A Unified Model of International Business Cycles and Trade*

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Abstract

We present a general, competitive open economy business cycles model with capital accumulation, trade in intermediate goods, production externalities in the intermediate and final goods sectors, and iceberg trade costs. Our main theoretical result shows that under appropriate parameter restrictions this model is isomorphic in terms of aggregate equilibrium predictions to dynamic versions of workhorse quantitative models of international trade: Eaton-Kortum, Krugman, and Melitz. The parameter restrictions apply on the overall scale of externalities, the split of externalities between factors of production, and the identity of sectors with externalities. Our quantitative exercise assesses whether various restricted versions of the general model — in forms they are typically considered in the literature — are able to resolve well-known aggregate empirical puzzles in the international business cycles literature. Our theoretical result on isomorphism between models provides insights on why dynamic versions of international trade models fail to resolve these puzzles in so many instances. We then additionally explore in what directions they need to be amended to provide a better fit with the data. We show that an essential feature is negative capital externalities in intermediate goods production. We thus provide a unified theoretical and quantitative treatment of the international business cycles and trade literatures in a general dynamic framework.

Key words: International business cycles; Dynamic trade models; Heterogeneous firms; Production externalities; Monopolistic competition; Export costs; Entry costs

JEL classifications: F12; F41; F44; F32

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1 Introduction

Are margins identified in the modern international trade literature important for international business cycles dynamics? Do features such as monopolistic competition with sunk cost of entry or heterogeneous firms with fixed cost of exporting matter quantitatively for the transmission of aggregate shocks in a dynamic open economy business cycles model? Do these margins change the aggregate predictions one gets from using a neoclassical business cycles model? If so, do they enable a better fit between the data and the model by resolving some well-known empirical puzzles such as the high international correlation of output compared to consumption, the positive cross-country correlations of investment and hours, and the low cyclicality of the real exchange rate? We provide a unified model of international business cycles and trade that can address these questions.

We provide such a unified treatment in steps. First, we formulate a general dynamic open economy model with endogenous labor supply where all sectors are competitive and some of the sectors feature external economies of scale. There are four sectors that produce intermediate, final, consumption, and investment goods respectively. External economies of scale are present in the intermediate and final goods sectors. The intermediate goods sector uses capital and labor to produce its goods, with aggregate productivity depending on the total amount of capital and labor employed in this sector (and taken by firms as given). The intermediate goods are internationally traded and trade is costly, which we model using iceberg costs. The intermediate goods, including imported ones, are combined into a final good using a standard Armington type aggregator. Aggregate productivity in the final goods sector depends on the total output in this sector (and is taken by producers as given). The final good is used in production of both investment and consumption goods. The production function for investment good additionally uses labor input as well. [1]

Second, we formulate general dynamic versions of three workhorse international trade models: Eaton-Kortum, Krugman, and Melitz.[2] In terms of intertemporal linkages, apart from trade in assets internationally, the dynamic Eaton-Kortum model features capital accumulation, while the dynamic Krugman and Melitz models feature a law of motion of

[1]Our set-up is general on international borrowing and lending, and we consider all three standard cases: financial autarky, bond economy, and a complete markets economy. As is somewhat obvious from this description, the canonical open economy real business cycles model, as in Backus *et al.* (1994) and Heathcote and Perri (2002) for instance, is a nested case with no iceberg trade costs, no externalities, and no labor as input in production of the investment good.

[2]We explain in detail later why our set-ups are more general than similar models in the literature and precisely how we generalize them. Here, we simply point out that these generalizations are needed to establish isomorphisms between the unified, competitive model and the three trade models.

differentiated varieties driven by firm entry and exit.

After formulating the general competitive dynamic model with production externalities and the three dynamic trade models, we then derive the main theoretical result. We show that, after appropriate re-labeling of variables and parameters, the three dynamic trade models are isomorphic in terms of aggregate implications to the general competitive dynamic model.[3] This isomorphism holds even though the three dynamic trade models have very different micro-foundations.

In terms of re-labeling of variables, our result on isomorphism is based on the similarity between the law of motion of physical capital in the general competitive model and the law of motion of the differentiated varieties in the dynamic Krugman and Melitz models. In terms of re-labeling of parameters, the isomorphism between the general competitive model and the Eaton-Kortum model is very direct. The re-labeling of parameters is more interesting and involved for the dynamic Krugman and Melitz models. For the standard dynamic Krugman model (as it appears in the literature), the elasticity of substitution between varieties simultaneously governs the capital share, the total scale of externalities, and the split between capital and labor externalities in production of intermediate goods in the corresponding unified model. Our generalization of the Krugman model fully relaxes this tight relationship between parameters of the corresponding unified model and, thus, establishes isomorphism between the two models.

The dynamic Melitz model, compared to the dynamic Krugman model, additionally features heterogeneous efficiencies of production of varieties and fixed costs of serving different markets. For the standard dynamic Melitz model with Pareto distribution of efficiencies (again, as it appears in the literature), a combination of the elasticity of substitution between varieties and the shape of Pareto distribution simultaneously govern all parameters of the production technology of intermediate goods in the corresponding unified model. Moreover, a key distinction of the dynamic Melitz model from the Krugman model is that it additionally features external economies of scale in the final goods sector, where intermediate goods are combined. Here, also, the elasticity of substitution between varieties and the shape of Pareto distribution govern the strength of economies of scale in production of the final good. Thus, again, the standard dynamic Melitz model implies tight links between key parameters of the corresponding unified model. We generalize the Melitz model to fully relax these tight links between parameters of the corresponding unified model and establish isomorphism between the two models.

Given the theoretical result, we then undertake a quantitative exercise. We first ana-

[3] More precisely, we show that the equilibrium system of equations that governs aggregate dynamics is the same across these variants.

lyze performance of different versions of the standard dynamic trade models — as they are often used in the literature without the generalizations we propose — in terms of business cycle moments.

We first study how these different versions of the standard dynamic trade models — as they are often used in the literature without the generalizations we propose — lead to differential aggregate implications in terms of business cycles moments. Our point of comparison is the standard open economy model that has no externalities and where country-specific productivity shocks drive the business cycle. We show that the dynamic versions of these standard trade models are not able to resolve the key empirical puzzles related to cross-country output, consumption, investment, and hours correlations that plague the standard business cycles model. We provide an interpretation based on our theoretical results: standard formulations and calibrations of these models lead to relatively small and positive production externalities, which are in turn tightly restricted in terms of splits across factors. This then leads to transmission mechanisms and aggregate second moments very similar to the standard competitive business cycles model with no externalities. In fact, we show that often the business cycle fit for the standard trade models is even worse than the standard competitive model without externalities.[4]

We next use the general model, which, because of the isomorphism, can be re-interpreted as a version of the generalized dynamic trade models (which again relax the tight restrictions on parameters governing externalities implied by the standard models), to explore if it is possible to achieve a better fit with the data. We show that an essential feature is negative capital externalities in intermediate goods production. As the standard dynamic trade models imply positive capital externalities in intermediate goods production, they do not provide a closer fit to the data.

What is the intuition behind the result that negative capital externalities help with resolving several international business cycle puzzles? First, note that the main empirical puzzles are associated with co-movement across countries in output, consumption, hours, and investment. In the standard model, the co-movement of consumption is counterfactually higher than output.[5] Moreover, while in the data labor hours and investment co-move positively, in the standard model with (at least some) risk-sharing, they co-move either weakly positively or, for investment, negatively. Second, it is critical to note that when there are negative capital externalities in production of intermediate goods, from the perspective of individual firms, it is as if the aggregate country-specific productivity

[4]As we explain more below, one driver of this is that positive externalities lead to a negative endogenous correlation in productivity across countries, dampening down the co-movement in output and making even more negative the co-movement in investment and labor.

[5]High co-movement of consumption across countries is not only due to perfect risk-sharing.

shock is less persistent with the same initial impact. This is because, in future, due to positive capital accumulation, the productivity shock faced by the firms is lower than the exogenous productivity shock. Third, note that since this feature is irrespective of the risk-sharing arrangements across countries, our finding applies independently of whether we assume complete financial markets or incomplete markets or financial autarky. For the sake of concreteness, we discuss below the case of complete financial markets.

Given this, how do agents, say at home, respond to a productivity shock that has the same initial size but is more transient? As is standard in competitive business cycle models, it is most useful to think through the labor supply response. As the shock is now more transient, compared to the no externality case, the substitution effect of wage increase is stronger than the income effect. This means that households supply more labor today. This, with the capital stock as given, then leads to a larger response of output. This helps with increasing output co-movement across countries. What should the households do with this increased income? While the initial effect on income is higher, in future, as the productivity process is more transient, income will be lower than in the model without externalities. Then through the usual intuition based on the permanent income hypothesis, while consumption rises today, due to the desire to smooth consumption over time, consumption rises by less than income, and, moreover, by a smaller amount than with no externalities. This smaller rise of consumption at home then helps with not counter-factually increasing consumption co-movement across countries and, in fact, helps with reducing consumption correlation.

Finally, why do cross-country investment and labor hours co-movements turn from negative to positive? First, given that consumption rises by less at home, investment increases by more. But this does not worsen international correlation in investment. An important feature now is that, while the country-specific productivity shocks themselves are uncorrelated in our experiments, negative capital externality leads to an endogenous positive correlation of productivities faced by the two countries. From the foreign country's perspective, starting from the next period, there is a positive effect on productivity, as typically there would be negative investment in the foreign country following a positive productivity shock in the home country. This positive effect on productivity faced by the foreign country then leads to increased labor hours and increased investment for very standard reasons. Moreover, this endogenous increase in productivity in the foreign country also leads to an increase in output, which helps further with increasing output co-movement across countries. Finally, consumption in the foreign country increases, but by less than it would with no externality.

In addition to assessing international correlations, we also explore the fit of the various models with the data in terms of domestic correlations of key open economy variables with output. We focus on cyclicality of exports, imports, real exchange rate, and the trade balance. We find again that dynamic versions of standard trade models lead to very similar moments as the standard competitive model.[6] Next, negative capital externalities in production help also with moving the model closer to the data in terms of generating less procyclical exports and the real exchange rate and a more countercyclical trade balance. That is, to meet the larger increase in investment demand that we discussed above, the home country imports more, as the investment good is produced using the final aggregate good that combines the domestic and foreign intermediate goods. Moreover, given the lower effect on relative consumption across countries we described above, the real exchange rate is now less procyclical. One exception is that negative capital externalities in production lead to a more procyclical imports, which makes the fit worse with the data as the standard business cycle model itself leads to imports that are more procyclical than the data. The reason imports become more procyclical is that the behavior of imports closely follow that of investment, and as we explained above, investment and output increases more sharply initially with negative capital externalities.

While negative capital externalities in production help with moving the model closer to the data in terms of cross-country co-movement of business cycle quantities, negative labor externalities do not uniformly do so. The main reason is that with negative labor externalities, while the productivity process faced by the home country is also less transient in future as typically there would be an increase in labor hours in future, the initial impact also shifts down. This is because, unlike capital stock, which is pre-determined today, labor hours respond positively today. This then looks basically like a productivity process for the home country that has shifted downwards at every point in time. Then, home households do not increase their hours initially. The effect is thus not as strong as with negative capital externalities in moving the correlation of hours and investment towards positive. In terms of the foreign country, there is again an endogenous correlation of productivity, as typically there would be a negative response of foreign labor hours, and so it does help qualitatively with generating a less negative response of foreign investment and hours. The main difference with negative capital externality is that consumption correlation actually increases, instead of decreasing. This is because consumption in the foreign country does not change its dynamic response, as there is not much difference in the response of investment in the foreign country. Finally, in the dynamic Melitz model,

[6] There is a subtle but important point on cyclicality of the trade balance depending crucially on whether the investment sector uses home labor or the final aggregate good in production. As we explain later in the paper, standard formulations of dynamic trade models imply the use of only home labor, which actually would counterfactually lead to a pro-cyclical net exports.

as we discussed above, there is an additional externality in the final goods sector, where intermediate goods are aggregated. We show that this externality behaves similarly to the labor externality in the intermediate good production technology, and so negative externality in this aggregator technology also does not uniformly improve the model fit.

Our paper is related to several strands of the literature. The most direct relation is to the vast literature on international real business cycle models, in which each country produces a unique tradeable good. This literature is represented, among others, by Backus, Kehoe and Kydland (1994), Heathcote and Perri (2002) and Fitzgerald (2012). In formulating a dynamic international business cycles model that incorporates the margins of the modern international trade literature, we are also clearly building on seminal trade contributions of Eaton and Kortum (2002), Krugman (1980), and Melitz (2003). In particular, Ghironi and Melitz (2005), Alessandria and Choi (2007), Fattal Jaef and Lopez (2014), and Eaton *et al.* (2016) also develop dynamic models similar to ours and assess how important international trade features are for aggregate dynamics and business cycles moments. Our first theoretical contribution is to formulate a general competitive model with production externalities that is isomorphic to various versions of such dynamic trade models. This result then helps to understand the quantitative findings of Alessandria and Choi (2007) and Fattal Jaef and Lopez (2014) that firm heterogeneity and costs of entry and exporting do not matter quantitatively for aggregate dynamics. Our second theoretical contribution is to generalize the dynamic trade models such that there is complete isomorphism between them and the general competitive model.

Our result about the isomorphism is related to a similar result in a static environment demonstrated in Kucheryavyy *et al.* (2017). Kucheryavyy *et al.* (2017) present a version of the Eaton-Kortum model with multiple manufacturing sectors that feature external economies of scale in production. They show that their model is isomorphic to generalized static versions of multi-industry Krugman and Melitz models. Here, we focus on dynamic versions of Eaton-Kortum, Krugman, and Melitz models that have only one manufacturing sector and additional "non-manufacturing" sectors: final aggregate, investment and consumption. Extension of the isomorphism from static to dynamic environments is non-trivial, adds several new features such as the split of externalities between labor and capital and the need to account for endogenous labor supply, and constitutes one of our main theoretical contributions. We then use the general model for quantitative evaluation of business cycle statistics and transmission mechanism.

Our paper is also related to the closed economy literature. In the closed economy endogenous growth literature, for instance, Romer (1986), growth is generated by increasing returns in production, where exernalities in the production function are modeled in the capital input. In our general open economy model, production externalities exist in both capital and labor. In closed-economy business cycle analysis, Benhabib and Farmer (1994) introduced production externalities to the standard neoclassical business cycles model to generate the possibility of multiple, bounded equilibrium. Also in a closed economy set-up, Bilbiie *et al.* (2012) discuss how firm dynamics and entry in a closed-economy model with monopolistic competition and sunk cost of entry (thus similar to the closed economy dynamic version of the Krugman model we develop in this paper) look similar to capital stock dynamics and investment in the standard competitive business cycles model. Our general model provides a similar interpretation as well, while additionally, showing formally how a competitive open economy set-up with different levels and types of production externalities is in fact isomorphic to various versions of monopolistic competition models with firm heterogeneity and costs of entry as well as exporting.

2 Unified Model of Trade and Business Cycles

We present a dynamic stochastic general equilibrium model with multiple countries and international trade. Time is discrete and the horizon is infinite. The world consists of N countries with countries indexed by n, i, and j.[7] Each country has four production sectors: intermediate, final aggregate, consumption, and investment. Intermediate goods are produced from capital and labor. Final aggregate is assembled from intermediate goods. Consumption good is produced directly from the final aggregate. Investment good is produced from the final aggregate and labor. All markets are perfectly competitive. Labor is perfectly mobile within a country between the sectors where it is used. Technology of production of intermediate goods and final aggregates features external economies of scale. There are three exogenous shocks in the economy — they are aggregate productivity shocks in the intermediate, final aggregate, and investment sectors. Only intermediate goods can be traded. Trade is subject to iceberg trade costs. International financial markets structure can be one of the three standard alternatives: financial autarky, bond economy, or complete markets.

We now describe the model in detail.

[7] In all our quantitative exercises we focus on the case of $N = 2$ as is standard in the business cycles literature. But there is nothing that prevents us from formulating the theoretical framework with any number of countries. Moreover, following the modern quantitative trade literature, we prefer to set up the environment in terms of a general N.

2.1 Intermediate Goods and International Trade

Output of a country-n's intermediate good producer that in period t employs $k_{X,nt}$ units of capital and $l_{X,nt}$ units of labor is given by $S_{X,nt} k_{X,nt}^{\alpha_{X,K}} l_{X,nt}^{\alpha_{X,L}}$, where $\alpha_{X,K} \geq 0$ and $\alpha_{X,L} \geq 0$ with $\alpha_{X,K} + \alpha_{X,L} = 1$, and

$$S_{X,nt} \equiv \Theta_{X,n} Z_{X,nt} K_{X,nt}^{\psi_{X,K}} L_{X,nt}^{\psi_{X,L}} \tag{1}$$

is aggregate productivity. The aggregate productivity consists of two parts: exogenous productivity, $\Theta_{X,n} Z_{X,nt}$, and endogenous productivity, $K_{X,nt}^{\psi_{X,K}} L_{X,nt}^{\psi_{X,L}}$. The term $Z_{X,nt}$ in the exogenous productivity part is an aggregate productivity shock, while the term $\Theta_{X,n}$ is a normalization constant that is introduced to later show isomorphisms between the current setup and dynamic versions of Eaton-Kortum, Krugman, and Melitz models. The endogenous productivity part captures external economies of scale in production of intermediates, and it is taken by firms as given. The terms $K_{X,nt}$ and $L_{X,nt}$ are the total amounts of country n's capital and labor used in production of intermediates. Parameters $\psi_{X,K}$ and $\psi_{X,L}$ drive the strength of external economies of scale. Perfect competition in production of intermediates implies that the total output of intermediates in country n in period t is given by

$$X_{nt} = S_{X,nt} K_{X,nt}^{\alpha_{X,K}} L_{X,nt}^{\alpha_{X,L}}.$$

Let $P_{X,nt}$ denote the price of country n's intermediate good in period t. Let W_{nt} and R_{nt} be the wage and capital rental rate in country n in period t. Again, due to perfect competition,

$$K_{X,nt} = \alpha_{X,K} \frac{P_{X,nt} X_{nt}}{R_{nt}} \quad \text{and} \quad L_{X,nt} = \alpha_{X,L} \frac{P_{X,nt} X_{nt}}{W_{nt}}.$$

Moreover,

$$P_{X,nt} = \frac{R_{nt}^{\alpha_{X,K}} W_{nt}^{\alpha_{X,L}}}{\widetilde{\Theta}_{X,n} Z_{X,nt} K_{X,nt}^{\psi_{X,K}} L_{X,nt}^{\psi_{X,L}}}, \tag{2}$$

where $\widetilde{\Theta}_{X,n} \equiv \alpha_{X,K}^{\alpha_{X,K}} \alpha_{X,L}^{\alpha_{X,L}} \Theta_{X,n}$.

Intermediate goods are the only traded goods, and trade in these goods is costly. Trade costs are of the iceberg nature: in order to deliver one unit of intermediate good to country n, country i needs to ship $\tau_{ni,t} \geq 1$ units of this good. To guarantee absence of arbitrage in the transportation of goods, we require that trade costs satisfy the triangle inequality: $\tau_{nj,t} \tau_{ji,t} \geq \tau_{ni,t}$ for any countries n, i, and j. This implies that the price of country i's intermediate good sold in country n is given by $\tau_{ni,t} P_{X,it}$.

2.2 Final Aggregates and Consumption Goods

Final aggregate is produced by combining intermediate goods imported from different counties. Let $X_{ni,t}$ denote the amount of intermediate good that country n buys from country i in period t. The total output of final aggregate in country n at time t, Y_{nt}, is given by

$$Y_{nt} = S_{Y,nt} \left[\sum_{i=1}^{N} \left(\omega_{ni} X_{ni,t} \right)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

where $\omega_{ni} \geq 0$ are exogenous importer-exporter specific weights, $\sigma > 0$ is an Armington elasticity of substitution between intermediate goods produced in different countries, and

$$S_{Y,nt} \equiv \Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}} \right)^{\psi_Y} \tag{3}$$

is aggregate productivity with $P_{Y,nt}$ being the price of the final aggregate.[8] As in production of intermediates, productivity in production of the final aggregate has two parts: exogenous productivity, $\Theta_{Y,n} Z_{Y,nt}$, and endogenous productivity, $\left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}} \right)^{\psi_Y}$ with ψ_Y driving the strength of external economies of scale in production of the final aggregate. The term $Z_{Y,nt}$ is an aggregate productivity shock. We do not put any restrictions on its correlation with the shock $Z_{X,nt}$ in the intermediate goods sector. The term $\Theta_{Y,n}$ is a normalization constant introduced for convenience. The endogenous part of $S_{Y,nt}$ captures external economies of scale in production of the final aggregate, and it is taken by firms as given. $\left(P_{Y,nt} Y_{nt} \right) / W_{nt}$ is the number of country-n's workers that produce the same value as the value of the final aggregate.[9]

Perfect competition in production of the final aggregate implies that the price of the final aggregate, $P_{Y,nt}$, is given by

$$P_{Y,nt} = \frac{\left[\sum_{i=1}^{N} \left(\tau_{ni,t} P_{X,it} / \omega_{ni} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}}}{\Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}} \right)^{\psi_Y}}, \tag{4}$$

[8] Recall that we assume that labor is perfectly mobile within a country between sectors where it is used. So, there is only one wage per country.

[9] The particular form in which the externality in production of the final aggregate is introduced is chosen to later show isomorphism with the Melitz model. This term appears in the Melitz model because of the fixed costs of serving markets that are paid in terms of the destination country labor.

and country n's share of expenditure on country i's intermediate good is given by

$$\lambda_{ni,t} = \frac{\left(\tau_{ni,t} P_{X,it} / \omega_{ni}\right)^{1-\sigma}}{\sum_{j=1}^{N} \left(\tau_{nj,t} P_{X,jt} / \omega_{nj}\right)^{1-\sigma}}. \tag{5}$$

Final aggregate in country n is used directly as the consumption good in this country as well as in the production process of the investment good, which we describe next.

2.3 Investment Goods

Let I_{nt} denote the total output of the investment good in country n in period t, and $P_{I,nt}$ the price of this good. Investment good is produced from labor and the final aggregate with the production technology given by

$$I_{nt} = \Theta_{I,n} Z_{I,nt} L_{I,nt}^{\alpha_I} Y_{I,nt}^{1-\alpha_I}, \tag{6}$$

where $0 \leq \alpha_I \leq 1$. Here $L_{I,nt}$ and $Y_{I,nt}$ are the total amounts of labor and final aggregate used in production of the investment good, $Z_{I,nt}$ is an exogenous aggregate productivity shock, and $\Theta_{I,n}$ is a normalization constant introduced for convenience. We do not put any restrictions on correlation of $Z_{I,nt}$ with the shocks $Z_{X,nt}$ and $Z_{Y,nt}$ in the intermediate and final goods sectors.[10]

Perfect competition in production of the investment good implies

$$L_{I,nt} = \alpha_I \frac{P_{I,nt} I_{nt}}{W_{nt}}, \quad \text{and} \quad Y_{I,nt} = (1-\alpha_I) \frac{P_{I,nt} I_{nt}}{P_{Y,nt}}.$$

Moreover,

$$P_{I,nt} = \frac{W_{nt}^{\alpha_I} P_{Y,nt}^{1-\alpha_I}}{\widetilde{\Theta}_{I,n} Z_{I,nt}}, \tag{7}$$

where $\widetilde{\Theta}_{I,n} \equiv \alpha_I^{\alpha_I} (1-\alpha_I)^{1-\alpha_I} \Theta_{I,n}$.

2.4 Households

Each country n has a representative household with the period-t utility function given by $U(C_{nt}, L_{nt})$, where C_{nt} and L_{nt} are the household's consumption and supply of labor in

[10]In the standard business cycles model, investment is made directly from the final good. This standard technology can be obtained from (6) by setting $\Theta_{I,n} = 1$, $Z_{I,nt} = 1$, and $\alpha_I = 0$. As we will see later, the technology for producing the investment good in the standard versions of Krugman and Melitz models corresponds to setting $\alpha_I = 1$ and having $\Theta_{I,n} Z_{I,nt} \neq 1$. These differing choices can have non-trivial implications for the cyclicality of net exports as we show later and therefore, we take a general approach.

period t. The household chooses consumption, supply of labor, investment, and holdings of financial assets (if allowed) so as to maximize the expected sum of discounted utilities, $E_t \sum_{s=0}^{\infty} \beta^s U(C_{n,t+s}, L_{n,t+s})$, subject to the budget constraint and the law of motion of capital, where $\beta \in (0,1)$ is the discount factor, and E_t denotes the expectation over the states of nature taken in period t. The law of motion of capital is given by

$$K_{n,t+1} = (1-\delta) K_{nt} + I_{nt},$$

where I_{nt} is the household's choice of investment in period t, and $\delta \in [0,1]$ is the capital depreciation rate. Depending on the international financial markets structure, households face different budget constraints. Below we consider three standard alternatives for international financial markets: financial autarky, bond economy, and complete markets.

2.4.1 Financial Autarky

In the case of financial autarky, there is no international trade in financial assets. Households in country n then face the following flow budget constraint

$$P_{Y,nt} C_{nt} + P_{I,nt} I_{nt} = W_{nt} L_{nt} + R_{nt} K_{nt}.$$

Observe that, since the consumption good is directly produced from the final aggregate (and there are no shocks in the consumption goods sector), the price of the consumption good is equal to the price of the final aggregate, $P_{Y,nt}$.

First-order conditions for the household's optimization problem are given by

$$P_{I,nt} = \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1(C_{n,t+1}, L_{n,t+1})}{U_1(C_{nt}, L_{nt})} \left[R_{n,t+1} + (1-\delta) P_{I,n,t+1} \right] \right\}, \tag{8}$$

$$-\frac{U_2(C_{nt}, L_{nt})}{U_1(C_{nt}, L_{nt})} = \frac{W_{nt}}{P_{Y,nt}}, \tag{9}$$

where $U_1(\cdot,\cdot)$ and $U_2(\cdot,\cdot)$ are derivatives of the utility function with respect to consumption and labor, correspondingly. Condition (8) is the standard Euler equation that equates the price of investment today with the expected price of investment tomorrow. Condition (9) equates the marginal rate of substitution between consumption and labor with real wage.

2.4.2 Bond Economy

We consider a bond economy where each country issues a non-state-contingent bond denominated in its consumption units. The representative households in each country chooses holdings of bonds of all countries. Holdings of country i's bond by country n are denoted by $B_{ni,t}$. The household's flow budget constraint is given by

$$P_{Y,nt}C_{nt} + P_{I,nt}I_{nt} + \sum_{i=1}^{N} P_{Y,it}\left(B_{ni,t} + \frac{b_{adj}}{2}B_{ni,t}^2\right)$$
$$= W_{nt}L_{nt} + R_{nt}K_{nt} + \sum_{i=1}^{N} P_{Y,it}\left(1 + r_{i,t-1}\right)B_{ni,t-1} + T_{nt}^B,$$

where $r_{i,t-1}$ is period-t return on country-i's bond, and $T_{nt}^B \equiv \frac{b_{adj}}{2}\sum_{i=1}^{N} P_{Y,it}B_{ni,t}^2$ is the bond fee rebate, taken as given by the household. Here b_{adj} is the adjustment cost of bond holdings, which is introduced to ensure stationarity. First-order conditions are given by conditions (8) and (9), plus an additional set of Euler equations:

$$P_{Y,it}\frac{U_1\left(C_{nt}, L_{nt}\right)}{P_{Y,nt}}\left(1 + b_{adj}B_{ni,t}\right) = \beta E_t\left\{\frac{U_1\left(C_{n,t+1}, L_{n,t+1}\right)}{P_{Y,n,t+1}}P_{Y,i,t+1}\left(1 + r_{it}\right)\right\},$$

for $i = 1, \dots, N$.

