# Dominant Currency Debt *

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**Abstract**

Why is the dollar the dominant currency of choice for debt contracts and what are its macroeconomic implications? We develop an international general equilibrium model where firms optimally choose the currency composition of their debt. We show that there exists a *dominant currency debt equilibrium*, in which all firms borrow in a single common currency. It is the currency of the country that effectively pursues expansionary monetary policy in global downturns, lowering real debt burdens of firms. We show that the dollar empirically fits this description, despite its short term safe haven properties. We provide broad modern and historical empirical support for our mechanism across time and currencies, including evidence using micro-level bond issuance data. We use our model to study the optimal monetary policy of the dominant currency central bank that reacts to global economic conditions.

**Keywords**: dollar debt, dominant currency, exchange rates, inflation

**JEL Classification Numbers**: E44, E52, F33, F34, F41, F42, F44, G01, G15, G32

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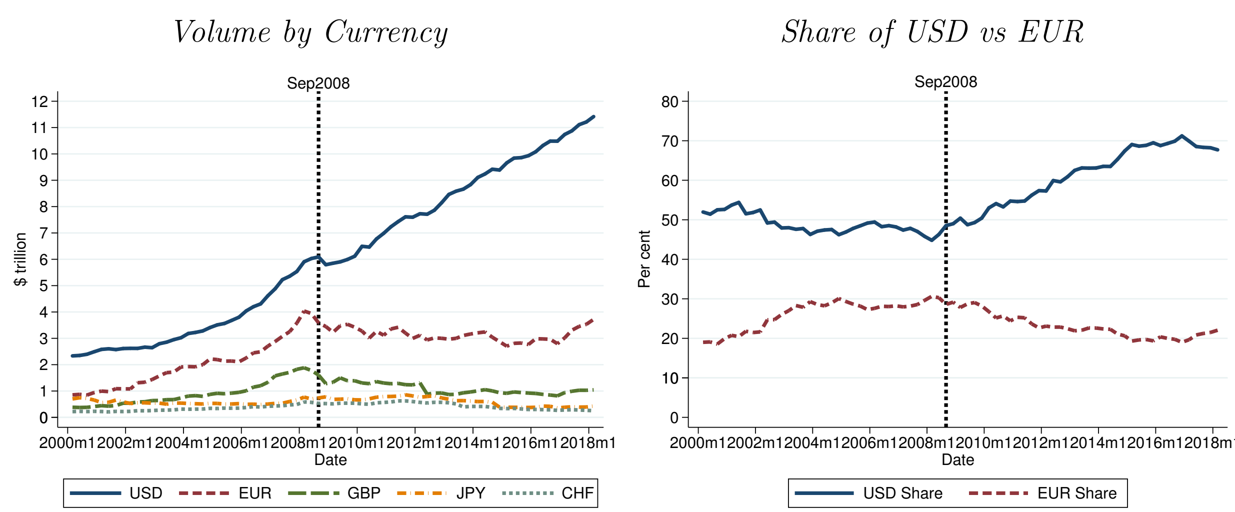
# 1 Introduction

The dollar is the most common currency of choice for debt contracts across the globe. According to the Bank for International Settlements, dollar-denominated credit to non-banks outside the United States amounts to around \$11.5 trillion. While the dominance of the dollar was in decline prior to 2008, it reinstated and strengthened its role in global debt markets since the Great Financial Crisis (Figure 1).[1]

**Figure 1: Currency Denomination of Foreign Currency Non-Bank Debt**




*Source: Bank for International Settlements (see BIS (2018) for details.)*

In this paper, we study how a single currency can become the most common currency of choice in denominating debt contracts in general equilibrium, why it is the dollar, why the dominance of the dollar might have declined and recovered in the last two decades.

We develop an international general equilibrium model with multiple countries where firms optimally choose the currency composition of their debt. All firms are exporters, prices are flexible and firms have fully diversified cash flows. Firms issue equity and nominal, defaultable debt, potentially in any currency. When firms have dollar debt, inflation in the

[1] Similar patterns were previously documented for debt issuance (see, for example, ECB (2017), Maggiori, Neiman and Schreger (2018), Aldasoro and Ehlers (2018)), and for global cross-border bond holdings (Maggiori, Neiman and Schreger (2018)), and also in other parts of the global financial system (Maggiori, Neiman and Schreger (2019)).

US and the exchange rate movements affect their real debt burdens. This is in essence the Fisherian debt-deflation channel applied to an international setting. When debt servicing costs are high relative to profits, firms face debt overhang. They cut production and reduce demand for intermediate inputs imported from other countries. This global debt overhang channel spreads debt overhang costs along the global value chain and serves as a key mechanism for international spillovers.

We model a central bank as a countercyclical monetary policy rule that attempts to ease financing conditions for firms in times when output gap is high, and vice versa. Central banks differ from each other in whether and how they are able to pursue inflation stabilisation policies that protect borrowers? real debt burdens. In the model, relative inflations between two countries determine the exchange rates through a relative purchasing power parity condition.

Our first main result is that there exists *dominant currency debt equilibrium*, defined as an equilibrium in which a single common currency is chosen as the currency in denominating all outstanding debt contracts, even though there are other currencies with almost identical characteristics. We call it the *dominant currency*. It is the currency of the country with the central bank that successfully pursues expansionary monetary policy that leads to currency depreciation in global downturns, and makes borrowing in this currency attractive ex-ante.

The mechanism underlying the dominant currency debt equilibrium is based on the Fisherian debt deflation channel. Firms issue nominal debt to maximize the tradeoff between tax shields and the risk of default. The latter is minimal for debt issued in currencies that co-move positively with firms' profits *at the horizons of their average debt maturity*: When profits drop, such currencies tend to depreciate as well, reducing firms' debt service costs. We use this observation to argue that the dollar is a natural candidate for the role of a dominant currency: Indeed, we show that while the dollar does negatively co-move with the stock markets in the short run (for horizons up to a year), this correlation turns positive for horizons of more than one year, and, in particular, for typical horizons of corporate debt maturity, which is around seven years globally.[2]

This pattern of correlations between the dollar and the stock market has direct implications for the maturity choices of firms for dollar denominated debt contracts. As the dollar co-movement with the stock market increases over longer horizons, our model predicts that the propensity to issue dollar-denominated debt increases with debt maturity. We use data at the bond issuance level in order to formally test this prediction and find strong support for our predictions across a variety of specifications and using a battery of fixed effects.

A key implication of our model is that changes in expectations of firms about the ability of a central bank to pursue inflation stabilization policies in bad times are a key determinant of the currency denomination choice of debt contracts. [3] While expectations about such counter-cyclical inflation stabilization policies are not directly observable, they can be directly backed out from the inflation risk premium (IRP). In fact, our model predicts that in the dominant currency debt equilibrium the dominant currency is always the currency with the highest inflation risk premium (IRP). Interestingly enough, historical estimates (see, for example, Hördahl and Tristani (2014)) show that the IRP was higher in the Eurozone compared to the US prior to the crisis, consistent with the rising share of euro-denominated debt during that period. However, the IRP was higher in the US than in the Eurozone after the crisis, consistent with the post-crisis rise in the dollar share of debt denomination.

Motivated by this evidence, we formally test our theoretical prediction about the link between the share of dollar debt and inflation expectations, as captured by the IRP. We find that the dynamics of IRP in the two countries explains about 80% of the variation in the share of dollar debt, with the signs of the regression coefficients consistent with our theory. Moreover, we show that the IRP is associated with the currency choice of debt issuance

[2]See subsection 7.2 and also Cortina, Didier and Schmukler (2018).

[3]The importance of accommodative monetary policy in helping reduce real debt burdens of firms and the differences across central banks in accomplishing this is also acknowledged by the ECB. See, for example, Praet (2016) and Cœuré (2019).

even at the quarterly level, controlling for year dummies. We interpret this fact as a strong evidence for a distinctive prediction of our theory that changes to the currency composition of debt can occur in high frequency. We also show that this inflation expectations channel sheds light on patterns of debt issuance in other major currencies such as the pound and the yen as well as the dynamics of shares of dollar- and pound-denominated debt during the interwar years as documented by Chiţu, Eichengreen and Mehl (2014). We also show that an extension of our basic model can be used to explain the distribution of local currency shares of corporate debt in a cross-section of emerging market economies.

Finally, our general equilibrium framework allows us to discuss the macroeconomic implications of a dominant currency debt equilibrium and the difference it makes for the global welfare when the Federal Reserve reacts to global versus domestic conditions. We run the following thought experiment: In the dominant currency debt equilibrium, with firms in the entire world issuing dollar debt, how should the Federal Reserve assign weights to output gaps of each country in order to maximize global welfare? We derive optimal weights analytically and show that they are *lower* for countries with volatile TFP shocks, high debt restructuring costs and countries that are more important in world trade. Limiting insurance given to those countries reduces their firms' leverage, improving global (and domestic) welfare.

**Roadmap.** The remainder of the paper is structured as follows. Section 2 provides an overview of the relevant literature. Section 3 describes the model. Section 4 derives the macroeconomic equilibrium with fixed leverage. Section 5 studies the dominant currency debt equilibrium. Section 6 shows how optimal monetary policy differs if the Federal Reserve maximizes global welfare. Section 7 provides empirical support for our theory. Section 8 concludes.

## 2 Literature Review

The role of the dollar as a key international currency has received a lot of attention in recent academic research. Dollar is omnipresent in all parts of the global financial system, including international trade invoicing (see Goldberg and Tille (2008), Casas, Díez, Gopinath and Gourinchas (2017)); global banking (Shin (2012), Ivashina, Scharfstein and Stein (2015), Aldasoro, Ehlers and Eren (2018)); corporate borrowing (Avdjiev, Bruno, Koch and Shin (2018), Bruno, Kim and Shin (2018), Bruno and Shin (2017), Bräuning and Ivashina (2017)); central bank reserve holdings (Bocola and Lorenzoni (2018)); and global portfolios (Maggiori, Neiman and Schreger (2018), Maggiori, Neiman and Schreger (2019)). Our paper belongs to the growing literature that studies the dominant role of the dollar in a general equilibrium framework.[4]

The most closely related to ours is the paper of Gopinath and Stein (2018), who show how the dollar can emerge as a key international currency in both trade invoicing and global banking, which in turn leads to emerging markets endogenously borrowing in dollars. The focus in Gopinath and Stein (2018) is on the interaction between the banking sector and invoicing decisions of exporters. By contrast, our goal is to characterize Fisherian debt deflation forces underlying the impact of nominal debt on the macroeconomy, as in Gomes, Jermann and Schmid (2016), but in an international setting.[5] The special role of the dominant currency arises from firms' demand for bonds with the optimal risk profile, linked to put-like policies pursued by the central bank (see Cieslak and Vissing-Jorgensen

[4]For example, Matsuyama, Kiyotaki and Matsui (1993), Rey (2001), Caballero, Farhi and Gourinchas (2008), Caballero and Krishnamurthy (2009), Devereux and Shi (2013), Chahrour and Valchev (2017), Mukhin (2017), Doepke and Schneider (2017), Farhi and Maggiori (2017), He, Krishnamurthy and Milbradt (2016), Bocola and Lorenzoni (2017), Wiriadinata (2018), and Jiang, Krishnamurthy and Lustig (2018).

[5]In our paper, we focus mostly on the dominant currency debt equilibrium in which all firms in all countries borrow only in dollars. In the real world, as Maggiori, Neiman and Schreger (2018) and Salomao and Varela (2018) show, the vast majority of firms in developed markets borrow in their local currency, and only large and productive firms issue bonds in foreign currency. See section subsection 7.4 for an extension of our model in which firms borrow in both local currency and the dollar.

(2017)). It is this risk profile that makes dollar-denominated debt endogenously safe in our model and hence also receive lower rates of return.[6]

Drenik, Kirpalani and Perez (2018) develop a model in which agents choose the currency denomination of bilateral contracts, and the government chooses the inflation rate. They show that a reduction in domestic political risk may lead to de-dollarization of emerging markets. While our results also imply that policy uncertainty discourages issuance in local currency, we argue that emerging markets may keep issuing in dollars if the US Federal reserve keeps convincing market participants in its superior ability to pursue aggressive policy in crisis times.

A large literature shows that global credit conditions, and, in particular, the US dollar, serve as an important mechanism for the international transmission and amplification of credit supply shocks.[7] In this paper, we are able to explicitly characterize the financial channel outlined in Avdjiev, Bruno, Koch and Shin (2018) and Bruno, Kim and Shin (2018), whereby shocks to balance sheets of firms with dollar debt impact their investment and exporting activities.

Numerous papers in international macroeconomics study how exchange rate pass-through into prices of real goods depends on price stickiness and the invoicing currency choice. See, for example, Engel (2006), Gopinath, Itskhoki and Rigobon (2010), Goldberg and Tille (2013), Corsetti and Pesenti (2015) and Casas, Díez, Gopinath and Gourinchas (2017). We highlight a potentially novel passthrough mechanism operating through the financial channel (see Avdjiev, Bruno, Koch and Shin (2018)). With dollar debt, a dollar appreciation shock puts leveraged firms in distress and increases their effective operational costs. Firms respond

[6]In particular, firms in our model issue dominant currency debt because they "reach for safety"; this leads to excessive leverage and can be destabilizing, as in Caballero and Krishnamurthy (2001, 2002); Caballero and Lorenzoni (2014); Caballero and Simsek (2018).

[7]See, for example, Aghion, Bacchetta and Banerjee (2000), Aghion, Bacchetta and Banerjee (2004), Aguiar (2005), Velasco and Chang (2006), Gourinchas and Obstfeld (2012), Schularick and Taylor (2013), Du and Schreger (2016a), Du and Schreger (2016b), Baskaya, di Giovanni, Kalemli-Ozcan and Ulu (2017), Avdjiev, Bruno, Koch and Shin (2018), Bruno, Kim and Shin (2018), and Miranda-Agrippino and Rey (2018).

to this by raising prices, consistent with the mechanism highlighted in Gilchrist, Schoenle, Sim and Zakrajšek (2017) and Malamud and Zucchi (2018). Importantly, this passthrough channel operates even when prices are fully flexible.

# 3 Model

## 3.1 Households and Exchange Rates

Time is discrete, indexed by $t = 0, 1, \cdots$. There are $N$ countries, indexed by $i = 1, \cdots, N$. Households work and consume. They maximize

$$E\left[\sum_{t=0}^{\infty} e^{-\beta t} U(C_{i,t}, N_{i,t})\right]$$

with

$$U(C_{i,t}, N_{i,t}) = \frac{C_{i,t}^{1-\gamma}}{1-\gamma} - \nu N_{i,t}$$

for some $\gamma \geq 2$,[8] where $N_{i,t}$ is the number of hours worked,[9] and

$$C_{i,t} = \left(\sum_j \int_0^1 (\tilde{C}_{i,t}(j,\omega))^{\frac{\eta-1}{\eta}} d\omega\right)^{\frac{\eta}{\eta-1}}$$

is the constant elasticity of substitution (CES) consumption aggregator, with the elasticity of substitution $\eta$. Here, $\tilde{C}_{i,t}(j,\omega)$ denotes the consumption of type-$\omega$ good imported from country $j$ into country $i$, with $\omega \in [0,1]$.

The price at which a country $j$ firms sell type$-\omega$ goods in country $i$ is denoted by $P_t^i(j,\omega)$.

[8] Condition $\gamma \geq 2$ is imposed for technical reasons and can be relaxed.

[9] The simplifying assumption that the inverse Frisch elasticity of labour is zero allows us to pin down equilibrium wage without the need keep track of equilibrium labour demand. It is made purely for technical reasons and can be removed at the cost of significant additional complexity of the calculations.

This price is always in the domestic, country-$i$ currency. The country-$i$ price index is defined by the standard formula

$$\mathcal{P}_{i,t} \equiv \left(\sum_j \int_0^1 P_t^i(j,\omega)^{1-\eta} d\omega\right)^{1/(1-\eta)} . \tag{1}$$

For simplicity, we assume that prices are fully flexible and the aggregate nominal price level follows an exogenous, country-specific stochastic process $\mathcal{P}_{i,t}$.

Households have access to a complete, frictionless financial market with a domestic, nominal pricing kernel $M_{i,t,\tau}$ *in the domestic currency*, for any $t < \tau$. The following lemma characterizes their optimal consumption choices.

**Lemma 3.1** *Optimal consumption demand is given by*

$$\tilde{C}_{i,t}(j,s) = (P_t^i(j,s))^{-\eta} (\mathcal{P}_{i,t})^{\eta} C_{i,t} ,$$

*consumption expenditures satisfy the inter-temporal Euler equation,*

$$e^{-\beta} C_{i,t+1}^{-\gamma} / C_{i,t}^{-\gamma} = M_{i,t,t+1} (\mathcal{P}_{i,t+1}/\mathcal{P}_{i,t})$$

*and equilibrium wages are given by*

$$w_{i,t} = \nu\, C_{i,t}^{\gamma} \mathcal{P}_{i,t} .$$

We denote by $\mathcal{E}_{i,j,t}$ the value of a unit of currency $i$ in the units of currency $j$. That is, when $\mathcal{E}_{i,j,t}$ goes up, currency $i$ appreciates relative to currency $j$. We select one reference currency (the US dollar; henceforth, the dollar), denoted by \$, and use $\mathcal{E}_{i,t} = \mathcal{E}_{i,\$,t}$ to denote the nominal exchange rate against the dollar. Due to assumed market completeness, consumers in different countries attain perfect risk sharing and pricing kernels $M_{i,t,t+1}$ and

$M_{j,t,t+1}$ of any two countries $i, j$ are linked through the no-arbitrage identity: $\frac{M_{i,0,t}}{M_{j,0,t}} = \frac{\mathcal{E}_{i,j,t}}{\mathcal{E}_{i,j,0}}$. For the sake of convenience, everywhere in the sequel we normalize the initial price levels $\mathcal{P}_{i,0}$ so that $\mathcal{E}_{i,0} = 1$ for all $i = 1, \cdots, N$.

## 3.2 Firms' Choices with an Exogenous Debt Overhang

### 3.2.1 Production

Each country's production sector is populated by a continuum of ex-ante identical firms, indexed by $\omega \in [0, 1]$, with firm $\omega$ producing type$-\omega$ goods. We use $(i, \omega)$ to denote the firm $\omega$ in country $i$. All firms use labour as well as goods[10] produced by other firms (domestic and foreign) as inputs in a standard Cobb-Douglas production technology: Absent debt overhang (to be defined below), the production function of an $(i, \omega)$ firm is given by

$$Y^*_{i,t}(\omega, L_t(i, \omega), X_t(i, \omega)) \;=\; Z_{i,t}(\omega)^{(\eta-1)^{-1}} e^{a_{i,t}} \, L_t(i, \omega)^{1-\alpha} \, X_t(i, \omega)^{\alpha}\,, \tag{2}$$

where, for each $i = 1, \cdots, N$,

- $Z_{i,t}(\omega) > 0$ is the firm $(i, \omega)$ idiosyncratic production shock that is drawn from the distribution with the density $\phi(z) = \ell z^{\ell-1}$ on $[0, 1]$ with a country specific parameter $\ell > 0$ and the cumulative distribution function

  $$\Phi(z) = \int_0^z \phi(x)dx = z^{\ell}. \tag{3}$$

  We assume that $Z_{i,t}$ are i.i.d. over time and across firms within a given country;

- $a_{i,t}$ is the country-$i$ productivity shock;

- $L_t(i, \omega)$ is labour hired by the $(i, \omega)$ firm at time $t$;

[10] For simplicity we assume that all goods are used both for production (as intermediate inputs) and consumption.

- $X_t(i,\omega)$ is the CES aggregator of goods used by the $(i,\omega)$ firm as inputs:[11]

$$X_t(i,\omega) \;=\; \left(\sum_{j=1}^{N}\int_0^1 (\tilde{X}_{t,(i,\omega)}(j,s))^{\frac{\eta-1}{\eta}}\,ds\right)^{\frac{\eta}{\eta-1}}\,.$$

Here, $\tilde{X}_{t,(i,\omega)}(j,s)$ is the demand of a $(i,\omega)$ firm for goods of a $(j,s)$-firm in country $j$. We assume that firms are taxed on profits at a tax rate $\tau$. As we show in the Appendix (see Lemma A.1), total nominal after tax profits of an $(i,\omega)$-firm measured in country-$i$ currency are given by

$$\Pi^*_{i,t}(\omega) \;=\; Z_{i,t}(\omega)\,\Omega_{i,t} \tag{4}$$

for some explicit, country-specific random variables $\Omega_{i,t},\ i = 1,\cdots,N$.

#### 3.2.2 Debt

In order to highlight the mechanisms through which debt affects real outcomes in our model, we first introduce debt exogenously.[12] We assume that all firms in each country $i$ have nominal debt. When firms receive a low draw of either the idiosyncratic shock $Z_{i,t}$ or the country-specific TFP shock $a_{i,t}$, they become distressed and produce less efficiently. This form of *debt overhang* is the key mechanism through which debt is linked to real outcomes in our model.

**Assumption 1 (Debt Overhang)** *Firms born at time $t-1$ live for one period, $[t-1,t]$ and are endowed with nominal debt whose random face value $\mathcal{B}_{i,t}$ in domestic currency is realized at time $t$.*

[11]For simplicity, we assume that the consumption aggregator coincides with the production aggregator. Without this assumption, we would need to separately consider consumer and producer price indices, which would complicate the analysis. Furthermore, we assume that an $(i,\omega)$ firm does not use its own goods for production, so that in the integral $\int_0^1 (\tilde{X}_{t,(i,\omega)}(i,s))^{\frac{\eta-1}{\eta}}ds$ we integrate only over $s \neq \omega$.

[12]In section 5, we endogenize the choice between debt and equity, as well as the choice of the compositions of currency denomination of debt.

- *Having observed the idiosyncratic shock realization, $Z_{i,t}$, and before starting production, the firm computes its optimal potential profits* (4)*:*
  - *If the after-tax profits* (4) *are sufficient to cover the debt servicing cost, the firm produces according to* (2)*.*
  - *If the after-tax profits are insufficient to cover the debt servicing cost, the firm enters a financial distress state, and is only able to produce at a fraction $\zeta^{(\eta-1)^{-1}} \in (0,1)$ of its capacity. That is, its production function is given by*

$$Y_{i,t}(\omega, L_t(i,\omega), X_t(i,\omega)) \ = \ Y^*_{i,t}(\omega, L_t(i,\omega), X_t(i,\omega))(1 \ + \ (\zeta^{(\eta-1)^{-1}} - 1)\mathbf{1}_{distress})$$

*and, similarly, total nominal profits are given by* $\Pi_{i,t} \ = \ \Pi^*_{i,t}(\omega)(1 \ + \ (\zeta - 1)\mathbf{1}_{distress})\,.$

The simple nature of debt overhang in Assumption 1 implies that profits in distress are identical to those in (4), but with $Z_{i,t}$ replaced by $\zeta Z_{i,t}$. Furthermore, Assumption 1 also implies that the firm enters a financial distress when $Z_{i,t}$ falls below the distress threshold

$$\Psi_{i,t} \ \equiv \ \frac{\mathcal{B}_{i,t}}{\Omega_{i,t}}\,. \tag{5}$$

In the sequel, we will also frequently refer to (5) as *leverage:* Indeed, $\Psi_{i,t}$ is the ratio of the face value of debt, $\mathcal{B}_{i,t}$, to the measure of country $i$ *nominal value of profits* as given by $\Omega_{i,t}$ (see (4)).

Assumption 1 allows us to capture two key features of the behaviour of financially constrained firms: When the debt service costs are high relative to profits, finally constrained firms often have problems paying their suppliers and employees, are thus often forced to fire employees and cut production. As a result, in such a distress state effective marginal costs surge, forcing firms to raise prices and cut production.[13] Both features are important for our results: In equilibrium, high prices hit global demand; firms respond by cutting their production and raising prices even further, spreading the debt overhang costs all over the world.

# 4 General Equilibrium

In our model, all goods are used both for consumption and for production. Furthermore, by assumption, households and firms in all countries are completely symmetric: They all have identical preferences and identical production functions, and hence only differ in productivity shocks $a_{i,t}$.[14] Under this condition, marginal rates of inter-temporal substitution are perfectly aligned across countries, so that

$$\bar{C}_t \equiv (C_{i,t}/C_{i,0})^{\gamma(\eta-1)}$$

is independent of $i$,[15] while nominal exchange rates are fully driven by inflation dynamics:

$$\mathcal{E}_{i,j,t} = \frac{\mathcal{P}_{i,t}^{-1}}{\mathcal{P}_{j,t}^{-1}}.$$

[13]See Gilchrist, Schoenle, Sim and Zakrajšek (2017) and Malamud and Zucchi (2018). In Section 5 we micro-found these costs by assuming that, in distress, debt-holders take over the firm, and are less efficient in running production. See Section G.1 in the Appendix, where we show how to introduce investment decisions and into the firm problem.

[14]This assumption is made purely for technical convenience and allows us to highlight the key mechanisms particularly clearly. In the Appendix, we show how our results can be extended to the case of heterogeneous firms. Household heterogeneity (such as, e.g., home bias in consumption) is another important direction for future research.

