# Over-reaction in Macroeconomic Expectations

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**Abstract**

We study the rationality of individual and consensus professional forecasts of macroeconomic and financial variables using the methodology of Coibion and Gorodnichenko (2015), which examines predictability of forecast errors from forecast revisions. We report two key findings: forecasters typically over-react to their individual news, while consensus forecasts under-react to average forecaster news. To reconcile these findings, we combine the diagnostic expectations model of belief formation from Bordalo, Gennaioli, and Shleifer (2018) with Woodford's (2003) noisy information model of belief dispersion. The forward looking nature of diagnostic expectations yields additional implications, which we also test and confirm. A structural estimation exercise indicates that our model captures important variation in the data, yielding a value for the belief distortion parameter similar to estimates obtained in other settings.

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## I. Introduction

According to the Rational Expectations Hypothesis, market participants form their beliefs about the future, and make decisions, on the basis of statistically optimal forecasts. A growing body of work tests this hypothesis using survey data on the anticipations of households and professional forecasters. The evidence points to systematic departures from statistical optimality, which take the form of predictable forecast errors. Such departures have been documented in the cases of forecasting inflation and other macro variables (Coibion and Gorodnichenko 2012, 2015, henceforth CG, Fuhrer 2017), the aggregate stock market (Bacchetta, Mertens, and Wincoop 2009, Amromin and Sharpe 2013, Greenwood and Shleifer 2014, Adam, Marcet, and Buetel 2017), the cross section of stock returns (La Porta 1996, Bordalo, Gennaioli, La Porta, and Shleifer 2017, henceforth BGLS), credit spreads (Greenwood and Hanson 2013, Bordalo, Gennaioli, and Shleifer 2018), and corporate earnings (DeBondt and Thaler 1990, Ben-David, Graham, and Harvey 2013, Gennaioli, Ma, and Shleifer 2016, Bouchaud, Kruger, Landier, and Thesmar 2017). Departures from optimal forecasts also obtain in controlled experiments (Hommes et al. 2004, Beshears et al. 2013, Frydman and Nave 2016, Landier, Ma, and Thesmar 2017).

Various relaxations of the Rational Expectations Hypothesis have been proposed to account for the data. In macroeconomics, the main approach builds on rational inattention and information rigidities (Sims 2003, Woodford 2003, Carroll 2003, Mankiw and Reis 2002, Gabaix 2014). This view maintains the rationality of individual inferences, but relaxes the assumption of common information or full information processing. This is often justified by arguing that acquiring or processing information entails significant material and cognitive costs. To economize on these costs, agents revise their expectations sporadically, or on the basis of selective news. As a consequence, expectations and decisions under-react to news relative to the case of unlimited information capacity. In a novel empirical test of these theories, CG (2015) study predictability of errors in consensus macroeconomic forecasts of inflation and other variables, and find evidence consistent with under-reaction.

In finance, in contrast, although there is some evidence of momentum and under-reaction (Cutler, Poterba, and Summers 1990, Jegadeesh and Titman 1993), the dominant puzzle is over-reaction to news. This puzzle has been motivated by the evidence that stock prices move too much relative to the movements in fundamentals both in the aggregate (Shiller 1981) and in the cross section (De Bondt and Thaler 1985). The leading psychological mechanism for over-reaction is Tversky and Kahneman's (1974) finding that, in reacting to news, people tend to overweight "representative" events (Barberis, Shleifer and Vishny 1998, Gennaioli and Shleifer 2010). For instance, exceptional past performance of a firm may cause overweighting of the probability that this firm is "the next google" because googles are representative of the group of well performing firms, even though they are objectively rare. This approach is not inconsistent with limited information processing, but stresses that people infer too much from the information they attend to, however limited, so that beliefs and decisions move too much with news (Augenblick and Rabin 2017, Augenblick and Lazarus 2017). BGLS (2017) look at the cross section of stock returns and analyst expectations of earnings growth and find support for over-reaction driven by representativeness.

This state of research motivates two questions. First, which departure from rational expectations is predominant, under- or over-reaction to news? Second, which mechanisms create these departures? Put differently, can one account for the main features in the data using a parsimonious model capturing precise cognitive mechanisms for under- and over-reaction?

This paper addresses these questions by studying the predictions of professional forecasters of 16 macroeconomic variables, which include and expand those considered by CG (2015). We use both the Survey of Professional Forecasters (SPF) and the Blue Chip Survey, which gives us 20 expectations time series in total (four variables appear in both surveys), including forecasts of real economic activity, consumption, investment, unemployment, housing starts, government expenditures, as well as multiple interest rates. We examine both consensus and individual level forecasts. SPF data are publicly available; Blue Chip data were purchased and hand-coded for the earlier part of the sample.

Section 3 describes the patterns of over- and under-reaction in different series. We follow CG's methodology of measuring a forecaster's reaction to news by their forecast revision, and of using this forecast revision to predict the forecast error, computed as the difference between the realization and the forecast. In this setting, under-reaction to news implies a positive correlation between forecast errors and forecast revisions, while over-reaction to news implies the opposite. Unlike CG, we examine not only consensus forecasts, defined as the average forecast across all analysts, but also individual ones. The consequences of aggregating forecasts turn out to be crucial for understanding their properties.

For the case of consensus forecasts, we confirm the CG findings of under-reaction: the average forecast revision positively predicts the average future forecast error for most series. At the individual level, however, the opposite pattern emerges: for most series, the forecast revision of the average forecaster negatively predicts the same forecaster's future error. In stark contrast to the consensus results, at the level of the individual forecaster over-reaction is the norm, under-reaction the exception. These results are robust to several potential sources of predictability, including forecaster heterogeneity, small sample bias, measurement error, nonstandard loss functions, and non-normality of shocks.

In Section 4 we propose a model that reconciles these seemingly contradictory findings. In our setup, agents must predict the future value of a state that follows an AR(1) process. Each agent observes a different noisy signal of the current value of this state. Forecaster-specific noise can capture either inattention or the fact that different forecasters have access to different data. As in Woodford (2003), these noisy signals are optimally evaluated using the Kalman filter. We allow for over-reaction by assuming that, in processing the signals, agents are swayed by the representativeness heuristic.

To formalize this heuristic we use the Gennaioli and Shleifer (2010) model, originally proposed to describe lab experiments on probabilistic judgments but later applied to social stereotypes (Bordalo, Coffman, Gennaioli, and Shleifer 2016), forecasts of credit spreads (BGS 2018), and forecasts of firm performance (BGLS 2017). In this approach, the representativeness of a future state is measured by the proportional increase in its probability in light of recent news. Agents exaggerate the probability of more representative states – states that have become *relatively* more likely – and underestimate the probability of others. Representativeness causes expectations to follow a modified Kalman filter that overweighs recent news. As in earlier work, we call expectations distorted by representativeness "diagnostic."

In this model, under-reaction in the consensus can be reconciled with over-reaction at the individual level, *but only* when each forecaster over-reacts to the news he receives. When each forecaster over-reacts to his own information, the econometrician detects a negative correlation between his forecast error and his earlier forecast revision. At the consensus level, however, the econometrician may still detect a positive correlation between the forecast error and the consensus revision provided the distortion caused by representativeness is not too strong. The reason is that, while over-reacting to their own signal, individual forecasters do not react to the signals observed by others. Because all signals are informative and on average correct about the state, the average forecast under-reacts to the average information. As a consequence, judging whether individuals under- or over-react to news on the basis of consensus forecasts is misleading. Even if all forecasters over-react, as they do under diagnostic expectations, consensus forecasts may point to under-reaction simply because different analysts over-react to different news.

In Section 5 we assess whether individual forecasts are consistent with a key prediction of diagnostic expectations, the "kernel of truth" property, which is the idea that expectations exaggerate true patterns in the data. This implies that belief updating should depend on the persistence of the series, distinguishing our model from mechanical models of extrapolation such as adaptive expectations.

In Section 5.1 we present cross-sectional tests. We show first that individual forecast revisions at different horizons are more positively correlated with each other for the more persistent variables. This finding is consistent with diagnostic expectations, but not with adaptive expectations, where the same updating rule is used for all series. We then show that the individual-level CG coefficients display less over-reaction for the more persistent series. In line with diagnostic expectations, higher persistence causes rational forecast revisions to be more volatile, reducing the scope for over-reaction.

In Section 5.2 we develop a time-series test of the kernel of truth. We model individual series as AR(2) processes to account for long term reversals of actuals, consistent with Fuster, Laibson, and Mendel (2010). We find that 12 out of 16 variables exhibit hump-shaped dynamics. In this setting, the kernel of truth property implies that beliefs should exaggerate not only short term response but also long term reversals. We find that this prediction is borne out in the data. The evidence is broadly consistent with the kernel of truth property of beliefs that is central to the diagnostic expectation mechanism.

In Section 6 we estimate the structural parameters of our baseline model using the simulated method of moments. We find the diagnostic parameter $\theta$ is significantly positive for 17 out of 20 series, with an average value of 0.6 that falls in the ballpark of estimates we obtained in other contexts using different methods (BGS 2018, BGLS 2017). We estimate a small but significantly negative $\theta$ for one series, unemployment. These results suggest that over-reaction is sizable: the predictable component of the forecast error is comparable to the size of the rational response to news.

This paper documents the prevalence of over-reaction to news in individual macroeconomic forecasts and reconciles this finding with under-reaction in the consensus using a model of diagnostic expectations. There have been other approaches to similar phenomena. One is adaptive expectations; we show that the diagnostic expectations model has better psychological foundations and fits the data better. Another approach is Natural Expectations (Fuster, Laibson, and Mendel 2010), which argues that forecasters form beliefs assuming that growth follows a simple AR(1) model. Forecast errors arise because agents neglect longer lags. The authors show that many macroeconomic variables are described by hump-shaped dynamics (which we confirm), so natural expectations systematically overreact to short term growth. Diagnostic expectations share some predictions with natural expectations, but also make distinctive predictions, which we show more closely describe the data.[2]

Predictable forecast errors may reflect model mis-specification, and not over-reaction to news. Even macro-econometricians find it difficult to find the best specification for many series. The evidence in support of the kernel of truth however suggests that forecasters pay attention to key features of reality such as persistence and reversals, and exaggerate them in their forecasts. More broadly, representativeness and mis-specification may be synergistic: in a complex world in which forecasters are considering different models, data representative of a certain model may induce the forecaster to attach excessive weight to it. In this sense, the difficulties of learning may help explain persistence of representativeness-induced errors.

Diagnostic expectations are also related to overconfidence, in the sense of overestimating the precision of private information, which implies an exaggerated reaction to private signals (Daniel, Hirshleifer, and Subrahmanyam 1998, Moore and Healy 2008). Overconfidence has been used to explain excess volatility in prices of both asset and goods (Barber and Odean 2001, Benigno and Kourantasias 2018). In independent work, Broer and Kohlhas (2018) explore the role of overconfidence in driving

[2] A large literature considers how incentives may distort professional forecasters' stated expectations. Ottaviani and Sorensen (2006) point out that if forecasters compete in an accuracy contest with particular rules (winner-take-all), they overweigh private information. In contrast, Fuhrer (2017) argues that in the SPF data, individual forecast revisions can be negatively predicted from past deviations relative to consensus. Kohlhas and Walther (2018) also offer a model of asymmetric loss functions. We discuss these issues in Sections 3.2 and 5.

individual over-reaction in forecasts for GDP and inflation. In Sections 4 and 6 we compare overconfidence and our model. At the same time, we stress that diagnostic expectations describe beliefs and over-reaction in a wide range of settings, both in the lab and in the field, including those where overconfidence can be ruled out (such as when information is common and public). Developing portable models that are applicable in very different domains is a key step in identifying robust departures from rationality.

## 2. The Data

*Data on Forecasts.* We collect forecast data from two sources: Survey of Professional Forecasters (SPF) and Blue Chip Financial Forecasts (Blue Chip).[3] SPF is a survey of professional forecasters currently run by the Federal Reserve Bank of Philadelphia. At a given point in time, around 40 forecasters contribute to the SPF anonymously. SPF is conducted on a quarterly basis, around the end of the second month in the quarter. It provides both consensus forecast data and forecaster-level data (identified by forecaster ID). Forecasters report forecasts for outcomes in the current and next four quarters, typically about the level of the variable in each quarter.

Blue Chip is a survey of panelists from around forty major financial institutions. The names of institutions and forecasters are disclosed. The survey is conducted around the beginning of each month. To match with the SPF timing, we use Blue Chip forecasts from the end-of-quarter month survey (i.e. March, June, September, and December). Blue Chip has consensus forecasts available electronically, and we digitize individual-level forecasts from PDF publications. Panelists forecast outcomes in the current and next four to five quarters. For variables such as GDP, they report (annualized) quarterly growth rates. For variables such as interest rates, they report the quarterly average level. For both SPF and Blue Chip, the median (mean) duration of a panelist contributing forecasts is about 16 (23) quarters.

Given the timing of the SPF and Blue Chip forecasts we use, by the time the forecasts are made in quarter $t$ (i.e. around the end of the second month in quarter $t$), forecasters know the actual values of

[3] Blue Chip provides two sets of forecast data: Blue Chip Economic Indicators (BCEI) and Blue Chip Financial Forecasts (BCFF). We do not use BCEI since historical forecaster-level data are only available for BCFF.

variables with quarterly releases (e.g. GDP) up to quarter $t-1$, and the actual values of variables with monthly releases (e.g. unemployment rate) up to the previous month.

Table 1 presents the list of variables we study, as well as the time range for which forecast data are available from SPF and/or Blue Chip. These variables cover both macroeconomic outcomes, such as GDP, price indices, consumption, investment, unemployment, government consumption, and financial variables, primarily yields on government bonds and corporate bonds. SPF covers most of the macro variables and selected interest rates (three month Treasuries, ten year Treasuries, and AAA corporate bonds). Blue Chip includes real GDP and a larger set of interest rates (Fed Funds, three month, five year, and ten year Treasuries, AAA as well as BAA corporate bonds). Relative to CG (2015), we add two SPF variables (nominal GDP and the 10Y Treasury rate) as well as the Blue Chip forecasts.[4]

**Table 1. List of Variables**

This table lists our outcome variables, the forecast source, and the period for which forecasts are available.

| Variable | SPF | Blue Chip | Abbreviation |
|---|---|---|---|
| Nominal GDP | 1968Q4--2014Q4 | N/A | NGDP |
| Real GDP | 1968Q4--2014Q4 | 1999Q1--2014Q4 | RGDP |
| GDP Price Deflator | 1968Q4--2014Q4 | N/A | PGDP |
| Real Consumption | 1981Q3--2014Q4 | N/A | RCONSUM |
| Real Non-Residential Investment | 1981Q3--2014Q4 | N/A | RNRESIN |
| Real Residential Investment | 1981Q3--2014Q4 | N/A | RRESIN |
| Federal Government Consumption | 1981Q3--2014Q4 | N/A | RGF |
| State & Local Government Consumption | 1981Q3--2014Q4 | N/A | RGSL |
| Housing Starts | 1968Q4--2014Q4 | N/A | HOUSING |
| Unemployment Rate | 1968Q4--2014Q4 | N/A | UNEMP |
| Fed Funds Rate | N/A | 1983Q1--2014Q4 | FF |
| 3M Treasury Rate | 1981Q3--2014Q4 | 1983Q1--2014Q4 | TB3M |
| 5Y Treasury Rate | N/A | 1988Q1--2014Q4 | TN5Y |
| 10Y Treasury Rate | 1992Q1--2014Q4 | 1993Q1--2014Q4 | TN10Y |
| AAA Bond Rate | 1981Q3--2014Q4 | 1984Q1--2014Q4 | AAA |
| BAA Bond Rate | N/A | 2000Q1--2014Q4 | BAA |

We use an annual forecast horizon. For GDP and inflation we look at the annual growth rate from quarter $t-1$ to quarter $t+3$. In SPF, the forecasts for these variables are in levels (e.g. level of GDP), so we transform them into implied growth rates. Actual GDP of quarter $t-1$ is known at the time of the

[4] Relative to CG, we do not use SPF forecasts for CPI inflation and industrial production index, as real time data are missing for these two variables for a period of time.

forecast, consistent with the forecasters' information sets. Blue Chip reports forecasts of quarterly growth rates, so we add up these forecasts in quarters $t$ to $t+3$. For variables such as the unemployment rate and interest rates, we look at the level in quarter $t+3$. Both SPF and Blue Chip have direct forecasts of the quarterly average level in quarter $t+3$. Appendix B provides a description of variable construction.

Consensus forecasts are computed as means from individual-level forecasts available at a point in time. We calculate forecasts, forecast errors, and forecast revisions at the individual level, and then average them across forecasters to compute the consensus.[5]

*Data on Actual Outcomes.* The values of macroeconomic variables are released quarterly but are often subsequently revised. To match as closely as possible the forecasters' information set, we focus on initial releases from Philadelphia Fed's Real-Time Data Set for Macroeconomists.[6] For example, for actual GDP growth from quarter $t-1$ to quarter $t+3$, we use the initial release of $\text{GDP}_{t+3}$ (available in quarter $t+4$) divided by the initial release of $\text{GDP}_{t-1}$ (available in quarter $t$, prior to when the forecasts are made). For financial variables, the actual outcomes are available daily and are permanent (not revised). We use historical data from the Federal Reserve Bank of St. Louis. In addition, we always study the properties of the actuals (mean, standard deviation, persistence, etc) using the same time periods as the corresponding forecasts. The same variable from SPF and Blue Chip may have slightly different actuals when the two datasets cover different time periods.

*Summary Statistics.* Table 2 below presents the summary statistics of the variables, including the mean and standard deviation for the actuals being forecasted, as well as the consensus forecasts, forecast errors, and forecast revisions at a horizon of quarter $t$+3. The table also shows statistics for the quarterly share of forecasters with no meaningful revisions,[7] and the quarterly share of forecasters with positive revisions.

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[5] There could be small differences in the set of forecasters who issue a forecast in quarter $t$, and those who revise their forecast at $t$ (these need to be present at $t-1$ as well). This issue does not affect our results, which are robust to considering only forecasters who have both forecasts and forecast revisions.

[6] When forecasters make forecasts in quarter $t$, only initial releases of macro variables in quarter $t-1$ are available.

[7] We categorize a forecaster as making no revision if he provides non-missing forecasts in both quarters $t-1$ and $t$, and the forecasts change by less than 0.01 percentage points. For variables in rates, the data is often rounded to the first decimal point, and this rounding may lead to a higher incidence of no-revision.

**Table 2. Summary Statistics**

Mean and standard deviation of main variables. All values are in percentages. Panel A shows the statistics for actuals, consensus forecasts, consensus errors and consensus revisions. Actuals are realized outcomes corresponding to the forecasts, and errors are actuals minus forecasts. Actuals are measured using the same time periods as when the corresponding forecasts are available. Revisions are forecasts of the outcome made in quarter *t* minus forecasts of the same outcome made in quarter *t-1*. Panel B shows additional individual level statistics. The forecast dispersion column shows the mean of quarterly standard deviations of individual level forecasts. The revision dispersion column shows the mean of quarterly standard deviations of individual level forecast revisions. Non-revisions are instances where forecasts are available in both quarter *t* and quarter *t-1* and the change in the value is less than 0.01 percentage points. The non-revision and up-revision columns show the mean of quarterly non-revision shares and up-revision shares. The final column of Panel B shows the fraction of quarters where less than 80% of the forecasters revise in the same direction.

Panel A. Consensus Statistics

| | | Actuals | | Forecasts | | Errors | | Revisions | |
|---|---|---|---|---|---|---|---|---|---|
| Variable | Format | mean | sd | mean | sd | mean | sd | mean | sd |
| Nominal GDP (SPF) | Growth rate from end of quarter *t*-1 to end of quarter t+3 | 6.19 | 2.90 | 6.43 | 2.30 | -0.24 | 1.75 | -0.14 | 0.71 |
| Real GDP (SPF) | | 2.56 | 2.31 | 2.73 | 1.38 | -0.17 | 1.74 | -0.18 | 0.64 |
| Real GDP (BC) | | 2.66 | 1.55 | 2.62 | 0.86 | 0.03 | 1.30 | -0.12 | 0.48 |
| GDP Price Index (SPF) | | 3.56 | 2.49 | 3.63 | 2.03 | -0.07 | 1.14 | 0.02 | 0.48 |
| Real Consumption (SPF) | | 2.85 | 1.46 | 2.53 | 0.76 | 0.32 | 1.15 | -0.05 | 0.51 |
| Real Non-Residential Investment (SPF) | | 4.90 | 7.35 | 4.41 | 3.68 | 0.49 | 5.86 | -0.26 | 1.78 |
| Real Residential Investment (SPF) | | 2.77 | 11.68 | 2.67 | 6.19 | 0.11 | 8.71 | -0.64 | 2.48 |
| Real Federal Government Consumption (SPF) | | 1.36 | 4.59 | 1.34 | 2.61 | 0.02 | 3.22 | 0.13 | 1.24 |
| Real State&Local Govt Consumption (SPF) | | 1.62 | 1.68 | 1.62 | 1.09 | 0.00 | 1.12 | 0.00 | 0.59 |
| Housing Start (SPF) | | 1.67 | 22.16 | 4.75 | 15.33 | -3.08 | 18.81 | -2.41 | 5.97 |
| Unemployment (SPF) | Average level in quarter *t*+3 | 6.38 | 1.55 | 6.38 | 1.43 | 0.00 | 0.76 | 0.06 | 0.33 |
| Fed Funds Rate (BC) | | 4.10 | 2.99 | 4.53 | 2.94 | -0.42 | 1.04 | -0.18 | 0.54 |
| 3M Treasury Rate (SPF) | | 3.98 | 2.86 | 4.54 | 2.93 | -0.56 | 1.15 | -0.21 | 0.52 |
| 3M Treasury Rate (BC) | | 3.76 | 2.73 | 4.28 | 2.72 | -0.52 | 1.02 | -0.18 | 0.51 |
| 5Y Treasury Rate (BC) | | 4.45 | 2.24 | 4.86 | 2.05 | -0.41 | 0.89 | -0.15 | 0.45 |
| 10Y Treasury Rate (SPF) | | 4.49 | 1.56 | 4.99 | 1.40 | -0.50 | 0.76 | -0.12 | 0.37 |
| 10Y Treasury Rate (BC) | | 4.42 | 1.56 | 4.86 | 1.38 | -0.44 | 0.75 | -0.13 | 0.39 |
| AAA Corporate Bond Rate (SPF) | | 7.26 | 2.4 | 7.74 | 2.52 | -0.47 | 0.85 | -0.11 | 0.39 |
| AAA Corporate Bond Rate (BC) | | 6.84 | 1.94 | 7.26 | 2.01 | -0.42 | 0.7 | -0.12 | 0.37 |
| BAA Corporate Bond Rate (BC) | | 6.30 | 1.08 | 6.75 | 0.95 | -0.45 | 0.68 | -0.14 | 0.31 |

Panel B. Additional Individual Level Statistics

| | | Forecasts | Revisions | | | |
|---|---|---|---|---|---|---|
| Variable | Format | Dispersion | Dispersion | non-rev share | up-rev share | Pr(<80% revise same direction) |
| Nominal GDP (SPF) | Growth rate from end of quarter *t*-1 to end of quarter t+3 | 0.59 | 1.13 | 0.02 | 0.45 | 0.79 |
| Real GDP (SPF) | | 0.63 | 0.94 | 0.02 | 0.43 | 0.74 |
| Real GDP (BC) | | 0.17 | 0.40 | 0.05 | 0.43 | 0.66 |

| | | | | | | |
|---|---|---|---|---|---|---|
| GDP Price Index (SPF) | | 0.52 | 0.75 | 0.05 | 0.49 | 0.79 |
| Real Consumption (SPF) | | 0.68 | 0.76 | 0.03 | 0.48 | 0.76 |
| Real Non-Residential Investment (SPF) | | 1.03 | 2.47 | 0.02 | 0.49 | 0.71 |
| Real Residential Investment (SPF) | | 2.09 | 4.24 | 0.03 | 0.45 | 0.83 |
| Real Federal Government Consumption (SPF) | | 1.38 | 2.25 | 0.06 | 0.52 | 0.87 |
| Real State&Local Govt Consumption (SPF) | | 1.45 | 1.28 | 0.10 | 0.48 | 0.93 |
| Housing Start (SPF) | | 5.46 | 8.61 | 0.00 | 0.39 | 0.68 |
| Unemployment (SPF) | Average level in quarter *t*+3 | 0.13 | 0.30 | 0.18 | 0.42 | 0.77 |
| Fed Funds Rate (BC) | | 0.33 | 0.48 | 0.22 | 0.30 | 0.68 |
| 3M Treasury Rate (SPF) | | 0.29 | 0.48 | 0.15 | 0.34 | 0.68 |
| 3M Treasury Rate (BC) | | 0.29 | 0.46 | 0.19 | 0.32 | 0.63 |
| 5Y Treasury Rate (BC) | | 0.15 | 0.42 | 0.12 | 0.35 | 0.61 |
| 10Y Treasury Rate (SPF) | | 0.09 | 0.38 | 0.10 | 0.35 | 0.65 |
| 10Y Treasury Rate (BC) | | 0.08 | 0.35 | 0.13 | 0.33 | 0.57 |
| AAA Corporate Bond Rate (SPF) | | 0.25 | 0.51 | 0.09 | 0.38 | 0.73 |
| AAA Corporate Bond Rate (BC) | | 0.22 | 0.47 | 0.12 | 0.34 | 0.71 |
| BAA Corporate Bond Rate (BC) | | 0.12 | 0.41 | 0.13 | 0.32 | 0.81 |

Several patterns emerge from Table 2. First, the average forecast error is about zero. Macro analysts do not seem to have asymmetric loss functions that systematically bias their forecasts in a given direction. Second, there is significant dispersion of forecasts and revisions at each point in time, as shown in Table 2 Panel B. Third, analysts frequently revise their forecasts (share of analysts with no revision is small), but they do so in different directions. As shown by the final column of Panel B, it is uncommon to have quarters where more than 80% forecasters revise in the same direction. This suggests that different forecasters observe or attend to different news, either because they are exposed to different information or because they use different models, or both. Berger, Erhmann, and Fratzscher (2011) show that the geographical location of forecasters influences their predictions of monetary policy decisions. Different forecasters may have personal contacts with the industry, policymakers, etc., which offers one explanation for the disagreement we see in the data.

### 3. Over-reaction vs. Under-reaction: Basic Tests

Many tests of the rational expectations hypothesis assess whether forecast errors can be predicted using information available at the time the forecast is made. Understanding whether departures from rational expectations are due to over- or under-reaction to information is more challenging, since the forecaster's full information set cannot be directly observed by the econometrician.

