Firm Growth and Promotion Opportunities*

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Abstract

We develop a model in which a firm makes a sequence of production decisions and has to motivate each of its employees to exert effort. The firm motivates its employees through incentive pay and promotion opportunities, which may differ across different cohorts of workers. We show that the firm benefits from reallocating promotion opportunities across cohorts, resulting in an optimal personnel policy that is seniority-based. Our main contribution is to highlight a novel time-inconsistent motive for firm growth: when the firm adopts an optimal personnel policy, it may pursue future growth precisely to create promotion opportunities for existing employees.

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1 Introduction

To meet the increased demand for explosives brought about by World War I, DuPont expanded its work force from 5,000 in 1914 to 85,000 in 1918. But executives were keenly aware that the war would not last forever, and they formed plans for post-war diversification in part to ensure their employees would continue to have jobs. In addition to entering into related chemical-based industries, they made investments in other industries to have "a place to locate some managerial personnel who might not be absorbed by the expansion into chemical-based industries." (Chandler, 1962, p. 90)

This example illustrates that good management requires planning ahead. It requires planning future production both to adapt to future business conditions, and to set up future career opportunities for current employees. Such opportunities are abundant in fast-growing firms and can be used to great effect in motivating workers. And in declining firms, they may be scarce or nonexistent (Bianchi et al., 2018). Production plans therefore affect the kinds of personnel policies the firm should adopt.

At the same time, the firm's personnel policies influence its future production plans, as many practitioners and management scholars have long argued. Barnard (1938), for example, points out that there is an "innate propensity for all organizations to expand... to grow seems to offer opportunity for the realization of all kinds of active incentives." (p. 159). Similarly, Jensen (1986) claims that using promotions to motivate employees "creates a strong organizational bias toward growth to supply the new positions that such promotion-based reward systems require." (p. 2) More recently, Bennett and Levinthal (2017) argues that firm growth "can have implications for the firm's competitive advantage as a result of the impact of firm growth on the firm's ability to motivate and incentivize its employees." (p. 2006) Such growth is fundamentally backward-looking in nature and is often derided as wasteful, but it may serve an important purpose. Production plans and personnel policies therefore interact in meaningful ways and should be designed together.

This paper is an attempt to understand how a firm's past production decisions impact its future production plans when firms motivate their employees through the use of long-term, career-based incentives. Existing economic theories are not well-suited to explore these issues, since they either focus on the forward-looking determinants of firm growth without accounting for long-term employee incentives (Lucas, 1978; Jovanovic, 1982; Hopenhayn, 1992; Ericson and Pakes, 1995) or they focus on long-term incentives for individual employees without exploring their implications for the size of the firm's workforce (see Rogerson (1985) and Spear and Srivastava (1987) for early contributions and Biais, Mariotti, and Rochet (2013) for a recent survey with a focus on financial contracts). We contribute to the existing literature by highlighting a novel time-inconsistent motivation for firm growth, and in doing so, we are able to assess when and why firms should pursue seemingly unprofitable growth strategies.

Model In our model, a single principal interacts repeatedly with a pool of employees. The interaction between the principal and each employee is a dynamic moral hazard problem with a limited-liability constraint. In each period, the principal assigns each employee to one of two jobs: a bottom job and a top job. In each job, the worker faces a moral hazard problem and must be provided incentives to exert effort. In the top job, the firm motivates the worker by paying a bonus for good performance. In the bottom job, the firm motivates the worker through a combination of bonuses for good performance and the prospect of being promoted to the top job.

Workers' promotion prospects depend both on how many new top positions there will be in the next period—which is determined by the firm's growth prospects—and on how many people are in line for these positions today—which is determined both by firm's production decisions today and by the personnel policies it has in place. The firm's problem is therefore to make its production plans and design its personnel policies jointly. In the model, the firm's profit-maximization problem can be decomposed into two steps. First, the firm chooses its pay and promotion policies to minimize its wage bill taking as given its production plan, which is a sequence of production decisions—the number of top and bottom positions it will have in each period. In the second step, the firm chooses its production plan.

Results and Implications Our first set of results shows how the firm optimally designs its personnel policies given its production plan. Because the prospect of getting promoted to the top job motivates employees in the bottom job, top positions serve as a free incentive instrument. The firm's production plan determines how many promotion opportunities will arise and when, and the firm's problem is to figure out how to allocate these opportunities across different cohorts of workers who began employment at different times. Allocating promotion opportunities to a cohort of workers allows the firm to reduce their wages while keeping them motivated. Under an optimal promotion policy, promotion opportunities are not wasted, in the sense that they cannot be reallocated in a way that reduces the firm's wage bill. We show that such policies can implemented through a modified first-in-first-out rule that favors workers with more seniority.

Our second set of results explores the implications of the firm's optimal incentive provision for its production decisions. Future production decisions determine the set of future opportunites available and therefore determine the wages the firm has to pay today. We show that the firm optimally introduces both static and dynamic distortions relative to a benchmark in which the firm takes wages as given. The static distortions arise because today's production decisions affect current workers' opportunities and hence the wages required to motivate them. For example, having more workers in the bottom job today means that they will have fewer promotion opportunites in the future. Moreover, if the firm hires more workers today, it can do so at a lower cost if it will have more opportunities in the future. This distortion leads to the motivation for firm growth highlighted by the many authors above, and it illustrates how firm growth may be *sequentially* inefficient without necessarily being inefficient.

Extensions The tools we develop to analyze optimal personnel policies also allow us to explore how firms should manage employees' careers when business conditions require the firm to downsize. An optimal personnel policy for a firm that has to make permanent cuts involves a first-in-last-out layoff policy and seniority-based severance payments: all laid-off workers are paid a severance payment upon dismissal, and less senior workers are dismissed first and receive a smaller severance payment. If the cuts the firm has to make are only temporary, then an optimal personnel policy entails seniority-based temporary layoffs: less-senior employees are laid off, but once the firm begins hiring again, it rehires them before it hires new employees. Finally, we extend our tools to analyze personnel policies in environments in which production plans are stochastic. Optimal personnel policies again resemble an internal labor market, and seniority-based promotion policies can be optimal.

Literature Review This paper contributes to the literature on internal labor markets (see Gibbons (1997), Gibbons and Waldman (1999b), Lazear (1999), Waldman (2012), and Lazear and Oyer (2013) for reviews of the theory on and evidence for internal labor markets). A particular feature of our model is that optimal personnel policies involve seniority-based promotion rules. The existing literature argues that seniority-based promotion policies can help motivate employees to invest in firm-specific human capital, (Carmichael, 1983) reduce rent-seeking behavior, (Milgrom and Roberts, 1988; Prendergast and Topel, 1996) and to better capture information rents related to its workers' abilities (Waldman, 1990). In our model, basing promotion decisions on seniority allows a firm that experiences uneven growth to better provide incentives by reallocating promotion opportunities across different cohorts of workers.

Our paper also contributes to the literature on the determinants of firm growth. Standard models of firm growth have not explored the effects of the size of a firm's workforce on its production plans (Jovanovic (1982), Hopenhayn (1992), Ericson and Pakes (1995), Albuquerque and Hopenhayn (2004), Clementi and Hopenhayn (2006)). One exception is Bennett and Levinthal (2017). In Bennett and Levinthal's (2017) model, workers exert unobservable effort to improve the production process. Two key assumptions are that the moral hazard problem is more severe in larger firms and that process improvement exhibits diminishing returns. As a result, smaller firms can motivate workers more cheaply and, at the same time, grow faster on average. Bennett and Levinthal (2017) does not consider dynamic incentive provision, and as a result, optimal production plans are time-consistent. In contrast, our focus on dynamic incentive provision leads to time-inconsistent production plans.

Finally, our paper contributes to the literature on dynamic moral hazard problems. In dynamic moral hazard settings, nontrivial dynamics can arise when there are contracting frictions.[1] The closest papers are Board (2011) and Ke, Li, and Powell (2018). In Board's (2011) model, firms hire one supplier in each period, and the focus of its analysis is on which supplier to utilize. The focus of our model is on the number of workers to hire in each period, and this is a choice variable of the firm and can change from period to period. Ke, Li, and Powell (2018) examines how organizational constraints affect firms' personnel policies in a stationary environment in which the size of the firm is constant. In this case, there are no gains to reallocating promotion opportunities across cohorts, and optimal personnel policies are seniority-blind. In our model, uneven growth leads to seniority-based personnel policies, and the need to provide long-term incentives leads the firm to adopt time-inconsistent production plans.

2 The Model

A firm interacts with a pool of risk-neutral workers in periods $t = 1, \dots, T$, where T may be infinite, and all players share a common discount factor $\delta \in (0, 1)$. The firm's labor pool consists of a large mass of identical workers, and the firm chooses a personnel policy, which we will describe below, to maximize its discounted profits.

Production requires two types of activities to be performed, and each worker can perform a single activity in each period. A worker performing activity $i \in \{1, 2\}$ in period t chooses an effort level $e_t \in \{0, 1\}$ at cost $c_i e_t$. A worker who chooses $e_t = 0$ is said to *shirk*, and a worker who

[1] Such dynamics are highlighted by DeMarzo and Fishman (2007) and Clementi and Hopenhayn (2006) in a financial contracting setting (see Biais, Mariotti, and Rochet (2013) for a survey), by Manove (1997), Li and Matouschek (2013), Halac (2012), Zhu (2012), Fong and Li (2017), Barron and Powell (2018), and Fudenberg and Rayo (2018) in an employment setting, by Boleslavsky and Kelly (2014) in a regulatory setting, by Padro i Miquel and Yared (2012) in a political economy setting, by Andrews and Barron (2016), and Urgun (2017) in a supplier allocation setting, and by Li, Matouschek, and Powell (2017), Lipnowski and Ramos (2017), and Guo and Horner (2017) in a delegation setting.

chooses $e_t = 1$ is said to *exert effort*. We refer to a worker who exerts effort as *productive*. A worker's effort is his private information, but it generates a publicly observable signal $y_{i,t} \in \{0, 1\}$ with $\Pr[y_{i,t} = 1| e_t] = e_t + (1 - q_i)(1 - e_t)$, that is, shirking in activity i is contemporaneously detected with probability q_i. If the firm employs masses $N_{1,t}$ and $N_{2,t}$ of productive workers in the two activities, revenues are $\theta_t f(N_{1,t}, N_{2,t})$, where θ_t is the firm's period-t *demand parameter*, and f is strictly concave. We will refer to $\theta = (\theta_1, \ldots, \theta_T)$ as a *demand path*.

In each period, the firm assigns each worker to an activity $i_t \in \mathcal{A} \equiv \{0, 1, 2\}$, where activity 0 is a non-productive activity. The worker accepts the assignment or rejects the assignment and exits the firm's labor pool, receiving an outside option that yields utility 0. If the worker accepts the assignment, he then exerts effort e_t, his signal $y_{i,t}$ is realized, and then the firm pays the worker an amount $W_t \geq 0$. That is, the worker is protected by a limited-liability constraint. At the end of each period, each worker exogenously exits the firm's labor pool with probability d and receives 0 in all future periods, and a group of new workers enters the firm's labor pool.

Define a worker's *employment history* to be a sequence $h^t = (0, \ldots, 0, A_\tau, \ldots, A_t) \in \mathcal{H}^t$, where $A_s \in \mathcal{A}$ specifies the activity he was assigned to in period s, and τ is the time at which he first enters the firm's labor pool. By convention, we say that a worker is assigned to activity 0 in each period before he is in the firm's labor pool. We will say that a worker who is assigned to activity 1 or 2 for the first time in period t is a *new hire in period t* and that he is a *cohort-t worker*. Define $L(h^t)$ to be the mass of workers with employment history h^t.

Before we define a contract between the firm and a worker, we pause to make two observations that will help simplify notation. First, if a worker is assigned to activity 1 or 2 and is not asked to exert effort this period, then we can instead assign him to activity 0 this period. Second, if a worker is assigned to activity 1 or 2 and is asked to exert effort, it is without loss of generality to pay him 0 in this period and in all future periods if his signal is equal to 0. This follows because when a worker's signal is 0, the worker must have shirked, and this is the harshest punishment possible.

Given these two observations, we can now define a contract between the firm and a worker. A *contract* is a sequence of *assignment policies* $P_{i,t} : \mathcal{H}^{t-1} \to [0, 1]$ specifying the probability the worker is assigned to activity i given employment history h^t, and a sequence of *wage policies* $W_t : \mathcal{H}^t \to [0, \infty)$ specifying the wage the worker receives at the end of period t given his history. A *personnel policy* is a set of contracts the firm has with each worker in its labor pool.

The firm's period-t profits are

$$\theta_t f\left(N_{1,t}, N_{2,t}\right) - \sum_{h^t \in H^t} W_t\left(h^t\right) L\left(h^t\right),$$

and each worker's period t utility is $W_t\left(h^t\right) - c_i e_t$. The firm's problem is to choose $\{W_t, P_{i,t}\}$ to maximize its expected discounted profits, and given the contract he faces, each worker chooses his acceptance decisions and effort decisions to maximize his expected discounted utility. Throughout most of the analysis, we will be focusing on contracts for which if a worker is ever assigned to activity 0 after he has been assigned to activity 1 or 2, he is assigned to activity 0 in all future periods. We will refer to such contracts as *full-effort contracts* because they motivate the worker to exert effort in every period they have been employed by the firm. In Section 7.2, we discuss situations in which the firm would optimally choose contracts that permit workers to shirk in some periods.

