Low Interest Rates, Market Power, and Productivity Growth*

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Abstract

We show theoretically that a low interest rate gives industry leaders a strategic advantage over followers, and this advantage becomes more dominant as the interest rate approaches zero. Consequently, as the interest rate declines, market structure becomes more monopolistic, and, for a sufficiently low interest rate, productivity growth slows. This prediction is tested through an analysis of excess returns for industry leaders relative to followers in response to a decline in interest rates. A decline in the ten year Treasury yield generates positive excess returns for leaders, and the magnitude of the excess returns rises as the yield approaches zero. The model provides a unified explanation for why the fall in long-term interest rates has been associated with rising market concentration, reduced dynamism, a widening productivity-gap between leaders and followers, and slower productivity growth.

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1 Introduction

Long term interest rates have fallen globally since the 1980s, reaching their lowest levels in recent years. A large body of recent research explores both the causes and consequences of low long-term interest rates (e.g., Summers (2014)). This study analyzes the consequences of long-term interest rates for the production side of the economy. The question addressed can be put as follows: suppose long-term interest rates fall due to demand-side issues, how does the supply side of the economy respond?

Traditionally, a lower interest rate is viewed as expansionary for the supply side of the economy. Consider a typical firm making an investment decision. A decline in the interest rate, all else equal, increases the net present value of future cash flows leading the firm to increase immediate investment. This mechanism explains why the supply-side relationship between economic growth and interest rates is typically negative in endogenous growth models.[1] However, these models typically do not take strategic competition and market structure into account. Is it reasonable to assume that a significant reduction in the long-term interest rate would have no impact on the competitiveness of an industry?

This study examines this question through a model in which competition is introduced between firms and market structure is endogenous. The model is rooted in the dynamic competition literature (e.g., Aghion, Harris, Howitt and Vickers (2001)) where two firms compete in an industry for market share by investing in productivity-enhancing technology. Firms decide whether to invest at each point in time. An investment increases the probability that a firm improves its productivity position relative to its competitor. Such a relative improvement occurs gradually; for example, a follower can only slowly reduce the size of the productivity gap between the leader and himself. The decision to invest in the model is only a function of the current productivity gap between the leader and the follower, which is the key state variable of the model. A larger productivity gap gives the leader a larger share of industry profits.

The solution to the model reveals two regions of market structure. If the productivity gap be-

[1] See e.g. Aghion and Howitt (1992), Klette and Kortum (2004), Grossman and Helpman (1991) and Romer (1990). The supply-side relationship between growth and interest rate is flat in exogenous growth models such as the Solow and Ramsey models.

tween the leader and the follower is small, then the industry is in a "competitive region" in which both firms invest in an effort to escape competition. If the productivity gap becomes large, the industry enters a "monopolistic region" in which the follower does not invest due to a "discouragement effect": the prospect of overtaking the leader in the future is too small relative to the cost of investment. If the productivity gap becomes large enough, even the leader stops investing in productivity enhancement as the perceived threat of being overtaken becomes too small. The model includes a continuum of industries, all of which feature the dynamic game between the leader and follower. The state variable of each market is random and is governed by the stochastic process induced by investment decisions. The model shows that aggregate productivity growth, in a steady-state, declines as the fraction of markets that are in the monopolistic region increases.

The key comparative static explored by the model is the effect of a lower interest rate on aggregate productivity growth. In any given industry, a decline in the interest rate has a *traditional effect* of inducing both the leader and the follower to increase investment in productivity enhancement. However, the investment response to a lower interest rate is stronger for the leader relative to the follower. Intuitively, both leaders and followers invest in order to raise productivity, thereby acquiring market power and achieving higher payoffs in the future. The leader is closer to high-payoff states than the follower is; hence, not only is the leader's incentive to invest stronger than that of the follower, but so is the leader's marginal gain in incentive to invest following a decline in the interest rate. A lower interest rate induces firms to be more patient, and more patience leads to a stronger investment response only if the firm can ultimately achieve the high payoffs associated with market leadership. Such high payoffs are more achievable for the leader.

The stronger investment response of the leader to a lower interest rate leads to a *strategic effect* of a decline in the interest rate. In particular, the steady-state average productivity gap between the leader and the follower increases when the interest rate falls due to the unequal investment responses. The increase in the average productivity gap in turn discourages the follower from investing. Due to the strategic effect, the expected time that an industry spends in the monopolistic region increases when the interest rate falls.

The key theoretical result of the model is that as the interest rate declines to zero, the strategic effect dominates the traditional effect; as a result, a given industry spends almost all of the time in the monopolistic region at a low enough interest rate. This implies that as the interest rate declines, the fraction of industries in the monopolistic region of the state space expands and aggregate productivity growth falls.

This induces an inverted-U shaped supply-side relationship between economic growth and the interest rate. Starting from a high level of the interest rate, growth increases as the interest rate declines because the traditional effect dominates the strategic effect. However, as the interest rate declines further, the endogenous investment response of the leader and follower causes the strategic effect to dominate, and economic growth begins to fall. The key theoretical result shows that this positive relationship between the interest rate and economic growth must happen before the interest rate hits zero.

An empirical test of the model focuses on the relationship between interest rates and industry concentration. Such a relationship is difficult to estimate given that industry concentration moves slowly in practice. However, the empirical analysis presented here takes advantage of the fact that stock prices are forward-looking and incorporate expectations regarding changes in industry concentration in response to interest rate shocks. Using CRSP-Compustat merged data from 1980 onward, we analyze the performance of a portfolio that goes long industry leaders and shorts industry followers in response to changes in the ten year Treasury rate. We call this the "leader" portfolio.

The model predicts that the leader portfolio earns excess returns in response to a decline in interest rates, and these excess returns become larger as the level of interest rates approaches zero. This cross-derivative is a prediction of the model that is reasonably distinct, and therefore offers a strong test of the model mechanism. The data reveal exactly the pattern predicted by the model: the leader portfolio exhibits higher returns in response to a decline in interest rates, and this response becomes stronger at a lower initial level of interest rates. The estimated effect is large in magnitude and robust to a number of tests.

The model is also useful in explaining a number of important macroeconomic trends in a single coherent framework. For example, the decline in the ten year interest rate globally has been associated with a rise in industry concentration, higher markups and corporate profit share, and a decline in business dynamism. These facts are well-documented in the literature.[2] The model is also consistent with the fact that market concentration increased initially as interest rates began to decline, but productivity growth slowed down later in time when long-term interest rates approached zero.[3]

The model also explains cross-sectional patterns in the productivity slowdown found in the literature. In the model, the slowdown in productivity growth is associated with a larger average productivity gap between the industry leader and followers. Using firm-level data from the OECD, Berlingieri and Criscuolo (2017) and Andrews et al. (2016) show that the productivity gap between the 90th versus 10th percentile firms within industries has been increasing since 2000. Moreover, the productivity gap between leaders and followers has risen most in industries where productivity growth has slowed down the most.[4]

The model provides an alternative explanation for "secular stagnation," or the observation that growth has slowed down as long-term rates have fallen toward zero. Current explanations of secular stagnation (e.g., Summers (2014)) focus almost exclusively on the demand side; in these explanations, frictions such as the zero lower bound on nominal interest rates or nominal rigidity generate a long and persistent slowdown in growth.[5] In the framework presented here, the initial decline in the interest rate may come from weakness on the demand side. However, the key point of departure is the insight that if the interest rate falls to a low enough level, then the rise in market competition may itself result in a constraining force on growth. In such a framework, one does not need to rely on financial frictions, a liquidity trap, nominal rigidities, or a zero lower bound to explain the persistent growth slowdown.

There has been work in the past that links interest rates to the *level* of productivity (e.g. Caballero, Hoshi and Kashyap (2008); and Gopinath, Kalemli-Ozcan, Karabarbounis and Villegas-

[2]See De Loecker and Eeckhout (2017), Barkai (2018), Autor, Dorn, Katz, Patterson and Van Reenen (2017), Gutiérrez and Philippon (2016, 2017), Grullon, Larkin and Michaely (2016), Davis and Haltiwanger (2014), Decker, Haltiwanger, Jarmin and Miranda (2016), Haltiwanger (2015), Hathaway and Litan (2015), and Andrews, Criscuolo and Gal (2016).

[3]The productivity growth slowdown started in 2005, well before the Great Recession, which suggests that structural as opposed to cyclical factors are behind the decline. This is consistent with the model presented here.

[4]Relatedly Gutiérrez and Philippon (2016, 2017) and Lee, Shin and Stulz (2016) show sharp decline of investment relative to operating surplus and that the investment gap is especially pronounced in concentrated industries. Furthermore, Cette, Fernald and Mojon (2016) show in two-variable VAR that a negative shock to long-term interest rates leads to a decline in productivity growth, which is consistent with our framework.

[5]See e.g., Krugman (1998), Eggertsson and Krugman (2012), Guerrieri and Lorenzoni (2017), Benigno and Fornaro (n.d.) and Eggertsson, Mehrotra and Robbins (2017).

Sanchez (2017)). This paper differs in its explicit modeling of the supply-side to investigate the relationship between interest rates and the *growth rate* of productivity.

This study also contributes to the endogenous growth literature. The framework on competition within an industry differs from the seminal work of Aghion et al. (2001) in that catch-up innovation by the follower happens step-by-step. The follower cannot leap-frog the leader instantly. This is a key assumption of the model, and it helps to explain many of the model's novel predictions relative to the existing literature. While leap-frogging does happen at times, especially during episodes of large technological disruptions, the model is closer to a more typical process of innovation which is gradual.

The derivation of our central theoretical result—that as the interest rate converges to zero, aggregate innovation must decline because the average distance between the leader and follower diverges—provides a technical contribution to the literature on dynamic patent races. Models of dynamic competition as stochastic games are notoriously difficult to analyze, and even seminal contributions in the literature either rely on numerical methods or impose significant restrictions on the state space to keep the analysis tractable. By deriving first-order approximations of the recursive value functions when the discount rate is small, we are able to provide sharp, analytical characterizations of the asymptotic equilibrium in the limiting case when discounting tends to zero, even as the ergodic subset of the state space becomes infinitely large. The techniques should be applicable to other stochastic games of strategic interactions with a large state space and low discounting.

2 Supply-side model with investment and strategic competition

2.1 Setup

Demand

The demand side of the model is intentionally simplistic. Time is continuous. There is a representative consumer who, at each instance, chooses consumption $Y(t)$ and supplies labor $L(t)$

according to the within-period utility function $U(t) = \ln Y(t) - L(t)$. The consumption good aggregated from differentiated goods is:

$$\ln Y(t) \equiv \int_0^1 \ln y\,(t;\nu)\,d\nu,$$

where ν is an index for markets, and $y\,(t;\nu)$ is aggregator of each duopoly market:

$$y\,(t;\nu) = \left[y_1\,(t;\nu)^{\frac{\sigma-1}{\sigma}} + y_2\,(t;\nu)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}.$$

$y_i\,(t;\nu)$ is the quantity produced by firm i of market ν.

We normalize the wage rate to one. Given the demand structure, the total revenue in each market is also equal to one: $P(t)Y(t) = p(t;\nu)y(t;\nu)$, where $P(t) \equiv \exp\left(\int_0^1 \ln p(t;\nu)d\nu\right)$ is the aggregate price index and $p\,(t;\nu) = \left[p_1\,(t;\nu)^{1-\sigma} + p_2\,(t;\nu)^{1-\sigma}\right]^{\frac{1}{1-\sigma}}$ is the price index for market ν.

Within-market competition

We now discuss the within-market dynamic game between duopolists. For expositional simplicity, we drop the market index ν and describe the game for a generic market.

Static block Over each time instance, the duopolists strategically compete in the product market. Let $z_1\,(t)\,, z_2\,(t) \in \mathbb{Z}_{>0}$ denote the productivity levels of the two market participants; the marginal cost of a firm with productivity z is λ^{-z}. Let $s\,(t) = |z_1\,(t) - z_2\,(t)| \in \mathbb{Z}_{\geq 0}$ be the state variable that captures the productivity gap of the two firms. When $s = 0$, the two participants are said to be neck-to-neck; when $s > 0$, one of the firm is a temporary leader (L) while the other is a follower (F).

The two firms compete a la Bertrand, and firm profits can be written as implicit functions of the state variable s (productivity gap). Let π_s denote the profit of the leader in a market with productivity gap s, and likewise let π_{-s} be the normalized profit of the follower in the market.[6]

[6] These profit functions π_s and π_{-s} can be written as $\pi_s = \frac{\rho_s^{1-\sigma}}{\sigma+\rho_s^{1-\sigma}}$ and $\pi_{-s} = \frac{1}{\sigma\rho_s^{1-\sigma}+1}$, where ρ_s is implicitly defined by $\rho_s = \lambda^{-s}\frac{(\sigma+\rho_s^{1-\sigma})\rho_s^{1-\sigma}}{\sigma\rho_s^{1-\sigma}+1}$. These expressions are derived in Appendix A; also see Aghion et al. (2001) and Atkeson and

Conditioning on the state variable, π_s and π_{-s} no longer depend on the time index and have the following properties.

Lemma 1. *Follower's flow profits π_{-s} are non-negative, weakly decreasing, and convex; leader's and joint profits π_s, π_{-s} are bounded, weakly increasing, and eventually concave in s (a sequence $\{a_s\}$ is eventually concave iff there exists $\bar{s}$ such that a_s is concave in s for all $s \geq \bar{s}$).*

It can be easily verfied that $\lim_{s\to\infty} \pi_{-s} = 0$ and $\lim_{s\to\infty} \pi_s = 1$ under the specified market structure. Nevertheless, our theoretical results applies to any sequence of profits that satisfy the technical properties in Lemma 1; hence, we let $\pi \equiv \lim_{s\to\infty} \pi_s$ denote the limiting total profits in each market as $s \to \infty$, and we derive our theory using the notation π.

A higher productivity gap s is associated with higher joint profits and more unequal profits between the leader and the follower. We interpret state s to be more competitive than state s' if $s < s'$ and more concentrated if $s > s'$. As an example of the market structure, the case of perfect substitutes within market ($\sigma = \infty$) under Bertrand competition generates profit $\pi_s = 1 - e^{-\lambda s}$ for leaders and $\pi_{-s} = 0$ for followers (e.g., see Peters (2016)).

Dynamic block Each firm can invest in order to improve its productivity, which evolves in step-increments. Investment $\eta_s \in [0, \eta]$ in each state s is bounded by η and carries a marginal cost c. Specifically, the firm can choose to pay a cost $c\eta_s$ in exchange for a Poisson rate η_s with which the firm's productivity improves by one.[7] Specifically, given investment decisions $\{\eta_s, \eta_{-s}\}$ over interval Δ at time t, state s transitions according to

$$s(t+\Delta) = \begin{cases} s(t)+1 & \text{with probability } \Delta \cdot \eta_s \\ s(t)-1 & \text{with probability } \Delta \cdot (\kappa + \eta_{-s}) \\ s(t) & \text{otherwise.} \end{cases}$$

Burstein (2008).

[7] The central results of the model are not dependent on the assumption that investment intensity is bounded with a constant marginal cost . In a numerical example below, investment is modeled as unbounded with a convex cost. The central results are similar. The bounded investment with a constant marginal cost allows for an analytical characterization of the equilibrium as the interest rate approaches zero.

The technology diffusion parameter κ is the exogenous Poisson rate that the follower catches up by one step; it can also be seen as the rate of patent expiration.

In the model, firms are forward looking; they invest not only for gains in the flow profits in higher states, but, more importantly, they invest in order to also enhance market positions, thereby enabling them to reach for even higher profits in the future. For the follower, closing the productivity gap by one step enables him to further close the gap in the future and eventually catch up with the leader. For the leader, widening the productivity gap brings higher profits, the option value to further increase the lead in the future, as well as the improved expected duration of market leadership, because it would now take the follower additional steps to catch up.

Firms discount future payoffs at interest rate r. We take r to be exogenous for now. Section 2.7 endogenizes r by closing the model in general equilibrium. Each firm's value $v_s(t)$ in state s at time t can be expressed as the expected present-discount-value of future profits net of investment costs:

$$v_s(t) = \mathbb{E}\left[\int_0^\infty e^{-r\tau}\left\{\pi(t+\tau) - c(t+\tau)\right\}\Big|s\right].$$

We look for a stationary symmetric Markov-perfect equilibrium such that the value functions and investment decisions depend on the state but not the time index. The HJB equations for firms in state $s \geq 1$ are

$$rv_s = \pi_s + (\kappa + \eta_{-s})(v_{s-1} - v_s) + \max_{\eta_s \in [0,\eta]} \{0, \eta_s (v_{s+1} - v_s - c)\} \tag{1}$$

$$\begin{aligned} rv_{-s} &= \pi_{-s} + \eta_s\left(v_{-(s+1)} - v_{-s}\right) + \kappa\left(v_{-(s-1)} - v_{-s}\right) && (2)\\ &\quad + \max_{\eta_{-s}\in[0,\eta]}\left\{0, \eta_{-s}\left(v_{-(s-1)} - v_{-s} - c\right)\right\}. && (3) \end{aligned}$$

In state zero, the HJB equation for either market participants is

$$rv_0 = \pi_0 + \eta_0(v_{-1} - v_0) + \max_{\eta_0\in[0,\eta]}\{0, \eta_0(v_1 - v_0 - c)\}.$$

Definition 1. (**Equilibrium**) Given interest rate r, a symmetric Markov-perfect equilibrium is a collection of value functions and investment decisions $\{\eta_s, \eta_{-s}, v_s, v_{-s}\}_{s=0}^{\infty}$ that satisfy the infinite collection of equations in (1) and (3). The collection of flow profits $\{\pi_s, \pi_{-s}\}_{s=0}^{\infty}$ are generated by strategic competition in the static block.

The key assumption embodied in the investment technology is that catching up is a gradual process: the productivity gap has to be closed step-by-step, and the follower cannot "leapfrog" the leader by overtaking leadership with one successful innovation. This assumption plays an important role in the results and is the key difference between the model presented here and the setup in Aghion et al. (2001). On the other hand, that κ is a state-independent constant is not a crucial assumption for the model presented here, as we discuss in Section 2.6.

Aggregation: Steady-state and productivity growth

In each market, firms engage in both static competition—by maximizing flow profits, taking the productivity gap as given—and dynamic competition—by strategically choosing investment in order to raise their own productivity and maximize the present discounted value of future payoffs. The state variable in each market follows a stochastic Markov process with transition rates governed by the investment decisions $\{\eta_s, \eta_{-s}\}_{s=0}^{\infty}$ of market participants. We define a steady-state equilibrium as one in which the distribution of productivity gaps in the entire economy, $\{\mu_s\}_{s=0}^{\infty}$, is time invariant. The steady-state distribution of productivity gap must satisfy the property that, over each time instance, the density of markets leaving and entering each state must be equal. This implies the following equations:

$$2\mu_0\eta_0 = (\eta_{-1} + \kappa)\,\mu_1, \tag{4}$$

$$\mu_s\eta_s = \left(\eta_{-(s+1)} + \kappa\right)\mu_{s+1} \quad \text{for all } s > 0, \tag{5}$$

(the number "2" on the left-hand-side of the first equation reflects the fact that a market leaves state zero if either participant makes a successful innovation).

Definition 2. (**Steady-State**) Given equilibrium investment $\{\eta_s, \eta_{-s}\}_{s=0}^{\infty}$, a steady-state is the distribution $\{\mu_s\}_{s=0}^{\infty}$ ($\sum \mu_s = 1$) over state space that satisfies equations (4) and (5).

The aggregate productivity is defined as the total cost of production relative to total value of output; since the wage rate is normalized to one, productivity can be measured as the inverse of the aggregate price index. The aggregate productivity growth rate at time t, defined as $g \equiv -d \ln P(t)/dt$, therefore measures the productivity growth rate of each market—aggregated from firm-level investment decisions—weighted by the distribution over the market structure.

Lemma 2. *In a steady state, the aggregate productivity growth rate is*

$$g = \ln \lambda \left(\sum_{s=0}^{\infty} \mu_s \eta_s + \mu_0 \eta_0 \right).$$

The lemma shows that aggregate growth can be simplified as a weighted average of the productivity growth of market leaders (recall λ is the proportional productivity increment for each successful investment). This result is a direct consequence of the fact that, in a steady-state, the growth rate of the productivity leaders is, on average, the same as that of followers.

2.2 Analysis of the equilibrium and steady state

We first analyze the equilibrium structure of the two-firm dynamic game in a generic market, again dropping the market index ν. We then aggregate market equilibrium to the economy and study aggregate comparative statics with respect to the interest rate r.

Equilibrium in each market

We impose the following regularity conditions.

Assumption 1. *1. The upper bound of investment, η, is sufficiently high: $\eta > \kappa$ and $2c\eta > \pi$. 2. $\pi_1 - \pi_0 > c\kappa > \pi_0 - \pi_{-1}$.*

The first assumption ensures that firms can scale up investment η_s to a sufficiently large amount if they choose to. The condition ($\eta > \kappa$) means that, if the follower does not invest and the leader invests as much as possible, then the productivity gap tends to widen on average. The condition ($2c\eta > \pi$) means that if both firms choose to invest as much as possible, the total flow payoff ($\pi_s + \pi_{-s} - 2c\eta$) is negative in any state.

The second parametric assumption rules out a trivial equilibrium in which firms do not invest even when they are state zero, generating a degenerate steady-state distribution with zero growth. When this assumption holds, there are always some firms who invest in a steady state, meaning market structure is always temporary and evolves stochastically as a result of the investment decisions. Leaders invest in order to stay ahead, whereas followers invest in order to close the productivity gap and eventually catch up and then become leaders themselves.

Because investment costs are linear in investment intensities, firms generically invest at either the upper or lower bound in any state. Investment effectively becomes a binary decision, and any interior investment decisions can be interpreted as firms playing mixed strategies. For exposition purposes, we focus on pure-strategy equilibria in which $\eta_s \in \{0, \eta\}$, but all of our results hold in mixed-strategy equilibria as well.

