Resolving the Missing Deflation Puzzle*

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Abstract

We propose a resolution of the missing deflation puzzle, i.e. the fact that inflation fell very little during the Great Recession against the backdrop of the large and persistent fall in GDP. Our resolution of the puzzle stresses the importance of nonlinearities in price and wage-setting using Kimball (1995) aggregation. We show that a nonlinear macroeconomic model with Kimball aggregation resolves the missing deflation puzzle. Importantly, the linearized version of the underlying nonlinear model fails to resolve the missing deflation puzzle. In addition, our nonlinear model reproduces the skewness and kurtosis of inflation observed in post-war U.S. data. All told, our results caution against the common practice of using linearized models to study inflation dynamics when the economy is exposed to large shocks.

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1. Introduction

A key feature of the recent Great Recession in the United States and other advanced economies was a very large, sharp and persistent fall in output with nearly 10 percent as deviation from its pre-crisis trend. Inflation, on the other hand, remained remarkably stable despite the large output contraction. Different measures of core inflation, like the core PCE index or the GDP deflator, which are the relevant benchmarks for macromodels without commodities, fell only gradually by a modest 1 percent during the same time window (see e.g. Christiano, Eichenbaum and Trabandt, 2015).

The fact that inflation fell very little during the recent recession has attracted considerable interest. Hall (2011) sparked the literature by arguing that inflation fell so little in face of the large contraction in demand that one might view inflation as being essentially exogenous to the economy. Specifically, Hall (2011) argues that popular DSGE models based on the simple New Keynesian Phillips curve, according to which prices are set on the basis of a markup over expected future marginal costs, "cannot explain the stabilization of inflation at positive rates in the presence of long-lasting slack". Similarly, Ball and Mazumder (2011) argue that Phillips curves estimated for the post-war pre-crisis period in the United States cannot explain the behavior of inflation during the years of the financial crisis from 2008 through 2010. They argue that the fit of the standard Phillips curve deteriorates sharply during the crisis. One of the reasons for this is that the labor share, a proxy for firms' marginal costs and the key driver of inflation in the New Keynesian model, declined dramatically during the crisis, but inflation nevertheless did fall only very little. A further challenge to the New Keynesian Phillips curve is raised by King and Watson (2012), who find a large discrepancy between actual inflation and inflation predicted by the workhorse Smets and Wouters (2007) model.

Our proposed resolution of the tension between the evolution of output and inflation during these crises is twofold. First, we argue that it is key to introduce real ridigities in price- and wage-setting. To do this, we follow Dotsey and King (2005) and Smets and Wouters (2007) and use the Kimball (1995) instead of the standard Dixit-Stiglitz (1977) aggregator. The Kimball aggregator introduces additional strategic complementarities in the price- and wage-setting, which lowers the sensitivity to prices and wages to the relevant wedges for a given degree of price- and wage-stickiness. As such, the Kimball aggregator is commonly used in New Keynesian models, see e.g. Smets and Wouters (2007), as it allows to simultaneously account for the *macroeconomic evidence* of a low

Phillips curve slope and the *microeconomic evidence* of frequent price changes.

Second, we argue that the standard procedure of linearizing all equilibrium equations around the steady state, except for the ZLB constraint on nominal policy rates, introduces large approximation errors when large shocks hit the economy like in the Great Recession. Hence, our analysis suggests that one ought to use the solution of the nonlinear model rather than the solution of the linearzed model. Implicit in the linearization procedure is the assumption that the linearized solution is accurate even far away from the steady state. However, recent work by Boneva, Braun, and Waki (2016) and Linde and Trabandt (2018) suggests that linearization may produce severely misleading results when large shocks hit the economy, implying that it is invalid to extrapolate decision rules far away from the steady state. Key here is the nonlinearity introduced by the Kimball aggregator, which implies that the demand elasticity for intermediate goods is state-dependent, i.e. the firms' demand elasticity is an increasing function of its relative price. In short, the demand curve is quasi-kinked. While the fully nonlinear model takes this state-dependency explicitly into account, a linear approximation replaces nonlinearity by a linear function.

At the end of the day, one of our main contributions is to show that the introduction of the zero lower bound (ZLB) and real ridigities reduces the elasticity between inflation and output in a large recession in a nonlinear framework and thus helps the model to account for the large output contraction and modest decline in inflation that we witnessed during the global financial crisis and the euro area sovereign debt crisis. Critical in this is to rely on the nonlinear solution: a linearized Phillips curve is associated with a notably larger decline in inflation for a comparable decline in output in a deep recession that triggers the ZLB to bind for a long time. Importantly, this finding is only partly driven by nonlinearities introduced by the ZLB. Even more important is to use the nonlinear solution of the model with the Kimball aggregators.

We establish this result in a variant of the Erceg, Henderson and Levin (2000) benchmark model amended with real ridigities. As the EHL model does not allow for endogenous capital and other real rigidities like habit formation in consumer preferences and investment adjustment costs, we provide support of our finding in this model by estimating a fully-fledged New Keynesian model with endogenous capital formation using Bayesian maximum likelihood techniques. By performing stochastic simulations of a nonlinear variant of this model, we can also establish two further important insights.

First, recent work – see for instance IMF (2016, 2017) – has been aimed towards understanding the absence of upward pressure of price and wage inflation during the recovery from the global financial crisis and the euro area sovereign debt crisis. The nonlinear formulation of our model offers an explanation for this phenonema. In a nutshell, our model implies a nonlinear relationship between price and wage inflation and the output gap. The slope of the price and wage Phillips curves is notably lower (higher) when the economy is in a recession (boom) when the economy is driven by demand or risk-premium shocks. Put differently, the price and wage Phllips curves in our nonlinear model with the Kimball aggregator are not linear for large fluctuations in demand but rather have a banana-type shape as in the seminal paper by Phillips (1958). Thus, while the resumption of growth following the crisis triggered the output gap to narrow eventually, our nonlinear model implies that wage and price inflation would remain subdued until until the level of economic activity relative to its potential had recovered sufficiently. This mechanism in our model offers one possible explanation of the subdued wage and price inflationary pressures in many advanced economies during the recovery from the crisis.

Second, our nonlinear model can be used to understand the positive skewness in post-war U.S. price inflation, i.e. that price inflation scares are much more common than deflationary episodes. We establish this result by comparing higher-order moments for inflation in the data and our model. Our nonlinear model delivers a strong positive skew in inflation along with a negative skew in the output gap. In addition, partly due to the introdution of the ZLB, and partly to the strong positive skew in price inflation, our estimated model also implies a positive skew for the federal funds rate that is in line with the data.

Recent research has examined possible resolutions to the missing deflation puzzle. Fratto and Uhlig (2017) and Lindé, Smets and Wouters (2016) find that the benchmark Smets and Wouters model relies on large offsetting positive price markup shocks to cope with the small fall in inflation in the face of a persistent fall in output observed during the Great Recession. Recent research has also emphasized that financial frictions may be responsible for the small elasticity between output and inflation witnessed during the crisis. Christiano, Eichenbaum and Trabandt (2015) use a model to show that the observed fall in total factor productivity and the rise in firms' cost to borrow funds for working capital played critical roles in accounting for the small drop in inflation that occurred during the Great Recession. Del Negro, Giannoni and Schorfheide (2015) show that the introduction of a financial accelerator together with a flattening of the Phillips curve can account for the small drop in inflation in the Great Recession. Gilchrist, Schoenle, Sim and Zakrajsek (2016) develop a model in which firms face financial frictions when setting prices in an environment with customer markets. Financial distortions create an incentive for financially constrained firms to raise prices in response to adverse financial or demand shocks in order to preserve internal liquidity and avoid accessing external finance. While financially unconstrained firms cut prices in response to these adverse shocks, the share of financially constrained firms is sufficiently large in their model to attenuate the fall in inflation in response to fluctuations in GDP. Gilchrist, Schoenle, Sim and Zakrajsek (2016) examine a micro data set which supports the implications of their model.

Our work is related to Arouba et al. (2017). Using asymmetric price and wage adjustment costs, the authors show that their model can produce skewness in inflation and output growth as observed in the data. However, their model cannot account for the skewness of the federal funds rate data while ours does. In contrast to these authors, our model does not rely on asymmetric adjustment costs but generates the skewness observed in macro data due to the Kimball (1995) aggregator. Most importantly, Arouba et al. (2017) do not study the implications of the zero lower bound while our paper focuses on the interplay of large shocks and the zero lower bound to characterize nonlinearities in price and wage Phillips curves.

More generally, the mechanism we identify in our paper offers an alternative, perhaps complementary and not mutually exclusive, channel to understand the same phenomena. Our resolution of the puzzle stresses the nonlinear influence of strategic complementarities and real rigidities in price-setting of firms. We find it attractive due to its simplicity and that it solves an important tension between micro- and macroevidence on price-setting behavior.

The paper is organized as follows. Section 2 presents the stylized New Keynesian model with stickiness and real rigidities in price- and wage-setting while Section 3 demonstrates the results based on the stylized model. Section 4 examines the robustness of our results in stochastic simulations of a nonlinear variant of an estimated benchmark New Keynesian model. Finally, we provide concluding remarks in Section 5.

2. A Stylized New Keynesian Model

The simple model we use is very similar to the Erceg, Henderson and Levin (2000) (EHL henceforth) model with gradual price and wage adjustment. We deviate from EHL in two ways. First, by allowing for Kimball (1995) aggregators in price and wage setting (with the standard Dixit and Stiglitz (1977) specification as a special case). Second, by including a discount factor, or more generally savings, shock. The complete specification of the nonlinear and linearized formulation of the model is provided in Appendix A.

2.1. Model

2.1.1. Firms and Price Setting

Final Goods Production The single final output good Y_t is produced using a continuum of differentiated intermediate goods $Y_t(f)$. Following Kimball (1995), the technology for transforming these intermediate goods into the final output good is

$$\int_0^1 G\left(\frac{Y_t(f)}{Y_t}\right) df = 1. \tag{1}$$

Following Dotsey and King (2005) and Levin, Lopez-Salido and Yun (2007), we assume that $G_Y(\cdot)$ is given by the following strictly concave and increasing function:

$$G_Y\left(\frac{Y_t(f)}{Y_t}\right) = \frac{\omega_p}{1+\psi_p}\left[\left(1+\psi_p\right)\frac{Y_t(f)}{Y_t} - \psi_p\right]^{\frac{1}{\omega_p}} - \left[\frac{\omega_p}{1+\psi_p} - 1\right], \tag{2}$$

where $\omega_p = \frac{\phi_p(1+\psi_p)}{1+\phi_p\psi_p}$. Here $\phi_p > 1$ denotes the gross markup of the intermediate goods firms. The parameter $\psi_p \leq 0$ governs the degree of curvature of the intermediate firm's demand curve.[1] In Figure 1 we show how relative demand is affected by the relative price under alternative assumptions about ψ_P, given a value for the gross markup of $\phi_p = 1.1$. When $\psi_p = 0$, the demand curve exhibits constant elasticity as under the standard Dixit-Stiglitz aggregator, implying a log-linear relationship between relative demand and relative prices. When $\psi_p < 0$ – as in e.g. Smets and Wouters (2007) – a firm instead faces a quasi-kinked demand curve, implying that a drop in its relative price only stimulates a small increase in demand. On the other hand, a rise in its relative price generates a larger fall in demand compared to the $\psi_p = 0$ case. Relative to the standard Dixit-Stiglitz aggregator, this introduces more strategic complementarity in price setting which causes intermediate firms to adjust prices by less to a given change in marginal cost. Finally, we notice that $G_Y(1) = 1$, implying constant returns to scale when all intermediate firms produce the same amount.

Firms that produce the final output good are perfectly competitive in both product and factor markets. Thus, final goods producers minimize the cost of producing a given quantity of the output index Y_t, taking as given the price $P_t(f)$ of each intermediate good $Y_t(f)$. Moreover, final goods producers sell units of the final output good at a price P_t, and hence solve the following problem:

$$\max_{Y_t, Y_t(f)} P_t Y_t - \int_0^1 P_t(f)\, Y_t(f)\, df \tag{3}$$

[1] The parameter used in Smets and Wouters (2007) to characterize the curvature of the Kimball aggregator can be mapped to our model using the following formula: $\epsilon_p = -\frac{\phi_p}{\phi_p - 1}\psi_p$.

subject to the constraint (1). The first order conditions can be written as

$$\begin{aligned}
\frac{Y_t(f)}{Y_t} &= \frac{1}{1+\psi_p}\left(\left(\frac{P_t(f)}{P_t}\frac{1}{\vartheta_t^p}\right)^{\frac{\phi_p}{1-\phi_p}(1+\psi_p)}+\psi_p\right), \\
P_t\vartheta_t^p &= \left(\int P_t\left(f\right)^{\frac{1+\psi_p\phi_p}{1-\phi_p}}df\right)^{\frac{1-\phi_p}{1+\psi_p\phi_p}}, \\
\vartheta_t^p &= 1+\psi_p-\psi_p\int\frac{P_t(f)}{P_t}df,
\end{aligned} \tag{4}$$

where ϑ_t^p denotes the Lagrange multiplier on the aggregator constraint (2). Note that for $\psi_p = 0$, this problem leads to the usual Dixit and Stiglitz (1977) expressions

$$\frac{Y_t\left(f\right)}{Y_t} = \left[\frac{P_t\left(f\right)}{P_t}\right]^{-\frac{\phi_p}{\phi_p-1}},\ P_t = \left[\int P_t\left(f\right)^{\frac{1}{1-\phi_p}}df\right]^{1-\phi_p}$$

Intermediate Goods Production A continuum of intermediate goods $Y_t(f)$ for $f \in [0,1]$ is produced by monopolistically competitive firms, each of which produces a single differentiated good. Each intermediate goods producer faces a demand schedule from the final goods firms through the solution to the problem in (3) that varies inversely with its output price $P_t\left(f\right)$ and directly with aggregate demand Y_t.

Aggregate capital (K) is assumed to be fixed, so that aggregate production of the intermediate good firm is given by

$$Y_t\left(f\right) = K\left(f\right)^{\alpha}N_t\left(f\right)^{1-\alpha}. \tag{5}$$

Despite the fixed aggregate stock $K \equiv \int K\left(f\right)df$, shares of it can be freely allocated across the f firms, implying that real marginal cost, $MC_t(f)/P_t$ is identical across firms and equal to

$$\frac{MC_t}{P_t} \equiv \frac{W_t/P_t}{MPL_t} = \frac{W_t/P_t}{\left(1-\alpha\right)K^{\alpha}{N_t}^{-\alpha}}, \tag{6}$$

where the determination of the aggregate labor-index N_t is discussed in Section 2.1.2.

The prices of the intermediate goods are determined by Calvo-Yun (1996) style staggered nominal contracts. In each period, each firm f faces a constant probability, $1-\xi_p$, of being able to reoptimize its price $P_t(f)$. The probability that any firm receives a signal to reset its price is assumed to be independent of the time that it last reset its price. If a firm is not allowed to optimize its price in a given period, it adjusts its price according to $\tilde{P}_t = \Pi P_{t-1}$,where Π is the steady-state (gross, i.e. $\Pi = 1+\pi$) inflation rate and $\tilde{P}_t$ is the updated price.

Given Calvo-style pricing frictions, firm f that is allowed to reoptimize its price ($P_t^{opt}\left(f\right)$) solves the following problem

$$\max_{P_t^{opt}(f)} \mathrm{E}_t \sum_{j=0}^{\infty} \left(\beta\xi_p\right)^j \varsigma_{t+j}\Lambda_{t,t+j} \left[\Pi^j P_t^{opt}(f) - MC_{t+j}\right] Y_{t+j}(f)$$

where $\Lambda_{t,t+j}$ is the stochastic discount factor (the conditional value of future profits in utility units, recalling that the household is the owner of the firms), and demand $Y_{t+j}(f)$ from the final goods firms is given by the equations in (4).

2.1.2. Households and Wage Setting

Labor Contractors Competitive labor contractors aggregate specialized labor inputs $N_{t,j}$ supplied by households into homogenous labor n_t which is hired by intermediate good producers. Labor contractors maximize profits

$$\max_{N_{t,j},N_t} W_t N_t - \int W_{t,j} N_{t,j} dj$$

where $W_{t,j}$ is the wage paid by the labor contractor to households for supplying type j labor. W_t denotes the wage paid to the labor contractor for homogenous labor.

Maximization of profits is subject to

$$\int G_N\left(\frac{N_{t,j}}{N_t}\right) dj = 1,$$

where

$$G_N\left(\frac{N_{t,j}}{N_t}\right) = \frac{\omega_w}{1+\psi_w}\left[(1+\psi_w)\frac{N_{t,j}}{N_t} - \psi_w\right]^{\frac{1}{\omega_w}} - \frac{\omega_w}{1+\psi_w} + 1$$

is the Kimball aggregator specification as used in Dotsey and King (1995) or Levin, Lopez-Salido and Yun (2007) adapted for the labor market. Note that $\omega_w = \frac{(1+\psi_w)\phi_w}{1+\phi_w\psi_w}$ where $\theta_w \geq 0$ denotes the net wage markup, $\phi_w \geq 1$ denotes the gross wage markup and $\psi_w \leq 0$ is the Kimball parameter that controls the degree of complementarities in wage setting.

If we let ϑ_t^w denote the multiplier on the labor contractor's constraint, optimization results in the following conditions:

$$\frac{N_{t,j}}{N_t} = \frac{1}{1+\psi_w}\left(\left[\frac{W_{t,j}}{W_t}\right]^{-\frac{\phi_w(1+\psi_w)}{\phi_w-1}} \left[\vartheta_t^w\right]^{\frac{\phi_w(1+\psi_w)}{\phi_w-1}} + \psi_w\right) \tag{7}$$

$$W_t\vartheta_t^w = \left[\int W_{t,j}^{-\frac{1+\psi_w+(\phi_w-1)\psi_w}{\phi_w-1}} dj\right]^{-\frac{\phi_w-1}{1+\psi_w+(\phi_w-1)\psi_w}} \tag{8}$$

$$\vartheta_t^w = 1+\psi_w - \psi_w\int \frac{W_{t,j}}{W_t} dj \tag{9}$$

Where equation (7) denotes the demand for labor, equation (8) is the aggregate wage index and equation (9) is the zero profit condition for labor contractors.

Note that for $\psi_w = 0$ we get the standard Dixit-Stiglitz expressions

$$\frac{N_{t,j}}{N_t} = \left[\frac{W_{t,j}}{W_t}\right]^{-\frac{\phi_w}{\phi_w - 1}}, W_t = \left[\int W_{t,j}^{\frac{1}{1-\phi_w}} dj\right]^{1-\phi_w}, \vartheta_t^w = 1$$

Households There is a continuum of households $j \in [0, 1]$ in the economy. Each household supplies a specialized type of labor j to the labor market. The j^{th} household is the monopoly supplier of the j^{th} type of labor service. The j^{th} household maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \varsigma_t \left\{ \ln C_{j,t} - \omega \frac{N_{j,t}^{1+\chi}}{1+\chi} \right\} \tag{10}$$

s.t.

$$P_t C_{j,t} + B_{j,t} = W_{j,t} N_{j,t} + R_t^k K + (1 + i_{t-1}) B_{j,t-1} + \Gamma_t - T_t + A_{j,t}$$

where the choice variables of the j^{th} household are consumption $C_{j,t}$ and risk-free government debt B_t. The j^{th} household also chooses the wage W_j subject to Calvo sticky prices as in Erceg, Henderson and Levin (2000, EHL). The household understands that when choosing W_j that it must supply the amount of labor N_j demanded by a labor contractor according to equation (7).

In principle, the presence of wage setting frictions implies that households have idiosyncratic levels of wealth and, hence, consumption. However, we follow EHL in supposing that each household has access to perfect consumption insurance. Because of the additive separability of the family utility function, perfect consumption insurance at the level of households implies equal consumption across households. Given this, we have simplified our notation and not include a subscript, j, on the j^{th} family's consumption (and bond holdings). Note that even though consumption is equal across households, consumption in response to shocks is not constant over time across households.

The variable ς_t is an exogenous shock to the discount factor. We assume that $\delta_t = \frac{\varsigma_t}{\varsigma_{t-1}}$ is exogenous with $\delta = 1$ in steady state. P_t denotes the aggregate price level. R_t denotes the gross nominal interest rate on bonds purchased in period $t-1$ which pay off in period t. R_t^k is the rental rate of capital that the households rents to goods producing firms. Note that the households capital stock K is fixed, i.e. we abstract from endogenous capital accummulation. T_t are lump-sum taxes net of transfers and Γ_t denotes the share of profits that the household receives. $A_{j,t}$ denotes the payments and receipts associated with the insurance associated with wage stickiness. $\omega > 0$ and $\chi \geq 0$ and $0 < \beta < 1$ are parameters.

Utility maximization for consumption and government bond holdings yields the standard consumption Euler equation (in a symmetric equilibrium):

$$1 = \beta E_t \left(\delta_{t+1} \frac{1+i_t}{1+\pi_{t+1}} \frac{C_t}{C_{t+1}} \right), \tag{11}$$

where $1+\pi_{t+1} = P_{t+1}/P_t$.

Wage Setting The household faces a standard standard monopoly (labor union) problem of selecting $W_{j,t}$ to maximize the welfare, (10) subject to the demand for labor (7). Following EHL, we assume that the household experiences Calvo-style frictions in its choice of $W_{j,t}$. In particular, with probability $1-\xi_w$ the j^{th} family has the opportunity to reoptimize its wage rate. With the complementary probability, the family must set its wage rate according to $W_{j,t} = \Pi_w W_{j,t-1}$where Π_w denotes the steady state gross rate of wage inflation. The households optimal choice for $\tilde{W}_{j,t}$ is to maximize

$$\max_{\tilde{W}_{j,t}} E_t \sum_{i=0}^{\infty} (\beta \xi_w)^i \varsigma_{t+i} \left\{ -\omega \frac{N_{j,t+i}^{1+\chi}}{1+\chi} + \Lambda_{t+i} \tilde{W}_{j,t} \Pi_w^i N_{j,t+i} \right\}$$

subject to labor demand:

$$N_{t+i,j} = \frac{1}{1+\psi_w} \left(\left[\frac{\tilde{W}_{j,t} \tilde{\Pi}_w^i}{W_{t+i} \vartheta_{t+i}^w} \right]^{-\frac{\phi_w(1+\psi_w)}{\phi_w - 1}} + \psi_w \right) N_{t+i}.$$

2.1.3. Monetary Policy

We assume that the central bank sets the nominal interest rate following a Taylor-type policy rule that is subject to the zero lower bound:

$$1+i_t = \max \left(1, (1+i) \left[\frac{1+\pi_t}{1+\pi} \right]^{\gamma_\pi} \left[\frac{Y_t}{Y_t^{pot}} \right]^{\gamma_x} \right) \tag{12}$$

where Y_t^{pot} denotes the level of output that would prevail if prices and wages were flexible.

In terms of fiscal policy, we assume that the government balances its budget using lump-sum taxes.

2.1.4. Aggregate Resource Constraint

It is straightfoward to show that the aggregate resource constraint is given by

$$C_t = Y_t = (p_t^*)^{-1} (w_t^*)^{-(1-\alpha)} K^\alpha l_t^{1-\alpha} \tag{13}$$

where

$$p_t^* \equiv \frac{1}{1+\psi_p}\int_0^1 \left(\left(\frac{P_t(f)}{P_t\vartheta_t^p}\right)^{\frac{\phi_p}{\phi_p-1}(1+\psi_p)} + \psi_p\right) df$$

$$w_t^* = \frac{1}{1+\psi_w}\int_0^1 \left(\left(\frac{W_{t,j}}{W_t\vartheta_t^w}\right)^{\frac{\phi_w}{\phi_w-1}(1+\psi_w)} + \psi_w\right) dj$$

where aggregate hours per capita supplied by the household l_t is given by $l_t = \int N_{t,j}dj$. The variables $p_t^* \geq 1$ and $w_t^* \geq 1$ denote the Yun (1996) aggregate price and wage dispersion terms. Both price- and wage dispersion, ceteris paribus, will lower output in the economy. In the technical appendix, we show how to develop recursive formulations of the sticky price and wage distortion terms p_t^* and w_t^*. Note, however, that p_t^* and w_t^* vanish when the model is linearized.

