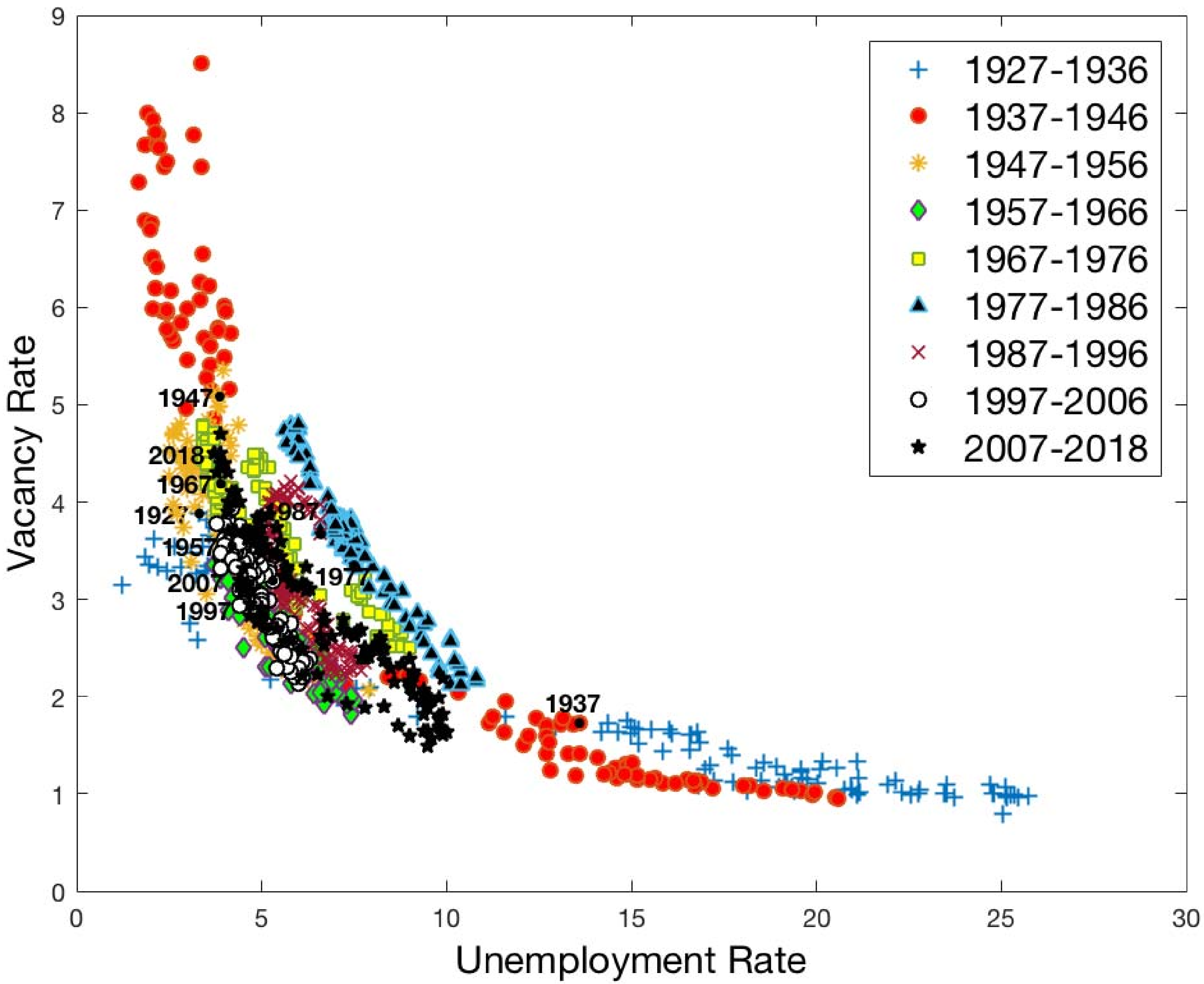

Vacancy Rate
Unemployment Rate
1927-1936
1937-1946
1947-1956
1957-1966
1967-1976
1977-1986
1987-1996
1997-2006
2007-2018
1947
2018
1967
1927
1987
1957
2007
1977
1997
1937

Introduction

Stylized long-run facts:

1. No secular movement in the Beveridge curve

$$u = \frac{\delta_t}{\delta_t + A_t p(v/u)}.$$

2. No secular movement along the Beveridge curve

$$u_t,\ v_t,\ v_t/u_t \text{ stationary.}$$

From the perspective of search theory, the above observations imply that the efficiency A_t of the search technology has not improved from 1926 to 2017. Telephone? Fax? Mobile phone? PC? Internet? All irrelevant!

Environment

Workers:

Firms:

Labor market:

Exogenous processes:

Environment

Workers:

- *population*: measure 1;
- *objective*: max pv of income $\{b_t, w_t\}$ discounted at rate r.

Firms:

Labor market:

Exogenous processes:

Environment

Workers:

Firms:

- *population*: positive measure;
- *technology*:

  maintain vacancies v_t at flow unit cost k_t;

  CRTS technology: 1 unit of labor –> $y_t z$ units of output;
- *objective*: max pv of income $y_t z - w_t$ discounted at rate r.

Labor market:

Exogenous processes:

Environment

Workers:

Firms:

Labor market:

- u_t and v_t come together through a matching fn $A_t M(u_t, v_t)$;
- u meets v at rate $A_t p(\theta_t)$, where $\theta_t = v_t/u_t$ and $p(\theta_t) = M(1, \theta_t)$;
- upon meeting u and v observe quality $z \sim F$, decide whether to match and bargain over terms of trade.

Exogenous processes:

Environment

Workers:

Firms:

Labor market:

Exogenous processes:

- search efficiency A_t grows at the rate g_A;
- production efficiency y_t grows at the rate g_y;
- unemp. benefit b_t grows at the rate g_b;
- vacancy cost k_t grows at the rate g_k.

Definition of a BGP

A BGP is a $\{R_t, S_t, G_t\}$ and a $\{\theta, h_{ue}, h_{eu}, u, g_z\}$ s.t.

1. Reservation quality R_t:

$$y_t R_t = b_t + A_t p(\theta)\gamma \int_{R_t} S_t(z) dF(z)$$

2. Surplus of a match S_t:

$$rS_t(z) = y_t(z - R_t) + \dot{S}_t(z)$$

3. Market tightness θ_t:

$$k_t = A_t \frac{p(\theta_t)}{\theta_t}(1-\gamma)\int_{R_t} S_t(z) dF(z)$$

Definition of a BGP

A BGP is a $\{R_t, S_t, G_t\}$ and a $\{\theta, h_{ue}, h_{eu}, u, g_z\}$ s.t.

