Borrowing to Save and Investment Dynamics

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Abstract

Existing literature on financial frictions argue that firms reduce investment in a crisis due to a lack of credit. However, U.S. public firms, which together accounted for 89 percent of the decline in investment during the Great Recession, experienced no drop in borrowing. Instead of investing, they borrowed to expand their stock of safe assets; that is, they borrowed to save. I model borrowing to save as an optimal portfolio choice when firms face gradually resolving uncertainty. In a quantitative general equilibrium model with heterogeneous firms, I show that this mechanism can simultaneously generate a sharp downturn and a slow recovery.

KEYWORDS: Financial Crises, Business Cycles, Uncertainty, Firm Heterogeneity
JEL CLASSIFICATION: D52, D53, E22, E32, E44, G11

1 Introduction

Table 1: Annual Growth Rates of Total Debt, Liquid Assets and Investment During the 2007-09 Financial Crisis

	All	Public	Non-Public
[1] Δ Debt	−1.4%	+0.6%	−8.2%
[2] Δ Investment	−6.1%	−8.3%	−2.9%
[3] Δ Liquid Assets	+2.0%	+4.3%	−1.9%
% of decline in aggregate investment		89.0%	11.0%

Note: This table presents the annual growth rates of total debt, liquid assets, and real investment between 2007 and 2009 for each of the three samples: all non-financial corporate businesses in the US (the Flow of Funds), publicly traded non-financial firms (Compustat), and others (the difference between the two). Total debt (the sum of short-term and long-term debt) captures the amount of funds actively raised by the public firms through debt contracts. Real investment is the sum of capital expenditures and acquisitions, less sales of property, plants and equipment. Liquid assets are the cash and marketable securities. See Appendix A for detailed data description.

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How does the supply of credit affect investment? Existing literature on financial frictions argue that firms reduce investment in a crisis due to a lack of credit. However, evidence from the Great Recession, as shown in Table 1, raises questions on the relevance of this channel. U.S. public firms – which together accounted for 89 percent of the decline in aggregate investment between 2007 and 2009 – experienced no drop in borrowing. By contrast, private firms were short of credit, but their contraction in real investment was much milder. Instead of investing, public firms borrowed to expand their stock of safe assets. Motivated by this evidence, this paper formalizes "borrowing to save" as an optimal portfolio choice when firms face gradually resolving uncertainty. In a quantitative general equilibrium model with heterogeneous firms, I evaluate the relevance of this channel in accounting for the sharp downturn in 2008 as well as the slow recovery thereafter. With its emphasis on firms' asset allocations, this paper challenges the conventional wisdom on how financial frictions drive business cycle fluctuations.

The Great Recession has led to a recent macro literature that uses quantitative business cycle models to explain how financial frictions affect investment in a crisis – whether it originates from a negative first-moment shock to the credit constraint (see, for example, the work of Jermann and Quadrini (2012); Perri and Quadrini (2018)), or a positive shock to the volatility of idiosyncratic productivity (Arellano, Bai, and Kehoe (2018)).[1] This paper contributes to the literature in two ways. First, existing studies predict a strong positive co-movement between firms' real activity and their ability to borrow, but this is not in line with the evidence in Table 1. This paper develops a model in which the shock structure plays an important role – as argued by Cao, Lorenzoni, and Walentin (2018) – that can rationalize public firms' borrowing to save behavior at a time when credit spreads are high and the risk-free rate is depressed. Through the lens of the model, I show that a tightening in financial conditions can have significant real impact by affecting not only how much firms can borrow, but also how they allocate the credit. Second, while existing studies can generate large fluctuations, they have limited explanatory power on the subject of the slow recovery. I show that the borrowing to save channel can simultaneously account for both, even without adjustment frictions.

In standard investment models with debt financing, firms issue one-period debt contracts and may default in equilibrium. Even if one introduces savings as an additional choice variable, firms would not borrow to accumulate safe assets, as the presence of external finance premium implies that marginal cost of borrowing is always at least as high as the marginal return on savings. As a result, firms only borrow to invest, and the level of borrowing and investment would always move in the same direction. Any savings or cash holdings are typically modeled as negative borrowing, or are financed by equity issues (Alfaro, Bloom,

[1] Closely related work also includes quantitative general equilibrium models which study the impact of financial frictions on households' consumption and employment in a credit crisis, such as Guerrieri and Lorenzoni (2017); Jones, Midrigan, and Philippon (2018).

and Lin (2018); Eisfeldt and Muir (2016)).

To generate the divergence between borrowing and investment at the firm-level, I utilize three key ingredients that are typically missing in the standard debt financing models. First, firms adjust their asset portfolios much more frequently than the level of borrowing, as observed in the data (see Table A.1 in Appendix). Second, firms face gradually resolving uncertainty in the form of sequential shocks. Third, recessions are driven by shocks with a negative first moment and a positive second moment (Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018)).

The model has a continuum of heterogeneous firms that produce differentiated products. In each period, these firms choose how much to borrow, and how to allocate funds between capital for production and a safe asset. Production is subject to an idiosyncratic productivity shock, followed by a demand shock that captures the gradual resolution of uncertainty. The two shocks are independent of each other, but both affect the profitability of a firm and hence its default probability.[2] Capital yields a higher expected return than the safe asset, but it is riskier and less liquid. Borrowing decision is risky, as it takes place before the firm learns about its productivity or demand, and it can only issue state-uncontingent debt. After observing the productivity shock, the firm cannot issue any new debt, but it can adjust their asset portfolios (at a cost) before the demand shock occurs. Firms default if they cannot pay for their debt at the end of a period, and the cost of default is the lost future expected profits.

In a typical debt financing model, firms issue one-period debt only to invest (and pay the operating costs), whereas in this model, they issue one-period debt for liquidity, but such liquidity could be used to either invest or accumulate assets, depending on the realization of uncertainty. The borrowing to save mechanism is best explained by backward induction. Upon observing their productivity draws, firms can adjust their asset portfolios by trading off growth against self-insurance. For each unit of debt issued, investing it in safe assets yields a weakly lower return than the cost of debt repayment, but it allows the firm to transfer resources from the repayment states to default states after the realization of demand. As firms are forward-looking, they choose how much to borrow before knowing their productivity draws, taking into account their expected asset allocations. If the productivity shock becomes more volatile, the magnitudes of both upside potential and downside risk increase. Importantly, as firms know that they are protected on the downside by the opportunity to adjust their asset portfolios before demand realization, they choose to continue borrowing despite the higher credit spreads associated with higher volatilities.

The calibrated theory can generate the observed behavior of public firms in a recession modeled as a second moment shock to idiosyncratic productivity and a negative first moment "financial shock" to firms' ability to borrow, à la Jermann and Quadrini (2012). The negative

[2]Having two independent shocks is a simplifying assumption, and not crucial for the mechanism. One can rewrite the model with only one shock, and firms receive an interim signal before its realization. As long as firms face gradually resolving uncertainty and can adjust their asset portfolio upon observing the signal, the mechanism still goes through.

financial shock increases the deadweight loss in default, and hence the credit spread per unit of borrowing. For any given level of debt, the likelihood of default increases as the repayment burden is higher. In response, firms channel a larger fraction of the debt into safe assets to prevent costly default. Without the second-moment shock, firms would decrease borrowing and investment simultaneously. An increase in the volatility of idiosyncratic productivity shocks would encourage firms to increase borrowing *ex ante* (i.e. before the productivity draw), as the upside potential becomes more attractive. At the same time, it also strengthens firms' incentive to save *ex post*, since a higher level of debt and higher credit spreads amplify the probability of default.[3].

Besides explaining the borrowing to save behavior of public firms, the model can also generate a slow recovery after transitory aggregate shocks. An obvious reason for firms' gradual adjustment in asset portfolios is the presence of adjustment costs. A more subtle and interesting reason relates asset choices to the rate of firm growth. Equity growth is slower in this model compared to one where firms only borrow to invest in capital, because firms loaded with safe assets make less profit conditional on survival. Moreover, since the accumulation of safe assets reduces default probabilities and hence the cost of borrowing, firms can sustain more debt and operate at a greater optimal scale. Slower growth and greater optimal scale both imply that it takes longer for firms to grow out of their borrowing constraints. As a result, compared to the standard models where firms only borrow to invest, the effects of financial frictions are more persistent in a recession, even in the absence of adjustment costs.

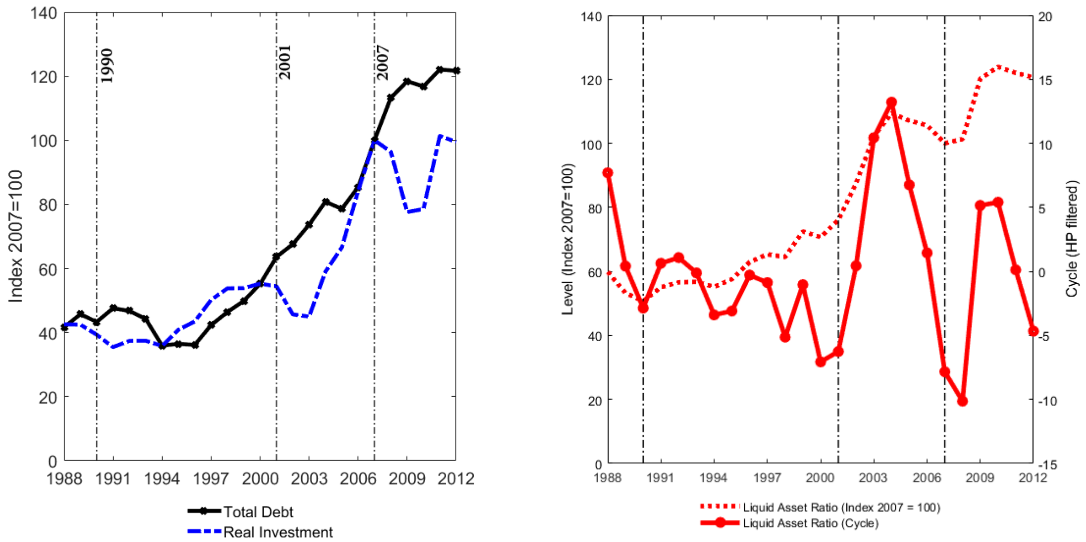
The quantitative model can successfully reproduce the main variables at both the micro and macro level; in other words, the aggregate implications are derived from a framework consistent with micro-level data. I use firm-level data on liquidity ratios, leverage ratios, credit spreads, and the time variation in the cross-sectional dispersion of investment rates, to calibrate the parameters governing the magnitude of financial frictions, the idiosyncratic productivity and demand shock processes. Subsequently I show that the resulting model predictions for the cross-sectional moments are consistent with micro-level data. At the macro level, the model can generate business cycle features of both real and financial variables, and provides a structural framework for analyzing aggregate investment fluctuations. I show that the borrowing to save channel can explain 28 percent of the total decline in investment during the Great Recession, and the rest is due to reductions in firm borrowing as financial conditions tighten. Since the model is calibrated to the sample of public firms, which were responsible for 89 percent of the decline in aggregate investment (public and private firms), a back-of-the-envelope calculation suggests that borrowing to save can explain about 25 percent of the decline in aggregate investment during the recession.

[3]The timing of the uncertainty shock is crucial to the borrowing to save mechanism: If the volatility of the demand shock increases, firms would unambiguously borrow less *ex ante*, as there is no opportunity to change their asset portfolios after all uncertainties are resolved (e.g. Arellano, Bai, and Kehoe (2018))

It is worth noting that on average borrowing and investment of U.S. public firms have diverged in each recession over the past 30 years, and the liquid asset ratio is strongly countercyclical, as shown in Figure 1. Hence, borrowing to save appears to be a cyclical phenomenon, although the impact was stronger in the Great Recession. Part of the reason was the magnitude of the financial shock and the heightened uncertainty at the beginning of the crisis, but the fact that a significant fraction of firms substituted from bank loans to bonds during the crisis also played an important role. Since bonds are less flexible than bank loans (Crouzet (2018); De Fiore and Uhlig (2015)), being forced to substitute to bonds when banks tighten credit supply exposes firms to higher default risks, which strengthen the incentive of firms loaded with debt to save rather than invest. I show that with debt substitution, the borrowing to save mechanism can explain almost 30 percent of the decline in aggregate investment, and the effect of the transitory shocks lasts longer.

Figure 1: Borrowing to Save Over the Business Cycle

(a) Debt Financing and Real Investment **(b)** Liquid Assets to Total Assets



Note: Panel (a) plots the mean levels of total debt and real investment between 1988 and 2012 for non-financial public firms in the US. Plot (b) plots the mean liquidity ratio and its cyclical component after applying an HP filter (with lambda = 100). The mean in each year is calculated after removing outliers, by winsorizing each variable of interest at the 1st and 99th percentiles. Data is from Compustat; see Appendix A for detailed description.

In analyzing why firms accumulate safe assets on their balance sheets, this paper is related to Kiyotaki and Moore (2012), which emphasizes the value of liquidity. In their model, firms issue equity to accumulate cash because it provides liquid funds that can be easily devout to invest when an opportunity arises. Guerrieri and Lorenzoni (2009) show why consumers hoard liquid assets in recessions, and how this amplifies aggregate volatilities. This paper shares the emphasis on liquidity, as the firms borrow for liquidity, and accumulate safe assets to prevent default. Differently from the existing literature, I focus on explaining why firms would borrow and save simultaneously when it is costly to do so.

This paper contributes to a growing body of work using heterogeneous firms to study business cycle fluctuations. Besides the papers aforementioned, Khan and Thomas (2013) show that shocks directly to the collateral constraint can lead to long-lasting recessions through disruptions on the real side when firms face investment adjustment costs. Buera and Moll (2015) share the focus on collateral constraints, and emphasize the importance of modeling heterogeneity in quantitative business cycle models. Unlike these studies, I build a model with endogenous firm defaults, which lead to fluctuations in credit spreads and investment dynamics. This feature is shared by Gilchrist, Sim, and Zakrajšek (2014) and Arellano, Bai, and Kehoe (2018), which study how financial frictions can generate large amplifications from uncertainty shocks. This paper differs in three ways. First, their amplification is via a contraction in the level of borrowing when uncertainty is high, whereas this paper emphasizes the allocation of credit to safe assets.[4] Second, model simulations here suggest that credit crises are best modeled with a combination of first- and second-moment shocks. Third, I show that unlike the credit contraction mechanism, borrowing to save can also generate a slow recovery. Thus this paper is complementary to the existing literature.

2 Adding Borrowing to Save to a Standard Debt Finance Model

This section presents a two-period partial equilibrium model of firm borrowing and asset allocation decisions, which can explain the borrowing to save behavior by the public firms. The model shares all the key features of a standard macro model with debt finance by firms (e.g. Covas and Den Haan (2012); Arellano, Bai, and Zhang (2012)), including a risk-neutral firm with decreasing returns to scale technology, endogenous default in equilibrium, inefficient liquidation, perfectly competitive lenders, and forward-looking debt prices determined before any shock realization.

In addition to the standard features aforementioned, the setting here adds two new features that generate the borrowing to save behavior. First, the firm experiences a sequence of two shocks, so uncertainty is resolved gradually rather than at once. Second, after the first shock is realized, the firm can adjust its asset portfolio, for a given liability structure. I present empirical evidence for these two assumptions, and I illustrate below how these two features give rise to a "risk taking *ex-ante*, precautionary savings *ex-post*" mechanism, which turns out to be a quantitatively important channel for the transmission of aggregate shocks for public firms.

[4]In the sovereign risk literature, Bianchi, Hatchondo, and Martinez (2018) show that it is optimal for governments to issue debt accumulate reserves to reduce rollover risk. However, their model set-up and mechanism for governments' precautionary savings differ from the set-up and mechanism for firms. In their model, governments issue debt for the purpose of accumulating reserves, whereas in my model, firms issue debt for liquidity, and save for precautionary reasons as profitable investment opportunities do not arise. Moreover, in the sovereign risk model, the assumptions of long-term debt and risk averse lenders are crucial for generating precautionary savings. By contrast, I show why risk-neutral firms choose to issue one-period debt and accumulate safe assets, despite that the marginal return on saving is dominated by the marginal cost of borrowing.

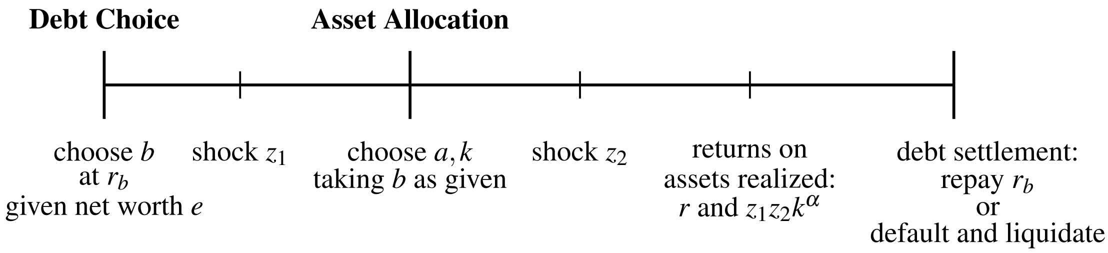


Figure 2: Timing in the static model

2.1 Model Setup

Production and asset allocation A firm with net worth e produces output (y) using a production technology that has decreasing returns to capital (k), subject to two productivity shocks z_1 and z_2 sequentially:

$$y = z_1 z_2 k^{\alpha}, \quad 0 < \alpha < 1.$$

Capital depreciates at rate $\delta \in [0,1]$. As in standard models of firm investment, decreasing returns to scale guarantees that firms have a finite optimal scale of production. The firm can finance investment in capital k from two sources: internal equity e, with which it is initially endowed, and external debt b. Besides investing in physical capital, the firm can also allocate its resources N into a safe asset a. Therefore, the resulting balance sheet constraint is:

$$k + a = e + b.$$

While the return on physical capital is subject to the two productivity shocks, the safe asset earns a constant return r, which is exogenously given here, and endogenously determined in Section 3.

Timing Figure 2 summarizes the timing of the firm's problem. At the beginning of the period, the firm chooses how much to borrow b at a repayment rate r_b, which is endogenously determined. The two productivity shocks z_1 and z_2 are independent and identically distributed, and importantly, they are realized sequentially.[5] After z_1 is observed and before z_2 is realized, the firm chooses how to allocate its resources between investment and savings, taking b and r_b as given. For simplicity, I assume here that the asset allocation decision only occurs after the realization of z_1, but this is not necessary for the borrowing to save mechanism. I show in the quantitative model in Section 3 that the firm can also choose asset allocation at the same time that the borrowing decision is made, as long as the firm can adjust its asset portfolio more easily than its liabilities as uncertainty resolves. This motivates the first key assumption of the model:

[5] One can also assume that the two shocks are correlated, or that z_1 could be a signal of the realization of z_2. Since signal extraction plays no role here, I assume for simplicity that the two shocks are independently distributed.

Assumption 1. (Portfolio adjustment) *The firm can adjust its asset portfolio, but not liabilities, after the first shock* z_1.

