Market-Beta and Downside Risk

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Abstract

The plain market-beta was a good predictor of stock returns not only during bull and ordinary markets, but also during bear markets and crashes. Thus, it was indeed a good measure of the hedge against market risk. This plain beta also predicted the subsequent down-beta (i.e., measured only on days when the stock market declined) better than the prevailing down-beta. Stocks with higher ex-ante down-betas did not earn a positive risk premium. We conclude that ex-ante down-betas were neither useful hedging nor useful risk-pricing measures.

Keywords: Market-Beta, Crash Risk. Down-Beta

JEL Codes: G11, G12.

As early as the 1950s, Markowitz (1959) recognized that market beta could be a poor hedge measure exactly when it is needed the most: in subsequent bear markets and crashes. In turn, such behavior could explain why ex-ante market betas did not associate positively with ex-post average returns, shown effectively in Frazzini and Pedersen (2014).

Markowitz thus proposed an alternative theory based on asymmetric market risk. The relevant risk measure was a "down-beta," i.e., a market-beta computed only from days on which the stock market declined. However, early empirical tests like those in Jahankhani (1976), Bawa and Lindenberg (1977), Menezes, Geiss, and Tressler (1980), and Harlow and Rao (1989), suffered from low power, because they formed portfolios that reduced down-beta variation, and because they calculated betas with monthly rates of return.

Ang, Chen, and Xing (2006) (ACX) reinvigorated the asymmetric-beta hypothesis with the first powerful hypothesis-tailored test, using individual stocks' down-betas calculated from daily frequency returns. They showed that stocks with higher down-betas also had higher average rates of return, albeit largely using ex-post returns to estimate down-betas (Section 2). Recognizing that the ex-post nature limited the usefulness of their betas, they then showed that current down-betas could also predict future down-betas, albeit without considering plain betas (Section 3.1). Finally, after discarding ex-ante high-volatility stocks, they showed that ex-ante down-betas could even predict some future stock return averages, albeit with few controls (Section 3.2 and 3.3).

With over 800 cites in Google Scholar and over 200 cites in Web of Science as of 2018, ACX have sparked an active and still growing body of research literature.[1] The usefulness of down-beta has again become an influential hypothesis, with Ang, Chen, and Xing (2006) as the seminal paper.

[1]For example, Bali, Demirtas, and Levy (2009) provided further evidence on a VaR and tail-risk measure. Harvey and Siddique (2000) explore coskewness, shown by ACX to be unrelated to down-beta. Huang et al. (2012) looked at extreme downside risk. The literature on other moments, quasi-moments, tail risk, jump risk, and the asymmetric return literature on other assets or funds is too broad to discuss in detail.

Our paper revisits and contrasts the hedging performance and risk premiums using various market-beta estimators.

It begins by showing that plain betas—i.e., the "all-days"[2] betas computed using all days regardless of market performance—can predict subsequent market-conditional performance quite well also in bear, extreme bear, and crashing market conditions. In bad times, low-beta stocks outperformed high-beta stocks, just as predicted. This is good news for the usefulness of plain beta as a measure of the hedge against market crashes.

Nevertheless, because stocks tend to come in a limited range of betas, the practical usefulness of hedging only with low-beta stocks remains limited. Only about 10-20% of the stock market capitalization (depending on the year) is in stocks with beta estimates of 0.5 or less, and almost none is in stocks with negative market betas. Consequently, a portfolio restricted only to long stocks is too limited in its ability to deliver a diversified market-neutral stock portfolio. It requires the use of shorting and/or other financial instruments to construct one.

Our paper then shifts to exploring the differences between inferences with ex-ante and ex-post risk betas. This requires some background discussion.

Most equilibrium asset-pricing theories are based on *known* risk metrics, with causal implications for asset prices (expected returns). In both real-world practice and empirical tests, the risk measures are not truly known but have to be assessed. Thus, the econometrician's measure can be different from the (representative) investors' measures.[3]

It is unclear whether the econometrician or the representative investor can better assess the ex-ante priced risk measure better. On the one hand, an econometrician's ex-ante best risk measure may be worse than the investors' measures, because the econometrician may

[2]"Unconditional beta" has sometimes been used to refer to a beta calculated over the entire sample, including future periods, and sometimes used to refer to non-instrumented betas. Our phrases "all-days beta" (and sometimes "plain beta") are meant to convey the "prevailing historical-only market-beta not measured only on up- or down-market days but on all days."

[3]Hansen and Richard (1987) discuss the theory of different information sets in the context of dynamic asset-pricing models. Unconditional tests may not be suitable to testing conditional optimizations.

not know as much as investors in the aggregrate. This is especially acute in models with strong investor-specific aspects (such as utility parameters, consumption, employment, or wealth). On the other hand, the investors' ex-ante risk measure may be worse than the econometrician's, because forecasting is a specialized skill and the aggregated investor construct may contain many less sophisticated investors.

If econometricians believe their estimates are worse than investors', an important related question is then whether the econometrician can use the observable *ex-post* risk measures as a stand-in to proxy for the investors' better ex-ante risk measures. If the ex-post measure is a short-term realized metric, rather than a very long-average metric, it further raises the concern that even if investors know the true risk measure, the (econometrician's) ex-post measure would be a mix of this true measure plus an error realization. For the case of market betas, the source of noise are realized stock returns. These are very noisy and difficult to predict. They are a potentially large error component for the beta estimates for individual stocks. Again, this is true even if investors knew the correct beta for each stock. Furthermore, when predicting future stock returns with betas in a Fama-Macbeth regression, the coefficient measured on the ex-post beta is the risk-premium. The measured risk premium would be a downward-biased estimate of the coefficient on the investors' true expected risk premium.

The standard practice in cross-sectional asset pricing model tests has been to avoid ex-post risk measures, primarily for fear that false hypotheses are accepted. Instead, at least since Fama and MacBeth (1973), the standard practice has been to conduct tests with ex-ante measures (and p-levels between 0.1% and 10%), yet to remain cognizant of the dangers of rejecting true hypotheses. If anything, modern test standards have become even more concerned with false positives (Harvey, Liu, and Zhu (2016)).

Avoiding all ex-post data has not been a hard rule, however. For example, the use of full-sample lead-lag regression to create instruments has a history in some financial economics contexts—even though such measures are known to have some look-ahead bias (Goyal and Welch (2008)).

A thought experiment can clarify how ex-post metrics could lead to absurd inferences. Stocks of companies that are taken over show average rates of return of 20-30%. This is not evidence that investors require higher average rates of return because of the danger that their stocks would be taken over. Instead, it is difficult to predict which stocks will be taken over and concomitant positive rates of return should be viewed as positive surprise realizations and not as (average) risk premia.

In our context, a more relevant illustration are Fama-Macbeth tests predicting future stock returns with plain all-days betas. In the ACX sample (equivalent to those in our Table 6 below, albeit without controls), the results are:

Predicting Stock Returns with	FM Gamma	(T-stat)
... Ex-Ante Market Betas	–0.3%/year	(–0.22)
... Contemporaneous Market Betas	+8.4%/year	(+3.84)

The first regression is the familiar plain beta puzzle result in Frazzini and Pedersen (2014). The ex-ante market-beta has not been positively associated with (future) average returns. The second regression is the equivalent of the test in Ang, Chen, and Xing (2006). The contemporaneous market-beta is positively associated with stock returns. Note how even the much more prominent plain beta puzzle (the lack of a strong positive association between beta and returns) in Frazzini-Pedersen disappears. The literature has settled on the Frazzini-Pedersen approach.

This is not to assert that the ACX ex-post beta measure is worse. It could even be correct if investors indeed knew both the true measure and the realization error. This is a strong assumption, however, and there is no empirical evidence that investors knew both. (Of course, there is also no evidence that they did not—investors' information sets are unobservable and perhaps even meaningless and unmeasurable for aggregate representative agent constructs.)

Our paper points out how the inference changes when we replace the ex-post beta measure with an ex-ante beta measure. This places the ACX evidence in perspective relative to other papers' ex-ante measures in the cross-sectional asset-pricing literature. With the ex-ante beta, our paper leaves the market-risk compensation puzzle in the state in which Frazzini and Pedersen (2014) left it: The all-days market-beta seems to be a good predictive measure of hedging contribution (Section II), yet it does not seem to earn investors higher average rates of return (Section IV).

Much of our paper's empirical reexamination is about the ex-ante versus ex-post properties not only of the plain beta, but also of the down-beta, the focus in ACX. Our paper shows that the all-days beta is a better predictor of the future down-beta than its own historical down-beta counterpart. This is not because the typical difference between measured up-beta and down-beta for the same stock is small. Indeed, the absolute up- minus down-beta spread is about 0.5 for an average stock in any given year. This is similar in magnitude to the cross-sectional heterogeneity in the all-days beta (s.d. of 0.7) itself.

Instead and unfortunately, even a large measured beta spread in one year can only predict a small measured beta spread in the following year. The auto-coefficient is about 0.1, meaning that an 0.5 spread between a stock's up- minus down-beta translates only into an 0.05 predicted spread in the following year. Although there is a link between historical and current spreads between up-beta and down-beta, this link is very weak.

The primary reason for the large typically observed realized spread in up- versus down-betas and the reason for the stark mean-reversion seems to be measurement (realization) error. The large *realized* differences in the spread are mostly spurious (or perhaps temporary) to begin with.

Thus, there are good reasons why the all-days beta performs so well predicting down-beta, while the down-beta itself does not. The up- and down-beta estimates are best pooled. Econometricians and investors can estimate all-days betas with about twice as many days and on a wider range of x-variables (the overall market rate of return in the market-model regression) than they can estimate down-betas. With much of the down-beta/up-beta being merely realization noise, more historical data is more useful than the isolation of the small persistent stock-specific differences between down-betas and all-days beta. If investors are not perfectly informed but still have to estimate down-beta (just like the econometrician), though perhaps with more precision, their down-beta estimator would still have to overcome this observational handicap. If it does not, investors, too, would rely on and price the all-days beta and not the down-days beta, even if all they cared about was down-beta.

Our paper can also show that some of the forecasting decline seems not to be due to measurement error, but due to to mean reversion in the true underlying up- minus down-beta spread itself. This further raises the challenge not only of predicting beta for the longer term, but of estimating the current spread from historical (and thus always already somewhat outdated) stock-return data. At the current state-of-the-art, predicting beta *much* better than our paper seems like a difficult or perhaps outright impossible challenge.[4]

Our paper then continues with an empirical investigation of how ex-ante and ex-post betas are associated with subsequent rates of return.

Fama and MacBeth (1973) were extensively concerned with error in beta estimates. Theoretically, any measurement error in betas results in attenuated market risk premia. The 8.4%/annum implied market risk premium on the ex-post all-days beta shown above must raise warning flags. Given that we know that the measurement error is high, the investors' implied true market risk premium has to be many times this 8.4%. A premium that is too high should reject the theory just like one that is too low.

More importantly, we find analogous empirical evidence for down-betas as for plain betas: The association between down-betas and stock returns flips from positive with ex-post betas (as in Ang, Chen, and Xing (2006)) to insignificant or negative with ex-ante betas. This is the case both in Fama and MacBeth (1973) and Fama and French (1993) style tests. The coefficient turns significantly negative if we include the years after Ang,

[4] Even the best estimators of beta, such as those in Ait-Sahalia, Kalnina, and Xiu (2014) and Bollerslev, Li, and Todorov (2016) improve just slightly over the much easier-to-use Levi and Welch (2017) beta estimator, which takes the true beta mean reversion into account.

Chen, and Xing (2006) was published. Today, the best inference is that stocks with high down-betas had starkly lower subsequent stock returns.

Finally, our paper turns to the evidence in Lettau, Maggiori, and Weber (2014) (LMW), which investigates the performance of down-beta across different asset classes. Unlike ACX, LMW do not use the realized ex-post down-beta, but full-sample down-betas as their proxy. Full-sample betas are also not ex-post data clean, but they lean less heavily on the assumption of perfect foreknowledge and on the identification of the *realized* measure as the investors' expectation. If the model is stable, full-sample betas can be the best beta estimates from an econometrician's perspective if the model is true. In contrast to our inference that the ACX positive results unambiguously disappear when ex-ante betas are used, our inference about the results in LMW remains ambiguous. Some of their associations vanish when we use ex-ante down-betas, while others remain. The most intriguing associations include sovereign bonds. Yet their time-series, originally from Borri and Verdelhan (2011), is short (1995 to 2011). It will be interesting to revisit the LMW evidence in a few years when more data will be available.

