Robots, Trade, and Luddism*

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Abstract

Technological change, from the advent of robots to expanded trade opportunities, tends to create winners and losers. When are such changes welcome? How should government policy respond? We provide a general analysis of optimal technology regulation in a second best world, with linear taxes on goods and a nonlinear tax on labor income. Our first result shows that, despite these limited instruments, productivity improvements are always welcome and valued in the same way as in a first best world. Our second set of results offers three optimal tax formulas, with minimal structural assumptions, involving sufficient statistics that can be implementing using existing reduced-form evidence on the distributional impact of robots and trade. Our final result is a comparative static exercise illustrating that while distributional concerns create a rationale for non-zero taxes on robots and trade, the magnitude of these taxes may decrease with improvements in new technologies. Hence, as the process of automation and globalization deepens, more inequality may best be met with lower luddism.

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1 Introduction

Robots and artificial intelligence technologies are on the rise. So are imports from China and other developing countries. Regardless of its origins, technological change creates opportunities for some workers, destroys opportunities for others, and generates significant distributional consequences, as documented in the recent empirical work of Autor, Dorn and Hanson (2013) and Acemoglu and Restrepo (2017) for the United States.

When should technological change be welcome? Should any policy response be in place? And if so, how should we manage new technologies? Should we become more luddites as machines become more efficient or more protectionist as trade opportunities expand? The goal of this paper is to provide a general second-best framework to help address these and other related questions.

In seeking answers to these questions one first needs to take a stand on the range of available policy instruments. Obviously, if the idilic lump-sum transfers are available, distribution can be done efficiently, without distorting production. Even in the absence of lump-sum transfers, if linear taxes are available on all goods and factors, production efficiency may hold, as in Diamond and Mirrlees (1971b). In both cases, zero taxes on robots and free trade are optimal. At another extreme, in the absence of any policy instrument, whenever technological progress creates at least one loser, a welfare criterion must be consulted and the status quo may be preferred.

Here, we focus on intermediate, and arguably more realistic, scenarios where tax instruments are available, but are more limited than those ensuring production efficiency. Our framework is designed to capture general forms of technological change. We consider two sets of technologies, which we refer to as "old" and "new". For instance, firms using the new technology may be producers of robots or traders that export some goods in exchange for others. Since we are interested in the optimal regulation of the new technology, we do not impose any restriction on the taxation of firms using that technology, e.g. taxes on robots or trade. In contrast, to allow for a meaningful trade-off between redistribution and efficiency, we restrict the set of taxes that can be imposed on firms using the old technology as well as on consumers and workers. In the economic environment that we consider, the after-tax wage structure can be influenced by tax policy, but not completely controlled.

Our first set of results focuses on the welfare impact of new technologies under the assumption that constrained, but optimal policies are in place. We offer a novel envelope result that generalizes the evaluation of productivity shocks in first-best environments, as in Solow (1957) and Hulten (1978), to distorted economies. Because of restrictions on the set of available tax instruments, marginal rates of substitution may not be equalized across agents and marginal rates of transformation may not be equalized between new and old technology firms. Yet, "Immiserizing Growth," as in Bhagwati (1958), never arises. Provided that governments can tax new technology firms, the welfare impact of technological progress can be measured in the exact same way as in first best environments (despite not being first best).

A direct implication of our envelope result is that even if new technologies tend to have a disproportionate effect on the wages of less skilled workers, and we care about redistribution, this does not create any new rationale for taxes and subsidies on innovation. In the case of terms-of-trade shocks, our envelope result states that such shocks are beneficial if and only if they raise the value of the trade balance at current quantities. In other words, the gains from international trade can still be computed by integrating below the demand curves for foreign goods. Distributional considerations may affect how much we trade, but not the mapping between observed trade flows and welfare, as in Arkolakis, Costinot and Rodríguez-Clare (2012).

Our second set of results characterizes the structure efficient taxes on old and new technology firms in general environments when the taxation of different labor factors is limited to non-linear income taxation, as in Mirrlees (1971). In a two-type setting, Naito (1999) has proven that governments seeking to redistribute income from high- to low-skill workers may have incentives to depart from production efficiency. Doing so manipulates relative wages, which cannot be taxed directly, and relaxes incentive compatibility constraints. Our analysis generalizes this result and , more significantly, goes beyond qualitative insights by deriving optimal tax formulas expressed in terms of sufficient statistics.

Specifically, we provide three optimal tax formulas using minimal structural assumptions. The sufficient statistics we identify provide insight into the optimal taxation of new technology and are, in principle, empirically measurable. In all three formulas, the change in the wage schedule plays a central role, as captured by the change in the wage distribution. This is intuitive, since the key motive for considering a distortion in production is its distributional effect across households. Our formula makes clear that this intuitively important object, which may be of empirical interest for descriptive reasons, is actually a sufficient statistic for optimal policy design. Given knowledge of this statistic, the underlying structure of the economy leading to the change in wages can be left in the background.

Our formulas are informative about how taxes should be set. We apply our formulas to robots and trade, using the reduced-form evidence of Acemoglu and Restrepo (2017) on the impact of robots and the evidence of Chetverikov, Larsen and Palmer (2016) on the impact of Chinese imports on inequality as illustrations. This exercise produces taxes on robots of 99% and on trade of 15%.[1]

We conclude our analysis with a comparative static exercise that asks: as progress in Artificial Intelligence and other areas makes for cheaper and better robots, should we tax them more? Or in a trade context, does hyper-globalization call for hyper-protection? Through a simple example, we show that in contrast to many popular discussions, improvements in new technologies may be associated with more robots and more trade, as well as more inequality caused by those improvements, but lower optimal taxes on robots and trade.