International trade in bonds allows unbalanced trade in intermediate goods. Define country n's trade balance TB_{nt} as the value of net exports of intermediate goods:

$$\text{TB}_{nt} \equiv P_{X,nt}X_{nt} - P_{Y,nt}Y_{nt},$$

and define country n's current account CA_{nt} as the change in this country's net financial assets position:[11]

$$\text{CA}_{nt} \equiv \sum_{i=1}^{N} P_{Y,it}\left(B_{ni,t} - B_{ni,t-1}\right).$$

[11] Using markets clearing conditions (described later), it can be shown that trade balance and current account can also be written as

$$\text{TB}_{nt} = W_{nt}L_{nt} + R_{nt}K_{nt} - P_{Y,nt}C_{nt} - P_{I,nt}I_{nt}, \text{ and } \text{CA}_{nt} = \text{TB}_{nt} + \sum_{i=1}^{N} r_{i,t-1}P_{Y,it}B_{ni,t-1}.$$

2.4.3 Complete Financial Markets

To introduce the household's budget constraint in the case of complete markets, we employ notation for the states of nature in period t, denoted by s_t, and history of states in period t, denoted by s^t. In each state with history s^t, countries trade a complete set of state-contingent nominal bonds denominated in the numeraire currency. Let $B_{n,t+1}\left(s^t, s_{t+1}\right)$ denote the amount of the nominal bond with return in state s_{t+1} that country n acquires in the state with history s^t. Assuming that there are no costs of trading currency or securities between countries, we can denote by $P_{B,t}\left(s^t, s_{t+1}\right)$ the international price of this bond in the state with history s^t. Country n's budget constraint is given by

$$\begin{aligned} &P_{Y,nt}\left(s^t\right) C_{nt}\left(s^t\right) + P_{I,nt}\left(s^t\right) I_{nt}\left(s^t\right) + A_{nt}\left(s^t\right) \\ &\qquad = W_{nt}\left(s^t\right) L_{nt}\left(s^t\right) + R_{nt}\left(s^t\right) K_{nt}\left(s^t\right) + B_{nt}\left(s^t\right), \end{aligned}$$

where

$$A_{nt}\left(s^t\right) \equiv \sum_{s_{t+1}} P_{B,t}\left(s^t, s_{t+1}\right) B_{n,t+1}\left(s^t, s_{t+1}\right)$$

is country n's net foreign assets position in period t. First-order conditions in the case of complete markets are given by conditions (8) and (9) (with the state-dependent notation added to them), plus an additional set of conditions:

$$\begin{aligned} P_{B,t}\left(s^t, s_{t+1}\right) &= \beta \frac{\pi_{t+1}\left(s^{t+1}\right)}{\pi_t\left(s^t\right)} \cdot \frac{P_{Y,nt}\left(s^t\right)}{P_{Y,n,t+1}\left(s^{t+1}\right)} \cdot \frac{U_1\left(C_{n,t+1}\left(s^{t+1}\right), L_{n,t+1}\left(s^{t+1}\right)\right)}{U_1\left(C_{nt}\left(s^t\right), L_{nt}\left(s^t\right)\right)}, \\ Q_{ni,t}\left(s^t\right) &= \kappa_{ni} \frac{U_1\left(C_{nt}\left(s^t\right), L_{nt}\left(s^t\right)\right)}{U_1\left(C_{it}\left(s^t\right), L_{it}\left(s^t\right)\right)}, \quad \text{for each } i, \end{aligned}$$

where $\pi_t\left(s^t\right)$ is the probability of history s^t occurring in period t,

$$Q_{ni,t}\left(s^t\right) \equiv \frac{P_{Y,nt}\left(s^t\right)}{P_{Y,it}\left(s^t\right)}$$

is the real exchange rate, and

$$\kappa_{ni} \equiv \left(\frac{U_1\left(C_{n0}\left(s^0\right), L_{n0}\left(s^0\right)\right) / P_{Y,n0}\left(s^0\right)}{U_1\left(C_{i0}\left(s^0\right), L_{i0}\left(s^0\right)\right) / P_{Y,i0}\left(s^0\right)} \right)^{-1}.$$

By dropping the state-dependent notation, we can write the conditions compactly as

$$P_{Y,nt}C_{nt} + P_{I,nt}I_{nt} + A_{nt} = W_{nt}L_{nt} + R_{nt}K_{nt} + B_{nt},$$
$$A_{nt} = \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1\left(C_{n,t+1}, L_{n,t+1}\right)}{U_1\left(C_{nt}, L_{nt}\right)} B_{n,t+1} \right\},$$
$$Q_{ni,t} = \kappa_{ni} \frac{U_1\left(C_{nt}, L_{nt}\right)}{U_1\left(C_{it}, L_{it}\right)}, \quad \text{for each } i. \tag{10}$$

Condition (10) is the standard Backus-Smith condition that says that the real exchange co-moves with the ratio of marginal utilities. As in the case of the bond economy, trade balance is defined as net exports of intermediate goods, and current account is defined as the change in net foreign assets position,

$$\text{TB}_{nt} = P_{X,nt}X_{nt} - P_{Y,nt}Y_{nt},$$
$$\text{CA}_{nt} = A_{nt} - A_{n,t-1}.$$

2.5 Market Clearing Conditions

The labor market clearing condition is given by

$$W_{nt}L_{X,nt} + W_{nt}L_{I,nt} = W_{nt}L_{nt} + a\text{TB}_{nt}, \quad \text{for } n = 1, \ldots, N, \tag{11}$$

where a is a constant. When $a = 0$, we have a standard labor market clearing condition. The extra term $a\text{TB}_{nt}$ is introduced to later show isomorphism with the Melitz model, for which $a > 0$, and for which this term appears only if trade is unbalanced. The rest of the market clearing conditions for the economy are standard. Since capital is used only in production of intermediate goods, we have

$$K_{X,nt} = K_{nt}, \quad \text{for } n = 1, \ldots, N.$$

The final aggregate is used in consumption and production of the investment good

$$C_{nt} + Y_{I,nt} = Y_{nt} \quad \text{for } n = 1, \ldots, N.$$

Demand for intermediate goods is equal to supply

$$\sum_{n=1}^{N} \tau_{ni,t} X_{ni,t} = X_{it}, \quad \text{for } i = 1, \ldots, N,$$

In the case of the bond economy and complete markets we also have the sets of bond market clearing conditions, which are given by

$$\sum_{n=1}^{N} B_{ni,t} = 0, \quad \text{for } i = 1, \ldots, N,$$

for the bond economy, and by

$$\sum_{n=1}^{N} A_{nt} = 0$$

for complete markets.

For convenience, the full set of equilibrium conditions is provided in Appendix A.1.

2.6 Discussion

The unified model described in this section is a generalization of the standard real business cycles model studied in the previous literature. For example, a two-country model studied by Heathcote and Perri (2002) can be obtained as a special case of the unified model by shutting down externalities, requiring that capital investment uses the final aggregate only (i.e., it does not use labor), leaving exogenous shocks only in production of intermediate goods, and dropping the additional term $a\text{TB}_{nt}$ in the labor market clearing condition. Formally, this requires setting $\psi_{X,K} = \psi_{X,L} = \psi_Y = 0$, $\alpha_I = 0$, $Z_{Y,nt} = Z_{I,nt} = 1$, $\Theta_{X,n} = \Theta_{Y,n} = \Theta_{I,n} = 1$, and $a = 0$. We further need to remove iceberg trade costs (i.e., set $\tau_{ni,t} = 1$) in order to obtain exactly the environment considered by Heathcote and Perri (2002).

3 Generalized Versions of the Standard Trade Models

We next present the key elements of generalized dynamic versions of the workhorse international trade models: Eaton-Kortum, Krugman, and Melitz. The focus of this section is to present the elements of these models that differ from their standard expositions, as they appear in the literature. Thus, our presentation omits all the derivations, which are provided in Appendix B. Perceiving isomorphisms between the unified, Eaton-Kortum, Krugman, and Melitz models, we use the same notation for parameters and variables of these models that map into each other. To mark some of the parameters and variables as being specific to a particular model, we use superscripts "EK" for the Eaton-Kortum model, "K" for the Krugman model, and "M" for the Melitz model.

3.1 Generalized Dynamic Version of the Eaton-Kortum Model

Household's problem is identical to the one in the unified model. Moreover, as in the unified model, the production side consists of intermediate, final, consumption, and investment goods. All markets are perfectly competitive. The intermediate goods sector here is different from the intermediate goods sector in the unified model — it consists of a continuum of varieties indexed by $\nu \in [0,1]$. Any country has a technology to produce any of the varieties $\nu \in [0,1]$. The production technology of variety ν in country n in period t is given by

$$x_{nt}(\nu) = S_{X,nt} z_n(\nu) k_{X,nt}(\nu)^{\alpha_{X,K}} l_{X,nt}(\nu)^{\alpha_{X,L}},$$

where $k_{X,nt}(\nu)$ and $l_{X,nt}(\nu)$ are capital and labor used in production of variety ν, $z_n(\nu)$ is the efficiency of production of variety ν, and $S_{X,nt} \equiv \Theta_{X,n} Z_{X,nt} K_{X,nt}^{\psi_{X,K}} L_{X,nt}^{\psi_{X,L}}$ is aggregate productivity. All terms of $S_{X,nt}$ have similar meanings as the corresponding terms of the aggregate productivity in the intermediate goods sector in the unified model given by expression (1). In particular, $K_{X,nt}$ and $L_{X,nt}$ denote total amounts of capital and labor used in production of all varieties in country n in period t.[12] As in the unified model, aggregate productivity $S_{X,nt}$ captures external economies of scale in the production of varieties and is taken by firms as given.

Efficiencies $z_n(\nu)$ are drawn from the Fréchet distribution given by its cumulative distribution function

$$Prob\left[z_{nt}(\nu) \leq z\right] = e^{-z^{-\theta^{EK}}}.$$

Varieties are traded. Trade is costly and is subject to iceberg trade costs $\tau_{ni,t}$.

Varieties are combined into the non-tradeable final aggregate:

$$Y_{nt} = S_{Y,nt}^{EK} \left[\int_0^1 \left[\sum_{i=1}^{N} \omega_{ni} x_{ni,t}(\nu) \right]^{\frac{\sigma^{EK}-1}{\sigma^{EK}}} d\nu \right]^{\frac{\sigma^{EK}}{\sigma^{EK}-1}},$$

where $x_{ni,t}(\nu)$ is the amount of variety ν that country n buys from country i in period t, $\omega_{ni} \geq 0$ are exogenous importer-exporter specific weights, and, similarly to the unified model,

$$S_{Y,nt}^{EK} \equiv \Theta_{Y,n}^{EK} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}} \right)^{\psi_Y}$$

[12] This production technology generalizes the production technology used in Kucheryavyy *et al.* (2017) by introducing capital in addition to labor as a factor of production and adding capital externality in addition to labor externality. This generalization is a natural extension of the static environment of Kucheryavyy *et al.* (2017) with no capital to the dynamic environment of the current paper with capital accumulation.

is aggregate productivity. All terms of $S^{\text{EK}}_{Y,nt}$ have similar meanings as the corresponding terms of the aggregate productivity in the final goods sector in the unified model given by expression (3). Production function for Y_{nt} implies that varieties produced by different countries are perfect substitutes in production of the final aggregate. Hence, producers of the final aggregate in country n buy each variety ν from the cheapest source (taking into account taste parameters ω_{ni}). We can then derive the price of the final aggregate

$$P_{Y,nt} = \frac{\left[\sum_{i=1}^{N}\left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{-\theta^{\text{EK}}}\right]^{-\frac{1}{\theta^{\text{EK}}}}}{\Theta_{Y,n} Z_{Y,nt}\left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}}\right)^{\psi_{Y}}}, \tag{12}$$

where $\Theta_{Y,n} \equiv \Gamma\left(\frac{\theta^{\text{EK}}+1-\sigma^{\text{EK}}}{\theta^{\text{EK}}}\right)^{\frac{1}{\sigma^{\text{EK}}-1}} \Theta^{\text{EK}}_{Y,n}$ with $\Gamma(\cdot)$ denoting the gamma-function, and

$$P_{X,it} \equiv \frac{R_{it}^{\alpha_{X,K}} W_{it}^{\alpha_{X,L}}}{\widetilde{\Theta}_{X,i} Z_{X,it} K_{X,it}^{\psi_{X,K}} L_{X,it}^{\psi_{X,L}}}, \tag{13}$$

with $\widetilde{\Theta}_{X,i} \equiv \alpha_{X,K}^{\alpha_{X,K}} \alpha_{X,L}^{\alpha_{X,L}} \Theta_{X,i}$. Price $P_{X,it}$ can be interpreted as the price of the output of varieties in country i in period t. The expenditure share of country n on varieties produced in country i is similar to the corresponding expression (5) in the unified model and is given by

$$\lambda_{ni,t} = \frac{\left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{-\theta^{\text{EK}}}}{\sum_{j=1}^{N}\left(\tau_{nj,t} P_{X,jt}/\omega_{nj}\right)^{-\theta^{\text{EK}}}}.$$

The final aggregate is used for consumption and investment. As in the unfed model, the consumption good is directly produced from the final good, and so the price of the consumption good in country n is $P_{Y,nt}$. The technology of production of the investment good is also assumed to be the same as in the unified model, i.e., it assumed to be given by expression (6). Hence, the price of the investment good is the same as in the unified model and is given by

$$P_{I,nt} = \frac{W_{nt}^{\alpha_I} P_{Y,nt}^{1-\alpha_I}}{\widetilde{\Theta}_{I,n} Z_{I,nt}} \tag{14}$$

The complete set of equilibrium conditions for the generalized Eaton-Kortum model is provided in Appendix B.1.

3.1.1 Discussion

A straightforward extension of the standard static version of the Eaton-Kortum model to a dynamic version with intertemporal investment decisions — along the lines of, for example, Eaton *et al.* (2016) — can be obtained from the generalized Eaton-Kortum model by shutting down externalities, requiring that capital investment uses the final aggregate only, and leaving shocks only in production of varieties. Formally, this is achieved by setting $\psi_{X,K} = \psi_{X,L} = \psi_{Y} = 0$, $\alpha_{I} = 0$, and $Z_{Y,nt} = Z_{I,nt} = 1$.

3.2 Generalized Dynamic Version of the Krugman Model

Production side of the Krugman model is different from the unified and Eaton-Kortum models: production of intermediate goods uses only labor, intermediate good producers are engaged in monopolistic competition and pay sunk costs of entry into the economy. We describe the Krugman model in the following subsection.

3.2.1 Production of Varieties, International Trade, and Final Aggregate

Each country i produces a unique set of varieties Ω_{it}, which is endogenously determined in every period t. Let M_{it} be the measure of this set. All varieties can be internationally traded. Let $p_{ni,t}(\nu)$ denote the price of variety $\nu \in \Omega_{it}$ produced by country i and sold in country n. Assuming iceberg trade costs and no arbitrage in international trade, we have that $p_{ni,t}(\nu) = \tau_{ni,t} p_{ii,t}(\nu)$.

Countries use varieties to produce non-traded final aggregates. Technology of production of the final aggregate in country n is given by the nested CES production function

$$Y_{nt} = S_{Y,nt} \left[\sum_{i=1}^{N} \left[M_{it}^{\phi_{Y,M} - \frac{1}{\sigma^K - 1}} \left[\int_{\nu \in \Omega_{it}} \left(\omega_{ni} x_{ni,t}(\nu) \right)^{\frac{\sigma^K - 1}{\sigma^K}} d\nu \right]^{\frac{\sigma^K}{\sigma^K - 1}} \right]^{\frac{\eta^K - 1}{\eta^K}} \right]^{\frac{\eta^K}{\eta^K - 1}}, \tag{15}$$

where $x_{ni,t}(\nu)$ is the amount of variety $\nu \in \Omega_{it}$ that country n buys from country i in period t, $\omega_{ni} \geq 0$ are exogenous importer-exporter specific weights, and

$$S_{Y,nt} \equiv \Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}} \right)^{\psi_{Y}}.$$

All terms of $S_{Y,nt}$ have the same meaning as in the corresponding definition (3) in the unified model. The nested CES structure of (15) implies that the elasticity of substitution between varieties produced in one country, given by σ^{K}, is different from the elasticity of substitution between varieties produced in different countries, given by η^{K}.[13] We assume that $\sigma^{K} > 1$ and $\eta^{K} > 1$. The term $M_{it}^{\phi_{Y,M}-\frac{1}{\sigma^{K}-1}}$ introduces correction for the love-of-variety effect, which is the only source of externalities in the standard Krugman model with CES preferences. As is discussed in Benassy (1996), parameter $\phi_{Y,M}$ governs the taste for variety in the Krugman model (the standard Krugman model implies that the strength of the taste for variety is $1/\left(\sigma^{K}-1\right)$). At the same time, as we shall see later, in the unified model, parameter $\phi_{Y,M}$ governs the strength of economies of scale induced by capital in production of intermediate goods. Having this parameter is critical for showing the full isomorphism with the unified model.

Assuming perfect competition in production of the final aggregate, we get the usual CES demand:

$$x_{ni,t}(\nu) = S_{Y,nt}^{\eta^{K}-1} M_{it}^{\left(\sigma^{K}-1\right)\left(\phi_{Y,M}-\frac{1}{\sigma^{K}-1}\right)} \omega_{ni}^{\sigma^{K}-1} \left(\frac{p_{ni,t}(\nu)}{P_{ni,t}}\right)^{-\sigma^{K}} \left(\frac{P_{ni,t}}{P_{Y,nt}}\right)^{-\eta^{K}} Y_{nt}, \tag{16}$$

$$P_{ni,t} = M_{it}^{-\left(\phi_{Y,M}-\frac{1}{\sigma^{K}-1}\right)} \left[\int_{\nu\in\Omega_{it}} \left(p_{ni,t}(\nu)/\omega_{ni}\right)^{1-\sigma^{K}} d\nu\right]^{\frac{1}{1-\sigma^{K}}}, \tag{17}$$

$$P_{Y,nt} = S_{Y,nt}^{-1} \left[\sum_{i=1}^{N} P_{ni,t}^{1-\eta^{K}}\right]^{\frac{1}{1-\eta^{K}}}. \tag{18}$$

Production of variety $\nu \in \Omega_{nt}$ requires only labor and is given by

$$x_{nt}(\nu) = S_{X,nt}^{K} l_{nt}(\nu), \tag{19}$$

where $l_{nt}(\nu)$ is the amount of labor used in production of variety ν, and $S_{X,nt}^{K} \equiv \Theta_{X,n} Z_{X,nt} L_{X,nt}^{\phi_{X,L}}$ is the aggregate productivity in production of varieties. The aggregate productivity $S_{X,nt}^{K}$ consists of two parts: exogenous productivity, $\Theta_{X,n} Z_{X,nt}$, and endogenous productivity, $L_{X,nt}^{\phi_{X,L}}$. Here $\Theta_{X,n}$ is a normalization constant, $Z_{X,nt}$ is an exogenous shock, and $L_{X,nt}$ is the total amount of labor allocated to production of varieties in country n in period t. The endogenous part of the aggregate productivity is an additional source of external economies of scale (on top of the love-of-variety effect) and is taken by firms as given. Having this

[13] A combination of the nested CES production technology with the monopolistic competition environment is also used in Alessandria and Choi (2007), Fattal Jaef and Lopez (2014), Feenstra *et al.* (2018), and Kucheryavyy *et al.* (2017), among others. As Kucheryavyy *et al.* (2017) show, interpreted through the lens of a competitive framework with external economies of scale, having $\eta^{K} \neq \sigma^{K}$ in the static environment allows one to separate the value of trade elasticity, given by $1-\eta^{K}$, from the strength of economies of scale induced by labor and given by $1/\left(\sigma^{K}-1\right)$. In the dynamic environment of the current paper, having $\eta^{K} \neq \sigma^{K}$ allows us to separate the trade elasticity, also given by $1-\eta^{K}$, from the share of labor used in production of the intermediate good, given by $1-1/\sigma^{K}$.

additional source of externality is critical for showing the full isomorphism with the unified model.

Producers of varieties ν are engaged in monopolistic competition. Hence, the price of variety $\nu \in \Omega_{it}$ is

$$p_{ni,t}(\nu) = \frac{\sigma^{\kappa}}{\sigma^{\kappa}-1} \cdot \frac{\tau_{ni,t} W_{it}}{S^{\kappa}_{X,it}},$$

the bilateral price index is $P_{ni,t} = \tau_{ni,t} P_{X,it}/\omega_{ni}$, where

$$P_{X,it} \equiv \frac{\sigma^{\kappa}}{\sigma^{\kappa}-1} \cdot \frac{W_{it}}{\Theta_{X,i} Z_{X,it} M_{it}^{\phi_{Y,M}} L_{X,it}^{\phi_{X,L}}}, \tag{20}$$

and the share of expenditure of country n on country i's varieties is

$$\lambda_{ni,t} = \frac{\left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{1-\eta^{\kappa}}}{\sum_{j=1}^{N} \left(\tau_{nj,t} P_{X,jt}/\omega_{nj}\right)^{1-\eta^{\kappa}}}. \tag{21}$$

Substituting expression for $P_{ni,t}$ into (18), we get

$$P_{Y,nt} = \frac{\left[\sum_{i=1}^{N} \left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{1-\eta^{\kappa}}\right]^{\frac{1}{1-\eta^{\kappa}}}}{\Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}}\right)^{\psi_{Y}}}. \tag{22}$$

Similarly to the price of intermediates in the generalized Eaton-Kortum model, $P_{X,it}$ here can be interpreted as the price of the output of varieties in country i in period t.

Let $\mathcal{X}_{nt}$ denote the value of total output of varieties in country n in period t, and D_{nt} denote the average profit of country n's producers of varieties Ω_{nt}. We have

$$\mathcal{X}_{nt} = \frac{\sigma^{\kappa}}{\sigma^{\kappa}-1} W_{nt} L_{X,nt}, \quad \text{and} \quad D_{nt} = \frac{1}{\sigma^{\kappa}} \cdot \frac{\mathcal{X}_{nt}}{M_{nt}}.$$

3.2.2 Entry and Exit of Producers of Varieties

In order to enter the economy, producer of a variety in country n in period t needs to pay sunk cost equal to $\frac{W_{nt}^{\alpha_I} P_{Y,nt}^{1-\alpha_I}}{\widetilde{\Theta}_{I,n} Z_{I,nt}}$, where $0 \leq \alpha_I \leq 1$, and $\widetilde{\Theta}_{I,n} Z_{I,nt}$ is an exogenous cost shifter. Paying this sunk cost involves hiring $L_{I,nt} = \alpha_I \frac{V_{nt}}{W_{nt}}$ units of labor and using $Y_{I,nt} = (1-\alpha_I) \frac{V_{nt}}{P_{Y,nt}}$ units of the final aggregate, where V_{nt} is the value of a variety in country n

in period t.[14]

In every period t, each country has an unbounded mass of prospective entrants (firms) into the production of varieties. Entry into the economy is free, and, therefore, the value of a variety is equal to the sunk cost of entry:

$$V_{nt} = \frac{W_{nt}^{\alpha_I} P_{Y,nt}^{1-\alpha_I}}{\widetilde{\Theta}_{I,n} Z_{I,nt}}. \tag{23}$$

Timing is as follows. Firms entering in period t start producing in the next period. At the end of each period t, an exogenous fraction δ of the total mass of firms (i.e., a fraction δ of M_{nt}) exits. The probability of exit is the same for all firms regardless of their age. Since exit occurs at the end of a period, any firm that entered into the economy produces for at least one period. Let $M_{I,nt}$ denote the number of producers of varieties that enter into the country n's economy in period t. Given the described process of entry and exit of firms, the law of motion of varieties is

$$M_{n,t+1} = (1-\delta)\, M_{nt} + M_{I,nt}. \tag{24}$$

All producers of varieties are owned by households. We turn next to their problem.

3.2.3 Households

Similarly to the unified model, households in country n maximize expected sum of discounted utilities, $E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_{nt}, L_{nt}\right)$, by choosing consumption C_{nt}, supply of labor L_{nt}, the number of new varieties $M_{I,nt}$, and holdings of financial assets (if allowed). Constraints faced by the households are the budget constraint and the law of motion of varieties given by (24). The specification of the budget constraint depends on the financial markets structure, as in Section 2. In the case of financial autarky the budget constraint is given by

$$P_{Y,nt} C_{nt} + V_{nt} M_{I,nt} = W_{nt} L_{nt} + D_{nt} M_{nt}.$$

The left-hand side of this expression contains household's expenditure in period t: the household spends its budget on consumption and entry of new firms. The right-hand side of this expression contains household's income in period t: it consists of labor income and profits of firms. In the case of the bond economy and complete markets the budget

[14] In Appendix B.2 we derive the sunk cost by introducing an R&D sector and specifying an invention process for new varieties. Labor and final aggregate needed to pay the sunk cost of entry are interpreted as the production factors used in the R&D sector for the invention of varieties.

constraints can be written by adding the expenditure and income from financial assets in the same manner as it is done in the unified model in Section 2.

3.2.4 Markets Clearing Conditions

All market clearing conditions are standard. Labor is used for production and invention of varieties,

$$L_{X,nt} + L_{I,nt} = L_{nt},$$

demand for varieties is equal to supply,

$$\sum_{n=1}^{N} \lambda_{ni,t} P_{Y,nt} Y_{nt} = \mathcal{X}_{it},$$

and the final aggregate is used for consumption and invention of varieties,

$$C_{nt} + Y_{I,nt} = Y_{nt}.$$

The complete set of equilibrium conditions for the generalized Krugman model is provided in Appendix B.2.

3.2.5 Discussion

A dynamic version of the standard Krugman model — which can be obtained by, for example, a straightforward extension of Bilbiie *et al.* (2012) to a multi-country environment — can be obtained from the generalized Krugman model by removing correction for the love of variety, shutting down external economies of scale, requiring that producers of varieties pay entry costs in terms of labor only, and removing the exogenous shock in production of the final aggregate. Formally, this is achieved by setting $\phi_{Y,M} = \frac{1}{\sigma^{\kappa} - 1}$, $\phi_{X,L} = \psi_Y = 0$, $\alpha_I = 1$, and $Z_{Y,nt} = 1$.

3.3 Generalized Dynamic Version of the Melitz Model

Production side of the Melitz model is similar to the production side of the Krugman model in using only labor in production of intermediate goods, featuring monopolistic competition, and having sunk costs of entry into the economy. Additional features of the Melitz model are heterogeneous firms with Pareto distribution of efficiencies of production and the requirement that firms pay fixed costs of serving markets.

3.3.1 Production of Varieties, International Trade, and Final Aggregate

In every period t, country i can produce any of the varieties from an endogenously determined set of varieties Ω_{it} with measure M_{it}. All varieties from the set Ω_{it} can be internationally traded, but not all of them are available in a particular country n. The subset of country-i's varieties available in country n is denoted by $\Omega_{ni,t}$ (with $\Omega_{ni,t} \subseteq \Omega_{it}$), and its measure is denoted by $M_{ni,t}$. Subsets of varieties $\Omega_{ni,t}$ are endogenously determined. Importantly, only a subset $\Omega_{ii,t}$ of the whole set of varieties Ω_{it} is available in the domestic market i, and, generally, some varieties from Ω_{it} are not available in any country (i.e., some varieties from Ω_{it} are not produced in period t). In general it can happen that some varieties from Ω_{it} are available in country $n \neq i$, but not in country i. In other words, generally it can be the case that $\Omega_{ni,t} \nsubseteq \Omega_{ii,t}$.

In order to sell in the country-n's market, a country-i's producer of a variety has to pay two types of costs: the usual per-unit iceberg trade costs $\tau^{\text{M}}_{ni,t}$ and fixed cost $\Phi_{ni,t} > 0$, which are paid in terms of country-n's labor. The fixed cost $\Phi_{ni,t}$ is an endogenous object. Its formal definition is introduced later.

As in the Krugman model, countries combine varieties to produce non-traded final aggregates using the nested CES technology,

$$Y_{nt} = \left[\sum_{i=1}^{N} \left[\int_{\nu \in \Omega_{ni,t}} \left(\omega_{ni} x_{ni,t} \left(\nu \right) \right)^{\frac{\sigma^{\text{M}}-1}{\sigma^{\text{M}}}} d\nu \right]^{\frac{\sigma^{\text{M}}}{\sigma^{\text{M}}-1} \cdot \frac{\eta^{\text{M}}-1}{\eta^{\text{M}}}} \right]^{\frac{\eta^{\text{M}}}{\eta^{\text{M}}-1}} . \tag{25}$$

Differently from the Krugman model, we do not add correction for the love-of-variety effect in (25) — the reasons for this are discussed below in Section 4.1 (also, in Appendix B.3 we introduce the correction for the love-of-variety effect and formally explore implications of this correction). Also, (25), differently from (15), does not have an exogenous shock and external economies of scale. The reason for this is that the structure of the Melitz model endogenously generates both the exogenous shock and externalities in production of the final aggregate — both of these components of production function come from the fixed costs of serving markets, which are introduced below.

Perfect competition in production of the final aggregate implies the usual expressions for the CES demand that are almost the same as the corresponding expressions (16)-(18) in the Krugman model, except for there is no term correcting for the love of variety.

Production technology of variety $\nu \in \Omega_{it}$ is given by $x_{it} \left(\nu \right) = S^{\text{M}}_{X,it} z_i \left(\nu \right) l_{it} \left(\nu \right)$, where $l_{it} \left(\nu \right)$ is the amount of labor used in production of ν, $z_i \left(\nu \right)$ is the efficiency of production of ν, and $S^{\text{M}}_{X,it} \equiv \Theta^{\text{M}}_{X,i} Z_{X,it} \left[L^{\text{M}}_{X,it} \right]^{\phi_{X,L}}$ is the aggregate productivity in production of varieties, with $L^{\text{M}}_{X,it}$ being the total amount of labor used in production of varieties in country i. As in the Krugman model, $S^{\text{M}}_{X,it}$ features external economies of scale and is taken by firms as given. Monopolistic competition in production of varieties implies that the price of variety $\nu \in \Omega_{ni,t}$ is given by

$$p_{ni,t}(\nu) = \frac{\sigma^{\text{M}}}{\sigma^{\text{M}} - 1} \cdot \frac{\tau^{\text{M}}_{ni,t} W_{it}}{S^{\text{M}}_{X,it} z_i(\nu)}.$$

3.3.2 Entry and Exit of Producers of Varieties

This part of the Melitz model is almost the same as the corresponding part of the Krugman model with one important difference that, upon entry, producer of a new variety in country n gets an idiosyncratic draw of efficiency of production, $z_n(\nu)$, from the Pareto distribution given by its cumulative distribution function with shape θ^{M} and minimal efficiency $z_{min,n}$,

$$G_n(z) \equiv Prob\left[z_n(\nu) \le z\right] = 1 - \left(\frac{z_{min,n}}{z}\right)^{\theta^{\text{M}}}.$$

As in the Krugman model, the expected value of entry (before drawing the efficiency of production) is denoted by V_{nt}. The sunk cost of entry is equal to $\frac{W^{\alpha_I}_{nt} P^{1-\alpha_I}_{Y,nt}}{\widetilde{\Theta}_{I,n} Z_{I,nt}}$. Assuming that entry is free, the sunk cost of entry is equalized with the expected value of entry in equilibrium,

$$V_{nt} = \frac{W^{\alpha_I}_{nt} P^{1-\alpha_I}_{Y,nt}}{\widetilde{\Theta}_{I,n} Z_{I,nt}}. \tag{26}$$

The number of producers of varieties entering into the country n's economy in period t is denoted by $M_{I,nt}$. The law of motion of varieties is $M_{n,t+1} = (1-\delta) M_{nt} + M_{I,nt}$. Since the probability of exit is the same for all varieties $\nu \in \Omega_{nt}$, the distribution of efficiencies of production of varieties $\nu \in \Omega_{nt}$ in any period t is given by $G_n(z)$.