4. Stationarity of UE, EU, u and θ_t:

$$
\begin{aligned}
A_t p(\theta)(1 - F(R_t)) &= h_{ue}, \\
G_t'(R_t)\dot{R}_t &= h_{eu}, \\
(1 - u)h_{eu} &= u h_{ue}, \\
\theta_t &= \theta.
\end{aligned}
$$

5. Distribution G_t of workers across z such that every quantile $z_t(x)$ grows at some constant rate g_z:

$$
(1 - u)G_t'(z_t(x))z_t(x)g_z + uA_t p(\theta)[F(z_t(x)) - F(R_t)] = (1 - u)G_t'(R_t)R_t g_z.
$$

Necessary Conditions for a BGP

N1 A BGP may exist only if F is Pareto with some coefficient α.

Sketch: The stationarity condition for the UE rate is:

$$A_t p(\theta)(1 - F(R_t)) = h_{UE}, \forall t \geq 0.$$

Differentiating with respect to t, we obtain

$$g_A = \frac{F'(R_t)}{1 - F(R_t)} R_t g_z$$

The differential equation for F has the unique solution

$$F(z) = 1 - \left(\frac{z_\ell}{z}\right)^\alpha.$$

Necessary Conditions for a BGP

N2 A BGP may exist only if g_b and g_k are equal to $g_y + g_z$.

Sketch: Combining the equilibrium conditions for R_t and θ, we obtain:

$$y_t R_t = b_t + \frac{\gamma}{1-\gamma}\theta k_t, \ \forall t \geq 0.$$

The condition above can only be satisfied if

$$g_b, g_k = g_y + g_z$$

Existence and Uniqueness of a BGP

Impose the necessary conditions on the fundamentals.

1. Reservation quality R_t grows at the constant rate $g_z = g_A/\alpha$:

$$y_t R_t = b_t + A_t p(\theta)\gamma \underbrace{\int_{R_t} S_t(z) dF(z)}_{\Phi y_t R_t^{-(\alpha-1)}}$$

2. Market tightness θ_t is constant

$$k_t = A_t \frac{p(\theta)}{\theta}(1-\gamma) \underbrace{\int_{R_t} S_t(z) dF(z)}_{\Phi y_t R_t^{-(\alpha-1)}}$$

Existence and Uniqueness of a BGP

Impose the necessary conditions on the fundamentals.

3. Quality distribution $G_t(z)$ grows at constant rate g_z and starts at

$$G_0(z) = 1 - \left(\frac{R_0}{z}\right)^{\alpha}.$$

4. Unemployment u is constant at

$$u = \frac{g_A}{g_A + A_0 p(\theta)[1 - F(R_0)]}.$$

5. UE and EU rates are both constant.

Existence and Uniqueness of a BGP

We have established the following.

Proposition 1: Take arbitrary growth rates $g_y > 0$ and $g_A > 0$.

A BGP exists if and only if:

(**a**) F is Pareto with coefficient α;

(**b**) g_b and g_k are equal to $g_y + g_A/\alpha$.

If a BGP exists, it is unique and such that:

(**i**) u, θ, h_{ue}, h_{eu} are constant over time;

(**ii**) G_t is Pareto truncated at R_t growing at rate $g_z = g_A/\alpha$;

(**iii**) labor productivity grows at rate $g_y + g_A/\alpha$.

Search on the Job

Workers search on the job with relative intensity $\rho \in [0,1]$.

Proposition 2: Take arbitrary growth rates $g_y > 0$ and $g_A > 0$.

A BGP exists if and only if:

(**a**) F is Pareto with coefficient α;

(**b**) g_b and g_k are equal to $g_y + g_A/\alpha$.

Any BGP is such that:

(**i**) u, θ, h_{ue}, h_{eu} are constant over time;

(**ii**) G_t is Fréchet truncated at R_t and grows at rate $g_z = g_A/\alpha$;

(**iii**) labor productivity grows at rate $g_y + g_A/\alpha$.

Population Growth

Population grows at rate g_N and matching fn is $A_t N_t^{\beta} M(u_t, v_t)$.

Proposition 3: Take arbitrary growth rates $g_y > 0$, $g_A > 0$, $g_N > 0$ such that the overall search efficiency improves over time, i.e. $g_A + \beta g_N > 0$.

A BGP exists if and only if:

(**a**) F is Pareto with coefficient α;

(**b**) g_b and g_k are equal to $g_y + (g_A + \beta g_N)/\alpha$.

Any BGP is such that:

(**i**) u, θ, h_{ue}, h_{eu} are constant over time;

(**ii**) G_t is Pareto truncated at R_t and grows at rate $g_z = (g_A + \beta g_N)/\alpha$;

(**iii**) labor productivity grows at rate $g_y + (g_A + \beta g_N)/\alpha$.

Back-of-the-Envelope Calculations

The theory raises some quantitative questions:

1. Can't infer growth of search efficiency from time trends of u, θ, h_{ue}, h_{eu}.

   a. Measure technological improvements in search process?

   b. Measure contribution to economic growth of improvements in search process?

2. Can't infer returns to scale in search from time trends or cross-sections of u, θ, h_{ue}, h_{eu}

   a. Measure returns to scale in search process?

   b. Measure the contribution to economic growth of returns to scale in search process?

Back-of-the-Envelope Calculations

Average number of applications per vacancy are

$$A_t N_t^{\beta} q(\theta_t).$$

- In the model, applications per vacancy grow at rate $\beta g_N + g_A$.
- In the data, applications per vacancy were 24 in 1981 (EOPP) and 45 in 2010 (Career Builder, SnagAJob).
- These observations suggest

$$\beta g_N + g_A = 2.2\%$$

Back-of-the-Envelope Calculations

Relative number of applications per vacancy in two markets of sizes N_1 and N_2 with the same search technology is

$$\frac{A_{1,t} N_{1,t}^{\beta} q(\theta_{1,t})}{A_{2,t} N_{2,t}^{\beta} q(\theta_{2,t})} = \left(\frac{N_{1,t}}{N_{2,t}} \right)^{\beta}.$$

- In the model, elasticity of applications per vacancy wrt size is β.
- In the data, elasticity of applications per vacancy wrt size is 0.52.
- These observations suggest $\beta = 0.52$ and

$$\beta g_N = 0.52 \cdot 1.1\% = 0.6\%$$

$$g_A = \underbrace{g_A + \beta g_N}_{2.2\%} - \underbrace{\beta g_N}_{0.6\%} = 1.6\%$$