[15]That is, real exchange rates equal one. This is a convenient property that allows us not to worry about foreign exchange risk premia. Indeed, absent financial frictions, our model would be at odds with the exchange rate disconnect puzzle (see, e.g., Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2017)) and would generate counter-factual behaviour of currency risk premia.

The function

$$G(\Psi) \;=\; \ell(\ell+1)^{-1}\left((\zeta-1)\Psi^{\ell+1}+1\right)\,,\;\; \Psi\in[0,1]\,,$$

will play an important role in the subsequent analysis. It captures the total drop in productivity of country$-i$ firms that are subject to debt overhang: Indeed, by (4), total profits of country $i$ firms are given by

$$\int_0^1 \Pi_{i,t}(\omega)d\omega \;=\; \Omega_{i,t}\int_0^1 (Z_{i,t}(\omega)\mathbf{1}_{Z_{i,t}(\omega)>\Psi_{i,t}}+\zeta Z_{i,t}(\omega)\mathbf{1}_{Z_{i,t}(\omega)<\Psi_{i,t}})d\omega \;=\; G(\Psi_{i,t})\,\Omega_{i,t}\,. \quad (6)$$

As we explain above (see (5)), distressed firms are those with idiosyncratic shock realizations $Z_{i,t}$ below the threshold $\Psi_{i,t}$. Thus, by (3), the fraction of distressed firms is given by measure$[\omega\;:\;Z_{i,t}(\omega)<\Psi_{i,t}] \;=\; \min\{\Psi_{i,t},1\}^{\ell}$. We will restrict our attention to *normal* equilibria in which a strictly positive fraction of firms in each country is not in distress, so that $\Psi_{i,t}<1$ for all $i=1,\cdots,N$, and $\bar{C}_t$ is monotonically increasing in $a_{j,t},\; j=1,\cdots,N$.[16] The following result shows explicitly how the global debt overhang influences equilibrium consumption.

**Theorem 4.1** *There exist constants $\xi_i,\;i=1,\cdots,N$ and $\alpha_*$ such that:*

- *total after-tax profits of an $(i,\omega)$ firm are given by*

$$\Pi_{i,t}(\omega) \;=\; Z_{i,t}(\omega)\,\Omega_{i,t},\;\textit{where}\;\Omega_{i,t} \;=\; \xi_i e^{a_{i,t}(\eta-1)}\bar{C}_t^{\hat{\eta}}\;,\;\textit{with}\;\hat{\eta}\;\equiv\;\frac{\gamma^{-1}}{\eta-1}+\alpha-1\,. \quad (7)$$

- *if $\zeta$ is not too small and all random variables have bounded support, then there exists*

[16] In the case of $\hat{\eta}>0$, there might exist a "non-economic equilibrium" with an unreasonable feature that consumption is decreasing in productivity shocks $a_{j,t}$. For the rest of the paper, we neglect this equilibrium and focus on the normal equilibrium.

*a unique normal equilibrium solution $\bar{C}_t$ to the equation*

$$\bar{C}_t^{1-\alpha} \;=\; \alpha_* \sum_{j=1}^{N} \xi_j e^{a_{j,t}(\eta-1)} \left(1 \;-\; \Delta_t \bar{C}_t^{-\hat{\eta}(\ell+1)}\right) , \tag{8}$$

*where*

$$\Delta_t \;\equiv\; (1-\zeta)\, \frac{\sum_{j=1}^{N} \xi_j e^{a_{j,t}(\eta-1)} \left(\frac{\mathcal{B}_{j,t}\mathcal{P}_{j,t}^{-1}}{\xi_j e^{a_{j,t}(\eta-1)}}\right)^{\ell+1}}{\sum_{j=1}^{N} \xi_j e^{a_{j,t}(\eta-1)}} \,.$$

*is the Global Debt Overhang Factor.*

We complete this section with two results that are crucial for understanding the real effects of debt overhang. First, since financial distress lowers production, debt overhang leads to and output gap and unemployment. Second, due to the input-output linkages, rising debt burdens in one country always transmit to other countries; we show that, under natural conditions, a rising debt burden in one country always leads to higher debt overhang costs in all other countries. The following two corollaries formalize this intuition.

**Corollary 4.2** *Denote by $\bar{L}_t(i)$ and $\bar{O}_t(i)$ the country $i$ equilibrium labour demand (employment) and output, respectively. Let also $\bar{L}_t^*(i)$ and $\bar{O}_t^*(i)$ denote the corresponding frictionless benchmarks absent debt overhang. Then, both the output gap and the employment gap are given by*

$$\frac{\bar{L}_t(i)}{\bar{L}_t^*(i)} \;=\; \frac{\bar{O}_t(i)}{\bar{O}_t^*(i)} \;=\; \frac{G(\Psi_{i,t})}{\ell(\ell+1)^{-1}} \;<\; 1\,. \tag{9}$$

**Corollary 4.3 (Default transmission)** *Suppose that $\hat{\eta} > 0$.*[17] *Then, a shock to debt bur-*

[17] This assumption is important. While firm profits are always increasing in productivity, $a_{i,t}$, they are be decreasing in consumption when $\hat{\eta} < 0$. The sign of $\hat{\eta}$ depends on the importance of labour in the Cobb-Douglas production function (2). A higher $\bar{C}_t$ leads to a higher equilibrium wage level, making production too costly when $\alpha$ is very low.

*den $\mathcal{B}_{j,t}$ of a country $j$ always leads to an increase in the fraction of distressed firms in all other countries $i \neq j$.*

# 5 Dominant Currency Debt

In this section, we study the firms' choice of leverage and the composition of currency denomination of their debt in general equilibrium.

We assume that firms finance themselves by issuing both equity and defaultable short-term nominal bonds in *any* of the $N$ currencies.[18] Each bond has a nominal face value of one currency unit, and the firm is required to pay a coupon of $c$ currency units per unit of outstanding debt.[19] We denote by $B_{j,t}(i)$ the stock of outstanding nominal debt at time $t$ of country $i$ firms, denominated in the currency of country $j$. We also denote by $B_t = (B_{j,t}(i))_{j=1}^N$ the $N \times N$ matrix of debt stocks in different currencies. As in Gomes, Jermann and Schmid (2016), we assume that coupon payments are shielded from taxes, so that

$$\mathcal{B}_{i,t+1}(B_t) \;=\; ((1-\tau)c+1)\sum_{j=1}^{N}\mathcal{E}_{j,i,t+1}B_{j,t}(i)$$

is the total debt servicing cost in country$-i$ currency, net of tax shields. The choice of firm leverage therefore depends on the trade-off between tax advantages and the distress costs.[20] Then, absent default, nominal distribution to shareholders at time $t+1$ is given by

$$\Pi_{t+1}(i,\omega) \;-\; \mathcal{B}_{i,t+1}(B_t)\,. \tag{10}$$

[18]We interpret this one period as a typical maturity of corporate debt, of the order of several years. See, for example, Cortina, Didier and Schmukler (2018). In particular, we do not address the known fact that US dollar tends to appreciate over short term during crises (see, for example, Maggiori (2013) and Farhi and Maggiori (2017)), making it unattractive for short-term borrowing. Below, we provide empirical evidence suggesting that firms are well aware of this risk profile, and tend to issue long-term dollar-denominated debt.

[19]Apart from the multiple currencies assumption, in modelling the financing side we closely follow Gomes, Jermann and Schmid (2016).

[20]For simplicity, as in Gomes, Jermann and Schmid (2016), we assume that tax shields are the only motivation for issuing debt. However, one could also interpret $\tau$ as reduced form of gains from debt issuance, such as alleviation of adverse selection costs.

If the cash flows (10) are non-positive, shareholders default on firm's debt. Upon default, debt-holders take over the firm and shareholders get zero.[21]

## 5.1 Optimal Leverage and Dominant Currency Debt

As we explain above, shareholders default whenever cash flows (10) are non-positive; that is, when $\Pi_{t+1}(i,\omega) \ \leq \ \mathcal{B}_{i,t+1}(B_t)$ . Thus, default occurs whenever $Z_{i,t+1}$ falls below the default threshold

$$\Psi_{i,t+1}(B_t) \ \equiv \ \frac{\mathcal{B}_{i,t+1}(B_t)}{\Omega_{i,t+1}} .$$

We assume that, upon default, debt-holders only recover a fraction $\rho_i$ of their promised value, $1+c$.[22] Thus, by direct calculation, the nominal price in country $i$ currency of one unit of debt denominated in currency $j$ is given by

$$\delta_i^j(B_t) \ = \ E_t\left[M_{i,t,t+1}\left(1-(1-\rho_i)\Phi(\Psi_{i,t+1}(B_t))\right)(1+c)\mathcal{E}_{j,i,t+1}\right] .$$

We assume that country $i$ firms face a proportional cost $q_i(j)$ of issuing in country $j$ currency for $i,j = 1,\cdots,N$[23] and are maximizing equity value plus the proceeds from debt issuance

[21] In the Appendix (Proposition C.1), we show that under the assumption of shareholders getting zero in default, firms always choose not to hedge foreign exchange risk. The intuition behind this result is straightforward. Hedging effectively plays a role of investment, and the firm only gets the payoff $X_{t+1}$ from this investment in good (survival) states, while paying the market price at time $t$ to get the payoff in all states. Thus, hedging is just a transfer of funds from shareholders to debt-holders, and firms optimally decide to minimize this transfer.

[22] We assume that $\rho_i$ is sufficiently small relative to productivity in default parameter, $\zeta$, so that debt holders can recover at most what they get from (inefficiently) running the firm net of (unmodelled) default costs paid to lawyers, etc. We assume that these costs go directly to the representative consumer and hence have no impact on equilibrium outcomes. There are big differences in these default costs across countries. See, Favara, Morellec, Schroth and Valta (2017).

[23] While we do not micro-found these costs, it is not difficult to do so. These costs may originate from underwriting costs, limited risk bearing capacity of intermediaries (in case of bank loans), or the actual debt placement costs incurred by the investment banks (such as locating bond investors). These costs differ drastically depending on the currency in which debt is issued. For example, according to Velandia and Cabral (2017), "... in the case of Mexico, the average bid- ask spread of the yield to maturity on outstanding

net of issuance costs:

$$\max_{B_t} \left\{ \sum_{j=1}^{N} \delta_i^j(B_t) B_{j,t}(1 - q_i(j)) + E_t[M_{i,t,t+1} \max\{\Pi_{t+1}(i,\omega) \ - \ \mathcal{B}_{i,t+1}(B_t), 0\}] \right\}.$$

Everywhere in the sequel, we will use $E_t^{\$}$ and $\mathrm{Cov}_t^{\$}$ to denote conditional expectation and co-variance under the dollar risk neutral measure with the conditional density $E_t[M_{\$,t,t+1}]^{-1} M_{\$,t,t+1}$. Furthermore, for each stochastic process $X_t$, we consistently use the notation

$$X_{t,t+1} \ \equiv \ \frac{X_{t+1}}{X_t}\,.$$

We need the following assumption ensuring that the leverage choice problem has a non-trivial solution.

**Assumption 2** *We have*

$$(1 - q_i(j))(1 + c) \ > \ (1 + c(1 - \tau)) \qquad \textit{and}$$
$$\bar{q}_i(j, \$) \ \equiv \ \frac{((1 - q_i(j))(1 + c) - (1 + c(1 - \tau)))}{(1 - \rho_i)(1 + c)[(1 - q_i(j)) + \ell(1 - q_i(\$))] - (1 + c(1 - \tau))} \ > \ 0$$

*for all* $i,\ j = 1, \cdots, N$. *Let also* $\bar{q}_i(\$) \equiv \bar{q}_i(\$, \$)$.

The first condition ensures that the cost $q_i(j)$ of issuing debt is less than the gains, measured by the value of tax shields, so that there is positive debt issuance. The second condition ensures that the recovery rate $\rho_i$ is sufficiently small: Otherwise, funding becomes so cheap for the firm that it may want to issue infinite amounts of debt. The following is true.

USD-denominated international bonds is 7 basis points, compared to 10 basis points for outstanding EUR-denominated bonds; and Mexico is an example with very liquid benchmarks on both currencies."

**Theorem 5.1** *Issuing only in dollars is optimal if and only if*

$$\frac{\bar{q}_i(j,\$)}{\bar{q}_i(\$)} - 1 \;\leq\; \frac{\mathrm{Cov}_t^{\$}\left((\mathcal{E}_{i,t,t+1}\Omega_{i,t+1})^{-\ell}, \mathcal{E}_{j,t,t+1}\right)}{E_t^{\$}\left[(\Omega_{i,t+1}\mathcal{E}_{i,t,t+1})^{-\ell}\right] E_t^{\$}[\mathcal{E}_{j,t,t+1}]} \tag{11}$$

*for all* $j = 1, \cdots, N$. *In this case, optimal dollar debt satisfies*

$$b_{\$,t}(i) \;=\; \mathcal{E}_{i,t}^{-1} B_{\$,t} \;=\; (1 + c(1-\tau))^{-1} \left(\frac{\bar{q}_i(\$)}{E_t^{\$}\left[(\Omega_{i,t+1}\mathcal{E}_{i,t,t+1})^{-\ell}\right]}\right)^{\ell^{-1}} . \tag{12}$$

Condition (11) shows that the incentives for issuing in dollars are determined by two forces: The effective cost of issuance, $\bar{q}_i(\$)$, and the risk profile of the dollar. Dollar capital and derivative markets are deep and liquid (see Moore, Sushko and Schrimpf (2016)); thus, the low effective cost of issuance, $\bar{q}_i(\$)$, is an obvious factor favoring the dollar as the dominant currency of choice for debt contracts. However, our main result does not rely on the dollar having low issuance costs: Theorem 5.1 implies that the dollar can arise as the dominant debt-denomination currency purely due to its risk profile. To understand the underlying mechanism, we note that, absent heterogeneity in effective issuance costs (that is, when $q_i(j)$ is independent of $j$), (11) takes the form

$$\mathrm{Cov}_t^{\$}\left((\mathcal{E}_{i,t+1}\Omega_{i,t+1})^{-\ell}, \mathcal{E}_{j,t,t+1}\right) \;\geq\; 0\;,\; j = 1, \cdots, N\,. \tag{13}$$

Here, $\mathcal{E}_{i,t+1}\Omega_{i,t+1}$ is the factor determining the value of profits of country $i$ firms in dollars (see (6)). Intuitively, at time $t$, firms deciding on the currency composition of their debt choose to issue in dollars if they anticipate the dollar to depreciate at times when their time $t+1$ profits are low; condition (13) provides a precise formalization of this intuition. Since $(\mathcal{E}_{i,t+1}\Omega_{i,t+1})^{-\ell}$ attains its largest value when dollar profits $\mathcal{E}_{i,t+1}\Omega_{i,t+1}$ are close to zero, covariance (13) overweights distress states: When $\ell$ is sufficiently high, (13) essentially requires dollar to depreciate against all its key competitors at times of major economic downturns. It is also important to note that condition (13) corresponds to the problem of a firm choosing between dollar debt and debt denominated in other key currencies such as, e.g., the euro, the yen, the franc and the pound. For an emerging markets firm choosing between local currency debt and dollar debt, heterogeneity in issuance costs may be as (if not more) important as the currency risk profile. However, even for the choice between dollar- and euro-denominated debt, ignoring differences in issuance costs puts the dollar at disadvantage: Existing evidence (see, e.g., Velandia and Cabral (2017)) suggests that issuing is dollars is significantly cheaper that issuing in euros.

## 5.2 Dominant Currency Debt in General Equilibrium

In this section, we combine the equilibrium characterization in Theorem 4.1 with the dominant currency debt condition of Theorem 5.1 to answer the question: When does dominant currency debt arise in general equilibrium? We will make the following simplifying assumption.

**Assumption 3** *We have*

- *issuing costs are independent of currency denomination:* $q_i(\$) = q_i(j)$ *for all* $i, j = 1, \cdots, N$.
- *TFP shocks satisfy* $a_{j,t} = a_t + \varepsilon^a_{j,t}$ *for some common shock* $a_t$ *and idiosyncratic TFP shocks* $\varepsilon^a_{j,t}$ *with small variance that are independent across countries and are also independent of* $a_t$.

As we explain above, in our model consumption is perfectly aligned across countries, real exchange rates equal one, and nominal exchange rates' changes are determined purely by the relative inflation rates: $\mathcal{E}_{i,t,t+1} = \mathcal{P}^{-1}_{i,t,t+1}\mathcal{P}_{\$,t,t+1}$. Thus, substituting the profits $\Omega_{i,t}$ from (7) and using Assumption 3, we get from (13) that issuing in dollar is the dominant currency debt is an equilibrium if and only if

$$\mathrm{Cov}_t^{\$}\left(\left(\bar{C}_{t+1}^{\hat{\eta}} e^{(\eta-1)a_{i,t+1}} \mathcal{P}_{\$,t,t+1}\right)^{-\ell}, \mathcal{P}_{j,t,t+1}^{-1} \mathcal{P}_{\$,t,t+1}\right) \;\geq\; 0 \tag{14}$$

for all $i, j = 1, \cdots, N$.

In our model, a central bank policy that leads to a positive shock to $\mathcal{P}_{i,t}$ has two main functions: First, it eases the real debt burden of firms borrowing in country $i$ currency; and second, it leads to a depreciation of country $i$ currency. Therefore, an increase (respectively, decrease) in $\mathcal{P}_{i,t}$ can be interpreted as monetary easing (respectively, tightening). [24] We assume a counter-cyclical monetary policy rule whereby the central bank eases (respectively, tightens) when employment or output falls (respectively, rises) relative to the frictionless benchmark (see (9)):

**Assumption 4** *Inflation rate in country $i$ is determined by*

$$\mathcal{P}_{i,t,t+1} \;=\; \left(1 - \bar{L}_{t+1}(i)/\bar{L}_{t+1}^{*}(i)\right)^{\phi_i} \; e^{\varepsilon_{i,t+1}}\,. \tag{15}$$

*Here, $\phi > 0,\; i = 1, \cdots, N$ is a country specific parameter that measures the responsiveness of domestic monetary policy to economic conditions, and $\varepsilon_{i,t+1}$ is a country-specific monetary policy shock with bounded support and variance $\sigma_{i,\varepsilon}^2$.*

We can now characterize the conditions when a dominant currency debt equilibrium emerges. The following theorem is the main result of this paper.

**Theorem 5.2** *Suppose that monetary policy uncertanties $\sigma_{i,\varepsilon},\; i = 1, \cdots, N$ are sufficiently small and that the indices $\phi_i$ are all pairwise different and $1 - \zeta$ is not too large. Then, there exists a Dominant Currency Debt equilibrium. The dominant debt currency is always the currency of the country with the highest index $\phi_i$.*

[24] There is ample evidence that monetary easing typically leads to a simultaneous currency depreciation. See, for example, Ferrari, Kearns and Schrimpf (2017).

In addition to inflation stabilization indices $\phi_i$, countries may also differ in the volatility of inflation shocks, $\sigma_{i,\varepsilon}$. Naturally, firms view this uncertainty as an additional and undesirable form of risk. The following is true.

**Proposition 5.3** *Absent heterogeneity in the indices $\phi_i$, $i = 1, \cdots, N$, firms always issue in the currency of the country with the lowest volatility of inflation shocks, $\sigma_{i,\varepsilon}$.*

While $\sigma_{i,\varepsilon}$ does represent idiosyncratic inflation volatility in our model, Proposition 5.3 holds true for any shocks to exchange rates that are unrelated to economic fundamentals; for example, monetary policy shocks or temporary demand pressures and liqudity shocks in currency markets. Proposition 5.3 suggests that, in addition to insufficient market liquidity (modelled by high issuance costs), the significant idiosyncratic volatility of emerging market currencies may serve as an additional important mechanism explaining why firms do not want to issue in these currencies, despite the fact that such currencies do tend to significantly depreciate during crises (see also Du, Pflueger and Schreger (2016)). As an illustration, consider a typical emerging market currency, the Argentinian Peso (ARS). During the period of November 1995-September 2018, the standard deviation of the monthly returns on the dollar index was 1.9%, while the standard deviation of monthly returns on the ARS/USD exchange rate was 7.1%. Furthermore, this volatility was almost entirely due to idiosyncratic shocks: Indeed, the $R^2$ of a regression of the monthly ARS/USD returns on the returns on the dollar index is only 0.0033.

# 6 Optimal Monetary Policy

Our general equilibrium framework allows us to discuss the macroeconomic implications of a dominant currency debt equilibrium and the role of the central bank of the dominant currency country as the world's central bank. In particular, we would like to understand the difference it makes for the global welfare when the central bank reacts to global versus domestic economic conditions.

An active stabilization policy of the dominant currency country lowers *ex-post* real debt burdens of firms through higher inflation and exchange rate depreciation. Therefore, it reduces the effective cost of issuing dominant currency debt, prompting firms to take higher leverage *ex-ante.* However, higher leverage also means higher distress costs in the face of more severe shocks. Even though active monetary policy in economic downturns is optimal ex-post when a crisis state is realized, it is never optimal ex-ante. Namely, expected welfare gains from reducing distress costs of firms are more than offset by the welfare costs of higher leverage. Central banks would prefer not to provide this insurance to firms ex-ante, but cannot credibly do so.

We do the following thought experiment. Assume that the global central bank optimally assigns weights on the output gaps of different countries in order to maximize global welfare, taking into account all spillovers arising from the interconnectedness of different countries due to global value chains.[25] We make the following assumption:

**Assumption 5** *There exist* $\chi_i \geq 0,\ i = 1, \cdots, N,$ *such that*

$$\mathcal{P}_{\$,t,t+1} \ = \ \prod_i \left(1 - \bar{L}_{t+1}(i)/\bar{L}^*_{t+1}(i)\right)^{\chi_i} \ .$$

The following is true.

**Proposition 6.1** *The welfare maximizing policy is to only react to output gap in countries with:*

- *low TFP variance of* $a_{i,t}$
- *low restructuring cost,* $1 - \zeta_i$

[25] Since all countries are symmetric, maximizing global welfare is equivalent to maximizing domestic welfare.

- *low importance in global trade, $\xi_i$*

The global central bank chooses weights to precisely limit the leverage of firms in countries where the adverse effects of leverage are the highest as shown in Proposition 6.1. That in turn reduces leverage ex-ante and improves global welfare. Our results have implications for the recent academic literature about the Global Financial Cycle (see, for example, Gourinchas and Rey (2007), Gourinchas, Govillot and Rey (2010), Rey (2013), Cerutti, Claessens and Rose (2017), Miranda-Agrippino and Rey (2018)). First, it might in fact be optimal for the dominant currency country to respond to global economic conditions; and second, the dynamics of global expectations about the monetary policy of the dominant currency CB might be as important as the monetary policy itself because it is these expectations that determine ex-ante leverage of firms.

# 7 Empirical Evidence

In this section, we provide evidence on the viability of the dollar as a dominant currency through the channels described in our model, as well as other evidence that is consistent with the predictions of our theory.

## 7.1 Why is the dollar the dominant currency of choice in international debt markets?

As we show above (see condition 13), firms prefer to issue in dollars if the dollar exchange rate co-moves positively[26] with the present value of their profits, which we proxy by their stock market value. As a proxy for the global dollar exchange rate, we use the trade-weighted dollar index against major currencies, including the Eurozone, Canada, Japan,

[26] In fact, given that issuing in dollars is cheaper than in any other currency, condition (11) implies that firms would issue all their debt in dollars even if this correlation were negative, but not too negative relative to the cost gain of issuing in dollars.

United Kingdom, Switzerland, Australia, and Sweden, obtained from the FRED database. For the stock market, we use S&P 500 and MSCI AC World prices.

Given that the dollar is the most common currency of denomination in international debt markets, the first prediction of our model is that the returns on the dollar index are positively correlated with the returns on the stock market indices at horizons corresponding to the weighted average corporate debt maturity, that is around 6-7 years (see subsection 7.2, Choi, Hackbarth and Zechner (2018), Cortina, Didier and Schmukler (2018)). To test this prediction, we first run the following regressions for the horizons of $h \in \{3, 12, 24, 36, 48, 60, 72, 84, 96, 108, 120\}$ months:[27]

$$Return\_USD_t^h = \alpha^h + \beta_h Return\_SP500_t^h + \epsilon_t^h \,. \tag{16}$$

Here, $Return\_USD_t^h$ and $Return\_SP500_t^h$ denote the rolling returns on the dollar index and the SP500 index over $h$ months, respectively. The left-hand panel of Figure 2 reports the results for the regression coefficient $\beta_h$ over different horizons together with the 95% confidence intervals for the period between December 1987 and September 2018.