In other words, leverage is chosen based on the expectation of z_1, whereas assets are chosen based on its realization. Since the latter clearly has more variation, and cash holdings respond to the latter, one testable implication of this assumption is that the variations in cash holdings are larger than the variations in leverage ratios. Using Compustat, I find that for non-financial firms with non-trivial debt amounts (book leverage above 5%), the coefficient of variation (standard deviation divided by the mean) for cash is consistently higher than the correlation of variation for debt, across all definitions of debt and all quartiles of firms, and the differences are significant at the 1% level (see Table A.1 in Appendix).

Debt settlement and pricing The cash flow of the firm after returns on both assets are realized is given by:

$$\pi(z_1,z_2) = z_1 z_2 k^{\alpha} + (1-\delta)k + (1+r)a. \tag{1}$$

Default happens when $\pi(z_1,z_2)$ falls below the amount of repayment $(1+r_b)b$, where r_b was agreed between the firm and the lender at the beginning of the period. Therefore, for every realization of z_1, the net worth based default rule implicitly defines an endogenous default threshold $\underline{z}_2$, whereby the firm defaults if $z_2 \leq \underline{z}_2(z_1)$, and repays otherwise:

$$z_1 \underline{z}_2 k^{\alpha} + (1-\delta)k + (1+r)a = (1+r_b)b. \tag{2}$$

In the dynamic setting, a net worth based default rule implies that the firm defaults when temporary declines in cash flow may leave the firm unable to repay its debt, even when the equity value of the firm is positive (Arellano, Bai, and Kehoe (2018); Gilchrist, Sim, and Zakrajšek (2014)).[6] Due to credit market frictions, the distressed firm cannot raise necessary external financing to honor its financial obligations. Consequently, insufficient balance sheet liquidity may result in default, even if the equity value of the firm is positive. To capture this effect in the two-period model, I assume that when a firm defaults, it loses an exogenous continuation value V in its end-of-period payoff.

In liquidation, the firm is shut down and the lenders can only seize a fraction χ of the firm's end-of-period resources $\pi(z_1,z_2)$; in other words, liquidation entails a deadweight loss when $\chi < 1$. This is a common assumption in many models in which the underlying financial friction is limited liability; it captures the fact that bankruptcy proceedings are typically costly and involve fire sales of assets and loss of sales (e.g. Jermann and Quadrini

[6] This class of models assumes that default is primarily "liquidity-driven", as opposed to "value-driven" whereby firms may find it optimal to continue operating with negative net worth, as long as the option value of equity is high enough to keep the firm alive. In "value-driven" models of risky debt, a firm defaults when the equity value of the firm falls below a certain threshold (e.g. Cooley and Quadrini (2001)). Empirically, which of these two views is more suitable for credit risk modeling is unclear, as there is sparse evidence on the role of insolvency versus illiquidity in triggering default. It may be unclear even to the firms and creditors at the time.

(2012); Covas and Den Haan (2012)). Therefore, the second key assumption of the model is the presence of financial frictions, which makes liquidation inefficient:

Assumption 2. (Financial frictions) *Default occurs if a firm's cash flow falls short of debt repayment. Upon default, the liquidation value of the firm is given by $\chi\pi$, where $0 \leq \chi < 1$.*

Recall that the cost of borrowing r_b determined at the beginning of the period is forward-looking and not contingent upon either z_1 and z_2. Assuming perfectly competitive lenders, then r_b is determined endogenously by equating the expected return from lending to the lenders' cost of funds:

$$\underbrace{\int_{z_1}\int_{\underline{z}_2(z_1)}(1+r_b)b\,dF(z_2)dF(z_1)}_{\text{Repay}}+\underbrace{\int_{z_1}\int^{\underline{z}_2(z_1)}\chi\pi(z_1,z_2)dF(z_2)dF(z_1)}_{\text{Default}}=(1+r)b, \tag{3}$$

where $\pi(z_1,z_2)$ is defined in (1) and the default threshold $\underline{z}_2$ is given by (2), which depends on the level of z_1 realized, and the cumulative distribution functions of z_1 and z_2 are denoted as $F(z_1)$ and $F(z_2)$.

2.2 The Firm's Problem

The firm's problem can be solved by backward induction in two steps. The first step is to solve for the optimal asset allocation before the realization of z_2, for any given debt level b and any realization of z_1. Define $s=\frac{a}{b+e}$ as the fractions of a firm's total resources allocated to the safe asset a. At the asset allocation stage, a firm with net worth e solves:

$$\begin{aligned}\max_{s(b,z_1,e)} \mathrm{E}_{z_2}\Pi(z_1,b,e) = & \int\limits_{\underline{z}_2(z_1,b,s,e)} \Big[z_1z_2(1-s)^{\alpha}N^{\alpha}+(1-\delta)(1-s)N+(1+r)sN-(1+r_b)b\Big]dF(z_2) \\ & + \int\limits^{\underline{z}_2(z_1,b,s,e)} -V\,dF(z_2),\end{aligned} \tag{4}$$

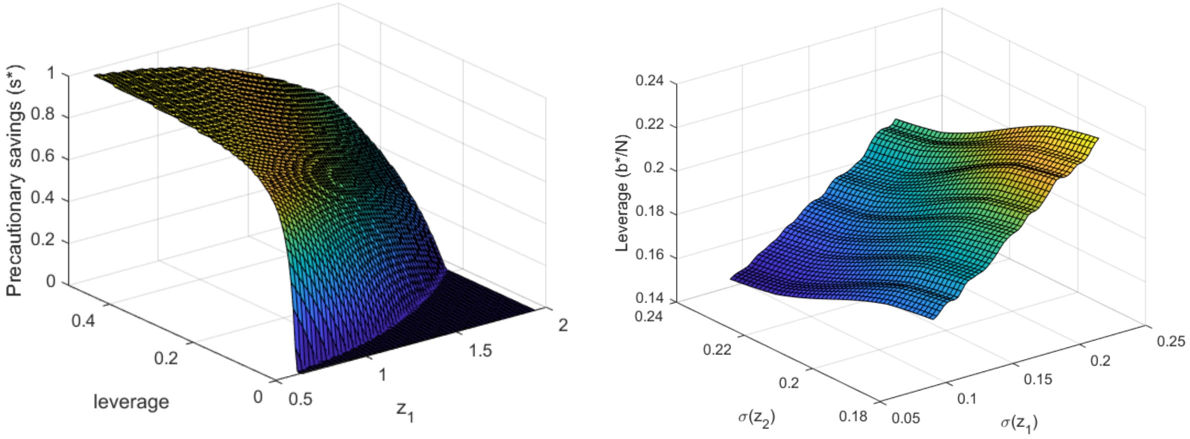
subject to the default threshold (2) for $\underline{z}_2$, the debt pricing equation (3) for r_b, and $N \equiv b+e$. In the second step, I substitute the optimal solution $s^*(b,z_1,e)$ into the firm's beginning-of-period problem, which determines the optimal amount of borrowing b^* *ex-ante*, by a firm with an internal finance level e:

$$\begin{aligned}\max_{b(e)} \mathrm{E}_{z_2,z_1}\Pi(e) = & \int\limits_{z_1}\int\limits_{\underline{z}_2(z_1,b,s^*,e)}\Big[z_1z_2(1-s^*(b,z_1))^{\alpha}N^{\alpha}+(1-\delta)(1-s^*(b,z_1))N \\ & +(1+r)s^*(b,z_1)N-(1+r_b)b\Big]dF(z_2)dF(z_1)+\int\limits_{z_1}\int\limits^{\underline{z}_2(z_1,b,s^*,e)} -V\,dF(z_2)dF(z_1),\end{aligned} \tag{5}$$

Figure 3: Precautionary savings (s^*) and borrowing (b^*) by firm e

(a) Precautionary Savings (Asset Allocation Stage)

(b) Optimal Leverage (Debt Choice Stage)



Note: This figure shows the numerical solutions to a firm's backward induction problem. First, it solves for the optimal asset allocation (equation 4), given any level of borrowing and realization of the first shock (panel (a)). Then it substitutes the optimal asset choices back to the beginning-of-period problem (equation 5) to determine the optimal level of borrowing (panel (b)).

subject to the default threshold (2) for $\underline{z}_2$ and the debt pricing equation (3) for r_b.

2.3 Mechanism

Since there is no closed-form solution, in the rest of this section, I use numerical simulations to illustrate the key forces at play.[7] Although it is only a two-period model, it contains the key ingredients to highlight the interaction between credit market frictions and asset choices that are crucial to explaining the propagation of financial shocks in the dynamic model in Section 3.

Borrowing to Save A risk-neutral firm with limited liability finds it optimal to accumulate safe assets with a lower return because (1) default is costly, and (2) firms cannot adjust the asset- and liability-sides of their balance sheets simultaneously at all times. In the absence of either condition, firms would not choose to issue costly debt to finance the accumulation of safe assets. I call this "precautionary savings" by firms, as they trade off higher profits conditional on survival against the higher probability of survival after the first shock has realized, for a given level of debt.

Panel (a) of Figure 3 plots s^*, the optimal amounts of "precautionary saving" as a fraction of a firm's total resources at the asset allocation stage (equation (4)), for different levels

[7] The key parameters pertaining to assumptions 1 and 2 are χ, V, the distributions of z_1 and z_2, and δ. Here I set χ equal to μ_χ (see Table 2), and V to be the loss of continuation value averaged across exiting firms in the quantitative model. I assume that z_1 and z_2 are drawn from independent lognormal distributions, and set $\mu_{z_1} = -0.05$, $\sigma_{z_1} = 0.25$, $\mu_{z_2} = -0.15$, and $\sigma_{z_2} = 0.2$, similar to the parameters governing the distributions of z and ψ in the quantitative model. I set $\delta = 0.2$ to capture two features of the quantitative model that are missing here: one is the illiquidity of capital; the other is the cost of operation as a function of capital $F_o k$. The results are qualitatively similar for a wide range of parameter values.

of debt b and realized values of the first productivity shock z_1. Despite that capital yields a higher expected return, the safe asset enables the firm to transfer resources from the repayment states to default states after the realization of z_2. Therefore, after the firm observes its z_1, the opportunity to adjust its asset portfolio allows it to trade-off higher profits conditional on survival against the higher probability of survival. For any given firm, the optimal fraction of safe assets s^* is decreasing in the realized level of z_1 and increasing in leverage ($\frac{b}{N}$), as the probability of default is higher when z_1 is low and b is high.

Panel (b) of Figure 3 plots the optimal level of borrowing (b^*) as a fraction of total assets (N), solved from the second step of the firm's problem (equation (5)). In standard firm financing models with defaultable debt, uncertainty is resolved at once, and the level of borrowing is decreasing in the level of uncertainty. By contrast, the most important observation from panel (b) is that the uncertainty levels of the two sequential shocks have asymmetric impact on the optimal level of debt: b^* is increasing in the volatility of the first shock, and decreasing in the volatility of the second shock.

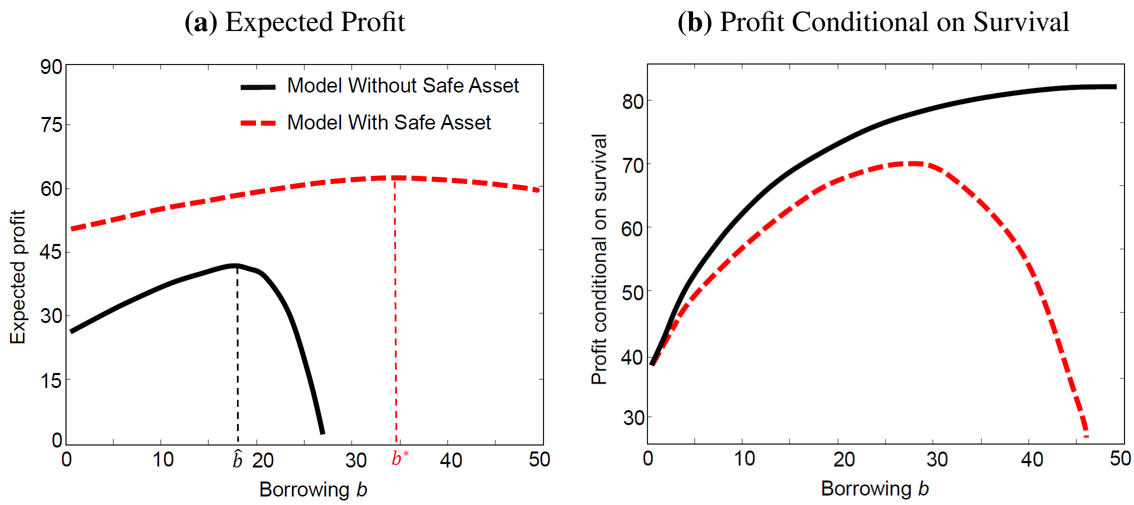
If the volatility of the first shock is high, there is a probability that the firm may receive a very favorable productivity draw, which encourages the firm to increase its level of borrowing *ex ante*, so that it will have enough funds for investment later on. Of course, there is also an equal probability that z_1 may be very low, but the firm can still insure itself against such risk by optimizing – or reoptimizing (in the case of the dynamic model) – its asset allocation after z_1 is observed. After z_2 is realized, the firm no long has the means to insure itself, so the external finance premium associated with default risk discourages firms from borrowing if the volatility of z_2 is high.

Therefore, the firm borrows to save as a "risk taking *ex ante*, and precautionary savings *ex post*" strategy. When the firm knows that uncertainty resolves gradually, it would increase borrowing *ex ante* to capture the potential upside risk, and after the first unfavorable shock is realized, it insures itself against potential default by accumulating safe assets. The higher level of initial borrowing, the greater the risk of default, and the larger fraction of assets saved at a given level of z_1 to prevent default.

Slower Rate of Firm Growth In standard models of financial frictions, as firms accumulate internal funds over time, they can quickly grow out of their financial constraint, especially the more productive firms. As a consequence, these firms will not be affected by financial frictions, as they no longer require external finance.

In this model, two related forces – as a result of the endogenous asset allocation problem – imply that firms do not grow out of the financial constraint as quickly as in standard models where firms only borrow to invest. First, the accumulation of safe assets in a firms portfolio reduces its default probability, which reduces the cost of borrowing and implies a greater optimal scale of operation. I illustrate this point in Panel (a) of Figure 4, where I plot

Figure 4: Slower Firm Growth in Model with Safe Assets



Note: Panel (a) plots the expected profit $E_z\pi$ as a function of the level of borrowing b, for a firm with net worth e, before any shock realization. The red dotted line shows the solutions in the model described in the section, whereas the black solid line indicates the solutions in a counterfactual model without the asset allocation stage. Panel (b) plots the firm's profit π as a function of debt, conditional on survival.

the expected profit of a firm as a function of the level of borrowing b in the model described above (with safe assets), compared to that in a counterfactual model where firm only borrows to invest. As the probability of default is lower in the model with safe assets, the expected profit at the beginning of the period $E_z\pi$ (red dotted line) is higher – for every level of borrowing – than in a counterfactual model without safe assets (black solid line). Since the firm chooses its leverage based on the expected profit, the optimal level of borrowing in this model b^* exceeds the level in a model without safe assets $\hat{b}$.

Second, equity growth is slower in the model with safe assets, since part of the net worth is invested in safe assets. I revisit this in the dynamic model, but Panel (b) of Figure 4 illustrates the reason in this static example: Conditional on survival, the firm with safe assets would make less profit at the end of the period. If the firm's problem were dynamic, it would reinvest its end-of-period profit. Therefore, it takes longer for the firm to build its net worth in the model with safe assets. To sum up, the key takeaway from Figure 4 is as follows: Slower growth and a greater optimal scale both imply that firms rely on debt financing for longer, and hence the impact of financial frictions would be longer lasting.

I come back to this point in Section 4, where I show that by adding safe assets to a firm's portfolio, its investment takes longer to recover after a negative shock that reduces the firm's net worth. Since there is no exogenous adjustment cost in the static model, it gives a cleaner illustration of how adding safe assets to a firm's portfolio can endogenously generate slower firm growth out of financial constraints.

3 A Quantitative Model in General Equilibrium

This section outlines the set-up of a quantitative dynamic model with the borrowing to save mechanism. The aim of the model is to examine whether it can explain the observed divergence between debt financing and investment by public firms in recessions, and to quantity the significance of this channel in explaining the sizable contraction in aggregate economic activity during the Great Recession.

The model features a representative household, continuums of intermediate goods and final goods firms, and perfectly competitive financial intermediaries. The household lends to the intermediate goods firms via the financial intermediaries, and is the owner of all firms. The final goods firms are competitive and have a technology that converts intermediate goods into a final good that is consumed by the household. The intermediate goods firms are the key agents, who optimize their asset portfolio and leverage by maximizing the equity value subject to the household's stochastic discount factor. They are monopolistically competitive and use capital to produce differentiated products. Crucially, as in the static model, they experience two independent idiosyncratic shocks sequentially: the first one is a productivity shock (z), and the second one is a demand shock (ψ) from the final goods firms, and they can reoptimize their assets – but not the liabilities – in between the shocks.[8]

There are two standard aggregate shocks in the model. The first is a financial shock modeled as an unexpected change in the recovery rate in default (Jermann and Quadrini (2012)). The second is the standard second-moment shocks considered in the literature – that is, shocks to the dispersion of the idiosyncratic productivity shock (e.g. Bloom (2009); Bachmann and Bayer (2013); Basu and Bundick (2017); Bloom, Floetotto, Jaimovich, Saporta-Eksten, and Terry (2018)). Both shocks are realized before the opportunity to reoptimize firms' asset portfolio. The timing of the uncertainty shock is crucial, as it determines whether the amount of borrowing is increasing or decreasing in volatility (Figure 3). Simulation results show that both shocks are required to successfully model the behavior of public firms in a recession.

3.1 Asset Portfolio Reoptimization

Figure 5 summarizes the timing of each intermediate good firm's problem. At the beginning of each period, shocks pertaining to the production and borrowing decisions are realized. This includes the level of idiosyncratic productivity (z), the recovery rate in default (χ), and the variance of innovations to the productivity process (σ_z). Before production, firms can reoptimize their allocation of assets between capital and safe assets, subject to the intra-period resource constraint:

$$\hat{k}+\hat{a}_f+g(\hat{k},k)\leq k+a_f, \tag{6}$$

[8] These two shocks play the roles of z_1 and z_2 in the static model.

