Reconciling VAR-based Forecasts with Survey Forecasts*

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Abstract

This paper proposes a novel Bayesian approach to model realized data and survey forecast of the same variable jointly in a vector autoregression (VAR). In particular, our method imposes a prior distribution on the consistency between the forecast implied by the VAR and the survey forecast for the same variable. When the prior is placed on unconditional forecasts from the VAR, the prior shapes the posterior of the reduced-form VAR coefficients. When the prior is placed on conditional forecasts (i.e. impulse responses), the prior shapes the posterior of the structural VAR coefficients. To implement our prior, we combine importance sampling with a maximum entropy prior for forecast consistency to obtain posterior draws of VAR parameters at low computational cost. We use two empirical examples to illustrate some potential applications of our methodology: (i) the evolution of tail risks for inflation in a time-varying parameter VAR model and (ii) the identification of credible forward guidance shocks using sign and forecast-consistency restrictions in a monetary VAR model.

Keywords: Vector Autoregression (VAR), Survey Forecasts, Bayesian VAR, Inflation risk, Forward Guidance
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1 Introduction

Changing expectations about the future economic outlook are important for understanding macroeconomic dynamics based on people's intertemporal decisions. Consumption and investment as well as price-setting all depend on agents' beliefs of the future. Many empirical studies use forecasts based on a vector autoregression (VAR) as proxies for expectations (e.g. Campbell and Shiller (1988) and Keating (1990)). However, using multi-step ahead VAR forecasts as proxies for long-term expectations crucially depends on the assumption that the current and past values included in the VAR correctly represent all information available to agents, which may not hold when the VAR includes only a small number of aggregate macro variables. Leduc and Sill (2013) argue that this problem can be alleviated by adding survey forecasts to the conventional VAR framework because they can provide a sufficient measure of expectations beyond that implied by the past behavior of macroeconomic aggregates.

However, simply adding survey data to the VAR without imposing consistency between the VAR forecast and the survey forecast can generate an internal inconsistency. As noted by Cogley (2005), if one variable in the VAR (e.g. long-term interest rate) is closely linked to future expected values of another variable (e.g. short-term interest rate) in the VAR, then within one model there exist two different forecasts for essentially the same variable. Previous work has sought to resolve this tension by imposing restrictions on the reduced-form model's coefficients. For example, Kozicki and Tinsley (2012) impose the internal consistency between expected inflation based on an autoregressive (AR) model of inflation with time-varying intercepts and survey-based measures of expected inflation. They express the parameters determining the evolution of survey forecasts for inflation as explicit functions of parameters governing the evolution of actual inflation. While strictly imposing internal consistency is theoretically appealing, the practice may overly restrict the process representing the evolution of survey forecasts because there are no independent parameters to govern it. In addition, there are often conceptual differences between the variable and the survey forecast available. For example, the Livingston Survey/the Survey of Professional Forecasters provides forecasts of GNP growth prior to 1992 and an alternative concept of GDP thereafter than is implied by the current vintage of GDP data available form the Bureau of Economic Analysis (BEA). Moreover, imposing strict consistency in practice can be computationally burdensome, especially beyond bivariate VAR models with no more than a couple of lags.

In this paper, we propose a Bayesian approach to address the internal consistency issue posed by including realized data and survey forecasts of the same variable in a VAR. In particular, we argue for constructing a non-degenerate prior for VAR coefficients that places greater mass on areas of the parameter space where the restrictions implied by forecast consistency are closer to being satisfied. We call this the *forecast-consistent prior.* Relative to this previous literature, we draw an important distinction between the unconditional and conditional forecasts from the VAR. When the forecast-consistent prior is placed unconditional forecasts, forecasts implied using observed data up to time t to forecast the VAR at $t+h$, then our prior shrinks the reduced-form VAR coefficients in the direction of satisfying cross-equation restrictions. However, when the forecast-consistent prior is placed on conditional forecasts, impulse response functions, then our prior informs the set of structural VAR models to be considered. From an implementation standpoint, our method is much less computationally burdensome compared to strictly imposing the coefficient restrictions as in Cogley (2005). In particular, we simply re-weight posterior draws obtained without using the forecast-consistent prior by importance sampling weights based on the forecast-consistent prior. These weights are informed by a hyperparameter which specifies the precision of our prior. In the limit, as the prior precision grows arbitrarily high, we can nest the strict imposition of cross-equation restrictions used in the previous literature as a special case.

We illustrate the implications of imposing unconditional forecast consistency for assessing inflation trail-risk in a time-varying parameter (TVP) VAR model. The model includes survey forecasts for inflation with realized inflation, a real activity measures and a monetary policy instrument as in Clark and Davig (2011). We then use the posterior estimates to compute time-varying tail risks to inflation, a topic that has received much attention since the recession of 2007-09 (Anene and D'Amico, 2017; Fleckenstein, Longstaff and Lustig, 2017; Hattori, Schrimpf and Sushko, 2016). Tail risks in in our model are quite comparable to estimates based on financial market data such as inflation swaps and options from Fleckenstein, Longstaff and Lustig (2017) although we do not use market data at all. Imposing forecast consistency restrictions reduces tail risks of inflation on average during the period of 2009:Q4-2015:Q4, reflecting the stability of inflation expectations in survey data in recent decades. The estimated deflation probabilities came down more rapidly after spiking up in 2008:Q3 when forecast consistency restrictions are imposed. Short-run inflation expectations from surveys bottom-out in 2009:Q1 and have not declined much since then. When forecast consistency restrictions are not imposed, deflation probabilities are less sensitive to this change in the survey forecast. Hence, imposing forecast consistency restrictions amplifies the influence of survey forecasts on the VAR-based forecast of future inflation and makes tail risks more sensitive to changes in survey forecasts.

We then use the conditional forecast consistent prior to identify credible forward guidance shocks. We augment to the Uhlig (2005) VAR model survey forecasts of short-term interest rates (3-month T-bill rate) one year ahead. Applying similar sign restrictions that Uhlig (2005) proposes to this augmented model, we find that – similar to Uhlig's finding for the effects of conventional monetary policy shocks – the effects of expansionary forward guidance shocks on output are ambiguous. In some instances, output actually contracts following what is typically thought of as an expansionary forward guidance shock. However, we also show that these sign-restrictions admit a wide-range of time paths for the actual federal funds rate. Many of these time paths deviate significantly from the path of rates predicted by forecasters following the forward guidance shock. We then impose the forecast consistent prior in addition to the basic sign restrictions on the impulse responses to elicit forward guidance shocks in which the path of actual rates is tilted toward the path forecasters expect. We call such shocks *credible* forward guidance shocks. We find that these shocks lead to a modest, but persistent expansion in output.

The rest of our paper is organized as follows. Section 2 illustrates the basic idea behind forecast consistent priors using a simple example of a bi-variate VAR(1) with one-period ahead survey forecasts. Section 3 provides details of our approach in a more general case with multi-horizon and multiple survey forecasts. Section 4 illustrates how to apply the methodology to impulse response functions from structural VAR models. Section 5 provides an empirical application of our unconditional forecast-consistent prior methodology to a TVP-VAR model to assess time-varying tail risk. Section 6 provides an empirical application of our conditional forecast-consistent prior methodology to identify credible forward guidance shocks in a structural VAR. Section 7 relates our work to the previous literature in detail and Section 8 concludes.

2 A simple example: VAR(1) with one-period ahead survey forecasts

Let's consider the following bivariate VAR(1) model for $y_t = [x_t\,,\, E_t^s(x_{t+1})]$. x_t is a variable of interest such as inflation or output growth and $E_t^s(x_{t+1})$ is the one-period ahead forecast from the available survey on x_t.

$$y_t = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} y_{t-1} + u_t = Ay_{t-1} + u_t\,,\; u_t \sim i.i.d.\mathcal{N}(0, \Sigma_u). \tag{1}$$

We assume the following Normal and Inverse-Wishart conjugate priors for ($\alpha = \text{vec}(A)$, Σ_u).

$$\begin{aligned}\Sigma_u &\sim IW(\Psi, d),\\ \alpha|\Sigma_u &\sim \mathcal{N}(\overline{\alpha}, \Sigma_u \otimes \Omega),\end{aligned} \tag{2}$$

where $\Psi, d, \overline{\alpha}$ and Ω are hyperparameters. Let's stack $[y_1, \cdots, y_T]'$ by Y and $[y_0, \cdots, y_{T-1}]$ by X. The ordinary least squares (OLS) estimator for A is $(X'X)^{-1}(X'Y)$ and the posterior mean of A is

$$\widehat{A} = (X'X + \Omega^{-1})^{-1}(X'Y + \Omega^{-1}\overline{A}), \tag{3}$$

where $\text{vec}(\overline{A}) = \overline{\alpha}$ and $\widehat{\alpha} = \text{vec}(\widehat{A})$. The posterior draws of Σ_u and α follow the Normal and Inverse-Wishart distributions.

$$\begin{aligned}\Sigma_u|y_0^T &\sim IW(\Psi + \widehat{u}'\widehat{u} + (\widehat{A} - \overline{A})'(\Omega)^{-1}(\widehat{A} - \overline{A}), T - 1 + d),\\ \alpha|\Sigma_u, y_0^T &\sim \mathcal{N}(\widehat{\alpha}, \Sigma_u \otimes (X'X + \Omega^{-1})^{-1}),\end{aligned} \tag{4}$$

where $\widehat{u} = Y - X\widehat{A}$.

Now we turn to imposing cross-equation restrictions implied by the internal consistency of forecasts. From the first equation of the VAR, we can generate the VAR-based forecast for x_{t+1} at time t.

$$E_t^{\text{VAR}}(x_{t+1}) = a_{11}x_t + a_{12}E_t^s(x_{t+1}). \tag{5}$$

Imposing the consistency restriction that $E_t^{\text{VAR}}(x_{t+1}) = E_t^s(x_{t+1})$ for all the values of x_t implies the following restrictions on VAR coefficients.

$$g(A) = e_1'A - e_2' = [0, 0] \;\Leftrightarrow\; [a_{11}, a_{12} - 1] = [0, 0]. \tag{6}$$

In this case, a_{21} and a_{22} remain free parameters. [1] By adopting the Bayesian approach, we can vary the tightness of these cross-equation restrictions to allow a possibility that survey forecasts may be an imperfect proxy for the VAR-based forecast. Depending on the degree of tightness, we can consider the three cases as follows:

1. **Strong Consistency**: $g(A) = [0, 0]$. Therefore, $a_{11} = 0$ and $a_{12} = 1$.

2. **Weak Consistency**: $g(A) \sim \mathcal{N}(0, (\lambda W)^{-1})$. In other words, the average one-period ahead survey forecast is consistent with the average VAR-based one-period ahead forecast. But they may deviate from each other from time to time. This notion of weak consistency can be modeled by a non-degenerate prior distribution for $g(A)$. The normal distribution is the maximum entropy prior for $g(A)$ under constraints on first and second moments. So the prior under weak consistency is the distribution maximizing entropy under the constraint that the first moment of $g(A)$ is zero and the second moment of $g(A)$ is $(\lambda W)^{-1}$. Our choice of the maximum entropy prior is motivated by the fact that it minimizes the amount of prior information on the moments of $g(A)$ other than the chosen moments translated to the prior distribution, in the information-theoretic sense.[2]

3. **No Consistency**: $g(A)$ is not restricted and we can assume a diffuse prior for it.

Strong consistency and no consistency can be regarded as limiting cases when λ approaches ∞ and 0, respectively. In terms of A, the prior density function of $g(A)$ can be treated like the likelihood function for A using observations satisfying these restrictions.[3] We use this property to calculate the posterior density of α that satisfies cross-equation restrictions as follows. First, we denote a prior density kernel for α that satisfies forecast consistency by $h(\alpha) = p(g(A(\alpha)))$. Second, we obtain the new posterior density of α that satisfies forecast consistency as follows:

$$p(\alpha|\Sigma_u, y_0, Y, g(A)) = \frac{h(\alpha)p(\alpha|y_0^T, \Sigma_u)}{\int h(\alpha)p(\alpha|y_0^T, \Sigma_u,)d\alpha}. \tag{7}$$

[1]Under this restriction, $u_{1,t}$ is a martingale difference series of unanticipated shocks to x_t while $u_{2,t}$ represents a news shock for x_{t+1} at time t.

[2]See Robert (2007); Cover and Thomas (2012).

[3]The maximum entropy prior for $g(A)$ is similar to the limited information likelihood for A in Kim (2002) if first and second moment restrictions for $g(A)$ are interpreted as nonlinear moment restrictions for A. We replace sample moment conditions for artificial observations satisfying weak consistency by population moment conditions as in Del Negro and Schorfheide (2004).

Using the above definition, we can simulate posterior draws of α from this new density simply by re-weighting existing posterior draws with no consistency. Define the following importance weight for α^i that is drawn from $p(\alpha|\Sigma_u, y_0, Y)$,

$$w(\alpha^i) = \frac{h(\alpha^i)}{\sum_{j=1}^{M} h(\alpha^j)}\ ,\ \alpha^j \sim p(\alpha|\Sigma_u, y_0, Y). \tag{8}$$

Now, we resample these draws according to new weights $[w(\alpha^1), \cdots, w(\alpha^M)]$.

