Currency Choice in Contracts*

Andres Drenik
Columbia University
ad337@columbia.edu

Rishabh Kirpalani
Pennsylvania State University
rishabh.kirpalani@psu.edu

Diego J. Perez
New York University
diego.perez@nyu.edu

October 2018

Abstract

We study a model in which agents choose the currency in which to denominate contracts, and the government chooses the inflation rate. The optimal choice of currency trades-off the price risk of each currency with how this risk covaries with the relative consumption needs of the agents signing the contract. When a larger share of private contracts are denominated in local currency, the government can use inflation to redistribute resources more effectively within the economy which, in turn, makes local currency more attractive as a unit of account for private contracts. The use of local currency is more likely when there is low domestic policy risk. Consistent with recent policy initiatives, policies that encourage the denomination of contracts exclusively in local currency can be desirable, since private agents do not internalize the complementarities between private actions and those of the government. We also use our model to explain observed hysteresis of dollarization that occurred in several Latin American countries, and the wide use of the dollar in international trade contracts.

*We thank seminar audiences at Columbia and the SED for valuable comments.

1 Introduction

One of the central roles of currency is to serve as a unit of account in credit contracts. While in most countries this role is exclusively fulfilled by the local currency, several countries also rely on a foreign currency (for example the dollar) to denominate domestic contracts. The coexistence of multiple currencies in denominating contracts is especially relevant in emerging economies, which are often subject to high levels of government policy risk. In this paper, we address two related questions on the role of currencies as units of account. First, what determines the currency choice of contracts among private agents? Second, how does this collective currency choice affect the government's conduct of monetary policy?

To answer these questions, we study a general equilibrium model in which agents choose the currency in which to denominate contracts, and the government chooses the inflation rate. These contracts involve the provision of a good in exchange for a future payment in some currency. The optimal choice of currency considers the price risk of each currency and how this risk covaries with the relative consumption needs of the agents signing the contract. The price of the local currency is chosen ex-post by a benevolent government and depends on the use of local currency in private contracts. A key feature of this model is a source of complementarities between the actions of private agents and those of the government. When a larger share of private contracts is denominated in local currency, the government can use inflation to redistribute resources more effectively within the economy which, in turn, makes local currency more attractive as a unit of account for private contracts. Local governments are also subject to exogenous policy risk which reduces the attractiveness of denominating contracts in local currency. We show that the set of equilibria depends crucially on the level of policy risk and multiple equilibria can emerge. We also ask if competitive equilibria are efficient and argue that there might be a role for policy to encourage private agents to denominate contracts exclusively in local currency. This is because private agents do not internalize the complementarities described above. This might help explain recent policy initiatives in many emerging economies aimed at discouraging or prohibiting the use of foreign currency in domestic contracts.

We begin by characterizing the optimal bilateral credit contract. Agents engage in credit contracts to exploit gains from trade of a special good. Credit contracts stipulate the amount of a special good that is provided at the date the contract is signed, in exchange for an amount of local and/or foreign currency to be paid in the future. Currencies serve only as units of account, since the actual payment in the future is made in terms of a numeraire good. Agents receive taste shocks which affect their marginal utility of consuming the numeraire good. This increases the desirability of currencies whose price (measured in terms of the numeraire good) covaries with these shocks. We assume that default is costly, which in turn reduces the desirability of currencies with higher price risk. The optimal currency choice features a trade-off between these two forces.

The government's optimal choice of inflation trades off the benefits of redistributing resources between agents with the costs of deviating from a target. The benefits of redistribution arise from the differences in taste shocks of creditors and debtors, and capture a need for discretion in monetary policy. The optimal inflation choice redistributes resources between creditors and debtors in an ex-post efficient way. For example, when debtors have a high marginal utility (relative to the creditors) the government chooses higher inflation to lower the burden of debt payments. The degree of redistribution that takes place depends positively on the use of local currency in private contracts. The government's inflation choices are also affected by the degree of policy risk, modeled as a stochastic inflation target, which is independent of the currency choice. The choice of inflation induces a distribution of local currency prices which affect the ex-ante benefits of local currency relative to the foreign one.

We fully characterize the set of equilibria for different levels of policy risk. For low levels of policy risk there is a unique equilibrium in which all contracts are denominated in local currency while for high levels of this risk, all contracts denominated in foreign currency. For intermediate levels of policy risk, there are three equilibria: two which involve exclusive use of either the local or foreign currency and a third interior one in which both the local and foreign currency are used. This characterization helps rationalize why countries with low levels of policy risk like the U.S., Europe, or Japan rely exclusively on their local currency as a unit of account in domestic contracts. In contrast, countries with high policy risk such as those in Latin America and Eastern Europe tend to partially or fully rely on foreign currency as a unit of account.

Both recently and historically, many countries have introduced policy initiatives to either encourage or discourage the use of foreign currency as a unit of account. In recent years there have been policy initiatives in a large number of emerging market economies that discourage the use of foreign currency as a unit of account. Two such examples are Brazil and Colombia that currently prohibit the denomination of bank deposits and loans in foreign currency. Other examples of similar initiatives include policies in Croatia, Hungary, and Poland which either heavily regulated or forced conversion of foreign currency housing loans to domestic currency.[1] In contrast, two decades ago Ecuador and El Salvador fully dollarized their domestic economies.

Our paper can help explain the prevalence of such policy initiatives. We study the problem of a social planner subject to the same constraints as private agents. We find

[1] Another example is Peru which in 2004 prohibited retail price setting in foreign currency.

that the optimal allocation calls for exclusive use of local currency if policy risk is low and exclusive use of foreign currency if policy risk is high. As a result, for regions of the parameter space in which there are multiple equilibria, allocations that involve use of both currencies are dominated by ones in which only one of the currencies is used. In particular, for relatively low values of policy risk the interior equilibrium is dominated by one in which only the local currency is used while for relatively high values of policy risk the interior equilibrium is dominated by one in which only the foreign currency is used. Moreover, the set of parameters under which the former is true is larger than the latter and this difference increasing in the variance of the process governing the taste shocks of the numeraire good.

We then use our model to shed light on the observed hysteresis in the share of foreign currency-denominated contracts. This pattern is most striking in many Latin American economies that still exhibit high levels of financial dollarization in spite of continued success in controlling inflation and inflation risk in the last decade. Figure 2 plots the evolution of deposit dollarization and annual inflation (capped at 100% per annum) for 4 developing countries from 1980 to 2007: Argentina, Bolivia, Peru and Uruguay. All economies went through episodes of rapid increases in the inflation rate, followed by a rapid increase in the fraction of deposits in US dollars. Surprisingly, even though inflation later stabilized, financial dollarization remained high and stable.

To address this empirical pattern, we enhance our baseline model by endowing debtors with claims on local and foreign currency that, as we show, can arise endogenously as a consequence of trading within a credit chain. In this model, currency choice exhibits hysteresis due to the fact that there are benefits of matching the currency of denomination of new debt contracts to the outstanding claims that back the debtors future payments. We illustrate this by showing that even if policy risk gets arbitrarily small, in equilibrium, foreign currency (dollars in this case) will still be used a unit of account. The reason is that it is optimal to match the currency of older contracts and only de-dollarize the claims that are backed with future income.

Next, we extend our model to study currency choice in international trade contracts that involve parties from two different countries. Gopinath (2015) documents that the dollar is widely used as a unit of account in international trade contracts. In particular, countries like Japan and Korea have low inflation risk, low domestic dollarization, and yet have a significant fraction of their international trade contracts denominated in dollars. We extend our model to allow for international trade contracts in which debtors and creditors are from two different countries and contracts can be set in three possible currencies: the currencies of the debtor or creditor country, and a foreign currency (which in this case stands for the dollar). Our model can rationalize the large use of dollars in international contracts relative to domestic contracts. In particular, we show that the range of policy risk for which a unique full dollar equilibrium exists in the model with international trade contracts is larger than the range that determines the unique full dollar equilibrium in the model with only domestic contracts. As a result, for levels of policy risk that are low enough to sustain a unique local currency equilibrium for domestic contracts, equilibria in which international contracts are denominated in foreign currency can exist. The reason is that the benefit for an agent to denominate contracts in the local currency of its trading partner is lower if the partner is from a different country. This is because the government only has incentives to respond to the taste shocks of its own citizens and not those from other countries. In contrast, for domestic contracts, the government responds to the taste shocks of both partners involved, thus raising the benefit of denominating in the local currency.

Finally, we extend our model to allow for strategic default. We show that the ability of private agents to partially insurance against taste shocks via default can increase the relative benefit of denominating in the local currency as it reduces the average cost for the government of providing insurance in states where there is no default.

Related literature. Our paper contributes to the literature that studies the coexistence of currencies in fulfilling the roles of money, and is closely related to papers that study the use of foreign currency as a unit of account in debt contracts. Doepke and Schneider (2017) study the general properties of the optimal unit of account in economies with credit chains. Ize and Levy-Yeyati (2003) and Rappoport (2009) study models to characterize equilibrium levels of financial dollarization. Other papers study the role of currency denomination of debt in models with financial frictions (see Caballero and Krishnamurthy (2003), Schneider and Tornell (2004) and Bocola and Lorenzoni (2017)).[2] These papers stress the use of both currencies in debt contracts given their differential hedging properties associated with exchange rate fluctuations. We contribute to this literature by developing a general equilibrium theory of the joint determination of currency choice in private contracts and government monetary policy that stresses the role of policy risk in the use of local currency as unit of account.

Other strands of literature have focused on the use of currencies for other purposes. Matsuyama et al. (1993) and Uribe (1997) study the use of a foreign currency as a means of payment. Other papers study the implications of full dollarization (for example, Alesina and Barro (2002), Gale and Vives (2002) and Arellano and Heathcote (2010)) or currency areas (for example,Neumeyer (1998), Chari et al. (2015), Aguiar et al. (2015)). A large literature has studied the effects of the currency denomination of prices. Some examples include Engel (2006), Gopinath et al. (2010), Gopinath (2015), and Mukhin (2018) in the

[2]Other papers study the optimal choice of currency for corporate debt (see, for example, Aguiar (2005) and Salomao and Varela (2017)) and sovereign debt (see, for example, Ottonello and Perez (2016)).

case of international prices and Drenik and Perez (2017) in domestic prices.

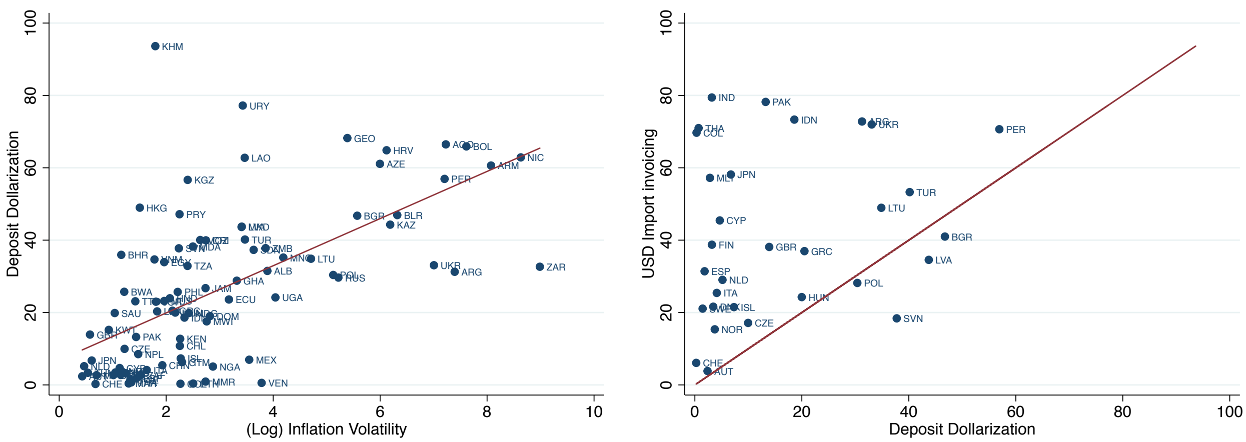
Finally, our paper contributes to a growing literature on the global role of the dollar (see, for example, Farhi and Maggiori (2017), Gopinath and Stein (2017), Chahrour and Valchev (2017), and Maggiori et al. (2018)). We contribute to this literature by jointly analyzing the use of the dollar both in domestic and international contracts.

The rest of the paper is organized as follows. Section 2 presents several empirical facts motivating our research question. Section 3 presents the model and characterizes the equilibrium. Section 4 analyzes the constrained efficient allocation of the economy. In the remaining sections we present extensions of our baseline model to analyze hysteresis in the currency of contracts (section 5), contracts in international trade (section 6) and defaultable contracts (section 7). We conclude in section 8.

2 Empirical motivation

In this section, we motivate our research question by presenting facts about dollarization and briefly summarize the previous literature that empirically studies the determinants of dollarization. Panel 1a of figure 1, shows the cross-country relationship between the share of deposits denominated in US dollars and the volatility of inflation . Financial dollarization is increasing in the volatility of domestic inflation, which is a symptom of high government policy risk. Ize and Levy-Yeyati (2003) show that financial dollarization across countries is positively affected by the relative risk of local currency (captured by the volatility of inflation) and foreign currency (captured by the volatility of the real exchange rate). Further empirical analysis conducted by Nicolo et al. (2003) and Rennhack and Nozaki (2006) supports this result. In addition, these papers find that measures of policy stability are negatively associated with financial dollarization. Our model is able to rationalize these facts by showing that countries with higher policy risk are more likely to use foreign currency and have higher inflation volatility.

A recent literature has shown that the US dollar is the main currency of invoicing in international trade. We find a similar positive relationship between dollarization in international trade contracts (measured by the share of imports of a country, from destinations other than the US, invoiced in US dollars) and the volatility of domestic inflation (see appendix C). We also find that dollarization is more prevalent if we focus on international contracts. In panel 4 of figure 1, we show the share of imports invoiced in US dollars against the share of bank deposits (which includes domestic and international deposits) denominated in US dollars. Most of the observations are above the 45 degree line, indicating the strong prevalence of dollarization in contracts across borders. In section 6 we extend our model to analyze the case of interational contracts and show that the use of foreign currency is more prevalent in international contracts than in domestic ones.



(a) Financial Dollarization and Inflation Volatility

(b) Financial versus International Trade Dollarization

Figure 1: Financial and Trade Dollarization

Notes: Financial dollarization is measured as the share of bank deposits denominated in US dollars. The source of this data is Levy-Yeyati (2006). Inflation volatility is measured as the standard deviation of annual inflation for the period 1980-2017. The source of this data is IFS. Trade Dollarization is computed as the share of imports, from destinations other than the US, invoiced in US dollars. The source of this data is Gopinath (2015).

Another important feature of dollarization is that it exhibits a large degree of hysteresis. This can be seen by analyzing the observed de-dollarization process of several Latin American countries. As shown in figure 2, these countries experienced high levels of policy risk (measured for example, by average levels of inflation and inflation volatility) and high levels of dollarization of private contracts during the 1990s. In early 2000s policy risk significantly subsided, yet the levels of dollarization of credit and deposits only decreased mildly. In section 5 we extend our model to analyze hysteresis. The model is able to predict this behavior since it is optimal to currency-match previously accumulated contracts when these are used to back newly issued debt.

3 Model

3.1 General Environment

There are two periods $1, 2$. The domestic economy is populated by two types of agents: citizens and a government. Citizens are further divided into sellers and buyers, with a unit measure of each.

Buyers have preferences over a special good produced by sellers. Buyers and sellers also value the consumption of a numeraire good which takes place at the end of period 2.

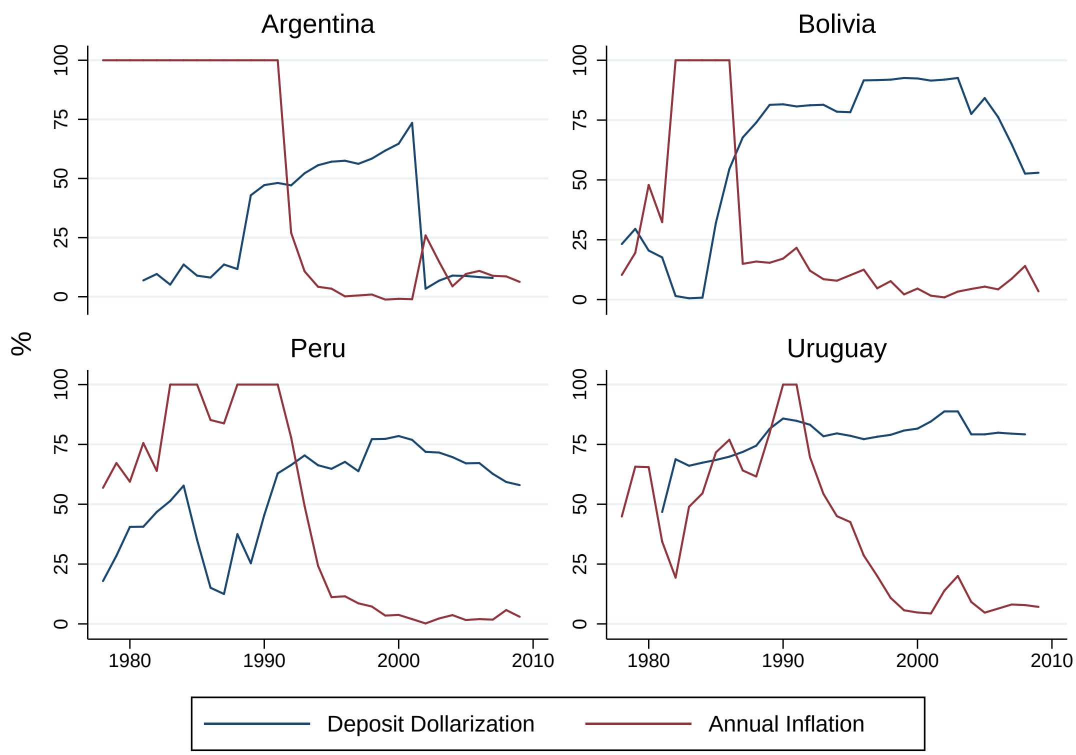


Figure 2: Persistence of Financial Dollarization

*Sources:*Levy-Yeyati (2006) and IFS.