2.2. Parameterization

Our benchmark calibration is fairly standard at a quarterly frequency. We set the discount factor $\beta = 0.9975$, and the steady state net inflation rate $\pi = .005$; this implies a steady state interest rate of $i = .0075$ (i.e., three percent at an annualized rate). We set the capital share parameter $\alpha = 0.3$ and the disutility of labor parameter $\chi = 0$.[2] As a compromise between the low estimate of ϕ_p in Altig et al. (2011) and the higher estimated value by Smets and Wouters (2007), we set $\phi_p = 1.1$. We set $\xi_p = 0.66$. To pin down the Kimball parameter ψ_p consider the log-linearized New Keynesian Phillips Curve in our model:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa_p \, \widehat{mc_t}, \tag{14}$$

where $\widehat{mc_t}$ denotes the log-deviation of marginal cost from its steady state. $\hat{\pi}_t$ denotes the log-deviation of gross inflation from its steady state. The parameter κ denotes the slope of the Phillips curve and is given by:

$$\kappa_p \equiv \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}\frac{1}{1-\phi_p\psi_p}. \tag{15}$$

The *macroeconomic* evidence suggest that the sensitivity of aggregate inflation to variations in marginal cost is very low, see e.g. Altig et al. (2011). To capture this, we set the Kimball parameter $\psi_p = -12.2$ so that the slope of the Phillips curve is $\kappa_p = 0.012$ given the values for β, ξ_p and ϕ_p discussed above.[3] This calibration allows us to match micro- and macroevidence about

[2] We will consider $\chi > 0$ in a future revision of this paper.

[3] The median estimates of the Phillips Curve slope in recent empirical studies by e.g. Adolfson et al. (2005), Altig et al. (2011), Galí and Gertler (1999), Galí, Gertler and López-Salido (2001), Lindé (2005), and Smets and Wouters (2003, 2007) are in the range of $0.009 - .014$.

firms' price setting behavior and is aimed to capture the resilience of core inflation, and measures of expected inflation, to a deep downturn such as the Great Recession.

For the parameters pertaining to the nominal wage setting frictions we assume that $\phi_w = 1.1$, $\xi_w = 0.75$, and $\psi_w = -6$. These parameter values correspond to those set and estimated in the medium-sized New Keynesian model discussed below. We use the standard Taylor (1993) rule parameters $\gamma_\pi = 1.5$ and $\gamma_x = .125$.

In order to facilitate comparison between the nonlinear and linear model, we specify processes for the exogenous shocks such that there is no loss in precision due to an approximation. In particular, discount factor evolves according to the following AR(1) process:

$$\delta_t - \delta = \rho_\delta \left(\delta_{t-1} - \delta\right) + \sigma_\delta \varepsilon_{\delta,t} \tag{16}$$

where $\delta = 1$. Our baseline parameterization adopts a persistence coefficient $\rho_\delta = 0.95$ in (16).

2.3. Solving the Model

We compute the linearized and nonlinear solutions using the Fair and Taylor (1983) method. This method imposes certainty equivalence on the nonlinear model, just as the linearized solution does by definition. In other words, the Fair and Taylor solution algorithm traces out the implications of not linearizing the equilibrium equations for the resulting multiplier without shock uncertainty. Hence, future shock uncertainty does not matter for neither the nonlinear nor the linearized model solution. All relevant information is captured by the current state of the economy, including the various contemporaneous shocks we allow for in the model.

An alternative approach would have been to compute solutions where uncertainty about future shock realizations matters for the dynamics of the economy following for instance Adam and Billi (2006, 2007) within a linearized framework and Fernández-Villaverde et al. (2015) and Gust, Herbst, López-Salido and Smith (2016) within a nonlinear framework. These authors have shown that allowing for future shock uncertainty can potentially have important implications for equilibrium dynamics. Importantly, none of the authors have considered a model with Kimball aggregation. Linde and Trabandt (2018) solve a standard NK model with sticky prices and Kimball aggregation under shock uncertainty using global methods. There, shock uncertainty implies that agents expect the ZLB to bind in 10 out of 100 quarters. Linde and Trabandt (2018) show that the effects of shock uncertainty on the global solution of the nonlinear model are quantitatively negligible. With the introduction of wage stickiness and Kimball aggregation in the labor market in the present paper (in addition to price stickiness and Kimball aggregation in the goods market as in Linde and Trabandt, 2018) should moderate the effect of shock uncertainty in the nonlinear model even further.[4]

As a practical matter, we feed the relevant equations in the nonlinear and log-linearized versions of the model to Dynare. Dynare is a pre-processor and a collection of MATLAB routines which can solve nonlinear models with forward looking variables, and the details about the implementation of the algorithm used can be found in Juillard (1996). We use the perfect foresight simulation algorithm implemented in Dynare using the 'simul' command.[5] The algorithm can easily handle the ZLB constraint: one just writes the Taylor rule including the max operator in the model equations, and the solution algorithm reliably calculates the model solution in fractions of a second. Thus, apart from gaining intuition about the mechanisms embedded into the models, there is no need anymore to linearize models in order to solve and simulate them.

3. Inflation Dynamics in the Stylized Model

In this section, we report our main results for the linearized and nonlinear solution of the model outlined in the Section above. As mentioned earlier, our aim is to study the joint output-inflation dynamics for large adverse demand shocks.

3.1. A Recession Scenario

We first study the effects of a large adverse demand shock. Following the literature on fiscal multipliers (e.g. Christiano, Eichenbaum and Rebelo, 2011), the particular shock we consider is a large positive shock to the discount factor δ_t. Specifically, we assume that $\varepsilon_{\delta,1} = 0.01$ in (16) so that δ_t increases from 1 to 1.01 in the first period and then gradually reverts back to steady state.

Figure 2 reports the linear and nonlinear solutions for a selected set of variables, assuming that the economy is in the deterministic steady state in period 0, and then the shock hits the economy in period 1. In A.5, we report effects for an extended set of variables. The left column of Figure 2 shows results when the ZLB is, hypothetically, not assumed to be binding, whereas the right column shows the effects when the ZLB binds. As is evident from the left column, the same-sized shock has a rather different impact on the economy depending on whether the model is linearized or solved in its original nonlinear form. For instance, we see that while output falls more in the

[4] In a future version of this paper we plan to solve the nonlinear and linearized model subject to shock uncertainty.
[5] The solution algorithm implemented in Dynare's simul command is the method developed in Fair and Taylor (1983).

nonlinear model, wage and price inflation falls notably less than in the linearized solution. So the linearized model features a larger elasticity between output and inflation compared to the nonlinear model.

In the right column in Figure 2, we report the effects of the same shock, but now assume that the central bank is constrained by the ZLB on the policy rate. Important insights about the differences between the linearized and nonlinear solutions can be gained. First, although the drop in the potential real rate is about the same in both models, the linearized model generates a much longer liquidity trap because inflation and expected inflation fall much more, which in turn causes the actual real interest rate to rise much more initially. The larger initial rise in the actual real interest rate — and thus in the gap between the actual and potential real rates — triggers a larger fall in the output gap (real GDP also falls more in the linearized model because the discount factor shock does not impact potential GDP). Even so, and perhaps most important, we see that price inflation falls substantially less in the nonlinear model. This suggests that the difference between the linearized and nonlinear solutions too a large extent is driven by the linearization of the pricing and wage block of the model.

It is also instructive to compare the solutions with and without imposing the ZLB. Comparing the linearized solutions, we find that imposing the ZLB results in a notably larger fall output (from -3.5 to almost -7 percent) and deflation in prices and wages (not shown). For the nonlinear solution, we find that imposing the ZLB (albeit admittedly so with a shorter duration compared to the linearized solution) does not affect the price and wage inflation paths much - they are essentially unaffected. The main impact of imposing the ZLB in the nonlinear model is — apart from the interest rate path — a somewhat deeper output contraction. According to United States congressional budget office (CBO), the output gap fell roughly by 6 percent during the great recession but PCE price inflation (4-quarter change) never fell below 1 percent. Our nonlinear solution is consistent with this fact, whereas the linearized model is associated with a notably lower inflation path which is counterfactual relative to the data.[6] In addition, we never experienced any persistent nominal wage deflation (at least not wage inflation measured with nominal compensation per hour).

The Kimball aggreggator is key for shrinking the sensitivity of inflation to the large adverse shock in economic activity in the nonlinear model. To show this, we solved our model under

[6] Figure A.3 in Appendix A.5 shows the paths in the linearized and nonlinear solutions when the size of the discount factor shock is set to give an identical fall in the output gap the first period. In this case, the fall in inflation is about twice as large in the linearized solution after one year.

the assumption that final good and labor services are aggregated with the standard Dixit-Stiglitz constant elasticity demand schedule ($\psi_p = \psi_w = 0$). Because this adjustment changes the slope coefficients in the linearized price and wage schedules (see e.g. eqs. 14 and 15 for prices), we adjusted ξ_p and ξ_w so that the linear model solutions are identical under both benchmark calibration with the Kimball aggregator and with our alternative Dixit-Stiglitz specification.[7] With this alternative aggregator in both wage and price aggregation, Figure A.4 in Appendix A.5 shows that price inflation would fall at least as much in the nonlinear model as in the linearized solution. So the kinked demand curve introduced by the Kimball aggregator is essential.

3.2. Phillips Curves

To understand the unconditional differences in dynamics implied by the linearized and nonlinear solutions, we now undertake stochastic simulations of the model for shocks to the stochastic discount factor δ_t. We solve and simulate the linearized and nonlinear solutions for a long sample of 10,000 periods contingent on exactly the same sequence of shocks $\{\varepsilon_{\delta,t}\}_{t=1}^{10{,}000}$ in (16). However, we use somewhat different standard deviations for the linearized model ($\sigma_\delta = 0.00125$) and the nonlinear model ($\sigma_\delta = 0.0015$), to ensure that the probability of hitting the ZLB is 10 percent in both model solutions. The left column in Figure 3 shows the paths with simulated data in the nonlinear model, whereas the right column shows the simulated data in the linearized model for the same set of variables as depicted in Figure 2. In Appendix A.5, we report results for an extended set of variables.

From the figure, we see noticeable differences in the behavior of nominal wage and price inflation between the linearized and nonlinear model. The simulated data from the linearized model is characterized by several episodes with substantial deflation, whereas the nonlinear model does not feature any larger (if any) periods with deflation in wages and prices. There are several episodes when price and wage inflation is persistently low, but no stretch with deflationary outcomes because the Kimball aggregator implies that firms (unions) become reluctant to change prices (wages) much when relative demand is low (i.e. when they are located in the upper left quadrant in Figure 1). On the other hand, in periods when relative demand is percieved to be high by agents, they are more willing to change their prices (i.e. they are located in the lower right quadrant in Figure 1). As a result, the nonlinear model produces episodes with more elevated wage and price inflation than the linearized solution in which household and firms are equally sensitive to the wage and price

[7] Notice that this requires increasing ξ_p and ξ_w to about 0.9, respectively.

markups.

While the results in Figure 3 are instructive to understand many features of the nonlinear model, it is not straightforward to connect the behavior of price and wage inflation to state of the business cycle. The relationship between actual price/wage inflation and some measure of resource utilization is traditionally referred to as a “Phillips curve”. Phillips (1958) drew this original relationship between the rate of wage inflation and the unemployment rate. More recently, researchers have extended his approach to the relationship between price inflation and the output gap. Thus, we use the simulated data in Figure 3 to produce bivariate scatterplots between price (and wage) inflation on the y-axis and the negative of the output gap on the x-axis. By using the negative of the output gap, we derive a traditional downward-sloping relationship as in Phillips (1958).

Figure 4 show the results. The left column shows the results for our benchmark calibration with the Kimball aggregator. As expected, we see that the relationship between the wages and prices and (minus) the output gap is characterized by a constant negative slope coefficient around the steady state of 2 percent (we do not allow for permanent productivity gains that would raise nominal wage inflation above price inflation in the steady state) in the linearized model (blue circles). However, when the economy hits the ZLB, which tends to happen when the output gap is around -2.5 percent for this shock, then the slope flattens somewhat, i.e. the output gap are more strongly affected by discount factor shocks than wage and price inflation when the economy is at the ZLB.

The nonlinear model with the Kimball aggregator in the left column (red crosses), on the other hand, features stronger responses of wage and prices when the output gap is elevated — lending support for inflation scares in booms (see e.g. Goodfriend, 1993) — but a much weaker relationship between the rate of change in prices and wages in recessions. Effectively, the price and wage schedules become much flatter (steeper) when the output gap is sufficiently negative (positive). This finding is very interesting as it — in addition to explain why inflation fell so little during the recession — offers a possible explanation why inflation rose so little when advanced economies recovered from the recession. Our simple modification of the basic New Keynesian model predicts that inflation pressures will remain low even if growth returns until the output gap is closed. This differs from the prediction of the simple linearized model which implies that inflation will start to rise notably when growth resumes. Another difference between the linearized and nonlinear solutions is that the output gap is more volatile in the linearized solution, mainly due to strong propagation of the ZLB constraint.

In the right column, we show the corresponding results with the Dixit-Stiglitz aggregator. Because we reparameterise ξ_p and ξ_w as discussed in Section 3.1 to the slopes of the wage and pricing schedules are unchanged, the linearized price and wage Phillips curves are unaltered. We will add and discuss the nonlinear Phillips curves with the Dixit-Stiglitz specification of the model in the next revision of the paper.

Finally, it is imperative to understand that the relationships in Figure 4 are contingent on the assumption that the discount factor shock is the single driver of business cycles. No other shocks are assumed to affect the economy. This is why we can derive such a clean relationship between prices and resource utilization. As we will see in the estimated model that we study next, this tight negative relationship ceases to exist in both the linearized and nonlinear model when different shocks affect the economy simultaneously.

4. An Estimated Medium-Sized New Keynesian Model

The benchmark EHL model studied so far is useful for highlighting many of the key factors affecting how real ridigities in price- and wage-setting may affect inflation dynamcis in a deep recession when solving the model nonlinearly. Specifically, we used it to demonstrate some of the benefits of taking nonlinearities into account as opposed to the traditional approach which entails log-linearizing the key model equations apart from the monetary policy rule.

However, this analysis was done in a stylized model with one shock and without allowing for endogenous capital accumulation. In this section we move on to a substantive analysis with the aim of examining the importance real ridigities in a nonlinear setting in a more quantitatively realistic model environment. Specifically, we specify and estimate a workhorse New Keynesian model with endogenous investment that closely follows the seminal model of Christiano, Eichenbaum and Evans (2005) but allows for variety of shocks as in Smets and Wouters (2003, 2007). The model is estimated in linearized form on U.S. data until 2007Q4.

Next, we use the estimated model to examine the properties of the nonlinear and linearized solutions along several dimensions. First, the paths of the linearized and nonlinear models are compared with the actual outcomes during the Great Recession following Christiano, Eichenbaum and Trabandt (2015). Second, we redo the Phillips curve analysis done in the benchmark model in Section 3.2. Third and finally, we examine the ability of the estimated model to capture the unconditional properties of price and wage inflation in terms of skewness and kurtosis.

4.1. Model

Following Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003, 2007), the model includes both sticky nominal wages and prices, where the Kimball (1995) aggregator is used to aggregate intermediate goods and labor to final output goods and effective labor input. It also features internal habit persistence in consumption, and embeds a $Q-$theory investment specification modified so that changing the level of investment (rather than the capital stock) is costly. We use the same shocks in the model as Smets and Wouters (2007) do when estimating the model.[8] The complete specification of the nonlinear and linearized formulation of the model is provided in Appendix B.

4.2. Estimation

We now proceed to discuss how the model is estimated on US data 1965Q1-2008Q2**.** We estimate a (log-)linearized variant of the model with Bayesian maximum likelihood techniques. To solve the system of linearized equations, we use the code package Dynare which provides an efficient and reliable implementation of the method proposed by Blanchard and Kahn (1980). We estimate a similar set of parameters as Smets and Wouters (2007) do.

4.2.1. Data

We use seven key macro-economic quarterly US time series as observable variables: the log difference of real GDP, real consumption, real investment and the log-difference of compensation per hour, log hours worked, the log difference of the GDP deflator, and the federal funds rate. Further details about the data are provided in Appendix B. The measurement equations are available in the technical appendix.

4.2.2. Estimation Methodology

Following Smets and Wouters (2003, 2007), we use Bayesian techniques U.S. data from 1965Q1 to 2008Q2. Bayesian inference starts out from a prior distribution that describes the available information prior to observing the data used in the estimation. The observed data is subsequently

[8] A difference with respect to the benchmark Smets and Wouters (2003, 2007) and CEE models is that we do not allow for indexation to past price- and wage-inflation of non-optimizing firms and wage-setters. This implies that prices (and wages) are kept unchanged when they are not re-optimized. By implication, there is no intrinsic persistence in the linearized price and wage Phillips curves (i.e. they are completely forward-looking). We adopt the no-indexation assumption as it is supported by microevidence on price and wage setting behavior.

used to update the prior, via Bayes' theorem, to the posterior distribution of the model's parameters which can be summarized in the usual measures of location (e.g. mode or mean) and spread (e.g. standard deviation and probability intervals).[9]

Some of the parameters in the model are kept fixed throughout the estimation procedure (i.e., are subject to infinitely strict priors). We choose to calibrate the parameters we think are weakly identified by the data that we use in the estimation. The parameters that are calibrated are set to values that are standard in the literature. We estimate 27 model parameters. Table 1 contains information about the priors and posterior distributions.

4.3. Role of Nonlinearities During the Great Recession

We solve the nonlinear and linearized model when subjecting both model versions to a positive risk premium shock. We compare the the resulting paths with the outcomes in the data following the methodology of Christiano, Eichenbaum and Trabandt (2015), CET henceforth. The risk premium shock enters the model in the optimality condition for bondholdings:

$$\lambda_t = \beta E_t \lambda_{t+1} \frac{\epsilon_{RP,t} R_t}{\Pi_{t+1}} \tag{17}$$

where λ_t denotes the lagrange multiplier on the household budget constraint, R_t is the gross nominal interest rate, Π_t is expected gross inflation and $\epsilon_{RP,t}$ denotes the risk premium shock used by Smets and Wouters (2003, 2007) which follows an AR(1) process that we have estimated when estimating the model.

The risk premium $\epsilon_{RP,t}$ in eq. (17) is assumed to rise in a uniform fashion for 16 quarters before gradually receding. The size of the risk-premium shock is set so that both the linearized and nonlinear models' output path roughly matches the "actual outcome" (discussed in detail below) during the crisis. Both CET and Lindé, Smets and Wouters (2016) argue that this was a key shock driving the Great Recession. Figure 5 depicts the results in the nonlinear and linearized model together with actual outcomes in the data. The gray area and black solid lines are computed using the methodology in CET, and implies that a pre-crisis trend is deducted from the actual data 2008Q3-2015Q2. The idea behind this procedure is an assessment how the economy would have evolved absent the large shocks associated with the Great Recession. For each variable, we fit a linear trend from date x to 2008Q2, where x =$\{1985Q1, 2003Q1\}$. To characterize what the data

[9] We refer the reader to Smets and Wouters (2003, 2007) for a more detailed description of the estimation procedure.

would have looked like absent the shocks that caused the financial crisis and Great Recession, we follow CET and extrapolate a trend line for each variable for the period 2008Q3-2015Q2. According to our model, all the nonstationary variables in the analysis are difference stationary. The CET linear extrapolation procedure implicitly assumes that the shocks in the estimation period were small relative to the drift terms in the time series. If we knew the correct value of x, the "target gaps" we compute (actual outcome in logs minus the fitted pre-crisis trend) would represent our estimates of the economic effects of the shocks that hit the economy in 2008Q3 and later. Now, since neither CET nor we know the correct value of x, we follow them and construct a min-max range for the target gaps using all the values of $x = \{1985Q1, 2003Q1\}$: The min-max ranges of the target gaps for all the variables correspond to the gray intervals displayed in Figure 5 and the black solid line is the mean of the obtained target gaps. The purpose behind the analysis is to assess whether, given plausible shocks, which model version that implies values of the endogenous variables in the post 2008Q2-period within the target gap ranges.

As can be seen from Figure 5, the elevated risk-premium shock excerts a significant adverse impact on the economy, in which economic activity dampens and inflation falls. As a result, the policy rate is driven towards a prolonged zero lower bound episode. The model matches well the decline in consumption, but the fall of investment is somewhat underestimated, probably because it lacks financial friction amplification mechanisms. Importantly, for the same-sized output response, inflation in the nonlinear model falls about 1 percent less than in the linearized solution, confirming the results in the stylized model.

4.4. Phillips Curves in the Estimated Model

To provide intution for the muted inflation response in the nonlinear solution following a positive risk-premium shock, Figure 6 shows a scatter plot of price inflation and the (negative of the) output gap in both the linearized and the nonlinear variant of the estimated medium-sized New Keynesian model. Following the procedure in Section 3.2 to simulate data from the model, the upper left scatter plot is generated by sampling only risk premium innovations from a normal distribution using the posterior mode and then simulating a long sample of 10,000 periods. Notice that the simulations are initiated at the steady state, and that the negative of the output gap is plotted on the x-axis, which means that a large positive number is associated with a deep recession.

As can be seen from the upper left plot in the figure, the linearized model is associated with a linear relationship between inflation and output gap when only risk-premium shocks are active, wheares the nonlinar model suggests a concave relationship. The relationship is all together linear because the risk-premium shocks are not large enough alone in the estimated model to drive the economy into a liquidity trap. Even so, the risk-premium shock is a key driver of business cycle dynamics in the estimated model, this difference between the linearized and nonlinear solution will have important implications for output and inflation dynamics.

Figure 6 also contains the implied price Phillips curves for the other six shock of the model: we run stochastic simulations for each of the other six shocks – one at a time – in both the linearized and nonlinear models and then use the simulated data to construct scatter plots for inflation (on the y-axis) and (the negative) output gap (on the x-axis). We use the estimated shock processes and the parameters reported in Table 1 in the simulations.[10] Hence, large movements in inflation and the output gap in a given subplot reflects that the simulated shock is an important driver of inflation and output gap dynamics according to our estimated model. In the last subplot (bottom right panel), we plot the results when all shocks are active simultaneously.

As can been seen from the figure, risk-premium and wage-markup shocks are key to understand inflation and output dynamics in the estimated model. For these two shocks, we obtain the most sizeable fluctuations in inflation and the output gap. Because these two shocks move the equilibrium far away from the steady state, the nonlinearities are also most evident for these two shocks. Given the estimated parameters, the wage markup shocks are the only source of fluctuations which have the potential to generate close to zero inflation or very mild deflationary episodes. It is striking how the dynamics of the price and wage markup shocks differ between the linearized and nonlinear solution. Other shocks which only cause moderate fluctuations in the output gap and inflation result in small differences between the linearized and nonlinear solutions because they do not generate any greater devations from the steady state.

Moreover, it is evident that there are no or small trade-offs in stabilizing output and inflation to fluctuations in the risk-premium, government spending, and the neutral technology shocks. By contrast, the investment-specific and markup shocks – especially the wage markup – create a noticable trade-off between inflation and output gap stabilization. Furthermore, when using all shocks in the simulation, the importance of the wage markup shock renders the Phillips curve completely flat or even upward-sloping, consistent with the empirical observation of no clear unconditional

[10] Because the way the markup shocks enter into linear and nonlinear equations differs for large shocks, we shrink the size of wage and price markup shocks in the nonlinear model so that the unconditional volatility of price and wage inflation is the same in both the linearized and nonlinear stochastic simulations. Since equally-sized markup shocks have notably larger effects on inflation in the nonlinear solution, we thus find the nonlinear solution moderates the models' dependency on large markup shocks, and hence mitigates the important critique against New Keynesian models by Chari, Kehoe and McGrattan (2010).

Phillips curve pattern in postwar US data.