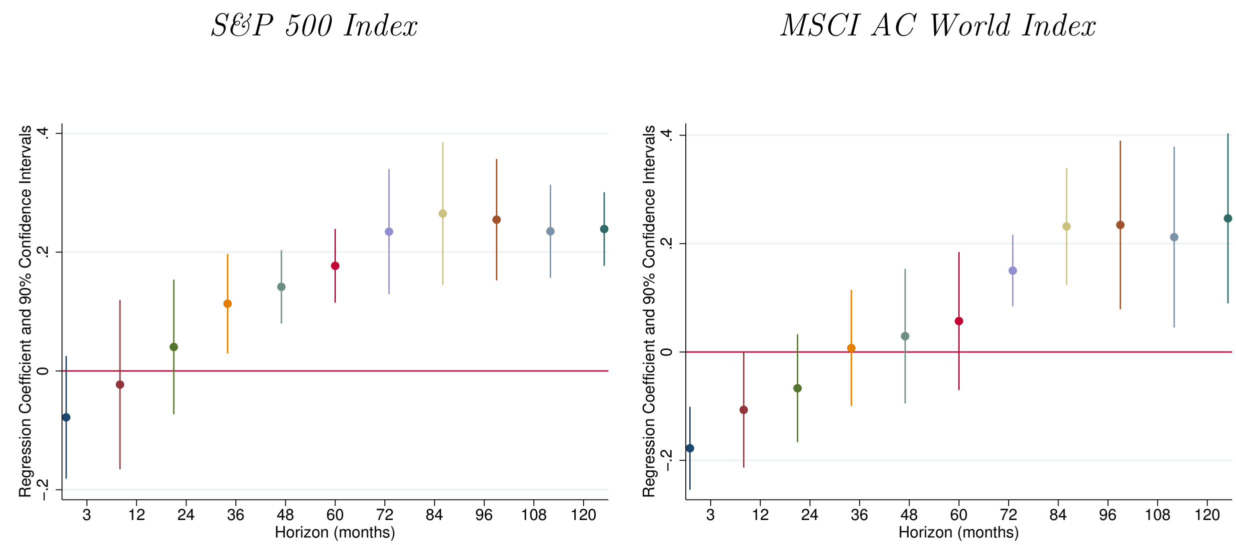
The results show a pattern of negative correlations at short horizons and positive and mostly increasing correlations at longer horizons. These findings together with condition (13) suggest that US firms are better off is they borrow in dollars rather than in other major international currencies if their debt maturity exceeds roughly two years, which is the case.

Next, we turn to the rest of the world and compute the correlation of the returns on the dollar index with the returns on the MSCI AC World Index. We follow the same procedure as before and run the following regressions, for the same horizons $h$ as above:

$$Return\_USD_t^h = \alpha^h + \beta_h Return\_MSCIACWorld_t^h + \epsilon_t^h \,. \tag{17}$$

[27]We control for autocorrelation at the respective horizons by using the Newey-West correction with the respective number of lags.

**Figure 2: Correlation of the USD index with Stock Market Indices**




*Notes:* The graph on the left-hand side reports the regression coefficients $\beta_h$ from the regressions (16). The graph on the right-hand side reports the regression coefficients from the regressions (17). The dots show the corresponding values of the $\beta_h$ coefficients while the lines show the 95% confidence intervals for these coefficients. Standard errors are corrected using the Newey-West procedure, with the number of lags equal to the horizon $h$ of returns in each respective regression.

The results from these regressions are aligned with the results for the S&P 500 stock index. While dollar is negatively correlated with the MSCI AC World Index at short horizons, this correlation turns positive at horizons longer than three years. Thus, condition (13) suggests that international firms with average debt maturities longer than three years (which is indeed the case for most firms; see subsection 7.2, and also Choi, Hackbarth and Zechner (2018), Cortina, Didier and Schmukler (2018)) are better off issuing debt denominated in dollars.

## 7.2 Debt currency and maturity choice

Our results in subsection 7.1 have direct implications for the link between debt maturity and the incentives to issue dollar-denominated debt. Namely, as the dollar comovement with the stock market increases over longer horizons, we expect that firms with longer maturity of debt have a preference for issuing this debt in dollars.

We use data at the bond issuance level in order to formally test the hypothesis that the propensity to issue dollar-denominated debt increases with debt maturity. We restrict the sample to non-financial corporations that issued bonds between 2000 and 2019.[28,29] For identification, we restrict the sample to only firms that issued bonds in multiple currencies in a given month and use fixed effects at the $Firm * Month$ level in order to test whether a non-financial corporate firm that issues bonds in different currencies in a given month has the longer maturity bonds denominated in dollars.

#### 7.2.1 Data

We use data from Dealogic where observations are at the ISIN level of bond issuance. In order to keep the timing of our analysis similar to the previous sections, we restrict the sample to bonds issued between January 2000 and February 2019. Our dataset includes a total of 102,159 bonds, issued by 23,992 firms that are headquartered in 110 different countries.

The dataset includes information on the identity of the firm, the country where it is headquartered, the industry as well as information on the bond such as the currency denomination, date of issuance, maturity date, issued amount denominated in the local currency of the firms' headquarters, and whether the bond is investment-grade.

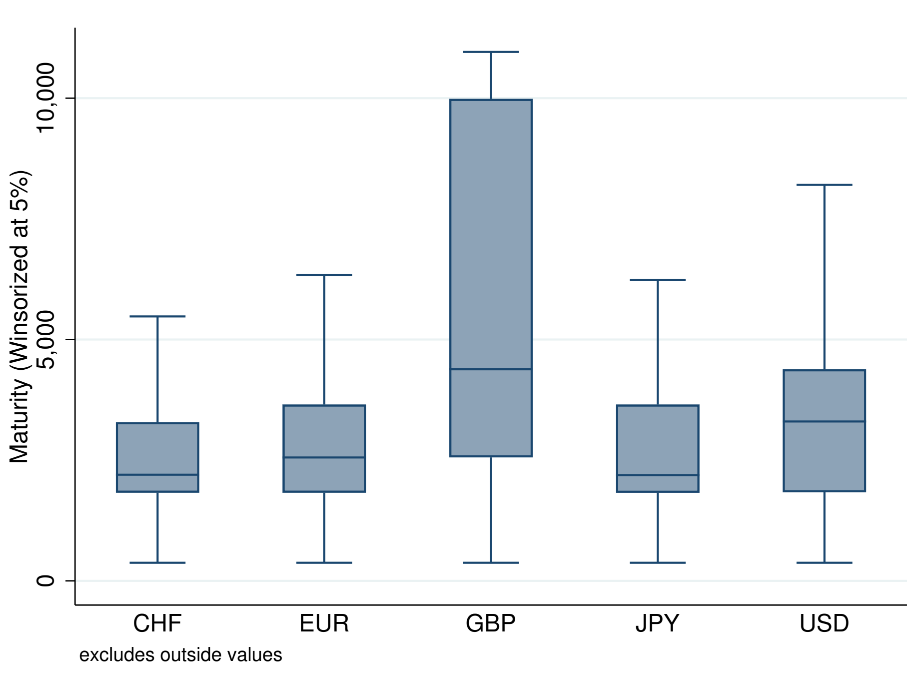
In the full sample, the mean of the winsorized maturity is 2,950 days, with a standard deviation of 2,646 days, the minimum value is 375 days and the maximum value is 10,958 days. We present the summary statistics regarding debt maturity by different currencies in Figure 3 in a box plot. As Figure 3 shows, maturities of bonds issued in pounds have the largest median, followed by bonds issued in dollars.[30]

[28] The dataset includes perpetual bonds as well. In order to include them in the analysis, we winsorize the maturity of the bonds at 5%, both at the lower and upper tail of the maturity distribution.

[29] We drop all observations for which the issuer is not a non-financial firm or the industry is coded as "Finance."

[30] We provide further discussion of pound-denominated debt in subsection 7.5.

**Figure 3: Summary Statistics of Maturity by Currency**




*Source: Dealogic, authors' calculations*
*Notes: The box plots the 25th and the 75th percentile on the outside and the the line inside the box is the median. The whiskers show the lower and upper adjacent values.*

#### 7.2.2 Results

Following our results in subsection 7.1 and the patterns in Figure 1, we test the following hypotheses using micro-level data on bond issuance.

*Hypothesis BI-1:* A longer debt maturity is associated with a higher propensity to issue dollar-denominated debt.

In our model, we take debt maturity as given. As we have shown previously, given debt maturity, firms' propensity to issue in dollars is be increasing in the correlation of the dollar with the stock market; hence, *Hypothesis BI-1* follows directly from the fact that the correlation of the dollar with the stock market is higher over longer horizons.

To measure the propensity to issue dollar-denominated debt, we use is $\mathbb{1}(USD)$, which is a dummy variable that takes the value 1 if the currency denomination of the bond is the dollar. Then, the independent variables of interest in our regressions are: $Maturity_w$, which is the winsorized and standardized value of maturity at 5% level; $\mathbb{1}(Maturity > 1y)$, which is a dummy variable takes the value 1 if non-winsorized maturity is greater than one year. According to our hypotheses, we expect a positive coefficients for these variables.

Other control variables we use are the size of the issuance and a dummy variable that is equal to one if the bond is investment-grade. Moreover, depending on the specification, we include $Industry$, $Country * Month$ and $Firm * Month$ fixed effects. We cluster the standard errors at the $Country * Year$ level.[31]

We run different linear regressions varying the fixed effects used and making different cuts at the sample in order to test the predictions of our theory. Table 1 presents the results.

The first three columns control for bond characteristics as well as industry and $Country * Month$ fixed effects. In column (1), we run the regression with the full sample. The coefficient on $Maturity_w$ suggests that a one standard deviation increase in maturity increases the likelihood of the currency denomination of the bond to be dollars by 2 percentage points. In column (2), in order to address worries that this result might be driven by the ease of issuing longer maturity bonds in dollars, we still use the full set of firms, but restrict the sample to bonds with maturity of 10 years or less. For this specification, we use the non-winsorized sample, and test whether $\mathbb{1}(Maturity > 1y)$ predicts a higher likelihood of dollar debt issuance. Indeed, dollar bond issuance is more likely for maturities of 1y-10y, in line with our findings in subsection 7.1. In column (3), we restrict the sample to bonds issued by non-US, non-Euro area, non-JP, non-GB, non-CHF firms and ones that are only

[31] In the benchmark specification, we use the country where the headquarters of the parent company of the issuer is located at. As a robustness check, we use the residence of the issuer instead. Our results are virtually unchanged.

issued in USD, EUR, JPY, GBP and CHF. We obtain the same result that longer maturities correspond to a higher likelihood of issuance in dollars.

For identification, we rely on firms that issue multiple bonds in at least two different currencies in a given month. This allows us to tightly identify that the same firm, which has access to multiple markets, chooses to issue the longer maturity bond in dollars as opposed to other currencies. In column (4), we repeat the results for a modification of the regression in column (1), but use $Firm * Month$ fixed effects instead in a sample that is restricted to firms that issued debt in multiple currencies in a given month. In column (5), we repeat column (3) in the new sample with $Firm * Month$ fixed effects. In both cases, the results corroborate *Hypothesis BI-1*.[32]

## 7.3 The share of dollar-denominated debt in the last two decades

### 7.3.1 Dollar debt and inflation risk premia

In our model, changes in exchange rates are fully determined by relative inflation dynamics in the two countries. A consequence of this result is Theorem 5.2, stating that firms should be issuing dollar debt only if they expect US to have the most counter-cyclical inflation over the horizon of their debt maturity.[33] These expectations about inflation cyclicality can be backed out from the inflation risk premium, given by the difference between inflation expectations under the risk-neutral and the physical measure:

$$IRP_{i,t} \;=\; \log\left(\frac{E_t^i[\mathcal{P}_{i,t,t+1}]}{E_t[\mathcal{P}_{i,t,t+1}]}\right) \;=\; \log\left(\frac{e^{r_{i,t}}\mathrm{Cov}_t(M_{i,t,t+1},\mathcal{P}_{i,t,t+1})}{E_t[\mathcal{P}_{i,t,t+1}]}\right).$$

[32]When we repeat column (2) using $Firm * Month$ fixed effects, we get a coefficient similar in magnitude, yet statistically insignificant. Note however that the power of these regressions is low due to a large number of fixed effects.

[33]While the perfect link between exchange rates and inflation relies on a strong form of PPP, Theorem 5.2 would still hold true even with large PPP deviations, as long as the relative inflation component of exchange rates contributed significantly to the covariance (13) over the horizons of debt maturity of a typical firm.

**Table 1:** Debt maturity and currency choice

| | (1) | (2) | (3) | (4) | (5) |
|---|---|---|---|---|---|
| Sample: | Full | Full & $< 10y$ | Partial & FC | Full$^{\dagger}$ | Partial & FC$^{\dagger}$ |
| | $\mathbb{1}(USD)$ | $\mathbb{1}(USD)$ | $\mathbb{1}(USD)$ | $\mathbb{1}(USD)$ | $\mathbb{1}(USD)$ |
| $Maturity_w$ | 0.0195*** | | 0.0203*** | 0.0405*** | 0.0786*** |
| | (0.00294) | | (0.00543) | (0.0148) | (0.0253) |
| $\mathbb{1}(Maturity > 1y)$ | | 0.0343*** | | | |
| | | (0.00742) | | | |
| Controls | X | X | X | X | X |
| Industry FE | X | X | X | | |
| Country*Month FE | X | X | X | | |
| Firm*Month FE | | | | X | X |
| Observations | 99,283 | 72,382 | 6,826 | 4,127 | 727 |
| R-squared | 0.744 | 0.714 | 0.646 | 0.409 | 0.531 |
| Mean of Dep. Var | 0.330 | 0.247 | 0.842 | 0.344 | 0.635 |

*Notes:* Standard errors clustered by $Country * Year$ in parantheses. *, **, *** denote significance at the 10, 5 and 1% level respectively. $\mathbb{1}(USD)$ is a dummy variable that takes the value 1 if the currency of the issued bond is the dollar. $Maturity_w$ is the standardized value of maturity winsorized at 5% and 95% levels. $\mathbb{1}(Maturity > 1y)$ is a dummy variable that is 1 if maturity is greater than 1 year. Controls include the local currency amount of the size of the issuance and a dummy variable regarding the status of investment-grade status of the bond. Full sample includes all observations. Partial & FC refers to observations where the nationality of the company is not the United States, a country in the euro area, Japan, the Great Britain or Switzerland, but the currency is either USD, EUR, JPY, GBP or CHF. $^{\dagger}$ means that the sample is further restricted only to firms that issued debt in multiple currencies in each month.

The covariance term, $\text{Cov}_t(M_{i,t,t+1}, \mathcal{P}_{i,t,t+1})$, reflects the basic intuition: Inflation risk premium is determined by market expectations about inflation cyclicality. The following is true.

**Proposition 7.1** *The inflation risk premium, $IRP_{i,t}$, has the largest value for the dominant currency country.*

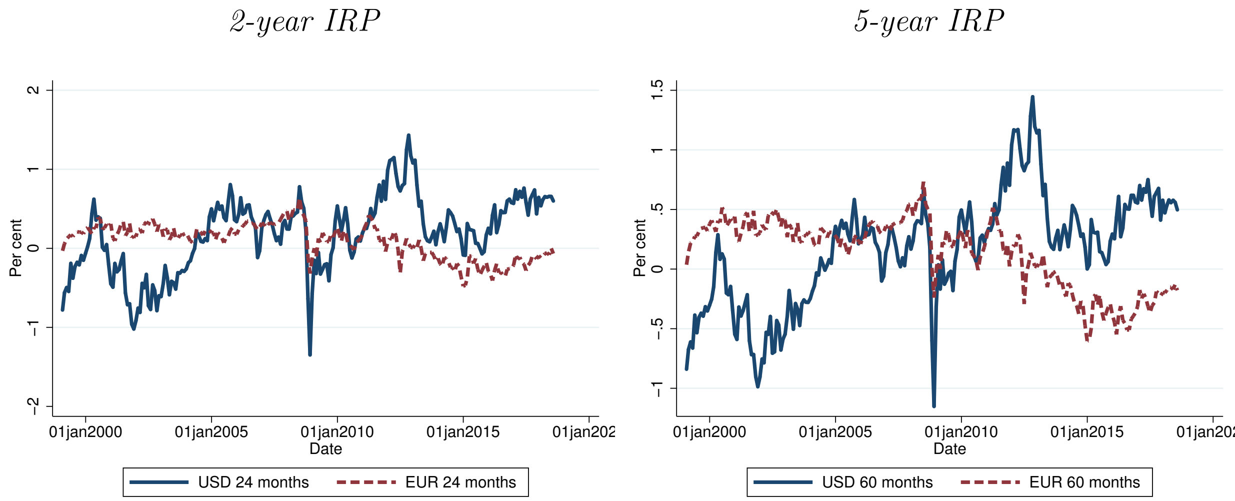
The key prediction of Proposition 7.1 is a direct link between the IRP and the dominancy of a currency. To test this prediction empirically, consider an extension of our benchmark model featuring time variation in the coefficients $\psi_{€,t}$ and $\psi_{\$,t}$, the respective abilities of the ECB and the Federal Reserve to pursue counter-cyclical stabilization policies. Importantly, what matters is not the actual value of these coefficients but the firms' expectations about them. In such an extended model, Theorem 5.2 implies that a change in the sign of $\psi_{\$,t}-\psi_{€,t}$ from positive to negative will immediately trigger a regime change, making the dollar lose its dominant currency status to euro. Proposition 7.1 predicts that this status change should be accompanied with a simultaneous change in the sign of $IRP_{\$,t} - IRP_{€,t}$ from positive to negative.

We compare the pre- and post-crisis trends in the shares of euro- and dollar-denominated debt (Figure 1) with the pre- and post-crisis dynamics of inflation risk premia in the US and the Eurozone.[34] Figure 1 and Figure 4 show a behaviour that is consistent with our key prediction: In the pre-crisis period, the inequality $IRP_{€,t} \geq IRP_{\$,t}$ held most of the time, and the fraction of euro-denominated debt was rising; post crisis, the relationship reverted to $IRP_{EUR_t} \leq IRP_{\$,t}$, and the fraction of euro-denominated debt started to fall.

We now investigate the dynamic link between IRP and debt issuance in different currencies. Clearly, the presence of any type of inertia in debt currency denomination (e.g., due to some implicit or explicit adjustment costs or smooth variations in beliefs) will make the transition between the euro- and dollar-dominance regimes smooth, with the share of dollar

[34] We use the estimates from Hördahl and Tristani (2014) for the inflation risk premia in the US and the Eurozone. Hördahl and Tristani (2014) use a joint macroeconomic and term structure model, combined with survey data on inflation and interest rate expectations, as well as price data on nominal and inflation index-linked bonds. To the best of our knowledge, Hördahl and Tristani (2014) is the only paper in the existing literature that applies the same rigorous methodology to recover IRP both for the US and the Eurozone. We report their estimates for the 2-year and 5-year horizons in Figure 4; the results are qualitatively similar for other horizons.

**Figure 4: Inflation Risk Premia in the US and the Eurozone**




*Source: Hördahl and Tristani (2014), authors' calculations.*

debt increasing in $IRP_{\$,t}$ and decreasing in $IRP_{€,t}$. Stated formally, we test the following hypothesis.

<u>*Hypothesis IRP-1:*</u> The share of dollar-denominated debt, $USD_t^{shr}$, is positively related to $IRP_{\$,t} - IRP_{€,t}$. More specifically, it is positively related to $IRP_{\$,t}$, while it is negatively related to $IRP_{€,t}$.

We test this hypothesis using linear regression. Table 2 presents the results of these tests. First, in column (1), we find a positive, statistically significant relationship between the share of dollar-denominated debt on the IRP differential, with an $R^2$ of 52.8%. In column (3), even after including year dummies, the positive sign remains positive and statistically significant. The estimates are also economically large: a 1 percentage point increase in the IRP differential between the dollar and the euro is associated with an increase in the share of dollar-denominated debt by about 10 percentage points.

Second, we run an alternative regression specification, including both inflation risk premia for the dollar and the euro separately as independent variables in the regression. In column (2) and (4), we report the results without and with year dummies, respectively. The results are again consistent with our hypothesis: The share of dollar-denominated debt is positively related to $IRP^{5Y}_{\$,t}$, while it is negatively related to $IRP^{5Y}_{€,t}$. Furthermore, the $R^2$ of the specification in column (2) is much higher than that in column (1), and the coefficient on the $IRP^{5Y}_{€,t}$ is economically very large: a 1 percentage point increase in the Euro IRP is associated with a drop in the share of dollar-denominated debt of about 22 percentage points. The negative coefficient remains economically significant even after the inclusion of year dummies in quarterly regressions in column (4).

**Table 2:** The debt currency choice and inflation risk premium

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| | $USD^{shr}_t$ | $USD^{shr}_t$ | $USD^{shr}_t$ | $USD^{shr}_t$ |
| $IRP^{5Y}_{\$,t} - IRP^{5Y}_{€,t}$ | 9.636*** | | 1.553** | |
| | (1.129) | | (0.718) | |
| $IRP^{5Y}_{\$,t}$ | | 2.079* | | 0.841 |
| | | (1.053) | | (0.688) |
| $IRP^{5Y}_{€,t}$ | | -22.71*** | | -4.863*** |
| | | (1.495) | | (0.967) |
| Year dummy | | | ✓ | ✓ |
| Freq. | Q | Q | Q | Q |
| Observations | 72 | 72 | 72 | 72 |
| R-squared | 0.528 | 0.786 | 0.986 | 0.989 |

*Notes:* Robust standard errors in parantheses. *, **, *** denote significance at the 10, 5 and 1% level respectively. $USD^{shr}_t$ refers to the share of dollar debt including both bank loans and debt securities. $IRP^{5Y}_{\$,t}$ and $IRP^{5Y}_{€,t}$ refer to the 5-year inflation risk premia for the dollar and the euro respectively, as measured by Hördahl and Tristani (2014). $Q$ refers to quarterly frequency since 2000.

In our model, IRP can be viewed as a barometer of market expectations of inflation counter-cyclicality, as captured by $\phi_{€,t}$. Hence, our results suggest that the significant rise in the post-crisis share of dollar-denominated debt may be due to declining expectations of inflation stabilization and increasing risk of deflation in the Euro area following the Global Financial Crisis in 2008 and the European Debt Crisis in 2011.

Consistent with Figure 1, Maggiori, Neiman and Schreger (2018) show that the share of dollar-denominated debt in cross-border corporate holdings has drastically increased in the post-crisis period compared to the euro. We argue that this pattern is to a large extent driven by the bond-supply channel of Figure 1, and bond investors hold what the firms issue to clear the markets in general equilibrium. That said, while bond investors might generally dislike holding nominal bonds with a high inflation risk premium, there is an opposite force in our model that increases the attractiveness of dollar-denominated bonds for lenders: The default probability of these bonds is lower because it is easier for firms to repay dollar debt due to lower real debt burdens in bad times.[35] However, in equilibrium the former dominates and hence the dominant debt currency has higher inflation risk premium.

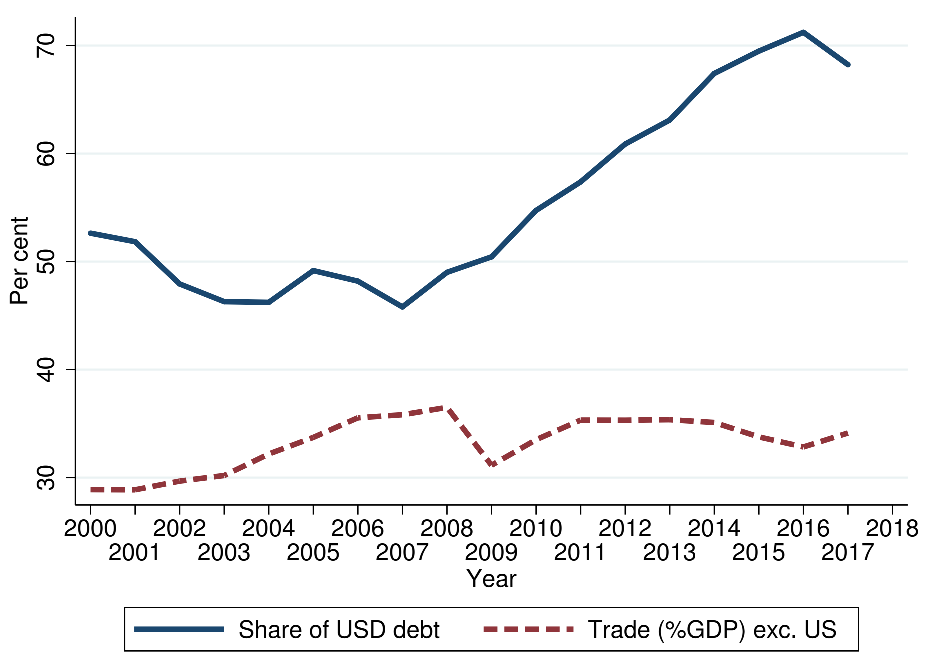
#### 7.3.2 Dollar debt and international trade

Our model also makes predictions about the relationship between international trade and dollar-denominated debt. As we show in the Appendix (see the proof of Proposition 6.1), in the dominant currency debt equilibrium of Theorem 5.2, an increase in the coefficient $\phi$ of monetary policy effectiveness of the dominant currency country's central bank is always associated with (i) more issuance of debt denominated in the dominant currency; and (ii) a drop in the conditional expectation of the amount of international trade. This is intuitive: An aggressive monetary policy provides incentives for firms to choose higher leverage, which ex post leads to more debt overhang and a drop in international demand. Thus, in the extended version of the model discussed in the previous section, shocks to $\psi_{\$,t}$ should move

[35] See also Kang and Pflueger (2015).

trade and the amount of debt denominated in the dominant currency in opposite directions. Figure 5 shows the joint dynamics of dollar-denominated debt and international trade over the last two decades. Consistent with our theory, the pre- and post-crisis trends in Figure 1 move-to-one with opposite trends in international trade.[36]

**Figure 5: Trade and Debt**




*Source: BIS, World Bank, FRED, authors' calculations*
*Notes:* $TotalTrade(\%GDP)_t^{exUS}$ is the total trade to world GDP, excluding the US.