Let $n+1$ be the first state in which the market leader chooses not to invest, $n+1 \equiv \min\{s|\eta_s < \eta\}$; likewise, let $k+1$ be the first state in which the market follower chooses not to invest, $k+1 \equiv \min\{s|\eta_{-s} < \eta\}$.

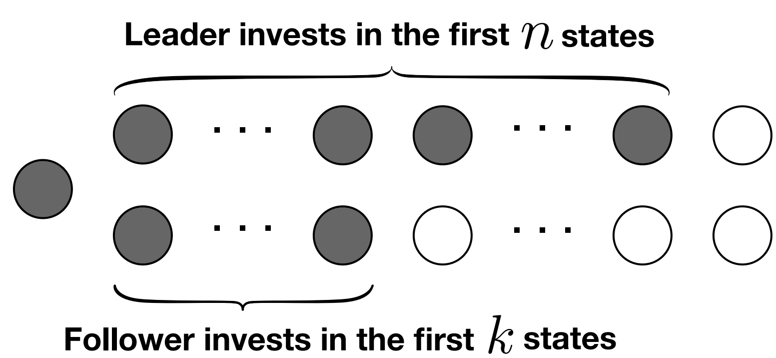
Lemma 3. *The leader invests in more states than the follower, $n \geq k$. Moreover, the follower does not invest in states $s = k+2, ..., n+1$.*

The lemma establishes that in any equilibrium, the leader must maintain investments in more states than the follower does, a structure we refer to as *leader dominance*. To understand this, note that the productivity gap closes at a slow rate κ if the follower does not invest in the state and at a faster rate $\eta + \kappa$ if the follower does. Firms are motivated to invest because of the high future flow payoffs after consecutive successful investments. The leader is motivated to invest in all states $s \leq n$ in order to reach state $n+1$, so that he can enjoy the flow payoff π_{n+1} without having to pay the investment costs in the state. The state $(n+1)$ is especially attractive if the follower does not invest in that state, because the leader can enjoy the payoff for a longer expected duration before the state stochastically transitions down to n, after which he has to incur investment cost again.

The follower, on the other hand, is also motivated by future payoffs. He incurs investment costs in exchange for the possibility of closing the gap and catching up with the leader, and for the possibility of eventually becoming the leader himself in the future so that he can enjoy the high flow payoffs. In other words, investment decisions for both forward-looking firms are motivated by high flow profits in the high states, and the incentive to reach these states is stronger for the leader because the leader is closer to those high-payoff states.

Another way to describe the intuition is to consider the contradiction brought by $n < k$. Suppose the leader stops investing before the follower does. In this case, the high flow payoff π_{n+1} is transient for the leader and market leadership is fleeting because of the high rate of downward state transition; this implies that the value for being a leader in state $n+1$ is low. However, because firms are forward-looking and their value functions depend on future payoffs, the low value in state $n+1$ "trickles down" to affect value functions in all states, meaning the incentive for the follower to invest—motivated by the dynamic prospect of eventually becoming the leader in state $n+1$—is low. This generates a contradiction to the presumption that follower invests more than the leader does.

Under the lemma, the structure of an equilibrium can be represented by the following diagram. States are represented by circles, going from state 0 on the very left and state $(n+1)$ on the very right. The coloring of a circle represents investment decisions: states in which the firm invests are represented by dark circles, while white ones represent those in which the firm does not invest. The top row represents the leader's investment decisions while the bottom row represents the follower's investment decisions.

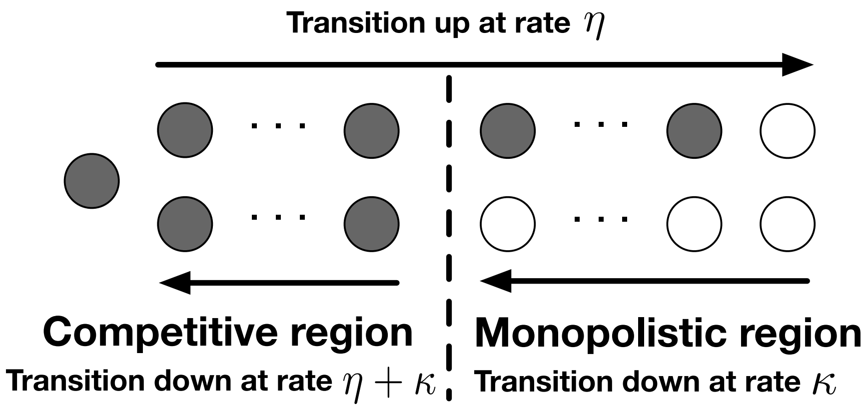


In the diagram, investment decisions are monotone for both firms: starting from state zero, they invest in consecutive states before reaching the respective cutoff state, k and n, and then cease investment from there on. This is a manifestation of two effects. First, when the follower is too far behind, the firm value is low and the marginal value of catching up by one step is not worth the investment cost. This is also known as the "discouragement effect" in the dynamic contest literature (Konrad (2012)). Second, the leader's strategy is monotone due to a "lazy monopolist" effect: when the leader is far ahead of the follower, he ceases investment because the marginal gain in value brought by advancing market position is no longer worth the investment cost.

Theoretically, because $\{\pi_s\}$ are not necessarily concave, investment decisions are not necessarily monotone in the state, and firms might resume investment after state $n+1$. However, we focus on monotone equilibria in the paper for two important reasons. First, given that market leaders do not invest in state $n+1$, the steady-state distribution of market structure never exceeds $n+1$, and investment decisions beyond state $n+1$ are irrelevant for characterizing the steady-state equilibrium. Second and most importantly, all equilibria follow the monotone structure when interest rate r is small, and our main result concerns the comparative statics of the economy as we take the interest rate r close to zero.

Analysis of the steady-state

The fact that the leader invests in more states than the follower enables us to partition the set of non-neck-to-neck states $s = 1, \ldots, n+1$ into two regions: one in which the follower invests ($s = 1, \ldots, k$) and the other in which the follower does not ($s = k+1, \ldots, n+1$). In the first region, the state transitions up with Poisson rate η and transitions down with rate $(\eta + \kappa)$. In expectation, the state s decreases over time in this region, and the market structure tends to move towards being more competitive. For this reason, we refer to this as the *competitive* region. Note that this label is not a reflection of the static profits, which can be very high for leaders in this region. Instead, the label reflects the fact that joint profits tend to decrease *dynamically*. In the second region, the downward transition happens at a lower rate (κ), and the market structure tends stay monopolistic and concentrated. We refer to this as the *monopolistic* region.



The aggregate productivity growth rate in the economy is a weighted average of the productivity growth in each market; hence, aggregate growth depends on both the investment decisions in each market structure as well as the distribution of market structure, which in turn is a function of the investment decisions. The following lemma shows that the aggregate growth rate can be characterized by the fraction of markets in each region.

Lemma 4. *In a steady-state induced by investment cut-off states (n, k), the aggregate productivity growth rate is*

$$g = \ln \lambda \left(\mu^C \cdot (\eta + \kappa) + \mu^M \cdot \kappa \right),$$

where $\mu^C \equiv \sum_{s=1}^{k} \mu_s$ is the fraction of markets in the competitive region and $\mu^M \equiv \sum_{s=k+1}^{n+1} \mu_s$ is the fraction of markets in the monopolistic region.

The lemma shows that steady-state growth is increasing in the fraction of markets in the competitive region and decreasing in the fraction of markets in the monopolistic region. In the competitive region, all firms invest. Consequently, productivity improvements are rapid, state transition rate is high, dynamic competition is fierce, leadership is contentious, and market power tends to decrease over time. On the other hand, only the leader is investing in the monopolistic region, and the rate of state transition and productivity growth is low.

The steady-state growth rate is increasing in k decreasing in $(n - k)$. Higher k implies that the follower invests in more states, thereby raising the steady-state fraction of markets in the competitive region; by contrast, higher $(n - k)$ expands the monopolistic region and reduces the fraction of markets in the competitive region, thereby reducing aggregate productivity growth.

The last result of this section provides a lower bound of steady-state growth rate.

Lemma 5. *If followers invest at all* ($k \geq 1$)*, then the steady-state aggregate productivity growth is bounded below by* $\ln \lambda \cdot \kappa$*, the step-size of productivity increments times the rate of technology diffusion.*

2.3 Comparative statics: $r \to 0$

Our key theoretical results concern the limiting behavior of aggregate steady-state variables as the interest rate declines toward zero. Conventional intuition suggests that, *ceteris paribus*, when firms discount future profits at a lower rate, the incentive to invest should increase because the cost of investment is lower relative to future benefits. This intuition holds in our model, and we formalize it into the following lemma.

Lemma 6. $\lim_{r \to 0} k = \lim_{r \to 0} (n - k) = \infty$.

The result suggests that, as the interest rate declines toward zero, firms in all states tend to raise investment. In the limit, as firms become arbitrarily patient, they sustain investment even when arbitrarily far behind or ahead: followers are less easily discouraged, and leaders are less lazy.

However, the fact that firms raise investment in all states does not translate into high aggregate investment and growth. These aggregate economic variables are averages of the investment and growth rate in each market, weighted by the steady-state distribution of market structure. A decline in the interest rate not only affects state-dependent investment decisions but also shifts the steady-state distribution of market structures. As Lemma 4 shows, a decline in the interest rate can boost aggregate productivity growth if and only if it expands the fraction of markets in the competitive region; conversely, if more markets are in the monopolistic region—for instance if n increases at a "faster" rate than k—aggregate productivity growth rate could slow down.

Our main result establishes that, as $r \to 0$, a slow down in aggregate productivity growth is inevitable and is accompanied by a decline in investment and a rise in market power.

Proposition 1. *As* $r \to 0$*,*

1. *The fraction of markets in the competitive region vanishes, and the monopoly region becomes absorb-*

ing:

$$\lim_{r\to 0} \mu^C = 0; \qquad \lim_{r\to 0} \mu^M = 1.$$

2. *The productivity gap between leaders and followers diverges:*

$$\lim_{r\to 0} \sum_{s=0}^{\infty} \mu_s s = \infty.$$

3. *Aggregate investment to GDP declines:*

$$\lim_{r\to 0} I \equiv \lim_{r\to 0} c \cdot \sum_{s=0}^{\infty} \mu_s \left(\eta_s + \eta_{-s}\right) = c\kappa.$$

4. *Aggregate productivity growth slows down:*

$$\lim_{r\to 0} g = \kappa \cdot \ln \lambda.$$

5. *Industry leaders take over the whole market, with high profit shares and markups:*

$$\lim_{r\to 0} \sum_{s=0}^{\infty} \mu_s \frac{\pi_s}{p_s y_s} = 1,$$

   where $p_s y_s$ is the total revenue of market s.

6. *Market dynamism declines, and leadership becomes permanently persistent:*

$$\lim_{r\to 0} \sum_{s=0}^{\infty} M_s \mu_s = \infty,$$

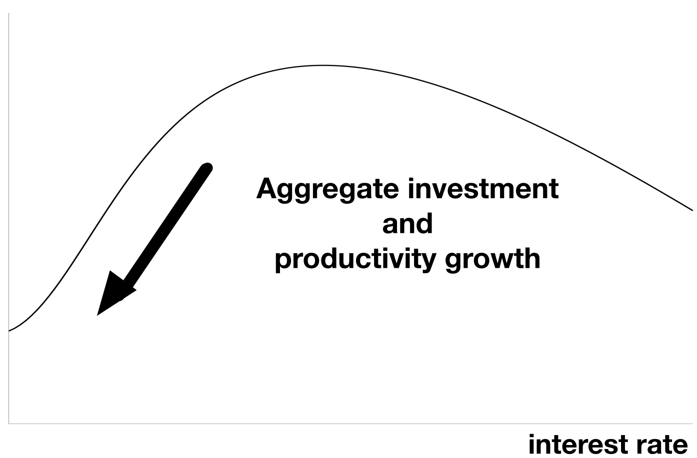
   where M_s is the expected duration before a leader in state s reaches state zero.

7. *Relative market valuation of leaders and followers diverges:*

$$\lim_{r\to 0} \frac{\sum_{s=0}^{\infty} \mu_s v_s}{\sum_{s=0}^{\infty} \mu_s v_{-s}} = \infty.$$

The proposition states that, as $r \to 0$, all markets are in the monopolistic region in a steady-state, and leaders almost surely stay permanently as leaders. Followers cease to invest, and leaders invest only to counteract the exogenous technology diffusion. As a result, aggregate investment and productivity growth decline and converge to their respective lower bounds governed by the parameter κ. This implies an inverted-U relationship between growth and the interest rate, as depicted in the figure below. At very high rates, few firms invest in any market structure, and aggregate productivity growth is low; at very low rates, most markets are in the monopolistic region, in which only leaders invest, and aggregate productivity is again low. The rise in market power also generates other implications: a rising leader-follower productivity gap, diverging relative market valuation of leaders, and a rising profit share of the leader.

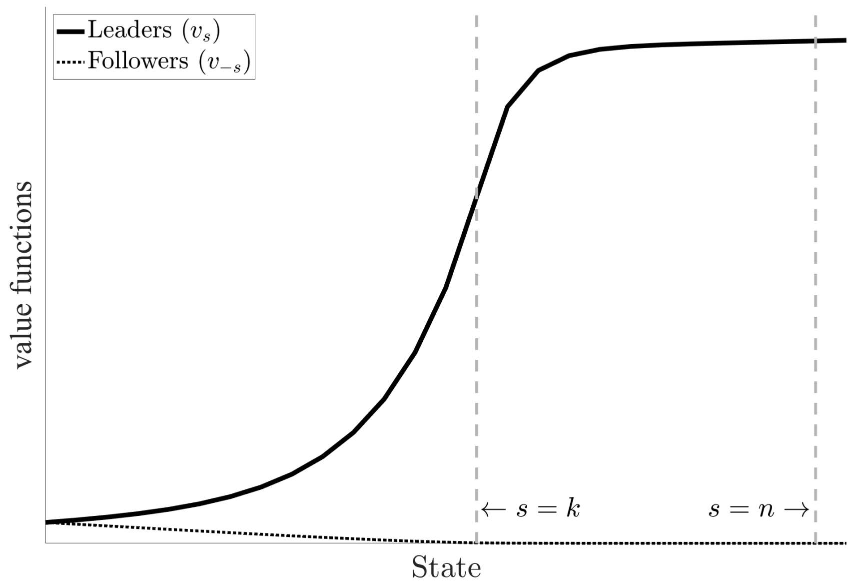
Steady-state growth rate: inverted-U



To understand this result, it is useful to first demonstrate the firm value functions, as shown in the figure below. The solid black curve represents the value function of the leader, whereas the dotted black curve represents the value function of the follower. The two dashed and gray vertical lines represent k and n, the last states in which the follower and the leader invest, respectively.

Following Lemma 2, both k and $(n-k)$ diverge to infinity as $r \to 0$: the follower invests in many states, and the leader invests in many *additional* states. Hence, the fraction of markets in the monopolistic region converges to one if and only if $(n-k)$ asymptotically dominates k, i.e., the leader raises investment at a "faster rate" than the follower does, as $r \to 0$. Intuitively, as the figure demonstrates, the value of leaders in the competitive region is small relative to the value in state $n+1$. Therefore, holding k constant, a patient leader must sustain investment in sufficiently

Value functions



many states $(n-k)$ beyond state k such that, in the state $(n+1)$ that he chooses not to invest, the expected duration of staying in the monopolistic region is sufficient long. As a leader becomes infinitely patient, even the distant threat of losing monopoly power is perceived to be imminent; consequently, leaders scale back investment only if they expect to never leave the monopolistic region.

The formal proof requires characterizing the asymptotic behavior of economic variables and is relegated to the appendix. In what follows, we provide the intuition for this result, which closely follows the proof, in four steps. Each step aims to explain a specific feature in the shape of value functions. We use $x \to y$ to denote $\lim_{r\to 0} x = y$. Note that the maximum flow profit a firm can earn is 1, which is the profit earned by a monopolist who charges infinite markup; hence $rv_s \leq 1$ for all s.

First, the leader's value in state $n+1$ must be asymptotically large; formally, rv_{n+1} is bounded away from zero (and $v_{n+1} \to \infty$). This is because the leader stops investment in state $n+1$ if and only if the marginal investment cost is higher than the change in value function, implying

$$c \geq v_{n+2} - v_{n+1} \geq \frac{\pi_{n+2} - rv_{n+1}}{r+\kappa}, \tag{6}$$

where the last inequality follows from solving the HJB equation (1) for state $n+2$ ($v_{n+2} = \frac{\pi_{n+2}+\kappa v_{n+1}}{r+\kappa}$). This in turn generates a lower bound for rv_{n+1}:

$$rv_{n+1} \geq \pi_{n+2} - c(r+\kappa) \to \pi - c\kappa.$$

Second, the value of follower in state $k+1$ must be very small even as $r \to 0$; formally, $v_{-(k+1)} \to$ constant. This is because even a patient follower finds the marginal change in value function $v_{-k} - v_{-(k+1)}$ not worth the investment cost, despite knowing that, if he gives up, the market structure tends to move in the leader's favor indefinitely, as the leader will continue to invest in many states beyond $k+1$. As $(n-k)$ grows large, $v_{-k} - v_{-(k+1)}$ is small if and only if v_{-k} is low.

Third, the value of being in the neck-to-neck state must be small ($rv_0 \to 0$). To see this, consider the total value of both firms in state 0, which can be written as a weighted average of joint flow payoffs across all states:

$$2v_0 = \sum_{s=0}^{\infty} \lambda_{s|0} \left(\frac{\pi_s + \pi_{-s} - c\left(\eta_s + \eta_{-s}\right)}{r} \right), \quad \sum_{s=0}^{\infty} \lambda_{s|0} = 1.$$

The term $\left(\frac{\pi_s+\pi_{-s}-c(\eta_s+\eta_{-s})}{r}\right)$ represents the "permanent value" of state s, i.e., the present-discount value of joint flow payoffs if the market stays in state s permanently. The joint value in state 0 can be written as a weighted average of the permanent values across all states; the weight $\lambda_{s|0}$ can be interpreted as the present-discounted fraction of time that the market is going to be in state s, given that the current state is zero.

This re-formulation demonstrates that, as $k \to \infty$, the market in state zero could expect to spend a significant fraction of time in the competitive region (states $s = 1, \ldots, k$), in which both firms invest as much as possible and the joint flow payoff is negative ($\pi_s + \pi_{-s} < 2c\eta$ under Assumption 1). In an equilibrium, k must diverge at a rate exactly consistent with an asymptotically small v_0 ($rv_0 \to 0$). Because firms are forward-looking, an asymptotically large v_0 can only be consistent with a slowly rising k as $r \to 0$, but this in turn implies that v_{-k} must be large which contradicts the earlier statement $rv_{-k} \to 0$. Conversely, the fact that v_0 must be non-negative (as firms can always guarantee at least zero payoff over every instance) imposes an upper bound on the rate at which k diverges.

Last and most importantly, asymptotically small v_0 and large v_{n+1} implies that the increase in leader's value as the state goes from $(k+1)$ to $(n+1)$ must be asymptotically large:

$$\lim_{r \to 0}(rv_{n+1} - rv_{k+1}) > 0.$$

Starting from state $k+1$, the leader keeps investing in additional states to consolidate market power. The firm value increases in the state, and the leader stops only when the value function is sufficiently high, as characterized by (6). The fact that the gain in value $(rv_{n+1} - rv_{k+1})$ is asymptotically large implies that, as a leader becomes infinitely patient, he must invest in sufficiently many states beyond k so that the chance diminishes of falling back into the competitive region, thereby causing the monopolistic region to become endogenously absorbing.

2.4 A numerical illustration

To demonstrate the model mechanics for how aggregate productivity growth slows down as the interest rate declines, we turn to a numerical illustration, parameterizing the investment decision as unbounded with convex marginal costs $c(\eta) = \eta/2$. We adopt this conduct this exercise for two reasons. First, the numerical model demonstrates that the key results in Proposition 1 survive beyond the bounded and constant-marginal-cost specification. Second, under this formulation, changes in investment intensity are smoothed out across states, thereby getting around the discreteness in many of the figures we show below. The HJB equations of the numerical model follow

$$rv_s = \max_{\eta \geq 0} \pi_s - \eta^2/2 + (\kappa + \eta_{-s})(v_{s-1} - v_s) + \eta (v_{s+1} - v_s)$$

$$rv_{-s} = \max_{\eta \geq 0} \pi_{-s} - \eta^2/2 + (\kappa + \eta)\left(v_{-(s-1)} - v_{-s}\right) + \eta_s \left(v_{-(s+1)} - v_{-s}\right)$$

$$rv_0 = \max_{\eta \geq 0} \pi_0 - \eta^2/2 + \eta_0 (v_{-1} - v_0) + \eta (v_1 - v_0).$$

We now provide demonstrations of the investment function $\{\eta_s\}$, the steady-state distribution $\{\mu_s\}$, and value functions as well as how these functions change in response to lower interest rates. We also provide numerical illustrations of how steady-state levels of productivity growth vary with interest rates. In generating these numerical plots, we parametrize the within-market demand aggregator using $\sigma = \infty$, the case in which two firms produce perfect substitutes.

Figures 1 through 3 illustrate the main outcomes of the model. The top panel in Figure 1 shows the investment functions of the leader and follower across states for a high interest rate. The figure illustrates the leader dominance of Lemma 3; the leader invests more in all states beyond the neck-to-neck state. The dotted lines show the investment functions of the leader and follower for a lower interest rate. Both the leader and follower invest more in all states when the interest rate is lower, which represents the traditional effect of lower interest rates on investment.

However, as the bottom panel demonstrates, the leader's investment response to a lower interest rate is stronger than the follower's response for all states. The stronger response of the leader's investment to lower interest rates is the driving force behind the strategic effect through which lower interest rates boost market concentration.

The top panel of Figure 2 shows that, following a decline in r, the steady-state distribution of market structure shifts to the right, and aggregate market power increases.

Why does the leader's investment respond more to a lower interest rate? The bottom panel of Figure 2 shows the leader's and follower's value functions before and after a decline in the interest rate. The change in the leader's value is larger than the change in the follower's value; this is the key driver behind the leader's stronger investment response following a drop in r. Finally, Figure 3 numerically verifies the central result of the Proposition above. For a low enough interest rate, a further decline in the interest rate leads to lower growth. Figure 3 also verifies that $g \to \kappa \cdot \ln \lambda$ in the numerical exercise with variable investment intensity.