CG (2015) address this problem with forecast revisions. Denote by $x_{t+h|t}$ the $h$-periods ahead forecast made at time $t$ about the future value $x_{t+h}$ of a variable. Denote by $x_{t+h|t-1}$ the forecast of the same variable in the previous period. The $h$-periods ahead forecast revision at $t$ is given by $FR_{t,h} = \left(x_{t+h|t} - x_{t+h|t-1}\right)$, or the one period change in the forecast about $x_{t+h}$. This revision captures the reaction to whichever news the forecasters have observed. The extent to which forecasters under- or over-react to information can then be assessed by estimating the regression:

$$x_{t+h} - x_{t+h|t} = \beta_0 + \beta_1 FR_{t,h} + \epsilon_{t,t+h}. \tag{1}$$

Under the Rational Expectations Hypothesis, the forecast error should be unpredictable using any current information, including the forecast revision itself, so $\beta_1 = 0$. When instead the forecast under-reacts to information, we expect $\beta_1 > 0$. To see why, suppose that positive information is received, leading to a positive forecast revision $FR_{t,h} > 0$. If the forecast under-reacts, the upward revision is insufficient, predicting a positive forecast error $\mathbb{E}_t\left(x_{t+h} - x_{t+h|t}\right) > 0$. The converse holds if negative information is received: the downward revision is insufficient, predicting a negative error. Under-reaction implies that the forecast error should be positively correlated with the forecast revision.

By the same logic, when the forecast over-reacts to information we should expect $\beta_1 < 0$. Indeed, over-reaction means that after positive information $FR_{t,h} > 0$ the forecast is too optimistic, so the forecast error is negative $\mathbb{E}_t\left(x_{t+h} - x_{t+h|t}\right) < 0$. On the other hand, after negative information $FR_{t,h} < 0$ it is too pessimistic, so the error is positive $\mathbb{E}_t\left(x_{t+h} - x_{t+h|t}\right) > 0$. That is, over-reaction implies that the forecast error should be negatively correlated with the forecast revision.

To test for Rational Inattention, CG's baseline estimate of Equation (1) uses *consensus* SPF forecasts. The consensus forecast $x_{t+h|t}$ is defined as the average of individual forecasters' predictions $x_{t+h|t} = \frac{1}{I}\sum_i x^i_{t+h|t}$, where $I > 1$ is the number of forecasters. Similarly, $FR_{t,h}$ is the $h$-periods ahead "consensus information" or forecast revision. CG estimate (1) for the GDP price deflator (PGDP_SPF) at a horizon $h = 3$ and find $\beta_1 = 1.2$, which is robust to a number of controls. They also run Equation (1) for 13 SPF variables by pooling forecast horizons from $h = 0$ to $h = 3$, and find qualitatively similar results, with 8 out of 13 variables exhibiting significantly positive $\beta_1$'s and the average coefficient being close to 0.7 (see Figure 1 Panel B of CG (2015)). The general message is that consensus forecasts of macroeconomic variables display under-reaction.

We estimate Equation (1) for our 20 series for the same baseline horizon $h = 3$, using consensus forecasts. Standard errors are Newey-West with the automatic bandwidth selection following Newey and West (1994).[8] The results are reported in columns (1) through (3) of Table 3, and confirm the findings of CG. The estimated $\beta_1$ is positive for 14 out of 20 series, statistically significant for 8 of them at the 5% confidence level, and for a further two series at the 10% level (and our point estimate for inflation forecasts coincides with CG's). While results for the other SPF series are not directly comparable (since CG pool across forecast horizons), the estimates lie in a similar range. The one exception is RGF_SPF (federal government spending) for which the estimated $\beta_1$ is negative and significant at the 5% level. Results from the Blue Chip survey align well with SPF where they overlap, but do not exhibit significant consensus over-reaction for the remaining (exclusively financial variables) series.

We stress that the various forecast series are not independent. For instance, nominal and real GDP growth are highly correlated; the different interest rate series are also closely connected. Nonetheless, the general message holds: for macro variables and short rates, under-reaction is common in the consensus forecast regressions, while such patterns are largely absent in long-term rates.

As mentioned above, insufficient updating of consensus beliefs may be due to aggregation issues, rather than to under-reaction to information by individual forecasters. As we saw in Table 2, individual

[8] We also perform sensitivity analysis on the kernel bandwidth selection for Newey-West standard errors. In Appendix C Table C.1, we present standard errors using lags from zero to eight, which cover the reasonable range given the length of our time series. The results are largely similar.

forecasters often revise in different directions, perhaps because they look at different data or use different models. Over-reaction of individual forecasters may thus be attenuated by heterogeneity and aggregation.

**Table 3. Error-on-Revision Regression Results**

This table shows coefficients from the CG (forecast error on forecast revision) regression. Coefficients are displayed for both consensus time-series regressions, and forecaster-level pooled panel regressions, together with standard errors and *p*-values. Standard errors are Newey-West for consensus time-series regressions, and clustered by both forecaster and time for individual level regressions.

| | Consensus | | | Individual | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | No fixed effects | | | With fixed effects | | |
| | $\beta_1$ | s.e. | *p*-val | $\beta_1^p$ | s.e. | *p*-val | $\beta_1^p$ | s.e. | *p*-val |
| Variable | (1) | (2) | (3) | (4) | (5) | (6) | (7) | (8) | (9) |
| Nominal GDP (SPF) | 0.48 | 0.22 | 0.03 | -0.26 | 0.07 | 0.00 | -0.30 | 0.06 | 0.00 |
| Real GDP (SPF) | 0.45 | 0.25 | 0.07 | -0.23 | 0.08 | 0.00 | -0.21 | 0.06 | 0.00 |
| Real GDP (BC) | 0.59 | 0.34 | 0.09 | 0.12 | 0.19 | 0.26 | -0.02 | 0.17 | 0.93 |
| GDP Price Index Inflation (SPF) | 1.21 | 0.21 | 0.00 | -0.07 | 0.10 | 0.46 | -0.16 | 0.07 | 0.03 |
| Real Consumption (SPF) | 0.18 | 0.22 | 0.41 | -0.34 | 0.11 | 0.00 | -0.39 | 0.10 | 0.00 |
| Real Non-Residential Investment (SPF) | 0.93 | 0.38 | 0.02 | 0.01 | 0.13 | 0.93 | -0.03 | 0.12 | 0.82 |
| Real Residential Investment (SPF) | 1.26 | 0.38 | 0.00 | -0.02 | 0.10 | 0.82 | -0.12 | 0.08 | 0.14 |
| Real Federal Government Consumption (SPF) | -0.44 | 0.23 | 0.05 | -0.62 | 0.07 | 0.00 | -0.63 | 0.06 | 0.00 |
| Real State & Local Govt Consumption (SPF) | -0.16 | 0.20 | 0.42 | -0.71 | 0.14 | 0.00 | -0.73 | 0.13 | 0.00 |
| Housing Start (SPF) | 0.45 | 0.31 | 0.14 | -0.25 | 0.09 | 0.01 | -0.28 | 0.08 | 0.00 |
| Unemployment (SPF) | 0.82 | 0.21 | 0.00 | 0.33 | 0.11 | 0.00 | 0.26 | 0.11 | 0.02 |
| Fed Funds Rate (BC) | 0.61 | 0.23 | 0.01 | 0.15 | 0.09 | 0.11 | 0.12 | 0.09 | 0.19 |
| 3M Treasury Rate (SPF) | 0.71 | 0.26 | 0.01 | 0.24 | 0.09 | 0.01 | 0.19 | 0.09 | 0.04 |
| 3M Treasury Rate (BC) | 0.67 | 0.25 | 0.01 | 0.20 | 0.09 | 0.02 | 0.16 | 0.08 | 0.06 |
| 5Y Treasury Rate (BC) | 0.05 | 0.22 | 0.84 | -0.12 | 0.10 | 0.23 | -0.19 | 0.10 | 0.05 |
| 10Y Treasury Rate (SPF) | -0.01 | 0.28 | 0.97 | -0.18 | 0.10 | 0.06 | -0.23 | 0.09 | 0.01 |
| 10Y Treasury Rate (BC) | -0.06 | 0.25 | 0.81 | -0.17 | 0.12 | 0.14 | -0.25 | 0.11 | 0.02 |
| AAA Corporate Bond Rate (SPF) | -0.01 | 0.24 | 0.97 | -0.21 | 0.08 | 0.00 | -0.26 | 0.07 | 0.00 |
| AAA Corporate Bond Rate (BC) | 0.21 | 0.21 | 0.31 | -0.17 | 0.07 | 0.00 | -0.22 | 0.06 | 0.00 |
| BAA Corporate Bond Rate (BC) | -0.14 | 0.28 | 0.62 | -0.28 | 0.10 | 0.00 | -0.34 | 0.10 | 0.00 |

To assess whether individual forecasters over- or under-react to their own information, we continue to follow the CG methodology, but perform the analysis at the individual analyst level. Here $FR_{t,h}^i = \left(x_{t+h|t}^i - x_{t+h|t-1}^i\right)$ is the analyst-level revision, and the $h$-periods ahead individual forecast error is $x_{t+h} - x_{t+h|t}^i$. For each variable, we then pool all analysts and estimate the regression:

$$x_{t+h} - x_{t+h|t}^i = \beta_0^p + \beta_1^p FR_{t,h}^i + \epsilon_{t,t+h}^i. \qquad (2)$$

Superscript $p$ on the coefficients recognizes that we are pooling individual level data. The logic of the test, however, does not change: $\beta_1^p > 0$ indicates that the average analyst under-reacts to his own information, while $\beta_1^p < 0$ indicates that the average analyst over-reacts.[9]

Columns (4) through (6) of Table 3 report the results of estimating Equation (2). Surprisingly, the picture is essentially reversed from the consensus: at the individual level, the average analyst appears to over-react to information, as measured by a negative $\beta_1^p$ coefficient. The estimated $\beta_1^p$ is negative for 14 out of the 20 series (13 out of 16 variables), and significantly negative for 9 series at the 5% confidence level, and for one other series at the 10% level. Except for short rates (Fed Funds and 3-months T-bill rate), all financial variables display over-reaction, consistent with Shiller's evidence of excess volatility. But many macro variables also display over-reaction, including nominal GDP, real GDP (in SPF, not in Blue Chip), real consumption, real federal government expenditures, real state and local government expenditures. GDP price deflator inflation, real GDP in Blue Chip, and non-residential investment display neither over-nor under-reaction ($\beta_1^p$ close to zero). Only the 3-months T-bill rate and unemployment rate display individual level under-reaction with positive and statistically significant $\beta_1^p$.

In columns (7) to (9), we also analyze regressions with forecaster fixed effects to account for possible time-invariant differences among analysts. Some analysts may be consistently overly-optimistic or overly-pessimistic, perhaps due to differences in their prior beliefs, contributing to positive correlations between forecast errors and revisions. Specifically, the overly optimistic analysts systematically receive bad news, leading to negative revisions and negative forecast errors, while the overly pessimistic analysts systematically receive good news, leading to positive revisions and positive forecast errors. In the data, the results with and without forecaster fixed effects are similar. With forecaster fixed effects, the estimated $\beta_1^p$ is negative for 17 series, and significantly negative for 13 series at the 5% confidence level. The message of Table 3 is clear: at the level of the individual forecaster, over-reaction is the norm.

[9] The individual level coefficient $\beta_1^p$ can in principle be different from the consensus coefficient $\beta_1$: to the extent that some information is forecaster specific, and that individuals do not react to information they do not possess, errors $\epsilon_{t,t+h}^i$ may be correlated across individuals over time. In Section 4 we formalize this intuition.

In sum, a fascinating picture emerges from these tests. At the consensus level, expectations typically under-react. At the individual level, they typically over-react. We conclude this section with a number of robustness checks. In Section 4, we present a model capable of reconciling these patterns.

### 3.1 Robustness Checks

Predictability of forecast errors might arise from features of the data unrelated to individuals' under- or over-reaction to news. We next show that our results are robust to many such confounds.

*Small Samples.* Our individual level estimates can face small sample problems. Finite-sample biases exist in time series regressions (Kendall 1954, Stambaugh 1999) and panel regressions with fixed effects (Nickell, 1981). In the baseline individual-level tests in Table 3, our panel regressions do not have fixed effects, which alleviates the concern (Hjalmarsson 2008). Adding fixed effects does not change the results much, indicating that the bias, even if present, is not severe. Moreover, the finite sample biases are stronger when the predictor variables are persistent. The predictor variable in the CG regressions, namely forecast revision, has low persistence in the data (about zero for most variables at the individual level, and less than 0.5 at the consensus level). Finally, simulation analyses in Appendix D show that, for parameter values and time frames relevant to our data, the coefficients do not have notable biases.

*Measurement Error.* Forecasts measured with noise can mechanically lead to negative predictability of forecast errors in Equation (2): a positive shock increases the measured forecast revision and decreases the forecast error. In our case, since professional forecasters directly report their forecasts, it is hard to think of literal "measurement error." Moreover, motivated by the fact that some series display an AR(2) structure, in Section 5 we regress the forecast error at $t + h$ on revisions of forecasts for *previous* periods $t + h - 1$ and $t + h - 2$ (Equation 13). In line with the predictions of the model (Proposition 3), but not with measurement error, we find strong predictability in these regressions as well (Table 6). Finally, in Section 6 we estimate our model without using information from the CG coefficients; we obtain estimates that indicate significant individual level over-reaction and generate CG regression coefficients very similar to the data.

*Heterogeneity among Forecasters.* Forecaster heterogeneity either in updating (e.g., heterogeneous signal to noise ratios), or in beliefs about long term means, may affect the predictability of forecast errors. To assess this problem, we perform forecaster level regressions, focusing on forecasters with at least 10 observations. Table C2 in Appendix C compares the median coefficient from forecaster level regressions to the coefficients from pooled individual level regressions from Table 3. The coefficients are very similar, so the observed over-reaction describes the median forecaster. On average across series, we estimate a negative $\beta_1^p$ for two thirds of the forecasters. In some series, nearly every forecaster over-reacts while in other series the distribution of $\beta_1^p$s is more balanced. We return to forecaster heterogeneity in Section 6, when we estimate our model.

*Asymmetric Loss Functions.* Another concern with our findings is that forecast errors reflect not cognitive limitations but analysts' biased incentives. Of course, an analyst's objective is difficult to observe. Here we discuss the implications of several analyst loss functions proposed in the literature.

With an asymmetric loss function (Capistran and Timmerman 2009), the over-reaction pattern in Table 3 may be generated by a combination of: i) a higher cost of over- than under-predicting, and ii) suitably time varying volatility (Pesaran and Weale 2006). In this case, an asymmetric loss function would also generate an average forecast error in the form of pessimism. In the data, however, forecasts are not systematically upward or downward biased on average. The consensus forecast errors are small and insignificant (Table 2, panel A). This is also true for individual forecast errors: we fail to reject that the average error is different from zero for about 60% of forecasters for the macroeconomic variables.[10]

Another source of bias in reported expectations is that individuals may follow consensus forecasts (Morris and Shin 2002, Fuhrer 2017). Let $\tilde{x}_{t+h|t}^i = \alpha x_{t+h|t}^i + (1-\alpha)\tilde{x}_{t+h|t}$, where $x_{t+h|t}^i$ is the individual rational forecast and $\tilde{x}_{t+h|t}$ is the average contemporaneous forecast with this bias (which coincides with the consensus without this bias). Our benchmark model has $\alpha = 1$ but for $\alpha < 1$ forecasters put weight on others' signals at the expense of their own. In this model, in line with intuition, following

[10] Some individual forecasters have average errors that are significantly different from zero for some series, but these average out in the population for nearly all series. For interest rates, average forecast errors tend to be negative, but this reflects the secular decline in rates over the time period we examine.

consensus forecasts leads to individual level under-reaction, namely positive individual level CG coefficients, contrary to our findings.[11]

Reputational incentives may also induce forecast smoothing. In response to news at $t$, forecasters may wish to minimize forecast revisions by taking into account the previous forecast $x^i_{t+h|t-1}$ as well as the future path $x^i_{t+h|t+j}$. To assess the relevance of this mechanism, note that forecast smoothing should reduce the current revision for the current quarter ($h = 0$), creating under-reaction. This prediction is contradicted by the data: negative predictability prevails even at this horizon (Appendix C, Table C3).

More generally, the similarity of our results across datasets suggests that distorted incentives cannot be the whole story. The SPF panelists are anonymous, the Blue Chip ones are not. Thus, forecasts in Blue Chip should be more affected by the above reputational incentives or by additional ones (e.g., individual forecasters may wish to distinguish themselves from others in order to prevail in a winner-take-all context, as in Ottaviani and Sorensen (2006)). However, in our data, when Blue Chip and SPF forecasts are available for the same series, they display very similar average forecast errors and revisions (see Table 2), they have similar CG coefficients (see Table 3), and they lead to similar model estimates (see Section 6). Analyst incentives do not seem a compelling explanation for our findings.

*Fat tailed shocks*. In our data both fundamentals and forecast revisions have high kurtosis. To see whether fat tailed shocks may, by themselves, create a false impression of over-reaction, in Appendix D we consider a learning setting with fat tailed fundamental shocks. Without normality, we can no longer use the Kalman filter, but instead need to use the particle filter (Liu and Chen, 1998; Doucet, de Freitas, and Gordon, 2001). We find that when forecasts are produced using the particle filter under rational expectations, individual forecast errors are not predictable from forecast revisions, and thus cannot explain the evidence. Moreover, in Section 6 we estimate a modified particle filter that allows for overreaction to news, and find that fat

[11] Formally, denote $\widetilde{FE}^i_{t+h,t} = x_{t+h} - \tilde{x}^i_{t+h|t}$ the forecast error and $\widetilde{FR}^i_{t+h,t} = \tilde{x}^i_{t+h|t} - \tilde{x}^i_{t+h|t-1}$ the forecast revision. It follows that $\widetilde{FE}^i_{t+h,t} = \alpha FE^i_{t+h,t} + (1-\alpha)FE_{t+h|t}$ and similarly $\widetilde{FR}^i_{t+h,t} = \alpha FR^i_{t+h,t} + (1-\alpha)FR_{t+h|t}$. Then $cov(\widetilde{FE}^i_{t+h,t}, \widetilde{FR}^i_{t+h,t}) > 0$ follows from $cov(FE^i_{t+h,t}, FR^i_{t+h,t}) = 0$ and $cov(FE_{t+h|t}, FR_{t+h|t}) > 0$ under noisy rational expectations, together with $cov(FE^i_{t+h,t}, FR_{t+h|t}), cov(FE_{t+h|t}, FR^i_{t+h,t}) > 0$.

tailed shocks do not significantly affect our quantitative estimates. Because fat tails do not appear to affect our results, we maintain the more tractable assumption of normality in our theoretical analysis.[12]

## 4. Diagnostic Expectations

We present a model that reconciles under-reaction of consensus expectations with over-reaction of individual level expectations. At each time $t$, the target of forecasts is a hidden state $x_{t+h}$ whose current value $x_t$ is not directly observed. What is observed instead is a noisy signal $s_t^i$:

$$s_t^i = x_t + \epsilon_t^i, \tag{3}$$

where $\epsilon_t^i$ is noise, i.i.d. normally distributed across forecasters and over time, with mean zero and variance $\sigma_\epsilon^2$. The hidden state $x_t$ evolves according to an AR(1) process with persistence $\rho$:

$$x_t = \rho x_{t-1} + u_t, \tag{4}$$

where $u_t$ is a normal shock with mean zero and variance $\sigma_u^2$. This AR(1) setting, also considered by CG (2015), yields convenient closed form predictions. In Section 6 we examine the AR(2) case.

This setup accommodates several interpretations. In CG (2015), unobservability of $x_t$ stems from rational inattention (Sims 2003, Woodford 2003). Forecasters could in principle observe $x_t$ but doing so is too costly, so they observe a noisy proxy for it and optimally use that proxy in their forecasts.[13] This rational inattention interpretation is not entirely convincing, since the job of professional forecasters is precisely to be attentive to, and to predict, the variables in question.

A more compelling story is that forecasters observe the same data, say GDP or interest rates, but differ in their interpretations because they have different pieces of other information. Think of the current GDP estimate or interest rate level as a noisy proxy for an unobservable persistent state. Due to individual

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[12] Apart from fat tails, skewness of shocks may also lead to systematically biased forecasts under Bayesian updating (Orlik and Veldkamp 2015). As we saw in Table 2, in our data forecasts are not biased on average.

[13] As CG show, the same predictions are obtained if rational inattention is modelled à la Mankiw and Reis (2002), where agents observe the same information but only sporadically revise their predictions.

expertise or contacts in the industry, a forecaster has personal information on that hidden state. This implies that the current GDP estimate or interest rate level is transformed into a forecaster-specific signal $s_t^i$. Even so, a Bayesian forecaster optimally filters noise in his own signal. In this sense, under both the rational inattention and the dispersed information interpretations, forecasters rationally update on the basis of noisy signals. We refer to both mechanisms as "Noisy Rational Expectations".

A Bayesian, or rational, forecaster enters period $t$ carrying from the previous period beliefs about the current state $x_t$ summarized by a probability density $f\left(x_t|S_{t-1}^i\right)$, where $S_{t-1}^i$ denotes the full history of signals observed by this forecaster.[14] In period $t$, the forecaster observes a new signal $s_t^i$. In light of this evidence, he updates his estimate of the current state using Bayes' rule:

$$f\left(x_t|S_t^i\right) = \frac{f\left(s_t^i|x_t\right)f\left(x_t|S_{t-1}^i\right)}{\int f\left(s_t^i|x\right)f\left(x|S_{t-1}^i\right)dx}. \tag{5}$$

Equation (5) iteratively defines the forecaster's beliefs. Given normal shocks, $f\left(x_t|S_t^i\right)$ is described by the Kalman filter. A rational forecaster estimates the current state at $x_{t|t}^i = \int xf\left(x|S_t^i\right)dx$ and forecasts future values using the AR(1) structure, so $x_{t+h|t}^i = \rho^h x_{t|t}^i$.

We allow beliefs to be distorted by Kahneman and Tversky's representativeness heuristic, as in our model of Diagnostic Expectations. In line with BGLS (2017), who apply Diagnostic Expectations to a (diagnostic) Kalman Filter, we define the representativeness of a state $x$ at $t$ as the likelihood ratio:

$$R_t(x) = \frac{f\left(x|S_t^i\right)}{f\left(x|S_{t-1}^i \cup \left\{x_{t|t-1}^i\right\}\right)}. \tag{6}$$

State $x$ is more representative at $t$ if the signal $s_t^i$ received in this period increases the probability of that state, relative to not receiving any news. Receiving no news means observing a signal equal to the ex-ante forecast, $s_t^i = x_{t|t-1}^i$, as described in the denominator of equation (6).

[14] Equation (5) assumes that forecasters observe only their individual signals. In reality they also observe common signals, such as public announcements and the past consensus of all other forecasters. In our analysis we focus on individual signals, which drive the difference between individual and consensus forecasts. We consider public signals in Corollary 1, and show that they do not alter the qualitative properties of the model.

Intuitively, the most representative states are those whose likelihood has increased the most in light of recent data. The forecaster then overweighs representative states by using the distorted posterior:

$$f^{\theta}\left(x_t|S_t^i\right) = f\left(x_t|S_t^i\right)R_t(x_t)^{\theta}\frac{1}{Z_t}, \tag{7}$$

where $Z_t$ is a normalization factor ensuring that $f^{\theta}\left(x_t|S_t^i\right)$ integrates to one. Parameter $\theta \geq 0$ denotes the extent to which beliefs are distorted by representativeness. For $\theta = 0$ beliefs are rational, described by the Bayesian conditional distribution $f\left(x_t|S_t^i\right)$. For $\theta > 0$ the diagnostic density $f^{\theta}\left(x_t|S_t^i\right)$ inflates the probability of representative states and deflates the probability of unrepresentative ones. Mistakes occur because states that have become relatively more likely may still be unlikely in absolute terms.

This formalization of representativeness as relative likelihood, and its effect on probability assessments, has been shown to unify well-known laboratory biases in probability assessments such as base rate neglect, the conjunction fallacy, and the disjunction fallacy (Gennaioli and Shleifer 2010). It has also been used to explain real world phenomena such as stereotyping (BCGS 2016), self-confidence (BCGS 2018), and expectation formation in financial markets (BGS 2018, BGLS 2017). Here we assess whether this same structure can shed light on errors in forecasting macroeconomic variables.

Equation (7) yields a very intuitive characterization of beliefs.

**Proposition 1** *The distorted density $f^{\theta}\left(x_t|S_t^i\right)$ is normal. In the steady state it is characterized by a constant variance $\frac{\Sigma\sigma_\epsilon^2}{\Sigma+\sigma_\epsilon^2}$ and by a time varying mean $x_{t|t}^{i,\theta}$ where:*

$$x_{t|t}^{i,\theta} = x_{t|t-1}^{i} + (1+\theta)\frac{\Sigma}{\Sigma+\sigma_\epsilon^2}\left(s_t^i - x_{t|t-1}^i\right), \tag{8}$$

$$\Sigma = \frac{-(1-\rho^2)\sigma_\epsilon^2 + \sigma_u^2 + \sqrt{[(1-\rho^2)\sigma_\epsilon^2 - \sigma_u^2]^2 + 4\sigma_\epsilon^2\sigma_u^2}}{2}. \tag{9}$$

In equations (8) and (9), $x_{t|t-1}^i$ refers to the rational forecast of the hidden state implied by the Kalman Filter. Diagnostic beliefs resemble rational beliefs. They have the same conditional variance $\Sigma$, and their mean $x_{t|t}^{i,\theta}$ updates past rational beliefs $x_{t|t-1}^{i}$ with "rational news" $s_t^i - x_{t|t-1}^i$, to an extent that increases in the signal to noise ratio $\Sigma/\sigma_\epsilon^2$. Because diagnostic expectations overweigh the impact of news by the multiplicative factor $\theta$ in Equation (8), they entail over-reaction to information.

Equation (8) also highlights that diagnostic expectations create excess volatility but not an average bias. Indeed, the discrepancy between rational and diagnostic expectations arises only in the presence of rational news, when $\left(s_t^i - x_{t|t-1}^i\right)$ is non-zero. Since rational news are zero on average, diagnostic expectations over-react when news arrive but then systematically revert to rationality.

In contrast to traditional departures from rationality such as adaptive expectations, diagnostic expectations are forward-looking in that they depend on the parameters of the true data generating process. They are characterized by the "kernel of truth" property: they exaggerate true patterns in the data. Positive news are objectively associated with improvement, but representativeness causes excess focus on the right tail, generating excessive optimism. As we show in Sections 5 and 6, the kernel of truth property offers testable predictions on how updating and forecast errors should change as the process becomes more persistent or when it is influenced by longer AR(2) lags. Critically, these predictions can be tested against conventional mechanical models of extrapolation such as adaptive expectations.