Our paper makes three distinct contributions to the existing literature. The first one is a new perspective on the welfare impact of technological progress in the presence of distortions. In a first best world, the impact of small productivity shocks can be evaluated, absent any restriction on preferences and technology, using a simple envelope argument as in Solow (1957) and Hulten (1978). With distortions, evaluating the welfare impact of productivity shocks, in general, requires additional information about whether such shocks aggravate or alleviate underlying distortions. In an environment with markups, for instance, this boils down to whether employment is reallocated towards goods with higher or lower markups, as in Basu and Fernald (2002), Arkolakis, Costinot, Donaldson and Rodríguez-Clare (Forthcoming), and Baeqee and Farhi (2017). If the aggravation of distortions is large enough, technological progress may even lower welfare, as discussed by Bhagwati (1971). Here, we follow a different approach. Our analysis builds on the idea that while economies may be distorted and tax instruments may be limited, the government may still have access to policy instruments to regulate the new technology. If so, we show that the envelope results of Solow (1957) and Hulten (1978) still hold, with direct implications for the measurement of the welfare gains from globalization and automation as well as for the taxation of innovation.

Our second contribution is a general characterization of the structure of optimal taxes in environments with restricted factor income taxation. In so doing, we fill a gap between the general analysis of Diamond and Mirrlees (1971b,a) and Dixit and Norman (1980), which assumes that linear taxes on all factors are available, and specific examples, typically with two goods and two factors, in which only income taxation is available, as in the original work of Naito (1999), and subsequent work by Guesnerie (1998), Spector (2001), and Naito (2006).[2] On the broad spectrum of restrictions on available policy instruments,

[1] To put these numbers in perspective, in the case of robots, a tax of near 100%, doubles its price, which is equivalent to taxing half of its rental earnings.

[2] In all three papers, like in Dixit and Norman (1980), the new technology is international trade. In another related trade application, Feenstra and Lewis (1994) study an environment where governments

one can also view our analysis as an intermediate step between the work of Diamond and Mirrlees (1971b,a) and Dixit and Norman (1980) and the trade policy literature, as reviewed for instance in Rodrik (1995), where it is common to assume that the only instruments available for redistribution are trade taxes.

Our third contribution is to offer a more specific application of our general formulas to the taxation of robots and trade. In recent work, Guerreiro, Rebelo and Teles (2017) have studied a model with both skilled and unskilled workers as well as robots, with a nested CES production function. Assuming factor-specific taxes are unavailable, they find a nonzero tax on robots to be generally optimal, in line with Naito (1999). Although we share the same rationale for finding nonzero taxes on robots, based on Naito (1999), our main goal is not to sign the tax on robots, nor to explore a particular production structure, but instead to offer tax formulas highlighting key sufficient statistics needed to determine the level of taxes, with fewer structural assumptions. In this way, our formulas provide a foundation for empirical work as well as the basis for novel comparative static results.[3] In another recent contribution, Hosseini and Shourideh (2018) analyze a multi-country Ricardian model of trade with input-output linkages and imperfect mobility of workers across sectors. Although sector-specific taxes on labor are not explicitly allowed, these missing taxes can be perfectly mimicked by the available tax instruments. By implication, their economy provides an alternative implementation but fits Diamond and Mirrlees (1971a,b) and Dixit and Norman (1980, 1986), where households face a complete set of linear taxes, including sector-specific taxes on labor. Production efficiency and free trade then follow, just as they did in Diamond and Mirrlees (1971a,b).[4]

2 Environment

Our model economy allows for a finite number of goods indexed by $i = 1, ..., N$ and a continuum of households indexed by their skill $\theta \in \Theta$ with distribution F. Importantly, we do not restrict heterogeneity to be one dimensional, as the classical Mirrleesian analysis does. Indeed, allowing for rich multidimensional heterogeneity is an important step

cannot subject different worker types to different taxes, but can offer subsidies to workers moving from one industry to another in response to trade. They provide conditions under which such a trade adjustment assistance program are sufficient to guarantee Pareto gains from trade, as in Dixit and Norman (1980).

[3] Our specific tax formulas in Section 5 relate to the work of Jacobs (2015) who considers the same tax instruments as in Naito (1999), in an environment without robots, featuring a continuum of workers and no general equilibrium effects.

[4] A separate line of work, e.g. Itskhoki (2008), Antras, de Gortari and Itskhoki (2017) and Tsyvinski and Werquin (2018), studies technological changes such as trade or robots, without considering taxes on these new technologies, but instead focusing on how the income tax schedule may respond to these changes.

in terms of generality.

2.1 Preferences

Households have identical and weakly separable preferences between goods and labor. The utility of household θ is given by

$$U(\theta) = u(C(\theta), n(\theta)),$$
$$C(\theta) = v(c(\theta)),$$

where $C(\theta)$ is the sub-utility that household θ derives from consuming goods, $n(\theta)$ is her labor supply, $c(\theta) \equiv \{c_i(\theta)\}$ is her vector of good consumption, and u and v are the utility functions associated with lower- and upper-level preferences, respectively.

2.2 Technology

There are two types of technologies, which we refer to as "old" and "new," each associated with a distinct production set. In our applications, the new technology may represent trade with the rest of the world or the production of machines, like robots. This dichotomy between new and old technology, as opposed to the consolidation into a single aggregate production set, allows us to consider the differential taxation of firms using these two technologies, and, in turn, allows for aggregate production inefficiency.