2.5 Transitional dynamics: productivity and market power

The analysis above illustrates that, starting from a high level of the interest rate, a declining interest rate is at first expansionary—measured by *steady-state* growth—and only becomes contractionary when r falls sufficiently low; yet, *steady-state* market power tends to rise when r declines at any level.

Something parallel is true for transitional dynamics after an unanticipated, permanent change in interest rates, though for different reasons. Starting from a steady-state, a decline in the interest rate which firms expect to be permanent immediately moves market participants to a new equilibrium, featuring higher investments given any market structure. The equilibrium distribution of market power starts to rise, although it moves slowly, as depicted in the top panel of Figure 4.

Over time, as the distribution of market structures converges to the new steady-state and as average market power increases, the equilibrium growth rate and investment eventually decline to the new steady-state level. Whether productivity growth is higher or lower in the new steady-state, relative to the initial one, depends on the level of the starting interest rate before the interest rate falls. If the starting interest rate is low (i.e. to the left of the peak in Figure 3), then the growth rate would be lower in the new steady-state. Productivity growth along the transitional path in such a case is depicted in the bottom panel of Figure 4.

2.6 State-dependent catch up

The no-leapfrogging assumption—that the follower has to catch up one step at a time no matter how far behind—is crucial for our results and is a key distinction between our model and the seminal work of Aghion et al. (2001). Intuitively, the leader invests in the monopolistic region for two reasons: 1) to capture rising flow profits and 2) to escape dynamic competition. By getting one step further ahead of the follower, the leader is able to secure his dynamic rents for longer duration. When the discount rate is low, it is the "escaping competition" motive that dominates; indeed, the results survive even if flow profits stay constant as a function of the state ($\pi_s = \bar{\pi}$ for all $s > 0$, $\pi_s = 0$ otherwise). Conversely, if the follower can close an arbitrarily large number of gaps with one successful innovation, such a prospect of leapfrogging kills the leader's dynamic incentive to get further ahead, leaving higher flow profits as the only reason for the leader to invest. If flow profits do not increase with the state, the leader does not invest at all.

That being said, our key result—Proposition 1—does not depend crucially on the constancy of exogenous catch up rate κ and the investment cost c. For instance, suppose investment costs and exogenous catch-up rates are both state-dependent functions and are increasing and bounded, with $\lim_{s\to\infty} c_s = \bar{c}$ and $\lim_{s\to\infty} \kappa_s = \bar{\kappa}$. Proposition 1 remains to hold as long as the upper bounds $\bar{\kappa}$ and $\bar{c}$ satisfy assumption 1. Intuitively, the proposition makes asymptotic predictions as $r \to 0$, and the state-to-state variations in κ_s and c_s do not affect firms behavior when they are patient.

2.7 Closing the model in general equilibrium

The focus of this paper is on how competition and productivity growth is impacted by changes in the interest rate. As such we have so far treated the interest rate as exogenous, and traced out the implied supply-side relationship between competition and interest rate, or growth and interest rate as shown in Figure 3.

It is relatively straightforward to close our model in general equilibrium by adding an Euler equation from the demand-side, following Aghion et al. (2001) and Benigno and Fornaro (n.d.). Doing so explicitly identifies where movement in interest rate might come from, i.e. shifts in the demand curve, and solves for equilibrium interest rate and growth rate as well. Specifically, the inter-temporal preferences

$$\int_0^\infty U(t)^{\frac{\theta-1}{\theta}} e^{-\rho t} dt$$

generate an Euler equation $g(t) = \dot{Y}(t) = \frac{1}{\theta}(r(t) - \rho)$, which is an upward-sloping relationship between aggregate growth rate g and interest rate r. Coupled with our supply-side, inverted-U relationship between g and r, the two curves pin down the level of growth rate and interest rate on a balanced growth path, along with a stationary distribution of productivity gaps across markets.

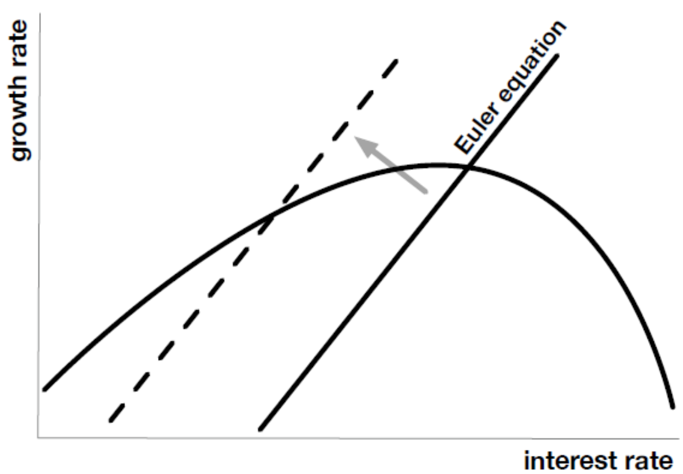
Interest rate shocks that we refer to in the main text can be simply seen as shocks to the consumer discount rate ρ (as in Krugman (1998)). An issue with this interpretation is that the interest rate is bounded below by the discount rate ρ, and any positive growth rate is incompatible with the interest rate being close to zero even as $\rho \to 0$. This issue is an artifact of the demand-side being frictionless; any incomplete markets and idiosyncratic risk would generate additional terms in the Euler equation that push down the real interest rate for any level of the growth rate. For instance, Benigno and Fornaro (n.d.) use idiosyncratic, uninsurable unemployment risk to microfound the following Euler equation

$$g(t) = \frac{1}{\theta}(r(t) - \rho + b).$$

The term $b \geq 0$ measures the severity of the unemployment risk under their specific microfoundation model, but it can be more broadly seen as a catch-all term for any shock on the demand side that pushes consumers towards saving more and consuming less, including changes in preferences, tightened borrowing constraints (e.g. Eggertsson and Krugman (2012)), or structural shifts such as an aging population and rising inequality (e.g. Summers (2014)).

The figure below shows the demand-side Euler equation as an upward sloping line. An inward shift in the demand curve lowers the interest rate and increases concentration. If the prevailing interest rates are low, i.e. economy is in the upward sloping region of the supply curve, then a fall in interest rate is also contractionary as productivity growth slows.

Supply and demand relationships in growth models



Hence, the model presents an alternative interpretation of "secular stagnation." As in traditional secular stagnation explanations, an initial inward shift in the demand curve can lower equilibrium interest rates to very low levels. However, "stagnation" is not due to monetary constraints such as the zero lower bound or nominal rigidities. Instead, a large fall in interest rates can make the economy more monopolistic for reasons laid out in the model, thereby lowering innovation and productivity growth.

3 Testing model predictions

This section examines whether the supply-side mechanism that links lower interest rates with increased concentration and market power is empirically relevant. It first shows that the model generates specific predictions on how the relative market values of leaders versus followers in an industry respond to a reduction in interest rates. It then tests these predictions using data from publicly listed firms in the United States.

3.1 Interest rate shocks: asymmetric effects on firm value

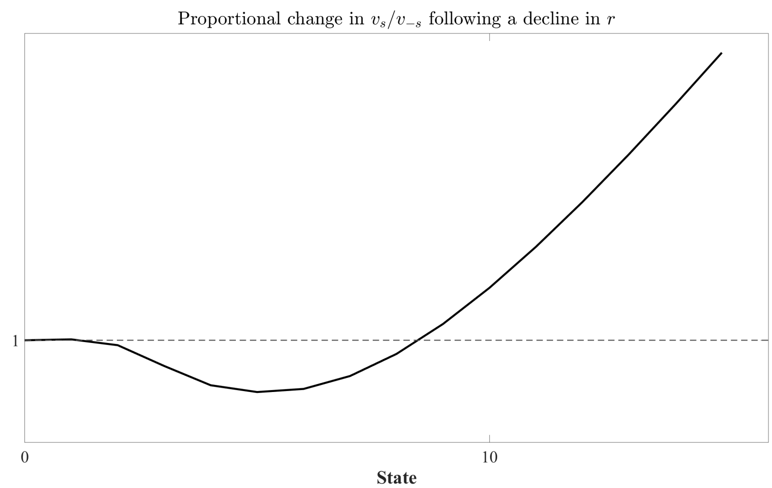
How does a fall in the interest rate affect the valuation of the industry leader versus follower in the model? A decline in r lowers the discount rate of future cash flows, raising market values of all firms. The focus of the empirical analysis below is on the *relative* valuation effects of a decline in the interest rate. Intuitively, the effect of a lower interest rate on firm value is especially pronounced for leaders relative to followers near the right tail of the monopolistic region where the gap between the leader and follower is large. In this region, the leader is expected to enjoy long periods of high rents while facing almost no threat of catch-up by the follower.

By contrast, the relative valuation gain for the leader over the follower is muted toward the left end of the monopolistic region and could even be a relative loss in the competitive region. Over these states, the follower's investment is especially responsive to a decline in the interest rate, which poses a significant threat to the leader of losing future rents in the new equilibrium. The figure below shows the effect of a decline in the interest rate on the valuation of the leader relative to the follower as a function of the state variable. More specifically, it plots $\frac{v_s'}{v_s} \Big/ \frac{v_{-s}'}{v_{-s}}$ where v_s' indicates the value function in state s after a decline in r.

Now consider aggregating the asymmetric response to the entire economy. Suppose a decline in the interest rate happens to a steady-state economy at time t_0, and let

$$\chi(t) \equiv \frac{\sum_{s=0}^{\infty} \mu_s(t)\, v_s(t)}{\sum_{s=0}^{\infty} \mu_s(t)\, v_{-s}(t)} \Big/ \frac{\sum_{s=0}^{\infty} \mu_s(0)\, v_s(0)}{\sum_{s=0}^{\infty} \mu_s(0)\, v_{-s}(0)}$$

track the over-time proportional changes in the *total* market valuation of all leaders relative to all

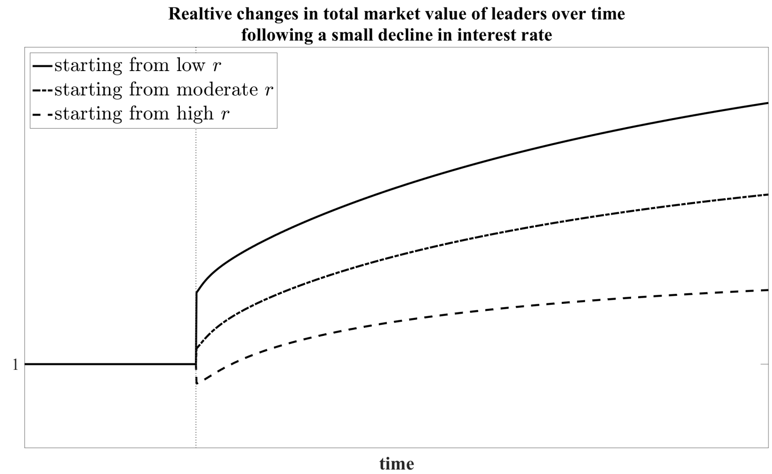


followers, as compared to the relative valuation in the pre-shock steady-state.

$\chi(t)$ is equal to one before t_0. Following a decline in interest rate at t_0, market valuation changes immediately, resulting in a discrete jump in $\chi(t_0)$, the magnitude of which is an average of the on-impact valuation response in each individual state, weighted by the distribution over market structure in the initial steady-state. If the initial level of the interest rate is high, the steady-state features significant mass of markets in the competitive region and small fractions of markets toward the right-end of the monopolistic region. Consequently, the average leader in the economy could experience losses in valuations relative to the average follower. In this case, χ falls below one at time t_0, as it places small weights over the states in which the leader experiences large valuation gains.

Alternatively, under low levels of initial r, a decline in r can significantly raise $\chi(t_0)$ above one because there are large fractions of markets with high monopoly power, and leaders in these states experience relatively large gains. When r is close to zero, the jump in χ must be above one. To demonstrate these intuitions, the figure below plots the time path of $\chi(\cdot)$ following a sudden and permanent decline in r, starting from three different levels of initial interest rates.

Note that, over any time interval $[t, t+\Delta]$, market structure is ergodic and the identity of the market leader does not necessarily stay constant as time passes. This is why at a very high interest rate, the relative valuation of the leader actually declines with a fall in the interest rate. Nevertheless, when the interest rate initially declines, leaders at time $t_0 - \epsilon$ tend to remain leaders at time



$t_0 + \epsilon$ for small enough $\epsilon > 0$. As a result, the on-impact asymmetric response in market valuations enables us to perfom an empirical test of the model mechanism through sudden changes in stock market returns. That is, starting from a moderate level of the interest rate and following an unexpected further decline in r, market leaders should experience significant excess returns relative to market followers. Furthermore, the excess return for a leader versus a follower when the interest rate falls should be higher when the initial level of the interest rate is lower. This last prediction is the strongest test of the model's dynamics. The next subsection implements a triple difference-in-differences specification to test this key prediction.

3.2 Empirical results

The data set for the analysis is the CRSP-Compustat merged data set from 1980 onward, which is used to compute excess returns for industry leaders versus followers in response to an interest rate shock. The 10-year treasury yield is used as the default measure of the long-run interest rate, and robustness tests using alternative duration of interest rate yields are also shown.[8] The 10-year yield has the advantage that it has the longest available historical data at the long-end of the yield curve. The Fama-French classification of industries is used as the default, but we also show robustness to industry definitions based on SIC codes.

[8] The nominal rate is used given the difficulty in measuring the real interest rate. Inflation expectations have been well-anchored during the time period analyzed, and so the nominal yield is the appropriate measure to use given the measurement error trade-off. Anchored inflation expectations since 1980 is the reason we focus on this time period.

"Industry leaders" are defined using different approaches. The default leader definition is based on size. Firms are classified as industry leaders if they are in the top 5 percent of firms in the industry based on market value at the beginning of when excess returns are computed. The results are robust to other classifications as well. Because the number of firms in an industry can vary significantly, the top 5 firms in an industry are also defined as leaders instead of the top 5 percent. In some specifications, EBITDA and sales are used to sort firms instead of market value. The results are robust to these alternative definitions.

The key empirical test implied by the model at the firm level can be written as,

$$R_{i,j,t} = \alpha_{j,t} + \beta_0 D_{i,j,t-1} + \beta_1 D_{i,j,t-1} * \Delta i_t + \beta_2 D_{i,j,t-1} * i_{t-1} + \beta_3 D_{i,j,t-1} * \Delta i_t * i_{t-1} + X_{i,j,t}\gamma + \varepsilon_{i,j,t} \tag{7}$$

where $R_{i,j,t}$ is the dividend and split-adjusted stock return of firm i in industry j from date $t-91$ days to t (i.e. one quarter growth), and $D_{i,j,t-1}$ is an indicator variable equal to 1 if firm i is in the top 5% of market capitalization in its industry j at date $t-91$. Firms with $D_{i,j,t-1}$=1 are called leaders while the rest are called followers. i_t is the nominal 10-year Treasury yield, with i_{t-1} being the interest rate 91 days prior and Δi_t being the change in the interest rate from date $t-91$ to t. The matrix X includes potential control variables such as firm leverage interacted with both the level of the interest rate and the change in the interest rate. All regressions are value-weighted and standard errors are dually clustered by industry and date. $\alpha_{j,t}$ are industry-date fixed effects.

The key coefficients of interest are β_1 and β_3. A negative estimate of β_1 implies that a decline in the interest rate leads to a larger increase in the stock return of industry leaders. A positive estimate of β_3 implies that this effect is stronger when the level of interest rates is lower. In other words, a negative estimate of β_1 and a positive estimate of β_3 signify that industry leaders experience higher excess returns when interest rates fall, and this effect is amplified when interest rates start from a low level. This is the key prediction of the model.

Table 1 shows the results of estimating (7) on our merged CRSP-Compustat data set from 1980 onwards. Only the relevant coefficients are displayed in the tables, but the actual regression includes all variables specified in the equation 7. Column (1) estimates equation (7) without interactions with the level of interest rate. The coefficient β_1 is negative and significant; leaders earn positive excess return when the interest rate falls.

Column (2) presents estimates from the full specification (7). The coefficient β_3 is positive and significant as predicted by the model. Excess returns for leaders are *higher* in response to a fall in the interest rate when the level of the interest rate is lower. This is succinctly captured by β_1 which reflects the increase in excess returns when interest rates fall near the zero lower bound (i.e., when $i_{t-1} \approx 0$). The excess return near the zero lower bound in column (2) (3.88) is three times the average excess return of 1.19 in column (1).

One concern with these results is that the measure of industry leaders is spuriously correlated with balance sheet factors that are more sensitive to interest rate movements. For example, perhaps leaders are more levered and a fall in the interest rate helps lower the interest burden. To test for this, and other related concerns, we include a number of firm level characteristics as controls by including all the interaction of the firm level characteristic with the change in interest rate as well as the level of the interest rate. We include the following firm-level characteristics, a firm's asset-liability ratio, debt-equity ratio, book-to-market ratio, and the percent of pre-tax income that goes to taxes. The number of observations decreases because we have to limit the sample to Compustat firms with the available data on firm financials. Column (3) shows that the inclusion of this extensive list of firm-level controls does not change the coefficients of interest materially.

Column (4) controls for another potentially spurious firm-level attribute. What if industry leaders are spuriously more cyclical? If a fall in the interest rate represents changing economic expectations, industry leaders might generally be more responsive to changing market conditions irrespective of the level of interest rate. To test for this possibility, the market beta of each firm is estimated using the historical data as of $t-1$ and then it is interacted with both the change in the interest rate and the level of the interest rate in column (4). As before, the main coefficients of interest are not materially affected. Table A1 in the appendix also shows robustness of the main findings to alternative definitions of industry leadership: top 5 instead of top 5 percent, SIC instead of Fama French industries, and sorting on EBITDA and sales instead of market value.

Table 2 performs a time-series version of the excess return test implemented in Table 1. In particular, the results are based on the following specification,

$$R_t = \alpha + \beta_0 i_{t-1} + \beta_1 \Delta i_t + \beta_2 \Delta i_t * i_{t-1} + \varepsilon_t \tag{8}$$

where R_t is the market-capitalization weighted average of returns for a stock portfolio that goes long industry-leader stocks and goes short industry-follower stocks from date $t-91$ to t. We refer to this portfolio as the "leader portfolio." Given that observations have overlapping differences, we compute standard errors using a Newey-West procedure with a maximum lag length of 60 days to account for built-in correlation. A negative estimate of coefficient β_1 would signify that a decline in interest rates boosts the return on the leader portfolio, while a positive estimate of β_2 would signify that the positive response of the return on the leader portfolio to a decline in interest rates is larger when the level of the interest rate is lower.

The estimates reported in columns (1) and (2) confirm earlier results. A decline in the interest rate is associated with positive returns for the leader portfolio, and this positive return response to a decline in the interest rate is larger in magnitude when the interest rate is lower. Column (3) shows that the results are not driven by the excess market return of the HML factor.[9] Column (4) shows that the results are driven by both positive and negative changes in interest rates. In particular, the excess return results are materially unchanged whether only positive changes in interest rates or only negative changes in interest rate are used.

There may be a concern that industry leaders tend to have a high price-to-earnings ratio and that such growth firms benefit disproportionately from a decline in long-term interest rates because more of their earnings are in the future. We test for this concern by constructing a "PE portfolio" that is long the top 5% of firms by PE in an industry and short the rest. However, including the PE portfolio return does not change the coefficients of interest, and the leader-minus-follower portfolio is itself negatively corrected with the PE portfolio.

The default specification constructs returns and interest rate changes at a quarterly frequency. This reflects our view that this is the appropriate frequency because it captures interest rate movements that are deemed more permanent. Figure 5 plots the histograms of interest rate changes

[9] We cannot control for the small minus big size factor because it is very highly correlated with the leader portfolio.

in the sample, from daily to annual frequency. On average interest rates went down during this time period. However, there is substantial variation with the change in the interest rate being positive on a high fraction of days. As already shown, the key findings are symmetric to whether the change in the interest rate is positive or negative.

As one moves from daily to annual frequency, the range of interest rate changes increases. This is another reason to focus on longer term differences; the market needs sufficient time to incorporate a large change in interest rates when forming expectations. Table A2 in the appendix repeats the core specification for interest rate changes at frequencies ranging from daily to annual. For reasons discussed above, the effect tends to be stronger when the interest rate change is computed over longer horizons.

Another robustness test concerns the exact interest rate used in the specification. For example, do the excess return results depend on whether the change in the interest rate is at the short versus the long end of the yield curve? Statistically this is a somewhat hard test to perform because interest rate movements along the yield curve tend to be highly correlated. Table A3 in the appendix shows the correlation matrix of quarterly changes in forward rates of varying non-overlapping durations. The correlations are generally quite high, leading to problems of collinearity in joint testing. The lowest correlation is in the range of 0.7 to 0.75 between change in 0-2 forward rate and longer term forward rates (e.g. 10-30).

Table 3 presents estimates of equation (8) using the forward rate of varying duration. The main takeaway is that the results shown above are similar for interest rate changes throughout the yield curve (columns (1) through (6)). When both the 0-2 and 10-30 forward rates are put in the specification together (columns (7) and (8)), both ends of the yield curve appear to be independently important, with some evidence that the longer end of the yield curve is more important.

Figure 6 shows the key result in a more non-parametric fashion. The top panel plots the following two variables against each other: (i) the rolling 2-year correlation of the interest rate change ($t - 91$ days to t) and the industry leader minus follower excess returns over the same period, and (ii) the 10-year treasury yield at $t - 91$. There is a strong positive correlation between the two variables. As the level of the interest rate declines, the correlation measured on the y-axis becomes more negative. In other words, the excess return on the leader portfolio is higher in response to an interest rate decline when the interest rate level is lower.