The consensus diagnostic forecast of $x_{t+h}$ at time $t$ is given by:

$$x_{t+h|t}^{\theta} = \int x_{t+h|t}^{i,\theta} di = \rho^h \int x_{t|t}^{i,\theta} di,$$

so that the diagnostic forecast error and revision are respectively given by $x_{t+h} - x_{t+h|t}^{\theta}$ and $x_{t+h|t}^{\theta} - x_{t+h|t-1}^{\theta}$. In Appendix A, we prove the following result.

**Proposition 2** *Under the Diagnostic Kalman Filter, the estimated coefficients of regression (2) at the consensus and individual level, $\beta_1$ and $\beta_1^p$, are given by:*

$$\frac{cov\left(x_{t+h} - x_{t+h|t}^{\theta}, x_{t+h|t}^{\theta} - x_{t+h|t-1}^{\theta}\right)}{var\left(x_{t+h|t}^{\theta} - x_{t+h|t-1}^{\theta}\right)} = (\sigma_\epsilon^2 - \theta\Sigma)g(\sigma_\epsilon^2, \Sigma, \rho, \theta) \quad (10)$$

$$\frac{cov\left(x_{t+h} - x^{i,\theta}_{t+h|t}, x^{i,\theta}_{t+h|t} - x^{i,\theta}_{t+h|t-1}\right)}{var\left(x^{i,\theta}_{t+h|t} - x^{i,\theta}_{t+h|t-1}\right)} = -\frac{\theta(1+\theta)}{(1+\theta)^2 + \theta^2\rho^2} \tag{11}$$

*where* $g(\sigma_\epsilon^2, \Sigma, \rho, \theta) > 0$ *is a function of parameters. Thus, for* $\theta \in (0, \sigma_\epsilon^2/\Sigma)$ *the Diagnostic Kalman Filter entails a positive consensus coefficient* $\beta_1 > 0$*, and a negative individual coefficient* $\beta_1^p < 0$*.*

When representative types are not too overweighed, $\theta < \sigma_\epsilon^2/\Sigma$, the diagnostic filter reconciles positive consensus coefficients with negative individual level coefficients, consistent with the patterns in Section 3. Intuitively, over-reaction of individual analysts to their own information implies a negative pooled coefficient $\beta_1^p < 0$. At the same time, analysts do not react at all to the information received by other analysts (which they do not observe). This effect can create under-reaction of consensus to average information if $\sigma_\epsilon^2/\Sigma$ is large enough. If information is very noisy, not using the signals observed by other forecasters entails a large loss of information. As long as individual forecasters discount news, consensus forecasts exhibit under‒reaction, even if each analyst discounts their own information too little.

In contrast to diagnostic expectations, Noisy Rational Expectations ($\theta = 0$) can generate under-reaction of consensus forecasts, $\beta_1 > 0$, but not over-reaction of individual analysts, $\beta_1^p < 0$. In that model, because forecasters optimally use the limited information at their disposal, their forecast error is uncorrelated with their own forecast revision. As is evident from Equations (9) and (10), when $\theta = 0$ there is no individual-level predictability, inconsistent with the evidence of Section 3.

Finally, Proposition 2 also illustrates the cross-sectional implications of the kernel of truth mentioned above: the predictability of forecast errors depends on the true parameters characterizing the data generating process $(\sigma_\epsilon^2, \Sigma, \rho, \theta)$. In particular, stronger persistence $\rho$ reduces individual over-reaction, in the sense that it pushes the pooled coefficient $\beta_1^p$ toward zero.

Table 4 summarizes the predictions of three departures from rational expectations for the tests of Section 3. These include: Noisy Rational Expectations (or Rational Inattention), Diagnostic Expectations, and Mechanical Extrapolation (adaptive expectations). We evaluate these models according to three predictions: 1) consensus level predictability, 2) individual level predictability, and 3) dependence of forecast revisions on the features of the data generating process.

**Table 4.** Model Comparison

| Model | Consensus | Individual | Updating |
|---|---|---|---|
| **Noisy Rational** | under-reaction | no predictability | depends on fundamentals |
| **Diagnostic** | consistent with under-reaction | over-reaction | depends on fundamentals |
| **Mechanical / Adaptive** | Undetermined | under-reaction for persistent series | does not depend on fundamentals |

The sign switch between consensus and individual coefficient we documented for 9 out of 20 series (and 8 out of 16 variables) is consistent with diagnostic expectations but not with noisy rational expectations. The evidence for 4 series out of 20 – the GDP price deflator, the investment variables, and the Federal Funds rate – is consistent with rational inattention, featuring $\beta_1 > 0$ and $\beta_1^p \approx 0$. Finally, the results for the 3-month T-bill rate (in SPF and Blue Chip) and the unemployment rate are consistent with neither Rational Inattention nor Diagnostic Expectations because they exhibit under-reaction at both the consensus and individual level, $\beta_1, \beta_1^p > 0$. This pattern may be accounted for by adaptive expectations.

Overall, most of the evidence is consistent with Diagnostic Expectations, but Rational Inattention or Adaptive Expectations may play a role for some series. We further assess these models in Section 5.

We conclude this Section by considering the possibility, relevant in many real world settings, that forecasters also observe public signals. Suppose that each analyst observes, in addition to the private signal $s_t^i$, a public signal $s_t = x_t + v_t$, where $v_t$ is also normal with variance $\sigma_v^2$. In this case, the diagnostic estimate uses both the private and the public signal according to their informativeness. We then obtain:

**Corollary 1** *Suppose that $\theta \in (0, \sigma_\epsilon^2/\Sigma)$. Then, increasing the precision $1/\sigma_v^2$ of the public signal while holding constant the total precision $(1/\sigma_\epsilon^2 + 1/\sigma_v^2)$ of the private and the public signals: i) leaves the pooled coefficient $\beta_1^p$ unchanged, and ii) lowers the consensus coefficient $\beta_1$.*

When a higher share of information comes from a public signal, the information of different forecasters is more correlated, so that individual forecasts incorporate a larger share of the information available to others. As a result, the consensus forecast exhibits less under-reaction, or possibly even over-reaction. This may explain why in financial market variables such as interest rates we detect less consensus under-reaction than in most other series: market prices act as public signals that correlate to a significant extent the information sets of different forecasters.

The results in this section describe the features of over−reaction implied by diagnostic expectations. It is useful to compare over−reaction in this specific setting to the concept of overconfidence, modeled as overweighting of private signals relative to public signals (Daniel et al. 1998).[15] Inflating the signal to noise ratio of private information can cause over-reaction, by boosting the Kalman gain closer to its upper bound of 1. In contrast, under diagnostic expectations, the Kalman gain is multiplied by $(1 + \theta)$ and so the reaction to information is not bounded by 1 (see Equation 8). In our estimation in Section 6, we find clear evidence for the latter for several series. This difference has important implications for consensus forecasts: Proposition 2 shows that consensus forecasts can over−react when the diagnostic Kalman gain is large, which cannot happen under overconfidence. Moreover, Corollary 1 shows that there is more consensus over−reaction when there is more public information, another result that cannot be obtained from overconfidence, which predicts more under-reaction when more information is public.

## 5. Kernel of Truth

We first run a cross sectional test based on the persistence of the different series, which allows us to compare Diagnostic Expectations with Adaptive Expectations. We then assess whether, for series that feature hump-shaped dynamics, beliefs over-react both to short-term news and to longer-term reversals.

### 5.1 Persistence Tests

Under Noisy Rational and Diagnostic Expectations forecast revision at $t$ satisfies:

$$x^i_{t+h|t} - x^i_{t+h|t-1} = \rho\left(x^i_{t+h-1|t} - x^i_{t+h-1|t-1}\right).$$

[15] As mentioned in the Introduction, diagnostic expectations describe beliefs in a wide range of settings, both in the lab and in the field, including those where overconfidence can be ruled out (such as when all information is public, for example in experimental illustrations of base rate neglect or social stereotypes, BCGS 2016).

The revision $h$ periods ahead reflects the forecast revision about the same variable $h-1$ periods ahead, adjusted by the persistence $\rho$ of the series. The idea is simple: when forecasts are forward looking, more persistent series should witness more correlated revisions across different forecast horizons.

Under Adaptive Expectations, in contrast, updating is mechanical and should not depend on the true persistence of the forecasted process. Formally, in this case:

$$x_{t+h|t}^{i} - x_{t+h|t-1}^{i} = \mu\left(x_{t+h-1|t}^{i} - x_{t+h-1|t-1}^{i}\right),$$

where $\mu$ is a positive constant independent of $\rho$. [16]

To assess this prediction, we fit an AR(1) for the actuals of each series and estimate $\rho$. The actuals have the same format as the forecast variables, and we use the exact time period for which the forecasts are available. We run the following individual level regression using forecast revisions for different horizons:

$$x_{t+3|t}^{i} - x_{t+3|t-1}^{i} = \gamma_{o}^{p} + \gamma_{1}^{p}\left(x_{t+2|t}^{i} - x_{t+2|t-1}^{i}\right) + \epsilon_{t+3}^{i},$$

and repeat the same specification at the consensus level. We then study the relationship between the slope coefficient $\gamma_1^p$ and the persistence $\rho$ of each series.

The results are reported in Figure 1 Panel A. At both the individual and the consensus level, the more persistent series display more correlated forecast revisions. While we only have 20 series, the correlation is statistically different from zero with a p-value less than 0.001. [17] In line with forward-looking models, forecasters see more persistent impact of news for more persistent series. The positive relationship between the slope coefficient $\gamma_1^p$ and the persistence $\rho$ of each series depends only on the first autocorrelation lag, and so holds also for series with richer dynamics than AR(1). The pattern is similar for consensus forecasts, shown in Figure 1 Panel B. This evidence is inconsistent with adaptive

[16] This formula is based on the Error-Learning model, a generalization of adaptive expectations for longer horizons (Pesaran and Weale 2006). This model postulates $x_{t+s|t}^{i} - x_{t+s|t-1}^{i} = \mu_s\left(x_t - x_{t|t-1}^{i}\right)$, so that $\mu = \mu_h/\mu_{h-1}$.

[17] The results in Figure 1 and 2 are similar if we exclude the Blue Chip series that are also available in SPF (e.g. real GDP, 3-month Treasuries, 10-year Treasuries, AAA corporate bond rate).

expectations, in which updating does not depend on persistence, in which case the line in Figure 1 should be flat.

Figure 1. Properties of Forecast Revisions and Actuals

In Panel A, the y-axis is the coefficient $\gamma_1^p$ from regression $x^i_{t+3|t} - x^i_{t+3|t-1} = \gamma_o^p + \gamma_1^p\left(x^i_{t+2|t} - x^i_{t+2|t-1}\right) + \epsilon^i_{t+3}$. The x-axis is the persistence measured from an AR(1) regression of the actuals corresponding to the forecasts. For each variable, the AR(1) regression uses the same time period as when the forecast data is available. In Panel B, the y-axis is the regression coefficient from the parallel specification using consensus forecasts.

Panel A. Individual Level Coefficients

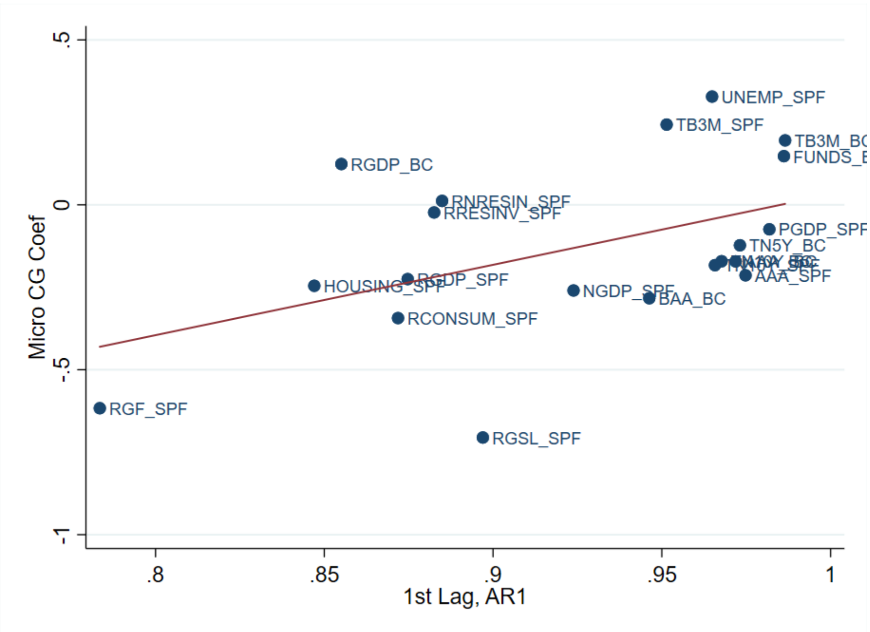

Panel B. Consensus Coefficients

Another approach is to assess the correlation between the persistence of a series and the CG coefficient of reaction to news. Diagnostic Expectations do not have clear predictions at the consensus level: the coefficient $(\sigma_\epsilon^2 - \theta\Sigma)g(\sigma_\epsilon^2, \Sigma, \rho, \theta)$ in Equation (10) can be either decreasing or increasing in persistence $\rho$, depending on the parameter values. On the other hand, Equation (11) says that the individual CG coefficient should increase, i.e. get closer to zero, as $\rho$ increases. When the series is more persistent, rational revisions become more volatile, which reduces the predictability of errors for a given level of noise. Of course, under Noisy Rational Expectations individual coefficients should be zero, so they should be uncorrelated with the persistence of fundamentals.

Figure 2 shows the correlation for the CG coefficient estimated from individual-level regressions. We find that the CG coefficient rises with persistence, which lends additional support for diagnostic expectations. The correlation is statistically different from zero with a $p$-value of 0.035.

Figure 2. CG Regression Coefficients and Persistence of Actual

Plots of individual level CG regression (forecast error on forecast revision) coefficients in the y-axis, against the persistence of the actual process in the x-axis.



### 5.2. Kernel of Truth in the Time Series

We now allow the forecasted series to be described by an AR(2) process. As shown by Fuster, Laibson and Mendel (2010), several macroeconomic variables follow hump-shaped dynamics with short-term momentum and longer-term reversals. Considering this possibility is relevant for two reasons. First, under the kernel of truth, forecasters should exaggerate true features of the data generating process, including the presence of long-term reversals. This also allows us to compare these approaches to the model of Natural Expectations proposed by Fuster, Laibson and Mendel (2010), in which agents forecast an AR(2) process "as if" it was AR(1) in changes.

#### 5.2.1 Diagnostic Expectations with AR(2) Processes

Suppose that forecasters seek to forecast an AR(2) process:

$$x_{t+3} = \rho_2 x_{t+2} + \rho_1 x_{t+1} + u_{t+3}. \tag{12}$$

If $\rho_2 > 0$ and $\rho_1 < 0$, the variable displays short-term momentum and long-term reversal. Each forecaster now observes two signals, one about the current state $s_{t,t}^i = x_t + \epsilon_t^i$ and another about the past state $s_{t-1,t}^i = x_{t-1} + v_t^i$. The presence of two signals implies that the current forecast revisions for $x_{t+1}$ and $x_{t+2}$ are not perfectly collinear, which is necessary for out test.

The diagnostic forecasts about $t + 1$ and $t + 2$ overweigh each signal (this is proved in Appendix A), so that forecast revisions are excessive. The diagnostic forecast of $x_{t+3}$ is then a linear combination of the forecasts of $x_{t+2}$ and $x_{t+1}$ with weights given by the autoregressive parameters $\rho_1$ and $\rho_2$:

$$x_{t+3|t}^{i,\theta} = \rho_2 x_{t+2|t}^{i,\theta} + \rho_1 x_{t+1|t}^{i,\theta}.$$

This formula suggests a way to test for overreaction, generalizing Equation (2) to AR(2). To do so, simply predict forecast errors in the long term using forecast revisions about shorter term:

$$x_{t+3} - x_{t+3|t}^i = \delta_0^p + \delta_2^p FR_{t,t+2}^i + \delta_1^p FR_{t,t+1}^i + \epsilon_{t,t+h}, \tag{12}$$

where $FR_{t,t+1}^i$ and $FR_{t,t+2}^i$ stand for the surveyed forecast revisions at for $t + 1$ and $t + 2$, respectively. Under diagnostic expectations, estimates of (12) satisfy the following property.

**Proposition 3.** *Under the Diagnostic Kalman filter, the estimated coefficients* $\hat{\delta}_1^p$ *and* $\hat{\delta}_2^p$ *in Equation* (12) *are proportional to the negative of the AR(2) coefficients:*

$$\hat{\delta}_1^p \propto -\rho_1\theta, \tag{13}$$

$$\hat{\delta}_2^p \propto -\rho_2\theta. \tag{14}$$

Once again, under rational expectations ($\theta = 0$) individual forecast errors cannot be predicted from any forecast revisions. Diagnostic expectations instead imply that the coefficients should be non-zero, with flipped signs relative to the data generating process. This is due to the kernel of truth. Over-reaction to short term news, $\rho_2 > 0$, implies that upward forecast revisions about $x_{t+2}$ lead to exaggerated optimism about $x_{t+3}$ and thus negative forecast errors. This yields $\hat{\delta}_2^p < 0$. On the other hand, over-reaction to long-term reversal, $\rho_1 < 0$, implies that upward forecast revisions about $x_{t+1}$ lead to exaggerated pessimism about $x_{t+3}$ and thus positive forecast errors. This yields $\hat{\delta}_1^p > 0$. [18]

Before moving to the data, we link this discussion to Natural Expectations, which have been proposed to account for expectations errors in AR(2) settings. In this model, forecasts are based on an AR(1) process in changes.[19] This implies that Natural Expectations exaggerate the short run persistence of the series and, similarly to Diagnostic Expectations, entail negative predictability of forecast errors at this horizon. On the other hand, Natural Expectations also dampen long-term reversals, unlike our prediction of over-reaction to long-term reversals ($\hat{\delta}_1^p > 0$). Thus, the two models predict overlapping but distinguishable patterns of predictable forecast errors.

In the remainder of the section, we test the predictions of Proposition 3.

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[18] Proposition 3 also implies that the tests of Section 3 may be biased toward finding under-reaction when the AR(2) process has $\rho_2 > 0$ and $\rho_1 < 0$. Positive news at $t$ may then trigger an upward revision of the forecasts for both $x_{t+1}$ and $x_{t+2}$. The former creates excess pessimism, the latter excess optimism. If the first effect is strong, the test of Section 3 may detect excess pessimism after good news, giving a false impression of under-reaction.

[19] Formally, forecasters use the rule $(x_{t+1} - x_t) = \varphi(x_t - x_{t-1}) + v_{t+1}$ with fitted coefficient $\varphi = (\rho_2 - \rho_1 - 1)/2$. For a stationary AR(2) process (i.e. if $\rho_2 - \rho_1 < 1$, $\rho_1 + \rho_2 < 1$ and $|\rho_1| < 1$) this implies that forecasters exaggerate short term momentum and dampen long term reversals. This model cannot be directly estimated using Equation (12) because it implies that the two forecast revisions are perfectly collinear.

### 5.2.2 AR(1) vs AR(2) Dynamics

As a first step, we assess which of our 16 variables is more accurately described by AR(2) rather than AR(1). We do not aim to find the unconstrained optimal ARMA($k, q$) specification, which is well known to be difficult. We only wish to capture the simplest longer lags and see whether expectations react to them as predicted by the model. We fit a quarterly AR(2) process for our 20 series. Figure 4 below plots the estimates for $\rho_1$ and $\rho_2$.[20] As before, the actuals have the same format as the forecast variables, and for each series the regression covers the time period when the forecast data are available.

The signs of coefficients point to a positive momentum at short horizons ($\rho_2 > 0$) for all series, and to long-run reversals ($\rho_1 < 0$) for most series, the remaining ones having $\rho_1$ approximately zero.[21] To assess which dynamics better describe the series, we compare the AR(2) estimates to the AR(1) estimates from Section 5.1. Table 6 shows the Bayesian Information Criterion (BIC) score associated with each fit.

For the majority of series, AR(2) is favored over AR(1). The tests favor AR(1) dynamics only for real consumption (SPF) and the BAA bond rate (BC), while for the 10-year Treasury rate series the tests are inconclusive.[22] In sum, hump shaped dynamics are a key feature of several series.

[20] Just like for the case of AR(1), for growth variables we run quarterly AR(2) regressions of growth from $t-1$ to $t+3$. For variables in levels, we run quarterly regressions in levels. We run separate regressions for the variables that occur both in SPF and BC, because they cover slightly different time periods.

[21] We check whether multicollinearity may affect our results in this Section, given that forecasts revisions at different horizons are often highly correlated. The standard issue with multicollinearity is the coefficients are imprecisely estimated, which we do not find to be the case. We also perform simulations to verify that the correlation among the right hand side variables by itself does not mechanically lead to the patterns we observe.

[22] The Akaike Information Criterion (AIC) yields similar results, except that it positively identifies the TN10Y (SPF) series as AR(2). To interpret the IC scores, recall that lower scores represent a better fit. The likelihood ratio $\frac{\Pr(AR2)}{\Pr(AR1)}$ is estimated as $exp\left[-\frac{BIC_{AR2}-BIC_{AR1}}{2}\right]$, so that $\Delta BIC_{2-1} = -2$ means the AR(2) model is 2.7 times more likely than the AR(1) model.

Figure 4. AR(2) Coefficients of Actuals

For each variable, the AR(2) regression uses the same time period as when the forecast data is available. The blue circles show the first lag and the red diamonds show the second lag. Standard errors are Newey-West, and the vertical bars show the 95% confidence intervals.

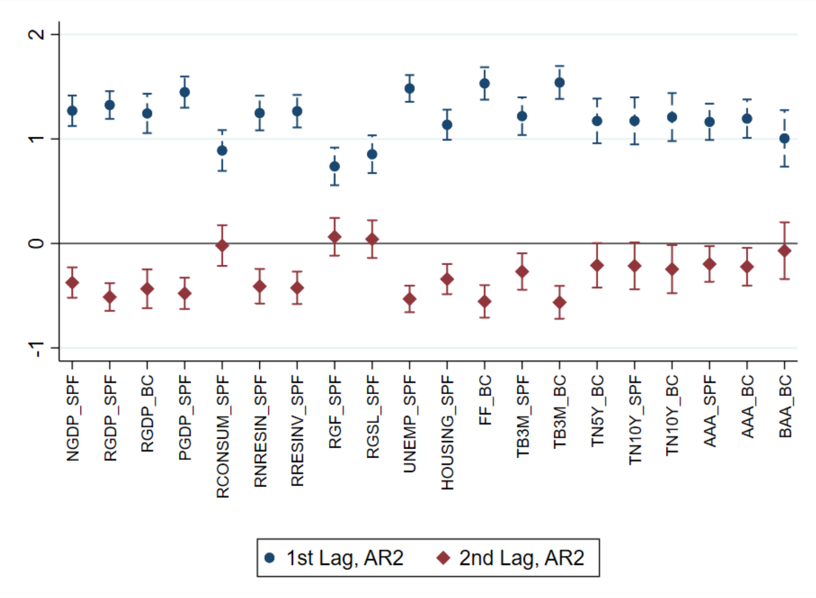


**Table 6**. BIC of AR(1) and AR(2) Regressions of Actuals

This table shows the BIC statistic corresponding to the AR(1) and AR(2) regressions of the actuals. The final column shows the specification that has a lower BIC (preferred).

| Variable | $BIC_{AR1}$ | $BIC_{AR2}$ | $\Delta BIC_{2\text{-}1}$ | model |
|---|---|---|---|---|
| Nominal GDP (SPF) | -1133.74 | -1149.13 | -15.39 | AR(2) |
| Real GDP (SPF) | -1120.33 | -1164.52 | -44.19 | AR(2) |
| Real GDP (BC) | -618.50 | -626.83 | -8.33 | AR(2) |
| GDP Price Index Inflation (SPF) | -1423.70 | -1456.90 | -33.20 | AR(2) |
| Real Consumption (SPF) | -924.47 | -911.66 | 12.82 | AR(1) |
| Real Non-Residential Investment (SPF) | -509.72 | -524.37 | -14.65 | AR(2) |
| Real Residential Investment (SPF) | -375.81 | -401.05 | -25.25 | AR(2) |
| Real Federal Government Consumption (SPF) | -560.97 | -553.12 | 7.85 | AR(1) |
| Real State&Local Govt Consumption (SPF) | -905.91 | -896.23 | 9.68 | AR(1) |
| Housing Start (SPF) | -250.88 | -265.89 | -15.01 | AR(2) |
| Unemployment (SPF) | 168.69 | 111.57 | -57.12 | AR(2) |
| Fed Funds Rate (BC) | 191.89 | 149.87 | -42.02 | AR(2) |
| 3M Treasury Rate (SPF) | 240.87 | 232.25 | -8.62 | AR(2) |
| 3M Treasury Rate (BC) | 163.27 | 118.76 | -44.51 | AR(2) |
| 5Y Treasury Rate (BC) | 126.30 | 123.51 | -2.79 | AR(2) |
| 10Y Treasury Rate (SPF) | 89.66 | 89.91 | 0.25 | AR(1) |
| 10Y Treasury Rate (BC) | 86.54 | 84.80 | -1.74 | AR(2) |
| AAA Corporate Bond Rate (SPF) | 129.84 | 118.64 | -11.20 | AR(2) |
| AAA Corporate Bond Rate (BC) | 86.05 | 84.72 | -1.32 | AR(2) |
| BAA Corporate Bond Rate (BC) | 58.33 | 61.79 | 3.46 | AR(1) |

#### 5.2.3 Empirical Tests of Over-Reaction with AR(2) dynamics

We next restrict the analysis to the series for which AR(2) is favored, and test the prediction of Proposition 3 by estimating Equation (12). Since our AR(2) series exhibit $\rho_2 > 0$ and $\rho_1 < 0$, under diagnostic expectations the estimated coefficient on medium term forecast revision should be negative, $\hat{\delta}_2^p < 0$, while the estimated coefficient on short term forecast revision should be positive, $\hat{\delta}_1^p > 0$.

Figure 5 shows, for each relevant series, the forecast error regression coefficients $\hat{\delta}_2^p$ and $\hat{\delta}_1^p$ obtained from estimating Equation (12) with pooled individual data. Table 7 reports these coefficients, together with their corresponding standard errors and $p$-values. In line with the predictions of the model, the signs of the coefficients indicate that the short-term revision positively predicts forecast errors ($\hat{\delta}_1^p > 0$ for all 15 series, 10 of which are statistically significant at the 5% level) while the medium-term revision negatively predicts them ($\hat{\delta}_2^p < 0$ for 12 out of 15 series, 8 of which are statistically significant at the 5% level). To further assess these results, we perform a test of joint significance for $\hat{\delta}_2^p < 0, \hat{\delta}_1^p > 0$. We resample the data using block bootstrap, and calculate the fraction of times when $\hat{\delta}_2^p < 0, \hat{\delta}_1^p > 0$ holds, as shown in the last column of Table 7. The probability is greater than 95% for 8 out of the 15 series.