Back-of-the-Envelope Calculations

Contribution of Declining Search Frictions

	Pareto coefficient		
1981-2010	$\alpha = 4$	$\alpha = 8$	$\alpha = 16$
labor productivity growth		1.9%	
cont. of search technology	0.4%	0.2%	0.1%
cont. of IRS in search	0.15%	0.07%	0.04%
cont. of declining search frictions	0.55%	0.27%	0.14%

Back-of-the-Envelope Calculations

Returns to Scale and Productivity across Cities

	$\alpha = 4$	$\alpha = 8$	$\alpha = 16$
0.5 million workers	0.91	0.95	0.98
1 million workers	1	1	1
10 million workers	1.34	1.16	1.08

Declining Search Frictions, Unemployment and Growth

Paolo Martellini
University of Pennsylvania

Guido Menzio
NYU and NBER

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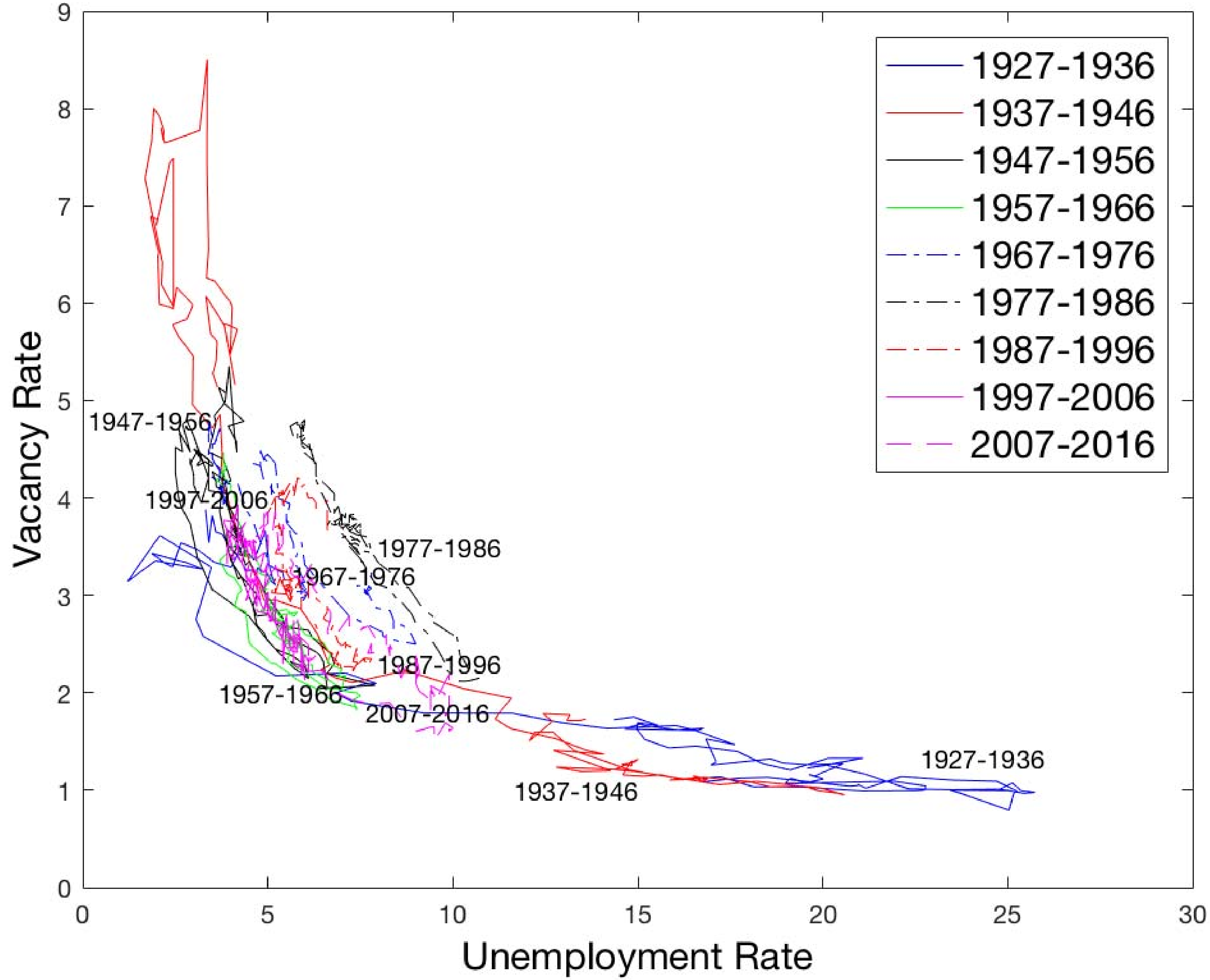
Minnesota Macro