[36] Indeed, regressions with yearly data of the share of dollar debt on differences in dollar and euro IRP and total trade to world GDP, excluding the US yields a positive and significant coefficient for IRP differences and a negative coefficient for trade, in line with our predictions.

## 7.4 Cross-sectional evidence: Local currency shares

In this section, we test the predictions of our model using a cross-section of emerging market economies for which data on corporate debt in different currencies are available.[37] To this end, we prove the following extension of Theorem 5.1 for the case when firms issue a mixture of local currency (LC) and dollar-denominated debt (see Theorem G.1 in the Appendix for a proof. Note that, while Proposition 7.2 is a partial equilibrium result, it still holds in general equilibrium when debt overhang costs are sufficiently small).

**Proposition 7.2** *Suppose that (1) $q_i(i) = q_i(\$)$ (that is, issuing in LC costs the same as issuing in dollars); (2) the variance of all shocks is sufficiently small; (3) inflation in all countries is determined by (15); and (4) issuing in both LC and dollars is optimal. Then,*

*(a) the fraction $\frac{B_{i,t}(i)}{B_{i,t}(\$)\mathcal{E}_{\$,i,t}}$ is monotone increasing in the covariance $\mathrm{Cov}_t(\varepsilon_{i,t+1}, \varepsilon_{\$,t+1})$ if and only if $B_{i,t}(i) \geq B_{i,t}(\$)\mathcal{E}_{\$,i,t}$;*

*(b) the fraction $\frac{B_{i,t}(i)}{B_{i,t}(\$)\mathcal{E}_{\$,i,t}}$ is always monotone decreasing in $\sigma_{i,\varepsilon}$.*

Items (a)-(b) of Proposition 7.2 directly translate into the following two empirical hypotheses.

*Hypothesis CS-1:* The local currency share of corporate debt is *higher* for countries in which domestic inflation correlates more with US inflation controlling for relevant factors.

Indeed, local currency debt partly replicates insurance properties of the dominant currency in downturns, while being a better hedge against domestic productivity shocks. In order to test this hypothesis, we proceed as follows. For each country $i$ in our sample, we estimate the following time series regression:

$$\pi_t^i = \gamma_0 + \gamma_1 \cdot Return\_MSCIACWorld_t + \Gamma \cdot Return\_DomesticStockIndex_t^i + \pi_t^{res,i}, \quad (18)$$

[37] Data are obtained from the Institute for International Finance (IIF) for period from 2005 Q1 to 2018 Q2. The countries in our sample are: Argentina, Brazil, Chile, China, Colombia, Czechia, Hong Kong, Hungary, India, Indonesia, Israel, Republic of Korea, Malaysia, Mexico, Poland, Russian Federation, Saudi Arabia, Singapore, South Africa, Thailand and Turkey.

where $\pi_t^i$ is the domestic monthly inflation rate in country $i$ and $Return\_MSCIACWorld_t$ is the monthly return on the MSCI AC World Index. $Return\_DomesticStockIndex_t^i$ is the monthly return on the domestic stock market index. $\pi_t^{res,i}$ are the residuals from this regression. We also run the following regression for the US:

$$\pi_t^{US} = \mu_0 + \mu_1 Return\_MSCIACWorld_t + \pi_t^{res,US}\,, \tag{19}$$

We then run the following regression in order to compute a proxy for the covariance $\mathrm{Cov}_t(\varepsilon_{i,t+1}, \varepsilon_{\$,t+1})$ between the residual domestic inflation and residual US inflation (see item (a) of Proposition 7.2),

$$\pi_t^{res,i} = \alpha + \beta \pi_t^{res,US} + \epsilon_t\,,$$

where $\pi_t^{res,i}$ is the residual domestic monthly inflation rate in country $i$ from (18) and $\pi_t^{res,US}$ is the residual monthly inflation rate in the US from (19). We denote the estimated slope coefficient by $\hat{\beta}_i^{\pi_t^{res,i},\pi_t^{res,US}}$.

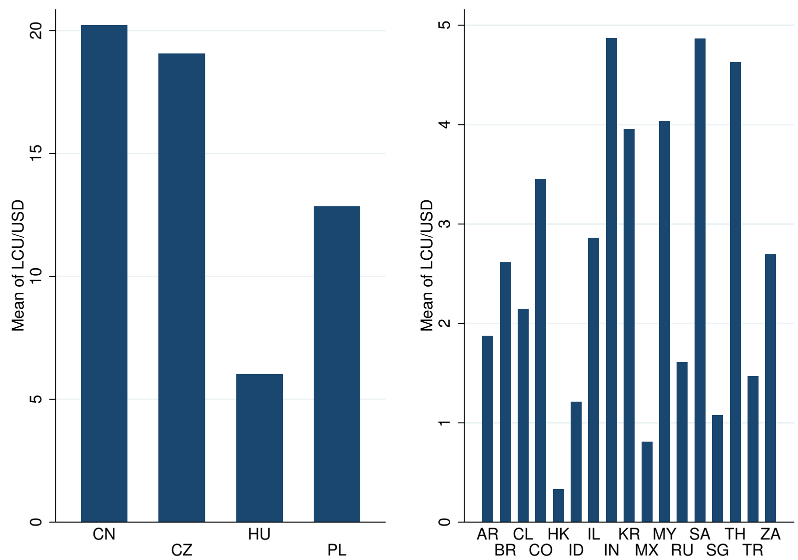
We then run the following cross-sectional regression:

$$\frac{L\bar{C}U}{USD}_i = \alpha_1 + \beta_1 \hat{\beta}_i^{\pi_t^{res,i},\pi_t^{res,US}} + X_i + \eta_i\,. \tag{20}$$

Here, $\frac{L\bar{C}U}{USD}_i$ is the average ratio of debt denominated in local currency to debt denominated in dollars for corporates in the countries in the dataset, and $X_i$ denote other control variables.

Figure 6 shows the mean of the debt ratio, $\frac{L\bar{C}U}{USD}_i$, for each country in our sample. The left-hand panel shows several outliers: China and the EU countries in the sample (Czechia, Hungary and Poland), while the right-hand panel shows the rest countries. We exclude outliers from our regressions and focus only on the sample of countries on the right-hand panel.

**Figure 6: Mean of the local currency to USD debt ratio by country**




*Source: IIF, authors' calculations*

Item (a) of Proposition 7.2 predicts that the coefficient $\beta_1$ in the regression (20) should be positive. The first three columns of Table 3 show that this is indeed the case. In column (1), we show the results of a univariate regression. In column (2), we add an additional control variable $\bar{kaopen}_i$ : a financial openness index obtained from Chinn and Ito (2006). In column (3), we take the predictions of the model literally as appear in item (a) of Proposition 7.2: $\beta_1 > 0$ for countries where $\frac{\bar{LCU}}{USD}_i > 1$ and exclude the two countries where $\frac{\bar{LCU}}{USD}_i < 1$, namely Hong Kong and Mexico. In all three columns, regressions corroborate *Hypothesis CS-1.* [38]

A second hypothesis is the following:

[38] All our results are qualitatively and quantitatively similar if we use raw domestic and US inflation rates, instead of residuals. Moreover, all results go through if we use the share of local currency debt in total debt instead of the ratio of local currency debt to USD debt.

**Table 3:** The cross section of the local currency to USD debt ratio

| | (1) | (2) | (3) | (4) |
|---|---|---|---|---|
| | $\frac{L\bar{C}U}{USD}_i$ | $\frac{L\bar{C}U}{USD}_i$ | $\frac{L\bar{C}U}{USD}_i$ | $\frac{L\bar{C}U}{USD}_i$ |
| $\hat{\beta}_i^{\pi_t^{res,i},\pi_t^{res,US}}$ | 3.951*** | 3.930*** | 3.713*** | |
| | (0.680) | (0.640) | (0.775) | |
| $ka\bar{o}pen_i$ | | -0.0108 | 0.102 | -0.334 |
| | | (0.327) | (0.413) | (0.349) |
| $\sigma_i^{\pi_t^{res,i}}$ | | | | -2.218 |
| | | | | (1.306) |
| Observations | 17 | 17 | 15 | 17 |
| R-squared | 0.537 | 0.537 | 0.409 | 0.217 |

*Notes:* Robust standard errors in parantheses. *, **, *** denote significance at the 10, 5 and 1% level respectively. $\frac{L\bar{C}U}{USD}_i$ is the mean share of local currency debt obtained from the IIF for each of the 17 emerging market economies between 2005 Q1 and 2018 Q2. $\hat{\beta}_i^{\pi_t^{res,i},\pi_t^{res,US}}$ is the estimated regression coefficient of a linear regression of residuals of monthly domestic inflation rate from (18) on the residuals of the US inflation rate from (19). $ka\bar{o}pen_i$ is the mean of Chinn-Ito financial openness index for each country. $\sigma_i^{\pi_t^{res,i}}$ is the standard deviation of the residuals of the monthly domestic inflation rate obtained from (18). In column (3), Hong Kong and Mexico are excluded.

*Hypothesis CS-2:* Firms in countries with more volatile domestic inflation tend to have less debt denominated in local currency.[39]

To test this hypothesis, we calculate the standard deviation of $\pi_t^{res,i}$ as a proxy for $\sigma_{\varepsilon,i}$ in Proposition 7.2, and then run the following cross-sectional regression:

$$\frac{L\bar{C}U}{USD}_i = \alpha_2 + \beta_2 \sigma_i^{\pi_t^{res,i}} + X_i + \eta_i\,. \tag{21}$$

Proposition 7.2, item (b) predicts that $\beta_2 < 0$. Column (4) of Table 3 shows the results of regression (21). Although the result is lacking statistical significance, the sign of the coefficient is consistent with our theoretical prediction.

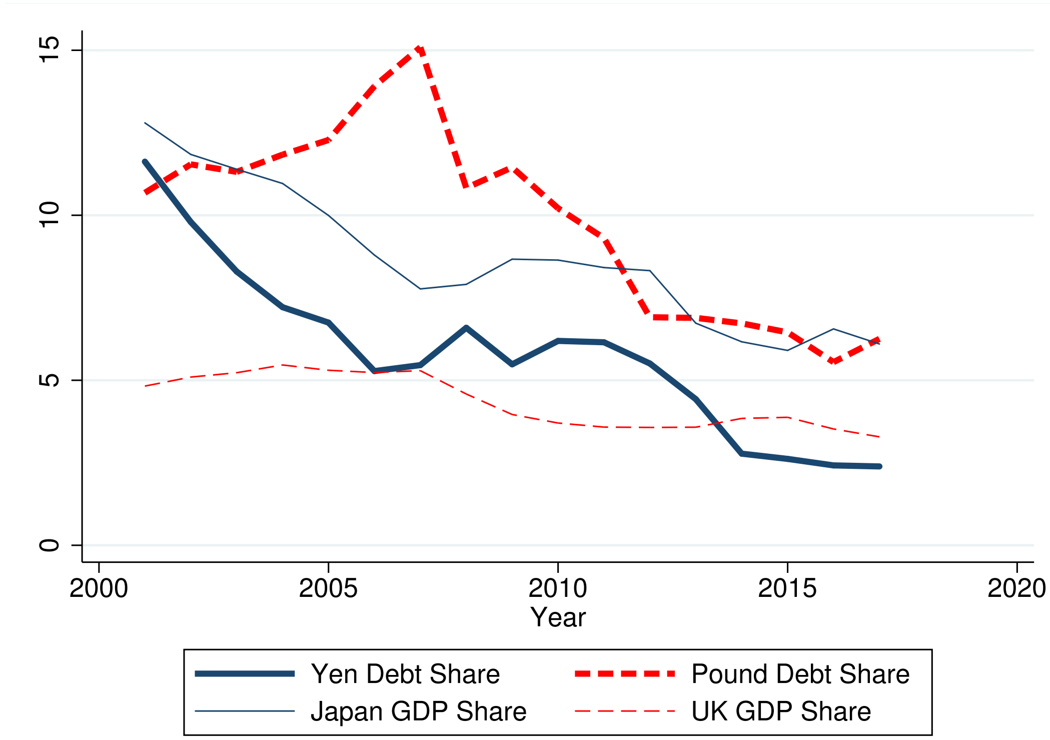
### 7.5 Further evidence: yen vs. pound, and pound vs. dollar in the interwar years

The inflation expectations channel discussed above can also be used to study patterns of debt issuance in other major currencies. Intuitively, one would expect that the share of international debt issued in the currency of a given country should be directly linked to the size of that country, as measured by the share of that country in the world GDP. However, this is generally not the case. For example, Figure 7 shows the dynamics of shares of Japan and the United Kingdom in the world GDP and together with the shares of their currencies in denominating debt issued by foreign firms. Surprisingly, despite the fact that Japan has both a higher world GDP share and a lower interest rate, the share of pound-denominated debt is significantly higher than that of yen-denominated debt.

Through the lens of our model, this could be explained by the fact that inflation in the UK was often close to and above the inflation target of the Bank of England, with firms seeing the real value of their pound-denominated debt decline more often (Figure 9). On the

[39] Farhi and Maggiori (2017) make a related prediction that firms in countries with a volatile nominal rate tend to issue dollars.

**Figure 7: Japan versus the United Kingdom**

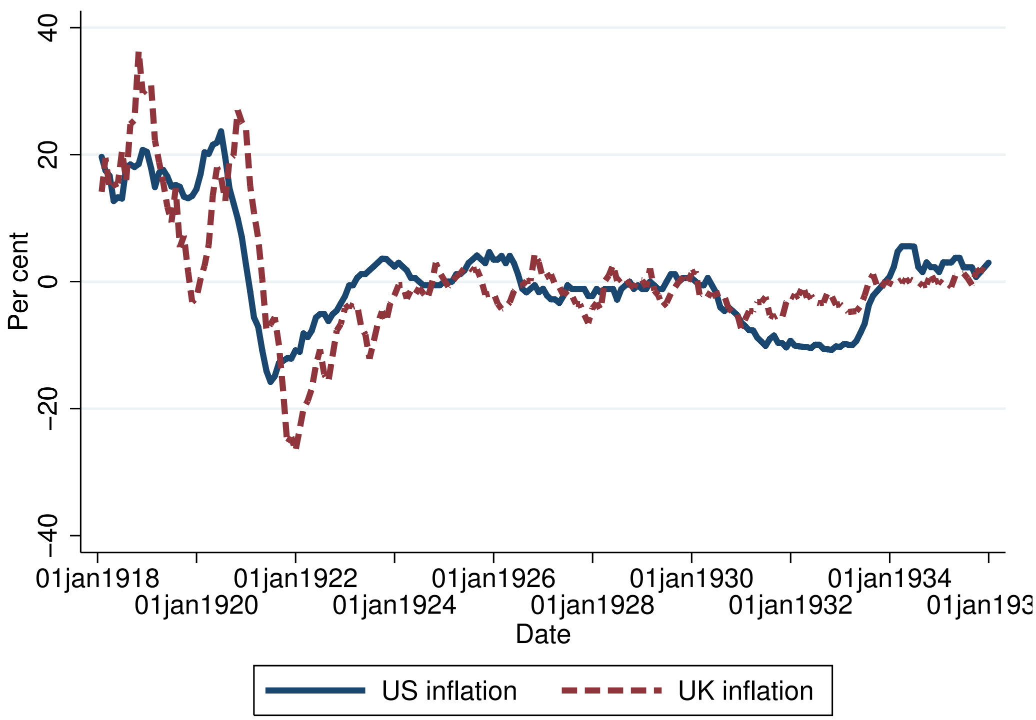



*Source: BIS, IMF WEO, authors' calculations*

other hand, firms borrowing in yen have seen the real values of their nominal debt increase as inflation in Japan constantly undershot the target set by the Bank of Japan. Our model predicts that such negative surprises should lead to a drop in the issuance of yen-denominated debt because rational firms update their expectations about deflation risk in Japan in bad times.

Similarly, one could use the expectations channel to shed light on the history of multiple, repeated switches between the pound and the dollar in their role of the main reserve currencies during the inter-war period (see Chiţu, Eichengreen and Mehl (2014)). Consider two currencies (the pound and the dollar) with sufficiently similar indices $\phi_{\$,t} \approx \phi_{GBP,t}$. Our model predicts that shocks to expectations about the difference $\phi_{\$,t} - \phi_{GBP,t}$ can lead to quick switches back and forth between different dominant currency debt equilibria, in which one currency repeatedly gains and then looses the dominant currency role to its competitor. Consistent with our theory, the pound started loosing its role as the dominant currency after the negative inflation surprise at the beginning of 1920s during the 1920-21 recession (Figure 8). At the same time, the dollar faced greater deflation during the Great Depression, with a the subsequent partial regaining of dominance by the pound, according to the evidence provided by Chiţu, Eichengreen and Mehl (2014).

**Figure 8: Historical Inflation Rates**




*Source: Global Financial Data, Office of National Statistics, BIS*

# 8 Conclusion

Motivated by the omnipresence of the dollar in the denomination of debt contracts globally, we develop a simple international general equilibrium model. Our main theoretical result offers a potential explanation of why the dollar is the dominant currency, despite the presence of rival currencies with deep and liquid debt markets. This happens if firms believe that the Federal Reserve is both able and willing to effectively pursue aggressive monetary policy in global economic downturns, generating more inflation than its peer central banks for horizons matching the average debt maturity of corporates which in turn reduces real debt burden of firms. The empirically observed behaviour of the dollar vis-a-vis other major currencies since 2000 is supportive of this mechanism. We show how the expectations of market participants underlying the dollar dominance can be backed out from the inflation risk premium, and provide further empirical evidence that is consistent with our theory.

What do our results imply for the future of the dollar? Many explanations of the dominant role of the dollar in the international monetary system feature arguments like inertia, size, network externalities, and market liquidity. All these arguments suggest that changes in the dominance status of a currency occur very slowly. By contrast, our results suggest that dollar can quickly lose its dominance if the expectations that the Federal Reserve is able to stimulate the economy and reduce real debt burdens of firm during global crises decline. As this relies on the beliefs of market participants, this may occur abruptly as historical evidence in Chiţu, Eichengreen and Mehl (2014) shows.

In our model, we have abstracted from many important features of the real world to highlight the main mechanism. Our model can be extended in multiple directions. First, addressing the interactions between the role of the dollar in trade and finance may shed important light on endogenous inflation dynamics and the role of the dollar in trade invoicing. Second, modelling the demand for safe assets would help in understanding the role of the dollar for financial intermediation and household balance sheets. We leave these important questions for future research.

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# A Consumption and Production Decisions

In the appendix, we consider a more general preference specification with potentially different weights $\theta_j$ defining the consumption bundle:

$$C_{i,t} \;=\; \left( \sum_j \theta_j \int_0^1 (\tilde{C}_{i,t}(j,\omega))^{\frac{\eta-1}{\eta}} d\omega \right)^{\frac{\eta}{\eta-1}} .$$

We will also define the respective price index

$$\mathcal{P}_{i,t} \;=\; \left( \sum_j \theta_j^{\eta} \int_0^1 (P_t^i(j,\omega))^{1-\eta} d\omega \right)^{(1-\eta)^{-1}} . \tag{22}$$

We also allow the idiosyncratic shock distributions to be heterogeneous across countries, with country-specific indices $\ell_j,\; j = 1, \cdots, N$.

**Proof of Lemma 3.1**. Given a chosen level of nominal consumption expenditures $\hat{C}_{i,t}$, the intra-temporal optimization problem of a household is given by

$$\max \left\{ \sum_j \theta_j \int_0^1 (\tilde{C}_{i,t}(j,\omega))^{\frac{\eta-1}{\eta}} d\omega \;:\; \sum_j \int \tilde{C}_{i,t}(j,\omega) P_t^i(j,\omega) d\omega \;=\; \hat{C}_{i,t} \right\} .$$

The first order condition gives

$$\theta_j \tilde{C}_{i,t}(j,\omega)^{-1/\eta} \;=\; \lambda^{-1/\eta} P_t^i(j,\omega) ,$$

where $\lambda$ is the Lagrange multiplier of the intra-temporal budget constraint. Thus,

$$\tilde{C}_{i,t}(j,\omega) \;=\; \lambda\, \theta_j^{\eta} P_t^i(j,\omega)^{-\eta}$$

and the budget constraint implies

$$\hat{C}_{i,t} \;=\; \lambda \sum_j \theta_j^\eta \int \tilde{(}P_t^i(j,\omega))^{1-\eta} d\omega$$

implying that

$$\lambda \;=\; \hat{C}_{i,t}\mathcal{P}_{i,t}^{\eta-1}\,.$$

so that

$$\tilde{C}_{i,t}(j,\omega) \;=\; \theta_j^\eta(\hat{C}_{i,t}/\mathcal{P}_{i,t})P_t^i(j,\omega)^{-\eta}\mathcal{P}_{i,t}^\eta\,.$$

and hence the consumption aggregator is given by

$$\begin{aligned}
C_{i,t} \;&=\; \left(\sum_j \theta_j \int_0^1 (\tilde{C}_{i,t}(j,\omega))^{\frac{\eta-1}{\eta}} d\omega\right)^{\eta/(\eta-1)} \\
&=\; \left(\sum_j \theta_j \int_0^1 (\theta_j^\eta(\hat{C}_{i,t}/\mathcal{P}_{i,t})P_t^i(j,\omega)^{-\eta}\mathcal{P}_{i,t}^\eta)^{\frac{\eta-1}{\eta}} d\omega\right)^{\eta/(\eta-1)} \\
&=\; \mathcal{P}_{i,t}^{-1}\hat{C}_{i,t}\,.
\end{aligned}$$

This is the known result about Dixit-Stiglitz aggregators: Consumption expenditures equal consumption aggregator times the price index.

Thus, the utility function of the households in terms of the nominal expenditures can be expressed as

$$E\left[\sum_{t=0}^\infty e^{-\beta t}\left(\frac{C_{i,t}^{1-\gamma}}{1-\gamma} \;-\; \nu N_{i,t}\right)\right]$$

under the inter-temporal budget constraint

$$E\left[\sum_{t=0}^{\infty} C_{i,t}\mathcal{P}_{i,t}M_{i,t}\right] \ \le\ W_{i,0} \ +\ E\left[\sum_{t=0}^{\infty} w_{i,t}N_{i,t}M_{i,t}\right],$$

where $W_{i,0}$ is household's initial wealth. That is, total present value (discounted with the market stochastic discount factor) of nominal expenditures should not exceed initial wealth plus the value of nominal wages. Intra-temporal optimization over $C_{i,t},\ N_{i,t}$ delivers

$$e^{-\beta t}C_{i,t}^{-\gamma} \ =\ \lambda\mathcal{P}_{i,t}M_{i,t},\ e^{-\beta t}\nu \ =\ \lambda\, w_{i,t}M_{i,t}\,,$$

which gives $w_{i,t} \ =\ \nu C_{i,t}^{\gamma}\mathcal{P}_{i,t}$, as well at the inter-temporal Euler equation $e^{-\beta}C_{i,t+1}^{-\gamma}/C_{i,t}^{-\gamma} \ =\ M_{i,t,t+1}\,(\mathcal{P}_{i,t+1}/\mathcal{P}_{i,t})$. Q.E.D.

Due to the assumed identical CES structure of consumption and production aggregators, for each time $t+1$, each firm $(i,\omega)$ faces the downward sloping demand for its goods sold in country $j$, and given by

$$D_{t+1}^{j}(i) \ =\ \mathcal{D}_{t+1}^{j}(i)(P_{t+1}^{j})^{-\eta}\,, \tag{23}$$

where the demand coefficients $\mathcal{D}_{t+1}^{j}(i)$ are determined in equilibrium. We will use

$$\bar{D}_{t+1}(i) \ =\ \sum_{j=1}^{N} D_{t+1}^{j}(i)$$

to denote the global demand for the goods produced by any given $(i,\omega)$ firm. Since all firms within each country are symmetric, this demand does not depend on $\omega$.

With flexible prices and CES demand, it is always optimal for each firm to set prices in different countries using the law of one price: $P_{t+1}^{j}(i) \ =\ P_{t+1}^{i}(i)/\mathcal{E}_{j,i,t+1}$. Therefore, global demand can be rewritten as $\bar{D}_{t+1}(i) \; = \; (P^i_{t+1}(i))^{-\eta}\tilde{\mathcal{D}}_t(i)$, with

$$\tilde{\mathcal{D}}_t(i) \;\equiv\; \sum_j \mathcal{D}^j_t(i)\,\mathcal{E}^{\eta}_{j,i,t}\,. \tag{24}$$

The following is true.