**Figure 5.** Coefficients in CG Regression AR(2) Version

This plot shows the coefficients $\delta_2^p$ (blue circles) and $\delta_1^p$ (red diamonds) from the regression in Equation (12). Standard errors are clustered by both forecaster and time, and the vertical bars shown the 95% confidence intervals.

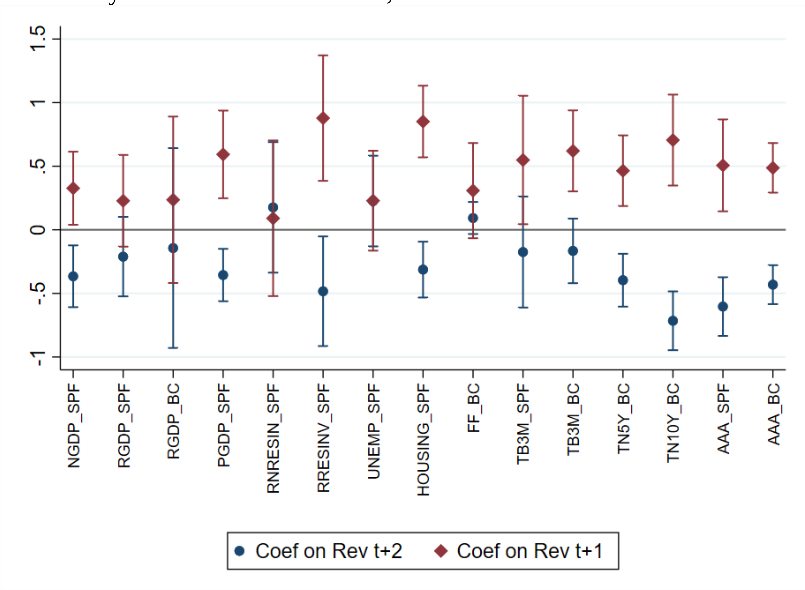

**Table 7.** Coefficients in CG Regression AR(2) Version

Coefficients $\delta_2^p$ and $\delta_1^p$ from the regression in Equation (12), together with the corresponding standard errors and *p*-values. The final column resamples the data using block bootstrap and shows the probability of $\delta_2^p < 0$ and $\delta_1^p > 0$.

| Variable | $\delta_2^p$ | s.e. | *p*-val | $\delta_1^p$ | s.e. | *p*-val | Prob $\delta_2^p < 0$ & $\delta_1^p > 0$ |
|---|---|---|---|---|---|---|---|
| Nominal GDP (SPF) | -0.37 | 0.12 | 0.00 | 0.33 | 0.15 | 0.03 | 0.99 |
| Real GDP (SPF) | -0.21 | 0.16 | 0.19 | 0.23 | 0.18 | 0.22 | 0.86 |
| Real GDP (BC) | -0.14 | 0.40 | 0.72 | 0.24 | 0.33 | 0.48 | 0.78 |
| GDP Price Index Inflation (SPF) | -0.36 | 0.11 | 0.00 | 0.59 | 0.18 | 0.00 | 0.99 |
| Real Non-Residential Investment (SPF) | 0.18 | 0.26 | 0.50 | 0.09 | 0.31 | 0.77 | 0.11 |
| Real Residential Investment (SPF) | -0.48 | 0.22 | 0.03 | 0.88 | 0.25 | 0.00 | 1.00 |
| Housing Start (SPF) | -0.31 | 0.11 | 0.01 | 0.85 | 0.14 | 0.00 | 1.00 |
| Unemployment (SPF) | 0.23 | 0.18 | 0.22 | 0.23 | 0.20 | 0.26 | 0.03 |
| Fed Funds Rate (BC) | 0.09 | 0.06 | 0.15 | 0.31 | 0.19 | 0.11 | 0.40 |
| 3M Treasury Rate (SPF) | -0.17 | 0.22 | 0.43 | 0.55 | 0.26 | 0.03 | 0.85 |
| 3M Treasury Rate (BC) | -0.17 | 0.13 | 0.20 | 0.62 | 0.16 | 0.00 | 0.92 |
| 5Y Treasury Rate (BC) | -0.40 | 0.11 | 0.00 | 0.46 | 0.14 | 0.00 | 1.00 |
| 10Y Treasury Rate (BC) | -0.72 | 0.12 | 0.00 | 0.71 | 0.18 | 0.00 | 1.00 |
| AAA Corporate Bond Rate (SPF) | -0.60 | 0.12 | 0.00 | 0.51 | 0.18 | 0.01 | 1.00 |
| AAA Corporate Bond Rate (BC) | -0.43 | 0.08 | 0.00 | 0.49 | 0.10 | 0.00 | 1.00 |

These results are consistent with kernel of truth but are harder to reconcile with Natural Expectations, where forecasters neglect longer lags (in the current setting, this means fitting an AR(1) model even for AR(2) series).[23] Overall, then, the AR(2) analysis confirms and perhaps strengthens the evidence for over-reaction in the data. Four of the seven series (PGDP_SPF, RRESINV_SPF, TN5Y_BC and TN10Y_BC) for which individual level forecast errors seemed unpredictable (Table 3), and thus consistent with Noisy Rational Expectations, show evidence of over-reaction in the AR(2) setting. In addition, the two series that seemed to display under-reaction at the individual level, unemployment and the 3-months T Bill rate, now show evidence of over-reaction to long-term reversals ($\hat{\delta}_1^p > 0$), albeit not significantly. In all these cases, it is possible that over-reaction to long term reversals moved the individual level coefficient in Table 4 close to zero or above, giving the false impression of rationality or under-reaction. Only for the variable RGDP_SPF, which displayed significant over-reaction under the AR(1) specification loses its significance at conventional level in the AR(2) case.

[23] Beshears et al. (2013) report results from a laboratory experiment in which subjects recognize reversals occurring within ten periods, but not in fifty periods. In our data reversals are fast, which is consistent with their findings.

## 6. Model Estimation

We next use the simulated method of moments to quantify $\theta$ and assess the performance of our model. In the baseline quantification, we assume that shocks are normal and that the macro series follow the better-fitting process among AR(1) or AR(2). We then present a sensitivity analysis. We first estimate $\theta$ under the assumption that all series follow an AR(1). The results are similar, which is reassuring given the well known difficulty of finding the proper AR specification. We next allow for fundamental shocks to be drawn from fat tailed distributions. Using the particle filter, we find that our results again remain stable. Finally, we estimate an overconfidence model, and show that diagnostic expectations better fit of the data quantitatively. Appendix E presents supporting material for these exercises.

The estimation exercises share the following general structure. First, we assume forecasters describe each series $k$ using the vector of estimated fundamental parameters ($(\rho_{1,k}, \sigma_{u,k})$ for the AR(1) specifications and $(\rho_{1,k}, \rho_{2,k}, \sigma_{u,k})$ for the AR(2) specifications). By separating the estimation of fundamental and expectations parameters, we minimize the degrees of freedom in fitting expectations data. Second, given these parameter values we use the simulated method of moments to estimate, for each expectations series, the series-specific measurement noise $\sigma_{\varepsilon,k}$ and the diagnostic parameter $\theta_k$. We initially take $(\theta_k, \sigma_{\varepsilon,k})$ to be common to all forecasters, but also estimate them at the forecaster level.

We estimate $\sigma_{\varepsilon,k}$ and $\theta_k$ by matching two moments of the expectations data: the variance of the forecast errors, $\sigma^2_{FE,k} = var_{i,t}\left(FE^i_{k,t}\right)$, and the variance of forecast revisions, $\sigma^2_{FR,k} = var_{i,t}\left(FR^i_{k,t}\right)$, computed across time and forecasters. We choose these moments because they can be measured directly from the data with reasonable precision and they are linked to the parameters of interest.[24] By the law of total variance, the variance of forecast errors $\sigma^2_{FE,k}$ is the sum of the: i) average cross sectional variance of errors, and the ii) over time variance of consensus errors. The first term is informative about measurement noise $\sigma_{\varepsilon,k}$, without which any cross sectional variance would be zero. The second term is informative about the over-reaction parameter $\theta_k$. A similar logic holds with respect to the total variance of forecast revisions.

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[24] In contrast, matching average forecast errors and revisions would not be informative about $\sigma_{\varepsilon,k}$ and $\theta_k$, as these sample moments are close to zero in our data (consistently with diagnostic but also rational expectations). Importantly, we do not use the CG coefficients in the estimation because we later use these moments to assess model performance.

We do not estimate the model using maximum likelihood for two reasons. First, because our model is simple and transparent, it is also likely to be misspecified. In this case, moment estimators are often more reliable. Second, fundamental shocks can be fat tailed, and estimating a non-normal model by maximum likelihood is problematic. The likelihood function cannot in fact be written in closed form. Numerical approximations methods must be used, and these may introduce additional noise in parameter estimates. Despite the limitation, our structural estimation exercise can be viewed as useful first step in assessing the ability of our model to account for variation in forecast errors and revisions in expectations data.

### 6.1 Baseline Estimation

We first explain the estimation procedure. In our baseline exercise we describe each series $k$ as either an AR(1) or an AR(2) process following Table 6, using the fundamental parameters $(\rho_{1,k}, \sigma_{u,k})$ or $(\rho_{1,k}, \rho_{2,k}, \sigma_{u,k})$ respectively (see Figure 4 and Appendix E, Table E1 for the estimates). In the following, we refer to this specification as the "baseline specification," which uses the AR(2) (respectively, AR(1)) version of the model to those series identified as AR(2) (respectively, AR(1)) according to Table 6, and. Next, for each series $x_t^k$ of actuals and given $(\theta_k, \sigma_{\varepsilon,k})$, we simulate time series of signals $s_t^{i,k} = x_t^k + \epsilon_t^{i,k}$ where $\epsilon_t^{i,k}$ is drawn from $\mathcal{N}(0, \sigma_{\varepsilon,k}^2)$ i.i.d. across time and forecasters. We then use $(\theta_k, \sigma_{\varepsilon,k})$ and $s_t^{i,k}$ to generate diagnostic expectations associated with each forecaster, using Equation (8) for AR(1) processes and its generalization Equation (E1) for AR(2) processes, for the exact period in which he forecasts a given series (we drop forecasters with less than ten observations). We compute the forecast revisions and forecast errors of each forecaster, as well as the model-implied variances of forecast errors $\widehat{\sigma_{FE,k}^2}$ and of forecast revisions $\widehat{\sigma_{FR,k}^2}$. Finally, we search through a grid of $(\theta_k, \sigma_{\varepsilon,k})$ to find parameter values that minimize the distance between model moments and data moments:

$$(\theta_k^*, \sigma_{\varepsilon,k}^*) = \underset{(\theta,\sigma_\epsilon)}{\operatorname{argmin}} \left(\sigma_{FE,k}^2 - \widehat{\sigma_{FE,k}^2}(\theta, \sigma_\epsilon)\right)^2 + \left(\sigma_{FR,k}^2 - \widehat{\sigma_{FR,k}^2}(\theta, \sigma_\epsilon)\right)^2.$$

To obtain confidence intervals for our estimates, we repeat the process using 60 bootstrap samples (with replacement) from the panel of forecasters.

Table 8 summarizes the estimation results. For 17 out of the 20 series, we estimate a significantly positive $\theta$, varying roughly between 0.2 and 1.5 (except for State & Local Government Consumption, which is an outlier). For the Federal Funds rate and the 3-month Treasury rate (BC), two closely related series, we estimate a $\theta$ of zero. For unemployment, we estimate a small but significant negative $\theta$.

Model estimation strengthens the finding of over-reaction. Our estimates of $\theta$ exhibit tight confidence intervals, with an average of 0.6. Estimates of standard deviation of noise $\sigma_\epsilon$, normalized by the standard deviation of shocks $\sigma_u$, show more variation across series and are less precisely estimated.

**Table 8.** SMM Estimates of $\theta$ and $\sigma_\epsilon$

This table shows the estimates of $\theta$ and $\sigma_\epsilon$ in the baseline specification of the model, as well as the 95% confidence interval based on block bootstrap (bootstrapping forecasters with replacement). The standard deviation of the noise $\sigma_\epsilon$ is normalized by the standard deviation of innovations in the actual process $\sigma_u$. Results for each series are estimated using the AR(1) or AR(2) version of the diagnostic expectations model based on the properties of the actuals according to Table 6.

| | $\theta$ | 95% CI | $\sigma_\epsilon/\sigma_u$ | 95% CI |
|---|---|---|---|---|
| Nominal GDP (SPF) | 0.21 | (0.06, 0.43) | 0.45 | (0.10, 1.08) |
| Real GDP (SPF) | 0.51 | (0.09, 0.87) | 0.79 | (0.34, 1.00) |
| Real GDP (BC) | 0.34 | (0.11, 0.58) | 1.39 | (0.58, 2.00) |
| GDP Price Index Inflation (SPF) | 0.45 | (0.12, 0.84) | 3.18 | (2.32, 4.00) |
| Real Consumption (SPF) | 1.56 | (0.95, 2.00) | 3.56 | (2.25, 4.00) |
| Real Non-Residential Investment (SPF) | 0.35 | (0.19, 0.57) | 1.46 | (1.03, 2.08) |
| Real Residential Investment (SPF) | 0.28 | (0.16, 0.45) | 1.37 | (0.82, 2.00) |
| Real Federal Government Consumption (SPF) | 1.18 | (0.8, 1.55) | 1.66 | (1.00, 2.40) |
| Real State & Local Govt Consumption (SPF) | 2.80 | (1.30, 3.90) | 4.81 | (3.74, 5.00) |
| Housing Start (SPF) | 1.00 | (0.54, 1.61) | 1.81 | (1.00, 3.36) |
| Unemployment (SPF) | -0.25 | (-0.67, -0.08) | 0.57 | (0.01, 1.01) |
| Fed Funds Rate (BC) | -0.02 | (-0.10, 0.06) | 1.17 | (0.77, 1.62) |
| 3M Treasury Rate (SPF) | 0.18 | (0.11, 0.21) | 1.11 | (0.93, 1.43) |
| 3M Treasury Rate (BC) | 0.01 | (-0.03, 0.09) | 1.86 | (1.44, 2.29) |
| 5Y Treasury Rate (BC) | 0.37 | (0.32, 0.42) | 2.19 | (1.84, 2.61) |
| 10Y Treasury Rate (SPF) | 0.59 | (0.50, 0.60) | 2.91 | (2.70, 3.00) |
| 10Y Treasury Rate (BC) | 0.29 | (0.21, 0.37) | 2.21 | (1.78, 2.87) |
| AAA Corporate Bond Rate (SPF) | 0.63 | (0.50, 0.79) | 4.60 | (3.95, 5.21) |
| AAA Corporate Bond Rate (BC) | 0.71 | (0.60, 0.85) | 4.85 | (4.10, 5.60) |
| BAA Corporate Bond Rate (BC) | 0.73 | (0.64, 0.80) | 2.63 | (2.30, 3.00) |

The estimates for $\theta$ are in line with BGS (2018), who obtain $\theta = 0.9$ for expectations data on credit spreads, and with BGLS (2017) who also obtain $\theta = 0.9$ for expectations data on firm level earnings' growth. In the current exercise the average estimate is a bit lower, but this may be due to the fact that here we allow for AR(2) specifications (if we assume an AR(1) structure for all series, we find an average $\theta$ of 0.81, see Section 6.3). To have a sense of the magnitude, a $\theta \approx 1$ means that forecasters' reaction to news is roughly twice as large as the rational expectations benchmark. In BGLS (2017), we find that this magnitude of $\theta$ can account for the observed 12% annual return spread between stocks analysts are pessimistic about and stocks they are optimistic about. This suggests that this magnitude of distortions can have sizable economic consequences.

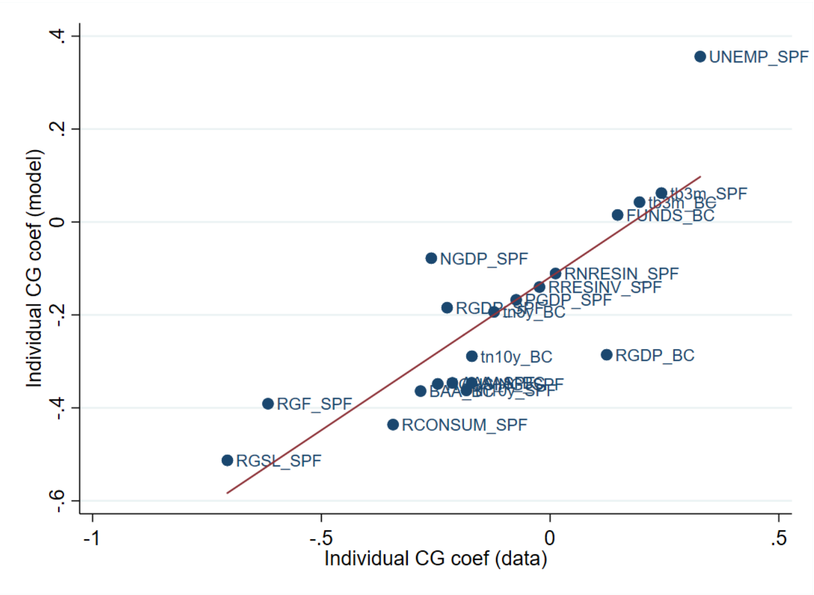
### 6.2 Model Performance

We first assess the ability of the model to match the target moments. Across different series $k$, the average absolute log difference between the variance of forecast errors in the data ($\sigma^2_{FE,k}$) and that in the simulated model ($\widehat{\sigma^2_{FE,k}}(\theta, \sigma_\epsilon)$) is 0.022, with a minimum of 0.001 for the Fed Funds Rate and a maximum of 0.207 for Real State and Local Government Consumption. Likewise, the variance of forecast revisions in the data ($\sigma^2_{FE,k}$) and that in the simulated model ($\widehat{\sigma^2_{FE,k}}(\theta, \sigma_\epsilon)$) is 0.028, with a minimum of 0.002 for Housing Starts and a maximum of 0.188 for Unemployment Rate (see Appendix E, Table E2).

Second, we assess the ability of the model to match the Coibion-Gorodnichenko coefficients, at the individual and consensus levels. We calculate the CG coefficients in the model using the estimated ($\theta$, $\sigma_\epsilon$) for each series, together with the actual process and its parameters, to generate model-based forecasts associated with each forecaster and each time period where the forecaster is available; we then run CG regressions using these model-based forecasts, and compare the results with CG regressions using survey data. Figure 6 shows the individual CG coefficients from the estimated model and those from the survey data. The correlation between the two sets of coefficients is high, about 0.83 (p-value of 0.00).

**Figure 6.** Individual CG Coefficients using Estimated $\theta$ and $\sigma_\epsilon$

The figure plots individual CG coefficients in the baseline specification of the model (with estimated $\theta$ and $\sigma_\epsilon$) in the y-axis, and CG coefficients in the survey data in the x-axis. Results for each series are estimated using the AR(1) or AR(2) version of the diagnostic expectations model based on the properties of the actuals according to Table 6.




For consensus CG coefficients, we also find a positive correlation between estimates from the model and in the survey data, but the correlation is lower than in the individual case (0.30 vs 0.83, see Appendix E Figure E1). The lower correlation reflects the fact that, unlike individual level coefficients, consensus coefficients are highly dependent on the magnitude of measurement noise $\sigma_{\epsilon,k}$, which is less precisely estimated as shown in Table 8.

### 6.3 Sensitivity Analysis and Overconfidence

We next assess the robustness of our results to alternative assumptions. We complement our baseline specification above with two other specifications: we first restrict all series to follow an AR(1) process, keeping the assumption of normal shocks; we then allow the fundamental shocks to be non-normal, as macro series are known to have fat tails. Table E3 reports the estimated target moments, Table E4 reports the $\theta_k$ estimates, and Table E5 assesses model performance in terms of reproducing individual and consensus CG coefficients.

We find a very high correlation between the distortions $\theta_k$ estimated under the different specifications, between 91% and 96%, and the average estimates for $\theta$ in the alternative specifications are also very similar (0.6 in the baseline specification, 0.81 for AR(1) and 0.74 for AR(1) with fat tails and particle filtering, see Table E4 for details). Our baseline estimates are robust to these alternative specifications. In terms of model performance, the baseline specification (which allows for AR(2)) seems to do a better job than the other ones. It achieves a lower value of the loss function (half as large as the next best performer for the median series, based on moments shown in Table E2 and E3), and it explains a larger share of variation in individual CG coefficients (see Table E5, panel A).

We also assess the ability of the model to capture observed heterogeneity in distortions across different analysts. To do so, we estimate distortion and noise coefficients $(\theta_k^i, \sigma_{\epsilon,k}^i)$ analyst by analyst. Table E6 in Appendix E reports the median estimate of $\theta_k^i$ and $\sigma_{\epsilon,k}^i$ across forecasters for each series, which confirms our previous results. The estimated $\theta_k^i$ are also generally positively correlated across series: Table E7 shows that individuals who over-react more in forecasting certain series also tend to over-react more in forecasting other series.[25]

Finally, we compare the performance of the diagnostic expectations model with the performance of a model of overconfidence in which analysts perceive their noisy signals to be more informative than in reality. To this end, we repeat the previous simulation procedures, but estimate parameters $(\alpha_k, \sigma_{\epsilon,k})$, where $\sigma_{\epsilon,k}$ is the actual volatility of the noise but forecasters perceive it to be $\alpha_k \sigma_{\epsilon,k}$ (see Appendix E.3). In other words, $\alpha_k < 1$ captures the potential under-estimation of noise, which would inflate the Kalman gain. To facilitate comparison, we focus on AR(1) fundamentals, for which both overconfidence and diagnostic expectations can be collapsed into a single Kalman gain. Table E8 shows that the diagnostic expectations model performs generally better than overconfidence. For 14 out of 20 series, it achieves a smaller loss than the overconfidence model, and its loss is about a half of the latter's loss for the median series. This is mainly due to the fact that the overconfidence model bounds the extent of over-reaction by forcing the Kalman gain to be at most one. The diagnostic Kalman gain is instead allowed to exceed one, which is supported by the data for seven series, see Figure E2.

Overall, our structural estimation exercise yields three results. First, diagnostic distortions in professional forecasters' expectations are sizable and in the ballpark of previous estimates obtained in different contexts. Representativeness is thus a promising candidate for a robust psychological distortion in expectation formation. Second, the estimated distortions are quite robust to alternative assumptions. Third, the diagnostic expectation model does a good job at capturing variation in the data.

[25] Here we take heterogeneity of $(\theta_k^i, \sigma_{\epsilon,k}^i)$ as given, but it would be interesting in future work to explore its sources.

## 7. Conclusion

Using data from both the Blue Chip Survey and the Survey of Professional Forecasters, we have investigated how professional forecasters react to information using the methodology of Coibion and Gorodnichenko (2015). We have found that while under-reaction is the norm for the consensus forecast, as previously shown by CG (2015), for individual forecasters the norm is over-reaction to information, in the sense of forecast errors being (negatively) predictable from forecast revisions. We showed that individual-level overreaction is robust to a wide range of possible confounds. We then applied a psychologically founded model of belief formation, diagnostic expectations, to these data. We showed that diagnostic expectations generate over-reaction in individual forecasts, but if different forecasters see different information and/or use different models, the consensus forecast may exhibit under-reaction. The model thus reconciles these seemingly opposite patterns in the data.

The kernel of truth property of diagnostic expectations yields several additional predictions as to when we would see over-reaction in forecasts, and by how much, as a function of the series' underlying dynamics. These predictions are supported in the data, consistent with forecasters being forward looking and their judgment distorted by representativeness. Thus, individual forecasts are better described by diagnostic expectations than by mechanical models of extrapolation, such as adaptive expectations, which have been criticized by Lucas (1976) precisely on the grounds that people are assumed to be entirely backward looking. In fact, diagnostic expectations can serve as a micro-foundation of extrapolation, and the latter may reflect the former at a crude level.

Our approach enables us to document and reconcile distinctive features of expectations data. At the most basic level, it reconciles individual and consensus forecast patterns. Perhaps more subtly, diagnostic expectations when extended to the AR(2) context enable us to model expectations for hump shaped series. In this setting, diagnostic expectations capture some features of Natural Expectations (Fuster et al. 2010), such as exaggeration of short term persistence, but also yield over-reaction to long term reversal, which seems to be a feature of the data. Finally, unlike overconfidence, diagnostic expectations can generate effective Kalman gains above 1, which also seem to describe several series.

The ubiquity of over-reaction in individual macroeconomic forecasts helps reconcile several findings in finance and macroeconomics. Financial economics has put together a lot of evidence of over-reaction in individual markets, such as housing, credit, and equities. It would be puzzling if macroeconomic forecasts were the opposite, but as we show this is likely to be a consequence of aggregation. The extent of individual over-reaction estimated from the data is sizable. In our estimates of the diagnostic parameter, the predictable component of individual forecast errors entailed by over-reaction is comparable in magnitude to the rational response to news.

Of course, predictable forecast errors can also be influenced by model mis-specification. In fact, representativeness and mis-specification may work in tandem: in a complex world in which forecasters consider different models, data that is representative of a given model may induce the forecaster to attach excessive weight to it, as in Barberis, Shleifer and Vishny (1998). In this sense, learning may help explain the persistence of representativeness-induced errors, and this may be a way to understand the variation in the strength $\theta$ of diagnostic distortions across series.

We leave at least two important problems to future work. We have stressed over-reaction in individual time series, which seems to be the norm in our data, but other studies have also found rigidity in expectations (e.g., Bouchaud, Kruger, Landier, and Thesmar 2017). In this paper we have combined over-reaction with *aggregate* rigidity by incorporating representativeness in a noisy information setting. The reconciliation of anchoring with over-reaction to information based on psychological foundations remains an open problem.

We have not addressed the basic question: what are the macroeconomic consequences of diagnostic expectations? One might think at first sight that what matters for aggregate outcomes is consensus expectations, so all one needs to know is that consensus expectations under-react. This view misses two critical points. First, over-reaction by individual forecasters can influence aggregate outcomes by magnifying dispersion in beliefs. Belief heterogeneity plays an important role in several macroeconomic and finance models. The ability of optimists to lever up may create asset price bubbles and financial fragility (Geanakoplos 2010), or misallocation across firms or sectors. Second, at key junctures news may be correlated across different agents, for instance when major innovations are introduced, or when repeated news in the same direction provide highly informative evidence of large and persistent changes. In these cases, individual over-reaction will entail aggregate over-reaction, as shown by our analysis of public signals. Such aggregate over-reaction has been documented in the cross section, where extremely positive consensus forecasts of long term earnings growth of fast growing firms predict poor returns and revisions of expectations going forward (BGLS 2017). Aggregate over-reaction is also found in the time series, where buoyant credit markets and extreme optimism about firms' performance predict slowdowns in investment and GDP growth (Greenwood and Hanson 2013, Lopez-Salido et al. 2017, Gulen et al. 2018). Whether diagnostic expectations can offer a coherent and micro-founded theory for macroeconomic phenomena such as investment booms or business cycles is an important open question for future work.