July 31, 2018



1927-1936
1937-1946
1947-1956
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1967-1976
1977-1986
1987-1996
1997-2006
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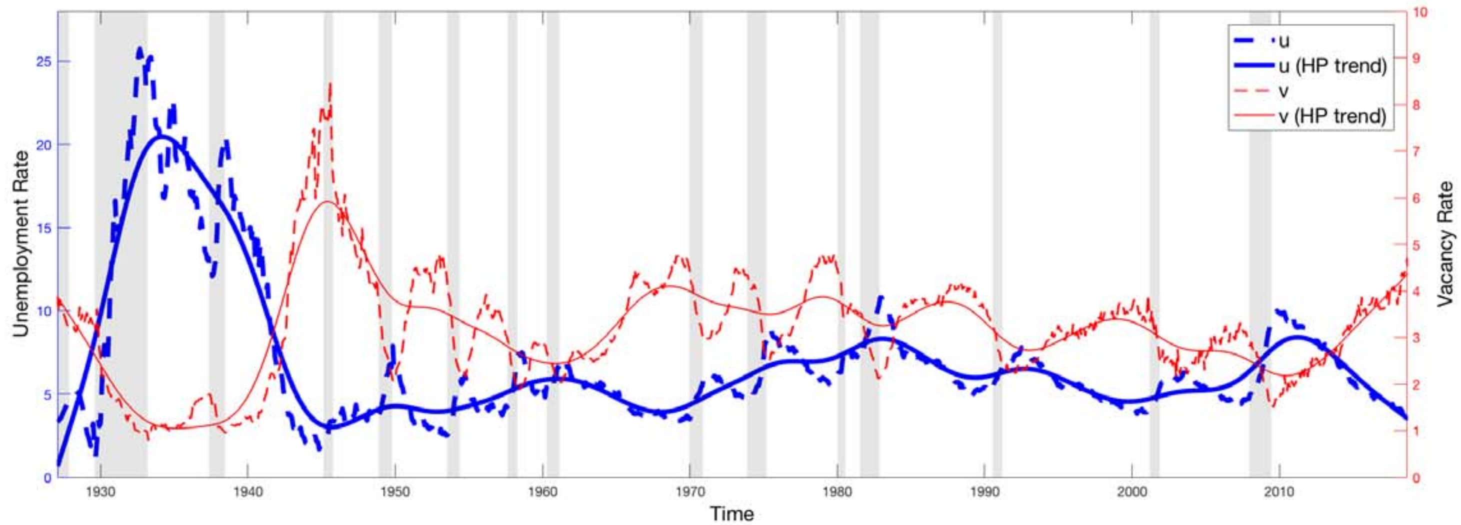
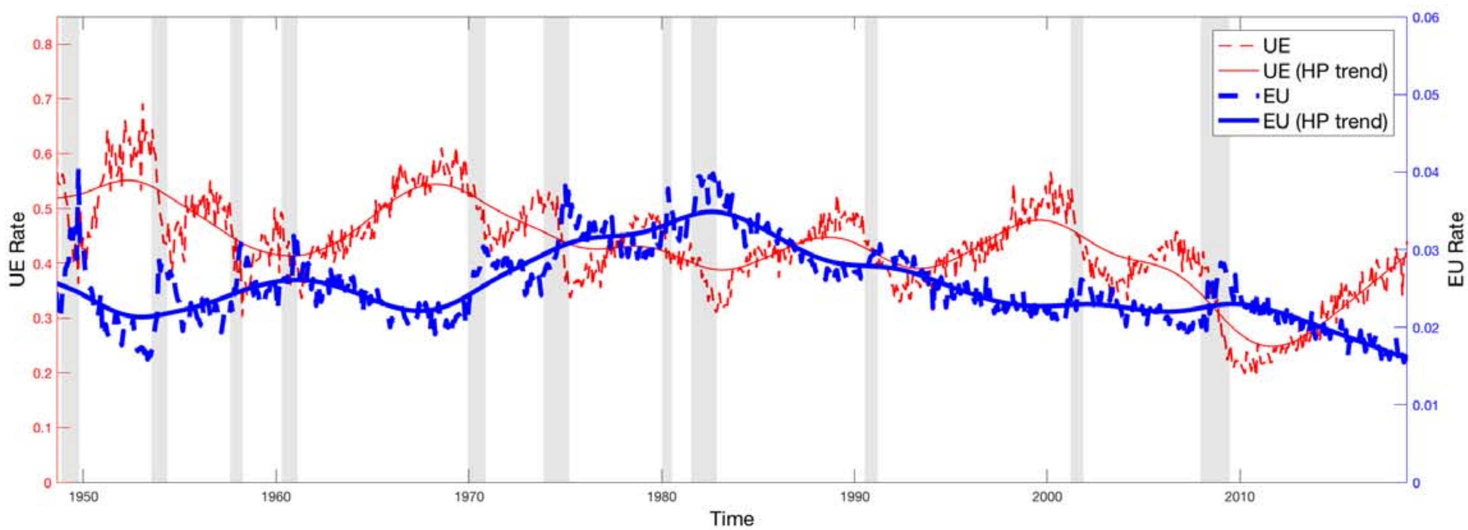
Vacancy Rate
Unemployment Rate



Unemployment Rate
Vacancy Rate
Time
u
u (HP trend)
v
v (HP trend)



UE Rate
EU Rate
Time
UE
UE (HP trend)
EU
EU (HP trend)

Introduction

Stylized long-run facts:

1. No secular movement in the Beveridge curve

$$u = \frac{\delta_t}{\delta_t + A_t p(v/u)}.$$

2. No secular movement along the Beveridge curve

$$u_t,\ v_t,\ v_t/u_t \text{ stationary.}$$

3. No secular movement in the UE and EU rates.

From the perspective of search theory, the above observations imply that the efficiency A_t of the search technology has not improved from 1926 to 2017. Telephone? Fax? Mobile phone? PC? Internet? All irrelevant!

Introduction

Modify the textbook search theory of u and v by introducing a distinction between meeting and matches.

- Identify conditions for a **Balanced Growth Path** in which Beveridge Curve, u, v, UE and EU rates are constant despite improving search technology.
- Under these conditions, improvements in search technology show up in labor productivity growth not u.
- Under the same conditions, returns to scale in search are not identifiable from u or UE data.
- Develop a strategy to measure improvements in search technology, returns to scale in search and their contribution to labor productivity growth.

Environment

Workers:

- *population*: measure 1;
- *endowment*: indivisible unit of labor;
- *objective*: max pv of income $\{b_t, w_t\}$ discounted at rate r.

Firms:

- *population*: positive measure;
- *technology*:

  maintain vacancies v_t at flow unit cost k_t;

  CRTS technology: 1 unit of labor –> $y_t z$ units of output;
- *objective*: max pv of income $y_t z - w_t$ discounted at rate r.

Environment

Labor market:

- u_t and v_t come together through a CRTS matching fn $A_t M(u_t, v_t)$;
- u meets v at rate $A_t p(\theta_t)$, where $\theta_t = v_t/u_t$ and $p(\theta_t) = M(1, \theta_t)$;
- v meets u at rate $A_t q(\theta_t)$, $q(\theta_t) = p(\theta_t)/\theta_t$;
- upon meeting, u and v observe match quality $\hat{z} \sim F(\hat{z})$ and decide whether to form the match;
- upon matching, u and v Nash bargain over the terms of trade.

Note: Technological improvement in the matching function needs to be Hicks-neutral for BGP (if not Hicks-neutral: u stationary and v trending).

Environment

Environment is non-stationary:

- declining search frictions: $A_t = A_0 \exp(g_A t)$
- general productivity growth: $y_t = y_0 \exp(g_y t)$
- increasing unemployment income: $b_t = b_0 \exp(g_b t)$
- increasing vacancy cost: $k_t = k_0 \exp(g_k t)$.

Balanced Growth Path

Initial State: a measure of unemployed workers u_0, and a distribution of employed workers across match qualities G_0

Rational Expectations Equilibrium: time-path for value and policy functions, tightness θ_t, unemployment u_t, employment G_t, UE and EU rate that satisfy optimality, market clearing and consistency conditions given (u_0, G_0).

Balanced Growth Path: Initial State and a Rational Expectations Equilibrium such that some variables are constant over time ($u,\ h_{UE,}\ h_{EU},\ \theta$) and others grow at a constant rate (G_t).