The bottom panel plots the coefficients $\{\beta_{2,j}\}$ of the specification:

$$R_t = \sum_{j=1}^{4} \beta_{1,j} D_t\{4(j-1) <= i_{t-1} <= 4j\} + \sum_{j=1}^{4} \beta_{2,j} D_t\{4(j-1) <= i_{t-1} <= 4j\}\Delta i_t + \varepsilon_t \tag{9}$$

R_t is defined as the market-capitalization weighted average of returns for a stock portfolio that goes long in leader stocks and goes short in follower stocks from date $t-91$ to t. $D_t\{4(j-1) <= i_{t-1} <= 4j\}$ is defined as an indicator variable equal to 1 when the interest rate 91 days prior is between $4(j-1)$ and $4j$ for $j = 1, 2, 3, 4$. Consistent with the top panel, the bottom panel shows that response of excess returns to a decline in the interest rate monotonically increases as the level of the interest rate declines.

Figure 7 plots the coefficients $\{\beta_{0,j}\}$ of the following specification:

$$R_{t+j} = \alpha_j + \beta_{0,j}\Delta i_t + \beta_{1,j}\Delta i_{t-1} + \beta_{2,j}\Delta i_t i_{t-1} + \varepsilon_t \tag{10}$$

R_{t+j} is defined as the market-capitalization weighted average of returns for a stock portfolio that goes long in leader stocks and goes short in follower stocks from date t to $t+j$. Δi_t is defined here as the change in the interest rate from date t to $t+91$. The coefficients $\beta_{0,j}$ can be interpreted as the effect of a change in interest rates from $t-91$ to t on the returns of the leader portfolio from time t to time $t+j$ when the level of interest rates at $t-1$ is equal to zero. In other words, the figure represents the impulse response function at a daily frequency of the leader portfolio return to a change in interest rates.

As the figure shows, the effect of a change in interest rates starts quickly but the full effect is not realized until about 90 days. Further, there is no evidence of reversal over the following quarter. The increase in the value of the leader portfolio is persistent.

3.3 Aggregate evidence

The previous section focused on firm-level predictions of the model in response to a decline in interest rates. This section discusses how the model unifies a number of important aggregate trends in a single coherent framework.

A central prediction of the model is that lower interest rates increase the share of industries that are in the monopolistic region of the state space. As a result, lower interest rates should be associated with a higher profit share, higher markups, and higher concentration. The left panel of Figure 8 plots the profit share of GDP against the 10-year nominal U.S. Treasury rate and shows that there is indeed a sharp rise in profit share as long-term interest rate moved towards zero. The right panel plots the share of market value that goes to the top-5 firms within an industry against the 10-year treasury rate. There is a clear trend toward an increase in market concentration as the long-term rate declines.

The model also shows that as interest rates decline, "business dynamism," defined as the likelihood of a follower overtaking the leader, declines. One proxy for firm dynamism uses establishment entry and exit information for the United States from 1985 to 2014.[10] Figure 9 shows that lower interest rates are associated with both a decline in the entry and exit rates of establishments in the United States. From the model's perspective, this decline in business dynamism reflects higher market power of industry leaders and the reduced incentives to enter new markets by followers.

To the best of our knowledge, the model presented here is the first to predict that as interest rates decline and move toward zero, the steady state average gap in productivity growth between leaders and followers widens. Using firm level productivity data from OECD countries, Berlingieri and Criscuolo (2017) provide evidence in support of this prediction. The two panels of Figure A1 summarize their findings. The left panel uses data from the OECD multi-prod data set and shows that the gap between the labor productivity of market leaders and followers increases as interest rates decline over the 2000s. Leaders are defined to be firms in the 90th percentile of the labor productivity distribution for a given 2-digit industry, and followers are defined to be firms

[10]The establishment entry and exit rates time series for the United States comes from the US Census' Business Dynamics Statistics database.

in the 10th percentile. The gap between leaders and followers has been increasing steadily from 2000 to 2014 as long term interest rates have fallen.

The right panel extracts the year fixed effects from a country by industry level panel containing this same gap. As it shows, there is a steady increase in the productivity gap between market leaders and followers across countries and industries. A similar result is in Andrews et al. (2016), who show a widening gap in labor productivity of frontier versus laggard firms in both manufacturing and services for OECD firms. Andrews et al. (2016) also show cross-sectional evidence supporting the predictions of the model. They show that industries in which the gap between leader and follower multi-factor productivity is rising the most are the same industries where sector-aggregate multi-factor productivity is falling the most. As in the model, low interest rates lead to a large gap between the leader and follower in productivity investment; as a result, total investment in productivity of the industry falls.

Finally, the model shows that as interest rates approach zero, productivity growth also slows down due to the accumulated rise in market concentration. This is also consistent with the growth slowdown globally. The left panel of Figure 10 plots log total factor productivity for OECD countries and the 10-year nominal U.S. Treasury rate against time. As is well-known in the literature, and apparent from the figure, there is a marked slowdown in productivity growth around 2005 - well before the Great Recession. This suggests that there is a common global factor, not related to the Great Recession, that may be responsible for the growth slowdown. The model suggests the the very low level of long-term rates is a possible factor. The right panel plots the OECD average productivity growth rate against the 10-year rate and shows the slowdown in productivity growth as the interest rate approaches zero.[11]

In summary, lower interest rates are associated with higher market concentration, a decline in business dynamism, a widening of the gap between the productivity of leaders and followers, and a slowdown in productivity growth. This is true both in the model and in the data.

[11] As in the model, investment as a share of GDP has also declined with lower interest rates. In related papers, Jones and Philippon (2016) show that increase in industry concentration is associated with lower firm investment and Gutiérrez and Philippon (2016) show that investment is not correlated with market valuation and profitability after 2000s.

4 Conclusion

The focus of this paper is on understanding how the supply-side of the economy responds to a reduction in long-term interest rates driven by demand-side forces. The existing literature in growth either assumes no supply-side response to declining interest rates, or a positive response driven by an increased incentive to invest in the face of a higher discounted present value of future profits. The point of departure from this literature lies in explicitly modeling competition within an industry and analyzing how lower interest rates effect the nature of competition. The model builds on the dynamic contests literature to show that in a fairly general set up and without relying on any financial or other forms of frictions, the effect of lower interest rates on growth in a low interest rate regime can be negative.

A reduction in long-term interest rates tends to make market structure less competitive within an industry. The reason is that while both the leader and follower within an industry increase their investment in response to a reduction in interest rates, the increase in investment is always stronger for the leader. As a result, the gap between the leader and follower increases as interest rates decline, making an industry less competitive and more concentrated. When interest rates are already low, this negative effect of lower interest rates on industry competition tends to lower growth and overwhelms the traditional positive effect of lower interest rates on growth. This produces a hump-shaped inverted-U supply-side relationship between growth and interest rates.

The model delivers a distinctive *upward* sloping supply-curve in a low interest rate regime. We believe that this insight is useful in understanding the slowdown in productivity growth in recent decades and the broader discussion regarding "secular stagnation." The slowdown in productivity growth is global as it shows up in almost all advanced economies. The slowdown started well before the Great Recession, suggesting that cyclical forces related to the crisis are unlikely to be the trigger. And the slowdown in productivity is highly persistent, lasting well over a decade. The long-run pattern suggests that explanations relying on price stickiness or the zero lower bound on nominal interest rates are unlikely to be the complete explanation.

This paper introduces the possibility of low interest rates as the common global "factor" that drives the slowdown in productivity growth. The mechanism that the theory postulates delivers a number of important predictions that are supported by empirical evidence. A reduction in long term interest rates increases market concentration and market power in the model. A fall in the interest rate also makes industry leadership and monopoly power more persistent. There is empirical support for these predictions in the data, both in aggregate time series as well as in firm-level panel data sets.

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Figure 1: Investment response to a decline in r

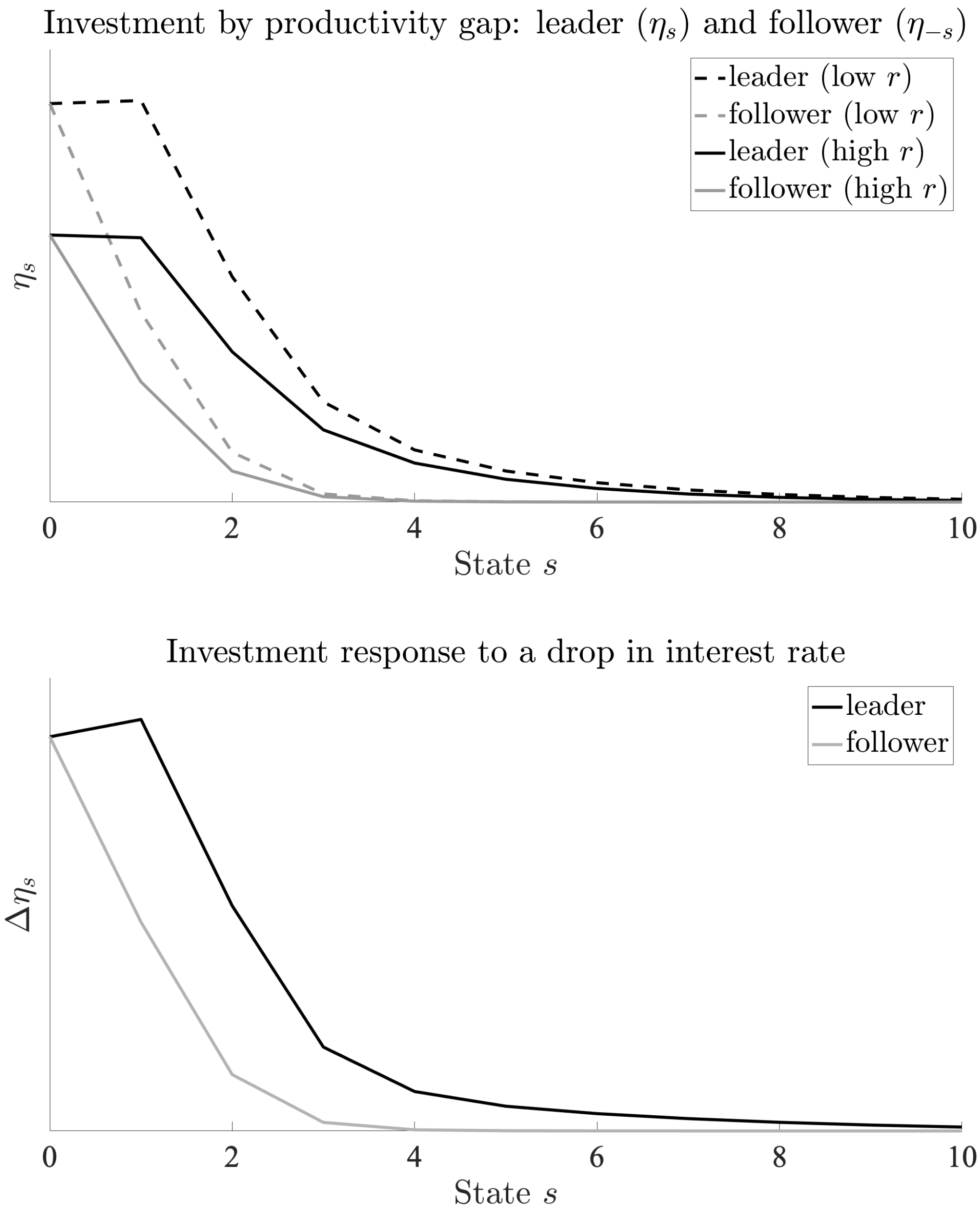


Figure 2: Response of steady-state distribution and value functions to a decline in r

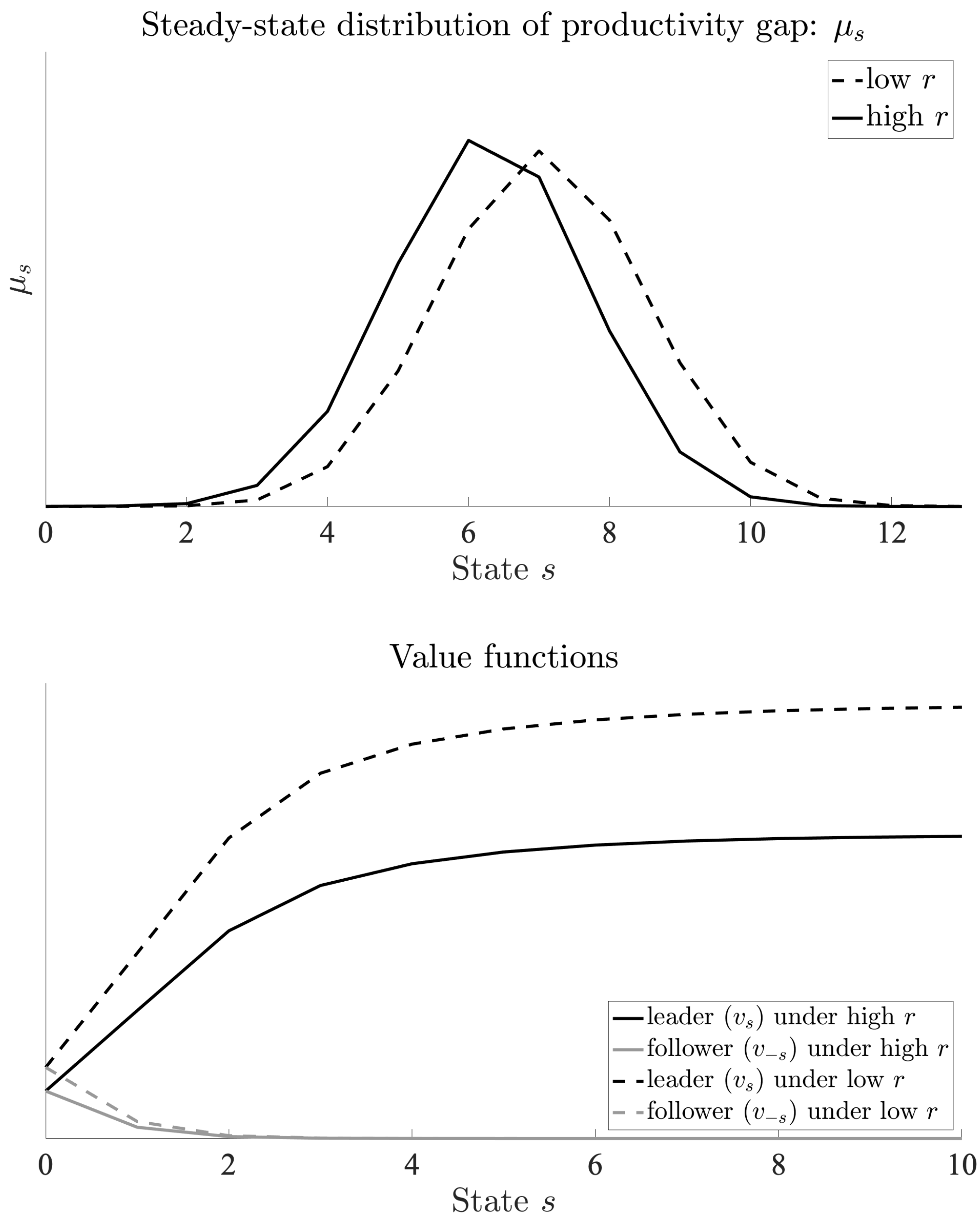


Figure 3: Steady-state growth

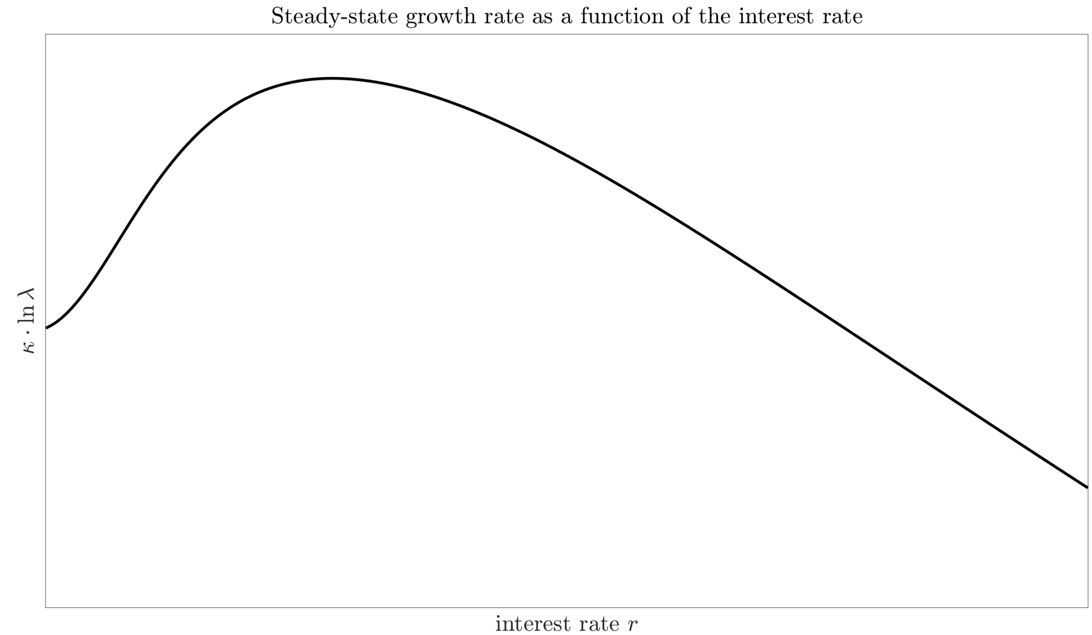
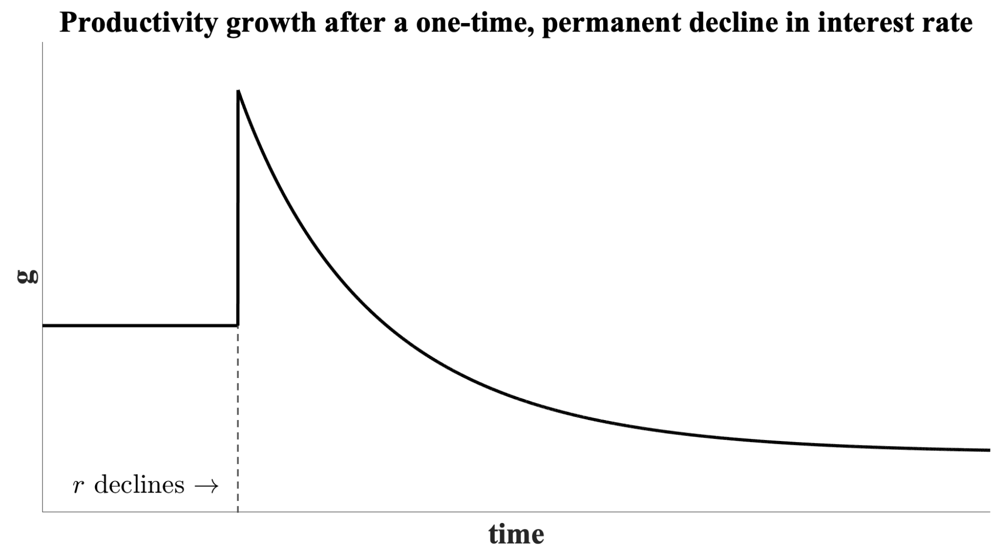


Figure 4: Time-path of markup and growth rate following a shock to r



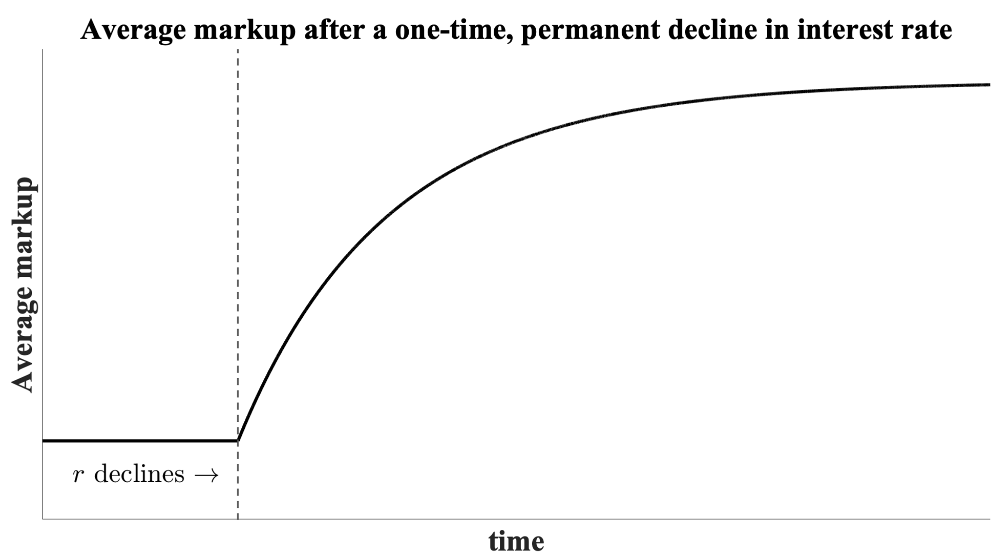
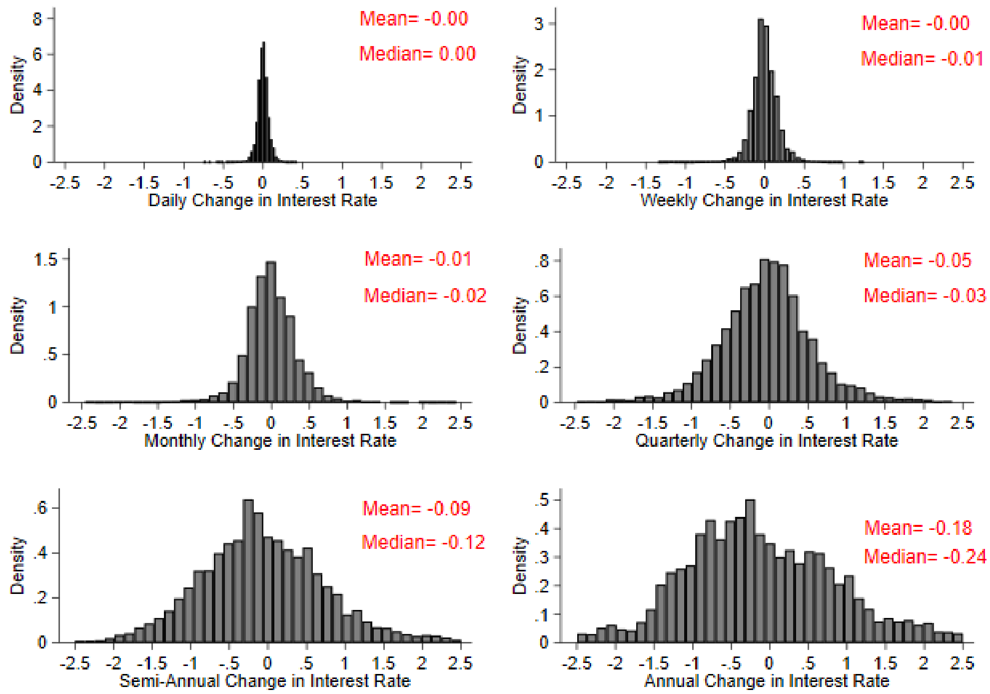


Figure 5

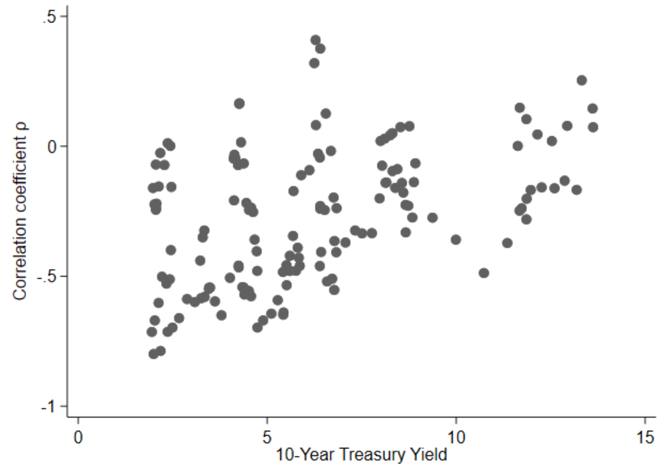
Distribution of Interest Rate Changes at Varying Frequencies

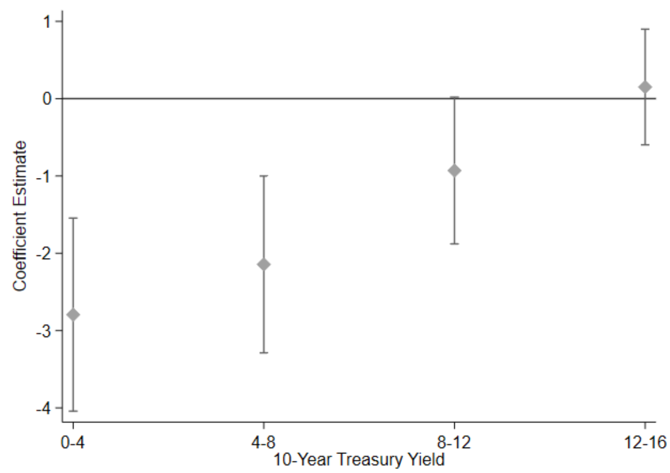


The panels plots the histograms of interest rate changes in our sample, from daily to annually.

