Quantitative Sovereign Default Models and the European Debt Crisis*

Luigi Bocola† **Gideon Bornstein**‡ **Alessandro Dovis**§

June 23, 2018

Abstract

A large literature has developed quantitative versions of the Eaton and Gersovitz (1981) model to analyze default episodes on external debt. In this paper, we study whether the same framework can be applied to the analysis of debt crisis in which domestic public debt plays a prominent role. We consider a model where a government can issue debt to both domestic and foreign investors, and we derive conditions under which their sum is the relevant state variable for default incentives. We then apply our framework to the European debt crisis. We show that matching the cyclicality of public debt ---rather than that of external debt--- allows the model to better capture the empirical distribution of interest rate spreads and gives rise to more realistic crises dynamics.

*Preliminary, comments welcomed. Prepared for 2018 International Seminar on Macroeconomics (ISOM) in Dublin, Ireland.

†Stanford University, Minneapolis Fed, and NBER

‡Northwestern University

§University of Pennsylvania and NBER

1 Introduction

The model of Eaton and Gersovitz (1981) is the benchmark model to study sovereign debt crises. In its simplest form, this model considers the decision problem of a government that faces income risk and issues non-contingent debt with the option to default. Its key prediction is that the default option is more valuable when the income of the government is low and/or when its debt obligations are high. Consequently, income flows and debt repayments are key variables to explain default episodes in this class of models.

Following the work of Arellano (2008) and Aguiar and Gopinath (2006), several researchers have applied this framework to the analysis of debt crises in emerging markets. The canonical approach in the literature consolidates the budget constraint of the government and that of the private sector, implicitly assuming that the government has enough instruments to control the saving behavior of private domestic agents. This literature has shown that quantitative versions of the Eaton and Gersovitz (1981) model can account for the joint behavior of net external debt, income, and interest rate spreads in emerging markets, most notably reproducing the coincidence of debt crises with reversals of current account deficits.

The literature is, however, typically silent on the role of public debt – in particular domestic public debt – because this variable is irrelevant for default incentives under the assumptions emphasized earlier. This is a limitation for the analysis of debt crises in which domestic public debt plays a prominent role.[1] This paper takes a step toward filling this gap in the literature by applying this class of models to the analysis of the recent debt crisis in Southern Europe. We first document that in the case of Portugal, Spain, and Italy, total public debt (domestic and foreign) outperforms standard indicators of external debt in accounting for the dynamics of interest rate spreads. We next show that we can reconcile the canonical default model with such evidence. Specifically, we derive theoretical conditions under which total public debt is a relevant state variable in the decision problem of the government. We then show how to calibrate this model to account for the behavior of income, public debt and interest rate spreads for Southern European economies.

We start by empirically studying the relation between spreads, income and different indicators of debt for Portugal, Spain, and Italy. We show that different indicators of indebtedness have a different dynamics in the data. In particular, total public debt increases substantially after 2008 and mirrors the dynamics of interest rate spreads. This is not true for other measures of indebtedness like external public debt and net external debt. Sim-

[1] In our paper, the distinction between domestic and external debt regards the residence of bondholders, as opposed to the jurisdiction under which the bonds were issued. Most of the oustanding debt securities of European countries was issued under domestic law.

ple statistical regressions confirm that total public debt is more tightly associated to the behavior of interest rate spreads on government debt securities than more traditional indicators of debt used in the literature for the countries we study.

Motivated by this evidence, we study theoretically the conditions under which total public debt is relevant for the default incentives of the government. We consider a framework that allows for a clear distinction between external and public debt. A benevolent government finances public consumption goods using tax revenues and issues government debt to maximize the utility of the representative agent in the economy. Government debt is non-contingent, but the government cannot commit to repay. Public debt can be held by domestic households and by foreign investors. Importantly, we assume that domestic households can also trade with foreign lenders, and that the government has no margin to interfere with private choices.

Our main result is that only total government debt (and not its external component) is a relevant state variable in the decision problem of the government if the following two conditions hold. First, the government cannot discriminate between domestic and foreign bondholders in default. Second, private financial markets are *sophisticated* in that domestic and foreign private investors can trade securities contingent on the state of the economy and on the government policies. Moreover, we show that the government problem in such environment is isomorphic to the one considered in Arellano (2008), with the exception that the resource constraint of the small open economy is replaced with the budget constraint of the government. While the assumption of sophisticated private financial markets is clearly extreme, it provides a useful theoretical benchmark to contrast with the standard approach in the literature.

Equipped with these results, we explore the effects of fitting the Arellano (2008) model to match the behavior of public debt. As mentioned earlier, total public debt has a different cyclicality than external debt: the former increases in recessions, while the latter declines. This represents a challenge for this class of models. In the canonical calibration, the model generates high levels of debt by having an "impatient" government with a discount rate that that is substantially above the market discount rate. This same feature, however, also implies that debt issuances decline in recessions: the government borrows most of the time up to the implicit limit induced by default risk, and because default incentives are higher in low income states, the government ends up borrowing less when income contracts and more when it expands.

To deal with this issue, we follow Bocola and Dovis (2016) and introduce a consumption commitment in the objective function of the government, which proxies for expenditures that are hard to change in the short run such as pensions and wages of public employees. This parameter allows one to control the precautionary motives of the government, and it gives additional flexibility to match the dynamics of debt in the data.

Calibrating the model to Spain, we show that this feature is critical to match standard empirical targets considered in the literature along with the level and the cyclicality of public debt.

We finally compare our parametrization to a more traditional one that targets the cyclicality of the trade balance. The main result from this analysis is that matching countercyclical debt issuances allows the model to reproduce more realistic behavior for interest rate spreads. In our parametrization, the non-homotheticity of the utility function generates sizable precautionary motives, with the government increasing debt only conditional on sufficiently bad income shocks. Because of this feature, the government is essentially not at risk of default in good times, and interest rate spreads gravitate most of the times around zero.[2] However, occasional jumps may occur after a sequence of bad income shocks and increasing debt dynamics. This behavior generates a distribution of interest rate spreads that is remarkably close to the one observed for countries in Southern Europe. Instead, we show that in the more traditional calibration, frontloading incentives generate a distribution of interest rate spreads that has a substantially lower mass around zero than the one observed in the data.

Our paper contributes to the literature on sovereign debt. Following the work of Eaton and Gersovitz (1981), the papers of Arellano (2008) and Aguiar and Gopinath (2006) were the first to analyze the quantitative performance of this class of models. Following this work, several papers have enriched the standard framework with more realistic features. Hatchondo and Martinez (2009), Chatterjee and Eyigungor (2012), and Hatchondo et al. (2015) introduce long term debt in the canonical framework. Arellano and Ramanarayanan (2012) consider an explicit maturity choice for debt issuances. Mendoza and Yue (2012) propose a model where default costs endogenously arise because of the negative effects of default on financial markets and international trade. Bianchi et al. (2012) apply the canonical default model to explain the coexistence of short term foreign assets and long term foreign debt in the government's balance sheet. All these studies focus on external debt. Our contribution is, instead, that of understanding the implications of fitting this class of models to public debt. In doing so, we deliberately study the most basic model in the literature, Arellano (2008). While this reduces the realism of our analysis, it helps the reader in understanding how different calibration targets affect the behavior of the bechmark model.