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**Appendix: for online publication only**

## A. Proofs

**Proposition 1.** The data generating process is $x_t = \rho x_{t-1} + u_t$, where $u_t \sim \mathcal{N}(0, \sigma_u^2)$ i.i.d. over time. Forecaster $i$ observes a noisy signal $s_t^i = x_t + \epsilon_t^i$, where $\epsilon_t^i \sim \mathcal{N}(0, \sigma_\epsilon^2)$ is i.i.d. analyst specific noise. Rational expectations are obtained iteratively:

$$f(x_t | S_t^i) = f(x_t | S_{t-1}^i) \frac{f(s_t^i | x_t)}{f(s_t^i)}$$

The rational estimate thus follows $f(x_t | S_t^i) \sim \mathcal{N}\left(x_{t|t}^i, \frac{\Sigma_{t|t-1}\sigma_\epsilon^2}{\Sigma_{t|t-1} + \sigma_\epsilon^2}\right)$ with

$$x_{t|t}^i = x_{t|t-1}^i + \frac{\Sigma_{t|t-1}}{\Sigma_{t|t-1} + \sigma_\epsilon^2}\left(s_t^i - x_{t|t-1}^i\right),$$

where $\Sigma_{t|t-1}$ is the variance of the prior $f(x_t | S_{t-1}^i)$. The variance of $f(x_{t+1} | S_t^i)$ is:

$$\Sigma_{t+1|t} \equiv var_t(\rho x_t + u_{t+1}) = \rho^2 \frac{\Sigma_{t|t-1}\sigma_\epsilon^2}{\Sigma_{t|t-1} + \sigma_\epsilon^2} + \sigma_u^2,$$

so that the steady state variance $\Sigma = \Sigma_{t+1|t} = \Sigma_{t|t-1}$ is equal to:

$$\Sigma = \frac{-(1-\rho^2)\sigma_\epsilon^2 + \sigma_u^2 + \sqrt{[(1-\rho^2)\sigma_\epsilon^2 - \sigma_u^2]^2 + 4\sigma_\epsilon^2\sigma_u^2}}{2}$$

Beliefs about the current state are then described by $f(x_t | S_t^i) \sim \mathcal{N}\left(x_{t|t}^i, \frac{\Sigma\sigma_\epsilon^2}{\Sigma + \sigma_\epsilon^2}\right)$, where:

$$x_{t|t}^i = x_{t|t-1}^i + \frac{\Sigma}{\Sigma + \sigma_\epsilon^2}\left(s_t^i - x_{t|t-1}^i\right)$$

Let us now construct diagnostic expectations. For $s_t^i = x_{t|t-1}^i$ we have $x_{t|t}^i = x_{t|t-1}^i = \rho x_{t-1|t-1}^i$, so that $f(x_t | S_{t-1}^i \cup \{x_{t|t-1}^i\}) \sim \mathcal{N}\left(\rho x_{t-1|t-1}^i, \frac{\Sigma\sigma_\epsilon^2}{\Sigma + \sigma_\epsilon^2}\right)$. In light of the definition of diagnostic expectations in Equation (7), we have that the diagnostic distribution $f^\theta(x_t | S_t^i)$ fulfils:

$$\ln f^\theta(x_t | S_t^i) \propto -\frac{\left(x_t - x_{t|t}^i\right)^2}{2\frac{\Sigma\sigma_\epsilon^2}{\Sigma + \sigma_\epsilon^2}} - \theta \frac{\left(x_t - x_{t|t}^i\right)^2 - \left(x_t - x_{t|t-1}^i\right)^2}{2\frac{\Sigma\sigma_\epsilon^2}{\Sigma + \sigma_\epsilon^2}}$$

$$= -\frac{1}{2\frac{\Sigma\sigma_\epsilon^2}{\Sigma + \sigma_\epsilon^2}}\left[x_t^2 - 2x_t\left(x_{t|t}^i + \theta\left(x_{t|t}^i - x_{t|t-1}^i\right)\right) + \left(x_{t|t}^i\right)^2(1+\theta) - \theta\left(x_{t|t-1}^i\right)^2\right]$$

Given the normalization $\int f^{\theta}(x|S_t^i)dx = 1$, we find $f^{\theta}(x_t|S_t^i) \sim \mathcal{N}\left(x_{t|t}^{i,\theta}, \frac{\Sigma\sigma_\epsilon^2}{\Sigma+\sigma_\epsilon^2}\right)$ with $x_{t|t}^{i,\theta} = x_{t|t}^i + \theta\left(x_{t|t}^i - x_{t|t-1}^i\right)$. Using the definition of the Kalman filter $x_{t|t}^i$ we can write:

$$x_{t|t}^{i,\theta} = x_{t|t-1}^i + (1+\theta)\frac{\Sigma}{\Sigma+\sigma_\epsilon^2}\left(s_t^i - x_{t|t-1}^i\right). \blacksquare$$

**Proposition 2.** Denote by $K = \Sigma/(\Sigma + \sigma_\epsilon^2)$ the Kalman gain. The rational consensus estimate for the current state is then equal to $\int x_{t|t}^i\, di \equiv x_{t|t} = x_{t|t-1} + K\left(x_t - x_{t|t-1}\right)$.

The consensus estimation error under rationality is then equal to $x_t - x_{t|t} = \frac{1-K}{K}(x_{t|t} - x_{t|t-1})$. The diagnostic filter for an individual analyst is equal to $x_{t|t}^{i,\theta} = x_{t|t}^i + \theta(x_{t|t}^i - x_{t|t-1}^i)$, which implies a consensus equation $x_{t|t}^{\theta} = x_{t|t} + \theta(x_{t|t} - x_{t|t-1})$. We thus have:

$$x_t - x_{t|t}^{\theta} = \left(\frac{1-K}{K} - \theta\right)\left(x_{t|t} - x_{t|t-1}\right).$$

Note, in addition, that the diagnostic consensus forecast revision is equal to:

$$x_{t|t}^{\theta} - x_{t|t-1}^{\theta} = (1+\theta)\left(x_{t|t} - x_{t|t-1}\right) - \theta\rho\left(x_{t-1|t-1} - x_{t-1|t-2}\right).$$

Therefore, the consensus CG coefficient is given by:

$$\beta = \frac{cov\left(x_{t+h} - x_{t+h|t}^{\theta}, x_{t+h|t}^{\theta} - x_{t+h|t-1}^{\theta}\right)}{var\left(x_{t+h|t}^{\theta} - x_{t+h|t-1}^{\theta}\right)}$$

$$= \left(\frac{1-K}{K} - \theta\right) \cdot \frac{cov\left[x_{t|t} - x_{t|t-1}, (1+\theta)\left(x_{t|t} - x_{t|t-1}\right) - \theta\rho\left(x_{t-1|t-1} - x_{t-1|t-2}\right)\right]}{var\left[(1+\theta)\left(x_{t|t} - x_{t|t-1}\right) - \theta\rho\left(x_{t-1|t-1} - x_{t-1|t-2}\right)\right]}.$$

Where we have that:

$$\begin{aligned} &cov\left[x_{t|t} - x_{t|t-1}, (1+\theta)\left(x_{t|t} - x_{t|t-1}\right) - \theta\rho\left(x_{t-1|t-1} - x_{t-1|t-2}\right)\right] \\ &\qquad = (1+\theta)var\left(x_{t|t} - x_{t|t-1}\right) - \theta\rho cov\left(x_{t|t} - x_{t|t-1}, x_{t-1|t-1} - x_{t-1|t-2}\right), \end{aligned}$$

and

$$\begin{aligned} &var\left[(1+\theta)\left(x_{t|t} - x_{t|t-1}\right) - \theta\rho\left(x_{t-1|t-1} - x_{t-1|t-2}\right)\right] \\ &\qquad = [(1+\theta)^2 + \theta^2\rho^2]var\left(x_{t|t} - x_{t|t-1}\right) \\ &\qquad - 2\theta(1+\theta)\rho cov\left(x_{t|t} - x_{t|t-1}, x_{t-1|t-1} - x_{t-1|t-2}\right). \end{aligned}$$

To compute the covariance between adjacent rational revisions, note that $x_{t|t} = x_{t|t-1} + K(x_t - x_{t|t-1})$ and $x_{t|t-1} = x_{t|t-2} + K(\rho x_{t-1} - x_{t|t-2})$ imply that:

$$x_{t|t} - x_{t|t-1} = (1-K)\rho\left(x_{t-1|t-1} - x_{t-1|t-2}\right) + Ku_t.$$

As a result,

$$cov\left(x_{t|t} - x_{t|t-1}, x_{t-1|t-1} - x_{t-1|t-2}\right) = (1-K)\rho \cdot var\left(x_{t|t} - x_{t|t-1}\right)$$

Therefore:

$$\beta = \left(\frac{1-K}{K} - \theta\right) \cdot \frac{(1+\theta) - \theta\rho^2(1-K)}{[(1+\theta)^2 + \theta^2\rho^2] - 2\theta(1+\theta)\rho^2(1-K)},$$

which is positive if and only if $1 - K > \theta K$, namely, $\theta < \sigma_\epsilon^2/\Sigma$.

Consider individual level forecasts. The coefficient (at the individual level) of regressing forecast error on forecast revision is equal to:

$$\beta^p = \frac{cov\left(x_{t+h} - x_{t+h|t}^{i,\theta}, x_{t+h|t}^{i,\theta} - x_{t+h|t-1}^{i,\theta}\right)}{var\left(x_{t+h|t}^{i,\theta} - x_{t+h|t-1}^{i,\theta}\right)} =$$

$$\frac{cov\left(x_{t|t} - x_{t|t}^{i,\theta}, x_{t|t}^{i,\theta} - x_{t|t-1}^{i,\theta}\right)}{var\left(x_{t|t}^{i,\theta} - x_{t|t-1}^{i,\theta}\right)}$$

where $x_{t|t}^{i,\theta} - x_{t|t-1}^{i,\theta} = (1+\theta)\left(x_{t|t}^i - x_{t|t-1}^i\right) - \theta\rho\left(x_{t-1|t-1}^i - x_{t-1|t-2}^i\right)$. Because at the individual level $cov\left(x_{t|t}^i - x_{t|t-1}^i, x_{t|t-1}^i - x_{t|t-2}^i\right) = 0$, we immediately have that:

$$\beta^p = -\frac{\theta(1+\theta)}{(1+\theta)^2 + \rho^2\theta^2}.$$

■

**Corollary 1.** Denote by $p_i$ the precision of the private signal, by $p$ the precision of the public signal, by $p_f$ the precision of the lagged rational forecast $x_{t|t-1}^i$. The diagnostic filter at time $t$ is:

$$x_{t|t}^{i,\theta} = x_{t|t-1}^i + (1+\theta)\frac{p_i}{p_i + p + p_f}\left(s_t^i - x_{t|t-1}^i\right) + (1+\theta)\frac{p}{p_i + p + p_f}\left(s_t - x_{t|t-1}^i\right).$$

The precision $p_f$ of the forecast depends on the sum of the precisions $(p_i + p)$ and hence stays constant as we vary the relative precision of the public versus private signal.

Denote the Kalman gains as $K_1 = \frac{p_i}{p_i+p+p_f}$ and $K_2 = \frac{p}{p_i+p+p_f}$, and $K = K_1 + K_2$. The consensus Kalman filter can then be written as $x_{t|t} = x_{t|t-1} + K\left(x_t - x_{t|t-1}\right) + K_2 v_t$, while the diagnostic filter can be written as $x_{t|t}^{\theta} = x_{t|t} + \theta\left(x_{t|t} - x_{t|t-1}\right)$. The consensus coefficient is then:

$$\frac{cov\left(x_{t+h} - x_{t+h|t}^{\theta}, x_{t+h|t}^{\theta} - x_{t+h|t-1}^{\theta}\right)}{var\left(x_{t+h|t}^{\theta} - x_{t+h|t-1}^{\theta}\right)} = \frac{\rho^{2h} cov\left(x_t - x_{t|t}^{\theta}, x_{t|t}^{\theta} - x_{t|t-1}^{\theta}\right)}{\rho^{2h} var\left(x_{t|t}^{\theta} - x_{t|t-1}^{\theta}\right)}.$$

Consider first the numerator. Denote by $FR_t \equiv x_{t|t} - x_{t|t-1}$ the revision of the rational forecast of $x_t$ between $t$ and $t-1$. Then:

$$x_t - x_{t|t}^{\theta} = \left(\frac{1-K}{K} - \theta\right) FR_t - \frac{K_2}{K} v_t,$$

$$x_{t|t}^{\theta} - x_{t|t-1}^{\theta} = (1+\theta) FR_t - \theta\rho FR_{t-1}.$$

The difference between $x_{t|t} = x_{t|t-1} + K\left(x_t - x_{t|t-1}\right) + K_2 v_t$ and $x_{t|t-1} = x_{t|t-2} + K\left(\rho x_{t-1} - x_{t|t-2}\right) + K_2 \rho v_{t-1}$ reads:

$$FR_t = (1-K)\rho FR_{t-1} + K u_t + K_2 (v_t - \rho v_{t-1}),$$

which in turn implies:

$$cov(FR_t, FR_{t-1}) = (1-K)\rho \cdot var(FR_t) - \rho K_2^2 \sigma_v^2. \qquad (A.1)$$

It is also immediate to find that:

$$var(FR_t) = \frac{K^2 \sigma_u^2 + [(1+\rho^2) - 2\rho^2(1-K)] K_2^2 \sigma_v^2}{1 - [(1-K)\rho]^2}.$$

The numerator of the CG coefficient is then equal to:

$$cov\left(x_t - x_{t|t}^{\theta}, x_{t|t}^{\theta} - x_{t|t-1}^{\theta}\right) = \left(\frac{1-K}{K} - \theta\right) cov[FR_t, (1+\theta) FR_t - \theta\rho FR_{t-1}] - \frac{K_2}{K} (1+\theta) K_2 \sigma_v^2$$

$$= \left(\frac{1-K}{K} - \theta\right) \left[[1 + \theta - \theta\rho^2(1-K)] var(FR_t) + \theta\rho^2 K_2^2 \sigma_v^2\right] - \frac{(1+\theta) K_2^2 \sigma_v^2}{K} \qquad (A.2)$$

The denominator of the CG coefficient equals:

$$var\left(x_{t|t}^{\theta} - x_{t|t-1}^{\theta}\right) = var[(1+\theta) FR_t - \theta\rho FR_{t-1}]$$
$$= [(1+\theta)^2 + \theta^2\rho^2] var(FR_t) - 2\theta(1+\theta)\rho cov(FR_t, FR_{t-1})$$

which implies that:

$$\frac{var\left(x_{t|t}^{\theta}-x_{t|t-1}^{\theta}\right)}{[(1+\theta)^2+\theta^2\rho^2]}+\frac{2\theta(1+\theta)\rho}{[(1+\theta)^2+\theta^2\rho^2]}cov(FR_t,FR_{t-1})=var(FR_t). \qquad (A.3)$$

Putting $(A.3)$ together with $(A.1)$ one obtains:

$$cov(FR_t,FR_{t-1})=$$
$$=\frac{(1-K)\rho var\left(x_{t|t}^{\theta}-x_{t|t-1}^{\theta}\right)}{\left[1-\frac{2\theta(1-K)(1+\theta)\rho^2}{[(1+\theta)^2+\theta^2\rho^2]}\right][(1+\theta)^2+\theta^2\rho^2]}-\frac{\rho K_2^2\sigma_v^2}{\left[1-\frac{2\theta(1-K)(1+\theta)\rho^2}{[(1+\theta)^2+\theta^2\rho^2]}\right]} \qquad (A.4)$$

Using Equations $(A.2)$ and $(A.4)$ we find:

$$cov\left(x_t-x_{t|t}^{\theta},x_{t|t}^{\theta}-x_{t|t-1}^{\theta}\right)$$
$$=\left(\frac{1-K}{K}-\theta\right)\left[(1+\theta)\frac{var\left(x_{t|t}^{\theta}-x_{t|t-1}^{\theta}\right)}{(1+\theta)^2+\theta^2\rho^2}\right.$$
$$\left.+\theta\rho\left(\frac{2(1+\theta)^2}{(1+\theta)^2+\theta^2\rho^2}-1\right)cov(FR_t,FR_{t-1})\right]-\frac{(1+\theta)K_2^2\sigma_v^2}{K}=$$

$$=\beta_\infty var\left(x_{t|t}^{\theta}-x_{t|t-1}^{\theta}\right)-K_2^2\sigma_v^2\left[\frac{\theta\rho^2\left(\frac{1-K}{K}-\theta\right)\left(\frac{2(1+\theta)^2}{(1+\theta)^2+\theta^2\rho^2}-1\right)}{\left[1-\frac{2\theta(1-K)(1+\theta)\rho^2}{(1+\theta)^2+\theta^2\rho^2}\right]}+\frac{(1+\theta)}{K}\right],$$

where $\beta_\infty$ is the consensus coefficient obtained when the public signal is fully uninformative, namely $\sigma_u^2\to\infty$ and thus $K_2\to 0$. On the other hand using equation (A.3) this can be rewritten as:

$$var\left(x_{t|t}^{\theta}-x_{t|t-1}^{\theta}\right)=\frac{[(1+\theta)^2+\theta^2\rho^2-2\theta(1+\theta)(1-K)\rho^2]K^2\sigma_u^2}{1-[(1-K)\rho]^2}+AK_2^2\sigma_v^2,$$

where $A$ is a suitable positive coefficient. The CG coefficient is then equal to:

$$\frac{cov\left(x_t-x_{t|t}^{\theta},x_{t|t}^{\theta}-x_{t|t-1}^{\theta}\right)}{var\left(x_{t|t}^{\theta}-x_{t|t-1}^{\theta}\right)}=\beta_\infty-\frac{\left[\frac{\theta\rho^2\left(\frac{1-K}{K}-\theta\right)\left(\frac{2(1+\theta)^2}{(1+\theta)^2+\theta^2\rho^2}-1\right)}{1-\frac{2\theta(1-K)(1+\theta)\rho^2}{(1+\theta)^2+\theta^2\rho^2}}+\frac{(1+\theta)}{K}\right]K_2^2\sigma_v^2}{\frac{[(1+\theta)^2+\theta^2\rho^2-2\theta(1+\theta)(1-K)\rho^2]K^2\sigma_u^2}{1-[(1-K)\rho]^2}+AK_2^2\sigma_v^2}.$$

For given total informativeness $K$, the above expression falls in the precision of the public signal, namely as $K_2^2$ grows, if and only if:

$$\left[\frac{\theta\rho^2\left(\frac{1-K}{K}-\theta\right)\left(\frac{2(1+\theta)^2}{(1+\theta)^2+\theta^2\rho^2}-1\right)}{1-\frac{2\theta(1-K)(1+\theta)\rho^2}{(1+\theta)^2+\theta^2\rho^2}}+\frac{(1+\theta)}{K}\right]>0.$$

A sufficient condition for this to hold is that $\left(\frac{1-K}{K}-\theta\right)>0$, which is equivalent to $\beta_{\infty}>0$.

■

**Proof of Proposition 3**

The diagnostic expectation at time $t$ about $t+3$ is given by:

$$x_{t+3|t}^{i,\theta}=x_{t+3|t}^{i}+\theta FR_{t+3|t}^{i},$$

where $FR_{t+3|t}^{i}=\left(x_{t+3|t}^{i}-x_{t+3|t-1}^{i}\right)$ The diagnostic forecast revision $FR_{t+3|t}^{i,\theta}=\left(x_{t+3|t}^{i,\theta}-x_{t+3|t-1}^{i,\theta}\right)$ is therefore equal to:

$$FR_{t+3|t}^{i,\theta}=(1+\theta)FR_{t+3|t}^{i}-\theta FR_{t+3|t-1}^{i}.$$

The diagnostic forecast error $FE_{t+3|t}^{i,\theta}\equiv x_{t+3}-x_{t+3|t}^{i,\theta}$ is equal to:

$$FE_{t+3|t}^{i,\theta}=u_{t+3}-\theta FR_{t+3|t}^{i},$$

where $u_{t+3}$ is white noise. We then have:

$$cov\left(FE_{t+3|t}^{i,\theta},FR_{t+3|t}^{i,\theta}\right)=-\theta cov\left(FR_{t+3|t}^{i},(1+\theta)FR_{t+3|t}^{i}-\theta FR_{t+3|t-1}^{i}\right)$$

$$=-\theta(1+\theta)var\left(FR_{t+3|t}^{i}\right)$$

$$var\left(FR_{t+3|t}^{i,\theta}\right)=(1+\theta)^{2}var\left(FR_{t+3|t}^{i}\right)+\theta^{2}var\left(FR_{t+3|t-1}^{i}\right).$$

As a result, the relationship between forecast error and forecast revision is equal to:

$$FE_{t+3|t}^{i,\theta}=-\frac{\theta(1+\theta)}{(1+\theta)^{2}+\theta^{2}\frac{var\left(FR_{t+3|t-1}^{i}\right)}{var\left(FR_{t+3|t}^{i}\right)}}FR_{t+3|t}^{i,\theta}+v_{t+3}$$

By plugging Equation (13) in the text, we obtain:

$$FE_{t+3|t}^{i}=-\frac{\rho_{2}\theta(1+\theta)}{(1+\theta)^{2}+\theta^{2}\frac{var\left(FR_{t+3|t-1}^{i}\right)}{var\left(FR_{t+3|t}^{i}\right)}}FR_{t+2|t}^{i}-\frac{\rho_{1}\theta(1+\theta)}{(1+\theta)^{2}+\theta^{2}\frac{var\left(FR_{t+3|t-1}^{i}\right)}{var\left(FR_{t+3|t}^{i}\right)}}FR_{t+1|t}^{i}+v_{t+3},$$

If $FR_{t+2|t}^{i}$ and $FR_{t+1|t}^{i}$ are not collinear, the above equation can be estimated and it satisfies the prediction of Proposition 3. To conclude the proof, we therefore need to prove non-collinearity. Recall that the state follows AR(2) dynamics:

$$x_{t+1} = ax_t + bx_{t-1} + u_{t+1},$$

At time $t$, the agent observes two signals, one about the current state, $s_t^i = x_t + \epsilon_t^i$, and one about the past state $z_t^i = s_{t-1,t}^i = x_{t-1} + v_t^i$. Signals $\epsilon_t^i$ and $v_t^i$ are normal with precision $\epsilon$ and $v$. At time t, the agent forms estimates about $x_t$ and $x_{t-1}$. He then combines them to forecast about $x_{t+k}$, $k \geq 1$.

To ease notation we drop superscripts $i$ from the noise and the signals and subscript $t$ from the signals. Conditional on the signals, the density of the current state $f(x_t, x_{t-1}|s_t, z_t)$ satisfies:

$$-\ln f \propto \epsilon(1-\varphi^2)(s_t - x_t)^2 + v(1-\varphi^2)(z_t - x_{t-1})^2 + \left(x_t - x_{t|t-1}\right)^2 p + \left(x_{t-1} - x_{t-1|t-1}\right)^2 q$$
$$- 2\varphi\sqrt{pq}\left(x_t - x_{t|t-1}\right)\left(x_{t-1} - x_{t-1|t-1}\right)$$

where $p$ is the precision of $x_t$, $q$ is the precision of $x_{t-1}$, and $\varphi$ is their correlation.

Maximizing the likelihood $f$ with respect to $x_t$ and $x_{t-1}$ yields the first order conditions:

$$-2\epsilon(1-\varphi^2)\left(s_t - x_{t|t}\right) + 2p\left(x_{t|t} - x_{t|t-1}\right) - 2\varphi\sqrt{pq}\left(x_{t-1|t} - x_{t-1|t-1}\right) = 0$$

$$-2v(1-\varphi^2)\left(z_t - x_{t-1|t}\right) + 2q\left(x_{t-1|t} - x_{t-1|t-1}\right) - 2\varphi\sqrt{pq}\left(x_{t|t} - x_{t|t-1}\right) = 0$$

which identify the conditional estimates (the Kalman filter):

$$x_{t|t} = \frac{(1-\varphi^2)\frac{\epsilon}{p}s_t + x_{t|t-1} + \varphi\sqrt{\frac{q}{p}}FR_{t-1|t}}{(1-\varphi^2)\frac{\epsilon}{p} + 1},$$

$$x_{t-1|t} = \frac{(1-\varphi^2)\frac{v}{q}z_t + x_{t-1|t-1} + \varphi\sqrt{\frac{p}{q}}FR_{t|t}}{(1-\varphi^2)\frac{v}{q} + 1},$$

Where $FR_{s|t}$ is the forecast revision at $t$ for $x_s$. This further implies that:

$$FR_{t|t} = \frac{(1-\varphi^2)\frac{\epsilon}{p}\left(s_t - x_{t|t-1}\right) + \varphi\sqrt{\frac{q}{p}}FR_{t-1|t}}{(1-\varphi^2)\frac{\epsilon}{p} + 1},$$

$$FR_{t-1|t} = \frac{(1-\varphi^2)\frac{v}{q}\left(z_t - x_{t-1|t-1}\right) + \varphi\sqrt{\frac{p}{q}}FR_{t|t}}{(1-\varphi^2)\frac{v}{q} + 1}.$$

These equations imply that, provided $\varphi < 1$, the forecast revisions $FR_{t|t}$ and $FR_{t-1|t}$ are linearly independent combinations of the news $s_t - x_{t|t-1}$ and $z_t - x_{t-1|t-1}$:

$$FR_{t|t} = \frac{\left[(1-\varphi^2)\frac{v}{q}+1\right]\frac{\epsilon}{p}\left(s_t - x_{t|t-1}\right) + \varphi\sqrt{\frac{1}{qp}}v\left(z_t - x_{t-1|t-1}\right)}{\left[(1-\varphi^2)\frac{v}{q}+1\right]\frac{\epsilon}{p}+\frac{v}{q}+1},$$

$$FR_{t-1|t} = \frac{\left[(1-\varphi^2)\frac{\epsilon}{p}+1\right]\frac{v}{q}\left(z_t - x_{t-1|t-1}\right) + \varphi\sqrt{\frac{1}{qp}}\epsilon\left(s_t - x_{t|t-1}\right)}{\left[(1-\varphi^2)\frac{\epsilon}{p}+1\right]\frac{v}{q}+\frac{\epsilon}{p}+1}.$$

Therefore, $FR^i_{t|t}$ and $FR^i_{t-1|t}$ are not collinear. Since $FR^i_{t+1|t} = aFR^i_{t|t} + bFR^i_{t-1|t}$ and $FR^i_{t+2|t} = (a^2 + b)FR^i_{t|t} + abFR^i_{t-1|t}$, we conclude that $FR^i_{t+2|t}$ and $FR^i_{t+1|t}$ are not collinear.