Figure 6

Leaders See Higher Returns from a Drop in Interest Rates as Interest Rate Goes to Zero

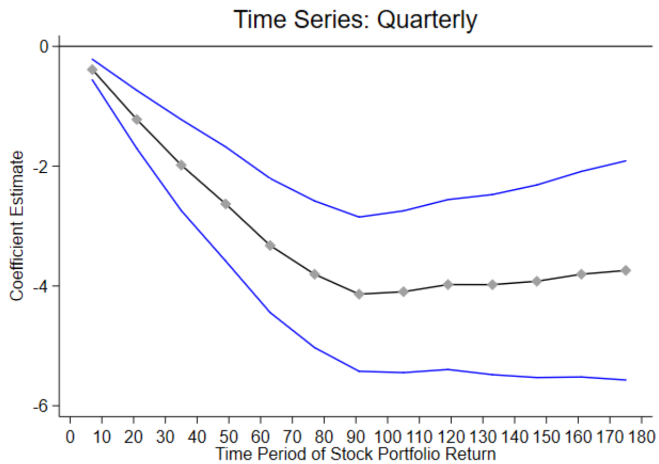




The top panel plots the rolling 2-year correlation of interest rate changes and top-5 portfolio returns against the 10-year Treasury yield 91 days prior. The bottom panel plot the coefficients $\{\beta_{2,j}\}$ of the specification $R_t = \sum_{j=1}^{4} \beta_{1,j} D_t\{4(j-1) <= i_{t-1} <= 4j\} + \sum_{j=1}^{4} \beta_{2,j} D_t\{4(j-1) <= i_{t-1} <= 4j\}\Delta i_t + \varepsilon_t$ at date t. R_t is defined as the market-capitalization weighted average of returns for a stock portfolio that goes long in leader stocks and goes short in follower stocks from date $t-91$ to t. $D_t\{4(j-1) <= i_{t-1} <= 4j\}$ is defined here as an indicator variable equal to 1 when the interest rate 91 days prior is between $4(j-1)$ and $4j$ for$j = 1, 2, 3, 4$. Leaders are defined as the firms in the top 5% of market capitalization in its FF industry on date $t-91$. i_t is defined as the nominal 10-year Treasury yield, with i_{t-1} being the interest rate 91 days prior and Δi_t being the change in the interest rate from date $t-91$ to t. Standard errors are Newey-West with a maximum lag length of 60 days prior.

Figure 7

Impulse Response of Changes in Interest Rate when Rate is Zero



The figue plots the coefficients $\{\beta_{0,j}\}$ of the specification $R_{t+j} = \alpha_j + \beta_{0,j}\Delta i_t + \beta_{1,j}\Delta i_{t-1} + \beta_{2,j}\Delta i_t i_{t-1} + \varepsilon_t$ at date t. R_{t+j} is defined as the market-capitalization weighted average of returns for a stock portfolio that goes long in leader stocks and goes short in follower stocks from date t to $t+j$. Δi_t is defined here as the change in the interest rate from date t to $t+91$. Leaders are defined as the firms in the top 5% of market capitalization in its FF industry on date t. Standard errors are Newey-West with a maximum lag length of 60 days prior.

Figure 8: Aggregate profit share, market concentration and interest rate

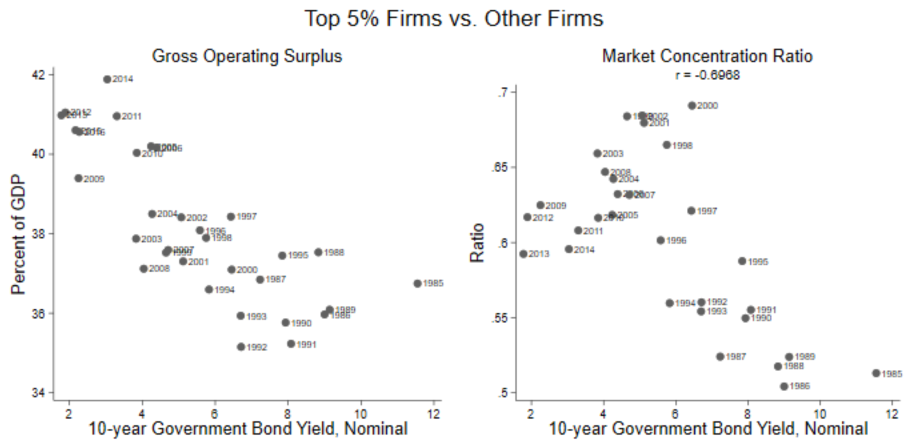


Figure 9: Business Dynamism

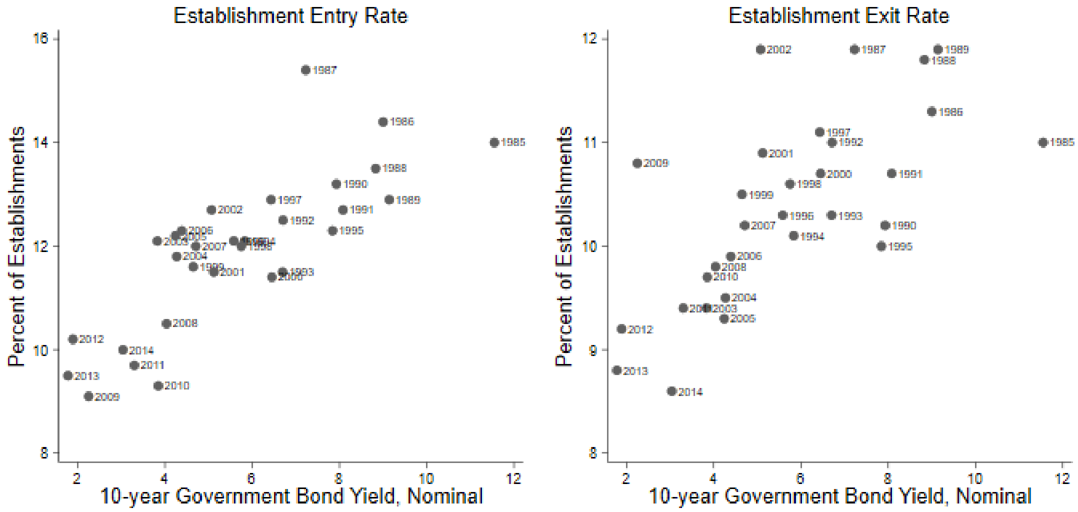


Figure 10: Productivity growth and interest rates

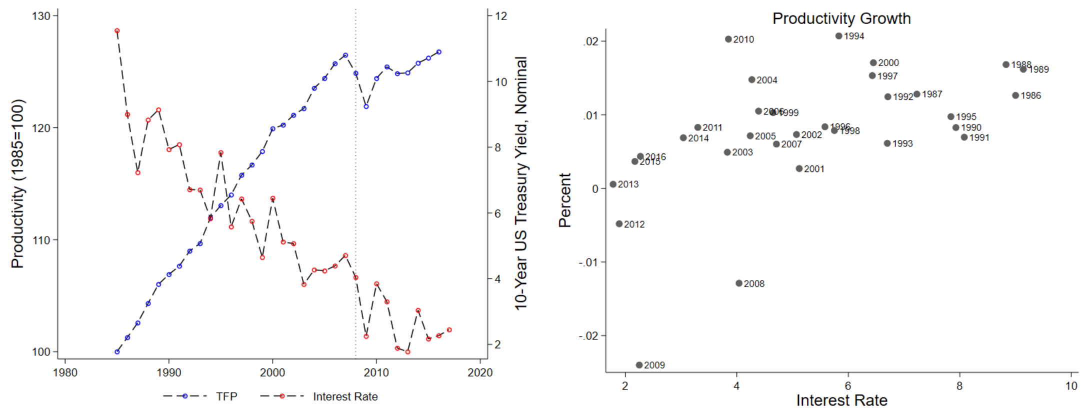


Table 1: Differential Interest Rate Responses of Leaders vs. Followers: Top 5 Percent

	Stock Return			
	(1)	(2)	(3)	(4)
Top 5 Percent=1 x Δi	-1.187***	-3.881**	-4.415***	-4.182***
	(0.260)	(1.113)	(0.893)	(0.529)
Top 5 Percent=1 x Δi x Lagged i		0.293**	0.346***	0.301***
		(0.095)	(0.079)	(0.045)
Firm β x Δi				14.10***
				(0.795)
Firm β x Δi x Lagged i				-1.260***
				(0.082)
Sample	All	All	All	All
Controls	N	N	Y	
Industry-Date FE	Y	Y	Y	Y
N	61,313,604	61,313,604	44,104,181	61,299,546
R-sq	0.403	0.403	0.415	0.409

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Regression results for the specification $\Delta \ln\left(P_{i,j,t}\right) = \alpha_{j,t} + \beta_0 D_{i,j,t} + \beta_1 D_{i,j,t}\Delta i_t + \beta_2 D_{i,j,t} i_{t-1} + \beta_3 D_{i,j,t}\Delta i_t i_{t-1} + X_{i,j,t}\gamma + \varepsilon_{i,j,t}$ for firm i in industry j at date t. $\Delta \ln\left(P_{i,j,t}\right)$ is defined here as the log change in the stock price for firm i in industry j from date $t-91$ to t (one quarter growth). $D_{i,j,t}$ is defined here as an indicator equal to 1 at date t when a firm i is in the top 5% of market capitalization in its industry j on date $t-91$. Firms with $D_{i,j,t}$=1 are called leaders while the rest are called followers. i_t is defined as the nominal 10-year Treasury yield, with i_{t-1} being the interest rate 91 days prior and Δi_t being the change in the interest rate from date $t-91$ to t. Controls X include a firm's asset-liability ratio, debt-equity ratio, book-to-market ratio, and percent of pre-tax income that goes to taxes. Industry classifications are the Fama-French industry classifications (FF). Standard errors are dually clustered by industry and date.

Table 2: Portfolio Returns Response to Interest Rate Changes: Top 5 Percent

	Portfolio Return				
	(1)	(2)	(3)	(4)	(5)
Δi_t	-1.150***	-3.819***	-2.268***	-3.657***	-3.001***
	(0.309)	(0.641)	(0.602)	(0.949)	(0.720)
i_{t-1}		0.0842	0.0336	0.160*	0.167*
		(0.050)	(0.044)	(0.071)	(0.069)
$\Delta i_t \times i_{t-1}$		0.294***	0.117*	0.328***	0.239*
		(0.059)	(0.056)	(0.081)	(0.096)
Excess Market Return			-0.168***		
			(0.023)		
High Minus Low			0.0371		
			(0.044)		
$(\Delta i_t > 0)$=1 $\times \Delta i_t$				0.341	
				(1.717)	
$(\Delta i_t > 0)$=1 $\times \Delta i_t \times i_{t-1}$				-0.102	
				(0.170)	
PE Portfolio Return					-0.207***
					(0.059)
N	9,016	9,016	9,016	9,016	7,402
R-sq	0.044	0.089	0.228	0.092	0.196

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Regression results for the specification $R_t = \alpha + \beta_0 i_{t-1} + \beta_1 \Delta i_t + \beta_2 \Delta i_t i_{t-1} + \varepsilon_t$ at date t. R_t is defined as the market-capitalization weighted average of returns for a stock portfolio that goes long in leader stocks and goes short in follower stocks from date $t-91$ to t. Leaders are defined as the firms in the top 5% of market capitalization in its FF industry on date $t-91$. i_t is defined as the nominal 10-year Treasury yield, with i_{t-1} being the interest rate 91 days prior and Δi_t being the change in the interest rate from date $t-91$ to t. Standard errors are Newey-West with a maximum lag length of 60 days prior. In column 4, the terms $(\Delta i_t > 0) = 1$ and $(\Delta i_t > 0) = 1$ x i_{t-1} were suppressed from the table. Their coefficients are 0.0222 (0.602) and -0.0616 (0.086), respectively.

Table 3: Portfolio Returns Response to Interest Rate Changes: Along the Yield Curve

	30-Year		2-Year		10-30 Forward		2-Year & 10-30 Fwd.	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Δi_t	-1.129**	-4.537***						
	(0.348)	(0.826)						
$\Delta i_t \times i_{t-1}$		0.362***						
		(0.077)						
$\Delta i_{t,0,2}$			-0.584*	-3.535***			-0.126	-2.066*
			(0.244)	(0.833)			(0.349)	(0.970)
$\Delta i_{t,0,2} \times i_{t-1}$				0.280***				0.145
				(0.069)				(0.080)
$\Delta i_{t,10,30}$					-1.084**	-4.165***	-0.938	-3.138**
					(0.354)	(0.835)	(0.523)	(1.043)
$\Delta i_{t,10,30} \times i_{t-1}$						0.334***		0.289**
						(0.080)		(0.107)
N	8,006	8,006	8,065	8,065	8,006	8,006	8,006	8,006
R-sq	0.036	0.078	0.021	0.063	0.030	0.066	0.031	0.084

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Regression results for the specification $R_t = \alpha + \beta_0 i_{t-1} + \beta_1 \Delta i_t + \beta_2 \Delta i_t i_{t-1} + \varepsilon_t$ at date t in columns 1-2 and $R_t = \alpha + \beta_0 i_{t-1} + \beta_{1,1} \Delta i_{t,0,2} + \beta_{1,2} \Delta i_{t,10,30} + \beta_{1,3} \Delta i_{t,0,10} + \beta_{2,1} \Delta i_{t,0,2} i_{t-1} + \beta_{2,2} \Delta i_{t,10,30} i_{t-1} + \beta_{2,3} \Delta i_{t,0,10} i_{t-1} + \varepsilon_t$ at date t in columns 3-8. R_t is defined as the market-capitalization weighted average of returns for a stock portfolio that goes long in leader stocks and goes short in follower stocks from date $t-91$ to t. Leaders are defined as the firms in the top 5% of market capitalization in its FF industry on date $t-J$. i_t is defined as the nominal 30-year Treasury yield, with i_{t-1} being the interest rate J days prior and Δi_t being the change in the interest rate from date $t-91$ to t. $i_{t,0,2}$, $i_{t,0,10}$ and $i_{t,10,30}$ are the 2-year and 10-year Treasury yield and 10 to 30 forward Treasury yield, respectively. Standard errors are Newey-West with a maximum lag length of 60 days prior. We cannot reject that the main and interaction coefficients in columns 7 and 8 are not equal.

A Appendix: Proofs (work in progress)

A.1 Properties of flow profits and steady-state growth rate

In this appendix section, we prove lemmas 1 and 2.

In the main text, we model market structure as Bertrand competition, generating a sequence of state-dependent flow profits $\{\pi_s, \pi_{-s}\}$ that satisfy properties outlined in Lemma 1, and that $\lim_{s\to\infty} \pi_s = 1$, $\lim_{s\to-\infty} \pi_s = 0$. Our theoretical results hold under any sequence of flow profits that satisfy Lemma 1; hence, our theory nests other market structures. We use $\pi \equiv \lim_{s\to 0} \pi_s$ to denote the limiting total profits in each market, and we exposit using the notation π.

Lemma 1: Follower's flow profits π_{-s} are non-negative, weakly decreasing, and convex; leader's and joint profits π_s, π_{-s} are bounded, weakly increasing, and eventually concave in s (a sequence $\{a_s\}$ is eventually concave iff there exists $\bar{s}$ such that a_s is concave in s for all $s \geq \bar{s}$).

Proof. Let δ_i be the market share of firm i. Under CES demand within each market, $\delta_i \equiv \frac{p_i y_i}{p_1 y_1 + p_2 y_2} = \frac{p_i^{1-\sigma}}{p_1^{1-\sigma} + p_2^{1-\sigma}}$. Under Bertrand competition, the price charged by a firm with productivity z_i must solve $p_i = \frac{\sigma(1-\delta_i)+\delta_i}{(\sigma-1)(1-\delta_i)} \lambda^{-z_i}$ (recall we normalize wage rate to 1). Aghion et al. (2001) and Atkeson and Burstein (2008) provides detailed derivations of these expressions.

Define ρ_s as an implicit function of the productivity gap: $\rho_s^\sigma = \lambda^{-s} \frac{\left(\sigma \rho_s^{\sigma-1} + 1\right)}{\sigma + \rho_s^{\sigma-1}}$. It can be verified that flow profits satisfy $\pi_s = \frac{1}{\sigma \rho_s^{\sigma-1} + 1}$ for any productivity gap s. Asymptotically, $\lim_{s\to\infty} \rho_s^\sigma \lambda^s = 1/\sigma$ and $\lim_{s\to-\infty} \rho_s^\sigma \lambda^s = \sigma$; hence, for large s, $\pi_s \approx \frac{1}{\sigma^{\frac{1}{\sigma}} \lambda^{-\frac{\sigma-1}{\sigma} s} + 1}$ and $\pi_{-s} \approx \frac{1}{\sigma^{\frac{2\sigma-1}{\sigma}} \lambda^{\frac{\sigma-1}{\sigma} s} + 1}$. The eventual concavity of π_s and $(\pi_s + \pi_{-s})$ as $s \to \infty$ is immediate. To show concavity of $-\pi_{-s}$ is a matter of algebra (to be added).

Lemma 2: In a steady state, the aggregate productivity growth rate is $g \equiv \ln \lambda \left(\sum_{s=0}^{\infty} \mu_s \eta_s + \mu_0 \eta_0\right)$.

Proof The expression $\left(\sum_{s=0}^{\infty} \mu_s \eta_s + \mu_0 \eta_0\right)$ tracks the weighted-average growth rate of the productivity frontier in the economy, i.e., the rate at which markets leave the current state s and move to state $s+1$. In a steady-state, the growth rate of frontier must be the same as the rate at which states fall down by one step, from $s+1$ to s; hence, aggregate growth rate g can also be written as $g = \ln\lambda\left(\sum_{s=1}^{\infty}\mu_s\left(\eta_{-s}+\kappa\right)\right)$.

To prove the expression formally, we proceed in two steps. First, we express aggregate productivity growth as a weighted average of productivity growth in each market. We then use the fact that, given homothetic within-market demand if a follower in state s improves productivity by one step (i.e. by a factor λ) and a leader in state $s-1$ improves also by one step, the net effect should be equivalent to one step improvement in the overall productivity of a single market.