There are recent papers that have studied the dynamics and interactions of domestic and foreign debt. Dovis et al. (2016) considers a theoretical model where the distribution of domestic and foreign debt is a key determinant of debt sustainability. D'Erasmo and Mendoza (2013) study a quantitative model in which the government has incentive

[2]The absence of spread between Southern European Eurozone members and Germany can also be accounted for by expectations of future bailouts. For instance, see Dovis and Kirpalani (2018).

to default on domestic debt because of distributional motives. We view our contribution as twofold. First, we find conditions under which only total public debt is relevant for the government problem and not its distribution between domestic and foreign investors. Second, and differently from D'Erasmo and Mendoza (2013), we emphasize how the different cyclicality of public and external debt impacts the calibration of the default model.

Finally, there are recent papers that have applied this class of models to the European debt crisis. See for example Salomao (2017), Bocola and Dovis (2016), and Arellano et al. (2018). Paluszynski (2016) emphasizes that this class of models has a hard time matching the debt crisis in these countries. Specifically, he shows that the model fits poorly because the Portuguese government had more external debt in 2008 than in 2011. Our analysis confirms this finding, but it also demonstrates that fitting this model to public debt rather than external debt substantially improves the fit of the model.

The paper is structured as follows. In Section 2 we briefly present the dynamics of interest rate spreads and various indicators of debt for Portugal, Italy, and Spain. Section 3 lays out our model and derives the main theoretical results. Section 4 calibrates the model to Spain, and it presents the main quantitative results. Section 5 concludes.

2 Empirics

Models of sovereign debt imply that equilibrium interest rate spreads are a function of the sovereign's income and its debt obligations. In this section, we study the relation between interest rate spreads, income, and three different debt indicators across Spain, Italy, and Portugal. The three debt indicators we consider are net external debt, public external debt, and total public debt. We find that total public debt outperforms both public external debt and net external debt in explaining variations in the behavior of spreads over time.

The standard approach when comparing sovereign debt models to the data is to aggregate all agents in the economy into one entity, a benevolent sovereign. As a result, the relevant debt obligations of the sovereign in the model correspond to the sum of debt obligations to foreigners of domestic households, firms, and the government. This is the first debt indicator we consider, net external debt in the economy.[3] In practice, however, researches use public external debt when comparing the model's prediction to the data. So the second debt indicator we consider is public external debt. The final debt indicator we consider is total public debt, which includes the obligations of the government to both domestic and foreign lenders.

Figure 1 presents the detrended output, the interest rate spread, and the three debt in-

[3] We define net external debt in the economy to be the negative of net international investment position.

dicators for Spain, Italy, and Portugal. The time period we consider is 2002Q1 to 2012Q2.[4] The final period of analysis was chosen so that we exclude the periods following the introduction of the Outright Monetary Transaction program by the European Central Bank.

As it can be seen in the figure, the three countries went through two recessions during this period. The first in 2008 and the second in 2011. While both recessions were of similar magnitudes in their effect on output, there is a substantial difference in the behavior of spreads between the two recessions. Following the 2008 recession, spreads went up from about 0% to 1-1.5% in the first quarter of 2009. Following the 2011 recession, spreads reached levels of about 5% in Spain and Italy, and about 12% in Portugal.

The last three panels of Figure 1 present the behavior of net external debt, public external debt, and total public debt. As it is apparent in the figure, the different debt indicators follow different dynamics. Following the 2008 recession, total public debt increases substantially in all three countries. The two indicators of external debt, instead, show little sign of change and, in the case of external public debt, they decline toward the end of the period.

To formally assess the relation between each debt indicator and interest rate spreads, we regress spreads on each debt indicator and on detrended output. We assume that spreads are approximated by the following equation

$$r_t = \alpha + \beta X_t + \varepsilon_t$$

where r_t is the spread at time t, and X_t is a set of covariates at time t. We consider specifications in which X_t includes only the level of debt, and ones in which it includes both the level of debt and detrended output. We use the different debt indicators, one at a time, as the level of debt in the regression. We run this regression separately for Spain, Italy, and Portugal. The results are reported in Table 1.

The first column presents the regression results when the only covariate is output. We can see that the R^2 varies from 0.3 in the case of Italy to 0.67 in Portugal. So variation in output goes a long way in explaining variations in the spread, as predicted by a standard default model.

The next three columns present the regression results when the only covariate is the level of debt in the economy. We find that the R^2 when using total public debt is between 2 to 7 times larger than when using the other two debt indicators. A similar result is obtained when considering the specification in which both output and the level of debt

[4] We obtain real GDP from OECD quarterly national accounts for the period 1960Q1-2012Q2. Detrended output is defined as the difference between log real GDP and a quadratic time trend. Interest rate spreads on bonds with a five years residual maturity are obtained from Markit. As for the three debt series, we obtain total public debt as a fraction of GDP from the OECD, public external debt from the Bruges data set, and the net international investment position from Eurostat.

