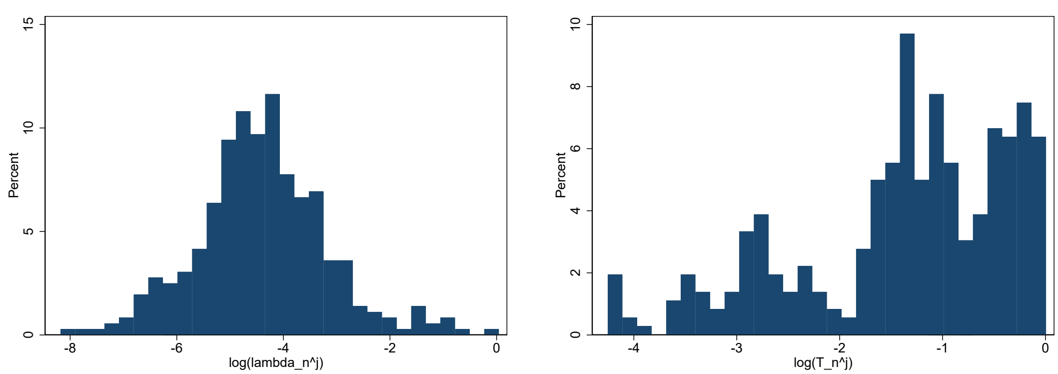
Table 1: Calibrated (common) parameters

Parameter	Value	Description
β_r	0.45	Elasticity of innovation
g_T	0.12	Growth of stock of knowledge
g_y	0.028	Growth of income per capita
θ	4.00	Trade elasticity
ρ	0.90	Discount factor

Note: Table contains values of common parameters.

We find that the efficiency of innovation λ_n^j and the stock of knowledge, $\hat{T}_n^j$ are heterogeneous across our sample of country-sectors. In particular, we find that λ_n^j ranges from 2.8×10^{-4} to 1.04, with mean 0.032 and standard deviation 0.10 (see Figure 4(a)). The stock of knowledge has a mean of roughly 0.37, with an standard deviation of 0.28 (see 4(b)). In Appendix F we show that there is dispersion in λ_n^j and T_n^j both across sectors and across countries.

Figure 4: The efficiency of innovation and the stock of knowledge



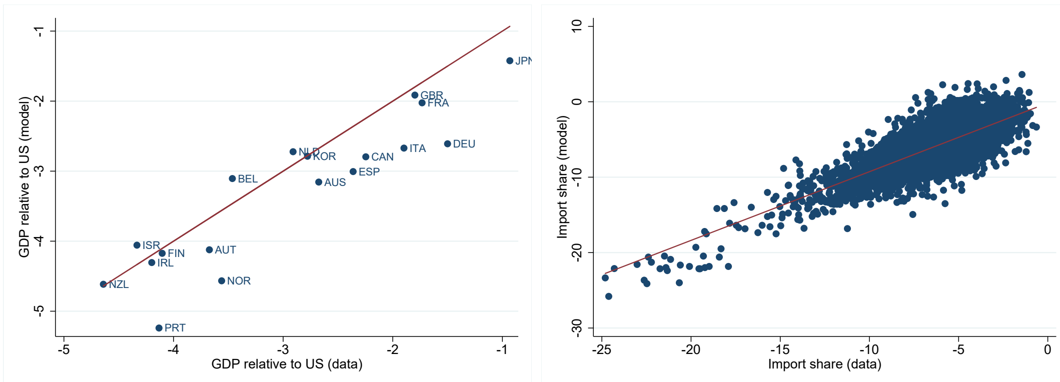
(a) Efficiency of innovation (λ_n^j)

(b) Stock of knowledge ($\hat{T}_n^j$)

Notes: This figure shows the dispersion of both the exogenous component of the efficiency of innovation of each country-sector (λ_n^j) and the endogenous component of the efficiency of innovation, which corresponds to the stock of knowledge of each country-sector ($\hat{T}_n^j$). Both variables are expressed taking logs.

Validation Our calibration strategy delivers relative income and trade flows that are broadly consistent with those observed in the data. The correlation between GDP in our model and in the data is around 0.98. The model does a good job at matching the dispersion of GDP per capita across countries, being roughly 0.3 in both cases (see Figure 5(a)). The correlation of trade flows in the model and in the data is roughly 0.8 (see Figure 5(b)).[17]

Figure 5: GDP and trade flows in the model and in the data



(a) GDP (relative to the US)

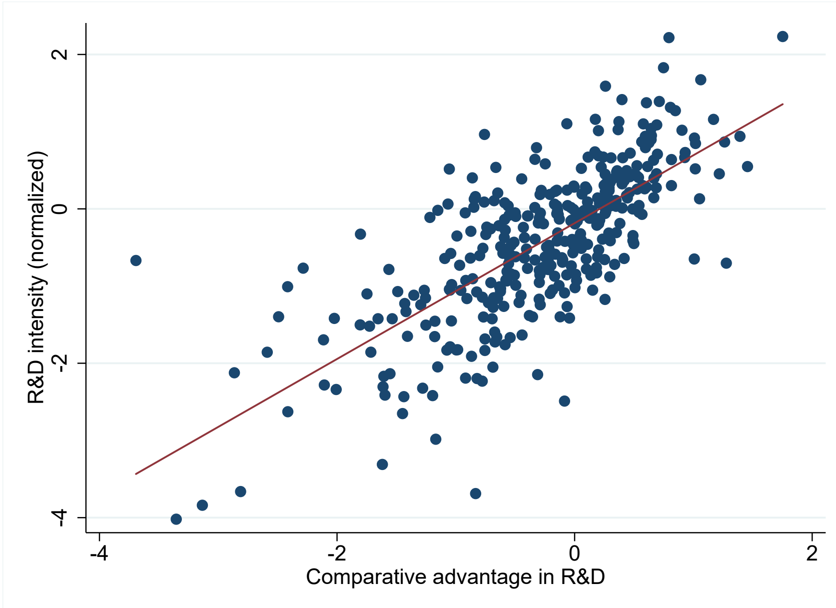
(b) Trade flows

Notes: This figure provides validation of our calibration on GDP (relative to the US) and trade flows, which are computed as $\log\left(\frac{\pi^j_{in}}{\pi^j_{nn}}\right)$.

Comparative advantage in innovation and R&D intensity Our calibration results imply that countries invest more in R&D in those sectors in which they have comparative advantage in innovation. The Y-axis of Figure 6 represents data on R&D intensity, computed as the ratio of R&D spending to world output, for each country-sector in our sample. The X-axis represents comparative advantage in innovation. Comparative advantage in innovation is calculated by obtaining double ratios of the efficiency of innovation $\lambda^j_n \hat{T}^j_n$, as $\frac{\lambda^j_n T^j_n / \frac{1}{M}\sum_n \lambda^j_n T^j_n}{\frac{1}{J}[\sum_j \lambda^j_n T^j_n / \frac{1}{M}\sum_n \lambda^j_n T^j_n]}$. Note that the efficiency of innovation in our model has two components. An exogenous component, determined by λ^j_n, and an endogenous component determined by the stock of knowledge, $\hat{T}^j_n$. The correlation between R&D intensity, s^j_n, and the exogenous component of the efficiency of innovation, λ^j_n, is around 0.60, and the R&D intensity and the stock of knowledge, $\hat{T}^j_n$, is roughly 0.64.

[17]The reason why we do not match exactly trade flows from the data is because we use gravity regressions at the sector level to obtain the trade costs, hence predicted trade flows are measured with error.

Figure 6: R&D intensity and comparative advantage in R&D



Notes: This figure shows differences between the country-specific cross-sector R&D intensity (measured as R&D as a percentage of world GDP) and comparative advantage in innovation. Both variables are in logs.

Decomposition of the sources of growth Our model allows us to decompose the main sources of knowledge around the world using Equation (21). The contribution of each country-sector ik to the stock of knowledge of each country-sector nj, T_{ni}^{jk}, can be expressed as

$$T_{ni}^{jk} = \frac{\varepsilon_{ni}^{jk}/g_T}{\varepsilon_{ni}^{jk} + gT} \lambda_i^k T_i^k \left(s_i^k\right)^{\beta_r}$$

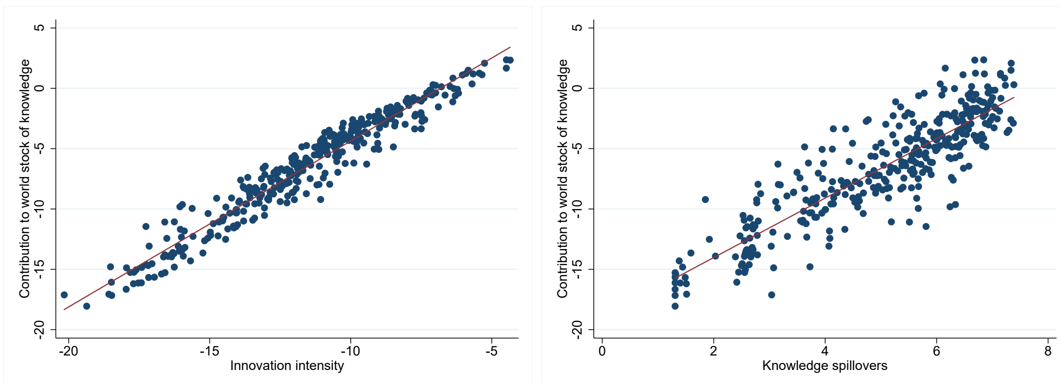
Summing across each country-sector nj, we obtain the contribution of each country-sector ik to the total stock of knowledge of the world. That is,

$$\sum_{n=1}^{M} \sum_{j=1}^{J} T_{ni}^{jk} = \underbrace{\lambda_i^k T_i^k \left(s_i^k\right)^{\beta_r}}_{\text{Innovation}} \underbrace{\sum_{n=1}^{M} \sum_{j=1}^{J} \frac{\varepsilon_{ni}^{jk}/g_T}{\varepsilon_{ni}^{jk} + gT}}_{\text{Knowledge spillovers}} \tag{37}$$

The first component of Equation (37) captures the effect of the innovation of country-sector ik on the world stock of knowledge. The second component captures the effect of the strength of knowledge diffusion of country-sector ik on the world stock of knowledge. Country-sectors that are more innovative contribute more to the stock of knowledge of the world if they are better at spreading knowledge.