3 A general case involving multi-horizon and multiple survey forecasts

Our methodology can be easily extended to a more general case involving multi-horizon and multiple survey forecasts. In this general case, consistency restrictions become nonlinear functions of VAR coefficients.

3.1 Multi-horizon survey forecasts

Survey forecasts typically involve multi-horizon forecasts as well as one-period ahead forecasts. We can derive implications of the consistency of longer-horizon forecasts for VAR coefficients similarly as we did above. Since the multi-step forecast in the VAR involves the repeated iteration of one-step ahead forecasts, the resulting cross-equation restriction shows up as a nonlinear function of all the VAR coefficients. For example, consider the following VAR(1) model with two-step ahead forecasts where $y_t = [x_t, E_t^s(x_{t+2})]'$. The VAR implied two-step ahead forecast is

$$E_t^{\mathrm{VAR}}(x_{t+2}) = (a_{11}^2 + a_{12}a_{21})x_t + a_{12}(a_{11} + a_{22})E_t^s(x_{t+2}). \tag{9}$$

The strong consistency implies that the following two nonlinear restrictions should hold for VAR coefficients.

$$\begin{aligned} g_1(A) &= a_{11}^2 + a_{12}a_{21} = 0, \\ g_2(A) &= a_{12}(a_{11} + a_{22}) - 1 = 0. \end{aligned} \tag{10}$$

We can put together the two restrictions into a vector of nonlinear equations $g(A)$ as in the previous section. Unlike the one-step ahead forecast case, solving for a_{11} and a_{12} as functions of a_{21} and a_{22} to impose the strong consistency is complicated because there might be cases of multiple solutions or no solution. This raises a particular challenge when we want to impose strong consistency restrictions as in Cogley (2005). However, this is not a serious issue for weak consistency because cross-equation restrictions hold only probabilistically and we do not need to solve nonlinear equations.

3.2 Multiple survey forecasts

In a more general case, the VAR may include multiple survey forecasts of different horizons. For instance, y_t may contain a long-horizon survey forecast as well as a short-horizon one as follows:

$$y_t = Ay_{t-1} + u_t\ ,\ y_t = [x_t, E_t^s(x_{t+1}), E_t^s(x_{t+2})]'. \tag{11}$$

In general, the number of cross-equation restrictions implied by consistency requirements is equal to the number of parameters describing the evolution of survey forecasts. So in the above three-variable VAR(1) with two survey forecasts, consistency requirements generate $2 \times 3 = 6$ cross-equations on 9 parameters in A. To generalize this, we look at the dimension of cross-equation restrictions from the following a VAR(P) with non-zero constants and n_s survey forecasts.

$$y_t = A_0 + A_1[y'_{t-1}, \cdots, y'_{t-p}] + u_t\ ,\ y_t = [x_{1,t}, \cdots, x_{n_x,t}, E_t^s(x_{1,t+1}), \cdots, E_t^s(x_{n_x,t+h})]'. \tag{12}$$

The number of equation (n_y) is equal to the sum of the number of x_t variables (n_x) and the number of survey forecasts ($n_s <= n_x \times h$). Consistency requirements will impose $n_s \times (n_s + n_x) \times (p+1)$ cross-equation restrictions on the $(n_x + n_s)^2 \times n_x(p+1)$ matrix $A = [A_0, A_1]$.

4 Imposing forecast consistency in impulse responses

While the methodology to this point has emphasized consistency between the unconditional VAR forecasts, the approach can also be applied to functions of conditional forecasts following an economic shock. Impulse responses to identified structural shocks represent one such conditional forecast that is of interest to macroeconomists. For illustration, consider the structural VAR using the notation used in Section 3.2 above:

$$By_t = BA_0 + BA_1[y'_{t-1}, \cdots, y'_{t-p}] + \varepsilon_t, \tag{13}$$

where ε_t are orthogonal structural shocks, each with a standard deviation of one. These structural shocks are related to the reduced form residuals u_t by way of the mapping $\varepsilon_t = B^{-1}u_t$ where $u_t \sim \mathcal{N}(0, \Sigma_u)$. It will be convenient to define the cross-equation restrictions associated with expectational consistency in terms of the companion form of the VAR:

$$Y_t = \mathcal{A}_0 + \mathcal{A}_1 Y_{t-1} + \mathcal{B}^{-1} U_t, \tag{14}$$

where $Y_t = [y_t', \cdots, y_{t-p-1}']'$, $U_t = [u_t; 0_{n_y(p-1)\times 1}]$, $\mathcal{A}_0 = [A_0; 0_{n_y(p-1)\times 1}]$, $\mathcal{A}_1 = [A_1; I_{n_y} \otimes I_{(p-1)}, 0_{n_y(p-1)\times 1}]$, and $\mathcal{B}^{-1} = [B^{-1}; 0_{n_y(p-1)\times n_y}]$.

Suppose that we are interested in the response to the structural innovation ε_t^1 occurring at time t. Then, the k-step ahead forecast of all the variables is given by :

$$E_{t|\varepsilon_t^1}^{VAR}(Y_{t+k}) = [I_{n_y p} + \mathcal{A}_1 + \cdots + \mathcal{A}_1^k]\mathcal{A}_0 + \mathcal{A}_1^{k+1} Y_{t-1} + \mathcal{A}_1^k \mathcal{B}^{-1}[1, 0, \cdots, 0]. \tag{15}$$

Define the k-step ahead impulse response of all the variables as the difference between this conditional forecast and the unconditional forecast made at time t:

$$\begin{aligned} IRF(k) &= E_{t|\varepsilon_t^1}^{VAR}(Y_{t+k}) - E_t^{VAR}(Y_{t+k}) \\ &= [I_{n_y p} + \mathcal{A}_1 + \cdots + \mathcal{A}_1^k]\mathcal{A}_0 + \mathcal{A}_1^{k+1} Y_{t-1} + \mathcal{A}_1^k \mathcal{B}^{-1}[1, 0, \cdots, 0] \\ &- [I_{n_y p} + \mathcal{A}_1 + \cdots + \mathcal{A}_1^k]\mathcal{A}_0 + \mathcal{A}_1^{k+1} Y_{t-1} \\ &= \mathcal{A}_1^k \mathcal{B}^{-1}[1, 0, \cdots, 0]. \end{aligned} \tag{16}$$

Let e_i be a $n_y p \times 1$ vector whose $i'th$ element is one and all other elements are zero. If the variable ordered in position N_x is the same variable being foretasted one-period ahead in survey data in position N_s, then the internal consistency of forecasts would require that:

$$e_{N_x}' IRF(k+1) = e_{N_s}' IRF(k). \tag{17}$$

Therefore, forecast consistency would require that:

$$e_{N_x}' \mathcal{A}_1^{k+1} \mathcal{B}^{-1}[1, 0, \cdots, 0] = e_{N_s}' \mathcal{A}_1^k \mathcal{B}^{-1}[1, 0, \cdots, 0]. \tag{18}$$

Notice that two features of this condition contrasts with the unconditional forecast consistency restrictions presented in Section 3. First, even if consistency is imposed in response to every structural shock, forecast consistency in the impulse responses imposes no restrictions on the VAR intercept term. This is because the intercept has no direct bearing on the impulse responses form the VAR. Of course, the unconditional forecasts depend crucially on A_0. Second, the forecast consistency criteria depends on the matrix B^{-1} which provides the identification of the structural shocks, or the mapping between reduced form VAR residuals and structural shocks. This suggests that the identification of structural shocks and forecast consistency are not independent.

Just as above, we will consider a weak-consistency prior denoted now by $g(A, B, h) \sim \mathcal{N}(0, (\lambda W)^{-1})$ that relaxes the strict requirement for Equation (18) to hold at every impulse response horizon h. To highlight the role that this prior can play in sharpening structural identification, it is useful to consider a fixed A (i.e. fix the reduced form parameters at the OLS estimates) and a prior only over alternative structural models $g(B, h|A) \sim \mathcal{N}(0, \lambda W^{-1})$. In either case, the horizon h is a hyperparameter that calibrates the horizon over which the prior applies to the impulse responses. In our application below, h aligns with the horizon over which impulse responses are plotted and displayed.

5 Time-varying inflation tail risks implied by TVP-VAR with survey forecasts of inflation

Kozicki and Tinsley (2012) estimate a time-varying parameter (TVP) AR(p) model for inflation together with survey forecasts. To efficiently use survey information on the estimation of time-varying coefficients, they impose consistency requirements that survey forecasts correspond to noisy observations of forecasts implied by the TVP-AR(p) model. After introducing a noise term to survey forecasts, they essentially impose weak consistency requirements in our classification. The prior co-variance matrix for cross-equation restrictions in our framework is comparable to the variance matrix of the noise term in their formulation. In Kozicki and Tinsley (2012), only constant terms ("inflation end-point") are allowed to drift over time while other coefficients in the AR(p) model remain time-invariant. Keeping AR coefficients time-invariant has some limitations in modeling possible time-variations in the relationship between inflation expectations and other variables. Changes in monetary policy regimes such as the adoption of explicit inflation target, if credible, will make inflation expectations less responsive to temporary shocks.[4] For this reason, we consider a more general TVP-VAR model in which not only constant terms but also other VAR coefficients drift over time. Our data include survey forecasts for short-term and long-term inflation and we impose weak consistency requirements for the model-based forecasts.

5.1 Data

As in Clark and Davig (2011), we use both 1-year and 10-year ahead forecasts for Consumer Price Index (CPI) inflation from the Survey of Professional Forecasters (SPF) as well as realized CPI inflation. Our measure of long-term inflation is the average expected inflation between the five quarters from now and the fourth quarter of ten years from the current year.[5] We include a measure of real economic activity (Chicago Fed National Activity Index, x_t) and a monetary policy indicator (the effective federal funds rate spliced together with Wu and Xia (2016) shadow rate (r_t) measure during the period when the short-term interest rate is constrained at the effective lower bound) on top of the three variables related to inflation. Figure 1 shows all the variables used in estimation. While the real economic activity measure does not show any trending behavior, all the other nominal variables exhibit downward trends since the early 1980s. The TVP-VAR model can accommodate time-varying trends in variables with the random-walk drifts of VAR coefficients.

[4] Bundick and Smith (2018) show that the announcement of a numerical inflation target by the Federal Open Market Committee (FOMC) in 2012 reduced the sensitivity of the long-horizon forward inflation to temporary surprises in the realized inflation. Although their finding is based on the high-frequency financial market data, Doh and Oksol (2018) find that the predictability of inflation including long-horizon inflation expectations from survey data also declined after the announcement of the explicit inflation target.

[5] The 10-year forecast for CPI inflation from the SPF moves the forecast window only at the first quarter of each year. For example, the 10-year forecast for CPI inflation from the SPF at each quarter of 1990 is the expected inflation averaged over 1990:Q1-1999:Q4. In contrast, the 1-year forecast from the SPF moves the forecast window each quarter. Therefore, simply taking the difference between the 10-year forecast and the 1-year forecast will leave some overlapping with realized inflation data depending on the quarter of the year. We make additional adjustments using realized CPI inflation data and nowcasts observations to ensure the long-term forward inflation component does not overlap with other inflation-related variables in the VAR.

5.2 TVP-VAR Model

We consider the following TVP-VAR(4) model with stochastic volatility for the five variables $y_t = [\pi_t^{S,L}, \pi_t^{S,S}, \pi_t, x_t, r_t]$:

$$
\begin{aligned}
y_t &= A_{0,t} + \sum_{j=1}^{4} A_{j,t} y_{t-j} + B^{-1} u_t \, , \; u_t \sim \mathcal{N}(0, \Sigma_{u,t}), \\
\Sigma_{u,t} &= \begin{pmatrix} \sigma_{1,t}^2 & 0 & \cdots & 0 \\ 0 & \sigma_{2,t}^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{5,t}^2 \end{pmatrix} \\
\tilde{y}_t &= \tilde{A}_{0,t} + \tilde{A}_{1,t} \tilde{y}_{t-1} + \tilde{u}_t, \\
\tilde{y}_t &= \begin{bmatrix} y_t \\ y_{t-1} \\ y_{t-2} \\ y_{t-3} \end{bmatrix}, \tilde{A}_{0,t} = \begin{bmatrix} A_{0,t} \\ 0_{(15\times 5)} \end{bmatrix}, \tilde{A}_{1,t} = \begin{pmatrix} A_{1,t} & A_{2,t} & A_{3,t} & A_{4,t} \\ I_5 & 0_{(5\times5)} & 0_{(5\times5)} & 0_{(5\times5)} \\ 0_{(5\times5)} & I_5 & 0_{(5\times5)} & 0_{(5\times5)} \\ 0_{(5\times5)} & 0_{(5\times5)} & I_5 & 0_{(5\times5)} \end{pmatrix}, \tilde{u}_t = \begin{bmatrix} B^{-1} u_t \\ 0_{(15\times 5)} \end{bmatrix}, \\
A_t &= [A_{0,t}, A_{1,t}, A_{2,t}, A_{3,t}, A_{4,t}]', \\
\mathrm{vec}(A_t) &= \mathrm{vec}(A_{t-1}) + \epsilon_t \, , \; \epsilon_t \sim \mathcal{N}(0, Q), \\
\ln(\sigma_{i,t}^2) &= \ln(\sigma_{i,t-1}^2) + e_{i,t} \, , \; e_{i,t} \sim \mathcal{N}(0, \sigma_{e,i}^2).
\end{aligned} \tag{19}
$$

$\pi_t^{S,L}$ and $\pi_t^{S,S}$ are weighted averages of quarterly forward inflation at different horizons as defined below while π_t is realized CPI quarterly inflation.[6]

$$
\begin{aligned}
\pi_t^{S,L} &= \frac{\sum_{j=5}^{l(t)} E_t^s(\pi_{t+j})}{l(t) - 4}, \\
\pi_t^{S,S} &= \frac{\sum_{j=1}^{4} E_t^s(\pi_{t+j})}{4}.
\end{aligned} \tag{20}
$$

[6] Since the forecast horizon of the 10-year forecast from the SPF changes only at the first quarter of each year, the number of quarterly forward inflation contained in the 10-year forecast varies depending on the quarter of year. In principle, we could treat four quarterly observations of the 10-year forecast of any given year as one quarterly observation for four different measures of long-term forecasts to keep the same forecast horizon for each measure of long-term forecasts. However, doing so will require us to run a mixed-frequency TVP-VAR that will substantially increase the dimension of parameters and latent variables. We leave this for the future research agenda.