Under the assumption that efficiencies of production of varieties are distributed Pareto, we can derive that the set of country-i's varieties available in country n is given by

$$\Omega_{ni,t} = \left\{\nu \in \Omega_{it} \,\middle|\, z_i(\nu) \ge z^*_{ni,t}\right\},$$

where $z^*_{ni,t}$ is given by

$$\left(\frac{z_{\text{min},i}}{z^*_{ni,t}}\right)^{\theta^{\text{M}}} = \frac{\theta^{\text{M}} + 1 - \sigma^{\text{M}}}{\theta^{\text{M}} \sigma^{\text{M}}} \cdot \frac{\mathcal{X}_{ni,t}}{W_{nt} \Phi_{ni,t} M_{it}},$$

with $\mathcal{X}_{ni,t}$ being the total value of varieties that country n buys from i in period t.

3.3.3 Fixed Costs of Serving Markets

At this point we need to introduce the formal definition of the fixed costs of serving market n by firms from market i, $\Phi_{ni,t}$. Let $L_{F,nt}$ be the total amount of country n's labor that is used to pay the fixed costs of serving its market. We posit that

$$\Phi_{ni,t} \equiv \left[M_{it}^{\frac{1}{\theta^{M}} - \phi_{F,M}} L_{F,nt}^{\vartheta - \phi_{F,L}} \right]^{\frac{1}{\vartheta}} F_{ni,t}, \tag{27}$$

where $F_{ni,t}$ is an exogenous part of the fixed costs, $\left[M_{it}^{\frac{1}{\theta^{M}} - \phi_{F,M}} L_{F,nt}^{\vartheta - \phi_{F,L}} \right]^{\frac{1}{\vartheta}}$ is an endogenous part of the fixed costs that is taken by firms as given, and

$$\vartheta \equiv \frac{1}{\sigma^{M} - 1} - \frac{1}{\theta^{M}}.$$

Under the assumption that $\theta^{M} > \sigma^{M} - 1$, we have that $\vartheta > 0$. The term $\left[M_{it}^{\frac{1}{\theta^{M}} - \phi_{F,M}} L_{F,nt}^{\vartheta - \phi_{F,L}} \right]^{\frac{1}{\vartheta}}$ corrects for the externality that arises due to interaction of love-of-variety and scale effects. Parameter $\phi_{F,M}$ governs the strength of capital externality in production of intermediate goods in the corresponding unified model, while parameter $\phi_{F,L}$ governs the strength of externality in production of the final aggregate in the corresponding unified model. The intuition is the following. If the market is served by a small set of large firms, then it is cheaper to serve this market, because average costs for each of the firms are lower. This is the scale effect. The scale effect goes against the love-of-variety effect: consumers of the final good gain from access to a larger set of varieties. The trade-off between these two effects is captured by ϑ. When $\vartheta = 0$, the two effects just offset each other. Given σ^{M}, larger θ^{M} implies larger ϑ. High θ^{M} implies lower variance of Pareto efficiencies. When variance of efficiencies is low, all firms look similar. And so they either all enter the market, or none of them enters. Conversely, low θ^{M} implies higher variance of Pareto productivities, which allows for the scale effect to kick in.

Under the assumption (27) on the form on fixed costs of serving markets, we can derive that the bilateral price index is $P_{ni,t} = \tau_{ni,t}^{M} P_{X,it}$, where

$$P_{X,it} = \frac{\sigma^{M}}{\sigma^{M} - 1} \cdot \frac{W_{it}}{z_{\min,i} \Theta_{X,i}^{M} Z_{X,it} M_{it}^{\phi_{F,M}} \left[L_{X,it}^{M} \right]^{\phi_{X,L}}} \tag{28}$$

is interpreted as the price of the output of varieties in country i in period t. The price of the final aggregate is

$$P_{Y,nt} = \left(\frac{\theta^{M}}{\theta^{M}+1-\sigma^{M}}\right)^{-\frac{1}{\sigma^{M}-1}+\phi_{F,L}} \left(\frac{P_{Y,nt}Y_{nt}}{\sigma^{M}W_{nt}}\right)^{-\phi_{F,L}} \left[\sum_{i=1}^{N}\left(F_{ni,t}^{\vartheta}\tau_{ni,t}^{M}P_{X,it}/\omega_{ni}\right)^{-\theta^{M}\xi}\right]^{-\frac{1}{\theta^{M}\xi}}, \quad (29)$$

where

$$\xi \equiv \frac{1}{\left(\frac{1}{\eta^{M}-1}-\frac{1}{\sigma^{M}-1}\right)\theta^{M}+1}, \quad (30)$$

and the share of expenditure of country n on country i's varieties is

$$\lambda_{ni,t} = \frac{\left(F_{ni,t}^{\vartheta}\tau_{ni,t}^{M}P_{X,it}/\omega_{ni}\right)^{-\theta^{M}\xi}}{\sum_{l=1}^{N}\left(F_{nl,t}^{\vartheta}\tau_{nl,t}^{M}P_{X,lt}/\omega_{nl}\right)^{-\theta^{M}\xi}}. \quad (31)$$

The value of total output of varieties in country n in period t is

$$\mathcal{X}_{nt} = \frac{\sigma^{M}}{\sigma^{M}-1}W_{nt}L_{X,nt}^{M},$$

and total average profits of country n's producers of varieties are

$$D_{nt} = \frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}\cdot\frac{\mathcal{X}_{nt}}{M_{nt}}.$$

The total amount of country n's labor used to serve its market is

$$L_{F,nt} = \frac{\theta^{M}+1-\sigma^{M}}{\theta^{M}\sigma^{M}}\cdot\frac{P_{Y,nt}Y_{nt}}{W_{nt}},$$

which can also be written as

$$L_{F,nt} = \vartheta\frac{P_{Y,nt}Y_{nt}}{\mathcal{X}_{nt}}L_{X,nt}^{M}.$$

If trade is balanced — for example, as is always the case under financial autarky — then $P_{Y,nt}Y_{nt} = \mathcal{X}_{nt}$ and so $L_{F,nt} = \vartheta L_{X,nt}^{M}$.

3.3.4 Household's Problem and Markets Clearing Conditions

The household's problem is identical to the one in the Krugman model. Labor market clearing condition is different from the corresponding condition in the Krugman model — it involves labor used for serving markets, $L_{F,nt}$,

$$L_{X,nt}^{M} + L_{F,nt} + L_{I,nt} = L_{nt}.$$

The other conditions are the same as in the Krugman model:

$$\sum_{n=1}^{N} \lambda_{ni,t} P_{Y,nt} Y_{nt} = \mathcal{X}_{it},$$
$$C_{nt} + Y_{I,nt} = Y_{nt}.$$

The complete set of equilibrium conditions for the generalized Melitz model is provided in Appendix B.3.

3.3.5 Discussion

There are no direct analogs in the existing literature of the generalized Melitz model. There are two important differences of the generalized Melitz model with the dynamic versions of the Melitz model described in, for example, Ghironi and Melitz (2005), Alessandria and Choi (2007), and Fattal Jaef and Lopez (2014). First, fixed costs of serving markets in the generalized Melitz model are payed in terms of the destination-country labor, while in the existing dynamic Melitz models the fixed costs are paid in terms of the source-country labor. Second, there are non-zero fixed costs of serving the domestic markets in the generalized Melitz model, while in the existing dynamic Melitz models there are no fixed costs of serving domestic markets. The presence of such costs in the generalized Melitz model creates the situation when in every period there are some firms that neither produce nor exit. These firms have too low efficiency of production to overcome fixed costs of serving markets, but had high enough efficiency of production to enter the economy at some point. In the existing dynamic Melitz models all firms that enter the economy produce for at least the domestic market. Quantitatively, the effects of the differences in these assumptions are small in the environment with two symmetric countries, which is traditionally the focus of the international business cycles literature (and which is studied in the quantitative part of the current paper). The benefit of the assumptions about fixed costs of serving markets made in the generalized Melitz model here is that these assumptions allow us to establish isomorphism with the unified model.[15]

If we shut down external economies of scale in production of varieties and in the fixed costs of serving markets (by setting $\phi_{X,L} = 0$, $\phi_{F,M} = \frac{1}{\theta^{M}}$, and $\phi_{F,L} = \vartheta$), and if we require that the sunk costs of entry into the economy are paid in terms of labor only (by setting

[15] The generalized Melitz model can be considered as an extension to a dynamic environment of the static version of the Melitz model described in Kucheryavyy *et al.* (2017), who make the same assumptions about fixed costs of serving markets as in the current paper. These assumptions allow Kucheryavyy *et al.* (2017) to establish isomorphism between a static multi-industry version of the Melitz model and a static multi-industry version of the Eaton-Kortum model with external economies of scale.

$\alpha_I = 1$), then the only essential differences between the generalized Melitz model and the version of the dynamic Melitz model presented in Ghironi and Melitz (2005) will be the differences in the assumptions about fixed costs of serving markets described in the previous paragraph. In Fattal Jaef and Lopez (2014), production technology for intermediate varieties uses capital together with labor. And, so, Fattal Jaef and Lopez (2014) model capital accumulation in addition to entry and exit of producers of varieties. The environment in Alessandria and Choi (2007) features sunk costs of entry into exporting markets, which create exporters hysteresis — the feature absent in the setup of the generalized Melitz model of the current paper.

4 Results

In this section we first formulate our main theoretical result: isomorphisms between the unified model of Section 2 and the models of Section 3. After that we describe the relationship between these models and relevant models in the literature. And then we explore quantitatively the ability of the unified model to match business cycle moments observed in the data.

In the rest of this paper, for brevity, when there is no risk of confusion, we refer to the generalized dynamic international trade models of Section 3 simply as "the Eaton-Kortum model", "the Krugman model", and "the Melitz model".

4.1 Theoretical Isomorphisms

The key results in establishing the link between the unified model of Section 2 and the models of Section 3 are the following three lemmas.

Lemma 1. *By an appropriate relabeling of variables and parameters, the price of country n's output of varieties in the Eaton-Kortum, Krugman, and Melitz models — given, correspondingly, by expressions* (13), (20), *and* (28) *— can be written as the price of country n's intermediates in the unified model given by expression* (2).

Proof. There is nothing to prove in the case of the Eaton-Kortum model: the price of output of varieties in the Eaton-Kortum model, given by (13), is identical to the price in expression (2).

In Appendices B.2 and B.3 we show that expressions (20) and (28) for prices in the

Krugman and Melitz models can be rewritten, correspondingly, as

$$P_{X,nt} = \frac{D_{nt}^{\frac{1}{\sigma^K}} W_{nt}^{1-\frac{1}{\sigma^K}}}{\widetilde{\Theta}_{X,n}^{K} Z_{X,nt} M_{nt}^{\phi_{Y,M}-\frac{1}{\sigma^K}} L_{X,nt}^{\phi_{X,L}+\frac{1}{\sigma^K}}} \quad \text{and} \quad P_{X,nt} = \frac{D_{nt}^{\frac{\sigma^M-1}{\sigma^M\theta^M}} W_{nt}^{1-\frac{\sigma^M-1}{\sigma^M\theta^M}}}{\widetilde{\Theta}_{X,n}^{M} Z_{X,nt} M_{nt}^{\phi_{F,M}-\frac{\sigma^M-1}{\sigma^M\theta^M}} L_{X,nt}^{\phi_{X,L}+\frac{\sigma^M-1}{\sigma^M\theta^M}}}, \tag{32}$$

where $\widetilde{\Theta}_{X,n}^{K}$ and $\widetilde{\Theta}_{X,n}^{M}$ are model-specific constants, and, in the case of the Melitz model,

$$L_{X,nt} \equiv \left(\frac{\sigma^M}{\sigma^M - 1} - \frac{1}{\theta^M}\right) L_{X,nt}^{M}. \tag{33}$$

By examining expressions (32), we see that they become identical to expression (2) for price in the unified model, if we (i) relabel variables D_{nt} as R_{nt} and M_{nt} as $K_{X,nt}$; (ii) map exponents of all variables in (32) to the corresponding exponents in (2); and (iii) multiply the amount of labor used in production of varieties in the Melitz model, $L_{X,nt}^{M}$, by $\left(\frac{\sigma^M}{\sigma^M-1} - \frac{1}{\theta^M}\right)$ to map it to the amount of labor used in production of intermediates in the unified model, $L_{X,nt}$.

Informally, the average firms' profit in country n and the measure of country n's varieties in the Krugman and Melitz models play the role of, correspondingly, return on capital in country n and the stock of country n's capital in the unified model. The adjustment to $L_{X,nt}^{M}$ in the Melitz model has to be done because in the Melitz model — differently from the other models — there is an extra use of the total labor available in the economy: to pay fixed costs of serving markets. The labor used to pay fixed costs of serving markets can be written as

$$L_{F,nt} = \left(\frac{1}{\sigma^M - 1} - \frac{1}{\theta^M}\right) L_{X,nt}^{M} - \frac{\theta^M + 1 - \sigma^M}{\theta^M \sigma^M} \cdot \frac{TB_{nt}}{W_{nt}}.$$

The sum of $L_{X,nt}^{M}$ and the first term on the right-hand side of the above expression gives the right-hand side of (33). The second term on the right-hand side of the above expression is mapped into the additional term on the right-hand side of the labor market clearing condition (11) in the unified model with

$$a = \frac{\theta^M + 1 - \sigma^M}{\theta^M \sigma^M}.$$

Mappings between exponents in expressions (32) and (2) are summarized in Table 1 and discussed later in this section. □

In order to formulate the next lemma, we need to introduce an additional assumption for the Melitz model:

Assumption 1. ***(Melitz)*** *(i)* $\left(F_{ni,t}/F_{nn,t}\right)^{\vartheta}\tau^{\mathrm{M}}_{ni,t}\geq 1$ *for all* n, i *and all* t*; and (ii)* $\left(F_{nl,t}F_{li,t}\right)^{\vartheta}\tau^{\mathrm{M}}_{nl,t}\tau^{\mathrm{M}}_{li,t}\geq \left(F_{ni,t}F_{nn,t}\right)^{\vartheta}\tau^{\mathrm{M}}_{ni,t}$ *for all* n, l, i *and all* t.

Observe that, since $\vartheta=\frac{1}{\sigma^{\mathrm{M}}-1}-\frac{1}{\theta^{\mathrm{M}}}$ and $\theta^{\mathrm{M}}>\sigma^{\mathrm{M}}-1$, we have that $\vartheta>0$. So the sufficient conditions to guarantee Assumption 1 are (i) $F_{ni,t}\geq F_{nn,t}$ for all n, i and all t; and (ii) $F_{nl,t}F_{li,t}\geq F_{ni,t}F_{nn,t}$ for all n, l, i and all t.

Lemma 2. *By an appropriate relabeling of variables and parameters, the price of country n's final aggregate in the Eaton-Kortum and Krugman models — given, correspondingly, by expressions* (12) *and* (22) *— can be written as the price of country n's final aggregate in the unified model given by expression* (4)*. Moreover, under Assumption 1, the price of country n's final aggregate in the Melitz model — given by expression* (29) *— can also be written as the price of country n's final aggregate in the unified model.*

Proof. Comparing expressions (12) and (22) for the Eaton-Kortum and Krugman models with expression (4) for the unified model, we see that they are almost identical. The only difference is in the exponents of the aggregators of the CES price indices.

One way to achieve a mapping between expression (29) for the Melitz model and (4) is by making two redefinitions in the Melitz model. First, we can redefine iceberg trade cost as

$$\tau_{ni,t}\equiv\left(\frac{F_{ni,t}}{F_{nn,t}}\right)^{\vartheta}\tau^{\mathrm{M}}_{ni,t}.$$

Assumption 1 guarantees that $\tau_{ni,t}$ defined this way are, indeed, iceberg trade costs that satisfy the no-arbitrage condition. Second, we can write $F^{-\vartheta}_{nn,t}=\Theta^{\mathrm{M}}_{Y,n}Z_{Y,nt}$ and define

$$\Theta_{Y,n}\equiv\left(\frac{\theta^{\mathrm{M}}}{\theta^{\mathrm{M}}+1-\sigma^{\mathrm{M}}}\right)^{\frac{1}{\sigma^{\mathrm{M}}-1}-\phi_{F,L}}\left[\sigma^{\mathrm{M}}\right]^{-\phi_{F,L}}\Theta^{\mathrm{M}}_{Y,n}. \tag{34}$$

Then we get expression for $P_{Y,nt}$ in the Melitz model that is almost identical to (4). Again, the only difference is in the exponents of the aggregators of the CES price indices. Mappings between these exponents across models are summarized in Table 1 and discussed later in this section. □

Lemma 3. *By an appropriate relabeling of variables and parameters, price of country n's investment good in the Eaton-Kortum model — given by expression* (14) *— and the value of a variety before entry in the economy in the Melitz and Krugman models — given, correspondingly, by expressions* (23) *and* (26) *— can be written as the price of country n's investment good in the unified model given by expression* (7)*.*

Proof. There is nothing to prove in the case of the Eaton-Kortum model. In the cases of the Krugman and Melitz models, all we need to do is to relabel the value of a variety before entry in the economy, V_{nt}, as the price of the investment good in the unified model, $P_{I,nt}$. After this relabeling, expressions (23) and (26) become identical to (7). □

Lemmas 1-3 lead to our main theoretical result formulated in the next proposition.

Proposition 1. *By an appropriate relabeling of variables and parameters in the Eaton-Kortum, Krugman, and Melitz models, and by making an additional Assumption 1 for the Melitz model, we can write the equilibrium system of equations in each of these models in a form identical to the equilibrium system of equations in the unified model. Thus, these models are isomorphic to each other in their aggregate predictions.*

Proof. Appendix B. □

This proposition says that, up to relabeling, the generalized versions of the Eaton-Kortum, Krugman, and Melitz models are essentially the same, despite having very different micro-foundations. In particular, under certain parameterizations, these models are identical to a standard international business cycles model extended to allow for external economies of scale in production and iceberg trade costs.

Parameter mappings between models are summarized in Table 1. Let us first consider the Krugman model. As one can see from Table 1, in the standard Krugman model, elasticity of substitution between varieties governs four out five key parameters of the corresponding unified model: the share of capital in production of intermediates, $\alpha_{X,K}$; strengths of economies of scale in production of intermediates, given by $\psi_{X,K}$ for capital and $\psi_{X,L}$ for labor; and trade elasticity, given by the (minus of) exponent of $\tau_{ni,t}$ in expression (5) for trade shares. Thus, the standard Krugman model implies tight links between key parameters of the corresponding unified model. The modeling assumptions of the generalized Krugman model of Section 3.2 allow us to break these tight links. To understand these modeling assumptions, observe that we can obtain the standard Krugman model as a special case of the generalized Krugman model by making several parameter restrictions. First, we need to set the elasticity of substitution between varieties produced in different countries equal to the elasticity of substitution between varieties produced in one country (i.e., assume that $\eta^K = \sigma^K$). Second, we need to remove the correction for the love-of-variety effect in the production technology for the final aggregate by setting $\phi_{Y,M} = \frac{1}{\sigma^K - 1}$. Third, we need to shut down external economies of scale in production of varieties by setting $\phi_{X,L} = 0$. And, fourth, we need to shut down external economies of scale and exogenous shocks in production of the final aggregate by setting $S_{Y,nt} = 1$. In

Model	$\alpha_{X,K}$	$\psi_{X,K}$	$\psi_{X,L}$	ψ_Y	α_I	Trade elasticity
Standard Eaton-Kortum	$\alpha_{X,K}$	0	0	0	0	θ^{EK}
Standard Krugman	$\frac{1}{\sigma^K}$	$\frac{1}{\sigma^K - 1} - \frac{1}{\sigma^K}$	$\frac{1}{\sigma^K}$	0	1	$\sigma^K - 1$
Standard Melitz	$\frac{\sigma^M - 1}{\sigma^M \theta^M}$	$\frac{1}{\sigma^M \theta^M}$	$\frac{\sigma^M - 1}{\sigma^M \theta^M}$	$\frac{1}{\sigma^M - 1} - \frac{1}{\theta^M}$	1	θ^M
Generalized Krugman	$\frac{1}{\sigma^K}$	$\phi_{Y,M} - \frac{1}{\sigma^K}$	$\phi_{X,L} + \frac{1}{\sigma^K}$	ψ_Y	α_I	$\eta^K - 1$
Generalized Melitz	$\frac{\sigma^M - 1}{\sigma^M \theta^M}$	$\phi_{F,M} - \frac{\sigma^M - 1}{\sigma^M \theta^M}$	$\phi_{X,L} + \frac{\sigma^M - 1}{\sigma^M \theta^M}$	$\phi_{F,L}$	α_I	$\theta^M \xi$

Notes: $\alpha_{X,K}$ is the capital share in production of intermediates in the unified model as well as the capital share in production of varieties in the standard Eaton-Kortum model. $\psi_{X,K}$ and $\psi_{X,L}$ are the scale elasticities of capital and labor in production of intermediates in the unified model. ψ_Y is the scale elasticity of real output of the final aggregate in production of the final aggregate in the unified model. σ^K and σ^M are the elasticities of substitution between varieties in the Melitz and Krugman models. θ^{EK} is the parameter of the Fréchet distribution in the Eaton-Kortum model. θ^M is the shape of Pareto distribution in the Melitz model. $\phi_{Y,M}$ is the correction for the love-of-variety effect in the generalized Krugman model. $\phi_{X,L}$ is the scale elasticity of labor in production of varieties in the generalized Krugman and Melitz models. $\phi_{F,M}$ and $\phi_{F,L}$ are the scale elasticities of total measure of varieties and total amount of labor in fixed costs of serving markets in the generalized Melitz model. α_I is the labor share in production of the investment good in the unified and Eaton-Kortum models as well as the labor share in the cost of entry into the economy in the Krugman and Melitz models. Trade elasticity in the unified model is given by the exponent of $\tau_{ni,t}$ in expression (5). η^K is the elasticity of substitution between varieties produced by different countries in the Krugman model. ξ is given by expression (30) in the Melitz model.

Table 1: Parameter mappings between models

the generalized Krugman model, trade elasticity is given by the (minus of) exponent of $\tau_{ni,t}$ in expression (21) for trade shares and is equal to $(\eta^K - 1)$. Thus, by assuming that $\eta^K \neq \sigma^K$, we break the link between parameter σ^K and trade elasticity. By introducing correction for the love-of-variety effect in the generalized Krugman model — by assuming that $\phi_{Y,M} \neq \frac{1}{\sigma^K - 1}$ — we break the tight link between parameter σ and the strength of economies of scale for capital. We can get any desired value of parameter $\psi_{X,K}$ in the unified model by varying $\phi_{Y,M}$. However, the correction for the love-of-variety effect does not break the link between parameter σ^K and the strength of economies of scale for labor. To break this last link, we directly introduce external economies of scale in the technology of production of varieties given by (19) — with the strength of these economies of scale given by parameter $\phi_{X,L}$. With this generalization we can get any desired level of the strength of economies of scale for labor in production of intermediates in the unified model.

Let us now turn to the Melitz model. Two parameters of the standard Melitz model — elasticity of substitution between varieties, σ^{M}, and the shape of Pareto distribution, θ^{M}, govern the five key parameters of the corresponding unified model: $\alpha_{X,K}$, $\psi_{X,K}$, $\psi_{X,L}$, ψ_{Y}, and trade elasticity. Thus, as it is the case with the standard Krugman model, the standard Melitz model implies tight links between these key parameters of the corresponding unified model. Again, the modeling assumptions of the generalized Melitz model of Section 3.3 allow us to break these tight links. In order to understand these assumptions, let us describe parameter restrictions that we need to make to obtain the standard Melitz model from the generalized Melitz model. First, the same as in the generalized Krugman model, we need to set $\eta^{M} = \sigma^{M}$. Second, we need to remove correction for the externality that arises due to interaction of scale and love-of-variety effects in the presence of the fixed costs of serving markets. This involves setting $\phi_{F,M} = \frac{1}{\theta^{M}}$ and $\phi_{F,L} = \vartheta$. Third, we need to shut down external economies of scale in production of varieties by setting $\phi_{X,L} = 0$. Relaxing these parameter restrictions allows us to have isomorphism between the generalized Melitz model and the unified model.

In the generalized Melitz model, we do not have correction for the love-of-variety effect in production technology of the final aggregate given by (25). External economies of scale in production of intermediate goods arise in the Melitz model due to the selection effect: everything else equal, increase in the number of varieties produced in country i, M_{it}, leads to an increase in the cut-off threshold for the minimal efficiency available in any country n, $z^{*}_{ni,t}$. This increase in the cut-off threshold leads to dropping of varieties with efficiencies smaller than $z^{*}_{ni,t}$. The number of varieties dropped is such that the total amount of varieties left available at any destination n, $M_{ni,t}$, is unchanged. Since the remaining varieties have higher average efficiency relative to the previously available set of varieties, the price of production of intermediate goods available in any country n, given by $\tau^{M}_{ni,t}P_{X,it}$, falls (with elasticity $1/\theta^{M}$ in the standard Melitz model) as M_{it} increases. We formally show in Appendix (B.3) that correction for the love-of-variety effect in (25) does not affect the elasticity $1/\theta^{M}$ with which price $P_{X,it}$ falls as M_{it} increases. In order to change this elasticity, we correct the selection effect by introducing external economies of scale with respect to the number of varieties M_{it} in the fixed costs of serving markets, $\Phi_{ni,t}$.

4.2 Quantitative Exercise

We now assess quantitatively the international business cycle implications of the dynamic trade models using our general, competitive model to provide perspectives on the transmission mechanisms that get altered compared to the standard international business cycle model. We also show in what direction the standard international business cycle model needs to be amended to provide a better fit with the data.

4.2.1 Calibration

Common Parameters	$\beta = 0.99$, $\gamma = 2$, $\delta = 0.025$, $\mu = 0.34$, $\sigma = 2$, $\tau_{ni,t} = 5.67$, $\omega_{ni} = 0.5$, $\Theta_{X,n} = \Theta_{I,n} = 1$, $\Theta_{Y,n} = 2.7$, $b_{adj} = 0.0025$ Productivity Process: $\begin{bmatrix} \log(Z_{X,1t}) \\ \log(Z_{X,2t}) \end{bmatrix} = \begin{bmatrix} \rho_{X,11} & \rho_{X,12} \\ \rho_{X,21} & \rho_{X,22} \end{bmatrix} \times \begin{bmatrix} \log(Z_{X,1,t-1}) \\ \log(Z_{X,2,t-1}) \end{bmatrix} + \begin{bmatrix} \varepsilon_{X,1t} \\ \varepsilon_{X,2t} \end{bmatrix}$, $\begin{bmatrix} \varepsilon_{X,1t} \\ \varepsilon_{X,2t} \end{bmatrix} \sim \mathcal{N}\left(\begin{bmatrix} 0 \\ 0 \end{bmatrix}, \begin{bmatrix} \sigma^2_{X,1} & \sigma_{X,12} \\ \sigma_{X,21} & \sigma^2_{X,2} \end{bmatrix} \right)$, with $\rho_{X,11} = \rho_{X,22} = 0.97$, $\rho_{X,12} = \rho_{X,21} = 0.025$, $\sigma_{X,1} = \sigma_{X,2} = 0.0073$, $\sigma_{X,12} = \sigma_{X,21} = 0.29$
IRBC	$\alpha_{X,K} = 0.36$, $\psi_{X,K} = \psi_{X,L} = \psi_Y = 0$, $\alpha_I = 0$, $a = 0$, $Z_{I,nt} = Z_{Y,nt} = 1$
Krugman	$\alpha_{X,K} = \frac{1}{3.8} \approx 0.26$, $\psi_{X,K} = \frac{1}{3.8-1} - \frac{1}{3.8} \approx 0.094$, $\psi_{X,L} = \frac{1}{3.8} \approx 0.26$, $\psi_Y = 0$, $\alpha_I = 1$, $a = 0$, $Z_{I,nt} = Z_{X,nt}$, $Z_{Y,nt} = 1$
Melitz	$\alpha_{X,K} = \frac{3.8-1}{3.8*3} \approx 0.25$, $\psi_{X,K} = \frac{1}{3.8*3} \approx 0.088$, $\psi_{X,L} = \frac{3.8-1}{3.8*3} \approx 0.25$, $\psi_Y = \frac{1}{3.8-1} - \frac{1}{3} \approx 0.024$, $\alpha_I = 1$, $a = \frac{3+1-3.8}{3.8*3} \approx 0.018$, $Z_{I,nt} = Z_{X,nt}$, $Z_{Y,nt} = [Z_{X,nt}]^{\frac{1}{3.8-1} - \frac{1}{3}} \approx [Z_{X,nt}]^{0.024}$

Table 2: Standard calibrations of models.

In this quantitative section we focus on the world economy that consists of two symmetric countries. We consider the following preferences:

$$U\left(C_{nt}, L_{nt}\right) = \frac{1}{1-\gamma}\left[C_{nt}^{\mu}\left(1-L_{nt}\right)^{1-\mu}\right]^{1-\gamma}.$$

We start with a calibration that we call "standard". It is summarized in Table 2. For this calibration we choose three sets of parameter values of the unified model that correspond to standard IRBC, Krugman, and Melitz models. We choose parameter values of the unified model corresponding to the Krugman and Melitz models so that in the Krugman and Melitz models almost all generalizations are shut down. We only allow for the nested CES production technology of the final aggregate. Formally, the implied parameterization for the Krugman model is $\phi_{Y,M} = \frac{1}{\sigma^{K}-1}$ and $\phi_{X,L} = 0$, but allowing for $\eta^{K} \neq \sigma^{K}$. Similarly, the implied parameterization for the Melitz model is $\phi_{F,M} = \frac{1}{\theta^{M}}$, $\phi_{F,L} = \vartheta$, and $\phi_{X,L} = 0$, but allowing for $\eta^{M} \neq \sigma^{M}$.