Old Technology. Let $y \equiv \{y_i\}$ denote the vector of total net output by old technology firms and let $n \equiv \{n(\theta)\}$ denote the schedule of their total labor demand. Positive values for y_i represent output, while negative y_i represent inputs. The production set associated with the old technology corresponds to all production plans (y, n) such that

$$G(y, n) \leq 0,$$

where G is some convex and homogeneous function of (y, n). Homogeneity of G implies constant returns to scale.

New Technology. Let $y^* \equiv \{y_i^*\}$ denote the vector of total net output by new technology firms. The production set associated with the new technology corresponds to all production plans y^* such that

$$G^*(y^*, \phi) \leq 0,$$

where G^* is some convex and homogeneous function of y^* and $\phi > 0$ is a productivity parameter. We assume that G^* is decreasing in ϕ so that an increase in ϕ corresponds to an improvement in the new technology.

Unlike the old technology, the new technology does not employ labor directly. Abstracting from labor in the new technology is convenient, as it implies that wages are determined by the old technology. New technology has an effect on wages, through its effect on the structure of production within the old technology, but not directly through employment. For given ϕ, one can show that the omission of labor from G^* is without loss of generality; thus, this assumption has no effect on our tax formulas. Essentially, the new technology sector can be thought of as the last stage of production, when taxation is imposed.[5] For changes in ϕ, as in our comparative static results, the assumption restricts attention to labor-neutral technological changes, which are well-suited to our two main applications trade and robots, allowing us to capture reduction in trade barriers or quality improvements in robots.[6] In the first case, new technology firms are traders who can export and import goods,

$$G^*(y^*;\phi) = \bar{p}(\phi) \cdot y^*,$$

where $\bar{p}(\phi) \equiv \{\bar{p}_i(\phi)\}$ denotes the vector of world prices and $\cdot$ denotes the inner product of two vectors. An increase in ϕ corresponds to a positive terms-of-trade shock, that may be due to a change in transportation costs or productivity in the rest of the world. In the second case, new technology firms may be robot-producers that transform a composite of all other goods in the economy, call it gross output, into robots.

Resource Constraint. For all goods, the demand by households is equal to the supply by old and new technology firms,

$$\int c(\theta)dF(\theta) = y + y^*.$$

[5]By defining the elements of the vector y and y^* appropriately, the production of a machine or robot may take place in the traditional G sector, combining labor and other inputs. This machine can then be required to be pass through the new technology sector G^* before returning to the traditional sector G as an input.

[6]The technological changes are labor-neutral only in the sense that that they do not induce changes in wages *for given prices p* faced by the old technology firms (or for a given production y). However, without government intervention, in general equilibrium, changes in ϕ generally induce changes in p (or output y) that affect equilibrium wages, for given $\{n\}$.

2.3 Prices and Taxes

Factors. Let $w \equiv \{w(\theta)\}$ denote the schedule of wages faced by firms. Because of income taxation, a household with ability θ and labor earnings $w(\theta)n(\theta)$ retains

$$w(\theta)n(\theta) - T(w(\theta)n(\theta)),$$

where $T(w(\theta)n(\theta))$ denotes its total tax payment. Crucially, the income tax schedule T is the same for all households. This rules out agent-specific lump-sum transfers, in contrast to the Second Welfare Theorem, as well as factor-specific linear taxes, in contrast to the analysis of Diamond and Mirrlees (1971b,a) and Dixit and Norman (1980).

Goods. Let $p^* \equiv \{p_i^*\}$ denote the vector of good prices faced by new technology firms. Because of ad-valorem taxes $t^* \equiv \{t_i^*\}$, these prices may differ from the vector of good prices $p \equiv \{p_i\}$ faced by old technology firms and households,

$$p_i = (1 + t_i^*)p_i^*, \text{ for all } i.$$

Production inefficiency arises if $t^* \neq 0$. In a trade context, an import tariff or an export subsidy on good i corresponds to $t_i^* > 0$, whereas an import subsidy or an export tax corresponds to $t_i^* < 0$.

Since demand and supply only depend on relative prices, we can normalize prices and taxes such that $p_1 = p_1^* = 1$ and $t_1^* = 0$. We maintain this normalization throughout.

2.4 Social Welfare

We consider a general social welfare criterion that depends on the distribution of individual well-beings, not the particular well-being of certain agents. Any consumption and labor supply schedule $(c, n) \equiv \{c(\theta), n(\theta)\}$ is associated with a utility schedule $U \equiv \{U(\theta)\}$. This induces a cumulative distribution μ_U over utilities levels $\mu_U(x) = \int_{U(\theta) \leq x} dF(\theta)$ for all x. The social welfare objective is assumed to be a function of μ_U,

$$W(\mu_U).$$

We assume W is increasing with respect to first-order stochastic dominant changes in μ_U. In what follows, we let $\bar{W}(c, n) \equiv W(\mu_{\{u(v(c(\theta)), n(\theta))\}})$ denote the social welfare associated with a given consumption and labor supply schedule.

When θ is one dimensional and higher-θ households achieve higher utility, as in the standard Mirrleesian setup, our welfare criterion nests the special case of a weighted utilitarian objective

$$\int U(\theta)\lambda(\theta)dF(\theta),$$

without any further restriction on the Pareto weights $\lambda(\theta) \geq 0$ of the households.

When θ is multidimensional, our assumption about W only restricts Pareto weights to be the same for all households θ earning the same wage, since these households obtain the same utility.