Preferences for the representative seller are given by

$$u_s = -x + \mathbb{E}\left[\theta_s c_s\right] - \mathbb{E}l\left(R_l\right),$$

where x is the special good produced by the seller, c_s is the seller's consumption of the numeraire good, and θ_s is a taste shock which measures the seller's marginal utility of consuming the numeraire good. We denote R_l as the price level in the economy, and $l\left(R_l\right)$ captures inflation costs (we define these in detail below). Preferences for the representative buyer are given by

$$u_b = (1+\lambda)\, x + \mathbb{E}\left[\theta_b c_b\right] - \mathbb{E}l\left(R_l\right)$$

where $1+\lambda$ is the valuation of the special good provided by a seller, c_b is the buyer's consumption of the numeraire good, and θ_b is the buyer's taste shock. The parameter $\lambda > 0$ governs the gains of trading the special good between sellers and buyers. The shocks θ_s and θ_b are independently drawn in period 2 from a distribution with bounded support $\left[\underline{\theta}, \overline{\theta}\right]$ and $\mathbb{E}\left[\theta_i\right] = 1$ for $i = s, b$. The fact that θ_s and θ_b are unknown in period 1 introduces uncertainty in the relative marginal utilities of the numeraire good and gives rise to gains from making relative consumption state-contingent. A high (low) value of θ_b relative to θ_s makes consumption of buyers, relative to sellers, more (less) desirable. As we will see, these taste shocks are a stilized way of generating value in having discretion in government policy. The differences in θ_s and θ_b can capture any reason for why it is socially and privately desirable to shift resources between different agents in the population.

The timing of the model is as follows:

1. In the first period sellers produce a special good for buyers in exchange for the promise of payment in period 2.

2. In the second period rthe taste shocks θ_s and θ_b are realized, the domestic government it chooses its policy which is the aggregate price level, all signed contracts are executed, and consumption of the numeraire good takes place.

Buyers and sellers are endowed with $y > 0$ units of the numeraire good, respectively. Next, we formally define a contract and discuss its properties.

3.2 Bilateral Contracts

A contract between a buyer and a seller consists of a provision of the special good (from the seller to the buyer) in exchange for the promise of future payment (from the buyer to the seller). We impose three important assumptions on the contracting environment. The first is that payments are non-contingent and in particular, cannot depend on the realization of the state (θ_s, θ_b). The second is that payments cannot be made directly in terms of the numeraire good. Instead, payments can only be made in two possible "units of account", which we will call *currencies*. One interpretation of this assumption is that goods are observable but *unverifiable,* as is commonly assumed in the incomplete contracts literature. We will denote the two possible currencies by l (local) and f (foreign). A payment b_l in currency l yields $b_l R_l$ units of the domestic numeraire good in period 2, while a payment b_f in currency f yields $b_f R_f$ units of the domestic numeraire good in period 2. In general, R_l and R_f are random variables from the perspective of private agents that are unknown at the time of the contract being signed. The third assumption is that we assume sufficiently high default costs so that contracts must be *default-free*. In other words, actual payments must equal promised payments in all states of the world. We relax this assumption in Section 7.

Formally, a bilateral contract signed in sub-period i is a the tuple (x, b_l, b_f), where x indicates the units of special good provided to the buyer and (b_l, b_f) are the units of local and foreign currency promised to be paid to the seller at date 2, respectively. Contracts must satisfy the following feasibility constraint

$$b_l R_l + b_f R_f \leqslant y \quad \text{for all} \quad R_l, R_f \tag{1}$$

where $\mathbf{R} \subset \mathbb{R}^2_+$ is the compact set of possible price realizations. This inequality states that for all possible price realizations, the income of the buyer should not exceed the promised repayment. This is equivalent to assuming that the consumption of the numeraire good in period 2 is non-negative. Agents are exposed to risk from uncertainty about aggregate prices. We adopt the notation convention that R_c is the price of a unit of currency c in terms of the numeraire good of the domestic economy. Therefore, a low (high) R_c indicates a high (low) level of domestic inflation in currency c. Prices in local currency R_l in this economy are endogenous and citizens take them as given. Prices in foreign currency R_f in this economy are exogenous, stochastic with support support $\left[\underline{R}_f, \overline{R}_f\right]$, and independent from the other random variables. We associate the foreign currency with stable currencies like the dollar or the euro, and interpret the risk in R_f as real exchange rate risk.[3]

Without loss of generality, we assume that in each bilateral meeting the buyer makes a take-it-or-leave-it offer to the seller. The seller is willing to participate in the contract as long as

$$-x + \mathbb{E}\left[\theta_s \left(b_l R_l + b_f R_f\right)\right] \geqslant 0 \tag{2}$$

where we normalize the seller's outside option to zero. The optimal contract for the buyer solves

$$\max_{x, b_l, b_f} (1+\lambda)x - \mathbb{E}\left[\theta_b \left(b_l R_l + b_f R_f\right)\right] \tag{3}$$

subject to (1), (2), and non-negativity constraints $b_l, b_f \geqslant 0$.

3.3 Competitive Equilibrium given Government Policy

Local and foreign currency constitute two different units of account that have price risk relative to the domestic numeraire good. The local currency is subject to endogenous inflation risk associated with government policy. From the perspective of citizens, the price level R_l is a random variable with cdf $G(R_l)$ and support $\left[\underline{R}_l, \overline{R}_l\right]$. In equilibrium, the shape of $G(R_l)$, and the bounds of the support, $\underline{R}_l$ and $\overline{R}_l$, are endogenous and depend on the choices of the government.

[3] In Appendix B.1 we show how risk in R_f can arise in a model with tradable and non-tradable goods and shocks to the relative demand of these goods.

Given the problem defined in the previous section, we can now characterize the optimal bilateral contract between a seller and buyer, taking the distribution of R_l and R_f as given.

Proposition 1. *In the optimal bilateral contract, the amount of special good is given by* $x = \mathbb{E}\left[\theta_s\left(b_l R_l + b_f R_f\right)\right]$*, while the payments satisfy*

1. *If* $\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_l}{\bar{R}_l}\right] < \mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_f}{\bar{R}_f}\right]$ *then* $b_l = 0$ *and* $b_f = \frac{y}{\bar{R}_f}$

2. *If* $\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_l}{\bar{R}_l}\right] = \mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_f}{\bar{R}_f}\right]$ *then* $b_l = \gamma\frac{y}{\bar{R}_l}$ *and* $b_f = \left(1-\gamma\right)\frac{y}{\bar{R}_f}$ *for any* $\gamma \in [0,1]$.

3. *If* $\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_l}{\bar{R}_l}\right] > \mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_f}{\bar{R}_f}\right]$ *then* $b_l = \frac{y}{\bar{R}_l}$ *and* $b_f = 0$.

All proofs are included in the Appendix. First notice that since preferences are linear and $\lambda > 0$, there are positive gains from trading as much of good x as possible. The limit on how much x can be traded is given by the fact that buyers need to be able to pay for that good in the following period. This implies that the feasibility constraint ((1)) will always be binding. Additionally, the state for which this constraint will bind is the one in which inflation in both currencies are at their lowest possible realizations (i.e., $R_l = \overline{R}_l$ and $R_f = \overline{R}_f$). If we substitute the participation and feasibility constraint into the objective and take derivatives with respect to b_l we obtain

$$\underbrace{\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_l}{\bar{R}_l}\right]}_{\text{Marginal benefit of local currency}(M_l)} - \underbrace{\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_f}{\bar{R}_f}\right]}_{\text{Marginal benefit of foreign currency}(M_f)}$$

The expression above represents the difference between the marginal benefit of setting the contract in local currency (first term) and the marginal benefit of setting it in foreign currency (second term). Since the objective is linear, these objects are constant and independent of the choice of b_l. The optimal contract calls for using the currency that has the largest marginal benefit. When the marginal benefit is the same in both currencies, any combination of local and foreign currency is optimal. Using the assumption that θ_s and θ_s have equal means we can rewrite the marginal benefit of currency c as

$$M_c \equiv \lambda\frac{\mathbb{E}\left[R_c\right]}{\bar{R}_c} + \text{cov}\left(\left(\theta_s\left(1+\lambda\right)-\theta_b\right), \frac{R_c}{\bar{R}_c}\right) \tag{4}$$

for $c = l, f$. The marginal benefit of each currency has two components: a price risk term and a covariance term. The ratio $\frac{\mathbb{E}[R_c]}{\bar{R}_c}$ denotes the price risk of denominating contracts in currency c. A higher (lower) value of $\frac{\mathbb{E}[R_c]}{\bar{R}_c}$ represents a lower (higher) risk indexing contracts in currency c. Note that it is the maximal values of R_c that determines price risk due the assumption that payments must feasible in all states of the world. The second term is the covariance of relative taste shocks and currency prices. The marginal benefit of denominating in the foreign currency is exogenous and only given by the price risk term since the covariance term is zero given our assumption of independence between R_f and the shocks θ_b, θ_s. Suppose first that θ_b, θ_s are deterministic. Then the optimal currency choice is determined exclusively by comparing the price risk in both currencies, $\frac{\mathbb{E}[R_l]}{\bar{R}_l} - \frac{\mathbb{E}[R_f]}{\bar{R}_f}$. In this case choosing the currency with the lowest price risk maximizes the gains from trade as it allows buyers to promise larger payments. In contrast suppose that the taste parameters are stochastic. Now the optimal currency choice also depends on the covariance between prices in local currency and the marginal utility (taste shocks). For example, if R_l is high in states in which the seller values consumption relatively more than the buyer (high θ_s relative to θ_b), denominating in local currency will be more attractive. As we will see in the next section a benevolent government will choose R_l so that this covariance term is positive. Finally, note that the optimal choice of x can be computed directly from the participation constraint (2).

3.4 Government

We consider a utilitarian government that controls monetary policy and chooses the price level of the domestic economy R_l in the second period to maximize the sum of the utilities of buyers and sellers. As mentioned earlier private agents also suffer inflation losses captured by $l(R_l)$. We assume that $l(R_l) = \frac{\psi}{2}\left(R_l - R^\dagger\right)^2$ where $R^\dagger$ denotes the price level target for the government. $R^\dagger$ is a random variable realized in period 2 and thus is stochastic at the time at which contracts are signed. We assume that $R^\dagger$ has bounded support $\left[\underline{R}^\dagger, \overline{R}^\dagger\right]$. Similar to our definitions of price risk, we will refer to $\frac{\mathbb{E}[R^\dagger]}{\overline{R}^\dagger}$ as policy risk. This is meant to capture all other sources of uncertainty in monetary policy that are unrelated to the economic developments in the domestic economy. The problem of the government is given by

$$\max_{R_l} [\theta_b C_b + \theta_s C_s] - 2l\left(R_l - R^\dagger\right)$$

where

$$C_b = y - R_l B_l - R_f B_f \tag{5}$$

is the average consumption of the buyer, B_l, B_f are the average levels of contracts denominated in local and foreign currency respectively, and

$$C_s = y + R_l B_l + R_f B_f \tag{6}$$

is the average consumption of the seller.[4] Given the functional form for $l(\cdot)$, the solution to this problem is

$$R_l = R^{\dagger} + \frac{1}{2\psi}(\theta_s - \theta_b) B_l \quad (7)$$

The optimal choice of inflation redistributes resources between sellers and buyers in an efficient way. When the buyers have a high marginal utility (relative to the sellers) the government chooses a higher inflation (lower R_l) to lower the burden of debt payments by the buyer and redistribute resources from buyers to sellers. The opposite occurs when the sellers have a high marginal utility relative to buyers. The level of redistribution depends positively on the use of local currency in private contracts B_l.

The government's choices of inflation affects the marginal benefit of local currency (M_l) (defined in equation (4)) in the first period. On the one hand, the redistribution that the government attains using monetary policy induces a positive covariance between relative marginal utilities and prices in local currency, thereby increasing the marginal benefit of local currency. The higher the use of local currency B_l, the higher the endogenous positive covariance for local currency. On the other hand, by reacting to taste shocks the government also affects the price risk of local currency. Recall that we defined price risk of local currency as the ratio $\frac{\mathbb{E}[R_l]}{\bar{R}_l}$. Given the optimal choice of R_l, we have that $\mathbb{E}[R_l] = \mathbb{E}\left[R^{\dagger}\right]$ and the maximal value of R_l is given by

$$\bar{R}_l = \bar{R}^{\dagger} + \frac{1}{2\psi}\left(\overline{\theta} - \underline{\theta}\right) B_l. \quad (8)$$

The higher the use of local currency B_l, the higher $\overline{R}_l$ and the lower $\frac{\mathbb{E}[R_l]}{\bar{R}_l}$ (or the higher the price risk of local currency). Throughout our baseline analysis we make the following parametric assumption.

Assumption 1. *Assume that*

$$\frac{var(\theta)}{\left(\overline{\theta} - \underline{\theta}\right)} > 1.$$

As mentioned previously, introducing taste shocks are a simple way of generating value for discretion in monetary policy. Thus the variance of these taste shocks captures the importance of discretion. Assumption 1 ensures that the value of discretion is sufficiently large. Denote $M_l(B_l)$ as the marginal benefit of denominating in local currency

[4] In this economy the choice of monetary policy is governed by redistributiontal concerns. Another relevant margin for the choice of inflation is the collection of seignorage revenues to finance the provision of public goods. We believe this type of consideration is also broadly captured by our setup since the collection of seigniorage to finance public expenditure ultimately achieves redistribution, in this case from taxpayers to the users of the public goods.

(defined in equation (4)), once we substitute in the optimal choice of R_l by the government. Assumption 1 also guarantees that $M_l(B_l)$ is increasing in B_l. In particular, it guarantees that the positive effect of higher B_l on the covariance more than offsets the effect of higher B_l on higher price risk of local currency. Therefore, under this assumption, the benefit of denominating contracts in local currency is increasing in B_l, thus generating complementarities in denomination choices.[5]

Given this we can now define a competitive equilibrium for this economy.

Definition 1. A competitive equilibrium is an allocation for private citizens (x, B_l, B_f) and an inflation choice for the government R_l such that, given R_l the allocations solve contracting problem defined in (3), and given B_l, R_l satisfies (7).

3.5 Equilibrium Characterization

We now provide a characterization of the set of competitive equilibria. The main objective of this exercise is to understand how the set of equilibria changes as we vary the level of policy risk $\frac{\mathbb{E}[R^\dagger]}{\overline{R}^\dagger}$. As we will show, for low levels of risk, there is a unique equilibrium in which all contracts are denominated in local currency. For intermediate levels of this risk there are three equilibria: two in which all contracts are completely denominated in either local or foreign currency and an interior equilibrium. Finally, for high enough levels of policy risk there is a unique equilibrium in which all contracts are denominated in the foreign currency.

To vary policy risk, we fix $\overline{R}^\dagger$ and vary $\mathbb{E}\left[R^\dagger\right]$. In particular, a higher value of $\mathbb{E}\left[R^\dagger\right]$ denotes a lower level of policy risk. The set of equilibria is characterized in the following proposition.

Proposition 2. *Suppose that Assumption 1 holds. Then, there exist thresholds* $\mu_1 = \frac{\mathbb{E}[R_f]}{\bar{R}_f}$ *and* $\mu_2 < \mu_1$ *such that:*

1. *If* $\frac{\mathbb{E}[R^\dagger]}{R^\dagger} > \mu_1$ *there exists a unique equilibrium in which* $B_l = \frac{y}{\bar{R}^*}$ *where* $\bar{R}^*$ *is the solution to*

$$\bar{R}_l^* = \bar{R}^\dagger + \frac{1}{2\psi}\left(\overline{\theta} - \underline{\theta}\right)\frac{y}{\bar{R}_l^*}.$$

2. *If* $\mu_2 < \frac{\mathbb{E}[R^\dagger]}{R^\dagger} \leqslant \mu_1$ *there exist three equilibria with* $B_l = \frac{y}{\bar{R}^*}$, $B_l = 0$ *and* $B_l \in \left(0, \frac{y}{\bar{R}^*}\right)$.

3. *If* $\frac{\mathbb{E}[R^\dagger]}{R^\dagger} \leqslant \mu_2$ *there exists a unique equilibrium in which* $B_l = 0$.

[5] In Appendix B.2, we study the effect of relaxing this assumption.

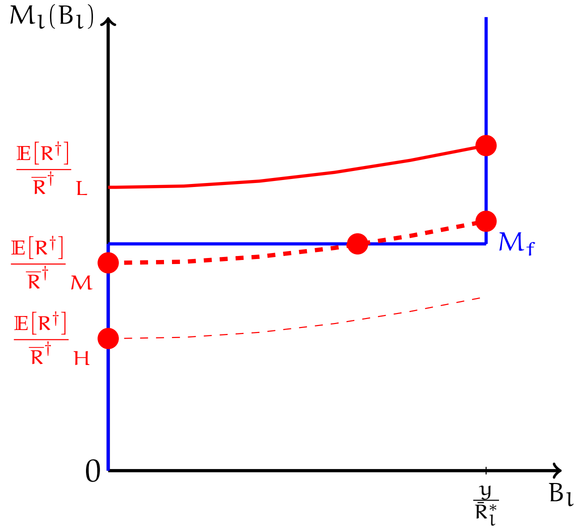


Figure 3: Characterization of Competitive Equilibrium

The thresholds μ_1 and μ_2 depend on parameters and are defined in the appendix. The figure below presents a graphical depiction of the set of equilibria. The blue line is the competitive equilibrium for a given government policy and thus, for a given M_l. When $M_l > M_f$ private agents denominate in local currency and when $M_l < M_f$ they denominate in foreign currency. The red lines depict the marginal benefit of local currency as a function of B_l for different values of policy risk. All lines are increasing since our assumption implies $M_l(B_l)$ is increasing. To understand the role of policy risk in the determination of equilibria it is useful to analyze how policy risk affects the marginal benefit of local currency. Note that when there are no contracts in local currency, the marginal benefit of local currency is given by policy risk, i.e., $M_l(0) = \lambda \frac{\mathbb{E}[R^\dagger]}{\overline{R}^\dagger}$. As we increase policy risk (decrease the ratio) the marginal benefit of local currency decreases for all possible values of B_l. When policy risk is lower than the price risk of foreign currency (case 1) the unique equilibrium only uses local currency, as shown at the intersection of the red and blue solid lines. This is because even when no contracts are set in local currency it is still worthwhile to denominate contracts in local currency if the price risk is low enough. As more contracts are signed the attractiveness of local currency increases as the government endogenously uses inflation to redistribute resources more effectively.