Our paper now proceeds as follows. Section I discusses the data and measures used in our paper, principally market betas. Like ACX and Unlike preceding literature, our own paper uses superior measures of market-beta. Unlike ACX, our paper focuses primarily on ex-ante and not ex-post measures of beta. Section II discusses the performance of ex-ante high-beta and low-beta stocks in bear markets. In particular, it also looks at the most extreme episodes: stock market crashes. Section III investigates all-days betas, up-betas and down-betas as predictors of their future counterparts. Section IV shows that, while ex-post down-betas associate positively with stock returns, ex-ante down-betas associate negatively. Section V shows that our key results are robust with respect to the use of other beta estimators. Section VI explores the evidence in Lettau, Maggiori, and Weber (2014). It shows that currencies primarily serve the role of "zero anchors" (i.e., having nothing to do with the stock market one way or another); and that replacing LMW's full-sample down-betas with ex-ante down-betas often but not always eliminates the power of down-betas to explain asset class cross-sectional rates of return. Section VII concludes.

I Market-Beta Measurement

All individual stock return data in our study are from CRSP. The value-weighted stock market rate of return and the risk-free rate are from Ken French's website. Both data sets begin in 1927 and end in 2016.

Much of our analysis is on a calendar-year basis. Each year, we estimate for each stock the OLS beta with daily stock return data. Individual stock returns entering the beta computation were winsorized at –25% and +25% on each day, and also at the 2nd and 98th percentiles (within each one-year estimation period). This reduces the effect of strong idiosyncratic return events on days when the market happened to increase or decrease. To be included, a stock had to have had more than 126 days of valid return data in the calendar interval. We view this OLS beta not as the best but as the simplest naive estimator, untainted by refinement search. Table 9 shows that our results are quite robust to using a variety of more sophisticated beta estimators instead of the simpler OLS estimates, including shrunk betas, longer-estimation period betas, and asynchronous-trading corrected betas.[5]

[Insert Table 1 here: **Sample Constitution**]

Table 1 offers basic descriptive statistics for our sample. We can calculate 348,629 non-overlapping betas from 87.1 million daily stock returns. Excluding the final year, we can calculate 316,587 calendar-year OLS betas for use as independent variables. The typical beta in our sample is 0.61. This is *not* due to asynchronous trading, but due to the fact that our beta average is equal-weighted, while the stock index in the market-model regressions is value-weighted.[6] The typical market-model time-series error has a standard deviation of about 0.15-0.20. The typical cross-sectional heterogeneity (s.d.) is about three times that, 0.55. In a Vasicek (1973) beta, the typical weight on the own OLS beta (our baseline beta) would be about 90%, with the rest on the cross-sectional market-average beta.

[5] Our first draft used the Levi and Welch (2017) beta. The results were very similar.

[6] Not reported, stocks with higher market-betas tend to be smaller and thus more likely to drop off the CRSP sample. Beta can predict disappearance. However, down-beta predicts this no better than up-beta, and both are completely subsumed by the plain beta.

II Low and High-Beta Portfolios in Down Stock Markets

Our first question is whether low market-beta stocks are good hedges in bear and crash markets.

[Insert Table 2 here: **Stock Performance by Ex-Ante Betas ($\hat{b}_{y-1}$) And Subsequent Market Conditions**]

In Table 2, we group stocks by ex-ante OLS beta into low ($\hat{b}_{y-1} < 0.5$), medium ($0.5 < \hat{b}_{y-1} < 1.0$) and high ($\hat{b}_{y-1} > 1.0$) categories. Each stock's beta i is calculated over a calendar year (y). We omit individual stock and/or portfolio subscripts, but they are always implicit. The average betas in these categories are about 0.2, 0.75, and 1.4, respectively. The bottom two rows in Table 2 show that the three categories had about an equal number of firms, but the low-beta category contained smaller firms than the medium and high beta categories.[7] This could be due to a greater non-synchronicity bias of smaller stocks or the fact that smaller stocks tend to be more idiosyncratic, while larger stocks tend to be more in tune with the overall stock market. We will return to this issue in Section V.

The main part of Table 2 describes the beta categories' relative performances in different ex-post market conditions over the following calendar year. The categorization breakpoints for market conditions are at the 1/8, 2/8, 6/8, and 7/8 quantiles, representing market daily rate of return cutoffs of about –0.9%, –0.4%, +0.5% and +0.9%. The most interesting categories are the "extreme bear" and "bear" categories—days on which the stock-market lost –1.4% (–0.9%) or more, with an average return of –2.4% (–1.8%). In the "extreme bear" category, the model would have expected low-beta stocks to lose about $\hat{b}_{y-1} \cdot m_y \approx 0.4\%$ (where m is the rate of return on the overall market). Instead, they lost approximately 0.8%, somewhat worse than predicted. However, this return is still far better than the –3.1% that high-beta stocks lost. In the "bear" category, low-beta stocks expected a loss of 0.3%, but lost an actual 0.55%. Again, this is much better than the 2.3% loss of their high-beta stock counterparts.

[7] With modest exceptions right after 1987 and 2001, these time patterns were fairly stable.

The slight underperformance of low-beta stocks (relative to $\hat{b}_{y-1} \cdot m_y$ expectations) is less a reflection of lower alphas than a reflection of misestimated betas. For example, the bull category predicted versus realized return averages are almost the negative of the bear category. Here, the model suggested a performance of 0.3% for low-beta firms, but they delivered about 0.55%. The ex-post beta for low-beta stocks tended to be higher (closer to 1) than our unshrunk ex-ante OLS beta estimates. We will revisit the issue of beta measurement in Section V.

[Insert Figure 1 here: **Performance of Value-Weighted Portfolios In Three Ex-Ante Beta ($\hat{b}_{y-1}$) Categories in Bear Markets (–1% or worse)**]

Figure 1 explores the performance of the low-beta category stocks by year. The effectiveness of the beta-based hedge today is about average. It was no better or worse in the past than it is today.

[Insert Figure 2 here: **Stock Return Performance During Market Crashes of 1,000 Largest Stocks Vs. Ex-Ante Beta $\hat{b}_{y-1}$**]

Another concern is that although low market-beta portfolios could have performed as expected in "bear" and "extreme bear" markets, maybe their hedge collapsed in the very worst crashing stock market conditions. Figure 2 shows the performance of the 1,000 largest stocks in the cross-section during the four most infamous crash episode days (1928, 1939, 1987, and 2008). The plot for the crash of 1987 shows that it is not clear whether medium-beta stocks performed better than high-beta stocks. However, it is clear that in all four episodes, low-beta stocks performed better than mid- or high-beta stocks.

In sum, even using our naive OLS beta estimates, we find that low-beta stocks were good hedges against subsequent market crashes and declines.

III Ex-Ante Beta Estimates as Predictors of Betas

We have now established that all-days betas worked well as hedging metrics for stocks in bear markets. Nevertheless, it could be that betas calculated only from stock-market-down days ($\hat{b}^{-}_{y-1}$) make even better predictors of their future counterparts than betas calculated from all days ($\hat{b}_{y-1}$]). As noted in the introduction, down-betas have been investigated prominently by Ang, Chen, and Xing (2006) and others.

A Descriptive Statistics for Non-Shrunk Betas

In our tests in Tables 4 and 5, we continue to work with our annual panel data set. This sample includes all CRSP stocks available from the NYSE, AMEX, and NASDAQ from 1926 to 2017 (over 1,081 months, about 90 million stock-day and 3 million stock-month rates of return). In our ACX replications in Section III below, data availability restrictions occasionally limit this to a more recent sample, typically from 1963 to 2017 (about 451 months, about 600 thousand stock month returns) For each stock, we calculate one annual beta and one annual rate of return.

[Insert Table 3 here: **Descriptive Statistics For Betas**]

Table 3 shows that the average up-beta estimate $\hat{b}^{+}_{y}$ was about 0.59, the typical all-days $\hat{b}_{y}$ was about 0.67, and the typical $\hat{b}^{-}_{y}$ was about 0.75. The typical $\hat{b}^{+}_{y}$ was also estimated from 133 days of daily returns, whereas the typical $\hat{b}^{-}_{y}$ was estimated from 115 days. The typical within-stock difference between $\hat{b}^{+}_{y}$ and $\hat{b}^{-}_{y}$, named $[\hat{b}^{-}_{y}–\hat{b}^{+}_{y}]$, was about –0.16, with a standard deviation of about 0.86. The absolute value of $[\hat{b}^{-}_{y}–\hat{b}^{+}_{y}]$ had a mean of about 0.55 with a standard deviation of 0.69. It appears that a large number of stocks had quite respectable spreads—large enough to potentially allow for differential pricing effects.

The "RMSE/day" row shows the root mean squared error of the difference between the market-model predicted stock return and the actual stock return. This also shows the effect of our volatility filter. The average daily RMSE was about 2.59% in the full sample (about 42% annualized), and about 1.78% in the low-volatility sample (29% annualized).

Panel B helps assess whether it is possible to explore the plain all-days betas and the up- and down-days betas simultaneously. It shows that the contemporaneous up- and down-betas together explain about 77% of the all-days beta.[8] Given the large number of data points, this suggests that we have no problems with degenerate multi-collinearity when we include three variables in later regressions (i.e., in which one coefficient on one beta takes on an excessively positive value and the coefficient on another beta takes an an excessively negative value). In sum, the three market-beta estimates ($\hat{b}_y$, $\hat{b}_y^+$, and $\hat{b}_y^-$) are highly correlated but are not so nearly collinear that it prevents their simultaneous use as predictors.

B Predicting Future Betas

[Insert Table 4 here: **Predicting Betas With Preceding Betas**]

Table 4 shows the results of regressions that predict betas with prevailing (i.e., historical) betas. Each specification is estimated in two forms. The first estimates are from very large simple pooled regressions ("Panel"). The second estimates are from Fama and MacBeth (1973) regressions ("FM"). The Fama-Macbeth technique is warranted not because of the serial uncorrelatedness of the dependent variable, as in typical stock return regressions. Instead, it is a device to reduce the weight of the larger number of stocks in later years. Neither set of regressions reports standard errors, because beta lead-lag associations among betas can be estimated quite accurately. The Panel standard errors are below 0.005 (heteroskedasticity-adjusted) in all cases. The Fama-Macbeth standard errors range from 0.1 to 0.2. (Neither Pooled nor Fama-Macbeth standard errors should be translated into probability statements.) Given the strong predictability, the reader can focus

[8] The up-beta and down-beta need not average to be the all-days beta. Conceptually, it would even be possible for a stock to have positive betas on both down days and up days and yet to have a negative beta when both up and down days are included.

[1] on the economic meaning of the coefficient estimates and [2] on the relative predictive power and signs of the different betas.

Predicting Plain Betas: The left-hand side in Panel A shows that the historical all-days OLS beta $\hat{b}_{y-1}$ can predict its future self $\hat{b}_y$ with an auto-coefficient of 0.69 (Panel) and 0.71 in the Fama-Macbeth regression, and an R^2 of about 47%. When the historical $\hat{b}^+_{y-1}$ and $\hat{b}^-_{y-1}$ are included separately instead of the historical $\hat{b}_{y-1}$, they offer no improvement. The last regressions show that Including all three beta estimates $\hat{b}^+_{y-1}$ and $\hat{b}^-_{y-1}$ cannot offer marginal positive explanatory power above and beyond the all-days $\hat{b}_{y-1}$ alone, either.

The right-hand side of the panel shows that these conclusions hold also for low-volatility stocks. The auto-coefficient estimates increase to 0.79, and the R^2 increases from 47% to 62%. But the key inference about the relative uselessness of $\hat{b}^+_{y-1}$ and $\hat{b}^-_{y-1}$ remains intact. For practical purposes, $\hat{b}^+_{y-1}$ and $\hat{b}^-_{y-1}$ can be ignored when trying to predict the future all-days beta $\hat{b}_y$.