For each country-sector ik, Figure 7 plots the contribution of each component of Equation (37) to the world stock of knowledge. Not surprisingly, we obtain that countries that are more innovative (Figure 7(a)) and better at spreading knowledge (Figure 7(b)) contribute more to the total stock of knowledge. Moreover, we find that, in our sample of analysis, those country-sectors that are more innovative are also better at spreading knowledge around the world.

Figure 7: Decomposition of sources of world stock of knowledge

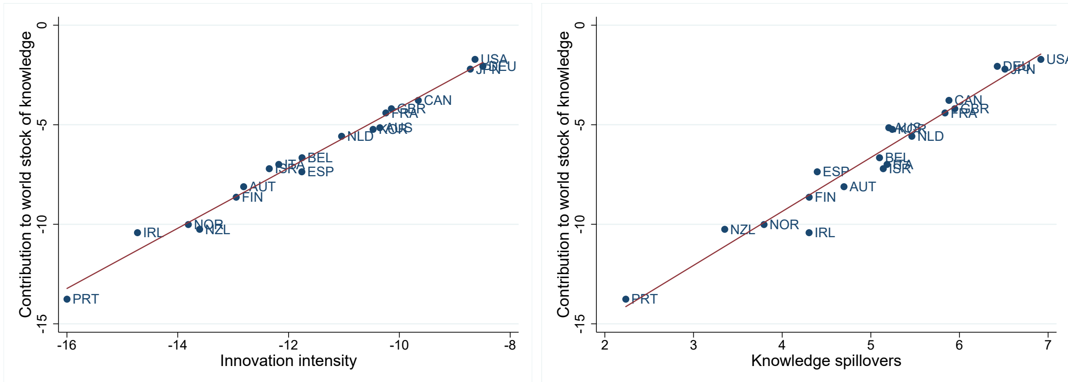


(a) Innovation (b) Diffusion

Notes: This figure shows the contribution of innovation an diffusion of each country i, sector k on the stock of knowledge of the world. The innovation component is $\lambda_i^k T_i^k \left(s_i^k\right)^{\beta_r}$. The diffusion component is $\sum_{n=1}^{M}\sum_{j=1}^{J}\frac{\varepsilon_{ni}^{jk}/g_T}{\varepsilon_{ni}^{jk}+gT}$. All variables are in logs.

Aggregating across all k sectors, we find that the United States, Germany and Japan are the main contributors to world knowledge, both because they are the most innovative countries and because they are better at spreading knowledge around the world. Instead, Portugal, Ireland, Norway and New Zealand are the countries that contribute the least to world knowledge (see Figure 8).

Figure 8: Decomposition of sources of world stock of knowledge (by country)



(a) Innovation (b) Diffusion

Notes: This figure shows the contribution of innovation an diffusion of each country i on the stock of knowledge of the world. The innovation component is $\sum_{k=1}^{J}\lambda_i^k T_i^k \left(s_i^k\right)^{\beta_r}$. The diffusion component is $\sum_{k=1}^{J}\sum_{n=1}^{M}\sum_{j=1}^{J}\frac{\varepsilon_{ni}^{jk}/g_T}{\varepsilon_{ni}^{jk}+g_T}$. All variables are in logs.

5.2 Counterfactual Analysis

We analyze the effect of trade liberalization on innovation, long-run growth and comparative advantage. In particular, we perform a uniform and permanent reduction of trade barriers of 25% (in terms of $d_{in}^j - 1$) across all sectors j and country-pairs i, n. All other parameters are kept fixed at their calibrated values. First, we describe briefly the algorithm that we develop to compute the counterfactual BGP. Then, we report our main results for our multi-country and multi-sector endogenous growth model featuring heterogeneous interlinkages in production and knowledge flows and analyze the role of knowledge spillovers by simulating two variations of our baseline model: (i) a model with (almost) negligible knowledge spillovers; and (ii) a model with knowledge spillovers that are homogeneous across countries and sectors..

5.2.1 The Algorithm

In our calibration procedure, we took the growth rate of the economy g_T, as given. However, in our endogenous growth model, g_T will change across counterfactuals when there are changes in trade costs. Changes in g_T and $\hat{T}_n^j$ are induced by changes in the innovation intensity, s_n^j, and by knowledge diffusion, as we explained in Section 4. Our algorithm to solve for the counterfactual equilibrium uses the properties of the Frobenius theorem. With calibrated values for $\{\gamma^j, \gamma^{jk}, \alpha^j, \beta_r, \lambda_n^j, \hat{T}_n^j, \varepsilon_{ni}^{jk}\}$ and the new value of trade costs d_{in}^j, we then compute the new static trade equilibrium. Then, we obtain a new optimal R&D intensity s_n^j and use the Frobenius theorem to derive the new growth rate, g_T, and associated stock of knowledge, $\hat{T}_n^j$. We repeat this process by iterating over Equation (21) until $g_T(t-1) = g_T(t)$. The associated eigenvector delivers the new $\hat{T}_n^j$.

5.2.2 Innovation, Growth and Comparative Advantage

In this section, we quantify the effect of trade liberalization on innovation, growth and comparative advantage according to the mechanisms exposed in Section 4.

R&D Reallocation and Specialization Effect In Section 4, we derived an analytical expression for the R&D allocation across country and sectors in two extreme cases—trade autarky and zero gravity. In this section, we examine whether our counterfactual experiment delivers the intuition mechanism explained previously. Following trade liberalization, R&D tends to reallocate more according to a country's endogenous comparative advantage in production, thus strengthening the specialization effects of trade liberalization.

Motivated by Equation (27), we examine the following regression using both the baseline and counterfactual results.

$$\log\left(\frac{s_n^j}{\sum_j s_n^j}\right) = \beta_0 + \beta_1 \log(ICA_n^j) + \beta_2 \log(PCA_n^j) + f_n + f_j + \mu_n^j, \tag{38}$$

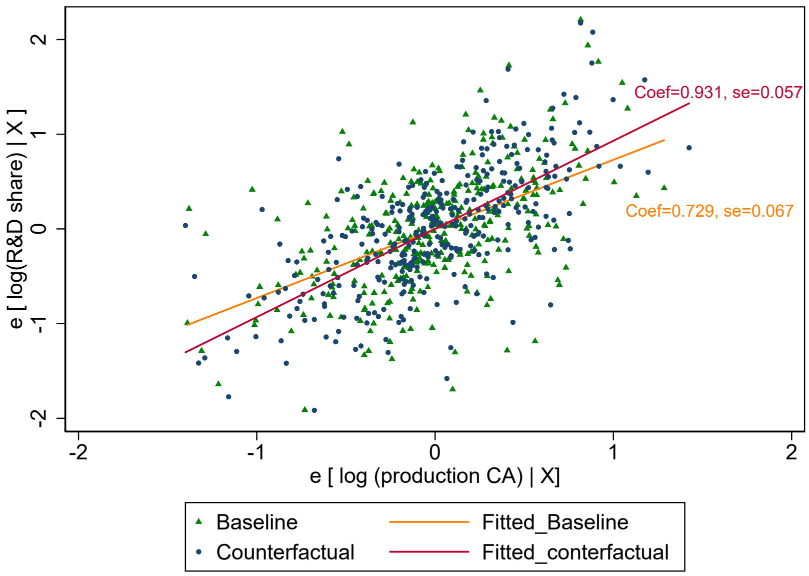
where ICA_n is the exogenous comparative advantage in innovation (i.e. based on λ_n^j) and PCA_n^j represents the comparative advantage in production (i.e. based on $T_n^j(c_n^j)^{-\theta}$), both are measured as in Hanson, Lind, and Muendler (2015) by applying the double normalization—first, we obtain a measure of a country's absolute advantage in a sector by normalizing λ_n^j or $T_n^j(c_n^j)^{-\theta}$ by the global mean for that sector, and then normalize absolute advantage by its country-wide mean.[18] The variables f_n and f_j are country- and sector-fixed effects, respectively. The sector fixed effects absorb the role of world demand and world aggregate technology as shown in Equation (27).

Figure 9 presents the post-estimation partial relationship between the R&D share and the production comparative advantage, controlling for other explanatory variables. First, as expected, within a country, sectors with higher production comparative advantage generally receive higher share of R&D both in the baseline case and after reducing the trade costs. However, in the counterfactual scenario this relationship becomes even tighter as manifested by the steeper fitted line and an increase in the coefficient of production comparative advantage, β_2, from 0.729 to 0.931.

[18] More precisely, $PCA = \frac{T_n^j(c_n^j)^{-\theta}/\frac{1}{M}\sum_n T_n^j(c_n^j)^{-\theta}}{\frac{1}{J}[\sum_j T_n^j(c_n^j)^{-\theta}/\frac{1}{M}\sum_n T_n^j(c_n^j)^{-\theta}]}$ and $ICA = \frac{\lambda_n^j/\frac{1}{M}\sum_n \lambda_n^j}{\frac{1}{J}[\sum_j \lambda_n^j/\frac{1}{M}\sum_n \lambda_n^j]}$.

This implies that production comparative advantage is now playing a even larger role in directing research efforts.

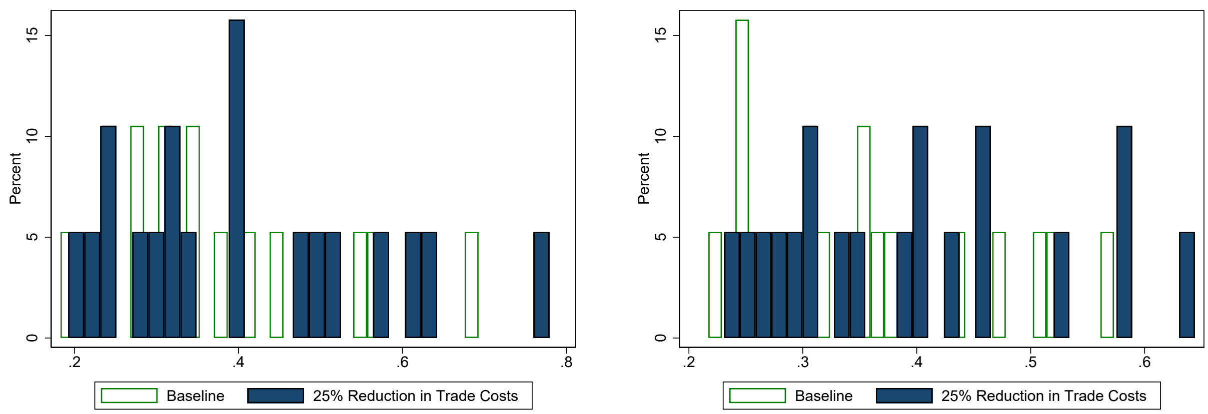
Figure 9: Reallocation of R&D, production comparative advantage and growth



Notes: This figure shows the partial residual plots of regression (38. The coefficients correspond to β_2 for the baseline and counterfactual cases, respectively. Standard errors associated with the estimated coefficients are also included.