It is important to understand that while many shocks do not generate any noticeable differences between the linearized and nonlinear solution when simulated one at a time, this does not imply that these shocks can propagate differently in a recession (or boom) in the nonlinear model relative to the linearized solution. To show this, Figure 7 shows the results corresponding to those in Figure 6 but conditional on an expected 8-quarter liquidity trap. To construct this figure, we first simulate a baseline scenario (negative demand/positive risk premium shock) which generates an anticipated 8-quarter liquidity trap from the first period the ZLB actually binds. Next, the first period policy rate reaches its lower bound, we create a counterfactual scenario by adding stochastically one of the shocks and compute the deviation from the baseline simulation. By repeating this procedure for each of the shocks separately and when all shocks are sampled simultaneously, we obtain a bivariate distribution for inflation and the output gap in both model variants conditional on a liquidity trap. The specific observation for which we plot the scenario minus baseline scenario difference in Figure 7 is one year after we have added the shock.[11]

As can be seen by comparing Figures 6 and 7, we see now notable differences between the linearized and nonlinear solutions for many of the shocks for which there were previously no differences (like monetary policy, government spending, neutral technology and and investment specific shocks). For instance, monetary policy is much less potent to affect inflation in a pro-longed liquidity trap than in normal times. The effects of monetary policy shocks on output are also somewhat moderated in the nonlinear model compared to the linearized solution. Negative investment-specific shocks, on the other hand, have notably more negative effects on the output gap in a liquidity trap. In constrast to wage markup shocks, price markup shocks have essentially no effect in a liquidity trap in the nonlinear model, capturing that firms are unwilling to respond to markup variations with the Kimball aggregator.

It is important to point out that the results in Figure 7 generally imply notably flatter Phillips curves for many shocks in a liquidity trap. Hence, the nonlinear model offers a possible explanation to the empirical observation that the sensitivity of inflation to economic activity has flattened in linearized models since the onset of the crisis (see e.g. Lindé, Smets and Wouters, 2016). In the nonlinear model, however, the reduced sensitivity is only temporary (contingent on a persistently negative output gap) and will rise once the economy recovers.

[11] We plot the observations after one year since habit formation and investment adjustment costs imply that for most shocks, output and inflation attain their peak effects with some delay.

4.5. Accounting for the Statistical Properties of Inflation

We have documented that the nonlinear model allows to account for the inflation dynamics during the Great Recession as well as the perceived reduction in the slope of the Phillips curve in the aftermath of the recession. We now turn to stochastic simulations to examine if the nonlinear solution helps to account for unconditional moments of inflation. It is well-known that inflation has a positive skew during the postwar period; i.e. there are episodes with inflation bursts and then there are episodes with very low and moderate rates of inflation but no long-lived deflationary episodes. This positive skew is a robust finding for different measures of inflation. The gray area in Figure 8 shows the kernel smoothed distribution for inflation measured by the PCE deflator for the sample 1965Q1-2017Q4.

The blue and red lines show the corresponding kernel smoothed distributions based on the stochastic simulations when all are shocks active, i.e. the observations plotted in the right bottom panel in Figure 6. As can be seen from Figure 8, the nonlinear model fits the unconditional statistical properties inflation markedly better than the linearized model. The linearized model features a completely symmetric normal distribution for inflation, with significant mass in deflationary territory. By contrast, the nonlinear solution features very little density in negative territory for inflation, in line with actual outcomes. It is striking how much better the nonlinear model captures unconditional U.S. inflation dynamics during the post-war period compared to the linearized model.

5. Conclusions

We have formulated a macroeconomic model which goes a long way towards accounting for the missing deflation puzzle, i.e. the empirical regularity that inflation fell so little in the United States against the backdrop of the large and persistent fall in output. Our resolution of the puzzle stresses the nonlinear influence of strategic complementarities and real rigidities in price-setting of firms.

Additional advantages of our proposed framework are that it (i) mitigates the tension between the macroeconomic evidence of a low Phillips curve slope and the microeconomic evidence of frequent price changes; (ii) allows us to explain the empirical positive skew in inflation without relying on a similar positive skew output (which is counterfactual); and (iii), helps us to explain the low rates of price and wage inflation in the last few years when the U.S. economy has recovered from the recession. In future work, it would be interesting to complement our macro approach with a study on firm-level data on prices and quantities as well as wages and labor market quantities.

References

Adam, Klaus, and Roberto M. Billi (2006), "Optimal Monetary Policy under Commitment with a Zero Bound on Nominal Interest Rates", *Journal of Money, Credit, and Banking* 38(7), 1877-1905.

Adam, Klaus, and Roberto M. Billi (2007), "Discretionary Monetary Policy and the Zero Lower Bound on Nominal Interest Rates", *Journal of Monetary Economics* 54(3), 728-752.

Adolfson, Malin, Stefan Laséen, Jesper Lindé and Mattias Villani (2005), "The Role of Sticky Prices in an Open Economy DSGE Model: A Bayesian Investigation", *Journal of the European Economic Association Papers and Proceedings* 3(2-3), 444-457.

Altig, David, Christiano, Lawrence J., Eichenbaum, Martin and Jesper Lindé (2011), "Firm-Specific Capital, Nominal Rigidities and the Business Cycle", *Review of Economic Dynamics* 14(2), 225-247.

Aruoba, S. Borağan, Pablo Cuba-Borda, and Frank Schorfheide (2017), "Macroeconomic Dynamics Near the ZLB: A Tale of Two Countries", *Review of Economic Studies*, forthcoming.

Aruoba, S. Borağan, Luigi Boccola, and Frank Schorfheide (2017), "Assessing DSGE Model Nonlinearities", *Journal of Economic Dynamics and Control* 83, 34-54.

Ascari, Guido and Tiziano Ropele (2007) "Optimal Monetary Policy Under Low Trend Inflation," *Journal of Monetary Economics* 54, 2568-2583.

Ball, Laurence, and Sandeep Mazumder (2011). "Inflation Dynamics and the Great Recession." *Brookings*

Papers on Economic Activity (Spring): 337–402.

Bernanke, Ben, Gertler, Mark and Simon Gilchrist (1999), "The Financial Accelerator in a Quantitative Business Cycle Framework", in John B. Taylor and Michael Woodford (Eds.), Handbook of Macroeconomics, North-Holland Elsevier Science, New York.

Bodenstein, Martin, James Hebden and Ricardo P. Nunes (2012), "Imperfect Credibility and the Zero Lower Bound on the Nominal Interest Rate", *Journal of Monetary Economics* 59(2), 135-149.

Boneva, Lena Mareen, Braun, R. Anton and Yuichiro Waki (2016), "Some unpleasant properties of loglinearized solutions when the nominal rate is zero," *Journal of Monetary Economics* 84, 216-232.

Braun, R. Anton, Lena Mareen Kvrber and Yuichiro Waki (2013), "Small and orthodox fiscal multipliers at the zero lower bound," Working Paper 2013-13, Federal Reserve Bank of Atlanta.

Christiano, Lawrence J., Martin Eichenbaum and Charles Evans (2005), "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy", *Journal of Political Economy* 113(1), 1-45.

Christiano, Lawrence, Martin Eichenbaum and Benjamin K. Johannsen (2016), "Does the New Keynesian Model Have a Uniqueness Problem?", manuscript, Northwestern University.

Christiano, Lawrence, Martin Eichenbaum and Sergio Rebelo (2011), "When is the Government Spending Multiplier Large?" *Journal of Political Economy* 119(1), 78-121.

Christiano, Lawrence, Martin Eichenbaum and Mathias Trabandt (2015), "Understanding the Great Recession", *American Economic Journal: Macroeconomics* 7(1), 110-167.

Christiano, Lawrence, Motto, Roberto and Massimo Rostagno (2008), "Shocks, Structures or Monetary Policies? The Euro Area and the US After 2001", *Journal of Economic Dynamics and Control* 32(8), 2476-2506.

Christiano, Lawrence J., Mathias Trabandt and Karl Walentin (2007), “Introducing financial frictions and unemployment into a small open economy model,” Sveriges Riksbank Working Paper Series No. 214.

Christiano, Lawrence J., Mathias Trabandt and Karl Walentin (2011), “DSGE Models for Monetary Policy Analysis,” chapter 7 in: Benjamin M. Friedman & Michael Woodford (eds.), Handbook of Monetary Economics, pp. 285-367 Elsevier.

Del Negro, Marco, Marc P. Giannoni, and Frank Schorfheide. 2015. "Inflation in the Great Recession and New Keynesian Models." American Economic Journal: Macroeconomics, 7(1): 168-96.

Dixit, Avinash K, and Joseph E. Stiglitz (1977), “Monopolistic Competition and Optimum Product Diversity”, *American Economic Review* 67(3), 297-308.

Dupor, Bill and Rong Li (2015), “The Expected Inflation Channel of Government Spending in the Postwar U.S.,” *European Economic Review* 74(1), 36-56.

Eggertsson, Gauti and Michael Woodford (2003), “The Zero Interest-Rate Bound and Optimal Monetary Policy”, *Brookings Papers on Economic Activity* 1, 139-211.

Erceg, Christopher, Guerrieri, Luca and Christopher Gust (2006), “SIGMA: A New Open Economy Model for Policy Analysis”, *Journal of International Central Banking* 2(1), 1-50.

Fair, Ray C. and John B. Taylor (1983), “Solution and Maximum Likelihood Estimation of Dynamic Rational Expectations Models”, *Econometrica* 51(4), 1169-1185.

Fernández-Villaverde, Jesús, Gordon, Grey, Guerrón-Quintana, Pablo and Juan F. Rubio-Ramírez (2015), “Nonlinear Adventures at the Zero Lower Bound”, *Journal of Economic Dynamics & Control* 57, 182–204.

Galí, Jordi and Mark (1999), “Inflation Dynamics: A Structural Econometric Analysis”, *Journal of Monetary Economics*, 44, 195-220.

Galí, Jordi, Gertler, Mark and David López-Salido (2001), “European Inflation Dynamics”, *European Economic Review*, 45, 1237-70.

Galí, Jordi, López-Salido, David and Javier Vallés (2007), “Understanding the Effects of Government Spending on Consumption”, *Journal of the European Economic Association* 5(1), 227-270.

Gilchrist, Simon, Raphael Schoenle, Jae Sim and Egon Zakrajšek (2016), “Inflation Dynamics: A Structural Econometric Analysis”, *American Economic Review*, forthcoming.

Goodfriend, Marvin (1993), “Interest Rate Policy and the Inflation Scare Problem: 1979-1992,” FRB Richmond Economic Quarterly 79(1), 1-23.

Gust, Christopher J., Edward P. Herbst, J. David Lopez-Salido, and Matthew E. Smith (2016). “The Empirical Implications of the Interest-Rate Lower Bound,” Board of Governors of the Federal Reserve System Finance and Economics Discussion Series 2012-83r.

Hall, Robert E. (2011), “The Long Slump”, *American Economic Review* 101, 431-469.

Hebden, J.S., Lindé, J., Svensson, L.E.O., 2009. Optimal monetary policy in the hybrid new Keynesian model under the zero lower bound constraint. Mimeo, Federal Reserve Board.

Iacoviello, Matteo, and Luca Guerrieri (2015), “OccBin: A Toolkit for Solving Dynamic Models with Occasionally Binding Constraints Easily”, *Journal of Monetary Economics* 70, 22–38.

Iacoviello, Matteo, and Luca Guerrieri (2016), “Collateral Constraints and Macroeconomic Asymmetries”, manuscript, Federal Reserve Board.

IMF (2016), "Global Disinflation in an Era of Constrained Monetary Policy", Chapter 2 i World Economic Outlook, October.

IMF (2017), "Recent Wage Dynamics in Advanced Economies: Drivers and Implications", Chapter 2 i World Economic Outlook, October.

Judd, Kenneth L., Maliar, Lilia and Serguei Maliar (2011), "Numerically stable and accurate stochastic simulation approaches for solving dynamic economic models," *Quantitative Economics* 2(2), 173-210.

Juillard, Michel (1996), "Dynare : A Program for the Resolution and Simulation of Dynamic Models with Forward Variables Through the Use of a Relaxation Algorithm," CEPREMAP Working Paper 9602.

Kimball, Miles S. (1995), "The Quantitative Analytics of the Basic Neomonetarist Model," *Journal of Money, Credit, and Banking* 27(4), 1241–1277.

King, Robert G., and Mark W. Watson (2012), "Inflation and Unit Labor Cost." *Journal of Money, Credit and Banking* 44 (S2): 111–49.

Klenow, Peter J. and Benjamin A. Malin (2010), "Microeconomic Evidence on Price-Setting", Chapter 6 in Benjamin M. Friedman and Michael Woodford (Eds.), Handbook of Monetary Economics, Elsevier, New York.

Keynes, John Maynard (1936), *The General Theory of Employment, Interest and Money*, London: Macmillan Press.

Lindé, Jesper (2005), "Estimating New Keynesian Phillips Curves: A Full Information Maximum Likelihood Approach", *Journal of Monetary Economics*, 52(6), 1135-49.

Lindé, Jesper, Frank Smets and Rafael Wouters (2016), "Challenges for Central Banks' Macro Models", Chapter 28 in John B. Taylor and Harald Uhlig (Eds.), Handbook of Macroeconomics Vol. 2, North-Holland Elsevier Science, New York.

Lindé, Jesper and Mathias Trabandt (2018), "Should We Use Linearized Models to Calculate Fiscal Mulipliers?", *Journal of Applied Econometrics, forthcoming.*

Nakata, Taisuke (2015), "Uncertainty at the Zero Lower Bound", *American Economic Journal: Macroeconomics* forthcoming.

Phillips, A. W. (1958), "The Relationship between Unemployment and the Rate of Change of Money Wages in the United Kingdom 1861-1957," *Economica* 25, 283–299.

Richter, Alexander and NathanielThrockmorton (2016), "Are nonlinear methods necessary at the zero lower bound?," Federal Reserve Bank of Dallas Working Papers 1606.

Smets, Frank and Raf Wouters (2003), "An Estimated Stochastic Dynamic General Equilibrium Model of the Euro Area", *Journal of the European Economic Association* 1(5), 1123-1175.

Smets, Frank and Raf Wouters (2007), "Shocks and Frictions in US Business Cycles: A Bayesian DSGE Approach", *American Economic Review* 97(3), 586-606.

Figure 1: Demand Curves -- Implications of Kimball vs. Dixit-Stiglitz Aggregators.