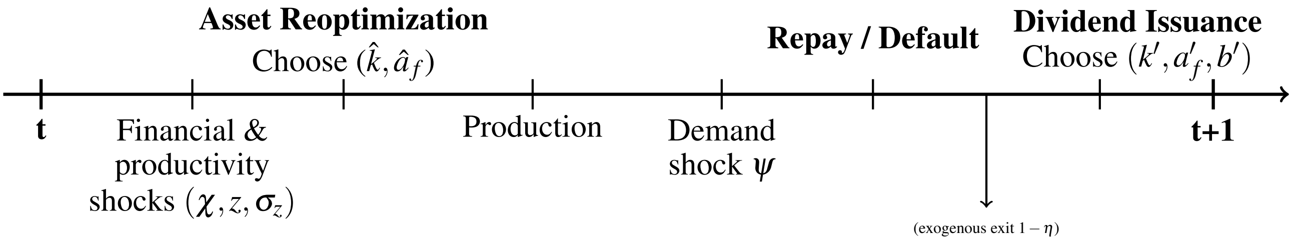


Figure 5: Overview and Timing of Intermediate Goods Firms' Problem

This constraint embodies Assumption 1, in that the firm cannot issue new debt to finance additional assets and the cost of adjusting capital $g(\hat{k}, k)$:

$$g(\hat{k}, k) = \frac{F_{k_1,t}}{2}\left(\frac{\hat{k}-k}{k}\right)^2 k, \tag{7}$$

where

$$F_{k_1,t} \equiv p_k^+ \times \mathbb{1}_{(\hat{k}-k)>0} + p_k^- \times \left(1 - \mathbb{1}_{(\hat{k}-k)<0}\right).$$

$F_{k_1,t}$ captures costly reversible investment, whereby the purchase price of capital p_k^+ is greater than the liquidation value of capital p_k^-. This friction is important for the quantitative implications of the model by increasing the riskiness of investment in capital. As a result, firms may not invest even if a favorable shock occurs, as it is very expensive to disinvest later on, especially if the future is uncertain and firms are highly leveraged.

Firms then produce output y from intra-temporal capital $\hat{k}$ using a decreasing returns to scale technology: $y = z\hat{k}^{\alpha}$ with $\alpha < 1$, and pay an operating cost that is proportional to their capital stock, $F_o\hat{k}$. Production is subject to an idiosyncratic productivity shock z, that follows an AR(1) process:

$$\log z' = \mu_z' + \rho_z \log z + \sigma_z \log \varepsilon_z'; \quad \varepsilon_z' \sim N(0,1). \tag{8}$$

I allow the variance of innovations to the productivity process σ_z to vary over time according to a two-state Markov chain. Setting $\mu_z' = -0.5\sigma_z^2$ ensures that the mean z across firms does not fluctuate with σ_z.

3.2 Demand from Final Goods Firms

Final goods firms buy the products from intermediate goods firms, and produce the final good Y via the technology:[9]

$$Y = \left(\int\int \psi y(z, \hat{k}, x)^{\frac{\zeta-1}{\zeta}} dF(\psi) d\mu(z, \hat{k}, x)\right)^{\frac{\zeta}{\zeta-1}}, \tag{9}$$

[9] In the model, the final goods producer has no value added, and hence this producer is a simple device to aggregate the output of the heterogeneous firms—referred to as intermediate goods firms—into a single value. Equivalently, one can think of these heterogeneous firms as final goods producers, and equation (9) reflects agents' preferences over these final goods.

where y denotes the intermediate goods produced by a firm with state $(z,\hat{k},x)$, $\zeta > 1$ is the elasticity of substitution across goods, and ψ is an i.i.d. idiosyncratic demand shock from a lognormal distribution with mean μ_ψ and standard deviation σ_ψ.[10] The final goods firms choose the intermediate goods to solve:

$$\max_{y(z,\hat{k},x)} Y - \int\int p(\psi,z,\hat{k},x)y(z,\hat{k},x)dF(\psi)d\mu(z,\hat{k},x), \tag{10}$$

subject to (9), and p is the price of intermediate good $y(z,\hat{k},x)$ relative to the price of the final good, which is the numeraire. This yields the demand for an intermediate good: $y = \left(\frac{\psi}{p}\right)^{\zeta} Y$.

Each intermediate firm decides on the price of its product after the demand shock has been realized. Since firms face demand curves with an elasticity larger than 1, they always choose prices to sell all of their output. Therefore, the price of intermediate good y follows $p = \psi\left(\frac{Y}{z\hat{k}^{\alpha}}\right)^{\frac{1}{\zeta}}$ and can be eliminated as a choice variable.

3.3 Debt Pricing

An intermediate goods firm can issue one-period, zero-coupon bonds b' to finance its operational expenses, and investment in capital and safe assets. In the subsequent period after all shocks have been realized, the firm decides whether to fulfill its debt obligations depending on its net worth (e.g. Gilchrist, Sim, and Zakrajšek (2014); Arellano, Bai, and Kehoe (2018)):

$$\begin{aligned} n' &= p'y' + p_k^-(1-\delta)\hat{k}' - F_o\hat{k}' + \hat{a}'_f - b' \\ &= \pi' - b', \end{aligned} \tag{11}$$

where $\pi' = p'y' + p_k^-(1-\delta)\hat{k}' - F_o\hat{k}' + \hat{a}'_f$ denotes the firm's current assets. A firm can only fully repay its debt if the price of its output $p(\psi)$ is high enough, such that $\pi' \geq b'$. Otherwise it is liquidated and its assets π' are passed onto creditors.

In general equilibrium, lenders receive deposits a'_h from the household and a'_f from firms, and subsequently use them to extend credit to firms. Asset prices are forward-looking and each lender faces perfect competition, so their expected total profits are driven down to zero in each period. Assuming that lenders cannot cross-subsidize firms, lenders must earn zero

[10]The assumption of demand shocks are i.i.d. is a simplification to reduce the number of state variables; it is not crucial to the mechanism. Even in the case of an AR(1) shock, whether a non-defaulting firm defaults tomorrow depends on the i.i.d. component of demand realized in the next period.

profit on each loan. The price of debt is thus given by:

$$q^b(\hat{k}',b',\hat{a}'_f,z,\mathbf{s}) = \mathrm{E}\left[\lambda(\mathbf{s},\mathbf{s}')\left(\underbrace{\int_{\varepsilon'_z}\int_{\underline{\psi}'} 1dF(\varepsilon'_z|\sigma_z)dF(\psi')}_{\text{repay}} + \underbrace{\int_{\varepsilon'_z}\int^{\underline{\psi}'} \frac{\chi'\pi'}{b'}dF(\varepsilon'_z|\sigma_z)dF(\psi')}_{\text{default}}\right)\middle| z,\mathbf{s}\right], \tag{12}$$

and the default threshold $\underline{\psi}'(z',\mathbf{s}')$ is implicitly defined by:

$$p(\underline{\psi}')y(z',\hat{k}') = b' + F_o\hat{k}' - \hat{a}'_f - p_k^-(1-\delta)\hat{k}'. \tag{13}$$

The aggregate financial shock is captured by χ', which follows an AR(1) process:

$$\log\chi' = \mu_\chi + \rho_\chi \log\chi + \sigma_\chi \log\varepsilon'_\chi; \quad \varepsilon'_\chi \sim N(0,1). \tag{14}$$

ε'_χ denotes the shock to recovery rate, or the efficiency of the financial intermediation process.

3.4 Intermediate Goods Firms' Recursive Problem

At the end of a period, non-defaulting firms that survive the exit shock pay their shareholders, the household, dividends d that must be nonnegative:[11]

$$\begin{aligned} d &= p(\psi)y(z) - F_o\hat{k} - g(k',\hat{k}) - \tilde{b} + \hat{a}_f - q^a a'_f + q^b b' \\ &= x - g(k',\hat{k}) + q^b b' - q^a a'_f \geq 0, \end{aligned} \tag{15}$$

where x is the non-defaulting firm's net liquid asset position: $x \equiv p(\psi)y(z,\hat{k}) - F_o\hat{k} + \hat{a}_f - b$. The price of the risk-free asset q^a follows:

$$q^a(\mathbf{s}) = \mathrm{E}\left[\lambda(\mathbf{s},\mathbf{s}')\middle|\mathbf{s}\right]. \tag{16}$$

The capital adjustment cost $\tilde{g}(k',\hat{k})$ has the same functional form as $g(\hat{k},k)$, which is the cost of reoptimizing a firm's asset portfolio in the middle of a period (7), except that $\tilde{g}(k',\hat{k})$ takes into account of capital depreciation (at the rate δ), which occurs at the end of a period:

$$\tilde{g}(k',\hat{k}) = \frac{F_{k_1,t}}{2}\left(\frac{k'-(1-\delta)\hat{k}}{\hat{k}}\right)^2\hat{k} \tag{17}$$

where $F_{k_1,t} \equiv p_k^+ \times \mathbb{1}_{(k'-(1-\delta)k)>0} + p_k^- \times \mathbb{1}_{(k'-(1-\delta)k)<0}$.

[11] This prevents firms from issuing equity instead of debt as a means to avoid costly default associated with debt financing. I assume that firms cannot issue equity here in order to focus on the relation between debt market frictions and firms' asset allocation. Nevertheless, this assumption is not necessary for the borrowing to save mechanism. A version of the model that simultaneously allows debt and equity financing can be found in Appendix B.2.

Let $V^0(x,\hat{k},z;\mathbf{s})$ denote the value of a non-defaulting firm before the exit shock $(1-\eta)$ in period t, and $V^1(x,\hat{k},z;\mathbf{s})$ denote its value after surviving the shock. The idiosyncratic states include the firm's productivity z, reoptimized level of capital $\hat{k}$, and its net liquid asset position x. The aggregate state $\mathbf{s}=(\chi,\sigma_z,a_h,\mu)$ includes the current aggregate shocks, household's savings, and the distribution over idiosyncratic states. The dynamic problem of a non-defaulting firm that survives the exit shock in period t consists of choosing the asset portfolio (a'_f,k') and debt level b' for the following period to maximize:

$$V^1(z,\hat{k},x;\mathbf{s}) = \max_{d,k',b',a'_f} \left\{ d + \mathrm{E}\left[\lambda(\mathbf{s},\mathbf{s}')\hat{V}^0(a'_f,k',b',z';\mathbf{s}') \middle| \mathbf{s} \right] \right\} \tag{18}$$

subject to the nonnegative dividend constraint (15), the cost of adjusting capital (17), the price of debt (12) and risk-free asset (16), and the aggregate law of motion $\mathbf{s}'=\Gamma(\mathbf{s})$, which I describe below. In the following period, after observing z' and $\mathbf{s}'$, the firm can reoptimize its asset portfolio by choosing $(\hat{a}'_f,\hat{k}')$ to maximize:

$$\hat{V}^0(a'_f,k',b',z';\mathbf{s}') = \max_{\hat{k}',\hat{a}'_f} \left[\int_{\underline{\psi'}}^{\infty} V^0(x',\hat{k}',z';\mathbf{s}')dF(\psi') \right] \tag{19}$$

subject to the intra-period resource constraint (6), capital adjustment cost (7), and the definition of default threshold $\underline{\psi}'$ (13). $V^0(x,\hat{k},z;\mathbf{s})$ is the value of a non-defaulting firm after debt settlement, before the exit shock, such that:

$$V^0(x',\hat{k}',z';\mathbf{s}') = (1-\eta)n(x',\hat{k}') + \eta V^1(x',\hat{k}',z';\mathbf{s}') \tag{20}$$

where $1-\eta$ is the exogenous exit rate, and $n(x',\hat{k}')$ is the realized net worth defined in (11). Firms hit by the exogenous exit shock must leave the economy immediately and any remaining profits are paid to the households as dividends.

There is a continuum of potential entrants, each endowed with $\hat{a}_0$, which is set to be a fixed fraction υ of the steady-state level of assets held by an average incumbent: $\hat{a}_0 = \frac{\upsilon}{N}\int(\hat{a}_f+\hat{k})d\mu_{ss}(z,x,\hat{k})$. The potential entrants first observe the aggregate shocks (γ^*,σ_z), and then each draws a signal s about its future productivity z' from a Pareto distribution with parameter ξ. The transition between signal and future productivity follows (8), with $z=s$. Entry decision takes place at the end of period t, and requires the payment of a fixed cost f_e. Firms can use their cash on hand $\hat{a}_0$ as well as debt financing to pay for f_e. Hence, becoming public is only feasible for a firm with signal s if $\max\left(q^b(s,b',a'_f,k';\mathbf{s})b'\right) \geq f_e - \hat{a}_0$. Among the feasible set, a subset of firms is randomly chosen to ensure that the number of exiting firms equals the number of entering firms (Arellano, Bai, and Kehoe (2018)). An entering firm solves the same dynamic problem as an incumbent firm, $V^1(s,0,x;\mathbf{s})$, with $x=-f_e+\hat{a}_0$.

3.5 Households

The representative household has a utility function $u(c)=\frac{c^{1-\theta}}{1-\theta}$, and solves a standard consumption-savings problem:[12]

$$W(\mathbf{s})=\max_{c,a_h'}\left\{u(c)+\beta\mathrm{E}\left[W(\mathbf{s}')\Big|\mathbf{s}\right]\right\}, \tag{21}$$

$$\text{subject to}: \quad c+q^a(\mathbf{s})a_h'\leq a_h+\int\left[d+F_o\hat{k}\right]d\mu(z,x,\hat{k})$$

The household receives payoff from the risk-free asset a_h and dividends d from the intermediate goods firms, and uses it to consume c, purchase new risk-free assets a_h'. Moreover, it receives the intermediate goods firms' fixed costs of operation $F_o\hat{k}$ as a lump-sum.

3.6 Recursive Competitive Equilibrium

Given an initial firm distribution μ_0, safe asset a_0, and aggregate shocks $(\chi_0,\sigma_{z,0})$, a recursive competitive equilibrium in this economy consists of (1) policy functions $d(z,x,\hat{k};\mathbf{s})$, $b'(z,x,\hat{k};\mathbf{s})$, $k'(z,x,\hat{k};\mathbf{s})$, $a_f'(z,x,\hat{k};\mathbf{s})$, $\hat{k}'(z,x,\hat{k};\mathbf{s})$, $\hat{a}_f'(z,x,\hat{k};\mathbf{s})$, $c(\mathbf{s})$, $a_h'(\mathbf{s})$; (2) value functions $V^1(z,x,\hat{k};\mathbf{s})$, $W(\mathbf{s})$; (3) prices $q^b(\hat{k}',\hat{a}_f',b',z;\mathbf{s})$, $q^a(\mathbf{s})$, such that for all t:

1. Given $\lambda(\mathbf{s},\mathbf{s}')$, $b'(z,x,\hat{k};\mathbf{s})$, $d(z,x,\hat{k};\mathbf{s})$, $k'(z,x,\hat{k};\mathbf{s})$, $a_f'(z,\psi,x,\hat{k};\mathbf{s})$, $\hat{k}'(z,x,\hat{k};\mathbf{s})$, $\hat{a}_f'(z,x,\hat{k};\mathbf{s})$ and $V^1(z,x,\hat{k};\mathbf{s})$ solve the intermediate goods firms problem (18);

2. $c(\mathbf{s})$, $a_h'(\mathbf{s})$, and $W(\mathbf{s})$ solve the household's problem (21);

3. Given $\lambda(\mathbf{s},\mathbf{s}')$, the financial intermediaries determine the optimal debt price $q^b(\hat{k}',\hat{a}_f',b',z;\mathbf{s})$ according to the zero profit condition (12);

4. Goods market clears:

$$c(\mathbf{s})=\left(\int\int\psi y(z,\hat{k},x)^{\frac{\zeta-1}{\zeta}}dF(\psi)d\mu(z,\hat{k},x)\right)^{\frac{\zeta}{\zeta-1}}-\int\tilde{g}\big(\hat{k},k'(z,x,\hat{k};\mathbf{s})\big)d\mu(z,x,\hat{k})$$

5. Financial market clears:

$$a_h+\int\hat{a}_f d\mu(z,x,\hat{k})=\int\tilde{R}^b d\mu(z,x,\hat{k})$$

such that intermediaries use the debt repayments by firms (including the seizure of assets in case of default) to pay household and firms on their safe asset holdings.

[12] Unlike the partial equilibrium framework in section 2 where firms are risk neutral, the owner of the firms is risk averse here, so in principle firms may save and borrow simultaneously even in the absence of Assumption 1, but this effect is not quantitatively significant under reasonable parameterization of household's risk aversion.

6. The measure of firms evolves according to $\mu' = \Gamma(x', z', \hat{k}; \sigma_z', \chi', \mathbf{s})$, such that

$$\Gamma(x', z', \hat{k}; \sigma_z', \chi', \mathbf{s}) = \int \Pi\big(x', z', \hat{k}', x, z, \hat{k} \big| \sigma_z, \mathbf{s}\big) d\mu(z, x, \hat{k}) + \mu_e(\sigma_z', \chi', \mathbf{s})$$

where $\Pi\big(x', z', \hat{k}', x, z, \hat{k} \big| \sigma, \mathbf{s}\big)$ is the probability that an incumbent firm with state $(x, z, \hat{k})$ transits to a state $(x', z', \hat{k}')$ in which it does not default. $\mu_e(\sigma_z', \chi', \mathbf{s})$ is the measure of new entrants in $t+1$, which equals to the total measure of exiting firms $\Omega(\sigma_z', \chi', \mathbf{s})$:

$$\Omega(\sigma_z', \chi', \mathbf{s}) = \int_{z', z, x, \hat{k}} \Big[\underbrace{F\Big(\underline{\psi}'(b', \hat{k}', \hat{a}_f')\Big)}_{\text{Default}} + \underbrace{(1-\eta)\Big(1 - F\big(\underline{\psi}'(b', \hat{k}', \hat{a}_f')\big)\Big)}_{\text{Exogenous exits}} \Big] d\mu(z, x, \hat{k}).$$

4 Quantitative Analysis

This section explores the quantitative implications of the model. I begin with a description of how the model is parameterized and an outline of the solution method, followed by a discussion on the optimal policy functions and the moments generated by the model. Then I analyze the effects of aggregate shocks in the model. In particular, I highlight which model ingredients are necessary for generating the diverging pattern of debt financing and investment observed during the recessions.

4.1 Calibration

There are two groups of parameters in the quantitative model. The first group are the exogenously calibrated parameters, most of which follow the commonly used calibrations in the literature. The second group are set in a moment-matching exercise. Each period reflects one quarter.

Predefined Parameters Starting from the household's preferences, the discount factor is set to $\beta = 0.99$, so the annual interest rate is 4%, and θ is set to one which implies that the household has log preferences in consumption. Next, $\{\alpha, \delta, \rho_z, p_k^-, \zeta\}$ govern the production side of the economy. The intermediate goods firms have a decreasing returns to scale technology with $\alpha = 0.7$, which is within the range of estimates in the literature. The quarterly depreciation rate δ is 0.025. The firm-specific productivity has a serial autocorrelation of $\rho_z = 0.95$, as in Alfaro, Bloom, and Lin (2018). Following the estimates in Bloom (2009), I set the liquidation value of capital p_k^- to be 0.57 while the purchased price of capital p_k^+ is normalized to one, so the investment resale loss is 43%. The elasticity of substitution in the final goods production function ζ is set to 5, which generates a 25% markup.

$\{\upsilon, \eta\}$ are the exogenously calibrated parameters governing the entry and exit dynamics. For simplicity, I set $\upsilon = 0.1$, and pick $\{f_e, \xi\}$ to match the relative investment rate and leverage of entrants. η is set to 0.98, so that 2 percent of non-defaulting intermediate firms are hit by the exogenous exit shock in every quarter, which is within the range used in the literature.