Aggregate productivity growth is a weighted average of productivity growth in each market:

$$\begin{aligned} g = -\frac{d\ln P}{d\ln t} &= -\frac{d\int_0^1 \ln p\left(\nu\right)d\nu}{d\ln t} \\ &= -\sum_{s=0}^{\infty}\mu_s \times \frac{d\left[\int_{z^F}\ln p\left(s,z^F\right)dF\left(z^F\right)\right]}{d\ln t}, \end{aligned}$$

where we use $\left(s,z^F\right)$ to index for markets in the second line. Now recognize that productivity growth rate in each market, $-\frac{d\ln p\left(s,z^F\right)}{d\ln t}$, is a function of only the productivity gap s and is invariant to the productivity of follower, z^F. Specifically, suppose the follower in market $\left(s,z^F\right)$ experiences an innovation, the market price index becomes $p\left(s-1,z^F+1\right)$. Similarly, if the leader experiences an innovation, the price index becomes $p\left(s+1,z^F\right)$. The corresponding log-changes in price indices are respectively

$$\begin{aligned} a_s^F &\equiv \ln p\left(s-1,z^F+1\right)-\ln p\left(s,z^F\right) \\ &= -\ln\lambda+\ln\left[\rho_{s-1}^{1-\sigma}+1\right]^{\frac{1}{1-\sigma}}-\ln\left[\rho_s^{1-\sigma}+1\right]^{\frac{1}{1-\sigma}}, \end{aligned}$$

$$\begin{aligned} a_s^L &= \ln p\left(s+1,z^F\right)-\ln p\left(s,z^F\right) \\ &= \ln\left[\rho_{s+1}^{1-\sigma}+1\right]^{\frac{1}{1-\sigma}}-\ln\left[\rho_s^{1-\sigma}+1\right]^{\frac{1}{1-\sigma}}, \end{aligned}$$

where ρ_s is the implicit function defined in the proof for Lemma 1. The log-change in price index is independent of z^F in either case. Hence, over time interval $[t,t+\Delta]$, the change in price index for markets with state variable s at time t follows

$$\Delta \ln p\left(s, z^F\right) = \begin{cases} a_s^L & \text{with probability } \eta_s \Delta, \\ a_s^F & \text{with probability } \left(\eta_{-s} + \kappa \cdot \mathbf{1}\left(s \neq 0\right)\right)\Delta. \end{cases}$$

The aggregate productivity growth can therefore be written as

$$g = -\mu_0 2\eta_0 a_0 - \sum_{s=1}^{\infty} \mu_s \times \left(\eta_s a_s^L + \left(\eta_{-s} + \kappa\right) a_s^F\right).$$

Lastly, note that if both leader and follower in a market experiences productivity improvements, regardless of the order in which these events happen, the price index in the market changes by a factor of λ^{-1}:

$$a_s^F + a_{s-1}^L = a_s^L + a_{s+1}^F = -\ln \lambda \quad \text{for all } s \geq 1.$$

Hence,

$$\begin{aligned} g &= -\mu_0 2\eta_0 a_0 - \sum_{s=1}^{\infty} \mu_s \times \left(\eta_s a_s^L + \left(\eta_{-s} + \kappa\right) a_s^F\right) \\ &= -\mu_0 2\eta_0 a_0 - \sum_{s=1}^{\infty} \mu_s \times \left(\eta_s a_s^L + \left(\eta_{-s} + \kappa\right)\left(-\ln \lambda - a_{s-1}^L\right)\right) \\ &= \ln \lambda \cdot \sum_{s=1}^{\infty} \mu_s \left(\eta_{-s} + \kappa\right) - \left(\sum_{s=1}^{\infty} \mu_s \times \left(\eta_s a_s^L - a_{s-1}^L \left(\eta_{-s} + \kappa\right)\right) + \mu_0 2\eta_0 a_0\right). \end{aligned}$$

Given that steady-state distribution $\{\mu_s\}$ must follow

$$\mu_s \left(\eta_{-s} + \kappa\right) = \begin{cases} \mu_{s-1}\eta_{s-1} & \text{if } s > 1, \\ 2\mu_0\eta_0 & \text{if } s = 1, \end{cases} \tag{11}$$

we know

$$\begin{aligned} & \sum_{s=1}^{\infty} \mu_s \times \left(\eta_s a_s^L - a_{s-1}^L \left(\eta_{-s} + \kappa\right)\right) + \mu_0 2\eta_0 a_0 \\ = & \sum_{s=1}^{\infty} \mu_s \eta_s a_s^L + \mu_0 2\eta_0 a_0 - \left(\sum_{s=1}^{\infty} \mu_s a_{s-1}^L \left(\eta_{-s} + \kappa\right)\right) \\ = & \ 0. \end{aligned}$$

Hence aggregate growth rate simplifies to $g = \ln\lambda \cdot \sum_{s=1}^{\infty} \mu_s \left(\eta_{-s} + \kappa\right)$, which traces the growth rate of productivity laggards. We can also apply substitutions in (11) again to express productivity growth as a weighted average of frontier growth:

$$g = \ln\lambda \cdot \left(\sum_{s=1}^{\infty} \mu_s \eta_s + 2\mu_0\eta_0\right).$$

A.2 Structure of Equilibrium

It is useful to first understand the structure of value functions given any sequence of (potentially non-equilibrium) investment decisions $\{\eta_s\}_{s=-\infty}^{\infty}$. The fact that firms are forward-looking implies that value function in each state can be written as a weighted average of flow payoffs in all ergodic states induced by the investment decisions, i.e.

$$v_s = \sum_{s'=-\infty}^{\infty} \lambda_{s'|s} \times PV_{s'}, \quad \text{where} \sum_{s'=-\infty}^{\infty} \lambda_{s'|s} = 1 \quad \text{for all } s. \tag{12}$$

The term $PV_{s'} \equiv \frac{\pi_{s'} - c\eta_{s'}}{r}$ represents the permanent value in state s', i.e. the present-discounted value of flow payoff in state s' if the firm stays in that state permanently; $s' > 0$ means the firm is a leader when the productivity gap is s', and $s' < 0$ means the firm is a follower when the productivity gap is $-s'$. In equilibrium, the firm value in state s can be written as a weighted average of the permanent value across all ergodic states. The weight $\lambda_{s'|s}$ can be interpreted as the present-discount fraction of time that the firm is going to be s' steps ahead of his competitor, given that he is currently s steps ahead. The weights $\left\{\lambda_{s'|s}\right\}_{s'=\infty}^{\infty}$ form a measure conditional on the current state s. When the current state s is high, the firm is expected to spend more time in higher indexed states, and the conditional distribution $\left\{\lambda_{s'|s+1}\right\}_{s'=\infty}^{\infty}$ first-order stochastically dominates $\left\{\lambda_{s'|s}\right\}_{s'=\infty}^{\infty}$ for all s.

Likewise, let $w_s \equiv v_s + v_{-s}$ be the joint value of leader and follower in state s. Following the same logic as in equation (12), we can rewrite w_s as a weighted average of the sum of permanent values of leader and follower in every state:

$$w_s = \sum_{s'=0}^{\infty} \tilde{\lambda}_{s'|s} \cdot \left(PV_{s'} + PV_{-s'}\right), \quad \text{where} \sum_{s'=0}^{\infty} \tilde{\lambda}_{s'|s} = 1. \tag{13}$$

The weights $\tilde{\lambda}_{s'|s}$ can be interpreted as the present-discounted fraction of time that the state variable is s', i.e. when either firm is s' steps ahead of the other, conditioning on the current gap being s; hence, $\tilde{\lambda}_{s'|s} = \lambda_{s'|s} + \lambda_{s'|-s}$. It is easy to verify that $\{\tilde{\lambda}_{s'|s+1}\}$ first order stochastically dominate $\{\tilde{\lambda}_{s'|s}\}$.

To understand the role of interest rate, note that the firm value in state s can be written as a weighted average of the permanent state payoff in state s and the firm value in neighboring states $s-1$ and $s+1$:

$$v_s = \frac{r}{r+\kappa+\eta_{-s}+\eta_s} \cdot PV_s + \frac{\kappa+\eta_{-s}}{r+\kappa+\eta_{-s}+\eta_s} v_{s-1} + \frac{\eta_s}{r+\kappa+\eta_{-s}+\eta_s} v_{s+1}$$

Holding investment decisions constant, a fall in interest rate r reduces the relative weight on the permanent value of state s, thereby reducing the difference in value functions across states. In fact, holding investment decisions fixed, if there is a state in which the leader chooses not to invest at all ($\eta_{\bar{s}} = 0$ for some $\bar{s}$), then $rv_s \to rv_0$ for all $s \leq \bar{s}$.

We now prove results that analyze the structure of equilibria. For expositional purposes, we assume firms play pure strategies (i.e. they invest at either lower or upper bounds $\eta_s \in \{0, \eta\}$); all of our claims hold for mixed strategy equilibria (i.e. those interior investment intensities).

Lemma 3. The leader invests in more states than the follower, $n \geq k$. Moreover, the follower does not invest in states $s = k+1, \cdots, n+1$. Recall $n+1$ is the first state in which market leaders choose not to invest, and $k+1$ is the first state in which followers choose not to invest: $n+1 \equiv \min\{s | s \geq 0, \eta_s < \eta\}$ and $k+1 \equiv \min\{s | s \leq 0, \eta_s < \eta\}$.

Proof Suppose $n < k$, i.e. leader invests in states 1 through n whereas follower invests in states 1 through at least $n+1$. Such equilibrium can only be supported by certain lower bounds on the value function of both leader and follower in state $n+1$. We reach for a contradiction, showing that, if $n < k$, then market power is too transient to support these lower bounds on value functions.

The HJB equation for the leader in state $n+2$ implies

$$\begin{aligned} rv_{n+2} &= \max_{\eta_{n+2}\in[0,\eta]} \pi_{n+2} + \eta_{n+2}\left(v_{n+3} - v_{n+2} - c\right) + \left(\eta_{-(n+2)} + \kappa\right)\left(v_{n+1} - v_{n+2}\right) \\ &\geq \pi_{n+2} + \left(\eta + \kappa\right)\left(v_{n+1} - v_{n+2}\right). \end{aligned}$$

The fact that leader does not invest in state $n+1$ implies $c \geq v_{n+2} - v_{n+1}$; combining with the previous inequality, we obtain

$$rv_{n+1} \geq \pi_{n+2} - c\left(\eta + \kappa + r\right).$$

The HJB equation for the follower in state $n+1$ implies

$$\begin{aligned} rv_{-(n+1)} &= \max_{\eta_{-(n+1)}\in[0,\eta]} \pi_{-(n+1)} + \left(\eta_{-(n+1)} + \kappa\right)\left(v_{-n} - v_{-(n+1)}\right) - c\eta_{-(n+1)} \\ &\geq \pi_{-(n+1)} + \kappa\left(v_{-n} - v_{-(n+1)}\right). \end{aligned}$$

The fact that follower chooses to invest in state $n+1$ implies $c \leq v_{-n} - v_{-(n+1)}$; combining with the previous inequality, we obtain

$$rv_{-(n+1)} \geq \pi_{-(n+1)} + c\kappa. \tag{14}$$

Combining this with the earlier inequality involving rv_{n+1}, we obtain an inequality on the joint value in state $n+1$:

$$rw_{n+1} \geq \pi_{n+2} + \pi_{-(n+1)} - c\left(\eta + r\right) \tag{15}$$

We now show that inequalities (14) and (15) cannot both be true. To do so, we construct alternative economic environments with value functions that dominate w_{n+1} and $v_{-(n+1)}$; we then show that these dominating value functions cannot satisfy both inequalities.

First, fix n and fix investment strategies (leader invests until state $n+1$ and follower invests at least through $n+1$); suppose for all states $1 \leq s \leq n+1$, follower's profits are equal to $\pi_{-(n+1)}$ and leader's profits are equal to π_{n+2}; two firms each earn $\frac{\pi_{-(n+1)}+\pi_{n+2}}{2}$ in state zero. The joint profits in this modified economic environment are independent of the state by construction; moreover, the joint flow profits always weakly dominate those in the original environment and strictly dominate in state zero ($\pi_{n+2} + \pi_{-(n+1)} \geq \pi_1 + \pi_{-1} > 2\pi_0$). Let $\hat{w}_s$ denote the value function in the modified environment; $\hat{w}_s > w_s$ for all $s \leq n+1$.

Consider the joint value in this modified environment but under alternative investment strategies. Let $\bar{n}$ index for investment strategies: leader invests in states 1 through $\bar{n}$ whereas the follower invests at least through $\bar{n}+1$. Let $\hat{w}_s^{(\bar{n})}$ denote the joint value in state s under investments indexed by $\bar{n}$; we argue that $\hat{w}_{\bar{n}+1}^{(\bar{n})}$ is decreasing in $\bar{n}$. To see this, note that the joint flow payoffs in all states 0 through $\bar{n}$ is constant by construction and is equal to $\left(\pi_{n+2} + \pi_{-(n+1)} - 2c\eta\right)$—total profits net of investment costs. The joint flow payoff in state $\bar{n}+1$ is $\left(\pi_{n+2} + \pi_{-(n+1)} - c\eta\right)$. Hence, the joint market value in state $\bar{n}+1$ under the investment strategies indexed by $\bar{n}$ is equal to

$$\hat{w}_{\bar{n}+1}^{(\bar{n})} = \frac{\pi_{n+2} + \pi_{-(n+1)} - 2c\eta\left(1 - \tilde{\lambda}_{\bar{n}+1|\bar{n}+1}^{(\bar{n})}/2\right)}{r},$$

where $\tilde{\lambda}_{\bar{n}+1|\bar{n}+1}^{(\bar{n})}$ is the present discount fraction of time that the market spends in state $\bar{n}+1$, conditioning on the current state is $\bar{n}+1$, and that firms follow investment strategies indexed by $\bar{n}$. The object $\tilde{\lambda}_{\bar{n}+1|\bar{n}+1}^{(\bar{n})}$ is decreasing in $\bar{n}$: the more states in which both firms invest, the less time that the market will spend in the state $\bar{n}+1$ in which only one firm (the follower) invests. Hence, $\hat{w}_{\bar{n}+1}^{(\bar{n})}$ is decreasing in $\bar{n}$, and that $\hat{w}_1^{(0)} \geq \hat{w}_{n+1}^{(n)} > w_{n+1}$. The same logic also implies $\hat{v}_0^{(0)} = \frac{1}{2}\hat{w}_0^{(0)} > \frac{1}{2}w_0 = v_0$.

The follower's value $\hat{v}_{-1}^{(0)}$, in the alternative environment, when investment strategies are indexed by zero (i.e. firms invest in states 0 and -1 only), is higher than $v_{-(n+1)}$. This is because

$$\begin{aligned}
\hat{v}_{-1}^{(0)} &= \frac{\pi_{-(n+1)} - c\eta + \kappa\hat{v}_0^{(0)}}{r+\kappa+\eta} \\
&> \frac{\pi_{-(n+1)} - c\eta + \kappa v_0}{r+\kappa+\eta} \\
&\geq \frac{\pi_{-(n+1)} - c\eta + \kappa v_{-n}}{r+\kappa+\eta} \\
&= v_{-(n+1)}.
\end{aligned}$$

We now show that the inequalities $r\hat{v}_{-1}^{(0)} \geq \pi_{-(n+1)} + c\kappa$ and $r\hat{w}_1^{(0)} \geq \pi_{n+2} + \pi_{-(n+1)} - c(\eta + r)$ cannot both hold. We can explicitly solve for the value functions from the HJB equations:

$$\hat{w}_0^{(0)} = \frac{\pi_{n+2} + \pi_{-(n+1)} - 2c\eta + 2\eta\hat{w}_1^{(0)}}{r + 2\eta}$$

$$\hat{w}_1^{(0)} = \frac{\pi_{n+2} + \pi_{-(n+1)} - c\eta + (\eta + \kappa)\hat{w}_0^{(0)}}{r + \eta + \kappa}$$

$$\hat{v}_{-1}^{(0)} = \frac{\pi_{-(n+1)} - c\eta + (\eta + \kappa)\hat{w}_0^{(0)}/2}{r + \eta + \kappa}$$

Solving for $\hat{w}_1^{(0)}$ and $\hat{v}_{-1}^{(0)}$, we obtain

$$r\hat{w}_1^{(0)} = \pi_{n+2} + \pi_{-(n+1)} - c\eta\left(1 + \frac{\eta + \kappa}{r + 3\eta + \kappa}\right)$$

$$(r + \eta + \kappa)\, r\hat{v}_{-1}^{(0)} = r\left(\pi_{-(n+1)} - c\eta\right) + (\eta + \kappa)\left(\frac{\pi_{n+2} + \pi_{-(n+1)}}{2} - c\eta\frac{r + 2\eta + \kappa}{r + 3\eta + \kappa}\right)$$

That $r\hat{v}_{-1}^{(0)} \geq \pi_{-(n+1)} + c\kappa$ implies

$$\begin{aligned} & (r + \eta + \kappa)\, r\hat{v}_{-1}^{(0)} \\ = \; & r\left(\pi_{-(n+1)} - c\eta\right) + (\eta + \kappa)\left(\frac{\pi_{n+2} + \pi_{-(n+1)}}{2} - c\eta\frac{r + 2\eta + \kappa}{r + 3\eta + \kappa}\right) \\ \geq \; & (r + \eta + \kappa)\left(\pi_{-(n+1)} + c\kappa\right) \end{aligned}$$

$$\Longrightarrow (\eta + \kappa)\left(\frac{\pi_{n+2} - \pi_{-(n+1)}}{2} - c\eta\frac{r + 2\eta + \kappa}{r + 3\eta + \kappa}\right) \geq (r + \eta + \kappa)\, c\kappa + c\eta r$$

Since $\frac{\pi_{n+2} - \pi_{-(n+1)}}{2} \leq \frac{\pi_{n+2}}{2} < c\eta$, it must be the case that

$$(\eta + \kappa)\, c\eta > (r + \eta + \kappa)\, c\kappa + c\eta r + (\eta + \kappa)\, c\eta\frac{r + 2\eta + \kappa}{r + 3\eta + \kappa}.$$

On the other hand, that $r\hat{w}_1^{(0)} \geq \pi_{n+2} + \pi_{-(n+1)} - c(\eta + r)$ implies $r \geq \eta\frac{\eta + \kappa}{r + 3\eta + \kappa}$; hence the previous inequality implies

$$\begin{aligned}(\eta+\kappa)\, c\eta &> (r+\eta+\kappa)\, c\kappa + (\eta+\kappa)\, c\eta \frac{\eta}{r+3\eta+\kappa} + (\eta+\kappa)\, c\eta \frac{r+2\eta+\kappa}{r+3\eta+\kappa} \\ &= (r+\eta+\kappa)\, c\kappa + (\eta+\kappa)\, c\eta,\end{aligned}$$

which is impossible; hence $n \geq k$.

We now show that the follower does not invest in states $s \in \{k+1, ..., n+1\}$. First, note

$$\begin{aligned}(r+\eta+\kappa)\,(v_{-s}-v_{-s-1}) &= \pi_{-s}-\pi_{-s-1}+\kappa\,(v_{-s+1}-v_s)+\eta\,(v_{-s-1}-v_{-s-2}) \\ &\quad +\max\{\eta\,(v_{-s+1}-v_{-s}-c)\,,0\}-\max\{\eta\,(v_{-s}-v_{-s-1}-c)\,,0\}\,.\end{aligned}$$

Suppose $v_{-s+1}-v_{-s} \geq (v_{-s}-v_{-s-1})$, then

$$(r+\eta+\kappa)\,(v_{-s}-v_{-s-1}) \geq \pi_{-s}-\pi_{-s-1}+\kappa\,(v_{-s+1}-v_{-s})+\eta\,(v_{-s-1}-v_{-s-2})$$

$$\Longrightarrow\ (r+\eta)\,(v_{-s}-v_{-s-1}) \geq \pi_{-s}-\pi_{-s-1}+\eta\,(v_{-s-1}-v_{-s-2})\,. \tag{16}$$

If $v_{-s+1}-v_{-s} < (v_{-s}-v_{-s-1})$, then

$$\begin{aligned}(r+\eta)\,(v_{-s}-v_{-s-1}) &< \pi_{-s}-\pi_{-s-1}+\eta\,(v_{-s-1}-v_{-s-2})+\max\{\eta\,(v_{-s+1}-v_{-s}-c)\,,0\}-\max\{\eta\,(v_{-s}-v_{-s} \\ &\leq \pi_{-s}-\pi_{-s-1}+\eta\,(v_{-s-1}-v_{-s-2})\,.\end{aligned}$$

Now suppose $\eta_{-k-1}=0$ but $\eta_{-s'}=\eta$ for some $s' \in \{k+2, ..., n+1\}$. This implies

$$v_{-(k-1)}-v_{-k} \geq c > v_{-k}-v_{-k-1} < v_{-s'+1}-v_{-s'},$$

implying there must be at least one $s \in \{k+2, ..., n+1\}$ such that $v_{-s+1}-v_{-s} \geq v_{-s}-v_{-s-1} < v_{-s-1}-v_{s-2}$. Inequalities (16) and (17) implies

$$(r+\eta)\,(v_{-s}-v_{-s-1}) \geq \pi_{-s}-\pi_{-s-1}+\eta\,(v_{-s-1}-v_{-s-2})$$

$$(r+\eta)\left(v_{-s-1}-v_{-s-2}\right)<\pi_{-s-1}-\pi_{-s-2}+\eta\left(v_{-s-2}-v_{-s-3}\right)$$

First inequality and $v_{-s}-v_{-s-1}<v_{-s-1}-v_{-s-2}$ implies $r\left(v_{-s}-v_{-s-1}\right)>\pi_{-s}-\pi_{-s-1}$; convexity in follower's profit functions further implies $r\left(v_{-s}-v_{-s-1}\right)>\pi_{-s-1}-\pi_{-s-2}$. Hence it must be the case that $\left(v_{-s-2}-v_{s-3}\right)>\left(v_{-s-1}-v_{-s-2}\right)$. Applying inequality (17) again,

$$(r+\eta)\left(v_{-s-2}-v_{-s-3}\right)<\pi_{-s-2}-\pi_{-s-3}+\eta\left(v_{-s-3}-v_{-s-4}\right).$$

That $r\left(v_{-s-2}-v_{-s-3}\right)>\pi_{-s-2}-\pi_{-s-3}$ further implies $\left(v_{-s-3}-v_{-s-4}\right)>\left(v_{-s-2}-v_{-s-3}\right)$. By induction, we can show

$$v_{s-1}-v_{s-2}<v_{s-2}-v_{s-3}<\cdots<v_{-n}-v_{-(n+1)}.$$

But

$$(r+\eta+\kappa)\left(v_{-n}-v_{-(n+1)}\right)\leq\pi_{-n}-\pi_{-(n+1)}+\kappa\left(v_{s+1}-v_{s}\right)+\eta\left(v_{-n+1}-v_{-n+1}\right)$$

$$\Longrightarrow\ (r+\eta)\left(v_{-n}-v_{-(n+1)}\right)\leq\pi_{-n}-\pi_{-(n+1)}$$

which is a contradiction, given convexity of the profit functions. Hence, we have shown $v_{-k}-v_{-(k+1)}\geq v_{-s}-v_{-s-1}$ for all $s\in\{k+1,...,n+1\}$, establishing that follower cannot invest in these states.