**Lemma A.1** *Let*

$$\bar{\alpha} \;=\; \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \;+\; \left(\frac{1-\alpha}{\alpha}\right)^{\alpha}.$$

*A country i firm optimally sets the price*

$$P^i_t(i) \;=\; \mathcal{P}_{i,t}\frac{\eta}{\eta-1}\bar{\alpha}(\nu C^{\gamma}_{i,t})^{1-\alpha} Z_{i,t}^{-(\eta-1)^{-1}}\, e^{-a_{i,t}} \tag{25}$$

*in the domestic market (in domestic currency) and sets prices in other countries using the law of one price; it hires labour*

$$L_t(i) \;=\; \bar{\mathcal{L}}_i(\nu C^{\gamma}_{i,t})^{-(\alpha+\eta(1-\alpha))}\mathcal{P}^{-\eta}_{i,t}\tilde{\mathcal{D}}_t(i)Z_{i,t}e^{a_{i,t}(\eta-1)}\,,$$

*where we have defined*

$$\bar{\mathcal{L}}_i \;=\; \left(\frac{\eta}{\eta-1}\bar{\alpha}\right)^{-\eta}\left(\frac{1-\alpha}{\alpha}\right)^{\alpha},$$

*and spends*

$$X_t(i,\omega) \;=\; \bar{\chi}(\nu C^{\gamma}_{i,t})^{(1-\alpha)(1-\eta)}\mathcal{P}^{-\eta}_{i,t}\,\tilde{\mathcal{D}}_t(i)Z_{i,t}(\omega)e^{a_{i,t}(\eta-1)}$$

*on intermediate goods, where we have defined*

$$\bar{\chi} \;=\; \left(\frac{\eta}{\eta-1}\bar{\alpha}\right)^{-\eta}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}.$$

*The demand of country $i$ firms for $(j,s)$ goods is given by*

$$\tilde{X}_{t,(i,\omega)}(j,s) \;=\; \theta_j^{\eta}(P_t^i(j,s))^{-\eta}\bar{\chi}(\nu C_{i,t}^{\gamma})^{(1-\alpha)(1-\eta)}\tilde{\mathcal{D}}_t(i)e^{a_{i,t}(\eta-1)}Z_{i,t} \tag{26}$$

*Total after tax profits of country $i$ firms are given by*

$$\Pi_{i,t} \;=\; \Omega_{i,t}Z_{i,t}$$

*with*

$$\Omega_{i,t} \;=\; \tilde{\mathcal{D}}_t(i)\mathcal{P}_{i,t}^{1-\eta}(1-\tau_i)\bar{\eta}\left((\nu C_{i,t}^{\gamma})^{1-\alpha}e^{-a_{i,t}}\right)^{1-\eta}. \tag{27}$$

**Proof of Lemma A.1**. For an $(i,\omega)$ firm, the global demand for its goods is linear in the vector $((P_t^j(i,\omega))^{-\eta})_{j=1}^N$, and the firm will be choosing that price vector. Given that vector, the firm will face a vector of demands,

$$D_t^j((i,\omega),P) \;=\; \mathcal{D}_t^j(i)\,P_t^j(i,\omega)^{-\eta},$$

and hence the nominal income in the domestic currency will be given by

$$\mathcal{I}((P_t^j)_{j=1}^N) \;=\; \sum_j \mathcal{D}_t^j(i)\,P_t^j(i,\omega)^{1-\eta}\mathcal{E}_{j,i,t}.$$

Thus, first, the objective of the firm is to maximize its income given the fixed demand:

$$\max\{\mathcal{I}((P_t^j)_{j=1}^N) : \sum_j \mathcal{D}_t^j(i)\, P_t^j(i,\omega)^{-\eta} \;=\; \bar{D}\}\,.$$

The first order conditions for this problem give

$$(1-\eta)\mathcal{D}_t^j(i)\, P_t^j(i,\omega)^{-\eta}\mathcal{E}_{j,i,t} \;+\; \lambda\eta\mathcal{D}_t^j(i)\, P_t^j(i,\omega)^{-\eta-1} \;=\; 0\,,$$

which gives

$$P_t^j(i,\omega)\mathcal{E}_{j,i,t} \;=\; \lambda\frac{\eta}{\eta-1}$$

implying that a flexible price monopolist facing CES demand always sets the prices satisfying the law of one price. Thus, total demand satisfies

$$\begin{aligned}\bar{D} \;&=\; \sum_j \mathcal{D}_t^j(i)\, P_t^j(i,\omega)^{-\eta} \;=\; \sum_j \mathcal{D}_t^j(i)\, (P_t^i(i,\omega)/\mathcal{E}_{j,i,t})^{-\eta} \;=\; P_t^i(i,\omega)^{-\eta}\sum_j \mathcal{D}_t^j(i)\, \mathcal{E}_{j,i,t}^{\eta} \\ &=\; P_t^i(i,\omega)^{-\eta}\tilde{\mathcal{D}}_t(i)\,.\end{aligned}$$

At the same time, the income is given by

$$\sum_j \mathcal{D}_t^j(i)\, P_t^j(i,\omega)^{1-\eta}\mathcal{E}_{j,i,t} \;=\; \sum_j \mathcal{D}_t^j(i)\, (P_t^i(i,\omega)/\mathcal{E}_{j,i,t})^{1-\eta}\mathcal{E}_{j,i,t} \;=\; P_t^i(i,\omega)^{1-\eta}\tilde{\mathcal{D}}_t(i)\,.$$

Given the production function (2), the firm matching the demand needs to produce

$$\bar{D} \;=\; Z_{i,t}(\omega)^{(\eta-1)^{-1}} e^{a_{i,t}}\, L_t(i,\omega)^{1-\alpha}\, X_t(i,\omega)^{\alpha} \tag{28}$$

with

$$X_t(i,\omega) \;=\; \left(\sum_{j=1}^{N} \theta_j \int_0^1 (\tilde{X}_{t,(i,\omega)}(j,s))^{\frac{\eta-1}{\eta}} ds\right)^{\frac{\eta}{\eta-1}}.$$

Hence, the first objective of the firm is to minimize the production cost:

$$\min\left(w_{i,t}L_{i,t}(\omega) + \sum_j \int_0^1 P_t^i(j,s)\tilde{X}_{t,(i,\omega)}(j,s)ds\right)$$

under the demand-matching constraint (28). Writing down the first order conditions gives

$$w_{i,t} \;=\; \lambda(1-\alpha)Z_{i,t}(\omega)^{(\eta-1)^{-1}} e^{a_{i,t}}\, L_t(i,\omega)^{-\alpha}\, X_t(i,\omega)^{\alpha}$$

so that

$$L_t(i,\omega) \;=\; \left(\lambda(1-\alpha)w_{i,t}^{-1} Z_{i,t}(\omega)^{(\eta-1)^{-1}} e^{a_{i,t}}\, X_t(i,\omega)^{\alpha}\right)^{\alpha^{-1}}$$

Similarly,

$$P_t^i(j,s) \;=\; \lambda\, \alpha Z_{i,t}(\omega)^{(\eta-1)^{-1}} e^{a_{i,t}}\, L_t(i,\omega)^{1-\alpha}\, X_t(i,\omega)^{\alpha-1} \frac{\eta}{\eta-1} X_t(i,\omega)^{\eta^{-1}} \theta_j \frac{\eta-1}{\eta} \tilde{X}_{t,(i,\omega)}(j,s)^{-1/\eta}$$

implying that

$$\tilde{X}_{t,(i,\omega)}(j,s) \;=\; \theta_j^{\eta}(P_t^i(j,s))^{-\eta}\Gamma$$

where

$$\Gamma \;=\; \left(\lambda\, \alpha Z_{i,t}(\omega)^{(\eta-1)^{-1}} e^{a_{i,t}}\, L_t(i,\omega)^{1-\alpha}\, X_t(i,\omega)^{\eta^{-1}+\alpha-1}\right)^{\eta}$$

Hence,

$$
\begin{aligned}
X_t(i,\omega) \;&=\; \left(\sum_{j=1}^{N}\theta_j\int_0^1 (\theta_j^{\eta}(P_t^i(j,s))^{-\eta}\Gamma)^{\frac{\eta-1}{\eta}}ds\right)^{\frac{\eta}{\eta-1}}\\
=\; \mathcal{P}_{i,t}^{-\eta}\Gamma \;&=\; \mathcal{P}_{i,t}^{-\eta}\left(\lambda\,\alpha Z_{i,t}(\omega)^{(\eta-1)^{-1}}e^{a_{i,t}}\,L_t(i,\omega)^{1-\alpha}\,X_t(i,\omega)^{\eta^{-1}+\alpha-1}\right)^{\eta}\\
=\; \mathcal{P}_{i,t}^{-\eta}&\left(\lambda\,\alpha Z_{i,t}(\omega)^{(\eta-1)^{-1}}e^{a_{i,t}}\,(\left(w_{i,t}^{-1}\lambda(1-\alpha)Z_{i,t}(\omega)^{(\eta-1)^{-1}}e^{a_{i,t}}\,X_t(i,\omega)^{\alpha}\right)^{\alpha^{-1}})^{1-\alpha}\,X_t(i,\omega)^{\eta^{-1}+\alpha-1}\right)^{\eta}\\
=\; \mathcal{P}_{i,t}^{-\eta}&X_t(i,\omega)(Z_{i,t}(\omega)^{(\eta-1)^{-1}}e^{a_{i,t}})^{\alpha^{-1}\eta}\lambda^{\alpha^{-1}\eta}\alpha^{\eta}(w_{i,t}^{-1}(1-\alpha))^{(\alpha^{-1}-1)\eta}\,.
\end{aligned}
$$

Hence,

$$
\begin{aligned}
\lambda \;&=\; \left(\mathcal{P}_{i,t}^{-\eta}(Z_{i,t}(\omega)^{(\eta-1)^{-1}}e^{a_{i,t}})^{\alpha^{-1}\eta}\alpha^{\eta}(w_{i,t}^{-1}(1-\alpha))^{(\alpha^{-1}-1)\eta}\right)^{-\alpha\eta^{-1}}\\
&=\; \mathcal{P}_{i,t}^{\alpha}(Z_{i,t}(\omega)^{(\eta-1)^{-1}}e^{a_{i,t}})^{-1}\alpha^{-\alpha}w_{i,t}^{1-\alpha}(1-\alpha)^{-(1-\alpha)}
\end{aligned}
$$

At the same time, (28) takes the form

$$
\begin{aligned}
\bar{D} \;&=\; Z_{i,t}(\omega)^{(\eta-1)^{-1}}e^{a_{i,t}}\,L_t(i,\omega)^{1-\alpha}\,X_t(i,\omega)^{\alpha}\\
&=\; Z_{i,t}(\omega)^{(\eta-1)^{-1}}e^{a_{i,t}}\,\left(w_{i,t}^{-1}\lambda(1-\alpha)Z_{i,t}(\omega)^{(\eta-1)^{-1}}e^{a_{i,t}}\,X_t(i,\omega)^{\alpha}\right)^{\alpha^{-1}(1-\alpha)}\,X_t(i,\omega)^{\alpha}\\
&=\; \left(w_{i,t}^{-1}\lambda(1-\alpha)\right)^{\alpha^{-1}-1}(Z_{i,t}(\omega)^{(\eta-1)^{-1}}e^{a_{i,t}})^{\alpha^{-1}}X_t(i,\omega)\\
&=\; \left((\mathcal{P}_{i,t}/w_{i,t})^{\alpha}(Z_{i,t}(\omega)^{(\eta-1)^{-1}}e^{a_{i,t}})^{-1}\alpha^{-\alpha}(1-\alpha)^{\alpha}\right)^{\alpha^{-1}-1}\\
&\times(Z_{i,t}(\omega)^{(\eta-1)^{-1}}e^{a_{i,t}})^{\alpha^{-1}}X_t(i,\omega)
\end{aligned}
$$

so that

$$
X_t(i,\omega) \;=\; \left(\frac{w_{i,t}}{\mathcal{P}_{i,t}}\right)^{1-\alpha}\bar{D}Z_{i,t}^{-(\eta-1)^{-1}}e^{-a_{i,t}}\left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha}
$$

and

$$L_t(i,\omega) \;=\; \left(\frac{\mathcal{P}_{i,t}}{w_{i,t}}\right)^{\alpha} \bar{D} Z_{i,t}^{-(\eta-1)^{-1}} e^{-a_{i,t}} \left(\frac{1-\alpha}{\alpha}\right)^{\alpha}$$

Thus, given the link between demand and prices, $\bar{D} = \tilde{\mathcal{D}}_t(i)\, P_t^i(i,\omega)^{-\eta}$, the maximization problem over prices is reduced to maximizing

$$\begin{aligned}
&- \left(w_{i,t} L_{i,t}(\omega) + X_t(i,\omega)\mathcal{P}_{i,t}\right) \;+\; \tilde{\mathcal{D}}_t(i)\, P_t^i(i,\omega)^{1-\eta} \\
&= \; -\left( w_{i,t} \left(\frac{\mathcal{P}_{i,t}}{w_{i,t}}\right)^{\alpha} \bar{D} Z_{i,t}^{-(\eta-1)^{-1}} e^{-a_{i,t}} \left(\frac{1-\alpha}{\alpha}\right)^{\alpha} + \left(\frac{w_{i,t}}{\mathcal{P}_{i,t}}\right)^{1-\alpha} \bar{D} Z_{i,t}^{-(\eta-1)^{-1}} e^{-a_{i,t}} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \mathcal{P}_{i,t} \right) \\
&+ \; \tilde{\mathcal{D}}_t(i)\, P_t^i(i,\omega)^{1-\eta} \\
&= \; -\bar{\alpha} w_{i,t}^{1-\alpha} (\mathcal{P}_{i,t})^{\alpha} \bar{D} Z_{i,t}^{-(\eta-1)^{-1}} e^{-a_{i,t}} \;+\; \tilde{\mathcal{D}}_t(i)\, P_t^i(i,\omega)^{1-\eta} \\
&= \; -\bar{\alpha} w_{i,t}^{1-\alpha} (\mathcal{P}_{i,t})^{\alpha} \tilde{\mathcal{D}}_t(i)\, P_t^i(i,\omega)^{-\eta} Z_{i,t}^{-(\eta-1)^{-1}} e^{-a_{i,t}} \;+\; \tilde{\mathcal{D}}_t(i)\, P_t^i(i,\omega)^{1-\eta}\,,
\end{aligned}$$

where we have defined

$$\bar{\alpha} \;=\; \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \;+\; \left(\frac{1-\alpha}{\alpha}\right)^{\alpha}.$$

Hence, the optimal price set by the firm is given by

$$P_t^i \;=\; \frac{\eta}{\eta-1} \bar{\alpha} w_{i,t}^{1-\alpha} (\mathcal{P}_{i,t})^{\alpha} Z_{i,t}^{-(\eta-1)^{-1}} e^{-a_{i,t}}\,,$$

and the total profit is given by

$$\begin{aligned}
&- \bar{\alpha} w_{i,t}^{1-\alpha} (\mathcal{P}_{i,t})^{\alpha} \tilde{\mathcal{D}}_t(i) \left(\frac{\eta}{\eta-1} \bar{\alpha} w_{i,t}^{1-\alpha} (\mathcal{P}_{i,t})^{\alpha} Z_{i,t}^{-(\eta-1)^{-1}} e^{-a_{i,t}}\right)^{-\eta} Z_{i,t}^{-(\eta-1)^{-1}} e^{-a_{i,t}} \\
&+ \; \tilde{\mathcal{D}}_t(i) \left(\frac{\eta}{\eta-1} \bar{\alpha} w_{i,t}^{1-\alpha} (\mathcal{P}_{i,t})^{\alpha} Z_{i,t}^{-(\eta-1)^{-1}} e^{-a_{i,t}}\right)^{1-\eta} \\
&= \; \bar{\eta} \left( w_{i,t}^{1-\alpha} (\mathcal{P}_{i,t})^{\alpha} Z_{i,t}^{-(\eta-1)^{-1}} e^{-a_{i,t}} \right)^{1-\eta} \tilde{\mathcal{D}}_t(i)
\end{aligned}$$

where we have defined

$$\bar{\eta} \equiv \frac{(\eta-1)^{\eta-1}\bar{\alpha}^{1-\eta}}{\eta^{\eta}}.$$

Q.E.D.

# B General Equilibrium

In our model, all goods are used both for consumption and for production. Total demand $D_t^i(j,s) = D_t^i(j)$ of country $i$ (that is, the join demand by households and firms) for any given good $s$ produced in country $j$ by the $(j,s)$ firms is thus given by the sum of consumers' and firms' demand:

$$D_t^i(j) = \tilde{C}_{i,t}(j) + \int_0^1 \tilde{X}_{t,(i,\omega)}(j)d\omega. \tag{29}$$

Since all $(j,s)$ firms set prices that are independent of $s$, we omit the $s$ index and denote these prices by $P_t^i(j) = P_t^i(j,s)$. By Lemma 3.1, consumers' demand is given by

$$\tilde{C}_{i,t}(j) = \theta_j^{\eta}(P_t^i(j))^{-\eta}\mathcal{P}_{i,t}^{\eta}C_{i,t}.$$

At the same time, country $i$ firms' demand can be decomposed into the demand of distressed and non-distressed firms. By the law of large numbers, formula (26) and Assumption 1 imply that total country $i$ firms' demand for $(j,s)$ goods can be rewritten as

$$\begin{aligned}
&\int_0^1 \tilde{X}_{t,(i,\omega)}(j,s)d\omega \\
&= \theta_j^{\eta}(P_t^i(j))^{-\eta}\mathcal{P}_{i,t}^{\eta-1}\frac{\bar{\chi}}{\bar{\eta}(1-\tau_i)}\Omega_{i,t}\int_0^1 (Z_{i,t}(\omega)\mathbf{1}_{Z_{i,t}(\omega)>\Psi_{i,t}} + \zeta_i Z_{i,t}(\omega)\mathbf{1}_{Z_{i,t}(\omega)<\Psi_{i,t}})d\omega,
\end{aligned}$$

where $\Psi_{i,t}$ is the distress threshold (5). Define

$$G_i(\Psi_{i,t}) \;=\; \ell_i(\ell_i+1)^{-1}\left((\zeta_i-1)\Psi_{i,t}^{\ell_i+1}+1\right) .$$

Then, by direct calculation

$$\int_0^1 (Z_{i,t}(\omega)\mathbf{1}_{Z_{i,t}(\omega)>\Psi_{i,t}} + \zeta_i Z_{i,t}(\omega)\mathbf{1}_{Z_{i,t}(\omega)<\Psi_{i,t}})d\omega \;=\; G_i(\Psi_{i,t})$$

and therefore, using (29), we can rewrite the coefficient $\mathcal{D}_t^i(j)$ in the demand schedule $D_t^i(j) \;=\; \mathcal{D}_t^i(j)(P_t^i(j))^{-\eta}$ (see (23)) as

$$\mathcal{D}_t^i(j) \;=\; (\mathcal{P}_{i,t})^{\eta}\hat{\mathcal{D}}_{i,t}\,, \tag{30}$$

with

$$\hat{\mathcal{D}}_{i,t} \;=\; \theta_j^{\eta}\left(C_{i,t}+G_i(\Psi_{i,t})\,\frac{\bar{\chi}}{\bar{\eta}(1-\tau_i)}\mathcal{P}_{i,t}^{-1}\Omega_{i,t}\right),\; i=1,\cdots,N\,. \tag{31}$$

Equation (31) defines the equilibrium system for global demand coefficients: demand of country $i$ firms for any given good depends on the global demand $\tilde{\mathcal{D}}_t(i)$ (see (24)) for country $i$ goods, as reflected in formula (27) for $\Omega_{i,t}$. Substituting (30) into (24), we get, after some algebra, that $\mathcal{P}_{j,t}^{-\eta}\tilde{\mathcal{D}}_t(j) \;=\; \sum_i \hat{\mathcal{D}}_{i,t}\,\widetilde{\mathcal{E}}_{i,j,t}^{\eta}$. Substituting real exchange rates, we get defining $c_{j,0} \;=\; C_{j,0}^{\gamma}$ that

$$\mathcal{P}_{j,t}^{-\eta}\tilde{\mathcal{D}}_t(j) \;=\; \theta_j^{\eta}(c_{j,0}^{-1}C_{j,t}^{\gamma})^{\eta}\bar{\mathcal{D}}_t\,, \tag{32}$$

where we have defined the *global demand factor*

$$\bar{\mathcal{D}}_t \;\equiv\; \sum_i \hat{\mathcal{D}}_{i,t}\,c_{i,0}^{\eta}C_{i,t}^{-\gamma\eta}\,. \tag{33}$$

Now, in order to derive the equilibrium system for global consumption, we need to compute price indices (1) and their response to debt overhang. By (25) and the Law of One Price, we get that the total contribution of country $j$ firms to country $i$ price index is given by

$$
\begin{aligned}
\int_0^1 (P_t^i(j,\omega))^{1-\eta} d\omega \;&=\; \int_0^1 (\mathcal{E}_{j,i,t} P_t^j(j,\omega))^{1-\eta} d\omega \\
=\; (\mathcal{P}_{i,t}^{-1} \tilde{\mathcal{E}}_{j,i,t})^{1-\eta}\, C_{j,t}^{\gamma(1-\alpha)(1-\eta)} &\left( \frac{\eta}{\eta-1} \nu_j^{1-\alpha} \bar{\alpha} e^{-a_{j,t}} \right)^{1-\eta} G_j(\Psi_{j,t})\,.
\end{aligned}
\tag{34}
$$

Therefore, debt overhang directly affects price level: Consistent with the evidence in Gilchrist, Schoenle, Sim and Zakrajšek (2017), financially constrained firms raise prices because their effective marginal cost of production is higher in distress.[40] Substituting (34) into formula (22) for the price index, we get

$$
1 \;=\; \sum_j \theta_j^\eta \left( C_{j,t}^{-\gamma} / C_{i,t}^{-\gamma} \frac{c_{j,0}}{c_{i,0}} \right)^{1-\eta} C_{j,t}^{\gamma(1-\alpha)(1-\eta)} \left( \frac{\eta}{\eta-1} \nu_j^{1-\alpha} \bar{\alpha} e^{-a_{j,t}} \right)^{1-\eta} G_j(\Psi_{j,t})\,. \tag{35}
$$

By assumption, all agents in all countries have identical preferences, markets are complete, and hence consumption is perfectly aligned across countries. As a result, *real exchange rates equal one for all country pairs* $i, j$ and hence nominal exchange rates move one-to-one with relative inflation. This is important: Absent financial frictions, our model would be at odds with the exchange rate disconnect puzzle (see, e.g., Gabaix and Maggiori (2015) and Itskhoki and Mukhin (2017)) and generate counter-factual behaviour of currency risk premia. Define $\bar{C}_t \;=\; (C_{i,t}^\gamma c_{i,0}^{-1})^{\eta-1}$. Then, (35) takes the form

$$
\bar{C}_t^{1-\alpha} \;\equiv\; \sum_{j=1}^N \theta_j^\eta (c_{j,0}^{1-\eta})^{(1-\alpha)} \left( \frac{\eta}{\eta-1} \nu_j^{1-\alpha} \bar{\alpha} e^{-a_{j,t}} \right)^{1-\eta} G_j(\Psi_{j,t})\,. \tag{36}
$$

Equation (36) characterizes equilibrium consumption in the presence of debt overhang. In the frictionless case, we have $G_j(\Psi_{j,t}) = \ell_j(\ell_j+1)^{-1}$ and hence the frictionless aggregate

[40] See also Malamud and Zucchi (2018).

consumption index, which we denote by $\bar{C}_{t,*}$, is a (non-linear) aggregate of total factor productivities. To solve for the equilibrium with debt overhang, we first need to derive the relationship between global consumption and production demands. To this end, we note that global production demand is proportional to the total exchange-rate weighted sum of domestic production demands, (33). Each domestic demand (31) is a sum of consumption demand $C_{i,t} = (c_{i,0}\bar{C}_t^{(\eta-1)^{-1}})^{\gamma^{-1}}$, and production demand. The latter is proportional to profits, $\Omega_{i,t}$, which are in turn proportional to the global demand factor (33). This leads to an equilibrium fixed point system, whose solution is reported in the following proposition.

**Proposition B.1** *We have* $\bar{\mathcal{D}}_t = d_* \bar{C}_t^{\frac{\gamma^{-1}-\eta}{\eta-1}}$ *for some* $d_* > 0$.

We will restrict our attention to equilibria in which a strictly positive fraction of firms in each country is not in distress. That is, $\Psi_{i,t} < 1$ for all $i = 1, \cdots, N$. Define

$$\xi_j \equiv d_*(1-\tau_j)\bar{\eta}(c_{j,0}^{1-\eta})^{(1-\alpha)} \left(\nu_j^{1-\alpha}\right)^{1-\eta} .$$

By direct calculation, we have

$$\Omega_{i,t} = \xi_i e^{a_{i,t}(\eta-1)} \bar{C}_t^{\hat{\eta}} , \ \hat{\eta} \equiv \frac{\gamma^{-1}}{\eta-1} + \alpha - 1 .$$

The following is true.