Finally, we define a *production path* to be a sequence $N = \left(N_{1,t}, N_{2,t}\right)_{t=1}^{T}$ that specifies the mass of productive workers in each activity in each period. We will say that a production path is *steady* if $N_{2,t+1} < (1-d)\left(N_{1,t} + N_{2,t}\right)$ and $N_{i,t+1} \geq N_{i,t}(1-d)$ for $i = 1, 2$. The first condition for a steady production path says that the number of top positions in the firm does not grow too fast—it ensures that, in each period, there are enough incumbent workers to fill all the activity 2 positions. The second condition says that the firm does not shrink too fast.

After we describe the main results, Section 6 discusses the role of several of the model's assumptions, including worker homogeneity, the monitoring structure, and deterministic demand parameters.

3 Preliminaries

Our analysis decomposes the firm's overall problem into two steps. First, given any production path N, we derive properties of optimal personnel policies that induce a mass $N_{i,t}$ of workers assigned to activity i to exert effort in period t. This section takes the production path as given, sets up the firm's cost-minimization program, and develops several intermediate results to simplify the analysis. In particular, we show that the firm's cost-minimization problem is equivalent to minimizing the rents that are paid to new hires, and an optimal solution can be implemented with an internal labor market.

The second step of the firm's problem is to choose an optimal production path N^* given the external environment it faces. Section 5 analyzes the second step of the firm's problem.

3.1 Cost-Minimization Problem

Recall that a personnel policy is a set of contracts the firm has with each worker in its labor pool, where each contract describes the assignment policy and the wage policy the worker is subject to. Given a production path N, we will say that a personnel policy *implements* N if, given the personnel policy, a mass $N_{1,t}$ and $N_{2,t}$ of workers exerts effort in activities 1 and 2 in period t. Denote a worker's *initial-hire history* by $n^t = (0, \ldots, 0, n_t)$, where $n_t \in \{1, 2\}$. The first lemma shows that the problem of characterizing cost-minimizing personnel policies can be simplified by focusing on a smaller class of personnel policies. All the proofs are in the appendix.

Lemma 1. *Given N, if there is an optimal personnel policy, there is an optimal personnel policy in which workers with the same employment history face the same wage and assignment policies.*

In order to specify the firm's program, define $c\left(h^t\right) = c_{A_t}$ if $A_t \in \{1, 2\}$ and 0 otherwise, and $q\left(h^t\right) = q_{A_t}$ if $A_t \in \{1, 2\}$. Denote by $w\left(h^t\right)$ the wage the worker receives if $y_{A_t,t} = 1$ and by $p_i\left(h^t\right)$ the probability the worker is assigned to activity i in the next period, conditional on remaining in the labor pool. Denote by $h^t A_{t+1} = (A_1, \ldots, A_{t+1})$ the concatenation of h^t with A_{t+1}. For all workers in the labor pool, we have

$$L\left(h^t i\right) = (1 - d)\, p_i\left(h^t\right) L\left(h^t\right).$$

Given a production path N, the firm's problem is to minimize its wage bill

$$\min_{w, p_i} \sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^{t-1} L\left(h^t\right) w\left(h^t\right)$$

subject to the following constraints.

Promise-Keeping Constraints. If we denote by $v\left(h^t\right)$ the worker's expected discounted payoffs at time t given employment history h^t, then workers' payoffs have to be equal to the sum of their current payoffs and their continuation payoffs:

$$v\left(h^t\right) = w\left(h^t\right) - c\left(h^t\right) + \delta\,(1 - d) \sum_{i \in \{1,2\}} p_i\left(h^t\right) v\left(h^t i\right). \tag{1}$$

Incentive-Compatibility Constraints. Productive workers prefer to exert effort in activity i if they cannot gain by shirking:

$$v\left(h^t\right) \geq (1 - q\left(h_t\right)) \left(w\left(h^t\right) + \delta\,(1 - d) \sum_{i \in \{1,2\}} p_i\left(h^t\right) v\left(h^t i\right) \right)$$

or equivalently

$$v\left(h^{t}\right) \geq \frac{1-q_{A_t}}{q_{A_t}} c_{A_t} \equiv R_{A_t}, \tag{2}$$

where we refer to the quantity R_{A_t} as the *incentive rent for activity* A_t. Note that these incentive-compatibility constraints imply that workers receive positive surplus in equilibrium, and they imply that workers' participation constraints are also satisfied. We therefore do not include workers' participation constraints in the firm's problem.

Flow Constraints. In each period, the firm employs a mass $N_{i,t}$ workers in activity i:

$$\sum_{h^t \mid A_t = i} L\left(h^{t}\right) = N_{i,t}, \text{ for } i \in \{1, 2\}. \tag{3}$$

Given these constraints, the firm maximizes its profits. For a given production path, the firm's discounted profits are equal to the total discounted surplus net of the rents it pays to workers. Given a production path, therefore, the firm's problem is to minimize these rents. Recall that a worker with employment history n^t is a worker who is first employed by the firm in period t.

Lemma 2. *Cost-minimizing personnel policies minimize the rents paid to new hires:*

$$\min \sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^t L\left(n^{t}\right) v\left(n^{t}\right)$$

subject to (1)*,* (2)*, and* (3)*.*

Lemma 2 shows that for a given production path, the firm's cost-minimization problem is equivalent to minimizing the present discounted value of the rents that are paid to new hires. It will be conceptually and expositionally convenient to decompose the rents paid to new hires into three components: the number of new hires, their necessary rents, and their excess rents. For the incentive-compatibility constraint to hold, the *necessary rents* that a new hire in period t must receive is R_{n_t}. We will refer to the quantity $v\left(n^{t}\right) - R_{n_t}$ as the *excess rents* paid to cohort-t workers. To reduce the rents that are paid to new hires, the firm therefore wants to reduce the number of new hires, as well as both the necessary and excess rents paid to new hires. We will now draw out the implications of this observation.

3.2 Internal Labor Markets

This section shows that internal labor markets are personnel policies that serve to minimize the number of new hires as well as the necessary rents for a given production path. The firm's cost-minimization problem involves choosing an assignment policy and a wage policy for each worker at every history and is not amenable to standard Lagrangian techniques. For ease of exposition, we will first focus our analysis on steady production paths, returning to "unsteady" production paths in Section 7.1. Recall that a production path N is steady if $N_{2,t+1} < (1-d)(N_{1,t} + N_{2,t})$ and $N_{i,t+1} \geq N_{i,t}(1-d)$ for $i = 1, 2$. In the lemma below, we show that we can focus on a narrower class of personnel policies without loss of generality.

Lemma 3. *Given a steady production path N, if there is a cost-minimizing personnel policy, there is a cost-minimizing personnel policy with the following three properties:*

(i.) $v\left(h^t\right) \leq R_2$ if $A_t = 1$ and $v\left(h^t\right) = R_2$ if $A_t = 2$.

(ii.) All new workers are assigned to activity 1.

(iii.) $p_2\left(h^t\right) = 1$ if $A_t = 2$ and $p_1\left(h^t\right) + p_2\left(h^t\right) = 1$ if $A_t = 1$.

The first part of Lemma 3 shows that in a cost-minimizing personnel policy, the firm does not gain by rewarding workers with rents exceeding R_2. Part 2 of the lemma shows that new hires are always assigned to activity 1 unless the firm has grown sufficiently rapidly that it must hire new workers directly into activity 2. The final part of the lemma shows that as long as the firm never shrinks abruptly, workers performing activity 2 will continue to do so, and workers performing activity 1 will either continue to perform activity 1 in the next period or will be "promoted" to activity 2.

Lemma 3 highlights several features that are consistent with Doeringer and Piore's description of internal labor markets: (1) there is a port of entry, (2) there is a well-defined career path, and (3) wages increase upon promotion. We will say that a personnel policy satisfying these three properties is an *internal labor market*. Lemma 3 immediately implies the following proposition.

Proposition 1. *If N is a steady production path, an internal labor market is an optimal personnel policy.*

Proposition 1 shows that in a relatively stable environment, a cost-minimizing personnel policy can be implemented as an internal labor market. This result generalizes Proposition 3 in Ke, Li, and Powell (2018). For the rest of this section, we will characterize further properties of cost-minimizing internal labor markets when N is a steady production path. We defer discussion of cost-minimizing personnel policies for other production paths to Section 7.1.

To develop some intuition for why an internal labor market is cost-minimizing, first notice that by setting $p_1\left(h^t\right) + p_2\left(h^t\right) = 1$ on the equilibrium path for all workers assigned to activity 1 or 2, the firm does not introduce unnecessary turnover, which in turn minimizes the total number of new hires. Next, to see why the firm optimally assigns new hires to activity 1, suppose the firm has an opening in activity 2. If it assigns a new hire to that opening, it would have to pay him at least R_2 in terms of first-period rents. If the firm instead assigns that new hire to activity 1, it can pay him weakly less in terms of first-period rents. Moreover, the firm can fill the opening with someone who is currently assigned to activity 1 by promoting him. This would make his future in the firm more, and the firm may be able to reduce his wages. Promotion opportunities therefore serve as a free incentive instrument the firm should allocate optimally.

4 Allocating Promotion Opportunities Across Cohorts

As we argued in the previous section, internal labor markets minimize the number of new hires and their necessary rents. The firm's cost-minimization problem therefore boils down to minimizing excess rents paid to new hires. Promotion opportunities serve as a free incentive instrument to motivate workers assigned to activity 1. These opportunities arrive at different times and should be allocated to different cohorts of workers in order to minimize their excess rents.

Recall that in an internal labor market, workers assigned to activity 2 remain assigned to activity 2, and their wages are independent of when they began employment. An internal labor market can therefore be summarized by the pre-promotion wages, promotion probabilities, and values, $w_{1,t}^{\tau}$, p_t^{τ}, and $v_{1,t}^{\tau}$ for cohort-τ workers assigned to activity 1 in period t for each τ and t. We begin by defining a class of promotion policies that will turn out to include an optimal promotion policy.

Definition 1. A promotion policy is a *modified first-in-first-out (modified FIFO)* policy if, for every t, there exist two thresholds $\tau_1(t)$ and $\tau_2(t)$ with $0 \leq \tau_1(t) \leq \tau_2(t) \leq t$ such that:

1. $p_t^{\tau} = \bar{p}_t$ for all $\tau \leq \tau_1(t)$ for some $\bar{p}_t \in (0, 1]$,
2. p_t^{τ} is decreasing in τ for $\tau_1(t) < \tau \leq \tau_2(t)$, and
3. $p_t^{\tau} = 0$ for all τ for $\tau_2(t) \leq \tau \leq t$.

In a modified FIFO policy, sufficiently new hires may not be promoted with positive probability, more senior workers are promoted with higher probability than less senior workers, and the promotion probability as a function of tenure is capped at some level $\bar{p}_t$. Two special cases of modified FIFO policies warrant special attention. The first are *strict FIFO* policies in which more-senior workers are always promoted first. The second special case are *seniority-blind* policies in which promotion opportunities are allocated evenly across cohorts. The following propsition describes cost-minimizing personnel policies.

Proposition 2. *Given a steady production path N, if there is a cost-minimizing personnel policy, an internal labor market with the following properties is cost-minimizing:* (1) $w_{1,t}^{\tau}$ *is weakly increasing in* t, (2) *if* $\tau < \tau'$, $w_{1,t}^{\tau} \geq w_{1,t}^{\tau'}$ *and* $v_{1,t}^{\tau} \geq v_{1,t}^{\tau'}$, *and* (3) *the promotion policy is a modified FIFO policy.*

The first part of Proposition 2 describes wage dynamics for a single cohort within the firm and shows that wages in activity 1 exhibit returns to tenure. This feature that wages are backloaded in a worker's career is familiar from models of optimal long-term contracts (Becker and Stigler, 1974; Lazear, 1979; Ray, 2002). The second part of the proposition compares wage and value dynamics across cohorts: wages and values are higher for earlier cohorts than for later cohorts. The last part of the proposition describes promotion dynamics and shows that promotion opportunities can be optimally allocated according to a modified FIFO policy. This implies that workers' promotion prospects are optimally weakly seniority-dependent.

To understand why promotion prospects may be strictly seniority-dependent, suppose the firm follows a seniority-blind promotion policy. Such a policy may allocate too many opportunities to later cohorts, in the sense that their first-period incentive constraints are slack and they are receiving excess rents, while the first-period incentive constraints for earlier cohorts may be binding. In this case, the firm would gain by reducing the promotion probability for newer workers in order to increase the promotion probability for older workers, allowing it to reduce their wages. Seniority-based promotion schemes are common in many settings, including the U.S. manufacturing sector where Bloom et al. (2018) finds that a sizeable share of firms base promotions at least in part on seniority.