Lemma 4: In a steady-state induced by investment cutoffs (n,k), the aggregate productivity growth rate is $g=\ln\lambda\cdot\left(\mu^{C}\left(\eta+\kappa\right)+\mu^{M}\kappa\right)$, where μ^{C} is the fraction of markets in the competitive region ($\mu^{C}=\sum_{s=1}^{k}\mu_{s}$) and μ^{M} is the fraction of markets in the monopolistic region ($\mu^{M}=\sum_{s=k+1}^{n+1}\mu_{s}$).

Proof. Given the cutoff strategies (n,k), aggregate productivity growth is (from Lemma 3)

$$g = \ln\lambda\cdot\left(\sum_{s=1}^{n}\mu_s\eta+2\mu_0\eta\right).$$

The steady-state distribution must follow

$$\mu_s\eta=\begin{cases}\mu_1\left(\eta+\kappa\right)/2 & \text{if } s=0\\ \mu_{s+1}\left(\eta+\kappa\right) & \text{if } 1\leq s\leq k-1\\ \mu_{s+1}\kappa & \text{if } k\leq s\leq n+1\\ 0 & \text{if } s>n+1\end{cases}$$

Hence we can rewrite the aggregate growth rate as

$$\begin{aligned} g &= \ln\lambda\cdot\left(2\mu_0\eta+\sum_{s=1}^{k-1}\mu_s\eta+\sum_{s=k-1}^{n}\mu_s\eta\right)\\ &= \ln\lambda\cdot\left(\mu_1\left(\eta+\kappa\right)+\sum_{s=2}^{k}\mu_s\left(\eta+\kappa\right)+\sum_{s=k}^{n+1}\mu_s\kappa\right)\\ &= \ln\lambda\cdot\left(\mu^C\left(\eta+\kappa\right)+\mu^M\kappa\right),\end{aligned}$$

as desired.

Lemma 5: If follower invests in state 1, then the steady-state aggregate productivity growth is bounded below by $\ln\lambda\cdot\kappa$, the step-size of productivity increments times the rate of technology diffusion.

Proof. Given $k \geq 1$, the fraction of markets in the competitive region can be written as

$$\begin{aligned}
\mu^C &= \sum_{s=1}^{k} \mu_s \\
&= \mu_1 + \underbrace{\mu_1 (1+\alpha)^{-1}}_{=\mu_2} + \cdots + \underbrace{\mu_1 (1+\alpha)^{-(k-1)}}_{=\mu_k} \\
&= \underbrace{\mu_0 \frac{\kappa+\eta}{2\eta}}_{=\mu_1} \frac{1-(1+\alpha)^{-k}}{1-(1+\alpha)^{-1}} \\
&\geq \mu_0 \frac{\kappa+\eta}{2\eta}
\end{aligned}$$

Aggregate growth rate can be re-written as

$$\begin{aligned}
g &= \ln\lambda \cdot \left[(1-\mu_0)\,\kappa + \mu^C \eta\right] \\
&\geq \ln\lambda \cdot \left[(1-\mu_0)\,\kappa + \mu_0 \frac{\kappa+\eta}{2}\right] \\
&\geq \ln\lambda \cdot \left[(1-\mu_0)\,\kappa + \mu_0 \kappa\right] \\
&= \ln\lambda \cdot \kappa,
\end{aligned}$$

as desired.

A.3 Asymptotic Results as $r \to 0$

Lemma A.1. $\Delta w_0 \equiv w_1 - w_0 = \frac{r w_0 + 2c\eta - 2\pi_0}{2\eta}$**;** Δw_0 **is bounded away from zero.**

Proof The equality from the HJB equation $r w_0 = 2\pi_0 - 2c\eta + 2\eta\,(w_1 - w_0)$. That Δw_0 is bounded away from zero follows from the fact that $r w_0 \geq 0$ and assumption 1 ($2c\eta > \pi \equiv \lim_{s\to 0} \pi_s + \pi_{-s} > 2\pi_0$). QED.

A.3.1 Mathematical Preliminaries

Consider the following recursive formulation of value functions:

$$ru_{s+1} = \lambda + p\left(u_s - u_{s+1}\right) + q\left(u_{s+2} - u_{s+1}\right)$$

The HJB equation states that, starting from state s, there's a Poisson rate p of moving up one state, and rate q of moving down; the flow payoff is λ and discount rate is r.

Fix a state s. Given u_s and $\Delta u_s \equiv u_{s+1} - u_s$, we can solve for all u_{s+t} with $t > 0$ as recursive functions of u_s and Δu_s; for instance,

$$u_{s+2} - u_{s+1} = \frac{ru_s - \lambda}{q} + \left(\frac{p+r}{q}\right)\Delta u_s,$$

$$u_{s+3} - u_{s+3} = \frac{ru_s - \lambda}{q} + \left(\frac{p+r}{q}\right)\left(u_{s+2} - u_{s+1}\right) + \frac{r\Delta u_s}{q},$$

and so on. The recursive formulation generically does not have a nice closed-form representation, as the number of terms quickly explodes as we expand out the recursion. However, as $r \to 0$, the value functions do emit asymptotic closed form expressions, as Proposition A.1 shows. In what follows, let $\sim$ denote asymptotic equivalence as $r \to 0$, i.e. $x \sim y$ iff $\lim_{r\to 0}\left(x - y\right) = 0$.

Proposition A.1. Let $\delta \equiv \frac{ru_s - \lambda}{q}$, $a \equiv p/q$, $b \equiv r/q$, then for all $t > 0$,

$$\begin{aligned}
u_{s+t} - u_s \quad \sim \quad & \left(\Delta u_s\right)\frac{1-a^t}{1-a} + \delta\frac{t - \frac{a-a^t}{1-a}}{1-a} \\
& + \Delta u_s \cdot b\frac{(t-1)\left(1+a^t\right)(1-a) - (2-a)\left(a^t - a\right)}{(1-a)^3} \\
& + \delta b\frac{1}{(1-a)^3}\left(\frac{(t-2)(t-1)}{2}(1-a) - (t-3)\,a^t - a\,(2-a)\,(t-1) + 2a\,(1-a)\right)
\end{aligned}$$

$$
\begin{aligned}
u_{s+t}-u_{s+t-1} \quad \sim \quad & \Delta u_s a^{t-1}+\delta\frac{1-a^{t-1}}{1-a} \\
& +\Delta u_s b\frac{\left((t-1)\left(1+a^t\right)-(t-2)\left(1+a^{t-1}\right)\right)(1-a)-(2-a)\left(a^t-a^{t-1}\right)}{(1-a)^3} \\
& +\delta b\frac{1}{(1-a)^3}\left(\frac{(t-2)(t-1)-(t-2)(t-3)}{2}(1-a)-(t-3)a^t+(t-4)a^{t-1}-a(2-a)\right)
\end{aligned}
$$

The following simplifications of the formulas will be useful if $\lim_{r\to 0} t \to \infty$:

1. when $a<1$:

$$u_{s+t}-u_{s+t-1}\sim \Delta u_s a^{t-1}+\frac{\delta}{1-a}+\frac{b\Delta u_s}{(1-a)^2};$$

   (a) if $r\Delta u_s \to 0$,

$$u_{s+t}-u_s \quad \sim \quad \Delta u_s\frac{1}{1-a}+\frac{t\delta}{1-a};$$

   (b) if $r\Delta u_s \not\to 0$,

$$r\left(u_{s+t}-u_s\right)\sim\frac{r\Delta u_s}{1-a}.$$

2. when $a>1$, $r\Delta u_s\to 0$, and $\Delta u_s+\frac{\delta}{a-1}\not\sim 0$,

$$r\left(u_{s+t}-u_s\right)\sim\left(\Delta u_s+\frac{\delta}{a-1}\right)\frac{ra^t}{a-1},$$

$$r\left(u_{s+t}-u_{s+t-1}\right)\sim\left(\Delta u_s+\frac{\delta}{a-1}\right)ra^{t-1}.$$

   If $\Delta u_s+\frac{\delta}{a-1}\sim 0$,

$$u_{s+t}-u_s\sim -\frac{b\delta}{(1-a)^4}\cdot a^{t+1}.$$

Suppose the flow payoffs are state-dependent $\{\lambda_s\}$, i.e.

$$ru_{s+1}=\lambda_{s+1}+p\left(u_s-u_{s+1}\right)+q\left(u_{s+2}-u_{s+1}\right)$$

If λ is an upper bound for $\{\lambda_s\}$, then the formulas provide asymptotic lower bounds for $u_{s+t}-u_{s+t-1}$ and u_{s+t} as functions of u_s and Δu_s. Conversely, if λ is a lower bound for $\{\lambda_s\}$, then the formulas provide asymptotic upper bounds for $u_{s+t} - u_{s+t-1}$ and u_{s+t}.

One can symmetrically characterize u_s and Δu_s as asymptotic functions of Δu_{s+t} and u_{s+t}.

Proof of Proposition A.1. The recursive formulation can be re-written as

$$u_{s+1} - u_s = \Delta u_s$$

$$\begin{aligned} u_{s+2} - u_{s+1} &= a\left(u_{s+1} - u_s\right) + \frac{r\left(u_{s+1} - u_s\right) + ru_s - v}{q} \\ &= a\Delta u_s + b\Delta u_s + \delta \end{aligned}$$

$$u_{s+2} - u_s = (1+a)\,\Delta u_s + b\Delta u_s + \delta$$

Likewise,

$$u_{s+3} - u_{s+2} = a^2\Delta u_s + (1+2a)\,b\Delta u_s + (1+a)\,\delta + o\left(r^2\right)$$

$$u_{s+3} - u_s = \left(1+a+a^2\right)\Delta u_s + (1+1+2a)\,b\Delta u_s + (1+1+a)\,\delta + b\delta + o\left(r^2\right)$$

Applying the formula iteratively, one can show that

$$u_{s+t+1} - u_{s+t} = a^t\Delta u_s + \delta\sum_{z=0}^{t-1} a^z + b\Delta u_s \sum_{z=1}^{t} za^{z-1} + b\delta\sum_{z=1}^{t-1}\sum_{m=1}^{z} ma^{m-1} + o\left(r^2\right)$$

$$u_{s+t+1} - u_s = \Delta u_s \sum_{z=0}^{t} a^z + \delta\sum_{z=0}^{t}\sum_{m=0}^{z-1} a^m + b\Delta u_s \sum_{z=1}^{t}\sum_{m=1}^{z} ma^{m-1} + b\delta\sum_{x=1}^{t-1}\sum_{z=1}^{x}\sum_{m=1}^{z} ma^{m-1} + o\left(r^2\right)$$

One obtains the Lemma by applying the following formulas for power series summation:

$$\sum_{z=0}^{t} a^z = \frac{1-a^{t+1}}{1-a}$$

$$\sum_{z=0}^{t}\sum_{m=0}^{z-1} a^m = \frac{t+1-\frac{a-a^{t+1}}{1-a}}{1-a}$$

$$\sum_{z=1}^{t}\sum_{m=1}^{z} ma^{m-1} \quad = \quad \frac{t\left(1+a^{t+1}\right)(1-a)-(2-a)\left(a^{t+1}-a\right)}{(1-a)^3}$$

$$\sum_{x=1}^{t-1}\sum_{z=1}^{x}\sum_{m=1}^{z} ma^{m-1} \quad = \quad \frac{1}{(1-a)^3}\left(\frac{t(t-1)}{2}(1-a)-(t-2)\,a^{t+1}-a(2-a)\,t+2a(1-a)\right).$$

The third, and fourth summations formulas follow because

$$\begin{aligned}
\sum_{m=1}^{z} ma^{m-1} &= \left(1+2a+3a^3+\cdots+za^{z-1}\right)\\
&= \left(\frac{1-a^z}{1-a}+a\frac{1-a^{z-1}}{1-a}+\cdots+a^{z-1}\frac{1-a}{1-a}\right)\\
&= \left(\frac{1+a+\cdots+a^{z-1}}{1-a}-\frac{za^z}{1-a}\right)\\
&= \left(\frac{1-a^z}{(1-a)^2}-\frac{za^z}{1-a}\right)
\end{aligned}$$

$$\begin{aligned}
\sum_{z=1}^{s}\sum_{m=1}^{z} ma^{m-1} &= \sum_{z=1}^{s}\left(\frac{1-a^z}{(1-a)^2}-\frac{za^z}{1-a}\right)\\
&= \frac{s}{(1-a)^2}-\frac{a-a^{s+1}}{(1-a)^2}-\frac{a}{1-a}\sum_{z=1}^{s} za^{z-1}\\
&= \frac{s}{(1-a)^2}-\frac{a-a^{s+1}}{(1-a)^2}-\frac{a}{1-a}\left(\frac{1-a^s}{(1-a)^2}-\frac{sa^s}{1-a}\right)\\
&= \frac{s(1-a)}{(1-a)^3}-\frac{a(1-a)-(1-a)\,a^{s+1}}{(1-a)^3}-\frac{a-a^{s+1}}{(1-a)^3}+\frac{sa^{s+1}(1-a)}{(1-a)^3}\\
&= \frac{s\left(1+a^{s+1}\right)(1-a)-(2-a)\left(a^{s+1}-a\right)}{(1-a)^3}
\end{aligned}$$

$$
\begin{aligned}
\sum_{x=1}^{s-1}\sum_{z=1}^{x}\sum_{m=1}^{z} m a^{m-1} &= \sum_{x=1}^{s-1}\frac{x\left(1+a^{x+1}\right)(1-a)-(2-a)\left(a^{x+1}-a\right)}{(1-a)^3}\\
&= \frac{1}{(1-a)^3}\left(\sum_{x=1}^{s-1} x(1-a)+xa^{x+1}(1-a)-(2-a)\left(a^{x+1}-a\right)\right)\\
&= \frac{1}{(1-a)^3}\left(\frac{s(s-1)}{2}(1-a)+a^2(1-a)\underbrace{\sum_{x=1}^{s-1} xa^{x-1}}_{=\frac{1-a^{s-1}}{(1-a)^2}-\frac{(s-1)a^{s-1}}{1-a}}\right.\\
&\qquad \left. -a(2-a)(s-1)-(2-a)a^2\frac{1-a^{s-1}}{1-a}\right)\\
&= \frac{1}{(1-a)^3}\left(\frac{s(s-1)}{2}(1-a)-(s-2)a^{s+1}-a(2-a)s+2a(1-a)\right)
\end{aligned}
$$

A.3.2 Proofs of Lemma 6: $\lim_{r\to 0} k = \lim_{r\to 0}(n-k) = \infty$

Recall n and k are the last states in which the leader and the follower, respectively, chooses to invest in an equilibrium. Both n and k are functions of the interest rate r. Also recall that we use $w_s \equiv v_s + v_{-s}$ to denote the total firm value of a market in state s.

We first prove $\lim_{r\to 0}(n-k) = \infty$**.**

Suppose k and $(n-k)$ are both bounded as $r \to 0$; let N be an upper bound for n, i.e. $N \geq n(r)$ for all r.

Consider the sequence of value functions $\hat{v}_s$ under alternative investment decisions: leader follows equilibrium strategies and invests in $n(r)$ states whereas follower does not invest at all. The sequence of value function dominates the equilibrium value functions ($\hat{v}_s \geq v_s$) for all $s \geq 0$, because:

1. The joint value is higher in every state $\hat{w}_s \geq w_s$, because flow payoffs are weakly higher and that the value functions are placing higher weights on higher states (which have higher flow payoffs). Hence the firm value in state zero is higher $\hat{v}_0 \geq v_0$.
2. The leader's value function can be written as a weighted average of flow payoffs in $s > 0$ and the value of being in state zero; the flow payoffs are the same for all $s > 0$, and $\hat{v}_0 \geq v_0$.

Furthermore when follower does not invest, the leader's value function always places higher weights in states with higher payoffs; hence $\hat{v}_s \geq v_s$ for all $s > 0$.

We now look for a contradiction. As $r \to 0$,

$$r\hat{v}_{N+1} = \frac{r\pi_{N+1} + \kappa r\hat{v}_N}{r+\kappa} \to r\hat{v}_N,$$

$$r\hat{v}_N = \frac{r\left(\pi_N - c\eta_N\right) + \kappa r\hat{v}_{N-1} + \eta_N r\hat{v}_{N+1}}{r+\kappa+\eta_N} \to r\hat{v}_{N-1},$$

and so on. By induction, $r\hat{v}_s \sim r\hat{v}_0$ for all $-N+1 \leq s \leq N+1$.

Also note that leader stops investing in state $n+1$ implies

$$\lim_{r\to 0} rv_{n+1} \geq \lim_{r\to 0} \pi_{n+2} - c\kappa,$$

thus $\lim_{r\to 0} r\hat{v}_0 \geq \lim_{r\to 0} \pi_{n+2} - c\kappa$.

Lastly, note $\Delta\hat{v}_0 \geq \Delta\hat{w}_0 = \frac{r\hat{w}_0 - (2\pi_0 - 2c\eta)}{2\eta} = \frac{r\hat{v}_0 - (\pi_0 - c\eta)}{\eta}$.

Putting these pieces together, we apply Proposition A.1 to compute a lower bound for $\Delta\hat{v}_n$ as a function of $\hat{v}_0$ and $\Delta\hat{v}_0$ (substituting $u_s = \hat{v}_0$, $u_{s+t} = \hat{v}_{n+1}$, $a = \kappa/\eta$, $b = r/\eta$, $\delta = \frac{r\hat{v}_0 - (\pi_{n+2} - c\eta)}{\eta}$):

$$\begin{aligned}
\lim_{r\to 0} \Delta\hat{v}_{n+1} &\geq \lim_{r\to 0} \left(\Delta\hat{v}_0 \left(\kappa/\eta\right)^n + \frac{r\hat{v}_0 - (\pi_{n+2} - c\eta)}{\eta} \frac{1 - (\kappa/\eta)^n}{1 - \kappa/\eta} \right) \\
&\geq \lim_{r\to 0} \frac{r\hat{v}_0 - (\pi_0 - c\eta)}{\eta} \left(\kappa/\eta\right)^n + \frac{r\hat{v}_0 - (\pi_{n+2} - c\eta)}{\eta} \frac{1 - (\kappa/\eta)^n}{1 - \kappa/\eta} \\
&\geq \lim_{r\to 0} \frac{\pi_{n+2} - c\kappa - (\pi_0 - c\eta)}{\eta} \left(\kappa/\eta\right)^n + \frac{\pi_{n+2} - c\kappa - (\pi_{n+2} - c\eta)}{\eta} \frac{1 - (\kappa/\eta)^n}{1 - \kappa/\eta} \\
&> \lim_{r\to 0} c \left(\kappa/\eta\right)^n + \frac{c\left(\eta - \kappa\right)}{\eta} \frac{1 - (\kappa/\eta)^n}{1 - \kappa/\eta} \\
&= c,
\end{aligned}$$

where the last inequality follows from assumption 1, that $\pi_{n+2} - \pi_0 \geq \pi_1 - \pi_0 > c\kappa$. But this is a contradiction to the claim that leader stops investing in state $n+1$ (i.e. $\Delta\hat{v}_{n+1} \leq c$ for any r).

Next, suppose $\lim_{r\to 0} k = \infty$ but $(n-k)$ remain bounded. Let $\epsilon \equiv 2c\eta - \lim_{s\to\infty} (\pi_s + \pi_{-s})$; $\epsilon > 0$ under assumption 1. The joint flow payoff $\pi_s + \pi_{-s} - 2c\eta$ is negative and bounded above by $-\epsilon$ in all states $s \leq k$. The joint market value in state 0 is

$$\begin{aligned} w_0 &= \sum_{s'=0}^{k} \tilde{\lambda}_{s'|0} \cdot (PV_{s'} + PV_{-s'}) + \sum_{s'=k+1}^{n+1} \tilde{\lambda}_{s'|0} \cdot (PV_{s'} + PV_{-s'}) \\ &\leq \frac{-\epsilon}{r} \cdot \left(\sum_{s'=0}^{k} \tilde{\lambda}_{s'|0} \right) + \sum_{s'=k+1}^{n+1} \tilde{\lambda}_{s'|0} \cdot (PV_{s'} + PV_{-s'}) . \end{aligned}$$

As $k \to \infty$ while $n-k$ remain bounded, the present-discount fraction of time that the market spends in states $s \leq k$ converges to 1 ($\sum_{s'=0}^{k} \tilde{\lambda}_{s'|0} \to 1$), implying that $\lim_{r \to 0} r w_0$ is negative. Since firms can always ensure non-negative payoffs by not taking any investment, this cannot be an equilibrium, reaching a contradiction. Hence $\lim_{r\to 0} (n-k) = \infty$.

To show $\lim_{r \to 0} k = \infty$, we first establish a few additional asymptotic properties of the model.

Lemma A.2. The following statements are true:

1. $r v_n \sim \pi - c\kappa$, where $\pi \equiv \lim_{s \to \infty} \pi_s$.
2. $v_{n+1} - v_n \sim c$.
3. $r(n-k) \sim 0$.
4. $rk \sim 0$.

Proof

1. The claim follows from the fact that if firm invests in state n but not in state $n+1$, then

$$v_{n+2} - v_{n+1} = \frac{\pi_{n+2} - r v_{n+1}}{r + \kappa} \leq c$$

$$v_{n+1} - v_n = \frac{\pi_{n+1} - r v_n}{r + \kappa} \geq c$$

implying

$$\pi - c\kappa = \lim_{r \to 0} (\pi_{n+2} - c\kappa) \geq \lim_{r \to 0} r v_n \geq \lim_{r \to 0} (\pi_{n+1} - c\kappa) = \pi - c\kappa,$$

as desired.

2. This claim follows from the previous one: $v_{n+1} - v_n = \frac{\pi_{n+1}}{r+\kappa} - \frac{rv_n}{r+\kappa} \sim \frac{\pi - rv_n}{\kappa} \sim c$.