Definition of a BGP

Joint value of a match:

$$V_t(z) = \max_{d \geq 0} \left[\int_t^{t+d} e^{-r(\tau - t)} y_\tau z \cdot d\tau \right] + e^{-rd}[U_{t+d} + 0].$$

Optimal break-up time d^*:

$$y_{t+d} z + \dot{U}_{t+d} \leq rU_{t+d},\ d \geq 0 \text{ with c.s.}$$

Reservation match quality R_t:

$$y_t R_t = rU_t - \dot{U}_t. \tag{E1}$$

- An existing match is maintained at date t iff $z > R_t$.
- A new potential matched is created at date t iff $z > R_t$.

Definition of a BGP

Surplus of a match:

$$S_t(z) = V_t(z) - [U_t + 0]. \tag{E2}$$

Value of unemployment to the worker

$$rU_t = b_t + A_t p(\theta)\gamma \int_{R_t} S_t(\hat{z}) dF(\hat{z}) + \dot{U}_t. \tag{E3}$$

Value of a vacancy to the firm

$$0 = A_t q(\theta)(1 - \gamma) \int_{R_t} S_t(\hat{z}) dF(\hat{z}) - k_t. \tag{E4}$$

Definition of a BGP

Stationarity of UE, EU and unemployment rates:

$$\begin{aligned} A_t p(\theta)(1 - F(R_t)) &= h_{UE}, \\ G_t'(R_t)\dot{R}_t &= h_{EU}, \\ (1 - u)h_{EU} &= uh_{UE}. \end{aligned} \tag{E5}$$

Definition of a BGP

Balanced growth of the distribution G_t of employed workers across matches of different quality:

The x-th quantile $z_t(x)$ of G_t grows at some constant rate g_z, i.e.

$$z_t(x) = z_0(x)e^{g_z t}.$$

The balanced growth condition for G_t is satisfied iff

$$(1-u)G_t'(z_t(x))z_t(x)g_z + uA_t p(\theta)[F(z_t(x)) - F(R_t)] = (1-u)G_t'(R_t)R_t g_z. \tag{E6}$$

Definition of a BGP

A BGP is a list $\{R_t, U_t, S_t\}$, $\{\theta, h_{UE}, h_{EU}, u, G_t, g_z\}$ such that for all $t \geq 0$:

i. R_t, U_t and S_t satisfy (E1), (E2) and (E3);

ii. θ satisfies (E4);

iii. $h_{UE,}$ h_{EU} and u satisfy (E5);

iv. G_t satisfies (E6) for some g_z.

Exogenous: A_0, y_0, b_0, k_0, g_A, g_y, g_b, g_k, F.

Endogenous: R_t, U_t, S_t, θ, h_{UE}, h_{EU}, u, G_t, g_z.

Necessary Conditions for a BGP

The stationarity condition for the UE rate is:

$$A_t p(\theta)(1 - F(R_t)) = h_{UE},\ \forall t \geq 0.$$

Differentiating with respect to t, we obtain

$$\dot{A}_t p(\theta)(1 - F(R_t)) - A_t p(\theta) F'(R_t)\dot{R}_t = 0, \forall t \geq 0.$$

$$\Leftrightarrow g_A(1 - F(R_t)) = F'(R_t) R_t g_z$$

The differential equation for F has the unique solution

$$F(z) = 1 - \left(\frac{z_\ell}{z}\right)^\alpha.$$

N1 A BGP may exist only if F is Pareto with coefficient α.

Necessary Conditions for a BGP

Combining the equilibrium conditions for R_t, U_t and θ, we obtain:

$$y_t R_t = b_t + \frac{p(\theta)}{q(\theta)} \frac{\gamma}{1-\gamma} k_t, \ \forall t \geq 0.$$

The above expression can only be satisfied if

$$g_b, g_k = g_y + g_z$$

N2 A BGP may exist only if g_b and g_k are equal to $g_y + g_z$.

Existence and Uniqueness of a BGP

The condition for the surplus can be written as:

$$S_t(z) = \int_t^{t+d} e^{-r(\tau-t)}[y_\tau z - y_\tau R_\tau]d\tau.$$

Solving the integral gives:

$$S_t(z) = y_t\left\{\frac{z}{r-g_y}\left[1-\left(\frac{R_t}{z}\right)^{\frac{r-g_y}{g_z}}\right] - \frac{R_t}{r-g_y-g_z}\left[1-\left(\frac{R_t}{z}\right)^{\frac{r-g_y-g_z}{g_z}}\right]\right\}.$$

Using the expression above and the fact that F is Pareto, we can solve for the expected surplus of a meeting between a firm and a worker

$$\int_{R_t} S_t(z)dF(z) = \Phi y_t R_t^{-(\alpha-1)}.$$

Existence and Uniqueness of a BGP

The condition for the reservation quality can be written as

$$y_t R_t = b_t + A_t p(\theta)\gamma \underbrace{\int_{R_t} S_t(z) dF(z)}_{\Phi y_t R_t^{-(\alpha-1)}}$$

The R_t that solves the condition exists and is such that

$$R_0 = b_0/y_0 + A_0 p(\theta)\gamma\Phi R_0^{-(\alpha-1)}. \tag{C1}$$

and

$$R_t = R_0 \exp(g_z t), \text{ with } g_z = g_A/\alpha.$$

Existence and Uniqueness of a BGP

The condition for the tightness of the market can be written as

$$k_t = A_t q(\theta)(1-\gamma) \underbrace{\int_{R_t} S_t(z)dF(z)}_{\Phi y_t R_t^{-(\alpha-1)}}$$

The θ that solves the condition for $t = 0$ is such that

$$k_0 = A_0 q(\theta)(1-\gamma)\Phi y_0 R_0^{-(\alpha-1)}, \tag{C2}$$

This θ also solves the condition for $t > 0$ since

$$\begin{aligned} g_k &= g_y + g_z \\ &= g_y + g_A/\alpha \\ &= g_y + g_A - (\alpha-1)g_A/\alpha. \end{aligned}$$

Existence and Uniqueness of a BGP

For $t = 0$, the condition for the balanced growth of G_t can be written as

$$(1-u)G_0'(z)zg_z = uA_0 p(\theta)[1-F(z)]$$

The solution for G that satisfies $G_0(R_0) = 0$, $G_0(\infty) = 1$ is

$$G_0(z) = 1 - \left(\frac{R_0}{z}\right)^{\alpha}.$$

The unemployment rate is

$$u = \frac{g_A}{g_A + A_0 p(\theta)[1-F(R_0)]}.$$

Existence and Uniqueness of a BGP

For $t \geq 0$, the condition is

$$(1-u)G_t'(ze^{g_z t})ze^{g_z t}g_z = uA_t p(\theta)[1-F(ze^{g_z t})].$$

Since $G_t(ze^{g_z t}) = G_0(z)$ and $G_t'(ze^{g_z t}) = G_0'(z)e^{-g_a t}$, we have

$$(1-u)G_0'(z)zg_z = uA_0 e^{g_A t}p(\theta)[1-F(z)]e^{-\alpha g_z t}.$$

The condition clearly holds because the LHS equals the RHS at $t = 0$ and they both grow at the same rate.