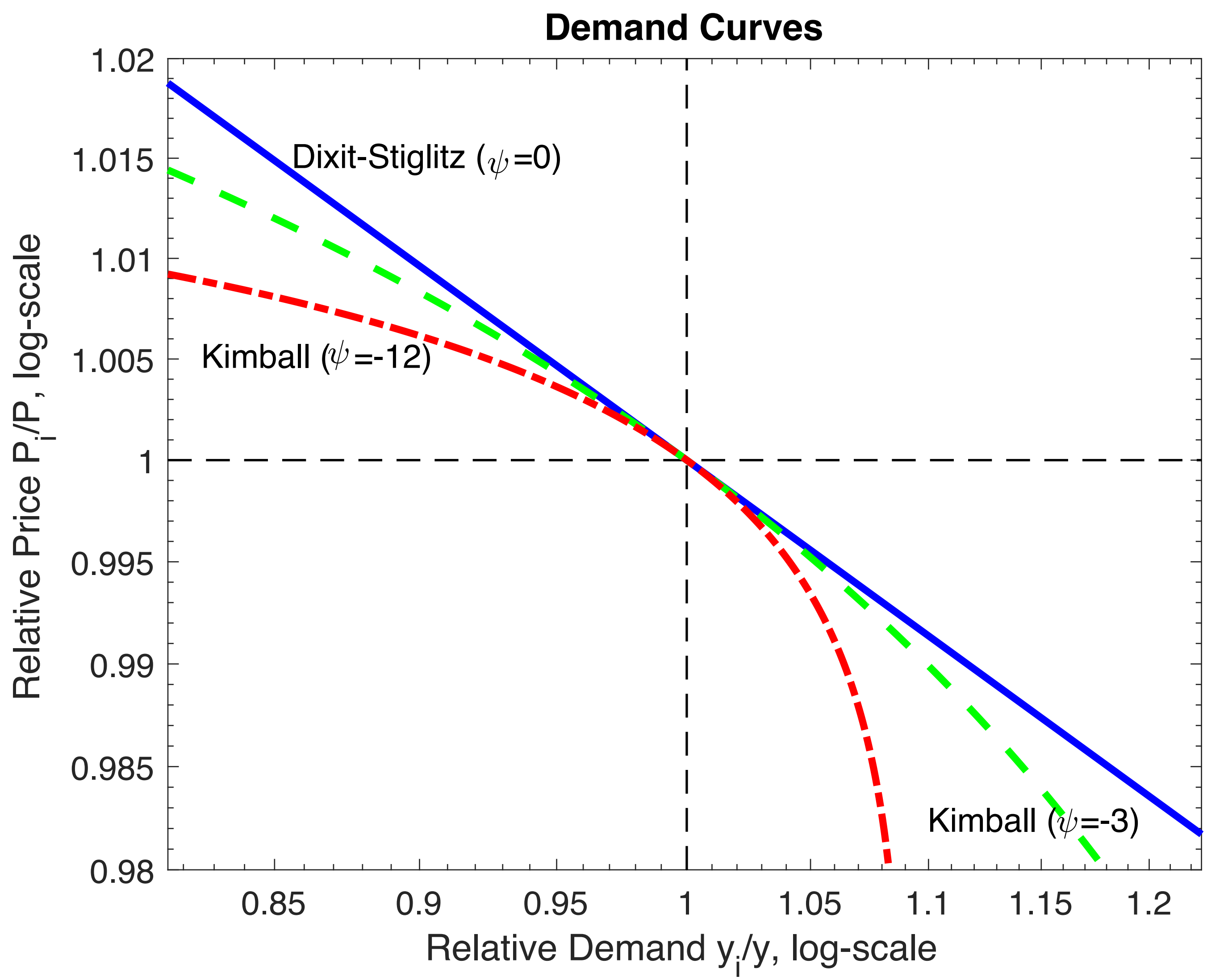


Figure 2: Impulse Responses to a 1% Discount Factor Shock

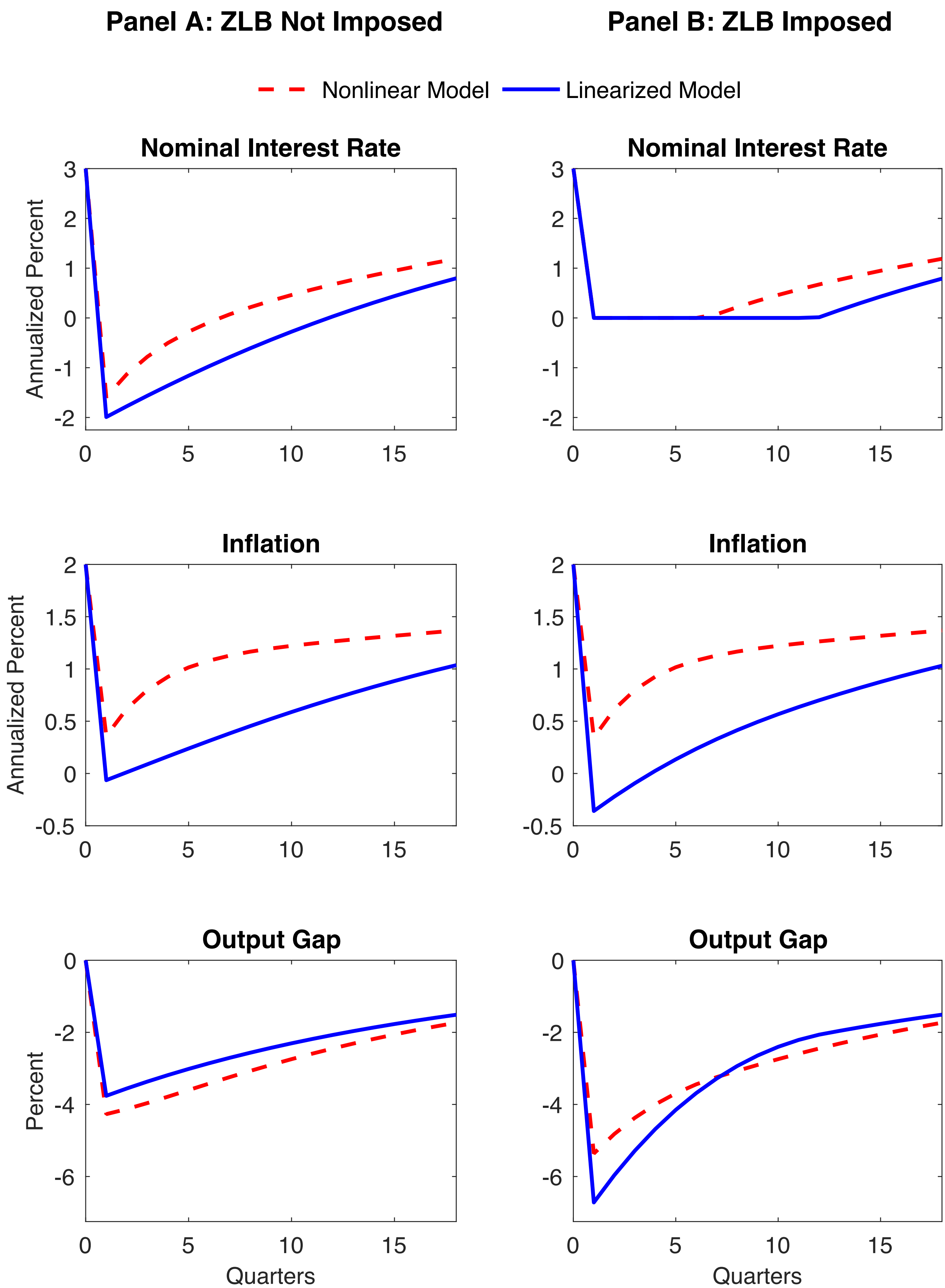


Figure 3: Stochastic Simulation of Nonlinear and Linearized Model

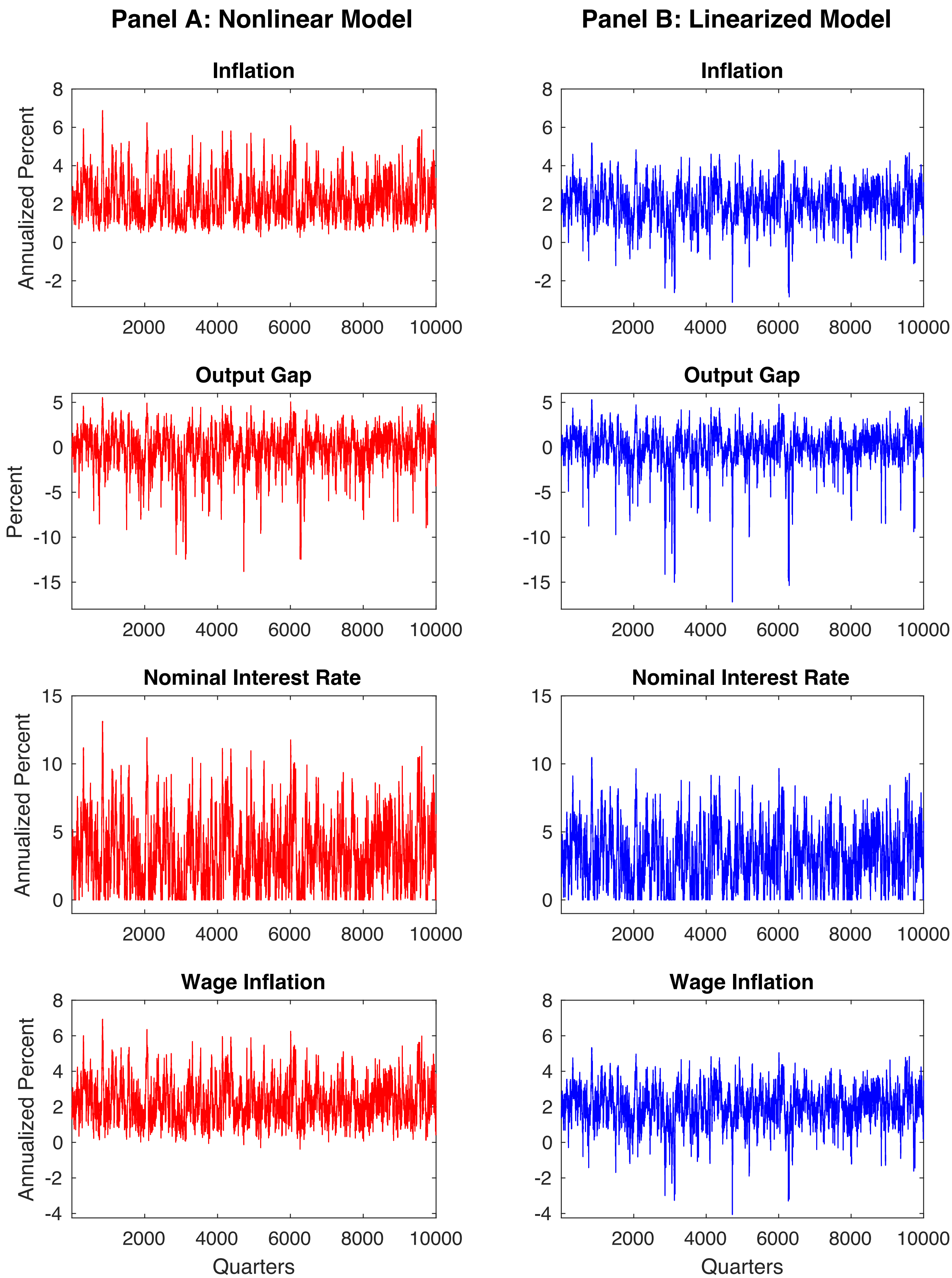


Figure 4: Price and Wage Phillips Curves

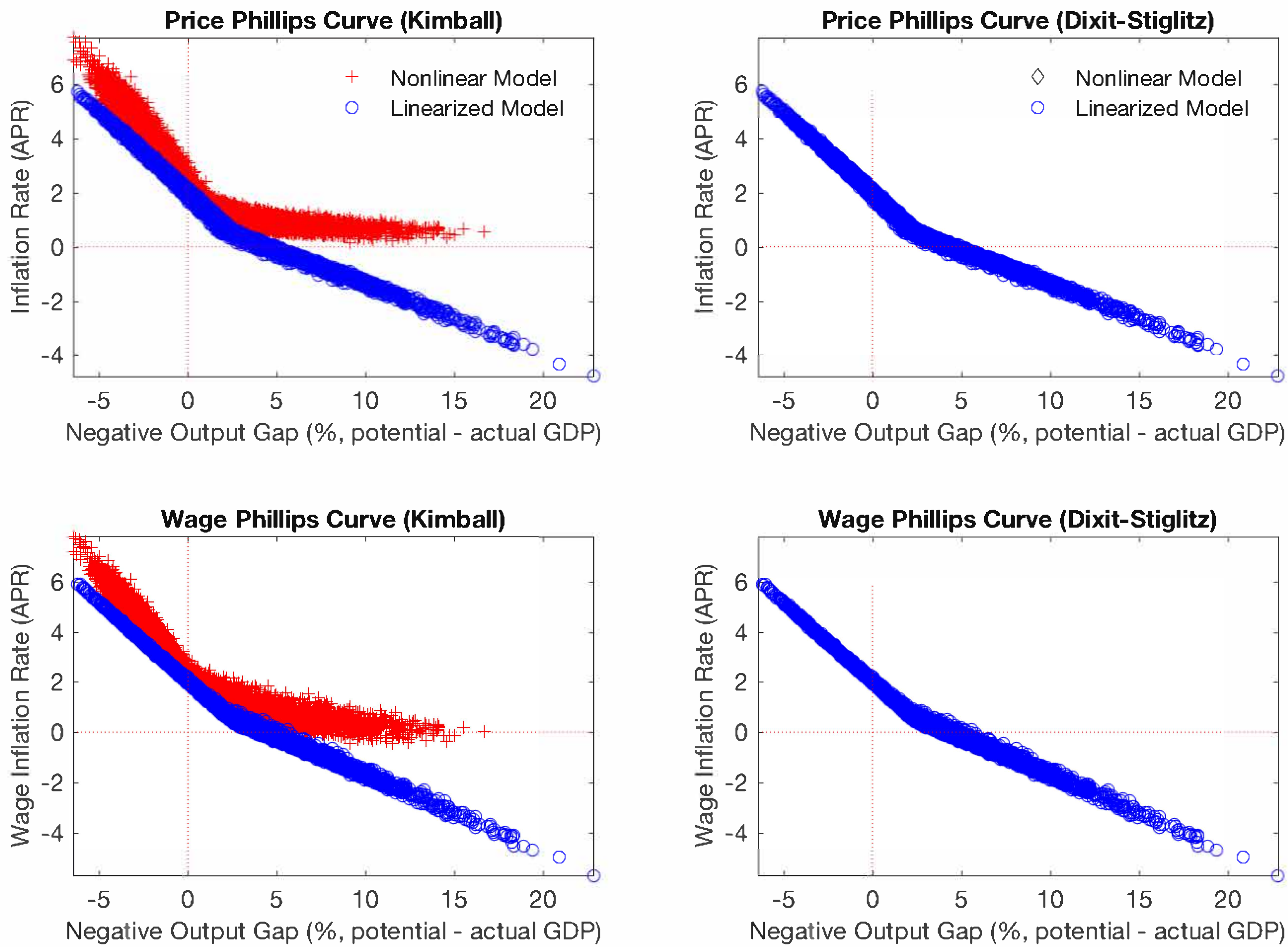
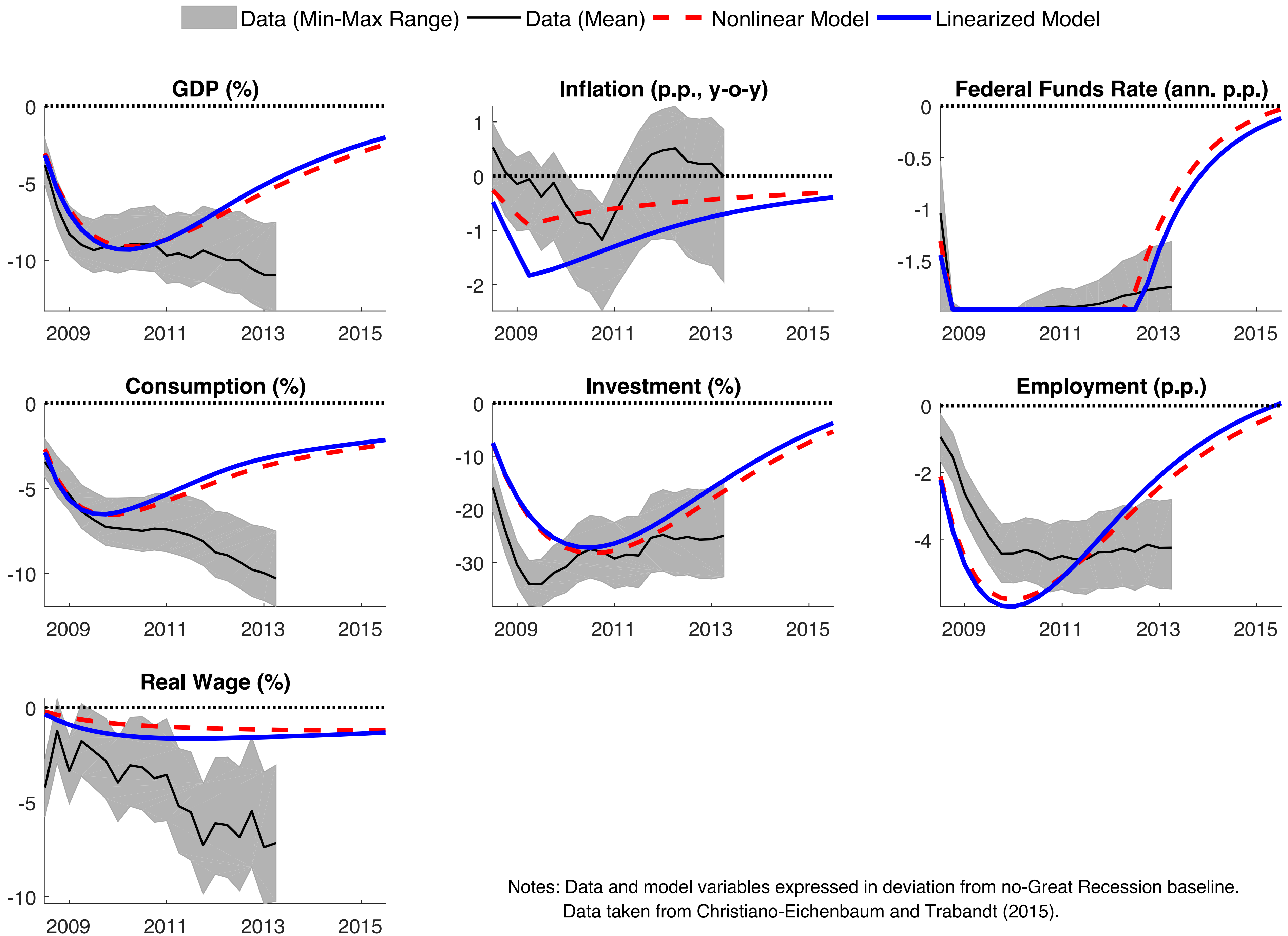


Figure 5: The U.S. Great Recession: Data vs. Estimated Medium-Sized Model



Notes: Data and model variables expressed in deviation from no-Great Recession baseline.
Data taken from Christiano-Eichenbaum and Trabandt (2015).

Figure 6: Phillips Curves in Estimated Medium-Sized Model

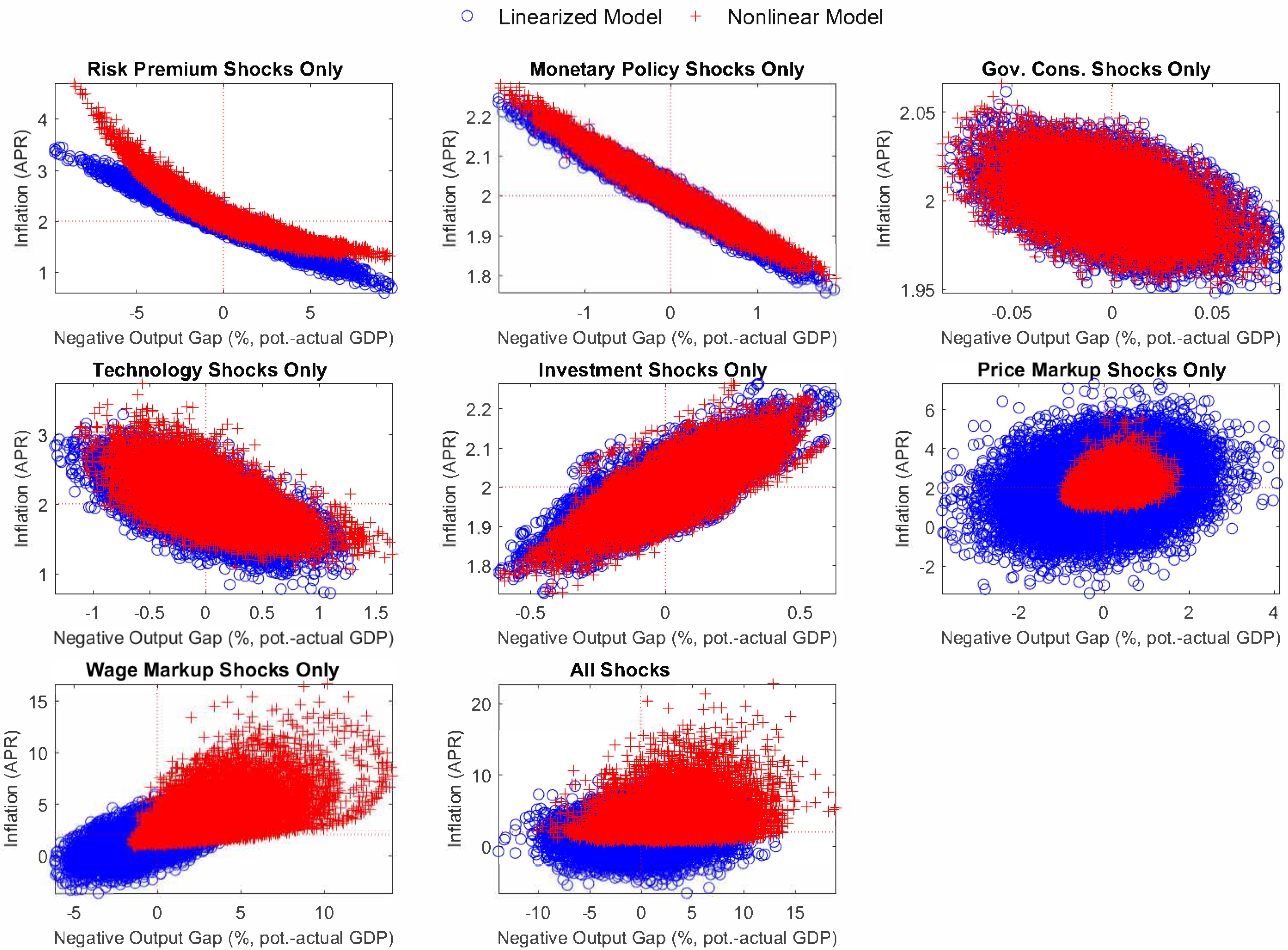
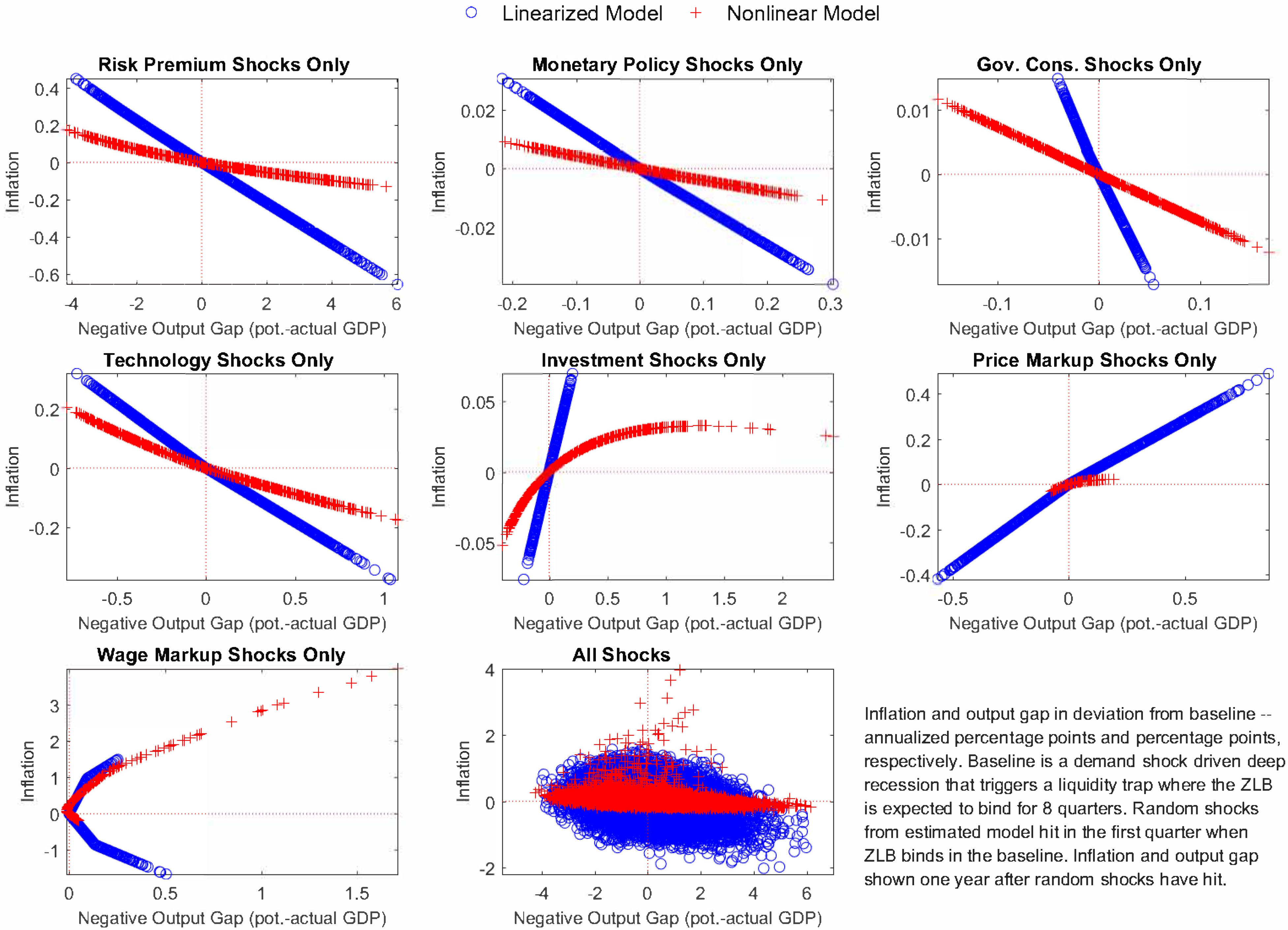


Figure 7: Phillips Curves in Estimated Model Conditional on an 8-Quarter Liquidity Trap



Inflation and output gap in deviation from baseline -- annualized percentage points and percentage points, respectively. Baseline is a demand shock driven deep recession that triggers a liquidity trap where the ZLB is expected to bind for 8 quarters. Random shocks from estimated model hit in the first quarter when ZLB binds in the baseline. Inflation and output gap shown one year after random shocks have hit.

Figure 8: Density of Inflation in Data vs. Stochastic Model Simulations

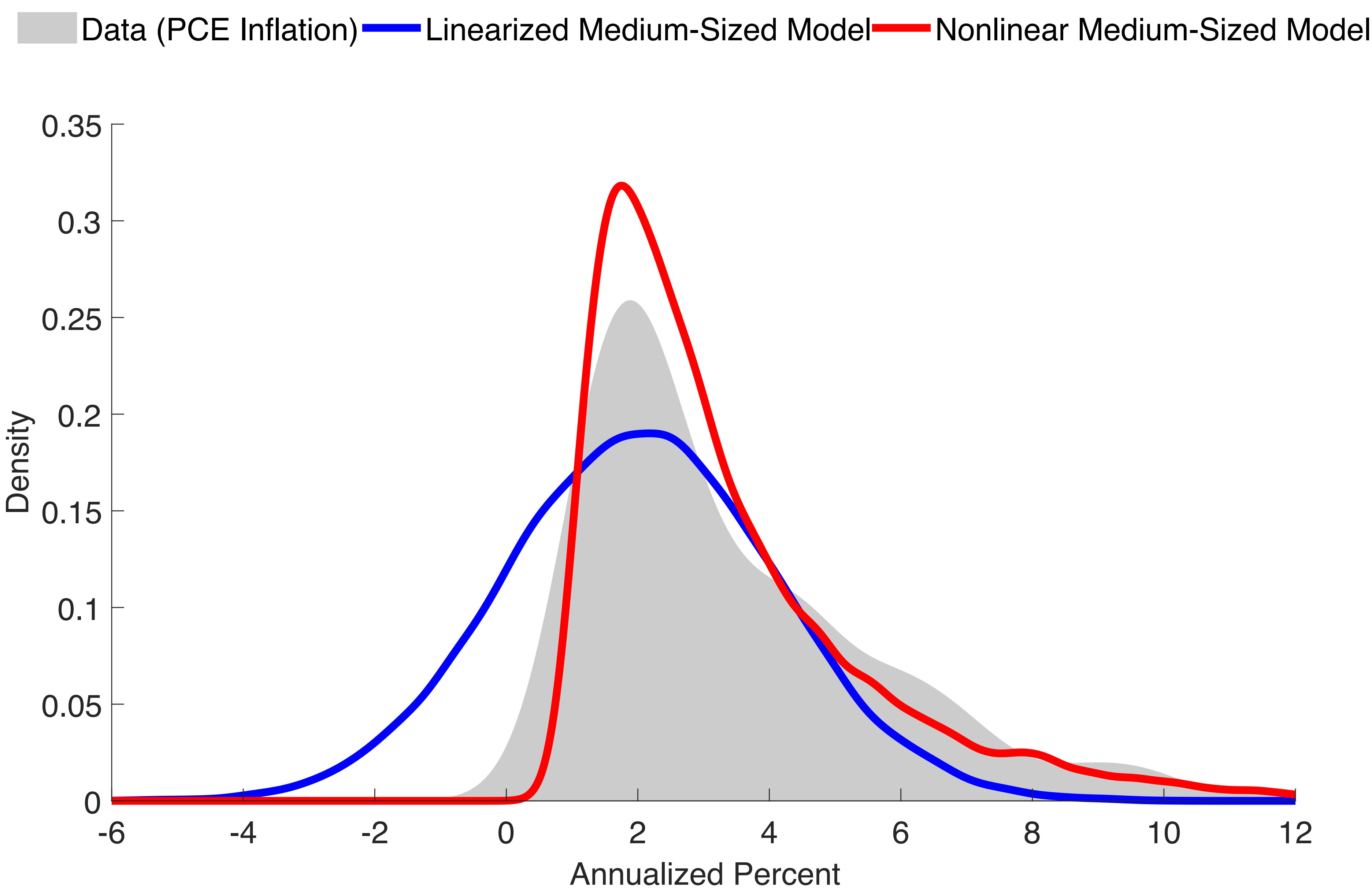


Table 1: Estimated Parameters of the Medium-Sized Model

	prior mean	post. mean	95% HPD interval		prior	pstdev
psie_est	6.000	12.5540	7.5826	17.7178	norm	4.0000
psiew_est	6.000	8.3103	1.9693	14.8010	norm	4.0000
Spp	4.000	4.8930	3.5291	6.2880	norm	1.0000
b	0.700	0.6677	0.6011	0.7319	beta	0.1500
sigmaa	1.000	0.7566	0.2244	1.4568	norm	0.7500
alfa	0.300	0.1956	0.1784	0.2129	norm	0.0750
idiff	3.000	2.7146	2.4296	2.9961	norm	0.2500
ydiff	1.750	1.5388	1.4465	1.6314	norm	0.2500
rhoR	0.800	0.7345	0.6789	0.7897	beta	0.1000
rpi	1.500	1.4428	1.2214	1.6744	norm	0.2500
ry	0.125	0.0666	0.0283	0.1072	norm	0.0500
rdy	0.250	0.3102	0.2181	0.4048	norm	0.1000
sigreps	0.200	0.2333	0.2080	0.2594	invg	Inf
sigtau	0.200	0.1679	0.1265	0.2106	invg	Inf
rhotau	0.700	0.7858	0.6825	0.8857	beta	0.1500
rhoMAtau	0.500	0.5071	0.2781	0.7221	beta	0.1500
sigzeth	0.200	0.4812	0.4026	0.5598	invg	Inf
rhozeth	0.700	0.9403	0.9052	0.9700	beta	0.1500
rhoMAzeth	0.500	0.8932	0.8331	0.9467	beta	0.1500
sigeps	1.000	0.5947	0.5320	0.6599	invg	Inf
rhoeps	0.700	0.9169	0.8827	0.9495	beta	0.1500
sigups	0.200	0.3614	0.2468	0.4849	invg	Inf
rhoups	0.700	0.4041	0.2616	0.5456	beta	0.1500
sigrp	0.200	0.6806	0.2937	1.1445	invg	Inf
rhorp	0.700	0.7818	0.6647	0.8931	beta	0.1500
sigg	1.000	1.7270	1.5437	1.9146	invg	Inf
rhog	0.700	0.9450	0.9083	0.9805	beta	0.1500

Appendix A.

