Persuasion Meets Delegation

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Principal – Agent Problem

- A principal wants to influence the decision of a biased agent

- Two instruments of influence
  - Delegation
  - Persuasion

- How are they related?

Preview of Main Result

- Two problems are equivalent under general assumptions
  - Explicit equivalence mapping between the two problems
  - Decisions and states are swapped in the two problems

Who Cares?

- Results in one problem to solve the other problem
- Reinterpretations of insights in the two problems
- Stepping stone for relations in other problems and extensions

Outline

- Persuasion and delegation problems
- Equivalence result
- Sketch of proof
- Application to monopoly regulation

A Problem

- Principal (she) and Agent (he)

- Agent makes a decision $x \in [0, 1]$

- State $\theta \in [0, 1]$ is uniformly distributed

Payoffs

- Agent's and Principal's payoffs are $U(\theta, x)$ and $V(\theta, x)$

- $\frac{\partial}{\partial x}U(\theta, x)$ and $\frac{\partial}{\partial x}V(\theta, x)$ are continuous in θ and x

- $\frac{\partial}{\partial x}U(\theta, x)$ is strictly increasing in θ and strictly decreasing in x

- A pair (U, V) is called a *primitive*

- $\mathcal{P}$ is the set of primitives that satisfy the above assumptions

Monotone Persuasion Problem

- Principal chooses a monotone partition Π of $[0, 1]$

- Π contains singletons $\{\theta\}$ and intervals (θ', θ'')
  - W.l.o.g., intervals are open
  - Π is fully identified by a set of singletons

- Π is a closed subset of $[0, 1]$ that contains 0 and 1

- Denote by $\mathbf{\Pi}$ the set of all such Π

Why Monotone Persuasion?

- Monotone partitions are widespread:
  - Credit ratings of financial institutions
  - Consumer ratings of services on Amazon, Yelp,...
  - Grade conversion schemes from 100-point to ABC scale

- Conditions for optimality of monotone partitions:
  Dworczak-Martini (2018), Kolotilin (2018)

- Characterization of optimal monotone partitions:
  Kolotilin and Li (2018)

Monotone Persuasion Problem

- Denote by $\mu_{\Pi}(\theta)$ the partition element that contains θ
  - Interpret $\mu_{\Pi}(\theta)$ as a message sent at state θ

- After observing $\mu_{\Pi}(\theta)$, Agent chooses

$$x_P^*(\theta, \Pi) \in \arg\max_{x \in [0,1]} \mathbb{E}[U_P(s, x) \mid s \in \mu_{\Pi}(\theta)]$$

- Principal's problem:

$$\max_{\Pi \in \mathbf{\Pi}} \mathbb{E}[V_P(\theta, x_P^*(\theta, \Pi))]$$

Balanced Delegation Problem

- Principal chooses a closed subset $\Pi \subset [0, 1]$ of decisions such that Π contains extreme decisions $\{0, 1\}$

- Denote by $\mathbf{\Pi}$ the set of all such delegation sets

- After privately observing θ, Agent chooses

$$x_D^*(\theta, \Pi) \in \arg\max_{x \in \Pi} U_D(\theta, x)$$

- Principal's problem

$$\max_{\Pi \in \mathbf{\Pi}} \mathbb{E}[V_D(\theta, x_D^*(\theta, \Pi))]$$

Why Balanced Delegation?

- A *balanced delegation problem* is a delegation problem with extra boundary conditions, which includes:
  - Standard delegation problems under general assumptions
  - Novel delegation problems with participation constraints

Including Standard Delegation

- $x \in \mathbb{R}$ and $\theta \in [0,1]$

- $U_D(\theta, x) \to -\infty$ and $V_D(\theta, x) \to -\infty$ as $x \to \pm\infty$

- *Lemma*: There exist $\underline{x} \in \mathbb{R}$ and $\overline{x} \in \mathbb{R}$ such that the delegation problem is the same if the principal chooses

  1. $\Pi \subset \mathbb{R}$,

  2. $\Pi \subset [\underline{x}, \overline{x}]$,

  3. $\Pi \subset [\underline{x}, \overline{x}]$ subject to $\{\underline{x}, \overline{x}\} \in \Pi$.