To understand why workers who are in a later cohort may optimally be promoted before a worker in an earlier cohort, suppose the firm follows a strict FIFO rule. The logic is the flip side of the logic above. Such a policy may allocate too many opportunities to early cohorts, so that they are receiving excess rents, while later cohorts are not. The firm would gain by reducing the promotion probability for early cohorts in order to increase the promotion probability for later cohorts, allowing it to reduce their wages. A modified FIFO, which bases promotions on an "interior" degree of seniority minimizes the excess rents that are paid to new hires by allocating promotion opportunities to transfer slack across cohorts' first-period incentive constraints.

An important constraint on the firm's problem is that, while it can allocate period-t promotion opportunities to workers who started working at the firm prior to period t, it cannot allocate period-t promotion opportunities to workers who will begin working at the firm later. This "irreversibility of time" constraint (Grenadier, Malenko, and Malenko, 2016) ensures that excess rents for cohort-t

workers are weakly decreasing in t: slack from later cohorts' first-period incentive constraints can be allocated to earlier cohorts but not vice versa.

Finally, we briefly discuss one implication of these results. In the agent's incentive constraint, promotions and bonus payments serve as substitute mechanisms to motivate workers, yet in the personnel policies described in Proposition 2, promotions and bonus payments are positively related across workers. Like with many dynamic contracting problems, optimal values are uniquely pinned down, although there are often many optimal solutions. We focus on optimal solutions with increasing wage profiles because they are consistent with evidence on tenure-wage profiles (Baker, Gibbs, and Holmström, 1994) but with the caveat that they are not uniquely optimal.

5 Personnel Policies and Production Paths

The previous section explored how differences in the firm's production path shape the characteristics of the personnel policies it puts in place. We now turn to the firm's full problem of choosing an optimal production path and implementing it with a cost-minimizing personnel policy in order to understand how dynamic incentive provision shapes the firm's optimal production path. We will show that incentive considerations lead the firm to distort production away from productive efficiency, and we will draw out some of the implications of this observation.

In order to define productive efficiency, consider an *exogenous wage benchmark* in which the firm takes a sequence of wages $\{w_{1,t}, w_{2,t}\}_{t=1}^{T}$ and demand parameters $\{\theta_t\}_{t=1}^{T}$ as given and chooses the number of workers it will assign to each activity in each period. The firm's problem is to

$$\max_{\{N_{1,t},N_{2,t}\}_{t=1}^{T}} \sum_{t=1}^{T} \theta_t f\left(N_{1,t}, N_{2,t}\right) - w_{1,t}N_{1,t} - w_{2,t}N_{2,t}.$$

Optimal production paths in the exogenous wage benchmark have two important features. First, for each activity in each period, the firm will assign more workers to that activity until the marginal revenue product of an additional worker equals the wage he is paid: $\theta_t \partial f\left(N_{1,t}, N_{2,t}\right) / \partial N_{i,t} = w_{i,t}$ for $i = 1, 2$. This is the standard productive efficiency condition from producer theory. The second feature of optimal production paths is that they are intertemporally independent: the optimal choice of $(N_{1,t}, N_{2,t})$ does not depend on θ_τ for any $\tau \neq t$. Both of these conclusions change when we consider dynamic incentives. The first subsection shows how the firm's optimal use of dynamic incentives leads to productive inefficiencies and describes how they evolve over the firm's life cycle. The second subsection shows how dynamic incentives lead to intertemporal linkages in the firm's optimal production path.

5.1 Dynamic Incentives and Productive Inefficiency

The firm's full problem is to choose an optimal production path $\{N_{1,t}, N_{2,t}\}_{t=1}^{T}$ given a demand path $\{\theta_t\}_{t=1}^{T}$ and given that it will put in place a cost-minimizing personnel policy satisfying (1) – (3). We describe the firm's full problem in detail in the appendix. Recall from Proposition 1 that optimal personnel policies have several key features: new hires are assigned to activity 1, they may be promoted to activity 2, no workers assigned to activity 2 are ever demoted, and wages in activity 2 do not depend on workers' employment histories. Choosing an optimal personnel policy involves choosing promotion probabilities and pre-promotion wages $\left(p_t^s, w_{1,t}^s\right)$ for each cohort s and each period t. In order to contrast optimal production paths with productive-efficient production paths, define the *wedge* $D_{i,t} = \theta_t \partial f\left(N_{1,t}, N_{2,t}\right)/\partial N_{i,t} - w_{i,t}^t$ as the difference between the marginal revenue productivity of activity i and the wage for a new hire assigned to activity i. The following proposition partially characterizes the firm's optimal production path in terms of these wedges.

Proposition 3. *If an optimal production path is steady, it has the following features:*

1. $D_{1,t} \geq 0$ *for all* t.

2. $\sum_{s=t}^{T} \delta^{s-t}\left(1-d\right)^{s-t} D_{2,s} \leq 0$ *for all* t.

3. *In the first period,* $D_{i,1} \geq 0$ *for* $i = 1, 2$*, and in the last period,* $D_{1,T} = 0$ *and* $D_{2,T} \leq 0$.

The first result shows that relative to productive efficiency, the firm optimally chooses too few bottom positions in each period. To see why this is true, consider a perturbation in which the firm reduces $N_{1,t}$ and therefore hires one fewer worker in period t. To keep future profits constant, suppose that tomorrow, when the firm has to fill whatever position this worker would have occupied, the firm hires a new worker and treats him as if he was hired at period t. This perturbation does not affect future profits, and it increases period-t profits by $-\theta_t \partial f/\partial N_{1,t} + w_{1,t}^t$. Any optimal production path must therefore have $D_{1,t} \geq 0$ in each period.

To see why this inequality may be strict, note that the cost of hiring an additional worker into the bottom position is not just the wages he receives in his first period of employment. It also includes the fact that some of his compensation will take the form of future promotion prospects. Hiring an additional worker today without adjusting the number of positions in the future, therefore, means that other workers must be promoted with a lower probability and thus have to be motivated with higher wages. This increase in wages illustrates the dynamic costs of hiring an additional worker in a promotion-based reward system.

A similar logic would seem to suggest that the firm might create more positions at the top because doing so allows it to reduce the wages it pays to workers at the bottom. This logic, however, is incomplete. Once the additional top position has been filled by a worker, he remain in that position in the future under an optimal personnel policy, and this reduces the promotion prospects for other workers in the future. Creating an additional top position therefore creates both dynamic benefits and dynamic costs.

The second result isolates the dynamic benefits while fixing the promotion prospects for other workers going forward. To see why the result is true, consider a perturbation in which the firm adds a "line" of top positions: it increases $N_{2,t}$ by ε, $N_{2,t+1}$ by $(1-d)\,\varepsilon$, $N_{2,t+2}$ by $(1-d)^2\,\varepsilon$, and so on. The direct benefits to the firm of this perturbation are the sum of the static benefits and costs described in the statement of the proposition. But there is an additional dynamic because this perturbation creates one more top position in period t. This means that for workers hired prior to t and who are in the bottom job, their promotion opportunities have increased. If they are being paid a positive wage, the firm can therefore do better by increasing their promotion prospects and reducing their wage, so there is a positive indirect benefit of creating an additional line of top positions in period t.

The last result shows that in the first period, the firm has too few positions both at the bottom and at the top. The result for the bottom job follows directly from part (1) of the proposition. The result for the top job follows because an additional worker hired in period 1 at the top reduces the promotion prospects for workers in the future, but the additional top position cannot be used as an incentive instrument for previous periods since this is the first period.

Finally, the result also shows that the firm has too many top positions in the last period ($D_{2,T} \leq 0$). The reason for this is the flip side of the reason for why the firm has too few top positions in the first period: an additional position creates opportunities for workers hired prior to T but does not constrain opportunities for workers after period T since it is the last period. Similarly, $D_{1,T} = 0$ because any worker hired in the last period has no opportunity to be promoted.

5.2 Intertemporal Linkages

We now explore how how dynamic incentives lead to intertemporal linkages in the firm's optimal production path. To do so, we first establish a lemma that imposes strong restrictions on optimal production paths.

Lemma 4. *In an optimal production path, workers receive no excess rents.*

Lemma 4 has several implications for the firm's optimal production path. In particular, it provides conditions under which the firm's optimal production path features intertemporal linkages in the sense that the firm's optimal production choices in period t depend on demand conditions in other periods. More precisely, we will say that the optimal production path $N^* = \left(N_{1,t}^*, N_{2,t}^*\right)_{t=1}^T$ features *local intertemporal independence* if, for each t, small changes in θ_τ do not affect $N_{i,t}^*$ for $t \neq \tau$, that is, $\partial N_{i,t}^* / \partial \theta_\tau = 0$ for all $t \neq \tau$. If the production path is not locally intertemporally independent, then even small changes in period τ demand conditions will have an impact on production in period t. Denote the set of demand paths $\theta = (\theta_1, \ldots, \theta_T)$ for which the optimal production path features local intertemporal independence as Θ^I.

To explore the conditions under which N^* has local intertemporal independence, we define the following relaxed problem:

$$\begin{aligned} \max_{\{N_{1,t}, N_{2,t}\}_{t=1}^T} \; & \theta_1 f\left(N_{1,1}, N_{2,1}\right) - \left(c_1 + R_1\right) N_{1,1} - \left(c_2 + R_2\right) N_{2,1} \\ + & \sum_{t=2}^T \delta^{t-1} \left[\theta_t f\left(N_{1,t}, N_{2,t}\right) - c_1 N_{1,t} - c_2 N_{2,t} - H_t R_1\right], \end{aligned}$$

where $H_t = N_{1,t} + N_{2,t} - (1-d)\left(N_{1,t-1} + N_{2,t-1}\right)$ is the mass of new hires in period t. Recall by Lemma 2 that the under a cost-minimizing personnel policy, the firm minimizes the rents it provides to new hires. The objective function above describes the firm's discounted profits imposing the restriction that each new hire receives the minimal amount of rents compatible with incentive compatibility, that is, workers hired into activity i receive R_i in their first period of employment. The solution to this problem, $\tilde{N}^* = \left(\tilde{N}_{1,t}^*, \tilde{N}_{2,t}^*\right)_{t=1}^T$, therefore, provides an upper bound to the firm's profits.

Lemma 5. *The solution to the relaxed problem is a solution to the full problem, $\tilde{N}^* = N^*$, if and only if under $\tilde{N}^*$, there exists an incentive-compatible personnel policy that gives each new hire no excess rents.*

Lemma 5 provides necessary and sufficient conditions for the solution to the relaxed problem to be a solution to the full problem. Moreover, as we show in Lemma 6 in the appendix, if the solution to the relaxed problem solves the full problem, then N^* has local intertemporal independence. Fully characterizing the set of $\tilde{N}^*$ which can be implemented with a personnel policy that gives no excess rents can be complicated. Instead, we provide a sufficient condition that is easy to check below. Define the average period-t promotion rate under a production path $\left(\tilde{N}_{1,t}, \tilde{N}_{2,t}\right)_{t=1}^T$ by $\tilde{p}_t = \left(\tilde{N}_{2,t+1} - (1-d)\tilde{N}_{2,t}\right) / \left((1-d)\tilde{N}_{1,t}\right)$, and define $\hat{p}$ to be the solution to the following equation

$$R_1 = -c_1 + \delta(1-d)\left(\hat{p} R_2 + (1-\hat{p}) R_1\right).$$

If the worker is paid a wage of 0 in each period he is assigned to activity 1, and he is promoted with probability $\hat{p}$, then his value for being assigned to activity 1 is exactly R_1. The following Corollary to Lemma 5 provides sufficient conditions in terms of $\tilde{p}_t$ and $\hat{p}$ for the optimal relaxed production path to be optimal.

Corollary 1. *The solution to the relaxed problem is a solution to the full problem if* $\tilde{p}_t < \hat{p}$ *for all* t.

Corollary 1 implies that when firms are limited in their promotion opportunities, their optimal production path is locally intertemporally independent. If we compare the optimal production path under two sequences of demand parameters that differ only in period t, then the optimal production paths also only differ in period t if they both satisfy the conditions in Corollary 1. To see why this is, note that a small increase in θ_t will lead the firm to increase the number of positions it has in period t. In turn, the firm can promote more workers who were hired prior to t and reduce their pre-promotion wages. The firm can therefore ensure new hires receive no excess rents simply by adjusting its personnel policy rather than by altering its production levels in any other period. If the demand parameter θ_t increase significantly, the firm cannot reduce its pre-promotion wages further and therefore has to adjust the number of positions in order to maintain the no excess rents condition. This can lead to both forward linkages and backward linkages, which we explore next.

Intertemporal linkages in production The previous corollary provides sufficient conditions under which an optimal production plan is locally intertemporally independent. The following corollary provides sufficient conditions under which there are intemporal linkages. Define $\Delta_t = N_{2,t} - (1-d)\,N_{2,t-1}$ to be the mass of new opportunities that arise in period t under production path $\{N_{1,t}, N_{2,t}\}_{t=1}^{T}$.