Existence and Uniqueness of a BGP

Finally, we check the stationarity of UE, EU and unemployment rates:

The UE rate

$$h_{UE} = A_t p(\theta)[1 - F(R_t)] = A_0 p(\theta)[1 - F(R_0)].$$

The EU rate

$$h_{EU} = G_t'(R_t) R_t g_z = g_A.$$

The unemployment rate

$$u = \frac{g_A}{g_A + A_0 p(\theta)[1 - F(R_0)]} = \frac{h_{EU}}{h_{UE} + h_{EU}}.$$

Existence and Uniqueness of a BGP

Given the necessary conditions (N1) and (N2), a BGP exists iff there is a solution to the system of date 0 conditions

$$R_0 = b_0/y_0 + A_0 p(\theta)\gamma\Phi R_0^{-(\alpha-1)}. \tag{C1}$$

$$k_0 = A_0 q(\theta)(1-\gamma)\Phi y_0 R_0^{-(\alpha-1)}, \tag{C2}$$

Simple algebra shows that there $\exists!$ (R_0, θ) that solves (C1)-(C2).

Existence and Uniqueness of a BGP

Proposition 1: Take arbitrary growth rates $g_y > 0$ and $g_A > 0$.

A BGP exists if and only if:

i. F is Pareto with coefficient α;

ii. g_b and g_k are $g_y + g_A/\alpha$;

iii. $\alpha > 1$ and $r > g_y + g_A/\alpha$.

If a BGP exists it is unique and such that:

i. u, h_{UE}, h_{EU}, θ are constant over time;

ii. G is a Pareto truncated at R_t and grows at the rate $g_z = g_A/\alpha$;

iii. labor productivity, output per capita grow at the rate $g_y + g_A/\alpha$.

Existence and Uniqueness of a BGP

Comments to Proposition 1:

1. Improvements in search technology increase the rate at which a worker meets a firm and, by increasing the reservation quality, lowers the probability that the firm and the worker choose to match rather than keep searching. When the match quality distribution is Pareto, the two effect offset each other and the UE rate remains constant.

Existence and Uniqueness of a BGP

Comments to Proposition 1:

1. Improvements in search technology increase the rate at which a worker meets a firm and, by increasing the reservation quality, lowers the probability that the firm and the worker choose to match rather than keep searching. When the match quality distribution is Pareto, the two effect offset each other and the UE rate remains constant.

2. Improvements in search technology do not affect the trend of unemployment, but they contribute to the growth of labor productivity:

$$\int_{R_t} y_t z \frac{F'(z)}{1 - F(R_t)} dz = \frac{\alpha}{\alpha - 1} y_t R_t .$$

The contribution depends on the tail coefficient α of the Pareto.

Existence and Uniqueness of a BGP

Comments to Proposition 1:

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2. Improvements in search technology do not affect the trend of unemployment, but they contribute to the growth of labor productivity.

3. Either you believe that there has been no secular improvement in search technology, or the conditions for a BGP are satisfied.

Existence and Uniqueness of a BGP

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1. Improvements in search technology increase the rate at which a worker meets a firm and, by increasing the reservation quality, lowers the probability that the firm and the worker choose to match rather than keep searching. When the match quality distribution is Pareto, the two effect offset each other and the UE rate remains constant.

2. Improvements in search technology do not affect the trend of unemployment, but they contribute to the growth of labor productivity.

3. Either you believe that there has been no secular improvement in search technology, or the conditions for a BGP are satisfied.

4. The conditions on g_k and g_b are not very restrictive. Satisfied as long as UB is proportional to output per capita and workers are needed to hire other workers.

Search on the Job

The baseline model assumes that workers only search when unemployed.

- assumption keeps the analysis simple
- assumption flies in the face of the observation that half of hires are poached directly from another firm.

We consider a more general environment, in which workers search for jobs both when unemployed and when employed. We let $\rho \in [0,1)$ denote the relative search intensity of employed workers.

Search on the Job

Joint value of a match:

$$V_t(z) = \max_{d \geq 0} \int_t^{t+d} e^{-r(\tau - t)} \mu_{\tau - t} \left[y_\tau z + A_\tau p(\theta) \rho \gamma \int_z (V_\tau(\hat{z}) - V_\tau(z)) dF(\hat{z}) \right] dx$$

$$+ e^{-rd} \mu_d U_{t+d}$$

where

$$\mu_x = \exp\left(\int_0^x -A_{t+s} p(\theta) \rho (1 - F(z)) ds \right).$$

Reservation match quality R_t:

$$y_t R_t + A_t p(\theta) \rho \gamma \int_{R_t} (V_t(\hat{z}) - V_t(R_t)) dF(\hat{z}) = rU_t - \dot{U}_t. \tag{E1}$$

Search on the Job

Surplus of a match:

$$S_t(z) = V_t(z) - U_t. \tag{E2}$$

Value of unemployment

$$rU_t = b_t + A_t p(\theta)\gamma \int_{R_t} S_t(\hat{z})dF(\hat{z}) + \dot{U}_t. \tag{E3}$$

Value of a vacancy

$$\begin{aligned} k_t = \; & A_t q(\theta)\frac{u}{u+\rho(1-u)}(1-\gamma)\int_{R_t} S_t(\hat{z})dF(\hat{z}) \\ & A_t q(\theta)\frac{\rho(1-u)}{u+\rho(1-u)}(1-\gamma)\int_{R_t}\left[\int_z (S_t(\hat{z}) - S_t(z))dF(\hat{z})\right]dG_t(z). \end{aligned} \tag{E4}$$

Search on the Job

Stationarity of UE, EU and unemployment rates:

$$\begin{aligned} A_t p(\theta)(1 - F(R_t)) &= h_{UE}, \\ G_t'(R_t)\dot{R}_t &= h_{EU}, \\ u h_{UE} &= (1 - u) h_{EU} \end{aligned} \tag{E5}$$

The balanced growth condition for G_t is satisfied iff

$$\begin{aligned} & (1 - u)[G_t(z_t(x)e^{g_z dt}) - G_t(z_t(x))] + uA_t p(\theta)[F(z_t(x)e^{g_z dt}) - F(R_t e^{g_z dt})]dt \\ = \quad & (1 - u)[G_t(R_t e^{g_z dt}) - G_t(R_t)] + (1 - u)\rho A_t p(\theta)[1 - F(z_t(x))]G_t(z_t(x)). \end{aligned} \tag{E6}$$

Search on the Job

The condition for the surplus can be written as:

$$rS_t(z) = y_t(z - R_t) - A_t p(\theta)\rho\gamma\left[S_t(z)(1 - F(z)) + \int_{R_t}^{z} S_t(\hat{z})dF(\hat{z}) \right] + \mathring{S}_t(z).$$

Solving for $S_t(z)$ seems hopeless....