Persuasion versus Delegation

What is the difference between these problems?

Main Result

The monotone persuasion problem and the balanced delegation problem are "equivalent".

Definition

Primitives (U_P, V_P) and (U_D, V_D) are equivalent if $\exists C$ such that

$$\mathbb{E}\Big[V_P(\theta, x_P^*(\theta, \Pi))\Big] = \mathbb{E}\Big[V_D(\theta, x_D^*(\theta; \Pi))\Big] + C \quad \text{for all } \Pi \in \mathbf{\Pi}.$$

Theorem

Consider primitives $(U_D, V_D) \in \mathcal{P}$ and $(U_P, V_P) \in \mathcal{P}$.

If, for all $(\theta_D, \theta_P) \in [0,1]^2$,

$$\left.\frac{\partial U_D(\theta_D, x)}{\partial x}\right|_{x=\theta_P} + \left.\frac{\partial U_P(\theta_P, x)}{\partial x}\right|_{x=\theta_D} = 0$$

and

$$\left.\frac{\partial V_D(\theta_D, x)}{\partial x}\right|_{x=\theta_P} + \left.\frac{\partial V_P(\theta_P, x)}{\partial x}\right|_{x=\theta_D} = 0,$$

then (U_D, V_D) and (U_P, V_P) are equivalent.

Corollary

Let $(U_D, V_D) \in \mathcal{P}$ be a balanced delegation primitive.

An equivalent monotone persuasion primitive $(U_P, V_P) \in \mathcal{P}$ is

$$U_P(\theta, x) = -\int_0^x \left.\frac{\partial U_D(s,t)}{\partial t}\right|_{t=\theta} \mathrm{d}s,$$

$$V_P(\theta, x) = -\int_0^x \left.\frac{\partial V_D(s,t)}{\partial t}\right|_{t=\theta} \mathrm{d}s.$$

Tractable Persuasion and Delegation Problems

- *Linear Persuasion*, as in Kamenica and Gentzkow (2011):

$$\frac{\partial}{\partial x}U_P(\theta, x) = \psi(\theta) + \eta(x) \text{ and } \frac{\partial}{\partial x}V_P(\theta, x) = \Lambda\psi(\theta) + \nu(x),$$

  where ψ is strictly increasing and η is strictly decreasing

- *Linear Delegation*, as in Amador and Bagwell (2013):

$$\frac{\partial}{\partial x}U_D(\theta, x) = b(x) + c(\theta) \quad \text{and} \quad \frac{\partial}{\partial x}V_D(\theta, x) = Ab(x) + d(\theta),$$

  where b is strictly decreasing and c is strictly increasing

- Linear Persuasion and Separable Delegation are equivalent

Auxiliary Problem

- Agent chooses between actions $a = 1$ and $a = 0$
- Agent has a private type $t \in [0, 1]$
- There is an unobservable state $s \in [0, 1]$
- s and t are independently and uniformly distributed

Payoffs

- Agent's and Principal's payoffs are $au(s,t)$ and $av(s,t)$
- $u(s,t)$ and $v(s,t)$ are continuous in s and t
- $u(s,t)$ is strictly increasing in s and strictly decreasing in t

Discriminatory Disclosure Problem

- Principal chooses a closed subset $\Pi \subset [0, 1]$ of cutoff tests such that Π contains 0 and 1

- Agent observes his type t and selects a cutoff test $y \in \Pi$ that reveals whether $s \geq y$ or $s < y$

- W.l.o.g, given selected y, Agent chooses $a = 1$ iff $s \geq y$

Discriminatory Disclosure Problem

- After privately observing t, Agent chooses a cutoff test

$$s^*(t, \Pi) \in \arg\max_{y \in \Pi} \mathbb{E}_s \Big[u(s,t) \cdot \mathbf{1}_{\{s \geq y\}} \Big]$$

- Principal's problem

$$\max_{\Pi \in \boldsymbol{\Pi}} \mathbb{E}_t \Big[\mathbb{E}_s \Big[v(s,t) \cdot \mathbf{1}_{\{s \geq s^*(t,\Pi)\}} \Big] \Big]$$