Corollary 2. *Suppose* $\tilde{N}^*$ *is a solution to the relaxed problem and there exists some* τ *such that*

$$\sum_{t=2}^{\tau+1} \delta^{t-1} \tilde{\Delta}_t^* R_2 - \sum_{t=1}^{\tau} \delta^{t-1} \tilde{N}_{1,t}^* c_1 + \delta^{\tau} \left(\tilde{N}_{1,\tau+1}^* - \tilde{H}_{\tau+1}^* \right) R_1 > \sum_{t=1}^{\tau} \delta^{t-1} \tilde{H}_t^* R_1,$$

where $\tilde{H}_t^*$ *is the mass of new hires into activity* 1 *in period* t*. Then the optimal production path* N^* *does not have local intertemporal independence.*

The conditions in the Corollary are sufficient conditions for there to be intertemporal linkages in the optimal production path. The conditions can be checked directly by calculating $\tilde{N}^*$, the solution to the relaxed problem. For each τ, the left-hand side of this inequality capture the mass of promotion opportunities for all periods up until τ. These opportunities have to be allocated to someone who started working at the firm at or before τ, and if there are too many opportunities, at least one cohort must receive excess rents. By Lemma 4, this cannot be optimal.

Intertemporal Linkages in a Two-Period Example In order to illustrate how intertemporal linkages can arise in an optimal production path, consider an example with $T = 2$ and with additively separable production within each period:

$$\theta_t f(N_{1,t}, N_{2,t}) = \theta_t [f_1(N_{1,t}) + f_2(N_{2,t})]$$

for some strictly increasing and strictly concave functions f_1 and f_2. By Lemma 4A in the appendix, an optimal production path solves the following problem:

$$\max_{\{N_{1,t},N_{2,t}\}_{t=1}^{2}} \sum_{t=1}^{2} \theta_t [f_1(N_{1,t}) + f_2(N_{2,t}) - c_1 N_{1,t} - c_2 N_{2,t}] - R_1 N_{1,1} - R_2 N_{2,1} - H_2 R_1$$

subject to the constraint that $\{N_{1,t}, N_{2,t}\}_{t=1}^{2}$ can be implemented with personnel policy that gives new hires no excess rents. This condition is easy to check when $T = 2$. In particular, it requires only that cohort-1 workers assigned to activity 1 do not have "too many" promotion opportunities, so that it is feasible for the firm to motivate them without providing them excess rents.

The following proposition describes the main properties of the optimal production path for this example.

Proposition 4. *There is a continuous and increasing function $\bar{\theta}_2(\theta_1)$ such that if $\theta_2 < \bar{\theta}_2(\theta_1)$, then N^* is locally intertemporally independent, that is, $\partial N_{i,1}^*/\partial\theta_2 = 0$ and $\partial N_{i,2}^*/\partial\theta_1 = 0$ for $i = 1, 2$. If $\theta_2 > \bar{\theta}_2(\theta_1)$, then there are forward and backward linkages, that is, $\partial N_{i,1}/\partial\theta_2 > 0$ for $i = 1, 2$, and $\partial N_{2,2}/\partial\theta_1 > 0$.*

This proposition illustrates the general points in Corollary 1 and Corollary 2. The first part of the proposition provides conditions on primitives so that the conditions in Corollary 1 hold, and the optimal production path is intertemporally independent. When demand does not grow very quickly, there will not be so many promotion opportunities that the firm will have to give new hires excess rents.

The second part of the proposition provides conditions on primitives so that the conditions in Corollary 2 hold. When the demand parameter in the second period is sufficiently large relative to the demand parameter in the first period, the optimal production path features intertemporal linkages. There are two implications of this result. When the firm expects better demand conditions in period 2 (i.e., when θ_2 increases), holding fixed θ_1, it adjusts by hiring more workers in the first period. This result holds because higher growth in the future creates promotion opportunities for workers today, which allows the firm to reduce the wages of workers currently assigned to activity 1, reducing the cost of hiring more workers in period 1.

Going in the other direction, the proposition also shows that there are lingering effects of past demand conditions. In particular, better demand conditions in the first period (i.e., higher values of θ_1) lead the firm to increase its size in the first period. The firm can do so at a lower cost if it expands workers' promotion opportunities by also increasing its size in the second period. One implication is that two firms facing the same demand conditions in the second period may operate at different sizes because they had different demand conditions in the past. The firm that had better demand conditions in the past will be larger precisely in order to provide promotion opportunities for the workers it hired in the past. This implication formalizes the idea, present in the early works of Barnard (1938), Jensen (1986), and Drucker (2009) that organizations may be biased towards growth in order to provide more opportunities for career advancement.

6 Discussion of Model Assumptions

Our discussion so far makes use of several simplifying assumptions that streamline the exposition. Specifically, we analyze a simple moral-hazard environment in which workers must receive rents in order to be motivated to exert effort, and we assume the firm puts in place a deterministic production path. Below, we discuss how the paper's results are affected by each of several assumptions.

Non-zero minimum wage: The model assumes that the minimum wage each worker must receive is $\underline{w} = 0$. Our main results continue to hold for an interval of minimum wage levels. If the minimum wage is sufficiently negative, then workers' participation constraints can be made binding in their first period of employment, implying that workers need not receive rents (Carmichael, 1985). In this case, there would be no need to back-load worker compensation across activities and therefore no need to make use of promotion-based incentives. Moreover, optimal production would be undistorted and intertemporally independent. As long as the minimum wage is not too low, so that workers must be given rents, the main features of optimal personnel policies remain. In particular, seniority-based promotions and intertemporal linkages may still arise.

In terms of optimal production paths, as long as the minimum wage is not too high, our results remain unchanged as long as $\theta_t \partial f(N_{1,t}, N_{2,t}) / \partial N_{1,t} > \underline{w}$ for all $(N_{1,t}, N_{2,t})$. If this inequality fails to hold, then the optimal production path will satisfy $\theta_t \partial f(N_{1,t}, N_{2,t}) / \partial N_{1,t} = \underline{w}$. In general, either new hires into activity 1 will receive rents R_1 or their marginal productivity will be equal to the minimum wage.

Worker heterogeneity: The model assumes that all workers are identical. One interpretation of our model is that the analysis is carried out on workers who are qualified to be promoted: even for qualified workers, promotion opportunities may be constrained by a firm's production path, and they may be allocated according to seniority. The logic of our analysis can be extended to allow for heterogeneity in how qualified workers are for activity 2, and it suggests that even if an older worker is less talented than a recently hired worker, he may nevertheless get promoted before the more talented worker.

Specifically, consider a three period model in which the optimal personnel policy features seniority-based promotions. In other words, cohort-1 workers are promoted at a higher rate than cohort-2 workers at the end of the second period. Now suppose that each cohort-2 worker is as productive as $\kappa > 1$ cohort-1 workers when assigned to activity 2. If promotion decisions depend entirely on worker ability, then all cohort-2 workers will be need to be promoted before cohort-1 workers. Under such a promotion policy, cohort-1 workers promotion opportunities are sigificantly reduced, and the firm must pay them more in terms of wages to keep them motivated. These extra wage payments may exceed the productivity gains associated with promoting cohort-2 workers when κ is close to 1. Promotion policies that depend only on worker ability may therefore be dominated by policies that take seniority into account.

Monitoring technology: The main model considers a monitoring technology in which a signal of $y_{i,t} = 0$ is perfectly indicative that the worker shirked. Another commonly studied monitoring technology is one in which a signal of $y_{i,t} = 1$ is perfectly indicative that the worker worked. For example, suppose $\Pr[y_{i,t} = 1 | e_t] = q_i e_t$. Under such a monitoring technology, there is no need to pay workers incentive rents: the firm can pay the worker 0 if $y_{i,t} = 0$ and a bonus of c/q_i if $y_{i,t} = 1$.[2] Since no incentive rents are required to motivate workers, optimal personnel policies are static, and optimal production paths would feature no distortions.

Stochastic production paths: Our main results for cost-minimizing personnel policies can be modified to allow $(N_{1,t}, N_{2,t})$ to be a stochastic process. In particular, suppose that $(N_{1,t}, N_{2,t})$ is a Markov process that takes on a countable number of values for each t and is steady along each path realization, in the sense that the firm never grows so fast that it needs to hire directly into activity 2, and the firm never shrinks so fast that it cannot assign all its incumbent workers to a productive activity. Because each worker is risk-neutral, his continuation payoffs in period t depend on his expected promotion probability, taking expectations over the continuation process for the

[2] Of course, if the minimum wage is sufficiently high, then this monitoring technology will also require workers to be given rents. In this case, as long as activity 2 requires more rents than activity 1, then many of the features of internal labor markets will arise. Additional features such as up-or-out promotions or firing on the equilibrium path might also be part of an optimal personnel policy (Fong and Li, 2017).

production path beginning in period $t+1$. In this case, optimal personnel policies again resemble an internal labor market, and it may still be strictly optimal to base promotions on seniority.

7 Unsteady Production and Partial Effort Contracts

In this section, we first characterize properties of cost-minimizing personnel policies in environments in which the production plan is unsteady. We next explore whether and why a firm might want to adopt a partial-effort contract: a contract in which some workers are not expected to exert effort in some periods. Finally, we show how our main model can be extended to analyze environments in which production paths are stochastic.

7.1 Unsteady Environments

We now explore the properties of cost-minimizing personnel policies for a wide class of production path. Our analysis above shows that if the firm can choose both the production path and the personnel policy at the same time, some of the cases we discuss below will not occur. Nevertheless, in some settings, the production path is either partially or completely inflexible (for example, in a bureaucracy).

The analysis in Section 5 presumed that the firm's production path N was a steady production path. That is, we assumed that for each t, $N_{2,t+1} \leq (1-d)(N_{1,t}+N_{2,t})$ and, for each i, $N_{i,t+1} \geq (1-d)N_{i,t}$. In this section, we explore characteristics of optimal personnel policies when N is not a steady production path. We will say that N experiences *breakneck growth at* $t+1$ if $N_{2,t+1} > (1-d)(N_{1,t}+N_{2,t})$, and we will say that N involves *deep downsizing in* i *at* $t+1$ if $N_{i,t+1} < (1-d)N_{i,t}$, and *deep downsizing for the firm overall* at $t+1$ if $N_{1,t+1}+N_{2,t+1} < (1-d)(N_{1,t}+N_{2,t})$.

For our analysis of optimal personnel policies under deep downsizing, we expand the number of activities an employee can be assigned to in each period to include a null activity in which the employee's effort has no impact on the firm's production. We therefore an employee's time-t activity assignment by $A_s \in \{0,1,2\}$, where $A_s = 0$ denotes the null activity, and we denote by $p_{0,t}^{\tau}$ the probability that a cohort-τ worker will be assigned to the null activity in $t+1$ conditional on remaining with the firm and having good performance in each period of employment.

7.1.1 Breakneck Growth

Suppose N experiences breakneck growth for the first time at $t+1$, that is, even if the firm promotes all workers assigned to activity 1 in period t, it must place some new hires at $t+1$ into activity 2. This implies that all workers hired prior to period $t+1$ must earn a continuation payoff of R_2

at the beginning of period $t+1$. We can then break the optimal personnel policy problem up into two problems.

We first solve for the optimal personnel policies in periods $1, \ldots, t$, treating t as effectively the last period of production but with the requirement that all incumbent workers at period t receive R_2 in continuation payoffs. For the second problem, we solve for optimal personnel policies in periods after $t+1$, and we take as given that all workers in cohorts prior to $t+1$ will initially be assigned to activity 2 and will therefore receive rents equal to R_2. In other words, the analysis can be carried out chunk-by-chunk, where each chunk starts with a period in which breakneck growth occurs and ends with the next period in which breakneck growth occurs. Within each chunk, the optimal personnel policy minimizes the rents that are paid to new hires assigned to activity 1, and the same type of analysis as in Section 5 can be applied, so the main results continue to hold.

7.1.2 Deep Downsizing

When firms go through periods of deep downsizing, managing personnel can be more complicated. In this section, we explore some features of personnel policies that might arise. If deep downsizing is permanent, in the sense that once there is deep downsizing in one period, there is deep downsizing in all future periods, the firm will never hire new workers, and it will shrink faster than by attrition alone. When this is the case, in order to motivate workers in their last period of employment, the firm has to pay severance pay to workers that it will not employ in the future. Proposition 5 describes optimal personnel policies in this case.

Proposition 5. *Suppose N satisfies $N_{1,t+1} < (1-d)\, N_{1,t}$, $N_{2,t+1} > (1-d)\, N_{2,t+1}$, and $N_{1,t+1} + N_{2,t+1} < (1-d)\,(N_{1,t} + N_{2,t})$ for all t. There is an optimal personnel policy in which (i.) laid-off workers receive severance pay, (ii.) if $\tau < \tau'$, then $p_{0,t}^{\tau} \leq p_{0,t}^{\tau'}$, and (iii.) conditional on being laid off, workers with more seniority receive greater severance pay.*

This proposition describes an optimal personnel policy for a firm that must downsize in every future period. The first part shows that when employees are laid off, they are paid severance pay in their last period of employment. Severance pay is necessary to maintain employees' incentives to exert effort in their last period of employment. The second part of the proposition shows that an optimal personnel policy exhibits a last-in-first-out pattern for layoffs: employees with more seniority are less likely to be laid off in each period. The final part shows that if employees of different cohorts are laid off in the same period, their severance payments are higher the longer they have been employed by the firm.