**Theorem B.2** *There exists at most one equilibrium solution* $\bar{C}_t$ *to the equation*

$$\bar{C}_t^{1-\alpha} = A \sum_{j=1}^{N} \frac{\xi_j}{1-\tau_j} e^{a_{j,t}(\eta-1)} G_j \left( \frac{\mathcal{B}_{j,t}\mathcal{P}_{j,t}^{-1}}{\xi_j e^{a_{j,t}(\eta-1)} \bar{C}_t^{\hat{\eta}}} \right)$$

*satisfying* $\max_i \Psi_{i,t} < 1$ *and such that* $\bar{C}_t$ *that is monotonically increasing in* $a_{j,t}$, $j = 1, \cdots, N$.[41]

[41]In the case of $\hat{\eta} > 0$, there might exist a "non-economic equilibrium" with an unreasonable feature that

**Proof of Proposition B.1**. By (32) and (27), we have

$$\Omega_{i,t} \;=\; \theta_i^{\eta}(C_{i,0}^{-\gamma}C_{i,t}^{\gamma})^{\eta}\bar{\mathcal{D}}_t\mathcal{P}_{i,t}(1-\tau)\bar{\eta}\left((\nu C_{i,t}^{\gamma})^{1-\alpha}e^{-a_{i,t}}\right)^{1-\eta},$$

so that

$$\begin{aligned}
\bar{\mathcal{D}}_t \;&=\; \sum_i \hat{\mathcal{D}}_{i,t}\,C_{i,0}^{\gamma\eta}C_{i,t}^{-\gamma\eta} \;=\; \sum_i \left(C_{i,t}+G_i(\Psi_{i,t})\,\frac{\bar{\chi}}{\bar{\eta}(1-\tau)}\mathcal{P}_{i,t}^{-1}\Omega_{i,t}\right)C_{i,0}^{\gamma\eta}C_{i,t}^{-\gamma\eta}\\
&=\; \sum_i \Bigg( (C_{i,0}^{\gamma}(\bar{C}_t)^{(\eta-1)^{-1}})^{\gamma^{-1}}\bar{C}_t^{-\eta/(\eta-1)}\\
&\quad + \theta_i^{\eta}G_i(\Psi_{i,t})\bar{\mathcal{D}}_t\bar{C}_t^{\alpha-1}(C_{i,0}^{\gamma(1-\eta)})^{(1-\alpha)}\left((\nu)^{1-\alpha}e^{-a_{i,t}}\right)^{1-\eta}\Bigg)\\
&=\; \bar{c}_0\bar{C}_t^{\frac{\gamma^{-1}-\eta}{\eta-1}} \;+\; \bar{\mathcal{D}}_t\bar{C}_t^{\alpha-1}\bar{\chi}\sum_i \theta_i^{\eta}G_i(\Psi_{i,t})(C_{i,0}^{\gamma(1-\eta)})^{(1-\alpha)}\left((\nu)^{1-\alpha}e^{-a_{i,t}}\right)^{1-\eta}.
\end{aligned}$$

Substituting from (36), we get with $\bar{c}_0 \;=\; \sum_i C_{i,0}$ that

$$\bar{\mathcal{D}}_t \;=\; \bar{c}_0\bar{C}_t^{\frac{\gamma^{-1}-\eta}{\eta-1}} \;+\; \bar{\mathcal{D}}_t\bar{C}_t^{\alpha-1}\bar{\chi}\,\bar{C}_t^{1-\alpha}(\bar{\alpha}^{1-\eta}\left(\frac{\eta}{\eta-1}\right)^{1-\eta})^{-1}$$

implying that

$$\bar{\mathcal{D}}_t \;=\; d_*\bar{C}_t^{\frac{\gamma^{-1}-\eta}{\eta-1}}$$

with

$$d_* \;\equiv\; \frac{\bar{c}_0}{1 \;-\; \bar{\chi}\,(\bar{\alpha}^{1-\eta}\left(\frac{\eta}{\eta-1}\right)^{1-\eta})^{-1}},$$

consumption is decreasing in productivity. For the rest of the paper, we neglect this equilibrium and only focus on the one that we call the "normal equilibrium;" that is, the equilibrium in which $\bar{C}_t$ is monotonically increasing in $a_{j,t}$ for all $j$.

and hence

$$
\begin{aligned}
\Omega_{i,t} &= \theta_i^{\eta}(C_{i,0}^{-\gamma}C_{i,t}^{\gamma})^{\eta} d_* \bar{C}_t^{\frac{\gamma^{-1}-\eta}{\eta-1}} \mathcal{P}_{i,t}(1-\tau)\bar{\eta}\left((\nu C_{i,t}^{\gamma})^{1-\alpha} e^{-a_{i,t}}\right)^{1-\eta} \\
&= \theta_i^{\eta}(C_{i,0}^{-\gamma})^{\eta} d_* \mathcal{P}_{i,t}(1-\tau)\bar{\eta}\left((\nu)^{1-\alpha}\right)^{1-\eta} e^{a_{i,t}(\eta-1)}\bar{C}_t^{\hat{\eta}}\,.
\end{aligned}
$$

Q.E.D.

**Proof of Theorem B.2**. We have

$$
\Psi_{i,t} = \frac{\mathcal{B}_{i,t}}{\Omega_{i,t}} = \frac{\mathcal{B}_{i,t}\mathcal{P}_{i,t}^{-1}}{\theta_i^{\eta}(C_{i,0}^{-\gamma})^{\eta} d_*(1-\tau)\bar{\eta}\left((\nu)^{1-\alpha}\right)^{1-\eta} e^{a_{i,t}(\eta-1)}\bar{C}_t^{\hat{\eta}}}
$$

and substituting this into equation (36) we get the required. Uniqueness follows by direct calculation. Q.E.D.

# C Leverage

**Proposition C.1** *Suppose that firms have a possibility of hedging foreign exchange risk by acquiring $h_t \ge 0$ units of a financial derivative contract with a payoff of $X_{t+1} \ge 0$ and a price of $E_t[M_{i,t,t+1}X_{t+1}]$ to be paid at time $t$. The firm always chooses $h_t = 0$.*

The intuition behind this result is straightforward. Hedging effectively plays a role of investment, and the firm only gets the payoff $X_{t+1}$ from this investment in good (survival) states, while paying the market price at time $t$ to get the payoff in all states. Thus, hedging is just a transfer of funds from shareholders to debt-holders, and firms optimally decide to minimize this transfer.[42]

[42]There is ample evidence that firms often choose not to hedge their foreign currency risk. See, for example, Bodnár (2006) who shows that only 4% of Hungarian firms with foreign currency debt hedge their currency risk exposure. Furthermore, according to Salomao and Varela (2018): "data from the Central Bank of Peru reveals that only 6% of firms borrowing in foreign currency employ financial instruments to hedge the exchange rate risk, and a similar number is found in Brazil." See also Niepmann and Schmidt-Eisenlohr (2017), Bruno and Shin (2017). While it is known that costly external financing makes hedging optimal (see, for example, Froot, Scharfstein and Stein (1993) and Hugonnier, Malamud and Morellec (2015)),

**Proof of Proposition C.1**. The maximization problem is

$$\max_{h_t}\Bigg\{-E_t[M_{t,t+1}X_{t+1}]h_t$$
$$+E_t\left[M_{t,t+1}\int_{\Omega_{i,t+1}Z_{i,t+1}>\mathcal{B}_{t+1}(B_t)-h_t(1-\tau)X_{t+1}}(\Omega_{i,t+1}Z_{i,t+1}-\mathcal{B}_{t+1}(B_t)+h_t(1-\tau)X_{t+1})\phi(Z_{i,t+1})dZ_{i,t+1}\right]\Bigg\}.$$

The derivative of this objective function with respect to $h_t$ is given by

$$-E_t[M_{t,t+1}X_{t+1}]\;+\;(1-\tau)E_t\left[M_{t,t+1}X_{t+1}\left(1-\Phi\left(\frac{\mathcal{B}_{t+1}(B_t)-h_t(1-\tau)X_{t+1}}{\Omega_{i,t+1}}\right)\right)\right]\;<\;0\,,$$

and hence $h_t=0$ is optimal. Q.E.D.

**Proof of Theorem 5.1**. Firm's problem is to maximize

$$\sum_j E_t\left[M_{i,t,t+1}\left[\left(1-(1-\rho_i)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell}\right)(1+c)\mathcal{E}_{j,i,t+1}\right]\right]B_{j,t}(1-q_i(j))$$
$$+\;E_t\left[M_{i,t,t+1}\left[-\mathcal{B}_{t+1}(B_t)\left(1-\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell}\right)+\Omega_{i,t+1}\ell(\ell+1)^{-1}\left(1-\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell+1}\right)\right]\right]$$

Differentiating, we get from the standard Kuhn-Tucker conditions that borrowing only in

Rampini, Sufi and Viswanathan (2014) show both theoretically and empirically that, in fact, more financially constrained firms hedge less.

dollars is optimal if and only if

$$
\begin{aligned}
& E_t\left[M_{i,t,t+1}\left[\left(1-(1-\rho_i)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell}\right)(1+c)\mathcal{E}_{j,i,t+1}\right]\right](1-q_i(j)) \\
& + E_t\left[M_{i,t,t+1}\left[\left(-\ell(1-\rho_i)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell-1}\Omega_{i,t+1}^{-1}\right)(1+c)\mathcal{E}_{\$,i,t+1}(1+c(1-\tau))\mathcal{E}_{j,i,t+1}\right]\right]B_{\$,t}(1-q_i(\$)) \\
& -(1+c(1-\tau))E_t\left[M_{i,t,t+1}\mathcal{E}_{j,i,t+1}\right] \\
& + E_t\left[M_{i,t,t+1}(\ell+1)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell}(1+c(1-\tau))\mathcal{E}_{j,i,t+1}\right. \\
& \left. -\ell\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell}(1+c(1-\tau))\mathcal{E}_{j,i,t+1}\right] \leq 0
\end{aligned}
$$

for all $j$ with the identity for $j = \$$. This inequality can be rewritten as

$$
\begin{aligned}
& E_t[M_{i,t,t+1}\mathcal{E}_{j,i,t+1}]((1-q_i(j))(1+c)-(1+c(1-\tau))) \\
\leq\ & E_t\left[M_{i,t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell}\mathcal{E}_{j,i,t+1}\right]((1-\rho_i)(1+c)[(1-q_i(j))+\ell(1-q_i(\$))]-(1+c(1-\tau)))
\end{aligned}
$$

At the same time, for the dollar debt we get

$$
\begin{aligned}
& E_t[M_{i,t,t+1}\mathcal{E}_{\$,i,t+1}]((1-q_i(\$))(1+c)-(1+c(1-\tau))) \\
=\ & E_t\left[M_{i,t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell}\mathcal{E}_{\$,i,t+1}\right]((1+\ell)(1-\rho_i)(1+c)(1-q_i(\$))-(1+c(1-\tau)))
\end{aligned}
$$

implying that

$$
B_{\$,t}(1+c(1-\tau)) = \left(\frac{E_t[M_{i,t,t+1}\mathcal{E}_{\$,i,t+1}]}{E_t\left[M_{i,t,t+1}\Omega_{i,t+1}^{-\ell}\mathcal{E}_{\$,i,t+1}^{1+\ell}\right]}\right)^{\ell^{-1}}
$$

and we get the Kuhn-Tucker conditions

$$\frac{\bar{q}_i(j,\$)}{\bar{q}_i(\$)}\frac{E_t[M_{i,t,t+1}\mathcal{E}_{j,i,t+1}]}{E_t\left[M_{i,t,t+1}\Omega_{i,t+1}^{-\ell}\mathcal{E}_{j,i,t+1}\mathcal{E}_{\$,i,t+1}^{\ell}\right]} \leq \frac{E_t[M_{i,t,t+1}\mathcal{E}_{\$,i,t+1}]}{E_t\left[M_{i,t,t+1}\Omega_{i,t+1}^{-\ell}\mathcal{E}_{\$,i,t+1}^{1+\ell}\right]}$$

Q.E.D.

# D Proof of Theorem 5.2

Substituting the debt servicing costs $\mathcal{B}_{\$,t}(B_{\$,t-1}) = ((1-\tau_\$)c+1)b_{\$,t-1}(j)\mathcal{P}_{\$,t-1,t}^{-1}\mathcal{P}_{j,t-1,t}$ into equation (9) for unemployment, and then using the assumed policy equation (15), we get that the US monetary policy satisfies the fixed point equation

$$\mathcal{P}_{\$,t-1,t} = \left((1-\zeta_\$)e^{-(\ell_\$+1)(\eta-1)a_{\$,t}}\left(\xi_\$^{-1}((1-\tau_\$)c+1)b_{\$,t-1}(\$)\,\mathcal{P}_{\$,t-1,t}^{-1}\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$} e^{\varepsilon_{\$,t+1}} \tag{37}$$

Indeed, the inflation policy is assumed to respond to unemployment, while the unemployment in turn responds to the inflation policy. The counter-cyclical policy assumption (that is, $\phi_\$ > 0$) implies that there is a unique solution to (37), given by

$$\mathcal{P}_{\$,t-1,t} = \left(\left((1-\zeta_\$)e^{-(\ell_\$+1)(\eta-1)a_{\$,t}}\left(\xi_\$^{-1}((1-\tau_\$)c+1)b_{\$,t-1}(\$)\,\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$} e^{\varepsilon_{\$,t+1}}\right)^{(1+(\ell_\$+1)\phi_\$)^{-1}}. \tag{38}$$

We have

**Lemma D.1** *Country $j$ firms' default threshold is given by*

$$\Psi_{j,t}(b_{t-1}) = \tilde{\psi}_j(b_{t-1})e^{-(\eta-1)a_{j,t}+\frac{-[(\eta-1)[-(\ell_\$+1)\phi_\$ a_{\$,t}]+\varepsilon_{\$,t}]}{1+(\ell_\$+1)\phi_\$}}\bar{C}_t^{-\frac{\hat{\eta}}{1+(\ell_\$+1)\phi_\$}},$$

*where we have defined*

$$\tilde{\psi}_j(b_{t-1}) \;=\; \frac{((1-\tau)c+1)b_{\$,t-1}(j)}{\xi_j((1-\zeta_\$)(\xi_\$^{-1}((1-\tau_\$)c+1)b_{\$,t-1}(\$))^{\ell_\$+1})^{\frac{\phi_\$}{1+(\ell_\$+1)\phi_\$}}}\,. \tag{39}$$

As above, we are only interested in equilibria in which $\Psi_{j,t} < 1$ for all $j$, which is equivalent to $\bar{C}_t^{\hat{\eta}} > (\bar{C}_t^M)^{\hat{\eta}}$, where

$$\bar{C}_t^M \;\equiv\; \max_j \left(\tilde{\psi}_j(b_{t-1})e^{\frac{-1}{1+(\ell_\$+1)\phi_\$}[(\eta-1)a_t+\varepsilon_{\$,t}]}\right)^{\frac{1+(\ell_\$+1)\phi_\$}{\hat{\eta}}}\,.$$

The equilibrium condition of Theorem 4.1 implies the following result.

**Proposition D.2** *Let $\bar{C}_{t,*}(a_t)$ be the frictionless consumption, solving (8) for the case with no debt overhang and exogenous monetary policy. All equilibria with active monetary policy (15) are then characterized by solutions $\bar{C}_t(b_{t-1}, a_t)$ to the equation*

$$\bar{C}_t^{1-\alpha} \;+\; A\sum_{j=1}^{N}\frac{\xi_j}{1-\tau}e^{a_{j,t}(\eta-1)}\frac{\ell(1-\zeta_j)}{\ell+1}\left(\Psi_{j,t}(b_{t-1})\right)^{\ell+1} \;=\; \bar{C}_{t,*}^{1-\alpha}\,,$$

*There is at most one economic equilibrium that is monotonically increasing in the common TFP shock $a_t$.*

We can now prove the characterization of the dominant currency debt equilibrium.

**Proof of Theorem 5.2**. By (38)

$$
\begin{aligned}
&\mathcal{P}_{j,t-1,t}^{-1}\mathcal{P}_{\$,t-1,t} \\
&= \left(\left((1-\zeta_\$)e^{-(\ell_\$+1)(\eta-1)a_t}\left(b_{\$,t-1}(\$)\,\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$}\right)^{(1+(\ell_\$+1)\phi_\$)^{-1}} \\
&\times\left((1-\zeta_j)e^{-(\ell+1)(\eta-1)a_t}\right. \\
&\times\left(b_{j,t-1}(\$)\left(\left((1-\zeta_\$)e^{-(\ell_\$+1)(\eta-1)a_t}\left(b_{\$,t-1}(\$)\,\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$}\right)^{-(1+(\ell_\$+1)\phi_\$)^{-1}}\right. \\
&\left.\left.\times\,\bar{C}_t^{-\hat{\eta}}\right)^{\ell+1}\right)^{-\phi_j} \\
&= \widetilde{b}_{t-1}\left(e^{-(\eta-1)a_t}\bar{C}_t^{-\hat{\eta}}\right)^{\frac{(\ell_\$+1)\phi_\$(1+\phi_j(\ell+1))}{1+(\ell_\$+1)\phi_\$}}\left(e^{(\eta-1)a_t}\bar{C}_t^{\hat{\eta}}\right)^{(\ell+1)\phi_j}.
\end{aligned}
$$

Thus, if $(\ell+1)\phi_j < (\ell_\$+1)\phi_\$$, then (14) always holds. Q.E.D.

**Proof of Proposition 6.1**. Since we assume that idiosyncratic TFP shocks are small, we will simply set $a_{j,t} = a_t$ is the future calculations. Defining

$$
\Delta_{i,t-1} \;\equiv\; \left(\frac{\bar{q}_i(\$)E_{t-1}\left[C_{t-1,t}^{-\gamma}\mathcal{P}_{\$,t-1,t}^{-1}\right]}{E_{t-1}\left[C_{t-1,t}^{-\gamma}\mathcal{P}_{\$,t-1,t}^{-\ell-1}(\bar{C}_t^{\hat{\eta}}e^{(\eta-1)a_t})^{-\ell}\right]}\right)^{\ell^{-1}}, \tag{40}
$$

equation (12) implies that

$$
b_{\$,t-1}(j) \;=\; \xi_j\Delta_{j,t-1}\,\mathcal{P}_{j,t-1}\,,
$$

and hence we get from (39) that

$$
\tilde{\psi}_j(b_{t-1}) \;=\; \frac{\Delta_{j,t-1}}{\left((1-\zeta_\$)(\Delta_{\$,t-1})^{\ell_\$+1}\right)^{\frac{\phi_\$}{1+(\ell_\$+1)\phi_\$}}}
$$

whereas, (38) takes the form

$$\mathcal{P}_{\$,t-1,t} \;=\; \left(\left((1-\zeta_\$)e^{-(\ell_\$+1)(\eta-1)a_t}\left(\Delta_{\$,t-1}\,\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$}\right)^{(1+(\ell_\$+1)\phi_\$)^{-1}} . \tag{41}$$

Thus, substituting (41) into (40), we get

$$\Delta_{\$,t-1}^{\frac{1}{1+(\ell_\$+1)\phi_\$}} \;=\; \Bigg(\bar{q}_i(\$)E_{t-1}\left[\bar{C}_t^{-(\eta-1)^{-1}}\left(\left((1-\zeta_\$)e^{-(\ell_\$+1)(\eta-1)a_t}\left(\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$}\right)^{-(1+(\ell_\$+1)\phi_\$)^{-1}}\right] /$$
$$E_{t-1}\Bigg[\bar{C}_t^{-(\eta-1)^{-1}}\left(\left((1-\zeta_\$)e^{-(\ell_\$+1)(\eta-1)a_t}\left(\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$}\right)^{-(\ell_\$+1)(1+(\ell_\$+1)\phi_\$)^{-1}}$$
$$\times\,(\bar{C}_t^{\hat{\eta}}e^{(\eta-1)a_t})^{-\ell_\$}\Bigg]\Bigg)^{\ell_\$^{-1}},$$

which gives

$$\Delta_{\$,t-1}^{\frac{1}{1+(\ell_\$+1)\phi_\$}} \;=\; \left(\frac{\bar{q}_i(\$)(1-\zeta_\$)^{\frac{\phi\ell_\$}{1+(\ell_\$+1)\phi}}\,E_{t-1}\left[\bar{C}_t^{-\frac{1}{\eta-1}+\frac{(\ell_\$+1)\phi}{1+(\ell_\$+1)\phi}\hat{\eta}}\,e^{\frac{(\ell_\$+1)\phi}{1+(\ell_\$+1)\phi}(\eta-1)a_t}\right]}{E_{t-1}\left[\bar{C}_t^{-\frac{1}{\eta-1}+\frac{(\ell_\$+1)\phi-\ell_\$}{1+(\ell_\$+1)\phi}\hat{\eta}}\,e^{\frac{(\ell_\$+1)\phi-\ell_\$}{1+(\ell_\$+1)\phi}(\eta-1)a_t}\right]}\right)^{\ell_\$^{-1}} .$$

Thus,

$$\Delta_{i,t-1} \;=\; \Delta_{\$,t-1}^{\frac{(\ell_\$+1)\phi_\$}{1+(\ell_\$+1)\phi_\$}}\Bigg(\bar{q}_i(\$)E_{t-1}\left[C_{t-1,t}^{-\gamma}\left(\left(\left((1-\zeta_\$)e^{-(\ell_\$+1)(\eta-1)a_t}\left(\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$}\right)^{(1+(\ell_\$+1)\phi_\$)^{-1}}\right)^{-1}\right]$$
$$\times\,E_{t-1}\left[C_{t-1,t}^{-\gamma}\left(\left(\left((1-\zeta_\$)e^{-(\ell_\$+1)(\eta-1)a_t}\left(\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$}\right)^{(1+(\ell_\$+1)\phi_\$)^{-1}}\right)^{-\ell-1}(\bar{C}_t^{\hat{\eta}}e^{(\eta-1)a_t})^{-\ell}\right]^{-1}\Bigg)^{\ell^{-1}},$$

Define

$$\tilde{C}_t \;\equiv\; \bar{C}_t^{\hat{\eta}} e^{(\eta-1)a_t}\,.$$

Then,

$$\begin{aligned}
&E_{t-1}\left[C_t^{-\gamma}\left(\left(\left(e^{-(\ell_\$+1)(\eta-1)a_t}\left(\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$}\right)^{(1+(\ell_\$+1)\phi_\$)^{-1}}\right)^{-1}\right]\\
&= \; E_{t-1}[C_t^{-\gamma}\tilde{C}_t\tilde{C}_t^{\hat{\ell}_\$}]
\end{aligned}$$

where we have defined

$$\hat{\ell}_\$ \;\equiv\; -\frac{1}{1+(\ell_\$+1)\phi_\$}\,.$$

Similarly,

$$\begin{aligned}
&E_{t-1}\left[C_t^{-\gamma}\left(\left(\left(e^{-(\ell_\$+1)(\eta-1)a_t}\left(\bar{C}_t^{-\hat{\eta}}\right)^{\ell_\$+1}\right)^{\phi_\$}\right)^{(1+(\ell_\$+1)\phi_\$)^{-1}}\right)^{-\ell-1}(\bar{C}_t^{\hat{\eta}}e^{(\eta-1)a_t})^{-\ell}\right]\\
&= \; E_{t-1}\left[C_t^{-\gamma}\tilde{C}_t^{\frac{(\ell_\$+1)\phi_\$(\ell+1)-\ell(1+(\ell_\$+1)\phi_\$)}{1+(\ell_\$+1)\phi_\$}}\right]\\
&= \; E_{t-1}[C_t^{-\gamma}\tilde{C}_t\tilde{C}_t^{\hat{\ell}_\$(\ell+1)}]\,.
\end{aligned}$$

Thus,

$$\begin{aligned}
&\tilde{\psi}_i(b_{t-1})\\
&= \; \bar{q}_i(\$)^{\ell^{-1}}\left(\frac{E_{t-1}[C_t^{-\gamma}\tilde{C}_t\tilde{C}_t^{\hat{\ell}_\$}]}{E_{t-1}[C_t^{-\gamma}\tilde{C}_t\tilde{C}_t^{\hat{\ell}_\$(\ell+1)}]}\right)^{\ell^{-1}}\,.
\end{aligned}$$