■

## B. Variable Definitions

For each variable, we report the source survey, the survey time, the survey question, and the definitions of forecast variable, revision variable, and actuals.

### 1. NGDP_SPF

- Variable: Nominal GDP. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of nominal GDP in the current quarter and the next 4 quarters.
- Forecast: Nominal GDP growth from end of quarter *t*-1 to end of quarter *t*+3: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$, where *t* is the quarter of forecast and x is the level of GDP in a given quarter; $x_{t-1}$ uses the initial release of actual value in quarter *t*-1, which is available by the time of the forecast in quarter *t*.
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$.
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro data: initial release of $x_{t+3}$ published in quarter *t*+4 and initial release of $x_{t-1}$ published in quarter *t*.

### 2. RGDP_SPF

- Variable: Real GDP. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of real GDP in the current quarter and the next 4 quarters.
- Forecast: Real GDP growth from end of quarter *t*-1 to end of quarter *t*+3: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$, where *t* is the quarter of forecast and x is the level of GDP in a given quarter; $x_{t-1}$ uses the initial release of actual value in quarter *t*-1, which is available by the time of the forecast in quarter *t*.
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$.

- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro data: initial realease of $x_{t+3}$ published in quarter *t*+4 and initial release of $x_{t-1}$ published in quarter *t*.

3. RGDP_BC

- Variable: Real GDP. Source: Blue Chip.
- Time: End of the middle month in the quarter/beginning of the last month in the quarter.
- Question: Real GDP growth (annualized rate) in the current quarter and the next 4 to 5 quarters.
- Forecast: Real GDP growth from end of quarter *t*-1 to end of quarter *t*+3: $F_t(z_t + z_{t+1} + z_{t+2} + z_{t+3})/4$, where *t* is the quarter of forecast and $z_t$ is the annualized quarterly GDP growth in quarter *t*.
- Revision: $\frac{F_t(z_t+z_{t+1}+z_{t+2}+z_{t+3})}{4} - \frac{F_{t-1}(z_t+z_{t+1}+z_{t+2}+z_{t+3})}{4}$.
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro data: initial realease of $x_{t+3}$ published in quarter *t*+4 and initial release of $x_{t-1}$ published in quarter *t*.

4. PGDP_SPF

- Variable: GDP price deflator. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of GDP price deflator in the current quarter and the next 4 quarters.
- Forecast: GDP price deflator inflation from end of quarter *t*-1 to end of quarter *t*+3: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$ , where *t* is the quarter of forecast and x is the level of GDP price deflator in a given quarter; $x_{t-1}$ uses the initial release of actual value in quarter *t*-1, which is available by the time of the forecast in quarter *t*.
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$.
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro data: initial realease of $x_{t+3}$ published in quarter *t*+4 and initial release of $x_{t-1}$ published in quarter *t*.

5. RCONSUM_SPF

- Variable: Real consumption. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of real consumption in the current quarter and the next 4 quarters.
- Forecast: Growth of real consumption from end of quarter *t*-1 to end of quarter *t*+3: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$ , where *t* is the quarter of forecast and x is the level of real consumption in a given quarter; $x_{t-1}$ uses the initial release of actual value in quarter *t*-1, which is available by the time of the forecast in quarter *t*.
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$.
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro data: initial realease of $x_{t+3}$ published in quarter *t*+4 and initial release of $x_{t-1}$ published in quarter *t*.

6. RNRESIN_SPF

- Variable: Real non-residential investment. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of real non-residential investment in the current quarter and the next 4 quarters.

- Forecast: Growth of real non-residential investment from end of quarter *t*-1 to end of quarter *t*+3: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$ , where *t* is the quarter of forecast and x is the level of real non-residential investment in a given quarter; $x_{t-1}$ uses the initial release of actual value in quarter *t*-1, which is available by the time of the forecast in quarter *t*.
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$.
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro data: initial realease of $x_{t+3}$ published in quarter *t*+4 and initial release of $x_{t-1}$ published in quarter *t*.

7. RRESIN_SPF

- Variable: Real residential investment. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of real residential investment in the current quarter and the next 4 quarters.
- Forecast: Growth of real residential investment from end of quarter *t*-1 to end of quarter *t*+3: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$ , where *t* is the quarter of forecast and x is the level of real residential investment in a given quarter; $x_{t-1}$ uses the initial release of actual value in quarter *t*-1, which is available by the time of the forecast in quarter *t*.
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$.
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro data: initial realease of $x_{t+3}$ published in quarter *t*+4 and initial release of $x_{t-1}$ published in quarter *t*.

8. RGF_SPF

- Variable: Real federal government consumption. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of real federal government consumption in the current quarter and the next 4 quarters.
- Forecast: Growth of real federal government consumption from end of quarter *t*-1 to end of quarter *t*+3: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$ , where *t* is the quarter of forecast and x is the level of real federal government consumption in a given quarter; $x_{t-1}$ uses the initial release of actual value in quarter *t*-1, which is available by the time of the forecast in quarter *t*.
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$.
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro data: initial realease of $x_{t+3}$ published in quarter *t*+4 and initial release of $x_{t-1}$ published in quarter *t*.

9. RGSL_SPF

- Variable: Real state and local government consumption. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of real state and local government consumption in the current quarter and the next 4 quarters.
- Forecast: Growth of real state and local government consumption from end of quarter *t*-1 to end of quarter *t*+3: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$ , where *t* is the quarter of forecast and x is the level of real state and local government consumption in a given quarter; $x_{t-1}$ uses the initial release of actual value in quarter *t*-1, which is available by the time of the forecast in quarter *t*.

- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$.
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro data: initial realease of $x_{t+3}$ published in quarter *t*+4 and initial release of $x_{t-1}$ published in quarter *t*.

10. UNEMP_SPF

- Variable: Unemployment rate. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of average unemployment rate in the current quarter and the next 4 quarters.
- Forecast: Average quarterly unemployment rate in quarter *t*+3: $F_t x_{t+3}$, where *t* is the quarter of forecast and x is the level of unemployment rate in a given quarter.
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$.
- Actual: $x_{t+3}$, using real time macro data: initial realease of $x_{t+3}$ published in quarter *t*+4.

11. HOUSING_SPF

- Variable: Housing starts. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of housing starts in the current quarter and the next 4 quarters.
- Forecast: Growth of housing starts from quarter *t*-1 to quarter *t*+3: $\frac{F_t x_{t+3}}{x_{t-1}} - 1$ , where *t* is the quarter of forecast and x is the level of housing starts in a given quarter; $x_{t-1}$ uses the initial release of actual value in quarter *t*-1, which is available by the time of the forecast in quarter *t*.
- Revision: $\frac{F_t x_{t+3}}{x_{t-1}} - \frac{F_{t-1} x_{t+3}}{F_{t-1} x_{t-1}}$.
- Actual: $\frac{x_{t+3}}{x_{t-1}} - 1$, using real time macro data: initial realease of $x_{t+3}$ published in quarter *t*+4 and initial release of $x_{t-1}$ published in quarter *t*.

12. FF_BC

- Variable: Federal funds rate. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of average federal funds rate in the current quarter and the next 4 quarters.
- Forecast: Average quarterly 3-month federal funds rate in quarter *t*+3: $F_t x_{t+3}$, where *t* is the quarter of forecast and x is the level of federal funds rate in a given quarter.
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$.
- Actual: $x_{t+3}$.

13. TB3M_SPF

- Variable: 3-month Treasury rate. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of average 3-month Treasury rate in the current quarter and next 4 quarters.
- Forecast: Average quarterly 3-month Treasury rate in quarter *t*+3: $F_t x_{t+3}$, where *t* is the quarter of forecast and x is the level of 3-month Treasury rate in a given quarter.
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$.
- Actual: $x_{t+3}$.

14. TB3M_BC

- Variable: 3-month Treasury rate. Source: Blue Chip.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of average 3-month Treasury rate in the current quarter and next 4 quarters.
- Forecast: Average quarterly 3-month Treasury rate in quarter *t*+3: $F_t x_{t+3}$, where *t* is the quarter of forecast and x is the level of 3-month Treasury rate in a given quarter.
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$.
- Actual: $x_{t+3}$.

15. TN5Y_BC

- Variable: 5-year Treasury rate. Source: Blue Chip.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of average 5-year Treasury rate in the current quarter and the next 4 quarters.
- Forecast: Average quarterly 5-year Treasury rate in quarter *t*+3: $F_t x_{t+3}$, where *t* is the quarter of forecast and x is the level of 5-year Treasury rate in a given quarter.
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$.
- Actual: $x_{t+3}$.

16. TN10Y_SPF

- Variable: 10-year Treasury rate. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of average 10-year Treasury rate in the current quarter and next 4 quarters.
- Forecast: Average quarterly 10-year Treasury rate in quarter *t*+3: $F_t x_{t+3}$, where *t* is the quarter of forecast and x is the level of 10-year Treasury rate in a given quarter.
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$.
- Actual: $x_{t+3}$.

17. TN10Y_BC

- Variable: 10-year Treasury rate. Source: Blue Chip.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of average 10-year Treasury rate in the current quarter and next 4 quarters.
- Forecast: Average quarterly 10-year Treasury rate in quarter *t*+3: $F_t x_{t+3}$, where *t* is the quarter of forecast and x is the level of 10-year Treasury rate in a given quarter.
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$.
- Actual: $x_{t+3}$.

18. AAA_SPF

- Variable: AAA corporate bond rate. Source: SPF.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of average AAA corporate bond rate in the current quarter and next 4 quarters.
- Forecast: Average quarterly AAA corporate bond rate in quarter *t*+3: $F_t x_{t+3}$, where *t* is the quarter of forecast and x is the level of AAA corporate bond rate in a given quarter.
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$.
- Actual: $x_{t+3}$.

19. AAA_BC

- Variable: AAA corporate bond rate. Source: Blue Chip.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of average AAA corporate bond rate in the current quarter and next 4 quarters.
- Forecast: Average quarterly AAA corporate bond rate in quarter *t*+3: $F_t x_{t+3}$, where *t* is the quarter of forecast and x is the level of AAA corporate bond rate in a given quarter.
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$.
- Actual: $x_{t+3}$.

20. BAA_BC

- Variable: BAA corporate bond rate. Source: Blue Chip.
- Time: Around the 3rd week of the middle month in the quarter.
- Question: The level of average BAA corporate bond rate in the current quarter and next 4 quarters.
- Forecast: Average quarterly BAA corporate bond rate in quarter *t*+3: $F_t x_{t+3}$, where *t* is the quarter of forecast and x is the level of BAA corporate bond rate in a given quarter.
- Revision: $F_t x_{t+3} - F_{t-1} x_{t+3}$.
- Actual: $x_{t+3}$.

## C. Robustness Checks

Table C1. Consensus CG Regressions
Kernel Bandwidth Selection for Newey-West Standard Errors

This table shows the standard errors and *t*-statistics (in brackets) in consensus time series CG regressions, for Newey-West standard errors with different lag lengths (0 to 8).

| Variable | | Kernel Lag Length $l$ (s.e. and [$t$]) | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | $\beta$ | $l = 0$ | $l = 1$ | $l = 2$ | $l = 3$ | $l = 4$ | $l = 5$ | $l = 6$ | $l = 7$ | $l = 8$ |
| Nominal GDP (SPF) | 0.48 | 0.24 | 0.27 | 0.29 | 0.30 | 0.29 | 0.29 | 0.28 | 0.27 | 0.26 |
| | | [1.99] | [1.79] | [1.66] | [1.62] | [1.62] | [1.65] | [1.72] | [1.79] | [1.87] |
| Real GDP (SPF) | 0.45 | 0.27 | 0.26 | 0.27 | 0.28 | 0.29 | 0.29 | 0.29 | 0.29 | 0.29 |
| | | [1.70] | [1.72] | [1.67] | [1.61] | [1.59] | [1.57] | [1.56] | [1.56] | [1.57] |
| Real GDP (BC) | 0.59 | 0.36 | 0.39 | 0.39 | 0.38 | 0.36 | 0.36 | 0.36 | 0.36 | 0.37 |
| | | [1.65] | [1.51] | [1.49] | [1.56] | [1.61] | [1.64] | [1.63] | [1.61] | [1.59] |
| GDP Price Index Inflation (SPF) | 1.21 | 0.25 | 0.31 | 0.35 | 0.39 | 0.41 | 0.43 | 0.44 | 0.44 | 0.44 |
| | | [4.87] | [3.90] | [3.42] | [3.12] | [2.95] | [2.84] | [2.78] | [2.74] | [2.73] |
| Real Consumption (SPF) | 0.18 | 0.24 | 0.26 | 0.28 | 0.29 | 0.29 | 0.30 | 0.31 | 0.31 | 0.31 |
| | | [0.78] | [0.71] | [0.66] | [0.64] | [0.62] | [0.61] | [0.60] | [0.59] | [0.58] |
| Real Non-Residential Investment (SPF) | 0.93 | 0.31 | 0.34 | 0.34 | 0.32 | 0.31 | 0.30 | 0.30 | 0.29 | 0.29 |
| | | [2.95] | [2.75] | [2.74] | [2.85] | [2.96] | [3.05] | [3.13] | [3.17] | [3.21] |
| Real Residential Investment (SPF) | 1.26 | 0.37 | 0.40 | 0.37 | 0.34 | 0.33 | 0.34 | 0.33 | 0.33 | 0.32 |
| | | [3.39] | [3.12] | [3.43] | [3.74] | [3.78] | [3.75] | [3.78] | [3.85] | [3.88] |
| Real Federal Government | -0.44 | 0.27 | 0.26 | 0.25 | 0.24 | 0.24 | 0.23 | 0.24 | 0.24 | 0.25 |
| Consumption (SPF) | | [-1.67] | [-1.72] | [-1.76] | [-1.82] | [-1.88] | [-1.89] | [-1.86] | [-1.82] | [-1.80] |
| Real Federal Government | -0.16 | 0.17 | 0.20 | 0.21 | 0.22 | 0.22 | 0.23 | 0.22 | 0.22 | 0.22 |
| Consumption (SPF) | | [-0.94] | [-0.81] | [-0.77] | [-0.75] | [-0.73] | [-0.72] | [-0.73] | [-0.73] | [-0.73] |
| Housing Start (SPF) | 0.45 | 0.28 | 0.30 | 0.32 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 | 0.34 |
| | | [1.61] | [1.50] | [1.41] | [1.35] | [1.33] | [1.32] | [1.33] | [1.34] | [1.35] |
| Unemployment (SPF) | 0.82 | 0.18 | 0.21 | 0.22 | 0.22 | 0.22 | 0.21 | 0.21 | 0.21 | 0.21 |
| | | [4.51] | [3.91] | [3.72] | [3.71] | [3.74] | [3.82] | [3.89] | [3.92] | [3.96] |
| Fed Funds Rate (BC) | 0.61 | 0.19 | 0.22 | 0.22 | 0.21 | 0.20 | 0.19 | 0.18 | 0.18 | 0.18 |
| | | [3.22] | [2.79] | [2.80] | [2.94] | [3.09] | [3.21] | [3.27] | [3.35] | [3.40] |
| Fed Funds Rate (BC) | 0.71 | 0.21 | 0.22 | 0.22 | 0.20 | 0.17 | 0.16 | 0.15 | 0.16 | 0.15 |
| | | [3.34] | [3.16] | [3.23] | [3.58] | [4.07] | [4.54] | [4.59] | [4.57] | [4.67] |
| 3M Treasury Rate (BC) | 0.67 | 0.18 | 0.20 | 0.20 | 0.18 | 0.16 | 0.15 | 0.14 | 0.14 | 0.13 |
| | | [3.62] | [3.28] | [3.37] | [3.68] | [4.04] | [4.38] | [4.63] | [4.88] | [5.10] |
| 5Y Treasury Rate (BC) | 0.05 | 0.21 | 0.22 | 0.21 | 0.17 | 0.15 | 0.14 | 0.13 | 0.12 | 0.11 |
| | | [0.22] | [0.20] | [0.22] | [0.26] | [0.31] | [0.34] | [0.36] | [0.39] | [0.41] |
| 10Y Treasury Rate (SPF) | -0.01 | 0.24 | 0.25 | 0.23 | 0.19 | 0.17 | 0.16 | 0.15 | 0.14 | 0.14 |
| | | [-0.04] | [-0.04] | [-0.04] | [-0.05] | [-0.06] | [-0.06] | [-0.06] | [-0.07] | [-0.07] |
| 10Y Treasury Rate (BC) | -0.06 | 0.22 | 0.23 | 0.20 | 0.17 | 0.15 | 0.14 | 0.14 | 0.13 | 0.12 |
| | | [-0.27] | [-0.26] | [-0.29 | [-0.35] | [-0.39] | [-0.41] | [-0.43] | [-0.46] | [-0.48] |
| AAA Corporate Bond Rate (SPF) | -0.01 | 0.23 | 0.24 | 0.23 | 0.23 | 0.22 | 0.23 | 0.23 | 0.23 | 0.23 |
| | | [-0.03] | [-0.03] | [-0.03] | [-0.04] | [-0.04] | [-0.03] | [-0.03] | [-0.03] | [-0.04] |
| AAA Corporate Bond Rate (BC) | 0.21 | 0.18 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 | 0.20 |
| | | [1.14] | [1.04] | [1.04] | [1.05] | [1.05] | [1.05] | [1.04] | [1.06] | [1.06] |
| BAA Corporate Bond Rate (BC) | -0.14 | 0.26 | 0.22 | 0.22 | 0.19 | 0.17 | 0.16 | 0.16 | 0.15 | 0.15 |
| | | [-0.53] | [-0.65] | [-0.66] | [-0.75] | [-0.81] | [-0.87] | [-0.90] | [-0.92] | [-0.94] |

## Table C2. Forecaster-by-Forecaster CG Regressions

Column "Pooled" shows the pooled panel CG regressions at the individual level (same as Table 3 column (4)). Column "By Forecaster (Median)" shows the median coefficient from forecaster-by-forecaster CG regressions; column "By Forecaster (%<0)" shows the fraction of forecasters where the coefficient is negative. For the forecaster-by-forecaster coefficients, we restrict to forecasters with at least 10 forecasts available.

| Variable | Pooled | By Forecaster | |
|---|---|---|---|
| | | Median | %<0 |
| Nominal GDP (SPF) | -0.26 | -0.14 | 0.63 |
| Real GDP (SPF) | -0.23 | -0.09 | 0.54 |
| Real GDP (BC) | 0.12 | 0.00 | 0.50 |
| GDP Price Index Inflation (SPF) | -0.07 | -0.11 | 0.57 |
| Real Consumption (SPF) | -0.34 | -0.20 | 0.83 |
| Real Non-Residential Investment (SPF) | 0.01 | -0.20 | 0.58 |
| Real Residential Investment (SPF) | -0.02 | -0.32 | 0.64 |
| Real Federal Government Consumption (SPF) | -0.62 | -0.43 | 0.95 |
| Real State&Local Govt Consumption (SPF) | -0.71 | -0.50 | 0.91 |
| Housing Start (SPF) | 0.33 | 0.24 | 0.35 |
| Unemployment (SPF) | -0.25 | -0.19 | 0.73 |
| Fed Funds Rate (BC) | 0.15 | 0.21 | 0.27 |
| 3M Treasury Rate (SPF) | 0.24 | -0.02 | 0.51 |
| 3M Treasury Rate (BC) | 0.20 | 0.20 | 0.28 |
| 5Y Treasury Rate (BC) | -0.12 | -0.18 | 0.82 |
| 10Y Treasury Rate (SPF) | -0.18 | -0.18 | 0.58 |
| 10Y Treasury Rate (BC) | -0.17 | -0.29 | 0.86 |
| AAA Corporate Bond Rate (SPF) | -0.21 | -0.35 | 0.85 |
| AAA Corporate Bond Rate (BC) | -0.17 | -0.28 | 0.84 |
| BAA Corporate Bond Rate (BC) | -0.28 | -0.34 | 0.95 |

## Table C3. Last Forecast Revision

The Table shows the pooled panel CG regressions at the consensus and individual levels (pooled panel regression) for horizon $h = 0$ (same as Table 3 columns 1, 2, 4, and 5).

| Variable | $\beta_1$ | *t*-stat | $\beta_1^p$ | *t*-stat |
|---|---|---|---|---|
| Nominal GDP (SPF) | -0.05 | -1.03 | -0.14 | -2.35 |
| Real GDP (SPF) | 0.06 | 1.01 | -0.06 | -1.15 |
| Real GDP (BC) | 0.16 | 1.04 | -0.05 | -0.54 |
| GDP Price Index Inflation (SPF) | -0.01 | -0.14 | -0.10 | -2.14 |
| Real Consumption (SPF) | -0.12 | -1.62 | -0.23 | -3.59 |
| Real Non-Residential Investment (SPF) | 0.03 | 0.50 | -0.06 | -0.85 |
| Real Residential Investment (SPF) | 0.23 | 3.74 | 0.04 | 0.99 |
| Real Federal Government Consumption (SPF) | -0.08 | -0.74 | -0.22 | -3.58 |
| Real State&Local Govt Consumption (SPF) | -0.18 | -2.84 | -0.26 | -3.33 |
| Housing Start (SPF) | 0.23 | 6.55 | 0.03 | 1.20 |
| Unemployment (SPF) | 0.42 | 5.95 | 0.09 | 2.09 |
| Fed Funds Rate (BC) | -0.03 | -0.89 | -0.11 | -2.25 |
| 3M Treasury Rate (SPF) | 0.17 | 7.30 | 0.00 | 0.21 |
| 3M Treasury Rate (BC) | 0.01 | 0.40 | -0.18 | -2.01 |

| | | | | |
|---|---|---|---|---|
| 5Y Treasury Rate (BC) | 0.12 | 3.27 | 0.00 | 0.04 |
| 10Y Treasury Rate (SPF) | 0.15 | 3.34 | -0.05 | -1.86 |
| 10Y Treasury Rate (BC) | 0.04 | 1.50 | -0.01 | -0.52 |
| AAA Corporate Bond Rate (SPF) | 0.07 | 1.29 | -0.10 | -2.15 |
| AAA Corporate Bond Rate (BC) | -0.10 | -2.46 | -0.16 | -4.74 |
| BAA Corporate Bond Rate (BC) | 0.04 | 1.26 | -0.09 | -3.43 |

## D. Non-Normal Shocks and Particle Filtering

In the main text, we assume that both the innovations of the latent process, $u_t$, and the measurement error for each expert, $\epsilon_t$, follow normal distributions. In this case, as all the posterior distributions are normal, the Kalman filter provides the closed form expression for the posterior densities. However, many processes for macro and financial variables may have heavy tails and more closely follow, for example, a $t$-distribution. In this appendix, we relax the normality assumption and verify the model predictions with fundamental shocks following fat tailed *t*-distributions.

In the non-normal case, while the point estimates of the Kalman filter still minimize mean-squared error (MSE), the mean and covariance estimates of the filter are no longer sufficient to determine the posterior distribution. Given that our formulation of diagnostic expectations involves a reweighting of the likelihood function, we require more than the posterior mean and variance to properly compute the diagnostic expectation distribution. Accordingly, we apply particle filtering to analyze expectations under non-normal shocks.

### D.1 Particle Filtering: Motivation and Set-Up

We start with the processes in Equations (3) and (4):

$$s_t^i = x_t + \epsilon_t^i, \quad x_t = \rho x_{t-1} + u_t$$

where $x_t$ is the fundamental process and $s_t^i$ is forecaster $i$'s noisy signal. In Section 3, the shocks to these processes are assumed to be normal. In the following, we analyze the case where the shock to the fundamental process $u_t$ follows a *t*-distribution.

Since the $t$-distribution is no longer conjugate to normal noise, one can no longer get closed form solutions. Instead, we draw from the posterior distribution in a Monte Carlo approach using the particle filter, a popular algorithm for simulating Bayesian inference on Hidden Markov Models (Doucet, de Freitas, and Gordon, 2001; Doucet and Johansen 2011). We first briefly describe this approach, then formulate the application to diagnostic expectations, and finally show simulation results for the CG forecast error on forecast revision regressions.

Particle filtering builds on the idea of importance sampling. Specifically, suppose we wish to estimate the expectation of $f(x)$, where $x$ is distributed according to $p$; we are not able to sample from $p$, or in general unable to compute its precise density, but can compute $p$ up to a proportionality constant: $p(x) = \frac{1}{Z}\tilde{p}(x)$, where $Z = \int \tilde{p}(x)\, dx$ is the (unknown) normalizing constant. If we can sample from an arbitrary density $q$, we can use the following importance sampling mechanism to indirectly sample from $p$: for each sample from $q$, $x_n$, compute the importance weight $w_n = \frac{\tilde{p}(x_n)}{q(x_n)}$ and resample from $x_n$according to probability proportional to the weights. It is easy to see that the average of the weights estimates the proportionality factor $Z$ : $\frac{1}{N}\sum_{n=1}^{N} w(x_n) \to \int \frac{\tilde{p}(x)}{q(x)} \cdot q(x)dx = \int \tilde{p}(x)\, dx = Z$ . Consequently, one can easily derive that the resampled $x_n$ converge in distribution to $p$: given any measurable function $\phi$, the expectation of $\phi(x)$for the resampled $x$ converges to $E_P\phi$:

$$\int \sum_{i=1}^{N} \phi(x_i)\ \frac{w(x_i)}{N}\frac{q(x_{1:N})}{Z}\ dx_{1:N} = \frac{1}{Z}\frac{1}{N}\sum_{i=1}^{N} \int \phi(x_i)\ \frac{\tilde{p}(x_i)}{q(x_i)} q(x_i) q(x_{-i})\, dx_{1:N} = \frac{1}{N}\sum_{i=1}^{N} E_p[\phi(x)] = E_p\phi$$

The algorithm above, called the sample-importance resample (SIR) algorithm, can be summarized in the following steps:

1. Sample $N$ particles from $q$, denoted as $x_{1:N}$
2. For each $x_i$, compute$w_i = \frac{\tilde{p}(x_i)}{q(x_i)}$.
3. Resample according to probability $\propto w_i$

For the hidden Markov Process model, the above idea generalizes to give us a quick algorithm to sample from the filtering density $p(x_n|s_{1:n})$. Like the Kalman filter, the idea is to proceed inductively, using the following forward equation:

$$p(x_n|s_{1:n}) = \frac{g(s_n|x_n)\, p(x_n|s_{1:n-1})}{p(s_n|\, s_{1:n-1})} = \frac{\int g(s_n|x_n)\, f(x_n|x_{n-1})\, p(x_{n-1}|s_{1:n-1}) ds_{1:n-1} dx_{n-1}}{p(s_n|s_{1:n-1})}$$

By induction, suppose that we have samples from the previous filtered distribution $p(x_{n-1}|s_{1:n-1})$. Now, given a (conditional) proposal $q(x_n|x_{n-1}, s_{1:n})$for each sample, the recursive equality above suggests the resampling weights: $w(x_n \mid x_{n-1}) = \frac{g(s_n|x_n) f(x_n|x_{n-1})}{q(x_n \mid x_{n-1}, s_{1:n})}$. For the base case, where we have only seen the data point $s_1$, our filtered density $p(x_1|s_1)$is the standard Bayesian posterior, which can be sampled via importance sampling.