Consistency requirements for $\pi_t^{S,L}$ and $\pi_t^{S,S}$ lead to 42 cross-equation restrictions $g(A)$:

$$\begin{aligned}
\pi_t^{S,L} - \frac{\sum_{j=5}^{l(t)} E_t^{\mathrm{VAR}}(\pi_{t+j})}{l(t)-4} &= e_1'\tilde{y}_t - e_3'\frac{[\sum_{h=5}^{l(t)}\sum_{j=0}^{h-1}\tilde{A}_{1,t}^j\tilde{A}_{0,t} + \sum_{h=5}^{l(t)}\tilde{A}_{1,t}^h\tilde{y}_t]}{l(t)-4}, \\
g(A)_L &= [-e_3'\frac{[\sum_{h=5}^{l(t)}\sum_{j=0}^{h-1}\tilde{A}_{1,t}^j\tilde{A}_{0,t}])}{l(t)-4}, e_1' - e_3'\frac{[\sum_{h=5}^{l(t)}\tilde{A}_{1,t}^h]}{l(t)-4}]' \\
\pi_t^{S,S} - \frac{\sum_{j=1}^{4} E_t^{\mathrm{VAR}}(\pi_{t+j})}{4} &= e_2'\tilde{y}_t - e_3'\frac{[\sum_{h=1}^{4}\sum_{j=0}^{h-1}\tilde{A}_{1,t}^j\tilde{A}_{0,t} + \sum_{h=1}^{4}\tilde{A}_{1,t}^h\tilde{y}_t]}{4}, \\
g(A)_S &= [-e_3'\frac{[\sum_{h=1}^{4}\sum_{j=0}^{h-1}\tilde{A}_{1,t}^j\tilde{A}_{0,t}]}{4}, e_2' - e_3'\frac{[\sum_{h=1}^{4}\tilde{A}_{1,t}^h]}{4}]', \\
g(A) &= [g(A)_L, g(A)_S]',
\end{aligned} \tag{21}$$

where e_i is a 20×1 vector whose ith element is 1 while all the other elements are zeros. To calculate VAR-implied forecasts in the above equation, we assume that agents do not take into account future drifts in parameters when they form expectations. In other words,

$$E_t^{\mathrm{VAR}}(\prod_{k=1}^{h} A_{t+k}) = A_t^h. \tag{22}$$

This “anticipated utility ”approximation works well for mean forecasts. Since we impose consistency requirements only on point forecasts from surveys, using the approximation should be even less problematic.

5.3 Priors

To determine priors for initial states $(A_{0,0}, A_{1,0}, A_{2,0}, A_{3,0}, A_{4,0}, \Sigma_{u,0})$ and parameters (Q, B, σ_e^2), we use a training sample from 1970:Q2 to 1981:Q2. Initial values of time-varying coefficients (A_0) and the covariance matrix of innovations to time-varying coefficients (Q) are calibrated from the Ordinary Least Squares (OLS) estimates of a time-invariant VAR(4) using the training sample data. The prior mean of A_0 are the OLS estimates of coefficients from the VAR(4) for the training sample data. The prior mean of Q is set to be proportional to the covariance matrix of the OLS estimates. More specifically, we rescale the covariance matrix of the OLS estimates to make the implied variation of the long-run mean of y_t plausible given the magnitude of the uncertainty about the the mean of y_t. More specifically, we generate A_{t+1} by adding one standard deviation shocks to all the components in A_t using the Cholesky decomposition of Q and calculate the implied variation of the long-run mean as follows:

$$\Delta \bar{y}_{t+1} = [1_{1\times 5}, 0_{1\times 15}]((I_{20} - \tilde{A}_{1,t+1})^{-1}\tilde{A}_{0,t+1} - (I_{20} - \tilde{A}_{1,t})^{-1}\tilde{A}_{0,t}). \quad (23)$$

We calibrate the prior mean of Q to match the implied $\Delta \bar{y}_{t+1}$ with the standard deviation of the constant mean of y_t under the assumption innovations to y_t are independently and identically distributed for the training-sample data. Similarly, priors for the initial values of stochastic volatilities ($\Sigma_{u,0}$) are calibrated from the estimated covariance matrix of the residuals in the time-invariant VAR(4). The prior for B is elicited by scaling up an identity matrix and the prior mean for σ_e^2 is set to 0.01, which implies small variations in the innovation to log volatility. As is common in the TVP-VAR literature, we assume that the initial values of latent states and other parameters are a-priori independent.

$$p(A_0, \Sigma_{u,0}, B, Q, \sigma_e^2) = p(A_0)p(\Sigma_{u,0})p(B)p(Q)p(\sigma_e^2). \quad (24)$$

To impose forecast consistency requirements, we need to calibrate hyperparameters such as λ and W that determine the maximum entropy prior for $g(A)$. Since the prior for $g(A)$ follows a normal distribution, W needs to match the inverse of the covariance matrix of $g(A)$. We simulate multiple draws of A from the prior distribution and compute $g(A)$ for each draw. From these multiple realizations of $g(A)$, we compute the covariance matrix and set W equal to it. For λ, there is no a-priori guideline to calibrate the value. As common in a hierarchical model, we estimate the TVP-VAR(4) model with different values of λ and pick the value that gives the best fit for the sample according to the marginal likelihood.

5.4 Posterior simulation

We obtain the posterior output of latent states and parameters using Gibbs sampling. While the joint posterior distribution of $(A^T, \Sigma_u^T, Q, B, \sigma_e^2)$ is difficult to characterize analytically, the distribution of one component conditional on all the other components is either analytically tractable or easy to simulate.[7] Given hyperparameters (λ, W), the joint posterior

[7]We use superscript to denote an array of observations up to the point at the superscript. For example, A^T represents an array of $[A_0, \cdots, A_T]$.

distribution of latent states and parameters can be obtained as the product of conditional posterior distributions as follows:

$$\begin{aligned}
p_{\lambda,W}(A^T, \Sigma_u^T, B, Q, \sigma_e^2|y^T) \propto\; & p(Q|A^T, \Sigma_u^T, B, \sigma_e^2, y^T)p(Q) \\
& \times p(\sigma_e^2|A^T, \Sigma_u^T, B, Q, y^T)p(\sigma_e^2) \\
& \times p(\Sigma_u^T|A^T, B, Q, \sigma_e^2, y^T)p(\Sigma_{u,0}) \\
& \times p(B|A^T, \Sigma_u^T, Q, \sigma_e^2, y^T)p(B) \\
& \times p_{\lambda,W}(A^T|\Sigma_u^T, B, Q, \sigma_e^2, y^T)p(A_0).
\end{aligned} \tag{25}$$

In the last step, we first simulate M draws of A^T from $p(A^T|\Sigma_u^T, B, Q, \sigma_e^2, y^T)p(A_0)$ and then resample them using importance-sampling weights given by

$$w(A^T(j)) = \frac{\exp^{-0.5g(A^T(j))'(\lambda W)g(A^T(j))}}{\sum_{k=1}^{M} \exp^{-0.5g(A^T(k))'(\lambda W)g(A^T(k))}}. \tag{26}$$

To choose the hyperparamter λ that controls the tightness of forecast consistency prior restrictions, we calculate the following marginal data density for different values of λ.[8]

$$p_(y^T|\lambda) = \int p(y^T|A^T, \Sigma_u^T, B)p(A^T, \Sigma_u^T, B)d(A^T, \Sigma_u^T, B). \tag{27}$$

5.5 Inflation Tail Risks

Since the Great recession of 2007-2009, deflation risk has received a lot of attention from financial markets and policymakers. Investors have used inflation-related derivatives to hedge against deflation risks and policymakers often justified the use of unconventional monetary policies such as large-scale asset purchases and forward guidance on the future path of the short-term interest rate to address impending deflation risks. Financial derivatives on inflation such as inflation swaps and options as well as survey forecasts provide information on future deflation risk. Fleckenstein, Longstaff and Lustig (2017) compute tail risks for inflation implied by estimating a no-arbitrage model for inflation swaps and options. Alternatively, we can use survey data on inflation to back out deflation probabilities. Survey data information can be combined with forecasts based on a time-series model of inflation as we do here using a TVP-VAR model.

[8] We calculate the inverse of the marginal likelihood using the harmonic mean of the likelihood implied by posterior draws. The details are explained in the appendix.

Table 1 compares estimated deflation probabilities from Fleckenstein, Longstaff and Lustig (2017) with those from the TVP-VAR model and the University of Michigan consumer survey. For the TVP-VAR model, we simulate future realization of inflation conditional on each draw of parameters assuming that they do not change in the future, which is consistent with our anticipated utility approximation.[9] For the University of Michigan survey, we calculate the percentage of respondents who anticipated prices would go down among all the respondents who provided answers on expected prices. To match the sample period in Fleckenstein, Longstaff and Lustig (2017), we calculate summary statistics for deflation probabilities for 2009:Q4-2015:Q4. Although the underlying source data are not overlapping, mean and median numbers are quite comparable given the magnitude of standard deviation in each measure. In terms of volatility and dispersion, estimates based on financial market data in Fleckenstein, Longstaff and Lustig (2017) rank first followed by the TVP-VAR estimates without forecast consistency restrictions. Overall, the TVP-VAR model with consistency restrictions shifts down the deflation probability with three quantile estimates (minimum, median, maximum) all smaller than the other estimates. This is also true for the estimated probability of high inflation (inflation greater than 4 percent averaged over the forecast horizon) in Table 2. Hence, imposing forecast consistency restrictions strengthens the influence of survey forecasts that have been stable over the recent decades in determining tail risks.

6 Identifying credible forward guidance shocks

The conventional textbook view in macroeconomics is that what matters for consumption and investment decisions is the entire path of expected future interest rates, not just interest rates today (Woodford, 2003). This notion drives the theoretical result that a central bank retains powerful ammunition to combat economic downturns in the face of constraints on the current policy rate through its ability to communicate a path of future short-term interest rates. So-called forward guidance relies on the central bank to deviate from the path of rates it would otherwise pursue in the future had the economy not experienced a downturn in the present. In short, as Krugman (1998) succinctly put it, the central bank has to credibly promise to be irresponsible in the future.

[9] We resample posterior draws using different values of λ, reflecting the varying degree of tightness in forecast consistency prior. Among the four values of λ (0, 0.1 , 1, 1.5) in which at least more than 100 posterior draws have non-negligible weights, $\lambda = 1.5$ has the highest marginal likelihood. Therefore, we use posterior draws resampled with λ equal to 1.5 in the later discussion.

In terms of econometric identification, the added condition that forward announcements be credible adds to the already formidable challenge of identifying monetary policy shocks. As is the case for even conventional monetary policy shocks, successful identification of forward guidance shocks requires disentangling the endogenous policy reaction from the exogenous policy surprise. But, what constitutes credible guidance on the part of the central bank? In this paper, we take the view that credible announcements should be believed by market participants and reflected in their revisions to the path of future short term interest rates. Moreover, if the central bank hopes to retain future credibility, then their announced path of policy rates should be realized on average. For instance, if the initial announcement leads to revised forecast for the future path of rates but then when the future arrives the central bank reneges on its previous commitment, then the public will likely be skeptical of any future forward guidance announcements. For this reason, credible forward guidance announcements – or, similarly, forward guidance announcements from a credible central bank – should result in a minimal deviation between the path of interest rates forecasters anticipate and the path of interest rates that is ultimately realized.

Following Uhlig (2005), our central question surrounds the response of output following a forward guidance shock while we impose restrictions on the response of survey interest rates and prices. The conditional forecast-consistent prior is ideal to aid in the identification of credible forward guidance shocks. This prior imposes weak consistency between the forecasted path of interest rates and the actual path of interest rates for finite values of the prior precision parameter λ. In this instance the tuning parameter λ has a somewhat structural interpretation. Specifically, higher values of λ will further increase the weight on structural shocks that are associated with greater credibility. Therefore, we expect λ to be monotonically related to the degree of credibility that underlies the identified forward guidance shock. In what follows, we apply the conditional forecast-consistent prior to a sign-restricted VAR along the lines of the one used by Uhlig (2005).