We now discuss some features of optimal personnel policies for a firm experiencing temporary deep downsizing, that is, the firm must downsize in one period, and there is a future period at which it will need to hire again. We will say that a worker is **permanently laid off** in period t if he is assigned to activity 0 in all future periods with probability 1. We say that a worker is **temporarily laid off** in period t if he is assigned to activity 0 in period $t+1$ and is assigned to activity 1 or 2 in a future period with positive probability. The next proposition partially characterizes optimal personnel policies when a firm experiences temporary deep downsizing.

Proposition 6. *Suppose there is a t_1 at which $N_{1,t_1+1} < (1-d)\, N_{1,t_1}$ and $N_{1,t_1+1} + N_{2,t+1} < (1-d)\,(N_{1,t_1} + N_{2,t_1})$ and there exists a $t_2 > t_1$ at which $N_{1,t_2+1} + N_{2,t_2+1} > (1-d)\,(N_{1,t_2} + N_{2,t_2})$. Then (a) no workers are permanently laid off in period t_1, and (b) $v^{\tau}_{1,t_2+k} \geq v^{t_2+k}_{1,t_2+k}$ for all $\tau < t_2$ and for all $k \geq 1$.*

The conditions for Proposition 6 imply that the firm must downsize at t_1, and at time t_2+1, it recovers and must hire workers into one of the two positions. This proposition shows that whenever this is the case, the firm favors rehiring laid-off workers. If, instead, the firm hired new workers, it would have to pay them rents in their first period of employment. By rehiring laid-off workers, the firm can allocate these rents to these workers and reduce the overall rents it has to pay. The second part of the proposition shows that temporarily laid off workers will be rehired before the firm hires a worker who has never worked for the firm in the past, and moreover, these workers receive higher continuation payoffs than new hires. The rationale for this result is similar to the logic underlying why seniority-based promotions can be optimal.

7.2 Partial Effort Contracts and Sabbaticals

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8 Conclusion and Discussion

In this paper, we develop a model of production and personnel management. We first show that optimal personnel policies resemble internal labor markets in which seniority plays an important role in promotion and wage decisions. Our main result sheds light on Baker, Jensen, and Murphy's (1988) observation that an "important problem with promotion-based reward systems is that they require organizational growth to feed the reward system." (p. 600) Indeed, in order to make use of promotion-based incentives, the firm has to grow faster than would be productively efficient. Yet, our model shows that doing so can be optimal ex ante, if not ex post. This time-inconsistency in the firm's production plans results from the optimal provision of long-term incentives to its employees.

One implication of our model is that firms may pursue inefficient growth in order to honor past promises. It may be difficult to distinguish such growth from empire building on the part of the executive team. Financial scholars suggest that firms should leverage up in order to commit their executives not to pursue inefficient growth strategies (Jensen, 1986). While such solutions might reduce CEO moral hazard, our model suggests that they might undermine the firm's ability to make efficient use of long-term incentives for its workforce.

The motive for inefficient growth we highlight abstracts from, but has implications for, the important strategic choices firms have to make when they decide to expand. Interwar DuPont, for example, pursued growth through diversification, expanding into other lines of business rather than expanding its existing business. One important issue they had to address was whether to expand organically or through acquisition. Our model suggests that organic growth may create additional career opportunities for existing employees that growth through acquisition might not. Future work examining the personnel implications of different ways of expanding can help improve our understanding of the dynamics of corporate strategy.

Our model is a first step in understanding the interaction between production plans and personnel policies, and it leaves out many factors. For instance, we assume employees are risk-neutral and make binary effort choices, and the firm has full commitment power. In addition, workers do not acquire human capital, and there is no uncertainty about their productivity.[3] Future work that incorporates these factors can improve our understanding of personnel policies that firms adopt in richer environments and how they interact with firm growth.

[3] Many papers examine how these different features affect personnel and supplier dynamics and hence firm-level productivity dynamics but do not speak directly to the dynamics of firm size. For papers emphasizing the role of supplier heterogeneity, see Board (2011), DeVaro and Waldman (2012), DeVaro and Morita (2013), Andrews and Barron (2016), Board, Meyer-ter-Vehn, and Sadzik (2017); for papers emphasizing human capital acquisition, see Gibbons and Waldman (1999, 2006); for papers emphasizing risk aversion and continuous effort, see Harris and Holmstrom (1982) and Holmstrom and Ricart i Costa (1986), Chiappori, Salanie, and Valentin (1999); for papers emphasizing lack of commitment, see Malcomson (1984), MacLeod and Malcomson (1988)

Appendix A: Cost-Minimizing Personnel Policies

Lemma 1. *Given N if there is an optimal personnel policy, there is an optimal personnel policy in which workers with the same employment history face the same wage and assignment policies.*

Proof of Lemma 1. If there is an optimal personnel policy in which two workers with the same employment history receive different wage and/or assignment policies, then we can consider an alternative assignment and wage policy that is a public randomization between these policies, and if both players are subject to this same alternative policy, their incentive constraints and the firm's flow constraints remain satisfied.■

Lemma 2. *Cost-minimizing personnel policies minimize the rents paid to new hires:*

$$\min \sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^t L\left(n^t\right) v\left(n^t\right)$$

subject to (1), (2), *and* (3).

Proof of Lemma 2. The PDV of the firm's wage bill, times δ is

$$\sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^t L\left(h^t\right) w\left(h^t\right).$$

For all workers who currently work in the firm in period t, that is for those for which $A_t \neq 0$, the flow constraint gives us

$$\begin{aligned} L\left(h^t 1\right) &= (1-d)\, p_1\left(h^t\right) L\left(h^t\right) \\ L\left(h^t 2\right) &= (1-d)\, p_2\left(h^t\right) L\left(h^t\right). \end{aligned}$$

In addition, for $i \in \{1,2\}$, we can write $N_{i,t} = \sum_{h^t | A_t = i} L\left(h^t\right)$. We can write the period-t wages paid to workers with employment history h^t as $L\left(h^t\right) w\left(h^t\right)$, which equals

$$\begin{aligned} & L\left(h^t\right) v\left(h^t\right) + L\left(h^t\right) c\left(h^t\right) - \delta\left(1-d\right) L\left(h^t\right)\left(p_1\left(h^t\right) v\left(h^t 1\right) + p_2\left(h^t\right) v\left(h^t 2\right)\right) \\ = \;& L\left(h^t\right) v\left(h^t\right) + L\left(h^t\right) c\left(h^t\right) - \delta L\left(h^t 1\right) v\left(h^t 1\right) - \delta L\left(h^t 2\right) v\left(h^t, 2\right), \end{aligned}$$

where the first equality plugs in the promise-keeping constraint for workers with employment history h^t, and the second equality plugs in the flow constraint.

The total wage bill is the sum of these expressions over time and over employment histories and is therefore

$$
\begin{aligned}
& \sum_{t=1,h^t}^{T} \delta^t L\left(h^t\right) w\left(h^t\right) \\
= & \sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^t \left(L\left(h^t\right) v\left(h^t\right) + L\left(h^t\right) c\left(h^t\right) - \delta L\left(h^t 1\right) v\left(h^t 1\right) - \delta L\left(h^t 2\right) v\left(h^t 2\right)\right) \\
= & \sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^t L\left(h^t\right) c\left(h^t\right) + \sum_{t=1}^{T} \sum_{h^t \in \mathcal{H}^t} \delta^t \left(L\left(h^t\right) v\left(h^t\right) - \delta L\left(h^t 1\right) v\left(h^t 1\right) - \delta L\left(h^t 2\right) v\left(h^t 2\right)\right) \\
= & \sum_{t=1}^{T} \delta^t \left(N_{1,t} c_1 + N_{2,t} c_2\right) + \sum_{t=1}^{T} \delta^t L\left(n^t\right) v\left(n^t\right),
\end{aligned}
$$

where recall that $L\left(n^t\right)$ are the new workers hired into the firm in period t. It follows that the firm's objective is simply to minimize

$$\sum_{t=1}^{\infty} \delta^t L\left(n^t\right) v\left(n^t\right),$$

which establishes the lemma.

Lemma 3. *Given a steady production path N, if there is an optimal personnel policy, there is an optimal personnel policy with the following three properties:*
(i.) $v\left(h^t\right) \leq R_2$ if $A_t = 1$ and $v\left(h^t\right) = R_2$ if $A_t = 2$.
(ii.) All new workers are assigned to activity 1.
(iii.) $p_2\left(h^t\right) = 1$ if $A_t = 2$ and $p_1\left(h^t\right) + p_2\left(h^t\right) = 1$ if $A_t = 1$.

Proof of Lemma 3. To establish part (i.), we will first show that for all h^t, we do not need to have both $w\left(h^t\right) > 0$ and $v\left(h^t\right) > R\left(A_t\right)$. To establish this intermediate result, there are two cases to consider. First, suppose the worker is a new hire in period t. In this case, if both $w\left(h^t\right) > 0$ and $v\left(h^t\right) > R\left(A_t\right)$, the firm can reduce the wage bill by reducing $w\left(h^t\right)$ without violating the incentive constraint. Second, if the worker was a new hire prior to period t, the firm can reduce $w\left(h^t\right)$ and increase $w\left(h^{t-1}\right)$ to maintain $v\left(h^{t-1}\right)$. This establishes the intermediate result and shows that it is without loss of generality to focus on personnel policies in which in each period, either the minimum wage constraint is binding or the IC constraint is binding. We will use this result to establish part (i.), but we do not make use of it in our other results.

For part (i.), there are two cases to consider. First, suppose $w\left(h^t\right) > 0$. Then by the previous result, we have $v\left(h^t\right) = R\left(h^t\right) \leq R_2$. Next, suppose $w\left(h^t\right) = 0$. We can then consider all histories that follow h^t. With probability 1, the workers must eventually receive a strictly positive wage, or else there must be some employment history following h^t at which his incentive constraint is violated. If $w\left(h^t\right) = 0$, we can write $v\left(h^t\right)$ as

$$v\left(h^t\right) = \sum_{h^\tau} \Pr\left[h^\tau \middle| h^t\right] \left(\sum_{s=0}^{\tau-t-1} \delta^s \left(-c_1\right) + \delta^{\tau-t} v\left(h^\tau\right)\right),$$

where h^τ is the first history following h^t such that $w\left(h^\tau\right) > 0$. We can again use the previous result to get

$$\begin{aligned} v\left(h^{t}\right) &= \sum_{h^{\tau}} \Pr\left[h^{\tau}\middle| h^{t}\right]\left(\sum_{s=0}^{\tau-t-1} \delta^{s}\left(-c_{1}\right)+\delta^{\tau-t} R\left(h^{\tau}\right)\right) \\ &< \sum_{h^{\tau}} \Pr\left[h^{\tau}\middle| h^{t}\right] R\left(h^{\tau}\right) \leq R_{2}, \end{aligned}$$

which establishes part $(i.)$.

For part $(ii.)$, note that part $(i.)$ implies that $v\left(h^{t}\right) \leq R_{2}$ if $A_{t}=1$. As a result, if a new worker is assigned to activity 2, it is better instead to assign them to activity 1 and promote an existing worker assigned to activity 1 to instead be assigned to activity 2. This would relax the existing workers' incentive constraints and reduce the wage bill.

Finally, for part $(iii.)$, suppose $p_{1}\left(h^{t}\right)+p_{2}\left(h^{t}\right)<1$. Because $N_{i,t+1} \geq (1-d) N_{i,t}$ for $i=1,2$, we must have that $L\left(n^{t+1}\right)>0$, so there must be positive hiring into either position 1 or position 2. We will construct a perturbation to the personnel policy in which any rents that would be paid out to new hires are paid out, instead, to currently employed workers. This perturbation will introduce slack into some current employees' incentive constraints, and it will not increase the total wage bill. If a positive mass of new workers is hired and assigned to activity 1, $L\left(\vec{0}1\right)$, let $\tilde{p}_{1}\left(h^{t}1\right)=p_{1}\left(h^{t}1\right)+\varepsilon$, and let $\tilde{L}\left(\vec{0}1\right)=L\left(\vec{0}1\right)-\varepsilon(1-d) L\left(h^{t}\right)$. This perturbation preserves the flow constraint, and it relaxes workers' incentive constraints in periods $s \leq t$ for those workers with history h^{t}. This perturbation therefore weakly decreases the firm's overall wage bill. A similar perturbation can be constructed if $L\left(\vec{0}2\right)>0$. Result (i) of this lemma implies that $v\left(h^{t}\right)=R_{2}$ if $A_{t}=2$, which implies that $p_{2}\left(h^{t}\right)=1$ if $A_{t}=2$.$\blacksquare$

Proposition 1. *If N is a steady production path, an internal labor market is an optimal personnel policy.*

Proof of Proposition 1. Follows directly from Lemma 3 and the definition of an internal labor market.