When the policy risk is intermediate (case 2) we have multiple equilibria. Multiplicity arises due to the complementarities between the private and government actions. As more contracts are set in local currency the government uses inflation to provide more insurance through better redistribution. One of the equilibria involves full use of foreign currency. If all private contracts are set in foreign currency, there are no incentives for the government to use inflation in order to redistribute. Therefore the marginal benefit of local currency is only given by policy risk which in this region is higher than the price risk of foreign currency. Another equilibrium involves full use of local currency. If all private contracts are denominated in local currency the government is incentivized to use inflation to redistribute efficiently, and this makes local currency more attractive than foreign currency. Finally, there is a third interior equilibrium at which the level of B_l is such that the marginal benefit of local and foreign currency are equal. In the Figure the three equilibria correspond to the three intersections of the blue and the middle red dashed line.

When the policy risk is high enough (case 3) the unique equilibrium involves full use of foreign currency. This equilibrium exists since the marginal benefit of local currency is completely determined by policy risk when all contracts are set in foreign currency, and policy risk is larger than price risk of foreign currency. The equilibrium is unique since even if all contracts are set in local currency, the government's use of inflation to redistribute does not compensate for the high levels of policy risk. In the Figure, this case corresponds to the intersection of the bottom dashed red line with the blue line.

This characterization helps rationalize observed variations in the use of foreign currency as a unit of account across countries. In particular, it offers a rationalization for why countries with low levels of policy risk like the U.S., Europe, or Japan rely exclusively on their local currency as a unit of account in domestic contracts. In contrast, countries with high policy risk such as those in Latin America and Eastern Europe tend to partially or fully rely on foreign currency as a unit of account.

4 Constrained Efficiency

We now consider the problem of a social planner who chooses allocations and contracts subject to the same constraints as private agents, and the same choice of monetary policy by the government in the second period. The utilitarian social planner solves

$$\max \mathbb{E}\left(-x + C_s + (1+\lambda)\,x + C_b - 2\psi l\left(R_l - R^{\dagger}\right)\right)$$

subject to the definitions of C_b and C_s in (5) and (6) respectively, the participation constraint for the seller, (2), the feasibility constraint (1), and the best reposes of the government (7), and (8). Note that we assumed that the participation constraint for the buyer is slack and we will check that it is satisfied ex-post.

Analogously to the competitive equilibrium, we can characterize the solution to the planner's problem given different values of policy risk.

Proposition 3. *Suppose that Assumption 1 holds. Then, there exists a threshold μ^{sp} with $\mu_2 < \mu^{sp} < \mu_1$*

1. *If* $\frac{\mathbb{E}[R^\dagger]}{R^\dagger} \geqslant \mu^{sp}$ *the solution to the Social Planner's problem is* $B_l^{sp} = \frac{y}{R^*}$

2. *If* $\frac{\mathbb{E}[R^\dagger]}{R^\dagger} \leqslant \mu^{sp}$ *the solution to the Social Planner's problem is* $B_l^{sp} = 0$

This result illustrates that an interior equilibrium can never be efficient. In particular, for policy risk in (μ_{sp}, μ_1), the full local currency equilibrium dominates the interior and full foreign currency equilibrium while for policy risk in (μ_2, μ_{sp}) the full foreign currency dominates the other two equilibria. In contrast, if policy risk is either very low or very high, the unique competitive equilibrium (full local in the former, full foreign in the latter) is constrained efficient.

The proof follows from the observation that Assumption 1 implies that the Social Planner's problem is strictly convex. As a result computing the solution of this problem involves comparing end points. The relative value of the end-points depends on whether policy risk is high or low. Intuitively, low policy risk increases the value of the full local currency equilibrium relative to the full foreign currency one while high policy risk does the opposite.

The reason for the inefficiency of some equilibria has to do with the fact that private agents do not internalize the effects of their currency choices on the government's actions. On one hand when private agents denominate a larger fraction of contracts in local currency, the government can provide more insurance against the taste shocks, which is welfare increasing. On the other hand, a larger stock of local currency also increases the price risk of local currency, which is welfare decreasing. The latter effect is stronger when policy risky is larger. Therefore, for high levels of policy risk, the efficient allocation involves full use of foreign currency and for low levels of policy risk, the efficient allocation involves full use of local currency.

The combination of the equilibrium characterization and the above result helps rationalize some of the policies described in the introduction. Consider a country with very low policy risk. The model predicts that contracts signed within the country will be denominated in local currency and it is efficient to do so. For slightly higher levels of policy risk, equilibria in which contracts are denominated in foreign currency exist but these are inefficient. Optimal policy would prescribe limits on how much contracts should be denominated in the foreign currency. This might help explain the prevalence of policies in a variety of Latin American countries, including Brazil, Colombia and Peru, that call for forced de-dollarization of contracts.[6] In contrast, for high enough levels of policy risk, optimal policy would encourage and incentivize the use of foreign currency. Examples of these types of policies are the forced dollarization adopted by Ecuador in the year 2000.

[6] In the case of Brazil and Colombia there are restrictions to denominating bank deposits or loans in foreign currency. In the case of Peru, the restrictions are less severe. While bank deposits and loans can be denominated in foreign currency, the government prohibits retail price setting in foreign currency.

It is worth comparing the relative sizes of the intervals (μ_{sp}, μ_1) and (μ_2, μ_{sp}). Given the definitions of these thresholds, it is easy to show that $\mu_{sp} - \mu_1 > \mu_2 - \mu_{sp}$. This implies that within the range of policy risk for which the economy is susceptible to multiple equilibria, the full use of local currency is the efficient outcome for a wider part of that range of policy risk. This asymmetry is due to the presence of the complementarities between actions of private agents and the government, through which a larger use of local currency in private contracts incentivizes the government to use inflation to redistribute resources efficiently and thus, makes local currency more attractive as a unit of account. Moreover, the relative size of these intervals is increasing in the variance of θ. This suggests that if there is a large need for discretion in monetary policy, for a large range of policy risk, optimal policy would prescribe a move away from foreign currency. As discussed earlier, such policies have been enacted in large number of countries.

5 Hysteresis

As discussed in section 2, a distinctive feature among many Latin American countries is the hysteresis of dollarization even after inflation risk stabilized. The model presented above suggests that the set of equilibria can change dramatically for small changes in policy risk around the threshold which might seem to be at odds with this observation. However, the above analysis ignores the fact that citizens might be part of credit chains and thus might also have endowments of obligations in both currencies. Here, we present a simple extension in which the buyer is endowed with claims $\left(\hat{b}_f, \hat{b}_l\right)$ payable to the buyer in the second period. In Appendix B.3, we present a model of a credit chain in which these endowments arise endogenously a consequence of trading within the chain. The optimal contract solves

$$\max_{b_l, b_f} (1+\lambda)\, x - \mathbb{E}\theta_b \left(R_l \left(b_l - \hat{b}_l\right) + R_f \left(b_f - \hat{b}_f\right)\right)$$

subject to (2) and the feasibility constraint

$$R_l \left(b_l - \hat{b}_l\right) + R_f \left(b_f - \hat{b}_f\right) \leqslant y, \ \forall \left(R_l, R_f\right)$$

Assumption 2. *Assume that*

$$\frac{1}{2\psi}\left(var\,(\theta) - \left(\overline{\theta} - \underline{\theta}\right)\frac{\mathbb{E}\left[R_f\right]}{\underline{R}_f}\right)\frac{y}{\overline{R}_l^*} < \overline{R}^{\dagger}\left(\frac{\mathbb{E}\left[R_f\right]}{\underline{R}_f} - 1\right).$$

This assumption requires an upper bound on the variance of θ. Note that this bound contains a free parameter $\underline{R}_f$, and which can be made arbitrarily small in order to satisfy this and Assumption 1.

Proposition 4. *Under Assumption 2, $b_f \geqslant \hat{b}_f$ and $b_l \geqslant \hat{b}_f$.*

The proposition says that even if policy risk is small, the optimal contract will still use a combination of foreign and local currency to denominate contracts. In particular, the optimal contract will feature currency matching of stocks but the flows will denominated in the currency with the largest marginal benefit. To illustrate this result, suppose that var $(\theta) = 0$. Then, we know from earlier that the optimal currency choice only involves comparing price risk. Notice that with existing obligations, the price level that makes the feasibility constraint bind will now depend on when $b_i \leqslant \hat{b}_i$ or not. In the former, the relevant price is $\underline{R}_i$ while in the latter it is $\bar{R}_i$. The difference in price risk is

$$\frac{\mathbb{E}\left[R_l\right]}{\tilde{R}_l} - \frac{\mathbb{E}\left[R_f\right]}{\tilde{R}_f}$$

where $\tilde{R}_i \in \left\{\underline{R}_i, \bar{R}_i\right\}$. Suppose that $b_f < \hat{b}_f$. Then the difference in price risk is

$$\frac{\mathbb{E}\left[R_l\right]}{\bar{R}_l} - \frac{\mathbb{E}\left[R_f\right]}{\underline{R}_f} < 0$$

which implies that $b_f < \hat{b}_f$ can never be part of an equilibrium contract. A similar argument holds for the local currency. This suggests that currency mismatch is costly and tightens the feasibility constraint. As a result, the optimal contract currency matches stocks and prices the flows in the currency with the largest marginal benefit. The argument in the appendix shows that the above argument generalizes as long as the variance of θ is not too large. If the variance is very large then it might be optimal to cannibalize the stocks of foreign currency.

6 Contracts in International Trade

One of the facts mentioned in section 2 is that there is extensive use of the dollar as a unit of account in international trade contracts. Perhaps surprisingly, trade involving countries with seemingly low political risk are often denominated in dollars. For example, Japan and Korea have low levels of inflation volatility and domestic dollarization, and yet have a significant fraction of trade contracts denominated in dollars. In this section, we study an extension of our baseline model that allows for international trade to help understand this. We incorporate international trade in our model by studying an economy in which agents from one country trade with agents from another country and contracts are set in any of the currencies of the involved countries or in a third, external currency. Our main result in this section shows that contracting with parties located in different countries, as opposed to signing contracts with domestic counter-parties, makes the use of a third, external currency, more likely.

There are two countries, denoted i and j, which are symmetric. In each country there is a continuum of buyers and sellers of equal size. A contract between a buyer and a seller consists of a provision of the special good in exchange for the promise of future payment. The first difference with the baseline model is that buyers in one country trade with sellers in the other country. The second difference is that we allow contracts to be set in three possible "units of account": currencies from country i and j, and the foreign currency f. The price levels of currencies i and j (denoted by R_i and R_j) are chosen by the governments of each country, whereas the price of foreign currency is exogenous.

Denote x_i to be the amount of special good provided by a seller from country j to a buyer of country i[7], and b_{ic} to be the promised payment of buyer from country i in currency c. The optimal private contract between a buyer of country i and a seller of country j solves

$$\max_{x_i, b_{ii} \geqslant 0, b_{ij} \geqslant 0, b_{if} \geqslant 0} (1+\lambda)\, x_i - \mathbb{E}\theta_{ib} \left(R_i b_{ii} + R_j b_{ij} + R_f b_{if}\right)$$

subject to the participation constraint

$$-x_i + \mathbb{E}\theta_{js} \left(R_i b_{ii} + R_j b_{ij} + R_f b_{if}\right) \geqslant 0,$$

and the feasibility constraint

$$\overline{R}_i b_{ii} + \overline{R}_j b_{ij} + \overline{R}_f b_{if} \leqslant y, \tag{9}$$

where θ_{ib} and θ_{js} denote the taste shocks of the buyer from country i and the seller from country j, respectively. The solution to this problem is characterized in Lemma 1 in the appendix and is similar to Proposition (1). Taking prices as given, agents write contracts using the currency that has the largest marginal benefit, allowing for combinations of two or three currencies whenever the buyer is indifferent.

Next, we revisit the government's problem. There are two utilitarian governments that control monetary policy and choose the price level of the local currencies in economies i and j. We assume that both countries have the same level of policy risk, $\frac{\mathbb{E}\left[R_i^{\dagger}\right]}{\overline{R}_i^{\dagger}} = \frac{\mathbb{E}\left[R_j^{\dagger}\right]}{\overline{R}_j^{\dagger}}$. Denote B_{ic} to be the aggregate promised payments in currency c of buyers of country i to sellers of country j. The problem of the government in country i is given by

[7] Note that we have suppressed the dependency on j since knowing that the buyer is i implies that the seller is j

$$\max_{R_i} [\theta_{ib} C_{ib} + \theta_{is} C_{is}] - 2\psi l \left(R_i - R_i^{\dagger} \right),$$

where

$$C_{ib} = y - R_i B_{ii} - R_j B_{ij} - R_f B_{if} \tag{10}$$

$$C_{is} = y + R_i B_{ji} + R_j B_{jj} + R_f B_{jf}$$

Given our functional form assumption for the inflation loss function, the solution to the problem of the government in country i is

$$R_i = R_i^{\dagger} + \frac{1}{2\psi} \left(\theta_{is} B_{ji} - \theta_{ib} B_{ii} \right), \tag{11}$$

and the largest feasible price level the government can implement is

$$\bar{R}_i = \bar{R}_i^{\dagger} + \frac{1}{2\psi} \left(\bar{\theta} B_{ji} - \underline{\theta} B_{ii} \right).$$

The problem of the government in country j is symmetric. We restrict attention to symmetric equilibria in which all international trade contracts are set in the same currency, i.e. $B_{jc} = B_{ic} \equiv B_c$ for all c. Note that we only restrict attention to symmetry within international contracts and not necessarily across governments' inflation choices. In Appendix B.4, we relax this assumption and also consider asymmetric equilibria.

Definition 2. A symmetric competitive equilibrium is an allocation for private citizens (x, B_i, B_j, B_f) and an inflation choice for governments R_i and R_j such that, given R_i and R_j, the allocations solve contracting problem defined in (9), and given B_b and B_s, R_b and R_s satisfy (11).

In the following proposition we argue that the use of the external currency is more likely in the economy with international contracts than in the baseline economy with domestic contracts. Recall that μ_2 is the threshold defined in the previous section such that if political risk is below μ_2, then there is unique equilibrium in which only the foreign currency is used as a unit of account.

Proposition 5. *Suppose that Assumption 1 holds. Then, there exists a threshold μ_2^I such that, if $\frac{\mathbb{E}\left[R_i^{\dagger}\right]}{\bar{R}_i^{\dagger}} = \frac{\mathbb{E}\left[R_j^{\dagger}\right]}{\bar{R}_j^{\dagger}} \leqslant \mu_2^I$ there exists a unique symmetric equilibrium in which $B_i = B_j = 0$. Furthermore, $\mu_2^I > \mu_2$.*

The threshold μ_2^I depends on parameters and is defined in the appendix. As in the baseline model, there exists a threshold μ_2^I such that if policy risk in country i and j is larger than this threshold, the unique equilibrium displays the use of the foreign currency as the sole unit of account. However, the most important result of this proposition is that

$\mu_2^I > \mu_2$, that is, the threshold obtained in the three country model is larger than the one found in the baseline model. This implies that for levels of policy risk such that $\mu_2^I > \frac{\mathbb{E}\left[R_i^\dagger\right]}{\bar{R}_i^\dagger} = \frac{\mathbb{E}\left[R_j^\dagger\right]}{\bar{R}_j^\dagger} > \mu_2$, there exists a unique foreign currency equilibrium in the model with international trade while there can exist equilibria with local currency in the model with only domestic contracts. This result suggests that we are more likely to see international trade contracts denominated in the foreign currency than domestic contracts.

The reason for this result is that in the case of international contracts, each government finds it optimal to use inflation to respond only to taste shocks of their own citizens and not those of the other country. That is, governments do not react to taste shocks of foreign buyers or sellers, and this implies that the covariance term in equation (4) is lower for a given aggregate exposure to the local currency. This in turn, lowers the marginal benefit of using local currencies of either country and makes the foreign external currency more attractive for private contracts.

While the proposition focuses on symmetric equilibria, in Appendix B.4 we argue that the uniqueness result generalizes for to all equilibria under a stronger parametric assumption.

7 Strategic Default

One assumption that we have maintained throughout is that private contracts are incomplete with respect to political and taste shocks. In particular, the only form of insurance against taste shocks comes in the form of ex-post government redistribution. Here we partially relax this assumption by allowing the buyer to default on its obligations in period 2. That default can allow for state contingency in the presence of incomplete markets is well known from the sovereign default, and bankruptcy literatures.

We now modify our baseline environment in two ways. First, we allow the buyer to fully default on its contract in period 2 after its taste shock is realized. If the buyer defaults, the seller receives nothing and the buyer's utility is

$$\theta_b y - \chi\left(R_f b_f + R_l b_l\right)$$

where $\chi\left(R_f b_f + R_l b_l\right)$ is the utility cost of default which depends on the *level* of defaulted debt. In particular this implies that a buyer who defaults on a larger stock of debt suffers a higher cost. One interpretation of this cost is that if there is exclusion after default, the exclusion time depends positively on the level of defaulted debt. See Kirpalani (2016) who shows the optimality of such punishments in a model with endowment risk, and Cruces and Trebesch (2013) who document in the sovereign default data that higher haircuts are associated with longer periods of exclusion.