We find that forward guidance shocks identified purely with sign restrictions have an ambiguous effect on output. And, in several instances, the median of the set of identified shocks which satisfy the sign restriction implies a contraction in output following what is thought to be an expansionary forward guidance shock. On the other hand, when we add to the sign restrictions the conditional forecast-consistent prior, we find expansionary effects on output for large values of λ. In other words, we find empirical evidence consistent with the notion that a credible forward guidance announcement of a lower forecasted path of interest rates leads to expansionary effects.

6.1 Data and VAR Model

To estimate the effects of forward guidance shocks on output, we closely follow the model specified in Uhlig (2005). In particular, we use monthly GDP as produced by Macroeconomic Advisers, the consumer price index, an index of commodity prices, the effective federal funds rate, non-borrowed reserves, total reserves, and 12-month ahead Blue Chip consensus economic forecasts for the 3-month Treasury bill rate. We take 100 times the natural log of all variables except for the federal funds rate and Blue Chip forecast for the 3-month Treasury bill rate. Unlike Uhlig (2005), we use the CPI directly as opposed to using the CPI and PPI to interpolate the quarterly GDP deflater to a monthly frequency. Also notable is the addition of survey forecasts of short-term interest rates. The inclusion of forecasted interest rates is central to our analysis of forward guidance shocks.

We model these time series as a VAR(3) where the number of lags is selected based on standard information criteria over the sample January 1994 to December 2007.[10] We begin our estimation in 1994 due to the chronology of communication from the Federal Open Market Committee (FOMC). Our interest lies in understanding the output effects of forward guidance, which has often been provided through press statements at the conclusion of FOMC meetings. Prior to 1994, the FOMC did not issue press statements, limiting the starting point of the sample range we consider. We end our sample in 2007 due to the dynamics of non borrowed reserves. In particular, beginning in January of 2008, this series begins to take on negative values. In addition to being peculiar from an economic standpoint, practically speaking this precludes using the natural log of this series after 2007. However, in a robustness check, we "correct" the non-borrowed reserves series to offset the fact some of the borrowing through the Fed's liquidity facilities appear to have been counted in borrowed reserves but not total reserves.[11]

6.2 Sign and shape restrictions on forward guidance shocks

A combination of sign and shape restrictions is employed to identify credible forward guidance shocks. Sign restrictions of the sort used by Uhlig (2005) restrict the responses of commodity prices, the price level, and forecasts of future interest rates for the first 6 months of the impulse response function. The notion that forward guidance shocks cause nominal interest rates and prices to move in opposite directions is shared by standard sticky-price

[10] We use pre-sample data beginning in October of 1993 to account for the three lags included in the VAR.

[11] Arithmetically, this is the only way that non-borrowed reserves could become negative (i.e. borrowed reserves must exceed total reserves).

models (Eggertsson and Woodford, 2003), models which attribute a large role to a "Fed information effect" (Nakamura and Steinsson, 2017), and models which dampen the output effects of forward guidance through a discounted Euler equation (McKay, Nakamura and Steinsson, 2016). Therefore, we consider structural rotations that result in inflation and future interest rates persistently moving in opposing directions to distinguish the monetary policy innovation from other macroeconomic shocks. Implicitly, this suggests that these other macroeconomic shocks would lead the Federal Reserve to track the nominal interest rate with inflation. But, depending on the reaction of the central bank, supply shocks could also cause inflation and nominal interest rates to move in opposite directions. However, according to the estimates in Barsky, Basu and Lee (2015) using U.S. data, both TFP shocks and TFP news shocks have historically caused inflation and nominal interest rates to comove. In addition, Kilian and Lewis (2011) show that the response of short-term interest rates to oil price shocks has been nil in the post-1987 period that includes our sample.[12] This suggests that we are on fairly solid ground by assuming that a monetary policy shock is the innovation that results in inflation and nominal interest rates persistently moving in opposite directions.

We make no explicit orthogonalization to distinguish conventional monetary policy shocks from forward guidance shocks. However, we note that by restricting our focus to policy shocks which result in persistent movements of expected future interest rates as implied by forecasters, we rule out short-lived monetary policy shocks that cause short-term rates to temporarily deviate from their implied rule. Specifically, since we impose our sign restrictions on the response of forecasts for one-year ahead rates for the first 6 months of the impulse response, deviations in the funds rate that die out in less than 18 months are explicitly disregarded. Along these lines, benchmark estimates of conventional monetary policy shocks result in movements in the funds rate tend to be pretty short-lived (Bernanke and Gertler, 1995). Therefore, our focus on one year ahead interest rate expectations offers some suggestive evidence that our sign restrictions are directing our focus towards forward guidance shocks as opposed to conventional monetary policy shocks.

In addition to qualitative sign restrictions, we also impose our conditional forecast-consistent prior over the shape of the impulse response functions. In particular, the VAR contains both the effective federal funds rate and the forecasted value of the 3-month Treasury Bill rate one year ahead. Historically, the federal funds rate has closely tracked the 3-month Treasury bill rate. Although the two instruments are highly correlated in both lev-

[12]The concern that the restrictions in place are consistent with both supply shocks and a monetary policy shocks applies to our application as well as the application in Uhlig (2005).

els and differences, they are not identical. Therefore, instead of imposing strong consistency, it is more appealing to impose weak consistency between the survey data on the 3-month Treasury bill rate and the VAR forecast of the funds rate. Specifically, we form a prior as follows:

$$g(B, h|A) = \frac{\partial}{\partial \varepsilon_t^{fg}} \mathbb{E}_{t+h}^{BC} R_{t+12+h}^{T-bill} - \frac{1}{3} \sum_{j=1}^{3} \left[\frac{\partial}{\partial \varepsilon_t^{fg}} ff_{t+9+h} \right]. \tag{28}$$

In the above equation, $\frac{\partial y_{t+h}}{\partial \varepsilon_t^{fg}}$ denotes the period h impulse response of y to a forward guidance shock that occurred in period t. $\mathbb{E}_{t+h}^{BC} R_{t+12+h}^{T-bill}$ denotes the Blue Chip consensus economics forecast for the 3-month Treasury bill 4-quarters (12 months) from now. In the monthly Blue Chip survey, forecasters report what they expect the 3-month T-bill to average over in the three months ending 4 quarters ahead. So, in December, forecasters report what they expect the yield on the 3-month Treasury bill to average in the three months ending December of the following year. To the extent that the yield on the 3-month Treasury bill rate closely tracks the federal funds rate, as has been the case historically, then this forecast should be linked with what the average federal funds rate over the three months ending in December. The average federal funds rate over the three months ending one year from now, conditional on a forward guidance shock, is given by $\frac{1}{3} \sum_{j=1}^{3} \left[\frac{\partial}{\partial \varepsilon_t^{fg}} ff_{t+9+h} \right]$.

As presented above, we assume that $g(B, h|A) \sim \mathcal{N}(0, \lambda^{-1})$ where λ tunes the precision over the conditional forecast-consistent prior. In this context, λ takes on more of a structural rather than statistical interpretation. In the context of credible forward guidance discussed above, higher values of λ imply greater credibility since it will tend to drive the posterior distribution of the VAR and survey forecasts of interest rates together. Unlike the use of unconditional forecast consistent priors, alternative values of λ result in structural models that fit the data equally well. To be clear, this issue arises in all structural VAR models that are not over-identified. For example, in the case of recursive short-run restrictions, there is no statistical criterion on which to select one candidate Cholesky ordering over another. Analogously in this application, there is no obvious statistical reason to prefer small versus large values of λ. Instead, we highlight how alternative values of λ affect the structural response of output. We then argue that given the different inference one could draw from alternative λ calibrations, the degree of credibility is a potentially salient factor shaping the estimated effects of forward guidance shocks. To our knowledge, this point has not been stressed in the empirical literature that estimates the effects of forward guidance. However, some theoretical models stress the importance of central credibility when assessing the effectiveness of forward guidance.

6.3 Implementation of sign and shape restrictions

We implement our sign restrictions as in Uhlig (2005). In particular, for given values of A and Σ, the reduced from VAR parameters, we draw a random orthonormal rotation matrix S such that $SS' = I$. Then we consider orthogonalized shocks defined by the factor matrix $F = CS$ where C is the lower triangular Cholesky factor of Σ. We then check whether the impulse response functions implied by A and F satisfy the sign restrictions. If so, then the impulse response function is retained, otherwise it is discarded and we proceed to another draw.

After accumulating M impulse responses which satisfy the sign restrictions, then we calculate the weight of each draw j according to

$$w(B(j)) = \frac{\exp^{-0.5 g(B(j),1:H)' \lambda g(B(j),1:H)}}{\sum_{k=1}^{M} \exp^{-0.5 g(B(k),1:H)' \lambda g(B(k),1:H)}}. \tag{29}$$

These weights form the importance sampling weights we use to simulate the posterior set of impulse responses. In practice, we set $M = 5000$ and find that even for very large values of λ, the effective sample size – defined as $\hat{M} = (\sum_{k=1}^{M} w(B(k))^2)^{-1}$ – is always above 300 and, in most instances, around 1000 suggesting the posterior weight is distributed across many draws. Finally, we scale the size each draws to have the same initial effect on forecasted interest rates before calculating the importance sampling weights. This prevents bigger shocks from being penalized simply due to the size of the forward guidance shock.

6.4 Impulse response functions

For our baseline analysis, we restrict attention to the case where A and Σ, the reduced from VAR parameters, are fixed at their OLS estimates. Therefore, in all graphs unless otherwise noted, the uncertainty bands reflect model uncertainty over the set of alternative structural VARs. While, in general, it is useful to account for both model and parameter uncertainty, it is worthwhile to initially focus on model uncertainty to illustrate how λ influences the identification of structural shocks.

Figure 3 shows the median and 68 percent posterior intervals of impulse response functions across the draws that satisfy the sign restrictions. That is to say that in this figure we set $\lambda = 0$. Per the imposed restrictions, forecasted interested rates decline and commodity prices along with the overall price level rise. In the months that follow the restricted periods, forecasted interested rates remain low and commodity prices remain elevated. The price level continues to gradually climb throughout the impulse response horizon. Recall that our sign restrictions leave the path of the actual federal funds rate unconstrained. However, along with forecasted interest rates, the federal funds rate declines one year after the forward guidance shock. Consistent with the decline in interest rates, non-borrowed and total reserves both rise 12 months after the shock. After this initial decline, the federal funds rate begins rising back to its pre-shock level and thereafter continues to rise, resulting in a persistent interest rate overshoot. In response to this path of realized interest rates, output initially declines a bit then remains near its pre-shock path. It is perhaps not too surprising that output fails to increase in response to this forward guidance shock. After all, forward guidance is premised on the idea that it is the entire path of interest rates that determines consumption and investment today. And the path of rates realized in this shock is not, on net, all that expansionary given the offsetting effects of the initial decline in expected rates and the subsequent reversal and interest rate overshoot.

How does this impulse response for the federal funds rate compare to the one year ahead forecast of short-term interest rates? The red-dashed line shows the VAR-implied forecast for the one year ahead short-term interest rate based on the impulse response of the federal funds rate. One year after the forward guidance shock, actual short-term interest rates are lowered by an amount similar to what forecasters anticipated. In other words, the initial shock leads to similar declines in both expected and realized short-term interest rates. However, in subsequent months, the path of actual short-term interest rates exceeds the path anticipated by forecasters and remains above forecasted interest rates for several years.

The identified response of output using only sign restrictions suggests that forward guidance may not be effective. However, there are reasons to question whether the resulting path of interest rates in the above shock scenario is consistent with credible, well-communicated forward guidance. Perhaps due to the lack of commitment on the part of the central bank, or a misinterpretation of the central bank's communication, realized short-term interest rates deviate persistently from the path of rates anticipated by forecasters. Therefore, the results in Figure 3 leave open the question of whether credible forward guidance is effective.

To understand the effects that credible forward guidance has on output, we now impose the conditional forecast-consistent prior in conjunction with the previously used sign restrictions. That is to say that we now consider $\lambda > 0$ whereas in Figure 3 we set $\lambda = 0$. Of course, this requires choosing a value for λ. Rather than picking just one value, we prefer to show how the response of output is influenced by the choice of λ. To first gain some intuition for how alternative values of λ will influence the posterior distribution of the impulse responses, it is instructive to examine two particular candidate models. Among the

5000 draws that satisfy the sign restriction, we isolate (i) the draw that comes the closest to satisfying the conditional forecast-consistent condition in Equation (28) and the draw that is the furthest from satisfying the conditional forecast-consistent condition in Equation (28). In other words, these are the two draws that register the highest and lowest probability on our prior, respectively. We refer to these as the "best" and "worst" draws from a forecast consistency standpoint.