We first choose a set of common parameter values for the three models. Most of these values are taken from the literature. Periods are interpreted as quarters. Values of parameters β, γ, δ, and μ are the same as in, for example, Heathcote and Perri (2002) and Ghironi and Melitz (2005). We follow the macro literature (as apposed to the international trade literature) and set the elasticity of substitution between intermediate goods in production of the final good to 2, i.e., we set $\sigma = 2$. This implies that the trade elasticity is equal to 1.[16] We choose the level of iceberg trade costs $\tau_{ni,t} = 5.67$ for $n \neq i$ to match the steady-state share of imports of intermediate goods of 0.15. Differently from Heathcote and Perri (2002), we do not have home bias in production of the final aggregate and set $\omega_{ni} = 0.5$ for all n and i.[17] Parameterization of the productivity process in the intermediate goods sector, $Z_{X,nt}$, is the same as in Heathcote and Perri (2002). We set the normalization constants in the intermediate goods and investment sectors to 1, $\Theta_{X,n} = \Theta_{I,n} = 1$. In order to get a plausible number for the fixed costs of serving markets in the Melitz model (which

[16] See, for example, Hillberry and Hummels (2013) on the choice between "macro" versus "micro" trade elasticity. Note however, that we do a sensitivity analysis with higher elasticity of substitution, in line with the international trade literature.

[17] In the case of two symmetric countries, the steady state prices of intermediate goods are the same across the two countries: $P_{X,1} = P_{X,2}$ (here we drop the time index t to emphasize that these are the steady state values of prices). Therefore, the steady state trade share — obtained from (5) by substituting steady state values of prices of intermediate goods — is simply

$$\lambda_{ni} = \frac{\left(\tau_{ni}/\omega_{ni}\right)^{1-\sigma}}{\left(\tau_{n1}/\omega_{n1}\right)^{1-\sigma} + \left(\tau_{n2}/\omega_{n2}\right)^{1-\sigma}}.$$

With the same values of taste parameters ω_{ni} across countries, the steady state trade share depends only on iceberg trade costs and parameter σ.

is discussed below), we set the normalization constant in the final aggregates sector to 2.7, $\Theta_{Y,n} = 2.7$. Finally, for the case of the bond economy, we choose a relatively low value of the bond holdings adjustment cost, $b_{adj} = 0.0025$.

The values of the remaining parameters are different for the IRBC, Krugman, and Melitz models. For the IRBC model, we set the same share of capital in production of intermediate goods as in Heathcote and Perri (2002), $\alpha_{X,K} = 0.36$, and require that investment is made in terms of the final good only (i.e., set $\alpha_I = 1$). The IRBC model does not have any externalities ($\psi_{X,K} = \psi_{X,L} = \psi_Y = 0$), it does not have productivity shocks in the investment and final aggregate sectors ($Z_{I,nt} = Z_{Y,nt} = 1$), and it does not have the additional term $a\mathrm{TB}_{nt}$ in the labor market clearing condition ($a = 0$).

For the parameterization corresponding to the Krugman model, we use the value of $\sigma^{\mathrm{K}} = 3.8$ from Bilbiie *et al.* (2012). This choice immediately implies values for all key parameters specific to the Krugman model: $\alpha_{X,K} = \frac{1}{\sigma^{\mathrm{K}}} \approx 0.26$, $\psi_{X,K} = \frac{1}{\sigma^{\mathrm{K}} - 1} - \frac{1}{\sigma^{\mathrm{K}}} \approx 0.094$, and $\psi_{X,L} = \frac{1}{\sigma^{\mathrm{K}}} \approx 0.26$ (see Table 1 for parameter mappings between the models). The standard Krugman model has neither externalities nor the productivity shocks in production of the final aggregate ($\psi_Y = 0$ and $Z_{Y,nt} = 1$), and it does not have the additional term $a\mathrm{TB}_{nt}$ in the labor market clearing condition ($a = 0$). Investment is made in terms of labor only ($\alpha_I = 0$). We follow Bilbiie *et al.* (2012) in setting the productivity shock in production of investment goods identical to the productivity shock in production of intermediate goods ($Z_{I,nt} = Z_{X,nt}$). The choice of the investment-sector normalization constant $\Theta_{I,n} = 1$ implies that the sunk entry cost into the economy in the Krugman model — given by $\widetilde{\Theta}_{I,n}^{-1}$ — is also equal to 1.[18] Finally, trade elasticity equal to 1 in the unified model implies that the elasticity of substitution between varieties from different countries in the Krugman model is equal to $\eta^{\mathrm{K}} = 2$.

Turning to the parameterization corresponding to the Melitz model, let us first consider fixed and variable costs of serving markets in the Melitz model. We assume that in the Melitz model $F_{12,t} = F_{11,t}$ and $F_{21,t} = F_{22,t}$ for all t. This implies that $\tau^{\mathrm{M}}_{ni,t} = \tau_{ni,t} = 5.67$. Following Ghironi and Melitz (2005), we further assume that the fixed costs of serving markets in the Melitz model are subject to the same shock as the production technology of varieties. Formally, we assume that $F_{nn,t} = f_{nn}/Z_{X,nt}$, where f_{nn} is a time-independent constant (defined below). We proved the part of Lemma 2 concerning the Melitz model by defining $F_{nn,t}^{-\vartheta} = \Theta^{\mathrm{M}}_{Y,n} Z_{Y,nt}$. This definition implies that $Z_{Y,nt} = [Z_{X,nt}]^{\vartheta}$ and $f_{nn} = \left[\Theta^{\mathrm{M}}_{Y,n}\right]^{-\frac{1}{\vartheta}}$.

[18] Bilbiie *et al.* (2012) also have the value of the sunk costs of entry into the economy equal to 1. As Bilbiie *et al.* (2012) note, this value does not affect any impulse-responses under CES preferences.

Using mapping (34), we find that the fixed costs of serving markets are given by

$$f_{nn} = \left(\frac{\theta^{M}}{\theta^{M} + 1 - \sigma^{M}} \right)^{\frac{\sigma^{M}-1}{\theta^{M}+1-\sigma^{M}}} \frac{1}{\sigma^{M}} \left[\Theta_{Y,n} \right]^{-\frac{1}{\vartheta}}. \tag{35}$$

Let us now discuss the choice of parameter values for the Melitz model. We use the same value of $\sigma^{M} = 3.8$ as Ghironi and Melitz (2005) (which is also the same as σ^{K}). We somewhat arbitrary choose $\theta^{M} = 3$ (which is close to the value of 3.4 used by Ghironi and Melitz (2005)). The choice of σ^{M} and θ^{M} implies that $\alpha_{X,K} = \frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}} \approx 0.25$, $\psi_{X,K} = \frac{1}{\sigma^{M}\theta^{M}} \approx 0.088$, $\psi_{X,L} = \frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}} \approx 0.25$, $\psi_{Y} = \frac{1}{\sigma^{M}-1} - \frac{1}{\theta^{M}} \approx 0.024$, and $Z_{Y,nt} \approx [Z_{X,nt}]^{0.024}$. Using expression (35) we get that the implied value of the fixed costs of serving markets in the Melitz model is $f_{nn} \approx 0.0059$.[19] The labor market clearing condition now features the additional term $a\mathrm{TB}_{nt}$ with $a = \frac{\theta^{M}+1-\sigma^{M}}{\sigma^{M}\theta^{M}} \approx 0.018$. As in the calibration corresponding to the Krugman model, $Z_{I,nt} = Z_{X,nt}$ and $\alpha_{I} = 1$. The implied sunk entry cost into the economy is equal to 1. Finally, the choice of $\sigma = 2$ in the unified model implies that in the Melitz model the elasticity of substitution between varieties from different countries is equal to

$$\eta^{M} = 1 + \left(\frac{1}{\sigma - 1} + \vartheta \right)^{-1} \approx 1.98.$$

4.2.2 Comparison Across Models

Moments across models for standard calibrations are presented in Table 3. Column 1 provides data moments from Heathcote and Perri (2002). Columns 2, 5, 8 present results for the standard IRBC model for three different financial market arrangements. Columns 3, 6, 9 and 4, 7, 10 present results for "standard" versions of the Krugman and Melitz models respectively. Comparing outcomes of the three models with moments in the data, we see that the Krugman and Melitz models perform no better than the standard IRBC model: the Krugman and Melitz models perform well (or even worse in output and hours cross country correlations and the cyclicality of trade balance) and fail in the same moments where the standard IRBC model performs well or fails.[20] This outcome was reported,

[19] Ghironi and Melitz (2005) calibrate fixed costs of serving foreign markets — f_X and f_X^* in their notation — to approximately 0.0084. But the reader should keep in mind that in Ghironi and Melitz (2005) firms do not pay fixed costs to serve domestic markets, while in the generalized Melitz model presented in the current paper firms pay fixed costs to serve both domestic and foreign markets.

[20] Note, as emphasized by Heathcote and Perri (2002), the IRBC model under financial autarky leads to international correlations closer to the data than under complete markets or the bond economy. Financial autarky, by construction, however cannot account for trade balance dynamics and the differential cyclicality of exports and imports. For our parameterization, even under financial autarky, the IRBC model does not

among others, by Alessandria and Choi (2007) and Fattal Jaef and Lopez (2014) for different versions of the Melitz model.

First, note that there is not much difference between the moments for the Krugman and Melitz models despite the fact that the Melitz model has a much richer firm-level dynamics than the Krugman model. From the point of view of the unified model, the Melitz model has three different features relative to the Krugman model: external economies of scale and shocks in production of the final aggregate as well as the additional term $a\text{TB}_{nt}$ in the labor market clearing condition. However, the standard calibration used for the Melitz model implies that these features have a quantitatively small impact. This follows from the fact that parameters responsible for these features are relatively small: $\psi_Y \approx 0.024$, $Z_{Y,nt} \approx [Z_{X,nt}]^{0.024}$, and $a \approx 0.018$. In the calibration for the Melitz model model we have values of three other parameters — $\alpha_{X,K}$, $\psi_{X,K}$, and $\psi_{X,L}$ — different from the calibration for the Krugman model. But again, this difference is small. The small values of ψ_Y, $Z_{Y,nt}$, and a as well as small differences between calibrations for the Krugman and Melitz models are implied by a small difference between the chosen values of $(\sigma^M - 1) = 2.8$ and $\theta^M = 3$ as well as our choice $\sigma^M = \sigma^K$. The chosen values of σ^M and θ^M are fairly standard, and the unified-model perspective allows us to see clearly the consequences of this choice. Table 2 shows how this standard calibration implies small differences between the Krugman and Melitz models.

Next, from the point of view of the standard IRBC model, the Krugman and Melitz models have several key modifications that could potentially have opposite or hard to understand effects on the performance of these models. Out of all new features of the Krugman and Melitz models (relative to the standard IRBC model), external economies of scale in production of intermediate goods and final aggregates are the most interesting. Before we focus on the role played by external economies of scale however, we consider a few more exercises to show that our comparisons across models are robust to various specifications. In exercises that we report below, for concreteness we focus on the complete markets benchmark.

First, one striking difference between the IRBC model and the Krugman and Melitz models in Table 3 is in terms of the cyclicality of the trade balance. The correlation of trade balance with output, counterfactually, switches from negative to positive in the Krugman and Melitz models. This change is however, due to the feature that trade models imply that the investment good (from the perspective of the general competitive model) is

generate positive international correlations in investment and labor. This is unlike the baseline specification in Heathcote and Perri (2002) and the reason is the differential calibration of the elasticity of substitution between intermediate goods in production of the final good (we use $\sigma = 2$ while Heathcote and Perri (2002) use $\sigma = 0.90$).

Moment	Data	Complete Markets			Bond Economy			Financial Autarky		
		IRBC	Krug	Mel	IRBC	Krug	Mel	IRBC	Krug	Mel
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\text{Corr}\left(\frac{\text{GDP}_1}{P_{Y,1}}, \frac{\text{GDP}_2}{P_{Y,2}}\right)$	0.58	0.14	0.03	0.04	0.16	0.08	0.08	0.29	0.16	0.17
$\text{Corr}(C_1, C_2)$	0.36	0.79	0.72	0.72	0.69	0.63	0.63	0.68	0.52	0.52
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	−0.48	−0.43	−0.44	−0.47	−0.40	−0.40	−0.02	−0.25	−0.25
$\text{Corr}(L_1, L_2)$	0.42	−0.51	−0.65	−0.65	−0.42	−0.57	−0.58	−0.23	−0.41	−0.41
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	−0.49	−0.49	0.48	0.49	−0.54	0.25	0.29			
$\text{Corr}\left(\frac{\mathcal{X}_{21}}{P_{Y,1}}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.32	0.38	0.86	0.87	0.29	0.79	0.81	0.91	0.85	0.85
$\text{Corr}\left(\frac{\mathcal{X}_{12}}{P_{Y,1}}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.81	0.95	0.50	0.50	0.96	0.70	0.69	0.91	0.85	0.85
$\text{Corr}\left(\text{ReR}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.13	0.53	0.56	0.57	0.48	0.50	0.53	0.60	0.62	0.62
$\frac{\text{Std}(\text{ReR})}{\text{Std}(\text{GDP}_1/P_{Y,1})}$	2.23	0.26	0.33	0.34	0.22	0.27	0.28	0.35	0.18	0.19

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = W_n L_n + R_n K_n$, $\mathcal{X}_{ni} = P_{X,ni} X_{ni}$, $\text{TB}_1 = P_{X,1} X_1 - P_{Y,1} Y_1$, $\text{ReR} = P_{Y,2}/P_{Y,1}$.

Table 3: Moments from standard calibrations and formulations of models

produced using home labor, as opposed to the final aggregate good (as is standard in international business cycle models). It is not directly related to production externalities.

To show this clearly, in Table 4 , in column 3, we first change the IRBC model such that investment is done in terms of home labor only (by setting $\alpha_I = 1$). Then in column 4 we on top add a shock to the investment sector that is perfectly correlated with the shock in the intermediate good sector (by setting $Z_{I,nt} = Z_{X,nt}$). Comparing with the benchmark results in column 2, it is clear that this change makes trade balance pro-cyclical in column 3 makes international correlation in investment negative, together with a procyclical net exports, in column 4.[21] To emphasize further the key role played by this assumption of

[21]Thus, it is not the case that simply adding investment to an international business cycles model ensures a counter cyclical trade balance by countervailing the consumption smoothing intuition in models without investment. It is critically important how the investment good is produced. If it is with labor input only, then even with investment in the model, while investment certainly increases with a positive productivity shock, it does not render net exports counter cyclical. The reason is that in such a case, the rise in imports, is much more muted. This is because now imports follow consumption closely (as investment good production does not use the foreign intermediate good), which is smoothed over time due to standard consumption smoothing incentives. This then plays a key role in making net exports procyclical, and the usual consumption smoothing intuition that applies in a model without investment continues to apply.

how the investment good is produced, we now change the Krugman and Melitz models in reverse. In columns 5 and 6 we now change these models such that the investment good is produced using the final aggregate good only (by setting $\alpha_I = 0$). It is clear that in this case, the trade balance is countercyclical. Finally, note that the Melitz model, other than a new externality, also introduces a shock to the final aggregate sector from the perspective of the general competitive model. In column 7 we shut down this shock (by setting $Z_{Y,nt} = 1$) and find that it does not affect the moments.

Having resolved the issue related to cyclicality of trade balance, we undertake one potentially important additional robustness exercise. In columns 8, 9, and 10 of Table 4 we show that the IRBC and the Krugman and Melitz models lead to very similar moments for key international business cycle variables even when we calibrate the model to a high trade elasticity. In particular, we follow the international trade literature here and set the elasticity of substitution between intermediate goods in production of the final good to 6, i.e., we set $\sigma = 6$. With this calibration, as is well-known, the fit of the IRBC model itself worsens significantly as international correlations become much weaker, with output correlations even turning negative.[22] But the differences across the three models for the key moments are still minor (and the fit still worse for the Krugman and Melitz models for output and hours correlation).[23]

Overall then, what is the main reason for the similar performance of the IRBC model on the one hand and the Krugman and Melitz models on the other hand, as shown in Tables 3 and 4? Our result on isomorphism is useful in answering this question. Note from above that in these restricted/standard versions of the Krugman and the Melitz models, the difference from the standard IRBC models is externalities that are highly restricted both in scale and in the split between capital and labor. Given the calibration in particular, where we follow the parameterization from the literature, not only are the extent and type of externalities highly restricted, they are also somewhat small overall. For instance, in the Krugman model, as $\psi_{X,K} = 0.094$ the positive externality on capital input in the intermediate good production is small. Moreover, since $\psi_{X,L} = 0.26$, the positive externality on labor input is higher, it is still not large enough to affect quantitatively as we show in detail later. Similar reasoning holds for the Melitz model, where the two externalities on the intermediate good production are relatively small, with $\psi_{X,K} = 0.088$, $\psi_{X,L} = 0.25$,

[22] That is generally, with the elasticity of substitution increasing, in the IRBC model, the cross-country correlation of output, investment, and labor decreases while that of consumption increases. If the productivity shock were to be much more persistent, essentially a random walk, then in the particular case of the bond economy only, this worsening of fit can be less severe.

[23] Of course, here again the trade balance is pro-cyclical for the Krugman and Melitz models as we report results from a standard specification of these models where investment good sector uses home labor to produce the investment good.

Moment	Data	Bench	Inv. labor: IRBC $Z_{I,n}=1$	Inv. labor: IRBC $Z_{I,n}=Z_{X,n}$	Inv. final aggregate: Krug	Inv. final aggregate: Mel	Inv. final aggregate: Mel $Z_{Y,n}=1$	$\sigma=6$: IRBC	$\sigma=6$: Krug	$\sigma=6$: Mel
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\text{Corr}\left(\frac{\text{GDP}_1}{P_{Y,1}}, \frac{\text{GDP}_2}{P_{Y,2}}\right)$	0.58	0.14	0.31	0.05	−0.01	−0.02	−0.02	−0.07	−0.22	−0.21
$\text{Corr}(C_1, C_2)$	0.36	0.79	0.75	0.71	0.69	0.67	0.67	0.88	0.78	0.78
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	−0.48	0.23	−0.33	−0.60	−0.63	−0.62	−0.76	−0.73	−0.73
$\text{Corr}(L_1, L_2)$	0.42	−0.51	−0.98	−0.58	−0.66	−0.67	−0.67	−0.71	−0.82	−0.82
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	−0.49	−0.49	0.59	0.56	−0.37	−0.38	−0.36	−0.44	0.20	0.24
$\text{Corr}\left(\frac{\mathcal{X}_{21}}{P_{Y,1}}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.32	0.38	0.94	0.92	0.50	0.48	0.50	−0.11	0.41	0.46
$\text{Corr}\left(\frac{\mathcal{X}_{12}}{P_{Y,1}}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.81	0.95	0.36	0.51	0.90	0.90	0.89	0.70	0.07	0.05
$\text{Corr}\left(\text{ReR}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.13	0.53	0.59	0.62	0.63	0.64	0.64	0.09	0.29	0.34
$\frac{\text{Std}(\text{ReR})}{\text{Std}(\text{GDP}_1/P_{Y,1})}$	2.23	0.26	0.67	0.30	0.35	0.38	0.37	0.13	0.18	0.18

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = W_n L_n + R_n K_n$, $\mathcal{X}_{ni} = P_{x,ni} X_{ni}$, $\text{TB}_1 = P_{X,1} X_1 - P_{Y,1} Y_1$, $\text{ReR} = P_{Y,2}/P_{Y,1}$. Column 2 corresponds to the standard calibration of the IRBC model and is identical to column 2 in Table 3. Columns 3 and 4 are for the case of investment done in terms of labor in otherwise standard IRBC model. For column 3 there is no shock in the investment sector, while for column 3 the shock to the investment sector is the same as the shock in the intermediate goods sector. Columns 5-7 are for the case of investment in terms of final aggregate in otherwise standard calibrations of Krugman and Melitz models. In column 7 there is no shock in the final aggregate sector. Columns 8-10 are for the case of $\sigma = 6$ in otherwise standard calibrations of IRBC, Krguman, and Melitz models.

Table 4: Robustness checks on comparisons across models. Complete markets.

and the externality on the final good production/aggregation technology similarly small as well, $\psi_Y = 0.024$. Table 2 shows how standard calibrations imply small differences between the Krugman and Melitz models for these externalities, as well as small overall differences from the standard IRBC model. Moreover, as we show later, positive externality, especially on the capital input, leads to a negative endogenous correlation in productivity across countries. This then further dampens down any co-movement in quantity variables across countries. In particular, it decreases the co-movement in output while making even more negative the correlation in investment and hours.

4.2.3 Potential Role of Negative Capital Externality

Given the results described above, we next use our general competitive model, which because of the isomorphism can then be re-interpreted as a version of the generalized dynamic trade models, to explore if it is possible to achieve a better fit with the data. The general model is particularly useful as we can independently vary both the overall scale and the split of externalities across capital and labor. We do comparative statics for all the three externalities: capital and labor input in the intermediate goods production technology and the externality on the final good production technology. But here we first focus on the role of capital externality as that turns out to be most crucial. This leads to one of our main insights: we show that an essential feature is negative capital externalities in intermediate goods production. This can be seen from the results in Table 5, where for comparison, we provide the moments from a model without any externality, as well as those with positive and negative externality.[24] As the standard dynamic trade models imply positive capital externalities in intermediate good production, they do not provide a closer fit, and in fact often a worse fit, to the data. What is the intuition for negative capital externalities helping with resolving several international business cycle puzzles, especially those that pertain to co-movement across countries?

Before going into the results, first, note that the main empirical puzzles are associated with co-movement across countries in output, consumption, hours, and investment, as is clear from Table 5. In the standard model, the co-movement of consumption is counterfactually higher than GDP.[25] Moreover, while in the data, labor hours and investment co-move positively, in the standard models, they co-move negatively. Second, it is critical to note that when there are negative capital externalities in production of intermediate goods, from the perspective of individual firms, it is as if the aggregate country-specific productivity shock is less persistent with the same initial impact. This is because, in future, due to positive capital accumulation, the productivity shock faced by the firms is lower than the exogenous productivity shock. Third, note that since this feature is irrespective of the risk-sharing arrangements across countries, our finding applies independently of whether we assume complete financial markets or incomplete markets or financial autarky. For concreteness, below when we present results we focus on the complete financial markets case and present all the results for the other two risk-sharing arrangements in the Appendix.

[24]Here, to keep the transmission mechanism and interpretation clear, we do not consider exogenously correlated shocks across countries. This is the reason why our benchmark moments are slightly different from the IRBC moments in Table 3. Moreover, through out next, investment is in terms of the final good.

[25]Note that high co-movement of consumption is not due to only perfect risk-sharing. This is also true even under financial autarky, as long as different countries produce different goods.

Moment	Data	Bench	$\psi_{X,K}$ 0.3	$\psi_{X,K}$ -1	$\psi_{X,L}$ 0.7	$\psi_{X,L}$ -1	ψ_{Y} 0.2	ψ_{Y} -1
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\text{Corr}\left(\frac{\text{GDP}_1}{P_{Y,1}}, \frac{\text{GDP}_2}{P_{Y,2}}\right)$	0.58	0.26	0.23	0.36	0.12	0.38	-0.02	0.39
$\text{Corr}\,(C_1, C_2)$	0.36	0.67	0.73	0.57	0.51	0.77	0.45	0.85
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	-0.12	-0.21	0.03	-0.22	-0.02	-0.51	0.30
$\text{Corr}\,(L_1, L_2)$	0.42	-0.02	-0.27	0.29	-0.07	0.04	-0.28	-0.01
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	-0.49	-0.42	-0.34	-0.48	-0.47	-0.37	-0.58	0.51
$\text{Corr}\left(\frac{\mathcal{X}_{21}}{P_{Y,1}}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.32	0.56	0.67	0.42	0.42	0.64	0.02	0.96
$\text{Corr}\left(\frac{\mathcal{X}_{12}}{P_{Y,1}}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.81	0.95	0.93	0.97	0.94	0.95	0.92	0.67
$\text{Corr}\left(\text{ReR}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.13	0.52	0.57	0.39	0.55	0.48	0.60	0.37
$\frac{\text{Std}\,(\text{ReR})}{\text{Std}\,(\text{GDP}_1/P_{Y,1})}$	2.23	0.26	0.31	0.18	0.28	0.25	0.32	0.17

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = W_n L_n + R_n K_n$, $\mathcal{X}_{ni} = P_{X,ni} X_{ni}$, $\text{TB}_1 = P_{X,1} X_1 - P_{Y,1} Y_1$, $\text{ReR} = P_{Y,2}/P_{Y,1}$.

Table 5: Moments from calibration with increasing and decreasing returns and uncorrelated shocks across countries. Complete markets.

We now provide an interpretation for the moments by analyzing in depth the transmission mechanism. For this we turn to an analysis of impulse response functions, where a 1% exogenous technology shock in the intermediate goods sector hits the home country. In order to focus on the main mechanism behind the results, we do not consider exogenously correlated shocks across the two countries. Figure 1 shows the first set of results where we vary only the externalities in capital input, $\psi_{X,K}$, for a model with complete markets. As we mentioned above, when there are negative (positive) capital externalities in production of intermediate goods, from the perspective of individual firms, it is as if the aggregate country-specific productivity shock is less (more) persistent with the same initial impact. This is because, in future, due to positive capital accumulation, the productivity shock faced by the firms is lower than the exogenous productivity shock under negative capital externalities. Given this, how do agents, say at home, respond to a productivity shock that has the same initial size but is more transient compared to the no externality case? As is standard in competitive business cycle models, it is most useful to think through the labor supply response. As the shock is now more transient, compared to the no externality case, the substitution effect of wage increases is stronger than the income effect. This means than that households supply more labor today. This, with the capital stock as given, then leads to a larger initial response of output. This helps with increasing output co-movement across countries.

What should households do with this increased income? While the initial effect on income is higher, in future, as the productivity process is more transient, income will be lower than in the model without externalities. Then through the usual intuition from the permanent income hypothesis, while consumption rises today, due to the desire to smooth consumption over time, consumption rises by less. This smaller rise of consumption at home then helps with not counterfactually increasing consumption co-movement across countries and in fact helps reduce the correlation in consumption across countries. We see these effects on output and consumption co-movement in Table 5.

Finally, why do cross-country investment and labor hours co-movement turn more positive, with investment and hours correlation in fact moving from negative to positive? First, given that consumption rises by less at home, investment increases by more. But this does not worsen international correlation in investment. An important feature now is that while the country-specific productivity shocks are uncorrelated in our experiments, negative capital externality leads to an endogenous positive correlation in the productivity faced by the two countries. In particular, from the foreign country's perspective, starting from the next period, there is a positive effect on productivity, as typically, there would be negative investment in the foreign country following a positive productivity shock in the home country. This positive effect on productivity faced by the foreign country then leads to increased labor hours and increased investment for very standard reasons. Moreover, note that this endogenous increase in productivity in the foreign country leads also to an increase in output, which helps further with increasing output co-movement across countries. Finally, consumption in the foreign country increases, but by less than it would with no externality.

In addition to assessing international correlations, we also explore the fit with the data in terms of domestic correlations of key open economy variables with output. We focus on cyclicality of exports, imports, real exchange rate, and the trade balance. As Table 5.shows, negative capital externalities in production help also with moving the model closer to the data in terms of generating less procyclical exports and the real exchange rate and a more countercyclical trade balance. That is, to meet the larger increase in investment demand that we discussed above, the home country imports more, as the investment good is produced using the final aggregate good that combines the domestic and foreign intermediate goods. Moreover, given the lower effect on relative consumption across countries we described above, the real exchange rate is now less procyclical. Figure 1 shows how the larger initial response of investment under negative capital externalities translate to a more countercyclical trade balance driven by a sharper negative effect initially.[26] One exception here is that negative capital externalities in production lead to a more procyclical imports, which makes the fit worse with the data as the standard business cycle model already leads to imports that are more procyclical than the data. The reason imports become more procyclical is that the behavior of imports closely follow that of investment as can be clearly seen in Figure 1. This is because with consumption smoothed over time, as is typical in business cycle models, investment response is comparatively larger to a productivity shock. As the investment good is produced with the final aggregate good, which uses the foreign intermediate good, it implies that the behavior of imports closely mirrors that of investment, wtih only the magnitude being smaller as determined by the import share. Then, given that we explained above how investment and output increases more sharply initially with negative capital externalities, imports follow a similar pattern, thereby increasing the pro-cyclicality.[27]

4.2.4 Varying Labor and Final Aggregator Externalities

We now conduct a similar exercise with labor and final aggregator externalities. While negative capital exernalities in production help with moving the model closer to the data in terms of co-movement across countries of business cycle quantities, negative labor externalities do not uniformly do so. We see this in Table 5, especially for co-movement of consumption. We show the transmission mechanisms underlying these results in Figure 2, where we vary only the externalities in labor input, $\psi_{X,L}$. The main reason is that with negative labor externalities, while the productivity process faced by the home country is also less transient in future as typically there would be an increase in labor hours in future, the initial impact also shifts down. This is because unlike capital stock which is pre-determined today, labor hours respond positively today as well. This then looks basically like a productivity process for the home country that has shifted downwards at every point in time. Then, home households do not increase their hours initially, which

[26] Over time, as is standard, trade balance switches to positive with investment increasing in the foreign country as it rebuilds its capital stock.