When leverage is small, we have

$$\bar{C}_t \;\approx\; \bar{C}_t^* \;=\; K\, e^{(\eta-1)(1-\alpha)^{-1}a_t}$$

for some $K > 0$, so that

$$\tilde{C}_t \;=\; K^{\hat{\eta}} e^{(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]}$$

and

$$C_t^{\gamma} \;=\; k\bar{C}_t^{(\eta-1)^{-1}} \;=\; kK^{(\eta-1)^{-1}} e^{(1-\alpha)^{-1}a_t}\,,$$

and hence, under the log-normal assumption, we get

$$\begin{aligned}
\frac{E_{t-1}[C_t^{-\gamma}\tilde{C}_t\tilde{C}_t^{\hat{\ell}_\$}]}{E_{t-1}[C_t^{-\gamma}\tilde{C}_t\tilde{C}_t^{\hat{\ell}_\$(\ell+1)}]} \;&=\; \frac{E_{t-1}\big[e^{[-(1-\alpha)^{-1}+(1+\hat{\ell}_\$)(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]]a_t}\big]}{E_{t-1}\big[e^{[-(1-\alpha)^{-1}+(1+\hat{\ell}_\$(\ell+1))(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]]a_t}\big]} \\
&=\; e^{-\hat{\ell}_\$\ell(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]\mu_{t-1}^a} \\
&\times e^{0.5[[-(1-\alpha)^{-1}+(1+\hat{\ell}_\$)(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]]^2-[-(1-\alpha)^{-1}+(1+\hat{\ell}_\$(\ell+1))(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]]^2](\sigma_{t-1}^a)^2} \\
&=\; e^{-\hat{\ell}_\$\ell(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]\mu_{t-1}^a}\, e^{-0.5\hat{\ell}_\$^2\tilde{\eta}^2\ell(\ell+2)(\sigma_{t-1}^a)^2}
\end{aligned}$$

where we have defined

$$\tilde{\eta} \;\equiv\; (\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]\,.$$

Thus, default probability is given by

$$\bar{q}_i(\$)\left(e^{-\hat{\ell}_\$(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]\mu_{t-1}^a}\, e^{-0.5\hat{\ell}_\$^2\tilde{\eta}^2(\ell+2)(\sigma_{t-1}^a)^2}\tilde{C}_t^{\hat{\ell}_\$}\right)^{\ell}$$

while losses due to debt overhang are given by

$$\bar{q}_i(\$)^{\ell^{-1}(\ell+1)}(1-\zeta_j)\left(e^{-\hat{\ell}_\$(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]\mu_{t-1}^a}\,e^{-0.5\hat{\ell}_\$^2\tilde{\eta}^2(\ell+2)(\sigma_{t-1}^a)^2}\tilde{C}_t^{\hat{\ell}_\$}\right)^{\ell+1}$$

Expected welfare is then

$$\begin{aligned}
&(1-\gamma)^{-1}E_{t-1}[C_t^{1-\gamma}] \;\approx\; (1-\gamma)^{-1}E_{t-1}[(C_t^*)^{1-\gamma}]\\
&-\;\sum_i \kappa_i E_{t-1}[(C_t^*)^{-\gamma}\bar{q}_i(\$)^{\ell^{-1}(\ell+1)}(1-\zeta_j)\left(e^{-\hat{\ell}_\$(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]\mu_{t-1}^a}\,e^{-0.5\hat{\ell}_\$^2\tilde{\eta}^2(\ell+2)(\sigma_{t-1}^a)^2}\tilde{C}_t^{\hat{\ell}_\$}\right)^{\ell+1}]\\
&=\;(1-\gamma)^{-1}E_{t-1}[(C_t^*)^{1-\gamma}]\\
&-\;\sum_i \kappa_i E_{t-1}[e^{-(1-\alpha)^{-1}a_t}\,\bar{q}_i(\$)^{\ell^{-1}(\ell+1)}(1-\zeta_j)\\
&\times\left(e^{-\hat{\ell}_\$(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]\mu_{t-1}^a}\,e^{-0.5\hat{\ell}_\$^2\tilde{\eta}^2(\ell+2)(\sigma_{t-1}^a)^2}e^{\hat{\ell}_\$\tilde{\eta}a_t}\right)^{\ell+1}]
\end{aligned}$$

for some constants $\kappa_i$. Thus, utility losses are given by

$$\begin{aligned}
&E_{t-1}[e^{-(1-\alpha)^{-1}a_t}\,\bar{q}_i(\$)^{\ell^{-1}(\ell+1)}(1-\zeta_j)\\
&\times\left(e^{-\hat{\ell}_\$(\eta-1)[\hat{\eta}(1-\alpha)^{-1}+1]\mu_{t-1}^a}\,e^{-0.5\hat{\ell}_\$^2\tilde{\eta}^2(\ell+2)(\sigma_{t-1}^a)^2}e^{\hat{\ell}_\$\tilde{\eta}a_t}\right)^{\ell+1}]\\
&\approx\; e^{-(1-\alpha)^{-1}\mu_{t-1}}(1-0.5\sigma_{t-1}^2((\ell+1)(\hat{\ell}_\$^2\tilde{\eta}^2+2\hat{\ell}_\$\tilde{\eta}(1-\alpha)^{-1})+(1-\alpha)^{-2}))\,,
\end{aligned}$$

Q.E.D.

**Proof of Proposition 7.1**. We need to compute

$$IRP_{i,t} \;=\; \frac{e^{r_{i,t}}\mathrm{Cov}_t(M_{i,t,t+1},\mathcal{P}_{i,t,t+1})}{E_t[\mathcal{P}_{i,t+1}]}\,.$$

We have

$$\mathcal{P}_{i,t,t+1} \;=\; \kappa_t\left(\left(e^{-(\eta-1)a_{\$,t}}\bar{C}_t^{-\hat{\eta}}\right)^{(\ell+1)\phi}e^{\varepsilon_{\$,t+1}}\right)^{(1+(\ell_\$+1)\phi_\$)^{-1}}$$

Now, when leverage is small, we have that consumption is close to the frictionless one,

$$\bar{C}_t \sim e^{a_t(\eta-1)(1-\alpha)^{-1}}$$

Since $\bar{C}_t \sim (C_t^{\gamma})^{\eta-1}$, we get that

$$C_t^{-\gamma} = \bar{C}_t^{-(\eta-1)^{-1}} \sim e^{-(1-\alpha)^{-1}a_t}.$$

whereas

$$e^{-(\eta-1)a_t}\bar{C}_t^{-\hat{\eta}} \sim e^{-(\eta-1)a_t}\bar{C}_t^{-(\frac{\gamma^{-1}}{\eta-1}+\alpha-1)} = e^{-(\eta-1)a_t}e^{-a_t(\eta-1)(1-\alpha)^{-1}(\frac{\gamma^{-1}}{\eta-1}+\alpha-1)} = e^{-a_t(1-\alpha)^{-1}\gamma^{-1}}.$$

Thus, ignoring the monetary shock, we get that

$$\mathcal{P}_{i,t,t+1} \sim \left(e^{-a_{t+1}(1-\alpha)^{-1}\gamma^{-1}}\right)^{(\ell+1)\phi(1+(\ell_\$+1)\phi_\$)^{-1}}.$$

At the same time,

$$M_{i,t,t+1} = e^{-\beta}C_{t,t+1}^{-\gamma}\mathcal{P}_{i,t,t+1}^{-1} \sim e^{-(1-\alpha)^{-1}a_{t+1}}\left(e^{a_{t+1}(1-\alpha)^{-1}\gamma^{-1}}\right)^{(\ell+1)\phi(1+(\ell_\$+1)\phi_\$)^{-1}}$$

Our goal is to prove that

$$IRP_{i,t}+1 = \frac{E_t[M_{i,t,t+1}\mathcal{P}_{i,t,t+1}]}{E_t[M_{i,t,t+1}]E_t[\mathcal{P}_{i,t,t+1}]}$$
$$= \frac{E_t[e^{-(1-\alpha)^{-1}a_{t+1}}]}{E_t[e^{-(1-\alpha)^{-1}a_{t+1}}\left(e^{a_{t+1}(1-\alpha)^{-1}\gamma^{-1}}\right)^{(\ell+1)\phi(1+(\ell_\$+1)\phi_\$)^{-1}}]E_t[\left(e^{-a_{t+1}(1-\alpha)^{-1}\gamma^{-1}}\right)^{(\ell+1)\phi(1+(\ell_\$+1)\phi_\$)^{-1}}]}$$

is monotone increasing in $\phi$. Define

$$x \;\equiv\; \gamma^{-1}(\ell+1)\phi(1+(\ell_\$+1)\phi_\$)^{-1},\; \tilde{a}_{t+1}=(1-\alpha)^{-1}a_{t+1}\,.$$

Then, we can rewrite it as

$$IRP_t(x)\;+\;1\;=\;\frac{E_t[e^{-\tilde{a}_{t+1}}]}{E_t[e^{-\tilde{a}_{t+1}(1-x)}]E_t[e^{-\tilde{a}_{t+1}x}]}\,.$$

Thus,

$$\frac{\partial}{\partial x}\log(IRP_t(x)\;+\;1)\;=\;\frac{E_t[e^{-\tilde{a}_{t+1}x}\tilde{a}_{t+1}]}{E_t[e^{-\tilde{a}_{t+1}x}]}\;-\;\frac{E_t[e^{-\tilde{a}_{t+1}(1-x)}\tilde{a}_{t+1}]}{E_t[e^{-\tilde{a}_{t+1}(1-x)}]}$$

Making a change of measure $d\tilde{P}\;=\;e^{-\tilde{a}_{t+1}x}/E_t[e^{-\tilde{a}_{t+1}x}]$, we can rewrite the required inequality as

$$\tilde{E}_t[\tilde{a}_{t+1}]\;>\;\frac{\tilde{E}_t[e^{-\tilde{a}_{t+1}(1-2x)}\tilde{a}_{t+1}]}{\tilde{E}_t[e^{-\tilde{a}_{t+1}(1-2x)}]}\,,$$

which is equivalent to $\tilde{C}ov_t(e^{-\tilde{a}_{t+1}(1-2x)},\tilde{a}_{t+1})<0$. Q.E.D.

# E Exorbitant Duty

Suppose that the DC CB follows the monetary policy

$$\mathcal{P}_{\$,t,t+1}\;=\;\prod_i\big(1-\bar{L}_{t+1}(i)/\bar{L}^*_{t+1}(i)\big)^{\chi_i}$$

This gives the fixed point equation

$$\mathcal{P}_{\$,t-1,t}\;=\;\prod_i\left((1-\zeta_\$)e^{-(\ell+1)(\eta-1)a_{i,t}}\left(((1-\tau_\$)c+1)\Delta_{i,t-1}\,\mathcal{P}^{-1}_{\$,t-1,t}\bar{C}_t^{-\hat{\eta}}\right)^{\ell+1}\right)^{\chi_i}$$

Thus,

$$\mathcal{P}_{\$,t-1,t} \;=\; q_* \prod_i (\Delta_{i,t-1}\tilde{C}_{i,t}^{-1})^{(\ell+1)\chi_i/(1+\bar{\chi})}$$

where

$$\tilde{C}_{i,t} \;=\; \bar{C}^{\hat{\eta}} e^{a_{i,t}(\eta-1)}$$

and where we still have

$$\Delta_{i,t-1} \;=\; \left(\frac{\bar{q}_i(\$)E_{t-1}\left[C_{t-1,t}^{-\gamma}\mathcal{P}_{\$,t-1,t}^{-1}\right]}{E_{t-1}\left[C_{t-1,t}^{-\gamma}\mathcal{P}_{\$,t-1,t}^{-\ell-1}(\bar{C}_t^{\hat{\eta}}e^{(\eta-1)a_{i,t}})^{-\ell}\right]}\right)^{\ell^{-1}}$$

$$= \;\left(\frac{\bar{q}_i(\$)E_{t-1}\left[C_{t-1,t}^{-\gamma}(\bar{C}^{\hat{\eta}}e^{\bar{a}_t(\eta-1)})^{\bar{\chi}/(1+\bar{\chi})}\right]}{E_{t-1}\left[C_{t-1,t}^{-\gamma}(\bar{C}^{\hat{\eta}}e^{\bar{a}_t(\eta-1)})^{\bar{\chi}(\ell+1)/(1+\bar{\chi})}(\bar{C}_t^{\hat{\eta}}e^{(\eta-1)a_{i,t}})^{-\ell}\right]}\right)^{\ell^{-1}} q_* \prod_j (\Delta_{j,t-1})^{(\ell+1)\chi_j/(1+\bar{\chi})}$$

where we have defined

$$\bar{\chi} \;=\; \sum_i (1+\ell)\chi_i,\;\; \bar{a}_t \;=\; \sum_i (1+\ell)\chi_i a_{i,t}/\bar{\chi}\,.$$

This gives the fixed point system for leverage. Defining

$$e^{Q_i} \;\equiv\; \left(\frac{E_{t-1}\left[C_{t-1,t}^{-\gamma}(\bar{C}^{\hat{\eta}}e^{\bar{a}_t(\eta-1)})^{\bar{\chi}/(1+\bar{\chi})}\right]}{E_{t-1}\left[C_{t-1,t}^{-\gamma}(\bar{C}^{\hat{\eta}}e^{\bar{a}_t(\eta-1)})^{\bar{\chi}(\ell+1)/(1+\bar{\chi})}(\bar{C}_t^{\hat{\eta}}e^{(\eta-1)a_{i,t}})^{-\ell}\right]}\right)^{\ell^{-1}} q_*(i),$$

we get

$$\log\Delta_{i,t-1} \;=\; Q_i \;+\; \sum_j (\ell+1)\chi_j(1+\bar{\chi})^{-1}\log\Delta_{j,t-1}\,.$$

Multiplying by

$$(\ell+1)\chi_j$$

and summing up, we get

$$\sum_j (\ell+1)\chi_j \log \Delta_{j,t-1} \;=\; \sum_j (\ell+1)\chi_j Q_j \;+\; \bar{\chi}\sum_j (\ell+1)\chi_j (1+\bar{\chi})^{-1}\log \Delta_{j,t-1}$$

so that

$$\sum_j (\ell+1)\chi_j (1+\bar{\chi})^{-1}\log \Delta_{j,t-1} \;=\; \sum_j (\ell+1)\chi_j Q_j\,.$$

Thus,

$$\log \Delta_{i,t-1} \;=\; Q_i \;+\; \sum_j (\ell+1)\chi_j Q_j\,.$$

Now, in the small shock approximation, we have

$$\begin{aligned}
&\log \bar{C}_t^{1-\alpha} \;\approx\; \log \bar{\kappa} \;+\; \log \sum_j \kappa_i e^{a_{j,t}(\eta-1)} \\
&\approx\; \bar{\mu}_{t-1}(\eta-1) \;+\; \log(1+\sum_j \kappa_j((a_{j,t}-\mu_{t-1})+0.5(a_{j,t}-\mu_{t-1})^2)) \\
&\approx\; (\eta-1)\tilde{a}_t \;+\; 0.5(\eta-1)^2 v_t\,,
\end{aligned}$$

so that

$$C_t^{-\gamma} \;=\; \bar{C}_t^{-(\eta-1)^{-1}} \;\approx\; e^{-(1-\alpha)^{-1}(\tilde{a}_t \;+\; 0.5(\eta-1)v_t)}$$

Define also

$$\tilde{a}_t \;=\; \sum_j \kappa_j a_{j,t},\;\; \tilde{v}_{t-1} \;=\; E_{t-1}[\sum_j \kappa_j (a_{j,t}-\mu_{t-1})^2 \;-\; \left(\sum_j \kappa_j (a_{j,t}-\mu_{t-1})\right)^2],$$

and note that since $v_t$ is of the order $\varepsilon^2$, only its expectation will matter for the approximate calculations below. Here, the weights $\kappa_j$ are normalized to add up to one. Thus,

$$\begin{aligned}
e^{Q_i \ell} \;&=\; q_*(i)^\ell \frac{E_{t-1}\left[C_{t-1,t}^{-\gamma}(\bar{C}^{\hat{\eta}} e^{\bar{a}_t(\eta-1)})^{\bar{\chi}/(1+\bar{\chi})}\right]}{E_{t-1}\left[C_{t-1,t}^{-\gamma}(\bar{C}^{\hat{\eta}} e^{\bar{a}_t(\eta-1)})^{\bar{\chi}(\ell+1)/(1+\bar{\chi})}(\bar{C}_t^{\hat{\eta}} e^{(\eta-1)a_{i,t}})^{-\ell}\right]} \\
&=\; q_*(i)^\ell \; E_{t-1}\left[e^{-(1-\alpha)^{-1}(\tilde{a}_t \;+\; 0.5(\eta-1)\tilde{v}_{t-1})}(e^{(1-\alpha)^{-1}\hat{\eta}((\eta-1)\tilde{a}_t \;+\; 0.5(\eta-1)^2\tilde{v}_{t-1})}e^{\bar{a}_t(\eta-1)})^{\bar{\chi}/(1+\bar{\chi})}\right] \\
&\times E_{t-1}\Bigg[e^{-(1-\alpha)^{-1}(\tilde{a}_t \;+\; 0.5(\eta-1)\tilde{v}_{t-1})}(e^{(1-\alpha)^{-1}\hat{\eta}((\eta-1)\tilde{a}_t \;+\; 0.5(\eta-1)^2\tilde{v}_{t-1})})^{(\bar{\chi}-\ell)/(1+\bar{\chi})} \\
&\times (e^{\bar{a}_t(\eta-1)})^{\bar{\chi}(\ell+1)/(1+\bar{\chi})}(e^{(\eta-1)a_{i,t}})^{-\ell}\Bigg]^{-1} \\
&=\; q_*(i)^\ell e^{(1-\alpha)^{-1}\hat{\eta}(\eta-1)^2 0.5\ell(1+\bar{\chi})^{-1}\tilde{v}_{t-1}} \frac{E_{t-1}[e^{\Gamma_1\tilde{a}_t+\Gamma\bar{a}_t}]}{E_{t-1}[e^{\Gamma_3(i)\tilde{a}_t+\Gamma_4(i)\bar{a}_t-(\eta-1)\ell a_{i,t}}]} \\
&=\; q_*(i)^\ell e^{\hat{\mu}_{i,t-1}} e^{(1-\alpha)^{-1}\hat{\eta}(\eta-1)^2 0.5\ell(1+\bar{\chi})^{-1}\tilde{v}_{t-1}} \\
&\times \exp\Bigg(0.5(\Gamma_1^2-\Gamma_3^2)\tilde{\sigma}_{t-1}^2 + 0.5(\Gamma^2-\Gamma_4^2(i))\bar{\sigma}_{t-1}^2 - 0.5(\eta-1)^2\ell^2\sigma_{i,t-1}^2 \\
&+ \sigma_{t-1}(\bar{a},\tilde{a})(\Gamma_1\Gamma-\Gamma_3(i)\Gamma_4(i)) + \sigma_{t-1}(\bar{a},a_i)(\eta-1)\ell\Gamma_4(i) + \sigma_{t-1}(\tilde{a},a_i)(\eta-1)\ell\Gamma_3(i)\Bigg)
\end{aligned}$$

where

$$\begin{aligned}
\Gamma_1 \;&=\; -(1-\alpha)^{-1} + (1-\alpha)^{-1}\hat{\eta}(\eta-1)\bar{\chi}/(1+\bar{\chi}) \\
\Gamma \;&=\; (\eta-1)\bar{\chi}/(1+\bar{\chi}) \\
\Gamma_3(i) \;&=\; -(1-\alpha)^{-1} + (1-\alpha)^{-1}\hat{\eta}(\eta-1)(\bar{\chi}-\ell)/(1+\bar{\chi}) \\
\Gamma_4(i) \;&=\; (\eta-1)(\ell+1)\bar{\chi}/(1+\bar{\chi})\,.
\end{aligned}$$

Furthermore,

$$\bar{Q} \;=\; \sum_i (\ell+1)\chi_i Q_i$$

and

$$\mathcal{P}_{\$,t-1,t} \;=\; q_* \tilde{C}_t^{-\bar{\chi}/(1+\bar{\chi})} e^{\bar{Q}}$$

We have

$$\mathcal{B}_{j,t}(B_{t-1}) \;=\; ((1-\tau)c+1) b_{\$,t-1}(j) \mathcal{P}_{\$,t-1,t}^{-1} \mathcal{P}_{j,t-1,t}$$

with

$$b_{\$,t-1}(j) \;=\; \xi_j \Delta_{j,t-1} \mathcal{P}_{j,t-1}$$

Thus,

$$\Psi_{j,t} \;=\; \frac{\mathcal{B}_{j,t}\mathcal{P}_{j,t}^{-1}}{\xi_j e^{a_{j,t}(\eta-1)} \bar{C}_t^{\hat{\eta}}} \;=\; \frac{\Delta_{j,t-1}\mathcal{P}_{\$,t-1,t}^{-1}}{e^{a_{j,t}(\eta-1)} \bar{C}_t^{\hat{\eta}}} \;=\; \frac{e^{Q_i} \tilde{C}_t^{\bar{\chi}/(1+\bar{\chi})}}{\tilde{C}_t} \;=\; e^{Q_i} \tilde{C}_t^{-(1+\bar{\chi})^{-1}} .$$

Expected welfare is then

$$(1-\gamma)^{-1} E_{t-1}[C_t^{1-\gamma}] \;\approx\; (1-\gamma)^{-1} E_{t-1}[(C_t^*)^{1-\gamma}] \;-\; \sum_i \kappa_i E_{t-1}[(C_t^*)^{-\gamma}(1-\zeta_j)\left(\Psi_{j,t}\right)^{\ell+1}] .$$

Thus, utility losses from country $i$ debt overhang are given by

$$E_{t-1}[(C_t^*)^{-\gamma}(1-\zeta_j)\left(\Psi_{i,t}\right)^{\ell+1}] \;=\; (1-\zeta_j) e^{Q_i(\ell+1)} E_{t-1}[(C_t)^{-\gamma} \tilde{C}_t^{-(1+\bar{\chi})^{-1}(\ell+1)}]$$

We have

$$
\begin{aligned}
C_t^{-\gamma} \tilde{C}_t^{-(1+\bar{\chi})^{-1}(\ell+1)} \;&\approx\; \big(e^{-(1-\alpha)^{-1}(\tilde{a}_t \;+\; 0.5(\eta-1)\tilde{v}_{t-1})}\big)\big(\bar{C}^{\hat{\eta}} e^{a_{i,t}(\eta-1)}\big)^{-(1+\bar{\chi})^{-1}(\ell+1)} \\
&=\; \big(e^{-(1-\alpha)^{-1}(\tilde{a}_t \;+\; 0.5(\eta-1)\tilde{v}_{t-1})}\big)\big(e^{(1-\alpha)^{-1}\hat{\eta}((\eta-1)\tilde{a}_t \;+\; 0.5(\eta-1)^2\tilde{v}_{t-1})} e^{a_{i,t}(\eta-1)}\big)^{-(1+\bar{\chi})^{-1}(\ell+1)} \\
&=\; e^{-\tilde{a}_t(1-\alpha)^{-1}(1+\hat{\eta}(\eta-1)(1+\bar{\chi})^{-1}(\ell+1)) - 0.5\tilde{v}_t(1-\alpha)^{-1}(\eta-1)(1+\hat{\eta}(\eta-1)(1+\bar{\chi})^{-1}(\ell+1))}\, e^{-a_{i,t}(\eta-1)(1+\bar{\chi})^{-1}(\ell+1)} \\
&=\; e^{-K_1(i)\tilde{a}_t - K_2(i)v_t - K_3(i)a_{i,t}}\,.
\end{aligned}
$$

Thus,

$$
\begin{aligned}
& E_{t-1}\big[(C_t)^{-\gamma}\tilde{C}_t^{-(1+\bar{\chi})^{-1}(\ell+1)}\big] \\
&\approx\; E_{t-1}\big[e^{-K_1(i)\tilde{a}_t - K_2(i)v_t - K_3(i)a_{i,t}}\big] \\
&\approx\; e^{-K_1(i)\tilde{\mu}_{t-1} - K_2(i)\tilde{v}_{t-1} - K_3(i)\mu_{t-1} + 0.5(K_1(i)^2\tilde{\sigma}_{t-1}^2 + K_3(i)^2\sigma_{i,t-1}^2 + 2K_1(i)K_3(i)\sigma_{t-1}(\tilde{a},a_i))}
\end{aligned}
$$

and hence the welfare loss is proportional to

$$
\begin{aligned}
& \sum_i (1-\zeta) e^{Q_i(\ell+1)} e^{-K_1(i)\tilde{\mu}_{t-1} - K_2(i)\tilde{v}_{t-1} - K_3(i)\mu_{t-1} + 0.5(K_1(i)^2\tilde{\sigma}_{t-1}^2 + K_3(i)^2\sigma_{i,t-1}^2 + 2K_1(i)K_3(i)\sigma_{t-1}(\tilde{a},a_i))} \\
&= \sum_i (1-\zeta) q_*(i)^{\ell+1} e^{\hat{\mu}_{i,t-1}(1+\ell^{-1})} e^{(1-\alpha)^{-1}\hat{\eta}(\eta-1)^2 0.5\ell(1+\bar{\chi})^{-1}\tilde{v}_{t-1}(1+\ell^{-1})} \\
&\times \exp\Bigg(\Bigg(0.5(\Gamma_1^2 - \Gamma_3^2)\tilde{\sigma}_{t-1}^2 + 0.5(\Gamma^2 - \Gamma_4^2(i))\bar{\sigma}_{t-1}^2 - 0.5(\eta-1)^2\ell^2\sigma_{i,t-1}^2 \\
&+ \sigma_{t-1}(\bar{a},\tilde{a})(\Gamma_1\Gamma - \Gamma_3(i)\Gamma_4(i)) + \sigma_{t-1}(\bar{a},a_i)(\eta-1)\ell\Gamma_4(i) + \sigma_{t-1}(\tilde{a},a_i)(\eta-1)\ell\Gamma_3(i)\Bigg)(1+\ell^{-1})\Bigg) \\
&\times e^{-K_1(i)\tilde{\mu}_{t-1} - K_2(i)\tilde{v}_{t-1} - K_3(i)\mu_{t-1} + 0.5(K_1(i)^2\tilde{\sigma}_{t-1}^2 + K_3(i)^2\sigma_{i,t-1}^2 + 2K_1(i)K_3(i)\sigma_{t-1}(\tilde{a},a_i))}
\end{aligned}
$$