Equivalence to Balanced Delegation

- Fix a type t and a cutoff $y \in \Pi$

- Agent's and Principal's payoffs (before learning s)

$$\mathbb{E}_s\Big[u(s,t) \cdot \mathbf{1}_{\{s \geq y\}}\Big] = \int_y^1 u(s,t)\mathrm{d}s := U_D(t,y)$$
$$\mathbb{E}_s\Big[v(s,t) \cdot \mathbf{1}_{\{s \geq y\}}\Big] = \int_y^1 v(s,t)\mathrm{d}s := V_D(t,y)$$

- (U_D, V_D) is an equivalent primitive of balanced delegation

Menu of Cutoff Tests and a Monotone Partition

- Menu $\Pi \in \mathbf{\Pi}$ defines a monotone partition Π of $[0, 1]$

- *Key observation:* Agent of type t is indifferent between:
  - observing a preferred cutoff test $s^*(t, \Pi)$
  - observing a monotone partition Π

Equivalence to Monotone Persuasion

- Agent's normal-form strategy maps $\mu_{\Pi}(s)$ and t to a

- W.l.o.g, Agent chooses a threshold type z, so $a = 1$ iff $t \leq z$

- Agent's and Principal's payoffs (before learning t):

$$\mathbb{E}_t[u(s,t) \cdot \mathbf{1}_{\{t \leq z\}}] = \int_0^z u(s,t)\mathrm{d}t := U_P(s,z)$$
$$\mathbb{E}_t[v(s,t) \cdot \mathbf{1}_{\{t \leq z\}}] = \int_0^z v(s,t)\mathrm{d}t := V_P(s,z)$$

- (U_P, V_P) is an equivalent primitive of monotone persuasion

Application: Monopoly Regulation

- x and q denote price and quantity

- Linear demand function: $q = 1 - x$

- Linear cost function cq, where $c \in [0, 1]$ is a private cost

- Marginal cost c has a positive **unimodal** density f

- Profit and welfare are given by

  $U(c, x) = (x - c)(1 - x)$ and $V(c, x) = U(c, x) + \frac{1}{2}(1 - x)^2$

- Regulator chooses a set $\Pi \subset [0, 1]$ of prices available for Monopolist, and Monopolist maximizes profit

Application: Monopoly Regulation

- Two versions:
  - Regulation without Monopolist's participation constraint (studied by Alonso and Matouschek 2008)
  - Regulation with Monopolist's participation constraint (new)