[27] Finally, while we focus in this paper on assessing implications for cross-country quantity correlations and within country cyclicality of open economy variables, we acknowledge that for one important moment that constitutes a long-standing puzzle in international business cycles, the volatility of the real exchange rate, negative capital externalities make the discrepancy worse between the model and the data. We see this in Table 5 and also in Figure 1. For this moment, the margins that get introduced from international trade features thus do help with enabling a closer fit with the data, as also seen in Table 3 (but as we mentioned above, this goes together with making the real exchange rate more procyclical, unlike the case in the data).

in turn means that the initial increase in investment and output also does not happen. The effect is thus not as strong before with negative capital externalities in moving the co-movement of hours and investment towards positive. In terms of the foreign country, there is again an endogenous correlation of productivity, as typically there would be a negative response of foreign labor hours, and so it does help qualitatively with generating a less negative response of foreign investment and hours, but the dynamic positive correlation of productivity that occurs with negative capital externality does not happen in this case as is clear in Figure 2. The main difference overall with negative capital externality is that consumption correlation actually increases, instead of decreasing.[28] This is mostly because consumption in the foreign country does not change its dynamic response, as there is realtively less difference in its investment and output paths. Finally, net exports also now become less countercylical, which worsens fit with the data.

Finally, we consider varying the externality in the production/aggregation technology of the final aggregate good. Note again that in terms of interpretation from trade models, this externality is a new feature of the dynamic Melitz model compared to the dynamic Krugman model. Negative externality here also does not uniformly help move the model closer to the data, as seen in Table 5. It for instance, increases co-movement in consumption.[29] We show detailed transmission mechanisms in Figure 3, where we vary only the externality in the final good aggregator technology, ψ_Y. For the home country, the effects are similar to that of negative labor externalities in the intermediate good production technology. Our modeling of this externality in terms of $\left(P_{Y,nt}Y_{nt}\right)/W_{nt}$, the number of country-$n$'s workers that produce the same value as the value of the final aggregate, suggests why this is the case. Moreover, making this externality negative leads to counterfactual effects on the trade balance, not only decreasing the counter cylicality as with the labor externality, but actually turning it from counter to procyclical.

5 Conclusion

We present a general, competitive open economy business cycles model with capital accumulation, production externalities, trade in intermediate goods, and iceberg trade costs. Our main theoretical result shows that models developed in the modern international trade literature that feature comparative advantage, monopolistic competition and cost

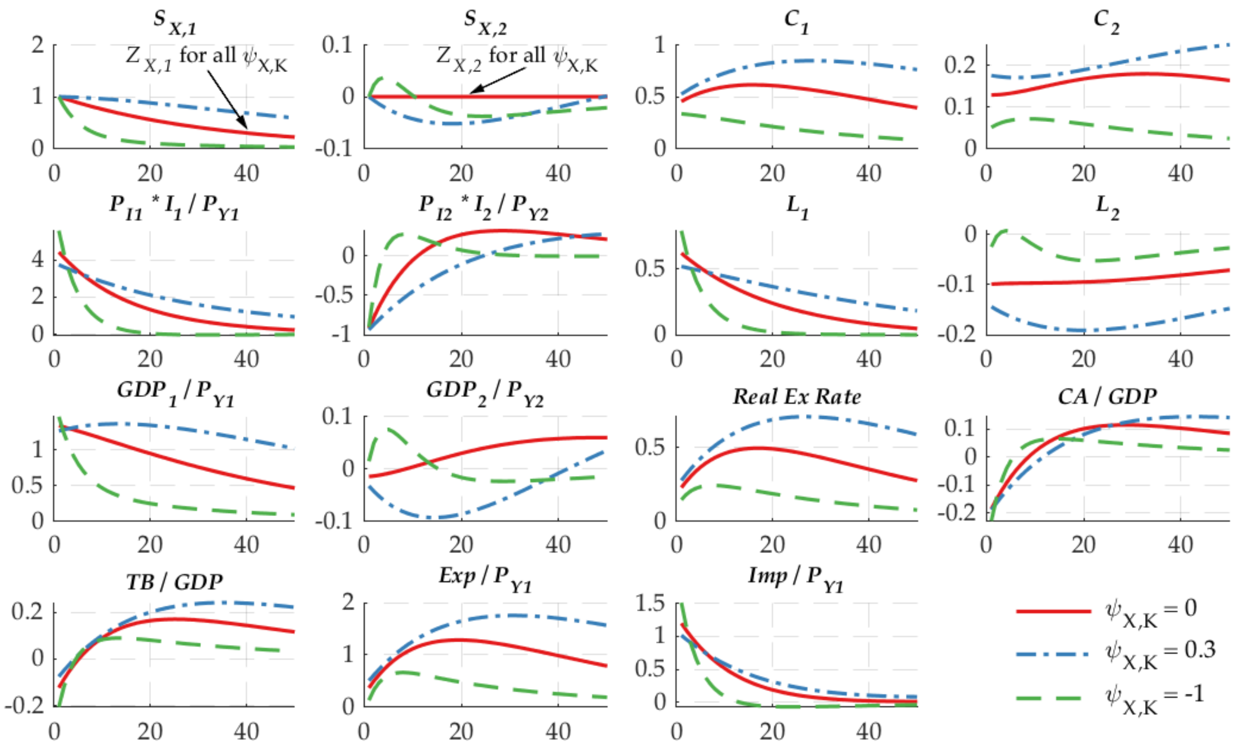
[28] Also note that the investment correlation is still negative. A further increase in the extent of negative labor externality can push this to positive. But regardless of the calibration, the consumption correlation increasing is a robust feature. Another issue is that the results are less robust than that of negative capital externality when we consider different risk-sharing arrangement across countries.

[29] Moreover, with this level of negative externality, the hours correlation is still negative.

of entry, and firm heterogeneity and cost of exporting are isomorphic, in terms of aggregate equilibrium, to versions of this competitive dynamic model under appropriate restrictions on the externalities. In particular, the restrictions apply on the overall scale of externalities, the split of externalities between the different factors of production, and the identity of the sectors with externalities.

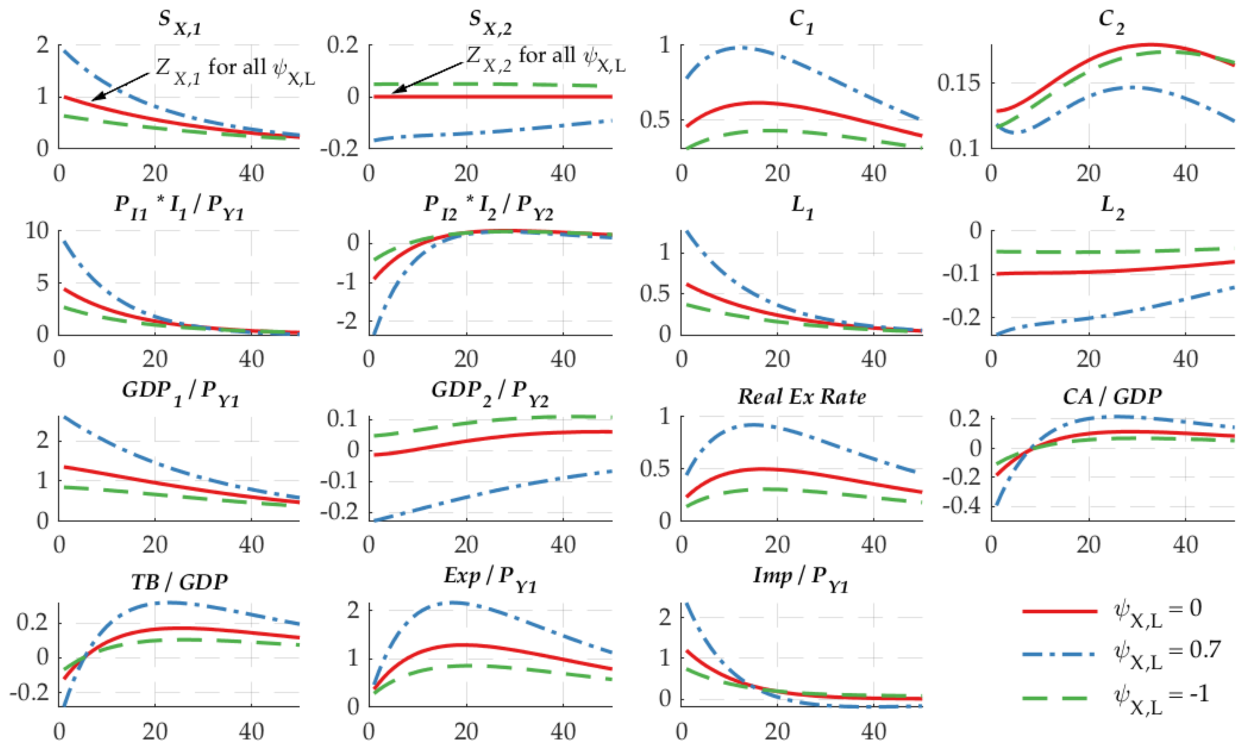
Our theoretical result shows that such isomorphism in terms of aggregate dynamics holds even though the dynamic new trade models have very different micro foundations. Our quantitative exercise then assesses whether various restricted versions of the general model, in forms they are often considered in the literature, are able to resolve the well-known aggregate empirical puzzles in international business cycles models. We provide insights on why they fail to do so in many instances and in what directions they need to be amended to generate the required co-movement across countries. A critical feature that is required is negative capital externalities in intermediate goods production.

In future work, we plan to extend the analysis in some key directions. It would be of interest to study, in our general framework, optimal trade policy to provide a unified treatment of normative issues that have been explored in various modern international trade models. It would also be worthwhile to use this model to delve further into the disconnect that has often been identified in the literature between the international business cycles and international trade fields, such as in estimation/calibration of trade elasticity.



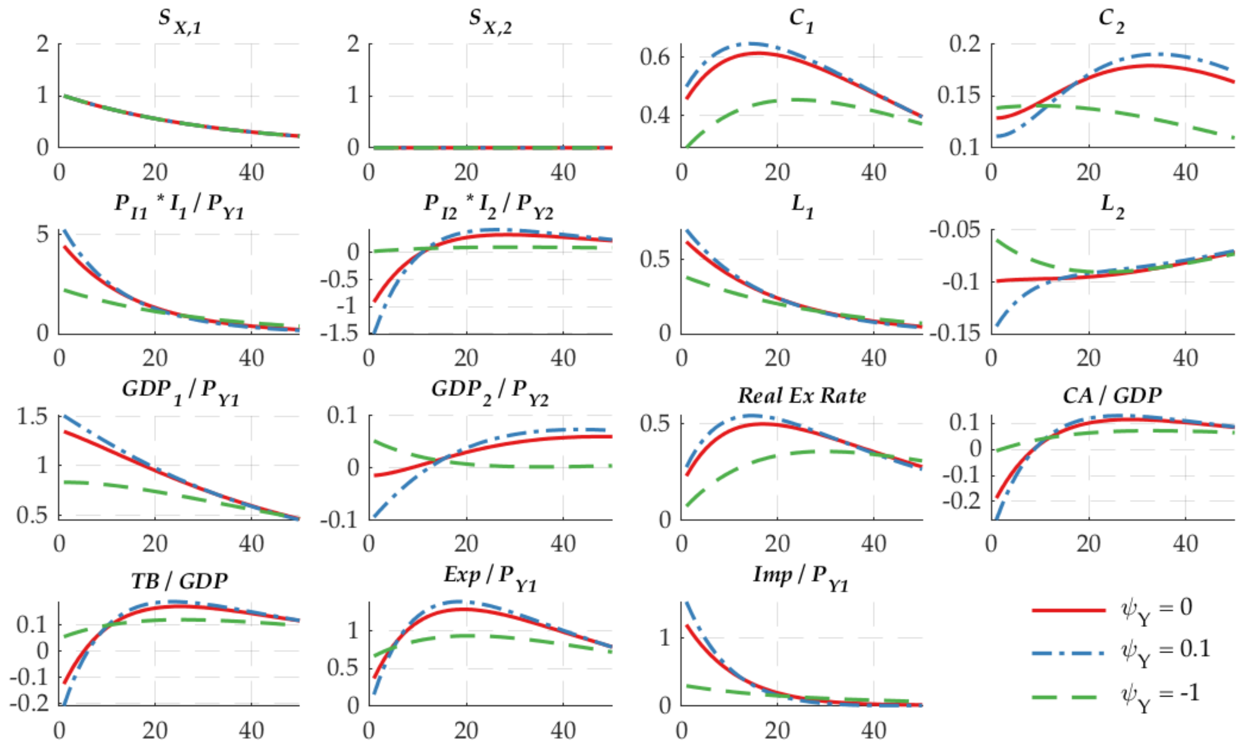
Notes: The plots show responses for 1% shock to the exogenous component of productivity in the intermediates sector in country 1, $Z_{X,1}$. All horizontal axes measure number of quarters after the shock. All vertical axes — except for the figure for the current account — measure percent deviation from steady state. The figure for the current account measures the number of percentage points. The case with $\psi_{X,K} = 0$ corresponds to the benchmark calibration of the unified model with no externalities and uncorrelated shocks (i.e., $\rho_{X,12} = \rho_{X,21} = 0$). Calibrations for the cases with $\psi_{X,K} = 0.3$ and $\psi_{X,K} = -1$ differ from the case with $\psi_{X,K} = 0$ only in having capital externality in the production of intermediates (with the corresponding value for $\psi_{X,K}$). All cases are for the complete markets economy. The red solid lines on the plots for $S_{X,1}$ and $S_{X,2}$ — in addition to responses of $S_{X,1}$ and $S_{X,2}$ for the case of $\psi_{X,K} = 0$ — also correspond to responses of $Z_{X,1}$ and $Z_{X,2}$ for all values of $\psi_{X,K}$.

Figure 1: Impulse-response functions for $Z_{X,1}$. Capital externalities in the intermediate goods sector. Complete markets.



Notes: The plots show responses for 1% shock to the exogenous component of productivity in the intermediates sector in country 1, $Z_{X,1}$. All horizontal axes measure number of quarters after the shock. All vertical axes — except for the figure for the current account — measure percent deviation from steady state. The figure for the current account measures the number of percentage points. The case with $\psi_{x,L} = 0$ corresponds to the benchmark calibration of the unified model with no externalities and uncorrelated shocks (i.e., $\rho_{X,12} = \rho_{X,21} = 0$). Calibrations for the cases with $\psi_{x,L} = 0.7$ and $\psi_{x,L} = -1$ differ from the case with $\psi_{x,L} = 0$ only in having labor externality in the production of intermediates (with the corresponding value for $\psi_{x,L}$). All cases are for the complete markets economy. The red solid lines on the plots for $S_{X,1}$ and $S_{X,2}$ — in addition to responses of $S_{X,1}$ and $S_{X,2}$ for the case of $\psi_{x,L} = 0$ — also correspond to responses of $Z_{X,1}$ and $Z_{X,2}$ for all values of $\psi_{x,L}$.

Figure 2: Impulse-response functions for $Z_{x,1}$. Labor externalities in the intermediate goods sector. Complete markets.



Notes: The plots show responses for 1% shock to the exogenous component of productivity in the intermediates sector in country 1, $Z_{X,1}$. All horizontal axes measure number of quarters after the shock. All vertical axes — except for the figure for the current account — measure percent deviation from steady state. The figure for the current account measures the number of percentage points. The case with $\psi_Y = 0$ corresponds to the benchmark calibration of the unified model with no externalities and uncorrelated shocks (i.e., $\rho_{X,12} = \rho_{X,21} = 0$). Calibrations for the cases with $\psi_Y = 0.2$ and $\psi_Y = -1$ differ from the case with $\psi_Y = 0$ only in having externality in production of the final aggregates (with the corresponding value for ψ_Y). All cases are for the complete markets economy.

Figure 3: Impulse-response functions for $Z_{x,1}$. Externality in the final aggregates sector. Complete markets.

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A Unified Model

A.1 Equilibrium Conditions

Equilibrium conditions of the unified model are given by:

$$
\begin{aligned}
& P_{I,nt} = \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1 \left(C_{n,t+1}, L_{n,t+1}\right)}{U_1 \left(C_{nt}, L_{nt}\right)} \left[R_{n,t+1} + (1-\delta) P_{I,n,t+1}\right] \right\}, \\
& -\frac{U_2 \left(C_{nt}, L_{nt}\right)}{U_1 \left(C_{nt}, L_{nt}\right)} = \frac{W_{nt}}{P_{Y,nt}}, \\
& K_{n,t+1} = (1-\delta) K_{nt} + I_{nt}, \\
& X_{nt} = \left(\Theta_{X,n} Z_{X,nt} K_{nt}^{\psi_{X,K}} L_{X,nt}^{\psi_{X,L}}\right) K_{nt}^{\alpha_{X,K}} L_{X,nt}^{\alpha_{X,L}}, \\
& Y_{nt} = \Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}}\right)^{\psi_Y} \left[\sum_{i=1}^{N} \left(\omega_{ni} \frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it}}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}}, \\
& I_{nt} = \Theta_{I,n} Z_{I,nt} L_{I,nt}^{\alpha_I} Y_{I,nt}^{1-\alpha_I}, \\
& W_{nt} L_{X,nt} + W_{nt} L_{I,nt} = W_{nt} L_{nt} + a \mathrm{TB}_{nt}, \\
& C_{nt} + Y_{I,nt} = Y_{nt}, \\
& \sum_{n=1}^{N} \lambda_{ni,t} P_{Y,nt} Y_{nt} = P_{X,it} X_{it}, \\
& \lambda_{ni,t} = \frac{\left(\tau_{ni,t} P_{X,it} / \omega_{ni}\right)^{1-\sigma}}{\sum_{j=1}^{N} \left(\tau_{nj,t} P_{X,jt} / \omega_{nj}\right)^{1-\sigma}}, \\
& K_{nt} = \alpha_{X,K} \frac{P_{X,nt} X_{nt}}{R_{nt}}, \\
& L_{X,nt} = \alpha_{X,L} \frac{P_{X,nt} X_{nt}}{W_{nt}}, \\
& L_{I,nt} = \alpha_I \frac{P_{I,nt} I_{nt}}{W_{nt}}, \\
& Y_{I,nt} = (1-\alpha_I) \frac{P_{I,nt} I_{nt}}{P_{Y,nt}}.
\end{aligned}
$$

The household's budget constraint in the case of financial autarky is given by

$$P_{Y,nt} C_{nt} + P_{I,nt} I_{nt} = W_{nt} L_{nt} + R_{nt} K_{nt},$$

in the case of the bond economy it is given by

$$P_{Y,nt} C_{nt} + P_{I,nt} I_{nt} + \sum_{i=1}^{N} P_{Y,it} B_{ni,t} = W_{nt} L_{nt} + R_{nt} K_{nt} + \sum_{i=1}^{N} P_{Y,it} \left(1 + r_{i,t-1}\right) B_{ni,t-1},$$

and in the case of complete markets it is given by

$$P_{Y,nt}C_{nt} + P_{I,nt}I_{nt} + A_{nt} = W_{nt}L_{nt} + R_{nt}K_{nt} + B_{nt},$$

with

$$A_{nt} = \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1\left(C_{n,t+1}, L_{n,t+1}\right)}{U_1\left(C_{nt}, L_{nt}\right)} B_{n,t+1} \right\}.$$

Additional conditions in the case of the bond economy are

$$\begin{aligned} P_{Y,it}\left(1 + b_{adj}B_{ni,t}\right) &= \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1\left(C_{n,t+1}, L_{n,t+1}\right)}{U_1\left(C_{nt}, L_{nt}\right)} P_{Y,i,t+1}\left(1 + r_{it}\right) \right\}, \\ &\text{for } i = 1, \ldots, N, \\ \sum_{n=1}^{N} B_{ni,t} &= 0, \end{aligned}$$

while in the case of complete markets they are

$$\begin{aligned} \frac{P_{Y,it}}{P_{Y,jt}} &= \kappa_{ij} \frac{U_1\left(C_{it}, L_{it}\right)}{U_1\left(C_{jt}, L_{jt}\right)}, \quad \text{for each } i \text{ and } j, \\ \sum_{i=1}^{N} A_{it} &= 0, \end{aligned}$$

where

$$\kappa_{ij} \equiv \left(\frac{U_1\left(C_{i0}, L_{i0}\right)/P_{Y,i0}}{U_1\left(C_{j0}, L_{j0}\right)/P_{Y,j0}} \right)^{-1}.$$

is found in the steady state.

A.2 Steady State

Given L_n, Y_n, R_n, W_n, $P_{X,n}$, $P_{I,n}$, $P_{Y,n}$, we can find the rest of the variables using the following conditions:

$$
\begin{aligned}
\lambda_{ni} &= \frac{\left(\tau_{ni} P_{X,i} / \omega_{ni}\right)^{1-\sigma}}{\sum_{j=1}^{N} \left(\tau_{nj} P_{X,j} / \omega_{nj}\right)^{1-\sigma}}, \\
X_i &= \frac{1}{P_{X,i}} \sum_{n=1}^{N} \lambda_{ni} P_{Y,n} Y_n, \\
K_n &= \alpha_{X,K} \frac{P_{X,n} X_n}{R_n}, \\
L_{X,n} &= \alpha_{X,L} \frac{P_{X,n} X_n}{W_n}, \\
I_n &= \delta K_n, \\
C_n &= \left(W_n L_n + R_n K_n - P_{I,n} I_n\right) / P_{Y,n}, \\
L_{I,n} &= \alpha_I \frac{P_{I,n} I_n}{W_n}, \\
Y_{I,n} &= \left(1 - \alpha_I\right) \frac{P_{I,n} I_n}{P_{Y,n}}.
\end{aligned}
$$

Conditions that determine L_n, Y_n, R_n, W_n, $P_{X,n}$, $P_{I,n}$, $P_{Y,n}$, are:

$$
\begin{aligned}
& L_n - L_{X,n} - L_{I,n} = 0, \\
& Y_n - C_n - Y_{I,n} = 0, \\
& R_n - \left(\frac{1}{\beta} - 1 + \delta\right) P_{I,n} = 0, \\
& -\frac{U_2\left(C_n, L_n\right)}{U_1\left(C_n, L_n\right)} - \frac{W_n}{P_{Y,n}} = 0, \\
& X_n - \Theta_{X,n} Z_{X,n} K_n^{\alpha_{X,K}+\psi_{X,K}} L_{X,n}^{\alpha_{X,L}+\psi_{X,L}} = 0, \\
& I_n - \Theta_{I,n} Z_{I,n} L_{I,n}^{\alpha_I} Y_{I,n}^{1-\alpha_I} = 0, \\
& Y_n - \Theta_{Y,n} \left(\frac{P_{Y,n} Y_n}{W_n}\right)^{\psi_Y} \left[\sum_{i=1}^{N} \left(\omega_{ni} \frac{\lambda_{ni} P_{Y,n} Y_n}{\tau_{ni} P_{X,i}}\right)^{\frac{\sigma-1}{\sigma}}\right]^{\frac{\sigma}{\sigma-1}} = 0.
\end{aligned}
$$

B Generalized Dynamic Versions of the Standard Trade Models

B.1 Generalized Dynamic Version of the Eaton-Kortum Model

Since the household's problem is identical to the one in the unified model of Section 2, it yields the same set of equilibrium conditions. Profit maximization problem of producer of variety ν implies

$$R_{nt}K_{X,nt}(\nu) = \alpha_{X,K}p_{nt}(\nu)\,x_{nt}(\nu)\,, \tag{36}$$

$$W_{nt}L_{X,nt}(\nu) = \alpha_{X,L}p_{nt}(\nu)\,x_{nt}(\nu)\,. \tag{37}$$

And the cost of production is

$$p_{nt}(\nu) = \alpha_{X,K}^{-\alpha_{X,K}}\alpha_{X,L}^{-\alpha_{X,L}}\frac{R_{nt}^{\alpha_{X,K}}W_{nt}^{\alpha_{X,L}}}{S_{X,nt}z_n(\nu)}.$$

In equilibrium,

$$K_{X,nt} = \int_0^1 k_{X,nt}(\nu)\,d\nu \quad \text{and} \quad L_{X,nt} = \int_0^1 l_{X,nt}(\nu)\,d\nu.$$

Denote the value of total output of varieties by $\mathcal{X}_{nt}$:

$$\mathcal{X}_{nt} \equiv \int_0^1 p_{nt}(\nu)\,x_{nt}(\nu)\,d\nu.$$

Integrating conditions (36)-(37) over ν, we get

$$R_{nt} = \alpha_{X,K}\frac{\mathcal{X}_{nt}}{K_{X,nt}} \quad \text{and} \quad W_{nt} = \alpha_{X,L}\frac{\mathcal{X}_{nt}}{L_{X,nt}}.$$

Let $\Omega_{ni,t} \subseteq [0,1]$ be the (endogenously determined) set of varieties that country n buys from i. We can write

$$Y_{nt} = S_{Y,nt}^{\text{EK}}\left[\sum_{i=1}^{N}\int_{\nu\in\Omega_{ni,t}}\left(\omega_{ni}x_{ni,t}(\nu)\right)^{\frac{\sigma^{\text{EK}}-1}{\sigma^{\text{EK}}}}d\nu\right]^{\frac{\sigma^{\text{EK}}}{\sigma^{\text{EK}}-1}}.$$

Demand for individual varieties $\nu \in \Omega_{ni,t}$ is given by

$$x_{ni,t}(\nu) = \left[S_{Y,nt}^{\text{EK}}\right]^{\sigma^{\text{EK}}-1}\omega_{ni}^{\sigma^{\text{EK}}-1}\left(\frac{p_{ni,t}(\nu)}{P_{Y,nt}}\right)^{-\sigma^{\text{EK}}}Y_{nt},$$

with the price index

$$P_{Y,nt} = \left[S_{Y,nt}^{\text{EK}}\right]^{-1}\left[\sum_{i=1}^{N}\left(P_{ni,t}/\omega_{ni}\right)^{1-\sigma^{\text{EK}}}\right]^{\frac{1}{1-\sigma^{\text{EK}}}},$$

where

$$P_{ni,t} \equiv \left[\int_{\Omega_{ni,t}} p_{ni,t}\left(\nu\right)^{1-\sigma^{\text{EK}}} d\nu\right]^{\frac{1}{1-\sigma^{\text{EK}}}}.$$

Producers of the final aggregate in country n buy each variety ν from the cheapest source. We can derive

$$P_{ni,t}^{1-\sigma^{\text{EK}}} = \Gamma\left(\frac{\theta^{\text{EK}}+1-\sigma^{\text{EK}}}{\theta^{\text{EK}}}\right) \frac{\left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{-\theta^{\text{EK}}}}{\left[\sum_{j=1}^{N}\left(\tau_{nj,t} P_{X,jt}/\omega_{ni}\right)^{-\theta^{\text{EK}}}\right]^{\frac{\theta^{\text{EK}}+1-\sigma^{\text{EK}}}{\theta^{\text{EK}}}}},$$

where

$$P_{X,it} \equiv \frac{R_{it}^{\alpha_{X,K}} W_{it}^{\alpha_{X,L}}}{\widetilde{\Theta}_{X,i} Z_{X,it} K_{X,it}^{\psi_{X,K}} L_{X,it}^{\psi_{X,L}}},$$

with $\widetilde{\Theta}_{X,i} \equiv \alpha_{X,K}^{\alpha_{X,K}} \alpha_{X,L}^{\alpha_{X,L}} \Theta_{X,i}$. Therefore

$$P_{Y,nt}^{1-\sigma^{\text{EK}}} = \Gamma\left(\frac{\theta^{\text{EK}}+1-\sigma^{\text{EK}}}{\theta^{\text{EK}}}\right) \left[S_{Y,nt}^{\text{EK}}\right]^{\sigma^{\text{EK}}-1} \sum_{i=1}^{N} \frac{\left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{-\theta^{\text{EK}}}}{\left[\sum_{j=1}^{N}\left(\tau_{nj,t} P_{X,jt}/\omega_{nj}\right)^{-\theta^{\text{EK}}}\right]^{\frac{\theta^{\text{EK}}+1-\sigma^{\text{EK}}}{\theta^{\text{EK}}}}},$$

which gives

$$P_{Y,nt} = \frac{\left[\sum_{i=1}^{N}\left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{-\theta^{\text{EK}}}\right]^{-\frac{1}{\theta^{\text{EK}}}}}{\Theta_{Y,n} Z_{Y,nt}\left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}}\right)^{\psi_Y}},$$

with $\Theta_{Y,n} \equiv \Gamma\left(\frac{\theta^{\text{EK}}+1-\sigma^{\text{EK}}}{\theta^{\text{EK}}}\right)^{\frac{1}{\sigma^{\text{EK}}-1}} \Theta_{Y,n}^{\text{EK}}$.