Search on the Job

We guess and we verify that

$$S_t(ze^{g_z t}) = S_0(z)e^{(g_y+g_z)t}.$$

This allows us to rewrite the condition for $S_t(z)$ as:

$$rS_t(z) = y_t(z - R_t) - A_t p(\theta)\rho\gamma\left[S_t(z)(1 - F(z)) + \int_{R_t}^{z} S_t(\hat{z})dF(\hat{z}) \right]$$

$$+ (g_y + g_z)S_t(z) - zg_z S_t'(z).$$

Evaluating at $t = 0$ and differentiating with respect to z, we obtain:

$$rS_0'(z) = y_0 + S_0'(z)[g_y - A_0 p(\theta)\rho\gamma(1 - F(z))] - zg_z S_0''(z).$$

We have now an ODE for $S_0'(z)$!

Search on the Job

The solution of the ODE for S' that satisfies smooth-pasting is

$$S_0'(z) = \frac{y_0}{g_z} \int_{R_0}^{z} \frac{1}{s} \exp\left\{ -\frac{1}{g_z} \left[\frac{\sigma}{\alpha}(F(z) - F(s)) + (r - g_y) \log(z/s) \right] \right\} ds.$$

The solution for the surplus S_0 that satisfies value matching is

$$S_0(z) = \int_{R_0}^{z} S_0'(x) dx.$$

Then, we show that:

- the expected surplus of a meeting between a firm and an unemployed worker grows at the constant rate $g_y - (\alpha - 1) g_z$;
- the expected surplus of a meeting between a firm and an employed worker grows at the constant rate $g_y - (\alpha - 1) g_z$.

Search on the Job

Proposition 2: Take arbitrary growth rates $g_y > 0$ and $g_A > 0$.

A BGP exists if and only if:

i. F is Pareto with coefficient α;

ii. g_b and g_k are $g_y + g_A/\alpha$;

iii. $\alpha > 1$ and $r > g_y + g_A/\alpha$.

Any BGP is such that:

i. u, h_{UE}, h_{EU}, θ are constant over time;

ii. G_t is a Fréchet truncated at R_t and grows at the rate $g_z = g_A/\alpha$;

iii. labor productivity, output per capita grow at the rate $g_y + g_A/\alpha$.

Population and Returns to Scale

Baseline model assumes constant population. Assumption is w.l.o.g. as long as the matching function has constant returns to scale.

We consider a more general environment in which the population might grow and the matching function may have non-constant returns to scale.

- Population:

$$N_t = N_0 \exp(g_N t);$$

- Matching function:

$$A_t N_t^{\beta} M(W_t, V_t) = A_t N_t^{\beta} \left[N_t M\left(\frac{W_t}{N_t}, \frac{V_t}{N_t} \right) \right].$$

* The overall efficiency of the search process is

$$\hat{A}_t = N_t^{\beta} A_t = \hat{A}_0 e^{(\beta g_N + g_A)t}.$$

Population and Returns to Scale

Proposition 3: Take arbitrary growth rates $g_y > 0$, $g_A > 0$, $g_N > 0$ such that the overall search efficiency improves over time, i.e. $g_A + \beta g_N > 0$.

A BGP exists if and only if:

i. F is Pareto with coefficient α;

ii. g_b and g_k are $g_y + (g_A + \beta g_N)/\alpha$;

iii. $\alpha > 1$ and $r > g_y + (g_A + \beta g_N)/\alpha$.

Any BGP is such that:

i. u, h_{UE}, h_{EU}, θ are constant over time;

ii. G_t is approximately a Fréchet truncated at R_t and grows at the rate $g_z = (g_A + \beta g_N)/\alpha$;

iii. labor productivity, output per capita grow at the rate $g_y + (g_A + \beta g_N)/\alpha$.

Population and Returns to Scale

Comments to Proposition 3:

Under the same conditions for which u, h_{UE}, h_{EU}, θ remain constant irrespective of rate of improvement in search technology:

1. u, h_{UE}, h_{EU} and θ remain constant as population grows irrespective of the returns to scale in the search process;

2. u, h_{UE}, h_{EU} and θ are independent of the size of different labor markets irrespective of the returns to scale in the search process.

Back-of-the-Envelope Calculations

The theory raises some quantitative questions:

1. Cannot infer growth of search technology from time trends of u, h_{UE}, h_{EU} and θ.
    a. How to measure improvements of search technology?
    b. How to measure the contribution of search technology to productivity growth?

2. Cannot infer returns to scale in search process from time-series or cross-sections of u, h_{UE}, h_{EU}, θ and population
    a. How to measure returns to scale in search process?
    b. How to measure the contribution of returns to scale in search to productivity growth and to differences in productivity between large and small markets?

Back-of-the-Envelope Calculations

Average number of applications per vacancy are

$$A_t N_t^{\beta} q(\theta_t).$$

- In the model, applications per vacancy grow at rate $\beta g_N + g_A$.
- In the data, applications per vacancy were 24 in 1981 (EOPP) and 45 in 2010 (Career Builder, SnagAJob).
- These observations suggest

$$\beta g_N + g_A = 2.2\%$$

Back-of-the-Envelope Calculations

Relative number of applications per vacancy in two markets of sizes N_1 and N_2 with the same search technology is

$$\frac{A_{1,t}N_{1,t}^{\beta}q(\theta_{1,t})}{A_{2,t}N_{2,t}^{\beta}q(\theta_{2,t})} = \left(\frac{N_{1,t}}{N_{2,t}}\right)^{\beta}.$$

- In the model, elasticity of applications per vacancy wrt size is β.
- In the data, elasticity of applications per vacancy wrt size is 0.52.
- These observations suggest $\beta = 0.52$ and

$$\beta g_N = 0.52 \cdot 1.1\% = 0.6\%$$

$$g_A = \underbrace{g_A + \beta g_N}_{2.2\%} - \underbrace{\beta g_N}_{0.6\%} = 1.6\%$$

Back-of-the-Envelope Calculations

Wage distribution L_t for workers hired from unemployment is s.t.

$$\lim_{w \to \infty} \frac{d \log[1 - L_t(w)]}{d \log w} = -\alpha.$$

- In the model, the right tail of L_t is Pareto with coefficient α.
- In the data, hard to measure $L_t(w)$, but let us suppose $\alpha \in [4, 16]$.