Predicting Down-Betas: Panel B is more interesting.[9] The historical $\hat{b}^-_{y-1}$ estimates predict $\hat{b}^-_y$ with an autocoefficient of 0.39 among all stocks and 0.59 among low-volatility stocks. Even more interestingly, the prevailing $\hat{b}^-_{y-1}$ estimates in one calendar year are not as good in predicting their own future $\hat{b}^-_y$ estimates as the prevailing all-days $\hat{b}_{y-1}$ estimates. In predicting $\hat{b}^-_y$, the prevailing $\hat{b}_{y-1}$ has a coefficient of 0.64 and 0.72 in the all-stocks and low-volatility stocks samples, respectively. Moreover, the own autocoefficient on down-beta is only about 0.04 when the all-days beta is included, too. There is very little use of the historical $\hat{b}^-_{y-1}$ estimates even on the margin, because the less noisy all-days OLS beta $\hat{b}_{y-1}$ estimates almost completely subsume all the $\hat{b}^-_{y-1}$s' explanatory power. The R^2 does not meaningfully increase with $\hat{b}^-_{y-1}$. It is good advice is to rely overwhelmingly on the all-days beta $\hat{b}_{y-1}$ when forecasting down-beta $\hat{b}^-_y$.

[9]Not shown in Table 4, the same conclusions hold for up-beta prediction.

There are two reasons for this. First, down-betas can be estimated with only half as many days as the all-days betas for any fixed time period. Not shown, if we estimate the all-days beta $\hat{b}_{y-1}$ with the same smaller number of days as the down-beta $\hat{b}^-_{y-1}$, randomly chosen throughout the year, the coefficient on the plain beta still remains higher than that of the down-beta. For example, the multivariate Panel coefficients become 0.30 for the all-days beta $\hat{b}_{y-1}$, 0.18 for the down-beta $\hat{b}^-_{y-1}$, and 0.08 for the up-beta $\hat{b}^+_{y-1}$. Second, the all-days market-model beta estimation regressions have a wider x-range (the market rates of return) and thus offer more stability in fitting the market-model line.

The right-hand side of the table shows that this advice (to rely on $\hat{b}_{y-1}$) holds true also for low-volatility stocks. The prevailing all-days beta $\hat{b}_{y-1}$ alone has higher coefficients and predicts the future down-beta better $\hat{b}^-_y$ than the lagged down-beta $\hat{b}^-_{y-1}$. Even on the margin, $\hat{b}^-_{y-1}$ seems largely irrelevant. If the intent is to forecast the future down-beta $\hat{b}^-_y$, the prevailing all-days beta $\hat{b}_{y-1}$ is best. The prevailing $\hat{b}^-_{y-1}$ can be ignored when trying to predict the future $\hat{b}^-_y$.

Not shown in the table, we spent a considerable amount of time searching for instruments to improve our prediction of the future $\hat{b}^-_y$. We could not find any. For example, if we also smooth in the cross-section with the stock's past average rate of return and log market cap, both the coefficients and the R^2 remain the same. The all-days beta remains the only meaningfully useful variable in predicting down-beta.

C Estimation Uncertainty or Time-Varying Up-Beta Down-Beta Spread?

[Insert Table 5 here: **Autocoefficient (Decay) of Up- Minus Down-Beta) ($[\hat{b}^-_y - \hat{b}^+_y]$)**]

We can explore the source of the prediction difficulties. Table 5 clarifies why the all-days beta performs so well, despite the large differences among most stocks' up and down-betas. For variety, this table also reports regressions starting only in 1962. The table shows regressions that predict $[\hat{b}_y^- - \hat{b}_y^+]$, the difference between the up-beta and the down-beta, with its own lagged value, $[\hat{b}_{y-1}^- - \hat{b}_{y-1}^+]$.

The results explain the core of the prediction problem: There is almost no stability in the $[\hat{b}_y^- - \hat{b}_y^+]$ spread. Its autocoefficient is less than 0.1. Recall from Table 3 that the up-down beta spread $[\hat{b}_y^- - \hat{b}_y^+]$ had a cross-sectional standard deviation of 0.86 (0.57 for low-volatility stocks) and a mean absolute spread of about 0.55 (0.40 for low-volatility stocks). In predicting the ex-post spread, a stock that had a $[\hat{b}_{y-1}^- - \hat{b}_{y-1}^+]$ spread of 0.6 in the last calendar year is likely to have a $[\hat{b}_y^- - \hat{b}_y^+]$ spread of under $0.1 \times 0.6 \approx 0.06$ this calendar year. The potential to explain this small remaining stable beta spread between $\hat{b}_y^+$ and $\hat{b}_y^-$ on the one hand has to be weighed against the efficiency gain from estimating an all-days $\hat{b}_y$ with roughly twice as many returns (and more stability due to a larger market-return spread) on the other hand, given any same historical estimation sample. The latter effect "wins."[10]

We can further diagnose the decay of the true up-down beta's spread. Is there a "regression to the mean" in the underlying unobserved true beta spread, too? This can be explored by examining the decay pattern in the auto-coefficients.

- If the only problem is an errors-in-variables problem in a time-constant true up- minus down-beta spread, then it should not matter whether $[\hat{b}_y^- - \hat{b}_y^+]$ is predicted with the one-year lagged estimated beta up-down spread ($[\hat{b}_{y-1}^- - \hat{b}_{y-1}^+]$) or the two-year lagged estimated beta up-down ($[\hat{b}_{y-2}^- - \hat{b}_{y-2}^+]$).

- If the underlying true up- minus down-beta spread wanders around randomly, too, then the coefficient on the twice lagged $[\hat{b}_{y-2}^- - \hat{b}_{y-2}^+]$ should be lower than the coefficient on the once lagged $[\hat{b}_{y-1}^- - \hat{b}_{y-1}^+]$.

[10] A simple Monte-Carlo experiment provides some over-the-envelope magnitudes. If the true underlying up-down spread measure stays constant for each stock, the measurement error standard deviation would have to be greater than 2.0 to explain such a low an auto-coefficient. The observed cross-sectional up-down spread of about 0.6 is thus only about one third as large as the measurement error. Put differently, a stock that has an observed up-down spread of 0.4 is more likely to have a true up-down spread of 0.1 and an error of 0.3.

The data show that the two-year-lagged $[\hat{b}^-_{y-2} - \hat{b}^+_{y-2}]$ coefficient is about 70-80% as large as the one-year $[\hat{b}^-_{y-1} - \hat{b}^+_{y-1}]$ coefficient. Thus, we know that the estimation problem is partly measurement noise in the beta and partly drift in the underlying time-varying beta spread itself.

Time variation in the underlying true betas has three consequences. The obvious one is that it limits our ability to forecast betas over long horizons. The perhaps less obvious one is that it also limits the usefulness of historical data. Even with an infinitely long time series of stock returns, the current beta can never be perfectly assessed. Both problems are faced by the econometrician and most likely also by investors. Third, with more recent returns being more useful, longer time-series for the down-beta cannot easily compensate for the reduced availability of more recent stock returns (relative to the all-days beta).

For the purposes of assessing the forecastability of down-beta, it does not matter greatly whether it is measurement error or mean reversion. Both make it difficult to predict the future down-beta. From what we can tell, the best estimator is always the prevailing all-days beta. (This is also the case if we use far longer prevailing sample lengths for the estimation.) To the extent that all betas (all-days or down-betas) remain imperfectly known, even if investors care only about ex-post downside market-risk, the best estimation advice we could offer to an individual investor with access only to the same stock return and stock attribute data as us, is still to rely overwhelmingly on the estimated prevailing all-days beta and largely ignore the estimated prevailing down-beta.

IV Betas as Predictors of Stock Returns

We have now established that if investors care about the ex-post down-beta and have to estimate it like us, they would likely not rely on ex-ante down-beta but on the ex-ante all-days beta. The all-days beta has intrinsic advantages when forecasting down-beta. We have also established that the true down-beta is very different from the realized down-beta. If investors knew the true down-beta but not the realization (noise), the realized down-beta would yield a greatly down-biased estimate of the true implied equity premium—the 8.4% estimated premium reported in the introduction would have to be greatly inflated

Pragmatically, it is still interesting to learn whether the ex-ante (down-) beta is priced (offers higher *abnormal* returns). Perhaps the residual component of its marginal forecastability, however small, is exactly the component that predicts future rates of return. Moreover, Sections 3.2 and 3.3 in Ang, Chen, and Xing (2006) provide some evidence using ex-ante down-betas to predict stock returns. We examine these in Section C.

To reexamine the results in ACX and to be able to reinterpret them, we construct a data set to replicate the original ACX data set as closely as possible. ACX graciously shared their code with us, making this easier. Their analysis contained only NYSE stocks from 1963–2001 with share codes 10 and 11. They used log-market rates of return to estimate market betas. For down- and up-betas, they cut the days sample by stocks that are above versus below the mean market return in the corresponding beta estimation window. They winsorize all betas and controls. They use a monthly panel frequency. In each month, they calculate both market-betas and an annual rate of return, both from the same 12 months of daily stock returns. Although their beta is contemporaneous with respect to this return, for ease of description, we sometimes call it "ex-post." They explore performance sometimes in the all-stocks data and sometimes in data that excludes higher volatility stocks. Thus, we provide analyses using only stocks with *ex-ante* market-model RMSEs (i.e., taken from the same lagged OLS regressions from which we calculate market-betas, and not from ex-post data) within the bottom 75%. Finally, we also create an "extended sample," which includes the full CRSP sample that we used in the previous section.

A Downside Risk in Size and Book-to-Market Portfolios

We do not explore the GMM specification in Section 2.5 (Table 6) of ACX. It suggests that down-return exposure helped explain the 25 Fama and French size and book-to-market portfolios. Ang, Chen, and Xing (2006) write that "Table 6 shows that the coefficient b_{m^-} is statistically significant at the 5% level. This indicates that, for pricing the size and book-to-market portfolios, the downside portion of market return plays a significant role, even in the presence of the standard market factor. This is true even when we allow for SMB and HML to be present in the model. This is a strong result because the SMB and HML factors are constructed specifically to explain the size and value premia of the 25 Fama-French portfolios." Although this is literally correct, the sign on this coefficient is the opposite of what the theory predicts. Stocks with higher down-beta exposure offered lower average rates of return, not higher average rates of return. Moreover, because these regressions also include the all-days beta, the summed coefficient suggests a net effect of just about zero.

B Predicting Rates of Return, Cross-Sectional Tests

[Insert Table 6 here: **Explaining Stocks' Rates of Return (r_y) in Fama-MacBeth Regressions**]

Table 6 turns to predicting rates of return. It replicates models in ACX Table 2.

With stock returns as the dependent variable, the cross-sectional return correlations require the use of the Fama-Macbeth method, i.e., of time-series averages of cross-sectional coefficients. The specification includes a Newey-West correction with 12 lags to account for the overlap. Table 6 reports the resulting Fama and MacBeth (1973) "gammas." The three panels in our table correspond to Models I, II, and V in ACX's Table 2.[11]

In each panel, the first row copies the numbers from the tables in Ang, Chen, and Xing (2006). The second row is our replication using the same specifications and data. Although we cannot replicate the ACX numbers perfectly, our own gamma estimates strongly coincide with their gamma esimtates. We consider our coefficient estimates to be successful replications.

The ex-post beta estimates associate strongly and positively with the (simultaneous) rates of return. Panel A (ACX Model I) shows that even the all-day market beta associates strongly positively with (ex-post) stock returns (as already mentioned in the introduction, though here with additional controls). The estimated (conditional) market-beta premium is about 19% per year. Panel B (ACX Model II) shows that all of the ex-post positive association is in the down-betas and not in the up-betas. In Panel C, with further controls, both the ex-post down-beta and the ex-post up-beta associate positively, but the down-beta association is three times as strong.