Since comparative advantage in production is endogenous in this model, as T_n^j endogenously evolves responding to changes in the trade costs and the associated movements of innovation allocation, we next examine how the dispersion in T_n^j changes after the liberalization. First, lumping all countries and sectors together, we find that the standard deviation of $\log(T_n^j)$ increases from 0.50 to 0.53. Second, when decomposing this increase in dispersion into country dimension and sector dimension, we find that in most countries the comparative advantage in T_n^j across sectors becomes more dispersed following a trade liberalization as most countries see the standard deviation of TCA_n^j across sectors increase (Figure 10(a)).

Figure 10: Distribution of $\sigma(\log TCA)$, baseline vs. counterfactual

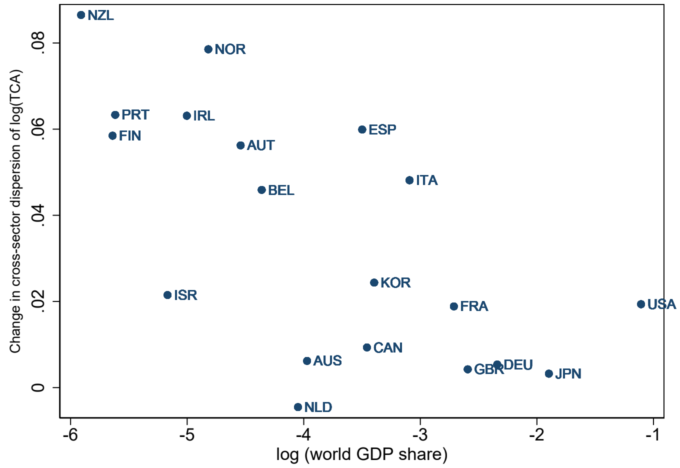


(a) Within-country across-sectors, $\sigma_n(\log TCA_n^j)$ (b) Within-sector across-countries, $\sigma_j(\log TCA_n^j)$

Notes: Figure (a) shows the country distribution graph of within-country cross-sector standard deviation of $\log(TCA_n^j)$, and Figure (b) shows the sectoral distribution graph of within-sector cross country standard deviation of $\log(TCA_n^j)$. Each is plotted for the baseline case (hollow green bars) and compared to the counterfactual scenario with 25% reduction in trade costs (solid navy bars).

Moreover, richer countries experience a lower increase in the cross-sector dispersion of comparative advantages in T_n^j (Figure 11). Similarly, for a given sector, countries also become become less similar in their T_n^j, as the comparative advantage in T_n^j across countries becomes more dispersed for all sectors in the counterfactual experiment (Figure 10(b)).

Figure 11: Change in cross-sector $\sigma(\log TCA)$ against GDP share, counterfactual – baseline



Notes: This figure shows how differences between the country-specific cross-sector dispersion in $\log TCA$ in the counterfactual and that in the baseline is related to the country's GDP share. The world GDP share is measured by nominal GDP in USD in 2005 as a share of the total GDP of the country sample.

The reallocation of R&D across sectors has both growth and level effects. As a result of the reallocation effect of R&D, growth jumps to a higher value in the new BGP. Growth of the stock of knowledge, g_T increases from 12% to 13.9%. The increase is exponential with the size of the trade liberalization, as Figure 12 exposes. After a 25% trade liberalization growth increases to 13.9%, whereas after a 50% trade liberalization it increases to 18.5%.

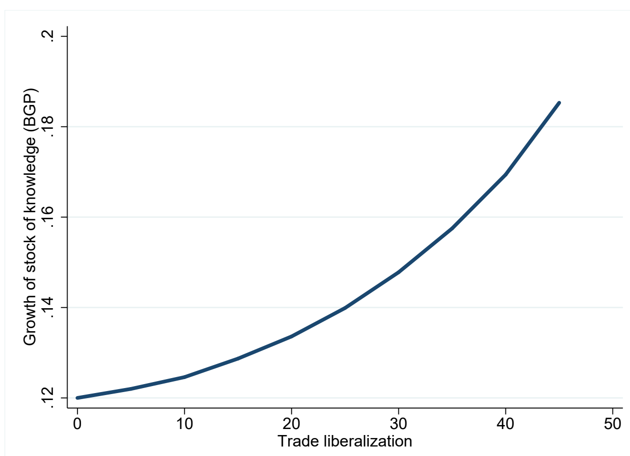


Figure 12: Effect of trade liberalization (% reduction on trade costs) of BGP growth

Notes: This figure shows the effect that the BGP growth rate of the stock of knowledge increases exponentially with the magnitude of the trade liberalization. The X-axis shows the particular drop in $d^j_{in} - 1$. Our counterfactual exercise considers a drop of 25% in trade costs.

After the trade liberalization, output is higher in every country. On average, output increases by 20%. The increase in output is heterogeneous across countries, with Japan experiencing the lowest increase (8%) and Germany experiencing the largest increase, 40%. The home trade of each country-sector nj share decreases on average by 15%, and sector productivity computed as $\frac{T_n^j}{\left(\pi_{nn}^j\right)^\theta}$, increases on average by 5%. Aggregating across countries, the average country experiences an increase in productivity of 12.5%. This increase is driven by both a decrease in the home trade share and an increase in the stock of knowledge.

The role of knowledge spillovers We study the role of knowledge diffusion on growth. We recalibrate our baseline model in two ways. First, we consider the case of homogeneous diffusion across all country-sector pairs, in which we set $\epsilon_{ni}^{jk} = \epsilon$, $\forall i, n, j, k$. Here ϵ is the average speed of diffusion estimated in the data. Second, we consider the case of no diffusion by setting the diffusion parameters ε_{ni}^{jk} to a very small value of 0.0001, for all $i \neq n$ and $k \neq j$ (we set $\varepsilon_{nn}^{jj} \to \infty$; that is, we assume instantaneous diffusion within the same country-sector pair).[19] These recalibrations do not affect the first-stage calibration that solved for the competitive equilibrium of the model. However, we need to recalibrate the second-stage parameters, β_r and λ_n^j, by using the same input-output linkage parameters $\{\alpha^j, \gamma^j, \gamma^{jk}\}$, estimated technology, T_n^j, R&D intensity, s_n^j, and growth rate, g_A, values than in the baseline model. We now obtain $\beta_r = 0.37$ in the case of homogeneous diffusion and $\beta_r = 0.10$ in the case of no diffusion. The effect of trade liberalization on growth rate is lower in the case of homogeneous diffusion or no diffusion. After trade liberalization, the growth rate increases to 0.1220 when there is homogeneous diffusion and to 0.1202 when there is no diffusion. Furthermore, the rate of increase of BGP growth with the size of trade liberalization is slower than in our baseline model with heterogeneous knowledge spillovers (see Figure 13).

[19]The Frobenius theorem is only valid if there is at least some diffusion across all country-sector pairs. Setting ε_{ni}^{jk} to a very low number allows us to make use of the properties of the Frobenius theorem while allowing for very slow to virtually no diffusion.

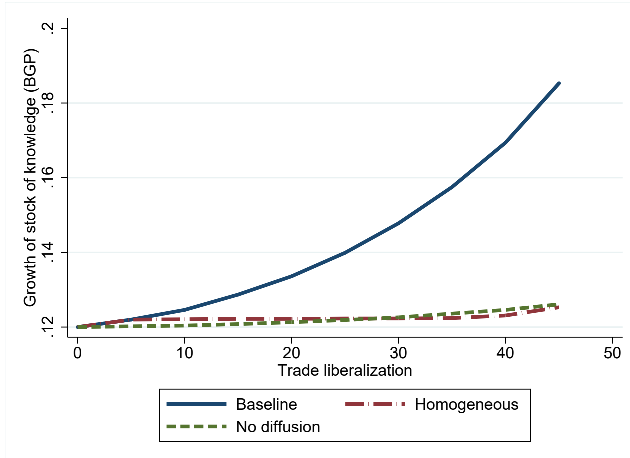


Figure 13: Effect of trade liberalization of BGP growth

Notes: This figure shows the different profile of BGP growth for different magnitude of the trade liberalization in three versions of our model: (i) baseline model, (ii) a model with (almost) negligible cross-country and cross-sector knowledge spillovers, and (iii) a model with homogeneous diffusion across countries and sectors.

5.2.3 Welfare Gains from Trade

We compute welfare gains from trade after a trade liberalization between the baseline and the counterfactual BGP. Welfare in our model is defined in equivalent units of consumption. We ignore transitional dynamics in this analysis. We can use equation (1) to obtain the lifetime utility in the initial BGP as

$$\bar{U}_i^* = \int_{t=0}^{\infty} e^{-\rho t} \frac{\left(\hat{C}_i^*\right)^{1-\gamma}}{1-\gamma} e^{g^*(1-\gamma)t} dt = \frac{\left(\hat{C}_i^*\right)^{1-\gamma}}{\rho - g^*(1-\gamma)},$$

and in the counterfactual BGP as

$$\bar{U}_i^{**} = \int_{t=0}^{\infty} e^{-\rho t} \frac{\left(\hat{C}_i^{**}\right)^{1-\gamma}}{1-\gamma} e^{g^{**}(1-\gamma)t} dt = \frac{\left(\hat{C}_i^{**}\right)^{1-\gamma}}{\rho - g^{**}(1-\gamma)}$$

with $*$ denoting the baseline BGP and $**$ denoting the counterfactual BGP.

Welfare gains are defined as the amount of consumption that the consumer is willing to give up in the counterfactual BGP to remain at the same level as in the initial BGP. We call this, λ_i, which is obtained as

$$\bar{U}_i^*(\lambda_i) = \bar{U}_i^{**}$$

$$\frac{\left(\hat{C}_i^* \lambda_i\right)^{1-\gamma}}{\rho - g^*(1-\gamma)} = \frac{\left(\hat{C}_i^{**}\right)^{1-\gamma}}{\rho - g^{**}(1-\gamma)}.$$

From here,

$$\lambda_i = \frac{\hat{C}_i^{**}}{\hat{C}_i^{*}} \left(\frac{\rho - g^*(1-\gamma)}{\rho - g^{**}(1-\gamma)} \right)^{\frac{1}{1-\gamma}}. \tag{39}$$

Welfare gains depend on changes in normalized consumption between the BGPs and the change in growth rates. From equation (74), normalized consumption in the BGP is equal to income per capita net of R&D expenditures.