2.5 Planning Problem

The government problem is to select a competitive equilibrium with taxes that maximizes social welfare. That is, the government chooses an allocation, $c \equiv \{c(\theta)\}$, $n \equiv \{n(\theta)\}$, $C \equiv \{C(\theta)\}$, $y \equiv \{y_i\}$, and $y^* \equiv \{y_i^*\}$, prices and wages, $p \equiv \{p_i\}$, $p^* \equiv \{p_i^*\}$, and $w \equiv \{w(\theta)\}$, as well as an income tax schedule, T, and taxes on new technology firms, $t^* \equiv \{t_i^*\}$, in order to maximize $W(\mu_{\{u(v(c(\theta)),n(\theta))\}})$ subject to the constraints that households maximize their utility, firms maximize their profits, markets clear, and prices satisfy the above non-arbitrage condition. The formal definition of a competitive equilibrium and the associated planning problem can be found in Appendix A.

3 When Is Technological Change Welcome?

Our first set of results focuses on the welfare impact of new technologies under the assumption that constrained, but optimal policies are in place.

3.1 An Envelope Result

Let $V(\phi)$ denote the value function associated with the government problem. As shown in Appendix B, it takes the general form,

$$V(\phi) = \max_{(c,n,y)\in\mathcal{Z}} \bar{W}(c,n) \tag{1a}$$

$$G^*(\int c(\theta)dF(\theta) - y;\phi) \leq 0, \tag{1b}$$

where the feasible set $\mathcal{Z}$ is independent of ϕ. Now consider a small productivity shock from ϕ to $\phi + d\phi$. By the Envelope Theorem, we have

$$\frac{dV}{d\phi} = \gamma \frac{\partial G^*}{\partial \phi}, \tag{2}$$

with $\gamma \geq 0$ the Lagrange multiplier associated with the resource constraint (1b).

This envelope condition can be thought of as a generalization of Hulten's (1978) Theorem. In spite of the economy not being first-best, the welfare impact of technological progress can be measured in the exact same way as in first-best environments. This observation leads to our first result.

Proposition 1. *Technological change increases social welfare, $dV/d\phi \geq 0$, if and only if it expands the production possibility of new technology firms, that is, if and only if $\partial G^*/\partial \phi \leq 0$.*

In the presence of distortions, it is well-known that technological progress may lower welfare. This is what Edgeworth (1884) and Bhagwati (1958) refer to as "Economic Damnification" and "Immesirizing Growth". How does Proposition 1 rule it out?

The critical assumption here is not that there are no distortions or, equivalently, that our planner has enough tax instruments to target any underlying distortion. In fact, the planning problem described above would also hold in an environment with externalities and other market imperfections, such as price and wage rigidities leading to labor market distortions. Proposition 1 instead follows from the assumption that, in spite of distortions and restrictions on the set of available instruments, our planner still has a tax instruments t^* to control the new technology firms, which is where technological change occurs.

Take the example of international trade: $G^*(y^*; \phi) = \bar{p}(\phi) \cdot y^*$. Equation (2) implies that a country gains from a terms-of-trade shock if and only if it raises the value of its net exports, evaluated at the initial quantities,

$$V'(\phi) \propto \bar{p}'(\phi) \cdot y^*.$$

Intuitively, if world prices $\bar{p}(\phi)$ change, the government always has the option to maintain domestic prices p unchanged by raising trade taxes by $\bar{p}'(\phi)$. Starting from a constrained optimum, the only first-order effect of such a policy change would be to raise tax revenues by $\bar{p}'(\phi) \cdot y^*$, regardless of whether or not the economy is first-best. Raising trade taxes by $\bar{p}'(\phi) \cdot y^*$, of course, may not be the optimal response, but the possibility of such a response is sufficient to evaluate the welfare impact of a terms-of-trade shock at the margin.[7]

[7]The previous observation does not depend on whether the country is a small open economy or not.

3.2 Implications

We now illustrate the usefulness of our envelope result through two applications.

Taxation of Innovation. Consider first the issue of whether governments may ever want to tax innovation because of its adverse distributional consequences. To shed light on this issue in the simplest possible way, suppose that there exists a set of feasible new technologies, Φ, that can be restricted by the government. The profit maximization problem of new technology firms is then given by

$$y^*, \phi \in \max_{\tilde{y}^*, \tilde{\phi} \in \bar{\Phi}} p^* \cdot \tilde{y}^*$$
$$G^*(\tilde{y}^*; \tilde{\phi}) \leq 0,$$

where $\bar{\Phi} \subset \Phi$ is the set of technologies allowed by the government. The government's problem, in turn, takes the general form,

$$\max_{(c,n,y) \in \mathcal{Z}, \phi \in \Phi} \bar{W}(c, n)$$
$$G^*(\int c(\theta) dF(\theta) - y; \phi) \leq 0,$$

By equation (2), the optimal technology $\bar{\phi}$ simply satisfies

$$\frac{\partial G^*(y^*; \bar{\phi})}{\partial \phi} = 0.$$

Provided that taxes of new technology firms, t^*, have been set such that they find it optimal to produce y^*, conditional on $\bar{\phi}$, they will also find it optimal to choose $\bar{\phi}$, if allowed to do so. It follows that the government does not need to affect the direction of innovation, in spite of its potential distributional implications.[8]

Valuation of New Technologies. Next we turn to the issue of evaluating the welfare gains from new technologies. Using our envelope result, one can follow, in a general equilibrium environment with distortions, the same steps used to compute equivalent

$\bar{p}(\phi)$ could itself be a function of y^*. The same result would hold. In fact, our envelope result provides the basis for a generalization of the results in Bagwell and Staiger (1999, 2001, 2012a,b): if trade taxes are unrestricted, the only rationale for a trade agreement is to correct terms-of-trade externalities.

[8]More generally, there may be externalities across firms that directly call for subsidizing, or taxing, innovation. In such environments, the implication of our envelope result is that distributional concerns do not add a new motive for taxing, or subsidizing, innovation.

and compensating variations in standard consumer theory.