The aggregate financial shock χ is a credit supply shock, which affects the fraction of assets seized by lenders when a firm defaults in the model. I obtain the autoregressive coefficient ρ_χ and standard deviation σ_χ from an AR(1) regression of the excess bond premium of Gilchrist and Zakrajšek (2012) between 1988 and 2010. This yields $\rho_\chi = 0.85$, which implies a half-life of four quarters, and $\sigma_\chi = 0.06$.

Parameters from Moment-Matching The last ten parameters are calibrated to jointly target moments in the data:

$$\left\{\sigma_H,\ \sigma_L,\ \pi_{HH},\ \pi_{LL},\ F_o,\ \mu_\chi,\ \mu_\psi,\ \sigma_\psi,\ f_e,\ \xi\right\}.$$

All data moments, except those on credit spreads, are constructed from the quarterly Compustat sample for US non-financial public firms between 1988 and 2012. Due to data limitation, the moments on credit spreads are constructed from a shorter sample between 1997 and 2012. Firm-quarter observations are included if total assets, cash and marketable securities, debt in current or long-term liabilities, capital expenditures, and net property, plant and equipment are non-missing. For the entrants, I consider only firms that start appearing in the database since 1988, and I use the first two years' data to construct the entrant-related moments.

The first four moments are computed from firms' investment rates, defined as $(I/K)_{i,t} = I_{i,t}/(0.5(K_{i,t} + K_{i,t-3}))$ for firm i at time t following Bloom (2009).[13] I first compute the interquartile range of investment rates across firms for each quarter, and subsequently calculate the mean, standard deviation, autocorrelation, and skewness of the IQR. The mean and difference in volatility levels affect the mean and standard deviation, respectively, of the IQR. Thus together they pin down σ_H and σ_L in the model. The autocorrelation and skewness are mostly affected by the transition probabilities π_{HH} and π_{LL}, such that high volatility shocks are relatively low probability events.

The next two moments are related to the balance sheet policies of public firms, focusing on their borrowing and saving behavior. The book value of leverage in the model corresponds to b/π, and is given total debt divided by asset in the data. The median leverage ratio is most affected by the mean recovery rate μ_χ in liquidation. The median cash-to-

[13] Capital stock $K_{i,t}$ is calculated by the perpetual inventory method: $K_{i,t} = (1-\delta)K_{i,t-3}(P_t/P_{t-3}) + I_{i,t}$, which is initialized using the net book value of capital, and $I_{i,t}$ is the net capital expenditures on plant, property and equipment. To remove seasonality in the data, the growth rates of investment are computed across four quarters.

Table 2: Parameters from Moment Matching

Parameter	*Description*	*Statistic (%)*	*Target*	*Moment*
$\sigma_H = 0.32,\ \sigma_L = 0.21$	Volatility levels	Invest IQR mean & std.	20 & 3.2	16 & 3.4
$\pi_{HH} = 0.86,\ \pi_{LL} = 0.95$	Transition probabilities	Invest IQR auto. & skew.	83 & 19	88 & 15
$F_o = 0.41$	Operating cost	Cash to asset median	7.3	6.9
$\mu_\chi = 0.71$	Financial shock (mean)	Leverage median	23	26
$\mu_\psi = -0.12$	Demand shock (mean)	Spread median	3.0	2.5
$\sigma_\psi = 0.19$	Demand shock (std.)	Spread median std.	1.5	0.9
$f_e = 10.61$	Entry cost	Entrants lev (rel.)	65	58
$\xi = 2.94$	Pareto distribution	Entrants invest (rel.)	1.8	1.4

Note: This table presents the parameters from the moment-matching. All data moments, except those on credit spreads, are constructed from the quarterly Compustat sample for US non-financial public firms between 1988 and 2012. Credit spreads data come from The Bank of America Merrill Lynch Indicators between 1997 and 2012. Firm-quarter observations are included if total assets, cash and marketable securities, debt in current or long-term liabilities, capital expenditures, and net property, plant and equipment are non-missing. For the entrants, I consider only firms that start appearing in the database since 1988, and I use the first two years' data to construct the entrant-related moments. See Appendix A for detailed variable definitions.

asset ratio, $\hat{a}_f/\pi$, is largely determined by the operating cost F_o. Next, parameters governing the demand shock, $\{\mu_\psi, \sigma_\psi\}$, have the most effect on the median and standard deviation of corporate bond spreads. Lacking firm-level spread data, I use the Bank of America Merrill Lynch corporate bond spread for a particular rating in each period as a proxy for the spread of firms with the same rating, and construct the median and standard deviation of the sample.[14]

The last two parameters $\{f_e,\ \xi\}$ govern the leverage and investment of the entrants relative to the incumbents. Since the entry decision here amounts to the decision of a firm to go public, I first compute the leverage and investment rates of firms whose initial public offering dates are within the sample period, in the periods when IPO took place. Then I calculate the median leverage (investment) of the entrants, relative to the median leverage (investment) of the incumbents. Table 2 reports the target moments in the data and the moment, as well as the parameters used in the moment-matching exercise.

4.2 Algorithm Overview

The numerical algorithm used to solve this model closely follows Khan and Thomas (2013) and Bachmann, Caballero, and Engel (2013). Here I provide an overview of the algorithm, and the detailed description is in Appendix B.1.

Recall that the aggregate state vector in the model is $\mathbf{s} = (\chi, \sigma_z, a_h, \mu)$. The evolution of the aggregate equilibrium is fully characterized by two mappings: $p = \Gamma_p(\chi, \sigma_z, a_h, \mu)$, and $\mu' = \Gamma_\mu(\chi, \sigma_z, a_h, \mu)$, where p is the marginal utility of the representative household. I follow Krusell and Smith (1998) and approximate the intractable cross-sectional distribution μ with

[14]The Compustat sample consists of firms with ratings between AAA and D. The Bank of America Merrill Lynch Indicators report the US corporate AAA, AA, A, BBB, BB, B, CCC (and below) yields since 1997. I construct the spread between each yield and the 10-year Treasury yield.

the current aggregate capital level K, the current aggregate debt level B, and the lagged uncertainty state σ_{-1}. I approximate the equilibrium mappings Γ_p and Γ_μ by the following log-linear rules $\hat{\Gamma}_p$, $\hat{\Gamma}_K$, and $\hat{\Gamma}_B$:

$$\begin{bmatrix} \log p \\ \log \hat{K}' \\ \log \hat{B}' \end{bmatrix} = \mathbf{A}(\sigma, \sigma_{-1}) + \mathbf{B}(\sigma, \sigma_{-1}) \begin{bmatrix} \log \hat{K} \\ \log \hat{B} \end{bmatrix} + \mathbf{C}(\sigma, \sigma_{-1}) \begin{bmatrix} \log \chi \\ \log z \end{bmatrix},$$

in which I allow the coefficients to depend not only on the current realization of uncertainty, but also on its value in the previous period. As usual with this procedure, the explicit forms chosen for equilibrium mappings are assumptions, and are verified that they are good approximations to the actual mapping (see Appendix B).

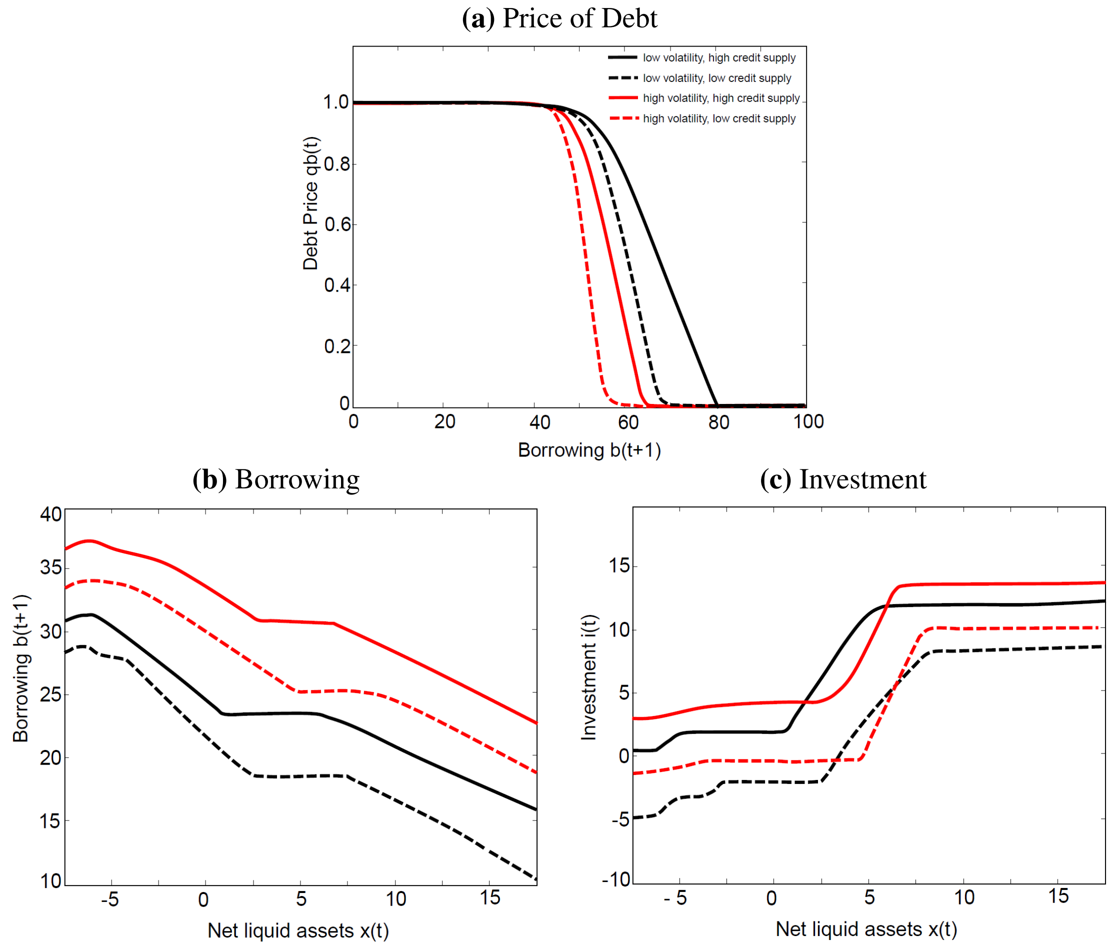
The algorithm begins with guessing the initial coefficients and initializing the forecast rules $\hat{\Gamma}_p^{(1)}$, $\hat{\Gamma}_K^{(1)}$, and $\hat{\Gamma}_B^{(1)}$. Then the model is solved with two loops. In the inner loop, I solve for the idiosyncratic firm problem using value function iteration on V^1 and Gauss-Hermitian numerical integration, taking as given $\hat{\Gamma}_p^{(1)}$, $\hat{\Gamma}_K^{(1)}$ and $\hat{\Gamma}_B^{(1)}$. The value functions in between grid points are interpolated using a multidimensional tensor product spline approximation. In the outer loop, based on the firm value $V^{1(1)}$ computed in the inner loop, I first simulate the economy for T periods, and find the equilibrium quantities and prices (p_t, K_t, B_t) for $t = 1, 2, ..., T$. Then the equilibrium mappings are updated using an OLS regression on a subset of the simulated data, resulting in $\hat{\Gamma}_p^{(2)}$, $\hat{\Gamma}_K^{(2)}$, and $\hat{\Gamma}_B^{(2)}$. The procedure is repeated until the approximated equilibrium mappings have converged.

4.3 Bond Pricing and Policy Functions

To illustrate the forces at play, I begin the quantitative analysis by studying how the policy functions shift with different aggregate conditions in a comparative static exercise. Figure 6 presents the bond pricing function as a function of borrowing (b'), and the decision rules for borrowing and investment as a function of its net liquid asset position (x), for a given level of productivity z and capital $\hat{k}$. Moreover, I show how each function shifts with the efficiency of financial intermediation χ, which is a proxy for credit market conditions, and aggregate volatility (σ_z), so there are four lines in each panel, which correspond to four pairs of aggregate conditions: high credit-low volatility (χ_H, σ_L), low credit-low volatility (χ_L, σ_L), high credit-high volatility (χ_L, σ_H), and low credit-high volatility (χ_L, σ_H). In the low (high) volatility regimes, I set σ_z equal to σ_L (σ_H). In the high credit supply regimes, χ equals to its mean μ_χ; in the low credit regimes, χ is set to two standard deviations below the mean.

Panel (a) of Figure 6 plots the price of debt as a function of leverage, for a median level of z and χ, holding capital level $\hat{k}$ constant. As expected, the price of debt is decreasing in leverage, as the probability of default increases with the level of debt. Moreover, the default mechanism implies that the price of debt is unambiguously increasing in the efficiency of financial intermediation, and decreasing in aggregate volatility. As a result, an aggregate state with low volatility and good credit conditions generates, for any given leverage, the lowest default probability and hence the highest price of debt.

Figure 6: Debt Pricing Function and Policy Functions



Note: This figure plots the debt pricing function q^b as a function of borrowing b' (panel (a)), and policy functions for borrowing b' (panel (b)) and investment $i = k' - (1-\delta)\hat{k}$ (panel (c)) as a function of net liquid assets $x \equiv py - F_o\hat{k} - b + \hat{a}_f$, at a given level of productivity z and capital $\hat{k}$. In each panel, I consider four pairs of aggregate conditions: high credit supply & low volatility (black solid line), low credit supply & low volatility (black dashed line), high credit supply & high volatility (red solid line), low credit supply & high volatility (red dashed line).

Panel (b) illustrates the states of the world in which firms do not cut back on debt financing, despite the lower debt prices. If firms know that they can adjust their asset portfolio as uncertainties resolve gradually, they may be willing to borrow more – despite the lower debt prices – when aggregate volatility is high (red lines in panel (b)). This is because firms are willing to take more risks by front-loading on debt, such that in case favorable conditions occur, they have sufficient funds to invest and grow. Importantly, this mechanism only works when the volatility shock hits early on. If, instead, the demand shock becomes more volatile, the optimal level of borrowing would be unambiguously lower. This is because firms would not have any means for self-insurance after the realization of the demand shock, and hence they would be less willing to take risks ex-ante.

Lastly, panel (c) shows, for different aggregate states, the investment function of a firm with an average productivity draw. Before the productivity draw, it would have borrowed more if aggregate uncertainty is high. Nevertheless, the actual level of capital invested after the productivity draw may not be higher in a more volatile state than a less volatile one: Unlike in panel (b), the red solid (dashed) line is not uniformly above the black solid (dashed) line in panel (c). Put differently, the aggregate states in which a firm borrows more may not be the ones in which it also invests more.

This arises because two forces affect investment in the model. One is the level of total credit available (leverage effect), and the other is the allocation of credit between investment and safe asset holdings (asset allocation effect). Although higher uncertainty ex-ante motivates firms to take on risks and issue more debt, they may end up channeling only a small fraction of the credit to investment at the asset reoptimization stage. This is often the case if the ex-ante high volatility does not lead to a very favorable productivity draw, but has driven the firm to take on a large amount of debt, which increases its default probability.

Holding the credit supply condition constant, which of the two effects dominates depends mostly on the productivity draw that the firm receives, and its net liquid asset position, so the level of investment may be higher or lower in the higher uncertainty states. In a high volatility state with low χ (bad credit conditions), which makes borrowing more expensive and default more likely, it is more likely that the asset allocation effect would dominate the leverage effect. As a result, investment could be hit hard at a time when borrowing remains relatively high (red dotted line in (b) and (c)).

4.4 Model Fit

Cross-Sectional Moments To examine the firm distribution generated by the model, Table 3 displays the cross-sectional moments, which were not explicitly targeted. For each period, I compute the medians within each asset class, and Table 3 reports the time series medians of the variables of interest.

Investment rates are concave in firm size, as in the data. Although smaller firms have higher incentives to invest due to the decreasing returns to scale assumption, their debt capacity is lower so their investment is constrained by their ability to borrow. Next consider the cash to asset ratio, which is decreasing in size in both the data and the model. Smaller firms hold proportionally more safe assets in the steady state as their default probabilities are higher. Having a safer asset portfolio thus lowers their cost of borrowing and hence increases their debt capacity. The model can also capture the cross-sectional patterns of leverage, whereby larger firms have higher levels of leverage than smaller firms.

Table 3: Cross-Sectional Moments

Asset Percentile	**Investment** (%)		**Cash/Asset** (%)		**Leverage** (%)	
	Data	Model	Data	Model	Data	Model
$0-25\%$	17	14	13	9.6	19	15
$25-50\%$	20	16	8.4	7.2	22	26
$50-75\%$	19	15	5.2	4.0	25	31
$75-100\%$	15	13	3.0	2.1	31	35

Note: This table presents the cross-sectional moments in the data and the corresponding moments generated by the model. For each variable, I first calculate the median for each group of firms (defined by size) for each quarter. The median of each time series is reported in the table. Data are from Compustat (1988Q1–2012Q4). See Appendix A for variable definitions.

Firm-Level Correlation Between Spreads and Cash Holdings An important firm-level moment related to this study is the correlation between bond spread and cash holdings. This moment was not targeted, and as a check on the model fit, I compute the correlation by calculating first the correlation across time for each firm, and then the median correlation across firms. The correlation coefficent is 33% in the data, and 47% in the model, and both are statistically significant at the 5 percent level. This echoes an important finding by Acharya, Davydenko, and Strebulaev (2012), that firms with higher cash holdings are robustly associated with higher credit spreads in the data. This finding may seem counterintuitive at first: If cash holdings reduce the risk of default, why do "safer" firms with higher cash holdings have higher spreads? The answer is that asset choices are endogenous in the model, and safe and liquid assets are more valuable for riskier firms. In contrast, Acharya, Davydenko, and Strebulaev (2012) show that exogenous variations in cash holdings are negatively (and significantly) related to bond spreads. To test this hypothesis in the model, I compute a counterfactual model in which firms can only choose how much to borrow, but not the allocation of assets. Each firm is endowed with an exogenous level of safe assets, and this level is randomly drawn from the distribution of safe assets in the baseline model. The correlation between cash-to-asset and spread becomes significantly negative (-19%) in the counterfactual model. Therefore, the positive correlation between cash holdings and credit spreads can be interpreted as evidence for precautionary savings by firms, and emphasizes the importance of modeling firm's asset allocation problem.

Business Cycle Moments Lastly, in Table 4 I report the business cycle moments of the firm balance sheet variables aggregate across all public firms. Specifically, I focus on the aggregate investment rate, total cash held by public firms to total assets, and total debt to total assets. Each series corresponds to the four-quarter rolling growth in the data as well as the model. I compute the correlation of each variable with the growth rate of output, as well as their standard deviations relative to output.

Table 4: Business Cycle Moments

	Corr. with Output (%)		Relative Std. Dev.	
	Data	Model	Data	Model
Investment (%)	71	94	2.7	5.1
Cash to Asset (%)	-14	-30	5.0	5.4
Leverage (%)	-18	-25	0.7	1.9

Note: This table reports the correlations of investment, cash to asset, and leverage with output, and the standard deviations of the series relative to output, in the data as well as the model. Each series corresponds to the four-quarter rolling growth. Data are from BEA (output) and Compustat (all other variables), aggregated across all non-financial public firms between 1988Q1 and 2012Q4. See Appendix A for variable definitions.