In static models or one-sector models of trade and innovation in which changes in trade costs do not have an effect on innovation, $g^* = g^{**}$ and $s_i^k = 0$. In that case, welfare gains from trade are computed as changes in the real wage. As in Caliendo and Parro (2015), we can obtain an expression for the real wage in country i as

$$\frac{W_i}{P_i} \propto \prod_{j=1}^{J} \left(\left(\frac{T_i^j}{\pi_{ii}^j} \right)^{\alpha^j/\theta} \prod_{k=1}^{J} \left(\frac{W_i}{P_i^k} \right)^{\alpha^j \gamma^{jk}} \right). \tag{40}$$

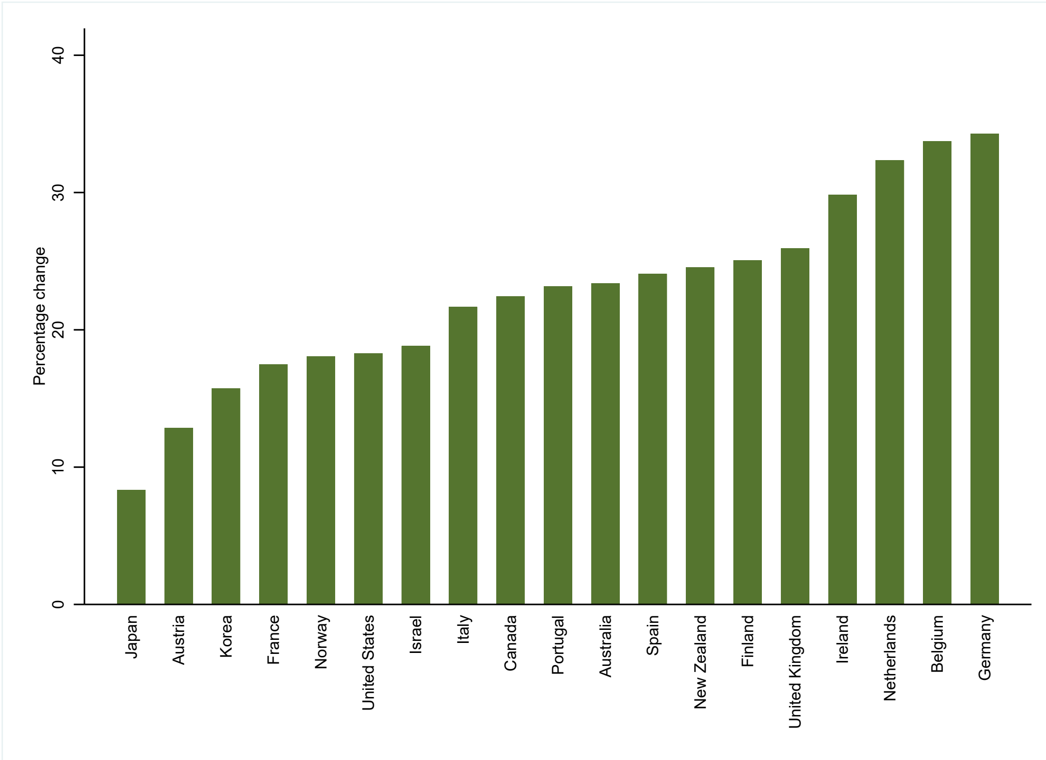
Note that this formula is the same as derived in Caliendo and Parro (2015) resembles the standard welfare formula in Arkolakis, Costinot, and Rodríguez-Clare (2012). In a one-sector version of our model, in which $j = 1$, γ^{jk}=0, and $\alpha^j = 1$, Equation (40) becomes

$$\frac{W_i}{P_i} \propto \left(\frac{T_i}{\pi_{ii}} \right)^{1/\theta}. \tag{41}$$

This is the standard formula for welfare gains from trade that has been used in the literature and depends on aggregate productivity, the home trade shares and the trade elasticity.

Welfare Results We compute welfare gains from trade using Equation (39). We find that welfare gains from trade are heterogeneous across countries, ranging from 8% to 34%, with a cross-country average gain of 23% and a standard deviation of 7% (see Figure 14). The gains are larger for smaller countries in general, which is consistent with the findings in Waugh (2010).

Figure 14: Welfare gains (%)



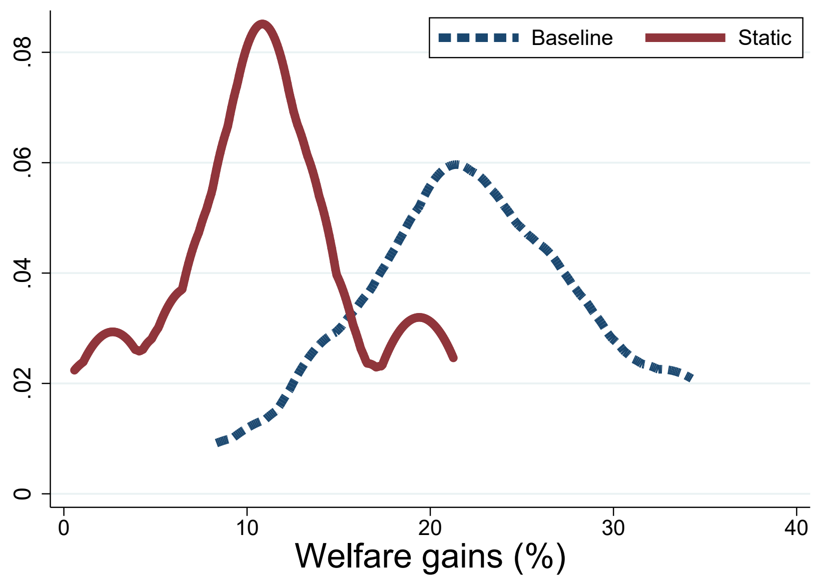
Notes: This figure shows welfare gains from trade in our multi-sector model with innovation and knowledge spillovers.

Welfare gains from trade can be divided into static and dynamic gains. Static gains correspond to those obtained in a model where the stock of knowledge, $\hat{T}^j_{nt}$, is not allowed to change over time. These are the gains that are obtained in standard static models of trade and are driven by increased specialization and comparative advantage. Dynamic gains take into account the effect of R&D and knowledge spillovers on the stock of knowledge. Both allow the stock of knowledge to increase over time. Higher innovation allows countries to increase their income per capita, which has an unambiguously positive effect on dynamic gains. Knowledge diffusion has two opposite effects on dynamic gains from trade. On the one hand, it increases the stock of knowledge of a country-sector as it can benefit from innovation created in other country-sectors. This has an additional effect on the efficiency of innovation, from equation (14), which reinforces the innovation channel. On the other hand, knowledge spillovers may generate convergence of comparative advantage over time, dampening the total welfare gains from trade that are driven by differences in comparative advantage.

To compute static welfare gains, we simulate our model keeping $\hat{T}^j_i$ constant across counterfactuals. Because we are analyzing only changes across BGPs, dynamic gains do not include the transition. We call them dynamic in that they reflect the gains that account for changes in the stock of knowledge across counterfactuals. Therefore, these gains are computed by letting $\hat{T}_i^j$ vary across counterfactuals.

Figure 15 compares welfare gains from trade in our baseline model to those static gains in which the stock of technology is kept constant across counterfactuals. The difference between the two gains is a measure of dynamic gains from trade. The cross-country distribution of static gains is shifted to the left, which implies that dynamic gains are positive in every country (see Table 2).

Figure 15: Welfare gains from trade (% change)



Notes: This figure compares gains from trade in our dynamic model with those of a static model in which $\hat{T}_n^j$ is kept constant across counterfactuals.

Finally, we compare welfare gains from trade in our baseline model to those in a model with homogeneous diffusion, no diffusion and a one-sector model with heterogeneous diffusion. The model that generates the lowest and least dispersed gains from trade is the one-sector model. Trade liberalization has no effect on innovation and growth in this case, making it a static model. Furthermore, the lack of input-output linkages does not allow for additional gains from trade of multi-sector models. With respect to our baseline, the cases of homogeneous or no diffusion also deliver lower and less disperse gains from trade (see Table 2).

Model	Mean	Std. Dev.	Min	Max
Baseline	22.63	6.90	8.33	34.27
Static	10.92	5.87	0.58	21.26
Homogeneous diffusion	16.80	6.75	7.60	32.90
No diffusion	18.76	6.80	6.23	30.57

Note: Table contains welfare gains from trade for three versions of the model: (i) baseline model, (ii) model with (almost) negligible cross-country cross-sector knowledge spillovers, and (iii) model with homogeneous knowledge spillovers across countries and sectors.

Table 2: Welfare gains from trade (% change)

6 Concluding Remarks

We develop a quantitative framework to study the effect of interlinkages among trade, knowledge flows and production on innovation, comparative advantage, growth and welfare. We distinguish between static gains from trade, which are driven by increased specialization, and dynamic gains from trade, which are driven by innovation and knowledge diffusion. Changes in trade barriers have a quantitatively important effect on innovation and welfare. After a trade liberalization, R&D reallocates toward sectors in which the country has a comparative advantage. Knowledge diffusion amplifies this effect, as comparative advantage is reallocated towards sectors with larger knowledge flows. Furthermore, knowledge spillovers allow sectors in a country to benefit for a larger pool of ideas, increasing dynamic welfare gains from trade. A one-sector version of our model delivers much smaller total gains in welfare and almost negligible, or even negative in some countries, dynamic welfare gains. This result reinforces the importance of modeling sectoral heterogeneity when studying the effect of trade liberalizations on innovation and welfare.

Our model can be extended to study other important issues in macroeconomics and international trade. If the production structure of the economy is assumed to be CES rather than Cobb-Douglas, a trade liberalization that changes technology and production costs will shift production shares across sectors, hence inducing structural change.