Figure 4 plots the best and worst draws. The green dashed-dotted line represents the best draw and in the red dashed line represents the worst draw. To visually understand what is behind these rankings, Figure 4 shows the cumulative forecast deviation in the bottom-left pane. By construction, the period 60 cumulative deviation is closer to zero for the best draw than for the worst draw. In other words, the best draw results in a path of the federal funds rate which closely mirrors the expected path of rates from forecasters. For the worst draw, the realized path of the funds rate meaningfully diverges from the path expected by forecasters in a direction that suggests policy ends up being more restrictive than survey expectations suggested. Although the response of output played no role in our selection, the best draw implies a persistent expansion in output, while the worst draw suggests that output declines following a reduction in the forecasted path of short-term interest rates. Given the path of interest rates that underly these alternative shocks, the resulting differences in their real effects may not be surprising. None the less, this Figure 4 offers some suggestive evidence that credible forward guidance has the potential to be more effective.

The impulse responses in Figure 4 make clear how the choice of λ will influence the posterior impulse responses. Higher values of λ will place greater weight on draws like the "best" one and lower weight on draws like the "worst" one. Notice that we continue to place no restriction on the response of output. Instead, as in Uhlig (2005), we continue to remain agnostic about the real effects of monetary policy. Figure 5 shows the the median response of output two years following the forward guidance shock for alternative values of λ. As the value of λ increases, so too does the peak of the median response of output. To the extent that larger values of λ isolate forward guidance shocks that are more credible and better communicated than smaller values of λ, then this provides some (suggestive) empirical evidence that clear communication and credibility are key determinants of the effectiveness of forward guidance.

Figure 6 shows the median and 68 percent posterior intervals of impulse response functions across the draws that satisfy the sign restrictions with $\lambda = 15000$. Though the precise value of λ is ordinal, larger values express a high-degree of prior precision over conditional forecast consistency. The resulting impulse response functions show that for this setting of λ, output persistently expands following an expansionary forward guidance shock. This shocks tends to generate paths of the federal funds rate which deviate from the expected path of professional forecasters by only a few basis points. Importantly, unlike the responses shown in Figure 3, after initially declining in line with expectations of forecasters the federal funds rate does not persistently overshoot for the life of the impulse response. Therefore, more of the anticipated accommodation remains in place which appears to support the persistent expansion of output.

6.5 Impulse response functions for alternative specifications

The finding that greater credibility, as calibrated through larger values of λ, leads to more expansionary output effects from forward guidance is a common finding across several alternative specifications. In most instances, the range of impulse responses suggests that output expands. And, in all instances the median response of output with a large value of λ exceeds the median responses when $\lambda = 0$.

- Figure 7 shows the 68 percent range of impulse responses when we don't scale the size each draw to have the same initial effect on forecasted interest rates in calculating the importance sampling weights. In particular, this penalizes bigger shocks simply due to the size of the forward guidance shock. This is visible in comparing the size of the initial movement in forecasted interest rates in Figures 7 and 6. Despite the initial movement in forecasted rates being smaller in this specification, this alternative weighting still suggests that more credible forward guidance shocks have a larger effect on output.

- On the one hand, the widespread use of forward guidance over the recent zero lower bound period provides motivation for including it in the sample. On the other hand, given the fact that one-year ahead expected rates from both interest-rate futures to professional surveys became truncated around 2010 according to Swanson and Williams (2014) is one motivation for limiting our attention to the pre-ZLB period. Therefore, we leave this full sample estimation as a robustness check. Figure 8 shows the 68 percent range of impulse responses when we extend the sample to include the zero lower bound period for an estimation sample of January 1994 through December 2015. For comparison, the median of the draws with $\lambda = 0$ is also shown. Over this extended sample the ranking of the median responses remains with larger values of λ implying more expansionary effects.[13]

- Figure 9 shows the 68 percent range of impulse responses when we also account for uncertainty in the reduced-form VAR parameters. Compared to the results previously shown, the range of responses is understandably wider as now both model and parameter uncertainty is taken into account. However, the ranking of the median responses remains with larger values of λ implying more expansionary effects.

7 Related literature

Our work is related to several strands of literature. From a methodological standpoint, our unconditional forecast-consistent prior is related to the literature on using informative priors to improve out-of-sample forecasting and the accuracy in the estimation of impulse response functions in VARs(Del Negro and Schorfheide, 2004; Giannone, Lenza and Primiceri, 2015, 2018). These papers shrink VAR coefficients toward a more parsimonious specification. In Del Negro and Schorfheide (2004), the parsimonious specification is a dynamic stochastic general equilibrium (DSGE) model while it is the long-run dynamics of macroeocnomic variables from a structural model in Giannone, Lenza and Primiceri (2018) or a combination of naive benchmark models in Giannone, Lenza and Primiceri (2015). Our model-consistent priors will also shrink VAR coefficients in the direction of a more parsimonious specification in which coefficients in equations for survey expectations are determined by coefficients in other equations via nonlinear restrictions. Our approach is more closely related to Del Negro and Schorfheide (2004) and Giannone, Lenza and Primiceri (2018) than Giannone, Lenza and Primiceri (2018) in that priors are derived from nonlinear restrictions based on economic theory. However, unlike Del Negro and Schorfheide (2004), we do not take a stance on an underyling equilibrium model that might have generated data used in the estimation of the VAR model. We only seek the internal consistency of forecasts inside the VAR model implied by the theory of rational expectations. Likewise, we do not take a stance on the long-run equilibrium relationship between different macroeconomic variables such as real output and real consumption in Giannone, Lenza and Primiceri (2018). Our nonlinear restrictions are derived from imposing the consistency of forecasts for the *same* variable. In this sense, our model-consistent priors for survey expectations are more flexible than the method used in the existing literature on imposing theory-based priors for VARs. The flexibility in our

[13] Due to the decrease in interest rate volatility over this sample, a much larger value of λ is needed over this sample for the forecast consistent prior to influence the posterior impulse responses.

approach is comparable to Frey and Mokinski (2016) who also impose the consistency of forecasts based on a time-series model with survey forecasts in a probabilistic way. However, their time-series model for inflation does not allow that survey forecasts can predict inflation above and beyond lagged inflation data. In their specification, survey nowcasts help inflation forecasting by reducing the estimation uncertainty of parameters in the time-series model only because survey nowcasts and actual inflation depend on lagged inflation data in a similar way. In contrast, our specification allows the possibility that survey forecasts contain forward-looking information on future inflation.

Our conditional forecast-consistent prior is related to the literature that restricts the sign or shape of impulse response functions to identify structural shocks. Uhlig (2005) seminal work on this topic argued that sign restrictions could be interpreted as making formal and transparent the a-priori theorizing that many macroeconomist do when assessing the success of a particular structural VAR. In this sense, sign restrictions are explicitly modeled priors overs the sign of the impulse responses that distinguish alternative structural shocks. Similarly, our approach uses a-priori reasoning over the shapes of the impulse responses for actual data and survey forecast of that data to identify structural shocks. In this regard, our implementation by way of importance sampling seems to be similar to the one used by Antolín-Díaz and Rubio-Ramírez (2018), who resample posterior draws of structural VARs to impose narrative sign restrictions. One main difference is that their importance sampling weights are determined by the probability distribution of exogenous shocks because narrative sign restrictions are imposed on the likelihood rather than prior distributions of parameters.

Our applications of these priors presented in the previous sections also relate to multiple branches of research. Several papers have recently focused on assessing the tail-risks to inflation with many using financial market data such as inflation swaps and options. Fleckenstein, Longstaff and Lustig (2017) estimate a dynamic asset pricing model in which trend and volatility of expected inflation are priced in inflation swaps and options. The model provides time-varying deflation risks that are identified from the dynamics of state variables determining the future evolution of inflation. Similarly, (Anene and D'Amico, 2017) and (Hattori, Schrimpf and Sushko, 2016) use derivative market data on inflation to back out the implied deflation probability. While Fleckenstein, Longstaff and Lustig (2017) focuses on the close correlation of deflation risk with other financial and macroeconomic variables such stock return and unemployment, (Anene and D'Amico, 2017) and (Hattori, Schrimpf and Sushko, 2016) study the impact of unconventional monetary policies on inflation tail risks. They find that monetary policy news related to unconventional monetary policies of the Federal Reserve significantly reduced inflation tail risks during the ZLB period. Our paper is complementary to these studies and highlights information from survey forecasts in calculating inflation tail risks. The relative stability of inflation expectations in the survey data compared to the volatile financial market data was often cited by policymakers as a mitigating factor for deflation risk during the ZLB period. Indeed, our estimates of deflation risks based on the TVP-VAR model with survey forecasts tend to lower than other estimates based on financial market data on average, consistent with these beliefs. Our results show that imposing forecast consistency restrictions between survey forecasts and VAR-based forecasts reduces deflation risks further by making inflation risks more sensitive to changes in survey forecasts. When survey forecasts for inflation started to bottom out in 2009:Q1, deflation risk from the TVP-VAR model with forecast consistency prior quickly came down and stayed at a low level thereafter. In contrast, deflation risks from the TVP-VAR model without forecast consistency prior came down more slowly. The marginal likelihood criterion favors the TVP-VAR model with forecast consistency prior, implying that data support deflation risks more tightly connected with variations in survey data.

Our VAR analysis on the effects of forward guidance on output relates to a growing literature that attempts to empirically assess the effects of unconventional monetary policy. Perhaps most related to our forward guidance application is the work of D'Amico and King (2017). They also use sign restrictions to elicit the effects of forward guidance from a VAR that includes actual and survey forecasts of the same data series. We view our analysis as largely complimentary to theirs, but there are two primary differences between our work and theirs. First, their work advances a literature that focuses on distinguishing between the Delphic and Odyssean channels of forward guidance announcements (see for example Campbell et al. (2012)). Meanwhile, we make no identifying restrictions regarding whether the forward guidance shocks we identify also contain a "Fed information" component. Instead, any output effects we identify should be interpreted through their lens as a joint or net effect of these two channels. A second distinction is our focus on central bank credibility in driving the macroeconomic responses we find, an issue largely outside the scope of their paper.

Our forward guidance application is also related to a more general literature asking whether monetary policy exacts any real effects under agnostic identification schemes. In particular, the work of Uhlig (2005) argued that the real effects of monetary policy are much more ambiguous when on only making sign restrictions. Follow-up work by Arias, Caldara and Rubio-Ramirez (2018) and Antolín-Díaz and Rubio-Ramírez (2018) argues that adding restrictions on the policy rule of the structural shocks themselves can overturn then Uhlig (2005) result. Our paper further informs both sides of this debate. First, we find that for forward guidance shocks, like conventional policy shocks in the application of Uhlig

(2005), result in ambiguous effects on real output when only identified via minimal sign restrictions. However, along the lines of the work of Arias, Caldara and Rubio-Ramirez (2018) and Antolín-Díaz and Rubio-Ramírez (2018), we find that adding further restrictions, in this case to the shape of the impulse response functions, sharpens the identification and leads to estimated forward guidance shocks which have the "conventional" effect on output. Finally, our work relates to the recent critique of Ramey (2016) who argues that when conventional policy shock identification schemes are applied to data from recent decades, the effects of monetary policy on the real economy are ambiguous. Since our sample focuses exclusively on the post 1994 period, our results suggest that some forms of monetary policy have retained their efficacy in recent decades.

8 Conclusion

Survey-based inflation forecasts for macroeconomic variables have been incorporated in VARs to accommodate forward-looking information that may not be included in the current and past values of inflation. However, adding survey-based forecasts into VARs without considering cross-equation restrictions on coefficients generates the internal inconsistency of forecasts. In this paper, we propose a novel Bayesian approach to put a prior distribution on cross-equation restrictions. This prior information also can be interpreted as a limited information likelihood for VAR parameters when we consider artificial observations that satisfy moment conditions for cross-equation restrictions. By combining importance sampling with a maximum entropy prior for cross-equation restrictions, we implement our approach without much computational burden. When applied to a TVP-VAR model for macroeconomic variables to calculate tail risks for inflation, imposing forecast consistency restrictions makes model-based forecasts of inflation more sensitive to movements in survey-based inflation expectations, with deflation probability jumping during the period of the financial crisis and recession in 2008-09 but subsiding more quickly since then. We also show that imposing weak-consistency across VAR-based and survey-based forecasts of interest rates leads to greater real effects from forward guidance, perhaps through a credibility channel.