Consider the following generalized version of our government's problem

$$V(\phi, D) = \max_{(c,n,y)\in\mathcal{Z}} \bar{W}(c, n) \tag{3a}$$

$$G^*(\int c(\theta)dF(\theta) - y; \phi) \leq D, \tag{3b}$$

The parameters ϕ and D play the same role here as prices and income in the utility maximization problem of a single consumer. In a trade context, D corresponds to a trade deficit, that is a transfer from the rest of the world. The welfare impact of a productivity shock from ϕ_0 to ϕ_1 can then be computed either as the transfer, $EV(\phi_0, \phi_1)$, that would be equivalent to the shock, $V(\phi_0, EV(\phi_0, \phi_1)) = V(\phi_1, 0)$, or as the transfer, $CV(\phi_0, \phi_1)$, required to compensate for the shock, $V(\phi_1, -CV(\phi_0, \phi_1)) = V(\phi_0, 0)$.

Let $y^*(\phi, D)$ denote the vector of output by new technology firms associated with the solution to (3). Equation (2) implies that

$$EV(\phi, \phi + d\phi) = CV(\phi, \phi + d\phi) = \frac{\partial G^*(y^*(\phi, 0); \phi)}{\partial \phi}. \tag{4}$$

This is the counterpart to Shephard's Lemma in standard consumer theory. And, like in standard consumer theory, this envelope condition can be integrated to compute the welfare impact of arbitrary productivity shocks, as described in Appendix (B.2). Gains from trade, for instance, can be computed by integrating (4) between the current productivity level and the one at which the economy reaches autarky, as in Arkolakis, Costinot and Rodríguez-Clare (2012).

Note that distortions will, in general, affect $y^*(\phi, D)$. So, the point here is not that distortions do not matter for the welfare consequences of globalization or automation. The point rather is that like in a first-best environment, the demand for goods produced using the new technology, either Chinese imports or robots, fully reveals the welfare gains associated with that technology. Concerns for redistribution and other potential sources of distortions affects how much we trade or how much we use robots, but not the mapping between quantities demanded, productivity shocks, and welfare.[9]

[9]The extent to which the previous result is useful in practice, of course, depends on whether constrained, but optimal policies are in place. The assumption that the government fully controls the new technology clearly is non-trivial. We view it, however, as a useful benchmark, not necessarily stronger than the opposite assumption, implicit in many papers, that governments cannot control the new technology at all. Galle, Rodríguez-Clare and Yi (2017), and Waugh and Lyon (2017) are recent welfare analysis of the so-called China shock that fall into this category. Brexit and the current debate about the renegotiation of NAFTA are stark reminders that trade policies are not set in stone.

4 How Should We Tax New Technologies?

Our next set of results characterizes the structure of taxes on new technology firms. Specifically, we provide optimal tax formulas expressed in terms of sufficient statistics and using minimal structural assumptions. The sufficient statistics we identify provide insight into the optimal taxation of new technology and are, at least in principle, empirically measurable.

Our formulas are informative about how taxes should be set. All our formulas coincide and hold at any optimum, for any given welfare function. Conversely, away from the optimum, each formula provides a different perspective and if any formulas does not hold, they indicate how welfare can be improved. In this way, the formulas can be applied even when the government is not optimizing.

4.1 Efficiency versus Redistribution

Our tax formulas are derived starting from any initial equilibrium, with taxes (t^*, T) and are based on a marginal change δt^* in the taxes on the new technology, potentially accompanied by adjustments in the nonlinear tax schedule δT. These marginal tax changes induce equilibrium marginal adjustments in prices δp, wages δw, quantities δy^* and labor δn. Our three formulas differ in whether the nonlinear tax is adjusted and how it is adjusted.

We first present an intermediate result that encompasses all cases, providing a condition that the marginal changes δt^*, δT and the marginal adjustments δp, δw, δy^* and δn must satisfy so that welfare is not improved by the variation. Let $z \in [0, 1]$ represent the percentile in the wage distribution; abusing notation slightly, we write $w(z)$, $n(z)$, etc.

Lemma 1. *A necessary condition for a feasible variation not to improve welfare is that*

$$\begin{aligned}(p - p^*) \cdot \delta y^* - \int \tau(z) w(z) \delta n(z)\, dz \\ = \int (\tilde{\lambda}(z) - 1)((1 - \tau(z)) n(z)\, \delta w(z) - c(z) \cdot \delta p - \delta T(x(z)))\, dz\end{aligned}$$

where $\tilde{\lambda}(z) = \frac{1}{\gamma} \frac{\partial \Phi}{\partial U(z)} u_{c_1}(z)$ and $\gamma > 0$ represents the marginal value of resources in the hand of the government.

Before discussing the terms in this expression, the fact that all variables and adjustments are expressed in terms of z shows that one can collapse heterogeneity and proceed *as if* there were of a single dimension of heterogeneity. In terms of its implementability, this aspect is crucial. It implies that researchers can focus empirical measurement on the quantile distribution (as is often done in practice) without modeling the underlying rich sources of heterogeneity.

Turning to the expression, the equality condition identified in the lemma trades off a distributional effects accruing directly to households with two fiscal externalities.

The distributional effect is captured by the first term on the right hand side. It evaluates the change in utility in monetary terms directly perceived by a household, weighted by $\tilde{\lambda}(\theta) - 1$. By an envelope argument, the marginal change in utility for θ is given by $\frac{1}{u_{c_1}(\theta)}\delta u(\theta) = (1-\tau(\theta))n(\theta)\,\delta w(\theta) - c(\theta)\cdot\delta p - \delta T(x(\theta))$. The terms $(1-\tau(\theta))n(\theta)\,\delta w(\theta) - \delta T(x(\theta))$ capture the change in income, due to both the change in before-tax income as well as the change in the tax schedule. The term $-c(\theta)\cdot\delta p$ adjusts this change in income by a household-specific measure of inflation. The total change in utility is a change in real income.