Unlike traditional business cycle studies, the data sample here consists of only public firms. Nonetheless, as expected, investment is strongly positively correlated with output, and approximately three times as volatile. Both cash to asset and leverage ratios are countercyclical, but the cash ratio is much more volatile than the leverage ratio. This validates the key assumption in the model, that it is easier for firms to adjust the asset-side of their balance sheets than the liability-side.

In the model, the cyclical patterns of borrowing and savings are similar to the patterns in the data, without exogenously imposing financing costs or returns to safe assets that are time-varying. The relative standard deviations are somewhat higher in the model than in the data, especially for investment and leverage. For investment in capital, there are two margins of adjustments: before any shock realization and before the demand shock. For leverage, it could be because that debt contracts are one-period and they are the only source of external financing, which are simplifying assumptions to facilitate computation. Nevertheless, the model is capable of generating the correct ranking of volatilities among these three variables.

4.5 Impulse Response Functions

Next I study the impulse response functions for firms' borrowing, savings, investment, and output at the aggregate level, in response to a combination of one-time financial and uncertainty shocks.[15] I illustrate the importance of the borrow-to-save mechanism by contrasting the responses in the baseline model to the ones in a counterfactual model without optimal asset choices. Then I show that the mechanism helps to generate a much slower recovery than in a standard debt finance model where firms only borrow to invest. Lastly, I show why both financial and uncertainty shocks are required to generate the public firms' financing and investment patterns observed in the data.

Borrowing to Save I simulate an economy of 3,000 firms for 10 years at the quarterly frequency, and I repeat the procedure 50,000 times. In each simulation the model is hit with

[15] See Appendix B for the computation of the impulse response functions.

an uncertainty shock ($\sigma_t = \sigma_H$) in the 5th quarter, and simultaneously, a financial shock of about three standard deviations, so the aggregate investment rate $(I/K)_t$ declines by approximately 25 percent upon impact. This matches the percentage decline in investment rate by the non-financial public firms between 2008Q4 and 2009Q1.[16] Figure 7 compares the impulse response functions in the baseline model (black solid lines) with those in a counterfactual model in which firms do not choose their asset portfolios endogenously (blue dotted lines). For the latter, the level of liquid assets held by each firm is exogenously fixed at the pre-shock level. The main difference between the two models is that investment and borrowing diverge in the baseline, but not in the counterfactual. This is the borrowing to save mechanism illustrated in Section 2.

An unexpected increase in the volatility of the idiosyncratic productivity shocks, *ceteris paribus*, increases borrowing by firms, if they have the opportunity to adjust their asset allocations. A negative financial shock affects firms' ability to borrow by decreasing the amount of assets that investors can seize upon default. This implies higher costs of borrowing, which reduce the default thresholds, for any given level of debt. Together with an increase in borrowing, firms' default probabilities increase as a result of the two shocks. With the opportunity to optimize their asset portfolios, firms increase holdings of liquid assets for insurance, as they trade-off higher profits against higher probabilities of survival. As illustrated in Figure 6, the financial shock plays an important role in determining which effect dominates quantitatively, and whether investment decreases despite the increase in borrowing.

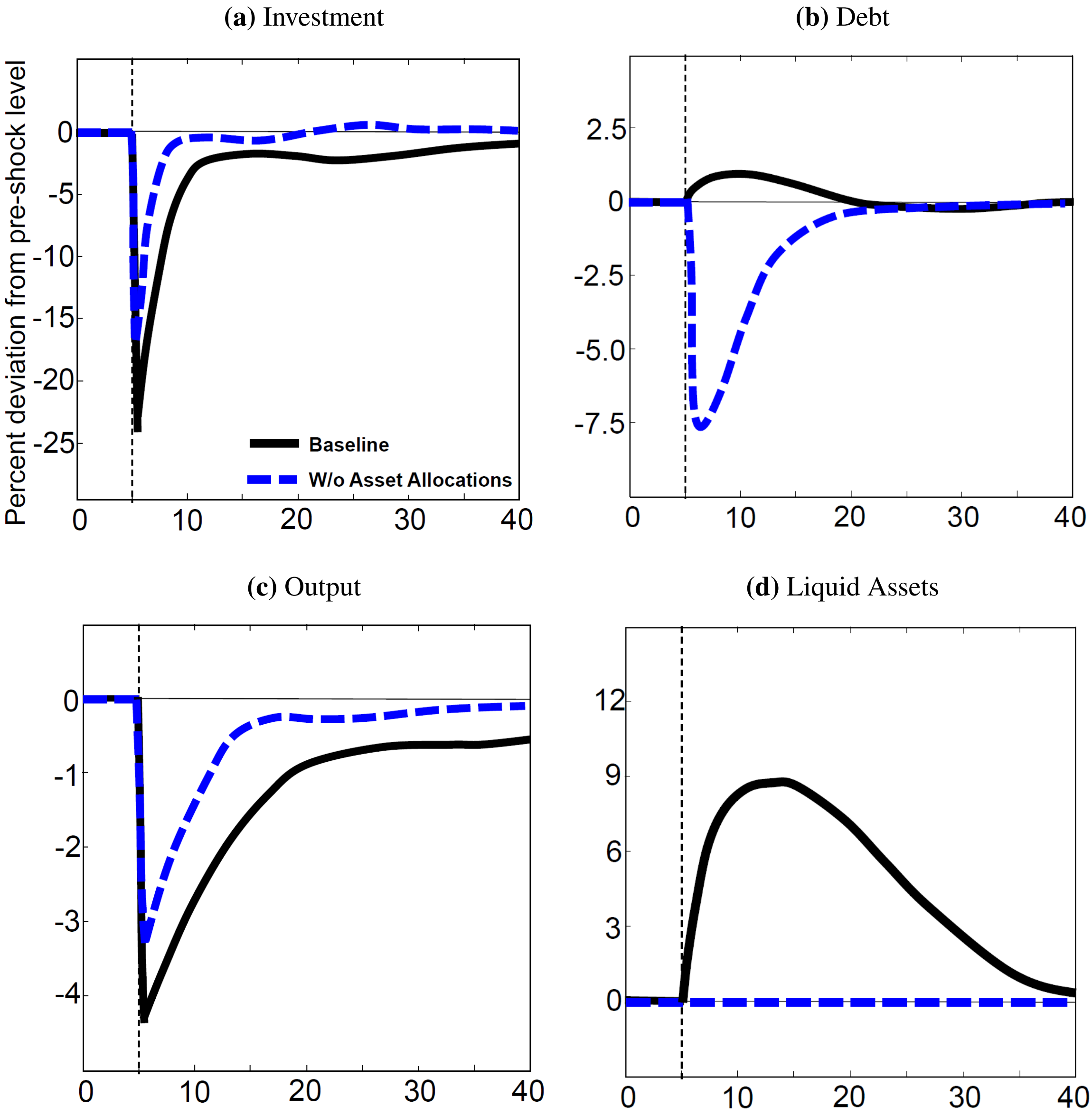
If firms do not have the opportunity to optimize their asset portfolios after the shocks, both investment and borrowing fall in response to higher uncertainty and credit tightening – as is the case in the counterfactual model – so the shocks propagate to the real side via a contraction in the quantity of credit. While this mechanism is quantitatively significant and matches the Flow of Funds data for US firms as a whole, it is not adequate for explaining the behavior of public firms during the recession (see Table 1).

Quantitatively, the decline in investment in the baseline model is 28% $(= 1 - \frac{17.3}{24})$ greater than in the counterfactual model, which is the size of the borrow-to-save mechanism in the model. Since the model is calibrated to the sample of public firms, which were responsible for 89% of the decline in aggregate investment (public and private firms), a back-of-the-envelope calculation suggests that borrowing to save can explain about 25%$(= 0.28 \times 0.89)$ of the decline in aggregate investment at the height of the recession.

Propagation Mechanism & Slow Recovery Figure 8 illustrates how adding optimal portfolio choice helps to generate a slower recovery from temporary shocks. While the half-lives of the volatility and financial shocks are about 4.5 quarters, the half-life of output is 7.5 quarters

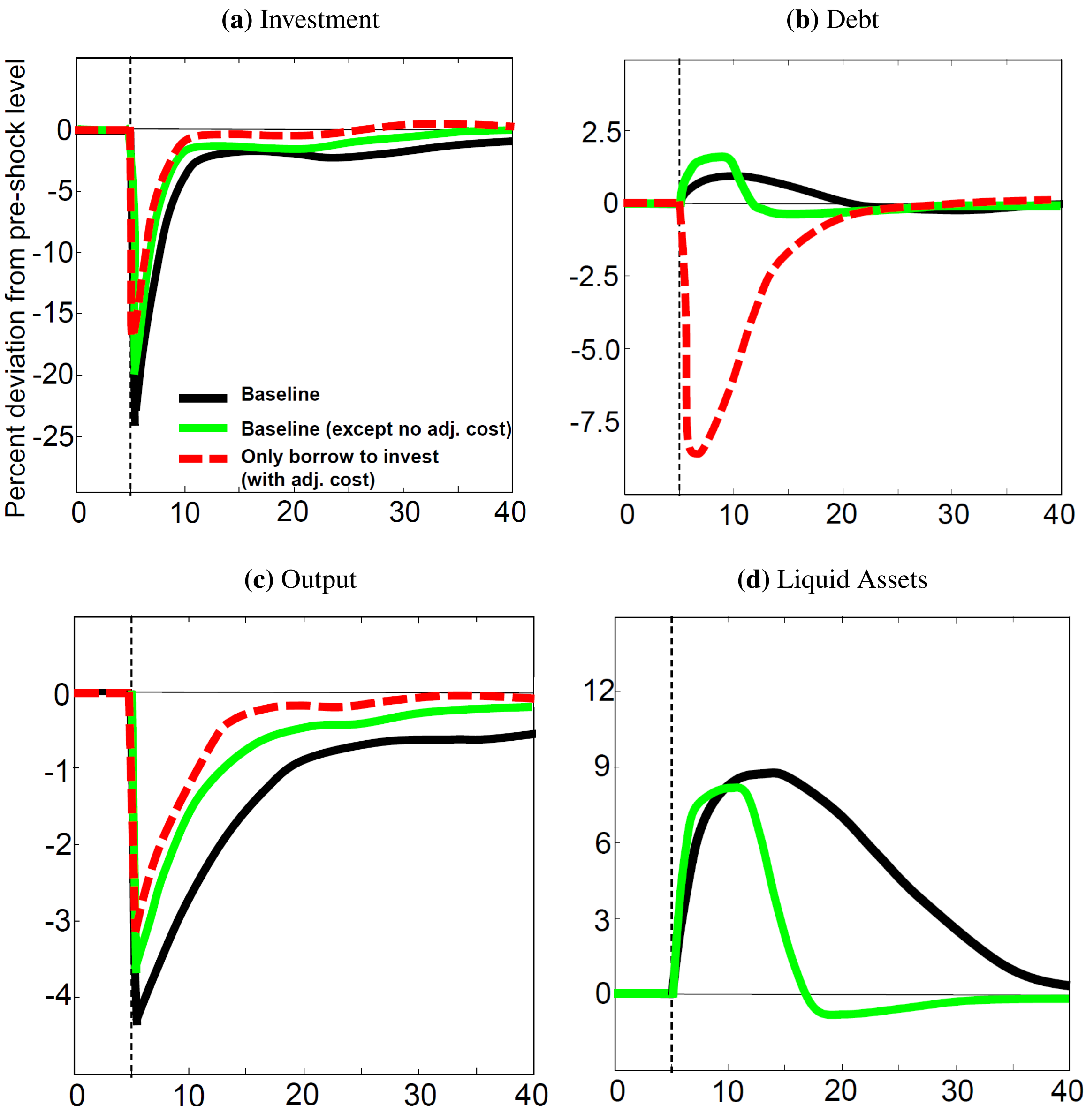
[16]This is calculated from the quarterly Compustat database for all non-financial corporate businesses.

Figure 7: Baseline Model
Impulse Response Functions to a Financial Shock
Combined with an Uncertainty Shock Before Asset Reoptimization



Note: This figure shows the impulse response functions to a temporary financial shock about three standard deviations in the 5th quarter, and simultaneously, the economy switches to the high volatility regime with $\sigma = \sigma_H$. The black solid lines depict the baseline model, whereas the blue dotted lines show the responses in a counterfactual model in which the level of liquid assets held by each firm is exogenously fixed at the pre-shock level.

Figure 8: Slower Recovery
Impulse Response Functions to a Financial Shock
Combined with an Uncertainty Shock Before Asset Reoptimization



Note: This figure shows the impulse response functions to a temporary financial shock about three standard deviations in the 5th quarter, and simultaneously, the economy switches to the high volatility regime with $\sigma = \sigma_H$. The black solid lines depict the baseline model. The green solid lines show the responses in a counterfactual model without adjustment costs (but firms still choose their allocations between safe assets and capital). The red dotted lines show the responses in a counterfactual model without portfolio choice, i.e. firms only borrow to invest in capital, as in a standard debt financing model.

in the baseline model with safe assets. In addition, I plot the impulse response functions from two counterfactual models: a version of the baseline model without adjustment costs, and another version in which firms do not have the option to accumulate safe assets; that is, only borrow to invest.[17] The slower rate of adjustment back to the pre-shock level is partly due to the additional adjustment frictions at the portfolio reoptimization stage. However, even in the model without adjustment costs (green solid lines), the transition is slower than the model without asset choices (red dotted lines).

This shows that the propagation of a negative credit shock is much slower in a model where firms hold safe assets. In both worlds (with or without safe assets), the negative financial shock reduces firms' profits and hence net worth. However, it takes longer for their net worth to grow back in the world with safe assets. I illustrate the reason for the slower growth of net worth in Figure 4 of Section 2: In any given period, a firm with safe assets make less profit than a firm with the same amount of capital but no safe asset. Since safe asset accumulation slows down equity growth, it takes longer for firms to grow out of their borrowing constraints, so the effect of a financial shock lasts for longer, even in the absence of external adjustment frictions.

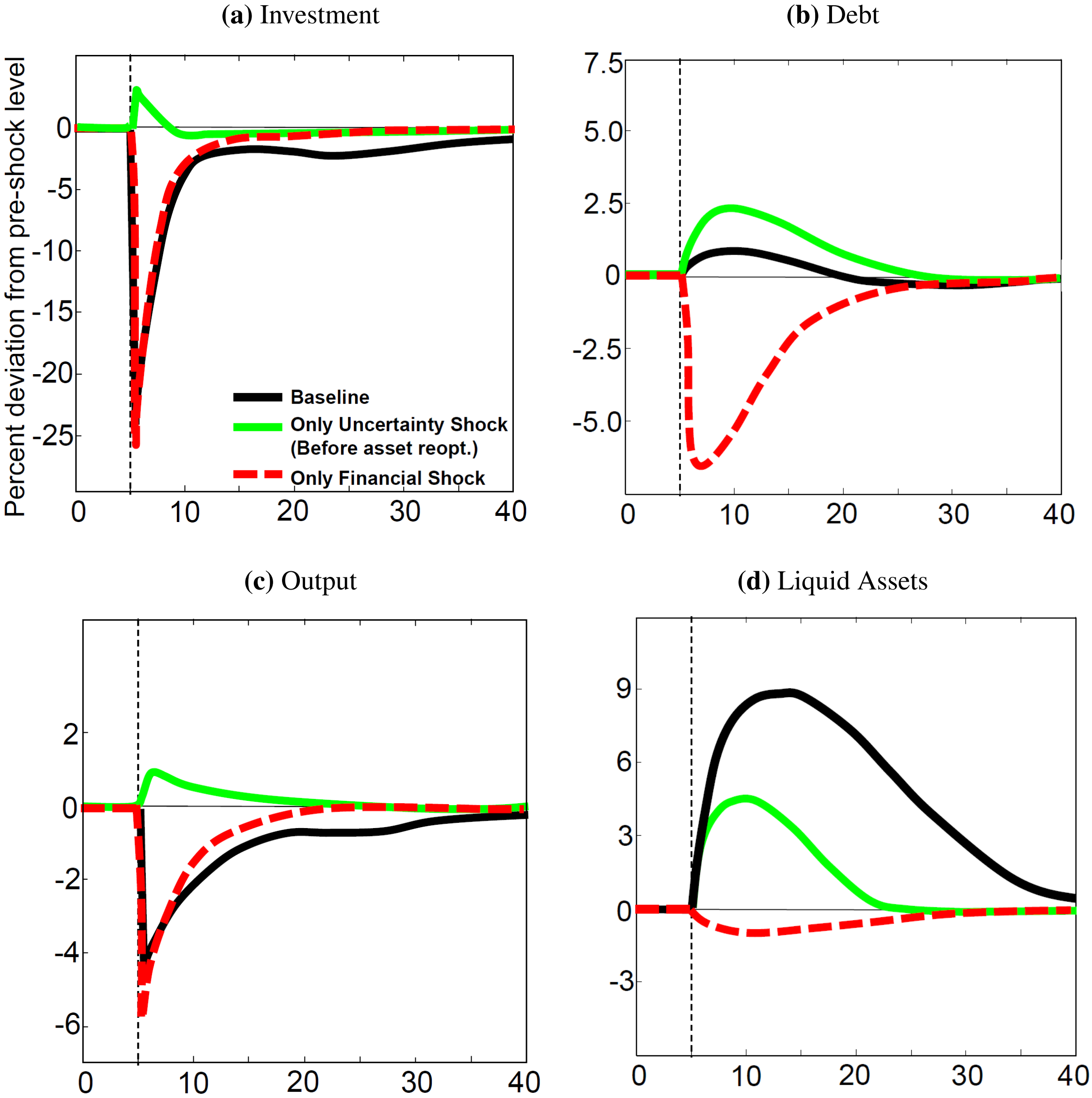
Why These Two Shocks? Figure 9 illustrates why a combination of first- and second-moment shocks is most suitable for mapping the model to data on public firms. An increase in the uncertainty level before asset reoptimization encourages firms to take more risks and increase borrowing. *Ceteris paribus*, a more indebted firm with a higher default probability also saves more. However, with more funds available, the level of investment in capital may or may not be lower. A negative financial shock ensures that there is a substantial reallocation from capital to safe assets, which dominates the effect of having more funds available for investment.

Nonetheless, for the borrowing to save mechanism, the negative first moment shock does not have to be a financial shock. For instance, if one replaces the idiosyncratic demand shock after asset reoptimization with an aggregate demand shock, and models the crisis as a reduction in demand combined with an increase in uncertainty, the borrowing to save mechanism would still be present, since the negative demand shock would also motivate firms to reallocate their assets from productive capital to safe assets. Here I chose to model the negative aggregate shock as a financial shock following the evidence from Adrian, Colla, and Shin (2012), that the Great Recession originated from a credit supply shock in the banking sector.