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Appendix

A.1. The derivation of the maximum entropy prior

Let π_0 be the Lebesgue measure on $\mathbb{R}$. Under the weak consistency, the prior $\pi(g)$ must satisfy the following two moment restrictions.

$$\int g\pi(g)dg = 0\ ,\ \int g^2\pi(g)dg = \lambda W^{-1}. \tag{30}$$

Maximizing the entropy of $\pi(g)$ under moment restrictions with respect to the reference measures π_0 is equivalent to minimizing the Kullbeck-Leibler distance between π and π_0 as follows:

$$\pi^\star(g) = \text{argmin}_{\pi(g)} \int \pi(g)\ln(\frac{\pi(g)}{\pi_0(g)})dg - \mu_1(\int g\pi(g)dg) - \mu_2(\int g^2\pi(g) - \lambda W^{-1}dg). \tag{31}$$

The first-order condition for this is $\pi^\star(g) = \pi_0(g)e^{1+\mu_1 g+\mu_2 g^2}$. Hence, $\pi^\star \propto e^{\mu_1 g+\mu_2 g^2}$ and $\pi^\star$ is the normal distribution whose mean and variance are equal to $-\frac{\mu_1}{2\mu_2}$ and $-\frac{1}{2\mu_2}$. Since $\pi^\star$ must satisfy the above two moment restrictions, $\mu_1 = 0$ and $\mu_2 = -\frac{W}{2\lambda}$.

$$\pi^\star(g) \propto e^{-\frac{Wg^2}{2\lambda}} \rightarrow g \sim \mathcal{N}(0, \lambda W^{-1}). \tag{32}$$

A.2. The calculation of the marginal likelihood

Since draws of VAR parameters from prior distributions are likely to have very low likelihood, calculating the marginal likelihood from prior draws is quite inefficient and practically infeasible. We calculate the marginal likelihood by using importance sampling for posterior draws. Let's notice that the posterior density of VAR parameters can be expressed as the ratio of the posterior density kernel to the marginal likelihood.

$$p(A^T, \Sigma_u^T, B|y^T, \lambda) = \frac{p(y^T|A^T, \Sigma_u^T, B)p(A^T, \Sigma_u^T, B|\lambda)}{\int p(y^T|A^T, \Sigma_u^T, B)p(A^T, \Sigma_u^T, B|\lambda)d(A^T, \Sigma_u^T, B)} = \frac{p(y^T|A^T, \Sigma_u^T, B)p(A^T, \Sigma_u^T, B|\lambda)}{p_\lambda(y^T)}. \tag{33}$$

Now, let's take the harmonic mean of the likelihood for posterior draws of VAR parameters.

$$\int \frac{1}{p(y^T|A^T,\Sigma_u^T,B)} p(A^T,\Sigma_u^T,B|\lambda,y^T)d(A^T,\Sigma_u^T,B) = \frac{\int p(A^T,\Sigma_u^T,B|\lambda)d(A^T,\Sigma_u^T,B)}{p_\lambda(y^T)}. \tag{34}$$

Since the tail of the inverse of the likelihood can be thick, we truncate the region of posterior draws that we calculate the harmonic mean of the likelihood to between the 16% quantile and the 84% quantile of VAR parameters in terms of the likelihood. Let's denote this region by χ. Then, we can compute the marginal likelihood as follows.

$$p_\lambda(y^T) = \int_\chi p(A^T,\Sigma_u^T,B|\lambda)d(A^T,\Sigma_u^T,B)[\int_\chi \frac{1}{p(y^T|A^T,\Sigma_u^T,B)} p(A^T,\Sigma_u^T,B|\lambda,y^T)d(A^T,\Sigma_u^T,B)]^{-1}. \tag{35}$$

When we have M draws of $(A^T(j),\Sigma_u^T(j),B)$ from $p(A^T,\Sigma_u^T,B|\lambda,y^T)$, the above calculation is straightforward because $p(A^T,\Sigma_u^T,B|\lambda)$ and $p(Y^T|A^T,\Sigma_u^T,B)$ are analytically known functions.

Table 1: Summary Statistics for Deflation Probabilities: 2009:Q4 - 2015:Q4

Horizon	Source	Mean	Standard Deviation	Minimum	Median	Maximum
1	Fleckenstein et al. (2017)	18.759	10.155	2.003	15.739	47.757
1	University of Michigan Survey	15.906	6.1249	6.06	14.1414	32.6531
1	TVP-VAR($\lambda = 0$)	12.39	7.04	2.72	10.36	25.98
1	TVP-VAR($\lambda = 1.5$)	7.28	4.35	1.80	6.74	17.76
2	Fleckenstein et al. (2017)	13.799	7.261	1.801	11.621	35.421
2	TVP-VAR ($\lambda = 0$)	15.69	5.47	8.38	14.98	29.06
2	TVP-VAR ($\lambda = 1.5$)	8.55	3.34	4.32	7.88	16.24

Notes: Deflation probabilities from Fleckenstein et al. (2017) are based on daily observations on inflation swaps and options from October 5, 2009 to October 28, 2015 while those from the University of Michigan survey are quarterly average values of monthly observations from October, 2009 to October, 2015.

Table 2: Summary Stastistics for High Inflation (>4 percent) Probabilities: 2009:Q4 - 2015:Q4

Horizon	Source	Mean	Standard Deviation	Minimum	Median	Maximum
1	Fleckenstein et al. (2017)	2.615	2.46	0.139	2.086	16.268
1	TVP-VAR ($\lambda = 0$)	1.33	2.33	0.04	0.43	10.92
1	TVP-VAR ($\lambda = 1.5$)	0.61	1.27	0	0.12	5.88
2	Fleckenstein et al. (2017)	3.089	2.394	0.295	2.702	15.315
2	TVP-VAR ($\lambda = 0$)	4.14	2.86	0.88	3.24	14.02
2	TVP-VAR ($\lambda = 1.5$)	2.29	2.16	0.42	1.36	9.32

Notes: Deflation probabilities from Fleckenstein et al. (2017) are based on daily observations on inflation swaps and options from October 5, 2009 to October 28, 2015.