There are two fiscal externalities. The term on the left hand side represents the change in revenues from the linear tax t^*, also equal to the marginal increase in the deadweight burden or "Harberger triangle". The second fiscal externality is represented by the last term on the right hand side, capturing the increase in revenue from the non-linear income tax schedule.

4.2 Three Tax Formulas

We now explore three feasible tax variations, each leading to an optimal tax formula. All variations involve a change in y_i^*, in any desired direction. The variations differ with respect to the nonlinear labor income tax schedule. In particular, we consider variations with

i. no change in in the income tax, $\delta T = 0$;

ii. no change in (the distribution of) labor supply, $\delta n = 0$;

iii. no change in (the distribution of) utility, $\delta U = 0$.

The first variation is straightforward and the most obvious to consider. The second variation adjusts the income tax schedule T to keep the distribution of labor supply remains unchanged, while the third does the same for utility. Note that in the canonical Mirrleesian case, where θ is one dimensional, the second and third variations ensure that labor supply and utility are unchanged for each household, respectively. When θ is multidimensional, there are not enough instruments to ensure this, but the second and third variations ensure that the distribution of labor and utilities are unchanged, respectively.

Our next result develops the optimality conditions derived from each of these variations.

Proposition 2. *A necessary condition for a feasible variation not to improve welfare is that*

$$p_i - p_i^* = \begin{cases} \int \left((\tilde{\lambda}(z) - 1)(1-\tau(z)) + \tau(z)\, \varepsilon^u(z) \right) n(z)\, \frac{dw(z)}{dy_i^*}|_{\delta T=0}\, dz - \int (\tilde{\lambda}(z)-1) c(z) \cdot \frac{dp}{dy}|_{\delta T=0}\, dz \\ \int \psi(z)(1-\tau(z)) w(z) n(z)\, \frac{d\omega(z)}{dy_i^*}|_{\delta n=0}\, dz \\ \int \tau(\theta) w(z) n(z)\, \frac{\epsilon(z)}{\epsilon(z)+1}\, \frac{1}{\omega(z)} \frac{d\omega(z)}{dy_i^*}|_{\delta U=0}\, dz \end{cases}$$

where $\omega(z) = w'(z)/w(z)$, $\varepsilon^u(z)$ is an uncompensated elasticity of labor supply, while $\epsilon(z)$ is a consumption-compensated elasticity of labor supply. The coefficient $\psi(z)$ is detailed in the appendix and depends only on private preferences and social weights, not technology. In the quasi-linear utility case it simplifies to $\psi(z) = \tilde{\Lambda}(z) - z$ where $\tilde{\Lambda}(z) = \int_0^z \tilde{\lambda}(\tilde{z}) d\tilde{z}$.

In all three formulas, the change in the wage schedule plays a central role, as captured by the change in the wage level w or wage growth ω. The precise statistic summarizing the effects on the wage schedule varies across each formula. In the first formula, it is the elasticity of the wage level w, whereas in the second and third equation it is the elasticity of the growth rate ω. Additionally, each formula requires the elasticity to capture the change in wages under the associated variation. Thus, the second equation holds the distribution of labor constant, the first and third include any effects from changes in labor supply.

The fact that the impact on wages appears as a key input into optimal taxes is intuitive, since the only motive for considering a distortion in production is its distributional effect across households. Our formula makes clear that this intuitively important object, which may be of empirical interest for descriptive reasons, is actually a sufficient statistic for optimal policy design. Given knowledge of this statistic, the underlying structure of the economy leading to the change in wages can be left in the background.

Each formula interacts information on wages in a different ways with other variables, placing different informational demands for its implementation. The first two formulas require information on welfare weights, whereas the third formula does not. The first and third formula require information on labor supply elasticities, whereas the second condition does not.

Interestingly, all three formulas provide a direct expression for the tax rate. This is unusual compared to the optimal linear tax literature (e.g. Diamond and Mirrlees, 1971a), which usually derives a system of simultaneous equations, with the entire set of tax rates on the left hand side. Our expression is more akin to the Pigouvian tax literature, which provides an explicit expression for the tax on each good in terms of its externality. Indeed, we favor a Pigouvian interpretation of these formulas, as correcting for distributional and fiscal externalities: if an extra unit of y_i^* is increased, then this has effects on the wage schedule that affect distribution and social welfare, as well the resources in the hands of the government; the tax asks agents to pay for these marginal effects.

All three formulas necessarily hold at an optimum. Indeed, even away from a full optimum, if the nonlinear tax schedule T is conditionally optimal given t^*, but t^* is not optimal, then all three formulas will not hold and in the same direction, providing the same diagnostic. Thus, when T is conditionally optimal in this way, any of the formulas conveys all the information of the others. However, if the current nonlinear tax schedule T is not conditionally optimal in this way, then each formula may provide different guidance and each one can be used to identify potential improvements.

The first formula follows from the simplest thought experiment and has a straightforward interpretation. On the one hand, if the tax schedule is unchanged the impact on household utility depends solely on the increase in retained income from the change in wages, $(1-\tau(z))n(z)dw(z)$. The formula evaluates the distributional value from the planner's perspective of paying out income in this pattern relative to the value it places on holding on to these funds, $\tilde{\lambda}-1$. Another distributional term is present, due to differential inflation across households, captured by the presence of $-c(z)\cdot dp$. This term drops out if households have homogenous preferences over consumption goods, since then they all experience the same inflation rate. On the other hand, the change in wages induces a change in labor supply, which impacts the planner's revenue through the nonlinear income tax, $\tau(z)n(z)\varepsilon^u(z)dw(z)$. The tax should equal the sum of the distributional and effiency effects.