This assumption implies that the buyer will default if

$$\theta_b > \chi$$

This implies that the buyer defaults when it has a high marginal utility of consumption. Therefore, the ability to default allows for partial insurance against taste shocks. This simple cutoff rule allows for a clean characterization of equilibria. We assume that $\underline{\theta} \leqslant \chi \leqslant \bar{\theta}$. Notice that if $\chi = \bar{\theta}$, the model collapses to our baseline model.

In this case the optimal contract solves

$$\max_{b_f, b_l} (1+\lambda)\, x - \mathbb{E}\left[\theta_b \left[y - (R_f b_f + R_l b_l)\right] \mid \theta_b \leqslant \chi\right] F\left(\theta_b \leqslant \chi\right)$$
$$+ \mathbb{E}\left[\theta_b y - \chi \left(R_f b_f + R_l b_l\right) \mid \theta_b > \chi\right] \left[1 - F\left(\theta_b \leqslant \chi\right)\right]$$

subject to the participation constraint of the seller,

$$-x + \mathbb{E}\left[\theta_s \left(R_f b_f + R_l b_l\right) b \mid \theta_b \leqslant \chi\right] F\left(\theta_b \leqslant \chi\right) \geqslant 0$$

and the feasibility constraint (1). The solution to this problem is similar to the baseline and is given in Lemma 2. The trade-offs are the same as in the baseline model, since the problem remains linear in debt choices. The difference is that with default, the covariance term associated with the marginal benefit is conditional on the states in which there is repayment. In addition the price level also affects the utility after default.

The problem for the government is given by

$$\max_{R_l} \left[\left(\theta_b \left[y - R_l B_l\right] + \theta_s \left(y + R_l B_l\right)\right) \mathbb{I}_{\theta_b \leqslant \chi} + \left(\theta_b y - \chi R_l B_l + \theta_s y\right) \mathbb{I}_{\theta_b > \chi}\right] - 2 \cdot \psi \frac{\left(R_l - R^{\dagger}\right)^2}{2}$$

Suppose $\theta_b \leqslant \chi$. In this case there is repayment and the optimal choice of R_l is

$$R_l = R^{\dagger} + \frac{1}{2\psi} \left(\theta_s - \theta_b\right) B_l$$

Suppose $\theta_b > \chi$. Then the choice of R_l is given by

$$R_l = R^{\dagger} - \frac{\chi}{2\psi} B_l$$

Since the default cost depends on the defaulted debt, the optimal price level will be chosen to mitigate these default costs. As in the baseline environment, define R^* to be the solution to

$$R^* = \bar{R}^\dagger + \frac{1}{2\psi}\left(\bar{\theta} - \underline{\theta}\right)\frac{y}{R^*}.$$

The definition of competitive equilibrium is identical to the baseline model.

Recall that in our baseline model there exists a threshold μ_2 such that for $\frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger} < \mu_2$ there is a unique equilibrium in which only the foreign currency is used a unit of account. Similarly, in the model with strategic default we can define an equivalent threshold $\mu_2(\chi)$ which coincides with original threshold if $\chi = \bar{\theta}$ (no default). The exact form of the threshold is stated in the appendix. To illustrate how strategic default affects currency choice, we focus on how the threshold $\mu_2(\chi)$ changes around $\chi = \bar{\theta}$. In particular we show that as χ decreases, the range of policy risk for which we have a unique foreign currency equilibrium decreases.

Proposition 6. *Suppose that Assumption 1 holds. Then, there exists a threshold $\mu_2(\chi)$ such if $\frac{\mathbb{E}[R^\dagger]}{R^\dagger} < \mu_2(\chi)$, there exists a unique equilibrium in which $B_l = 0$. Furthermore, $\lim_{\chi\uparrow\bar{\theta}} \mu_2'(\chi) > 0$.*

This proposition says that in a neighborhood around $\chi = \bar{\theta}$, the introduction of default makes the use of local currency *more* attractive. As in the baseline model the marginal benefit of choosing a particular currency depends on its price risk and its covariance of its price with relative marginal utilities. We show that the relative price risk of local and foreign currency is independent of χ while the covariance term associated with local currency is increasing in χ. The reason for this is that default eliminates the states of the world (high values of θ_b) in which providing insurance is very costly for the government (since it requires larger deviations from the target). Therefore, the introduction of default reduces the average inflation cost conditional on no-default thus allowing for greater insurance. However, there is a countervailing force since the set of states in which insurance is provided also decreases. We show that for χ close to $\bar{\theta}$, the first effect dominates.

8 Conclusion

This paper develops a framework to study the optimal choice of currency in the denomination of private credit contracts in general equilibrium. A key feature of the studied economy is a source of complementarities between the actions of private agents and those of the government. When more private contracts are denominated in local currency, the government has more incentives to use inflation to redistribute resources efficiently within the economy which, in turn, makes local currency even more attractive as a unit of account for private contracts. We argue that the degree of policy risk determines the type of equilibria that emerge. For low policy risk, the unique equilibrium only uses local currency, whereas for high levels of policy risk, the unique equilibrium only uses foreign currency. For intermediate levels of policy risk both equilibria co-exist with an additional interior equilibrium. Our constrained efficiency analysis argues that, for the majority of the parameter space in which the economy is vulnerable to multiple equilibria, the efficient outcome involves exclusive use of local currency. This result can help rationalize various policy initiatives aimed at de-dollarizing domestic economies. We also use our model to shed light on the observed hysteresis in the dollarization of contracts, and also to analyze the case of international debt contracts, in which dollars are even more prevalent.

References

AGUIAR, M. (2005): "Investment, devaluation, and foreign currency exposure: The case of Mexico," *Journal of Development Economics*, 78, 95 – 113. 5

AGUIAR, M., M. AMADOR, E. FARHI, AND G. GOPINATH (2015): "Coordination and Crisis in Monetary Unions," *The Quarterly Journal of Economics*, 130, 1727–1779. 5

ALESINA, A. AND R. J. BARRO (2002): "Currency Unions," *The Quarterly Journal of Economics*, 117, 409–436. 5

ARELLANO, C. AND J. HEATHCOTE (2010): "Dollarization and financial integration," *Journal of Economic Theory*, 145, 944–973. 5

BOCOLA, L. AND G. LORENZONI (2017): "Financial crises and lending of last resort in open economies," Tech. rep., National Bureau of Economic Research. 5

CABALLERO, R. J. AND A. KRISHNAMURTHY (2003): "Excessive Dollar Debt: Financial Development and Underinsurance," *The Journal of Finance*, 58, 867–893. 5

CHAHROUR, R. AND R. VALCHEV (2017): "International medium of exchange: Privilege and duty," Tech. rep., Boston College Department of Economics. 6

CHARI, V. V., A. DOVIS, AND P. J. KEHOE (2015): "Rethinking Optimal Currency Areas," Manuscript, University of Minnesota. 5

CRUCES, J. J. AND C. TREBESCH (2013): "Sovereign Defaults: The Price of Haircuts," *American Economic Journal: Macroeconomics*, 5, 85–117. 22

DOEPKE, M. AND M. SCHNEIDER (2017): "Money as a Unit of Account," *Econometrica*, 85, 1537–1574. 5

DRENIK, A. AND D. J. PEREZ (2017): "Pricing in Multiple Currencies in Domestic Markets," Tech. rep. 6

ENGEL, C. (2006): "Equivalence Results for Optimal Pass-through, Optimal Indexing to Exchange Rates, and Optimal Choice of Currency for Export Pricing," *Journal of the European Economic Association*, 4, 1249–1260. 5

FARHI, E. AND M. MAGGIORI (2017): "A model of the international monetary system," *The Quarterly Journal of Economics*, 133, 295–355. 6

GALE, D. AND X. VIVES (2002): "Dollarization, Bailouts, and the Stability of the Banking System," *The Quarterly Journal of Economics*, 117, 467–502. 5

GOPINATH, G. (2015): "The international price system," Tech. rep., National Bureau of Economic Research. 4, 5, 7, 51

GOPINATH, G., O. ITSKHOKI, AND R. RIGOBON (2010): "Currency Choice and Exchange Rate Pass-Through," *American Economic Review*, 100, 304–36. 5

GOPINATH, G. AND J. C. STEIN (2017): "Banking, Trade, and the Making of a Dominant Currency," . 6

IZE, A. AND E. LEVY-YEYATI (2003): "Financial dollarization," *Journal of International Economics*, 59, 323–347. 5, 6

KIRPALANI, R. (2016): "Endogenously Incomplete Markets with Equilibrium Default," . 22

LEVY-YEYATI, E. (2006): "Financial dollarization: evaluating the consequences," *economic Policy*, 21, 62–118. 7, 8

MAGGIORI, M., B. NEIMAN, AND J. SCHREGER (2018): "International currencies and capital allocation," Tech. rep., National Bureau of Economic Research. 6

MATSUYAMA, K., N. KIYOTAKI, AND A. MATSUI (1993): "Toward a theory of international currency," *The Review of Economic Studies*, 60, 283–307. 5

MUKHIN, D. (2018): "An Equilibrium Model of the International Price System," Tech. rep., Society for Economic Dynamics. 5

NEUMEYER, P. A. (1998): "Currencies and the allocation of risk: The welfare effects of a monetary union," *American Economic Review*, 246–259. 5

NICOLO, G. D., P. HONOHAN, AND A. IZE (2003): "Dollarization of the Banking System : Good or Bad?" . 6

OTTONELLO, P. AND D. PEREZ (2016): "The currency composition of sovereign debt," in *2016 meeting papers, Society for Economic Dynamics*. 5

RAPPOPORT, V. (2009): "Persistence of dollarization after price stabilization," *Journal of Monetary Economics*, 56, 979–989. 5

RENNHACK, R. AND M. NOZAKI (2006): "Financial Dollarization in Latin America," in *Financial Dollarization*, Springer, 64–96. 6

SALOMAO, J. AND L. VARELA (2017): "Exchange Rate Exposure and Firm Dynamics," Tech. rep. 5

SCHNEIDER, M. AND A. TORNELL (2004): "Balance Sheet Effects, Bailout Guarantees and Financial Crises," *The Review of Economic Studies*, 71, 883–913. 5

URIBE, M. (1997): "Hysteresis in a simple model of currency substitution," *Journal of Monetary Economics*, 40, 185–202. 5

A Omitted Proofs

Proof of Proposition 1

First note that the participation constraint of problem (3) is binding in the optimum. To see this, suppose it is not binding, then increasing infinitesimally x and leaving all remaining variables unchanged is feasible and has associated a strictly higher objective function. This implies that at the optimum the participation constraint is binding. Solving for x using the participation constraint yields the first result of the proposition. Once we substitute the optimal value of x in the problem we obtain the following re-formulated problem:

$$\max_{b_l \geqslant 0, b_f \geqslant 0} \mathbb{E}\left[\left((1+\lambda)\,\theta_s - \theta_b\right)\left(R_l b_l + R_f b_f\right)\right]$$

subject to the feasibility constraint

$$\overline{R}_l b_l + \overline{R}_f b_f \leqslant y.$$

Solving for b_f using the feasibility constraint and substituting in the objective problem yields the following problem:

$$\max_{b_l \in \left[0, \frac{y}{\overline{R}_l}\right]} \mathbb{E}\left[\left((1+\lambda)\,\theta_s - \theta_b\right)\left(R_l b_l + \frac{R_f}{\overline{R}_f}\left(y - \overline{R}_l b_l\right)\right)\right]$$

The objective is linear in b_l and the slope is $\mathbb{E}\left[(\theta_s(1+\lambda)-\theta_b)\left(R_l - \frac{R_f}{\bar{R}_f}\bar{R}_l\right)\right]$. Therefore, the solution is $b_l = \frac{y}{\bar{R}_l}$ when the slope is positive, $b_l = 0$ when the slope is negative and any $b_l \in \left[0, \frac{y}{\bar{R}_l}\right]$ when the slope is zero. Q.E.D.

Proof of Proposition 2

The following definitions will be useful for this proof. Define,

$$\mathcal{H}(B) \equiv (1+\lambda) M_2(B) - M_1(B)$$

where

$$\begin{aligned} M_2(B) &\equiv \mathbb{E}\left[\theta_s\left(R_l(B) - \frac{R_f}{\bar{R}_f}\bar{R}_l(B)\right)\right] \\ &= \bar{R}^{\dagger}\left(\frac{\mathbb{E}\left[R^{\dagger}\right]}{\bar{R}^{\dagger}} - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\right) + \frac{1}{2\psi}\left(\mathrm{var}(\theta) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right) B_l \end{aligned}$$

and

$$\begin{aligned} M_1(B) &\equiv \mathbb{E}\left[\theta_b\left(R_l(B) - \frac{R_f}{\bar{R}_f}\bar{R}_l(B)\right)\right] \\ &= \bar{R}^{\dagger}\left(\frac{\mathbb{E}\left[R^{\dagger}\right]}{\bar{R}^{\dagger}} - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\right) - \frac{1}{2\psi}\left(\mathrm{var}(\theta) + \frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right) B_l \end{aligned}$$

where we have used the best response of the government

$$R_l(B) = R^{\dagger} + \frac{1}{2\psi}(\theta_b - \theta_s) B_l$$

$$\tilde{y} \equiv \frac{\mathbb{E}[R_f]}{\bar{R}_f} y$$

It will also be useful to compute

$$M_1'(B) = -\frac{1}{2\psi}\left(\mathrm{var}(\theta) + \frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right)$$

and

$$M_2'(B) = \frac{1}{2\psi}\left(\mathrm{var}(\theta) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right)$$

Notice that the function $\mathcal{H}(B)$ is useful for characterizing the set of equilibria in this model. There are three types of equilibria that can exist. First, an equilibrium with $B_l = 0$ exists iff $\mathcal{H}(B) \leqslant 0$. Next, an equilibrium in which $B_f = 0$ can exist iff $\mathcal{H}\left(\frac{y}{\bar{R}^*}\right) \geqslant 0$, where

$\frac{y}{\bar{R}^*}$ corresponds to the maximal feasible value of B_l and $\bar{R}_l^*$ solves

$$\bar{R}_l^* = \bar{R}^\dagger + \frac{1}{2\psi}\left(\overline{\theta} - \underline{\theta}\right)\frac{y}{\bar{R}_l^*}$$

or

$$\bar{R}_l^* = \frac{\bar{R}^\dagger + \sqrt{\left(\bar{R}^\dagger\right)^2 + 2\frac{y}{\psi}\left(\overline{\theta} - \underline{\theta}\right)}}{2}$$

Finally, an interior equilibrium exists iff there exists some $B_l \in \left(0, \frac{y}{\bar{R}^*}\right)$ such that $\mathcal{H}\left(B_l\right) = 0$.

Define $\mu_1 \equiv \frac{\mathbb{E}[R_f]}{\bar{R}_f}$. We will show that if $\frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger} - \frac{\mathbb{E}[R_f]}{\bar{R}_f} > 0$, then there is a unique equilibrium in which $B_l = \frac{y}{\bar{R}^*}$.We need to show that $\mathcal{H}\left(\frac{y}{\bar{R}_l^*}\right) \geqslant 0$. Notice that under Assumption 1, $M_2\left(B\right) > 0$. Therefore, for any B.

$$\mathcal{H}\left(B\right) \geqslant M_2\left(B\right) - M_1\left(B\right) = \frac{\mathrm{var}\left(\theta\right)}{\psi} > 0$$

and in particular, $\mathcal{H}\left(\frac{y}{\bar{R}_l^*}\right) > 0$. Moreover, given our characterization of equilibria above, this also implies that we have a unique equilibrium.