However, the third and fourth rows in each panel show that *ex-ante* down-betas are either insignificant or negatively correlated with future average rates of return. There is no significant positive predictive association of any of the three *ex-ante* beta measures with subsequent rates of return. The ex-ante all-days beta and ex-ante down-beta both have statistically significant negative coefficients in the extended sample in Panels A and C.[12]

[Insert Table 7 here: **All-Days Betas and Full-Sample Betas in Fama-Macbeth Regressions**]

Table 7 extends Panel C of Table 6. The first line in Table 7 Panel A and the first two lines in Panel C repeat the last three lines from Table 6. Recall that they showed that lagged

[11] Not for publication: We report ACX Model V (instead of Model VI), because Model V did not require a Pastor-Stambaugh liquidity beta and thus allowed us to retain more observations.

[12] The data requirements in Panel C imply that the extended sample is primarily extended forward to 2016, not backward to 1927.

down-betas $\hat{b}^{-}_{y-1}$ associate not positively but negatively with future rates of return. This is the case in both the ACX and extended sample. Table 7 then examines (a) whether the down-beta $\hat{b}^{-}$ matters when in competition with the plain-beta $\hat{b}$, and (b) how the inference changes with the use of betas estimated either from the full sample (i.e., for each stock, one beta is estimated from all available daily rates of return) or from a surrounding time-series (four years before, contemporaneous, four years after) . These are akin to the beta estimation method used in Lettau, Maggiori, and Weber (2014). The pure full-sample estimator suits LMW better, because they have a shorter sample period. The 9-year beta series istherefore more similar in spirit. Full-sample betas (subscripted with a star) are less affected by stock return realization noise than ex-post betas, and contain not only ex-post but also ex-ante information.

Panel A shows that down-beta $\hat{b}^{-}_{y}$ has a solid association with contemporaneous rates of return. The FM gamma on $\hat{b}^{-}_{y}$ is about 8% per annum in the ACX specification, 6% per annum if the sample is extended, too; and 3% per annum *extra* when the all-days beta is included, too. (Recall that in their specification, an annual rate of return is regressed on a beta estimate.) The real surprise is the all-days beta $\hat{b}_{y}$. Its coefficient is between five and ten times larger than that on the down-beta $\hat{b}^{-}_{y}$! On down days, it suggests a *conditional-on-6-controls* market beta premium of $0.145 + 0.033 \approx 18\%$ per annum.

Panel B shows that the inference on down-beta is never good news for the down-beta hypothesis when we use long time horizons to estimate betas, liberally mixing ex-ante and ex-post stock returns in the estimation. The coefficient sign reverses when betas are estimated with stock returns over the "Full sample." The incremental contribution of down-beta does not suggest an additional risk premium. Another curious result emerges, too: the unconditional beta $\hat{b}_{y}$ is large and positive. When betas are estimated over the 9-year period surrounding the stock return that is to be explained, the down-beta is never statistically significantly positive.

Panel C is the cleanest ex-ante proxy. It shows no reliable positive marginal predictive association between $\hat{b}^{-}_{y-1}$ and stock returns. The total effect when $\hat{b}_{y-1}$ is included is also always negative, too.

Not shown, we also tried to instrument betas with prevailing ex-ante average rates of return (momentum), market cap, and other variables. The thus-fitted beta estimates could not predict stock returns any better either.

C Predicting Rates of Return, Time-Series Tests

Ang, Chen, and Xing (2006) did provide some stock return tests based on ex-ante down-betas. These are found in their Tables 9 and 10. They are easiest to explain in the context of classic Fama and French (1993) regressions (FF).

Fama-French regressions explain the rates of return on zero-investment test portfolios in terms of other zero-investment portfolios. All investment weights must be known, because portfolios are formed before the rates of return themselves become available. The dependent variables in these regressions are then a time-series of monthly rates of return on a test portfolio. The independent variables are equivalent time-series on known factor portfolios. We rely on the factor portfolios provided by Fama-French themselves.

In our case, the test portfolio is formed by sorting stocks based on ex-ante $\hat{b}^{-}_{y-1}$. The long leg in our test portfolios are the stocks in the highest quartile of *ex-ante* down-beta at the end of the preceding month. The short leg are those in the lowest quartile.[13] Both legs are equal-weighted.

[13] If one sorts stocks based on the difference between the down-beta and the plain beta, the test becomes one of whether stocks with higher down-betas have worse or better performance than stocks with higher all-days betas. Because stocks with higher betas have negative alphas, the test can then become one whether stocks with higher down-betas have less negative alphas. A negative alpha for a down-beta portfolio is not necessarily a great comfort from the perspective of the theory. It shows only that its alpha is not as negative as that of the plain beta portfolio. This also serves as a warning: it is possible to obtain a marginal positive coefficient on $\hat{b}^{-}_{y-1}$ when one includes both $\hat{b}^{-}_{y-1}$ and $\hat{b}_{y-1}$, but this may not indicate that $\hat{b}^{-}_{y-1}$ itself has a positive premium. It may merely state that $\hat{b}^{-}_{y-1}$ has a less negative premium than $\hat{b}_{y-1}$.

[Insert Table 8 here: **Fama-French Regressions Explaining Monthly Rates of Return $r_{[p,m]}$ on Zero-Investment Test Portfolios Formed By Quartiles of Stocks' Ex-Ante Down-Beta Estimates ($\hat{b}^{-}_{p,m-1}$)**]

Model 6 in Table 8 Panel A explains why ACX can show evidence that portfolios formed from ex-ante down-betas had increasing unadjusted average rates of return. There was indeed a positive association between the portfolios' ex-ante down-betas and their subsequent rates of return. The unadjusted +29.1bp/month rate of return (alpha) in our table is fully responsible for their unadjusted stock return spread (from 0.69% to 0.92%) in their Table 9B. The 46.0bp/month (SMB and HML adjusted) alpha in our Model 7 is fully responsible for their *size- and book-to-market adjusted* stock return spread (from –0.25% to +0.25) in their Table 10.

The remaining regression models in Table 8 dissect these findings.

1. Panel B shows that the ACX findings no longer hold when the sample is extended. With the exception of a trivial 6.8bp/month in Model 6, all (unadjusted) alphas turn negative. The reported positive coefficients in the low-volatility sample of Ang, Chen, and Xing (2006) have become obsolete.

2. Models 3 through 5 and Models 8 through 10 show that their lack of control for the market factor was an important contributor to their reported findings. This is because:

    (a) Zero-investment portfolios spread (sorted) by down-beta also had large spreads in all-days betas. Ex-post, Table 8 shows that the test portfolio's realized all-days net beta spread was about 0.5.

    (b) The stock market had an average excess rate of return of about 50bp/month in both the ACX and the extended samples.

    Ergo, in a sample in which the equity premium averaged 50bp/mo, any portfolio with a market beta of about 0.5 should have offered an extra rate of return of about

25bp/mo. This is the case even if there is absolutely no premium to be earned for taking on more beta exposure risk. (Of course, the asset-pricing puzzle is not why high beta portfolios performed better on average in times in which the stock market increased, but why they did not perform better *on average* overall, given the extra beta risk.)

Panel B Models 3 and 4 and Models 8 to 10 show that the Fama-French-adjusted alphas of the test portfolios were strongly negative in the extended CRSP sample—of course after controling for the strong realized performance of the equity market.[14] Stocks with higher ex-ante down-betas did not offer higher but lower abnormal rates of return. At about –60bp per month, this underperformance was stark. Although we have no explanations of why stocks that performed especially badly in down markets earned subsequently even lower rates of return (just as stocks with high betas performed badly in Frazzini and Pedersen (2014)), the evidence is surely not supportive of the conjecture that investors earned a positive risk premium when they formed portfolios consisting of high down-beta stocks.

V Alternative Beta Measures

[Insert Table 9 here: **Performance of and Key Results for Alternative Beta Estimators**]

We investigated whether our reported results are robust to using different beta estimators. We explored a good number of these, but the results were always very similar. Table 9 shows the results using some of them:

2-Year OLS: An OLS beta-estimator that uses two years of historical stock returns instead of one.

Dimson: The Dimson (1979) (DMS) beta estimator to adjust for infrequent trading. It is essentially the average coefficient over surrounding days.

[14]Not shown but important, the reduction in the negative alpha in Table 8, Panel B, Models 5 and 10, is attributable to all three additional factors, MOM (about 30bp), RMW (about 20bp), and CMA (about 30bp).

Vasicek: The Vasicek (1973) (VCK) beta estimator. It shrinks each stock's beta towards the cross-sectional mean beta according to the OLS time-series estimation error. Very efficient if the underlying beta is stable.

Levi-Welch: The Levi and Welch (2017) (LW) beta estimator shrinks the Vasicek beta a second time to improve prediction when the underlying beta is itself mean-reverting. The shrinkage factor is based on linear empirical predictive associations. The all-days beta is the Vasicek beta, best shrunk by another 20%; the up- and down-betas are best shrunk by 40%.[15] The first version of our paper used the LW estimator as its main estimator.

Panel A shows that the beta estimates are highly correlated. The DMS beta is a little more unusual. The VCK and LW estimators differ only in an affine way and therefore have perfect correlation.

Panel B shows the RMSE when each beta estimator has to predict each future beta estimate. This RMSE is simply the square root of the squared difference between the ex-ante beta calculated one way and the ex-post beta calculated another way. The LW estimator tends to outperform other estimators when predicting future betas, regardless of how these future betas are themselves calculated. The exception is that the VCK beta was better at predicting the 2-year OLS beta than the LW beta (which came in second).

The DMS beta is a particularly bad estimator in this context: If the econometrician wants to assess the future DMS beta, any of the other beta estimators are better choices than the prevailing DMS beta itself. Thus, even if non-synchronicity is a serious problem, it is unlikely that the DMS beta would be helpful *in this CRSP data context*. (It may work better with other data.)

Not shown, regardless of the beta estimator used, very few stocks have negative betas. Of course, this does not imply that taking market beta into consideration is not useful. It is

[15] Specifically, $0.133 + 0.8 \cdot b_{\text{VCK}}$ and $0.2 + 0.6 \cdot b_{\text{VCK}}^{-/+}$.

quite possible to buy portfolios of stocks with higher or lower market betas, and thus better or worse performance in down-markets.

Panel C is the most interesting panel. It shows key results from earlier tables with other estimators. Because we report the base OLS 1-year estimator here, too, it is easiest for the reader to refer back to the full explanations in the earlier table for more details.

A quick glance reveals that all of our results are very robust to variations in estimators: (1) Low-beta stocks always outperform high-beta stocks in down markets; (2) the all-days beta $\hat{b}_{y-1}$ is always a far better predictor of the future down-beta $\hat{b}^{-}_{y}$ than the prevailing down-beta $\hat{b}^{-}_{y-1}$ itself; (3) when ex-ante or full-sample betas are used to predict stock returns, the down-beta does not have a positive marginal coefficient; and (4) a zero-investment test portfolio that is long high down-beta stocks and short low down-beta stocks has a negative alpha.

We also explored beta estimators instrumented with lagged variables (based on full-sample regressions), but none of these performed markedly better than the LW estimator above.

VI Asset Classes (Lettau, Maggiori, and Weber (2014))

Lettau, Maggiori, and Weber (2014) "...follow Ang, Chen, and Xing (2006) in allowing a differentiation in unconditional and downside risk...[and]...show that expected returns in currency, equity, commodity, sovereign bond and option markets can be explained by a simple beta that measures the downside risk of assets in these asset classes" (p. 199). LMW not only post their data, but graciously shared their code with us as well.

In contrast to ACX, which uses the one-year ex-post market-beta, LMW estimate one full-sample market-beta for each of their asset portfolios. The drawbacks are that the LMW beta estimates still rely on return data that investors would not have known, and that there is no time-series variation in their key independent variable, $\hat{b}^{-}_{\star}$. The advantages are that full-sample beta estimates are efficient if betas are unchanging (especially over their relatively short sample period), that knowing the mix of ex-ante and ex-post average constant beta could be more plausible than knowing a future one-year beta, and that the full-sample beta largely avoids the issue that the dependent variable and a part of the independent variable are measured from the same stock returns.

Our focus is on LMW's Tables V and VI.[16] For our primary comparison, we estimate Fama-Macbeth regressions (a) without their a-priori CAPM restrictions, similar to some estimates in their appendix; and (b) with conditional betas estimated from the preceding rolling 36 months instead of the full-sample betas.