It is important to point out that the timing of the uncertainty shock is crucial to this mechanism. If the increase in uncertainty happens after firms have the chance to reoptimize

[17] This is slightly different from the counterfactual exercise in Figure 7, in which firms still hold safe assets in their portfolio, although the levels are fixed at the pre-shock levels.

Figure 9: Impulse Response Functions to Different Shocks



Note: This figure shows the impulse response functions in three versions of the model: the black solid lines denote the baseline model with a one-period financial shock (of three standard deviations) and volatility shock; the green lines denote a counterfactual model with a one-period financial shock (of three standard deviations) only; the red dotted lines denote another counterfactual model with only the volatility shock.

their asset portfolios – or, if there is no asset allocation problem altogether – then firms would be discouraged from risk-taking, resulting in lower debt issuance and output simultaneously, echoing the findings in Arellano, Bai, and Kehoe (2018). The asymmetric impact of the volatilities of the first and second shock was demonstrated in Panel (b) of Figure 3.

5 Extended Model: Adding Debt Substitution

In this paper, I argue that the borrow-to-save mechanism has a cyclical nature, and is not specific to the Great Recession. Recall from Figure 1 that borrowing and investment by the US public firms have diverged in each recession since 1990. Nonetheless, the impact of the mechanism was much more significant during the Great Recession than in earlier crises. Part of the reason was the magnitude of the financial shock and the heightened uncertainty at the beginning of the crisis. Moreover, as shown by Becker and Ivashina (2014) and Adrian, Colla, and Shin (2012), a significant fraction of public firms substituted from bank loans to bonds during the crisis. I illustrate in this section that, due to the trade-offs between loans and bonds, adding debt substitution into the model can amplify the borrow-to-save mechanism, resulting in a longer-lasting recession.

5.1 Adding Debt Substitution to the Quantitative Model

The model set-up remains the same as in Section 3, with two exceptions. First, the intermediate good firms now choose not only the level of debt, but also the composition of debt. There are two types of debt: bank debt (b) and market debt (m). I model the trade-off between bank debt (b) and market debt (m) and the debt settlement process following Crouzet (2018). Specifically, on the one hand, bank debt is more flexible than market debt as only the former can be restructured when a firm is at the risk of default. On the other hand, the cost of intermediation per unit of lending is always higher for bank lenders (γ^b) than for market lenders (γ^m), due to the more costly bank-specific activities such as screening and borrowing.

Second, here I model the Great Recession as an asymmetric "financial" shock to the supply of bank debt, but not market debt, following the argument of Adrian, Colla, and Shin (2012). Bond issuance is only indirectly affected, as public firms took advantage of their access to the bond market and issued bonds in large quantities. Specifically, I define the wedge between the intermediation costs as $\gamma^* = \gamma^b - \gamma^m$, and use this instead of the recovery rate in default χ as the financial shock in the extended model. Similar to χ, γ^* follows an AR(1) process:

$$\log \gamma^{*'} = \bar{\gamma}^* + \rho_\gamma \log \gamma^* + \sigma_\gamma \varepsilon'_\gamma; \quad \varepsilon'_\gamma \sim N(0,1).$$

The main difference from the baseline model is that now the firm has three options at the debt settlement stage (see Figure 5): full repayment, debt restructuring, or default. If it chooses to restructure its debt to avoid default, it enters a debt-renegotiation process with the bank lenders. Following the debt restructuring procedure in Crouzet (2018), two sets of possible equilibria arise, such that restructuring may occur in one (R-contract) and never occurs in the other (NR-contract), and the latter arises when the stake of the flexible creditors, b', is too small for restructuring to bring about sufficient gains for the firm to avoid default on market debt.

As a result, instead of having one debt price in the baseline (12), there are four debt prices (two for each type of debt) in the extended model. If $\frac{m'}{1-\chi} > \frac{b'}{\chi}$ (NR-contract), restructuring never occurs, and the debt prices are:

$$q^b_{NR} + \gamma^b = \mathrm{E}\left[\lambda(\mathbf{s},\mathbf{s}')\left(\int_{\varepsilon'_z}\int_{\underline{\psi}'_{NR}} dF(\varepsilon'_z|\sigma_z)dF(\psi') + \int_{\varepsilon'_z}\int^{\underline{\psi}'_{NR}} \frac{\chi\pi'}{b'} dF(\varepsilon'_z|\sigma_z)dF(\psi')\right)\middle| z,\mathbf{s}\right],$$

$$q^m_{NR} + \gamma^m = \mathrm{E}\left[\lambda(\mathbf{s},\mathbf{s}')\left(\int_{\varepsilon'_z}\int_{\bar{\psi}'_{NR}} dF(\varepsilon'_z|\sigma_z)dF(\psi') + \int_{\varepsilon'_z}\int^{\underline{\psi}'_{NR}} \frac{\chi\pi' - b'}{m'} dF(\varepsilon'_z|\sigma_z)dF(\psi')\right)\middle| z,\mathbf{s}\right].$$

If $\frac{m'}{1-\chi} \leq \frac{b'}{\chi}$ (R-contract), restructuring may occur, and the debt prices are:

$$q^b_{R} + \gamma^b = \mathrm{E}\left[\lambda(\mathbf{s},\mathbf{s}')\left(\int_{\varepsilon'_z}\int_{\bar{\psi}'_{R}} dF(\varepsilon'_z|\sigma_z)dF(\psi') + \int_{\varepsilon'_z}\int^{\bar{\psi}'_{R}} \frac{\chi\pi'}{b'} dF(\varepsilon'_z|\sigma_z)dF(\psi')\right)\middle| z,\mathbf{s}\right],$$

$$q^m_{R} + \gamma^m = \mathrm{E}\left[\lambda(\mathbf{s},\mathbf{s}')\left(\int_{\varepsilon'_z}\int_{\underline{\psi}'_{R}} dF(\varepsilon'_z|\sigma_z)dF(\psi')\right)\middle| z,\mathbf{s}\right].$$

π' denotes the firm's current assets, as previously defined, and the expressions for the four thresholds $(\underline{\psi}'_{NR}, \bar{\psi}'_{NR}, \underline{\psi}'_{R}, \bar{\psi}'_{R})$ are in Appendix B.3. Importantly, the timing of an intermediate goods firm's problem is exactly the same as in the baseline model (see Figure 5), and so is the problem of the entrants. The value functions of the incumbent intermediate goods firms are written out in Appendix B.3.

5.2 Calibration and Model Fit

Compared to the baseline model, the additional parameters are $\{\bar{\gamma}^*, \sigma_\gamma, \rho_\gamma, \gamma^m\}$, whereas $\{\rho_\chi, \sigma_\chi\}$, which were externally calibrated, no longer apply here. Among the new parameters, γ^m is exogenously calibrated; $\{\sigma_\gamma, \rho_\gamma\}$ are estimated from data; and $\bar{\gamma}^*$ is calibrated with the rest of the parameters to jointly target moments.

Since the financial shock in the extended model is an unexpected tightening of credit supplied by banks, I use data from the Federal Reserve's Senior Loan Officer Opinion Survey of Bank Lending Practices (SLOOS) to calibrate $\{\sigma_\gamma, \rho_\gamma\}$. This survey queries participating banks to report whether they have changed their standards during the survey period. To identify the component of the change in lending standards that is orthogonal to the determi-

Table 5: Parameters from Moment Matching in Extended Model

Parameter	*Description*	*Statistic (%)*	*Target*	*Moment*
$\sigma_H = 0.28, \quad \sigma_L = 0.19$	Volatility levels	Invest IQR mean & std.	15 & 2.7	13 & 2.9
$\pi_{HH} = 92, \quad \pi_{LL} = 93$	Transition probabilities	Invest IQR auto. & skew.	87 & -4.8	93 & 1.2
$F_o = 0.48$	Operating cost	Cash to asset median	10	7.6
$\chi = 0.64$	Recovery rate	Leverage median	25	29
$\mu_\psi = -0.17$	Demand shock (mean)	Spread median	3.3	2.8
$\sigma_\psi = 0.23$	Demand shock (std.)	Spread std.	1.8	1.4
$\bar{\gamma}^* = 0.03$	Wedge (mean)	Frac of bonds (median)	58	64
$f_e = 10.42$	Entry cost	Entrants lev (rel.)	65	61
$\xi = 2.87$	Pareto distribution	Entrants invest (rel.)	1.8	1.5

Note: This table presents the parameters from the moment-matching in the extended model. All data moments, except those on credit spreads, are constructed from the quarterly Compustat sample for US non-financial public firms with data available on debt compositions from Capital IQ between 2006 and 2014. Credit spreads data come from The Bank of America Merrill Lynch Indicators. Firm-quarter observations are included if total assets, cash and marketable securities, debt in current or long-term liabilities, capital expenditures, net property, plant and equipment, and debt compositions are non-missing. See Appendix A for detailed variable definitions.

nants of loan demand, I estimate a VAR(1) specification with quarterly data on macroeconomic variables—including log real GDP, log GDP deflator, the shadow federal funds rate of Wu and Xia (2016) —and the net percent of banks reporting tightening standards between 2006Q1 and 2014Q4. The credit variable is ordered after the macro variables. This yields $\rho_\gamma = 0.94$ and $\sigma_\gamma = 0.07$, which are within the range reported in the literature.

As a proxy for the intermediation cost of market debt, I use existing estimates of underwriting fees for corporate bond issuances, and set $\gamma^m = 0.01$, as in Crouzet (2018). Instead of measuring analogously intermediation costs of banks – for example from operating expenses reported in income statements of commercial banks – I calibrate the wedge in intermediation costs $\bar{\gamma}^*$ by matching the average fraction of bank debt among non-financial corporations in the US.[18] All targeted moments in the extended model are reported in Table 5. The data moments are computed from a subset of firms included in the baseline model calibration, including all non-financial US firms with data available on debt compositions from Capital IQ. As the focus is on the Great Recession, the sample period is between 2006Q1 and 2014Q4.

To check the model fit, Table 6 reports the cross-sectional moments in the model and the data. Compared to Table 3, the new moment here is the fraction of bonds for each asset class. In the data, smaller firms (<50%) have lower fractions of bonds than larger firms, which is also the case in the model, since smaller firms are more likely to default so would value the flexibility of bank loans more than larger firms. Similar to the baseline model, the extended model can also capture the cross-sectional patterns of investment rates, cash to asset ratios, and leverage ratios.

[18] It is difficult to obtain a clean measure of banks' intermediation cost from the data directly as operating expenses of banks can be associated with other non-lending activities.

Table 6: Cross-Sectional Moments in Extended Model

Asset Percentile	**Investment** (%) Data	Model	**Cash/Asset** (%) Data	Model	**Leverage** (%) Data	Model	**Bond Frac.** (%) Data	Model
$0-25\%$	13	12	15	11	21	25	48	54
$25-50\%$	18	14	13	8.4	23	26	41	57
$50-75\%$	18	13	8.2	7.1	26	29	65	64
$75-100\%$	16	11	6.9	6.3	28	31	87	71

Note: This table presents the cross-sectional moments in the data and the corresponding moments generated by the model. For each variable, I first calculate the median for each group of firms (defined by size) for each quarter. The median of each time series is reported in the table. Data are from Compustat and Capital IQ (2006Q1–2014Q4). See Appendix A for variable definitions.

5.3 Amplification of the Borrow to Save Mechanism

Figure 10 presents the impulse response functions in the extended model, for firms' borrowing, savings, investment, and output in response to a combination of one-time financial and uncertainty shocks. Contrary to the baseline model where the financial shock is an unexpected tightening of all firms' borrowing capacity, here the financial shock is unexpected increase in the wedge between bank and bond intermediation costs. This facilitates the switch from bank finance to bond finance, as we observed during the crisis (see Figure 2 in Adrian, Colla, and Shin (2012)). The financial shock is approximately two standard deviations in size, such that the decline in investment in the first quarter is similar to the decline in the data between 2008Q4 and 2009Q1.

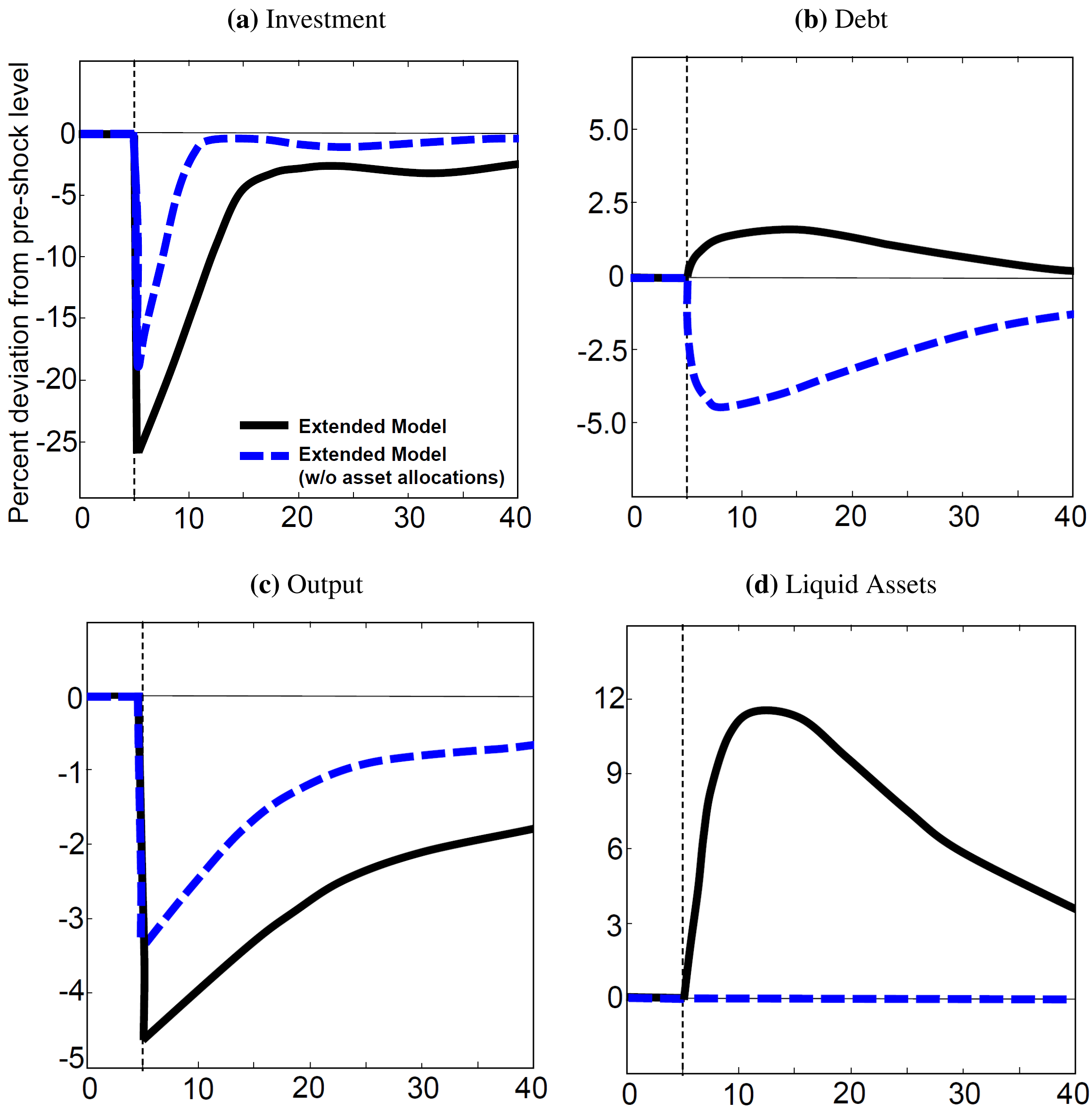
Like in Figure 7, Figure 10 compares the impulse responses in the extended model as described (black solid lines) with those in a counterfactual model (blue dotted lines) holding the fraction of liquid assets constant at the pre-shock level, i.e. firms cannot choose their asset portfolios optimally. If firms can adjust the compositions of both their assets and liabilities (extended model), both of their borrowings and liquid asset holdings increase by more than if they can only adjust the composition of their assets (baseline model). As the financial shock only directly affects the bank lenders and firms can substitute into bonds, it is straightforward to see why firms would increase borrowings by more when uncertainty resolves gradually. The impact on liquid asset holdings is more subtle, and arises because of the interaction between debt substitution and asset reallocation. In the extended model, when the cost of bank debt increases unexpectedly, a significant fraction of firms switch from a NR-contract with mixed debt to an R-contract with only bonds, and consequently, they lose the ability to restructure debt when they are at risk of default. As a result, debt substitution amplifies the importance of precautionary savings.[19]

[19]The switch to R-contract resembles the outcome in Crouzet (2018), but the consequence of the switch has different implications here. In Crouzet (2018), firms reduce their level of borrowing by more when they substitute into bonds than when they cannot substitute, as they take into account that they no longer have the option to restructure debt. In this model, firms decrease their borrowing by less when they substitute into bonds. This difference arises because of the shock structure here. When firms know that they can reoptimize their asset portfolios, they prefer to switch to bonds and maintain their level of debt *ex ante* than to cut borrowing straightaway. Unlike the setting in Crouzet (2018), this framework shows

Figure 10: Extended Model (With Debt Substitution)

Impulse Response Functions to a Financial Shock

Combined with an Uncertainty Shock Before Asset Reoptimization



Note: This figure shows the impulse response functions to a temporary shock to the relative supply of bank credit (modeled by the wedge γ^*) in the 5th quarter, and simultaneously, the economy switches to the high volatility regime with $\sigma = \sigma_H$. The size of the financial shock is approximately two standard deviations. The black solid lines depict the responses in the extended model (with debt substitution as well as asset optimization); the blue dotted lines show the responses in a counterfactual version of the extended model, with debt substitution but no asset optimization.

Overall, the counterfactual (without borrowing to save) here can account for two-thirds of the decline in the extended model, compared to 72 percent in the baseline model. This shows that adding debt substitution can amplify the impact of the borrowing to save channel. Recall that public firms accounted for 89% of the total decline, this extended model suggests that the borrowing to save channel can explain up to 30% of the total decline in investment at the height of the recession. Since firms have a higher fraction of safe assets in the extended model than in the baseline, firm growth and the transition back to the pre-shock level are also slower than in the baseline model. The aggregate shocks have a half-life of about 11 quarters, whereas the half-life of output – when firms choose their asset portfolios optimally – is about 19 quarters.

6 Concluding Remarks

This paper argues that fluctuations in investment are not necessarily driven by fluctuations in the quantity of credit. Evidence from the Compustat sample shows that the borrowing and investment decisions of U.S. public firms, on average, diverged in each recession over the past 30 years. Instead of investing, they borrow to finance the accumulation of safe assets. Existing models of financial frictions cannot account for this, as firms only borrow to invest in these models, so changes in investment and borrowing are typically one-for-one. Since the sample of firms accounted for 70 percent of the total investment of the universe of U.S. non-financial corporate businesses reported in the Flow of Funds in 2007, and almost 90 percent of its decline between 2007 and 2009, it is important to study the investment behaviors of public firms for understanding a great deal of business cycle fluctuations.