Next, define

$$\mu_2 \equiv \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f} - \frac{1}{2\psi}\frac{y}{\bar{R}^\dagger\bar{R}^*}\left(\frac{(2+\lambda)}{\lambda}\mathrm{var}\left(\theta\right) - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right)$$

Notice that $\mu_2 < \mu_1$. We show that for $\frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger} \in (\mu_2, \mu_1]$, there exist three equilibria. First, we show an equilibrium exists in which $B_l = 0$. We know from above that for this equilibrium to exist it must be that $\mathcal{H}\left(0\right) \leqslant 0$. Using the expressions, we derived earlier

$$\mathcal{H}\left(0\right) = \lambda\bar{R}^\dagger\left[\frac{\mathbb{E}\left[R^\dagger\right]}{\bar{R}^\dagger} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\right] \leqslant 0$$

which is implied by our assumption. Next, we want to show that there exists an interior equilibrium, i.e. there exists a B such that $\mathcal{H}\left(B\right) = 0$ or

$$(1+\lambda) = \frac{M_1\left(B\right)}{M_2\left(B\right)}$$

Using our definitions, we have

$$\frac{M_1(B)}{M_2(B)} = \frac{\bar{R}^{\dagger}\left(\frac{\mathbb{E}[R^{\dagger}]}{\bar{R}^{\dagger}} - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\right) - \frac{1}{2\psi}\left(\text{var}(\theta) + \frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right)B_l}{\bar{R}^{\dagger}\left(\frac{\mathbb{E}[R^{\dagger}]}{\bar{R}^{\dagger}} - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\right) + \frac{1}{2\psi}\left(\text{var}(\theta) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right)B_l}$$

We know this is equal to 1 at $B = 0$. Lets consider the slope which has the sign

$$\begin{aligned}
& M_2(B)\, M_1'(B) - M_1(B)\, M_2'(B) \\
= & \left[\bar{R}^{\dagger}\left(\frac{\mathbb{E}[R^{\dagger}]}{\bar{R}^{\dagger}} - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\right) + \frac{1}{2\psi}\left(\text{var}(\theta) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right)B_l\right]\left(-\frac{1}{2\psi}\left(\text{var}(\theta) + \frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right)\right) \\
& - \left[\bar{R}^{\dagger}\left(\frac{\mathbb{E}[R^{\dagger}]}{\bar{R}^{\dagger}} - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\right) - \frac{1}{2\psi}\left(\text{var}(\theta) + \frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right)B_l\right]\frac{1}{2\psi}\left(\text{var}(\theta) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right) \\
= & -\bar{R}^{\dagger}\left(\frac{\mathbb{E}[R^{\dagger}]}{\bar{R}^{\dagger}} - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\right)\frac{1}{\psi}\text{var}(\theta) \\
> & 0
\end{aligned}$$

Therefore, since the ratio equals one at $B_l = 0$ and in strictly increasing, there is a unique solution at some B^*. B^* solves

$$B^* = \frac{2\psi\lambda\bar{R}^{\dagger}\left(\frac{\mathbb{E}[R_f]}{\bar{R}_f} - \frac{\mathbb{E}[R^{\dagger}]}{\bar{R}^{\dagger}}\right)}{\left((2+\lambda)\,\text{var}(\theta) - \lambda\frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right)}$$

For this to be strictly interior a necessary and sufficient condition is

$$B^* < \frac{y}{\bar{R}^*}$$

or

$$\frac{2\psi\lambda\bar{R}^{\dagger}\left(\frac{\mathbb{E}[R_f]}{\bar{R}_f} - \frac{\mathbb{E}[R^{\dagger}]}{\bar{R}^{\dagger}}\right)}{\left((2+\lambda)\,var(\theta) - \lambda\frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right)} < \frac{y}{\bar{R}^*}$$

or

$$\bar{R}^{\dagger}\left(\frac{\mathbb{E}[R^{\dagger}]}{\bar{R}^{\dagger}} - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\right) + \frac{1}{2\psi}\frac{y}{\bar{R}^*}\left(\frac{(2+\lambda)}{\lambda}var(\theta) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right) > 0$$

or

$$\frac{\mathbb{E}[R^{\dagger}]}{\bar{R}^{\dagger}} > \mu_2$$

Finally, given the monotonicity of the ratio $\frac{M_1}{M_2}$, it follows that if there is an interior equilibrium, there also must exist an equilibrium with full use of local currency, since it must be that

$$\frac{M_1\left(\frac{y}{\bar{R}^*}\right)}{M_2\left(\frac{y}{\bar{R}^*}\right)} > \frac{M_1(B^*)}{M_2(B^*)} = 1+\lambda$$

which implies that $\mathcal{H}\left(\frac{y}{\bar{R}^*}\right) > 0$.

Finally, assume that $\frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger} \leqslant \mu_2$. Given the above analyses, it is straightforward to see that in this case there is a unique equilibrium in which $B_l = 0$. In particular, in this interval, it must be that $\mathcal{H}(B) \leqslant 0$. Q.E.D.

Proof of Proposition 3

Given that both the participation constraint and the feasibility constraint will bind, we can write the planner's problem is

$$\max_{B_l}\left(\mathbb{E}\left([(1+\lambda)\theta_s-\theta_b]\left(\left(R_l-\frac{R_f}{\bar{R}_f}\bar{R}_l\right)B_l+\frac{R_f}{\bar{R}_f}y\right)\right)+2y\right)-2\psi l\left(R_l-R^\dagger\right)$$

subject to (7), and (8). Given our previous definitions, it will be useful to define the planning problem as follows:

$$SP(B) \equiv \max_B\left[(1+\lambda)(M_2(B)B+\tilde{y})-(M_1(B)B+\tilde{y})+2y-2\psi\mathbb{E}l\left(R_l(B)-R^\dagger\right)\right]$$

where $\tilde{y} = \frac{\mathbb{E}[R_f]}{\bar{R}_f}y$ subject to

$$R_l(B) = R^\dagger + \frac{1}{2\psi}(\theta_s-\theta_b)B$$

The first order condition is

$$\begin{aligned} SP'(B) &= \left[(1+\lambda)\left[M_2(B)+M_2'(B)B\right]-M_1(B)-M_1'(B)B-2\psi\mathbb{E}l'\left(R_l(B)-R^\dagger\right)R_l'(B)\right] \\ &= [(1+\lambda)M_2(B)-M_1(B)+\Delta(B)B] \end{aligned}$$

where

$$\Delta(B) \equiv (1+\lambda)M_2'(B)-M_1'(B)-\mathbb{E}(\theta_s-\theta_b)R_l'(B)$$

Next, lets check the second order condition of the planner's problem. First, we have

$$\begin{aligned} \Delta'(B) &= h''(M_2(B)B+x)\left[M_2'(B)B+M_2(B)\right]M_2'(B)+h'(M_2(B)B+x)M_2''(B) \\ &\quad -\mathbb{E}M_1''(B)-\mathbb{E}(\theta_s-\theta_b)R_l''(B)=0 \end{aligned}$$

which implies that

$$\begin{aligned}
SP''(B) =& (1+\lambda) M_2'(B) - M_1'(B) + \Delta(B) \\
=& 2(1+\lambda) M_2'(B) - 2M_1'(B) - \mathbb{E}(\theta_s - \theta_b) R_l'(B) \\
=& 2\left((1+\lambda)\frac{1}{2\psi}\left(\text{var}(\theta) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}(\overline{\theta} - \underline{\theta})\right) + \frac{1}{2\psi}\left(\text{var}(\theta) + \frac{\mathbb{E}[R_f]}{\bar{R}_f}(\overline{\theta} - \underline{\theta})\right)\right) - \frac{1}{\psi}\text{var}(\theta) \\
>& 2\left(\frac{1}{2\psi}\left(\text{var}(\theta) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}(\overline{\theta} - \underline{\theta})\right) + \frac{1}{2\psi}\left(\text{var}(\theta) + \frac{\mathbb{E}[R_f]}{\bar{R}_f}(\overline{\theta} - \underline{\theta})\right)\right) - \frac{1}{\psi}\text{var}(\theta) \\
\geqslant& \frac{1}{\psi}\text{var}(\theta)
\end{aligned}$$

where the inequality in line 3 follows from Assumption 1. Therefore, the Planner's problem is strictly convex which implies that computing the solution involves comparing end points $B_l = 0$ and $B_l = \frac{y}{\bar{R}^*}$. Note that the maximal feasible level of B_l depends only on parameters and thus is identical across both the competitive equilibrium and the Planning problem.

Define
$$\mu^{sp} = \frac{\mathbb{E}[R_f]}{\bar{R}_f} - \frac{1}{2\psi}\frac{y}{\bar{R}^\dagger \bar{R}^*}\left(\frac{(1+\lambda)}{\lambda}\text{var}(\theta) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}(\overline{\theta} - \underline{\theta})\right)$$

We have

$$SP(0) = \lambda\frac{\mathbb{E}[R_f]}{\bar{R}_f}y + 2y$$

and

$$\begin{aligned}
SP\left(\frac{y}{\bar{R}^*}\right) &= \left[(1+\lambda)\left(M_2\left(\frac{y}{\bar{R}^*}\right)\frac{y}{\bar{R}^*} + \tilde{y}\right) - \left(M_1\left(\frac{y}{\bar{R}^*}\right)\frac{y}{\bar{R}^*} + \tilde{y}\right) + 2y - \psi\mathbb{E}\left(\frac{1}{2\psi}(\theta_s - \theta_b)\frac{y}{\bar{R}^*}\right)^2\right] \\
&= \left[\lambda\frac{\mathbb{E}[R_f]}{\bar{R}_f}y + 2y + (1+\lambda) M_2\left(\frac{y}{\bar{R}^*}\right)\frac{y}{\bar{R}^*} - M_1\left(\frac{y}{\bar{R}^*}\right)\frac{y}{\bar{R}^*} - \psi\mathbb{E}\left(\frac{1}{2\psi}(\theta_s - \theta_b)\frac{y}{\bar{R}^*}\right)^2\right]
\end{aligned}$$

Thus to compare the above two terms, we need to compute the sign of

$$\begin{aligned}
&(1+\lambda) M_2\left(\frac{y}{\bar{R}^*}\right)\frac{y}{\bar{R}^*} - M_1\left(\frac{y}{\bar{R}^*}\right)\frac{y}{\bar{R}^*} - \psi\mathbb{E}\left(\frac{1}{2\psi}(\theta_s - \theta_b)\frac{y}{\bar{R}^*}\right)^2 \\
=& \left[\left(\frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger} - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\right) + \frac{1}{2\psi}\frac{y}{\bar{R}^\dagger \bar{R}^*}\left(\frac{(1+\lambda)}{\lambda}\text{var}(\theta) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}(\overline{\theta} - \underline{\theta})\right)\right]\lambda\bar{R}^\dagger\frac{y}{\bar{R}^*}
\end{aligned}$$

which immediately implies the result given threshold μ^{sp}. Lets now check that the participation constraint of the buyer is satisfied. The buyer's payoff is

$$\begin{aligned}
&(1+\lambda)\, x + y - \mathbb{E}\theta_s \left[R_l B_l + R_f B_f\right] \\
=&y + \mathbb{E}\left[(1+\lambda)\,\theta_b - \theta_s\right]\left[R_l B_l + R_f B_f\right] \\
\geqslant&y + \lambda \mathbb{E}\left[R_f\right] \frac{y}{\bar{R}_f} \\
>&0
\end{aligned}$$

which implies that the participation constraint is satisfied. Finally, it is easy to see that $\mu^{sp} < \mu_1$ and a simple computation implies that

$$\mu^{sp} - \mu_2 = \frac{1}{2\lambda\psi}\frac{y}{\bar{R}^*}\mathrm{var}\,(\theta) > 0$$

which proves that $\mu_2 < \mu^{sp} < \mu_1$. Q.E.D.

Proof of Proposition 4

As before, we can substitute the participation and feasibility constraint to write the contracting problem as

$$\max_{b_l}\,(1+\lambda)\left(\mathbb{E}\theta_s\left(\left(R_l - \frac{R_f}{\tilde{R}_f}\tilde{R}_l\right)(b_l - \hat{b}_l) + \frac{R_f}{\tilde{R}_f}y\right)\right) - \mathbb{E}\theta_b\left(\left(R_l - \frac{R_f}{\tilde{R}_f}\tilde{R}_l\right)(b_l - \hat{b}_l) + \frac{R_f}{\tilde{R}_f}y\right)$$

where $\tilde{R} = \left\{\overline{R}, \underline{R}\right\}$ depending on whether $b \geqslant \hat{b}$. The first order condition is

$$(1+\lambda)\,\mathbb{E}\left[\theta_s\left(R_l - \frac{R_f}{\tilde{R}_f}\tilde{R}_l\right)\right] - \mathbb{E}\left[\theta_b\left(R_l - \frac{R_f}{\tilde{R}_f}\tilde{R}_l\right)\right] \geqslant 0$$

First, suppose that $b_l < \hat{b}_l$. Then the foc is

$$\underline{R}_l\left[(1+\lambda)\,\mathbb{E}\left[\theta_s\left(\frac{R^\dagger + \frac{1}{2\psi}(\theta_s - \theta_b)\,B_l}{\underline{R}^\dagger} - \frac{R_f}{\bar{R}_f}\right)\right] - \mathbb{E}\left[\theta_b\left(\frac{R^\dagger + \frac{1}{2\psi}(\theta_s - \theta_b)\,B_l}{\underline{R}^\dagger} - \frac{R_f}{\bar{R}_f}\right)\right]\right]$$

$$
\begin{aligned}
&(1+\lambda)\left[\left(\frac{\mathbb{E}\left[R^{\dagger}\right]+\frac{1}{2\psi}\operatorname{var}(\theta)\,B_l}{\underline{R}^{\dagger}}-\frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\right)\right]-\left(\frac{\mathbb{E}\left[R^{\dagger}\right]-\frac{1}{2\psi}\operatorname{var}(\theta)\,B_l}{\underline{R}^{\dagger}}-\frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\right)\\
\geqslant&(1+\lambda)\left(\frac{\mathbb{E}\left[R^{\dagger}\right]}{\underline{R}^{\dagger}}-\frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\right)-\left(\frac{\mathbb{E}\left[R^{\dagger}\right]}{\underline{R}^{\dagger}}-\frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\right)\\
>&0
\end{aligned}
$$

so that $b_l < \hat{b}_l$ can never be part of an equilibrium. Next, suppose that $b_f < \hat{b}_f$. Then the foc is

$$
\begin{aligned}
&\bar{R}_l\left[(1+\lambda)\,\mathbb{E}\left[\theta_s\left(\frac{R^{\dagger}+\frac{1}{2\psi}\left(\theta_s-\theta_b\right)B_l}{\overline{R}^{\dagger}+\frac{1}{2\psi}\left(\overline{\theta}-\underline{\theta}\right)B_l}-\frac{R_f}{\underline{R}_f}\right)\right]-\mathbb{E}\left[\theta_b\left(\frac{R^{\dagger}+\frac{1}{2\psi}\left(\theta_s-\theta_b\right)B_l}{\overline{R}^{\dagger}+\frac{1}{2\psi}\left(\overline{\theta}-\underline{\theta}\right)B_l}-\frac{R_f}{\underline{R}_f}\right)\right]\right]\\
=&\bar{R}_l\left[(1+\lambda)\left(\frac{\mathbb{E}\left[R^{\dagger}\right]+\frac{1}{2\psi}\operatorname{var}(\theta)\,B_l}{\overline{R}^{\dagger}+\frac{1}{2\psi}\left(\overline{\theta}-\underline{\theta}\right)B_l}-\frac{\mathbb{E}\left[R_f\right]}{\underline{R}_f}\right)-\left(\frac{\mathbb{E}\left[R^{\dagger}\right]-\frac{1}{2\psi}\operatorname{var}(\theta)\,B_l}{\overline{R}^{\dagger}+\frac{1}{2\psi}\left(\overline{\theta}-\underline{\theta}\right)B_l}-\frac{\mathbb{E}\left[R_f\right]}{\underline{R}_f}\right)\right]\\
=&\bar{R}_l\left[(1+\lambda)\left(\frac{\mathbb{E}\left[R^{\dagger}\right]}{\overline{R}^{\dagger}+\frac{1}{2\psi}\left(\overline{\theta}-\underline{\theta}\right)B_l}-\frac{\mathbb{E}\left[R_f\right]}{\underline{R}_f}\right)-\left(\frac{\mathbb{E}\left[R^{\dagger}\right]}{\overline{R}^{\dagger}+\frac{1}{2\psi}\left(\overline{\theta}-\underline{\theta}\right)B_l}-\frac{\mathbb{E}\left[R_f\right]}{\underline{R}_f}\right)\right]\\
&+\bar{R}_l\left[(1+\lambda)\left(\frac{\frac{1}{2\psi}\operatorname{var}(\theta)\,B_l}{\overline{R}^{\dagger}+\frac{1}{2\psi}\left(\overline{\theta}-\underline{\theta}\right)B_l}\right)-\left(\frac{\frac{1}{2\psi}\operatorname{var}(\theta)\,B_l}{\overline{R}^{\dagger}+\frac{1}{2\psi}\left(\overline{\theta}-\underline{\theta}\right)B_l}\right)\right]\\
=&\bar{R}_l\lambda\left[\left(\frac{\frac{1}{2\psi}\operatorname{var}(\theta)\,B_l}{\overline{R}^{\dagger}+\frac{1}{2\psi}\left(\overline{\theta}-\underline{\theta}\right)B_l}\right)-\left(\frac{\mathbb{E}\left[R_f\right]}{\underline{R}_f}-\frac{\mathbb{E}\left[R^{\dagger}\right]}{\overline{R}^{\dagger}+\frac{1}{2\psi}\left(\overline{\theta}-\underline{\theta}\right)B_l}\right)\right]
\end{aligned}
$$

For the model to display hysteresis we need the expression above to be less than zero. The sign of the expression above is equal to the sign of

$$
\begin{aligned}
\frac{1}{2\psi}\left(\operatorname{var}(\theta)-\left(\overline{\theta}-\underline{\theta}\right)\frac{\mathbb{E}\left[R_f\right]}{\underline{R}_f}\right)B_l-\overline{R}^{\dagger}\left(\frac{\mathbb{E}\left[R_f\right]}{\underline{R}_f}-\frac{\mathbb{E}\left[R^{\dagger}\right]}{\overline{R}^{\dagger}}\right)&<\\
\frac{1}{2\psi}\left(\operatorname{var}(\theta)-\left(\overline{\theta}-\underline{\theta}\right)\frac{\mathbb{E}\left[R_f\right]}{\underline{R}_f}\right)B_l-\overline{R}^{\dagger}\left(\frac{\mathbb{E}\left[R_f\right]}{\underline{R}_f}-1\right)&<0
\end{aligned}
$$

where the last inequality follows from Assumption 2. Q.E.D.

Proof of Proposition 5

The proof of this proposition requires the following lemma.