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Appendix

A Model Equations

The endogenous variables are, for each $i = 1 \dots M$ and $n = 1 \dots M$

$$\{\pi^j_{int}, T^j_{it}, c^j_{it}, W_{it}, P^j_{nt}, X^j_{nit}, X^j_{nt}, P_{nt}, Y_{nt}, \Phi^j_{nt}, C_{nt}, s^j_{nt}, V^j_{nt}, \Pi^j_{nt}\}$$

The corresponding equations are as follows:

(1) Probability of Imports

$$\pi^j_{nit} = T^j_{it} \frac{\left(c^j_{it} d^j_{ni}\right)^{-\theta}}{\Phi^j_{nt}}, \tag{42}$$

(2) Import shares

$$X^j_{nit} = \pi^j_{nit} X^j_{nt}. \tag{43}$$

(3) Cost of production

$$c^j_{nt} = \Upsilon^j W^{\gamma^j}_{nt} \prod_{k=1}^{J} (P^k_{nt})^{\gamma^{jk}}. \tag{44}$$

(4) Intermediate good prices in each sector

$$P^j_{nt} = A^j \left(\Phi^j_{nt}\right)^{-1/\theta}. \tag{45}$$

(5) Cost distribution

$$\Phi^j_{nt} = \sum_{i=1}^{M} T^j_{it} \left(d^j_{ni} c^j_{it}\right)^{-\theta}. \tag{46}$$

(6) Price index

$$P_{nt} = \prod_{j=1}^{J} \left(\frac{P^j_{nt}}{\alpha^j}\right)^{\alpha^j}. \tag{47}$$

(7) Labor market clearing condition

$$W_{nt}L_{nt} = \sum_{j=1}^{J} \gamma^j \sum_{i=1}^{M} \pi_{int}^j X_{it}^j. \tag{48}$$

(8) Sector production

$$X_{nt}^j = \sum_{k=1}^{J} \gamma^{kj} \sum_{i=1}^{M} X_{it}^k \pi_{int}^k + \alpha^j P_{nt} Y_{nt}. \tag{49}$$

(9) Income

$$P_{nt}C_{nt} = W_{nt}L_n + \frac{\sum_{j=1}^{J}\sum_{i=1}^{M} \pi_{int}^j X_{it}^j}{1+\theta}. \tag{50}$$

(10) Resource constraint

$$Y_{nt} = C_{nt} + \sum_{j=1}^{J} R_{nt}^j. \tag{51}$$

(11) Innovation

$$\dot{T}_{nt}^j = \sum_{i=1}^{M}\sum_{k=1}^{J} \varepsilon_{ni}^{jk} \int_{-\infty}^{t} e^{-\varepsilon_{ni}^{jk}(t-s)} \alpha_s^k T_s^k \left(s_{is}^k\right)^{\beta^k} ds. \tag{52}$$

(12) R&D expenditures

$$\beta^j \lambda_{nt}^j V_{nt}^j \left(s_{nt}^j\right)^{\beta^j - 1} = P_{nt}\bar{Y}_t. \tag{53}$$

(13) Value of an innovation

$$V_{nt}^j = \int_{t}^{\infty} \left(\frac{P_{nt}^j}{P_{ns}^j}\right) e^{-\int_t^s r_{nu} du} \frac{\Pi_{ns}^j}{T_{ns}^j} ds, \tag{54}$$

(14) Profits

$$\Pi_{nt}^j = \frac{1}{(1+\theta)} \sum_{i=1}^{M} X_{it}^j \pi_{int}^j. \tag{55}$$

(15) Trade balance

$$\sum_{k=1 k\neq j}^{J} X_{nt}^k \sum_{i=1 i\neq n}^{M} \pi_{nit}^k = \sum_{i=1 i\neq n}^{M} \sum_{k=1 k\neq j}^{J} \pi_{int}^k X_{it}^k. \tag{56}$$

B Derivations on the BGP

Here, we derive an expression for the growth rate of the economy along the BGP. We remove time subscripts when we refer to variables on the BGP.

First, note that through technology diffusion, the level of knowledge-related productivity, T_n^j, grows at the same rate for every country n and sector j. Therefore, we can pick country M and sector J's technology level to normalize every T_n^j. Normalized variables are denoted with a hat. In particular, $\hat{T}_n^j = \frac{T_n^j}{T_M^J}$.

From equation (48), X_i^j is normalized as $\hat{X}_i^j = \frac{X_i^j}{W_M}$ for all j. Hence, expenditures grow at a constant rate for all sectors, since π_{in}^j is constant in the BGP (see equations (42) and (46)). From equations (48) and (50), $P_n Y_n$ grow at the rate of W_M. Note that $g_{w_n} = g_w$ for all n.

From equation 51, by which consumption, C_n and final output, Y_n grow at the same constant rate on the BGP together with the fact that output in each country grows at the same rate, hence $\frac{Y_n}{Y}$ is constant in the BGP implies that the fraction of world output that is invested into R&D, s_n^j is constant on the BGP. This result, together with equation (16), implies that $\frac{V_n^j T_n^j}{W_M}$ is constant along the BGP. We then have from equation (15):

$$\hat{V}_n^j = \left(\frac{1}{r - g_T/\theta + g_T}\right) \frac{\sum_{i=1}^{M} \pi_{in}^j \hat{X}_i^j}{(1+\theta)},$$

in which $\hat{V}_n^j = \frac{V_n^j T_n^j}{P_n Y_n}$, $\hat{X}_i^j = \frac{X_i^j}{W_M}$, and $\hat{Y}_n = \frac{P_n Y_n}{W_M}$, with W_M being the nominal wage in the numeraire country M. We impose $r - g_T/\theta + g_T > 0$. From equation (11), π_{in}^j is constant along the BGP.

To derive an expression for the BGP growth rate of the real output per capita, Y_n, we start from the fact that $\frac{W_n}{P_n Y_n}$ is constant in steady-state. Hence,

$$g_{Y_n} = g_w - g_{P_n}.$$

Using equation (47),

$$g_{P_n} = \sum_{j=1}^{J} \alpha^j g_{p_n^j}.$$

We then derive the expression for $g_{p_n^j}$ from equations (44), (45) and (46). First, we rewrite equation (44) as

$$\frac{c_n^j}{W_n} = \prod_{k=1}^{J} \left(\frac{p_n^k}{W_n}\right)^{\gamma_n^{jk}}.$$

In growth rates, it becomes

$$g_{\tilde{c}_n^j} = \sum_{k=1}^{J} \gamma_n^{jk} g_{\tilde{p}_n^k}, \tag{57}$$

where $\tilde{c}_n^j = \frac{c_n^j}{W_n}$ and $\tilde{p}_n^k = \frac{p_n^k}{W_n}$. From equation (46),

$$g_{\Phi_n^j} = g_T - \theta g_{c_n^j} = g_T - \theta g_{c_i^j}.$$

Hence, $g_{c_n^j} = g_{c^j}$ for all n. Normalizing by wages,

$$g_{\tilde{\Phi}_n^j} = g_T - \theta g_{\tilde{c}_n^j}, \tag{58}$$

where $\tilde{\Phi}_n^j = \frac{\Phi_n^j}{W_n^{-\theta}}$

Combining equation (45) and (58) implies that

$$g_{\tilde{p}_n^k} = -\frac{1}{\theta} g_T + g_{\tilde{c}^k}. \tag{59}$$

Substitution into (57) and using $\sum_{k=1}^{J} \gamma^{jk} = 1 - \gamma^j$, we get

$$g_{\tilde{c}^j} = -\frac{(1-\gamma^j)}{\theta} g_T + \sum_{k=1}^{J} \gamma^{jk} g_{\tilde{c}^k}. \tag{60}$$

We can express the previous expression in matrix form so that

$$\begin{bmatrix} g_{\tilde{c}^1} \\ g_{\tilde{c}^2} \\ \vdots \\ g_{\tilde{c}^J} \end{bmatrix} = -\frac{1}{\theta} g_T \begin{bmatrix} 1-\gamma^1 \\ 1-\gamma^2 \\ \vdots \\ 1-\gamma^J \end{bmatrix} + \begin{bmatrix} \gamma^{11} & \gamma^{12} & \dots & \gamma^{1J} & \\ \gamma^{21} & \gamma^{22} & \dots & \gamma^{2J} & \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \gamma^{J1} & \gamma^{J2} & \dots & \gamma^{JJ} & \end{bmatrix} \begin{bmatrix} g_{\tilde{c}^1} \\ g_{\tilde{c}^2} \\ \vdots \\ g_{\tilde{c}^J} \end{bmatrix} \tag{61}$$

From here

$$\begin{bmatrix} g_{\tilde{c}^1} \\ g_{\tilde{c}^2} \\ \vdots \\ g_{\tilde{c}^J} \end{bmatrix} = -\frac{g_T}{\theta} (I-A)^{-1} \begin{bmatrix} 1-\gamma^1 \\ 1-\gamma^2 \\ \vdots \\ 1-\gamma^J \end{bmatrix} \tag{62}$$

where

$$A = \begin{bmatrix} \gamma^{11} & \gamma^{12} & \dots & \gamma^{1J} \\ \gamma^{21} & \gamma^{22} & \dots & \gamma^{2J} \\ \vdots & \vdots & \vdots & \ddots \\ \gamma^{J1} & \gamma^{J2} & \dots & \gamma^{JJ} \end{bmatrix}$$

Therefore, the cost of production c_n^j can be normalized as

$$\hat{c}_n^j = \frac{c_n^j}{W_M (T_M^J)^{-\frac{1}{\theta}\Lambda_j}}, \tag{63}$$

where Λ_j is the jth entry of the vector $\Lambda = (I - A)^{-1} \begin{bmatrix} 1-\gamma^1 \\ 1-\gamma^2 \\ \vdots \\ 1-\gamma^J \end{bmatrix}$.

With this, we can obtain an expression for the growth rate of real output as

$$g_{Y_n} = g_w - \sum_{j=1}^{J} \alpha^j g_{p_n^j}.$$

From Equation (59), we have

$$g_{Y_n} = g_w - \sum_{j=1}^{J} \alpha^j \left(\frac{-1}{\theta} g_T + g_{c^j} \right).$$

Based on Equation (63), the above equation becomes

$$g_{Y_n} = g_w - \sum_{j=1}^{J} \alpha^j \left(\frac{-1}{\theta} g_T + g_w - \Lambda_j g_T \right).$$

Therefore,

$$g_{Y_n} = \frac{1}{\theta} \left(1 + \sum_{j=1}^{J} \alpha_j \Lambda_j \right) g_T = g_y, \forall n. \tag{64}$$

Note that in a one-sector economy in which $\gamma^{jk} = 0, \forall n, k$ and $\gamma^j = 1, \forall j$, the growth rate is

$$g_y = -\frac{1}{\theta} g_T.$$

as in Eaton and Kortum (1996, 1999). With multiple sectors, however, the growth rate of the economy is amplified by the input-output linkages.