The first formula generalizes the formula found in the political economy of trade literature initiated by Grossman and Helpman (1994). The key difference between the class of problems that we consider and those in this literature is that we allow for endogenous labor supply and nonlinear income taxation. Redistribution is possible without distortions, such as trade protection, but not costless due to elastic labor supply.

Turning to the second formula, there is now only one term on the right-hand side, rather than two, reflecting the fact that the variation behind this formula is engineered to ensure that labor is unchanged. The remaining term on the right-hand side is purely distributional, evaluating winners and losers. Although the expression is simple, who wins and who loses, is no longer mechanical, but incorporates the adjustment in the tax schedule required to keep labor unchanged. This adjustment is shaped by the change in relative wages, as captured by $d\omega$, explaining its role. The formula shows that only changes in relative wages matter and proportional changes in all wages have no effect. The changes in relative wages captured by $d\omega$ are weighted by distributional concerns summarized in $\psi(z)$.

Intuitively, only relative wages matter for incentives explaining the central role played by ω. To see this more formally, consider the incentive compatibility constraint

$$u(C(z), n(z)) \geq u(C(z'), n(z')\tfrac{w(z')}{w(z)})$$

where $C(z)$ is the optimized level for $v(c(z))$ given expenditures $w(z)n(z) - T(w(z)n(z))$. The presence of $\frac{w(z')}{w(z)}$ reflects the fact that a household of type z that earns the same amount as one of type z' must work $\hat{n}(z, z')$ where $w(z)\hat{n}(z, z') = w(z')n(z')$. Indeed, the first order condition for z' evaluated at $z' = z$ gives

$$u_C C'(z) + u_n(z)w(z)\left(n'(z) + n(z)\omega(z)\right) = 0.$$

For a given $n(z)$ and $C(0)$ one can use this differential equation to solve for $C(z)$ and back out the implied tax schedule T. This highlights the role of $\omega(z)$. Adjusting $C(0)$ to maintain resource feasibility, one sees that changes in $\omega(z)$ dictate changes in $C(z)$ and, thus, in household utility (recall, labor is unchanged). The distributional consequences of the implied change in utility are dictated by the change in ω.

Interestingly, the elasticity of labor does not enter the formula. The intuition for this is that labor is kept fixed, so the degree to which labor is elastic never enters the picture. In this sense, the adjustment in the tax schedule undoes the change in wages. On the other hand, both of the first two formulas require information on the welfare weights, $\tilde{\lambda}$.

Turning to the third formula, we first note the absence of any welfare weight terms. This is due to the fact that the variation behind this formula ensures that the distribution of utility is unchanged. Thus, only efficiency considerations are involved, captured by the labor fiscal externality, so that only the last term from the lemma remains. The relative simplicity of the fiscal externality term is also remarkable. Tedious calculations in the appendix surprisingly turn up a simple expression for the change in labor supply, given by $\frac{\epsilon(z)}{\epsilon(z)+1}\frac{1}{\omega(z)}d\omega(z)$.

For purposes of implementation, the fact that social welfare evaluations are absent in the formula should be extremely welcome. It implies that an empirical evaluation is, in principle, feasible. In contrast, the first two formulas requires coming up with a welfare criterion, with all the subjectivity that this entails. In some cases one might imagine a benevolent planner, with some set social welfare function. Or simply wish to evaluate social welfare from the perspective of one's own private social welfare function. In these cases, the first two formulas can be easily implemented. In most cases, in reality, social welfare weights are often best interpreted as a proxy for a social and political negotiation or struggle.

How is it possible that distributional issues from relative wage changes are naturally to be at the center of the issue, yet social welfare weights do not make an appearance in this formula? No welfare criterion is required because the formula effectively detects the presence or not of a first-order dominant improvement in the distribution of utilities (i.e. a Pareto improvement in the one dimensional case). Unless the formula holds an improvement can be easily constructed.[10] This is true whether or not the original equilibrium and taxes are efficient.

The existing nonlinear income tax plays a crucial role, as evidenced by the presence of the marginal tax rates $\tau(z)$ in the formula. All other things the same, higher marginal taxes $\tau(z)$ potentiate fiscal externalities and demand larger t_i^*. To take an extreme case, if marginal taxes were zero, $\tau(z)$, then the formula says that $t_i^* = 0$. This makes sense. The welfare theorems hold, so the absence of taxation leads to a Pareto optimum that cannot be improved upon.

As mentioned earlier, the formula does not require the nonlinear tax T to be optimized conditional on t^*. However, if it is not optimized, and the planner has some given welfare function, then we may be in a situation where the formula may not detect a Pareto improvement, yet the planner can improve the social welfare function by a change in t^*. For example, suppose the extreme case with $\tau(z) = 0$ together with a planner with a strong preference for equality, such as a Rawlsian criterion. Clearly $\tau(z) = 0$ is not optimal for such a planner. In this case, the planner enjoys many ways of improving welfare. The planner could change the nonlinear tax schedule only, or it could also entertain changes in the linear tax t^*, evaluating them along the lines of the first or second formula, for instance. More generally, when the planner is not optimizing, there is no reason for the three formulas to coincide since each represents a different variation that could be performed.