Lemma 1. *In the optimal bilateral contract, the amount of special good is given by*

$$
x_i=\mathbb{E}\left[\theta_{js}\left(R_i b_{ii}+R_j b_{ij}+R_f b_{if}\right)\right]
$$

Additionally, for any currency c*, the optimal payments are given by* $b_{ic} = \gamma_c \frac{y}{\bar{R}_c}$ *with* $\gamma_c \in [0,1]$, $\sum_{k=i,j,f} \gamma_k = 1$, *and* $\gamma_c = 0$ *if*

$$\mathbb{E}\left[\left(\theta_{js}(1+\lambda) - \theta_{ib}\right)\left(\frac{R_c}{\bar{R}_c}\right)\right] < \max_{k=i,j,f} \mathbb{E}\left[\left(\theta_{js}(1+\lambda) - \theta_{ib}\right)\left(\frac{R_k}{\bar{R}_k}\right)\right]$$

Proof. We can use the same argument used in the baseline model to show that the participation constraint of problem (9) is binding in the optimum. We then solve for x using the participation constraint. Once we substitute x in the problem we obtain the following re-formulated problem:

$$\max_{b_i \geqslant 0, b_j \geqslant 0, b_f \geqslant 0} \mathbb{E}\left[\left((1+\lambda)\,{}_j\theta_s - {}_i\theta_b\right)\left(R_{ii}b_i + R_{ji}b_j + R_{fi}b_f\right)\right]$$

subject to the feasibility constraint

$$\overline{R}_{ii}b_i + \overline{R}_{ji}b_j + \overline{R}_{fi}b_f \leqslant y.$$

This is a linear problem whose solution involves corners. We solve this by supposing $b_c = 0$ and then the problem is the same as (3), which we solve using proposition (1). We do this for $c = i, j, f$ and then compare the objective function in each of the three cases. Comparing the values yields the results stated in the proposition. Q.E.D.

Proof of Proposition 5. We restrict attention to symmetric equilibria in which $B_{jc} = B_{ic} \equiv B_c$ for $c = i, j, f$. The proof of the proposition proceeds in two steps. First, we show the existence of an equilibrium with $B_i = 0$, $B_j = 0$ and $B_f = \frac{y}{\bar{R}_f}$. Second, we show its uniqueness.

In order for $B_i = 0$, $B_j = 0$ and $B_f = \frac{y}{\bar{R}_f}$ to be an equilibrium, the marginal value of signing the contract in currency f has to be larger than the marginal values of doing it in currency i and j:

$$\frac{\mathbb{E}\left[\left(\theta_{js}(1+\lambda) - \theta_{ib}\right)R_f\right]}{\bar{R}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}(1+\lambda) - \theta_{ib}\right)R_i\right]}{\bar{R}_i} \tag{12}$$

and

$$\frac{\mathbb{E}\left[\left(\theta_{js}(1+\lambda) - \theta_{ib}\right)R_f\right]}{\bar{R}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}(1+\lambda) - \theta_{ib}\right)R_j\right]}{\bar{R}_j}. \tag{13}$$

These conditions ensure that contracts between buyers from country i and sellers from country j are set in currency f. We also need conditions for which contracts between buyers from country j and sellers from country i are set in currency f, but these are equivalent to the previous ones given the symmetry across countries. After substituting in the governments' best responses and evaluating these expressions at $B_i = 0$, $B_j = 0$ and $B_f = \frac{y}{\bar{R}_f}$, these optimality conditions simplify to $\mu_1 = \frac{\mathbb{E}(R_f)}{\bar{R}_f} > \frac{\mathbb{E}\left(R_i^\dagger\right)}{\bar{R_i}^\dagger} = \frac{\mathbb{E}\left(R_j^\dagger\right)}{\bar{R_j}^\dagger}$. These are identical to the conditions obtained in the baseline model.

Now we show the conditions under which this equilibrium is unique in the set of symmetric equilibria. For this to be a unique equilibrium, it must also be true that the above inequalities hold for all prices R_i consistent with $B_i \in \left[0, \frac{y}{\bar{R}_i^*}\right]$. Note that imposing symmetry in the currency choices of international contracts yields the following optimal choice of inflation for the government of country i

$$R_i = R_i^\dagger + \frac{1}{2\psi}\left(\theta_{is} - \theta_{ib}\right) B_i.$$

Additionally, the minimum level of inflation (maximum level of R) is the same as in the baseline economy: $\bar{R}_i = \bar{R}_i^\dagger + \frac{1}{2\psi}\left(\bar{\theta} - \underline{\theta}\right) B_i$. We obtain symmetric expressions for R_j. Replacing the government's choice of inflation in inequality (12) yields

$$\frac{\mathbb{E}\left[\left({}_j\theta_s\left(1+\lambda\right) - {}_i\theta_b\right) R_f\right]}{\bar{R}_f} > \frac{\mathbb{E}\left[\left({}_j\theta_s\left(1+\lambda\right) - {}_i\theta_b\right)\left(R_i^\dagger + \frac{1}{2\psi}\left({}_i\theta_s - {}_i\theta_b\right) B_i\right)\right]}{\bar{R}_i^\dagger + \frac{1}{2\psi}\left(\bar{\theta} - \underline{\theta}\right) B_i}$$

or equivalently

$$\bar{R}_i^\dagger\left(\frac{\mathbb{E}\left[R_i^\dagger\right]}{\bar{R}_i^\dagger} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\right) + \frac{1}{2\psi}\left(\frac{\mathrm{var}\left(\theta\right)}{\lambda} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\bar{\theta} - \underline{\theta}\right)\right) B_i < 0.$$

To check if this inequality holds for all B_i we need to sign the expression $\left(\frac{\mathrm{var}(\theta)}{\lambda} - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\bar{\theta} - \underline{\theta}\right)\right)$. If it is negative then we know this holds for all B_i since $\frac{\mathbb{E}\left[R_i^\dagger\right]}{\bar{R}_i^\dagger} < \frac{\mathbb{E}[R_f]}{\bar{R}_f}$. If it is positive, we need to establish that

$$\frac{\mathbb{E}\left[R_i^\dagger\right]}{\bar{R}_i^\dagger} < \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f} - \frac{1}{\bar{R}_i^\dagger 2\psi}\left(\frac{\mathrm{var}\left(\theta\right)}{\lambda} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\bar{\theta} - \underline{\theta}\right)\right)\frac{y}{R^*}.$$

Replacing the government's choice of inflation in inequality (13) yields

$$\frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right)-\theta_{ib}\right)R_f\right]}{\bar{R}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right)-\theta_{ib}\right)\left(R_i^{\dagger}+\frac{1}{2\psi}\left(\theta_{js}-\theta_{jb}\right)B_j\right)\right]}{\bar{R}_j^{\dagger}+\frac{1}{2\psi}\left(\bar{\theta}-\underline{\theta}\right)B_j}$$

or equivalently

$$\bar{R}_j^{\dagger}\left(\frac{\mathbb{E}\left[R_j^{\dagger}\right]}{\bar{R}_j^{\dagger}}-\frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\right)+\frac{1}{2\psi}\left(\frac{(1+\lambda)}{\lambda}\mathrm{var}\left(\theta\right)-\frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\bar{\theta}-\underline{\theta}\right)\right)B_j<0.$$

As before, we need to sign the expression $\left(\frac{(1+\lambda)\mathrm{var}(\theta)}{\lambda}-\frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\bar{\theta}-\underline{\theta}\right)\right)$. If it is negative then we know this holds for all B_j since $\frac{\mathbb{E}\left[R_j^{\dagger}\right]}{\bar{R}_j^{\dagger}}<\frac{\mathbb{E}[R_f]}{\bar{R}_f}$. If it is positive then we need

$$\frac{\mathbb{E}\left[R_j^{\dagger}\right]}{\bar{R}_j^{\dagger}}<\frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}-\frac{1}{\bar{R}_j^{\dagger}2\psi}\left(\frac{(1+\lambda)\mathrm{var}\left(\theta\right)}{\lambda}-\frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\bar{\theta}-\underline{\theta}\right)\right)\frac{y}{R^*}.$$

Since both inequalities need to hold simultaneously the cutoff value of policy risk below which the equilibrium with $B_b = 0$, $B_s = 0$ and $B_f = \frac{y}{\bar{R}_f}$ is the unique symmetric equilibrium is given by

$$\mu_2^I=\max\left\{\frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}-\frac{1}{\bar{R}_j^{\dagger}2\psi}\left(\frac{(1+\lambda)\mathrm{var}\left(\theta\right)}{\lambda}-\frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\bar{\theta}-\underline{\theta}\right)\right)\frac{y}{R^*},\right.$$
$$\left.\frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}-\frac{1}{\bar{R}_i^{\dagger}2\psi}\left(\frac{\mathrm{var}\left(\theta\right)}{\lambda}-\frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\bar{\theta}-\underline{\theta}\right)\right)\frac{y}{R^*}\right\}$$

It is easy to see that $\mu_2^I<\mu_2$ since $\left(\frac{(1+\lambda)\mathrm{var}(\theta)}{\lambda}-\frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\bar{\theta}-\underline{\theta}\right)\right)<\left(\frac{(2+\lambda)\mathrm{var}(\theta)}{\lambda}-\frac{\mathbb{E}[R_f]}{\bar{R}_f}\left(\bar{\theta}-\underline{\theta}\right)\right)$.
Q.E.D.

Proof of Proposition 6

The proof requires the following lemma whose proof follows directly from the first order conditions of the contracting problem. Define

$$\Omega\equiv\mathbb{E}\left[\left[(1+\lambda)\,\theta_s-\theta_b\right]\left[\frac{R_l}{\bar{R}_l}-\frac{R_f}{\bar{R}_f}\right]\mid\theta_b\leqslant\chi\right]F\left(\theta_b\leqslant\chi\right)-\mathbb{E}\left[\chi\left[\frac{R_l}{\bar{R}_l}-\frac{R_f}{\bar{R}_f}\right]\mid\theta_b>\chi\right]\left[1-F\left(\theta_b\leqslant\chi\right)\right]$$

Lemma 2. *In the optimal bilateral contract, the amount of special good is given by*

$$x = \mathbb{E}\left[\theta_s\left(b_l R_l + b_f R_f\right) \mid \theta_b \leqslant \chi\right] F\left(\theta_b \leqslant \chi\right)$$

while the payments satisfy

1. *If* $\Omega < 0$, *then* $b_l = 0$ *and* $b_f = \frac{y}{\bar{R}_f}$
2. *If* $\Omega = 0$ *then* $b_l = \gamma \frac{y}{\bar{R}_l}$ *and* $b_f = (1-\gamma)\frac{y}{\bar{R}_f}$ *for any* $\gamma \in [0,1]$.
3. *If* $\Omega > 0$ *then* $b_l = \frac{y}{\bar{R}_l}$ *and* $b_f = 0$.

Proof of Proposition 6. First, from Lemma 2, we know that in order for an equilibrium with $B_l = 0$ to exist it must be that

$$\mathbb{E}\left[\left[(1+\lambda)\theta_s - \theta_b\right]\left[\frac{R^\dagger}{\bar{R}^\dagger} - \frac{R_f}{\bar{R}_f}\right] \mid \theta_b \leqslant \chi\right] F\left(\theta_b \leqslant \chi\right)$$
$$-\mathbb{E}\left[\chi\left[\frac{R^\dagger}{\bar{R}^\dagger} - \frac{R_f}{\bar{R}_f}\right] \mid \theta_b > \chi\right]\left[1 - F\left(\theta_b \leqslant \chi\right)\right] < 0$$

or

$$\left(\mathbb{E}\left[\left[(1+\lambda)\theta_s - \theta_b\right] \mid \theta_b \leqslant \chi\right] F\left(\theta_b \leqslant \chi\right) - \mathbb{E}\left[\chi \mid \theta_b > \chi\right]\left[1 - F\left(\theta_b \leqslant \chi\right)\right]\right)\left[\frac{\mathbb{E}\left[R^\dagger\right]}{\bar{R}^\dagger} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\right] < 0$$

Notice that if $\chi = \bar{\theta}$ then

$$\left(\mathbb{E}\left[\left[(1+\lambda)\theta_s - \theta_b\right] \mid \theta_b \leqslant \chi\right] F\left(\theta_b \leqslant \chi\right) - \mathbb{E}\left[\chi \mid \theta_b > \chi\right]\left[1 - F\left(\theta_b \leqslant \chi\right)\right]\right) = \lambda$$

and thus by continuity it is positive for χ close to $\bar{\theta}$. Therefore the expression is negative iff $\frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger} - \frac{\mathbb{E}[R_f]}{\bar{R}_f} < 0$. To show that this equilibrium is unique we need to show that

$$\mathbb{E}\left[\left[(1+\lambda)\theta_s - \theta_b\right]\left[\frac{R_l\left(B_l\right)}{\bar{R}_l\left(B_l\right)} - \frac{R_f}{\bar{R}_f}\right] \mid \theta_b \leqslant \chi\right] F\left(\theta_b \leqslant \chi\right)$$
$$-\mathbb{E}\left[\chi\left[\frac{R_l\left(B_l\right)}{\bar{R}_l\left(B_l\right)} - \frac{R_f}{\bar{R}_f}\right] \mid \theta_b > \chi\right]\left[1 - F\left(\theta_b \leqslant \chi\right)\right] < 0$$

for all values of $0 \leqslant B_l \leqslant \frac{y}{R^*}$ where R^* is defined as earlier. We can substitute the policy function for the government and rearrange the expression to obtain

$$\frac{\mathbb{E}\left[R^\dagger\right]}{\bar{R}^\dagger} < \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f} - \frac{1}{2\psi}\left[\left(\frac{\Phi(\chi)}{\Lambda(\chi)} F_\theta(\chi) - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\bar{\theta} - \underline{\theta}\right)\right) + \frac{\chi^2}{\Lambda(\chi)}\left[1 - F_\theta(\chi)\right]\right]\frac{B_l}{\bar{R}^\dagger}$$

where

$$\Lambda(\chi) \equiv [[(1+\lambda) - \mathbb{E}[\theta_b \mid \theta_b \leqslant \chi]] F_\theta(\chi) - \chi [1 - F_\theta(\chi)]]$$

$$\Phi(\chi) \equiv \left[(1+\lambda)(\text{var}(\theta) + 1 - \mathbb{E}[\theta_b \mid \theta_b \leqslant \chi]) - \left(\mathbb{E}[\theta_b \mid \theta_b \leqslant \chi] - \mathbb{E}\left[\theta_b^2 \mid \theta_b \leqslant \chi\right]\right)\right]$$

Notice that as $\chi \to \bar{\theta}$, the term in square brackets multiplying $\frac{B}{\bar{R}^\dagger}$ converges to

$$\frac{2+\lambda}{\lambda}\text{var}(\theta) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}(\bar{\theta} - \underline{\theta})$$

which is positive as a consequence of Assumption 1. Therefore, by continuity, for χ close to $\bar{\theta}$,

$$\left(\frac{\Phi(\chi)}{\Lambda(\chi)} F_\theta(\chi) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}(\bar{\theta} - \underline{\theta})\right) + \frac{\chi^2}{\Lambda(\chi)}[1 - F_\theta(\chi)] \geqslant 0$$

Next, we can define a threshold

$$\mu_2(\chi) \equiv \frac{\mathbb{E}[R_f]}{\bar{R}_f} - \frac{1}{2\psi}\left[\left(\frac{\Phi(\chi)}{\Lambda(\chi)} F_\theta(\chi) - \frac{\mathbb{E}[R_f]}{\bar{R}_f}(\bar{\theta} - \underline{\theta})\right) + \frac{\chi^2}{\Lambda(\chi)}[1 - F_\theta(\chi)]\right] \frac{y}{\bar{R}^\dagger R^*}$$

The previous argument establishes that for any $\frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger} < \mu_2(\chi)$, there exists a unique equilibrium in which $B_l = 0$. Next, notice that

$$\mu_2'(\chi) = -\left[\frac{\Lambda(\chi)[\Phi(\chi) f_\theta(\chi) + \Phi'(\chi) F_\theta(\chi)] - \Phi(\chi) F_\theta(\chi) \Lambda'(\chi)}{\Lambda(\chi)^2} \frac{1}{2\psi} \frac{y}{\bar{R}^\dagger R^*}\right.$$
$$\left. + \frac{\Lambda(\chi)\left[2\chi[1 - F_\theta(\chi)] - \chi^2 f_\theta(\chi)\right] - \Lambda'(\chi)\chi^2[1 - F_\theta(\chi)]}{\Lambda(\chi)^2}\right] \frac{1}{2\psi} \frac{y}{\bar{R}^\dagger R^*}$$

where

$$\Lambda'(\chi) = [(1+\lambda) - \mathbb{E}[\theta_b \mid \theta_b \leqslant \chi]] f_\theta(\chi) - \frac{\partial \mathbb{E}[\theta_b \mid \theta_b \leqslant \chi]}{\partial \chi} F_\theta(\chi) - [1 - F_\theta(\chi)] + \chi f_\theta(\chi),$$

$$\Phi'(\chi) = \left[(1+\lambda)\left(-\frac{\partial \mathbb{E}[\theta_b \mid \theta_b \leqslant \chi]}{\partial \chi}\right) - \left(\frac{\partial \mathbb{E}[\theta_b \mid \theta_b \leqslant \chi]}{\partial \chi} - \frac{\partial \mathbb{E}\left[\theta_b^2 \mid \theta_b \leqslant \chi\right]}{\partial \chi}\right)\right]$$

and,

$$\frac{\partial \mathbb{E}[\theta_b \mid \theta_b \leqslant \chi]}{\partial \chi} = \frac{f_\theta(\chi)}{F_\theta(\chi)}(\chi - \mathbb{E}[\theta_b \mid \theta_b \leqslant \chi]),$$

$$\frac{\partial \mathbb{E}\left[\theta_b^2 \mid \theta_b \leqslant \chi\right]}{\partial \chi} = \frac{f_\theta(\chi)}{F_\theta(\chi)}\left[\chi^2 - \mathbb{E}\left[\theta_b^2 \mid \theta_b \leqslant \chi\right]\right]$$

Taking the limit as $\chi \uparrow \bar{\theta}$ yields

$$\mu_2'(\bar{\theta}) = \left[\frac{f_\theta(\bar{\theta}) \left[(1+\lambda)^2 \operatorname{var}(\theta) + (\bar{\theta} - 1) \left[1 + \lambda - \bar{\theta} \right] \right]}{\lambda} + \frac{\chi^2 f_\theta(\bar{\theta})}{\lambda} \right] \frac{1}{2\psi} \frac{y}{\bar{R}^\dagger R^*} > 0$$

which establishes the desired result. Q.E.D.