Note that

$$
\tilde{v}_{t-1} \;=\; \sum_j \kappa_j \sigma_{j,t-1}^2 \;-\; \tilde{\sigma}_{t-1}^2\,.
$$

Denoting

$$\tilde{q}_i \;\equiv\; \kappa_i(1-\zeta)q_*(i)^{\ell+1}\,,$$

we get that the volatility part of the welfare loss is (up to an additive constant) approximately given by

$$\begin{aligned}
&\sum_i \tilde{q}_i \Bigg( (1-\alpha)^{-1}\hat{\eta}(\eta-1)^2 0.5\ell(1+\bar{\chi})^{-1}(\sum_j \kappa_j\sigma^2_{j,t-1} \;-\; \tilde{\sigma}^2_{t-1})(1+\ell^{-1}) \\
&+\; \Bigg(0.5(\Gamma_1^2-\Gamma_3^2)\tilde{\sigma}^2_{t-1} + 0.5(\Gamma^2-\Gamma_4^2(i))\bar{\sigma}^2_{t-1} - 0.5(\eta-1)^2\ell^2\sigma^2_{i,t-1} \\
&+\sigma_{t-1}(\bar{a},\tilde{a})(\Gamma_1\Gamma-\Gamma_3(i)\Gamma_4(i)) + \sigma_{t-1}(\bar{a},a_i)(\eta-1)\ell\Gamma_4(i) + \sigma_{t-1}(\tilde{a},a_i)(\eta-1)\ell\Gamma_3(i)\Bigg)(1+\ell^{-1}) \\
&-0.5(\sum_j \kappa_j\sigma^2_{j,t-1} \;-\; \tilde{\sigma}^2_{t-1})(1-\alpha)^{-1}(\eta-1)(1+\hat{\eta}(\eta-1)(1+\bar{\chi})^{-1}(\ell+1)) \\
&+\; 0.5(K_1(i)^2\tilde{\sigma}^2_{t-1} + K_3(i)^2\sigma^2_{i,t-1} + 2K_1(i)K_3(i)\sigma_{t-1}(\tilde{a},a_i))\Bigg) \\
&=\; \sum_j \Xi_j\sigma^2_{j,t-1} \;+\; \bar{\Xi}\bar{\sigma}^2_{t-1} \;+\; \tilde{\Xi}\tilde{\sigma}^2_{t-1} \;+\; \hat{\Xi}\tilde{\sigma}_{t-1}(\tilde{a}_t,\bar{a}_t) \;+\; \sigma_{t-1}(\bar{a}_t,\hat{a}_t) \;+\; \sigma_{t-1}(\tilde{a}_t,\;\check{a}_t)
\end{aligned}$$

Here,

$$
\begin{aligned}
\Xi_j &= -\tilde{q}_j 0.5(\eta-1)^2\ell^2(1+\ell^{-1}) + 0.5\tilde{q}_j K_3(j)^2 \\
&+ \kappa_j \sum_i \tilde{q}_i \left((1-\alpha)^{-1}\hat{\eta}(\eta-1)^2 0.5\ell(1+\bar{\chi})^{-1}(1+\ell^{-1}) - (1-\alpha)^{-1}(\eta-1)(1+\hat{\eta}(\eta-1)(1+\bar{\chi})^{-1}(\ell+1))\right) \\
\bar{\Xi} &= \sum_i \tilde{q}_i 0.5(\Gamma^2 - \Gamma_4^2(i))(1+\ell^{-1}) \\
\tilde{\Xi} &= \Big( -\sum_i \tilde{q}_i (1-\alpha)^{-1}\hat{\eta}(\eta-1)^2 0.5\ell(1+\bar{\chi})^{-1}(1+\ell^{-1}) \\
&+ \sum_i \tilde{q}_i 0.5(\Gamma_1^2 - \Gamma_3^2 + K_1(i)^2)(1+\ell^{-1}) \\
&+ \sum_i \tilde{q}_i 0.5(1-\alpha)^{-1}(\eta-1)(1+\hat{\eta}(\eta-1)(1+\bar{\chi})^{-1}(\ell+1))\Big) \\
\hat{\Xi} &= \sum_i \tilde{q}_i(\Gamma_1\Gamma - \Gamma_3(i)\Gamma_4(i))(1+\ell^{-1}) \\
\hat{a}_t &= \sum_i \tilde{q}_i(\eta-1)\ell\Gamma_4(i)(1+\ell^{-1})a_{i,t} \\
\check{a}_t &= \sum_i \tilde{q}_i[(\eta-1)\ell\Gamma_3(i)(1+\ell^{-1}) + K_1(i)K_3(i)]a_{i,t}\,.
\end{aligned}
$$

Out interest is in the dependence on the coefficients defining $\bar{a}_t$. This is the only place the exorbitant duty coefficients enter the welfare. This part of welfare can be rewritten as

$$
\bar{\Xi}\bar{\sigma}_{t-1}^2 \;+\; \sigma_{t-1}(\bar{a}_t, \hat{\Xi}\tilde{a}_t + \hat{a}_t)\,.
$$

Here,

$$
\begin{aligned}
\hat{\Xi}\tilde{a}_t + \hat{a}_t &= \sum_i \left(\hat{\Xi}\kappa_i + \tilde{q}_i(\eta-1)\ell\Gamma_4(i)(1+\ell^{-1})\right) a_{j,t} \\
&= \sum_i \left(\hat{\Xi}\kappa_i + \tilde{q}_i(\eta-1)\ell(\eta-1)(\ell+1)\bar{\chi}/(1+\bar{\chi})(1+\ell^{-1})\right) a_{j,t}
\end{aligned}
$$

with

$$\begin{aligned}
\hat{\Xi} &= \sum_i \tilde{q}_i(\Gamma_1\Gamma - \Gamma_3(i)\Gamma_4(i))(1+\ell^{-1}) \\
&= (\eta-1)\bar{\chi}/(1+\bar{\chi})(1-\alpha)^{-1}\sum_i \tilde{q}_i\Bigg((-1+\hat{\eta}(\eta-1)) \\
&- (-1+\hat{\eta}(\eta-1)(\bar{\chi}-\ell)/(1+\bar{\chi}))(\ell+1)\Bigg)(1+\ell^{-1}) \\
&= (\eta-1)\bar{\chi}/(1+\bar{\chi})(1-\alpha)^{-1}\sum_i \tilde{q}_i\left(\ell+\hat{\eta}(\eta-1)\frac{1-\ell(\bar{\chi}-(\ell+1))}{1+\bar{\chi}}\right)(1+\ell^{-1})
\end{aligned}$$

and

$$\begin{aligned}
\bar{\Xi} &= \sum_i \tilde{q}_i 0.5(\Gamma^2 - \Gamma_4^2(i))(1+\ell^{-1}) \\
&= -((\eta-1)\bar{\chi}/(1+\bar{\chi}))^2\sum_i \tilde{q}_i 0.5\ell(\ell+2)
\end{aligned}$$

Our result follows then from the following general lemma.

**Lemma E.1** *Consider the minimization problem*

$$\min_A\{\mathrm{Cov}_{t-1}(\sum_i A_i a_{i,t}, \sum_i \Psi_i a_{i,t}) - 0.5\mathrm{Var}_{t-1}(\sum_i A_i a_{i,t})\}$$

*over the unit simplex*

$$A_i \geq 0,\ \sum_i A_i = 1\,.$$

*then, the solution to this problem is given by the following: there exists a threshold* $\Psi_*$ *such that* $A_i = 0$ *if and only if* $\Psi_i/\sigma_i\ >\ \Psi_*$.

$a$

**Proof**. We have

$$\mathrm{Cov}_{t-1}(\sum_i A_i a_{i,t}, \sum_i \Psi_i a_{i,t}) \;=\; \sum_i A_i(\Psi_i \sigma_i^2 + \rho\sigma_i \sum_{j\neq i} \sigma_j \Psi_j) \;=\; \sum_i A_i \sigma_i (\Psi_i \sigma_i (1-\rho) + \rho\bar{\Psi})$$

with

$$\bar{\Psi} = \sum_j \sigma_j \Psi_j\,.$$

Thus, the first order Kuhn-Tucker condition takes the form

$$\Psi_i \sigma_i (1-\rho) + \rho\bar{\Psi} \;-\sum_{j\neq i} \sigma_i \sigma_j \rho A_j \;-\; \lambda \;\geq\; 0$$

when the constraint $A_i \geq 0$ binds, and and

$$\Psi_i \sigma_i (1-\rho) + \rho\bar{\Psi} \;-\; \sigma_i^2 A_i - \sum_{j\neq i} \sigma_i \sigma_j \rho A_j \;-\; \lambda \;=\; 0$$

when $A_i > 0$. Here, $\lambda$ is the Lagrange multiplier for the constraint $\sum_i A_i = 1$. That is, $A_i = 0$ for all countries for which

$$\Psi_i \sigma_i (1-\rho) + \rho\bar{\Psi} \;-\; \sigma_i \sum_j \sigma_j \rho A_j \;-\; \lambda \;\geq\; 0\,,$$

while the interior solution is for

$$\Psi_i \sigma_i (1-\rho) + \rho\bar{\Psi} \;-\; \sigma_i^2 (1-\rho) A_i - \sigma_i \sum_j \sigma_j \rho A_j \;-\; \lambda \;=\; 0$$

This gives

$$A_i \;=\; \frac{\Psi_i \sigma_i (1-\rho) + \rho\bar{\Psi} - \sigma_i \bar{A} - \lambda}{\sigma_i^2 (1-\rho)}$$

Q.E.D.

# F Additional Results

# G Mixture of LC and \$

We first state the following extension of the Theorem 5.1 for the case of firms borrowing both in local currency and in dollars.

**Theorem G.1** *Suppose that $q_i(i) = q_i(\$)$. Then, issuing in a mixture of local currency and dollars is optimal if and only if*

$$\frac{\bar{q}_i(j,\$)}{\bar{q}_i(\$)} - 1 \;\leq\; \frac{\mathrm{Cov}_t^{\$}\left(\left(\frac{\Omega_{i,t+1}}{\mathcal{B}_{t+1}(B_t)}\right)^{-\ell}, \mathcal{E}_{j,t,t+1}\right)}{E_t^{\$}\left[\left(\frac{\Omega_{i,t+1}}{\mathcal{B}_{t+1}(B_t)}\right)^{-\ell}\right] E_t^{\$}[\mathcal{E}_{j,t,t+1}]}$$

*for all $j = 1, \cdots, N$.*

**Proof of Theorem G.1**. The standard Kuhn-Tucker conditions that borrowing only in LC and dollars is optimal if and only if

$$\begin{aligned}
&E_t\left[M_{i,t,t+1}\left[\left(1-(1-\rho_i)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell}\right)(1+c)\mathcal{E}_{j,i,t+1}\right]\right](1-q_i(j))\\
&+E_t\left[M_{i,t,t+1}\left[\left(-\ell(1-\rho_i)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell-1}\Omega_{i,t+1}^{-1}\right)(1+c)\mathcal{E}_{j,i,t+1}\right]\mathcal{B}_{t+1}(B_t)\right](1-q_i(\$))\\
&-(1+c(1-\tau))E_t\left[M_{i,t,t+1}\mathcal{E}_{j,i,t+1}\right]\\
&+E_t\left[M_{i,t,t+1}(\ell+1)\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell}(1+c(1-\tau))\mathcal{E}_{j,i,t+1}\right.\\
&\left.-\ell\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell}(1+c(1-\tau))\mathcal{E}_{j,i,t+1}\right]\leq 0
\end{aligned}$$

for all $j$ with the identity for $j = i, \$$. This inequality can be rewritten as

$$\bar{q}_i(j,\$)\frac{E_t[M_{i,t,t+1}\mathcal{E}_{j,i,t+1}]}{E_t\left[M_{i,t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell}\mathcal{E}_{j,i,t+1}\right]} \;\leq\; 1 \;=\; \bar{q}_i(\$)\frac{E_t[M_{i,t,t+1}\mathcal{E}_{\$,i,t+1}]}{E_t\left[M_{i,t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)^{\ell}\mathcal{E}_{\$,i,t+1}\right]}$$

and the first claim follows.

For the second claim, we get the system

$$\begin{aligned}
1 \;&=\; \bar{q}_i(\$)\frac{E_t[M_{i,t,t+1}\mathcal{E}_{\$,i,t+1}]}{E_t\left[M_{i,t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)\mathcal{E}_{\$,i,t+1}\right]}\\
1 \;&=\; \bar{q}_i(\$)\frac{E_t[M_{i,t,t+1}]}{E_t\left[M_{i,t,t+1}\left(\frac{\mathcal{B}_{t+1}(B_t)}{\Omega_{i,t+1}}\right)\right]}
\end{aligned}$$

whereby

$$\mathcal{B}_{t+1}(B_t) \;=\; (1+c(1-\tau))\left(B_t(i) \;+\; B_t(\$)\mathcal{E}_{\$,i,t+1}\right)$$

Thus, we get the system

$$\begin{aligned}
E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}]B_t(i) \;+\; E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}\mathcal{E}_{\$,i,t+1}]B_t(\$) \;&=\; \tilde{q}_i(\$)E_t[M_{i,t,t+1}]\\
E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}\mathcal{E}_{\$,i,t+1}]B_t(i) \;+\; E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}\mathcal{E}_{\$,i,t+1}^2]B_t(\$) \;&=\; \tilde{q}_i(\$)E_t[M_{i,t,t+1}\mathcal{E}_{\$,i,t+1}]
\end{aligned}$$

where we have defined

$$\tilde{q}_i(\$) = \bar{q}_i(\$)/(1+c(1-\tau))\,.$$

Thus,

$$\begin{pmatrix} B_t(i)\\ B_t(\$)\end{pmatrix} \;=\; \tilde{q}_i(\$)\Delta_t^{-1}\begin{pmatrix} E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}\mathcal{E}_{\$,i,t+1}^2] & -E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}\mathcal{E}_{\$,i,t+1}]\\ -E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}\mathcal{E}_{\$,i,t+1}] & E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}]\end{pmatrix}\begin{pmatrix} E_t[M_{i,t,t+1}]\\ E_t[M_{i,t,t+1}\mathcal{E}_{\$,i,t+1}]\end{pmatrix}.$$

where

$$\Delta_t \;=\; E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}\mathcal{E}_{\$,i,t+1}^2]E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}] - (E_t[M_{i,t,t+1}\Omega_{i,t+1}^{-1}\mathcal{E}_{\$,i,t+1}])^2$$

Thus,

$$\frac{B_t(i)}{B_t(\$)\mathcal{E}_{t,\$,i}} \;=\; \frac{-\mathrm{Cov}_t^{\$}(\Omega_{i,t+1}^{-1}\mathcal{E}_{t,t+1,\$,i},\; \mathcal{E}_{t,t+1,\$,i}^{-1})}{\mathrm{Cov}_t^{\$}(\Omega_{i,t+1}^{-1},\; \mathcal{E}_{t,t+1,\$,i}^{-1})}\,.$$

Thus,

$$\frac{B_t(i)}{B_t(\$)\mathcal{E}_{t,\$,i}} \;=\; \frac{-\mathrm{Cov}_t^{\$}\left(\left(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}}\mathcal{P}_{\$,t,t+1}\right)^{-1}, \mathcal{P}_{i,t,t+1}^{-1}\mathcal{P}_{\$,t,t+1}\right)}{\mathrm{Cov}_t^{\$}\left(\left(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}}\mathcal{P}_{i,t,t+1}\right)^{-1}, \mathcal{P}_{i,t,t+1}^{-1}\mathcal{P}_{\$,t,t+1}\right)}\,.$$

Let now $\tilde{a}_{i,t+1} \;\equiv\; \log(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}}) - \beta\tilde{a}_{\$,t+1}$ where $\tilde{a}_{\$,t+1} = \log(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{\$,t+1}})$ and where $\beta$ is such that $\tilde{a}_{i,t+1}$ and $\tilde{a}_{\$,t+1}$ are uncorrelated.

Recall also that we assume that

$$\log\mathcal{P}_{i,t,t+1} \;=\; -\hat{\alpha}_i\tilde{a}_{\$,t+1} - \alpha_i\tilde{a}_{i,t+1} \;+\; \varepsilon_{i,t+1},\; \log\mathcal{P}_{\$,t,t+1} \;=\; -\tilde{\alpha}_{\$}\hat{a}_{\$,t+1} + \varepsilon_{\$,t+1}$$

where $\varepsilon_{i,t+1} \sim N(0,\sigma_{\varepsilon,i}^2)$. We also allow $\sigma_{\varepsilon,i,\$} \;\equiv\; \mathrm{Cov}_t(\varepsilon_{i,t+1},\varepsilon_{\$,t+1}) \;\neq 0$. Then, to the first order in variance, the measure change is irrelevant and

$$\begin{aligned} &-\;\mathrm{Cov}_t^{\$}\left(\left(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}}\mathcal{P}_{\$,t,t+1}\right)^{-1}, \mathcal{P}_{i,t,t+1}^{-1}\mathcal{P}_{\$,t,t+1}\right) \\ &\approx\; -\mathrm{Cov}_t(-\tilde{a}_{i,t+1} - \beta\tilde{a}_{\$,t+1} \;+\alpha_{\$}\tilde{a}_{\$,t+1} \;-\; \varepsilon_{\$,t+1}, -\alpha_{\$}\tilde{a}_{\$,t+1} \;+\; \varepsilon_{\$,t+1} + \alpha_i\tilde{a}_{i,t+1} + \hat{\alpha}_i\tilde{a}_{\$,t+1} \;-\; \varepsilon_{i,t+1}) \end{aligned}$$

whereas

$$\mathrm{Cov}_t^{\$}\left(\left(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}}\mathcal{P}_{i,t,t+1}\right)^{-1}, \mathcal{P}_{i,t,t+1}^{-1}\mathcal{P}_{\$,t,t+1}\right)$$
$$\approx \ \mathrm{Cov}_t(-\tilde{a}_{i,t+1}-\beta\tilde{a}_{\$,t+1}+\alpha_i\tilde{a}_{i,t+1}+\hat{\alpha}_i\tilde{a}_{\$,t+1} \ - \ \varepsilon_{i,t+1}, -\alpha_{\$}\tilde{a}_{\$,t+1} \ + \ \varepsilon_{\$,t+1}+\alpha_i\tilde{a}_{i,t+1}+\hat{\alpha}_i\tilde{a}_{\$,t+1} \ - \ \varepsilon_{i,t+1})$$

In the small variance approximation, we that's get

$$\frac{B_t(i)}{B_t(\$)\mathcal{E}_{t,\$,i}} \ \approx \ \frac{\sigma_{\varepsilon,\$}^2-\sigma_{\varepsilon,i,\$}+\alpha_i\sigma_c^2(i)+\alpha_{\$}^2\sigma_c^2(\$)-(\alpha_{\$}+\alpha_i\alpha_{\$})\sigma_c(i,\$)}{\sigma_{\varepsilon,i}^2-\sigma_{\varepsilon,i,\$}+(1-\alpha_i)(\alpha_{\$}\sigma_c(i,\$)-\alpha_i\sigma_c^2(i))}$$

where $\sigma_c(i)^2 = \mathrm{Var}_t[\log(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}})]$ and $\sigma_c(i,\$) = \mathrm{Cov}_t[\log(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{i,t+1}}), \log(\bar{C}_{t+1}^{\hat{\eta}}e^{(\eta-1)a_{\$,t+1}})]$.
Q.E.D.

## G.1 Investment and Debt Overhang

In this section, we assume that the firm can pay a cost of

$$h_i(1+\beta^{-1})^{-1}k_{i,t}^{\beta^{-1}+1},$$

to increase the lower bound of the support of the idiosyncratic shock distribution. Namely, upon having selected $k_{i,t}$, the firm gets $Z_{i,t}$ drawn from the distribution with the density $\phi(z) \ = \ (1+k_{i,t})\ell z^{\ell-1}$ on $[q_{i,t}, 1]$ with $q_{i,t} = (1-(1+k_{i,t})^{-1})^{\ell^{-1}}$.

The firm is then solving

$$-\, h_i\Omega_{i,t}(\beta+1)^{-1}k_i^{\beta+1}+k_i\Omega_{i,t}\int_{\Psi_{i,t}}^1 z\phi(z)dz$$
$$= \ -h_i\Omega_{i,t}(\beta^{-1}+1)^{-1}k_i^{\beta^{-1}+1}+(k_i+1)\Omega_{i,t}\ell(\ell+1)^{-1}(1-\Psi_{i,t}^{\ell+1})\,.$$

Solving the optimization problem gives

$$k_{i,t} = \left(h_i^{-1}\ell(\ell+1)^{-1}(1-\Psi_{i,t}^{\ell+1})\right)^{\beta} .$$

In particular, absent debt overhang,

$$k_{i,t} \;=\; k_i^* \;=\; \left(h_i^{-1}\ell(\ell+1)^{-1}\right)^{\beta} .$$

Note that here $1+k_i = (1-q_i^{\ell})^{-1}$.

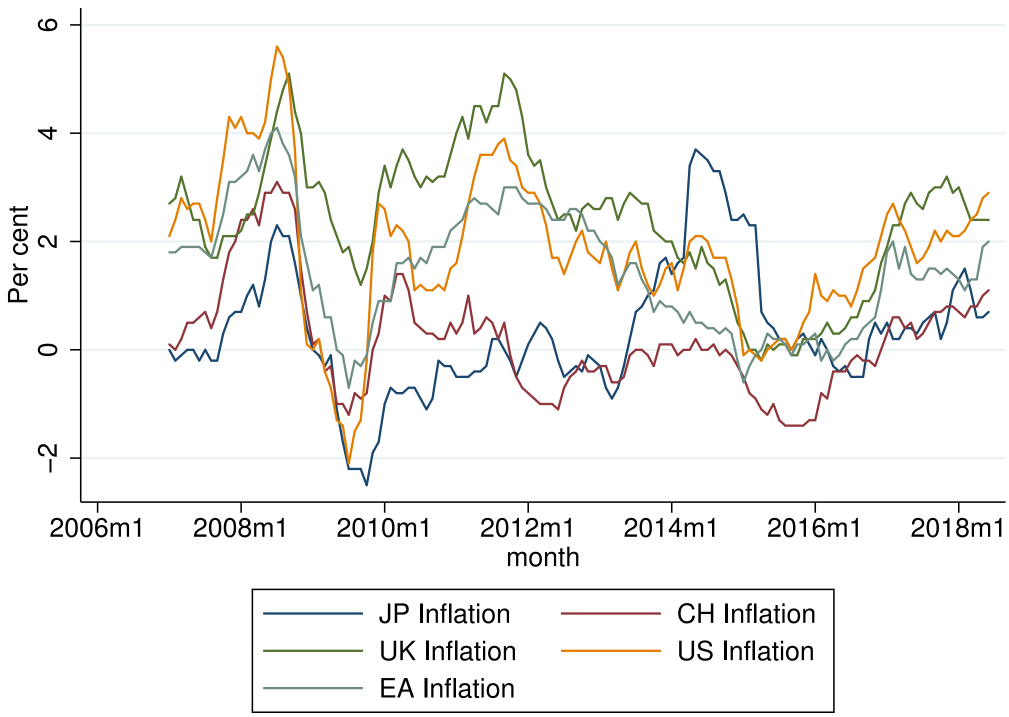
Then, redefining

$$\begin{aligned} G_i(\Psi_{i,t}) \;=\; & \ell(\ell+1)^{-1}\left(1+\left(h_i^{-1}\ell(\ell+1)^{-1}(1-\Psi_{i,t}^{\ell+1})\right)^{\beta}\right) \\ \times & \left((\zeta-1)\Psi_{i,t}^{\ell+1}+1-\zeta\left(\left(1-\left(1+\left(h_i^{-1}\ell(\ell+1)^{-1}(1-\Psi_{i,t}^{\ell+1})\right)^{\beta}\right)^{-1}\right)\right)^{\ell^{-1}+1}\right) , \end{aligned}$$

we get the same equilibrium equation.

## G.2 Data Appendix

**Figure 9: Inflation rates by Currency**




*Source: National data*