Participation Constraint

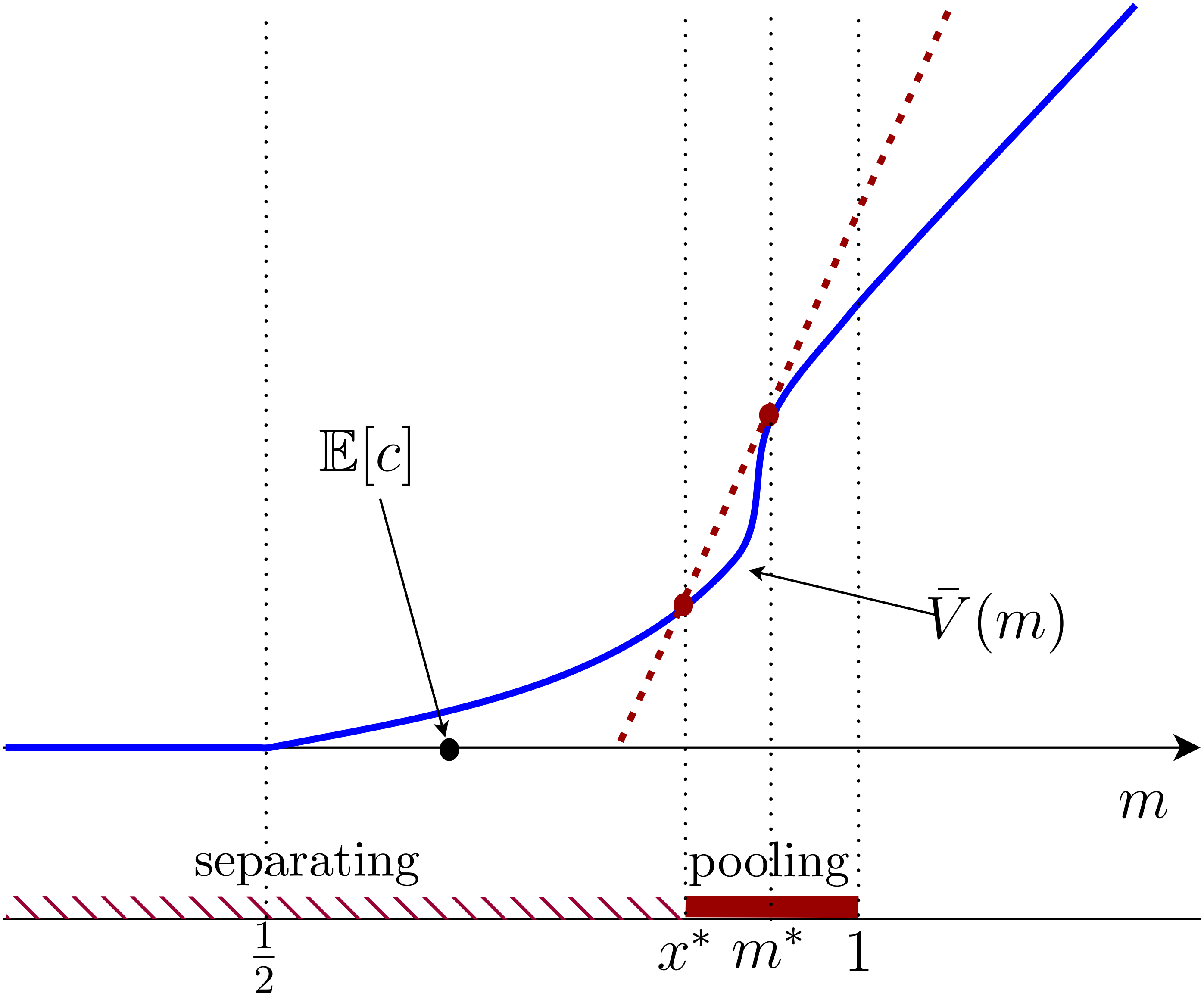
- Monopolist can always choose to produce zero quantity, equivalently set price $x = 1$, so $1 \in \Pi$
- Selling at zero price is less profitable than not producing, so, w.l.o.g., $0 \in \Pi$
- Defining $\theta = F(c)$ yields a balanced delegation problem

Equivalent Persuasion Problem

Principal's payoff from a message $\mu_{\Pi}(\theta)$ is

$$\bar{V}(m) = \int_0^{2m-1} (m - c) \mathrm{d}F(c),$$

where $m = \mathbb{E}[s|s \in \mu_{\Pi}(\theta)]$ and $\theta \sim U[0, 1]$.