C Model Equations (Normalized) along the BGP

In what follows, we report the equations of the model after normalizing the endogenous variables so that they are constant in the BGP. We follow the results obtained in Appendix B.

(1) Probability of imports

$$\pi_{ni}^{j} = \hat{T}_{i}^{j} \frac{\left(\hat{c}_{i}^{j} d_{ni}^{j}\right)^{-\theta}}{\hat{\Phi}_{n}^{j}}, \tag{65}$$

where $\hat{T}_n^j = \frac{T_n^j}{T_M^J}$ and $\hat{\Phi}_n^j = \frac{1}{T_M^J} \frac{\Phi_n^j}{(W_M)^{-\theta}(T_M^J)^{\Lambda^j}}$ with Λ^j defined in Appendix **??**.

(2) Import shares

$$\hat{X}_{ni}^{j} = \pi_{ni}^{j} \hat{X}_{n}^{j}. \tag{66}$$

(3) Cost of production

$$\hat{c}_{n}^{j} = \Upsilon^{j} \hat{W}_{n}^{\gamma^{j}} \prod_{k=1}^{J} (\hat{P}_{n}^{k})^{\gamma^{jk}}. \tag{67}$$

(4) Intermediate good prices in each sector

$$\hat{P}_{n}^{j} = B \left(\hat{\Phi}_{n}^{j}\right)^{-1/\theta}. \tag{68}$$

(5) Cost distribution

$$\hat{\Phi}_{n}^{j} = \sum_{i=1}^{M} \hat{T}_{i}^{j} \left(d_{ni}^{j} \hat{c}_{i}^{j}\right)^{-\theta}. \tag{69}$$

(6) Price index

$$\hat{P}_{n} = \prod_{j=1}^{J} \left(\frac{\hat{P}_{n}^{j}}{\alpha^{j}}\right)^{\alpha^{j}}. \tag{70}$$

(7) Labor market clearing condition

$$\hat{W}_{n} L_{n} = \sum_{j=1}^{J} \gamma^{j} \sum_{i=1}^{M} \pi_{in}^{j} \hat{X}_{i}^{j}. \tag{71}$$

(8) Sector production

$$\hat{X}_n^j = \sum_{k=1}^{J} \gamma^{kj} \sum_{i=1}^{M} \pi_{in}^k \hat{X}_i^k + \alpha^j \hat{Y}_n. \tag{72}$$

where $\hat{Y}_n = \frac{P_n Y_n}{W_M}$.

(9)Income

$$\hat{C}_n = \hat{W}_n L_n + \frac{\sum_{j=1}^{J} \sum_{i=1}^{M} \pi_{in}^j \hat{X}_i^j}{1+\theta}. \tag{73}$$

(10) Resource constraint

$$\hat{Y}_n = \hat{C}_n + \sum_{k=1}^{J} s_n^k \hat{\bar{Y}}. \tag{74}$$

with

$$\hat{\bar{Y}} = \sum_{m=1}^{M} \hat{Y}_m$$

(11) Innovation

$$g_T = \sum_{i=1}^{M} \sum_{k=1}^{J} \frac{\varepsilon_{ni}^{jk}}{g_T + \varepsilon_{ni}^{jk}} \lambda_i^k \frac{\hat{T}_i^k}{\hat{T}_n^j} \left(\frac{1}{r - g_y + g_T} \beta_r \lambda_i^k \frac{1}{(1+\theta)} \frac{\sum_{n=1}^{M} \pi_{ni}^k \hat{X}_n^k}{\hat{Y}_n} \right)^{\frac{\beta_r}{1-\beta_r}}.$$

(12) R&D expenditures

$$\beta_r \lambda_n^j \hat{V}_n^j \left(s_n^j \right)^{\beta_r - 1} = \hat{\bar{Y}}. \tag{75}$$

(13) Value of an innovation

$$\hat{V}_n^j = \left(\frac{1}{r - g_y + g_T} \right) \hat{\Pi}_n^j, \tag{76}$$

(14) Profits

$$\hat{\Pi}_n^j = \frac{1}{(1+\theta)} \sum_{i=1}^{M} \hat{X}_i^j \pi_{in}^j. \tag{77}$$

(15) Trade balance

$$\sum_{k=1 k\neq j}^{J} \hat{X}_{nt}^k \sum_{i=1 i\neq n}^{M} \pi_{nit}^k = \sum_{i=1 i\neq n}^{M} \sum_{k=1 k\neq j}^{J} \pi_{int}^k \hat{X}_{it}^k. \tag{78}$$

D Existence and Uniqueness of the Equilibrium

Here we provide sufficient conditions for the existence and uniqueness of the equilibrium in our model. We follow the methodology developed in Allen, Arkolakis, and Li (2015) to study uniqueness of the trade block of the model, given T_n^j. We then prove uniqueness of R&D intensity, s_n^j, given g_T and the trade equilibrium. Finally, we use the Frobenius theoremon the growth block of the model to prove uniqueness of g_T and T_n^j given the static trade equilibrium.

Notation

We start by rewriting equation (77) in matrix form as

$$\hat{\mathbf{\Pi}} = \pi_{\mathbf{diag}} \hat{\mathbf{X}}, \tag{79}$$

where $\hat{\mathbf{\Pi}}$ and $\hat{\mathbf{X}}$ are $M \times J$ dimensional vectors, with $((n-1) \times M + j)^{th}$ elements equal to $\hat{\Pi}_n^j$ and $\hat{X}_n^j$, respectively; and $\pi_{\mathbf{diag}}$ a block diagonal matrix of $J * J$ sub-blocks, in which the j^{th} main diagonal block is an $M * M$ matrix with (i, n) element equal to $\frac{\pi_{ni}^j}{(1+\theta)}$ and off-diagonal blocks equal to zero.

Substituting equation (74) into equation (73) we have

$$\hat{X}_n^j = \sum_{k=1}^{J} \gamma^{kj} \sum_{i=1}^{M} \pi_{in}^k \hat{X}_i^k + Y_n \alpha^j.$$

where $\mathbf{Y}^\alpha$ is a $M * J$ vector, whose $((j-1) * M + n)^{th}$ element is $\alpha^j Y_n$ or

$$\hat{\mathbf{X}} = \mathbf{A}_X \hat{\mathbf{X}} + \mathbf{Y}^\alpha,$$

where $\mathbf{A}_X$ is a square block matrix with $J \times J$ sub-blocks, and each sub-block is an $M \times M$ matrix, and the (n,i) element in the (j,k) sub-block is equal to $\gamma^{kj} \pi_{in}^k$; or

$$\mathbf{A}_X = \mathbf{\Gamma}^{kj} \pi_{diag}, \tag{80}$$

where $\mathbf{\Gamma^{kj}}$ is a $J \times J$ block matrix, and the (k,j) sub-block is an $M \times M$ square matrix with all elements equal to γ^{kj}.

$$\hat{\mathbf{X}} = (\mathbf{I} - \mathbf{A}_X)^{-1} \mathbf{Y}^\alpha. \tag{81}$$

Existence and uniqueness of the solution

In order to prove the existence and uniqueness of our counterfactual solution, we go take following steps.

Step 1

We start by proving that s_n^j is unique, given $\{Y_n, T_n^j, W_n, g_T\}$.

Proof. When $\{Y_n, T_n^j, W_n, g_T\}$ are given, we can obtain $\{X_n^j, V_n^j\}$ and π_{diag} from the static trade block. Using (75), (76), (79) and (81), we derive an expression for optimal R&D investment as

$$(s_n^j)^{1-\beta_r}\bar{Y} = \beta_r \lambda_n^j \hat{T}_n^j \hat{V}_n^j$$

or

$$(\mathbf{s})^{1-\beta_r} = \mathbf{\Omega}_s \mathbf{Y}^{\alpha}, \tag{82}$$

where

$$\mathbf{\Omega}_s = \mathbf{B}_v \pi_{diag}(\mathbf{I} - \mathbf{A}_X)^{-1},$$

and

$\mathbf{B}_v$ is a diagonal matrix with the $M \times (j-1) + n^{th}$ diagonal elements given by

$$B_v(M \times (j-1) + n, M \times (j-1) + n) = \frac{\beta_r \lambda_n^j T_n^j}{\bar{Y}} \left(\frac{1}{r - g_y + g_T} \right)$$

Therefore,

$$s_n^j = \left(\sum_{i,k} \Omega_s(M \times (j-1) + n, M \times (k-1) + i) \alpha_k Y_i \right)^{\frac{1}{1-\beta_r}}. \tag{83}$$

■

Step 2

We prove that Y_n is unique when s_n^j is a function of $\{Y_n,\ T_n^j,\ W_n\}$, and $\{T_n^j,\ W_n,\ g_T\}$ are given.

Proof. From (71), (73), (74), (81) and (79), we derive a function $f(Y_n)$ and prove that it satisfies the gross-substitution property. From equation ((73)),

$$Y_n - \sum_m Y_m \sum_j s_n^j = C_n$$

Then,

$$\mathbf{C}_n = \hat{\mathbf{W}}^{\mathbf{j}} + \sum_{j=1}^{J} \hat{\Pi}_n^j$$

$$\mathbf{C}_n = \mathbf{I}_M^{1J} \Gamma_\theta \hat{\mathbf{\Pi}} = \Omega_c \mathbf{Y}^\alpha, \tag{84}$$

where

$$\Omega_c = \mathbf{I}_M^{1J} \Gamma_\theta \pi_{diag} (\mathbf{I} - \mathbf{A}_x)^{-1}$$

and $\mathbf{I}_M^{1J}$ is a $1 \times J$ block matrix and each sub-block is an $M \times M$ unit matrix. $(\mathbf{I} - \mathbf{A}_x)^{-1}$ is the Leontief inverse matrix, which is strictly positive if $\mathbf{A}_x$ is strictly positive. (See Peterson and Olinick (1982)). Therefore, Ω_c and Ω_s are strictly positive too.

$\hat{\mathbf{W}}^{\mathbf{j}}$ is a $M \times J$ vector, whose $((j-1) \times M + n)^{th}$ element is $(1+\theta)\gamma^j \hat{\Pi}_n^j$. Γ_θ is a diagonal matrix with the $((j-1) \times M + n)^{th}$ diagonal element equals to $(1+\theta)\gamma^j + 1$.