Denote $X_{nt} \equiv \mathcal{X}_{nt}/P_{X,nt}$. After some manipulations, the set of equilibrium conditions that are common across all financial market structures can be written as

$$
\begin{aligned}
&P_{I,nt} = \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1\left(C_{n,t+1}, L_{n,t+1}\right)}{U_1\left(C_{nt}, L_{nt}\right)} \left[R_{n,t+1} + (1-\delta)\, P_{I,n,t+1}\right] \right\}, \\
&-\frac{U_2\left(C_{nt}, L_{nt}\right)}{U_1\left(C_{nt}, L_{nt}\right)} = \frac{W_{nt}}{P_{Y,nt}}, \\
&K_{n,t+1} = (1-\delta)\, K_{nt} + I_{nt}, \\
&X_{nt} = \left(\Theta_{X,n} Z_{X,nt} K_{nt}^{\psi_{X,K}} L_{X,nt}^{\psi_{X,L}}\right) K_{nt}^{\alpha_{X,K}} L_{X,nt}^{\alpha_{X,L}}, \\
&Y_{nt} = \Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}}\right)^{\psi_Y} \left[\sum_{i=1}^{N} \left(\omega_{ni} \frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it}}\right)^{\frac{\theta^{EK}}{\theta^{EK}+1}}\right]^{\frac{\theta^{EK}+1}{\theta^{EK}}}, \\
&I_{nt} = \Theta_{I,n} Z_{I,nt} L_{I,nt}^{\alpha_I} Y_{I,nt}^{1-\alpha_I}, \\
&L_{X,nt} + L_{I,nt} = L_{nt}, \\
&C_{nt} + Y_{I,nt} = Y_{nt}, \\
&\sum_{n=1}^{N} \lambda_{ni,t} P_{Y,nt} Y_{nt} = P_{X,it} X_{it}, \\
&\lambda_{ni,t} = \frac{\left(\tau_{ni,t} P_{X,it} / \omega_{ni}\right)^{-\theta^{EK}}}{\sum_{j=1}^{N} \left(\tau_{nj,t} P_{X,jt} / \omega_{nj}\right)^{-\theta^{EK}}}, \\
&K_{nt} = \alpha_{X,K} \frac{P_{X,nt} X_{nt}}{R_{nt}}, \\
&L_{X,nt} = \alpha_{X,L} \frac{P_{X,nt} X_{nt}}{W_{nt}}, \\
&L_{I,nt} = \alpha_I \frac{P_{I,nt} I_{nt}}{W_{nt}}, \\
&Y_{I,nt} = (1-\alpha_I) \frac{P_{I,nt} I_{nt}}{P_{Y,nt}}, \\
&C_{nt} + I_{nt} = Y_{nt}.
\end{aligned}
$$

Conditions, that are specific to different financial market structures, are identical to the ones in the unified model.

B.2 Generalized Dynamic Version of the Krugman Model

Production of Varieties, International Trade, and Final Aggregate. The profit maximization problem of producer of variety $\nu \in \Omega_{it}$ is given by

$$\max_{p_{ii,t}(\nu),x_{ni,t}(\nu),l_{it}(\nu)} \sum_{n=1}^{N} p_{ii,t}(\nu)\,\tau_{ni,t} x_{ni,t}(\nu) - W_{it} l_{it}(\nu)$$

s.t.

$$x_{ni,t}(\nu) = S_{Y,nt}^{\eta^{\kappa}-1} M_{it}^{(\sigma^{\kappa}-1)\left(\phi_{Y,M}-\frac{1}{\sigma^{\kappa}-1}\right)} \omega_{ni}^{1-\sigma^{\kappa}} \tau_{ni,t}^{-\sigma^{\kappa}} p_{ii,t}(\nu)^{-\sigma^{\kappa}} P_{ni,t}^{\sigma^{\kappa}-\eta^{\kappa}} P_{Y,nt}^{\eta^{\kappa}} Y_{nt},$$

$$\sum_{n=1}^{N} \tau_{ni,t} x_{ni,t}(\nu) = S_{X,it}^{\kappa} l_{it}(\nu), \tag{38}$$

This gives the monopolist's price

$$p_{ii,t}(\nu) = \frac{\sigma^{\kappa}}{\sigma^{\kappa}-1} \cdot \frac{W_{it}}{S_{X,it}^{\kappa}},$$

and the bilateral price index

$$\begin{aligned}
P_{ni,t} &= M_{it}^{-\left(\phi_{Y,M}-\frac{1}{\sigma^{\kappa}-1}\right)} \left[\int_{\nu\in\Omega_{it}} \left(p_{ni,t}(\nu)/\omega_{ni}\right)^{1-\sigma^{\kappa}} d\nu\right]^{\frac{1}{1-\sigma^{\kappa}}} \\
&= M_{it}^{-\left(\phi_{Y,M}-\frac{1}{\sigma^{\kappa}-1}\right)} \left[M_{it}\left[\frac{\sigma^{\kappa}}{\sigma^{\kappa}-1} \cdot \frac{\tau_{ni,t} W_{it}}{\omega_{ni} S_{X,it}^{\kappa}}\right]^{1-\sigma^{\kappa}}\right]^{\frac{1}{1-\sigma^{\kappa}}} \\
&= \tau_{ni,t} P_{X,it}/\omega_{ni},
\end{aligned}$$

where

$$P_{X,it} \equiv \frac{\sigma^{\kappa}}{\sigma^{\kappa}-1} \cdot \frac{W_{it}}{\Theta_{X,i} Z_{X,it} M_{it}^{\phi_{Y,M}} L_{X,it}^{\phi_{X,L}}}.$$

From here we can find total demand of country n for country i's varieties:

$$\begin{aligned}
\mathcal{X}_{ni,t} &= \int_{\nu\in\Omega_{it}} \tau_{ni,t} p_{ii,t}(\nu)\, x_{ni,t}(\nu)\, d\nu \\
&= S_{Y,nt}^{\eta^{\kappa}-1} \left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{1-\sigma^{\kappa}} P_{ni,t}^{\sigma^{\kappa}-\eta^{\kappa}} P_{Y,nt}^{\eta^{\kappa}} Y_{nt} \\
&= \lambda_{ni,t} P_{Y,nt} Y_{nt},
\end{aligned}$$

where

$$\lambda_{ni,t} \equiv \frac{\left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{1-\eta^{\kappa}}}{\sum_{j=1}^{N} \left(\tau_{nj,t} P_{X,jt}/\omega_{nj}\right)^{1-\eta^{\kappa}}}$$

is the expenditure share.

Next, multiplying both sides of (38) on price $p_{ii,t}(\nu)$ gives

$$\sum_{n=1}^{N} p_{ni,t}(\nu)\, x_{ni,t}(\nu) = \frac{\sigma^{K}}{\sigma^{K}-1} \cdot W_{it} l_{it}(\nu).$$

Integrating both sides of this expression over $\nu \in \Omega_{it}$, we get

$$\mathcal{X}_{it} = \frac{\sigma^{K}}{\sigma^{K}-1} W_{it} L_{X,it},$$

where $\mathcal{X}_{it}$ is the total value of output of all varieties in country i.

Profit of producer of variety $\nu \in \Omega_{it}$ is given by

$$D_{it}(\nu) = \sum_{n=1}^{N} p_{ni,t}(\nu)\, x_{ni,t}(\nu) - W_{it} l_{it}(\nu).$$

Let $D_{it} \equiv \frac{1}{M_{it}} \int_{\nu\in\Omega_{it}} D_{it}(\nu)\, d\nu$ be the average profit of country i's producers of varieties Ω_{it}. Integrating both sides of the above expression over $\nu \in \Omega_{it}$, we get

$$D_{it} = \frac{\mathcal{X}_{it} - W_{it} L_{X,it}}{M_{it}} = \frac{1}{\sigma^{K}} \cdot \frac{\mathcal{X}_{it}}{M_{it}}.$$

Invention of Varieties, Entry and Exit of Producers of Varieties. Varieties are invented in the R&D sector. The invention process uses labor and final aggregate. Specifically, a combination of l_I units of labor and y_I units of the final aggregate results in $\Theta_{I,n} Z_{I,nt} l_I^{\alpha_I} y_I^{1-\alpha_I}$ new varieties, where $0 \leq \alpha_I \leq 1$, and $\Theta_{I,n} Z_{I,nt}$ is an exogenous productivity in the R&D sector. Assuming perfect competition in the R&D sector and letting V_{nt} be the value of an invented variety, we get that invention of one variety requires $\alpha_I \frac{V_{nt}}{W_{nt}}$ units of labor and $(1-\alpha_I)\frac{V_{nt}}{P_{Y,nt}}$ units of the final aggregate. Perfect competition also implies that $V_{nt} = \frac{W_{nt}^{\alpha_I} P_{Y,nt}^{1-\alpha_I}}{\widetilde{\Theta}_{I,n} Z_{I,nt}}$, where $\widetilde{\Theta}_{I,n} \equiv \alpha_I^{\alpha_I} (1-\alpha_I)^{1-\alpha_I} \Theta_{I,n}$.

In every period t each country has an unbounded mass of prospective entrants (firms) into the production of varieties. All varieties invented in a particular country in period t are sold to these prospective entrants in the same period. A producer of a variety enters into the economy by buying this variety from the R&D sector. Entry into the economy is free, and so any entrant pays for the variety its value V_{nt}.

Let $M_{I,nt}$ denote the number of varieties that are invented in country n in period t (which is also the number of firms that enter into the economy). The total amount of labor and final aggregate used in the R&D sector are, respectively,

$$L_{I,nt} = \alpha_I \frac{V_{nt} M_{I,nt}}{W_{nt}}, \quad \text{and} \quad Y_{I,nt} = (1-\alpha_I) \frac{V_{nt} M_{I,nt}}{P_{Y,nt}}.$$

From here we also get that

$$M_{I,nt} = \Theta_{I,n} Z_{I,nt} L_{I,nt}^{\alpha_I} Y_{I,nt}^{1-\alpha_I}.$$

Households. Here we describe only financial autarky. Derivations for bond economy and complete markets can be done in a similar way. The problem of country n's households is

$$\max_{C_{nt}, L_{nt}, M_{I,nt}, M_{n,t+1}} E_0 \sum_{t=0}^{\infty} \beta^t U(C_{nt}, L_{nt})$$

s.t.

$$P_{Y,nt} C_{nt} + V_{nt} M_{I,nt} = W_{nt} L_{nt} + D_{nt} M_{nt},$$
$$M_{n,t+1} = (1-\delta) M_{nt} + M_{I,nt}.$$

First-order conditions for this problem imply:

$$V_{nt} = \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1(C_{n,t+1}, L_{n,t+1})}{U_1(C_{nt}, L_{nt})} \left[D_{n,t+1} + (1-\delta) V_{n,t+1} \right] \right\},$$
$$-\frac{U_2(C_{nt}, L_{nt})}{U_1(C_{nt}, L_{nt})} = \frac{W_{nt}}{P_{Y,nt}}.$$

Equilibrium System of Equations Let us manipulate the expression for $P_{X,nt}$ to bring it to a form isomorphic to the price of the intermediate good in the unified model. We have

$$P_{X,nt} = \frac{\sigma^K}{\sigma^K - 1} \cdot \frac{W_{nt}}{\Theta_{X,it} Z_{X,nt} M_{nt}^{\phi_{Y,M}} L_{X,nt}^{\phi_{X,L}}} = \left(1 - \frac{1}{\sigma^K}\right)^{-1} \frac{D_{nt}^{\frac{1}{\sigma^K}} W_{nt}^{1-\frac{1}{\sigma^K}}}{Z_{X,nt} M_{nt}^{\phi_{Y,M}} L_{X,nt}^{\phi_{X,L}} D_{nt}^{\frac{1}{\sigma^K}} W_{nt}^{-\frac{1}{\sigma^K}}}.$$

Using the facts that $D_{nt} = \frac{1}{\sigma^K} \cdot \frac{\mathcal{X}_{nt}}{M_{nt}}$ and $W_{nt} = \left(1 - \frac{1}{\sigma^K}\right) \frac{\mathcal{X}_{nt}}{L_{X,nt}}$, we get

$$P_{X,nt} = \frac{D_{nt}^{\frac{1}{\sigma^K}} W_{nt}^{1-\frac{1}{\sigma^K}}}{\widetilde{\Theta}_{X,n}^K Z_{X,nt} M_{nt}^{\phi_{Y,M} - \frac{1}{\sigma^K}} L_{X,nt}^{\phi_{X,L} + \frac{1}{\sigma^K}}},$$

where $\widetilde{\Theta}^{\kappa}_{X,n} \equiv \left(\frac{1}{\sigma^{\kappa}}\right)^{\frac{1}{\sigma^{\kappa}}} \left(1-\frac{1}{\sigma^{\kappa}}\right)^{1-\frac{1}{\sigma^{\kappa}}} \Theta_{X,n}$. Let $X_{nt} \equiv \mathcal{X}_{nt}/P_{X,nt}$ be the real output of varieties. By substituting the expressions for D_{nt} and W_{nt} into the above expression for $P_{X,nt}$, we get

$$X_{nt} = \left(\Theta_{X,n} Z_{X,nt} M_{nt}^{\phi_{Y,M}-\frac{1}{\sigma^{\kappa}}} L_{X,nt}^{\phi_{X,L}+\frac{1}{\sigma^{\kappa}}}\right) M_{nt}^{\frac{1}{\sigma^{\kappa}}} L_{X,nt}^{1-\frac{1}{\sigma^{\kappa}}}.$$

Next, we have

$$\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it}/\omega_{ni}} = S_{Y,nt}^{\eta^{\kappa}-1} \left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{-\eta^{\kappa}} P_{Y,nt}^{\eta^{\kappa}} Y_{nt},$$

which gives

$$\left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{\eta^{\kappa}} = S_{Y,nt}^{\eta^{\kappa}-1} \left(\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it}/\omega_{ni}}\right)^{-1} P_{Y,nt}^{\eta^{\kappa}} Y_{nt}.$$

Taking both sides to the power of $\frac{1-\eta^{\kappa}}{\eta^{\kappa}}$, we get

$$\left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{1-\eta^{\kappa}} = S_{Y,nt}^{(\eta^{\kappa}-1)\frac{1-\eta^{\kappa}}{\eta^{\kappa}}} \left(\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it}/\omega_{ni}}\right)^{\frac{\eta^{\kappa}-1}{\eta^{\kappa}}} P_{Y,nt}^{1-\eta^{\kappa}} Y_{nt}^{\frac{1-\eta^{\kappa}}{\eta^{\kappa}}}.$$

Summing over i and using the fact that

$$P_{Y,nt}^{1-\eta^{\kappa}} = S_{Y,nt}^{-(1-\eta^{\kappa})} \sum_{i=1}^{N} P_{ni,t}^{1-\eta^{\kappa}} = S_{Y,nt}^{-(1-\eta^{\kappa})} \sum_{i=1}^{N} \left(\tau_{ni,t} P_{X,it}/\omega_{ni}\right)^{1-\eta^{\kappa}},$$

we get

$$Y_{nt} = S_{Y,nt} \left[\sum_{i=1}^{N} \left(\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it}/\omega_{ni}}\right)^{\frac{\eta^{\kappa}-1}{\eta^{\kappa}}}\right]^{\frac{\eta^{\kappa}}{\eta^{\kappa}-1}}.$$

Combining all expressions and definitions, we get the equilibrium system in isomorphic form:

$$
\begin{aligned}
&V_{nt} = \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1\left(C_{n,t+1}, L_{n,t+1}\right)}{U_1\left(C_{nt}, L_{nt}\right)} \left[D_{n,t+1} + (1-\delta) V_{n,t+1}\right] \right\}, \\
&-\frac{U_2\left(C_{nt}, L_{nt}\right)}{U_1\left(C_{nt}, L_{nt}\right)} = \frac{W_{nt}}{P_{Y,nt}}, \\
&M_{n,t+1} = (1-\delta) M_{nt} + M_{I,nt}, \\
&X_{nt} = \left(\Theta_{X,n} Z_{X,nt} M_{nt}^{\phi_{Y,M} - \frac{1}{\sigma^K}} L_{X,nt}^{\phi_{X,L} + \frac{1}{\sigma^K}}\right) M_{nt}^{\frac{1}{\sigma^K}} L_{X,nt}^{1-\frac{1}{\sigma^K}}, \\
&Y_{nt} = \Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}}\right)^{\psi_Y} \left[\sum_{i=1}^{N} \left(\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it}}\right)^{\frac{\eta^K - 1}{\eta^K}}\right]^{\frac{\eta^K}{\eta^K - 1}}, \\
&M_{I,nt} = \Theta_{I,n} Z_{I,nt} L_{I,nt}^{\alpha_I} Y_{I,nt}^{1-\alpha_I}, \\
&L_{X,nt} + L_{I,nt} = L_{nt}, \\
&C_{nt} + Y_{I,nt} = Y_{nt}, \\
&\sum_{n=1}^{N} \lambda_{ni,t} P_{Y,nt} Y_{nt} = P_{X,it} X_{it}, \\
&\lambda_{ni,t} = \frac{\left(\tau_{ni,t} P_{X,it}\right)^{1-\eta^K}}{\sum_{l=1}^{N} \left(\tau_{nl,t} P_{X,lt}\right)^{1-\eta^K}}, \\
&M_{nt} = \frac{1}{\sigma^K} \cdot \frac{P_{X,nt} X_{nt}}{D_{nt}}, \\
&L_{X,nt} = \left(1 - \frac{1}{\sigma^K}\right) \cdot \frac{P_{X,nt} X_{nt}}{W_{nt}}, \\
&L_{I,nt} = \alpha_I \frac{V_{nt} M_{I,nt}}{W_{nt}}, \\
&Y_{I,nt} = (1-\alpha_I) \frac{V_{nt} M_{I,nt}}{P_{Y,nt}}, \\
&P_{Y,nt} C_{nt} + V_{nt} M_{I,nt} = W_{nt} L_{nt} + D_{nt} M_{nt}.
\end{aligned}
$$

B.3 Generalized Version of the Melitz Model

In order to show what role the love-of-variety effect plays in the Melitz model, let us introduce correction for this effect in the technology of production of final aggregate. Assume that the final aggregate technology is given by

$$
Y_{nt} = \left[\sum_{i=1}^{N} \left[M_{ni,t}^{\phi_{Y,M} - \frac{1}{\sigma^M - 1}} \left[\int_{\nu \in \Omega_{ni,t}} \left(\omega_{ni} x_{ni,t}(\nu)\right)^{\frac{\sigma^M - 1}{\sigma^M}} d\nu\right]^{\frac{\sigma^M}{\sigma^M - 1}}\right]^{\frac{\eta^M - 1}{\eta^M}}\right]^{\frac{\eta^M}{\eta^M - 1}},
$$

where $M_{ni,t}^{\phi_{Y,M}-\frac{1}{\sigma^M-1}}$ is the correction term for the love-of-variety effect with the strength of the effect given by parameter $\phi_{Y,M}$. Denote, for convenience, $\widetilde{\phi}_{Y,M} \equiv \phi_{Y,M} - \frac{1}{\sigma^M-1}$. Demand for individual varieties is given by

$$x_{ni,t}(\nu) = M_{ni,t}^{(\sigma^M-1)\widetilde{\phi}_{Y,M}} \omega_{ni}^{\sigma^M-1} \left(\frac{p_{ni,t}(\nu)}{P_{ni,t}}\right)^{-\sigma^M} \left(\frac{P_{ni,t}}{P_{Y,nt}}\right)^{-\eta^M} Y_{nt}.$$

$$P_{ni,t} = M_{ni,t}^{-\widetilde{\phi}_{Y,M}} \left[\int_{\nu\in\Omega_{ni,t}} \left(p_{ni,t}(\nu)/\omega_{ni}\right)^{1-\sigma^M} d\nu\right]^{\frac{1}{1-\sigma^M}},$$

$$P_{Y,nt} = \left[\sum_{i=1}^{N} P_{ni,t}^{1-\eta^M}\right]^{1-\eta^M}.$$

The profit that producer of variety $\nu \in \Omega_{it}$ can earn in market n is given by

$$\begin{aligned} D_{ni,t}(\nu) &= \frac{1}{\sigma^M} p_{ni,t}(\nu)\, x_{ni,t}(\nu) - W_{nt}\Phi_{ni,t} \\ &= \frac{1}{\sigma^M} M_{ni,t}^{(\sigma^M-1)\widetilde{\phi}_{Y,M}} \left(\frac{\sigma^M}{\sigma^M-1} \cdot \frac{\tau_{ni,t}^M W_{it}}{\omega_{ni} S_{X,it}^M z_i(\nu)}\right)^{1-\sigma^M} P_{ni,t}^{\sigma^M-\eta^M} P_{Y,nt}^{\eta^M} Y_{nt} - W_{nt}\Phi_{ni,t}. \end{aligned}$$

As long as $D_{ni,t}(\nu) \geq 0$, variety $\nu \in \Omega_{it}$ will be sold in country n. Condition $D_{ni,t}(\nu) = 0$ gives the cutoff efficiency $z_{ni,t}^*$ such that only producers with $z_i(\nu) \geq z_{ni,t}^*$ serve market n. After some algebra, we get

$$\frac{z_{ni,t}^*}{z_{\min,i}} = M_{ni,t}^{-\widetilde{\phi}_{Y,M}} \left(\frac{\sigma^M}{\sigma^M-1} \cdot \frac{\tau_{ni,t}^M W_{it}}{\omega_{ni} S_{X,it}^M z_{\min,i}}\right) \left(\frac{P_{Y,nt} Y_{nt}}{\sigma^M W_{nt}\Phi_{ni,t}}\right)^{\frac{1}{1-\sigma^M}} P_{ni,t}^{-\frac{\sigma^M-\eta^M}{\sigma^M-1}} P_{Y,nt}^{-\frac{\eta^M-1}{\sigma^M-1}}.$$

With Pareto distribution of efficiencies of production, we have that

$$M_{ni,t} = M_{it} \int_{z_{ni,t}^*}^{\infty} dG_i(z) = M_{it}\left(1 - G_i\left(z_{ni,t}^*\right)\right) = M_{it} \left(\frac{z_{ni,t}^*}{z_{\min,i}}\right)^{-\theta^M}.$$

This gives

$$\left(\frac{z_{ni,t}^*}{z_{\min,i}}\right)^{1-\widetilde{\phi}_{Y,M}\theta^M} = M_{it}^{-\widetilde{\phi}_{Y,M}} \left(\frac{\sigma^M}{\sigma^M-1} \cdot \frac{\tau_{ni,t}^M W_{it}}{\omega_{ni} S_{X,it}^M z_{\min,i}}\right) \left(\frac{P_{Y,nt} Y_{nt}}{\sigma^M W_{nt}\Phi_{ni,t}}\right)^{\frac{1}{1-\sigma^M}} P_{ni,t}^{-\frac{\sigma^M-\eta^M}{\sigma^M-1}} P_{Y,nt}^{-\frac{\eta^M-1}{\sigma^M-1}}. \tag{39}$$

Next, let us find the bilateral price indices. We have

$$
\begin{aligned}
P_{ni,t}^{1-\sigma^M} &= M_{ni,t}^{(\sigma^M-1)\phi_{Y,M}-1} M_{it} \int_{z_{ni,t}^*}^{\infty} \left(\frac{\sigma^M}{\sigma^M-1} \cdot \frac{\tau_{ni,t}^M W_{it}}{\omega_{ni} S_{X,it}^M z} \right)^{1-\sigma^M} dG_i(z) \\
&= \theta^M z_{\min,i}^{\theta^M} \left(\frac{\sigma^M}{\sigma^M-1} \cdot \frac{\tau_{ni,t}^M W_{it}}{\omega_{ni} S_{X,it}^M} \right)^{1-\sigma^M} M_{ni,t}^{(\sigma^M-1)\phi_{Y,M}-1} M_{it} \int_{z_{ni,t}^*}^{\infty} z^{\sigma^M-\theta^M-2} dz \\
&= \frac{\theta^M}{\theta^M+1-\sigma^M} \left(\frac{\sigma^M}{\sigma^M-1} \cdot \frac{\tau_{ni,t}^M W_{it}}{\omega_{ni} z_{\min,i} S_{X,it}^M} \right)^{1-\sigma^M} M_{ni,t}^{(\sigma^M-1)\phi_{Y,M}-1} M_{it} \left(\frac{z_{ni,t}^*}{z_{\min,i}} \right)^{\sigma^M-\theta^M-1} \\
&= \frac{\theta^M}{\theta^M+1-\sigma^M} \left(\frac{\sigma^M}{\sigma^M-1} \cdot \frac{\tau_{ni,t}^M W_{it}}{\omega_{ni} z_{\min,i} S_{X,it}^M} \right)^{1-\sigma^M} M_{it}^{(\sigma^M-1)\phi_{Y,M}} \left(\frac{z_{ni,t}^*}{z_{\min,i}} \right)^{(\sigma^M-1)\left(1-\theta^M\phi_{Y,M}\right)} .
\end{aligned}
\tag{40}
$$

In order to ensure that the right-hand side of this expression is positive, we need to make the technical assumption that $\theta^M > \sigma^M - 1$.

Without risk of confusion, let us redefine constant ϑ in definition (27) of $\Phi_{ni,t}$ to be $\vartheta \equiv \phi_{Y,M} - \frac{1}{\theta^M}$. Without correction for the love-of-variety effect (i.e., when $\phi_{Y,M} = 1/\left(\sigma^M - 1\right)$), we have the same definition of ϑ as in the main text. Substituting the expression (39) for the cutoff threshold into (40) and using the definition of $\Phi_{ni,t}$, we get:

$$
P_{ni,t}^{1-\sigma^M} = \frac{\theta^M}{\theta^M+1-\sigma^M} \left(\tau_{ni,t}^M P_{X,it}/\omega_{ni} \right)^{-\frac{\theta^M}{1-\widetilde{\phi}_{Y,M}\theta^M}} \left(\frac{P_{Y,nt} Y_{nt}}{\sigma^M L_{F,nt}^{\frac{\vartheta-\phi_{F,L}}{\vartheta}} W_{nt} F_{ni,t}} P_{ni,t}^{\sigma^M-\eta^M} P_{Y,nt}^{\eta^M-1} \right)^{\frac{\vartheta\theta^M}{1-\widetilde{\phi}_{Y,M}\theta^M}} ,
$$

where

$$
P_{X,it} \equiv \frac{\sigma^M}{\sigma^M-1} \cdot \frac{W_{it}}{z_{\min,i} S_{X,it}^M M_{it}^{\phi_{F,M}}}.
$$

Solving for $P_{ni,t}$, we get

$$
P_{ni,t}^{1-\eta^M} = \left(\frac{\theta^M}{\theta^M+1-\sigma^M} \right)^{\left(1-\widetilde{\phi}_{Y,M}\theta^M\right)\xi} \left(\tau_{ni,t}^M P_{X,it}/\omega_{ni} \right)^{-\theta^M\xi} \left(\frac{P_{Y,nt} Y_{nt}}{\sigma^M L_{F,nt}^{\frac{\vartheta-\phi_{F,L}}{\vartheta}} W_{nt} F_{ni,t}} P_{Y,nt}^{\eta^M-1} \right)^{\vartheta\theta^M\xi} ,
$$

where

$$
\xi \equiv \frac{1}{\left(\frac{1}{\eta^M-1} - \phi_{Y,M} \right) \theta^M + 1}.
$$

This allows us to find expression for the price index,

$$P_{Y,nt} = \left(\frac{\theta^{M}}{\theta^{M}+1-\sigma^{M}}\right)^{-\left(\frac{1}{\theta^{M}}-\widetilde{\phi}_{Y,M}\right)} \left(\frac{P_{Y,nt}Y_{nt}}{\sigma^{M}W_{nt}}\right)^{-\vartheta} L_{F,nt}^{\vartheta-\phi_{F,L}} \left[\sum_{i=1}^{N}\left(F_{ni,t}^{\vartheta}\tau_{ni,t}^{M}P_{X,it}/\omega_{ni}\right)^{-\theta^{M}\xi}\right]^{-\frac{1}{\theta^{M}\xi}}.$$

Next, bilateral trade flows are given by:

$$\begin{aligned}\mathcal{X}_{ni,t} &= M_{it}\int_{\Omega_{ni,t}} p_{ni,t}(\nu)\, x_{ni,t}(\nu)\, d\nu \\ &= \left(\frac{P_{ni,t}}{P_{Y,nt}}\right)^{1-\eta^{M}} P_{Y,nt}Y_{nt}.\end{aligned}$$

Substituting expressions for price indices, we get

$$\mathcal{X}_{ni,t} = \lambda_{ni,t}P_{Y,nt}Y_{nt},$$

where

$$\lambda_{ni,t} = \frac{\left(\Phi_{ni,t}^{\vartheta}\tau_{ni,t}^{M}P_{X,it}/\omega_{ni}\right)^{-\theta^{M}\xi}}{\sum_{l=1}^{N}\left(\Phi_{nl,t}^{\vartheta}\tau_{nl,t}^{M}P_{X,lt}/\omega_{nl}\right)^{-\theta^{M}\xi}}.$$

Let us now find profits. For this, we need to have the expression for $z_{ni,t}^{*}$. After some algebra, we get

$$\left(\frac{z_{ni,t}^{*}}{z_{\min,i}}\right)^{1-\widetilde{\phi}_{Y,M}\theta^{M}} = \frac{\tau_{ni,t}^{M}P_{X,it}/\omega_{ni}}{P_{ni,t}} M_{it}^{\frac{\phi_{Y,M}-\phi_{F,M}}{\vartheta}\left(\frac{1}{\sigma^{M}-1}-\vartheta\right)} \left(\frac{\mathcal{X}_{ni,t}}{\sigma^{M}W_{nt}F_{ni,t}}\right)^{\frac{1}{1-\sigma^{M}}} L_{F,nt}^{\frac{\vartheta-\phi_{F,L}}{\vartheta(\sigma^{M}-1)}}.$$