Below we state the nonlinear and linearized equilibrium conditions of the stylized model in 2.[A.1]

A.1. Nonlinear Equilibrium Equations

The nonlinear equilibrium equations can be written as:

$$
\begin{aligned}
\text{Marginal utility (n1)} &: \quad c_t^{-1} = \lambda_t \\
\text{Euler equation (n2)} &: \quad \lambda_t = \beta E_t \delta_{t+1} \frac{R_t}{\Pi_{t+1}} \lambda_{t+1} \\
\text{Resource Constraint(n3)} &: \quad c_t = y_t \\
\text{Production (n4)} &: \quad y_t = \left(p_t^*\right)^{-1} \left(w_t^*\right)^{-(1-\alpha)} k^\alpha l_t^{1-\alpha} \\
\text{Non.lin. pricing 1 (n5)} &: \quad s_t = \frac{\left(1+\psi_p\right)\left(1+\theta_p\right)}{1+\psi_p+\theta_p\psi_p} \lambda_t y_t \vartheta_t^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} mc_t \\
&\quad +\beta\xi_p E_t \delta_{t+1} \left(\Pi/\Pi_{t+1}\right)^{-\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} s_{t+1} \\
\text{Non.lin. pricing 2 (n6)} &: \quad f_t = \lambda_t y_t \vartheta_t^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} + \beta\xi_p E_t \delta_{t+1} \left(\Pi/\Pi_{t+1}\right)^{-\frac{\left(1+\psi_p+\psi_p\theta_p\right)}{\theta_p}} f_{t+1} \\
\text{Non.lin. pricing 3 (n7)} &: \quad a_t = \frac{\psi_p\theta_p}{1+\psi_p+\theta_p\psi_p} y_t\lambda_t + \beta\xi_p E_t\delta_{t+1}\left(\Pi/\Pi_{t+1}\right) a_{t+1} \\
\text{Non.lin.pricing 4 (n8)} &: \quad s_t = f_t \tilde{p}_t - a_t \tilde{p}_t^{1+\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} \\
\text{Zero profit condition prices (n9)} &: \quad \vartheta_t = 1+\psi-\psi\Delta_{t,2} \\
\text{Aggregate price index (n10)} &: \quad \vartheta_t = \Delta_{t,3} \\
\text{Overall price dispersion (n11)} &: \quad p_t^* = \frac{\vartheta_t^{\frac{1+\theta_p}{\theta_p}(1+\psi)}}{1+\psi_p} \Delta_{t,1}^{-\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} + \frac{\psi_p}{1+\psi_p} \\
\text{Price dispersion 1 (n12)} &: \quad \Delta_{t,1}^{-\frac{\left(1+\theta_p\right)\left(1+\psi_p\right)}{\theta_p}} = \left(1-\xi_p\right)\tilde{p}_t^{-\frac{\left(1+\theta_p\right)\left(1+\psi_p\right)}{\theta_p}} + \xi_p\left[\left(\Pi/\Pi_t\right)\Delta_{t-1,1}\right]^{-\frac{\left(1+\theta_p\right)\left(1+\psi_p\right)}{\theta_p}} \\
\text{Price dispersion 2 (n13)} &: \quad \Delta_{t,2} = \left(1-\xi_p\right)\tilde{p}_t + \xi_p\left(\Pi/\Pi_t\right)\Delta_{t-1,2} \\
\text{Price dispersion 3 (n14)} &: \quad \Delta_{t,3}^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}} = (1-\xi_p)\tilde{p}_t^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}} + \xi_p\left(\left(\Pi/\Pi_t\right)\Delta_{t-1,3}\right)^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}} \\
\text{Marginal cost (n15)} &: \quad (1-\alpha)\, mc_t = w_t k^{-\alpha}\left[\left(w_t^*\right)^{-1} l_t\right]^\alpha \\
\text{Taylor rule (n16)} &: \quad R_t = \max\left(1, R\left[\Pi_t/\Pi\right]^{\gamma_\pi}\left[\frac{y_t}{y}/\frac{y_t^{pot}}{y^{pot}}\right]^{\gamma_x}\right)
\end{aligned}
$$

[A.1] All derivations and the closed form steady state are provided in the technical appendix which is available at https://sites.google.com/site/mathiastrabandt

$$\text{Wage inflation (n17)} \quad : \quad \Pi_t^w = \Pi_t \frac{w_t}{w_{t-1}}$$

$$\text{Non.lin. wage setting 1 (n18)} \quad : \quad s_t^w = f_t^w \tilde{w}_t - a_t^w \tilde{w}_t^{1+\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}$$

$$\text{Non.lin. wage setting 2 (n19)} \quad : \quad f_t^w = (w_t^*)^{-1} l_t \lambda_t w_t \vartheta_{w,t}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \beta \xi_w \delta_{t+1} E_t \left[\frac{\Pi_w}{\Pi_{w,t+1}} \right]^{-\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}} f_{t+1}^w$$

$$\text{Non.lin. wage setting 3 (n20)} \quad : \quad a_t^w = \frac{\theta_w \psi_w}{\psi_w + \theta_w \psi_w + 1} (w_t^*)^{-1} l_t \lambda_t w_t + \beta \xi_w \delta_{t+1} E_t \frac{\Pi_w}{\Pi_{w,t+1}} a_{t+1}^w$$

$$\text{Non.lin. wage setting 4 (n21)} \quad : \quad s_t^w = \frac{(1+\psi_w)(1+\theta_w)}{1+\psi_w+\theta_w\psi_w} (w_t^*)^{-1} l_t \lambda_t mrs_t \vartheta_{w,t}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}$$
$$+\beta \xi_w \delta_{t+1} E_t \left[\frac{\Pi_w}{\Pi_{w,t+1}} \right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} s_{t+1}^w$$

$$\text{Zero profit condition wages (n22)} \quad : \quad \vartheta_t^w = 1 + \psi_w - \psi_w \Delta_{t,2}^w$$

$$\text{Agg. wage index (n23)} \quad : \quad \vartheta_t^w = \Delta_{t,3}^w$$

$$\text{Overall wage disperision (n24)} \quad : \quad w_t^* = \frac{\left[\vartheta_t^w\right]^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}}{1+\psi_w} \left[\Delta_{t,1}^w\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \frac{\psi_w}{1+\psi_w}$$

$$\text{Wage dispersion 1 (n25)} \quad : \quad \left[\Delta_{t,1}^w\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} = (1-\xi_w) \left[\tilde{w}_t\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}$$
$$+\xi_w \left[\frac{\Pi^w}{\Pi_t^w} \Delta_{t-1,1}^w \right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}$$

$$\text{Wage dispersion 2 (n26)} \quad : \quad \Delta_{t,2}^w = (1-\xi_w) \tilde{w}_t + \xi_w \left(\Pi^w / \Pi_t^w\right) \Delta_{t-1,2}^w$$

$$\text{Wage dispersion 3 (n27)} \quad : \quad \left[\Delta_{t,3}^w\right]^{-\frac{1+\psi_w+\psi_w\theta_w}{\theta_w}} = (1-\xi_w) \tilde{w}_t^{-\frac{1+\psi_w+\psi_w\theta_w}{\theta_w}}$$
$$+\xi_w \left(\left(\Pi^w / \Pi_t^w\right) \Delta_{t-1,3}^w \right)^{-\frac{1+\psi_w+\psi_w\theta_w}{\theta_w}}$$

$$\text{Marg. rate of subst. (n28):} \quad mrs_t = 1/\lambda_t$$

Flex-price and flex-wage (potential) economy is the version of the model when prices and wages are flexible, i.e. $\xi_p = \xi_w = 0$.

$$\text{Euler equation, pot. econ (n29)} \quad : \quad \left(c_t^{pot}\right)^{-1} = \beta E_t \delta_t rr_t^{pot} \left(c_{t+1}^{pot}\right)^{-1}$$

$$\text{Leisure/labor, pot. econ (n30)} \quad : \quad 1 = \left(c_t^{pot}\right)^{-1} \frac{1}{1+\theta_w} w_t^{pot}$$

$$\text{Wage, pot. econ (n31)} \quad : \quad \frac{1-\alpha}{1+\theta_p} \left(k^{pot}\right)^{\alpha} = w_t^{pot} \left(l_t^{pot}\right)^{\alpha}$$

$$\text{Res. constraint, pot. econ (n32)} \quad : \quad c_t^{pot} = y_t^{pot}$$

$$\text{Production, pot. econ (n33)} \quad : \quad y_t^{pot} = \left(k^{pot}\right)^{\alpha} \left(l_t^{pot}\right)^{1-\alpha}$$

Note that we do not remove the monopoly power distortion in the product and labor market, so the steady state level of output is below its efficient level in both the actual and potential economy.

Hence, we have 33 equations in the following 33 unknows:

$$c_t\ \lambda_t\ w_t\ R_t\ \Pi_t\ y_t\ p_t^*\ w_t^*\ l_t\ s_t\ \vartheta_t\ mc_t\ f_t\ a_t\ \tilde{p}_t\ \Delta_{t,1}\ \Delta_{t,2}\ \Delta_{t,3}$$
$$\Pi_t^w\ s_t^w\ f_t^w\ \tilde{w}_t\ a_t^w\ \vartheta_t^w\ \Delta_{t,1}^w\ \Delta_{t,2}^w\ \Delta_{t,3}^w\ mrs_t$$
$$c_t^{pot}\ rr_t^{pot}\ l_t^{pot}\ w_t^{pot}\ y_t^{pot}$$

The variable δ_t is exogenous. We specify a process for δ_t in eq. (16). In the above equations we have set $\omega = \zeta_{h,t}^{1/\kappa_w} = 1$.

A.2. Steady State

$$\begin{aligned}
R &= \Pi/\beta \\
\tilde{p} &= 1 \\
\Delta_1 &= 1 \\
\Delta_2 &= 1 \\
\Delta_3 &= 1 \\
p^* &= 1 \\
w^* &= 1 \\
\vartheta &= 1 \\
\delta &= 1 \\
l &= n \\
mc &= \frac{1}{1+\theta_p}
\end{aligned}$$

Notice that we have not made use of the first order condition for capital inputs by firms in the set of equilibrium equations. That first order condition simply pins down the rental rate on capital:

$$r^k = mc\alpha \left(\frac{l}{k}\right)^{1-\alpha}$$

Now, returns of bonds and capital must be the same, i.e.:

$$\begin{aligned}
R/\Pi - 1 &= 1/\beta - 1 = r^k = mc\alpha \left(\frac{l}{k}\right)^{1-\alpha} \\
\frac{k}{l} &= \left(\frac{1-\beta}{mc\alpha\beta}\right)^{\frac{1}{\alpha-1}}
\end{aligned}$$

$$w = (1-\alpha)\, mc \left(\frac{k}{l}\right)^{\alpha}$$
$$y/l = \left(\frac{k}{l}\right)^{\alpha}$$

$$c/y = 1$$
$$c/l = c/y \times y/l$$

$$\begin{aligned}
l &= \frac{1}{c/l}\frac{1}{1+\theta_w}w \\
y &= y/l * l \\
c &= c/l * l \\
\lambda &= 1/c
\end{aligned}$$

$$\begin{aligned}
s &= \frac{1}{1-\beta\xi_p}\frac{\left(1+\psi_p\right)\left(1+\theta_p\right)}{1+\psi_p+\theta_p\psi_p}\lambda y mc \\
f &= \frac{1}{1-\beta\xi_p}\lambda y \\
a &= \frac{1}{1-\beta\xi_p}\frac{\psi_p\theta_p}{1+\psi_p+\theta_p\psi_p}y\lambda
\end{aligned}$$

$$\begin{aligned}
\tilde{w} &= 1 \\
\Delta_1^w &= 1 \\
\Delta_2^w &= 1 \\
\Delta_3^w &= 1 \\
\vartheta^w &= 1 \\
\Pi^w &= \Pi
\end{aligned}$$

$$\begin{aligned}
f^w &= \frac{1}{1-\beta\xi_w}n\lambda w \\
a^w &= \frac{1}{1-\beta\xi_w}\frac{\theta_w\psi_w}{\psi_w+\theta_w\psi_w+1}n\lambda w \\
s^w &= \frac{1}{1-\beta\xi_w}\frac{\left(1+\psi_w\right)\left(1+\theta_w\right)}{1+\psi_w+\theta_w\psi_w}n\lambda mrs \\
s^w &= f^w - a^w
\end{aligned}$$

Using the last equation it follows that:

$$mrs = \frac{1}{1+\theta_w} w$$

$$\begin{aligned}
rr^{pot} &= \frac{1}{\beta} \\
mc^{pot} &= \frac{1}{1+\theta_p} \\
\frac{k^{pot}}{l^{pot}} &= \left(\frac{1-\beta}{mc^{pot}\alpha\beta} \right)^{\frac{1}{\alpha-1}} \\
w^{pot} &= \frac{1-\alpha}{1+\theta_p} \left(\frac{k^{pot}}{l^{pot}} \right)^{\alpha} \\
y^{pot}/l^{pot} &= \left(\frac{k^{pot}}{l^{pot}} \right)^{\alpha}
\end{aligned}$$

$$c^{pot}/y^{pot} = 1$$

$$c^{pot}/l^{pot} = c^{pot}/y^{pot} \times y^{pot}/l^{pot}$$

$$n^{pot} = \frac{1}{c^{pot}/l^{pot}} \frac{1}{1+\theta_w} w^{pot}$$

$$\begin{aligned}
c^{pot} &= c^{pot}/l^{pot} * l^{pot} \\
y^{pot} &= y^{pot}/l^{pot} * l^{pot}
\end{aligned}$$

A.3. Linearized Equilibrium Equations

Equations (n9) – (n14) can be expressed in log-linearized form as follows:

$$\hat{\vartheta}_t = 0, \hat{p}_t^* = 0, \hat{\Delta}_{t,1} = 0, \hat{\Delta}_{t,2} = 0, \hat{\Delta}_{t,3} = 0, \widehat{\tilde{p}}_t = \frac{\xi_p}{1-\xi_p} \hat{\Pi}_t$$

After some tedious math, eqs. (n5) – (n8) can be written in log-linear form as:

$$\begin{aligned}
\text{log.lin. pricing 1 (n5)} &: \quad \hat{s}_t = \left(1-\beta\xi_p\right) \left[\hat{y}_t + \hat{\lambda}_t + \widehat{mc}_t\right] + \beta\xi_p E_t \left[\hat{\delta}_{t+1} + \frac{1+\theta_p}{\theta_p} \left(1+\psi_p\right) \hat{\Pi}_{t+1} + \hat{s}_{t+1}\right] \\
\text{log.lin. pricing 2 (n6)} &: \quad \hat{f}_t = \left(1-\beta\xi_p\right) \left(\hat{y}_t + \hat{\lambda}_t\right) + \beta\xi_p E_t \left[\hat{\delta}_{t+1} + \frac{\left(1+\psi_p+\psi_p\theta_p\right)}{\theta_p} \hat{\Pi}_{t+1} + \hat{f}_{t+1}\right] \\
\text{log.lin. pricing 3 (n7)} &: \quad \hat{a}_t = \left(1-\beta\xi_p\right) \left(\hat{y}_t + \hat{\lambda}_t\right) + \beta\xi_p E_t \left[\hat{\delta}_{t+1} - \hat{\Pi}_{t+1} + \hat{a}_{t+1}\right] \\
\text{log.lin. pricing 4 (n8)} &: \quad \hat{s}_t = \frac{1+\psi_p+\theta_p\psi_p}{1+\psi_p} \hat{f}_t - \frac{\psi_p\theta_p}{1+\psi_p} \hat{a}_t + \frac{\xi_p\left(1-\psi_p-\psi_p\theta_p\right)}{1-\xi_p} \hat{\Pi}_t
\end{aligned}$$

Premultiply eqs. (n6) and (n7) by $\frac{1+\psi_p+\theta_p\psi_p}{1+\psi_p}$ and $\frac{\psi_p\theta_p}{1+\psi_p}$. Add eq. (n7) and substract eq. (n6) from (n5):

$$\begin{aligned}\hat{s}_t - \frac{1+\psi_p+\theta_p\psi_p}{1+\psi_p}\hat{f}_t + \frac{\psi_p\theta_p}{1+\psi_p}\hat{a}_t &= \left(1-\beta\xi_p\right)\widehat{mc}_t \\ +\beta\xi_p E_t\left(\hat{s}_{t+1} - \frac{1+\psi_p+\theta_p\psi_p}{1+\psi_p}\hat{f}_{t+1} + \frac{\psi_p\theta_p}{1+\psi_p}\hat{a}_{t+1} + \left(1-\psi_p-\psi_p\theta_p\right)\Pi_{t+1}\right)\end{aligned}$$

Use equation (n8) to get

$$\begin{aligned}\frac{\xi_p\left(1-\psi_p-\psi_p\theta_p\right)}{1-\xi_p}\hat{\Pi}_t &= \left(1-\beta\xi_p\right)\widehat{mc}_t \\ &+\beta\xi_p E_t\left(\frac{\xi_p\left(1-\psi_p-\psi_p\theta_p\right)}{1-\xi_p} + \left(1-\psi_p-\psi_p\theta_p\right)\right)\hat{\Pi}_{t+1}\end{aligned}$$

Or

$$\hat{\Pi}_t = \beta E_t\hat{\Pi}_{t+1} + \frac{\left(1-\xi_p\right)\left(1-\beta\xi_p\right)}{\xi_p}\frac{1}{1-\left(1+\theta_p\right)\psi_p}\widehat{mc}_t$$

The coefficient $\frac{1}{1-(1+\theta_p)\psi_p}$ is identical to the one in Levin, Lopez-Salido and Yun (2007).

The nonlinear wage setting equations can be log-linearized to obtain:

$$\begin{aligned}\hat{f}_t^w &= \left(1-\beta\xi_w\right)\left(\hat{n}_t+\hat{\lambda}_t+\hat{w}_t\right) + \beta\xi_w E_t\left(\hat{\delta}_{t+1} + \frac{1+\psi_w+\theta_w\psi_w}{\theta_w}\hat{\Pi}_{w,t+1} + \hat{f}_{t+1}^w\right) \\ \hat{a}_t^w &= \left(1-\beta\xi_w\right)\left(\hat{n}_t+\hat{\lambda}_t+\hat{w}_t\right) + \beta\xi_w E_t\left(\hat{\delta}_{t+1} - \hat{\Pi}_{w,t+1} + \hat{a}_{t+1}^w\right) \\ \hat{s}_t^w &= \left(1-\beta\xi_w\right)\left(\hat{n}_t+\hat{\lambda}_t+\widehat{mrs}_t\right) + \beta\xi_w E_t\left(\hat{\delta}_{t+1} + \frac{\left(1+\theta_w\right)\left(1+\psi_w\right)}{\theta_w}\hat{\Pi}_{w,t+1} + \hat{s}_{t+1}^w\right) \\ \hat{s}_t^w &= \frac{1+\psi_w+\theta_w\psi_w}{1+\psi_w}f_t^w - \frac{\theta_w\psi_w}{1+\psi_w}\hat{a}_t^w + \left(1-\psi_w-\theta_w\psi_w\right)\hat{\tilde{w}}_t\end{aligned}$$

Also:

$$\hat{\tilde{w}}_t = \frac{\xi_w}{1-\xi_w}\hat{\Pi}_{w,t}$$

So that

$$
\begin{aligned}
\frac{1+\psi_w+\theta_w\psi_w}{1+\psi_w}\hat{f}_t^w &= \frac{1+\psi_w+\theta_w\psi_w}{1+\psi_w}\left(1-\beta\xi_w\right)\left(\hat{n}_t+\hat{\lambda}_t+\hat{w}_t\right) \\
&\quad +\frac{1+\psi_w+\theta_w\psi_w}{1+\psi_w}\beta\xi_w E_t\left(\hat{\delta}_{t+1}+\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}\hat{\Pi}_{w,t+1}+\hat{f}_{t+1}^w\right) \\
\frac{\theta_w\psi_w}{1+\psi_w}\hat{a}_t^w &= \frac{\theta_w\psi_w}{1+\psi_w}\left(1-\beta\xi_w\right)\left(\hat{n}_t+\hat{\lambda}_t+\hat{w}_t\right)+\frac{\theta_w\psi_w}{1+\psi_w}\beta\xi_w E_t\left(\hat{\delta}_{t+1}-\hat{\Pi}_{w,t+1}+\hat{a}_{t+1}^w\right) \\
\hat{s}_t^w &= \left(1-\beta\xi_w\right)\left(\hat{n}_t+\hat{\lambda}_t+\widehat{mrs}_t\right)+\beta\xi_w E_t\left(\hat{\delta}_{t+1}+\frac{\left(1+\theta_w\right)\left(1+\psi_w\right)}{\theta_w}\hat{\Pi}_{w,t+1}+\hat{s}_{t+1}^w\right) \\
\hat{s}_t^w &= \frac{1+\psi_w+\theta_w\psi_w}{1+\psi_w}f_t^w-\frac{\theta_w\psi_w}{1+\psi_w}\hat{a}_t^w+\left(1-\psi_w-\theta_w\psi_w\right)\frac{\xi_w}{1-\xi_w}\hat{\Pi}_{w,t}
\end{aligned}
$$

Substract first and add second equation to third equation and substitute last equation to get:

$$
\begin{aligned}
\left(1-\psi_w-\theta_w\psi_w\right)\frac{\xi_w}{1-\xi_w}\hat{\Pi}_{w,t} &= \left(1-\beta\xi_w\right)\left(\widehat{mrs}_t-\hat{w}_t\right) \\
&\quad +\beta\xi_w E_t\left(\frac{\left(1+\theta_w\right)\left(1+\psi_w\right)}{\theta_w}\hat{\Pi}_{w,t+1}+\hat{s}_{t+1}^w\right) \\
&\quad +\frac{\theta_w\psi_w}{1+\psi_w}\beta\xi_w E_t\left(-\hat{\Pi}_{w,t+1}+\hat{a}_{t+1}^w\right) \\
&\quad -\frac{1+\psi_w+\theta_w\psi_w}{1+\psi_w}\beta\xi_w E_t\left(\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}\hat{\Pi}_{w,t+1}+\hat{f}_{t+1}^w\right)
\end{aligned}
$$

Or :

$$
\begin{aligned}
\left(1-\psi_w-\theta_w\psi_w\right)\frac{\xi_w}{1-\xi_w}\hat{\Pi}_{w,t} &= \left(1-\beta\xi_w\right)\left(\widehat{mrs}_t-\hat{w}_t\right) \\
&\quad +\beta\xi_w\left(1-\psi_w-\theta_w\psi_w\right)\left(1+\frac{\xi_w}{1-\xi_w}\right)E_t\hat{\Pi}_{w,t+1}
\end{aligned}
$$

Or:

$$
\hat{\Pi}_{w,t}=\beta E_t\hat{\Pi}_{w,t+1}+\frac{\left(1-\xi_w\right)\left(1-\beta\xi_w\right)}{\xi_w}\frac{1}{1-\left(1+\theta_w\right)\psi_w}\left(\widehat{mrs}_t-\hat{w}_t\right)
$$

where

$$
\begin{aligned}
\widehat{mrs}_t &= -\hat{\lambda}_t \\
&= \hat{c}_t
\end{aligned}
$$

So, the set of log-linearized equilibrium equations (with the exception of the nonlinear Taylor rule) can be written as:

$$
\begin{aligned}
\text{Euler equation (l1)} &: \quad 0 = E_t\left[\hat{\delta}_{t+1} + \hat{R}_t - \hat{\Pi}_{t+1} - \hat{c}_{t+1} + \hat{c}_t\right] \\
\text{Resource constraint (l2)} &: \quad \hat{c}_t = \hat{y}_t \\
\text{Production (l3)} &: \quad \hat{y}_t = (1-\alpha)\,\hat{l}_t \\
\text{Marginal cost (l4)} &: \quad \widehat{mc}_t = \hat{w}_t + \alpha \hat{l}_t \\
\text{Marg. rate of subst.(l5)} &: \quad \widehat{mrs}_t = \hat{c}_t \\
\text{Taylor rule (l6)} &: \quad \hat{R}_t = \max\left((1-R)/R, \gamma_\pi \hat{\Pi}_t + \gamma_x\left[\hat{y}_t - \hat{y}_t^{pot}\right]\right) \\
\text{Wage inflation (l7)} &: \quad \hat{\Pi}_t^w = \hat{\Pi}_t + \hat{w}_t - \hat{w}_{t-1} \\
\text{Price Phillips Curve (l8)} &: \quad \hat{\Pi}_t = \beta E_t \hat{\Pi}_{t+1} + \frac{\left(1-\xi_p\right)\left(1-\beta\xi_p\right)}{\xi_p} \frac{1}{1-\left(1+\theta_p\right)\psi_p} \widehat{mc}_t \\
\text{Wage Phillips Curve (l9)} &: \quad \hat{\Pi}_t^w = \beta E_t \hat{\Pi}_{t+1}^w + \frac{\left(1-\xi_w\right)\left(1-\beta\xi_w\right)}{\xi_w} \frac{1}{1-\left(1+\theta_w\right)\psi_w}\left(\widehat{mrs}_t - \hat{w}_t\right) \\
\text{Euler Equation, pot. econ (l10)} &: \quad 0 = E_t\left[\hat{\delta}_{t+1} + \widehat{rr}_t^{pot} - \hat{c}_{t+1}^{pot} + \hat{c}_t^{pot}\right] \\
\text{Leisure/Labor,pot. econ (l11)} &: \quad \hat{c}_t^{pot} = \hat{w}_t^{pot} \\
\text{Wage, pot. econ (l12)} &: \quad \hat{w}_t^{pot} = -\alpha \hat{l}_t^{pot} \\
\text{Res. constraint, pot. econ (l13)} &: \quad \hat{c}_t^{pot} = \hat{y}_t^{pot} \\
\text{Production, pot. econ (l14)} &: \quad \hat{y}_t^{pot} = (1-\alpha)\,\hat{l}_t^{pot}
\end{aligned}
$$

where hat variables denote percent deviations from steady state. Breve variables are absolute deviations from steady state.