3. The previous claims show $rv_n \sim \pi - c\kappa$ and $\Delta v_n \sim c$. We apply Proposition A.1 to iterate backwards and obtain

$$\begin{aligned} \lim_{r\to\infty} r\left(v_k - v_n\right) & \geq \lim_{r\to\infty} -\frac{r^2}{\kappa^2} \frac{rv_n - (\pi - c\eta)}{\left(1 - \eta/\kappa\right)^4} \left(\eta/\kappa\right)^{n-k+1} \\ & \sim -\frac{r^2}{\kappa^2} \frac{c\left(\eta - \kappa\right)}{\left(1 - \eta/\kappa\right)^4} \left(\eta/\kappa\right)^{n-k+1} \end{aligned}$$

Since $\left|\lim_{r\to 0} r\left(v_k - v_n\right)\right| \leq \pi$, it must be the case that $\lim_{r\to 0} r^2 \left(\eta/\kappa\right)^{n-k+1}$ remain bounded; therefore $r\left(n-k\right) \sim 0$.

4. We apply Proposition A.1 to find a lower bound for $w_k - w_0$:

$$\begin{aligned} \lim_{r\to 0} r\left(w_k - w_0\right) & \geq \lim_{r\to 0} \left(\Delta w_0 + \frac{rw_0 - (\pi - 2c\eta)}{a-1}\right) \frac{ra^k}{a-1} \\ & \geq \lim_{r\to 0} \left(\frac{2c\eta - \pi}{a-1}\right) \frac{ra^k}{a-1}. \end{aligned}$$

Since $r\left(w_k - w_0\right)$ stays bounded, it must be the case that ra^k is bounded; therefore $rk \sim 0$.

Lemma A.3. $rv_{-k} \sim r\Delta v_{-k} \sim rv_{-n} \sim \Delta v_{-n} \sim 0$**.**

Proof. First, note that follower does not invest in state $k+1$ implies $c \geq \Delta v_{-(k+1)}$. We apply Proposition A.1 to find an upper bound for $\left(v_{-n} - v_{-k}\right)$ as a function of rv_{-k} and $\Delta v_{-(k+1)}$:

$$v_{-n} - v_{-k} \leq \lim_{r\to 0} \left(-\Delta v_{-(k+1)} \frac{\eta}{\eta - \kappa} + (n-k) \frac{rv_{-k}}{\eta - \kappa}\right).$$

Hence, $v_{-n} - v_{-k} \leq -c\frac{\eta}{\eta-\kappa}$ and $r\left(v_{-n} - v_{-k}\right) \sim 0$.

Let $m = \text{floor}(k + \frac{n-k}{2})$. That the follower does not invest in state m implies that $c \geq \Delta v_{-m}$.

Proposition A.1. provides a lower bound for $v_{-n} - v_{-(n-1)}$ as a function of rv_{-m} and $\Delta v_{-(m+1)}$:

$$\begin{aligned}\lim_{r\to 0}\left(v_{-(n+1)} - v_{-n}\right) &\geq \lim_{r\to 0} -\Delta v_{-(m+1)}\left(\kappa/\eta\right)^{n-m} + \frac{rv_{-m} - \pi_{-m}}{\eta-\kappa}\\ &= \lim_{r\to 0}\frac{rv_{-m}}{\eta-\kappa},\end{aligned}$$

where the equality follows from $\lim_{m\to\infty}\pi_{-m} \to 0$. Hence, since the LHS is non-positive, it must be the case that $\lim_{r\to 0}\Delta v_{-n} = \lim_{r\to 0} rv_{-m} = 0$. But since $rv_{-n} \leq rv_{-m}$, it must be that $rv_{-n} \sim rv_{-k} \sim 0$. That $r\Delta v_{-k} \sim 0$ follows directly from the HJB equation for state k.

We now prove $\lim_{r\to 0} k = \infty$.

We first show that, if k is bounded, both rw_k and $r\Delta w_k$ must be asymptotically zero in order to be consistent with $rv_{-k} \sim 0$. Specifically, we use the fact that $0 \leq \pi_{-s}$ for all $0 \leq s \leq k$ and apply Proposition A.1 (simplification 1a, substituting $u_s \equiv v_{-k+1}$, $u_{s+t} = v_0$, $t = k+1$, $\Delta u_s = \Delta v_{-k}$, $a = \frac{\eta}{\eta+\kappa}$, $b = \frac{r}{\eta+\kappa}$, $\delta = \frac{rv_{-(k+1)} - (-c\eta)}{\eta+\kappa}$) to find an asymptotic upper bound for rv_0:

$$\begin{aligned}\lim_{r\to 0} rv_0 &= \lim_{r\to 0} r\left(v_0 - v_{-(k+1)}\right)\\ &\leq \lim_{r\to 0}\frac{r}{1-\kappa/\eta}\left(\Delta v_{-(k+1)} + k\frac{rv_{-(k+1)} + c\eta}{\eta}\right)\end{aligned}$$

If k is bounded, the last expression converges to zero, implying that $rv_0 \sim rw_0 \sim 0$. Lemma A.1 further implies that $\Delta w_0 \sim c$. Upper-bounds for rw_k and $r\Delta w_k$ can be found, as functions of Δw_0 and rw_0, using Proposition A.1 (simplification 2, substituting $u_s \equiv w_0$, $u_{s+t} = w_k$, $t = k$, $\Delta u_s = \Delta w_0$, $a = \frac{\eta+\kappa}{\eta}$, $b = \frac{r}{\eta}$, $\delta = \frac{rw_0 - (-2c\eta)}{\eta}$):

$$\lim_{r\to 0}\left(rw_k - rw_0\right) \leq \lim_{r\to 0}\left(\Delta w_0 + \frac{rw_0 + 2c\eta}{\kappa}\right)\frac{\eta}{\kappa} r\left(\frac{\eta+\kappa}{\eta}\right)^k$$

$$\lim_{r\to 0}\left(r\Delta w_k\right) \leq \lim_{r\to 0}\left(\Delta w_0 + \frac{rw_0 + 2c\eta}{\kappa}\right) r\left(\frac{\eta+\kappa}{\eta}\right)^{k-1}$$

If k is bounded, the RHS of both inequalities converge to zero, implying $rw_k \sim r\Delta w_k \sim 0$.

We now look for a contradiction. Suppose $r\Delta w_k \sim 0$; we apply Proposition A.1 (simplification

1a, substituting $u_s \equiv w_k$, $u_{s+t} = w_{n+1}$, $t = n+1-k$, $\Delta u_s = \Delta w_k$, $a = \frac{\kappa}{\eta}$, $b = \frac{r}{\eta}$, $\delta = \frac{rw_k-(\pi_k-c\eta)}{\eta}$) and obtain $\frac{rw_k-(\pi_k-c\eta)}{\eta-\kappa}$ as an asymptotic upper bound for $w_{n+1} - w_n$ (noting that π_k is a lower bound for π_s for all $n \geq s \geq k$). Lemma A.2 part 2 further implies that

$$\lim_{r \to 0} \frac{rw_k - (\pi_k - c\eta)}{\eta - \kappa} \geq c$$

$$\iff \lim_{r \to 0} rw_k \geq \pi - c\kappa > 0. \tag{18}$$

The last inequality follows from assumption 1 ($\pi_1 - \pi_0 \geq c\kappa$), that firms is state 0 has incentive to invest when sufficiently patient. QED.

A.3.3 Proof of Proposition 1.

The steady-state distribution follows:

$$\mu_s \eta = \begin{cases} \mu_1 (\eta + \kappa)/2 & \text{if } s = 0 \\ \mu_{s+1} (\eta + \kappa) & \text{if } 1 \leq s \leq k-1 \\ \mu_{s+1} \kappa & \text{if } k \leq s \leq n+1 \\ 0 & \text{if } s > n+1 \end{cases}$$

We can rewrite μ_s as a function of μ_{n+1} for all s. Let $\alpha \equiv \kappa/\eta$, then

$$\mu_s = \begin{cases} \mu_{n+1} \alpha^{n+1-s} & \text{if } n+1 \geq s \geq k \\ \mu_{n+1} \alpha^{n+1-k} (1+\alpha)^{k-s} & \text{if } k-1 \geq s \geq 0 \end{cases}$$

The fraction of markets in the competitive and monopolistic regions can be written, respectively, as

$$\mu^M = \mu_{n+1} \sum_{s=k+1}^{n+1} \alpha^{n+1-s} = \mu_{n+1} \frac{1-\alpha^{n-k+1}}{1-\alpha}$$

$$\mu^C = \mu_{n+1} \alpha^{n+1-k} \sum_{s=1}^{k} (1+\alpha)^{k-s} = \mu_{n+1} \alpha^{n-k} \left((1+\alpha)^k - 1 \right)$$

Also note that $\mu_0 = \mu_{n+1}\alpha^{n+1-k}(1+\alpha)^k$ and $1 = \mu_0 + \mu^C + \mu^M$. We now show $\lim_{r\to 0}\alpha^{n-k}(1+\alpha)^k = 0$, which is a sufficient condition for $\mu^M \to 1$, $\mu^C \to 0$, and $g \to \kappa \cdot \ln\lambda$.

To proceed, we first find a lower bound for Δw_k by applying simplification 2 of Proposition A.1 (substituting $u_s \equiv w_0$, $u_{s+t} = w_k$, $t = k$, $\Delta u_s = \Delta w_0$, $a = \frac{\eta+\kappa}{\eta}$, $b = \frac{r}{\eta}$, $\delta = \frac{rw_0-(\pi-2c\eta)}{\eta}$):

$$\lim_{r\to 0} r\Delta w_k \geq C_2 \equiv \lim_{r\to 0}\left(\Delta w_0 + \frac{rw_0-(\pi-2c\eta)}{\kappa}\right) r\left(\frac{\eta+\kappa}{\eta}\right)^k. \tag{19}$$

Next, we apply simplification 1b of Proposition A.1 to obtain (substituting $u_s \equiv w_k$, $u_{s+t} = w_n$, $t = n-k$, $\Delta u_s = \Delta w_k$, $a = \frac{\kappa}{\eta}$, $b = \frac{r}{\eta}$; we show below that $\lim_{r\to 0} r\Delta w_k > 0$ hence this simplification applies):

$$\begin{aligned} r(w_n - w_k) &\sim \frac{r\Delta w_k}{(\eta-\kappa)/\eta} \\ \iff \pi - c\kappa - rw_k &\sim \frac{r\Delta w_k}{(\eta-\kappa)/\eta} \end{aligned} \tag{20}$$

Simplification 1 of Proposition A.1 also provides asymptotic bounds for Δw_n (substituting $u_s \equiv w_k$, $u_{s+t} = w_n$, $t = n-k$, $\Delta u_s = \Delta w_k$, $a = \frac{\kappa}{\eta}$, $b = \frac{r}{\eta}$; the upper bound is obtained using $\delta = \frac{rw_k-(\pi_k-c\eta)}{\eta}$ and the lower bound is obtained using $\delta = \frac{rw_k-(\pi-c\eta)}{\eta}$):

$$\lim_{r\to 0}\left[\Delta w_k\left((\kappa/\eta)^{n-k} + \frac{r\eta}{(\eta-\kappa)^2}\right) + \frac{rw_k + c\eta - \pi_k}{\eta-\kappa}\right] \geq \lim_{r\to 0}\Delta w_n$$

and

$$\lim_{r\to 0}\Delta w_n \geq \lim_{r\to 0}\left[\Delta w_k\left((\kappa/\eta)^{n-k} + \frac{r\eta}{(\eta-\kappa)^2}\right) + \frac{rw_k + c\eta - \pi}{\eta-\kappa}\right].$$

Since $\lim_{r\to 0}\pi_k = \pi$, the lower and upper bounds coincide asymptotically. Furthermore, Lemma A.2 shows $\Delta w_n \sim c$; hence,

$$c \sim \Delta w_k\left((\kappa/\eta)^{n-k} + \frac{r\eta}{(\eta-\kappa)^2}\right) + \frac{rw_k + c\eta - \pi}{\eta-\kappa}$$

Applying asymptotic equivalence (20), we obtain

$$c \quad \sim \quad c + \Delta w_k \left((\kappa/\eta)^{n-k} + \frac{r\eta}{(\eta-\kappa)^2} \right) - \frac{r\eta \Delta w_k}{(\eta-\kappa)^2}$$

$$\iff \; 0 \sim \Delta w_k \, (\kappa/\eta)^{n-k}$$

Inequality (19) implies

$$0 \geq \lim_{r \to 0} \left(\Delta w_0 + \frac{r w_0 - (\pi - 2c\eta)}{\kappa} \right) \left(\frac{\eta + \kappa}{\eta} \right)^k (\kappa/\eta)^{n-k}$$

Given $\Delta w_0 \geq 0$, $r w_0 \geq 0$, and $2c\eta - \pi > 0$, the inequality can hold if and only if

$$\lim_{r \to 0} \left(\frac{\eta + \kappa}{\eta} \right)^k (\kappa/\eta)^{n-k} = 0,$$

as desired.

B Appendix: Additional Tables and Figures

Figure A1: Widening productivity gap between leaders and followers

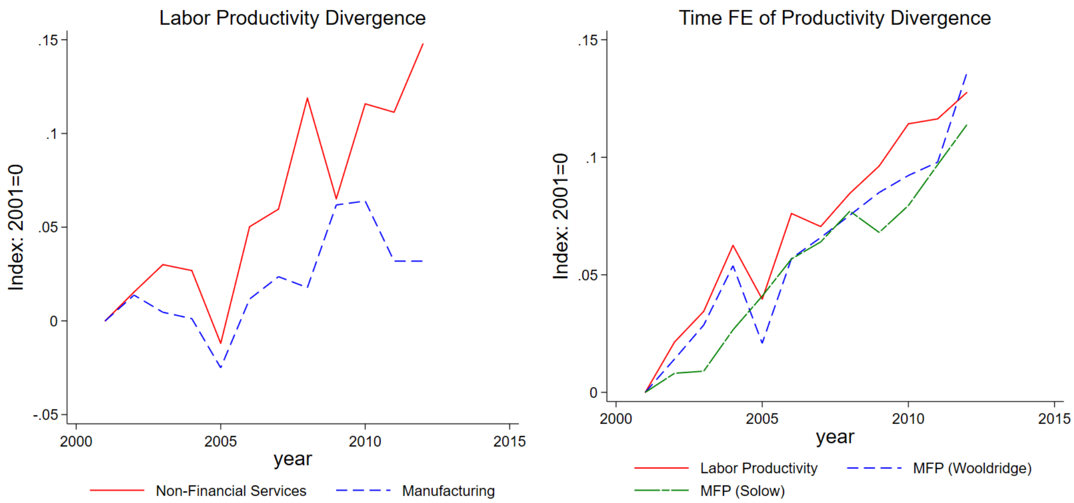


Figure A2: Top 5% average wage-employment ratio and interest rates

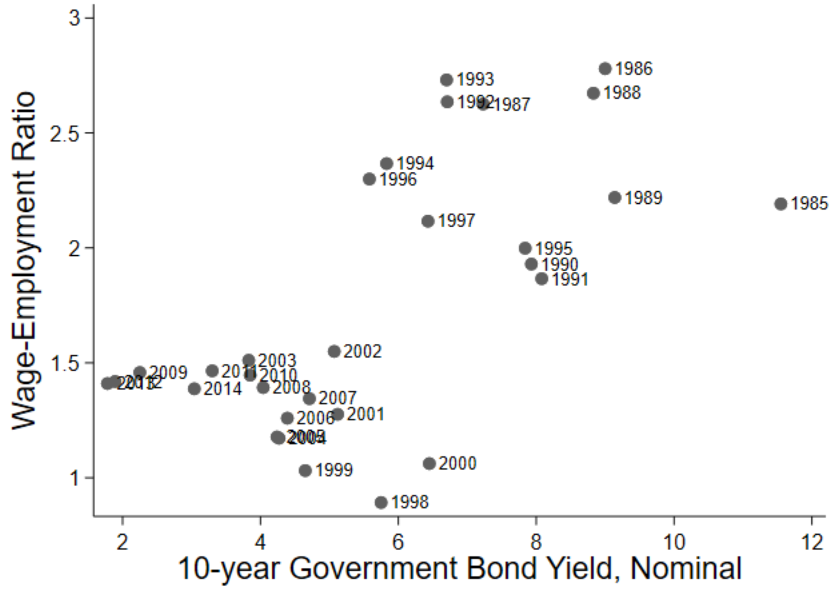
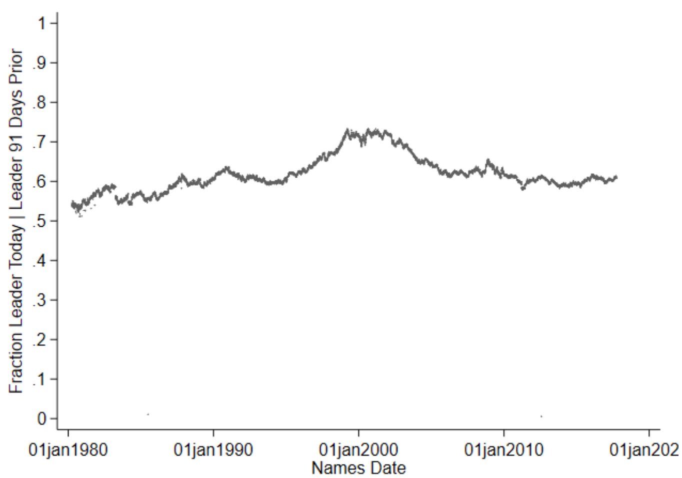


Figure A3: Fraction of Leaders who were Leaders a Quarter Prior



The panels plots the time series of the proportion of firms who are leaders today, given they were leaders 91 days prior.

Table A1: Differential Interest Rate Responses of Leaders vs. Followers: Robustness Checks

	Top 5		SIC		EBITDA		SALES	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Top 5 Percent=1 x Δi	-1.106***	-3.847**	-1.204***	-3.903***	-1.501***	-4.805***	-1.205**	-3.684**
	(0.273)	(1.220)	(0.222)	(0.936)	(0.287)	(1.077)	(0.350)	(1.325)
Top 5 Percent=1 x Δi x Lagged i		0.303**		0.293***		0.372***		0.277*
		(0.105)		(0.081)		(0.092)		(0.112)
Sample	All	All	All	All	All	All	All	All
Industry-Date FE	Y	Y	Y	Y	Y	Y	Y	Y
N	61,313,604	61,313,604	61,277,070	61,277,070	38,957,740	38,957,740	48,247,714	48,247,714
R-sq	0.403	0.403	0.403	0.404	0.427	0.428	0.411	0.412

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Regression results for the specification $\Delta \ln\left(P_{i,j,t}\right) = \alpha_{j,t} + \beta_0 D_{i,j,t} + \beta_1 D_{i,j,t}\Delta i_t + \beta_2 D_{i,j,t} i_{t-1} + \beta_3 D_{i,j,t}\Delta i_t i_{t-1} + X_{i,j,t}\gamma + \varepsilon_{i,j,t}$ for firm i in industry j at date t. The definitions are the same as in Table 2 except for $D_{i,j,t}$. In columns 1 and 2, leaders are chosen by the top 5 number of firms by market capitalization within an industry and date. In columns 3 and 4, leaders are chosen by the top 5% of firms by market capitalization within an industry and date, where we change the definition of industry to be the 2-digit Standard Industry Classification (SIC) codes. In columns 5 and 6, leaders are chosen by the top 5% of firms by earnings before interest, taxes, depreciation, and amortization (EBITDA) within an industry and date. In columns 7 and 8, leaders are chosen by the top 5% of firms by sales within an industry and date. Standard errors are dually clustered by industry and date.

Table A2: Portfolio Returns Response to Interest Rate Changes: Top 5 Percent, Different Frequencies

	Yearly		Semi-Yearly		Monthly		Weekly		Daily	
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
Δi_t	-1.061**	-5.570***	-1.188***	-4.594***	-1.000***	-2.365***	-0.964***	-1.846***	-0.839***	-1.244***
	(0.403)	(1.134)	(0.345)	(0.764)	(0.196)	(0.463)	(0.171)	(0.309)	(0.170)	(0.208)
i_{t-1}		0.381**		0.149		0.0273		0.00928		0.00327**
		(0.134)		(0.080)		(0.019)		(0.005)		(0.001)
$\Delta i_t \times i_{t-1}$		0.493***		0.385***		0.150***		0.0984**		0.0470
		(0.106)		(0.073)		(0.040)		(0.035)		(0.027)
Sample	All	All	All	All	All	All	All	All	All	All
N	9,037	9,037	8,962	8,962	9,081	9,081	9,099	9,099	9,080	9,080
R-sq	0.024	0.095	0.040	0.101	0.036	0.050	0.032	0.039	0.019	0.020

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Regression results for the specification $R_t = \alpha + \beta_0 i_{t-1} + \beta_1 \Delta i_t + \beta_2 \Delta i_t i_{t-1} + \varepsilon_t$ at date t. R_t is defined as the market-capitalization weighted average of returns for a stock portfolio that goes long in leader stocks and goes short in follower stocks from date $t-91$ to t. Leaders are defined as the firms in the top 5% of market capitalization in its FF industry on date $t-J$. i_t is defined as the nominal 10-year Treasury yield, with i_{t-1} being the interest rate J days prior and Δi_t being the change in the interest rate from date $t-91$ to t. For columns 1 and 2, $J = 364$; columns 3 and 4, $J = 28$; columns 5 and 6, $J = 7$; columns 7 and 8, $J = 1$, where 1 is one trading day. Standard errors are Newey-West with a maximum lag length of 60 days prior.

Table A3: Correlation Table of Forward Rates

Variables	0-2	2-3	3-5	5-7	7-10	10-30
0-2	1.00					
2-3	0.88	1.00				
3-5	0.82	0.93	1.00			
5-7	0.71	0.84	0.88	1.00		
7-10	0.70	0.82	0.83	0.86	1.00	
10-30	0.75	0.84	0.89	0.92	0.93	1.00

Correlation table of forward rates. P-values in parentheses.