Figure 1: SAMPLE DATA 1970:Q2-2017:Q4

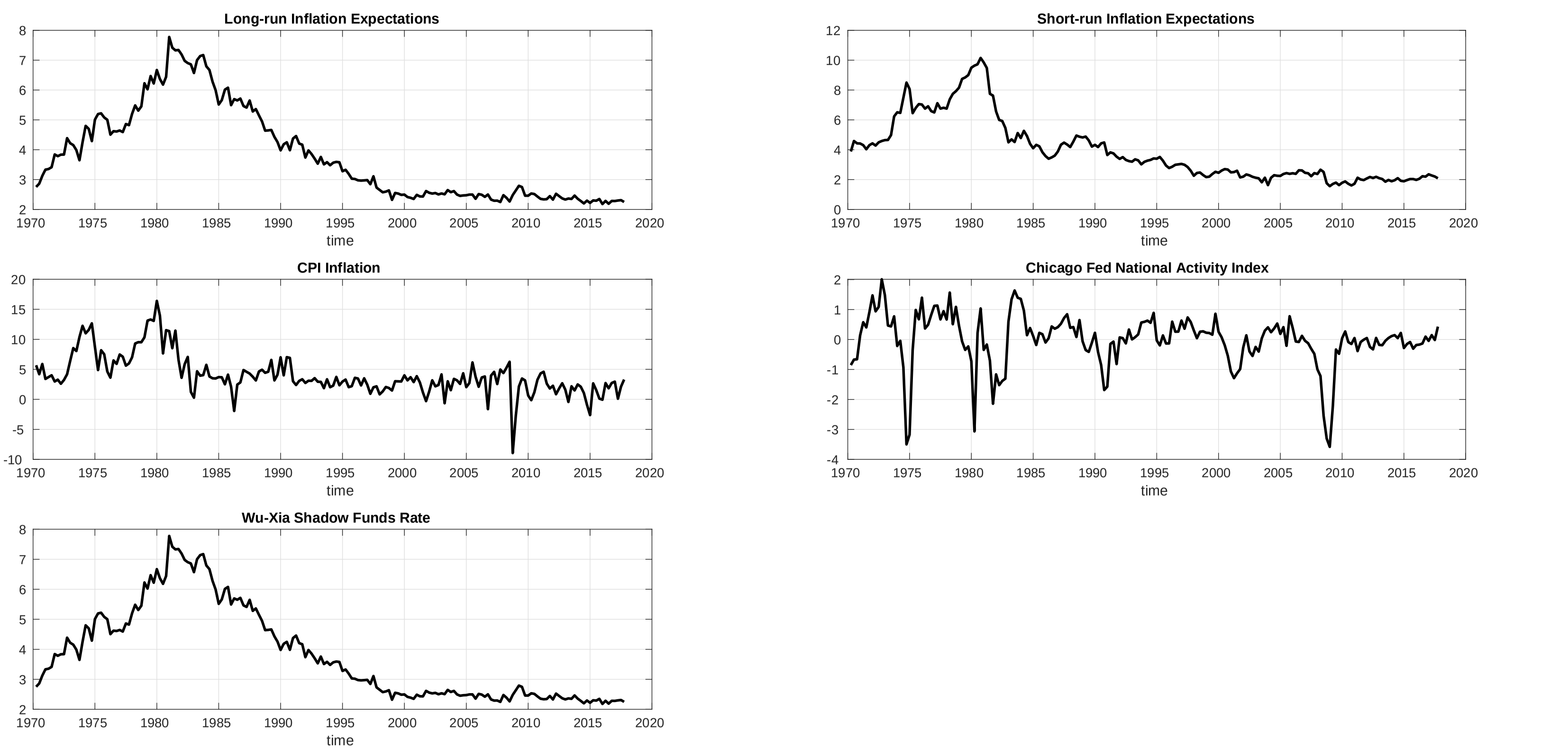


Figure 2: Inflation Tail Risks

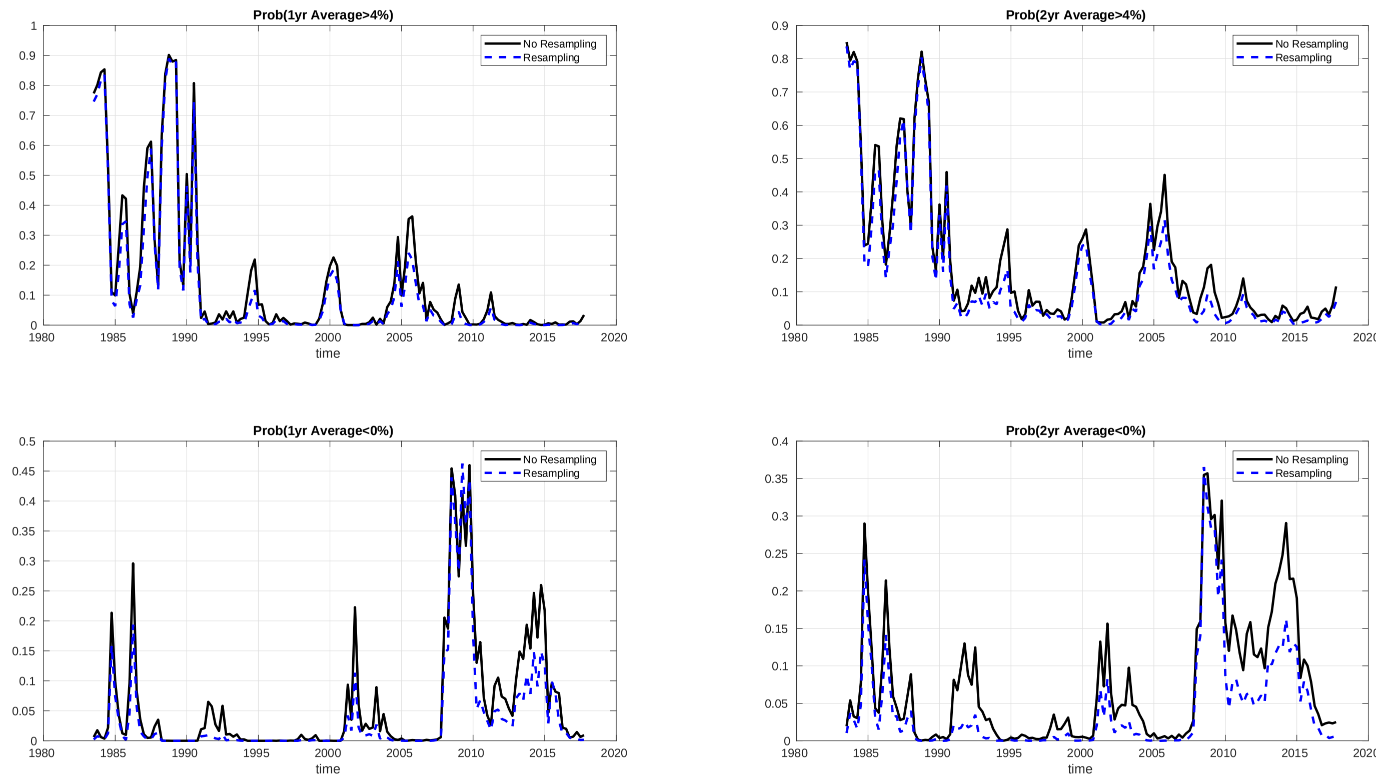
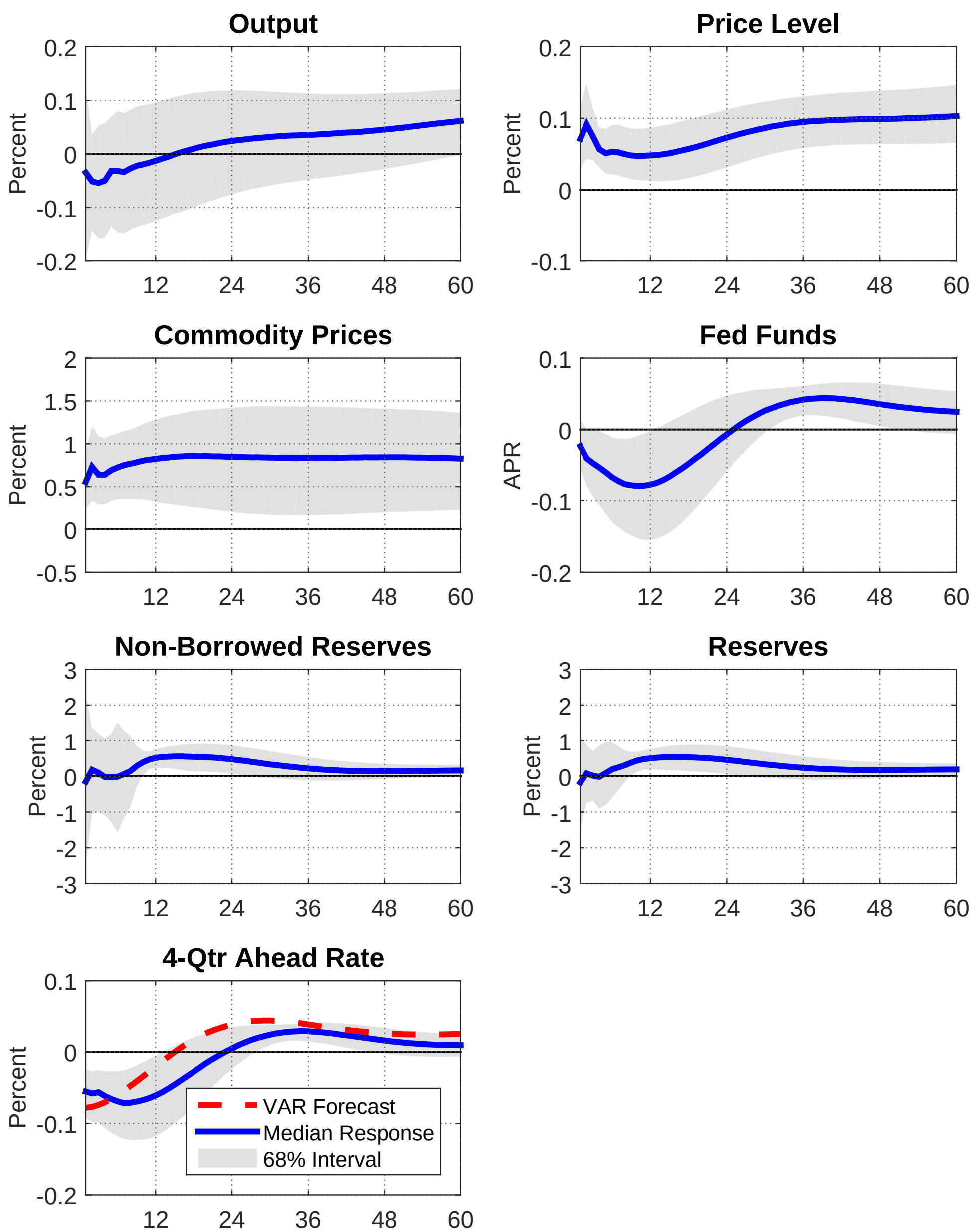
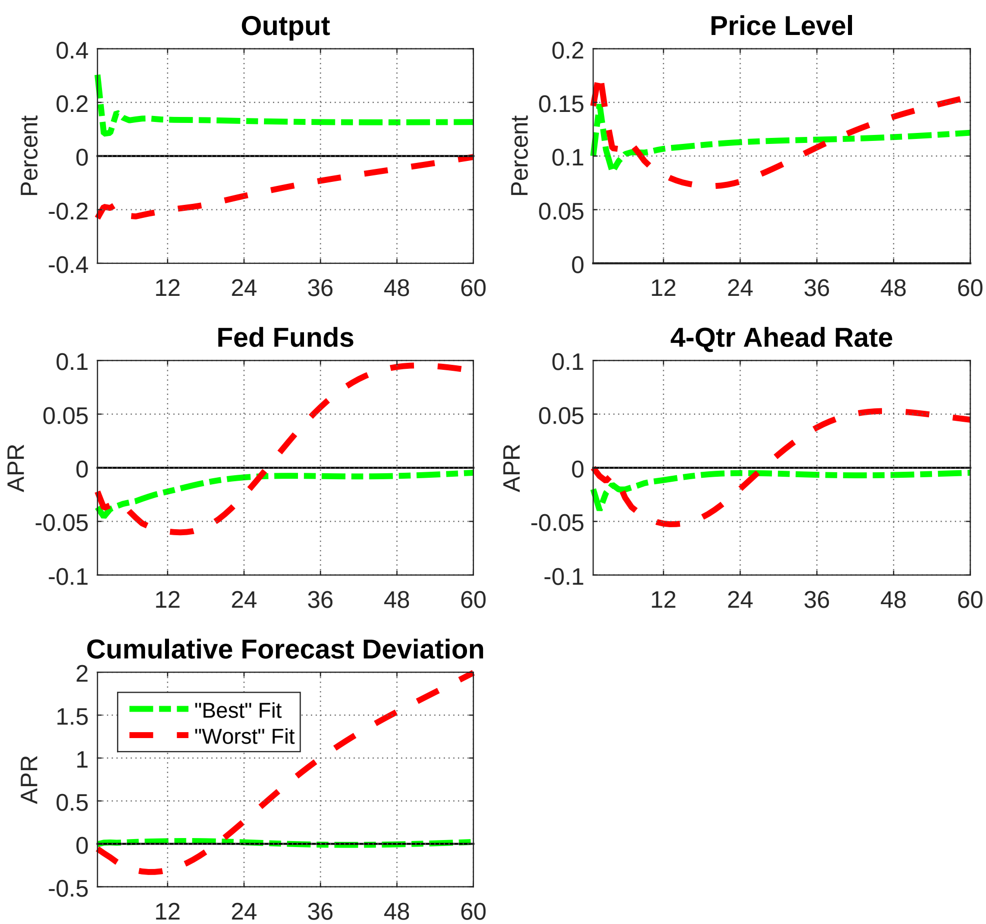


Figure 3: Forward Guidance Shock: Sign Restrictions Only



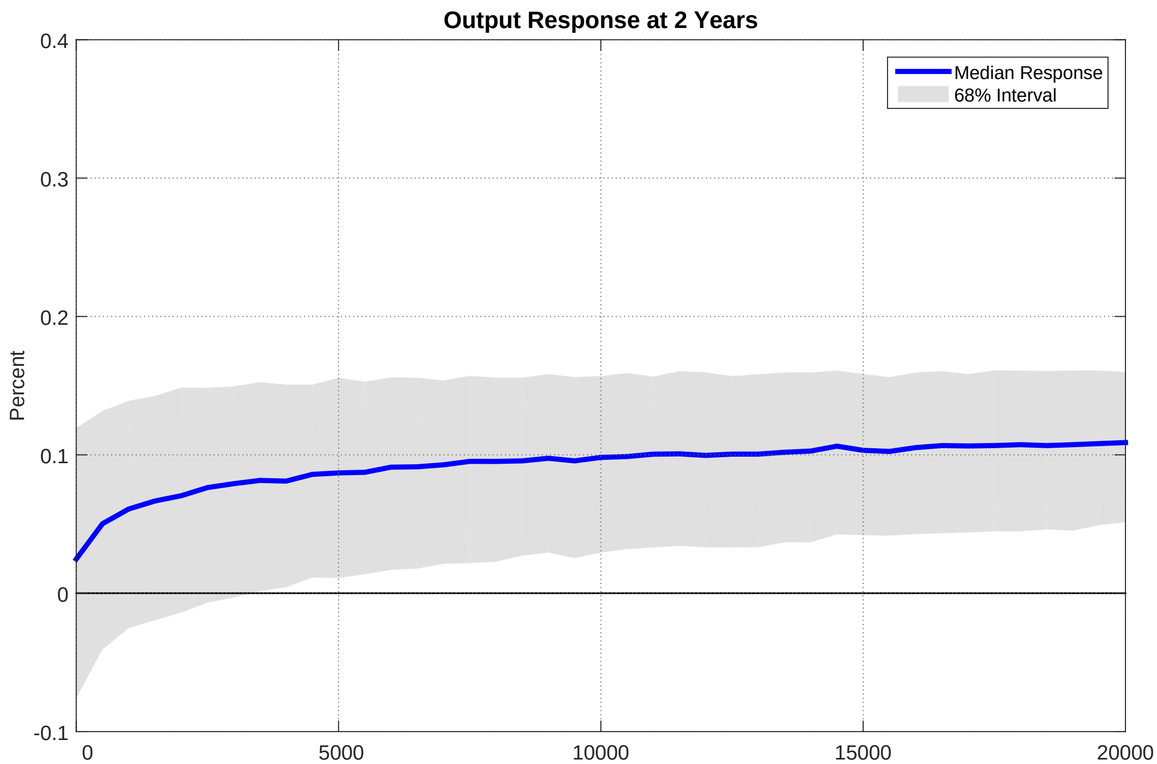
Notes: This figure shows the impulse responses to an identified forward guidance shock using only sign restrictions. The solid blue line is the median response and the shaded region is the 68% interval among structural VAR models. The red line shows the VAR-implied response of future short-term interest rates.

Figure 4: Forward Guidance Shocks: "Best" and "Worst" Fit



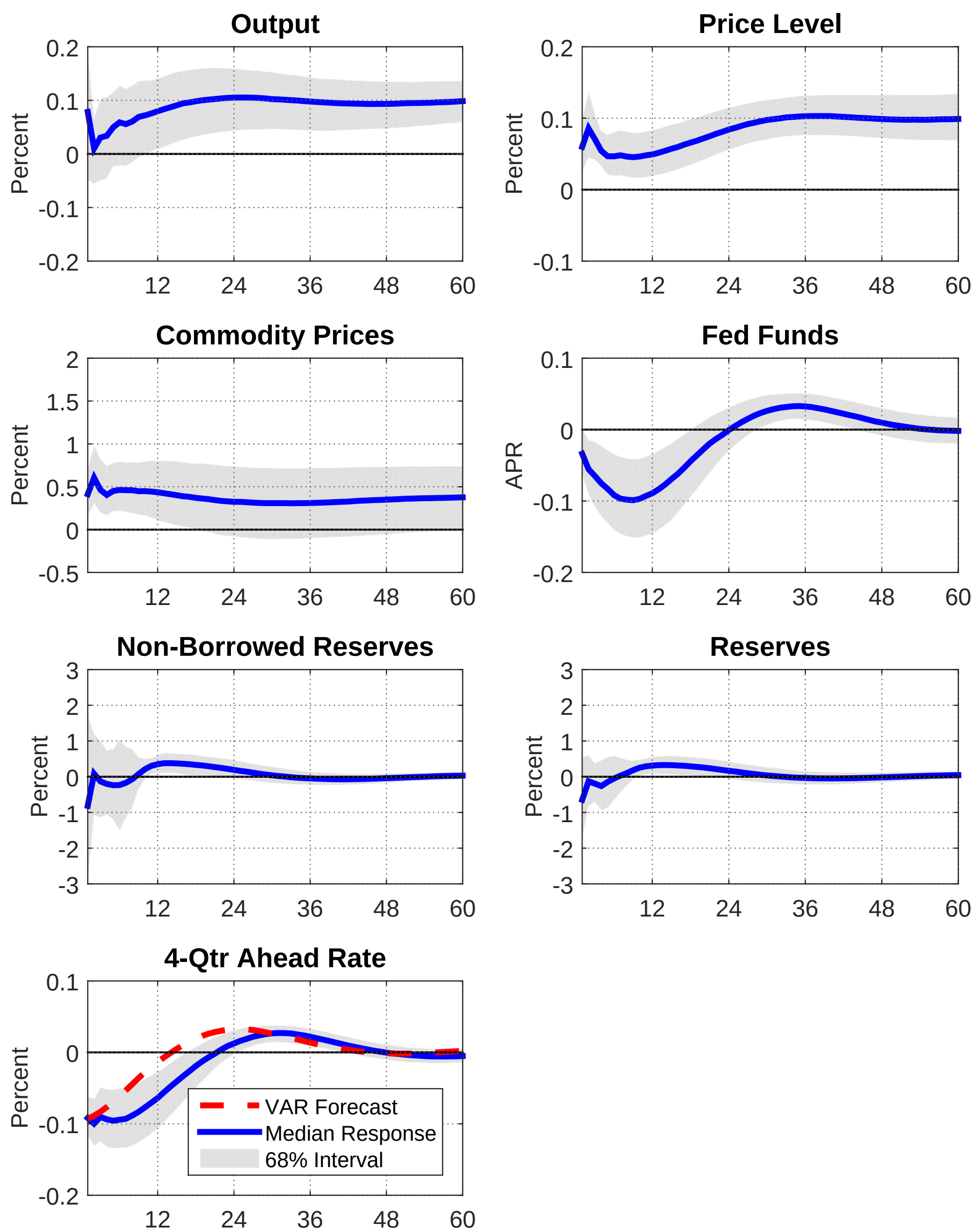
Notes: This figure shows the impulse responses to an identified forward guidance shock for the "best" and "worst" fitting models. The "best" fitting model is the draw that comes the closest to satisfying the conditional forecast-consistent condition and the "worst" fitting model is the draw that is the furthest from satisfying this condition.