Next, we have

$$\begin{aligned}\frac{\tau_{ni,t}^{M}P_{X,it}/\omega_{ni}}{P_{ni,t}} &= \left(\frac{\theta^{M}}{\theta^{M}+1-\sigma^{M}}\right)^{\frac{1-\widetilde{\phi}_{Y,M}\theta^{M}}{\theta^{M}}} \left(\frac{P_{ni,t}}{P_{Y,nt}}\right)^{(1-\eta^{M})\vartheta} \left(\frac{P_{Y,nt}Y_{nt}}{\sigma^{M}W_{nt}F_{ni,t}}\right)^{\vartheta} L_{F,nt}^{-\left(\vartheta-\phi_{F,L}\right)} \\ &= \left(\frac{\theta^{M}}{\theta^{M}+1-\sigma^{M}}\right)^{\frac{1-\widetilde{\phi}_{Y,M}\theta^{M}}{\theta^{M}}} \left(\frac{\mathcal{X}_{ni,t}}{\sigma^{M}W_{nt}F_{ni,t}}\right)^{\vartheta} L_{F,nt}^{-\left(\vartheta-\phi_{F,L}\right)},\end{aligned}$$

which allows us to find

$$\left(\frac{z_{ni,t}^{*}}{z_{\min,i}}\right)^{\theta^{M}} = \left[\frac{\theta^{M}+1-\sigma^{M}}{\theta^{M}\sigma^{M}}\cdot\frac{\mathcal{X}_{ni,t}}{W_{nt}F_{ni,t}}\right]^{-1} M_{it}^{\frac{\phi_{Y,M}-\phi_{F,M}}{\vartheta}} L_{F,nt}^{\frac{\vartheta-\phi_{F,L}}{\vartheta}},$$

and

$$M_{ni,t} = \left(\frac{z^*_{ni,t}}{z_{\min,i}}\right)^{-\theta^{\mathrm{M}}} M_{it} = \left(\frac{\theta^{\mathrm{M}} + 1 - \sigma^{\mathrm{M}}}{\theta^{\mathrm{M}}\sigma^{\mathrm{M}}} \cdot \frac{\mathcal{X}_{ni,t}}{W_{nt}F_{ni,t}}\right) \left[M_{it}^{\frac{1}{\theta^{\mathrm{M}}}-\phi_{F,M}} L_{F,nt}^{\vartheta-\phi_{F,L}}\right]^{-\frac{1}{\vartheta}}.$$

To get average profits of country i from exports to n, we need to calculate the following expression:

$$\begin{aligned}
D_{ni,t} &= \frac{1}{\sigma^{\mathrm{M}}} \cdot \frac{\mathcal{X}_{ni,t}}{M_{it}} - W_{nt}\left[M_{it}^{\frac{1}{\theta^{\mathrm{M}}}-\phi_{F,M}} L_{F,nt}^{\vartheta-\phi_{F,L}}\right]^{\frac{1}{\vartheta}} F_{ni,t}\frac{M_{ni,t}}{M_{it}} \\
&= \frac{1}{\sigma^{\mathrm{M}}} \cdot \frac{\mathcal{X}_{ni,t}}{M_{it}} - \frac{\theta^{\mathrm{M}} + 1 - \sigma^{\mathrm{M}}}{\theta^{\mathrm{M}}\sigma^{\mathrm{M}}} \cdot \frac{\mathcal{X}_{ni,t}}{M_{it}} \\
&= \frac{\sigma^{\mathrm{M}} - 1}{\theta^{\mathrm{M}}\sigma^{\mathrm{M}}} \cdot \frac{\mathcal{X}_{ni,t}}{M_{it}}.
\end{aligned}$$

Hence, total average profits of country i are

$$D_{it} = \sum_{n=1}^{N} D_{ni,t} = \frac{\sigma^{\mathrm{M}} - 1}{\sigma^{\mathrm{M}}\theta^{\mathrm{M}}} \cdot \frac{\mathcal{X}_{it}}{M_{it}},$$

where $\mathcal{X}_{it}$ is total output of intermediates in country i. We can find that, as in the Krugman model,

$$\mathcal{X}_{it} = \frac{\sigma^{\mathrm{M}}}{\sigma^{\mathrm{M}} - 1} W_{it} L^{\mathrm{M}}_{X,it}.$$

The amount of country n's labor that country i uses to serve country n's market is

$$L_{F,ni,t} = \left[M_{it}^{\frac{1}{\theta^{\mathrm{M}}}-\phi_{F,M}} L_{F,nt}^{\vartheta-\phi_{F,L}}\right]^{\frac{1}{\vartheta}} F_{ni,t}M_{ni,t} = \frac{\theta^{\mathrm{M}} + 1 - \sigma^{\mathrm{M}}}{\theta^{\mathrm{M}}\sigma^{\mathrm{M}}} \cdot \frac{\mathcal{X}_{ni,t}}{W_{nt}}.$$

Hence, the total amount of country n's labor used to serve its market is

$$\begin{aligned}
L_{F,nt} &= \sum_{i=1}^{N} L_{F,ni,t} = \frac{\theta^{\mathrm{M}} + 1 - \sigma^{\mathrm{M}}}{\theta^{\mathrm{M}}\sigma^{\mathrm{M}}} \cdot \sum_{i=1}^{N} \frac{\mathcal{X}_{ni,t}}{W_{nt}} \\
&= \frac{\theta^{\mathrm{M}} + 1 - \sigma^{\mathrm{M}}}{\theta^{\mathrm{M}}\sigma^{\mathrm{M}}} \cdot \frac{P_{Y,nt}Y_{nt}}{W_{nt}}.
\end{aligned}$$

This allows us to write

$$P_{Y,nt} = \left(\frac{\theta^{\mathrm{M}}}{\theta^{\mathrm{M}} + 1 - \sigma^{\mathrm{M}}}\right)^{-\frac{1}{\sigma^{\mathrm{M}}-1}} L_{F,nt}^{-\phi_{F,L}} \left[\sum_{i=1}^{N} \left(F_{ni,t}^{\vartheta}\tau^{\mathrm{M}}_{ni,t}P_{X,it}/\omega_{ni}\right)^{-\theta^{\mathrm{M}}\xi}\right]^{-\frac{1}{\theta^{\mathrm{M}}\xi}}.$$

or

$$P_{Y,nt} = \left(\frac{\theta^{M}}{\theta^{M}+1-\sigma^{M}}\right)^{-\frac{1}{\sigma^{M}-1}+\phi_{F,L}} \left(\frac{P_{Y,nt}Y_{nt}}{\sigma^{M}W_{nt}}\right)^{-\phi_{F,L}} \left[\sum_{i=1}^{N}\left(F_{ni,t}^{\vartheta}\tau_{ni,t}^{M}P_{X,it}/\omega_{ni}\right)^{-\theta^{M}\xi}\right]^{-\frac{1}{\theta^{M}\xi}}.$$

Also, we can write

$$L_{F,nt} = \frac{\theta^{M}+1-\sigma^{M}}{\theta^{M}\sigma^{M}}\cdot\frac{\mathcal{X}_{nt}}{W_{nt}}\cdot\frac{P_{Y,nt}Y_{nt}}{\mathcal{X}_{nt}} = \left(\frac{1}{\sigma^{M}-1}-\frac{1}{\theta^{M}}\right)\frac{P_{Y,nt}Y_{nt}}{\mathcal{X}_{nt}}L_{X,nt}^{M}.$$

Equilibrium System of Equations In order to write the equilibrium system in the isomorphic form, we need to do transformations of some of the equilibrium conditions. Define trade deficit as the value of net exports of varieties,

$$TB_{nt} \equiv \mathcal{X}_{nt} - P_{Y,nt}Y_{nt}.$$

We can write

$$\begin{aligned} L_{F,nt} &= \left(\frac{1}{\sigma^{M}-1}-\frac{1}{\theta^{M}}\right)\frac{P_{Y,nt}Y_{nt}}{\mathcal{X}_{nt}}L_{X,nt}^{M} = \left(\frac{1}{\sigma^{M}-1}-\frac{1}{\theta^{M}}\right)\frac{\mathcal{X}_{nt}-\mathrm{TB}_{nt}}{\mathcal{X}_{nt}}L_{X,nt}^{M} \\ &= \left(\frac{1}{\sigma^{M}-1}-\frac{1}{\theta^{M}}\right)L_{X,nt}^{M} - \left(\frac{1}{\sigma^{M}-1}-\frac{1}{\theta^{M}}\right)\frac{\mathrm{TB}_{nt}}{\mathcal{X}_{nt}}L_{X,nt}^{M}. \end{aligned}$$

Using expression $\mathcal{X}_{nt} = \frac{\sigma^{M}}{\sigma^{M}-1}W_{nt}L_{X,nt}^{M}$, we can write

$$\left(\frac{1}{\sigma^{M}-1}-\frac{1}{\theta^{M}}\right)\frac{\mathrm{TB}_{nt}}{\mathcal{X}_{nt}}L_{X,nt}^{M} = \frac{\theta^{M}+1-\sigma^{M}}{\theta^{M}\sigma^{M}}\cdot\frac{\mathrm{TB}_{nt}}{W_{nt}}.$$

Define

$$L_{X,nt} \equiv L_{X,nt}^{M} + \left(\frac{1}{\sigma^{M}-1}-\frac{1}{\theta^{M}}\right)L_{X,nt}^{M} = \left(\frac{\sigma^{M}}{\sigma^{M}-1}-\frac{1}{\theta^{M}}\right)L_{X,nt}^{M}.$$

With this definition the labor market clearing condition can be written as

$$L_{X,nt} + L_{I,nt} = L_{nt} + \frac{\theta^{M}+1-\sigma^{M}}{\theta^{M}\sigma^{M}}\cdot\frac{\mathrm{TB}_{nt}}{W_{nt}}.$$

Next, rewrite condition for $\mathcal{X}_{nt}$,

$$\mathcal{X}_{nt} = \frac{\sigma^{M}}{\sigma^{M}-1}W_{nt}L_{X,nt}^{M} = \frac{1}{1-\frac{\sigma^{M}-1}{\sigma^{M}\theta^{M}}}\cdot W_{nt}L_{X,nt}.$$

Manipulate the expression for $P_{X,nt}$,

$$P_{X,nt} = \frac{\sigma^M}{\sigma^M - 1} \cdot \frac{W_{nt}}{z_{min,n}\Theta^M_{X,n} Z_{X,nt} M_{nt}^{\phi_{F,M}} \left[L^M_{X,nt}\right]^{\phi_{X,L}}}$$

$$= \frac{\sigma^M}{\sigma^M - 1} \cdot \left(\frac{\sigma^M}{\sigma^M - 1} - \frac{1}{\theta^M}\right)^{\phi_{X,L}} \frac{D_{nt}^{\frac{\sigma^M-1}{\sigma^M\theta^M}} W_{nt}^{1-\frac{\sigma^M-1}{\sigma^M\theta^M}}}{z_{min,n}\Theta^M_{X,n} Z_{X,nt} M_{nt}^{\phi_{F,M}} L_{X,nt}^{\phi_{X,L}} D_{nt}^{\frac{\sigma^M-1}{\sigma^M\theta^M}} W_{nt}^{-\frac{\sigma^M-1}{\sigma^M\theta^M}}}.$$

Using the facts that $D_{nt} = \frac{\sigma^M - 1}{\sigma^M\theta^M} \cdot \frac{\mathcal{X}_{nt}}{M_{nt}}$ and $W_{nt} = \left(1 - \frac{\sigma^M - 1}{\sigma^M\theta^M}\right) \frac{\mathcal{X}_{nt}}{L_{X,nt}}$, we get

$$P_{X,nt} = \frac{D_{nt}^{\frac{\sigma^M-1}{\sigma^M\theta^M}} W_{nt}^{1-\frac{\sigma^M-1}{\sigma^M\theta^M}}}{\widetilde{\Theta}^M_{X,n} Z_{X,nt} M_{nt}^{\phi_{F,M}-\frac{\sigma^M-1}{\sigma^M\theta^M}} L_{X,nt}^{\phi_{X,L}+\frac{\sigma^M-1}{\sigma^M\theta^M}}},$$

where

$$\widetilde{\Theta}^M_{X,n} \equiv \left(\frac{\sigma^M - 1}{\sigma^M\theta^M}\right)^{\frac{\sigma^M-1}{\sigma^M\theta^M}} \left(1 - \frac{\sigma^M - 1}{\sigma^M\theta^M}\right)^{1-\frac{\sigma^M-1}{\sigma^M\theta^M}} \left(\frac{\sigma^M}{\sigma^M - 1} - \frac{1}{\theta^M}\right)^{-1-\phi_{X,L}} \Theta^M_{X,n} z_{min,n}.$$

Let $X_{nt} \equiv \mathcal{X}_{nt}/P_{X,nt}$ be the real output of varieties. By substituting expressions for D_{nt} and W_{nt} into the above expression for $P_{X,nt}$ we get

$$X_{nt} = \left(\Theta_{X,n} M_{nt}^{\phi_{F,M}-\frac{\sigma^M-1}{\sigma^M\theta^M}} L_{X,nt}^{\phi_{X,L}+\frac{\sigma^M-1}{\sigma^M\theta^M}}\right) M_{nt}^{\frac{\sigma^M-1}{\sigma^M\theta^M}} L_{X,nt}^{1-\frac{\sigma^M-1}{\sigma^M\theta^M}},$$

where

$$\Theta_{X,n} \equiv \left(\frac{\sigma^M}{\sigma^M - 1} - \frac{1}{\theta^M}\right)^{-1-\phi_{X,L}} \Theta^M_{X,n} z_{min,n}.$$

Next, we have

$$\sum_{i=1}^{N} \left(F^{\vartheta}_{ni,t} \tau^M_{ni,t} P_{X,it}/\omega_{ni}\right)^{-\theta^M\xi} = \left(\frac{\theta^M}{\theta^M + 1 - \sigma^M}\right)^{-\left(\frac{1}{\sigma^M-1}-\phi_{F,L}\right)\theta^M\xi} \left(\frac{P_{Y,nt}Y_{nt}}{\sigma^M W_{nt}}\right)^{-\phi_{F,L}\theta^M\xi} P_{Y,nt}^{-\theta^M\xi},$$

and so

$$\lambda_{ni,t} = \left(\frac{\theta^M}{\theta^M + 1 - \sigma^M}\right)^{\left(\frac{1}{\sigma^M-1}-\phi_{F,L}\right)\theta^M\xi} \left(\frac{P_{Y,nt}Y_{nt}}{\sigma^M W_{nt}}\right)^{\phi_{F,L}\theta^M\xi} \left(F^{\vartheta}_{ni,t} \tau^M_{ni,t} P_{X,it}/\omega_{ni}\right)^{-\theta^M\xi} P_{Y,nt}^{\theta^M\xi},$$

which gives

$$\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{F_{ni,t}^{\vartheta} \tau_{ni,t}^{M} P_{X,it}/\omega_{ni}} = \left(\frac{\theta^{M}}{\theta^{M}+1-\sigma^{M}}\right)^{\left(\frac{1}{\sigma^{M}-1}-\phi_{F,L}\right)\theta^{M}\xi} \left(\frac{P_{Y,nt} Y_{nt}}{\sigma^{M} W_{nt}}\right)^{\phi_{F,L}\theta^{M}\xi} \times \left(F_{ni,t}^{\vartheta} \tau_{ni,t}^{M} P_{X,it}/\omega_{ni}\right)^{-(1+\theta^{M}\xi)} P_{Y,nt}^{1+\theta^{M}\xi} Y_{nt}.$$

Taking both sides to the power of $\frac{\theta^{M}\xi}{1+\theta^{M}\xi}$, we get

$$\left(\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{F_{ni,t}^{\vartheta} \tau_{ni,t}^{M} P_{X,it}/\omega_{ni}}\right)^{\frac{\theta^{M}\xi}{1+\theta^{M}\xi}} = \left[\left(\frac{\theta^{M}}{\theta^{M}+1-\sigma^{M}}\right)^{\left(\frac{1}{\sigma^{M}-1}-\phi_{F,L}\right)\theta^{M}\xi} \left(\frac{P_{Y,nt} Y_{nt}}{\sigma^{M} W_{nt}}\right)^{\phi_{F,L}\theta^{M}\xi} Y_{nt}\right]^{\frac{\theta^{M}\xi}{1+\theta^{M}\xi}} \times \left(F_{ni,t}^{\vartheta} \tau_{ni,t}^{M} P_{X,it}/\omega_{ni}\right)^{-\theta^{M}\xi} P_{Y,nt}^{\theta^{M}\xi}.$$

Summing over i and doing some algebra, we get

$$Y_{nt} = \left(\frac{\theta^{M}}{\theta^{M}+1-\sigma^{M}}\right)^{\frac{1}{\sigma^{M}-1}-\phi_{F,L}} \left[\sigma^{M}\right]^{-\phi_{F,L}} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}}\right)^{\phi_{F,L}} \left[\sum_{i=1}^{N} \left(\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{F_{ni,t}^{\vartheta} \tau_{ni,t}^{M} P_{X,it}/\omega_{ni}}\right)^{\frac{\theta^{M}\xi}{1+\theta^{M}\xi}}\right]^{\frac{1+\theta^{M}\xi}{\theta^{M}\xi}}.$$

Let us redefine iceberg trade costs as

$$\tau_{ni,t} \equiv \left(\frac{F_{ni,t}}{F_{nn,t}}\right)^{\vartheta} \tau_{ni,t}^{M}.$$

Under Assumption 1, the redefined iceberg trade costs $\tau_{ni,t}$ satisfy $\tau_{ni,t} \geq 1$ for all n, i, and t, and they also satisfy the triangle inequality.[30] Using the definition of $\tau_{ni,t}$, we can write the expression for the final aggregate as

$$Y_{nt} = \left(\frac{\theta^{M}}{\theta^{M}+1-\sigma^{M}}\right)^{\frac{1}{\sigma^{M}-1}-\phi_{F,L}} \left[\sigma^{M}\right]^{-\phi_{F,L}} F_{nn,t}^{-\vartheta} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}}\right)^{\phi_{F,L}} \left[\sum_{i=1}^{N} \left(\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it}/\omega_{ni}}\right)^{\frac{\theta^{M}\xi}{1+\theta^{M}\xi}}\right]^{\frac{1+\theta^{M}\xi}{\theta^{M}\xi}}.$$

Let us write $F_{nn,t}^{-\vartheta} = \Theta_{Y,n}^{M} Z_{Y,nt}$, where $Z_{Y,nt}$ is supposed to be the same exogenous shock as

[30] In Assumption 1 we use the definition of ϑ from the main text, i.e., we use $\vartheta \equiv \frac{1}{\sigma^{M}-1} - \frac{1}{\theta^{M}}$. Formally speaking, for the purposes of the current appendix we need to modify Assumption 1 and use the definition $\vartheta \equiv \phi_{Y,M} - \frac{1}{\theta^{M}}$. This slight abuse of notation should not create confusion.

in the unified model. Define

$$\Theta_{Y,n} \equiv \left(\frac{\theta^{M}}{\theta^{M}+1-\sigma^{M}}\right)^{\frac{1}{\sigma^{M}-1}-\phi_{F,L}} \left[\sigma^{M}\right]^{-\phi_{F,L}} \Theta^{M}_{Y,n}.$$

Then we can write

$$Y_{nt} = \Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt}Y_{nt}}{W_{nt}}\right)^{\phi_{F,L}} \left[\sum_{i=1}^{N} \left(\frac{\lambda_{ni,t}P_{Y,nt}Y_{nt}}{\tau_{ni,t}P_{X,it}/\omega_{ni}}\right)^{\frac{\theta^{M}\xi}{1+\theta^{M}\xi}}\right]^{\frac{1+\theta^{M}\xi}{\theta^{M}\xi}}.$$

Combining all expressions and definitions, we get the equilibrium system in isomorphic form (for the case of financial autarky):

$$V_{nt} = \beta E_t \left\{ \frac{P_{Y,nt}}{P_{Y,n,t+1}} \cdot \frac{U_1\left(C_{n,t+1}, L_{n,t+1}\right)}{U_1\left(C_{nt}, L_{nt}\right)} \left[D_{n,t+1} + (1-\delta) V_{n,t+1}\right] \right\},$$

$$-\frac{U_2\left(C_{nt}, L_{nt}\right)}{U_1\left(C_{nt}, L_{nt}\right)} = \frac{W_{nt}}{P_{Y,nt}},$$

$$M_{n,t+1} = (1-\delta) M_{nt} + M_{I,nt},$$

$$X_{nt} = \left(\Theta_{X,n} M_{nt}^{\phi_{F,M} - \frac{\sigma^M - 1}{\sigma^M \theta^M}} L_{X,nt}^{\phi_{X,L} + \frac{\sigma^M - 1}{\sigma^M \theta^M}} \right) M_{nt}^{\frac{\sigma^M - 1}{\sigma^M \theta^M}} L_{X,nt}^{1 - \frac{\sigma^M - 1}{\sigma^M \theta^M}},$$

$$Y_{nt} = \Theta_{Y,n} Z_{Y,nt} \left(\frac{P_{Y,nt} Y_{nt}}{W_{nt}} \right)^{\phi_{F,L}} \left[\sum_{i=1}^{N} \left(\frac{\lambda_{ni,t} P_{Y,nt} Y_{nt}}{\tau_{ni,t} P_{X,it} / \omega_{ni}} \right)^{\frac{\theta^M \xi}{1+\theta^M \xi}} \right]^{\frac{1+\theta^M \xi}{\theta^M \xi}},$$

$$M_{I,nt} = \Theta_{I,n} Z_{I,nt} L_{I,nt}^{\alpha_I} Y_{I,nt}^{1-\alpha_I},$$

$$L_{X,nt} + L_{I,nt} = L_{nt} + \frac{\theta^M + 1 - \sigma^M}{\theta^M \sigma^M} \cdot \frac{TB_{nt}}{W_{nt}},$$

$$C_{nt} + Y_{I,nt} = Y_{nt},$$

$$\sum_{n=1}^{N} \lambda_{ni,t} P_{Y,nt} Y_{nt} = P_{X,it} X_{it},$$

$$\lambda_{ni,t} = \frac{\left(\tau_{ni,t} P_{X,it} / \omega_{ni}\right)^{-\theta^M \xi}}{\sum_{l=1}^{N} \left(\tau_{nl,t} P_{X,lt} / \omega_{nl}\right)^{-\theta^M \xi}},$$

$$M_{nt} = \frac{\sigma^M - 1}{\sigma^M \theta^M} \cdot \frac{P_{X,nt} X_{nt}}{D_{nt}},$$

$$L_{X,nt} = \left(1 - \frac{\sigma^M - 1}{\sigma^M \theta^M}\right) \frac{P_{X,nt} X_{nt}}{W_{nt}},$$

$$L_{I,nt} = \alpha_I \frac{V_{nt} M_{I,nt}}{W_{nt}},$$

$$Y_{I,nt} = (1-\alpha_I) \frac{V_{nt} M_{I,nt}}{P_{Y,nt}},$$

$$P_{Y,nt} C_{nt} + V_{nt} M_{I,nt} = D_{nt} M_{nt} + W_{nt} L_{nt}.$$

Let us discuss the role that the strength of the love-of-variety effect — given by parameter $\phi_{Y,M}$ — plays in the generalized Melitz model. The love-of-variety effect impacts the above system in two places. First, it impacts the trade elasticity, which is given by the exponent of $\tilde{\tau}_{ni,t}$ in the expression for trade shares $\lambda_{ni,t}$ and is equal to $\theta^M \xi$ with

$$\xi = \frac{1}{\left(\frac{1}{\eta^M - 1} - \phi_{Y,M}\right) \theta^M + 1}. \tag{41}$$

Second, if we remove labor externality in the fixed costs of serving markets by assuming that $\phi_{F,L} = \vartheta$, then the strength of economies of scale in production of the final aggregate will be given by $-\vartheta$ with $\vartheta = \phi_{Y,M} - \frac{1}{\theta^M}$. Importantly, not all combinations of the trade elasticity and the strength of economies of scale in production of the final aggregate in the unified model can be mapped into a valid trade elasticity $\theta^M \xi$ in the generalized Melitz model, if we keep parameter restriction that $\phi_{F,L} = \vartheta$. For example, the value $\psi_Y = \frac{1}{\eta^M - 1}$ can be used in the unified model, but not in the corresponding Melitz model. Indeed, having $\psi_Y = \frac{1}{\eta^M - 1}$ in the unified model implies that in the corresponding generalized Melitz model we need to have $\vartheta = -\psi_Y = -\frac{1}{\eta^M - 1}$ and $\phi_{Y,M} = -\vartheta + \frac{1}{\theta^M} = \frac{1}{\eta^M - 1} + \frac{1}{\theta^M}$. But this, in turn, implies that the denominator in expression (41) for ξ is zero. In other words, having $\psi_Y = \frac{1}{\eta^M - 1}$ in the unified model implies a non-valid value for ξ in the corresponding generalized Melitz model.

If we relax parameter restriction that $\phi_{F,L} = \vartheta$, and, thus, allow for labor externalities in the fixed costs of serving markets, then the only place where parameter $\phi_{Y,M}$ impacts the equilibrium system in the generalized Melitz model is the trade elasticity. Then any combination of trade elasticity and the strength of economies of scale in production of the final aggregate in the unified model can be mapped into the corresponding parameters in the generalized Melitz model. Thus, we can have isomorphism. However, in this case, the trade elasticity in the generalized Melitz model is governed by two free parameters: η^M and $\phi_{Y,M}$. So, one of these parameters is redundant for the purposes of isomorphism. It makes more economic sense to adjust parameter η^M — elasticity of substitution between varieties produced in different countries — rather than $\phi_{Y,M}$ to change the trade elasticity. Hence, parameter $\phi_{Y,M}$ is not needed in this case. This is why we choose to not to have correction for the love-of-variety in the generalized Melitz model in the main text, i.e., in the main text we have $\phi_{Y,M} = \frac{1}{\sigma^M - 1}$, $\vartheta \equiv \frac{1}{\sigma^M - 1} - \frac{1}{\theta^M}$, and

$$\xi = \frac{1}{\left(\frac{1}{\eta^M - 1} - \frac{1}{\sigma^M - 1}\right)\theta^M + 1}.$$

C Additional Tables with Moments

Moment	Data	Bench	Inv. labor		Inv. final aggregate			$\sigma = 6$		
			IRBC $Z_{I,n} = 1$	IRBC $Z_{I,n} = Z_{X,n}$	Krug	Mel	Mel $Z_{Y,n} = 1$	IRBC	Krug	Mel
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\text{Corr}\left(\frac{\text{GDP}_1}{P_{Y,1}}, \frac{\text{GDP}_2}{P_{Y,2}}\right)$	0.58	0.29	0.30	0.14	0.21	0.21	0.21	0.18	0.13	0.13
$\text{Corr}\left(C_1, C_2\right)$	0.36	0.68	0.42	0.49	0.59	0.58	0.58	0.60	0.46	0.45
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	−0.02	−0.07	−0.20	−0.17	−0.19	−0.18	−0.12	−0.25	−0.26
$\text{Corr}\left(L_1, L_2\right)$	0.42	−0.23	−0.91	−0.37	−0.36	−0.36	−0.36	−0.32	−0.41	−0.41
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	−0.49									
$\text{Corr}\left(\frac{\mathcal{X}_{21}}{P_{Y,1}}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.32	0.91	0.90	0.85	0.90	0.91	0.90	0.81	0.77	0.78
$\text{Corr}\left(\frac{\mathcal{X}_{12}}{P_{Y,1}}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.81	0.91	0.90	0.85	0.90	0.91	0.90	0.81	0.77	0.78
$\text{Corr}\left(\text{ReR}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.13	0.60	0.59	0.64	0.63	0.63	0.63	0.64	0.64	0.64
$\frac{\text{Std}\left(\text{ReR}\right)}{\text{Std}\left(\text{GDP}_1/P_{Y,1}\right)}$	2.23	0.35	0.29	0.18	0.37	0.39	0.38	0.10	0.05	0.06

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = W_n L_n + R_n K_n$, $\mathcal{X}_{ni} = P_{X,ni} X_{ni}$, $\text{TB}_1 = P_{X,1} X_1 - P_{Y,1} Y_1$, $\text{ReR} = P_{Y,2}/P_{Y,1}$. Column 2 corresponds to the standard calibration of the IRBC model and is identical to column 2 in Table 3. Columns 3 and 4 are for the case of investment done in terms of labor in otherwise standard IRBC model. For column 3 there is no shock in the investment sector, while for column 3 the shock to the investment sector is the same as the shock in the intermediate goods sector. Columns 5-7 are for the case of investment in terms of final aggregate in otherwise standard calibrations of Krugman and Melitz models. In column 7 there is no shock in the final aggregate sector. Columns 8-10 are for the case of $\sigma = 6$ in otherwise standard calibrations of IRBC, Krguman, and Melitz models.

Table 6: Robustness checks. Financial autarky

Moment	Data	Bench	Inv. labor: IRBC $Z_{I,n}=1$	Inv. labor: IRBC $Z_{I,n}=Z_{X,n}$	Inv. final aggregate: Krug	Inv. final aggregate: Mel	Inv. final aggregate: Mel $Z_{Y,n}=1$	$\sigma=6$: IRBC	$\sigma=6$: Krug	$\sigma=6$: Mel
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
$\text{Corr}\left(\frac{\text{GDP}_1}{P_{Y,1}}, \frac{\text{GDP}_2}{P_{Y,2}}\right)$	0.58	0.16	0.34	0.08	0.04	0.01	0.02	0.01	−0.09	−0.09
$\text{Corr}\left(C_1, C_2\right)$	0.36	0.69	0.69	0.60	0.61	0.58	0.59	0.69	0.62	0.63
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	−0.47	0.35	−0.29	−0.58	−0.61	−0.60	−0.72	−0.66	−0.66
$\text{Corr}\left(L_1, L_2\right)$	0.42	−0.42	−0.97	−0.51	−0.59	−0.60	−0.60	−0.57	−0.72	−0.73
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	−0.49	−0.54	0.57	0.42	−0.48	−0.48	−0.47	−0.48	−0.01	0.03
$\text{Corr}\left(\frac{\mathcal{X}_{21}}{P_{Y,1}}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.32	0.29	0.96	0.88	0.39	0.37	0.39	−0.14	0.28	0.33
$\text{Corr}\left(\frac{\mathcal{X}_{12}}{P_{Y,1}}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.81	0.96	0.47	0.70	0.93	0.93	0.93	0.75	0.33	0.31
$\text{Corr}\left(\text{ReR}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.13	0.48	0.57	0.60	0.59	0.61	0.60	−0.03	0.16	0.23
$\frac{\text{Std}\left(\text{ReR}\right)}{\text{Std}\left(\text{GDP}_1/P_{Y,1}\right)}$	2.23	0.22	0.62	0.24	0.30	0.32	0.31	0.12	0.16	0.16

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = W_n L_n + R_n K_n$, $\mathcal{X}_{ni} = P_{X,ni} X_{ni}$, $\text{TB}_1 = P_{X,1} X_1 - P_{Y,1} Y_1$, $\text{ReR} = P_{Y,2}/P_{Y,1}$. Column 2 corresponds to the standard calibration of the IRBC model and is identical to column 2 in Table 3. Columns 3 and 4 are for the case of investment done in terms of labor in otherwise standard IRBC model. For column 3 there is no shock in the investment sector, while for column 3 the shock to the investment sector is the same as the shock in the intermediate goods sector. Columns 5-7 are for the case of investment in terms of final aggregate in otherwise standard calibrations of Krugman and Melitz models. In column 7 there is no shock in the final aggregate sector. Columns 8-10 are for the case of $\sigma = 6$ in otherwise standard calibrations of IRBC, Krguman, and Melitz models.