Thus, we have 14 equations in the following 14 unknows:

$$
\begin{gathered}
\hat{R}_t \ \hat{\Pi}_t \ \hat{c}_t \ \hat{y}_t \ \hat{l}_t \ \hat{w}_t \ \widehat{mc}_t \ \widehat{mrs}_t \ \hat{\Pi}_t^w \\
\hat{c}_t^{pot} \ \widehat{rr}_t^{pot} \ \hat{l}_t^{pot} \ \hat{w}_t^{pot} \ \hat{y}_t^{pot}
\end{gathered}
$$

The variable $\hat{\delta}_t$ is exogenous, and derived from the δ_t-process in eq. (16).

A.4. Canonical Representation of Log-linearized Model

Assuming linear production in labor effort (i.e. $\alpha = 0$), the log-linearized equations can be expressed as follows:

$$
\begin{aligned}
\text{Euler equation (l1)} &: \quad \hat{y}_t = E_t \hat{y}_{t+1} - \left(\hat{R}_t - E_t \hat{\Pi}_{t+1} - \widehat{rr}_t^{pot} \right) \\
\text{Marginal cost (l4)} &: \quad \widehat{mc}_t = \hat{w}_t \\
\text{Marg. rate of subst.(l5)} &: \quad \widehat{mrs}_t = \hat{y}_t \\
\text{Taylor rule (l6)} &: \quad \hat{R}_t = \max \left((1-R)/R, \gamma_\pi \hat{\Pi}_t + \gamma_x \left[\hat{y}_t - \hat{y}_t^{pot} \right] \right) \\
\text{Wage inflation (l7)} &: \quad \hat{\Pi}_t^w = \hat{\Pi}_t + \hat{w}_t - \hat{w}_{t-1} \\
\text{Price Phillips Curve (l8)} &: \quad \hat{\Pi}_t = \beta E_t \hat{\Pi}_{t+1} + \frac{\left(1-\xi_p\right)\left(1-\beta\xi_p\right)}{\xi_p} \frac{1}{1-(1+\theta_p)\psi_p} \widehat{mc}_t \\
\text{Wage Phillips Curve (l9)} &: \quad \hat{\Pi}_t^w = \beta E_t \hat{\Pi}_{t+1}^w + \frac{(1-\xi_w)(1-\beta\xi_w)}{\xi_w} \frac{1}{1-(1+\theta_w)\psi_w} \left(\widehat{mrs}_t - \hat{w}_t \right) \\
\text{Potential Real Rate (l10)} &: \quad \widehat{rr}_t^{pot} = -E_t \hat{\delta}_{t+1} \\
\text{Potential Output (l11)} &: \quad \hat{y}_t^{pot} = 0
\end{aligned}
$$

Or, more compactly:

$$
\begin{aligned}
\text{Euler equation (l1c)} &: \quad \hat{y}_t = E_t \hat{y}_{t+1} - \left(\hat{R}_t - E_t \hat{\Pi}_{t+1} - \widehat{rr}_t^{pot} \right) \\
\text{Price Phillips Curve (l2c)} &: \quad \hat{\Pi}_t = \beta E_t \hat{\Pi}_{t+1} + \kappa_p \hat{w}_t \\
\text{Wage Phillips Curve (l3c)} &: \quad \hat{\Pi}_t^w = \beta E_t \hat{\Pi}_{t+1}^w + \kappa_w \left[\hat{y}_t - \hat{w}_t \right] \\
\text{Taylor rule (l4c)} &: \quad \hat{R}_t = \max \left\{ -\log R, \gamma_\pi \hat{\Pi}_t + \gamma_x \hat{y}_t \right\} \\
\text{Wage inflation (l5c)} &: \quad \hat{\Pi}_t^w = \hat{\Pi}_t + \hat{w}_t - \hat{w}_{t-1}
\end{aligned}
$$

where

$$
\widehat{rr}_t^{pot} = -E_t \hat{\delta}_{t+1},
$$

and

$$
\begin{aligned}
\kappa_p &= \frac{\left(1-\xi_p\right)\left(1-\beta\xi_p\right)}{\xi_p} \frac{1}{1-(1+\theta_p)\psi_p}, \\
\kappa_w &= \frac{(1-\xi_w)(1-\beta\xi_w)}{\xi_w} \frac{1}{1-(1+\theta_w)\psi_w}.
\end{aligned}
$$

A.5. Additional results in the Stylized model

In this section, we report some additional results in the stylized model.

First, Figures A.1 and A.2 show the results of an extended set of variables for the results depicted in Figure 1. Next, Figure A.3 reports results when we scale the discount factor shock in the linearized and nonlinear models to produce the same-sized output contraction in the first period. Figure A.4 reports the results when the Dixit-Stiglitz aggregator is used instead of the Kimball Aggregator. Finally, Figures A.5 and A.6 report results for the stochastic simulations for an extended set of variables.

Figure A.1: Impulse Responses to a 1% Discount Factor Shock (With Zero Lower Bound)

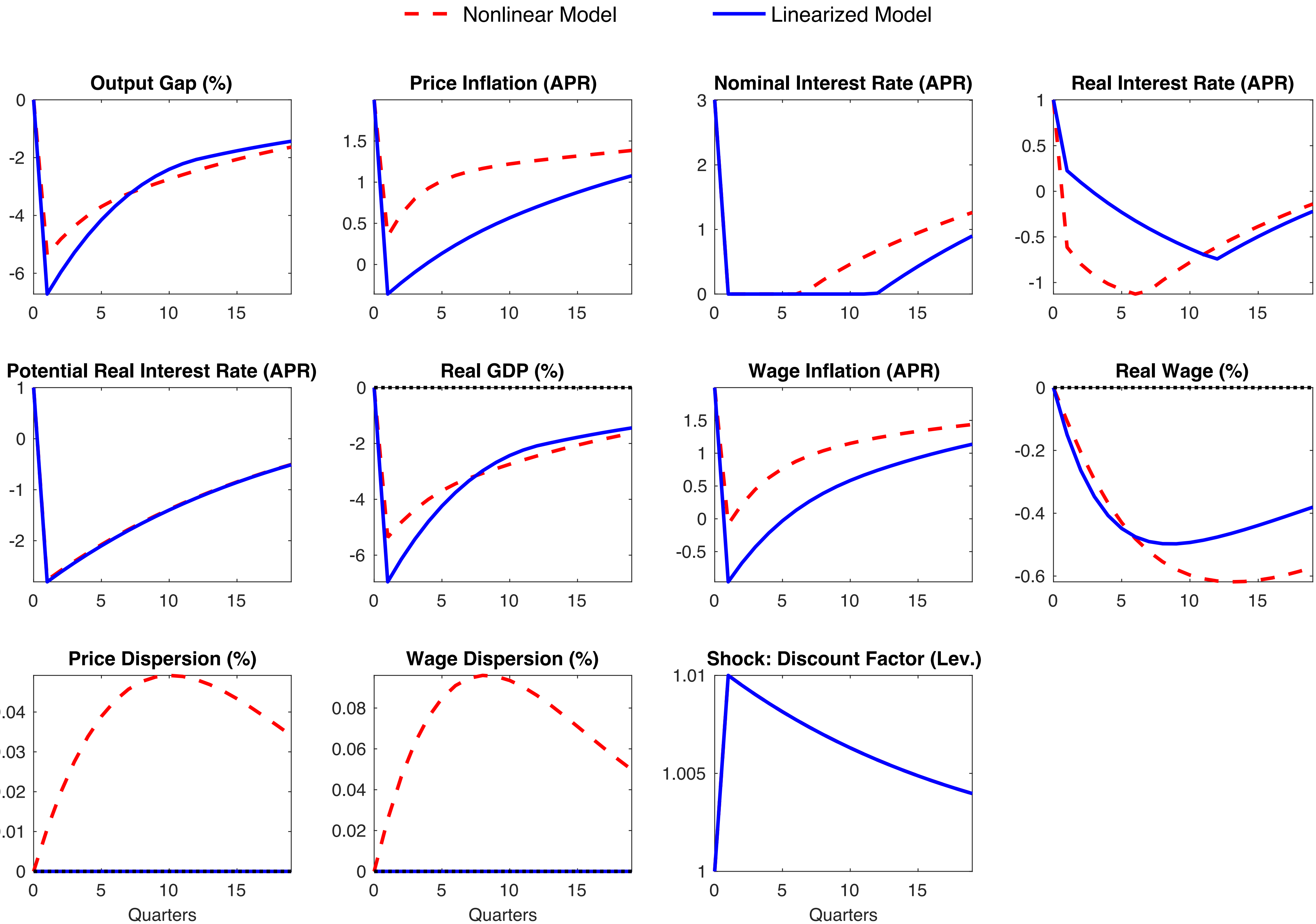


Figure A.2: Impulse Responses to a 1% Discount Factor Shock (Without Zero Lower Bound)

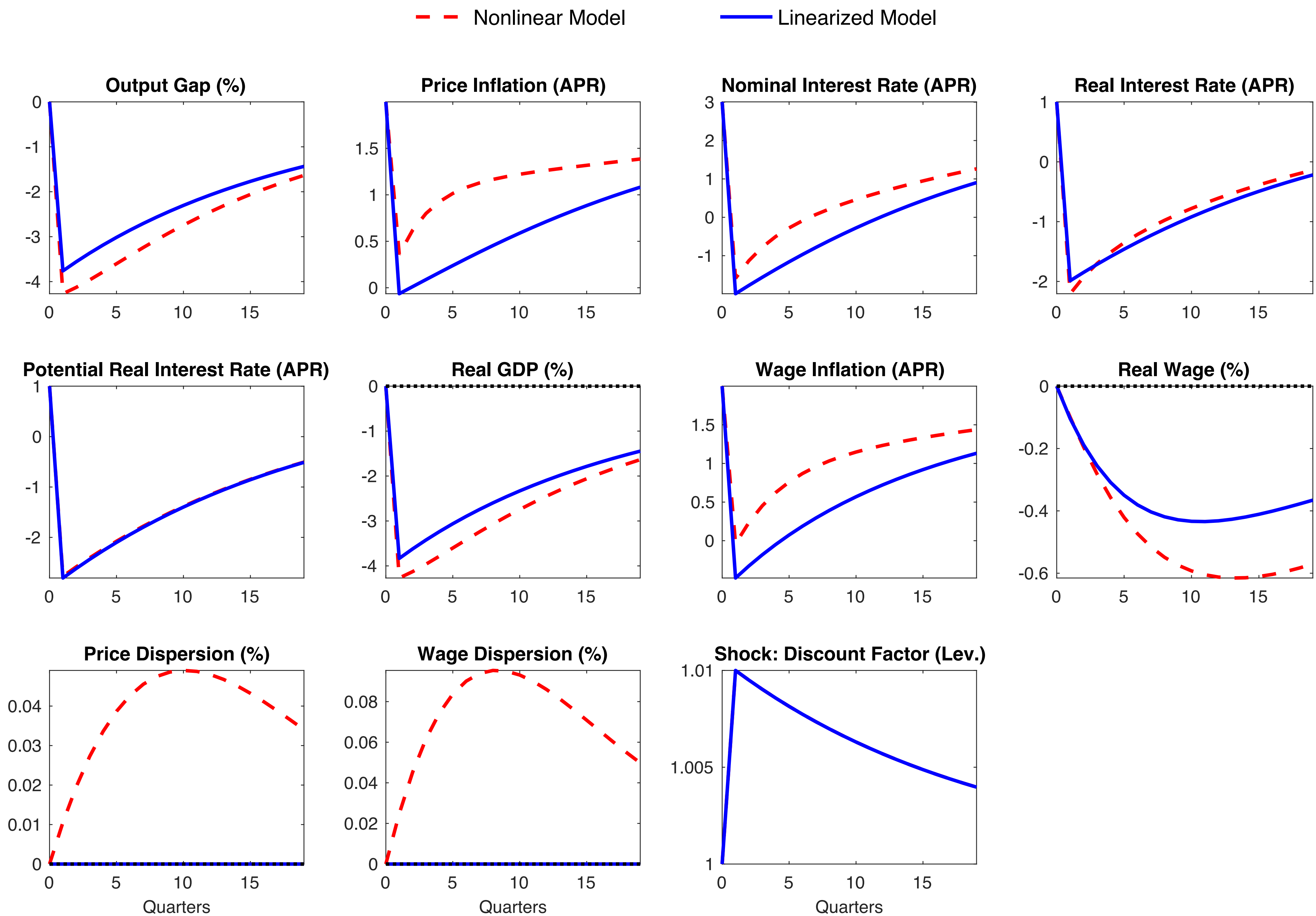


Figure A.3: Impulse Responses to a 1% Discount Factor Shock (Same Output Response)

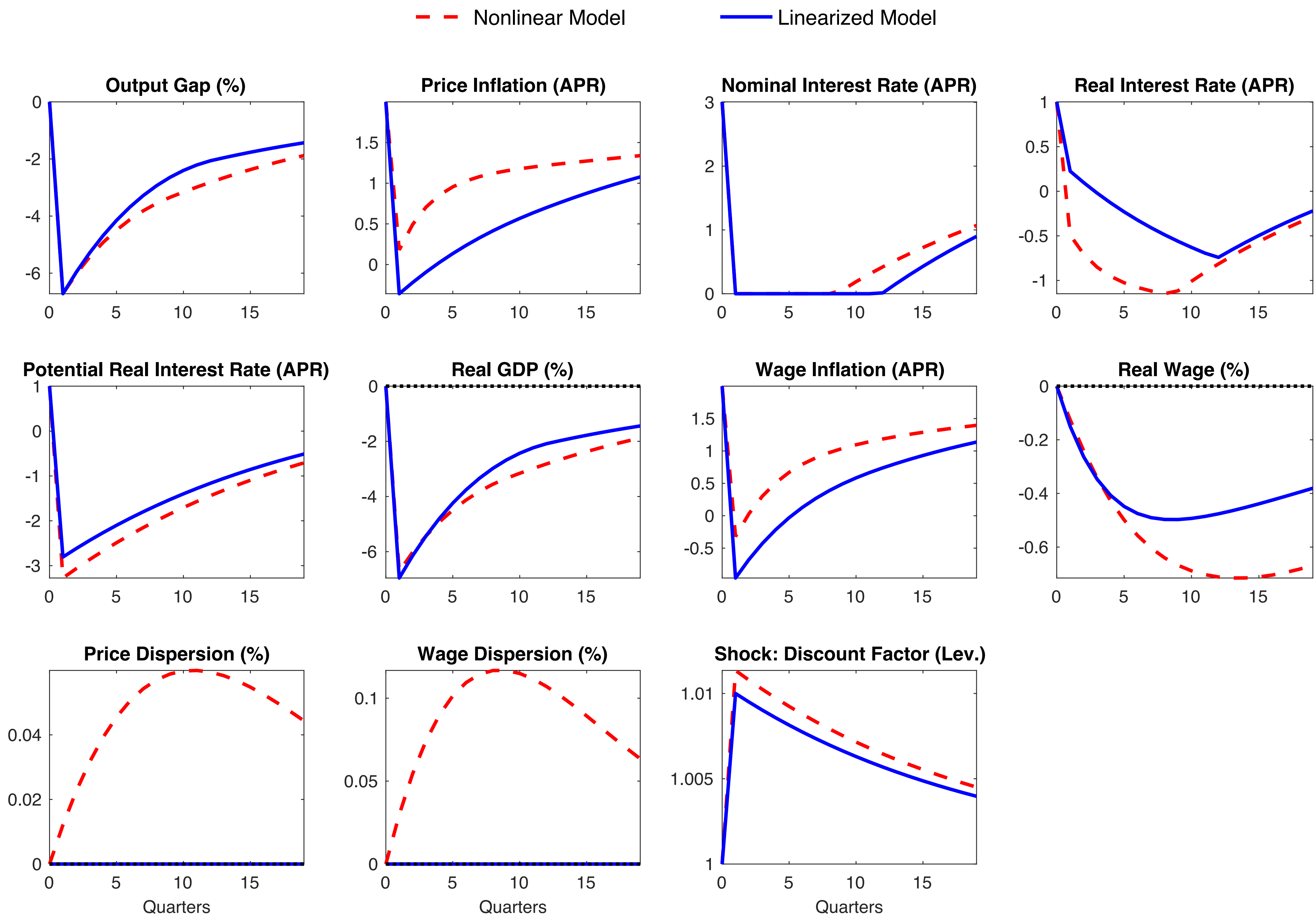


Figure A.4: Impulse Responses to a 1% Discount Factor Shock (Dixit-Stiglitz)