B Additional Results and Extensions

B.1 TNT Model with Endogenous Real Exchange Rate Risk

This section shows that the presence of the exogenous risk of the price of foreign currency $\frac{\mathbb{E}[R_f]}{\bar{R}_f}$ can arise in an extension of our model with tradable and non-tradable goods and shocks to the relative demand of these goods in the domestic economy. Suppose the numeraire good in our model is a composite of tradable and non-tradable goods, $c = c_T^{\alpha} c_N^{1-\alpha}$, where c_T (c_N) is the domestic consumption of tradables (non-tradables), and α is a stochastic parameter that captures shocks to the relative demand of these goods. The equivalent good in the foreign country is given by $c^* = (c_T^*)^{\alpha^*} (c_N^*)^{1-\alpha^*}$. We assume that α^* is deterministic. We also normalize the endowments $y_T = y_N = y_T^* = y_N^* = y$. Consistent with our baseline model we denote the price of the local (foreign) currency in terms of the domestic composite by R_l (R_f). Additionally we normalize the price of the foreign currency in terms of the foreign composite good to 1. The exchange rate e is defined as as the price of the local currency in terms of the foreign currency. Let p_T denote the price of the tradable goods in the domestic economy in terms of the local currency and p_T^* denote the price of the tradable goods in the foreign economy in terms of the foreign currency.

Given the Cobb-Douglas structure, p_T and p_T^* are given by

$$p_T = \frac{1}{R_l} \alpha \left(\frac{c_N}{c_T} \right)^{1-\alpha} \text{ and } p_T^* = \alpha^* \left(\frac{c_N^*}{c_T^*} \right)^{1-\alpha^*}.$$

In this model, the law of one price for tradable goods holds. Market clearing in all goods implies that the exchange rate e is given by

$$e = \frac{p_T}{p_T^*} = \frac{\alpha}{\alpha^*} \frac{1}{R_l}.$$

Therefore,

$$R_f = e R_l = \frac{\alpha}{\alpha^*}.$$

In this model we can generate fluctuations in the real exchange rate (the price of the foreign currency in terms of the domestic composite good, R_f) by assuming a stochastic process for α.

B.2 Relaxing Assumption 1

Recall that Assumption 1 ensures that the value of discretion in monetary policy is sufficiently high. To understand what would happen if this were not true we consider the extreme case in which there is no value to discretion, i.e. $\text{var}(\theta) = 0$. In this case we show that there is a unique symmetric equilibrium in which either the local currency is exclusively used or both currencies are used simultaneously. Next, we show that in general this equilibrium is inefficient as it features too little use of the foreign currency.

Assumption 3. *Assume that* $\text{var}(\theta) = 0$, $\theta_b > \theta_s$, *and* $(1+\lambda)\,\theta_s > \theta_b$

The next proposition shows that under this assumption there is a unique symmetric competitive equilibrium.

Proposition 7. *Suppose that Assumption 3 holds. If* $\frac{\mathbb{E}[R_f]}{\bar{R}_f} \geqslant \frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger}$, *there exists a unique equilibrium in which* $B_l = 0$. *If* $\frac{\mathbb{E}[R_f]}{\bar{R}_f} < \frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger}$, *there exists a unique equilibrium with* $B_l > 0$.

Proof. The bilateral contracting and government's problem is identical to baseline. In particular, the policy functions for the government is given by (7) and (8). Suppose first that $\frac{\mathbb{E}[R_f]}{\bar{R}_f} \geqslant \frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger}$. Then, there exists an equilibrium in which $B_l = 0$. To see that this a unique equilibrium notice that $\frac{\mathbb{E}[R^\dagger]+\frac{1}{2\psi}(\theta_s-\theta_b)B_l}{\bar{R}^\dagger+\frac{1}{2\psi}(\theta_s-\theta_b)B_l}$ is decreasing in B_l. Next, suppose that $\frac{\mathbb{E}[R_f]}{\bar{R}_f} < \frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger}$. Then it must be that $B_l > 0$ and the fact that $\frac{\mathbb{E}[R^\dagger]+\frac{1}{2\psi}(\theta_s-\theta_b)B_l}{\bar{R}^\dagger+\frac{1}{2\psi}(\theta_s-\theta_b)B_l}$ is decreasing in B_l implies that the equilibrium is unique. If B_l is interior then the unique equilibrium is the solution to the following system of equations

$$R_l = R^\dagger + \frac{1}{2\psi}\left(\theta_s - \theta_b\right) B_l$$

$$\frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f} = \frac{\mathbb{E}\left[R_l\right]}{\bar{R}_l}$$

$$\bar{R}_l = \bar{R}^\dagger + \frac{1}{2\psi}\left(\theta_s - \theta_b\right) B_l$$

We can combine these equations to get

$$B_l = \frac{\bar{R}^\dagger\left(\frac{\mathbb{E}[R_f]}{\bar{R}_f} - \frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger}\right)}{\left[1 - \frac{\mathbb{E}[R_f]}{\bar{R}_f}\right]\frac{1}{2\psi}\left(\theta_s - \theta_b\right)}$$

Therefore,

$$\bar{R}_l = \bar{R}^\dagger \left(1 + \frac{\left(\frac{\mathbb{E}[R_f]}{\bar{R}_f} - \frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger} \right)}{\left[1 - \frac{\mathbb{E}[R_f]}{\bar{R}_f} \right]} \right)$$

To check that the solution is interior we need to check that $B_l \leqslant \frac{y}{\bar{R}_l}$ or

$$\frac{\bar{R}^\dagger \left(\frac{\mathbb{E}[R_f]}{\bar{R}_f} - \frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger} \right)}{\left[1 - \frac{\mathbb{E}[R_f]}{\bar{R}_f} \right] \frac{1}{2\psi} (\theta_s - \theta_b)} \leqslant \frac{y \left[1 - \frac{\mathbb{E}[R_f]}{\bar{R}_f} \right]}{\bar{R}^\dagger \left(1 - \frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger} \right)}$$

Otherwise the unique solution is $B_l = \frac{y}{\bar{R}_l}$. Q.E.D.

Next, we study the social planner's problem. The planning problem is identical to the baseline environment.

$$\max_{B_l, B_f} \mathbb{E} \left[[(1+\lambda)\theta_s - \theta_b] (R_l B_l + R_f B_b) - 2l \left(R_l - R^\dagger \right) \right]$$

subject to (1), (7), and (8).

Proposition 8. *Suppose that Assumption 3 holds. If* $\frac{\mathbb{E}[R_f]}{\bar{R}_f} \geqslant \frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger}$*, the competitive equilibrium is efficient and* $B_l^{sp} = 0$*. If* $\frac{\mathbb{E}[R_f]}{\bar{R}_f} < \frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger}$*, and the solution the planning problem is interior, the competitive equilibrium is inefficient and* $B_l^{sp} < B_l^{ce}$*.*

Proof. The first order condition of the planning problem is

$$[(1+\lambda)\theta_s - \theta_b] \left(\bar{R}^\dagger \left[\frac{\mathbb{E}\left[R^\dagger\right]}{\bar{R}^\dagger} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f} \right] + \frac{1}{\psi} (\theta_s - \theta_b) B_l \left[1 - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f} \right] \right) - \frac{1}{2\psi} (\theta_s - \theta_b)^2 B_l$$

If $\frac{\mathbb{E}[R_f]}{\bar{R}_f} \geqslant \frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger}$, the expression above is negative and thus $B_l^{sp} = 0$. If $\frac{\mathbb{E}[R_f]}{\bar{R}_f} < \frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger}$, then if the solution is interior it satisfies,

$$B_l^{sp} = \frac{\bar{R}^\dagger \left[\frac{\mathbb{E}[R_f]}{\bar{R}_f} - \frac{\mathbb{E}[R^\dagger]}{\bar{R}^\dagger} \right]}{\left(2 \left[1 - \frac{\mathbb{E}[R_f]}{\bar{R}_f} \right] - \frac{(\theta_s - \theta_b)}{[(1+\lambda)\theta_s - \theta_b]} \right) \frac{1}{2\psi} (\theta_s - \theta_b)}$$

Therefore,

$$\frac{B_l^{sp}}{B_l^{ce}} = \frac{\left[1 - \frac{\mathbb{E}[R_f]}{\bar{R}_f} \right]}{\left(2 \left[1 - \frac{\mathbb{E}[R_f]}{\bar{R}_f} \right] - \frac{(\theta_s - \theta_b)}{[(1+\lambda)\theta_s - \theta_b]} \right)} < 1$$

since

$$0 < 1 - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f} - \frac{(\theta_s - \theta_b)}{\left[(1+\lambda)\,\theta_s - \theta_b\right]}$$

which follows from Assumption (3). Q.E.D.

The key implication of relaxing Assumption 1, is that in general, since there are no benefits of discretion, the efficient allocation will always prescribe greater use of the foreign currency. In contrast if Assumption 1 holds, then we show that private agents can underestimate the covariance channel and thus we can have situations in which the efficient allocations calls for greater use of local currency.

B.3 Model of a Credit Chain

We now present a simple credit chain model that endogenizes the stocks of foreign and local currency in Section 5.

Suppose that citizens are further divided into one of I sub-types $\mathcal{I} \in \{1, 2, ..., I\}$ with continuum of each. A citizen of type i has preferences over a special good produced by type $i+1$ and produces a special good valued by type $i-1$. All types also value the consumption of the numeraire good which takes place at the end of period 2. Preferences for the representative citizen type i are given by

$$u_i = (1+\lambda)\,{}_{i}x_{i+1} - {}_{i-1}x_i + \mathbb{E}\left[\theta_i c_i\right]$$

where ${}_{i}x_{i+1}$ is the special good produced by a citizen of type $i+1$ for a citizen of type i and ${}_{i-1}x_i$ is the special good produced by a citizen of type i for a citizen of type $i-1$. We assume that ${}_{0}x_1 = {}_{I}x_{I+1} = 0$ so that type 1 does not produce a special good for any other type and type I does not consume a special good. As in the baseline we assume that $\theta_i \in \left[\overline{\theta}, \underline{\theta}\right]$ is independent across sub-types and that $\mathbb{E}\left[\theta_i\right] = 1$.

The timing of the model is as follows:

1. The first period $t = 1$ is divided into $I-1$ sub-periods in which trade takes place sequentially:
   (a) In sub-period 1, citizens of type 2 produces a special good for citizens of type 1 in exchange for the promise of payment in period 2.
   (b) Similarly, in sub-period i, citizens of type $i+1$ produce a special good for citizens of type i in exchange for the promise of payment in period 2
2. The second period $t = 2$ is divided into three sub-periods:

(a) In sub-period 1, the type of the domestic government is realized and it chooses its policy which is the aggregate price level

(b) In sub-period 2, endowments for all citizens are realized

(c) In sub-period 3, all signed contracts are executed in the order in which they were signed and finally, consumption of the composite good takes place.

Assume that all citizens are endowed with y units of the numeraire good. The definition of a bilateral contract between i and $i+1$ is identical to Section 5. Note that in this contract $i+1$ is the seller and and i is the buyer. Given the structure of the credit chain, $\left(\hat{b}_f, \hat{b}_l\right)$ is the promised payment to type i from types $i-1$.

We can then use Propositions 2 and 4 to characterize the bilateral contract.

Proposition 9. *In the optimal bilateral contract, the amount of special good is given by* $x = \mathbb{E}\left[\theta_s\left(b_l R_l + b_f R_f\right)\right]$*, while the payments satisfy*

1. *If* $\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_l}{\bar{R}_l}\right] < \mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_f}{\bar{R}_f}\right]$ *then* $b_l = \hat{b}_l$ *and* $b_f = \hat{b}_f + \frac{y}{\bar{R}_f}$

2. *If* $\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_l}{\bar{R}_l}\right] = \mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_f}{\bar{R}_f}\right]$ *then* $b_l = \hat{b}_l + \gamma\frac{y}{\bar{R}_l}$ *and* $b_f = \hat{b}_f + \left(1-\gamma\right)\frac{y}{\bar{R}_f}$ *for any* $\gamma \in [0,1]$.

3. *If* $\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_l}{\bar{R}_l}\right] > \mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_f}{\bar{R}_f}\right]$ *then* $b_l = \hat{b}_l + \frac{y}{\bar{R}_l}$ *and* $b_f = \hat{b}_f$.

The result follows immediately from Propositions 2 and 4. In particular, the optimal contract will feature currency matching of stocks and will denominate the flows in the currency with the largest marginal benefit. As a corollary of this Proposition we can compute the aggregate stock of local currency which will be needed for the government's problem. Let $\left(B_{fi}, B_{li}\right)$ denote the aggregate stock of local and foreign currency obligations in the contract signed between $i+1$ and i.

Corollary 1. *In equilibrium,*

1. *If* $\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_l}{\bar{R}_l}\right] < \mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_f}{\bar{R}_f}\right]$ *then* $B_{li} = 0$ *and* $B_{fi} = i\frac{y}{\bar{R}_f}$

2. *If* $\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_l}{\bar{R}_l}\right] = \mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_f}{\bar{R}_f}\right]$ *then* $B_{li} = \sum_{j=1}^{i}\gamma_j\frac{y}{\bar{R}_l}$ *and* $B_{fi} = \sum_{j=1}^{i}\left(1-\gamma_j\right)\frac{y}{\bar{R}_l}$

3. *If* $\mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_l}{\bar{R}_l}\right] > \mathbb{E}\left[\left(\theta_s\left(1+\lambda\right)-\theta_b\right)\frac{R_f}{\bar{R}_f}\right]$ *then* $B_{li} = i\frac{y}{\bar{R}_l}$ *and* $B_{fi} = 0$.

Next, the government's problem is

$$\max_{R_l}\sum_i\left[\theta_i C_i\right] - I\cdot l\left(R_l - R^{\dagger}\right)$$

where

$$C_i = y - R_l \left(B_{li} - B_{li-1}\right) - R_f \left(B_{fi} - B_{fi-1}\right)$$

The best response of the government is given by

$$R_l = R^{\dagger} - \frac{1}{I\psi} \sum_{i=1}^{I} \theta_i \left(B_{li} - B_{li-1}\right)$$

and so

$$\bar{R}_l = \overline{R}^{\dagger} - \max_{\theta} \left(\frac{1}{I\psi} \sum_{i=1}^{I} \theta_i \left(B_{li} - B_{li-1}\right) \right)$$

Given this lets understand how the optimal contract changes in the credit chain. Recall that the marginal benefit of denominating the contract signed between i and $i+1$ in currency l is

$$\lambda \frac{\mathbb{E}\left[R_l\right]}{\bar{R}_l} + \operatorname{cov}\left(\left(\theta_{i+1}\left(1+\lambda\right) - \theta_i\right), \frac{R_l}{\bar{R}_l} \right)$$

Using the best response of the government the previous equation for the contract signed between 1 and 2 can be written as

$$\begin{aligned} &\frac{1}{\bar{R}_l} \left(\lambda \mathbb{E}\left[R^{\dagger}\right] + \operatorname{cov}\left(\left(\theta_2\left(1+\lambda\right) - \theta_1\right), R_l\right) \right) \\ =&\frac{1}{\bar{R}_l} \left(\lambda \mathbb{E}\left[R^{\dagger}\right] + \frac{1}{I\psi} \left(\left[\left(2+\lambda\right) B_{l1} - \left(1+\lambda\right) B_{l2}\right]\right) \operatorname{var}\left(\theta\right) \right) \end{aligned}$$

Recall the expression for marginal benefit in the baseline model

$$\frac{1}{\bar{R}_l} \left(\lambda \mathbb{E}\left[R^{\dagger}\right] + \frac{1}{2\psi} \left(\left[\left(\operatorname{var}\left(\theta\right)\left(2+\lambda\right)\right)\right]\right) B_{l1} \right)$$

since $B_{l2} = 0$. On comparing the two we see that the term in parenthesis is smaller in the environment with the credit chain. In particular the covariance term is smaller owing to the fact that each government policy will respond less to individual shocks within the chain. However the the term $\bar{R}_l$ also changes and the direction of this change is ambiguous. So it is hard to say anything generally about the set of equilibria.

For illustrative purposes suppose only sub-type one is endowed with y units of the numeraire good while all other sub-types are endowed with zero units of the good. In this case

$$\bar{R}_l = \overline{R}^{\dagger} + \frac{1}{I\psi} \left(\overline{\theta} - \underline{\theta}\right) \left(B_{l1}\right)$$

while in the baseline

$$\bar{R}_l = \overline{R}^{\dagger} + \frac{1}{2\psi}\left(\overline{\theta} - \underline{\theta}\right)\left(B_{l1}\right)$$

which implies that in the credit chain, the price risk of local currency is lower. Thus, we have two competing effects. In particular, as $I \to \infty$, the marginal benefit converges to

$$\lim_{I\to\infty} \frac{\left(\lambda\mathbb{E}\left[R^{\dagger}\right] + \frac{1}{I\psi}\left(\left[B_{l1}\right]\right)\text{var}\left(\theta\right)\right)}{\overline{R}^{\dagger} + \frac{1}{I\psi}\left(\overline{\theta} - \underline{\theta}\right)\left(B_{l1}\right)} = \frac{\lambda\mathbb{E}\left[R^{\dagger}\right]}{\overline{R}^{\dagger}}$$

so the the optimal currency choice only involves comparing the price risk.

B.4 Model with International Trade Contracts: Generalized Result

This section shows that the result in Proposition 5 can be generalized to show uniqueness among the entire set of equilibria, and not just symmetric equilibria, under the following parametric assumption.

Assumption 4. *Assume that*

$$var(\theta) \geqslant \lambda \geqslant \left(\overline{\theta} - \underline{\theta}\right).$$

We can now prove a generalization of Proposition 5.