Figure 1: Data

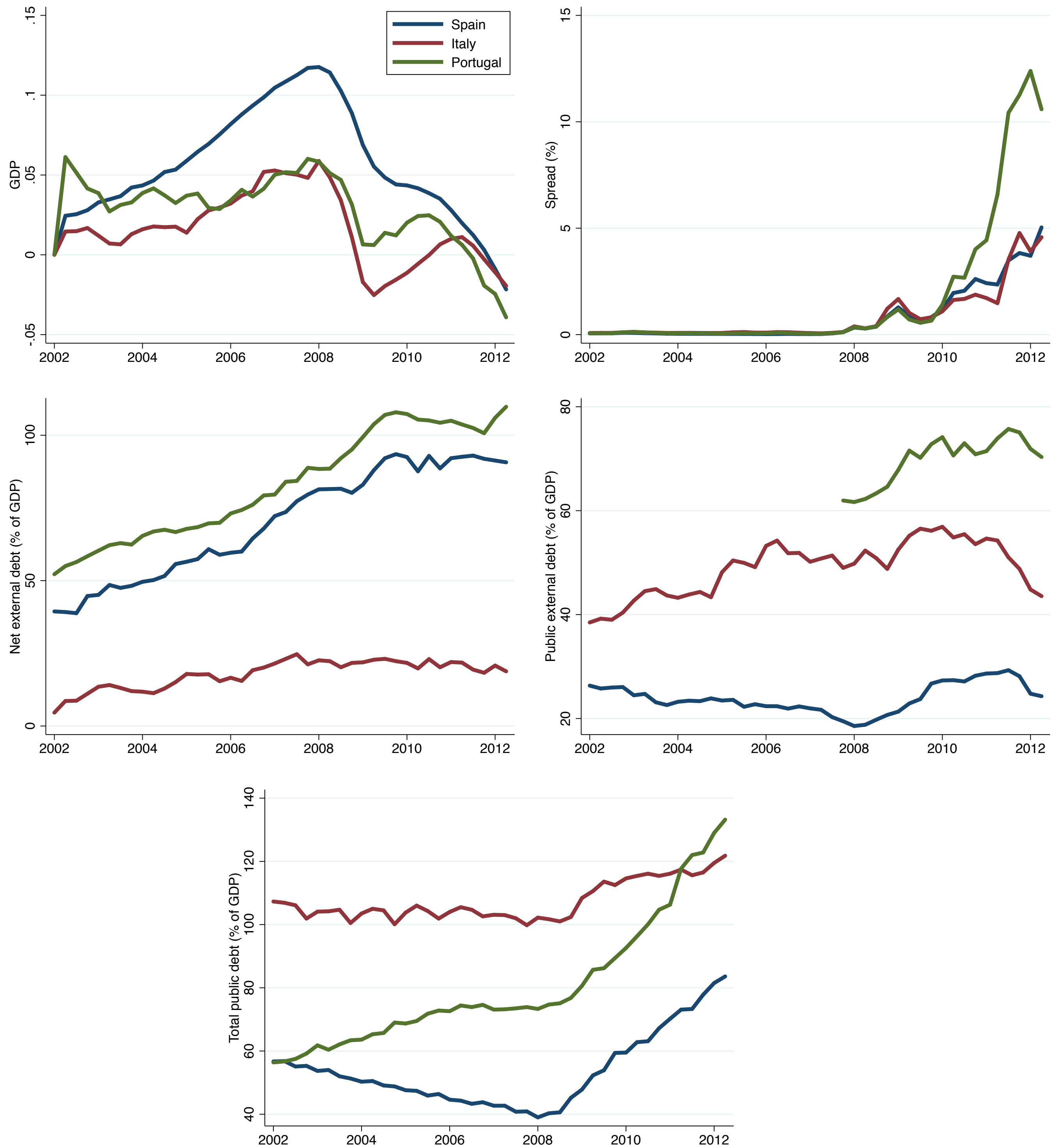


Table 1: Regression results

Dependent variable: *spread*

Country		Output only	Debt only			Output and debt		
			PED	NED	TPD	PED	NED	TPD
Spain	Debt		0.25*** (0.05)	0.05*** (0.01)	0.10*** (0.01)	0.06 (0.14)	0.05*** (0.01)	0.17*** (0.01)
	Output	-0.23*** (0.06)				-0.19 (0.13)	-0.23*** (0.03)	0.28*** (0.03)
	R^2	0.39	0.30	0.46	0.79	0.40	0.85	0.91
Italy	Debt		0.04 (0.03)	0.08*** (0.02)	0.17*** (0.03)	0.03 (0.03)	0.09*** (0.02)	0.18*** (0.03)
	Output	-0.31*** (0.08)				-0.31** (0.09)	-0.31*** (0.08)	0.03 (0.06)
	R^2	0.30	0.02	0.10	0.67	0.31	0.41	0.67
Portugal	Debt		0.54*** (0.13)	0.11*** (0.03)	0.15*** (0.02)	-0.03 (0.17)	0.02 (0.02)	0.13*** (0.02)
	Output	-1.21*** (0.19)				-1.33*** (0.27)	-1.12*** (0.21)	-0.25 (0.16)
	R^2	0.67	0.34	0.34	0.83	0.66	0.67	0.84

Notes: NED: Net External Debt; PED: Public External Debt; TPD: Total Public Debt. *** - significant at a 1% level, ** - significant at a 2% level.

is used, the three last columns of the table. This result indicates that out of the three debt indicators considered, total public debt is most tightly related to interest rate spreads.

3 Model

In this section, we show that if default is non-discriminatory and private financial markets are "sophisticated" then only total public debt is relevant for the government problem. We say that private financial markets are sophisticated if private agents (domestic and foreign) can trade securities contingent on the state of the economy and the government policies, in particular government default. This view contrast with the standard way to think about sovereign (or better external) debt in emerging market economies. In these models, private markets are extremely unsophisticated: there are no private markets. In this case, total external debt is the relevant variable for the government problem and the interest rate spreads.

3.1 Environment

We follow most of the literature and consider a model of a government that receives a stochastic flow of tax revenues and borrows to smooth across time and states (public) consumption. Alternatively, we could have considered a tax smoothing model in the Ramsey tradition where the government issues debt to smooth the costs of distortionary taxes. As shown in Aiyagari et al. (2002), under certain conditions these two problems admit the same representation.[5]

Time is indexed by $t = 1, 2, ...$ The state of the economy is $s_t \in S$. Let $s^t = (s_0, s_1, ..., s_t)$ and $\mu\left(s^t|s_0\right)$ be the probability of drawing a history s^t. The economy is populated by a benevolent government, domestic consumers, and foreign lenders. The domestic consumers have preferences over private consumption good, $c\left(s^t\right)$, and public consumption good, $g\left(s^t\right)$, given by

$$\sum_t \sum_{s^t} \beta^t \mu\left(s^t|s_0\right) U\left(c\left(s^t\right), g\left(s^t\right)\right) \tag{1}$$

where

$$U(c, g) = u(c) + \omega(g)$$

The consumer receives an income $y(s_t)$ in each period.

Foreign lenders have a large endowment in each period and evaluate repayments with a stochastic discount factor, SDF, $M(s_{t+1}, s_t)$.

The government is benevolent and maximizes the utility of the domestic consumer.[6] In each period it receives tax revenues $T(s_t) = \tau Y(s_t)$ and it can borrow from domestic and foreign agent by issuing uncontingent one-period debt. If the government defaults on its debt, it is excluded from international capital markets for a random period, and it has a probability of re-entering capital markets equal to ζ. While in default, the government suffers a loss in tax revenues equal to χ_t. This is motivated by evidence that sovereign defaults lead to severe financial and output disruptions (Hébert and Schreger, 2017; Bocola, 2016), and they should therefore imply a loss in fiscal revenues for the government.

3.2 Private equilibrium

We start by setting up the equilibrium by describing a private equilibrium for a given policy rule, $\pi = (\delta, B') = \mathbf{p}(S, s)$. The policy rule depends on the aggregate state that the government confronts when it chooses its policy: $S = (B_H, B_F, A(\cdot), d)$ where B_H is domestic holdings of government debt, B_F are foreign holdings of government debt, $A(\cdot)$

[5] See also Pouzo and Presno (2014).

[6] The results of this section holds even if we allow for the government to have a different discount factor than consumers. In particular, we could allow for the government discount factor to be lower than the one for the domestic consumers.

is the portfolios of Arrow securities held by domestic agents conditional on the policy chosen by the government, and d is an indicator variable that captures the ability of the government to issue new debt, $d = 1$ if new debt can be issued and $d = 0$ if no new debt can be issued.

We can now set up the recursive problem for the private agents. The endogenous aggregate state that private agents face is (S, π). The problem for an individual household with financial wealth a is

$$v(a, S, s, \pi) = \max_{c, a'(\cdot)} u(c) + \beta \sum_{s'} \mu(s'|s) \sum_{\pi'} \Pr(\pi' = \mathbf{p}(S', s') | S', s') \, v(\tilde{a}(S', s', \pi'), S', s', \pi') \tag{2}$$

subject to

$$c + \sum_{s'} \sum_{\pi'} Q(s', \pi', S, s, \pi) \, a(S', s', \pi') + q(S, s, \pi) \, b' \leqslant (1 - \tau) Y(s) + a$$

where

$$\tilde{a}(S', s', \pi') = a(S', s', \pi') + b' \delta(S', s' | \pi')$$

and

$$S' = \mathbf{S}'(S, \pi, s)$$

where $\mathbf{S}'$ is the equilibrium law of motion for the aggregate state.