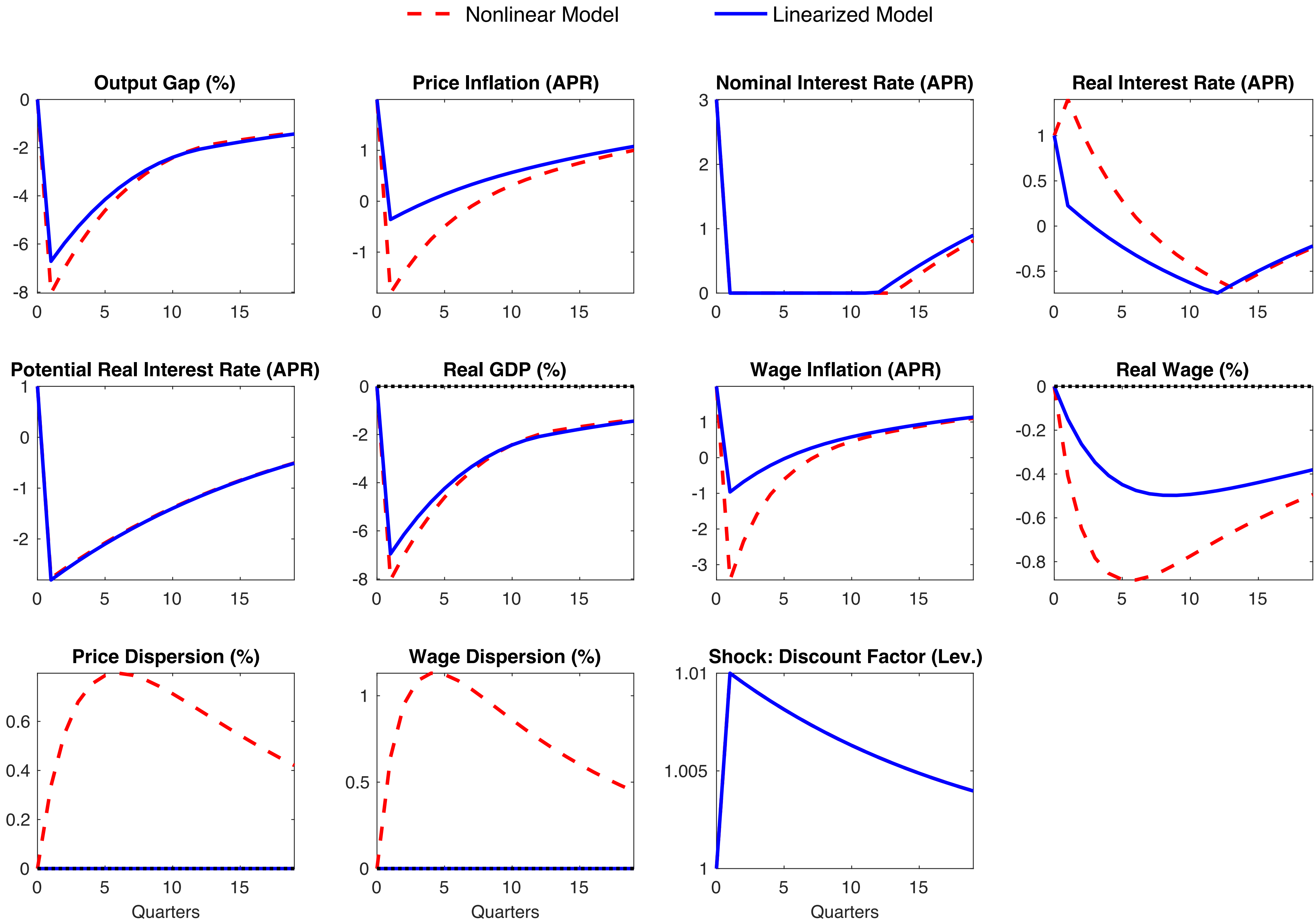


Figure A.5: Stochastic Simulation of Linearized Model when ZLB Binds

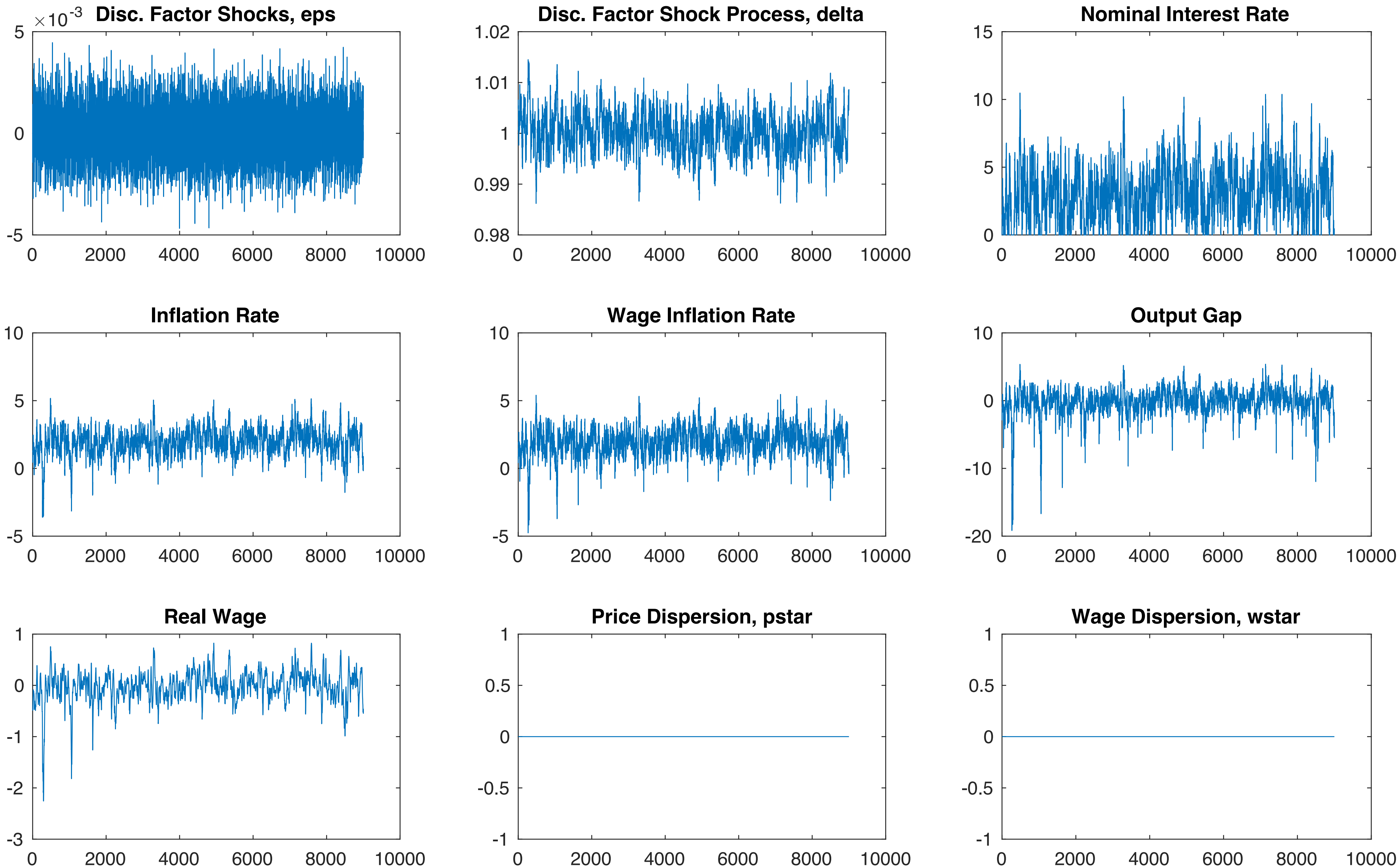
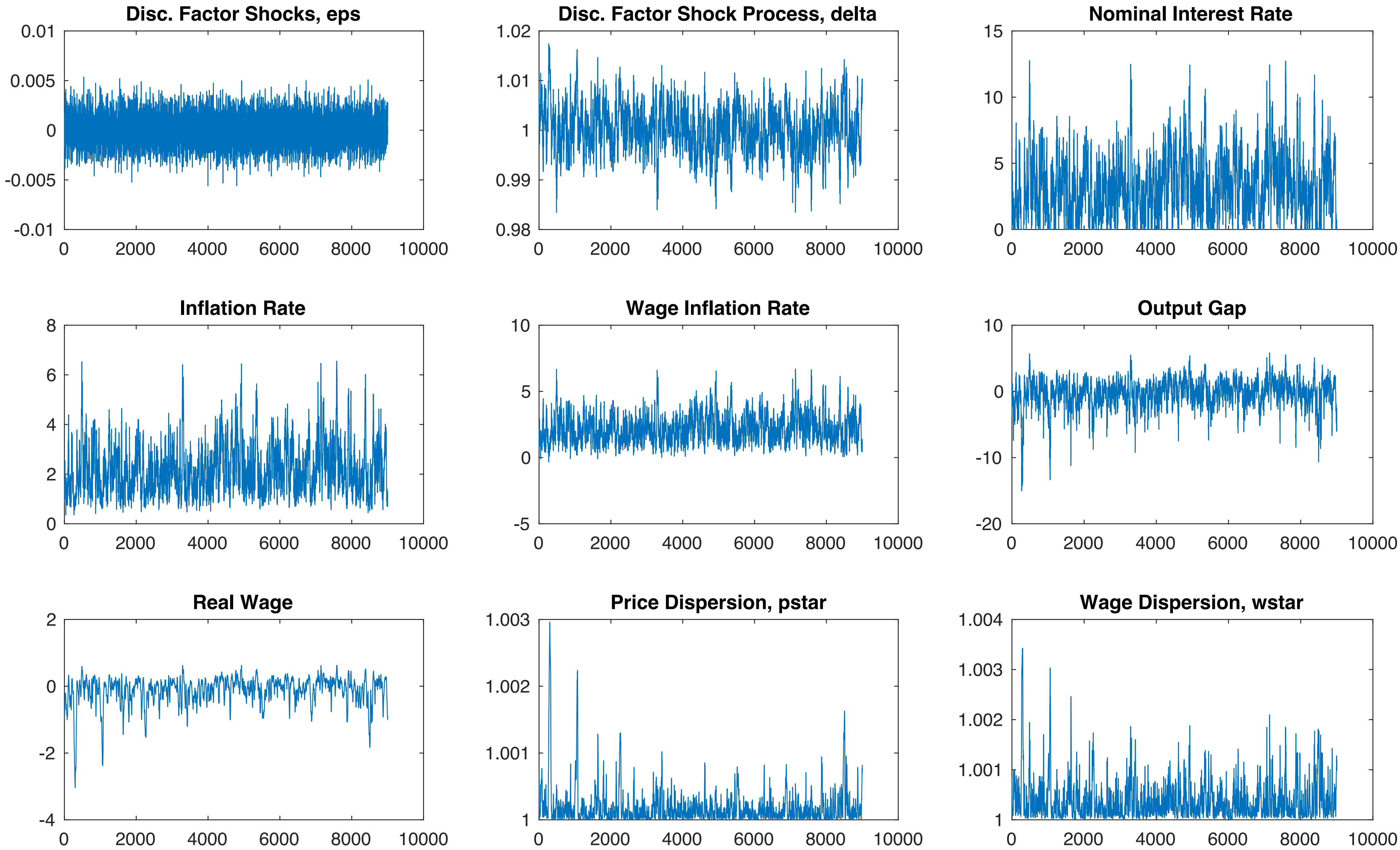


Figure A.6: Stochastic Simulation of Nonlinear Model When ZLB Binds



Appendix B. The Workhorse New Keynesian DSGE Model

In this appendix, we present the nonlinear and linearized equations in the workhorse model. The formulation of the model is identical to baseline specification with Calvo wage stickiness in Christiano, Eichenbaum and Trabandt (2016), but is adapted for the case of Kimball aggregation in goods and labor markets and multiple shocks.

B.1. Nonlinear Equilibrium Equations

The scaled nonlinear equilibrium equilibrium equations can be written as follows:

$$
\begin{aligned}
\text{Cons. FOC (n1)} &: \quad \lambda_t = \zeta_{c,t}(c_t - bc_{t-1}/\mu_t)^{-1} - \iota_c \beta b E_t \zeta_{c,t+1}\left(c_{t+1}\mu_{t+1} - bc_t\right)^{-1} \\
\text{Bond. FOC (n2)} &: \quad \lambda_t = \epsilon_{RP,t}\beta E_t \lambda_{t+1} R_t / \left(\pi_{t+1}\mu_{t+1}\right) \\
\text{Invest. FOC (n3)} &: \quad 1 = p_{k',t}\Upsilon_t^{\eta}\left[1 - S_t - S_t'\mu_t\mu_{i,t} i_t/i_{t-1}\right] \\
&\quad +\iota_i \beta E_t \lambda_{t+1}/\lambda_t p_{k',t+1}\Upsilon_{t+1}^{\eta} S_{t+1}'\left(i_{t+1}/i_t\right)^2 \mu_{i,t+1}\mu_{t+1} \\
\text{Capital FOC (n4)} &: \quad \lambda_t = \beta E_t \lambda_{t+1} R_{t+1}^k/(\pi_{t+1}\mu_{t+1}) \\
\text{Capital return (n5)} &: \quad R_t^k = \pi_t/(\mu_{i,t} p_{k',t-1})\left(u_t a'\left(u_t\right) - a(u_t) + (1-\delta)p_{k',t}\right) \\
\text{LOM capital (n6)} &: \quad k_t = (1-\delta)/(\mu_t\mu_{i,t})k_{t-1} + \Upsilon_t^{\eta}\left(1 - S_t\right) i_t \\
\text{Cost. minim. (n7)} &: \quad 0 = a'\left(u_t\right) u_t k_{t-1}/(\mu_{i,t}\mu_t) - \alpha/(1-\alpha) w_t\left[\nu R_t + 1 - \nu\right]\left(w_t^*\right)^{-1} l_t \\
\text{Marginal cost (n8)} &: \quad mc_t = \tau_t^{1/\kappa}\left(\mu_{i,t}\mu_t\right)^{\alpha} w_t\left[\nu R_t + 1 - \nu\right]\left(u_t k_{t-1}/\left(\left(w_t^*\right)^{-1} l_t\right)\right)^{-\alpha}/(\epsilon_t\left(1-\alpha\right)) \\
\text{Production (n9)} &: \quad y_t = \left(p_t^*\right)^{-1}\left[\epsilon_t\left(u_t k_{t-1}/(\mu_t\mu_{i,t})\right)^{\alpha}\left(\left(w_t^*\right)^{-1} l_t\right)^{1-\alpha} - \phi\right] \\
\text{Resources (n10)} &: \quad y_t = g_t + c_t + i_t + a\left(u_t\right) k_{t-1}/(\mu_{i,t}\mu_t) \\
\text{Real GDP (n11)} &: \quad \mathcal{Y}_t = g_t + c_t + i_t
\end{aligned}
$$

$$
\begin{aligned}
\text{Taylor rule (n12)} &: \quad \ln\left(R_t^{not}\right) = \rho_R \ln\left(R_{t-1}^{not}\right) + \ln\epsilon_{r,t} \\
&\quad +\left(1-\rho_R\right)\left[\begin{array}{c} \ln(R) + r_\pi \ln\left(\left(\pi_t^4\right)^{0.25}/\pi_t^T\right) \\ +r_y \ln\left(\mathcal{Y}_t/\mathcal{Y}_t^f\right) + r_{\Delta y}\ln\left(\frac{\mathcal{Y}_t/\mathcal{Y}_t^f}{\mathcal{Y}_{t-4}/\mathcal{Y}_{t-4}^f}\right)\end{array}\right] \\
\text{Ann. Infl. (n13)} &: \quad \pi_t^4 = \pi_t\pi_{t-1}\pi_{t-2}\pi_{t-3} \\
\text{ZLB (n14)} &: \quad R_t = \max(1, R_t^{not}) \\
\text{Cap. util. cost. (n15)} &: \quad a(u_t) = 0.5\sigma_b\sigma_a\left(u_t\right)^2 + \sigma_b\left(1-\sigma_a\right)u_t + \sigma_b\left((\sigma_a/2) - 1\right)
\end{aligned}
$$

$$\text{Cap. util. deriv. (n16)} \quad : \quad a'(u_t) = \sigma_b \sigma_a u_t + \sigma_b (1 - \sigma_a)$$

$$\text{Invest. adj. cost (n17)} \quad : \quad S_t = 0.5 \exp\left[\sqrt{S''}\left(\mu_t \mu_{i,t} i_t / i_{t-1} - \mu \cdot \mu_i\right)\right]$$
$$+0.5 \exp\left[-\sqrt{S''}\left(\mu_t \mu_{i,t} i_t / i_{t-1} - \mu \cdot \mu_i\right)\right] - 1$$

$$\text{Inv. adj. deriv. (n18)} \quad : \quad S'_t = 0.5\sqrt{S''} \exp\left[\sqrt{S''}\left(\mu_t \mu_{i,t} i_t / i_{t-1} - \mu \cdot \mu_\Psi\right)\right]$$
$$-0.5\sqrt{S''} \exp\left[-\sqrt{S''}\left(\mu_t \mu_{i,t} i_t / i_{t-1} - \mu \cdot \mu_i\right)\right]$$

$$\text{Non.lin. pricing 1 (n19)} \quad : \quad s_t = \frac{\left(1+\psi_p\right)\left(1+\theta_p\right)}{1+\psi_p+\theta_p\psi_p} \lambda_t y_t \vartheta_t^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} mc_t$$
$$+\beta \xi_p E_t \left(\pi/\pi_{t+1}\right)^{-\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} s_{t+1}$$

$$\text{Non.lin. pricing 2 (n20)} \quad : \quad f_t = \lambda_t y_t \vartheta_t^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} + \beta \xi_p E_t \left(\pi/\pi_{t+1}\right)^{-\frac{\left(1+\psi_p+\psi_p\theta_p\right)}{\theta_p}} f_{t+1}$$

$$\text{Non.lin. pricing 3 (n21)} \quad : \quad a_t = \frac{\psi_p \theta_p}{1+\psi_p+\theta_p\psi_p} y_t \lambda_t + \beta \xi_p E_t \left(\pi/\pi_{t+1}\right) a_{t+1}$$

$$\text{Non.lin.pricing 4 (n22)} \quad : \quad s_t = f_t \tilde{p}_t - a_t \tilde{p}_t^{1+\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)}$$

$$\text{Zero profit condition prices (n23)} \quad : \quad \vartheta_t = 1 + \psi_p - \psi_p \Delta_{t,2}$$

$$\text{Aggregate price index (n24)} \quad : \quad \vartheta_t = \Delta_{t,3}$$

$$\text{Overall price dispersion (n25)} \quad : \quad p_t^* = \frac{\vartheta_t^{\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)}}{1+\psi_p} \Delta_{t,1}^{-\frac{1+\theta_p}{\theta_p}\left(1+\psi_p\right)} + \frac{\psi_p}{1+\psi_p}$$

$$\text{Price dispersion 1 (n26)} \quad : \quad \Delta_{t,1}^{-\frac{\left(1+\theta_p\right)\left(1+\psi_p\right)}{\theta_p}} = \left(1-\xi_p\right) \tilde{p}_t^{-\frac{\left(1+\theta_p\right)\left(1+\psi_p\right)}{\theta_p}} + \xi_p \left[\left(\pi/\pi_t\right) \Delta_{t-1,1}\right]^{-\frac{\left(1+\theta_p\right)\left(1+\psi_p\right)}{\theta_p}}$$

$$\text{Price dispersion 2 (n27)} \quad : \quad \Delta_{t,2} = \left(1-\xi_p\right) \tilde{p}_t + \xi_p \left(\pi/\pi_t\right) \Delta_{t-1,2}$$

$$\text{Price dispersion 3 (n28)} \quad : \quad \Delta_{t,3}^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}} = (1-\xi_p) \tilde{p}_t^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}} + \xi_p \left(\left(\pi/\pi_t\right) \Delta_{t-1,3}\right)^{-\frac{1+\psi_p+\psi_p\theta_p}{\theta_p}}$$

$$\text{Non.lin. wage setting 1 (n29)} \quad : \quad s_t^w = f_t^w \tilde{w}_t - a_t^w \tilde{w}_t^{1+\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}$$

$$\text{Non.lin. wage setting 2 (n30)} \quad : \quad f_t^w = (w_t^*)^{-1} l_t \lambda_t w_t \vartheta_{w,t}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \beta \xi_w E_t \left[\frac{\Pi_w}{\Pi_{w,t+1}}\right]^{-\frac{1+\psi_w+\theta_w\psi_w}{\theta_w}} f_{t+1}^w$$

$$\text{Non.lin. wage setting 3 (n31)} \quad : \quad a_t^w = \frac{\theta_w \psi_w}{\psi_w + \theta_w \psi_w + 1} (w_t^*)^{-1} l_t \lambda_t w_t + \beta \xi_w E_t \frac{\Pi_w}{\Pi_{w,t+1}} a_{t+1}^w$$

$$\text{Non.lin. wage setting 4 (n32)} \quad : \quad s_t^w = \frac{(1+\psi_w)(1+\theta_w)}{1+\psi_w+\theta_w\psi_w} (w_t^*)^{-1} l_t \lambda_t mrs_t \vartheta_{w,t}^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}$$
$$+\beta \xi_w \delta_t E_t \left[\frac{\Pi_w}{\Pi_{w,t+1}}\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} s_{t+1}^w$$

$$
\begin{aligned}
\text{Zero profit condition wages (n33)} &: \quad \vartheta_t^w = 1 + \psi_w - \psi_w \Delta_{t,2}^w \\
\text{Agg. wage index (n34)} &: \quad \vartheta_t^w = \Delta_{t,3}^w \\
\text{Overall wage disperision (n35)} &: \quad w_t^* = \frac{\left[\vartheta_t^w\right]^{\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}}}{1+\psi_w} \left[\Delta_{t,1}^w\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} + \frac{\psi_w}{1+\psi_w} \\
\text{Wage dispersion 1 (n36)} &: \quad \left[\Delta_{t,1}^w\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} = (1-\xi_w)\left[\tilde{w}_t\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \\
&\quad +\xi_w \left[\frac{\Pi^w}{\Pi_t^w} \Delta_{t-1,1}^w\right]^{-\frac{(1+\theta_w)(1+\psi_w)}{\theta_w}} \\
\text{Wage dispersion 2 (n37)} &: \quad \Delta_{t,2}^w = (1-\xi_w)\tilde{w}_t + \xi_w \left(\Pi^w/\Pi_t^w\right)\Delta_{t-1,2}^w \\
\text{Wage dispersion 3 (n38)} &: \quad \left[\Delta_{t,3}^w\right]^{-\frac{1+\psi_w+\psi_w\theta_w}{\theta_w}} = (1-\xi_w)\tilde{w}_t^{-\frac{1+\psi_w+\psi_w\theta_w}{\theta_w}} \\
&\quad +\xi_w \left(\left(\Pi^w/\Pi_t^w\right)\Delta_{t-1,3}^w\right)^{-\frac{1+\psi_w+\psi_w\theta_w}{\theta_w}} \\
\text{Wage inflation (n39)} &: \quad \pi_{w,t} = w_t \mu_t \pi_t / w_{t-1} \\
\text{Marg. rate of subst (n40)} &: \quad mrs_t{=}\omega\zeta_h^{1/\kappa_w}/\lambda_t
\end{aligned}
$$

The parameters ι_c and ι_i allow to parameterize the model such that either internal or external habit in consumption as well as investment adjustment costs can be considered. The parameter ν parameterizes working capital.

The equations for the flexible price - flexibe wage economy are:

$$
\begin{aligned}
\text{Cons. FOC (n1f)} &: \quad \lambda_t^f = \zeta_{c,t}(c_t^f - bc_{t-1}^f/\mu_t)^{-1} - \iota_c \beta b E_t \zeta_{c,t+1}\left(c_{t+1}^f \mu_{t+1} - bc_t^f\right)^{-1} \\
\text{Bond. FOC (n2f)} &: \quad \lambda_t^f = \epsilon_{RP}\beta E_t \lambda_{t+1}^f R_t^f / \left(\pi_{t+1}^f \mu_{t+1}\right) \\
\text{Invest. FOC (n3f)} &: \quad 1 = p_{k',t}^f \Upsilon_t^\eta \left[1 - S_t^f - S_t'^f \mu_t \mu_{i,t} i_t^f / i_{t-1}^f\right] \\
&\quad +\iota_i \beta E_t \lambda_{t+1}^f/\lambda_t^f p_{k',t+1}^f \Upsilon_{t+1}^\eta S_{t+1}'^f \left(i_{t+1}^f / i_t^f\right)^2 \mu_{i,t+1}\mu_{t+1} \\
\text{Capital FOC (n4f)} &: \quad \lambda_t^f = \beta E_t \lambda_{t+1}^f R_{t+1}^{k,f}/(\pi_{t+1}^f \mu_{t+1}) \\
\text{Capital return (n5f)} &: \quad R_t^{k,f} = \pi_t^f/(\mu_{i,t} p_{k',t-1}^f)\left(u_t^f a'\left(u_t^f\right) - a(u_t^f) + (1-\delta)p_{k',t}^f\right) \\
\text{LOM capital (n6f)} &: \quad k_t^f = (1-\delta)/(\mu_t \mu_{i,t}) k_{t-1}^f + \Upsilon_t^\eta \left(1 - S_t^f\right) i_t^f \\
\text{Cost. minim. (n7f)} &: \quad 0 = a'\left(u_t^f\right) u_t^f k_{t-1}^f/(\mu_{i,t}\mu_t) - \alpha/(1-\alpha) w_t^f \left[\nu R_t^f + 1 - \nu\right] l_t^f \\
\text{Marginal cost (n8f)} &: \quad mc_t^f = \tau^{1/\kappa}\left(\mu_{i,t}\mu_t\right)^\alpha w_t^f \left[\nu R_t^f + 1 - \nu\right]\left(u_t^f k_{t-1}^f / l_t^f\right)^{-\alpha} / (\epsilon_t (1-\alpha)) \\
\text{Production (n9f)} &: \quad y_t^f = \epsilon_t \left(u_t^f k_{t-1}^f/(\mu_t \mu_{i,t})\right)^\alpha l_t^{1-\alpha,f} - \phi \\
\text{Resources (n10f)} &: \quad y_t^f = g_t + c_t^f + i_t^f + a\left(u_t^f\right) k_{t-1}^f/(\mu_{i,t}\mu_t)
\end{aligned}
$$

$$
\begin{aligned}
\text{Real GDP (n11f)} &: \quad \mathcal{Y}_t^f = g_t + c_t^f + i_t^f \\
\text{Taylor rule (n12f)} &: \quad \ln\left(R_t^{not,f}\right) = \rho_R \ln\left(R_{t-1}^{not,f}\right) \\
&\quad + (1-\rho_R)\left[\ln R + r_\pi \ln\left(\left(\pi_t^{4,f}\right)^{0.25}/\pi_t^T\right)\right] + \ln \epsilon_{r,t} \\
\text{Ann. Infl. (n13f)} &: \quad \pi_t^{4,f} = \pi_t^f \pi_{t-1}^f \pi_{t-2}^f \pi_{t-3}^f \\
\text{ZLB (n14f):} & \quad R_t^f = \max(1, R_t^{not,f}) \\
\text{Cap. util. cost. (n15f)} &: \quad a(u_t^f) = 0.5\sigma_b\sigma_a\left(u_t^f\right)^2 + \sigma_b(1-\sigma_a)u_t^f + \sigma_b((\sigma_a/2)-1) \\
\text{Cap. util. deriv. (n16f)} &: \quad a'(u_t^f) = \sigma_b\sigma_a u_t^f + \sigma_b(1-\sigma_a) \\
\text{Invest. adj. cost (n17f)} &: \quad S_t^f = 0.5\exp\left[\sqrt{S''}\left(\mu_t\mu_{i,t} i_t^f / i_{t-1}^f - \mu\cdot\mu_i\right)\right] \\
&\quad +0.5\exp\left[-\sqrt{S''}\left(\mu_t\mu_{i,t} i_t^f / i_{t-1}^f - \mu\cdot\mu_i\right)\right] - 1 \\
\text{Inv. adj. deriv. (n18f)} &: \quad S_t^{\prime f} = 0.5\sqrt{S''}\exp\left[\sqrt{S''}\left(\mu_t\mu_{i,t} i_t^f / i_{t-1}^f - \mu\cdot\mu_\Psi\right)\right] \\
&\quad -0.5\sqrt{S''}\exp\left[-\sqrt{S''}\left(\mu_t\mu_{i,t} i_t^f / i_{t-1}^f - \mu\cdot\mu_i\right)\right] \\
\text{Marginal cost (n19f)} &: \quad mc_t^f = 1/(1+\theta_p) \\
\text{Marg. rate of subst (n20f)} &: \quad mrs_t^f = \omega\zeta_h^{1/\kappa_w}/\lambda_t^f = w_t^f/(1+\theta_w) \\
\text{Wage inflation (n21f)} &: \quad \pi_{w,t}^f = w_t^f \mu_t \pi_t^f / w_{t-1}^f
\end{aligned}
$$

As Smets and Wouters (2007), we assume that the price markup shock and the wage markup shock do not affect the potential (flex. price, flex. wage) economy. We also assume that the risk premium shock does not affect the potential economy.