Proposition 10. *Under Assumption 4, there exists a threshold* μ_2^I *such that, if* $\frac{\mathbb{E}\left[R_i^{\dagger}\right]}{\bar{R}_i^{\dagger}} = \frac{\mathbb{E}\left[R_j^{\dagger}\right]}{\bar{R}_j^{\dagger}} \leqslant \mu_2^I$ *there exists a unique equilibrium in which* $B_{ii} = B_{ji} = B_{ij} = B_{jj} = 0$. *Furthermore,* $\mu_2^I > \mu_2$.

Proof. The proof is symmetric to that of Proposition 5. First, we show the existence of an equilibrium with $B_{ii} = B_{ji} = 0$, $B_{ij} = B_{jj} = 0$ and $B_{if} = B_{jf} = \frac{y}{\bar{R}_f}$. Second, we show this equilibrium is unique.

In order for the above allocation to be part of an equilibrium, the marginal value of signing the contract in currency f has to be larger than the marginal values of doing it in currency i and j:

$$\frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right) - \theta_{ib}\right)R_f\right]}{\bar{R}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right) - \theta_{ib}\right)R_i\right]}{\bar{R}_i} \tag{14}$$

and

$$\frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right) - \theta_{ib}\right)R_f\right]}{\bar{R}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right) - \theta_{ib}\right)R_j\right]}{\bar{R}_j}. \tag{15}$$

These conditions ensure that contracts between buyers from country i and sellers from country j are set in currency f. We also need conditions for which contracts between buyers from country j and sellers of country i are set in currency f, but these are identical to the ones in the proof of Proposition 5. After substituting in the governments' best responses and evaluating these expressions at $B_{ii} = B_{ji} = 0$, $B_{ij} = B_{jj} = 0$ and $B_{if} = B_{jf} = \frac{y}{\bar{R}_f}$, these optimality conditions simplify to $\mu_1 = \frac{\mathbb{E}(R_f)}{\bar{R}_f} > \frac{\mathbb{E}\left(R_i^\dagger\right)}{\bar{R_i}^\dagger} = \frac{\mathbb{E}\left(R_j^\dagger\right)}{\bar{R_j}^\dagger}$. These are identical to the conditions obtained in the baseline model.

Now we show the conditions under which this equilibrium is unique. For this to be a unique equilibrium, it must also be true that the previous inequalities hold for prices R_i consistent with all possible $B_{ii}, B_{ji} \in \left[0, \frac{y}{\bar{R}_i^*}\right]$. The optimal choice of inflation for the government of country i is given by

$$R_i = R_i^\dagger + \frac{1}{2\psi}\left(\theta_{is} B_{ji} - \theta_{ib} B_{ii}\right).$$

Additionally, the minimum level of inflation (maximum level of R) is the same as in the baseline economy: $\overline{R}_i = \bar{R}_i^\dagger + \frac{1}{2\psi}\left(\bar{\theta} B_{ji} - \underline{\theta} B_{ii}\right)$. We obtain symmetric expressions for R_j. Replacing the government's choice of inflation in inequality (14) yields

$$\frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right) - \theta_{ib}\right) R_f\right]}{\bar{R}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right) - \theta_{ib}\right)\left(R_i^\dagger + \frac{1}{2\psi}\left(\theta_{is} B_{ji} - \theta_{ib} B_{ii}\right)\right)\right]}{\bar{R}_i^\dagger + \frac{1}{2\psi}\left(\bar{\theta} B_{ji} - \underline{\theta} B_{ii}\right)}$$

or equivalently

$$\bar{R}_i^\dagger\left(\frac{\mathbb{E}\left[R_i^\dagger\right]}{\bar{R}_i^\dagger} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\right) + \frac{1}{2\psi}\left(\left[1 - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\bar{\theta}\right]{}_jB_i + \left[\frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\underline{\theta} + \frac{\mathrm{var}\left(\theta\right)}{\lambda} - 1\right]{}_iB_i\right) < 0 \quad (16)$$

Similarly, replacing the government's choice of inflation in inequality (15) yields

$$\frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right) - \theta_{ib}\right) R_f\right]}{\bar{R}_f} > \frac{\mathbb{E}\left[\left(\theta_{js}\left(1+\lambda\right) - \theta_{ib}\right)\left(R_j^\dagger + \frac{1}{2\psi}\left(\theta_{js} B_{ij} - \theta_{jb} B_{jj}\right)\right)\right]}{\bar{R}_j^\dagger + \frac{1}{2\psi}\left(\bar{\theta} B_{ij} - \underline{\theta} B_{jj}\right)}$$

or equivalently

$$\bar{R}_j^\dagger \left(\frac{\mathbb{E}\left[R_j^\dagger\right]}{\bar{R}_j^\dagger} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f} \right) + \frac{1}{2\psi} \left[\left(\frac{(1+\lambda)}{\lambda} \operatorname{var}(\theta) + 1 - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f} \bar{\theta} \right) B_{ij} + \left(\frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f} \underline{\theta} - 1 \right) B_{jj} \right] < 0. \tag{17}$$

Inequalities (16) and (17) should hold for any feasible $\mathbf{B} \equiv \left\{B_{ii}, B_{ij}, B_{ji}, B_{jj}\right\}$. Since both inequalities are linear in $\mathbf{B}$, it suffices to show that they hold for all combinations of extremum values. The extreme values are computed by solving a non-linear equation for the maximum values of R_i and R_j. We start with inequality (16). We first check the case in which $B_{ji} = 0$ and $B_{ii} = \frac{y}{R_1^*}$. Here R_1^* solves $R_{I1}^* = \bar{R}_i^\dagger - \frac{1}{2\psi}\underline{\theta}\frac{y}{R_{I1}^*}$. We take the largest root of this equation which is given by $R_1^* = \frac{\bar{R}_i^\dagger + \sqrt{\left(\bar{R}_i^\dagger\right)^2 - \frac{2}{\psi}\underline{\theta} y}}{2}$. Substituting these values in (16) yields the following inequality

$$\bar{R}_i^\dagger \left(\frac{\mathbb{E}\left[R_i^\dagger\right]}{\bar{R}_i^\dagger} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f} \right) + \frac{1}{2\psi} \left[\frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f} \underline{\theta} + \frac{\operatorname{var}(\theta)}{\lambda} - 1 \right] \frac{y}{R_1^*} < 0. \tag{18}$$

Second we check the other case in which $B_{ji} = \frac{y}{R_2^*}$ and $B_{ii} = 0$. Here R_2^* is the largest root that solves $R_{I2}^* = \bar{R}_i^\dagger + \frac{1}{2\psi}\bar{\theta}\frac{y}{R_{I2}^*}$, which is given by $R_2^* = \frac{\bar{R}_i^\dagger + \sqrt{\left(\bar{R}_i^\dagger\right)^2 + \frac{2}{\psi}\bar{\theta} y}}{2}$. Substituting these values in in (16) yields the following inequality

$$\bar{R}_i^\dagger \left(\frac{\mathbb{E}\left[R_i^\dagger\right]}{\bar{R}_i^\dagger} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f} \right) + \frac{1}{2\psi} \left[1 - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f} \bar{\theta} \right] \frac{y}{R_2^*} < 0. \tag{19}$$

Finally we also check the case in which both B_{ji}, B_{ii} are at their maximum values. In this case $B_{ji} = B_{ii} = \frac{y}{R^*}$,where R^* is defined as in the baseline model. Substituting these values in in (16) yields the following inequality

$$\bar{R}_i^\dagger \left(\frac{\mathbb{E}\left[R_i^\dagger\right]}{\bar{R}_i^\dagger} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f} \right) + \frac{1}{2\psi} \left(\frac{\operatorname{var}(\theta)}{\lambda} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f} \left(\bar{\theta} - \underline{\theta}\right) \right) \frac{y}{R^*} < 0 \tag{20}$$

We follow a symmetric approach with inequality (17). We first check the case in which $B_{ij} = 0$ and $B_{jj} = \frac{y}{R_1^*}$. Substituting these values in (17) yields the following inequality

$$\bar{R}_j^\dagger \left(\frac{\mathbb{E}\left[R_j^\dagger\right]}{\bar{R}_j^\dagger} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f} \right) + \frac{1}{2\psi} \left(\frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f} \underline{\theta} - 1 \right) \frac{y}{R_1^*} < 0. \tag{21}$$

Second we check the other case in which $B_{ij} = \frac{y}{R_2^*}$ and $B_{jj} = 0$. Substituting these values into (17) yields the following inequality

$$\bar{R}_j^{\dagger}\left(\frac{\mathbb{E}\left[R_j^{\dagger}\right]}{\bar{R}_j^{\dagger}} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\right) + \frac{1}{2\psi}\left(\frac{(1+\lambda)}{\lambda}\text{var}\left(\theta\right) + 1 - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\bar{\theta}\right)\frac{y}{R_2^*} < 0. \tag{22}$$

Finally we also check the case in which both B_{jj}, B_{ij} are at their maximum values. In this case $B_{jj} = B_{ij} = \frac{y}{R^*}$,where R^* is defined as in the baseline model. Substituting these values in (17) yields the following inequality

$$\bar{R}_j^{\dagger}\left(\frac{\mathbb{E}\left[R_j^{\dagger}\right]}{\bar{R}_j^{\dagger}} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\right) + \frac{1}{2\psi}\left(\frac{(1+\lambda)}{\lambda}\text{var}\left(\theta\right) - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\bar{\theta} - \underline{\theta}\right)\right)\frac{y}{R^*} < 0. \tag{23}$$

Now we need to show that inequalities (18) - (23) are satisfied for values of policy risk such that $\frac{\mathbb{E}\left[R_i^{\dagger}\right]}{\bar{R}_i^{\dagger}} = \frac{\mathbb{E}\left[R_j^{\dagger}\right]}{\bar{R}_j^{\dagger}} \leqslant \mu_2$. First note that (21) always holds since the second term is negative. Additionally, if (22) holds then (19) is also satisfied. Finally, if (23) holds then (20) is also satisfied. This leaves us with (18), (22) and (23). It is worth noting that $R_{I2}^* > R^* > R_{I1}^*$. Also recall that $\frac{\mathbb{E}\left[R_i^{\dagger}\right]}{\bar{R}_i^{\dagger}} = \frac{\mathbb{E}\left[R_j^{\dagger}\right]}{\bar{R}_j^{\dagger}} \leqslant \mu_2$ implies that

$$\bar{R}_j^{\dagger}\left(\frac{\mathbb{E}\left[R_j^{\dagger}\right]}{\bar{R}_j^{\dagger}} - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\right) + \frac{1}{2\psi}\left(\frac{(2+\lambda)}{\lambda}\text{var}\left(\theta\right) - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\bar{\theta} - \underline{\theta}\right)\right)\frac{y}{R^*} < 0. \tag{24}$$

It then follows that $\frac{\mathbb{E}\left[R_i^{\dagger}\right]}{\bar{R}_i^{\dagger}} = \frac{\mathbb{E}\left[R_j^{\dagger}\right]}{\bar{R}_j^{\dagger}} \leqslant \mu_2$ (or equivalently if (24) holds) then (23) is satisfied. Additionally, note that if we use the assumption that $var(\theta) > \lambda$ then (24) implies (22). Finally, we show that (24) implies (18). To show this we must have that

$$\left(\bar{R}_i{}^{\dagger} + \sqrt{\bar{R}_i{}^{\dagger 2} - 2\frac{1}{\psi}\underline{\theta}y}\right)\left(\frac{(2+\lambda)}{\lambda}var\left(\theta\right) - \frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\overline{\theta} - \underline{\theta}\right)\right) >$$
$$\left(\bar{R}_i{}^{\dagger} + \sqrt{\bar{R}_i{}^{\dagger 2} + 2\frac{1}{\psi}\left(\overline{\theta} - \underline{\theta}\right)y}\right)\left(\frac{var\left(\theta\right) - \lambda}{\lambda} + \underline{\theta}\frac{\mathbb{E}\left(R_f\right)}{\bar{R}_f}\right).$$

This can be rewritten as

$$\frac{\text{var}(\theta)}{\lambda}\left[\left(\bar{R}_i{}^{\dagger}+\sqrt{\bar{R}_i{}^{\dagger 2}-2\frac{1}{\psi}\underline{\theta}y}\right)(2+\lambda)-\left(\bar{R}_i{}^{\dagger}+\sqrt{\bar{R}_i{}^{\dagger 2}+2\frac{1}{\psi}\left(\overline{\theta}-\underline{\theta}\right)y}\right)\right]$$
$$-\frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left[\left(\bar{R}_i{}^{\dagger}+\sqrt{\bar{R}_i{}^{\dagger 2}-2\frac{1}{\psi}\underline{\theta}y}\right)\left(\overline{\theta}-\underline{\theta}\right)\right]$$
$$+\left(\bar{R}_i{}^{\dagger}+\sqrt{\bar{R}_i{}^{\dagger 2}+2\frac{1}{\psi}\left(\overline{\theta}-\underline{\theta}\right)y}\right)\left[1-\frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\underline{\theta}\right]>0 \tag{25}$$

Lets first consider the term

$$\left(\bar{R}_i{}^{\dagger}+\sqrt{\bar{R}_i{}^{\dagger 2}-2\frac{1}{\psi}\underline{\theta}y}\right)(2+\lambda)-\left(\bar{R}_i{}^{\dagger}+\sqrt{\bar{R}_i{}^{\dagger 2}+2\frac{1}{\psi}\left(\overline{\theta}-\underline{\theta}\right)y}\right)$$
$$\geqslant\left(\bar{R}_i{}^{\dagger}+\sqrt{\bar{R}_i{}^{\dagger 2}-2\frac{1}{\psi}\underline{\theta}y}\right)(2+\lambda)-\left(\bar{R}_i{}^{\dagger}+\sqrt{\bar{R}_i{}^{\dagger 2}-2\frac{1}{\psi}\underline{\theta}y}+\sqrt{2\frac{1}{\psi}\overline{\theta}y}\right)$$
$$=(1+\lambda)\,\bar{R}_i{}^{\dagger}-\sqrt{2\frac{1}{\psi}\overline{\theta}y}+(1+\lambda)\sqrt{\bar{R}^{\dagger 2}-2\frac{1}{\psi}\underline{\theta}y}$$
$$\geqslant 0$$

if $\bar{R}^{\dagger}-\sqrt{2\frac{1}{\psi}\overline{\theta}y}\geqslant 0$, which is a condition we need for R_1^* to be well-defined. Given this, the first two lines of (25) are greater than

$$\frac{\mathbb{E}\left[R_f\right]}{\bar{R}_f}\left(\left(\lambda-\left(\overline{\theta}-\underline{\theta}\right)\right)\bar{R}^{\dagger}+\bar{R}^{\dagger}-\sqrt{2\frac{1}{\psi}\overline{\theta}y}+\left(1+\lambda-\left(\overline{\theta}-\underline{\theta}\right)\right)\sqrt{\bar{R}^{\dagger 2}-2\frac{1}{\psi}\underline{\theta}y}\right)$$

which is positive if $\lambda\geqslant\left(\overline{\theta}-\underline{\theta}\right)$. Hence, we showed that (18) - (23) are satisfied for values of policy risk such that $\frac{\mathbb{E}\left[R_i^{\dagger}\right]}{\bar{R}_i^{\dagger}}=\frac{\mathbb{E}\left[R_j^{\dagger}\right]}{\bar{R}_j^{\dagger}}\leqslant\mu_2$. Finally, the cutoff value μ_2^I is defined as the smallest cutoff value such that (18) - (23) are satisfied. Q.E.D.

C Figures

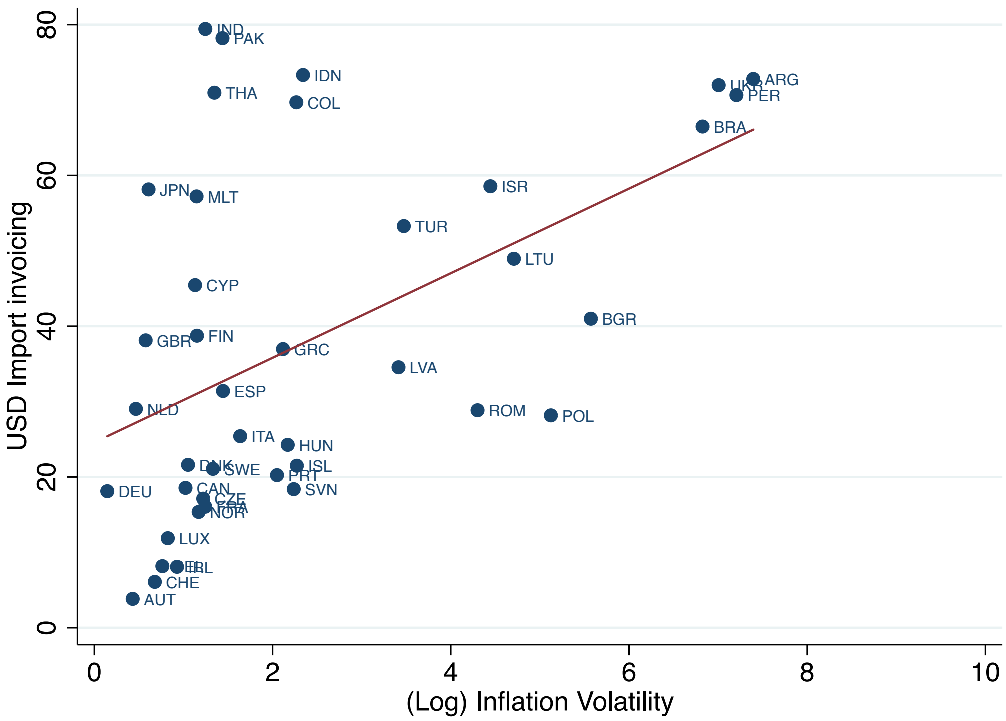


Figure 4: Trade Dollarization and Inflation Volatility
Sources: Gopinath (2015) and IFS.