[Insert Table 10 here: **Fama-MacBeth Regressions Explaining Asset Class Portfolios, as in LMW**]

Table 10 compares the performance of the full-sample betas in LMW's Table V to the performance of ex-ante known rolling betas. The data are 435 months from 1974 to 2010, six currency portfolios, six currency portfolios of developed countries only,[17] and six Fama-French equity portfolios, two size-sorted times three book-to-market sorted portfolios. We can perfectly replicate the LMW results. However, when we replace their full-sample down-betas with ex-ante down-betas, the Fama-Macbeth gamma coefficients shrink by a factor of 10 and the statistical significance disappears. For example, in their Table 5 Model 6, with a restricted constant and restricted coefficient on the plain-beta, they have a coefficient on the full-sample down- minus plain-beta of 1.41 with a T-statistic of 1.8. In contrast, the ex-ante down- minus plain-beta in an unconstrained Fama-Macbeth regression has a coefficient of 0.12 with a T-statistic of 0.8.

[16] These tables report Fama-Macbeth regressions to obtain standard errors that reflect the residual covariances in monthly stock returns. The dependent returns are explained by each stock's fixed all-days beta times the realized rate of return in this month, plus a coefficient on the difference between the down-beta and beta that is actually estimated. In effect, together with their intercept restriction, they estimate whether the excess-down-beta can help explain the CAPM residual. An alternative is to restrict the coefficient on beta (the market premium) to be its unconditional mean. The coefficient on the estimated coefficient on $\hat{b}_{\star}^{-} - \hat{b}_{\star}$ remains the same, but its standard error changes depending on the method.

[17] Countries are considered developed if they are included in the MSCI World Market Equity Index.

There are two possible interpretations: First, LMW are correct that betas are largely time-invariant,[18] full-sample betas are more efficient estimators, and ex-ante betas have too little power to measure the down-beta over 36 months (usually only about 12-20 of which are negative). Second, LMW are not correct, because down-betas are time-varying, investors and the markets do know subsequently realized betas, and ex-ante down-beta is not useful. Whatever the reason, the test that currencies with higher *ex-ante* down-betas have subsequently higher asset returns is not successful.

The last part of Table 10 expands on the observation in LMW that "...the CAPM cannot price the returns of these asset classes [because] within each asset class there is little dispersion in betas," but across asset classes, this dispersion is much larger. Currencies in LMW matter as assets primarily because they have nothing to do with the stock market. Risk-free assets, cash, or uncorrelated coin-flips could largely serve the same role as currencies when testing the pricing of the six equity portfolios (or indeed pricing models across asset classes). Currencies are primarily repeated "zero anchors," which force the market-model line to go through zero. When the currencies are replaced with zero anchors, the LMW full-sample down- minus plain beta coefficient equivalent is 1.27 with a T-statistic of 1.4 (rather than 1.41 with a T-statistic of 1.8), and the ex-ante unconstrained equivalent is –0.10 with a T-statistic of –0.5 (rather than 0.12 with a T-statistic of 0.8).

[Insert Table 11 here: **Fama-Macbeth Regressions Explaining Asset Class Portfolios in Additional Sets and with Different Specifications**]

Table 11 repeats the same analyses for the different asset test sets in LMW's Table 6 and the remaining test set combinations. The "6 equities" line confirms for these six portfolios that down-betas have a positive effect when estimated over the full sample and often a negative effect when estimated with ex-ante data—as in the ACX full-sample evidence for all stocks (not portfolios) in Table 7. In general, without a zero anchor, full-sample down-betas matter positively. Yet when down-betas are estimated in the ex-ante window,

[18]Time invariance of down-betas can be rejected for the six equity portfolios and for the highest-risk portfolios in each asset class.

only test sets including the sovereign bond portfolios matter reliably. With a zero anchor, down-betas again matter with the sovereign bond portfolios. There are also good hints that the down-betas may matter when commodities are included—except that these hints largely disappear when we add the residual uncertainty of the beta-estimating market-model regressions as an extra control variable (not shown).

[Insert Table 12 here: **Inspection of Six Sovereign Bond Portfolios**]

Sovereign bonds are interesting enough that they are worth some further investigation. It is of concern that the sovereign bond return sample begins only in 1995 (i.e., after the crash of 1987) and contains fewer than 200 months. Remarkably, the average all-day sovereign bond betas in this sample are almost 1. The sovereign bonds behaved more like U.S. equities than like other bonds!

Table 12 shows that the high market-betas were driven by two exceptional months: August 1998 (the Russian financial crisis) and October 2008 (the global financial crisis). Without these two months, the sovereign bond betas are in a more ordinary range, much closer to 0. It is left to the reader to judge whether these two events were unrepresentative outliers or representative indicators of differences in behavior during market declines. We reserve judgment on this point.

Our interpretation is that the LMW across-asset class evidence is more intriguing than the ACX within-equities evidence, although it is also typically weaker with ex-ante beta estimates than with full-sample beta estimates, and although the most intriguing evidence was driven by two unusual episodes. More time and data will tell.

VII Conclusion

Plain betas predict stock returns quite well during bear markets and crashes. That is, there is no evidence that the hedge provided by portfolios of stocks with low ex-ante betas tends to collapse during downturns and/or stock market crashes. This plain all-days market-beta is also better in predicting future down-beta than the lagged down-beta itself. In contrast to the *ex-post* down-beta, the *ex-ante* down-beta does not have a positive (and often a negative association) with future rates of return.

If investors know the true down-beta but not the realization noise, then the coefficient estimate of 8.4% oer annum suggests a market-premium that is far too high (because of its downward bias). If investors do not know but have to estimate the down-beta, they still suffer the handicap that all-days beta can be estimated with twice as many days as the down-beta. Without empirical evidence for such advance knowledge, a more standard asset pricing test for whether beta matters for *...the cross-section of expected returns* (Harvey, Liu, and Zhu (2016)) would rely on ex-ante all-days betas. Although all-day betas can predict future down-betas, stocks with high ex-ante betas and down-betas unfortunately still did not have higher subsequent average rates of return.

Finally, the evidence in Lettau, Maggiori, and Weber (2014) remains ambiguous, with some inferences changing and some inferences remaining when ex-ante down-betas are used.

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Table 1: Sample Constitution

Panel A: Overall Sample

	All Stocks	Low-Volatility Stocks
Years	1927-2016	1927-2016
Daily CRSP Observations	90.5 million	
...with valid stock returns	87.1 million	65.6 million
Not calculable betas (< 126 trading days)	37,461	
Calculated Calendar-Year Betas	348,629	261,471
Forecastable	316,587	222,827
1927 predicting 1928	555	443
2015 predicting 2016	6,674	5,575

Panel B: Statistics on 348,629 Calendar Year OLS Betas

	Percentiles				Moments	
	25	50	75	%pos	mean	sd
days/stock	252	252	253		249	23
avg ts beta	0.25	0.61	1.02	92%	0.67	0.60
se ts beta (σ_{ts})	0.10	0.15	0.25		0.20	0.18
avg xs beta	0.53	0.63	0.80		0.67	0.18
sd xs beta (σ_{xs})	0.50	0.55	0.64		0.56	0.10
$\sigma^2_{xs}/(\sigma^2_{xs}+\sigma^2_{ts})$	0.83	0.92	0.97		0.87	0.15

Interpretation: Each market-beta is calculated from one calendar year of daily stock returns using Ken French's daily market and risk-free time series, with a minimum requirement of 126 days. Low volatility sample exclude stock-years in the highest quartile of rmse in the beta estimation regression. Daily stock returns were winsorized at –25% and +25% and at the 2nd and 98th percentile within each beta regression throughout the study. Note: Table 9 shows key results with other beta estimates. Tables 6-8 follow Ang, Chen, and Xing (2006) more closely, with resulting modest (unimportant) variation.

Interpretation: The mean beta is far below 1, because the regressions are not value-weighted but equal-weighted. It is *not* because of non-synchronicity. The time-series estimation uncertainty is about half the cross-sectional heterogeneity. In a Vasicek estimator, about 90% of the weight would be on the OLS estimate.

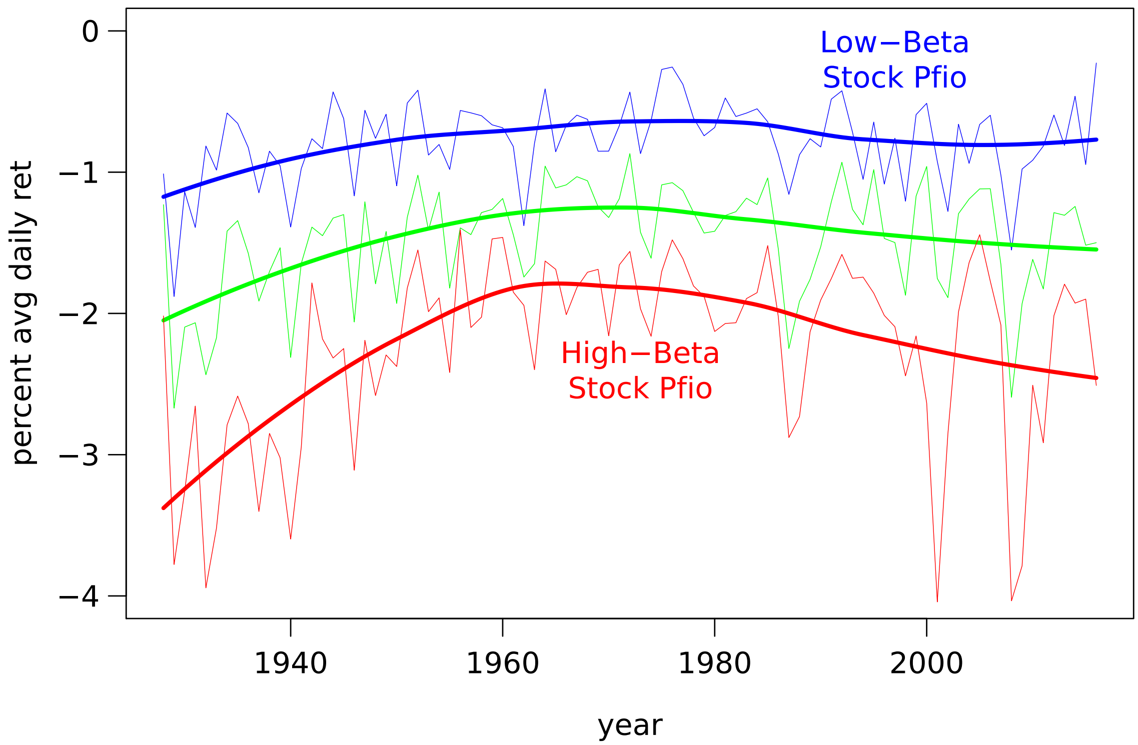
Table 2: Stock Performance by Ex-Ante Betas ($\hat{b}_{y-1}$) And Subsequent Market Conditions