No- arbitrage conditions from the foreign lenders require that

$$Q(s', \pi', S, s, \pi) = \sum_{s'} M(s', s) \Pr(\pi' = \mathbf{p}(\mathbf{S}'(S, \pi, s), s')) \tag{3}$$

$$q(S, s, \pi) = \sum_{s'} M(s', s) \, \delta(\mathbf{S}'(S, \pi, s), s') \tag{4}$$

We can then define a private equilibrium given a policy rule $\mathbf{p}(S, s)$ as the household value v and associated policy rules, prices for Arrow securities and government debt, and the law of motion for aggregate quantities such that i) the household problem solves (2), ii) the prices for Arrow securities and government debt are given by (3) and (4), and iii) the representativeness condition.

Implementability conditions From the first order conditions of the household's problem and the no-arbitrage conditions we obtain that for all $S' = \mathbf{S}(S, s, \pi)$

$$\Pr(\pi' = \mathbf{p}(S', s') | s') \, \beta \frac{u'(c(S', s', \pi'))}{u'(c(S, s, \pi))} = \Pr(\pi' = \mathbf{p}(S', s') | s') \, M(s', s)$$

For policies along the equilibrium path we have that $\Pr\left(\pi' = \mathbf{p}\left(S', s'\right) | s'\right) > 0$ and so it must be that

$$\beta \frac{u'\left(c\left(S', s', \pi'\right)\right)}{u'\left(c\left(S, s, \pi\right)\right)} = M\left(s', s\right) \tag{5}$$

For policies off the equilibrium path, instead, $\Pr\left(\pi' = \mathbf{p}\left(S', s'\right) | s'\right) = 0$ and so the risk sharing condition (5) does not necessarily hold. In what follows, we select the holdings of Arrow securities assuming that the government can tremble and choose all the possible policies with a small probability and take the limit of such probability to zero. With this selection, the risk sharing condition (5) must hold for any policies π', on and off path. This implies that the marginal utility of (private) consumption for the domestic household is constant across different policies. The domestic consumer and the foreign agent perfectly shares both the exogenous risk, s, and the endogenous policy (default) risk, π'. This implies a restriction to the portfolios of Arrow securities that can be chosen in equilibrium. Take two states with the same output but different policies. Suppose π_A has repayment and π_B has default. Budget feasibility requires that

$$A\left(S', \pi_B\right) = A\left(S', \pi_A\right) + B'_H \tag{6}$$

Thus, for each state s' and S', and all policies π_A and π_B the portfolio of Arrow securities must satisfy (6).

3.3 Government and equilibrium

We now examine the problem of the government:

$$V\left(S, s\right) = \max_{\pi = (\delta, B'), g} u\left(c\left(S, s, \pi\right)\right) + \omega\left(g\right) + \beta \sum_{s'} \mu\left(s' | s\right) V\left(\mathbf{S}'\left(S, s, \pi\right), s'\right) \tag{7}$$

subject to the government budget constraint if $d = \delta = 1$

$$g + B_H + B_F \leqslant \tau Y\left(s\right) + q\left(S, s, \pi\right) B'$$

and if $\delta = 0$ or $d = 0$

$$g = \tau Y\left(s\right) - \chi\left(s\right)$$

where $\chi\left(s\right)$ is a default cost, and the state next period is given by

$$S' = \left(\alpha\left(S, s, \pi\right) B', \left(1 - \alpha\left(S, s, \pi\right)\right) B', A'\left(S, s, \pi\right)\left(s', \pi'\right), d'\right)$$

where $\alpha\left(S, s, \pi\right)$ is the (possibly state varying) share of total debt held domestically, $A'\left(S, s, \pi\right)$ is the policy function of the households, and the transition for d is such that if $\delta = 1$ then

$d = 1$ while if $\delta = 0$ then $d = 1$ with probability ζ and $d = 0$ with complementary probability.

We can then define a recursive equilibrium for this model:

Definition. Given a share rule $\alpha(S, s, \pi)$, an equilibrium is a value for the government V and the associated policy rule $\mathbf{p}(S, s)$, the household value ν and associated policy rules, prices for Arrow securities and government debt, and the law of motion for aggregate quantities such that i) the government problem and the policy rules solve (7), ii) the household problem solves (2), iii) the prices for Arrow securities and government debt are given by (3) and (4), and iv) the representativeness condition.

Note that given total government debt, the split between domestic and foreign debt is not determined in equilibrium, hence the need to index equilibria by the selection rule $\alpha(S, s, \pi)$.

3.4 Primal Markov problem

We next show that, from period 1 onward, debt issuances and default decisions are function only of the total public debt and exogenous shocks and do not depend on the composition of the holdings of sovereign debt. In particular, from period 1 onward, the government solves

$$W(B, s) = \max\left\{W^r(B, s), W^d(B, s)\right\} \tag{8}$$

where W^r is the value from government consumption if there is repayment,

$$W^r(B, s) = \max_{G, B'} \omega(G) + \beta \sum_{s'} \mu(s'|s) V(B', s') \tag{9}$$

subject to the budget constraint

$$G + B \leqslant \tau Y(s) + q(s, B') B',$$

W^d is the value from government consumption if there is default,

$$W^d = \omega(\tau Y(s) - \chi(s)) + \beta \sum_{s'} \mu(s'|s)\left[(1-\zeta) W^d(s') + \zeta W(0, s')\right], \tag{10}$$

and the pricing schedule is given by

$$q(s, B') = \sum_{s'} M(s', s) \delta(S'(s')) \tag{11}$$

where $\delta(B', s') = 1$ if and only if $W^r(B, s) \geqslant W^d(B, s)$. In period 0, given S_0, the government solves

$$\max \left\{ \begin{array}{c} \max_{\pi=(\delta=1,B')} V(A_0(\pi), s) + \omega(\tau Y(s) - B + q(s, B')B') + \beta W(B', s'); \\ V(A_0(\pi|\delta=0), s) + W^d(s) \end{array} \right\} \tag{12}$$

where $V(A, s)$ is the unique solution to the following functional equation:

$$V(A, s) = \max_{c, A(\cdot)} u(c) + \beta \sum_{s'} V\left(A\left(s'\right), s'\right)$$

subject to

$$c + \sum_{s'} M\left(s', s\right) A\left(s'\right) \leqslant (1-\tau) Y(s) + A$$

The next proposition states the main result of this section:

Proposition. *Given an initial state S_0, the equilibrium outcome $\left\{B\left(s^t\right), \delta\left(s^t\right), q\left(s^t\right)\right\}$ can be recovered as the solution to the quasi-recursive problem in (8)–(12).*

The proof is straightforward. The risk sharing condition (5) implies that the consumption profile of the domestic household does not depend on the government policies from period 1 onward both on and off the equilibrium path. From period 1 onward, the domestic households are insured from government default as a consequence of (5), and we have that (6) holds. Thus, along the equilibrium path, when a government contemplates whether to default or not, the changes in the private net-foreign asset positions are not affected from this choice. Hence the government can ignore the private sector in its default and debt issuance choices. Thus from period 1 onward, government policies solve (8).