Back-of-the-Envelope Calculations

Contribution of Declining Search Frictions

	Pareto coefficient		
1981-2010	$\alpha = 4$	$\alpha = 8$	$\alpha = 16$
labor productivity growth		1.9%	
cont. of search technology	0.4%	0.2%	0.1%
cont. of IRS in search	0.15%	0.07%	0.04%
cont. of declining search frictions	0.55%	0.27%	0.14%

Back-of-the-Envelope Calculations

Returns to Scale and Productivity across Cities

	$\alpha = 4$	$\alpha = 8$	$\alpha = 16$
0.5 million workers	0.91	0.95	0.98
1 million workers	1	1	1
10 million workers	1.34	1.16	1.08

LEFTOVERS

Search on the Job: Existence of a BGP

The condition for the reservation quality can be written as

$$y_t R_t = b_t + A_t p(\theta)\gamma \underbrace{\int_{R_t} S_t(\hat{z})dF(\hat{z})}_{\left[\int_{R_0}^{z} S_0(\hat{z})dF(\hat{z})\right] e^{(g_y-(\alpha-1)g_z)t}}$$

The R_t that solves the condition exists and is such that

$$y_0 R_0 = b_0 + A_0 p(\theta)\gamma \int_{R_0}^{z} S_0(\hat{z})dF(\hat{z}). \tag{C1}$$

and

$$R_t = R_0 \exp(g_z t), \text{ with } g_z = g_A/\alpha.$$

Search on the Job: Existence of a BGP

The condition for the tightness of the market can be written as

$$\begin{aligned} k_t \;=\; & A_t q(\theta)\frac{u}{u+\rho(1-u)}(1-\gamma)\int_{R_t} S_t(\hat{z})dF(\hat{z}) \\ & A_t q(\theta)\frac{\rho(1-u)}{u+\rho(1-u)}(1-\gamma)\int_{R_t}\left[\int_z (S_t(\hat{z})-S_t(z))dF(\hat{z})\right]dG_t(z). \end{aligned}$$

The θ that solves the condition for $t = 0$ is such that

$$\begin{aligned} k_0 \;=\; & A_0 q(\theta)\frac{u}{u+\rho(1-u)}(1-\gamma)\int_{R_0} S_0(\hat{z})dF(\hat{z}) \\ & A_0 q(\theta)\frac{\rho(1-u)}{u+\rho(1-u)}(1-\gamma)\int_{R_0}\left[\int_z (S_0(\hat{z})-S_0(z))dF(\hat{z})\right]dG_0(z). \end{aligned} \tag{C2}$$

Search on the Job: Existence of a BGP

The condition for the tightness of the market can be written as

$$\begin{aligned} k_t = \; & A_t q(\theta) \frac{u}{u + \rho(1-u)} (1-\gamma) \int_{R_t} S_t(\hat{z}) dF(\hat{z}) \\ & A_t q(\theta) \frac{\rho(1-u)}{u + \rho(1-u)} (1-\gamma) \int_{R_t} \left[\int_z (S_t(\hat{z}) - S_t(z)) dF(\hat{z}) \right] dG_t(z). \end{aligned}$$

The same θ also solves the condition for $t > 0$ since

$$\begin{aligned} g_k &= g_y + g_z \\ &= g_y + g_A/\alpha \\ &= g_y + g_A - (\alpha - 1) g_A/\alpha. \end{aligned}$$

Search on the Job: Existence of a BGP

For $t = 0$, the condition for balanced growth of G_t is

$$(1-u)G_0'(z)zg_z = uA_0p(\theta)[1-F(z)] + (1-u)A_0p(\theta)\rho[1-F(z)]G_0(z).$$

The solution to the PDE satisfying $G_0(R_0) = 0,\ G_0(\infty) = 1$ is

$$G_0(z) = \frac{e^{-A_0p(\theta)\rho(1-F(z))/g_A} - e^{-A_0p(\theta)\rho(1-F(R_0))/g_A}}{1-e^{-A_0p(\theta)\rho(1-F(R_0))/g_A}}.$$

The unemployment rate is

$$u = \frac{g_A}{g_A + A_0p(\theta)[1-F(z)]}.$$

Search on the Job: Existence of a BGP

For $t \geq 0$, the condition is

$$\begin{aligned}(1-u)G_t'(ze^{g_z t})ze^{g_z t}g_z &= uA_t p(\theta)[F(ze^{g_z t}) - F(R_0 e^{g_z t})] \\ &+(1-u)\rho A_t p(\theta)[1-F(ze^{g_z t})]G_t(ze^{g_z t}).\end{aligned}$$

Since $G_t(ze^{g_z t}) = G_0(z)$ and $G_t'(ze^{g_z t}) = G_0'(z)e^{-g_a t}$, we can rewrite it as

$$\begin{aligned}(1-u)G_0'(z)zg_z &= uA_0 e^{g_A t}p(\theta)[F(z) - F(R_0)]e^{-\alpha g_z t} \\ &+(1-u)\rho A_0 e^{g_A t}p(\theta)[1-F(z)]G_0(z)e^{-\alpha g_z t}.\end{aligned}$$

The condition clearly holds, as the LHS and RHS are equal at $t = 0$ and grow at the same rate.

Search on the Job: Existence of a BGP

Given the necessary conditions (N1) and (N2), a BGP exists iff there is a solution to the system of date 0 conditions

$$y_0 R_0 = b_0 + A_0 p(\theta)\gamma \int_{R_0}^{z} S_0(\hat{z})dF(\hat{z}). \tag{C1}$$

$$\begin{aligned} k_0 = \; & A_0 q(\theta)\frac{u}{u+\rho(1-u)}(1-\gamma)\int_{R_0} S_0(\hat{z})dF(\hat{z}) \\ & A_0 q(\theta)\frac{\rho(1-u)}{u+\rho(1-u)}(1-\gamma)\int_{R_0}\left[\int_{z}(S_0(\hat{z})-S_0(z))dF(\hat{z})\right]dG_0(z). \end{aligned} \tag{C2}$$

Some algebra shows that there $\exists$ (R_0, θ) that solves (C1)-(C2).