Proposition 2. *Given a steady production path N, if there is a cost-minimizing personnel policy, an internal labor market with the following properties is cost-minimizing:* (1) *$w_{1,t}^{\tau}$ is weakly increasing in t,* (2) *if $\tau<\tau'$, $w_{1,t}^{\tau} \geq w_{1,t}^{\tau'}$ and $v_{1,t}^{\tau} \geq v_{1,t}^{\tau'}$, and* (3) *the promotion policy is a modified FIFO policy.*

Proof of Proposition 2. First, note that for any t, in any optimal personnel policy, it must be the case that $v\left(h^{t}\right) \geq v\left(n^{t}\right)$, that is, new hires receive lower rents than incumbent workers. Suppose to the contrary that $v\left(h^{t}\right)<v\left(n^{t}\right)$. We can then "switch" the future history of a worker with employment history h^{t} with a new worker. This switch preserves the total wage bill and relaxes the incentive constraints of workers whose employment histories are consistent with h^{t}.

Now, suppose $\tau_{1}<\tau_{2}$. We can write the rents workers that each cohort receives in period t if they are assigned to activity 1 as follows:

$$\begin{aligned} v_{1,t}^{\tau_{1}} &= w_{1,t}^{\tau_{1}}-c_{1}+\delta(1-d)\left(p_{t}^{\tau_{1}} R_{2}+\left(1-p_{t}^{\tau_{1}}\right) v_{1,t+1}^{\tau_{1}}\right) \\ v_{1,t}^{\tau_{2}} &= w_{1,t}^{\tau_{2}}-c_{1}+\delta(1-d)\left(p_{t}^{\tau_{2}} R_{2}+\left(1-p_{t}^{\tau_{2}}\right) v_{1,t+1}^{\tau_{2}}\right). \end{aligned}$$

Take a rent $v_{1,t}^{\tau_1}$. We can always reduce $w_{1,t}^{\tau_1}$ by ε and increase $v_{1,t+1}^{\tau_1}$ by $\varepsilon / \left[\delta(1-d)\left(1-p_t^{\tau_1}\right)\right]$, maintaining rents $v_{1,t}^{\tau_1}$, unless either $w_{1,t}^{\tau_1}=0$ or $v_{1,t+1}^{\tau_1}=R_2$. We can do this similar for the τ_2 cohort. Let $\bar{w}_{1,t}^1$ and $\bar{w}_{1,t}^2$ denote the resulting activity-1 wages at which this procedure terminates and $\bar{v}_{1,t+1}^{\tau_1}$ and $\bar{v}_{1,t+1}^{\tau_2}$ the resulting continuation payoffs. There are four cases to consider: (*i.*) $\bar{w}_{1,t}^1=0$, $\bar{w}_{2,t}^1=0$; (*ii.*) $\bar{w}_{1,t}^1>0$, $\bar{w}_{1,t}^2>0$; (*iii.*) $\bar{w}_{1,t}^1=0$, $\bar{w}_{1,t}^2>0$; and (*iv.*) $\bar{w}_{1,t}^1>0$, $\bar{w}_{1,t}^2=0$.

The first observation is that case (*iv.*) is impossible, because it would imply that $v_{1,t}^{\tau_1}>v_{1,t}^{\tau_2}$. If $\bar{w}_{1,t}^1>0$, this implies that $\bar{v}_{1,t+1}^{\tau_1}=R_2$, and as a result, it must be the case that cohort-τ_1's continuation payoff weakly exceeds cohort-τ_2's continuation payoff, and so do the wages.

Next, in case (*ii.*), $\bar{v}_{1,t+1}^{\tau_1}=\bar{v}_{1,t+1}^{\tau_2}=R_2$, so both cohorts have the same continuation payoffs. Moreover, if $v_{1,t}^{\tau_1} \leq v_{1,t}^{\tau_2}$, it must be the case that $\bar{w}_{1,t}^{\tau_1} \leq \bar{w}_{1,t}^{\tau_2}$. Define $\tilde{p}_t=\left(L_{1,t} p_t^{\tau_1}+L_{2,t} p_t^{\tau_2}\right) /\left(L_{1,t}+L_{2,t}\right)$, where $L_{i,t}$ is the mass of cohort-i workers assigned to activity 1 in period t. Promoting both cohorts at rate $\tilde{p}_t$ maintains the flow constraints, and it does not affect $v_{1,t}^{\tau_1}$ or $v_{1,t}^{\tau_2}$, so such a personnel policy is optimal if the original personnel policy is optimal, and it satisfies property (*ii.*) of the proposition. It also satisfies property (*i.*), which means that after period t, both cohorts earn wages $\hat{w}=c_1+(1-\delta(1-d)) R_2$, which must weakly exceed $\bar{w}_{1,t}^{\tau_1}$ and $\bar{w}_{1,t}^{\tau_2}$, or else $v_{1,t}^{\tau_1}$ or $v_{1,t}^{\tau_2}$ would exceed R_2. Moreover, property (*iii.*) is satisfied because $\bar{v}_{1,t+1}^{\tau_1}=\bar{v}_{1,t+1}^{\tau_2}=R_2$.

In case 3, $\bar{v}_{1,t+1}^{\tau_2}=R_2$, which necessarily exceeds $\bar{v}_{1,t+1}^{\tau_1}$. If $p_t^{\tau_1} \leq p_t^{\tau_2}$, then properties (*i.*) and (*ii.*) are automatically satisfied. We can then decrease $p_t^{\tau_1}$ by ε, increase $p_t^{\tau_2}$ by $L_{1,t} \varepsilon / L_{2,t}$. This perturbation does not affect $v_{1,t}^{\tau_2}$, since $\bar{v}_{1,t+1}^{\tau_2}=R_2$, and in order to maintain $v_{1,t}^{\tau_1}$, we increase $\tilde{v}_{1,t+1}^{\tau_1}$. We can keep doing this until either $\tilde{p}_t^{\tau_1}=0$ or $\tilde{p}_t^{\tau_2}=0$. Now, suppose $p_t^{\tau_1}>p_t^{\tau_2}$. Then choose $\tilde{p}_t^{\tau_1}=\tilde{p}_t^{\tau_2}=\left(L_{1,t} p_t^{\tau_1}+L_{2,t} p_t^{\tau_2}\right) /\left(L_{1,t}+L_{2,t}\right)$. This construction maintains cohort-τ_2's continuation payoff. Increase $\bar{v}_{1,t+1}^{\tau_1}$ to $\tilde{v}_{1,t+1}^{\tau_1}$ which maintains the same continuation payoff for cohort-τ_1. This construction satisfies properties (*i.*) and (*ii.*). Further, we can alter this construction just as we did in the proof of case 2 in order to construct an optimal personnel policy that satisfies property (*iii.*).

Finally, consider case 1. Set $\tilde{p}_t^{\tau_1}=\tilde{p}_t^{\tau_2}=\left(L_{1,t} p_t^{\tau_1}+L_{2,t} p_t^{\tau_2}\right) /\left(L_{1,t}+L_{2,t}\right)$, and choose $\tilde{v}_{1,t+1}^{\tau_1}$ and $\tilde{v}_{1,t+1}^{\tau_2}$ to maintain the same continuation payoffs for both cohorts. Since $v_{1,t}^{\tau_1} \leq v_{1,t}^{\tau_2}$, it must be the case that $\tilde{v}_{1,t+1}^{\tau_1} \leq \tilde{v}_{1,t+1}^{\tau_2}$. This establishes properties (*i.*) and (*ii.*), and we can use a similar argument as above to construct an optimal personnel policy that satisfies property (*iii.*).■

Appendix B: Optimal Production

This appendix characterizes the firm's problem of choosing an optimal production path. We will first describe the firm's full problem of choosing an optimal production path and a cost-minimizing personnel policy that implements that production path. To do so, define z_t^s to be the fraction of cohort-s workers who are still assigned to activity 1 in period t. Using the result of Lemma 3, it is without loss of generality to set $v_{2,t}^s=R_2$ for every t and every cohort s.

The firm's problem is to

$$\max_{\left\{N_{i,t}, H_t, w_{1,t}^s, z_t^s, v_{1,t}^s, p_t^s\right\}_{t=1}^T} \sum_{t=1}^T \delta^{t-1}\left[\theta_t f\left(N_{1,t}, N_{2,t}\right)-w_{2,t} N_{2,t}-\sum_{s=1}^t H_s w_{1,t}^s z_t^s\right]$$

subject to a flow constraint for $N_{1,t}$ ensuring that the new hires in period t plus those new hires prior to t who remain in activity 1 in period t sum up to $N_{1,t}$:

$$H_t + \sum_{s=1}^{t-1} H_s z_t^s = N_{1,t}.$$

The firm must also satisfy a flow constraint for $N_{2,t}$,

$$N_{2,t-1}(1-d) + \sum_{s=1}^{t-1} H_s z_{t-1}^s p_{t-1}^s = N_{2,t},$$

ensuring that workers previously assigned to activity 2 and who remain, as well as other workers who were promoted prior to $t-1$, sum up to $N_{2,t}$. The fraction z_t^s must evolve according to

$$z_{t+1}^s = z_t^s (1 - p_t^s)(1-d),$$

and the firm must also satisfy (IC) and (IR) for each worker in each period.

Define $D_{i,t} = \theta_t \partial f(N_{1,t}, N_{2,t}) / \partial N_{i,t} - w_{i,t}^t$ to be the firm's period-t wedge in activty i. The following propsition characterizes the evolution of these wedges.

Proposition 3. *If an optimal production path is steady, it has the following features:*

1. $D_{1,t} \geq 0$ *for all* t.
2. $\sum_{s=t}^{T} \delta^{s-t} (1-d)^{s-t} D_{2,s} \leq 0$ *for all* t.
3. *In the first period,* $D_{i,1} \geq 0$ *for* $i = 1, 2$*, and in the last period,* $D_{1,T} = 0$ *and* $D_{2,T} \leq 0$.

Proof of Proposition 3. To prove the first claim, consider a perturbation in which the firm reduces $N_{1,t}^*$ by ε. To keep future profits constant, suppose that in $t+1$, when the firm has to fill the εz_{t+1}^t positions these workers would have occupied, the firm hires εz_{t+1}^t new workers and treats them as if they were hired at period t. This perturbation does not affect future profits, and it increases period-t profits by $\left(-\theta_t \partial f / \partial N_{1,t} + w_{1,t}^t\right) \varepsilon \leq 0$, or else N^* is not optimal.

To establish the second claim, we can write the Lagrangian for the firm's constrained maximization problem as

$$\begin{aligned} \mathcal{L} \quad = \quad & \sum_{t=1}^{T} \delta^t \left[\theta f_t (N_{1,t}, N_{2,t}) - w_{2,t} N_{2,t} - \sum_{s=1}^{t} H_s w_{1,t}^s z_t^s \right] \\ & + \sum_{t=1}^{T} \delta^t \mu_t \left[H_t + \sum_{s=1}^{t-1} H_s z_t^s - N_{1,t} \right] \\ & + \sum_{t=1}^{T} \delta^t \eta_t \left[N_{2,t} - N_{2,t-1}(1-d) - \sum_{s=1}^{t-1} H_s z_{t-1}^s p_{t-1}^s \right], \end{aligned}$$

where $\delta^t \mu_t$ is the Lagrange multiplier for the flow constraint for $N_{1,t}$ and $\delta^t \eta_t$ is the Lagrange multiplier for the flow constraint for $N_{2,t}$. The optimality conditions for the problem are

$$\frac{\partial F_t}{\partial N_{1,t}} = w_{1,t}^t + \sum_{\tau=t+1}^{T} \delta^{\tau-1} z_\tau^t \left(w_{1,\tau}^t - \mu_\tau + p_\tau^t \eta_\tau \right) \tag{1}$$

and

$$\frac{\partial F_t}{\partial N_{2,t}} = w_{2,t}^t - \eta_t + \delta \left(1-d\right) \eta_{t+1}, \tag{2}$$

where μ_t and η_t can be shown to be nonnegative.

Consider a perturbation in which the firm increase $N_{2,t}$ by ε, $N_{2,t+1}$ by $(1-d)\,\varepsilon$, $N_{2,t+2}$ by $(1-d)^2\,\varepsilon$, and so on. By (2), this increases the firm's profits by $\sum_{s=t}^{T} \delta^{s-t} (1-d)^{s-t} D_{2,s}^* + \eta_t = 0$ for all t. Since $\eta_t \geq 0$, the second part of the proposition follows.

Finally, for the third part of the proposition, note that $\eta_{T+1} = 0$, so $D_{2,T}^* \leq 0$ and by the first part, $D_{1,T}^* \leq 0$. Finally, to see why $D_{2,1}^* \geq 0$, note that $\eta_1 = 0$, so that

$$0 = D_{2,1}^* + \sum_{s=2}^{T} \delta^{s-2} (1-d)^{s-2} D_{2,s}^* = D_{2,1}^* - \eta_2.$$

This completes the proof.■

Lemma 4. *In an optimal production path, workers receive no excess rents.*

Proof of Lemma 4. In order to get a contradiction, suppose that $v\left(n^t\right) > R_1$ for some n^t with $n_t = 1$. Then it must be the case that $w_{1,t}^t = 0$ or else the firm could reduce $w_{1,t}^t$ while still satisfying the incentive constraint for new hires in period t. We will construct a perturbation that holds fixed the firm's profits for all periods $\tau > t$ and under which the firm produces more in period t without paying any additional wages.