Period 0 is different because the asset holdings of the domestic households do not necessarily satisfy (6) and thus a default decision can affect domestic private wealth and so the distribution of government debt between domestic and foreign agent is relevant. To account for this, we have a slightly modified problem in (12).

3.5 Discussion

The representation in (8) looks identical the set-up in canonical models used to study sovereign default for emerging market economies. There is, however, an important difference in the interpretation of the state variable B. Canonical models that follow Eaton and Gersovitz (1981) consider the polar opposite of the case considered here: Private agents cannot borrow from foreign lenders.[7] The canonical sovereign debt model admits

[7] Alternatively, there is no distinction between private foreign liabilities and public foreign liabilities and all external debt is ultimately a liability of the government.

the representation in (8) but the relevant statistic is total external debt not total public debt.

Clearly, we need strong assumptions to compare these two cases. In particular, by assuming that private financial markets are sophisticated we obtain that there are no spillover from government decisions to the private sectors and the distribution of public debt holdings is not relevant. This contrasts with the findings of two large and growing literatures. One literature documents the pass-through of government interest spreads on private interest rates. See Bocola (2016) and Arellano et al. (2017). Another literature emphasizes that the composition of holdings of government debt is an important determinant of interest rate spreads and debt sustainability more broadly. See Perez (2015), Gennaioli et al. (2014), Broner et al. (2014), Chari et al. (2016), Dovis et al. (2016), D'Erasmo and Mendoza (2013). We are not advocating that these features are not relevant but we are abstracting from these to make the comparison in the starkest way possible.

Which of these two polar opposite is more relevant? This may depend on the particular application. In particular, it depends on the sophistication of financial markets and ability of governments to interfere with private domestic contracts. See Arellano et al. (2016) for a documentation of how different governments have different ability to interfere with private contracts. In Europe, private external financial markets are fairly advanced and sophisticated and EU regulations require free mobility of capital within member states. Thus we think our model is a useful benchmark to study the European debt crisis.

4 Calibration and workings of the model

In this section we perform two different calibrations of the model in (8) using Spanish data for the 1999Q1-2012Q2 period. Our calibration maps the variable B to total public debt, while the more traditional calibration maps it to total external debt. We then compare the performance of these two different approaches.

4.1 Calibration

We make the following functional form assumptions. We follow Bocola and Dovis (2016) and assume that the preferences for public good consumption are given by

$$\omega(g) = \frac{\left(g - \underline{g}\right)^{1-\sigma} - 1}{1-\sigma} \tag{13}$$

where $\sigma \geqslant 0$ and $\underline{g} \geqslant 0$ is a subsistence level for public consumption good. One can interpret $\underline{g}$ as capturing the components of public spending that are hardly modifiable by

Table 2: **Model parameters**

	Our calibration	Traditional calibration	
r^*	0.017	0.017	Annual risk free rate of 4%
σ	2.000	2.000	Conventional value
ζ	0.050	0.050	Cruces and Trebesch (2013)
ρ_y	0.945	0.945	Estimates of equation (14)
σ_y	0.025	0.025	Estimates of equation (14)
β	0.950	0.950	Method of Simulated Moments
$\tilde{d}$	0.051	0.034	Method of Simulated Moments
d_{ratio}	2.000	2.000	Method of Simulated Moments
$\underline{g}$	0.676	0.00	Method of Simulated Moments

Notes: We reparametrize the $d(.)$ function in equation (15) in order to make our results more easily interpretable. The parameter $\tilde{d}$ stands for the percentage loss in output after a default when output is at its median values. The parameter d_{ratio} is the ratio between default cost over output for top income vs. median income.

the government in the short run, such as wages of public employees and pensions. The presence of $\underline{g}$ breaks the homotheticity of the government preferences and it is critical to account for the data in our interpretation of the model as we will make clear later.

We further assume that foreign lenders are risk neutral so $M(s', s) = \Pr(s'|s) / (1 + r^*)$ where r^* is the risk-free real interest rate. The output process, $Y = \exp\{y\}$, is an AR(1) process

$$y' = \rho_y y + \sigma_y \varepsilon', \qquad \varepsilon' \sim \mathcal{N}(0, 1). \tag{14}$$

The default costs are parametrized following Chatterjee and Eyigungor (2012),

$$\chi(Y) = \max\{0, d_0 \tau Y + d_1 (\tau Y)^2\}. \tag{15}$$

The convexity of $\chi(Y)$ gives enough flexibility to match the volatility of interest rate spreads, see Chatterjee and Eyigungor (2012).

We select the model parameters to match a set of moments summarizing the behavior of public finances and interest rates. We set r^* so that the annual risk free real interest rate is four percent. We set $\zeta = 0.05$, a value that implies an average exclusion from capital markets of 5.1 years following a default, in line with the evidence in Cruces and Trebesch (2013). We then set $\sigma = 2$. The parameters of the output process in equation (14) are estimated on detrended log real GDP for Spain over the 1999Q1-2012Q2 period.[8]

The remaining parameters, β, d_0, d_1, and $\underline{g}$, are chosen to match a set of empirical targets. We include in the set of empirical targets statistics that summarize the behavior of

[8] We estimate a quadratic time trend on log real GDP for the 1960Q1-2012Q2 period. Real GDP data are from OECD.

outstanding debt and interest rate spreads. In both calibrations, we consider the sample mean of the debt services to output ratio, and the mean and standard deviation of yields differential between a Spanish and a German Tresury bill with a residual maturity of one year.[9] These are standard targets for quantitative sovereign debt models. The two calibrations differ, instead, in the measure of debt cyclicality. In our calibration, we target the correlation between the debt to GDP ratio and detrended real GDP, while in the traditional calibration we substitute this moment with the cyclicality of the trade balance over GDP, $\text{corr}(TB/Y, \ln Y)$.

We solve the model on a grid of points for $(\beta, d_0, d_1, \underline{g})$, and select the parameterization that minimizes a weighted distance between sample moments and their model implied counterparts. Model implied moments are computed on a long simulations $(T = 100,000)$, and we weight the distance between a sample moment and its model counterpart by the inverse of the standard deviation of the sample moment squared. Table 2 reports the value for model's parameters..

Table 3: **Calibration targets**

Statistic	Data		Our calibration	Traditional calibration
	Estimate	(S.E.)		
Average spread	0.32	(0.28)	0.30	0.53
Spread volatility	0.88	(0.29)	0.95	0.55
Average debt service/GDP	8.43	(0.60)	8.60	8.69
Debt service cyclicality	-0.78	(0.47)	-0.61	-
Trade balance cyclicality	-0.69	(0.34)	-	-0.14

Notes: Data moments computed using GMM, standard errors using Newey-West with 8 lags. Weighting matrix is a diagonal with inverse of standard errors squared.