Hence, we can define a scaffold function $f_n(Y) = g_n(Y, s^j)$:

$$f_n(Y) = C_n - Y_n + \sum_j s_n^j \sum_m Y_m.$$

First, we prove that $f_n(Y)$ satisfies gross-substitution condition ($\frac{\partial f_n(Y)}{\partial Y_m} \geq 0$, $n \neq m$) using (81) and (83).

$$\begin{aligned}\frac{\partial f_n(Y)}{\partial Y_m} &= \sum_{k,j} \Omega_c(M \times (j-1) + n, M \times (k-1) + m)\alpha^k + \sum_j \frac{\partial s_n^j}{\partial Y_m} + s_n^j \\ &= \sum_{k,j} \Omega_c(M \times (j-1) + n, M \times (k-1) + m)\alpha^k\end{aligned}$$

$$+\frac{1}{1-\beta_r} \sum_j (s_n^j)^{\frac{\beta_r}{1-\beta_r}} \left(\sum_{m,k} \alpha_k \Omega_s(M \times (j-1) + n, M \times (k-1) + m) \right) + s_n^j > 0, m \neq n$$

Second, it is easy to show that

$$\frac{\partial g_n(Y, s^j)}{\partial s_i^k} = \bar{Y} > 0, \text{ if } i = n;$$

$$\frac{\partial g_n(Y, s^j)}{\partial s_i^k} = 0, \text{ if } i \neq n;$$

Finally, using Theorem 3 in Allen, Arkolakis, and Li (2015), we can prove that there is at most one unique solution to $f_n(Y) = 0$. ■

Step 3 We prove the existence and uniqueness of $\hat{W}_n$ in equation (71), when s_n^j and Y_n are function of $\{W_n,\ T_n^j\}$; and T_n^j is given. (see Appendix in Allen, Arkolakis, and Li (2015), pages 36-39)

Step 4 Finally, we prove the existence and uniqueness of g_T and $\hat{T}_n^j$ in (C). We can define

$$\Delta(g_T) = \frac{\varepsilon_{ni}^{jk}}{g_T + \varepsilon_{ni}^{jk}} \lambda_i^k \frac{\hat{T}_i^k}{\hat{T}_n^j} \left(s_i^k\right)^{\beta_r} \tag{85}$$

If $\Delta(g_T)$ is indecomposable, there exists a unique positive balanced growth rate of stock of knowledge $g_T > 0$ given research intensity s_n^j. Associated with g_T there is a vector T defined up to a scalar multiple (See Eaton and Kortum (2010)).

Proof. The Frobenius theorem guarantees that there is a unique solution g_T and $\hat{\mathbf{T}}$ to equations specified in (C). Here $\hat{T}_n^j$ is the $(M \times (j-1) + n)^{th}$ element in $\hat{\mathbf{T}}$. g_T is the largest eigenvalue of an $M \times J$ by $M \times J$ matrix $\Delta(g_T)$ and $\hat{\mathbf{T}}$ is the correspondent strictly positive eigenvector. The (nj, ik) element of $\Delta(g_T)$ is equal to $\frac{\varepsilon_{ni}^{jk}}{g_T+\varepsilon_{ni}^{jk}} \lambda_i^k \frac{\hat{T}_i^k}{\hat{T}_n^j} \left(s_i^k\right)^{\beta_r}$

■

E Data Description and Calculation

This appendix describes the data sources and the construction of various variables for the paper. Nineteen countries are included in our analysis based on data availability (developed OECD countries: Australia, Austria, Belgium, Canada, Finland, France, Germany, Israel, Italy, Japan, Korea, the Netherlands, New Zealand, Norway, Poland, Portugal, Spain, the United Kingdom, and the United States. The model is calibrated for 2005. Eighteen tradable sectors and one aggregate nontradable sector are under consideration and reported in Table 3.

Bilateral trade flows at the sectoral level Bilateral trade data at the sectoral level (expenditure by country n of sector j goods imported from country i, X_{ni}^j) are obtained from the OECD STAN Bilateral Trade Dataset. Values are reported in thousands of U.S. dollars at current prices. Sectors are recorded at the ISIC (rev. 3) 2-3 digit level and are aggregated into the 19 sectors as listed in Table 3. We use the importer reported exports in each sector as the bilateral trade flows because it is generally considered to be more accurate than the exporter reported exports.

Value added and gross production Domestic sales in sector j, X_{nn}^j, are estimated based on the *domestic* input-output table provided by the OECD STAN database, which contains data at the ISIC 2-digit level that can be easily mapped into our 19 sectors. OECD provides separate IO tables for domestic output and imports. We sum up the values for a given row before the column "Direct purchases abroad by residents (imports)" to obtain X^j_{nn}. We compare this way of estimating the domestic expenditure on domestic product with an alternative calculation based on $X^j_{nn} = Y^j_n - \sum_{i \neq n}^{M} X^j_{in}$, where both gross production of country n in sector j, Y^j_n, and the total exports from n to i in sector j, $\sum_{i \neq n}^{M} X^j_{in}$, are from the OECD STAN Database for Structural Analysis. The first method proves to be superior, as the second generates a number of negative observations for some country-sectors. However, data are missing for India, for which we use the INDSTAT (2016 version) provided by United Nations Industrial Development Organization (UNIDO).

Trade barriers and gravity equation variables Data for variables related to trade costs used in gravity equations (such as geographic distance and common border dummies) at the country-pair level are obtained from the comprehensive geography database compiled by CEPII. The WTO's RTA database provides information on regional trade agreements. The currency union indicator is obtained from Rose (2004) and was updated to reflect Euro-area membership.

Factor shares and final consumption shares In our analysis, we used the U.S. factor shares in 2005 for all countries. Data on the share of materials from sector k used in the production in sector j, γ^{jk}, as well as the labor share of production in sector j, γ^j, come from the Input-Output Database maintained by OECD STAN. The I-O table gives the value of the intermediate input in row k required to produce one dollar of final output in column j. We then divide this value by the value of gross output of sector j to obtain γ^{jk}. Similarly, the labor share is calculated as the ratio of value added to gross output, as capital input does not exist in the model. In addition, the final consumption expenditure shares of each sector, α^j_n also come from the I-O matrix.

R&D data R&D expenditures at the country-sector level are obtained from the OECD database of Business Enterprise R&D expenditure by industry (ISIC Rev 3). Since R&D data for several sectors in other countries are missing, we obtain estimates of these missing observations using the following approach. First, we run a regression using existing country-sector specific R&D and patent data from USPTO for 2005:

$$\log(R^j_n) = \beta_0 + \beta_1 \log\left(PS^j_n\right) + \mu_n + \gamma_j + \varepsilon^j_n, \tag{86}$$

where R_n^j is the R&D dollar expenditure of country i in sector j and PS_n^j is the patent stock of country i in sector j. μ_i and γ_j are country and sector fixed effects. This relation is built on the observations that (i) in the steady state, R&D expenditure should be a constant ratio of R&D stock and (ii) innovation input (R&D stock) is significantly positively related to innovation output (patent stock). In fact, the coefficient β_1 is large and significant at 99% and the R^2 is close to 0.90. Assuming that the relationship captured by equation (86) holds for China, India, and Sweden, we can obtain the fitted value of their sectoral level R&D expenditure:

$$\log(\widehat{R}_n^j) = \widehat{\beta}_0 + \widehat{\beta}_1 \log\left(PS_n^j\right) + \widehat{\mu}_n + \widehat{\gamma}_j.$$

For these three countries, we have information on all the right-hand-side variables except for the country fixed effects, $\widehat{\mu}_n$. This allows us to compute the *share* of R&D in a given sector for each country as

$$\widehat{r}_n^j = \frac{\widehat{R}_n^j}{\sum_j \widehat{R}_n^j} = \frac{(PS_n^j)^{\widehat{\beta_1}} \exp(\widehat{\mu}_n) \exp(\widehat{\gamma}_j)}{\sum_j (PS_n^j)^{\widehat{\beta}} \exp(\widehat{\mu}_n) \exp(\widehat{\gamma}_j)} = \frac{(PS_n^j)^{\widehat{\beta_1}} \exp(\widehat{\gamma}_j)}{\sum_j (PS_n^j)^{\widehat{\beta_1}} \exp(\widehat{\gamma}_j)}.$$

Second, we obtain the aggregate R&D expenditure as a percentage of GDP, $R\&D/GDP_n^{WB}$, for each country from the World Bank World Development Indicator database. The country-sector specific R&D can then be estimated as $s_n^j = \widehat{r}_n^j \times R\&D/GDP_n^{WB}$. For the countries with missing sectors, we estimate the fitted value using the same procedure. To maintain consistency across countries, we correct the OECD data-generated total R&D with the World Bank total R&D.

$$s_n^j = R\&D/GDP_n^{WB} \times \frac{R_n^{j,OECD}}{\sum_j R_n^{j,OECD}}$$

This estimated s_n^j is the R&D intensity used in our quantitative analysis.

Table 3: List of Industries

Sector	ISIC	Industry Description
1	C01T05	Agriculture, Hunting, Forestry and Fishing
2	C10T14	Mining and Quarrying
3	C15T16	Food products, beverages and tobacco
4	C17T19	Textiles, textile products, leather and footwear
5	C20	Wood and products of wood and cork
6	C21T22	Pulp, paper, paper products, printing and publishing
7	C23	Coke, refined petroleum products and nuclear fuel
8	C24	Chemicals and chemical products
9	C25	Rubber and plastics products
10	C26	Other non-metallic mineral products
11	C27	Basic metals
12	C28	Fabricated metal products, except machinery and equipment
13	C29	Machinery and equipment, nec
14	C30T33X	Computer, Electronic and optical equipment
15	C31	Electrical machinery and apparatus, n.e.c.
16	C34	Motor vehicles, trailers and semi-trailers
17	C35	Other transport equipment
18	C36T37	Manufacturing n.e.c. and recycling
19	C40T95	Nontradables