Table 7: Robustness checks. Bond economy

Moment	Data	Bench	$\psi_{X,K}$		$\psi_{X,L}$		ψ_{Y}	
			0.3	-1	0.7	-1	0.2	-1
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\text{Corr}\left(\frac{\text{GDP}_1}{P_{Y,1}}, \frac{\text{GDP}_2}{P_{Y,2}}\right)$	0.58	0.29	0.26	0.36	0.14	0.43	0.24	0.46
$\text{Corr}\,(C_1, C_2)$	0.36	0.68	0.67	0.65	0.56	0.76	0.64	0.77
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	-0.02	-0.20	0.25	-0.11	0.10	-0.06	0.14
$\text{Corr}\,(L_1, L_2)$	0.42	-0.23	-0.57	0.20	-0.25	-0.17	-0.26	-0.13
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	-0.49							
$\text{Corr}\left(\frac{\mathcal{X}_{21}}{P_{Y,1}}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.32	0.91	0.91	0.92	0.89	0.93	0.92	0.85
$\text{Corr}\left(\frac{\mathcal{X}_{12}}{P_{Y,1}}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.81	0.91	0.91	0.92	0.89	0.93	0.92	0.85
$\text{Corr}\left(\text{ReR}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.13	0.60	0.61	0.57	0.66	0.53	0.62	-0.08
$\frac{\text{Std}\,(\text{ReR})}{\text{Std}\,(\text{GDP}_1/P_{Y,1})}$	2.23	0.35	0.36	0.33	0.38	0.31	0.44	0.05

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = W_n L_n + R_n K_n$, $\mathcal{X}_{ni} = P_{X,ni} X_{ni}$, $\text{TB}_1 = P_{X,1} X_1 - P_{Y,1} Y_1$, $\text{ReR} = P_{Y,2}/P_{Y,1}$.

Table 8: Moments from calibration with decreasing returns and correlated shocks. Financial autarky.

Moment	Data	Bench	$\psi_{X,K}$ 0.3	$\psi_{X,K}$ -1	$\psi_{X,L}$ 0.7	$\psi_{X,L}$ -1	ψ_{Y} 0.2	ψ_{Y} -1
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\text{Corr}\left(\frac{\text{GDP}_1}{P_{Y,1}}, \frac{\text{GDP}_2}{P_{Y,2}}\right)$	0.58	0.16	0.12	0.29	-0.19	0.37	-0.22	0.46
$\text{Corr}\,(C_1, C_2)$	0.36	0.69	0.67	0.66	0.46	0.76	0.47	0.83
$\text{Corr}\left(\frac{P_{I,1} I_1}{P_{Y,1}}, \frac{P_{I,2} I_2}{P_{Y,1}}\right)$	0.30	-0.47	-0.61	-0.16	-0.64	-0.32	-0.78	0.11
$\text{Corr}\,(L_1, L_2)$	0.42	-0.42	-0.71	0.11	-0.57	-0.28	-0.68	-0.23
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	-0.49	-0.54	-0.55	-0.53	-0.63	-0.48	-0.68	0.30
$\text{Corr}\left(\frac{\mathcal{X}_{21}}{P_{Y,1}}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.32	0.29	0.29	0.29	0.00	0.44	-0.28	0.88
$\text{Corr}\left(\frac{\mathcal{X}_{12}}{P_{Y,1}}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.81	0.96	0.96	0.97	0.94	0.97	0.92	0.82
$\text{Corr}\left(\text{ReR}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.13	0.48	0.51	0.36	0.57	0.40	0.60	0.04
$\frac{\text{Std}\,(\text{ReR})}{\text{Std}\,(\text{GDP}_1/P_{Y,1})}$	2.23	0.22	0.23	0.16	0.26	0.18	0.28	0.12

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = W_n L_n + R_n K_n$, $\mathcal{X}_{ni} = P_{x,ni} X_{ni}$, $\text{TB}_1 = P_{X,1} X_1 - P_{Y,1} Y_1$, $\text{ReR} = P_{Y,2}/P_{Y,1}$.

Table 9: Moments from calibration with decreasing returns and correlated shocks. Bond economy.

Moment	Data	Bench	$\psi_{X,K}$ 0.3	$\psi_{X,K}$ -1	$\psi_{X,L}$ 0.7	$\psi_{X,L}$ -1	ψ_{Y} 0.2	ψ_{Y} -1
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\text{Corr}\left(\frac{\text{GDP}_1}{P_{Y,1}}, \frac{\text{GDP}_2}{P_{Y,2}}\right)$	0.58	0.14	0.09	0.28	-0.25	0.41	-0.24	0.43
$\text{Corr}\,(C_1, C_2)$	0.36	0.79	0.81	0.72	0.51	0.89	0.57	0.93
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	-0.48	-0.63	-0.17	-0.68	-0.29	-0.78	0.09
$\text{Corr}\,(L_1, L_2)$	0.42	-0.51	-0.80	0.07	-0.64	-0.39	-0.71	-0.41
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	-0.49	-0.49	-0.44	-0.52	-0.61	-0.40	-0.67	0.49
$\text{Corr}\left(\frac{\mathcal{X}_{21}}{P_{Y,1}}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.32	0.38	0.49	0.31	0.04	0.60	-0.24	0.96
$\text{Corr}\left(\frac{\mathcal{X}_{12}}{P_{Y,1}}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.81	0.95	0.93	0.97	0.92	0.96	0.91	0.70
$\text{Corr}\left(\text{ReR}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.13	0.53	0.59	0.38	0.62	0.44	0.63	0.32
$\frac{\text{Std}\,(\text{ReR})}{\text{Std}\,(\text{GDP}_1/P_{Y,1})}$	2.23	0.26	0.31	0.17	0.30	0.22	0.32	0.16

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = W_n L_n + R_n K_n$, $\mathcal{X}_{ni} = P_{X,ni} X_{ni}$, $\text{TB}_1 = P_{X,1} X_1 - P_{Y,1} Y_1$, $\text{ReR} = P_{Y,2} / P_{Y,1}$.

Table 10: Moments from calibration with decreasing returns and correlated shocks. Complete markets.

Moment	Data	Bench	$\psi_{X,K}$ 0.3	$\psi_{X,K}$ -1	$\psi_{X,L}$ 0.7	$\psi_{X,L}$ -1	ψ_{Y} 0.2	ψ_{Y} -1
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\text{Corr}\left(\frac{\text{GDP}_1}{P_{Y,1}}, \frac{\text{GDP}_2}{P_{Y,2}}\right)$	0.58	0.42	0.40	0.43	0.51	0.38	0.42	0.41
$\text{Corr}(C_1, C_2)$	0.36	0.44	0.45	0.40	0.51	0.41	0.44	0.43
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	0.41	0.37	0.44	0.51	0.37	0.41	0.41
$\text{Corr}(L_1, L_2)$	0.42	0.41	0.34	0.45	0.51	0.36	0.41	0.40
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	-0.49							
$\text{Corr}\left(\frac{\mathcal{X}_{21}}{P_{Y,1}}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.32	0.93	0.92	0.93	0.94	0.92	0.94	0.85
$\text{Corr}\left(\frac{\mathcal{X}_{12}}{P_{Y,1}}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.81	0.93	0.92	0.93	0.94	0.92	0.94	0.85
$\text{Corr}\left(\text{ReR}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.13	0.54	0.55	0.53	0.50	0.55	0.54	0.34
$\frac{\text{Std}(\text{ReR})}{\text{Std}(\text{GDP}_1/P_{Y,1})}$	2.23	0.31	0.32	0.31	0.29	0.32	0.37	0.06

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = W_n L_n + R_n K_n$, $\mathcal{X}_{ni} = P_{X,ni} X_{ni}$, $\text{TB}_1 = P_{X,1} X_1 - P_{Y,1} Y_1$, $\text{ReR} = P_{Y,2}/P_{Y,1}$.

Table 11: Moments from calibration with decreasing returns and uncorrelated shocks. Financial autarky.

Moment	Data	Bench	$\psi_{X,K}$ 0.3	$\psi_{X,K}$ -1	$\psi_{X,L}$ 0.7	$\psi_{X,L}$ -1	ψ_Y 0.2	ψ_Y -1
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
$\text{Corr}\left(\frac{\text{GDP}_1}{P_{Y,1}}, \frac{\text{GDP}_2}{P_{Y,2}}\right)$	0.58	0.30	0.28	0.37	0.23	0.32	0.01	0.44
$\text{Corr}\,(C_1, C_2)$	0.36	0.45	0.43	0.43	0.42	0.41	0.26	0.56
$\text{Corr}\left(\frac{P_{I,1}I_1}{P_{Y,1}}, \frac{P_{I,2}I_2}{P_{Y,1}}\right)$	0.30	-0.09	-0.17	0.05	-0.12	-0.08	-0.49	0.34
$\text{Corr}\,(L_1, L_2)$	0.42	0.18	0.10	0.35	0.13	0.24	-0.15	0.33
$\text{Corr}\left(\frac{\text{TB}_1}{\text{GDP}_1}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	-0.49	-0.50	-0.51	-0.50	-0.51	-0.50	-0.61	-0.16
$\text{Corr}\left(\frac{\mathcal{X}_{21}}{P_{Y,1}}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.32	0.41	0.39	0.38	0.38	0.39	-0.05	0.80
$\text{Corr}\left(\frac{\mathcal{X}_{12}}{P_{Y,1}}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.81	0.97	0.97	0.98	0.96	0.97	0.93	0.86
$\text{Corr}\left(\text{ReR}, \frac{\text{GDP}_1}{P_{Y,1}}\right)$	0.13	0.45	0.45	0.35	0.47	0.41	0.56	-0.10
$\frac{\text{Std (ReR)}}{\text{Std}\,(\text{GDP}_1/P_{Y,1})}$	2.23	0.20	0.20	0.15	0.22	0.18	0.26	0.13

Notes: Data moments are from Heathcote and Perri (2002), Table 2. All series have been Hodrick-Prescott filtered with a smoothing parameter of 1600. Time index is dropped from notation for brevity. $\text{GDP}_n = W_n L_n + R_n K_n$, $\mathcal{X}_{ni} = P_{X,ni} X_{ni}$, $\text{TB}_1 = P_{X,1} X_1 - P_{Y,1} Y_1$, $\text{ReR} = P_{Y,2} / P_{Y,1}$.

Table 12: Moments from calibration with decreasing returns and uncorrelated shocks. Bond economy.

D Additional Impulse-Response Functions

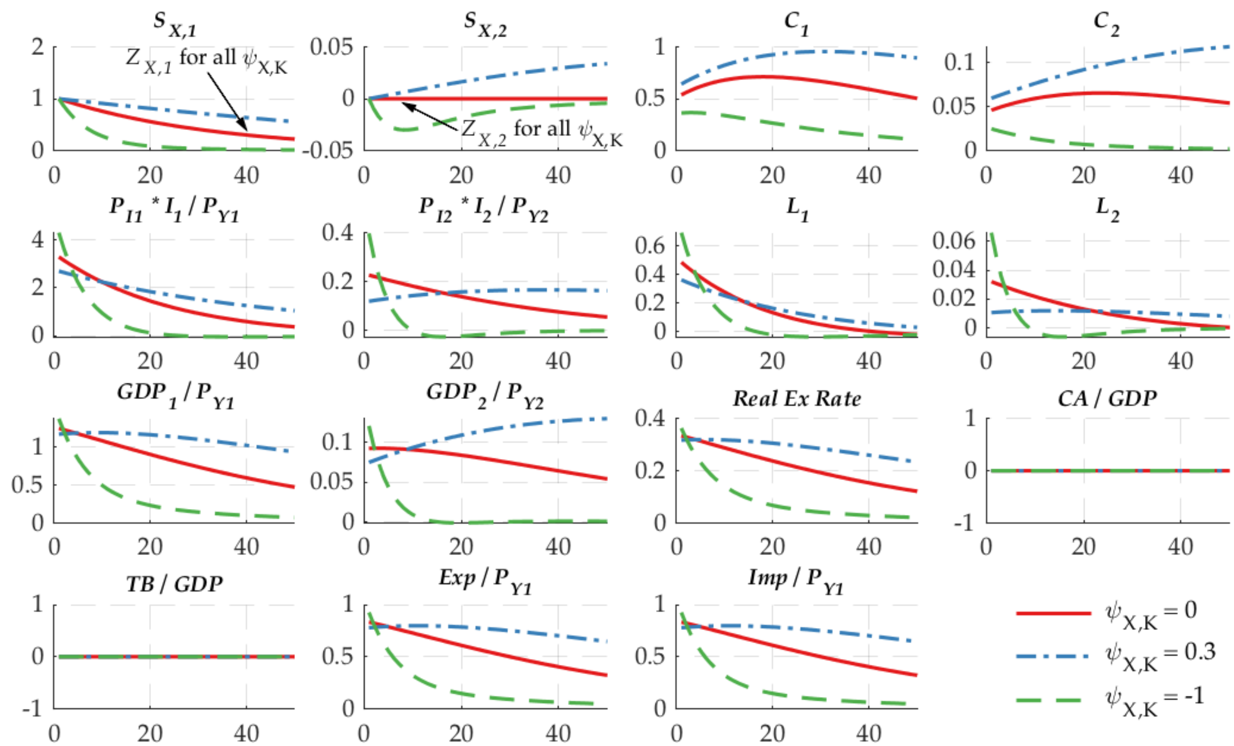

Figure 4: Impulse-response functions for $Z_{x,1}$. Capital externalities in the intermediate goods sector. Financial autarky.

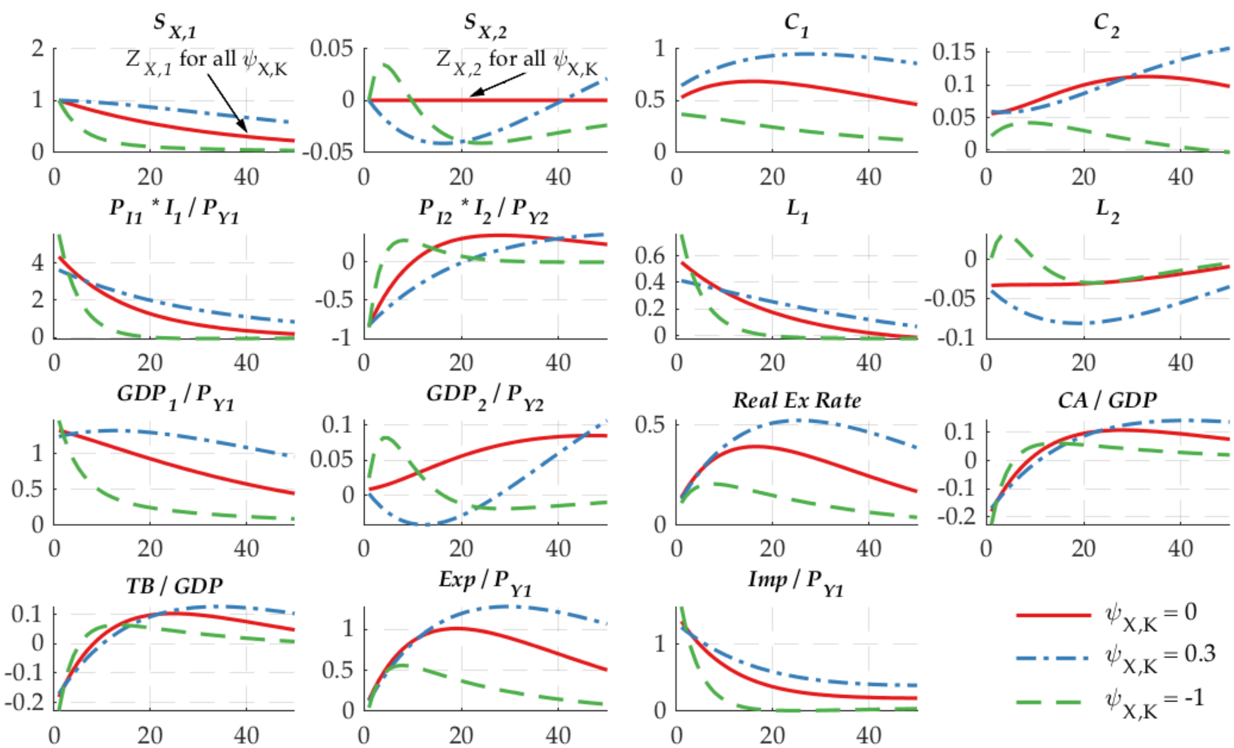

Figure 5: Impulse-response functions for $Z_{x,1}$. Capital externalities in the intermediate goods sector. Bond economy.

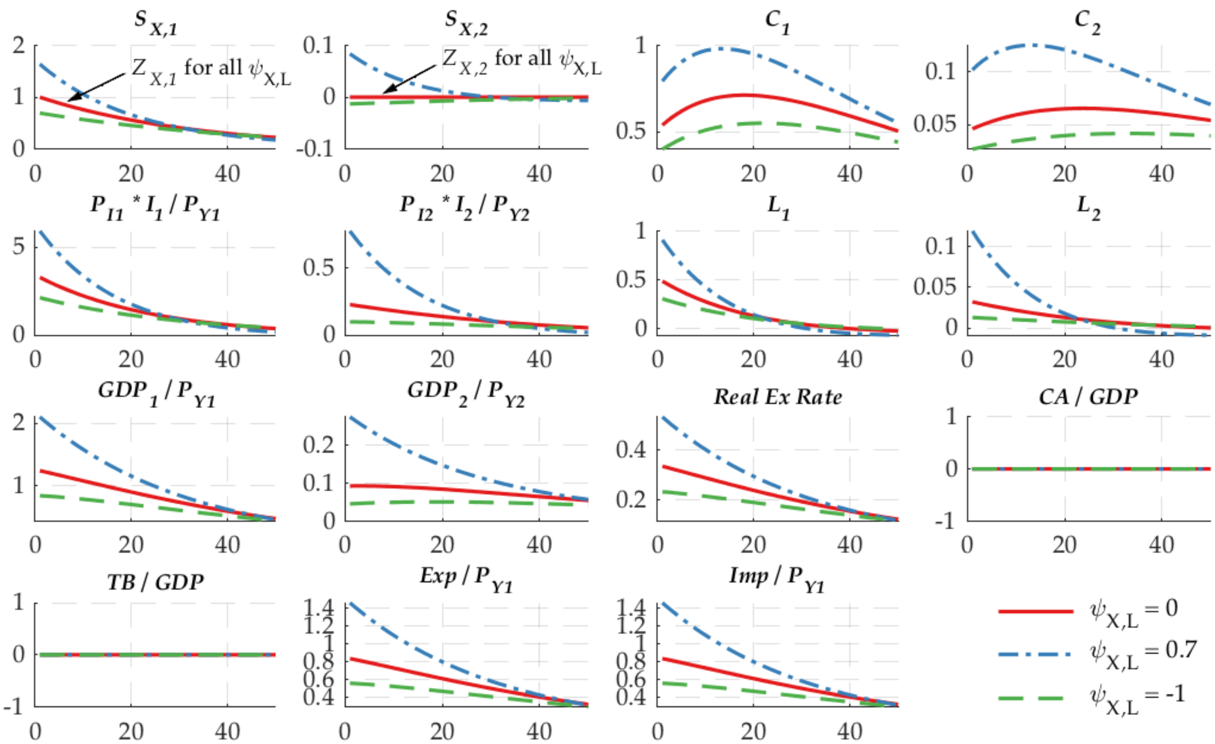

Figure 6: Impulse-response functions for $Z_{x,1}$. Labor externalities in the intermediate goods sector. Financial autarky.

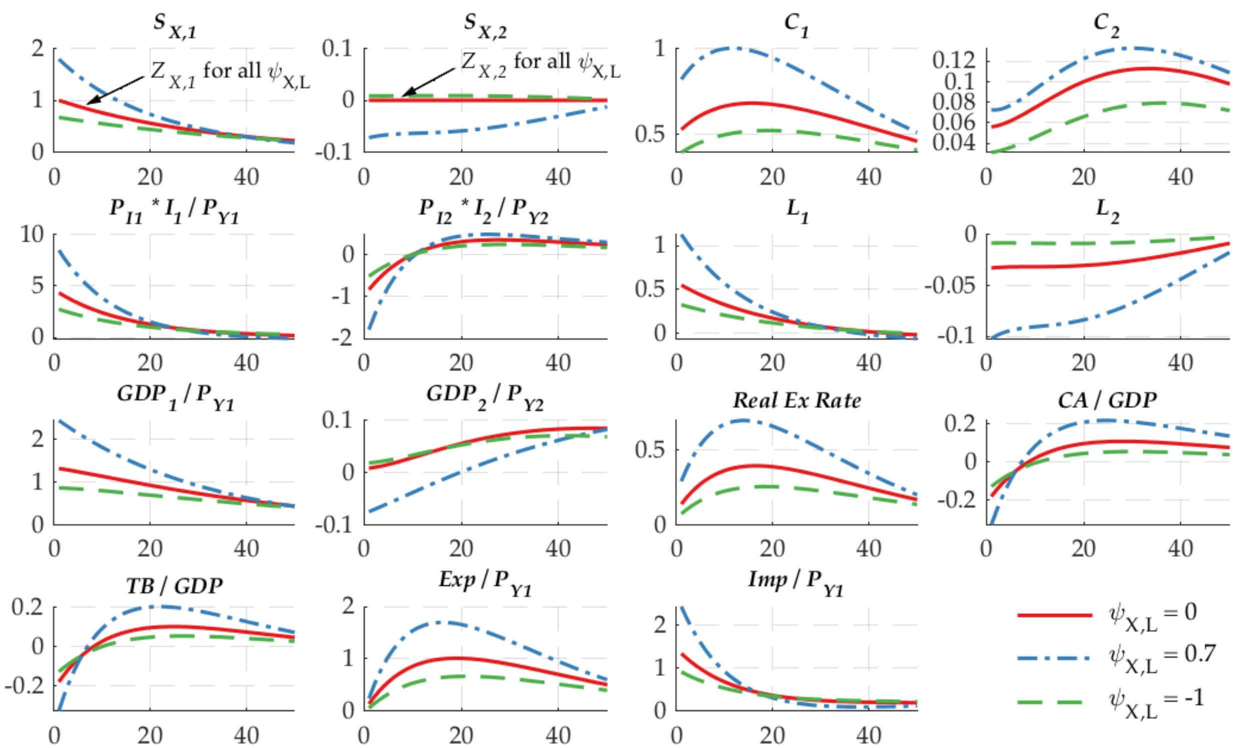

Figure 7: Impulse-response functions for $Z_{x,1}$. Labor externalities in the intermediate goods sector. Bond economy.

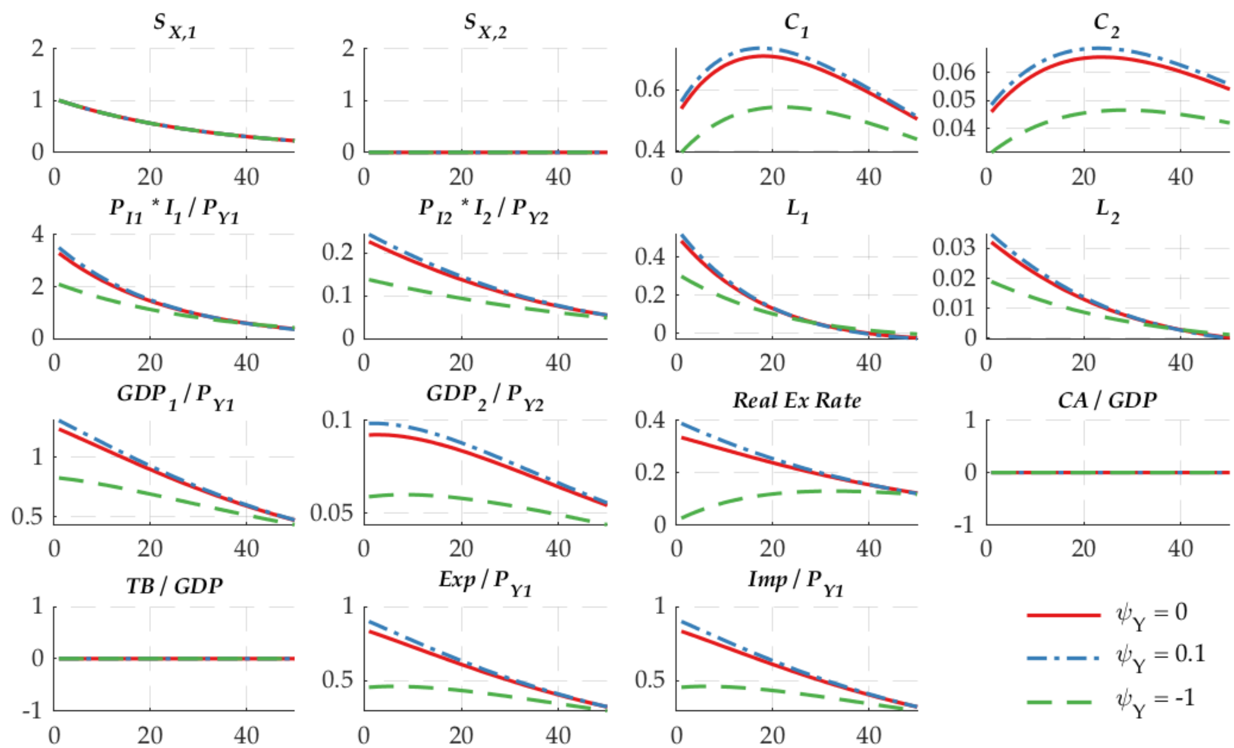

Figure 8: Impulse-response functions for $Z_{X,1}$. Externality in the final aggregates sector. Financial autarky.

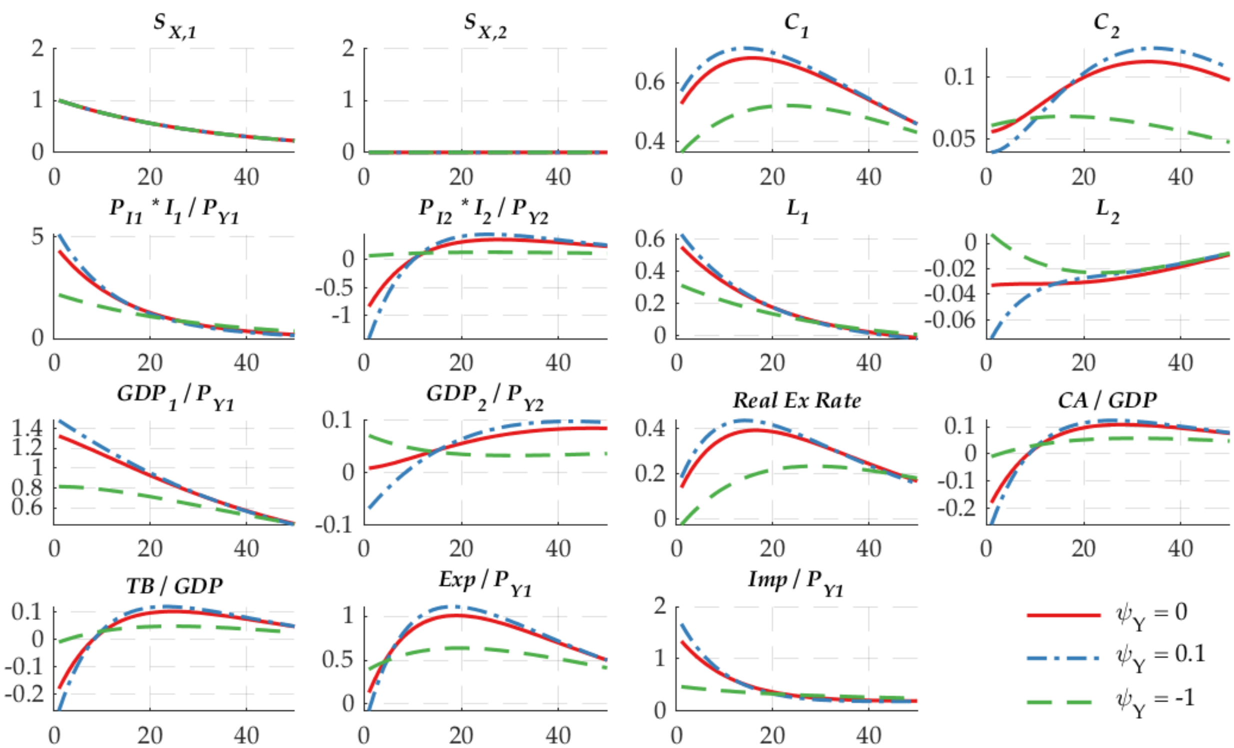

Figure 9: Impulse-response functions for $Z_{X,1}$. Externality in the final aggregates sector. Bond economy.