The first and second columns of Table 3 show that our version of the model has good in sample fit. As in the data, the government debt to output ratio (B/Y) is negatively correlated with output. Interest rate spreads are typically close to zero in model simulations, with an annualized average value of 0.30% relative to the 0.32% observed in the sample and they are as volatile as in the data. The canonical calibration does not fit as well on the targeted models, and it produces a coefficient of variation for interest rate spreads of roughly one, substantially below that of the data. Aguiar et al. (2016) discuss why traditional calibration of sovereign debt models deliver little volatility in interest rate spreads. As we explain later in this section, matching countercyclical debt issuances helps

[9] We decided to keep average debt services constant across the two calibrations in order to isolate the effects that different debt cyclicality has on the behavior of the model. The empirical target is constructed using public debt. The average net foreign debt for Spain, however, is comparable to average public debt, see Section 2. Thus, we do not expect our results to change drammatically if we were to use a more appropriate average external debt for the traditional calibration.

the model in obtaining more volatile spreads.

4.2 Workings of the model

We now contrast the workings of our calibration with the standard one that matches a negative trade balance where the government borrowing more when hit by positive output shocks.

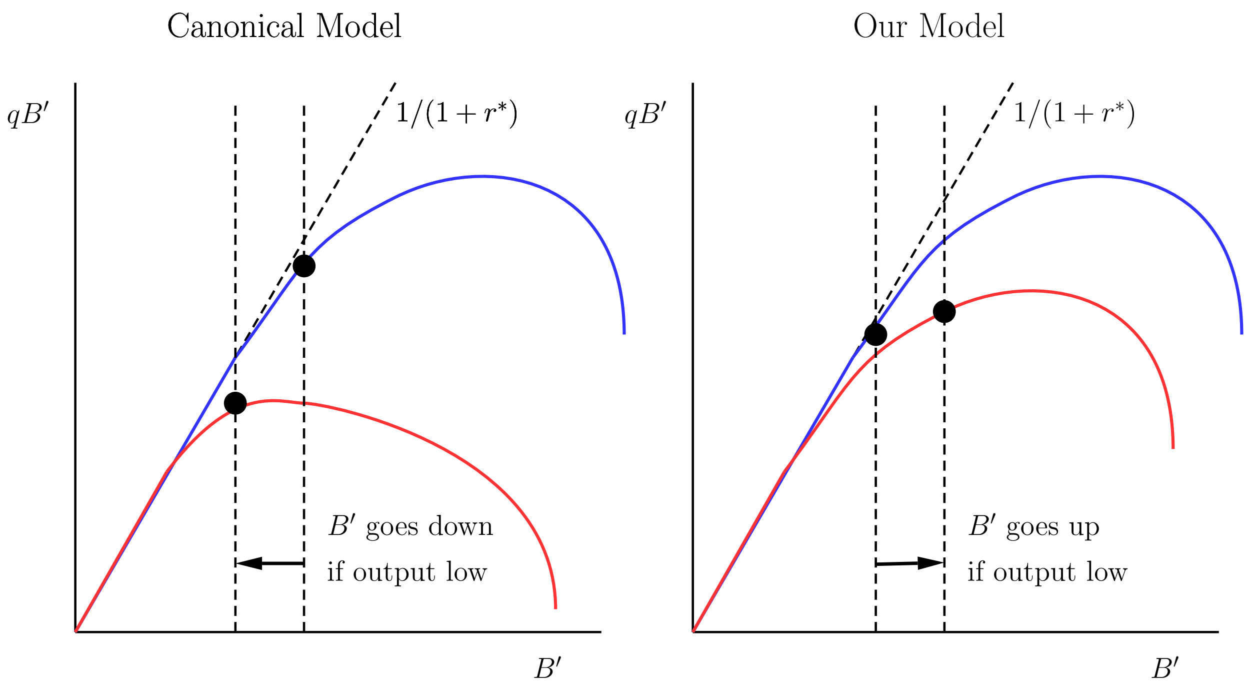
We start considering the standard calibration. How can the model generate a procyclical debt issuance in a pure exchange economy? Arellano (2008) shows that it takes a combination of i) high degree of impatience relative to international risk free rate, ii) weak precautionary saving motives, and iii) convex default costs and persistent output. If the government is sufficiently impatient and it has weak precautionary saving motives then the borrowing government behaves myopically. That is, it uses the debt market to frontload consumption rather than to smooth it across states of the world. If the output costs of default are sufficiently convex, this implies that the government is borrowing more in good times because it can credibly promise to repay in more states of the world tomorrow. This is because if output is high today it is likely it will be higher next period as well. If default is very costly in states of high output then the government will find it more attractive to repay in good times.

The left panel of Figure 2 illustrates this mechanism by plotting the revenues from debt issuances, $q(s, B')B'$, as a function of the face value of debt, B', for a high (blue line) and a low (red line) current realization of output. When output is high, the risk of future default is small, and the government is able to raise more resources by issuing debt. Conversely, low income states are associated with tighter pricing. Thus the revenues schedule shifts in restricting the ability of the government to front-load consumption. Because of high impatience and low precautionary motives, the government finds it optimal to borrow a lot in a high output (the optimal debt choices is depicted with a circle on the schedule $q\left(s, B'\right)B'$). In a low output state instead, the government is forced to cut its indebtedness because such large debt would not be credibly repaid. Hence, the impatience of the government coupled with the endogenous borrowing limits implied by default risk leads to procyclical debt issuances.[10]

The right panel of Figure 2 plots the revenue schedules, $q(s, B')B'$, for two realizations of current output and the respective optimal debt issuances for our baseline calibration. The revenue schedules in the two calibrations are qualitatively similar: they both define

[10]These features are more prominent in the typical calibration for emerging market economies that matches higher level of average spreads. In those calibrations, the government is most of the times at risk of a default, with interest rate spreads being far away from zero even when output is above average. For example, in Chatterjee and Eyigungor (2012) the correlation between net exports (equal to the government surplus) and output is -0.44, while annualized interest rate spreads are on average 8%.

Figure 2: **The cyclicality of debt issuances**



Notes: The filled dots represent the optimal debt issuances in the high and low output state.

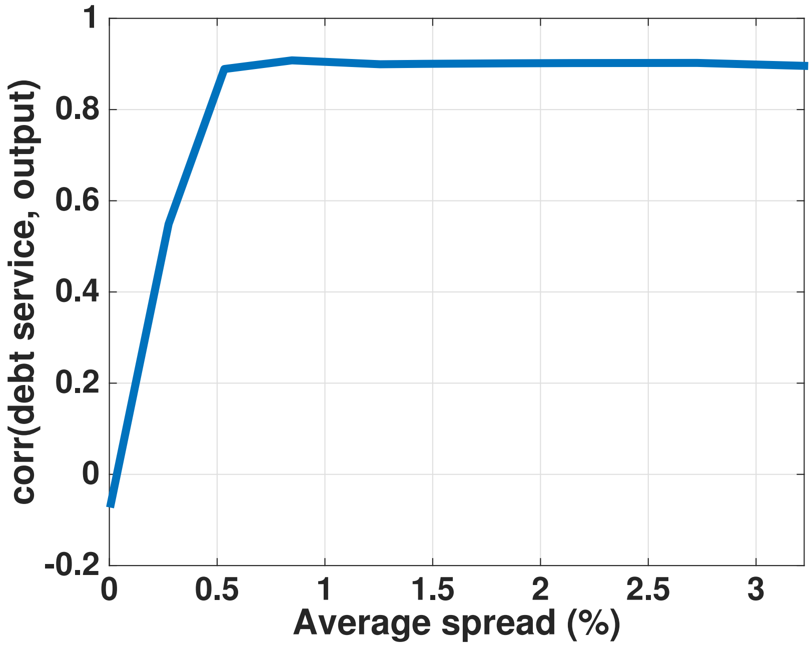
a Laffer curve for debt issuances, and they both shift inward when the economy is hit by a negative output shocks. The two calibrations differ in the debt choices made by the government. In our parametrization, the government uses the debt markets mostly for consumption smoothing because the presence of the subsistence level $\underline{g}$ generates stronger precautionary savings motives. The government reduces (or keeps constant) its debt in high output states while it borrows in the face of bad output shocks.