Motivated by such evidence, this paper proposes an incomplete-markets model to explain why firms would borrow to save at a time when credit spreads are high and the risk-free rate is depressed. Instead of borrowing to invest, firms borrow for liquidity, but such liquidity could be used to either invest or increase assets, depending on the realization of their uncertainty. Since firms are loaded with debt, any financial shock that increases their costs of borrowing and hence default probabilities will incentivize them to accumulate safe assets to avoid costly default. Moreover, adding safe assets to firms' portfolios implies that conditional on survival, firms earn less profit and hence equity growth is slower. Hence with borrowing to save, the effects of financial frictions and financial shocks are amplified, and more persistent, in recessions.

The quantitative framework developed in this paper can be extended in several dimensions. For example, I show how adding debt substitution, as we observed during the Great Recession, can amplify the borrowing to save mechanism due to the trade-off of bonds and loans. Finally, a missing element in the analysis is policy. An interesting extension would

that the bond market can act as a "spare tire" when the banking sector is impaired (results available upon request).

be to examine the role of monetary policy that may potentially change the mix of assets held by the firms.

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Appendix for Borrowing to Save and Investment Dynamics

For Online Publication

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A Data Appendix

A.1 Data Sources and Variable Definitions

- **Table 1**: Data for **all** US non-financial corporate businesses are from the (annual) Financial Accounts Z.1: *Debt* is the sum of commercial papers, municipal securities, corporate bonds, loans, depository institution loans n.e.c., and other loans and advances; *Real Investment* is total capital expenditures; *Liquid Assets* are the sum of checkable deposits and currency, total time and saving deposits, money market fund shares, commercial paper, Treasury securities, municipal securities, mutual fund shares, and security repurchase agreements. Data for **public** firms are from the annual Compustat database. The sample consists of US non-financial firms (excluding SIC codes 6000-6999) with non-missing and positive total assets, and non-missing firm characteristic variables considered in the study between 1988 and 2012. *Debt* is the sum of debt in current liabilities and long-term debt; *Real Investment* is the sum of capital expenditures and acquisitions, less sales of property, plants and equipment; *Liquid Assets* are cash and marketable securities. The series for **non-public** firms are constructed as the difference between the two samples. The annual growth rate of each series is computed as the difference in logs between 2007 and 2009.

- **Figure 1**: This figure plots the mean levels of *Debt*, *Real Investment*, and *Liquid Asset Ratio* from the Compustat sample described above, after winsorizing each variable at the 1st and 99th percentiles. The cyclical component of the *Liquid Asset Ratio* is extracted from an HP filter (with lambda = 100).

- **Tables 2 & 3**: All data moments, except those on credit spreads, are constructed from the quarterly Compustat database for all US non-financial public firms between 1988

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and 2012. Firm-year observations are included if total assets, cash and marketable securities, debt in current or long-term liabilities, capital expenditures, and net property, plant and equipment are non-missing. Following Bloom (2009), *Investment* for firm i at time t is defined as $(I/K)_{i,t} = I_{i,t}/(0.5(K_{i,t} + K_{i,t-3}))$, where capital $K_{i,t}$ is calculated by the perpetual inventory method: $K_{i,t} = (1-\delta)K_{i,t-3}(P_t/P_{t-3}) + I_{i,t}$, initialized using the net book value of capital. *Leverage* is the ratio of total debt (the sum of debt in current liabilities and long-term debt) to total assets. *Cash to Asset* is the ratio of cash and marketable securities to total assets. Credit spreads data are from The Bank of America Merrill Lynch Indicators between 1997 (the earliest available) and 2012. This dataset gives the bond spread for each rating in a particular quarter. To obtain the data moments on *Spread*, I first merge the Compustat sample with the Capital IQ database to obtain firm ratings, and then merge it with the BofAML data on the spread for each rating.

- **Table 4**: Data for *Investment*, *Cash to Asset*, and *Leverage* are from Compustat (defined above), aggregated across all non-financial public firms with non-missing and positive total assets, and non-missing firm characteristic variables between 1988 and 2012. Data for output are from the BEA. I calculate the year-on-year growth rates of investment, cash to asset, and leverage, respectively, and compute the correlations of each with the growth rate of output for *Correlations with Output*, or the ratio of each to the growth rate of output for *Relative Standard Deviations*.

- **Tables 5 & 6**: The construction of variables follows the same procedure as for Table 2, but the sample is different. The sample here is a subset of the sample for Table 2: I include all quarterly observations in Compustat with data available on debt compositions from Capital IQ between 2006 and 2014. Firm-quarter observations are included if total assets, cash and marketable securities, debt in current or long-term liabilities, capital expenditures, net property, plant and equipment, and debt compositions are non-missing. *Fraction of Bonds* is the ratio of bonds to the sum of bonds and loans.

A.2 Evidence for Assumption 1 (Portfolio Adjustment)

To test the validity of assumption 1, I compute, for the median firm of each quartile of firms by assets, the coefficient of variation (standard deviation divided by the mean) for: (i) cash as a proportion of total assets (column (1)), (ii) total capital expenditures as a proportion of total assets (columns (2)), (iii) total debt as a proportion of total assets (columns (3)), using the annual Compustat dataset for US non-financial firms with non-missing and positive total assets, book leverage above 5 percent, and non-missing firm characteristic variables between 1988 and 2012. The results are reported in Table A.1, which shows that the coefficient of variation (standard deviation divided by the mean) for cash is consistently higher than the correlation of variation for debt, across all definitions of debt and all quartiles of firms, and the differences (columns (4) and (5)) are significant at the 1 percent level.

Table A.1: Variations in Cash Holdings, Capital Expenditures, and Debt Financing

Asset Percentile	Cash (1)	Capex (2)	Debt (3)	Cash−Debt (4)	Capex−Debt (5)
$[0,100]$	0.6688	0.5537	0.3455	**0.3233**	**0.2081**
$[0,25]$	0.7128	0.7683	0.5005	**0.2122**	**0.2678**
$[25,50]$	0.5272	0.5248	0.3387	**0.1885**	**0.1861**
$[50,75]$	0.5628	0.4179	0.2450	**0.3179**	**0.1730**
$[75,100]$	0.6665	0.3558	0.2047	**0.4619**	**0.1511**

Note: The data sample includes all Compustat firm-year observations from 1988 to 2012 with positive values for the book value of total assets, except for financial firms (SIC code 6000-6999) and firms with book leverage below 5% or missing firm characteristic information (on cash, investment, or debt). Columns (1)–(3) report the coefficient of variation (standard deviation divided by the mean) for cash-to-asset, capital expenditures-to-asset, and debt-to-asset, respectively, for the median firm in the corresponding sample. Column (4) reports the difference between cash-to-asset (1) and debt-to-asset (3). Column (5) reports the difference between capital expenditures-to-asset (2) and debt-to-asset (3). Each row reports the statistics for the sample defined by the asset percentile. Differences significant at 1 percent are in bold.

B Model Appendix

B.1 Computation

The model is solved using the algorithm developed by Krusell and Smith (1998), in two loops: an inner and an outer loop. The algorithm begins with guessing the initial coefficients and initializing the forecast rules $\hat{\Gamma}_p^{(1)}$, $\hat{\Gamma}_K^{(1)}$, and $\hat{\Gamma}_B^{(1)}$. In the inner loop, I solve each firm's optimization problem iteratively until convergence, taking as given $\hat{\Gamma}_p^{(1)}$, $\hat{\Gamma}_K^{(1)}$ and $\hat{\Gamma}_B^{(1)}$. In the outer loop, based on the converged decisions from the inner loop and an initial distribution of firms $\mu_0(z,x,\hat{k})$, I simulate the economy for T periods and obtain firms' borrowing, default, investment, and saving decisions in each period. Based on the simulated data and the market clearing conditions, I update the forecast mappings to obtain $\hat{\Gamma}_p^{(2)}$, $\hat{\Gamma}_K^{(2)}$, and $\hat{\Gamma}_B^{(2)}$. The procedure is repeated until the approximate mappings converge.

I discretize the state space of the idiosyncratic firm problem before solving the inner loop. As noted in the main text, I assume a two-state Markov process for uncertainty σ, and the intractable cross-sectional distribution μ is approximated with the current aggregate capital level K, the current aggregate debt level B, and the lagged uncertainty state σ_{-1}, so the aggregate state vector $\mathbf{s}$ is approximated by $(\chi,\sigma,a_h,\sigma_{-1},B,K)$. Using the Gaussian quadrature method, the exogenous aggregate state variable χ is discretized into 4 grid points, and the exogenous idiosyncratic productivity process z is discretized into 5 grid points for each level of volatility σ_{-1}. I discretize the endogenous idiosyncratic state x into 15 endogenous grids that depend on the shock z and the aggregate state vector $\mathbf{s}$, and the other endogenous state $\hat{k}$ has 40 gridpoints. The demand shock ψ, which is not a state variable, is discretized into 100 gridpoints for evaluating the integrals in the firm's value. Given this discretization, the value functions are computed with Howard's improvement step. The value functions in between grid points are interpolated using a multivariate tensor product spline approximation.

In the outer loop, I simulate an economy for $T = 2,000$ quarters. For each period t, each firm's policy functions $(b',k',a'_f,\hat{k}',\hat{a}'_f)$ must be consistent with the market clearing, which occurs when the consumption level C implied by the market-clearing marginal utility price $\tilde{p}$ is equal to $1/\tilde{p}$. To ensure this when the excess demand function may contain discontinuities, I employ the market-clearing algorithm outlined in the Appendix of Bloom et al. (2018). First, define a grid for the marginal utility price $\tilde{p}$, and compute firms' policy functions and consumption level C for each of the 30 grid points of $\tilde{p}$. For prices outside the initial grid, I approximate the consumption values $\tilde{C}$ with shape-preserving piecewise cubic interpolation, and I use this to define a continuous excess demand function $e(\tilde{p}) = 1/\tilde{p} - \tilde{C}(\tilde{p})$, which is used to solve for the market-clearing price p^* that generates $e(p^*) = 0$. The next period's firm distribution μ' and aggregate capital K' are updated in a way that is consistent with the construction of the excess demand function (see Bloom et al. (2018)).

These simulated series are conditioned upon the equilibrium mappings $\hat{\Gamma}_p^{(1)}$, $\hat{\Gamma}_K^{(1)}$, and $\hat{\Gamma}_B^{(1)}$. To update the mapping coefficients $\mathbf{A}(\sigma,\sigma_{-1})$, $\mathbf{B}(\sigma,\sigma_{-1})$, and $\mathbf{C}(\sigma,\sigma_{-1})$ for each discrete pair (σ,σ_{-1}), I run the following OLS regressions on a subset of the simulated data, after discarding the first 100 quarters to remove the influence of the initial conditions:

$$\begin{bmatrix} \log p \\ \log \hat{K}' \\ \log \hat{B}' \end{bmatrix} = \mathbf{A}(\sigma,\sigma_{-1}) + \mathbf{B}(\sigma,\sigma_{-1}) \begin{bmatrix} \log \hat{K} \\ \log \hat{B} \end{bmatrix} + \mathbf{C}(\sigma,\sigma_{-1}) \begin{bmatrix} \log \chi \\ \log z \end{bmatrix}.$$

The procedure is repeated until the forecast mapping has converged, which is when the change in the accuracy of the forecast system is less than a predefined tolerance (see Den Haan (2010)).

Having obtained the solution to the model, I compute the impulse responses to an aggregate uncertainty shock, and simultaneously, an aggregate financial shock, by simulating an economy for $N = 3000$ (firms) and $T = 40$ (quarters). To remove the effect of sampling variation associated with the idiosyncratic technology shock, I repeat the procedure $M = 50,000$ times. The aggregates evolve normally until the shock period $T_{\text{shock}} = 5$. In the shock period, the economy switches to the high uncertainty state, and simultaneously, I impose a negative aggregate financial shock of approximately three standard deviations. The aggregate shocks are then allowed to die out according to the specified laws of motion over the remainder of the impulse response horizon. Denote the period t response of an aggregate variable X to the aggregate shocks as $\hat{X}_t$. It is computed as the percentage deviations from the pre-shock level:

$$\hat{X}_t = 100 \times \log \left[\frac{\bar{X}_t}{\bar{X}_{T_{\text{shock}}-1}} \right], \quad \text{for } t = 1,2,\ldots,T,$$

with $\bar{X}_t = \frac{1}{N}\frac{1}{M}\sum_{m=1}^{M}\sum_{n=1}^{N} X_{m,n,t}$ denoting the average level of the series in period t, and $\bar{X}_{T_{\text{shock}}-1} = \frac{1}{N}\frac{1}{M}\sum_{m=1}^{M}\sum_{n=1}^{N} X_{m,n,T_{\text{shock}}-1}$ denoting the average level of the series in the pre-shock period.

As usual with the Krusell-Smith approach, I check the accuracy of the forecasting system using two standard metrics: Table B.1 reports the R^2 of each predictive regression for p, $\hat{K}', \hat{B}'$ (for each pair (σ,σ_{-1})) and the percentage root mean standard errors of the predictions. As shown in the table, the forecasting rules are fairly accurate, as evidenced by the R^2 close to 1 and the root mean standard errors not higher than 0.524%. According to these metrics, the solution of the model is likely to be a good approximation of the model's true rational expectations equilibrium.

Furthermore, I also consider the dynamic forecast accuracy metrics following Den Haan (2010). Specifically, the forecasted value of p for period t is used to forecast a value for period $t+1$, and I iterate this procedure forwards to any desired horizon, using only the

Table B.1: Internal Accuracy Statistics for the Approximate Equilibrium Mappings

	$\log p$		$\log \hat{K}'$		$\log \hat{B}'$	
(σ, σ_{-1})	R^2	RMSE(%)	R^2	RMSE(%)	R^2	RMSE(%)
$(1,0)$	0.9975	0.27	0.9891	0.45	0.9954	0.31
$(0,1)$	0.9981	0.21	0.9925	0.37	0.9967	0.29
$(0,0)$	0.9748	0.52	0.9913	0.39	0.9921	0.37
$(1,1)$	0.9962	0.31	0.9874	0.48	0.9881	0.46

Note: Each row in the table above displays the performance of the equilibrium mapping conditional upon a subsample of the data characterized by a given (σ, σ_{-1}). RMSE represents the root mean squared error of the indicated rule's one-period ahead forecasts, and the R^2 measure is computed from the log-linear regression on the appropriate subsample of simulated data.

realized values of exogenous aggregate states. Table B.2 reports the Den Haan accuracy statistics for p at various horizons, including the mean and maximum errors in the dynamic forecasts, which are given by:

$$\varepsilon_{\max} = 100 \times \max \left| \log(p_t^{DH}) - \log(p_t) \right|$$
$$\varepsilon_{\text{mean}} = 100 \times \frac{1}{T-1} \sum_{t=2}^{T} \left| \log(p_t^{DH}) - \log(p_t) \right|.$$

The average forecasting error for price at a horizon of 3 years is 0.72%. This is fairly comparable to the dynamic forecast errors in other investment models with multiple shocks. For instance, Khan and Thomas (2013) report a mean dynamic forecast error of 0.8%; Bloom et al. (2018) report a mean error of 0.63%.

Table B.2: Dynamic Forecast Accuracy Statistics

Den Haan statistic (%)	3 Years	4 Years	5 Years	6 Years
$\varepsilon_{\text{mean}}$	0.72	0.85	0.94	1.03
$\varepsilon_{\max}$	1.50	1.72	1.98	2.23

Note: This table reports the Den Haan (2010) accuracy statistics for the forecasting system for the market-clearing marginal utility price p. The statistics, $\varepsilon_{\max}$ and $\varepsilon_{\text{mean}}$, are based on forward iteration of the forecasting system for marginal utility to a specific future horizon (3/4/5/6 years), substituting n-period ahead forecasts as inputs for $n+1$-period ahead forecasts.

B.2 Adding Equity Financing

In order to focus on debt financing, the model in the main text is abstract from equity financing. This section assesses how equity financing affects the borrowing to save channel. Firms can now raise two types of external financing: debt and equity. To motivate trade-offs between equity and debt, I assume that equity issuance is costly, such that the value of existing shares is reduced by more than the amount of newly issued shares. This is known as "equity dilution cost" and is typically modeled as a constant marginal cost of equity issuance (see, for instance, Cooley and Quadrini (2001)). The loss in the value of existing shares associated with the amount e of newly issued equity is given by:

$$\bar{\xi}(e) = e + \xi \max\{e, 0\} \tag{B.1}$$

where ξ measures the degree of frictions in the stock market, and e denotes either the value of newly issued shares ($e > 0$) or the value of share repurchases ($e < 0$), which are equivalent to dividend payments. Firms face a minimum dividend constraint, which captures the prevalence of dividend-smoothing policies in the data:

$$d \geq \underline{d} \geq 0. \tag{B.2}$$

The firm's dividend can now be written as:

$$\begin{aligned} d &= p(\psi)y(z(\sigma_{-1})) - F_o\hat{k} - b + \hat{a}_f - \upsilon g(k', \hat{k}) + q^b b' - q^a a'_f + e \\ &= x - \upsilon g(k', \hat{k}) + q^b b' - q^a a'_f + e. \end{aligned}$$

The firm's recursive problem (after the exogenous exit shock) now becomes:

$$V^1(z, \hat{k}, x; \mathbf{s}) = \max_{d, e, k', b', a'_f} \left\{ d - \bar{\xi}(e) + \mathrm{E}\left[\lambda(\mathbf{s}, \mathbf{s}')\hat{V}^0(a'_f, k', b', z'; \mathbf{s}') \middle| \mathbf{s} \right] \right\},$$

subject to (B.1), (B.2), the cost of adjusting capital (17), the price of debt (12) and risk-free rate (16), and the aggregate law of motion $\mathbf{s}' = \Gamma(\mathbf{s})$.