Figure 5: Output Effects of Forward Guidance: The Role of Credibility



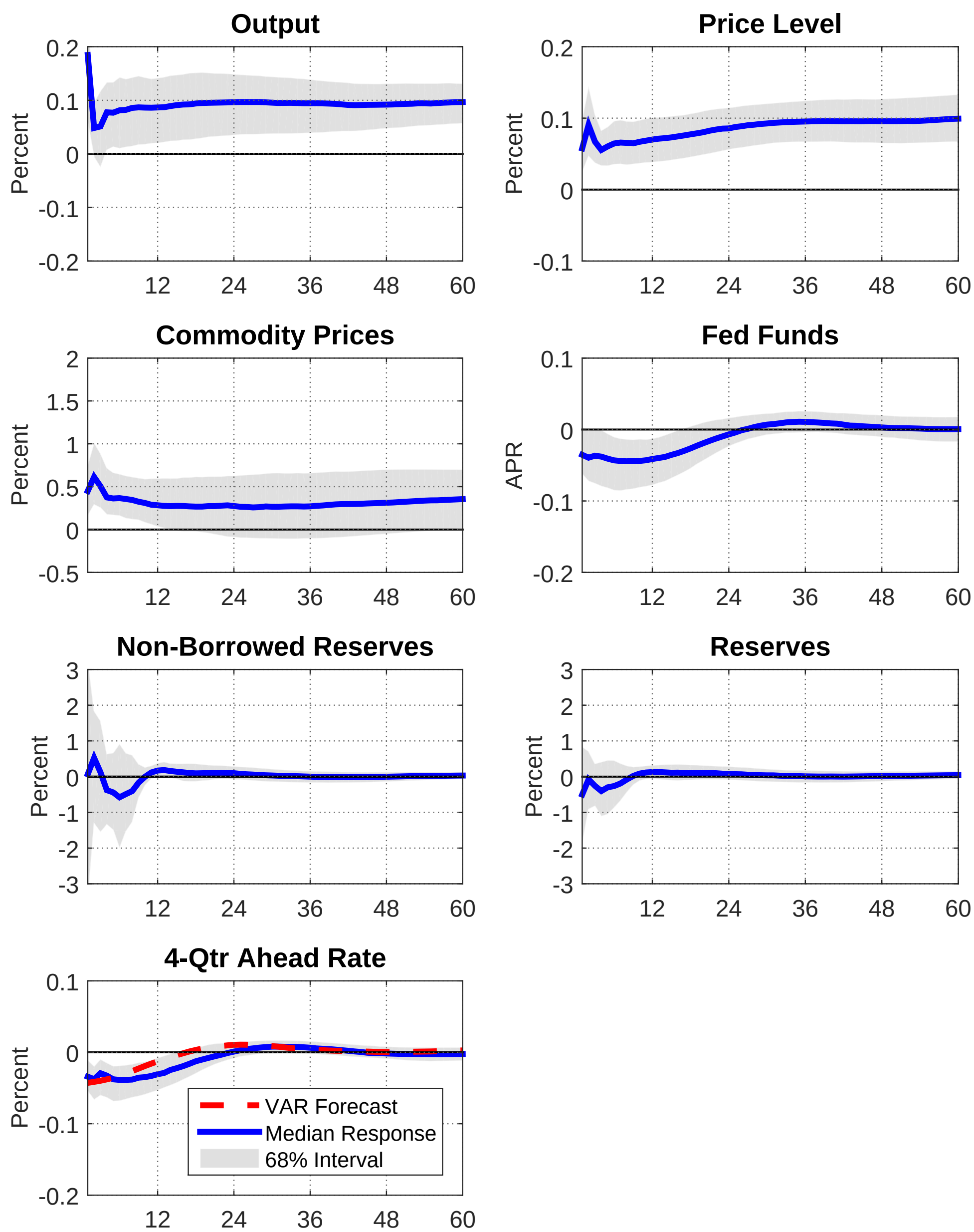
Notes: This figure shows the peak median output response across all impulse response horizons and the median output response after 2 years for alternative values of λ. Larger values of λ tilt the distribution of models towards those implying the actual path of rates matches the expected path of rates from survey forecasters.

Figure 6: Forward Guidance Shock: Sign and Shape Restrictions



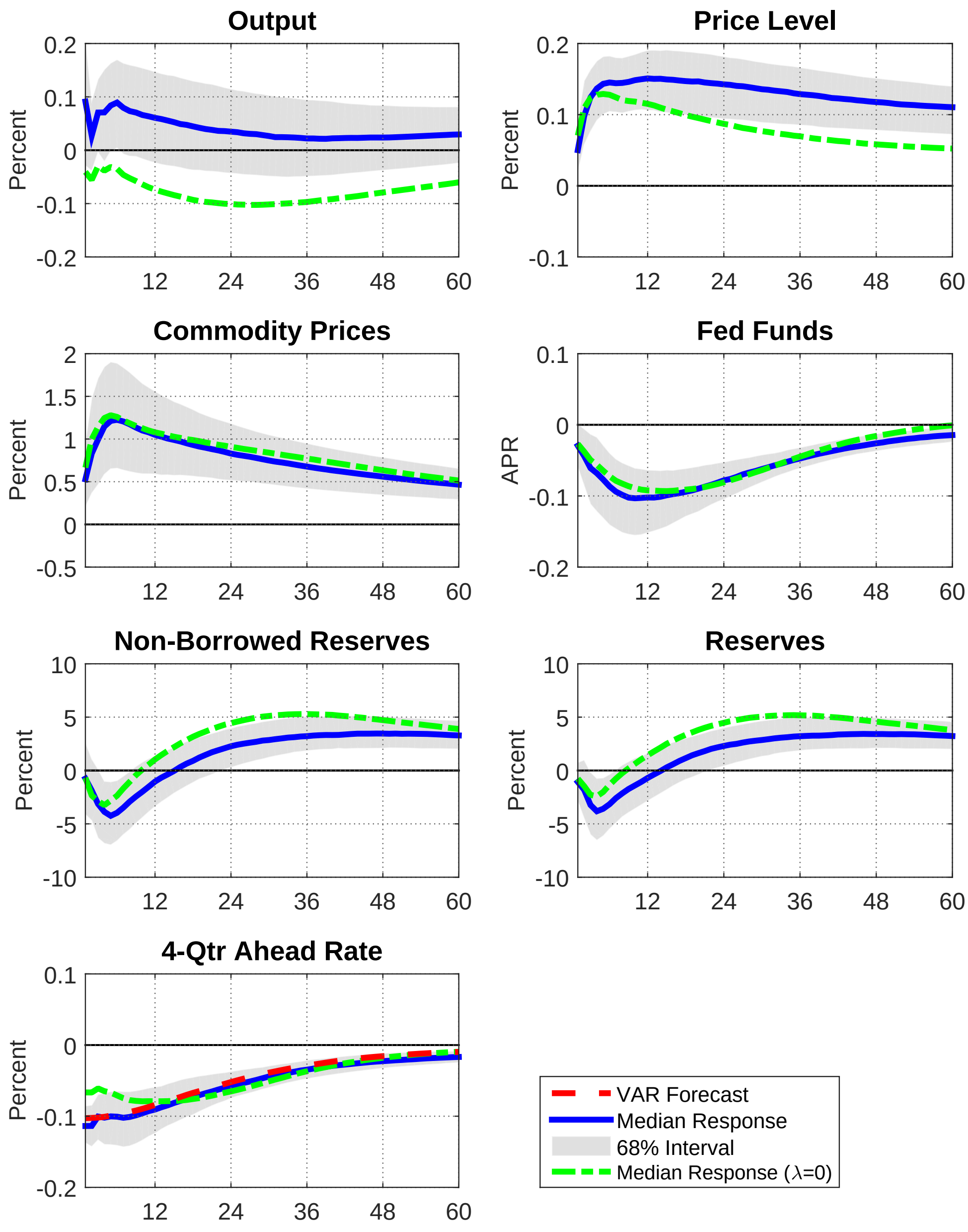
Notes: This figure shows the impulse responses to an identified forward guidance shock using sign restrictions as well as our conditional forecast-consistent prior which places some restrictions on the shape of likely impulse responses. The solid blue line is the median response and the shaded region is the 68% interval among structural VAR models. The red line shows the VAR-implied response of future short-term interest rates. For this figure, $\lambda = 15000$.

Figure 7: FORWARD GUIDANCE SHOCK:IMPORTANCE SAMPLING WEIGHTS BASED UNSCALED SHOCK SIZE



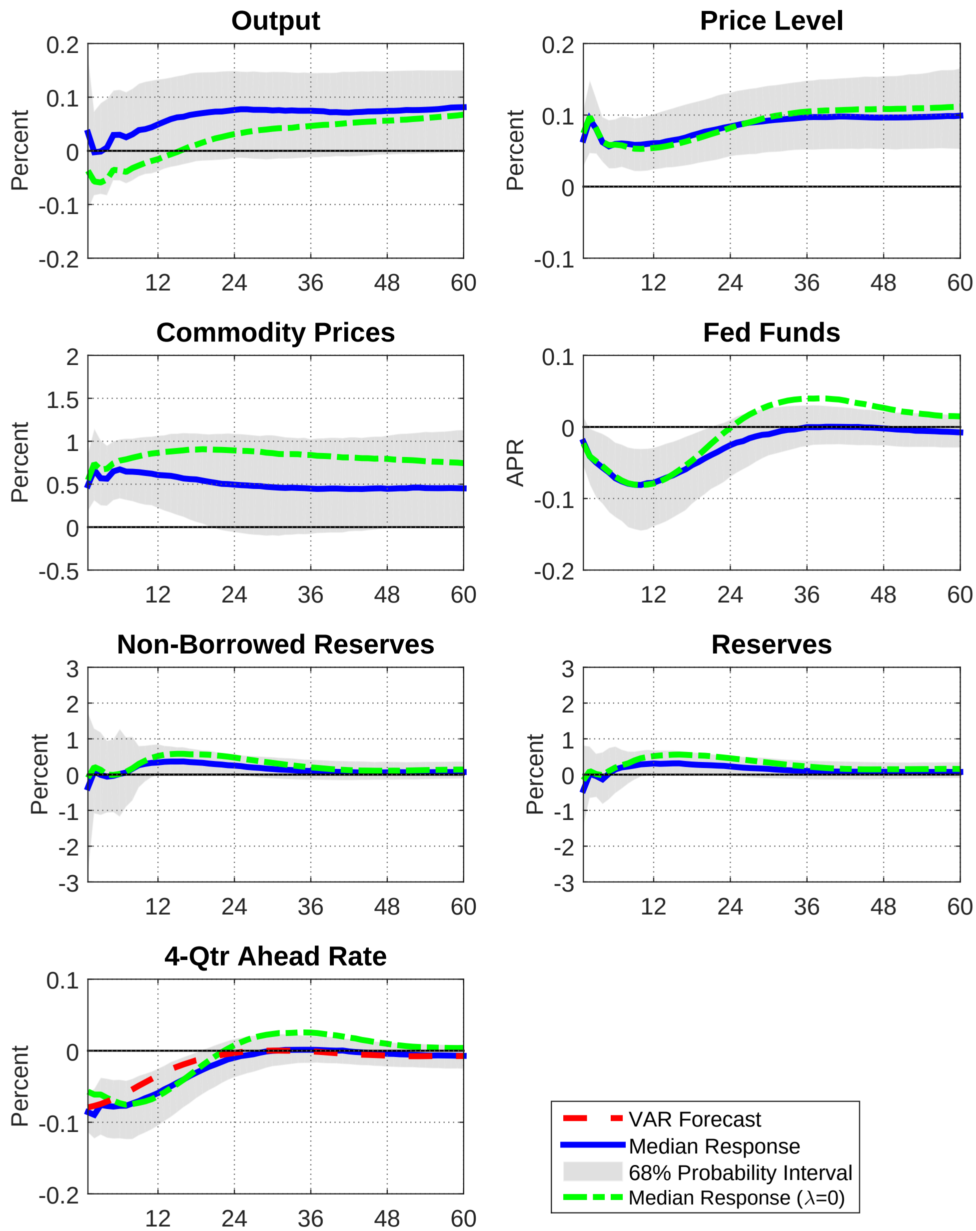
Notes: This figure shows the impulse responses to an identified forward guidance shock using sign restrictions as well as our conditional forecast-consistent prior which places some restrictions on the shape of likely impulse responses. In this figure, the importance sampling weights are calculated without normalizing the size of the initial response of forecasted rates. The solid blue line is the median response and the shaded region is the 68% interval among structural VAR models. The red line shows the VAR-implied response of future short-term interest rates. For this figure, $\lambda = 15000$.

Figure 8: Forward Guidance Shock: Including ZLB Sample



Notes: This figure shows the impulse responses to an identified forward guidance shock using sign restrictions as well as our conditional forecast-consistent prior which places some restrictions on the shape of likely impulse responses. The solid blue line is the median response and the shaded region is the 68% interval among structural VAR models. The red line shows the VAR-implied response of future short-term interest rates. The green dashed-dotted line shows the median impulse responses for $\lambda = 0$. For this figure, $\lambda = 90000$.

Figure 9: Forward Guidance Shock: Model And Parameter Uncertainty



Notes: This figure shows the impulse responses to an identified forward guidance shock using sign restrictions as well as our conditional forecast-consistent prior which places some restrictions on the shape of likely impulse responses. The solid blue line is the median response and the shaded region is the 68% probability interval across both structural VAR model uncertainty and parameter uncertainty. The red line shows the VAR-implied response of future short-term interest rates. The green dashed-dotted line shows the median impulse responses for $\lambda = 0$. For this figure, $\lambda = 15000$.