Suppose the firm hires ε additional workers into the bottom job in period t. Let the firm promote these workers with probability 1 in period τ, where τ is the first period in which existing new hires in period t are promoted with positive probability, and let the firm pay these workers $\tilde{w}_{1,t'}^t = 0$ for all $t \leq t' \leq \tau$. Notice that these workers' incentive constraints are satisfied. To see this, note that by Lemma 3, existing new hires can be paid $w_{1,t'}^t = 0$ for all $t \leq t' \leq \tau$ and have their incentive constraints satisfied in periods t to τ. The additional new workers are promoted with probability 1 in period τ, so their continuation payoffs in each period prior to τ are weakly higher than it is for existing new hires.

For the existing new hires, in period τ, reduce their promotion probability in period τ by ε/H_t and fire these workers with the same probability. This perturbation preserves the flow constraints and continues to satisfy the incentive constraints for existing new hires. Notice that it increases production in period t by $\theta_t f\left(N_{1,t}+\varepsilon, N_{2,t}\right) - \theta_t f\left(N_{1,t}, N_{2,t}\right) > 0$, and it preserves the firm's wage bill, which contradicts the claim that $v\left(n^t\right) > R_1$.■

This lemma has several important implications. Define the set $\mathcal{N}$ to be the set of production paths for which there exists a personnel policy that satisfies (IC), (IR), and $(Flow)$ and gives new hires no excess rents. Define the

$$\tilde{\pi}\left(N\right) = \sum_{t=1}^{T} \left(\theta_t f\left(N_{1,t}, N_{2,t}\right) - c_1 N_{1,t} - c_2 N_{2,t} - H_t R_1\right) - N_{2,1} R_2$$

where $H_t = N_{1,t} + N_{2,t} - (1-d)(N_{1,t-1} + N_{2,t-1})$ for $t > 1$ and $H_1 = N_1$, and let $\pi(N)$ be the profits associated with the cost-minimizing personnel policy given N. Define $\tilde{\pi}^* = \max_{N \in \mathcal{N}} \tilde{\pi}(N)$, and define $\pi^* = \max_N \pi(N)$ to be the firm's maximized profits.

The first lemma shows that these two profit levels coincide and therefore shows that the problem of choosing an optimal production path boils down to maximizing the objective $\tilde{\pi}(N)$ subject to the constraint that new rents can be given no excess rents. The lemma therefore shows how we can simplify both the objective function and the constraint set for the problem when solving for the optimal production path.

Lemma 4A. $\pi^* = \tilde{\pi}^*$.

Proof of Lemma 4A. For all N, we have that $\tilde{\pi}(N) \geq \pi(N)$, because the optimal personnel policy under an exogenously given N may pay excess rents to new hires. First, we will show that $\tilde{\pi}^* \geq \pi^*$. To see why this is the case, note that by Lemma 4, $\tilde{\pi}(N^*) = \pi(N^*)$ and that at N^*, there exists a feasible personnel policy that gives new hires no excess rents. This means that $\tilde{\pi}^* \geq \tilde{\pi}(N^*) = \pi(N^*) = \pi^*$.

Next, we will show that $\pi^* \geq \tilde{\pi}^*$. To see this, let $\tilde{N}^*$ maximize $\tilde{\pi}\left(\tilde{N}\right)$ subject to the constraint that $\tilde{N} \in \mathcal{N}$. At this $\tilde{N}^*$, there exists a feasible personnel policy that generates profits $\tilde{\pi}\left(\tilde{N}^*\right)$ and gives no excess rents to new hires. We therefore have $\tilde{\pi}^* = \tilde{\pi}\left(\tilde{N}^*\right) = \pi(N^*) \leq \pi^*$, completing the proof.■

The next set of results provides necessary and sufficient conditions for the optimal production path to be locally intertemporally independent.

Lemma 5. *The solution to the relaxed problem is a solution to the full problem, $\tilde{N}^* = N^*$, if and only if under $\tilde{N}^*$, there exists an incentive-compatible personnel policy that gives each new hire no excess rents.*

Proof of Lemma 5. First, suppose $\tilde{N}^*$ solves the full problem. Then the associated personnel policy must be incentive compatible, and it must give each new hire into activity i rents R_i by Lemma 4. Conversely, suppose there exists an incentive-compatible personnel policy under $\tilde{N}^*$ that gives each new hire into activity i rents R_i. Then the associated personnel policy satisfies all the constraints of the full problem, and since the firm's profit under $\tilde{N}^*$ and the associated personnel policy is an upper bound on the firm's profits, $\tilde{N}^*$ solves the full problem.■

Lemma 6. *Given a demand path θ, the solution to the relaxed problem is a solution to the full problem, $\tilde{N}^* = N^*$, if and only if θ is in the closure of Θ^I.*

Proof of Lemma 6. TBA.

Corollary 1. *The solution to the relaxed problem is a solution to the full problem if $\tilde{p}_t < \hat{p}$ for all t.*

Proof of Corollary 1. Suppose $\left\{\tilde{N}_{1,t}, \tilde{N}_{2,t}\right\}_{t=1}^{T}$ is a solution to the relaxed problem and that $\tilde{p}_t < \hat{p}$ for all t. Consider the following personnel policy. In each period t, all workers who have not yet been promoted are promoted with probability $\tilde{p}_t$, and they receive a wage $w_{1,t}$ satisfying

$$R_1 = \tilde{w}_{1,t} - c_1 + \delta(1-d)(\tilde{p}_t R_2 + (1-\tilde{p}_t) R_1).$$

Since $\tilde{p}_t < \hat{p}$ for each t, the wage $\tilde{w}_{1,t} \geq 0$. This personnel policy satisfies incentive compatibility in each period and provides new hires with no excess rents.■

Corollary 2. *Suppose $\tilde{N}^*$ is a solution to the relaxed problem and there exists some τ such that*

$$\sum_{t=2}^{\tau+1} \delta^{t-1} \tilde{\Delta}_t^* R_2 - \sum_{t=1}^{\tau} \delta^{t-1} \tilde{N}_{1,t}^* c_1 + \delta^\tau \left(\tilde{N}_{1,\tau+1}^* - \tilde{H}_{\tau+1}^* \right) R_1 > \sum_{t=1}^{\tau} \delta^{t-1} \tilde{H}_t^* R_1,$$

where $\tilde{H}_t^*$ is the mass of new hires into activity 1 in period t. Then the optimal production path N^* does not have local intertemporal independence.

Proof of Corollary 2. Let N be a production path, and abusing notation slightly, define H_1 to be the mass of new hires into the bottom job in the first period. Under an optimal personnel policy, we can bound the rents that are paid to new hires from below. To do so, we look at all cohorts hired prior to period τ. To calculate the discounted sum of the rents paid to these workers, the total rents they must receive can be decomposed into three parts.

$$\begin{aligned} & H_1 v_1 + \delta H_2 v_2 + \cdots + \delta^\tau H_\tau v_\tau \\ \geq \quad & \underbrace{\sum_{t=2}^{\tau+1} \delta^{t-1} \Delta_t R_2}_{\Delta_t \text{ promoted at } t-1 \text{ and get } R_2} \quad - \underbrace{\sum_{t=2}^{\tau} \delta^{t-1} N_{1,t} c_1}_{\text{effort costs}} + \delta^\tau \underbrace{\left(N_{1,\tau+1} - H_{\tau+1} \right)}_{\text{not promoted before } \tau+1} R_1. \end{aligned}$$

The first part of the right-hand side of this inequality is the rents that accrue to these workers when they are promoted to activity 2. The second part is the payoffs workers receive when they are assigned to activity 1 up to time τ, where notice that they always receive nonnegative wages. The third part is the continuation payoffs for these workers when they remain assigned to activity 1 in period $\tau + 1$, where notice that this continuation payoff must weakly exceed R_1.

Now, suppose to the contrary that N^* is locally intertemporally independent. By Lemma 5, $N^* = \tilde{N}^*$. Moreover, by Lemma 4, all new hires receive no excess rents, so $v_t = R_1$ for all t. Then the inequality above implies that

$$\sum_{t=2}^{\tau+1} \delta^{t-1} \tilde{\Delta}_t^* R_2 - \sum_{t=1}^{\tau} \delta^{t-1} \tilde{N}_{1,t}^* c_1 + \delta^\tau \left(\tilde{N}_{1,\tau+1}^* - \tilde{H}_{\tau+1}^* \right) R_1 \leq \sum_{t=1}^{\tau} \delta^{t-1} \tilde{H}_t^* R_1.$$

If this condition fails to hold, then it must be the case that $N^* \neq \tilde{N}^*$.■

Proposition 4. *There is a continuous and increasing function $\bar{\theta}_2(\theta_1)$ such that if $\theta_2 < \bar{\theta}_2(\theta_1)$, then N^* is locally intertemporally independent, that is, $\partial N_{i,1}^* / \partial \theta_2 = 0$ and $\partial N_{i,2}^* / \partial \theta_1 = 0$ for $i = 1, 2$. If $\theta_2 > \bar{\theta}_2(\theta_1)$, then there are forward and backward linkages, that is, $\partial N_{i,1} / \partial \theta_2 > 0$ for $i = 1, 2$, and $\partial N_{2,2} / \partial \theta_1 > 0$.*

Proof of Proposition 4. First, note that because $T = 2$, there is no need to give excess rents to new hires in period 2 because there are no future promotion opportunities for them. Moreover, any worker hired at $t = 1$ directly into activity 2 will also not receive excess rents. The only additional constraint that needs to be checked, therefore, is that the firm can give cohort-1 workers assigned to activity 1 no excess rents. This constraint can be written as $N_{2,2} \leq (1 - d) \hat{p} N_{1,1} + (1 - d) N_{2,1}$.

To see why this additional constraint is necessary, note that if it is not satisfied, cohort-1 workers must be promoted at a rate exceeding $\hat{p}$, which means that even if they receive $w_{1,1}^1 = 0$, their rents will exceed R_1. This constraint is also sufficient, since if it is satisfied, cohort-1 workers assigned to activity 1 will be promoted at rate $(N_{2,2} - (1-d) N_{2,1}) / ((1-d) N_{1,1}) < \hat{p}$, and their first-period incentive constraints can be satisfied with a positive wage $w^1_{1,1} > 0$.

The firm's problem is

$$\max_{\{N_{1,t},N_{2,t}\}_{t=1}^{T}} \theta_1 f(N_{1,1}, N_{2,1}) - (c_1 + R_1) N_{1,1} - (c_2 + R_2) N_{2,1}$$
$$+ \sum_{t=2}^{T} [\theta_t f(N_{1,t}, N_{2,t}) - c_1 N_{1,t} - c_2 N_{2,t} - H_t R_1],$$

subject to the constraint that $N_{2,2} \geq (1-d) N_{1,1}\hat{p} + (1-d) N_{2,1}$. Let μ be the Lagrange multiplier on this constraint. The Kunh-Tucker conditions for the firm's problem are

$$\begin{aligned}
\theta_1 f_1' \left(N^*_{1,1}\right) &= c_1 + (1 - \delta(1-d)) R_1 - \mu^* (1-d) \hat{p} \\
\theta_1 f_2' \left(N^*_{2,1}\right) &= c_2 + R_2 - \delta(1-d) R_1 - \mu^* (1-d) \\
\theta_2 f_1' \left(N^*_{1,2}\right) &= c_1 + R_1 \\
\theta_2 f_2' \left(N^*_{2,2}\right) &= c_2 + R_1 + \mu^*,
\end{aligned}$$

as well as $\mu^* \geq 0$, $N^*_{2,2} \geq (1-d) N^*_{1,1}\hat{p} + (1-d) N^*_{2,1}$, and complementary slackness.

Suppose the constraint is slack. Then the associated solution, N^*, in fact solves the constrained maximization problem if $N^*_{2,2} > (1-d) N^*_{1,1}\hat{p} + (1-d) N^*_{2,1}$, or

$$f_2'^{-1}\left(\frac{c_2 + R_1}{\theta_2}\right) > (1-d)\hat{p} f_1'^{-1}\left(\frac{c_1 + (1-\delta(1-d)) R_1}{\theta_1}\right) + (1-d) f_1'^{-1}\left(\frac{c_2 + R_2 - \delta(1-d) R_1}{\theta_1}\right).$$

Since f_1 and f_2 are strictly concave, the left-hand side of this inequality is increasing in θ_2, and the right-hand side is increasing in θ_1. Given θ_1, define $\bar{\theta}_2(\theta_1)$ so that this inequality holds with equality. This function is increasing in θ_1, and it is continuous. Moreover, for all $\theta_2 > \bar{\theta}_2(\theta_1)$, the inequality is satisfied, and therefore N^* solves the firm's relaxed and full problems and is locally intertemporally indepndent.