In total, we have 68 equations determining the following 68 unkown variables:

$\lambda_t, c_t, R_t, \pi_t, p_{k',t}, S_t, S_t', i_t, R_t^k, u_t, a'(u_t), a(u_t), k_t, w_t, w_t^*, l_t, mc_t, y_t, p_t^*, \mathcal{Y}_t,$ (20)

$R_t^{not}, s_t, \vartheta_t, f_t, a_t, \tilde{p}_t, \Delta_{t,2}, \Delta_{t,3}, \Delta_{t,1}, s_t^w, f_t^w, \tilde{w}_t, a_t^w, \Delta_{t,2}^w, \Delta_{t,3}^w, \Delta_{t,1}^w, \pi_{w,t}, \vartheta_t^w, mrs_t, \pi_t^4,$ (20)

$\lambda_t^f, c_t^f, R_t^f, \pi_t^f, p_{k',t}^f, S_t^f, S_t^{\prime f}, i_t^f, R_t^{k,f}, u_t^f, k_t^f, w_t^f, l_t^f, mc_t^f, y_t^f, \mathcal{Y}_t^f, R_t^{not,f}, a(u_t^f), a'(u_t^f), \pi_{w,t}^f, \pi_t^{4,f},$ (21)

$\tau_t, \epsilon_t, \Upsilon_t, \zeta_{h,t}, \epsilon_{rp,t}, \epsilon_{r,t}, g_t.$ (7)

Note that the shocks to the price markup and the wage markup are scaled by the inverse slopes of the price and wage Phillips curves

$$
\begin{aligned}
\kappa_p &= \frac{(1-\xi_p)(1-\beta\xi_p)}{\xi_p}\frac{1}{1-(1+\theta_p)\psi_p}, \\
\kappa_w &= \frac{(1-\xi_w)(1-\beta\xi_w)}{\xi_w}\frac{1}{1-(1+\theta_w)\psi_w}.
\end{aligned}
$$

This way, after log-linearization, both shocks enter additively separable with a unit coefficient in the respective Phillips curves. The same scaling is done in Smets and Wouters (2007) and in many other empirical papers. Note that both shocks equals unity in steady state so that the scaling does not affect the steady state. The scaling gets rid of the high negative correlation between the estimated standard deviations of the markup shocks and the estimated slope parameters. The process for the eight shocks are provided below.

In addition, we scale the investment specific technology shock by the factor

$$\eta = \frac{1}{\frac{i}{k}} = \frac{1}{1 - \frac{1-\delta}{\mu\mu_i}}$$

such that the investment specific technology shock enters the law of motion for capital with a unit coefficient. The same scaling was used in Adolfson, Laséen, Lindé and Villani (2007).

B.2. Exogenous Processes

We assume the following seven stochastic shocks drive the dynamics in the economy:

$$\begin{aligned}
\text{Price Markup (nS1)} &: \quad \ln \tau_t = (1-\rho_\tau)\ln \tau + \rho_\tau \ln \tau_{t-1} + \sigma_\tau \varepsilon_{\tau,t}/100 - \rho_\tau^{MA}\sigma_\tau \varepsilon_{\tau,t-1}/100 \\
\text{Stat. Neutr. Tech. (nS2)} &: \quad \ln \epsilon_t = (1-\rho_\epsilon)\ln \epsilon + \rho_\epsilon \ln \epsilon_{t-1} + \sigma_\epsilon \varepsilon_{\epsilon,t}/100 \\
\text{Stat. Invest. Tech. (nS3)} &: \quad \ln \Upsilon_t = (1-\rho_\Upsilon)\ln \Upsilon + \rho_\Upsilon \ln \Upsilon_{t-1} + \sigma_\Upsilon \varepsilon_{\Upsilon,t}/100 \\
\text{Lab. Pref. (nS4)} &: \quad \ln \zeta_{h,t} = (1-\rho_{\zeta_h})\ln \zeta_h + \rho_{\zeta_h} \ln \zeta_{h,t-1} + \sigma_{\zeta_h} \varepsilon_{\zeta_h,t}/100 - \rho_{\zeta_h}^{MA}\sigma_{\zeta_h} \varepsilon_{\zeta_h,t-1}/100 \\
\text{Risk Premium (nS5)} &: \quad \ln \epsilon_{rp,t} = (1-\rho_{\epsilon_{rp}})\ln \epsilon_{rp} + \rho_{\epsilon_{rp}} \ln \epsilon_{rp,t-1} + \sigma_{\epsilon_{rp}} \varepsilon_{\epsilon_{rp},t}/100 \\
\text{Monetary Policy (nS6)} &: \quad \ln \epsilon_{r,t} = (1-\rho_{\epsilon_r})\ln \epsilon_r + \rho_{\epsilon_r} \ln \epsilon_{r,t-1} + \sigma_{\epsilon_r} \varepsilon_{\epsilon_r,t}/100 \\
\text{Gov. Cons. (nS7)} &: \quad \ln g_t = (1-\rho_g)\ln g + \rho_g \ln g_{t-1} + \sigma_g \varepsilon_{g,t}/100
\end{aligned}$$

The equations above (nonlinear form) and below (log linearized form) also contain the shocks $\mu_{i,t}, \mu_{z,t}, \zeta_{c,t}, \pi_t^T$ and $\mu_t = \mu_{z,t}\mu_{i,t}^{\alpha/(1-\alpha)}$ but the variance of these four shocks are set to nil.

B.3. Measurement Equations

The have the following measurement equations of the estimated model:

$$\begin{aligned}
\pi_t^{obs} &= 400 \ln \pi_t \\
\pi_{w,t}^{obs} &= 400 \ln \pi_{w,t} \\
R_t^{obs} &= 400 \left(R_t - 1\right) \\
\Delta GDP_t^{obs} &= 400 \left(\ln \mu_t + \ln \mathcal{Y}_t - \ln \mathcal{Y}_{t-1}\right) \\
\Delta c_t^{obs} &= 400 \left(\ln \mu_t + \ln c_t - \ln c_{t-1}\right) \\
\Delta i_t^{obs} &= 400 \left(\ln \mu_t + \ln \mu_{i,t} + \ln i_t - \ln i_{t-1}\right) \\
l_t^{obs} &= 100 \left(\ln l_t - \ln l\right)
\end{aligned}$$

B.4. Log-linearized Equilibrium Equations

The log-linearized equations of the sticky price - sticky wage economy read as follows:

$$\begin{aligned}
\text{Cons. FOC (l1)} \quad &: \quad \hat{\lambda}_t = \frac{\mu}{\mu - \iota_c b\beta} \left[\hat{\zeta}_{c,t} - \frac{\iota_c \beta b}{\mu} E_t \hat{\zeta}_{c,t+1}\right] \\
& \quad - \frac{b\mu}{\left(\mu - b\beta\iota_c\right)\left(\mu - b\right)} \left[\hat{\mu}_t - \iota_c \beta E_t \hat{\mu}_{t+1} + \frac{\mu^2 + \iota_c b^2 \beta}{b\mu} \hat{c}_t - \iota_c \beta E_t \hat{c}_{t+1} - \hat{c}_{t-1}\right] \\
\text{Bond. FOC (l2)} \quad &: \quad \hat{\lambda}_t = \hat{\epsilon}_{RP,t} + E_t \hat{\lambda}_{t+1} + \hat{R}_t - E_t \hat{\pi}_{t+1} - E_t \hat{\mu}_{t+1} \\
\text{Invest. FOC (l3)} \quad &: \quad 0 = \hat{p}_{k',t} + \eta \hat{\Upsilon}_t - S''\left(\mu\mu_i\right)^2 \left[\begin{array}{c} \left(\widehat{i}_t - \widehat{i}_{t-1}\right) - \iota_i \beta \left(\widehat{i}_{t+1} - \widehat{i}_t\right) \\ + \left(\hat{\mu}_t + \hat{\mu}_{i,t}\right) - \iota_i \beta \left(\hat{\mu}_{t+1} + \hat{\mu}_{i,t+1}\right) \end{array}\right] \\
\text{Capital FOC (l4)} \quad &: \quad \hat{\lambda}_t = E_t \hat{\lambda}_{t+1} + \hat{R}_{t+1}^k - E_t \hat{\pi}_{t+1} - E_t \hat{\mu}_{t+1} \\
\text{Capital return (l5)} \quad &: \quad \hat{R}_t^k = \hat{\pi}_t - \hat{\mu}_{i,t} - \hat{p}_{k',t-1} + \frac{\beta}{\mu\mu_i} \left(\sigma_b \sigma_a \hat{u}_t + (1-\delta) \hat{p}_{k',t}\right)
\end{aligned}$$

$$\text{LOM capital (l6)} \quad : \quad \hat{k}_t = \frac{1-\delta}{\mu\mu_i}\left(\hat{k}_{t-1} - \hat{\mu}_t - \hat{\mu}_{i,t}\right) + 1/\eta\widehat{i}_t + \hat{\Upsilon}_t, \eta = \frac{1}{\frac{i}{k}} = \frac{1}{1-\frac{1-\delta}{\mu\mu_i}}$$

$$\text{Cost. minim. (l7)} \quad : \quad \left(1+\sigma_a\right)\hat{u}_t = \frac{\nu R}{\nu R + 1 - \nu}\hat{R}_t + \hat{w}_t + \hat{l}_t - \hat{k}_{t-1} + \hat{\mu}_{i,t} + \hat{\mu}_t$$

$$\text{Marginal cost (l8)} \quad : \quad \widehat{mc}_t = \frac{1}{\kappa}\hat{\tau}_t + \hat{w}_t + \frac{\nu R}{\nu R + 1 - \nu}\hat{R}_t - \alpha\left(\hat{u}_t + \hat{k}_{t-1} - \hat{\mu}_{i,t} - \hat{\mu}_t - \hat{l}_t\right) - \hat{\epsilon}_t$$

$$\text{Production (l9)} \quad : \quad \hat{y}_t = \left(1+\theta_p\right)\left(1-profy\right)\left[\hat{\epsilon}_t + \alpha\left(\hat{u}_t + \hat{k}_{t-1} - \hat{\mu}_t - \hat{\mu}_{i,t}\right) + \left(1-\alpha\right)\hat{l}_t\right]$$

$$\text{Resources (l10)} \quad : \quad y\hat{y}_t = g\hat{g}_t + c\hat{c}_t + i\widehat{i}_t + k/(\mu_i\mu)\sigma_b\hat{u}_t$$

$$\text{Real GDP (l11)} \quad : \quad \mathcal{Y}\widehat{\mathcal{Y}}_t = g\hat{g}_t + c\hat{c}_t + i\widehat{i}_t$$

$$\text{Taylor rule (l12)} \quad : \quad \hat{R}_t^{not} = \rho_R\hat{R}_{t-1}^{not} + \left(1-\rho_R\right)\left[\begin{array}{c} r_\pi\left(\hat{\pi}_t^4/4 - \hat{\pi}_t^T\right) + r_y\left(\widehat{\mathcal{Y}}_t - \widehat{\mathcal{Y}}_t^f\right) \\ +r_{\Delta y}\left(\left(\widehat{\mathcal{Y}}_t - \widehat{\mathcal{Y}}_t^f\right) - \left(\widehat{\mathcal{Y}}_{t-4} - \widehat{\mathcal{Y}}_{t-4}^f\right)\right) \end{array}\right] + \hat{\epsilon}_{r,t}$$

$$\text{Ann. Infl. (l13)} \quad : \quad \hat{\pi}_t^4 = \hat{\pi}_t + \hat{\pi}_{t-1} + \hat{\pi}_{t-2} + \hat{\pi}_{t-3}$$

$$\text{ZLB (l14)} \quad : \quad \hat{R}_t = \max(-\ln R, \hat{R}_t^{not})$$

$$\text{Price Phillips Curve (l15)} \quad : \quad \hat{\pi}_t = \beta E_t\hat{\pi}_{t+1} + \frac{\left(1-\xi_p\right)\left(1-\beta\xi_p\right)}{\xi_p}\frac{1}{1-\left(1+\theta_p\right)\psi_p}\widehat{mc}_t$$

$$\text{Wage Phillips Curve (l16)} \quad : \quad \hat{\pi}_t^w = \beta E_t\hat{\pi}_{t+1}^w + \frac{\left(1-\xi_w\right)\left(1-\beta\xi_w\right)}{\xi_w}\frac{1}{1-\left(1+\theta_w\right)\psi_w}\left(\widehat{mrs}_t - \hat{w}_t\right)$$

$$\text{Wage inflation (l17)} \quad : \quad \hat{\pi}_t^w = \hat{\pi}_t + \hat{w}_t - \hat{w}_{t-1} + \hat{\mu}_t$$

$$\text{Marg. Rate of Subst. (l18)} \quad : \quad \widehat{mrs}_t = \frac{1}{\kappa_w}\hat{\zeta}_{h,t} - \hat{\lambda}_t$$

The equations of the flexible price - flexible wage economy are:

$$\begin{aligned}
\text{Cons. FOC (l1f)} &: \quad \hat{\lambda}_t^f = \frac{\mu}{\mu - \iota_c b\beta}\left[\hat{\zeta}_{c,t} - \frac{\iota_c \beta b}{\mu}E_t\hat{\zeta}_{c,t+1}\right] - \frac{b\mu}{(\mu - b\beta\iota_c)(\mu - b)}\left[\hat{\mu}_t - \iota_c\beta E_t\hat{\mu}_{t+1}\right] \\
&\quad - \frac{b\mu}{(\mu - \iota_c b\beta)(\mu - b)}\left[\frac{\mu^2 + \iota_c b^2\beta}{b\mu}\hat{c}_t^f - \iota_c\beta E_t\hat{c}_{t+1}^f - \hat{c}_{t-1}^f\right] \\
\text{Bond. FOC (l2f)} &: \quad \hat{\lambda}_t^f = 0 * \hat{\epsilon}_{RP,t} + E_t\hat{\lambda}_{t+1}^f + \hat{R}_t^f - E_t\hat{\pi}_{t+1}^f - E_t\hat{\mu}_{t+1} \\
\text{Invest. FOC (l3f)} &: \quad 0 = \hat{p}_{k',t}^f + \eta\hat{\Upsilon}_t - S''(\mu\mu_i)^2\left[\begin{array}{c} \left(\widehat{i}_t^f - \widehat{i}_{t-1}^f\right) - \iota_i\beta\left(\widehat{i}_{t+1}^f - \widehat{i}_t^f\right) \\ + \left(\hat{\mu}_t + \hat{\mu}_{i,t}\right) - \iota_i\beta\left(\hat{\mu}_{t+1} + \hat{\mu}_{i,t+1}\right) \end{array}\right] \\
\text{Capital FOC (l4f)} &: \quad \hat{\lambda}_t^f = E_t\hat{\lambda}_{t+1}^f + \hat{R}_{t+1}^{k,f} - E_t\hat{\pi}_{t+1}^f - E_t\hat{\mu}_{t+1} \\
\text{Capital return (l5f)} &: \quad \hat{R}_t^{k,f} = \hat{\pi}_t^f - \hat{\mu}_{i,t} - \hat{p}_{k',t-1}^f + \frac{\beta}{\mu\mu_i}\left(\sigma_b\sigma_a\hat{u}_t^f + (1-\delta)\hat{p}_{k',t}^f\right) \\
\text{LOM capital (l6f)} &: \quad \hat{k}_t^f = \frac{1-\delta}{\mu\mu_i}\left(\hat{k}_{t-1}^f - \hat{\mu}_t - \hat{\mu}_{i,t}\right) + 1/\eta\widehat{i}_t^f + \hat{\Upsilon}_t \\
\text{Cost. minim. (l7f)} &: \quad (1+\sigma_a)\,\hat{u}_t^f = \frac{\nu R}{\nu R + 1 - \nu}\hat{R}_t^f + \hat{w}_t^f + \hat{l}_t^f - \hat{k}_{t-1}^f + \hat{\mu}_{i,t} + \hat{\mu}_t \\
\text{Marginal cost (l8f)} &: \quad 0 = 0 * \frac{1}{\kappa}\hat{\tau}_t + \hat{w}_t^f + \frac{\nu R}{\nu R + 1 - \nu}\hat{R}_t^f - \alpha\left(\hat{u}_t^f + \hat{k}_{t-1}^f - \hat{\mu}_{i,t} - \hat{\mu}_t - \hat{l}_t^f\right) - \hat{\epsilon}_t \\
\text{Production (l9f)} &: \quad \hat{y}_t^f = (1+\theta_p)(1 - profy)\left[\hat{\epsilon}_t + \alpha\left(\hat{u}_t^f + \hat{k}_{t-1}^f - \hat{\mu}_t - \hat{\mu}_{i,t}\right) + (1-\alpha)\,\hat{l}_t^f\right] \\
\text{Resources (l10f)} &: \quad y\hat{y}_t^f = g\hat{g}_t + c\hat{c}_t^f + i\widehat{i}_t^f + k/(\mu_i\mu)\sigma_b\hat{u}_t^f \\
\text{Real GDP (l11f)} &: \quad \mathcal{Y}\widehat{\mathcal{Y}}_t^f = g\hat{g}_t^f + c\hat{c}_t^f + i\widehat{i}_t^f \\
\text{Taylor rule (l12f)} &: \quad \hat{R}_t^{not,f} = \rho_R\hat{R}_{t-1}^{not,f} + (1-\rho_R)\,r_\pi\left(\hat{\pi}_t^{4,f} - \hat{\pi}_t^T\right) + \hat{\epsilon}_{r,t} \\
\text{Ann. Infl. (l13f)} &: \quad \hat{\pi}_t^{4,f} = 0.25\left(\hat{\pi}_t^f + \hat{\pi}_{t-1}^f + \hat{\pi}_{t-2}^f + \hat{\pi}_{t-3}^f\right) \\
\text{ZLB (l14f)} &: \quad \hat{R}_t^f = \max(-\ln R, \hat{R}_t^{not,f}) \\
\text{Wage inflation (l15f)} &: \quad \hat{\pi}_t^{w,f} = \hat{\pi}_t^f + \hat{w}_t^f - \hat{w}_{t-1}^f + \hat{\mu}_t \\
\text{Marg. Rate of Subst. (l16f)} &: \quad w_t^f = 0 * \frac{1}{\kappa_w}\hat{\zeta}_{h,t} - \hat{\lambda}_t^f
\end{aligned}$$

where $profy$ denotes the steady state profits to output ratio.

In total, we have 34 equations determining 34 endogenous variables

$$\hat{\lambda}_t, \hat{c}_t, \hat{R}_t, \hat{\pi}_t, \hat{p}_{k',t}, \widehat{i}_t, \hat{R}_t^k, \hat{u}_t, \hat{k}_t, \hat{w}_t, \hat{l}_t, \widehat{mc}_t, \hat{y}_t, \widehat{\mathcal{Y}}_t, \hat{R}_t^{not}, \widehat{mrs}_t, \hat{\pi}_t^w, \hat{\pi}_t^4, \quad (18)$$

$$\hat{\lambda}_t^f, \hat{c}_t^f, \hat{R}_t^f, \hat{\pi}_t^f, \hat{p}_{k',t}^f, \widehat{i}_t^f, \hat{R}_t^{k,f}, \hat{u}_t^f, \hat{k}_t^f, \hat{w}_t^f, \hat{l}_t^f, \hat{y}_t^f, \widehat{\mathcal{Y}}_t^f, \hat{R}_t^{not,f}, \hat{\pi}_t^{w,f}, \hat{\pi}_t^{4,f}. \quad (16)$$

In addition, we have the 7 exogenous shocks that are described above in eqs. (nS1) – (nS7).

B.5. Steady State

The steady state is computed as follows.

$$
\begin{aligned}
& \text{Impose } u = 1, \text{ solve for } \sigma_b \text{ later} \\
a(1) &= 0 \\
\mu &= \mu / \left(\mu_i\right)^{\alpha/(1-\alpha)} \\
S &= 0 \\
S' &= 0 \\
& \text{Impose } \pi, \text{ ``drop'' Taylor rule, i.e. } R = R \\
R &= \pi\mu/\beta \\
p_{k'} &= 1 \\
w^* &= 1 \\
p^* &= 1 \\
R^k &= \pi\mu/\beta \\
mc &= 1/(1+\theta_p) \\
\pi_w &= \pi\mu \\
\sigma_b &= R^k \mu_i p_{k'}/\pi - (1-\delta) p_{k'} \\
\bar{r}^k &= \sigma_b \\
a'(1) &= \sigma_b \\
a(u) &= 0.5\sigma_b\sigma_a + \sigma_b\left(1-\sigma_a\right) + \sigma_b\left(\left(\sigma_a/2\right) - 1\right)
\end{aligned}
$$

$$
\begin{aligned}
kl &= \left[\left(\alpha\left(\mu_i\mu\right)^{1-\alpha} mc/\sigma_b\right)\right]^{1/(1-\alpha)} \\
w &= \frac{(1-\alpha)\, mc}{\left(\mu_i\mu\right)^{\alpha}\left(\nu R+1-\nu\right)}\left(kl\right)^{\alpha} \\
& \quad \text{Assume some } l \text{ and solve for } \omega \text{ later} \\
k &= k * l \\
& \quad \text{Steady state profits are} \\
Prof &= Py - MC(y+\phi) \text{ solve for } \phi \\
\phi &= \left(\frac{1-mc}{mc}\right) y - \frac{Prof/P}{mc} \\
& \quad \text{substitute in production function and rewrite} \\
y &= \frac{mc}{1-profy}\left(kl/(\mu\mu_i)\right)^{\alpha} l \text{ for given } profy = \frac{Prof}{Py} \\
\phi &= \left(kl/(\mu\mu_{\Psi})\right)^{\alpha} l - y \\
i &= \left[1-(1-\delta)/(\mu\mu_i)\right] k \\
& \quad \text{Assume } g \text{ equals share } \eta_g \text{ of } y \\
c &= (1-\eta_g)y - i \\
g &= \eta_g y \\
\lambda &= (c - bc/\mu)^{-1} - \iota_c \beta b \left(c\mu - bc\right)^{-1} \\
\mathcal{Y} &= g + c + i
\end{aligned}
$$

$$
\begin{aligned}
\tilde{p} &= 1 \\
\Delta_1 &= 1 \\
\Delta_2 &= 1 \\
\Delta_3 &= 1 \\
\vartheta &= 1 \\
s &= \frac{1}{1-\beta\xi_p}\frac{\left(1+\psi_p\right)\left(1+\theta_p\right)}{1+\psi_p+\theta_p\psi_p}\lambda y mc \\
f &= \frac{1}{1-\beta\xi_p}\lambda y \\
a &= \frac{1}{1-\beta\xi_p}\frac{\psi_p\theta_p}{1+\psi_p+\theta_p\psi_p} y\lambda \\
\tilde{w} &= 1
\end{aligned}
$$

$$
\begin{aligned}
\Delta_1^w &= 1 \\
\Delta_2^w &= 1 \\
\Delta_3^w &= 1 \\
\vartheta^w &= 1 \\
f^w &= \frac{1}{1-\beta\xi_w} l\lambda w \\
a^w &= \frac{1}{1-\beta\xi_w}\frac{\theta_w\psi_w}{\psi_w+\theta_w\psi_w+1} n\lambda w \\
s^w &= \frac{1}{1-\beta\xi_w}\frac{(1+\psi_w)(1+\theta_w)}{1+\psi_w+\theta_w\psi_w} n\lambda mrs \\
s^w &= f^w - a^w
\end{aligned}
$$

Using the last equation it follows that:

$$
mrs = \frac{1}{1+\theta_w} w
$$

Finally,

$$
\omega = mrs\lambda
$$

B.6. Data

The model is estimated using seven key macro-economic time series: real GDP, consumption, investment, hours worked, price and wage inflation and a short-term interest rate. The Bayesian estimation methodology is extensively discussed by Smets and Wouters (2003). GDP, consumption and investment were taken from the U.S. Department of Commerce – Bureau of Economic Analysis data-bank. Real gross domestic product is expressed in billions of chained 2009 dollars.Nominal personal consumption expenditures and fixed private domestic investment are deflated with their respective deflators. Inflation is the first difference of the log of the personal consumption expenditures price deflator. Hours (non-farm business sector for all persons) are from the Bureau of Labor Statistics. Wage inflation is the first difference of the log index of hourly compensation (nonfarm business sector, all persons). The aggregate real variables are expressed per capita by dividing with the population size aged 16 or older. All series are seasonally adjusted. The interest rate is the Federal Funds Rate. Consumption, investment, GDP, wages, and hours are expressed in 100 times log. As noted in the measurement equations, the quantities that are observed in growth rates (output, consumption and investment) , interest rate and inflation rate are all expressed on an annualized (400 times first log difference) basis in the estimation.

The solid blue line in Figure B.1 shows the data for the full sample, which spans 1965Q1–2008Q2. The figure also includes a red-dotted line, which is the one-step ahead predicted estimates from the Kalman filter for the Posterior median. As we can tell from the figure, the model fits the data quite well.

Figure B.1: One-Step Ahead Predicted (red) vs Actual (blue) in Estimated Model