Ex-Post Market		Ex-Ante Market Beta $\hat{b}_{\text{i,y-1}}$ Low (<0.5)	Medium	High (>1)
Extreme Bear	$\hat{b}_{\text{i,y-1}}$	$\overline{\hat{b}_{\text{i,y-1}}} = 0.18$	$\overline{\hat{b}_{\text{i,y-1}}} = 0.74$	$\overline{\hat{b}_{\text{i,y-1}}} = 1.44$
(–17.50% to –1.40%)	$\hat{b}_{\text{i,y-1}} \cdot r_{m,y}$	–0.39%	–1.68%	–3.26%
Mean –2.37%	$r_{i,y}$	–0.81%	–1.92%	–3.10%
	N	2,178,175	1,568,829	1,340,412
Bear	$\hat{b}_{\text{i,y-1}}$	$\overline{\hat{b}_{\text{i,y-1}}} = 0.18$	$\overline{\hat{b}_{\text{i,y-1}}} = 0.74$	$\overline{\hat{b}_{\text{i,y-1}}} = 1.44$
(–17.50% to –0.93%)	$\hat{b}_{\text{i,y-1}} \cdot r_{m,y}$	–0.30%	–1.28%	–2.49%
Mean –1.79%	$r_{i,y}$	–0.55%	–1.41%	–2.35%
	N	4,086,752	2,975,580	2,535,285
Bearish	$\hat{b}_{\text{i,y-1}}$	$\overline{\hat{b}_{\text{i,y-1}}} = 0.17$	$\overline{\hat{b}_{\text{i,y-1}}} = 0.74$	$\overline{\hat{b}_{\text{i,y-1}}} = 1.44$
(–0.93% to –0.43%)	$\hat{b}_{\text{i,y-1}} \cdot r_{m,y}$	–0.11%	–0.48%	–0.93%
Mean –0.65%	$r_{i,y}$	–0.09%	–0.43%	–0.83%
	N	4,086,197	3,027,795	2,576,878
Neutral	$\hat{b}_{\text{i,y-1}}$	$\overline{\hat{b}_{\text{i,y-1}}} = 0.17$	$\overline{\hat{b}_{\text{i,y-1}}} = 0.74$	$\overline{\hat{b}_{\text{i,y-1}}} = 1.45$
(–0.43% to +0.52%)	$\hat{b}_{\text{i,y-1}} \cdot r_{m,y}$	0.01%	0.04%	0.09%
Mean 0.06%	$r_{i,y}$	0.14%	0.12%	0.12%
	N	16,368,870	12,339,650	10,620,528
Bullish	$\hat{b}_{\text{i,y-1}}$	$\overline{\hat{b}_{\text{i,y-1}}} = 0.17$	$\overline{\hat{b}_{\text{i,y-1}}} = 0.74$	$\overline{\hat{b}_{\text{i,y-1}}} = 1.44$
(0.52% to 0.94%)	$\hat{b}_{\text{i,y-1}} \cdot r_{m,y}$	0.12%	0.52%	1.01%
Mean 0.70%	$r_{i,y}$	0.31%	0.57%	0.91%
	N	4,272,900	3,164,288	2,672,191
Bull	$\hat{b}_{\text{i,y-1}}$	$\overline{\hat{b}_{\text{i,y-1}}} = 0.18$	$\overline{\hat{b}_{\text{i,y-1}}} = 0.74$	$\overline{\hat{b}_{\text{i,y-1}}} = 1.43$
(0.94% to 15.8%)	$\hat{b}_{\text{i,y-1}} \cdot r_{m,y}$	0.29%	1.23%	2.39%
Mean 1.74%	$r_{i,y}$	0.56%	1.31%	2.21%
	N	4,351,034	3,168,270	2,628,395
Fraction of Firms (avg over years):		35%	33%	32%
Market Cap Percentile (avg over years):		18%	44%	38%

Explanations: Tabulated market-betas (columns) are based on the previous calendar years. Tabulated returns (rows) are based on subsequent days' stock market rates of return, with stock market rate-of-return breakpoints chosen based on unconditional percentiles. The sample consists of all CRSP stocks from 1927 to 2015, as described in Table 1. The first categorized return is in 1928/01/03, the last in 2015/12/31.

Interpretation: On bad market days, low-beta stocks, by-and-large, did not perform much worse than expected. Their "hedge" did not collapse. Low-beta stocks were good insurance.

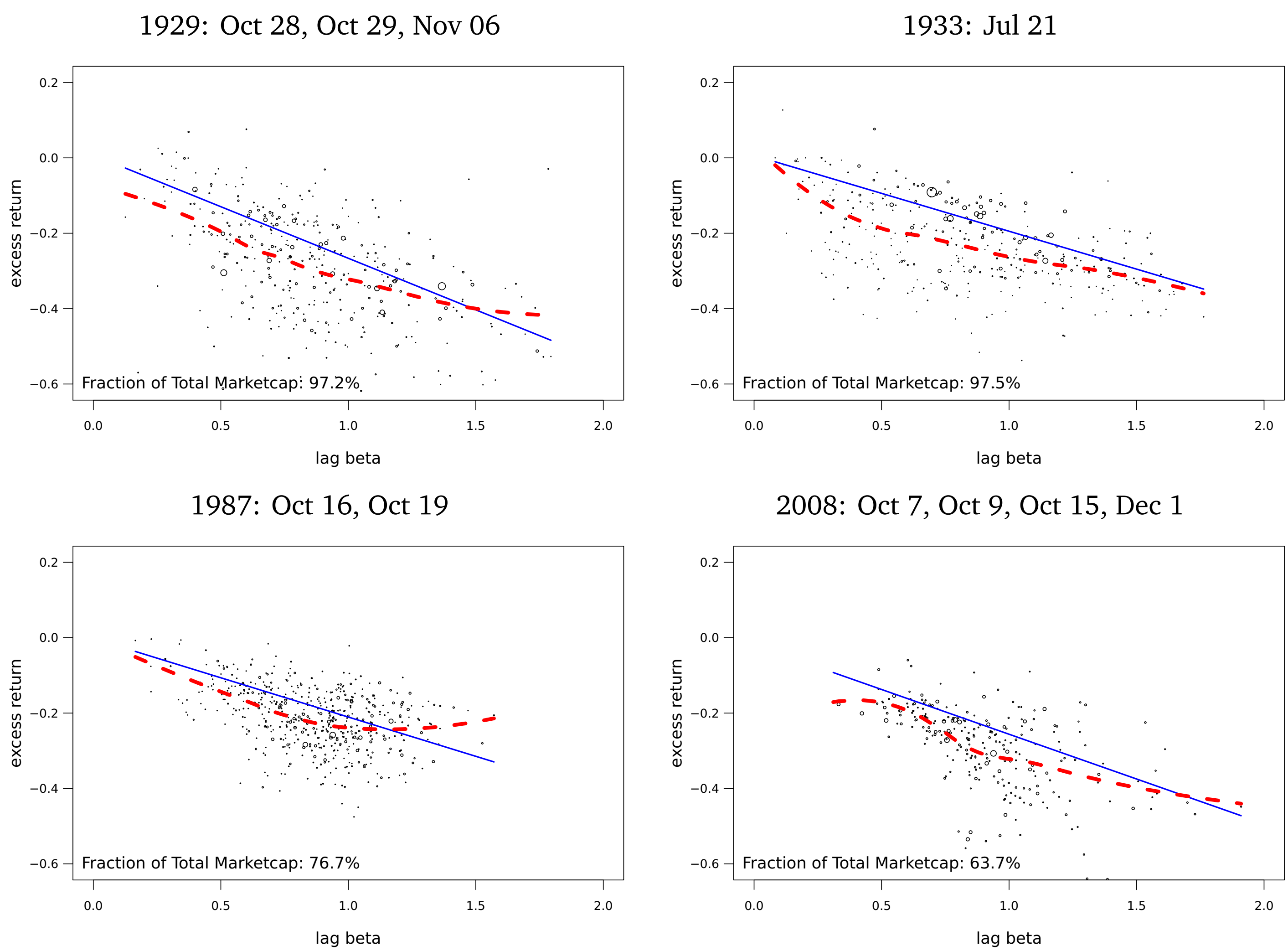
Figure 1: Performance of Value-Weighted Portfolios In Three Ex-Ante Beta ($\hat{b}_{y-1}$) Categories in Bear Markets (–1% or worse)



Explanations: This figure shows the average annual portfolio performances of low-beta stocks ($\hat{b}_y < 0.5$, in blue), mid-beta stocks ($0.5 < \hat{b}_y < 1.0$, in green), and high-beta stocks ($\hat{b}_y > 1.0$, in red) for value-weighted portfolios, but only on those days on which the stock market declined by at least 1%.

Interpretation: Low-beta stocks performed better than high-beta stocks on market-down days. Their hedge has worked well throughout the sample period.

Figure 2: Stock Return Performance During Market Crashes of 1,000 Largest Stocks Vs. Ex-Ante Beta $\hat{b}_{y-1}$



Explanations: Market and individual stock returns were compounded over multi-day episodes. The plots contain only the largest 1,000 stocks by market cap at the end of the ex-ante beta calculation period. The size of the points indicates their market caps. The blue line is the **fitted** expected rate of return given the market rate of return and lagged market beta. The red dashed line is an equal-weighted loess fit through the actual return performance points.

Interpretation: In all four episodes, lower-beta stocks outperformed higher-beta stocks. Their hedge did not collapse. However, in 1987, high-beta stocks with betas above 1 did not underperform stocks with beta of about 1.

Table 3: Descriptive Statistics For Betas

Panel A: Summary Statistics

	Variable	All Stocks Mean	All Stocks SD	"Low-Volatility" Stocks Mean	"Low-Volatility" Stocks SD
(All Days)	$\hat{b}_y$	0.67	0.60	0.67	0.54
	#Days	249	23	253	16
(Pos-Market Days)	$\hat{b}_y^+$	0.59	0.80	0.61	0.64
	#Days	133	17	135	15
(Neg-Market Days)	$\hat{b}_y^-$	0.75	0.79	0.72	0.62
	#Days	115	16	116	14
(Delta Beta)	$[\hat{b}_y^- - \hat{b}_y^+]$	–0.16	0.86	–0.11	0.57
(Abs Delta Beta)	$\lvert[\hat{b}_y^- - \hat{b}_y^+]\rvert$	0.55	0.69	0.40	0.43
	(Lagged) RMSE/Day	2.59%	1.82%	1.78%	0.76%
	Number	348,629		241,240	
	Years	1927 to 2016			

Panel B: Contemporaneous Regressions (Multicollinearity Diagnostic)

All Stocks	$\hat{b}_y$	=	0.15	+ $0.40 \times \hat{b}_y^+$	+ $0.39 \times \hat{b}_y^-$
	(SE)		(0.001)	(0.001)	(0.001)
$DF = 348,626$ $R^2 = 77\%$ RMSE $= 0.287$					
Only Low-Volatility Stocks	$\hat{b}_y$	=	0.08	+ $0.45 \times \hat{b}_y^+$	+ $0.43 \times \hat{b}_y^-$
	(SE)		(0.001)	(0.001)	(0.001)
$DF = 241,237$ $R^2 = 86\%$ RMSE $= 0.201$					

Explanations: These are calendar-year OLS market-betas. The OLS "$\hat{b}_y^+$" are estimated only from days on which the stock market increased; the OLS "$\hat{b}_y^-$" are estimated only from days on which the stock market decreased. Table 1 describes the sample. Delta beta is the difference between "$\hat{b}_y^+$" and "$\hat{b}_y^-$." "Low Volatility" always refers to stocks that had an RMSE in the (ex-ante) OLS market model calendar year estimation below the 75th percentile.

Interpretation: Panel A shows that there is a good spread between $\hat{b}_y^+$ and $\hat{b}_y^-$ for most stocks. Panel B shows that the set $(\hat{b}_y^-, \hat{b}_y^+)$ is highly correlated—but not degeneratively multicollinear—with the plain all-days $\hat{b}_y$.

Table 4: Predicting Betas With Preceding Betas

Panel A: Coefficients Predicting Plain All-Days Betas $\hat{b}_y$ with Various Lagged Betas ($\hat{b}_{y-1}$, $\hat{b}^-_{y-1}$, $\hat{b}^+_{y-1}$)

Dependent Variable↓	Reg Method	All Stocks $\hat{b}_{y-1}$	All Stocks $\hat{b}^+_{y-1}$	All Stocks $\hat{b}^-_{y-1}$	All Stocks R^2	Low-Volatility Stocks $\hat{b}_{y-1}$	Low-Volatility Stocks $\hat{b}^+_{y-1}$	Low-Volatility Stocks $\hat{b}^-_{y-1}$	Low-Volatility Stocks R^2
All-Mkt ($\hat{b}_y$)	Panel	0.69			47%	0.79			62%
"Plain Beta"	FM	0.71				0.79			
	Panel		0.26	0.28	35%		0.35	0.35	53%
	FM		0.29	0.31			0.35	0.37	
	Panel	0.74	–0.04	–0.01	47%	0.85	–0.04	–0.02	62%
	FM	0.82	–0.06	–0.06		0.91	–0.07	–0.06	

Panel B: Coefficients Predicting Down-Betas $\hat{b}^-_y$ with Various Lagged Betas ($\hat{b}_{y-1}$, $\hat{b}^-_{y-1}$, $\hat{b}^+_{y-1}$)

Dependent Variable↓	Reg Method	All Stocks $\hat{b}_{y-1}$	All Stocks $\hat{b}^+_{y-1}$	All Stocks $\hat{b}^-_{y-1}$	All Stocks R^2	Low-Volatility Stocks $\hat{b}_{y-1}$	Low-Volatility Stocks $\hat{b}^+_{y-1}$	Low-Volatility Stocks $\hat{b}^-_{y-1}$	Low-Volatility Stocks R^2
Neg-Mkt ($\hat{b}^-_y$)	Panel	0.64			24%	0.72			40%
"Down Beta"	FM	0.65				0.72			
	Panel			0.39	15%			0.56	30%
	FM			0.48				0.59	
	Panel		0.21	0.30	18%		0.27	0.39	35%
	FM		0.23	0.33			0.27	0.39	
	Panel	0.67	–0.06	0.04	24%	0.74	–0.07	0.06	40%
	FM	0.69	–0.07	0.03		0.74	–0.07	0.05	

Explanations: These are (auto-)coefficients explaining calendar-year market-betas with lagged market-betas. "Low Volatility" always refers to stocks that had an RMSE in the (ex-ante) OLS market model calendar year estimation below the 75th percentile. The "Panel" lines report simple pooled regression coefficient estimates. The "FM" lines report Fama-Macbeth style coefficient estimates. Not shown, the panel standard errors range from 0.001 to 0.002. Newey-West heteroskedasticity corrections increase this range to 0.001 to 0.005. Intercepts are included but not reported.