F Figures

Figure 16: Efficiency of innovation by country

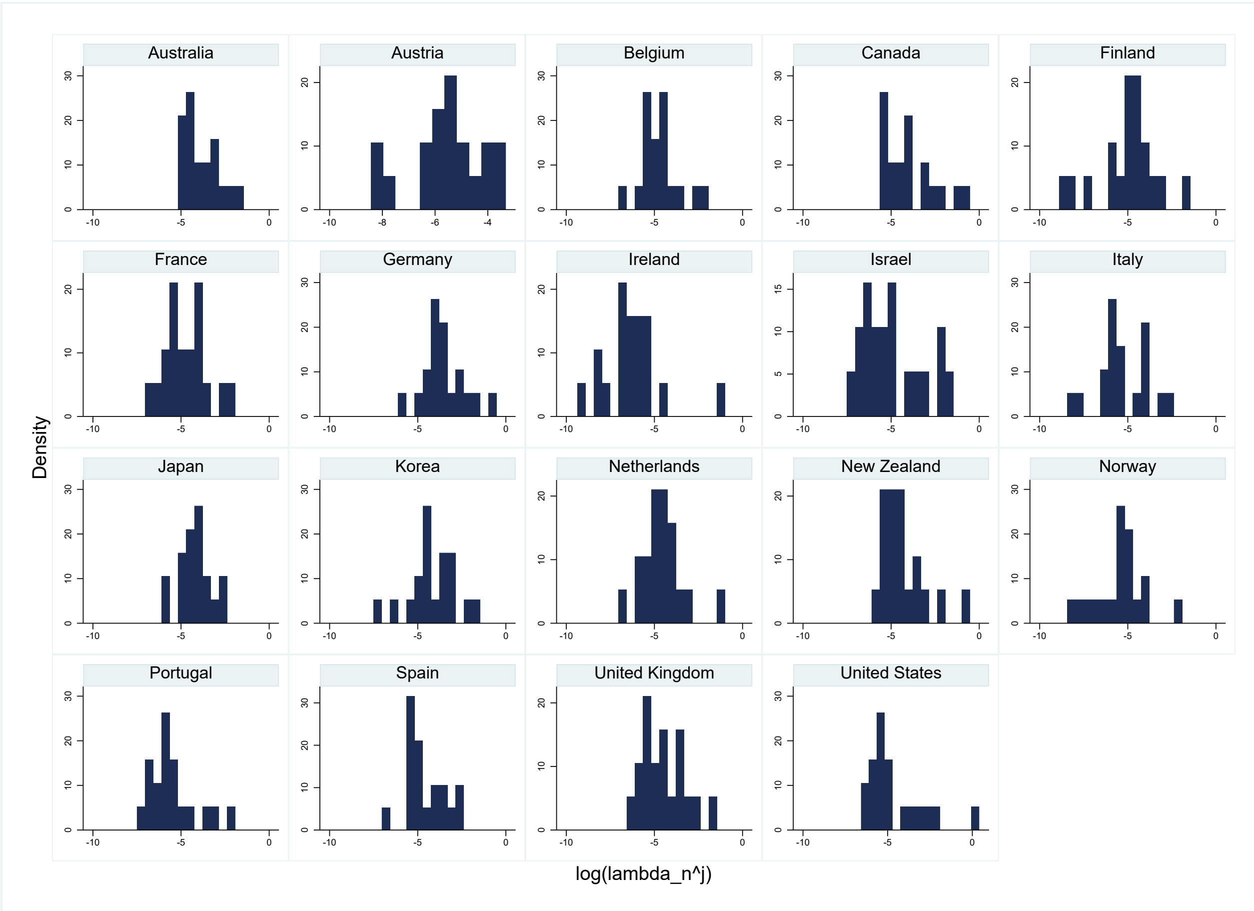

Figure 17: Efficiency of innovation by sector

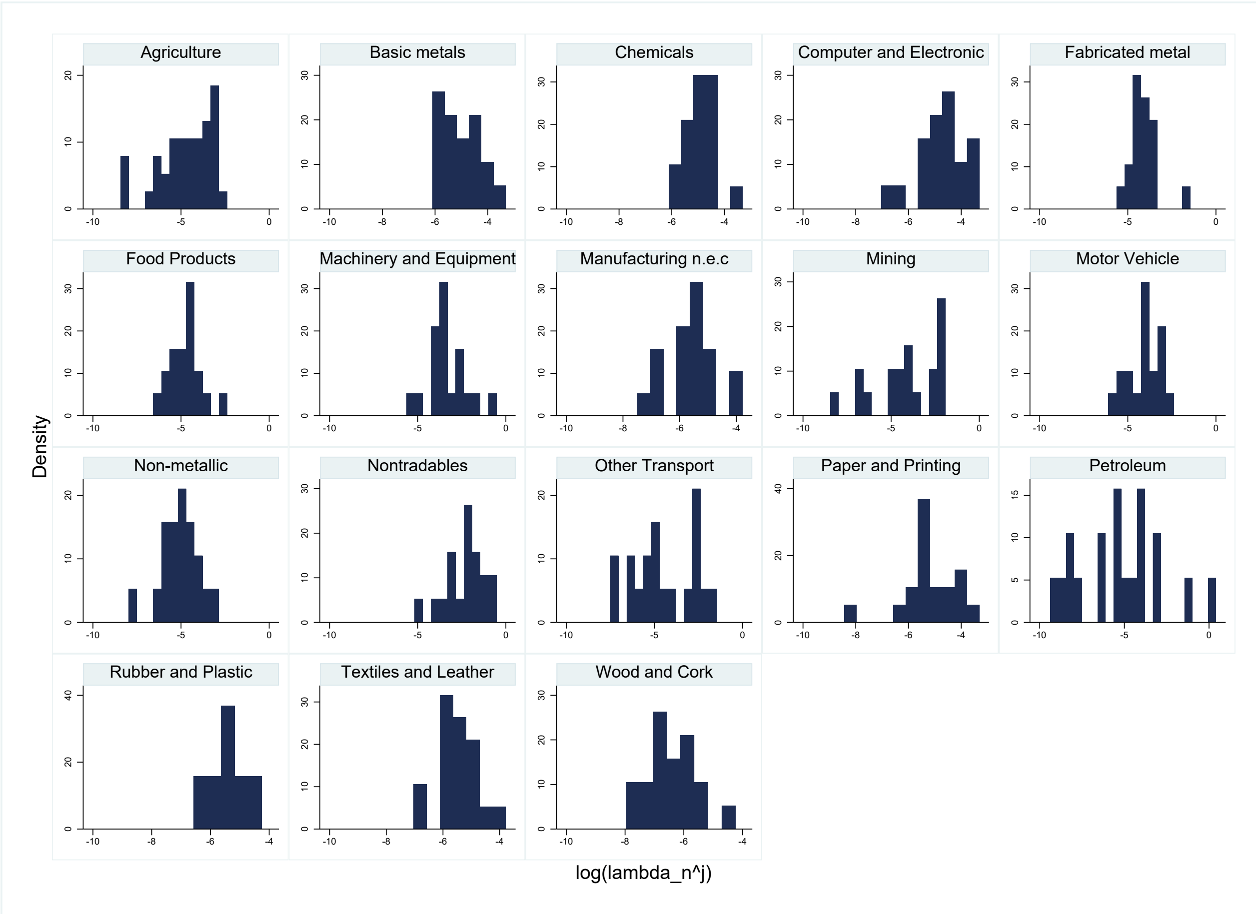

Figure 18: Stock of knowledge by country

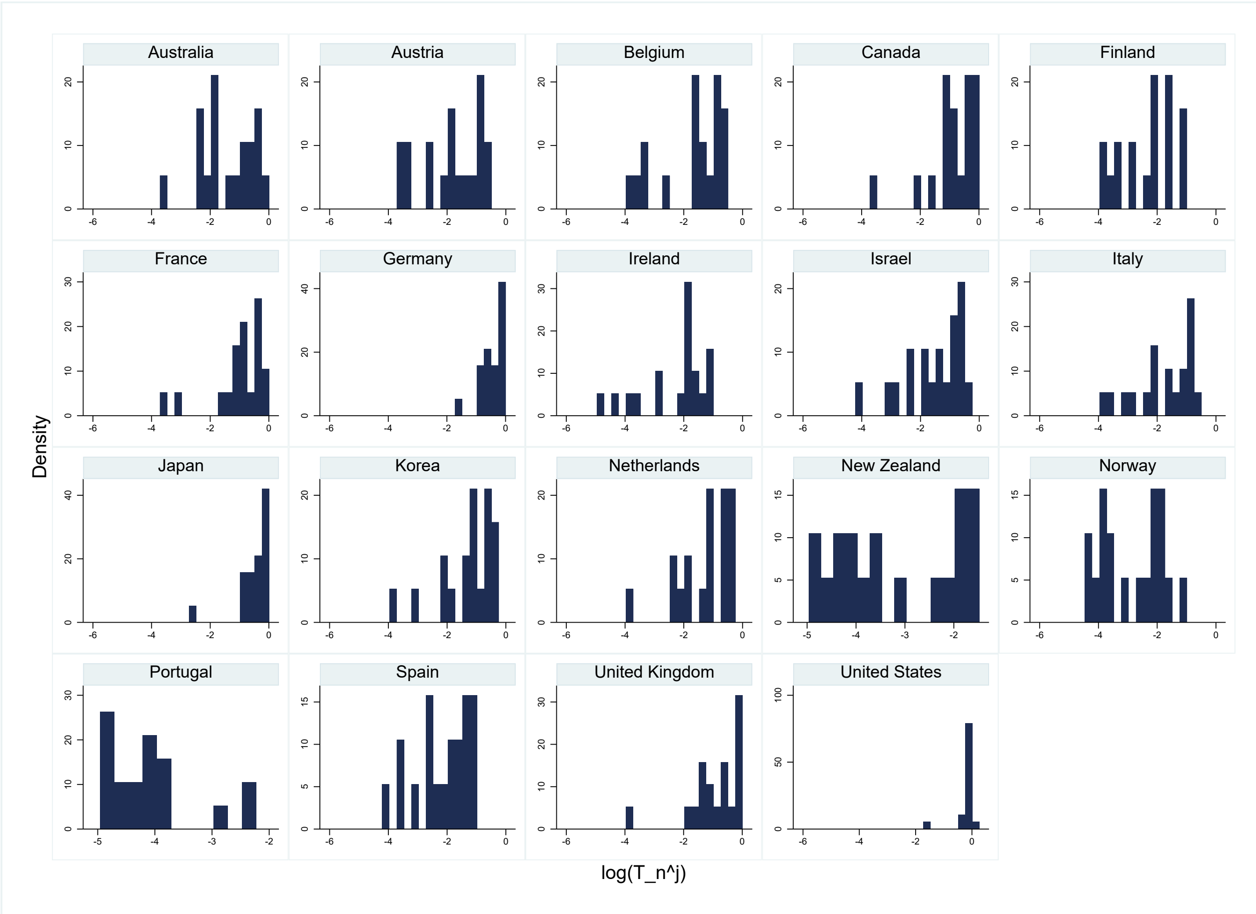

Figure 19: Stock of knowledge by sector