The presence of the subsistence level of public consumption good, $\underline{g}$, in the government's preferences (13) is critical to jointly replicate the countercyclicality of debt issuances and positive average spreads. We show this in Figure 3 by plotting the value of these two statistics generated by varying β and imposing $\underline{g} = 0$. When β is close enough to $1/(1+r^*)$, the model can generate a negative correlation between the debt to GDP ratio and output but with essentially zero spreads and no debt in equilibrium. The idea is that for high β, the government does not want to frontload consumption anymore and even small precautionary saving motives incentivize the government to accumulate assets.

4.3 Behavior of spreads

We now discuss the implications for the distribution of spreads. Our calibration generates interest rate spreads that are on average close to zero, and they jump to substantially

Figure 3: Average spreads and debt cyclicality



Notes: To generate this figure we set $\underline{g} = 0$ and solve the model for different values of β. We simulate artificial data under these different parametrizations, and we report the combination of average spreads and correlation between $\mathrm{corr}(B', y)$ across simulations.

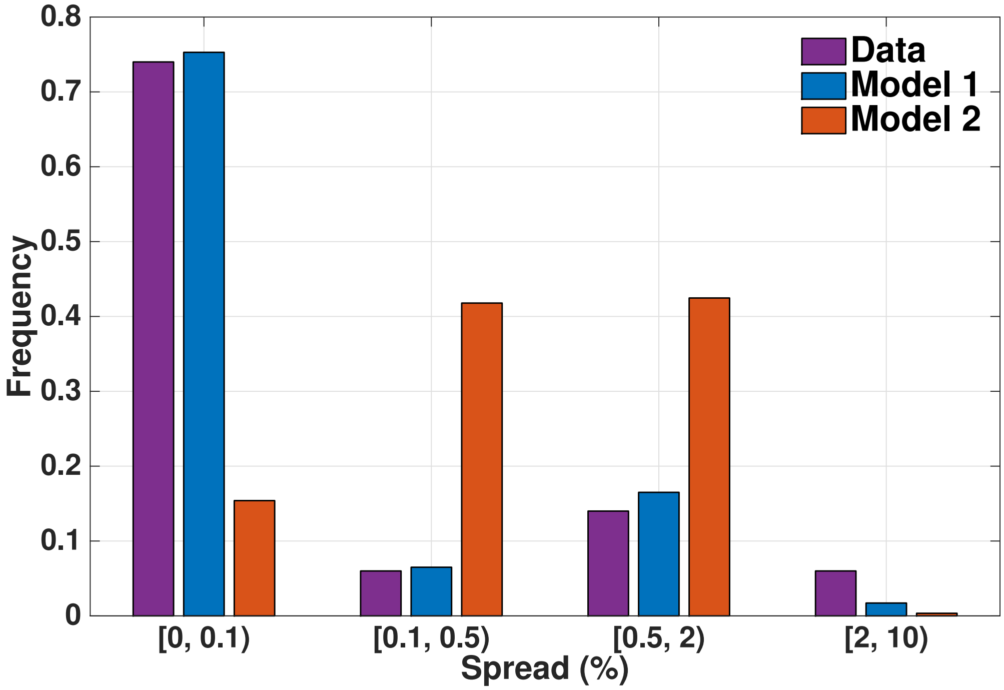
positive values only conditional on sufficiently low income realizations or high inherited debt. Figure 4 plots the distribution of spreads generated by our calibration (Model 1) together with the data for Spain. Our calibration matches very well the entire distribution of spreads despite targeting only the first two moments. In particular, it is notable that the model matches the probability that spreads are less than 10 basis points, about 70%.

Figure 4 also reports the distribution of spreads obtained under the canonical calibration (Model 2). The distribution is more centered around the mean unlike in the data. Spreads are lower than 10 basis points only 15% of the times (70% in the data). As explained in the previous section, in this calibration, the government acts in a myopic way and so the government is most of the times at risk of a default, with interest rate spreads being positive even when output is above average.

4.4 Default events

We now contrast the typical dynamics leading to a default under our calibration and the canonical one. The typical path for income, the ratio of debt to income, B'/Y, and spreads are reported in Figure 5. Blue lines are for our calibration while red lines correspond to the canonical one. In both cases the driving force leading to a default is a sequence of negative income shocks. Our specification needs larger negative shocks to induce a

Figure 4: Distribution of interest rate spreads



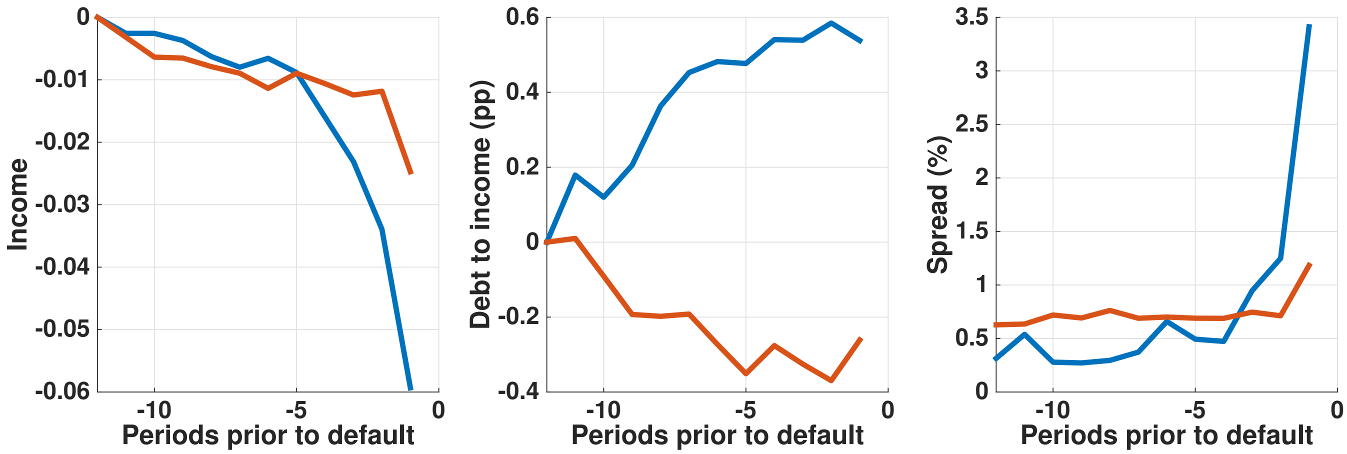
default.