Interpretation: When competing with the lagged (plain all-days) beta ($\hat{b}_{y-1}$), neither lagged up-beta ($\hat{b}_{y-1}$) nor down-beta ($\hat{b}^-_{y-1}$) can improve the prediction of either plain betas ($\hat{b}_y$) or down betas ($\hat{b}^-_y$).

Table 5: Autocoefficient (Decay) of Up- Minus Down-Beta) ($[\hat{b}^{-}_{y}-\hat{b}^{+}_{y}]$)

Panel A: All Stocks

Dependent Variable↓	Reg Method	Constant	Independent (Lag 1Y and 2Y) $[\hat{b}^{-}_{y-1}-\hat{b}^{+}_{y-1}]$	$[\hat{b}^{-}_{y-2}-\hat{b}^{+}_{y-2}]$	D.F.	R^2
All Years	Panel	0.140	0.069		316,585	0.4%
$[\hat{b}^{-}_{y}-\hat{b}^{+}_{y}]$	FM	0.113	0.086			
1962–	Panel	0.151	0.065		287,580	0.4%
	FM	0.158	0.070			
1962–	Panel	0.148		0.048	259,936	0.2%
	FM	0.155		0.056		
1962–	Panel	0.139	0.062	0.044	259,648	0.6%
	FM	0.143	0.067	0.050		

Panel B: Only Low Volatility Stocks

Dependent Variable↓	Reg Method	Constant	Independent (Lag 1Y and 2Y) $[\hat{b}^{-}_{y-1}-\hat{b}^{+}_{y-1}]$	$[\hat{b}^{-}_{y-2}-\hat{b}^{+}_{y-2}]$	D.F.	R^2
All Years	Panel	0.095	0.092		240,239	1.0%
$[\hat{b}^{-}_{y}-\hat{b}^{+}_{y}]$	FM	0.086	0.100			
1962–	Panel	0.103	0.087		215,220	0.9%
	FM	0.117	0.080			
1962–	Panel	0.102		0.061	197,427	0.5%
	FM	0.116		0.059		
1962–	Panel	0.093	0.085	0.053	197,292	1.3%
	FM	0.106	0.077	0.051		

Explanations: These are (auto-)coefficients, explaining the difference in calendar-year market-betas of up-betas minus down-betas ($[\hat{b}^{-}_{y}-\hat{b}^{+}_{y}]$) with up to two lags of themselves. "Low Volatility" always refers to stocks that had an RMSE in the (ex-ante) OLS market model calendar year estimation below the 75th percentile. The "Panel" lines report simple pooled regression coefficient estimates. The "FM" lines report Fama-Macbeth style coefficient estimates. The panel standard errors range from 0.001 to 0.002. Newey-West heteroskedasticity corrections increase this range to 0.001 to 0.005. (The Fama-Macbeth standard errors, which should not be used for inference, range from 0.1 to 0.2.) Intercepts are included but not reported.

Interpretation: The up-down beta difference is not very persistent. The auto-decay pattern suggests that both errors-in-variables and underlying mean-reverting changes in the difference play roles.

Table 6: Explaining Stocks' Rates of Return (r_y) in Fama-MacBeth Regressions

Panel A: ACX, Table 2, Model I (with 6 controls)

	Sample	Coef on $\hat{b}$ [T]	Stock-Mos	Mos
Original	ACX	+0.177 [+8.19]		
Our Replication	ACX	+0.188 [+6.16]	542,528	451
Ex-Ante Betas	ACX	–0.025 [–1.77]	521,655	439
Ex-Ante Betas	Extd	–0.059 [–4.89]	2,271,797	637

Panel B: ACX, Table 2, Model II (without controls)

	Sample	Coef on $\hat{b}^-$ [T]	Coef on $\hat{b}^+$ [T]	Stock-Mos	Mos
Original	ACX	+0.069 [+7.17]	–0.029 [–4.85]		
Our Replication	ACX	+0.080 [+5.92]	–0.015 [–1.69]	609,068	451
Ex-Ante Betas	ACX	+0.005 [+0.45]	–0.006 [–0.75]	561,714	439
Ex-Ante Betas	Extd	–0.001 [–0.14]	–0.019 [–3.15]	2,875,367	1,081

Panel C: ACX, Table 2, Model V (with 6 controls)

	Sample	Coef on $\hat{b}^-$ [T]	Coef on $\hat{b}^+$ [T]	Stock-Mos	Mos
Original	ACX	+0.062 [+6.00]	+0.020 [+2.33]		
Our Replication	ACX	+0.088 [+6.10]	+0.002 [+0.22]	542,528	451
Ex-Ante Betas	ACX	–0.009 [–1.56]	–0.005 [–0.78]	521,655	439
Ex-Ante Betas	Extd	–0.022 [–3.53]	–0.020 [–3.63]	2,271,797	637

Explanations: These Fama-Macbeth regressions (with Newey-West corrections) mimick those in Table 2 of Ang, Chen, and Xing (2006) as closely as possible. Thus, they use their measurement choices. For more details, refer to ACX. As in ACX, the six controls are log-size, bk-mkt, past ret, std dev, coskewness, and cokurtosis. The controls (except past returns) are mostly contemporaneous with the dependent variable. The betas and controls are winsorized at the 1% and 99% level in each month. The "ACX" sample uses 1963–2001, NYSE, code 10,11 stock returns. The extended sample uses stock returns from 1927 to 2016, all CRSP exchanges, code 10,11 sample in Panel B, and stock returns from July 1963 to 2016 in Panels A and C.

Interpretation: With ex-post betas, our replications come very close to the coefficients reported in ACX. They suggest that (ex-post) down-betas carried a large positive premium. This premium vanishes when ex-post betas are replaced with ex-ante betas.

Table 7: All-Days Betas and Full-Sample Betas in Fama-Macbeth Regressions

Panel A: Contemporaneous Betas (as in Ang, Chen, and Xing (2006))

		All Stocks, Gamma on			Low-Vol Stocks, Gamma on		
Sample	Mos	$\hat{b}^-_y$	$\hat{b}^+_y$	$\hat{b}_y$	$\hat{b}^-_y$	$\hat{b}^+_y$	$\hat{b}_y$
ACX	451	+0.088***	+0.002		+0.079***	+0.013	
Extd	649	+0.054***	–0.000		+0.069***	+0.012	
ACX	451	+0.030*	–0.056***	+0.213***	+0.032**	–0.045***	+0.219***
Extd	649	+0.013	–0.037***	+0.150***	+0.033***	–0.024*	+0.145***

Panel B: Full-Sample and 9-Year-Sample Betas (as in Lettau, Maggiori, and Weber (2014))

			All Stocks, Gamma on			Low-Vol Stocks, Gamma on		
$\hat{b}$ Years	Sample	Mos	$\hat{b}^-_\star$	$\hat{b}^+_\star$	$\hat{b}_\star$	$\hat{b}^-_\star$	$\hat{b}^+_\star$	$\hat{b}_\star$
Full	ACX	451	–0.047*	+0.099***		–0.040*	+0.067***	
	Extd	649	–0.008	+0.113***		–0.017	+0.090***	
	ACX	451	–0.087**	+0.059*	+0.087	–0.071*	+0.035	+0.068
	Extd	649	–0.138***	–0.007	+0.273***	–0.102***	+0.007	+0.178***
-4...+4	ACX	439	+0.014	+0.086**		+0.029	+0.058**	
	Extd	589	+0.026	+0.069**		+0.004	+0.067**	
	ACX	439	–0.057	+0.026	+0.146**	–0.021	+0.014	+0.114
	Extd	589	–0.061	+0.002	+0.170**	–0.067	+0.006	+0.146**

Panel A: Ex-Ante Betas

		All Stocks, Gamma on			Low-Vol Stocks, Gamma on		
Sample	Mos	$\hat{b}^-_{y-1}$	$\hat{b}^+_{y-1}$	$\hat{b}_{y-1}$	$\hat{b}^-_{y-1}$	$\hat{b}^+_{y-1}$	$\hat{b}_{y-1}$
ACX	439	–0.009	–0.005		–0.007	–0.010	
Extd	637	–0.022***	–0.020***		–0.014**	–0.016**	
ACX	439	+0.002	+0.011	–0.038	+0.013	+0.005	–0.044*
Extd	637	–0.005	–0.004	–0.050*	+0.002	+0.001	–0.046*

(t-statistics in parentheses. *: 2.0<$|t|$<2.6; **: 2.6<$|t|$<3.3; ***: $|t|$>3.3.)

(Table continues)

(Table 7 continues.)

Explanations: This extends Table 6 Panel C (i.e., with six controls, though unreported coefficients). They add the plain all-days beta and consider betas akin to those in Lettau, Maggiori, and Weber (2014). These "full-sample betas" use one single time-invariant beta estimate for each stock, estimated over the entire sample. The "9-year sample" uses 4 years before, the contemporaneous year, and 4 years after when estimating betas (from daily stock returns). The controls are always held constant relative to the dependent variable (the rate of return) that is to be explained. Following ACX, these controls are mostly contemporaneous with the dependent variable.

Interpretation: The *ex-post* down-beta $\hat{b}_y^-$ can no longer positively predict stock returns in the extended sample without volatility restriction. The down-beta coefficients are typically zero or negative when ex-ante or full-sample betas are used, and insignificant if 9-year betas are used.

Table 8: Fama-French Regressions Explaining Monthly Rates of Return $r_{[p,m]}$ on Zero-Investment Test Portfolios Formed By Quartiles of Stocks' Ex-Ante Down-Beta Estimates ($\hat{b}^{-}_{p,m-1}$)

Panel A: ACX Sample, 439 Months

			All Stocks					Low-Volatility Stocks		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Alpha	0.143	0.260	–0.153	–0.057	–0.030	0.291	0.460***	0.031	0.159	0.036
(T-stat)	(0.84)	(1.82)	(–1.31)	(–0.52)	(–0.27)	(1.91)	(3.34)	(0.29)	(1.48)	(0.34)
Mkt–RF			0.58	0.47	0.42			0.51	0.45	0.43
SMB		0.35		0.23	0.28		0.08		–0.04	0.08
HML		–0.48		–0.23	–0.06		–0.48		–0.24	–0.11
Mom					0.02					0.07
RMW					0.11					0.32
CMA					–0.40					–0.20
$\overline{R}^2$	na	33%	53%	61%	65%	na	21%	50%	54%	59%

Panel B: Extended CRSP Sample

			All Stocks					Low-Volatility Stocks		
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Alpha	–0.137	–0.270	–0.587***	–0.552***	–0.087	0.068	–0.200	–0.448***	–0.507***	–0.301**
(T-stat)	(–0.85)	(–1.78)	(–5.16)	(–5.01)	(–0.78)	(0.44)	(–1.48)	(–5.30)	(–6.31)	(–3.26)
Mkt–RF			0.69	0.68	0.52			0.79	0.73	0.63
SMB		0.59		0.26	0.33		0.61		0.25	0.30
HML		0.01		–0.21	–0.22		0.35		0.12	–0.04
Mom					–0.15					–0.04
RMW					–0.55					0.11
CMA					–0.55					–0.22
$\overline{R}^2$	na	13%	50%	54%	72%	na	23%	71%	74%	70%
Mos	1,081	1,081	1,081	1,081	643	1,081	1,081	1,081	1,081	643

(t-statistics in parentheses. *: $2.0<|t|<2.6$; **: $2.6<|t|<3.3$; ***: $|t|>3.3$.)