$\mathbb{E}[c]$
$\bar{V}(m)$
m
separating
pooling
$\frac{1}{2}$
x^* m^* 1

Solution

- Under unimodal f, $\Pi = [0, x^*] \cup \{1\}$ is optimal

- *Upper censorship* in the persuasion problem

- *Price cap* in the regulation problem

Regulation without Participation Constraint

- Extend profit U_D and welfare V_D to the domain $[0, 2]$ of prices

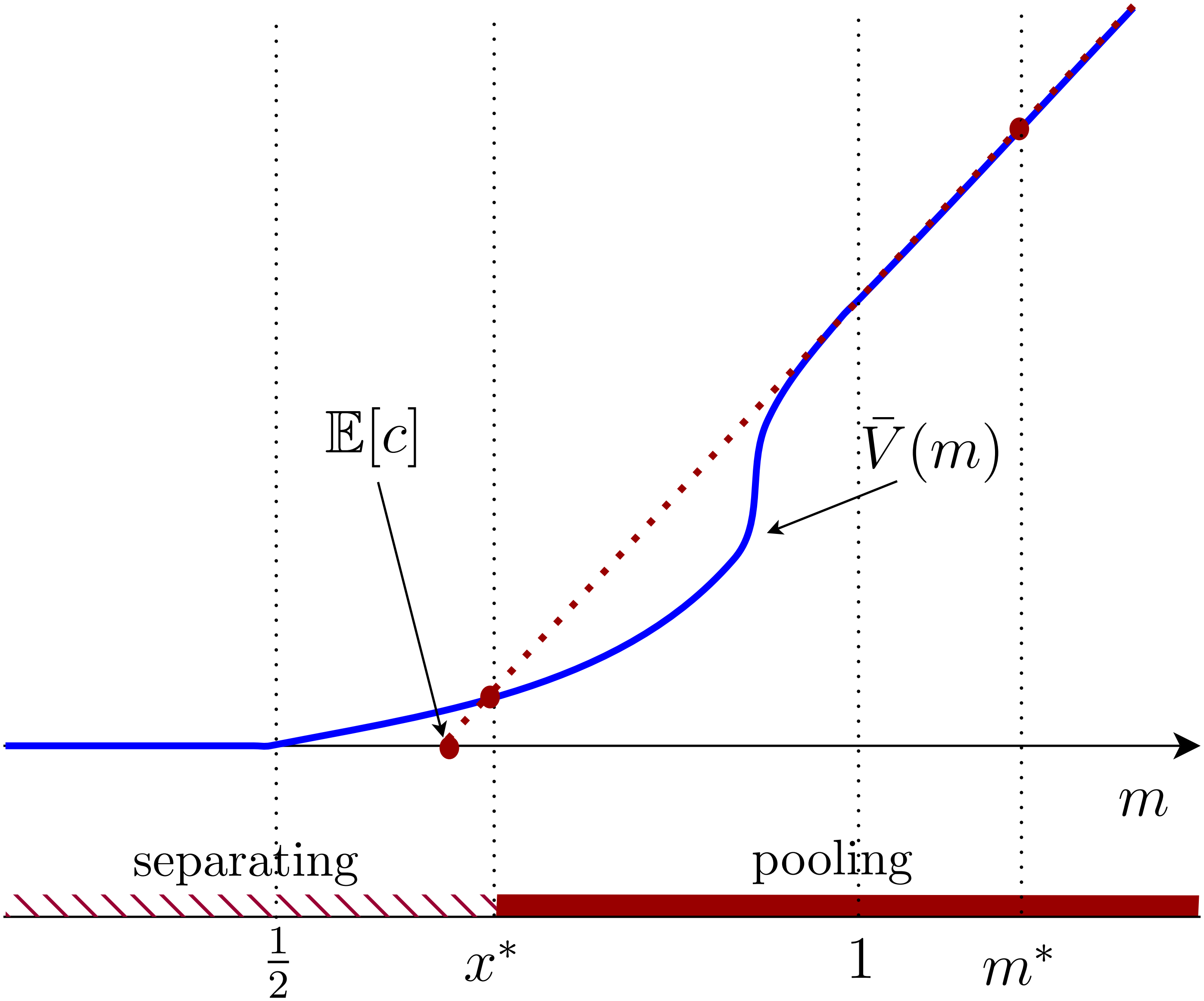
- *Lemma*: If $\Pi \subset [0, 2]$ is optimal, then $\Pi \cup \{0, 2\}$ is optimal.

Equivalent Persuasion Problem

Principal's payoff from a message $\mu_{\Pi}(\theta)$ is

$$\bar{V}(m) = \int_0^{2m-1} (m-c)\mathrm{d}F(c),$$

where $m = \mathbb{E}[s|s \in \mu_{\Pi}(\theta)]$ and $\theta \sim U[0,2]$.



$\mathbb{E}[c]$
$\bar{V}(m)$
m
separating
pooling
$\frac{1}{2}$
x^*
1
m^*

Discussion

- Monopoly regulation with and without participation constraint is solved using a single result from the persuasion literature

- Price cap is optimal in both versions of the problem

- Price cap is higher with the participation constrained

Conclusion

- The monotone persuasion problem and the balanced delegation problem are equivalent

- Both are equivalent to a discriminatory disclosure problem with an informed Agent who chooses between two actions

- Insights and results in one problem can be used to understand and solve the other problem

- Novel delegation problems with participation constraints and new results for standard delegation problems