Next, suppose $\theta_2 < \bar{\theta}_2(\theta_1)$. Then it must be the case that $N^*_{2,2} = (1-d) N^*_{1,1}\hat{p} + (1-d) N^*_{2,1}$. The firm's optimality conditions can be combined to give us

$$\theta_1 f_1' \left(N^*_{1,1}\right) + (1-d)\hat{p}\theta_2 f_2' \left(N^*_{2,2}\right) = c_1 + (1 - \delta(1-d)) R_1 + (1-d)\hat{p}(c_2 + R_1) \tag{1}$$

and

$$\theta_1 f_2' \left(N^*_{2,1}\right) + (1-d)\theta_2 f_2' \left(N^*_{2,2}\right) = c_2 + R_2 - \delta(1-d) R_1 + (1-d)(c_2 + R_2). \tag{2}$$

Note that the right-hand sides of these equations do not depend on θ_1 or θ_2.

We will first show that $\partial N^*_{i,1}/\partial\theta_2 > 0$. Differentiating (1) with respect to θ_2, we get

$$\theta_1 f_1'' \left(N^*_{1,1}\right) \frac{\partial N^*_{1,1}}{\partial \theta_2} = -(1-d)\hat{p}\left[f_2'\left(N^*_{2,2}\right) + \theta_2 f_2''\left(N^*_{2,2}\right)\frac{\partial N^*_{2,2}}{\partial\theta_2}\right].$$

In order to get a contradiction, suppose $f_2'\left(N^*_{2,2}\right) + \theta_2 f_2''\left(N^*_{2,2}\right)\partial N^*_{2,2}/\partial\theta_2 < 0$. Since $f_1'' < 0$, this implies that $\partial N^*_{1,1}/\partial\theta_2 < 0$. A similar argument establishes that $\partial N^*_{2,1}/\partial\theta_2 < 0$. But since $N^*_{2,2} = (1-d) N^*_{1,1}\hat{p} + (1-d) N^*_{2,1}$, it must be the case that $\partial N^*_{2,2}/\partial\theta_2 < 0$, but this contradicts the assumption that $f_2'\left(N_{2,2}^*\right)+\theta_2 f_2''\left(N_{2,2}^*\right)\partial N_{2,2}^*/\partial\theta_2<0$. It must therefore be the case that $\partial N_{i,1}^*/\partial\theta_2>0$.

Next, we will snow that $\partial N_{2,2}^*/\partial\theta_1>0$. Differentiating (1) with respect to θ_1, we get

$$f_1'\left(N_{1,1}^*\right)+\theta_1 f_1''\left(N_{1,1}^*\right)\frac{\partial N_{1,1}^*}{\partial\theta_1}=-\left(1-d\right)\hat{p}\theta_2 f_2''\left(N_{2,2}^*\right)\frac{\partial N_{2,2}^*}{\partial\theta_1},$$

and differentiating (2) with respect to θ_1, we get

$$f_1'\left(N_{2,1}^*\right)+\theta_1 f_2''\left(N_{2,1}^*\right)\frac{\partial N_{2,1}^*}{\partial\theta_1}=-\left(1-d\right)\hat{p}\theta_2 f_2''\left(N_{2,2}^*\right)\frac{\partial N_{2,2}^*}{\partial\theta_1}.$$

In order to get a contradiction, suppose $\partial N_{2,2}^*/\partial\theta_1<0$. Then it must be the case that $f_i'\left(N_{i,1}^*\right)+\theta_1 f_i''\left(N_{i,1}^*\right)\partial N_{i,1}^*/\partial\theta_1<0$ for $i=1,2$. But then we must have $\partial N_{i,1}^*/\partial\theta_1>0$ for $i=1,2$, and since $N_{2,2}^*=\left(1-d\right)N_{1,1}^*\hat{p}+\left(1-d\right)N_{2,1}^*$, $\partial N_{2,2}^*/\partial\theta_1>0$, which is a contradiction. It must therefore be the case that $\partial N_{2,2}^*/\partial\theta_1>0$.■

Appendix C: Unsteady Production and Partial Effort Contracts

Proposition 5. *Suppose N satisfies $N_{1,t+1}<\left(1-d\right)N_{1,t}$, $N_{2,t+1}>\left(1-d\right)N_{2,t+1}$, and $N_{1,t+1}+N_{2,t+1}<\left(1-d\right)\left(N_{1,t}+N_{2,t}\right)$ for all t. There is an optimal personnel policy in which (i.) laid-off workers receive severance pay, (ii.) if $\tau<\tau'$, then $p_{0,t}^{\tau}\leq p_{0,t}^{\tau'}$, and (iii.) conditional on being laid off, workers with more seniority receive greater severance pay.*

Proof of Proposition 5. Using a similar argument as in the proof of Proposition 2, we may assume that $v_{1,t}^{\tau}$ is decreasing in τ. That is, later-cohort workers value being assigned to activity 1 in period t more than newer-cohort workers. Given an optimal personnel policy, we now construct an optimal personnel policy with the desired properties by specifying $w_{1,t}^{\tau}, p_{0,t}^{\tau}, p_{2,t}^{\tau}, v_{1,t+1}^{\tau}$, and $v_{0,t+1}^{\tau}$. To do this, we proceed in three steps.

First, we will assign promotion opportunities in each period to workers so that workers with positive promotion probabilities all receive the same continuation payoff if they are not promoted, and the average rate of promotion for workers satisfies the flow constraint for activity 2, that is, $p_{2,t}=\left[N_{2,t+1}-\left(1-d\right)N_{2,t}\right]/\left[\left(1-d\right)N_{1,t}\right]$. In particular, it can be shown that there exists a k such that for all $\tau>k$, we have $p_{2,t}^{\tau}=0$, and for all $\tau,\tau'\leq k$, promotion probabilities will satisfy the following two sets of equations. First, for all $\tau,\tau'\leq k$

$$\frac{1-p_{2,t}^{\tau}}{1-p_{2,t}^{\tau'}}=\frac{v_{1,t}^{\tau}+c_1-\delta\left(1-d\right)R_2}{v_{1,t}^{\tau'}+c_1-\delta\left(1-d\right)R_2},$$

which ensures that, if they receive a wage of 0 this period, workers promoted with positive probability receive the same continuation conditional on not being promoted. Second, the flow constraint for activity 2 is satisfied $\sum_{\tau=1}^{k}N_{1,t}^{\tau}p_{2,t}^{\tau}=p_{2,t}N_{1,t}$. These two sets of equations pin down k and $p_{2,t}^{\tau}$ for all τ. Given the associated promotion probabilities, we can write, for each τ,

$$v_{1,t}^{\tau}=-c_1+\delta\left(1-d\right)\left(p_{2,t}^{\tau}R_2+\left(1-p_{2,t}^{\tau}\right)\tilde{v}_{t+1}^{\tau}\right).$$

Notice that our construction ensures that $\tilde{v}^{\tau}_{t+1} = \tilde{v}^{\tau'}_{t+1}$ for all $\tau, \tau' \leq k$, and $\tilde{v}^{\tau}_{t+1} \geq \tilde{v}^{\tau'}_{t+1}$ for all $k \leq \tau \leq \tau'$.

In the second step, we construct wages $w^{\tau}_{1,t}$ and continuation payoffs $\hat{v}^{\tau}_{t+1}$ for each cohort to guarantee that each cohort is promoted with the same probability as in the previous step, they receive the same payoffs $v^{\tau}_{1,t}$, and $\hat{v}^{\tau}_{t+1} \leq R_2$ for all τ. That is,

$$w^{\tau}_{1,t} = \max\left\{v^{\tau}_{1,t} + c_1 - (1 - \delta(1-d)) R_2, 0\right\}.$$

This implies that we can write

$$v^{\tau}_{1,t} = w^{\tau}_{1,t} - c_1 + \delta(1-d)\left(p^{\tau}_{2,t} R_2 + \left(1 - p^{\tau}_{2,t}\right)\hat{v}^{\tau}_{t+1}\right).$$

Notice that this construction implies there is some k' such that $w^{\tau}_{1,t} = 0$ for all $\tau \geq k'$.

Finally, we construct severance probabilities $p^{\tau}_{0,t}$ so that workers with the least seniority are laid off first, and we construct severance values $v^{\tau}_{0,t+1}$ so that the incentive constraints for laid-off workers remain satisfied. To this end, let $v^{\tau}_{0,t+1} = \hat{v}^{\tau}_{t+1} = v^{\tau}_{1,t+1}$, and write $p^{\tau}_{0,t} = \left(1 - p^{\tau}_{2,t}\right)\rho^{\tau}_{t}$, where ρ^{τ}_{t} is the probability of being laid off conditional on not being promoted. The flow constraint for activity 1 requires that the number of workers who are laid off is equal to the number of workers the firm has to get rid of, or

$$\sum_{\tau=1}^{t} p^{\tau}_{0,t} N^{\tau}_{1,t} = (1-d)(N_{1,t} + N_{2,t}) - (N_{1,t+1} + N_{2,t+1}).$$

This constraint implies there exists a k'' such that $\rho^{\tau}_{t} = 1$ for all $\tau > k''$, and $\rho^{\tau}_{t} = 0$ for all $\tau < k''$. This constructed policy satisfies all the conditions in the statement of the proposition.■

Proposition 6. *Suppose there is a t_1 at which $N_{1,t_1+1} < (1-d) N_{1,t_1}$ and $N_{1,t_1+1} + N_{2,t+1} < (1-d)(N_{1,t_1} + N_{2,t_1})$ and there exists a $t_2 > t_1$ at which $N_{1,t_2+1} + N_{2,t_2+1} > (1-d)(N_{1,t_2} + N_{2,t_2})$. Then (a) no workers are permanently laid off in period t_1, and (b) $v^{\tau}_{1,t_2+k} \geq v^{t_2+k}_{1,t_2+k}$ for all $\tau < t_2$ and for all $k \geq 1$.*

Proof of Proposition 6. Suppose $L\left(n^{t_2+1}\right) > 0$ and there is a positive mass of workers who worked for the firm by t_2 but are assigned to activity 0 in period t_2 and receive payoff v^{τ}_{0,t_2+1} for some $\tau < t_2$. Suppose such workers are permanently laid off. There are then two cases: either $v^{\tau}_{0,t_2+1} \geq v^{t_2+1}_{i,t_2+1}$ or $v^{\tau}_{0,t_2+1} < v^{t_2+1}_{i,t_2+1}$.

In the first case, consider an alternative personnel policy in which the firm does not hire the new worker and instead rehires the old worker and treats him the way the firm would have treated the new worker but pays him an additional $v^{\tau}_{0,t_2+1} - v^{t_2+1}_{i,t_2+1}$ in period $t_2 + 1$. This new personnel policy still satisfies the flow constraint for activity i, and it satisfies the promise-keeping constraint and the incentive constraints for the re-hired worker, and it pays out less in rents to new hires, so it reduces the overall wage bill.

In the second case in which $v^{\tau}_{0,t_2+1} < v^{t_2+1}_{i,t_2+1}$, similarly consider an alternative personnel policy in which the firm does not hire the new worker and instead rehires the old worker and treats him exactly the same ways as the firm would have treated the new worker. This new personnel policy is again feasible and reduces the overall wage bill because it pays out less in rents to new hires. This establishes property (a).

For part (b), if it is ever the case that $v^{\tau}_{1,t_2+k} < v^{t_2+k}_{1,t_2+k}$, then we can instead give the new worker initial rents of v^{τ}_{1,t_2+k} and give the cohort-τ worker period t_2+k rents of $v^{t_2+k}_{1,t_2+k}$. The associated personnel policy relaxes the cohort-τ worker's incentive constraint for all periods $t \leq t_2+k$, and it reduces the initial rents of the cohort-t_2+k worker while maintaining their incentive constraint. It therefore reduces the firm's overall wage bill.■

Proposition 7. *Under Assumptions 1-4, any full-effort contract is strictly suboptimal.*

Proof of Proposition 7. In this setting, the wages in period 3 are fixed and equal to $w_i^* = 2c_i$ for $i = 1, 2$. Suppose the firm does not hire new workers in period 2. Under a full-effort contract, it is impossible to motivate cohort-1 workers assigned to activity 1 in both periods if the firm sets wages for activity 1 equal to zero, since for such workers, $v^1_{1,1} \leq -c_1 - c_1 + (2c_2 - c_2)$ with equality if the worker is promoted with probability 1 in period 3. By Assumption 3, $v^1_{1,1} < 0$.

Next, we show that the firm can set the wages for activity 1 to be zero in both periods 1 and 2 if it hires new workers in period 2 and does not ask for effort from cohort-1 workers. Specifically, notice that the second-period growth rate of the firm is sufficiently high to ensure that even if the firm hires all new workers in period 2, it will be able to promote all cohort-1 and cohort-2 workers. In this case, cohort-1 workers receive $v^1_{1,1} = -c_1 + (2c_2 - c_2) > 0$, and similarly, cohort-2 workers receive $v^2_{1,2} = -c_1 + (2c_2 - c_2) > 0$. This shows that full-effort contracts are strictly suboptimal.■

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