The dynamics of debt to GDP ratios in the two specifications is markedly different by construction. In our specification, we target a positive commovement between debt issuances and GDP. Thus, as the economy is hit by negative income shocks, the government is issuing more debt to keep its consumption away from the subsistence level $\underline{g}$. The canonical calibration, instead, targets a countercyclical trade balance which implies procyclical debt issuances and so debt declines along the path leading to a default.

The most salient difference between the two paths is the behavior of interest rate spreads. In our model, spreads are less than 50 basis points until 4 quarters prior to default, even if income is below mean in all the periods. The spread then jumps and reaches about 3.50% in the quarter before default. Under the canonical calibration, spreads are higher in all periods, between 50 basis points and 1%, and they do not exhibit a substantial jump in the period before default. Consistently with Figure 4, spreads are clustered around their unconditional mean of 0.53%.

To understand why spreads do not reach higher levels in the canonical calibration, note that spreads in the model are driven by current income and newly issued debt. In the canonical calibration, a negative income shock increases spreads directly but it decreases them indirectly via the reduction in newly issued debt associated with it. This indirect effect moderates the impact of a negative income shock to spreads. In contrast, in our model, indebtedness increases with a negative income realization. Thus, both ef-

Figure 5: Typical outcome path leading to a default



Notes: The blue lines report default events under our calibration while red lines report rhe same experiment in the traditional calibration.

fects increase the spreads and the model can generate spreads significantly higher than the mean when hit by a sequence of negative income shocks.

5 Conclusion

This paper uses a standard quantitative sovereign default model to study the recent debt crisis in Southern Europe. We first document that in the case of Portugal, Spain, and Italy, total public debt outperforms standard indicators of external debt in accounting for the dynamics of interest rate spreads. Motivated by this evidence, we derive theoretical conditions under which total public debt is a relevant state variable in the decision problem of the government, while most of the existing literature focuses on external debt. We then show that a calibrated version of the model can account for the behavior of income, public debt, and interest rate spreads for Southern European economies. Compared to the traditional calibration in the literature that targets the behavior of external debt, our approach improves the fit of the model regarding interest rate spreads and produces more realistic crises dynamics.

We purposely analyzed the most basic default model to illustrate the mechanism in a transparent way. Extending the model to account for the joint behavior of external and public debt is a promising avenue for future research. One may also want to extend the model by introducing other realistic features can increase the fit of the model like long term debt, risk averse lenders, partial recovery of debt. We leave these for future research.

References

AGUIAR, M., S. CHATTERJEE, H. COLE, AND Z. STANGEBYE (2016): "Quantitative Models of Sovereign Debt Crises," Forthcoming, Handbook of Macroeconomics, vol. 2. 16

AGUIAR, M. AND G. GOPINATH (2006): "Defaultable Debt, Interest Rates and the Current Account," *Journal of International Economics*, 69, 64–83. 2, 4

AIYAGARI, S. R., A. MARCET, T. J. SARGENT, AND J. SEPPÄLÄ (2002): "Optimal taxation without state-contingent debt," *Journal of Political Economy*, 110, 1220–1254. 9

ARELLANO, C. (2008): "Default Risk and Income Fluctuations in Emerging Economies," *The American Economic Review*, 98, 690–712. 2, 3, 4, 17

ARELLANO, C., A. ATKESON, AND M. WRIGHT (2016): "External and public debt crises," *NBER Macroeconomics Annual*, 30, 191–244. 14

ARELLANO, C., Y. BAI, AND L. BOCOLA (2017): "Sovereign default risk and firm heterogeneity," Tech. rep., National Bureau of Economic Research. 14

ARELLANO, C., Y. BAI, AND S. LIZARAZO (2018): "Sovereign Risk Contagion," Tech. rep., unpublished manuscript, Federal Reserve Bank of Minneapolis. 5

ARELLANO, C. AND A. RAMANARAYANAN (2012): "Default and the Maturity Structure in Sovereign Debt," *Journal of Political Economy*, 120, 187–232. 4

BIANCHI, J., J. C. HATCHONDO, AND L. MARTINEZ (2012): "International reserves and rollover risk," Tech. rep., National Bureau of Economic Research. 4

BOCOLA, L. (2016): "The Pass-Through of Sovereign Risk," *Journal of Political Economy*, 124, 879–926. 9, 14

BOCOLA, L. AND A. DOVIS (2016): "Self-fulfilling debt crises: A quantitative analysis," Tech. rep., National Bureau of Economic Research. 3, 5, 14

BRONER, F., A. ERCE, A. MARTIN, AND J. VENTURA (2014): "Sovereign debt markets in turbulent times: Creditor discrimination and crowding-out effects," *Journal of Monetary Economics*, 61, 114–142. 14

CHARI, V., A. DOVIS, AND P. J. KEHOE (2016): "On the optimality of financial repression," *Manuscript, University of Minnesota*. 14

CHATTERJEE, S. AND B. EYIGUNGOR (2012): "Maturity, Indebtedness and Default Risk," *The American Economic Review*, 6, 2674–2699. 4, 15, 17

CRUCES, J. J. AND C. TREBESCH (2013): "Sovereign defauls: the price of haircuts," *American Economic Journal: Macroeconomics*, 5, 85–117. 15

D'ERASMO, P. AND E. MENDOZA (2013): "Distributional Incentives in an Equilibrium Model of Domestic Sovereign Default," NBER working paper, No. 19477. 4, 5, 14

DOVIS, A., M. GOLOSOV, AND A. SHOURIDEH (2016): "Political economy of sovereign debt: A theory of cycles of populism and austerity," Tech. rep., National Bureau of Economic Research. 4, 14

DOVIS, A. AND R. KIRPALANI (2018): "Reputation, Bailouts, and Interest Rate Spread Dynamics," . 4

EATON, J. AND M. GERSOVITZ (1981): "Debt with potential repudiation: Theoretical and empirical analysis," *The Review of Economic Studies*, 48, 289–309. 1, 2, 4, 13

GENNAIOLI, N., A. MARTIN, AND S. ROSSI (2014): "Sovereign default, domestic banks, and financial institutions," *The Journal of Finance*, 69, 819–866. 14

HATCHONDO, J. C. AND L. MARTINEZ (2009): "Long Duration Bonds and Sovereign Defaults," *Journal of International Economics*, 79, 117–125. 4

HATCHONDO, J. C., L. MARTINEZ, AND C. SOSA PADILLA (2015): "Debt Dilution and Sovereign Default Risk," Forthcoming, Journal of Political Economy. 4

HÉBERT, B. AND J. SCHREGER (2017): "The costs of sovereign default: Evidence from argentina," *American Economic Review*, 107, 3119–45. 9

MENDOZA, E. G. AND V. Z. YUE (2012): "A General Equilibrium Model of Sovereign Default and Business Cycles," *Quarterly Journal of Economics*, 127, 889–946. 4

PALUSZYNSKI, R. (2016): "Learning about Debt Crises," *University of Minnesota Manuscript*. 5

PEREZ, D. (2015): "Sovereign debt, domestic banks and the provision of public liquidity," *Manuscript, New York University*. 14

POUZO, D. AND I. PRESNO (2014): "Optimal taxation with endogenous default under incomplete markets," . 9

SALOMAO, J. (2017): "Sovereign debt renegotiation and credit default swaps," *Journal of Monetary Economics*, 90, 50–63. 5