As the model is in general equilibrium, the addition of equity financing also affects the household's problem, who solves:

$$W(a_h; \mathbf{s}) = \max_{c, a'_h} \left\{ u(c) + \beta \mathrm{E}\left[W(a'_h; \mathbf{s}') \middle| \mathbf{s} \right] \right\},$$

subject to a new budget constraint:

$$c + q^a(\mathbf{s})a'_h + \int p_s s' d\mu(z, x, \hat{k}) \leq a_h + \int \left[(d + \tilde{p}_s)s + F_o\hat{k} \right] \mu(z, x, \hat{k})$$

where p_s is the ex-dividend value of equity, $\tilde{p}_s$ is the current market value of equity, and $0 \leq s \leq 1$ is the fraction of outstanding shares owned by the household. The equity valuation terms are linked by the accounting identity: $\tilde{p}_s = p_s - \bar{\xi}(e)$, where $\bar{\xi}(e) \geq e$ is the cost of issuing new shares. Equity prices are pinned down by household's first order conditions:

$$p_s(z,\hat{k},x;\mathbf{s}) = \mathrm{E}\Big[\lambda(\mathbf{s},\mathbf{s}')\Big(d' - \bar{\xi}(e') + p_s'(z',\hat{k}',x';\mathbf{s}')\Big)\Big|\mathbf{s}\Big].$$

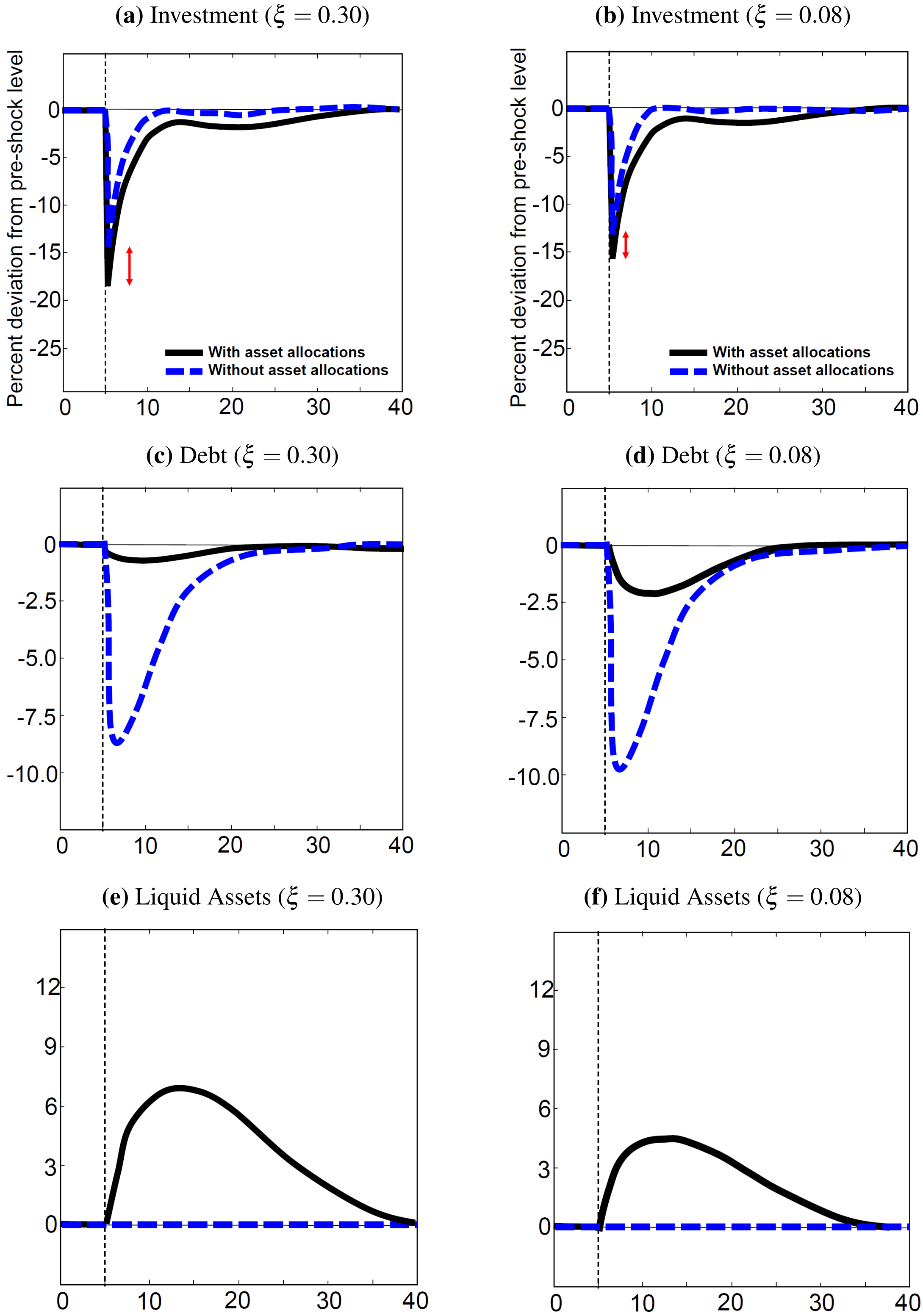
The estimates of the cost of seasoned equity issuance vary substantially in the literature, from a low of 0.08 (Gomes (2001)) to a high of 0.30 (Cooley and Quadrini (2001)). I recalibrate the baseline model plus equity financing (without debt substitution), following the procedure described in Section 4, and use each of these estimates for ξ in turn. For each calibration, I introduce the same aggregate shocks (financial plus uncertainty) of the same magnitudes as described in Section 4. Figure B.1 displays the impulse response functions, for high (left panels) and low costs (right panels) of equity issuance, respectively. To quantify the borrowing to save channel, I perform the same counterfactual exercise as in the baseline model, fixing exogenously the level of liquid assets held by each firm at the steady state level.

In the baseline model, a negative financial shock would expose firms to higher default risks due to the higher costs of borrowing. As long as firms rely on debt as the only external source of financing for growth, the default risks can only be partially mitigated by the accumulation of safe assets. This also implies that if firms have an alternative external source of financing, such as equity, they can substitute from debt to equity when the debt markets are impaired. As a result, the need for precautionary savings may be lower as equity helps to alleviate the incomplete-market frictions.

Comparing the impulse response functions in the baseline model (Figure 7 of the main text) and in the extended model (Figure B.1), debt falls in the model with equity, as firms prefer to issue equity – instead of borrowing – for liquidity when the cost of borrowing is high. Liquid assets still rise in response to the shocks, but to a lesser degree. With equity substitution, aggregate investment also falls by less compared to the baseline model, in response to shocks of the same magnitudes. Nevertheless, the borrowing to save mechanism is still present. As long as equity issuance is costly, firms would continue to borrow for liquidity *ex ante*, and save for self-insurance *ex post*. Under the calibrations of $\xi = 0.08$ and $\xi = 0.30$, borrowing to save can explain 21 percent and 16 percent, respectively, of the total decline in investment.

Figure B.1: Extended Model with Equity Financing

Impulse Response Function of **Investment** to a Financial Shock
Combined with an Uncertainty Shock Before Asset Reoptimization



Note: This figure shows the impulse response functions to a temporary financial shock about three standard deviations in the 5th quarter, and simultaneously, the economy switches to the high volatility regime with $\sigma = \sigma_H$. The magnitudes of the shocks are the same in both panels (as well as in the baseline model in the main text). The black solid lines depict the model with the borrowing to save channel, whereas the blue dotted lines show the responses in a counterfactual model in which the level of liquid assets held by each firm is exogenously fixed at the steady state level.

B.3 Extended Model with Debt Substitution

B.3.1 Default Thresholds

The trade-off between bank debt (b') and market debt (m') results in two sets of debt contracts – NR-contract and R-contract – such that restructuring only arises in the latter case. Following Crouzet (2018), two additional assumptions are made, which together determine whether a firm repays, restructures, or defaults at the end of each period: (1) bank debt is more senior than market debt; (2) if a firm renegotiates for a lower amount of repayment with bank lenders, it enters a two-stage Nash bargaining game and moves first. Given these assumptions, I outline below the default thresholds and lenders' payoffs in each type of contracts.

NR-contract If $\frac{m'}{1-\chi} > \frac{b'}{\chi}$, the firm repays its liabilities in full if $\pi' \geq b' + m'$; partially defaults (i.e. repays the more senior bank debt but defaults on market debt) if $\frac{b'}{\chi} \leq \pi' < b' + m'$; and defaults on both types of debt if $\pi' < \frac{b'}{\chi}$. Hence one can define a pair of thresholds for the demand shock ($\overline{\psi}'_{NR}$, $\underline{\psi}'_{NR}$)—conditional on $\mathbf{s}'$ and $(\hat{k}', b', m', a'_f, z')$—such that the firm defaults fully in the next period if $\psi' < \underline{\psi}'_{NR}$, and defaults partially if $\underline{\psi}'_{NR} \leq \psi' < \overline{\psi}'_{NR}$:

$$\overline{\psi}'_{NR}(b', m', \hat{k}', \hat{a}'_f, z') = \frac{b' + m' + F_o\hat{k}' - p_k^-(1-\delta)\hat{k}' - \hat{a}'_f}{\left(Y'\left(z'\hat{k}'^{\alpha}\right)^{\zeta-1}\right)^{\frac{1}{\zeta}}}, \tag{B.3}$$

$$\underline{\psi}'_{NR}(b', \hat{k}', \hat{a}'_f, z') = \frac{\frac{b'}{\chi} + F_o\hat{k}' - p_k^-(1-\delta)\hat{k}' - \hat{a}'_f}{\left(Y'\left(z'\hat{k}'^{\alpha}\right)^{\zeta-1}\right)^{\frac{1}{\zeta}}}.$$

The payoffs to the bank and market lender ($\tilde{\mathscr{R}}'_{b,NR}$ and $\tilde{\mathscr{R}}'_{m,NR}$) are :

$$\tilde{\mathscr{R}}'_{b,NR} = \begin{cases} b' & \text{if } \psi' \geq \underline{\psi}'_{NR} \\ \chi\pi' & \text{if } \psi' < \underline{\psi}'_{NR}, \end{cases}$$

and

$$\tilde{\mathscr{R}}'_{m,NR} = \begin{cases} m' & \text{if } \psi' \geq \overline{\psi}'_{NR} \\ \chi\pi' - b' & \text{if } \underline{\psi}'_{NR} \leq \psi' < \overline{\psi}'_{NR} \\ 0 & \text{if } \psi' \leq \underline{\psi}'_{NR}. \end{cases}$$

R-contract If $\frac{m'}{1-\chi} \leq \frac{b'}{\chi}$, one can also define a pair of thresholds for the prices ($\overline{\psi}'_R$, $\underline{\psi}'_R$)—conditional on $\mathbf{s}'$ and $(\hat{k}', b', m', a'_f, z')$—such that the firm defaults if $\psi' < \underline{\psi}'_R$, and restructures if $\underline{\psi}'_R \le \psi' < \overline{\psi}'_R$, and repays if $\psi' > \overline{\psi}'_R$:

$$\overline{\psi}'_R(b', \hat{k}', \hat{a}'_f, z') = \frac{\frac{b'}{\chi} + F_o\hat{k}' - p^-(1-\delta)\hat{k}' - \hat{a}'_f}{\left(Y'\left(z'\hat{k}'^{\alpha}\right)^{\zeta-1}\right)^{\frac{1}{\zeta}}}, \tag{B.4}$$

$$\underline{\psi}'_R(m', \hat{k}', \hat{a}'_f, z') = \frac{\frac{m'}{1-\chi} + F_o\hat{k}' - p^-(1-\delta)\hat{k}' - \hat{a}'_f}{\left(Y'\left(z'\hat{k}'^{\alpha}\right)^{\zeta-1}\right)^{\frac{1}{\zeta}}}.$$

The payoffs to the bank and market lender in an R-contract ($\tilde{\mathscr{R}}'_{b,R}$ and $\tilde{\mathscr{R}}'_{m,R}$) are given by:

$$\tilde{\mathscr{R}}'_{b,R} = \begin{cases} b' & \text{if } \psi' \ge \overline{\psi}'_R \\ \chi\pi' & \text{if } \psi' < \overline{\psi}'_R, \end{cases}$$

and

$$\tilde{\mathscr{R}}'_{m,R} = \begin{cases} m' & \text{if } \psi' \ge \underline{\psi}'_R \\ 0 & \text{if } \psi' < \underline{\psi}'_R. \end{cases}$$

B.3.2 Intermediate Good Firms' Recursive Problem

The definition of dividend in the model with two types of debt contracts is given by:

$$d_l = \begin{cases} p(\psi)y(z) - g(k', \hat{k}) - b - m + \hat{a}_f - q^a a'_f + q^b_l b' + q^m_l m', \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{if firm repays both } b \text{ and } m \\ \\ p(\psi)y(z) - g(k', \hat{k}) - b_R - m + \hat{a}_f - q^a a'_f + q^b_l b' + q^m_l m', \\ \qquad\qquad\qquad\qquad\qquad\qquad \text{if firm restructures } b \text{ and repays } m \end{cases} \tag{B.5}$$

where the subscript $l \in \{NR, R\}$ denotes whether the firm chooses a NR-contract or an R-contract for the next period, which has implications for the prices of debt, and b_R is the restructured amount of bank debt, $b_R = \chi\pi$, as shown in Crouzet (2018), and $\pi = p(\psi)z\hat{k}^{\alpha} - F_o\hat{k} + p^-(1-\delta)\hat{k} + \hat{a}_f$. The net liquid asset position of the firm (x) is now given by:

$$x \equiv \begin{cases} p(\psi)y(z) - F_o\hat{k} - b - m + \hat{a}_f, & \text{if firm repays both } b \text{ and } m \\ p(\psi)y(z) - F_o\hat{k} - b_R - m + \hat{a}_f, & \text{if firm restructures } b \text{ and repays } m \end{cases}$$

so the firm's dividend (B.5) can be rewritten as: $d = x - g(k', \hat{k}) + q^b b' + q^m m' - q^a a'_f$.

The intermediate goods firms' problem can be written recursively, following the sequence of events outlined in the timeline (Figure 5 of main text). Let $V^1(x, \hat{k}, z; \mathbf{s})$ denote the value function of the firm at the dividend issuance stage (after the exit shock), $V^0(x, \hat{k}, z; \mathbf{s})$ denote the value function of at the debt settlement stage, and $\hat{V}^0(x, \hat{k}, z; \mathbf{s})$ denote the value function of at the asset reallocation stage.

Asset reallocation stage Upon observing the productivity and financial shocks, and given the amount and composition of its liabilities (b,m), a firm can reoptimize its asset portfolio by maximizing:

$$\hat{V}^0(k,a_f,z;\mathbf{s}) = \max_{\hat{k},\hat{a}_f} V^0(x,\hat{k},z;\mathbf{s}) \tag{B.6}$$

subject to the intra-period resource constraint, and capital adjustment cost, both of which are defined in the main text. Again, the constraint here implies that the firm cannot issue additional debt (of either type) during the asset allocation stage. If the firm chooses not to reallocate its assets, it proceeds to production and subsequently debt settlement with value function $V^0(x,\hat{k},s;\mathbf{s})$, with $\hat{k}=k$, and $\hat{a}_f = a_f$ in the net liquid asset position x.

Debt settlement stage This is where the extended model differs mostly from the baseline model. If the firm had chosen an R-contract in the previous period, with $\frac{m}{1-\chi} \leq \frac{b}{\chi}$, it has three options at this period's debt settlement stage: liquidation, restructuring, and full payment of its liabilities, depending on the price of the good, and hence the net worth of the firm; in other words,

$$V^0(x,\hat{k},z;\mathbf{s}) = \begin{cases} V_P^0(x,\hat{k},z;\mathbf{s}) & \text{if} \quad \psi \geq \overline{\psi}_R \\ V_R^0(x_R,\hat{k},z;\mathbf{s}) & \text{if} \quad \underline{\psi}_R \leq \psi < \overline{\psi}_R \\ 0 & \text{if} \quad \psi \leq \underline{\psi}_R. \end{cases} \tag{B.7}$$

where V_P^0 and V_R^0 denote, respectively, the value function of a firm that repays and restructures its liabilities today. The thresholds for repayment and restructuring are defined in (B.4). By assumption, firm that liquidates exits with its resources being passed to its creditors, so its continuation value in liquidation is 0. Moreover, the firm knows that with probability $1-\eta$ that it is not going to survive until the next period and with probability η it survives and has value V^1 (defined below). Thus, today's value of the firm—depending on if the firm repays or restructures its liabilities—is either:

$$V_P^0(z,\hat{k},x;\mathbf{s}) = (1-\eta)n + \eta V^1(z,\hat{k},x;\mathbf{s}), \tag{B.8}$$

or

$$V_R^0(z,\hat{k},x_R;\mathbf{s}) = (1-\eta)n_R + \eta V^1(z,\hat{k},x_R;\mathbf{s}), \tag{B.9}$$

where $n_{(R)}$ is the realized net worth, $n_{(R)} = \pi - b_{(R)} - m$.

If the firm had chosen an NR-contract, it would only have two options at debt settlement: liquidation or full repayment, as both partial and full defaults involve the seizure of assets by the creditors. Therefore,

$$V^0(z,\hat{k},x;\mathbf{s}) = \begin{cases} V_P^0(z,\hat{k},x;\mathbf{s}) & \text{if} \quad \psi \geq \overline{\psi}_{NR} \\ 0 & \text{if} \quad \psi \leq \overline{\psi}_{NR}, \end{cases} \tag{B.10}$$

and $V_P^0(z,\hat{k},x;\mathbf{s})$ is also defined by (B.8). The thresholds for partial and full default are defined in (B.3).

Dividend issuance stage Firms that do not default in period t and survive the exit shock can choose between a NR-contract and an R-contract – depending on the relative amounts of b' and m' chosen – with value functions $V_{i,NR}^1$ and $V_{i,R}^1$ respectively:

$$V^1(z,\hat{k},x;\mathbf{s}) = \max\left\{V_{NR}^1(z,\hat{k},x;\mathbf{s}), V_R^1(z,\hat{k},x;\mathbf{s})\right\}. \tag{B.11}$$

The optimization problem for the firm that chooses a NR-contract ($\frac{b'}{\chi} < \frac{m'}{1-\chi}$) takes the following form:

$$\begin{aligned} V_{NR}^1(z,\hat{k},x;\mathbf{s}) &= \max_{k',b',m',a_f'} \left\{ d_{NR} + \mathrm{E}\left[\lambda(\mathbf{s},\mathbf{s}')\hat{V}^0(k',a_f',\psi',z';\mathbf{s}')\right]\right\} \\ &= \max_{k',b',m',a_f'} \left\{ d_{NR} + \mathrm{E}\left[\lambda(\mathbf{s},\mathbf{s}')\max_{\hat{k}',\hat{a}_f'} V^0(z',\hat{k}',x';\mathbf{s}')\right]\right\}, \end{aligned} \tag{B.12}$$

subject to (B.5), (B.7), (B.8), the non-negative dividend constraint, the capital adjustment costs, the NR-contract debt prices described in the main text, and $\mathbf{s}' = \Gamma(\mathbf{s})$. For a firm that chooses an R-contract ($\frac{b'}{\chi} \geq \frac{m'}{1-\chi}$), the Bellman equation becomes:

$$\begin{aligned} V_R^1(\hat{k},x,z;\mathbf{s}) &= \max_{k',b',m',a_f'} \left\{ d_R + \mathrm{E}\left[\lambda(\mathbf{s},\mathbf{s}')\hat{V}^0(k',a_f',\psi',z';\mathbf{s}')\right]\right\} \\ &= \max_{k',b',m',a_f'} \left\{ d_R + \mathrm{E}\left[\lambda(\mathbf{s},\mathbf{s}')\max_{\hat{k}',\hat{a}_f'} V^0(\hat{k}',x',z';\mathbf{s}')\right]\right\} \end{aligned} \tag{B.13}$$

subject to (B.5), (B.10), (B.8), the non-negative dividend constraint, the capital adjustment costs, the R-contract debt prices described in the main text, and $\mathbf{s}' = \Gamma(\mathbf{s})$.