The particle filter algorithm refers to this extension of the SIR algorithm to the sequential setting. The procedure is as follows:

1. At time n = 1, generate $N$i.i.d. samples from a default proposal $q$.
2. Compute for each sample the weights $w(x_i) = \frac{\mu(x_i)\, g(s_1 \mid x_i)}{q(x_i)}$
3. Resample according to the weights, and store the sample.
4. For $n \geq 2$: for each $x_{n-1}^i$ in the sample, propose $x_n^i$ according to $q(x_n|x_{n-1} = x^i{}_{n-1}, s_{1:n})$
5. Compute for each $x_n^i$ the weights $w(x_n{}^i) = \frac{g(s_n|x_n{}^i)\, f(x_n{}^i|x_{n-1}{}^i)}{q(x_n|x_{n-1}=x^i{}_{n-1}, s_{1:n})}$
6. Resample according to the weights, save as $x_n^i$.

Finally, we need to specify the proposal density $q(x_n|x_{n-1} = x^i{}_{n-1}, s_{1:n})$. It is well-known that the optimal proposal density should be the conditional distribution $p(x_n|x_{n-1} = x^i{}_{n-1}, s_n)$. If the latent Markov process is a simple AR(1) process with normal innovation, one can analytically derive the optimal proposal density $p(x_n|x_{n-1} = x^i{}_{n-1}, s_n)$.

$$x_n|x_{n-1}, s_n \sim N(\frac{\sigma_\epsilon^2}{\sigma_\epsilon^2 + \sigma_u^2} \rho\, x_{n-1} + \frac{\sigma_u^2}{\sigma_\epsilon^2 + \sigma_u^2}\, s_n, \frac{\sigma_\epsilon^2 \sigma_u^2}{\sigma_\epsilon^2 + \sigma_u^2}) = N(\bar{\mu}, \bar{\Sigma})$$

While this result is only precise for normal processes, we shall still use $\bar{\mu}, \bar{\Sigma}$as location and scale parameters for our proposal, which is now $t$-distributed. If the original innovations have $d$degrees of freedom, our proposal will have $\frac{d+2}{2}$degrees of freedom, which have much thicker tails.

### D.2 Application to Diagnostic Expectations

To analyze the case of diagnostic expectations, we only need to re-adjust the resampling weights by a simple likelihood ratio, as given by the following proposition:

**Proposition D1** *Let $s^*(s_{1:n-1})$ be the predictive expectation of $s_n$ given $s_{1:n-1}$. The representativeness $R(x_n|s_{1:n}) = \frac{p(x_n|\, s_{1:n})}{p(x_n|\, s_{1:n-1}, s^*)}$ can be simplified to the likelihood ratio $\frac{g(s_n|x_n)}{g(s^*|x_n)}$, up to a proportionality constant independent of $x_n$.*

*Proof.* By Bayes' rule: $R(x_n|s_{1:n}) = \frac{p(x_n|\, s_{1:n})}{p(x_n|\, s_{1:n-1}, s^*)} = \frac{p(s_n|s_{1:n-1}, x_n) \cdot p(x_n|\, s_{1:n-1})}{p(s_n \mid s_{1:n-1})} \cdot (\frac{p(s^* \mid s_{1:n-1}) \cdot p(x_n|\, s_{1:n-1})}{p(s^*|\, s_{1:n-1})})^{-1}$.

Due to the Markov property, $p(s_n|s_{1:n-1}, x_n) = g(s_n|x_n)$ and $p(s_n = s^* |s_{1:n-1}, x_n) = g(s^*|x_n)$. Plugging this in, we obtain:

$$R(x_n\,|s_{1:n}) = \frac{g(s_n \mid x_n) \cdot p(x_n|s_{1:n-1})}{p(s_n|s_{1:n-1})} \cdot (\frac{g(s^*|x_n) \cdot p(x_n \mid s_{1:n-1})}{p(s^*|s_{1:n-1})})^{-1} = \frac{g(s_n|x_n)}{g(s^*\,|x_n)} \cdot \frac{p(s^*|s_{1:n-1})}{p(s_n|s_{1:n-1})}$$

The latter term $\frac{p(s^*|s_{1:n-1})}{p(s_n|s_{1:n-1})}$ is constant with respect to $x_n$, as desired.

As we have assumed that $g$ is a normal density, the likelihood ratio simplifies to:

$$R(x_n|s_{1:n}) \propto exp\left(-\frac{(x_n - s_n)^2}{2{\sigma_\epsilon}^2} + \frac{(x_n - s^*)^2}{2{\sigma_\epsilon}^2}\right) = exp\left(\frac{(s_n - s^*)x_n}{{\sigma_\epsilon}^2}\right)$$

Hence, if the observed signal $s_n$ is greater than $s^*$(a positive news), the forecaster puts an exponentially heavier weight on larger values of $x_n$, and for negative news, he overweights smaller values of $x_n$, which is in line with over-reaction to most recent news.

With the particle filter, we get the exponential reweighting by multiplying the original weights $w(x_n{}^i) = \frac{g(s_n|x_n{}^i)\, f(x_n{}^i|x_{n-1}{}^i)}{q(x_n|x_{n-1}=x^i{}_{n-1},s_{1:n})}$ with the extra exponential factor $exp(\frac{(s_n - s^*)x_n}{\sigma_\epsilon{}^2})$. As with the basic particle filter algorithm discussed above, we need to specify our proposal density $q$ to target regions of high density. We would like to target $\tilde{q} \propto exp(\frac{(s_n - s^*)x_n}{\sigma_\epsilon{}^2})p(x_n|x_{n-1,}s_n)$, which we estimate by first assuming the normal model. Given that $x_n|x_{n-1}, s_n \sim N(\bar{\mu}, \bar{\Sigma})$ in the normal model, the diagnostic expectation is given by a shift of the posterior density by $\frac{\theta \cdot \bar{\Sigma} \cdot (s_n - s^*)}{\sigma_\epsilon{}^2}$. Thus we set the location and scale parameter of our proposals as $\mu_{diag} = \bar{\mu} + \frac{\theta \cdot \bar{\Sigma}(s_n - s^*)}{\sigma_\epsilon{}^2}$, $\Sigma_{diag} = \bar{\Sigma}$, where $\bar{\mu}, \bar{\Sigma}$ are the location and scale parameters for our original proposal. As before, we have $df_q = \frac{df + 2}{2}$ to ensure that our proposal has heavier tails than the target distribution. To summarize, the algorithm is as follows:

1. From the original particle filter, estimate $s^* = \rho\mu_{n-1}$, with $\mu_{n-1}$ our predictive mean $E[x_{n-1}|\, s_{1:n-1}]$, estimated by the mean of our particles $x^i{}_{n-1}$.
2. Propose according to a $t$-distribution with location parameter $\mu_{diag} = \bar{\mu} + \frac{\theta \cdot \bar{\Sigma}(s_n - s^*)}{\sigma_\epsilon{}^2}$, $\Sigma_{diag} = \bar{\Sigma}$, $df_q = \frac{df + 2}{2}$.
3. For each proposal, resample with weights$w_{diag}(x_n|x_{n-1,}s_n) = \frac{g(s_n|x_n{}^i)\, f(x_n{}^i|x_{n-1}{}^i)}{q(x_n|x_{n-1}=x^i{}_{n-1},s_{1:n})} exp(\frac{(s_n - s^*)x_n}{\sigma_\epsilon{}^2})$

**D.3 Results**

In the simulations below, we set $\rho = 0.9, \sigma_u = 0.2, \sigma_\epsilon = 0.2$, and $0 \leq \theta \leq 1.5$. We find that the basic qualitative characteristics of diagnostic expectations are robust to fat tails. As Figure D1 shows, the diagnostic expectation over-reacts to news, relative to the rational benchmark.

We then check the results of the CG forecast error on forecast revision regressions. Figure D2 shows the distribution of bootstrapped regression coefficients. Panel A first checks the case with normal shocks, the particle filter simulation agrees with the predicted coefficients $-\frac{\theta(1+\theta)}{(1+\theta)^2 + \theta^2\,\rho^2}$ using the Kalman filter. Panel B then shows the case where the shocks are heavy-tailed. We see that the coefficients for the heavy-tailed shocks are more negative compared to the predicted values for the normal case. Specifically, as the rational posterior exhibits heavier tail, the exponential reweighting of the diagnostic expectation results in greater mass located on the extreme values of the exponential weight, and hence greater shift in the diagnostic expectation. This effect is only present for diagnostic expectations — for rational expectations i.e. $\theta = 0$, we do not observe a divergence between normal and fat tailed distributions.

**Figure D1. Response to News under Rational and Diagnostic Expectations**

This plot shows the belief distribution in response to news, with fat tailed fundamental shocks and particle filtering. The black line plots the distribution with no news. The dashed red line plots the distribution in response to news with rational expectations. The dotted blue line plots the distribution in response to news with diagnostic expectations.

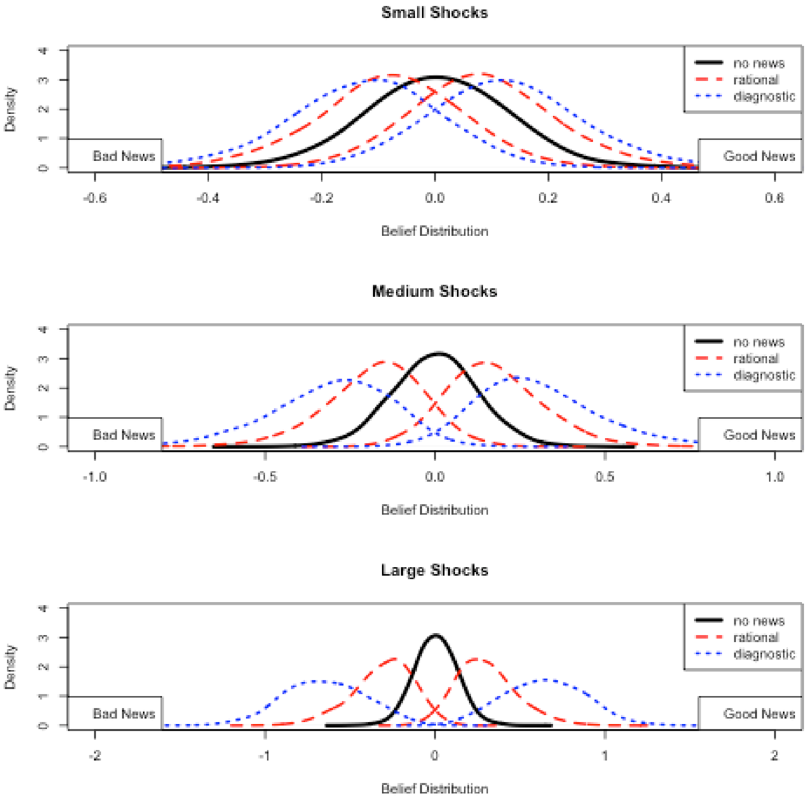
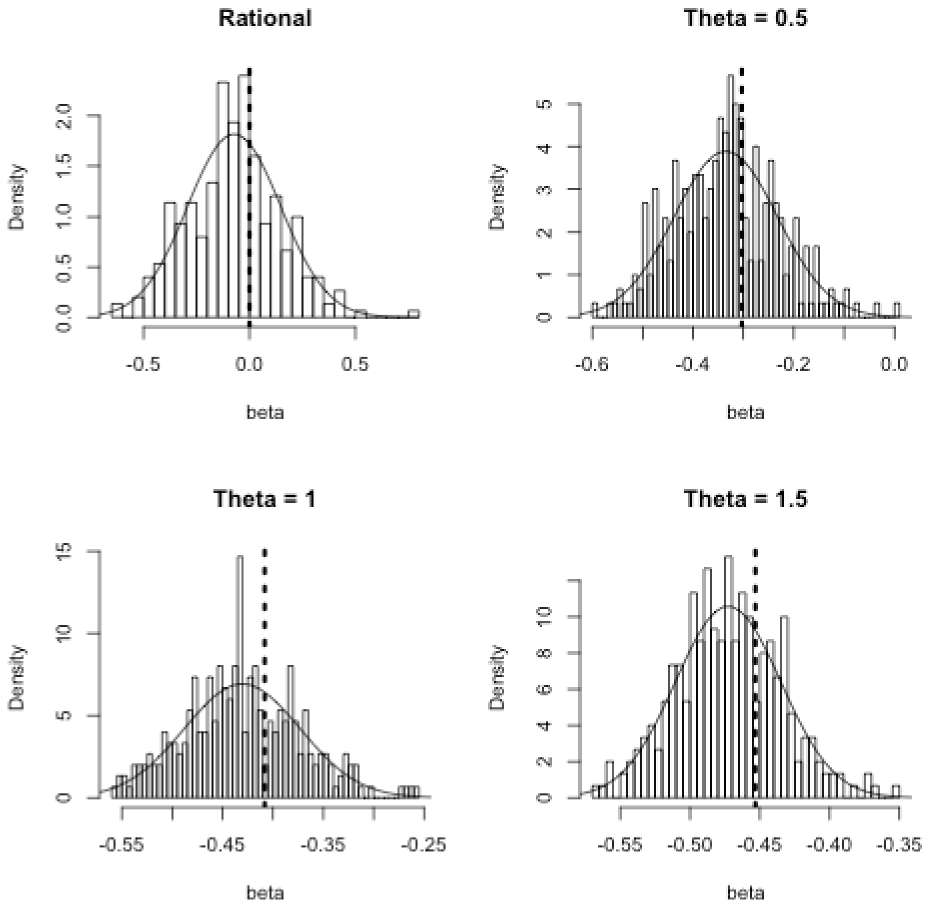


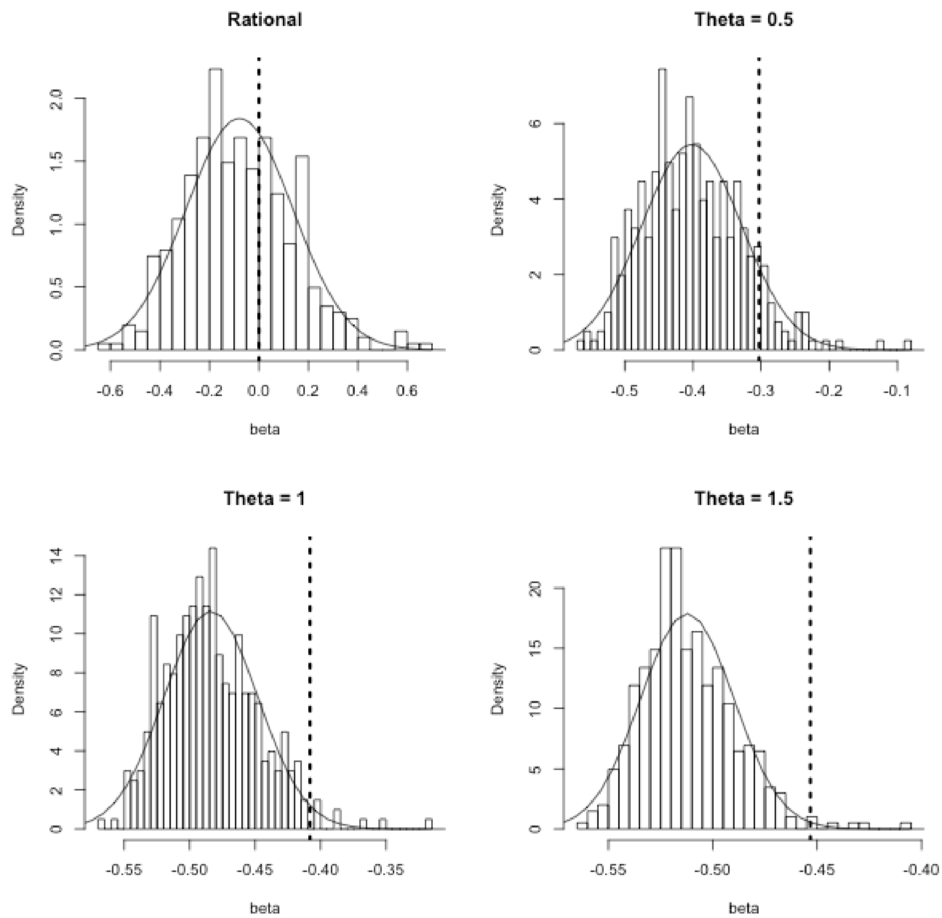
**Figure D2. Individual CG Coefficients with Normal and Fat Tailed Shocks**

This plot shows the distribution of coefficients from individual level (pooled panel) CG regressions. Panel A analyzes the case for normal shocks and Panel B analyzes the case for fat tailed shocks, both using the particle filter. Each simulation has 80 time periods and each plot shows the coefficients from 300 simulations. The dashed vertical line indicates $-\frac{\theta(1+\theta)}{(1+\theta)^2+\theta^2\rho^2}$, which is the coefficient predicted by normal shocks and Kalman filtering.

Panel A. Normal Shocks



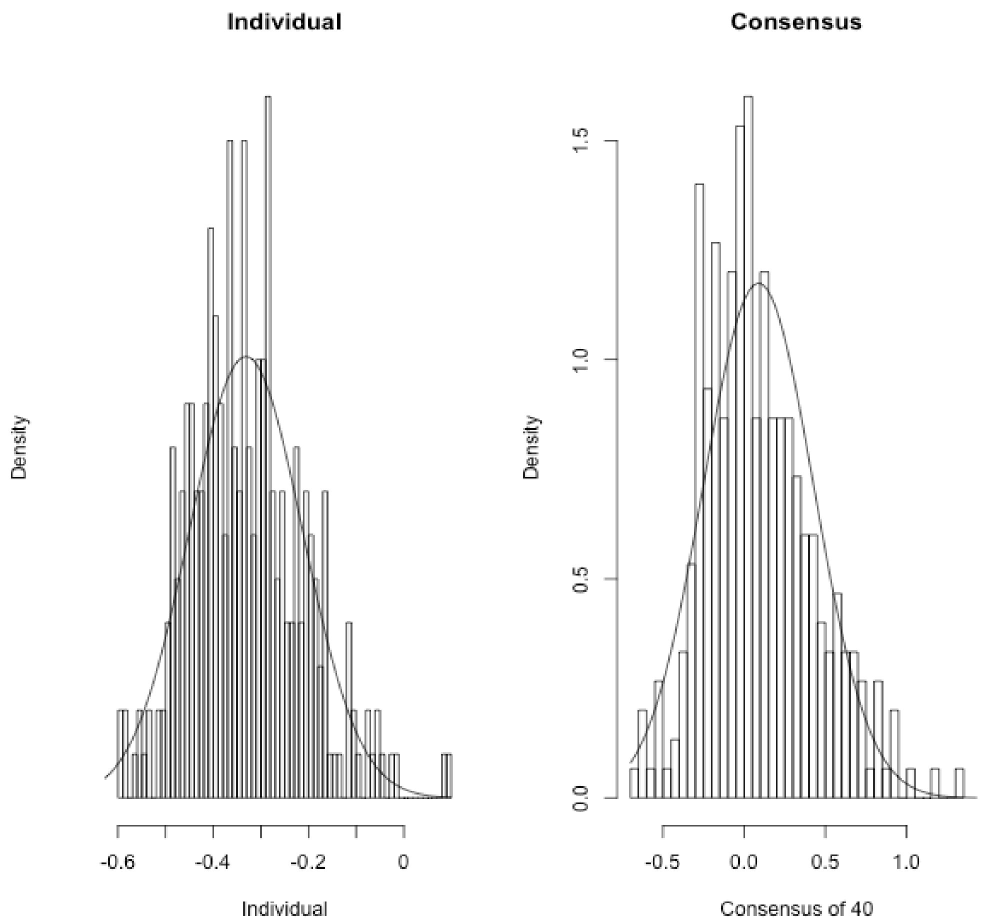
Panel B. Fat Tailed Shocks, df = 2.5




Finally, Figure D3 replicates the results for the contrast between regressions using individual and consensus forecasts. The general qualitative result is that there is much less over-reaction in consensus forecasts. On average, we get slight under-reaction in consensus forecasts. Under-reaction occurs when the noise ${\sigma_\epsilon}^2$is sufficiently high and individual over-reaction parameter $\theta$ is sufficiently low. Figure D3 plots the case where $\sigma_\epsilon = 1, \theta = 0.1$, which shows robustly positive consensus regression coefficients for 40 forecasters and 80 time periods.

**Figure D3. Individual vs. Consensus Diagnostic Expectations**

This plot shows the distribution of coefficients from individual level (pooled panel) and consensus CG regressions, using fat tailed shocks and particle filtering. The left panel shows the coefficients from pooled individual level regressions, and the right panel shows the coefficients from consensus regressions. Each draw has 40 forecasters and 80 time periods; there are 300 draws.



## E. Model Estimation: supporting information

Kalman inference for AR(1) processes was described in the text, see Equations (8,9). We now describe Kalman inference for an AR(2) process. The state variable is a vector $\vec{x}_t = (x_t, x_{t-1})$ which evolves according to $\vec{x}_t = A\vec{x}_{t-1} + W_t$, with transition matrix $A = \begin{bmatrix} \rho_1 & \rho_2 \\ 1 & 0 \end{bmatrix}$ and disturbance $W_t = \begin{bmatrix} u_t & 0 \\ 0 & 0 \end{bmatrix}$ with $u_t \sim \mathcal{N}(0, \sigma_u^2)$ i.i.d. across time. The observation equation is $s_t = C\vec{x}_t + \epsilon_t$ with $C = [1 \quad 0]$ and $\epsilon_t \sim \mathcal{N}(0, \sigma_\epsilon^2)$ i.i.d. across time. The Kalman filter can then be written:

$$x_{t|t}^{i,\theta} = x_{t|t-1}^{i} + (1+\theta)\frac{\Sigma_{11}}{\Sigma_{11} + \sigma_\epsilon^2}\left(s_t^i - \rho_1 x_{t-1|t-1}^i - \rho_2 x_{t-2|t-1}^i\right), \qquad \text{(E1)}$$

where $\Sigma_{11}$ is the first entry of the steady state variance matrix of beliefs at $t-1$ about $x_t$, which is given by the following condition:

$$\Sigma = \mathrm{A}\Sigma\mathrm{A}^T + W - A\Sigma\mathrm{C}(\mathrm{C}^T\Sigma\mathrm{C} + \sigma_\epsilon^2)^{-1}\mathrm{C}^T\Sigma\mathrm{A}^T$$

where $W = \begin{bmatrix} \sigma_u^2 & 0 \\ 0 & 0 \end{bmatrix}$. The above expression does not have a closed form solution. One can solve for $\Sigma$ by numerically solving for the unique root of a polynomial, or iterating the above equation until the value stabilizes. In practice, we solve for the root and confirm that the above condition is satisfied. Once we have the value of $\Sigma$, one can iterate equation (E1) to generate forecasts for our SMM estimation procedure.