Explanations: The test portfolio is long in the quartile of stocks with the highest downbeta and short in the quartile of stocks with the lowest downbeta. Each specification is the output from a single time-series regression, with factors obtained from Ken French's website. "Low Volatility" always refers to stocks that had an RMSE in the (ex-ante) OLS market model calendar year estimation below the 75th percentile.

Interpretation: The portfolio of high-downbeta stocks underperformed the portfolio of low-downbeta stocks. This is inconsistent with the ACX down-beta model.

Table 9: Performance of and Key Results for Alternative Beta Estimators

Panel A: Contemporaneous Correlations Among Betas

	1-Year OLS	2-Year OLS	Dimson	Vasicek	Levi-Welch
1-Year	100				
2-Year	92	100			
Dimson	82	77	100		
Vasicek	97	92	82	100	
Levi-Welch	97	92	82	100	100

Panel B: Predicting RMSE Proxying Future Betas With Prevailing Betas

	Predicted				
Predictor (Lagged)	1-Year OLS	2-Year OLS	Dimson	Vasicek	Levi-Welch
1-Year	0.470	0.246	0.580	0.420	0.411
2-Year	0.442	0.270	0.556	0.383	0.367
Dimson	0.570	0.438	0.628	0.531	0.524
Vasicek	0.424	**0.219**	0.545	0.353	0.332
Levi-Welch	**0.416**	0.234	**0.536**	**0.332**	**0.283**
Best Estimator	LW	VCK	LW	LW	LW

(continues)

Panel C: Key Results With Different Beta Estimators

			Reported 1-Year OLS	2-Year OLS	Dimson	Vasicek	Levi-Welch
Tbl 2	Extreme Bear	Low $\hat{b}_{y-1}$	–0.81%	–0.79%	–0.77%	–0.80%	–0.76%
	Average Realized	Med $\hat{b}_{y-1}$	–1.92%	–1.93%	–1.76%	–1.92%	–1.94%
	Performance	High $\hat{b}_{y-1}$	–3.10%	–3.13%	–2.82%	–3.17%	–3.29%
Tbl 4	FM Predicting	$\hat{b}_{y-1}$	0.74	0.76	0.63	0.62	0.46
	Downbeta ($\hat{b}^{-}_{y}$)	$\hat{b}^{+}_{y-1}$	–0.07	–0.10	–0.04	–0.05	–0.05
	Low-Vol Stocks	$\hat{b}^{-}_{y-1}$	0.05	0.05	0.04	0.08	0.08
Tbl 7	FM Predicting	$\hat{b}_{y-1}$	0.273***	Same	0.214***	0.279***	0.349***
Extd Full	Future Returns	$\hat{b}^{-}_{y-1}$	–0.138***	Same	–0.090***	–0.142***	–0.236***
Sample	w/ controls	$\hat{b}^{+}_{y-1}$	0.002	Same	–0.025	0.008	0.014
Tbl 7	FM Predicting	$\hat{b}_{y-1}$	–0.044*	–0.035	–0.021	–0.047*	–0.059*
ACX Ex-Ante	Future Returns	$\hat{b}^{-}_{y-1}$	+0.013	+0.006	+0.001	+0.015	+0.025
Low Vltlty	w/ controls	$\hat{b}^{+}_{y-1}$	+0.005	+0.004	+0.007	+0.005	+0.008
Tbl 8 (9)	FF3 Model, Low Vol	α	–0.507***	–0.540***	–0.420***	–0.504***	–0.504***

Explanations: This table shows the performance of and key results with different beta estimators. The Dimson estimator is from Dimson (1979). The Vasicek estimator is from Vasicek (1973). The LW estimator is from Levi and Welch (2017). Panel B calculates the RMSE when a prevailing beta of one type is used to proxy one-for-one for a future beta of another type. The details of the regressions in Panel C are easiest to understand by referring back to the detailed descriptions in the original tables, which also contain the same numbers as those reported in the "Reported 1-year OLS" column in Panel C.

Interpretation: All results from earlier tables are very robust with respect to the beta estimator.

Table 10: Fama-MacBeth Regressions Explaining Asset Class Portfolios, as in LMW

	Constant	Lambda (on $\hat{b}$)	Lambda– (on $\hat{b}^{-} - \hat{b}$)	T-stat		
	Coef [SE]	Coef [SE]	Coef [SE]	Fixed	CAPM	Months
LMW Table 5, Model (2): 6 Currencies						
Replication, Full-Sample Betas	Fixed @ 0.0	Fixed @ 0.39	2.18 [0.79]	2.8**	2.8**	435
36-mo Ex-Ante Betas	Fixed @ 0.0	Fixed @ 0.46	0.29 [0.29]	1.0	1.1	399
36-mo Ex-Ante Betas, Unfixed	–0.01 [0.14]	0.61 [0.85]	0.31 [0.25]	1.3		399
LMW Table 5, Model (4): Developed 6 Currencies						
Replication, Full-Sample Betas	Fixed @ 0.0	Fixed @ 0.39	2.34 [1.06]	2.2*	2.2*	435
36-mo Ex-Ante Betas	Fixed @ 0.0	Fixed @ 0.46	–0.29 [0.46]	–0.6	–0.6	399
Unfixed, 36-mo Ex-Ante Betas	–0.07 [0.20]	0.23 [0.90]	0.00 [0.35]	0.0		399
LMW Table 5, Model (6): 6 Currencies + 6 Equities						
Replication	Fixed @ 0.0	Fixed @ 0.39	1.41 [0.80]	1.8	3.5***	435
36-mo Ex-Ante Betas	Fixed @ 0.0	Fixed @ 0.46	0.04 [0.24]	0.2	–0.2	399
Unfixed, 36-mo Ex-Ante Betas	0.18 [0.10]	0.28 [0.25]	0.12 [0.15]	0.8		399
6 Zero Anchors + 6 Equities						
6 risk-free assets ($R_{i,t}{=}r_{f,t} \Leftrightarrow XR_{i,t}{=}0.0$; $\hat{b}_y{=}\hat{b}^{-}_y{=}0$)						
Full-Sample Betas	Fixed @ 0.0	Fixed @ 0.39	1.27 [0.90]	1.4	2.8**	435
Unfixed, 36-mo Ex-Ante Betas	0.02 [0.01]	0.44 [0.24]	–0.10 [0.22]	–0.5		399
6 cash holdings ($R_{i,t}{=}0 \Leftrightarrow XR_{i,t}{=}-r_{f,t}$; $\hat{b}_y{=}\hat{b}^{-}_y{=}0$)						
Full-Sample Betas	Fixed @ 0.0	Fixed @ 0.39	1.27 [0.90]	1.4	2.8**	435
Unfixed, 36-mo Ex-Ante Betas	–0.47 [0.01]	0.90 [0.24]	–0.15 [0.22]	–0.7		399

(t-statistics in parentheses. *: $2.0<|t|<2.6$; **: $2.6<|t|<3.3$; ***: $|t|>3.3$.)

Explanations: The first three groups mimic the models in LMW Table 5. The last two rows in each group use ex-ante market-beta estimates that are based on the most recent prevailing (rolling) 36 months of returns. The last two groups replace the currency portfolio with alternative zero anchors, either with investments in the risk-free asset ($R_{i,t}{=}r_{f,t}$) or with six “under-the-mattress” cash holdings ($R_{i,t}{=}0$). Both anchors have betas and down-betas of zero. (Not shown, the inference is the same if the replacement portfolio is a repeated simple fair-coin-toss gamble.) The “fixed” T-stat holds the to-be-realized excess rate of return on the market at the average, while the "CAPM" T-stat uses the realized value. (The latter is equivalent to explaining CAPM residuals.) The coefficient is the same, but the T-statistic changes.

Interpretation: Down-betas lose significance when they are calculated from a-priori asset return data. The inference is almost identical when we replace currency portfolios with portfolios that are simply uncorrelated with the stock market.

Table 11: Fama-Macbeth Regressions Explaining Asset Class Portfolios in Additional Sets and with Different Specifications

					w/Currencies			w/Zeros	
Test Set	MOS	LMW	Full	Ante	LMW	Full	Ante	Unc	Ante
6 Currencies	435	T5 (M2)	2.8**	1.3					
6 Dvlpd Currencies	435	T5 (M4)	2.2*	0.0	na	2.8**	1.4	2.2	1.0
6 Equities	435	na	1.4	–1.1	T5 (M6)	1.8	0.8	1.4	–0.7
5 Commodities	420	na	2.5*	0.9	T6 (M2)	2.7**	1.6	2.5*	2.4*
6 Sovereigns	183	na	2.0*	2.0*	T6 (M6)	2.0*	2.0*	2.0*	1.2
5 Commodities, 6 Equities	420	na	2.6**	1.1	T6 (M4)	2.8**	1.7	2.6**	1.2
6 Equities, 6 Sovereigns	183	na	1.9	2.4*	T6 (M8)	1.9	2.8**	1.9	2.2*
5 Commodities, 6 Sovereigns	168	na	1.7	0.4	na	1.8	0.9	1.7	0.1
All Three Asset Sets	168	na	1.7	1.5	na	1.7	1.7	1.7	1.2

(t-statistics in parentheses. *: 2.0<$|t|$<2.6; **: 2.6<$|t|$<3.3; ***: $|t|$>3.3.)

Explanations: The “LMW” columns refer to the table and model in Lettau, Maggiori, and Weber (2014). The “Full” columns show our replication of the T-statistics in the LMW *full-sample* down-beta estimates from the Fama-Macbeth regressions, with their zero restriction on the intercept and their market excess rate of return to the *average* market premium. The “Ante” columns use *rolling* 36-month betas instead (and thus have 36 fewer months). The regressions are analogous to those in Table 10. Thus, e.g., the 2.8,1.3 result in the first line here can also be seen in Table 10.

Interpretation: Currencies, commodities, and equities have significance with full-sample betas but typically lose it with ex-ante rolling betas. Sovereigns sometimes perform even better with ex-ante betas. For the most part, currencies could be replaced with any market-uncorrelated savings or gambling vehicle.

Table 12: Inspection of Six Sovereign Bond Portfolios

			Credit Rating					
		XMKT	High	2	3	4	5	Low
Average Return			0.19	0.37	0.57	0.62	0.94	1.23
all months	$\hat{b}_y$		0.24	0.29	0.51	0.24	0.39	0.71
	$\hat{b}_y^-$		0.92	0.89	1.78	0.84	1.07	1.97
	$\hat{b}_y^- - \hat{b}_y$		0.69	0.60	1.27	0.60	0.68	1.26
Russian Crisis	1998/08	–18%	–20%	–26%	–41%	–14%	–16%	–26%
Financial Crisis	2008/10	–21%	–13%	–7%	–19%	–14%	–20%	–37%
excl. 2 mos	$\hat{b}_y$		0.10	0.18	0.28	0.13	0.26	0.49
	$\hat{b}_y^-$		–0.08	0.07	0.34	–0.21	–0.09	0.43
	$\hat{b}_y^- - \hat{b}_y$		–0.18	–0.11	0.05	–0.34	–0.34	–0.06

Explanations: The sovereign bond portfolio returns are originally from Borri and Verdelhan (2011). The six portfolios are ordered by nations' credit ratings. Their returns are quoted in U.S. dollars. The table shows overall full-sample statistics with and without two extreme months. XMKT is the monthly excess rate of return on the stock market in the two crises.

Interpretation: Betas and down-betas of sovereign bonds are unusually high compared to other types of bonds, because these sovereign bond portfolios fell as much or more than equities in the two crisis episodes.