**Table E1. Estimates of AR(1) and AR(2) Parameters for Fundamentals**

This table shows the autocorrelation and standard deviation parameters of the fundamental processes, for both AR(1) and AR(2) specifications. The parameters are estimated for the same time period when the corresponding forecasts are available.

| | AR(1) | | AR(2) | | |
|---|---|---|---|---|---|
| | $\rho$ | $\sigma_u$ | $\rho_1$ | $\rho_2$ | $\sigma_u$ |
| Nominal GDP (SPF) | 0.92 | 1.08 | 1.27 | -0.37 | 1.00 |
| Real GDP (SPF) | 0.87 | 1.12 | 1.33 | -0.51 | 0.96 |
| Real GDP (BC) | 0.86 | 0.77 | 1.24 | -0.43 | 0.69 |
| GDP Price Index Inflation (SPF) | 0.98 | 0.49 | 1.45 | -0.48 | 0.43 |
| Real Consumption (SPF) | 0.87 | 0.72 | 0.89 | -0.02 | 0.72 |
| Real Non-Residential Investment (SPF) | 0.88 | 3.43 | 1.25 | -0.41 | 3.14 |
| Real Residential Investment (SPF) | 0.88 | 5.68 | 1.27 | -0.42 | 5.01 |
| Real Federal Government Consumption (SPF) | 0.78 | 2.83 | 0.74 | 0.06 | 2.82 |
| Real State&Local Govt Consumption (SPF) | 0.90 | 0.77 | 0.85 | 0.04 | 0.77 |

| | | | | | |
|---|---|---|---|---|---|
| Housing Start (SPF) | 0.85 | 11.80 | 1.14 | -0.34 | 11.12 |
| Unemployment (SPF) | 0.96 | 0.37 | 1.48 | -0.53 | 0.31 |
| Fed Funds Rate (BC) | 0.99 | 0.50 | 1.53 | -0.55 | 0.42 |
| 3M Treasury Rate (SPF) | 0.95 | 0.58 | 1.22 | -0.27 | 0.55 |
| 3M Treasury Rate (BC) | 0.99 | 0.45 | 1.54 | -0.56 | 0.37 |
| 5Y Treasury Rate (BC) | 0.97 | 0.44 | 1.17 | -0.21 | 0.42 |
| 10Y Treasury Rate (SPF) | 0.97 | 0.38 | 1.17 | -0.21 | 0.37 |
| 10Y Treasury Rate (BC) | 0.97 | 0.38 | 1.21 | -0.25 | 0.37 |
| AAA Corporate Bond Rate (SPF) | 0.97 | 0.38 | 1.16 | -0.20 | 0.36 |
| AAA Corporate Bond Rate (BC) | 0.97 | 0.33 | 1.19 | -0.22 | 0.32 |
| BAA Corporate Bond Rate (BC) | 0.95 | 0.37 | 1.01 | -0.07 | 0.37 |

**Table E2.** Variance of Forecast Errors and Forecast Revisions: Data and Model Baseline Specification

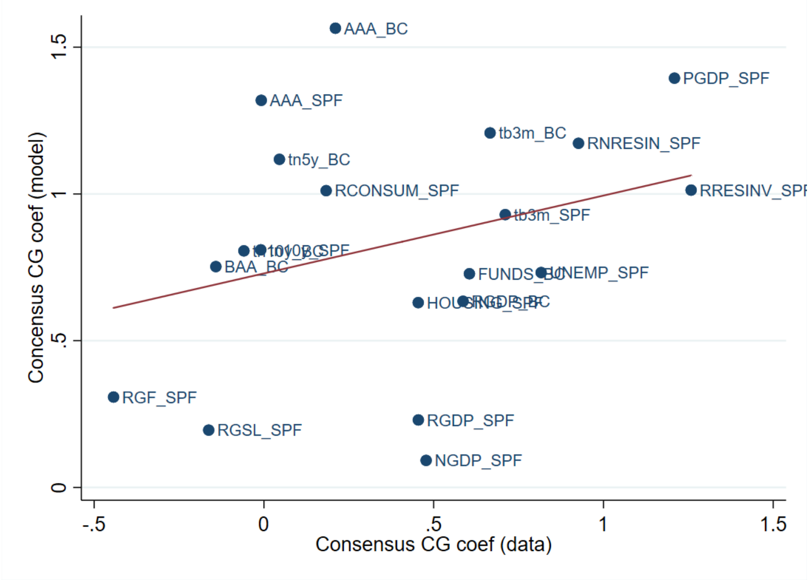
This table shows forecast error variance, $\sigma_{FE}^2$, and forecast revision variance $\sigma_{FR}^2$ in the data and in the estimated model, as well as the absolute log difference between them. The model is estimated using either the AR(2) version or the AR(1) version, based on properties of the fundamental process shown in Table 6.

| | Forecast Error Variance $\sigma_{FE}^2$ | | | Forecast Revision Variance $\sigma_{FR}^2$ | | |
|---|---|---|---|---|---|---|
| | Data | Model | Log Dif | Data | Model | Log Dif |
| Nominal GDP (SPF) | 4.67 | 4.60 | 0.016 | 1.91 | 1.83 | 0.042 |
| Real GDP (SPF) | 4.58 | 4.53 | 0.012 | 1.60 | 1.64 | 0.023 |
| Real GDP (BC) | 1.89 | 1.89 | 0.003 | 0.39 | 0.39 | 0.005 |
| GDP Price Index Inflation (SPF) | 2.53 | 2.45 | 0.032 | 1.03 | 1.08 | 0.047 |
| Real Consumption (SPF) | 2.03 | 1.97 | 0.029 | 0.85 | 0.90 | 0.061 |
| Real Non-Residential Investment (SPF) | 42.38 | 42.56 | 0.004 | 9.63 | 9.88 | 0.025 |
| Real Residential Investment (SPF) | 98.67 | 97.18 | 0.015 | 24.29 | 24.70 | 0.017 |
| Real Federal Government Consumption (SPF) | 15.89 | 15.99 | 0.006 | 6.03 | 6.07 | 0.007 |
| Real State&Local Govt Consumption (SPF) | 4.14 | 3.37 | 0.207 | 2.60 | 2.73 | 0.046 |
| Housing Start (SPF) | 488.41 | 499.82 | 0.023 | 133.61 | 133.32 | 0.002 |
| Unemployment (SPF) | 0.75 | 0.73 | 0.026 | 0.21 | 0.17 | 0.188 |
| Fed Funds Rate (BC) | 1.38 | 1.38 | 0.001 | 0.61 | 0.60 | 0.013 |
| 3M Treasury Rate (SPF) | 1.42 | 1.42 | 0.003 | 0.49 | 0.48 | 0.003 |
| 3M Treasury Rate (BC) | 1.33 | 1.34 | 0.005 | 0.52 | 0.51 | 0.005 |
| 5Y Treasury Rate (BC) | 0.98 | 0.99 | 0.007 | 0.41 | 0.41 | 0.009 |
| 10Y Treasury Rate (SPF) | 0.68 | 0.68 | 0.011 | 0.27 | 0.27 | 0.012 |
| 10Y Treasury Rate (BC) | 0.70 | 0.70 | 0.008 | 0.28 | 0.28 | 0.008 |
| AAA Corporate Bond Rate (SPF) | 0.87 | 0.88 | 0.009 | 0.37 | 0.37 | 0.014 |
| AAA Corporate Bond Rate (BC) | 0.81 | 0.80 | 0.017 | 0.40 | 0.39 | 0.021 |
| BAA Corporate Bond Rate (BC) | 0.63 | 0.63 | 0.002 | 0.27 | 0.27 | 0.003 |

Figure 6 in the text showed the model-predicted individual level CG coefficients were strongly correlated with those estimated in the pooled regressions. Figure E1 shows the corresponding predictions for the consensus CG coefficients.

**Figure E1.** Consensus CG Coefficients using Estimated $\theta$ and $\sigma_\epsilon$

The figure plots consensus CG coefficients in the baseline specification of the model (with estimated $\theta$ and $\sigma_\epsilon$) in the y-axis, and CG coefficients in the survey data in the x-axis. Results for each series are estimated using the AR(1) or AR(2) version of the diagnostic expectations model based on the properties of the actuals according to Table 6.




## E.1 Alternative Specifications: AR(1) and Particle Filtering

We present here the results of the specification where series are assumed to follow an AR(1) with normal shocks (denoted AR(1)), as well as an AR(1) specification where we allow for non-normal shocks (denoted AR(1) particle). The particle filter procedure used for estimating the latter case is explained in detail in Appendix D.

**Table E3.** Variance of Forecast Errors and Forecast Revisions
AR(1) and AR(1) Particle Specifications

This table shows forecast error variance, $\sigma^2_{FE}$, and forecast revision variance $\sigma^2_{FR}$ in the data and in the estimated model. The model is estimated using the AR(1) version as well as AR(1) with non-normal fundamental shocks and particle filtering.

| | Forecast Error Variance $\sigma^2_{FE}$ | Forecast Revision Variance $\sigma^2_{FR}$ |
|---|---|---|

| | Data | AR1 | AR1 Particle | Data | AR1 | AR1 Particle |
|---|---|---|---|---|---|---|
| Nominal GDP (SPF) | 4.67 | 4.76 | 4.73 | 1.91 | 1.97 | 2.10 |
| Real GDP (SPF) | 4.58 | 5.13 | 5.21 | 1.60 | 1.65 | 1.74 |
| Real GDP (BC) | 1.89 | 1.87 | 1.90 | 0.39 | 0.39 | 0.42 |
| GDP Price Index Inflation (SPF) | 2.53 | 2.45 | 2.63 | 1.03 | 1.00 | 1.02 |
| Real Consumption (SPF) | 2.03 | 1.97 | 1.90 | 0.85 | 0.90 | 0.83 |
| Real Non-Residential Investment (SPF) | 42.38 | 42.07 | 41.84 | 9.63 | 9.73 | 9.80 |
| Real Residential Investment (SPF) | 98.67 | 101.81 | 103.96 | 24.29 | 24.57 | 28.26 |
| Real Federal Government Consumption (SPF) | 15.89 | 15.99 | 16.78 | 6.03 | 6.07 | 6.69 |
| Real State&Local Govt Consumption (SPF) | 4.14 | 3.37 | 3.55 | 2.60 | 2.73 | 2.50 |
| Housing Start (SPF) | 488.41 | 498.63 | 517.97 | 133.61 | 141.43 | 127.69 |
| Unemployment (SPF) | 0.75 | 0.75 | 0.75 | 0.21 | 0.21 | 0.22 |
| Fed Funds Rate (BC) | 1.38 | 1.35 | 1.36 | 0.61 | 0.60 | 0.61 |
| 3M Treasury Rate (SPF) | 1.42 | 1.41 | 1.45 | 0.49 | 0.48 | 0.51 |
| 3M Treasury Rate (BC) | 1.33 | 1.32 | 1.39 | 0.52 | 0.51 | 0.55 |
| 5Y Treasury Rate (BC) | 0.98 | 0.97 | 0.95 | 0.41 | 0.40 | 0.39 |
| 10Y Treasury Rate (SPF) | 0.68 | 0.68 | 0.68 | 0.27 | 0.27 | 0.27 |
| 10Y Treasury Rate (BC) | 0.70 | 0.71 | 0.69 | 0.28 | 0.28 | 0.28 |
| AAA Corporate Bond Rate (SPF) | 0.87 | 0.79 | 0.80 | 0.37 | 0.39 | 0.32 |
| AAA Corporate Bond Rate (BC) | 0.81 | 0.79 | 0.80 | 0.40 | 0.41 | 0.40 |
| BAA Corporate Bond Rate (BC) | 0.63 | 0.63 | 0.66 | 0.27 | 0.27 | 0.27 |

**Table E4**. Estimates of $\theta$ for AR(1) and AR(1) particle specifications

This table shows estimates of $\theta$ as well as the 95% confidence interval based on block bootstrap (bootstrapping forecasters with replacement). The model is estimated using the AR(1) version as well as AR(1) with non-normal fundamental shocks (particle filtering).

| | AR1 | 95% CI | AR1 particle | 95% CI |
|---|---|---|---|---|
| Nominal GDP (SPF) | 0.64 | (0.45, 0.80) | 0.68 | (0.37, 1.00) |
| Real GDP (SPF) | 0.82 | (0.60, 1.15) | 1.10 | (0.58, 1.84) |
| Real GDP (BC) | 0.37 | (0.30, 0.50) | 0.37 | (0.26, 0.58) |
| GDP Price Index Inflation (SPF) | 0.97 | (0.60, 1.40) | 0.40 | (0.26, 0.58) |
| Real Consumption (SPF) | 1.56 | (0.95, 2.00) | 1.60 | (0.63, 2.38) |
| Real Non-Residential Investment (SPF) | 0.43 | (0.30, 0.50) | 0.41 | (0.27, 0.56) |
| Real Residential Investment (SPF) | 0.38 | (0.30, 0.50) | 0.33 | (0.26, 0.58) |
| Real Federal Government Consumption (SPF) | 1.18 | (0.80, 1.55) | 1.01 | (0.66, 1.38) |
| Real State&Local Govt Consumption (SPF) | 2.80 | (1.30, 3.90) | 3.04 | (1.28, 5.00) |
| Housing Start (SPF) | 0.68 | (0.50, 0.95) | 0.42 | (0.24, 0.55) |
| Unemployment (SPF) | 0.46 | (0.40, 0.50) | 0.46 | (0.42, 0.58) |
| Fed Funds Rate (BC) | 0.62 | (0.50, 0.70) | 0.46 | (0.37, 0.58) |
| 3M Treasury Rate (SPF) | 0.43 | (0.40, 0.50) | 0.27 | (0.26, 0.37) |
| 3M Treasury Rate (BC) | 0.57 | (0.50, 0.70) | 0.31 | (0.26, 0.37) |
| 5Y Treasury Rate (BC) | 0.54 | (0.40, 0.60) | 0.56 | (0.47, 0.58) |

| | | | | |
|---|---|---|---|---|
| 10Y Treasury Rate (SPF) | 0.59 | (0.50, 0.60) | 0.56 | (0.47, 0.58) |
| 10Y Treasury Rate (BC) | 0.55 | (0.50, 0.60) | 0.54 | (0.47, 0.58) |
| AAA Corporate Bond Rate (SPF) | 0.76 | (0.70, 0.90) | 0.63 | (0.47, 0.74) |
| AAA Corporate Bond Rate (BC) | 1.10 | (0.90, 1.30) | 1.10 | (0.84, 1.24) |
| BAA Corporate Bond Rate (BC) | 0.73 | (0.64, 0.80) | 0.46 | (0.38, 0.55) |

**Table E5.** CG Coefficients: Data vs Model

This table shows regressions of CG coefficients in the data (LHS) on CG coefficients in the estimated model (RHS) across different series. The model is estimated using the baseline version (primarily AR(2)), the AR(1) version, and AR(1) with non-normal fundamental shocks (particle filtering). Panel A uses individual CG coefficient from forecaster-level panel regressions. Panel B uses consensus CG coefficient from time series regressions of consensus forecasts.

Panel A. Individual CG

| | Data CG (Individual) | | |
|---|---|---|---|
| | (1) | (2) | (3) |
| Model CG (Baseline) | 1.043*** | | |
| | (0.168) | | |
| Model CG (AR1) | | 0.772*** | |
| | | (0.154) | |
| Model CG (AR1 particle) | | | 0.706*** |
| | | | (0.152) |
| Constant | 0.0832* | -0.0294 | -0.0336 |
| | (0.0397) | (0.0433) | (0.0443) |
| Observations | 20 | 20 | 20 |
| R-squared | 0.686 | 0.605 | 0.561 |

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

Panel B. Consensus CG

| | Data CG (Consensus) | | |
|---|---|---|---|
| | (1) | (2) | (3) |
| Model CG (Baseline) | 0.345 | | |
| | (0.260) | | |
| Model CG (AR1) | | 0.138 | |
| | | (0.214) | |
| Model CG (AR1 particle) | | | 0.342 |
| | | | (0.264) |
| Constant | 0.102 | 0.288 | 0.165 |
| | (0.222) | (0.174) | (0.195) |
| Observations | 20 | 20 | 20 |
| R-squared | 0.092 | 0.020 | 0.077 |

Robust standard errors in parentheses
*** p<0.01, ** p<0.05, * p<0.1

## E.2 Forecaster Level Results

Table 8 in Section 6 presents the pooled estimates of the latent parameters $\theta_k$ and $\sigma_{\epsilon,k}$ that were allowed to vary by series $k$ but not by individual forecaster. We also estimate the model at the individual level, and obtain estimated parameters $(\theta_k^i, \sigma_{\epsilon,k}^i)$ for each forecaster and a given series. Table E6 shows the median estimates of these parameters at the individual level in the baseline specification of our model. Results are similar using other specifications.

**Table E6.** Model Estimation Results by Forecaster

This table shows the median of individual-level $\theta^i$ and $\sigma_\epsilon^i$ (normalized by $\sigma_u$) estimates, as well as the CG coefficients in the model with estimated $\theta^i$ and $\sigma_\epsilon^i$ . For the model CG coefficients, we use the forecaster level estimates ($\theta^i$, $\sigma_\epsilon^i$), together with the fundamental process and its parameters, to generate model-implied forecasts for each forecaster and each time period where the forecaster is available; we then run panel CG regressions and consensus CG regressions using the model-based forecasts. Results for each series are estimated using the AR(1) or AR(2) version of the diagnostic expectations model based on the properties of the actuals according to Table 6.

| | Median $\theta^i$ | Median $\sigma_\epsilon^i/\sigma_u$ | Individual CG | Consensus CG |
|---|---|---|---|---|
| Nominal GDP (SPF) | 0.32 | 1.08 | -0.20 | 0.29 |
| Real GDP (SPF) | 0.69 | 0.78 | -0.26 | 0.10 |
| Real GDP (BC) | 0.63 | 1.43 | -0.30 | 0.35 |
| GDP Price Index Inflation (SPF) | 0.59 | 3.42 | -0.25 | 1.06 |
| Real Consumption (SPF) | 0.64 | 2.71 | -0.36 | 0.95 |
| Real Non-Residential Investment (SPF) | 0.44 | 1.55 | -0.15 | 1.15 |
| Real Residential Investment (SPF) | 0.42 | 1.68 | -0.22 | 0.90 |
| Real Federal Government Consumption (SPF) | 0.73 | 1.71 | -0.36 | 0.11 |
| Real State&Local Govt Consumption (SPF) | 0.91 | 4.50 | -0.47 | 0.38 |
| Housing Start (SPF) | 1.37 | 2.11 | -0.42 | 0.60 |
| Unemployment (SPF) | -0.17 | 0.67 | 0.26 | 1.00 |
| Fed Funds Rate (BC) | -0.01 | 1.24 | -0.04 | 0.61 |
| 3M Treasury Rate (SPF) | 0.21 | 1.60 | 0.04 | 1.18 |
| 3M Treasury Rate (BC) | -0.03 | 1.87 | 0.01 | 1.08 |
| 5Y Treasury Rate (BC) | 0.37 | 2.49 | -0.21 | 1.10 |
| 10Y Treasury Rate (SPF) | 0.47 | 2.55 | -0.35 | 0.68 |
| 10Y Treasury Rate (BC) | 0.26 | 2.74 | -0.30 | 0.88 |
| AAA Corporate Bond Rate (SPF) | 0.63 | 5.21 | -0.36 | 1.20 |
| AAA Corporate Bond Rate (BC) | 0.76 | 5.20 | -0.35 | 1.47 |
| BAA Corporate Bond Rate (BC) | 0.69 | 2.50 | -0.36 | 0.70 |

Table E7 shows that there is a consistent correlation between individual level estimates of $\theta^i$ across series.

**Table E7.** Rank Correlations for $\theta^i$

This table shows the rank correlation for forecaster-level estimates of $\theta^i$ across different series, and $p$-value in parenthesis. Panel A shows results for series and forecasters in SPF. Panel B shows results for series and forecasters in Blue Chip. $\theta^i$ for each series is estimated using the AR(1) or AR(2) version of the diagnostic expectations model based on the properties of the actuals according to Table 6.

Panel A: SPF Series

| | NGDP | RGDP | PGDP | RCONSUM | RNRESINV | RRESINV | RGF | RGSL | HOUSING | UNEMP | tb3m | tn10y |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| RGDP | 0.48 | | | | | | | | | | | |
| | (0.000) | | | | | | | | | | | |
| PGDP | -0.04 | 0.00 | | | | | | | | | | |
| | (0.747) | (0.976) | | | | | | | | | | |
| RCONSUM | -0.20 | -0.28 | -0.11 | | | | | | | | | |
| | (0.128) | (0.030) | (0.393) | | | | | | | | | |
| RNRESINV | 0.41 | 0.34 | -0.20 | -0.11 | | | | | | | | |
| | (0.001) | (0.008) | (0.127) | (0.382) | | | | | | | | |
| RRESINV | 0.29 | 0.13 | -0.07 | -0.01 | 0.25 | | | | | | | |
| | (0.023) | (0.326) | (0.571) | (0.919) | (0.048) | | | | | | | |
| RGF | -0.01 | -0.26 | -0.33 | 0.35 | 0.08 | 0.25 | | | | | | |
| | (0.938) | (0.043) | (0.010) | (0.005) | (0.539) | (0.047) | | | | | | |
| RGSL | 0.00 | -0.19 | -0.17 | 0.50 | 0.04 | -0.21 | 0.42 | | | | | |
| | (0.984) | (0.139) | (0.199) | (0.000) | (0.745) | (0.100) | (0.001) | | | | | |
| HOUSING | 0.08 | -0.03 | -0.09 | 0.02 | 0.18 | 0.45 | 0.02 | -0.03 | | | | |
| | (0.518) | (0.822) | (0.487) | (0.862) | (0.170) | (0.000) | (0.899) | (0.823) | | | | |
| UNEMP | -0.18 | -0.10 | 0.04 | 0.11 | -0.07 | -0.01 | 0.11 | -0.12 | 0.03 | | | |
| | (0.159) | (0.443) | (0.754) | (0.388) | (0.581) | (0.913) | (0.392) | (0.367) | (0.814) | | | |
| tb3m | 0.15 | 0.22 | -0.01 | -0.29 | 0.18 | 0.07 | -0.29 | -0.17 | 0.04 | 0.03 | | |
| | (0.233) | (0.087) | (0.944) | (0.023) | (0.158) | (0.609) | (0.023) | (0.182) | (0.732) | (0.791) | | |
| tn10y | 0.09 | -0.23 | -0.03 | 0.16 | -0.03 | 0.28 | 0.39 | 0.08 | -0.09 | 0.00 | -0.13 | |
| | (0.495) | (0.076) | (0.846) | (0.206) | (0.799) | (0.025) | (0.002) | (0.542) | (0.489) | (0.998) | (0.332) | |
| AAA | 0.15 | 0.13 | 0.21 | -0.27 | 0.29 | 0.14 | -0.19 | -0.19 | 0.04 | -0.02 | 0.36 | -0.22 |
| | (0.249) | (0.300) | (0.102) | (0.032) | (0.021) | (0.295) | (0.132) | (0.147) | (0.745) | (0.898) | (0.004) | (0.081) |

Panel B: Blue Chip Series

| | RGDPBC | FFBC | tb3mBC | tn5yBC | tn10yBC | AAABC |
|---|---|---|---|---|---|---|
| FFBC | 0.13 | | | | | |
| | (0.306) | | | | | |
| tb3mBC | 0.10 | 0.54 | | | | |
| | (0.450) | (0.000) | | | | |
| tb5yBC | 0.15 | 0.45 | 0.37 | | | |
| | (0.243) | (0.000) | (0.003) | | | |
| tn10yBC | -0.32 | 0.02 | -0.01 | 0.02 | | |
| | (0.010) | (0.876) | (0.956) | (0.863) | | |
| AAABC | -0.12 | 0.08 | -0.03 | 0.15 | 0.20 | |
| | (0.346) | (0.530) | (0.808) | (0.247) | (0.122) | |
| BAABC | -0.05 | 0.09 | 0.07 | 0.12 | -0.13 | 0.12 |
| | (0.722) | (0.480) | (0.592) | (0.332) | (0.302) | (0.352) |

### E.3 Overconfidence

We now estimate a model of overconfidence as described in Section 6.3. Here the agent underestimates the standard deviation of the noise in his signal by a factor of $\alpha$, where $\alpha < 1$. He then substitutes the deflated standard deviation of the noise into the Kalman filter update equation. Formally, setting $\widehat{\sigma^2_{\epsilon,\alpha}} = \alpha^2 \sigma^2_{\epsilon}$, $\alpha < 1$, the overconfidence Kalman update is given by the following two equations:

$$\widehat{\Sigma_\alpha} = \frac{-(1-\rho^2)\,\widehat{\sigma^2_{\epsilon,\alpha}} + \sigma^2_u + \sqrt{\left[(1-\rho^2)\,\widehat{\sigma^2_{\epsilon,\alpha}} - \sigma^2_u\right]^2 + 4\,\widehat{\sigma^2_{\epsilon,\alpha}}\sigma^2_u}}{2}$$

$$x_{i,t|t} = x_{i,t|t-1} + \frac{\widehat{\Sigma_\alpha}}{\widehat{\Sigma_\alpha} + \widehat{\sigma^2_{\epsilon,\alpha}}}(s^i_t - x_{i,t|t-1})$$

One can easily derive that the Kalman gain is a decreasing function of $\alpha$, which needs to be bounded above by 1. Intuitively, no matter how overconfident the agent is, he can only give at most full weight to the most recent observation. Extrapolating beyond the noisy signal is only possible for diagnostic agents.

Table E8 presents the results for the target moments $\sigma_{FE,k}$ and $\sigma_{FR,k}$. For comparison, we also include the estimates from the AR(1) version of the diagnostic expectations model.

**Table E8.** Variance of Forecast Errors and Forecast Revisions
Diagnostic Expectations vs Overconfidence

This table shows forecast error variance, $\sigma^2_{FE}$, and forecast revision variance $\sigma^2_{FR}$ in the data and in the estimated model. Results from the AR(1) version of the diagnostic expectations model and the over-confidence model are reported.

| | Forecast Error Variance $\sigma^2_{FE}$ | | | Forecast Revision Variance $\sigma^2_{FR}$ | | |
|---|---|---|---|---|---|---|
| | Actual | DE AR(1) | OC | Actual | DE AR(1) | OC |
| Nominal GDP (SPF) | 4.67 | 4.76 | 5.18 | 1.91 | 1.97 | 1.69 |
| Real GDP (SPF) | 4.58 | 5.13 | 5.73 | 1.60 | 1.65 | 0.88 |
| Real GDP (BC) | 1.89 | 1.87 | 1.94 | 0.39 | 0.39 | 0.36 |
| GDP Price Index Inflation (SPF) | 2.53 | 2.45 | 2.52 | 1.03 | 1.00 | 1.00 |
| Real Consumption (SPF) | 2.03 | 1.97 | 2.03 | 0.85 | 0.90 | 0.86 |
| Real Non-Residential Investment (SPF) | 42.38 | 42.07 | 44.41 | 9.63 | 9.73 | 8.30 |
| Real Residential Investment (SPF) | 98.67 | 101.81 | 99.50 | 24.29 | 24.57 | 24.49 |
| Real Federal Government Consumption (SPF) | 15.89 | 15.99 | 16.13 | 6.03 | 6.07 | 6.19 |
| Real State&Local Govt Consumption (SPF) | 4.14 | 3.37 | 3.73 | 2.60 | 2.73 | 2.89 |
| Housing Start (SPF) | 488.41 | 498.63 | 503.24 | 133.61 | 141.43 | 123.56 |
| Unemployment (SPF) | 0.75 | 0.75 | 0.83 | 0.21 | 0.21 | 0.17 |

| | | | | | | |
|---|---|---|---|---|---|---|
| Fed Funds Rate (BC) | 1.38 | 1.35 | 1.42 | 0.61 | 0.60 | 0.57 |
| 3M Treasury Rate (SPF) | 1.42 | 1.41 | 1.41 | 0.49 | 0.48 | 0.49 |
| 3M Treasury Rate (BC) | 1.33 | 1.32 | 1.34 | 0.52 | 0.51 | 0.52 |
| 5Y Treasury Rate (BC) | 0.98 | 0.97 | 0.99 | 0.41 | 0.40 | 0.42 |
| 10Y Treasury Rate (SPF) | 0.68 | 0.68 | 0.68 | 0.27 | 0.27 | 0.28 |
| 10Y Treasury Rate (BC) | 0.70 | 0.71 | 0.68 | 0.28 | 0.28 | 0.27 |
| AAA Corporate Bond Rate (SPF) | 0.87 | 0.79 | 0.87 | 0.37 | 0.39 | 0.38 |
| AAA Corporate Bond Rate (BC) | 0.81 | 0.79 | 0.81 | 0.40 | 0.41 | 0.41 |
| BAA Corporate Bond Rate (BC) | 0.63 | 0.63 | 0.63 | 0.27 | 0.27 | 0.27 |

Figure E2 plots the effective Kalman gains under our AR(1) model, namely the estimated $(1+\theta)\frac{\Sigma(\sigma_\epsilon^2)}{\Sigma(\sigma_\epsilon^2)+\sigma_\epsilon^2}$, against those in the overconfidence model, namely the estimated $\frac{\Sigma(\alpha,\sigma_\epsilon^2)}{\Sigma(\alpha,\sigma_\epsilon^2)+\sigma_\epsilon^2}$.

**Figure E2.** Model Kalman Gains for Diagnostic Expectations (AR (1)) and Overconfidence

The figure plots model implied Kalman gains in the AR(1) version of the diagnostic expectations model on the x-axis, and model implied Kalman gains in the overconfidence model on the y-axis.