Conversely, if the nonlinear tax schedule is optimized given t_i^* then all three formulas should coincide. This suggests that in this case one can intuitively interpret the absence of welfare weights in the last formula as reflecting a *revealed preference* on the part of the planner, with $\tau(z)$ somehow revealing $\tilde{\lambda}(z)$. Intuitively, the underlying preferences of the government for the utility of different groups of agents get revealed by the marginal tax rate $\tau(\theta)$ that they face after controlling for the distortionary cost of redistribution, as

[10]It should also be noted that the information required to implement the change in the tax schedule under this variation is relatively low: only the change in the wage schedule is required, and no information on preferences comes into play.

measured by the labor supply elasticity $\epsilon(\theta)$.[11] While useful and perhaps reassuring, this interpretation falls short and does no do full justice to the origin of the formula, nor to the fact that it holds even when the nonlinear tax is not optimal.

4.3 Quantitative Implications

How large should taxes on robots or trade be? The key input into the formulas in Proposition 2 is the elasticity of relative wages with respect to the number of machines. We now discuss, using two examples, how estimates of this key elasticity can be obtained from existing empirical work, as well as the policy implications.

Robot Example: Acemoglu and Restrepo (2017). Our first example focuses on robots. Using a difference-in-difference strategy, Acemoglu and Restrepo (2017) have estimated the effect of industrial robots, defined as "an automatically controlled, reprogrammable, and multipurpose [machine]" on different quantiles of the wage distribution between 1990 and 2007 across US commuting zones. Using our notation, their estimates can be interpreted as the semi-elasticity of wages with respect to robots, $\eta_{AR}(\theta) = \frac{d \ln w(\theta)}{d|y_m|}$, where $|y_m|$ is expressed as number of robots per thousand workers. The elasticity that we are interested in can then be approximated by

$$\frac{d \ln \omega(\theta)}{d \ln |y_m|} \simeq \frac{|y_m|}{\Delta \ln w(\theta)} \times \Delta \eta_{AR}(\theta),$$

where $\Delta z(\theta)$ denotes the change between a given variable $z(\theta)$ between two consecutive quantiles of the wage distribution. Figure 1a reports the local elasticity of relative wages (solid line) using $|y_m| \simeq 1.2$ as the number of robots per thousand workers in the United States in 2007.

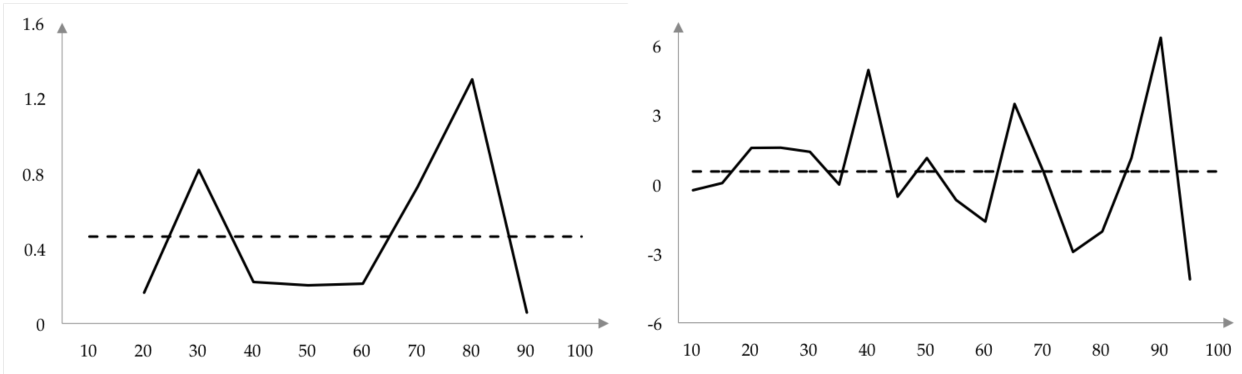
We first implement our formula abstracting away from any heterogeneity in labor supply and relative wage elasticities, $\epsilon(\theta) = \epsilon$ and $\frac{d \ln \omega(\theta)}{d \ln |y_m|} = \frac{d \ln \omega}{d \ln |y_m|}$.[12] In this case, we can further simplify the formula of Proposition (2) into

$$t_m^* = \frac{\int x(\theta) dF(\theta)}{p_m^* |y_m|} \frac{\epsilon}{\epsilon + 1} \frac{d \ln \omega}{d \ln |y_m|} \bar{\tau},$$

where $\bar{\tau} \equiv \int \tau(\theta) \frac{x(\theta)}{\int x(\theta') dF(\theta')} dF(\theta)$ denotes the average marginal income tax rates. In

[11] This is the idea behind Werning's (2007) test of whether an income tax schedule is Pareto optimal. Namely, it is if the inferred Pareto weights are all positive.

[12] In the next subsection, we provide micro-foundations on preferences and technology such that both elasticities are indeed constant across agents of different types.



(a) Robots (Acemoglu and Restrepo, 2017) **(b)** Chinese Imports (Chetverikov, Larsen and Palmer, 2016)

Figure 1: Elasticity of relative wages, $\frac{d \ln \omega(\theta)}{d \ln y_m}$, across quantiles of US wage distribution.

Figure 1a, the average of the local elasticity, $\frac{d \ln \omega}{d \ln |y_m|}$, across deciles (dashed line) is around 0.5. Graetz and Michaels (2018) estimate that the share of robot services in total capital services is around 0.64%, which leads to a ratio of total labor earnings to spending on robots around 245, whereas Guner, Kaygusuz and Ventura (2014) point toward an average marginal income tax rate around 0.1 for the United States. For a labor supply elasticity of $\epsilon = 0.1$, in the low range of the micro-estimates reviewed in Chetty (2012), we therefore obtain an optimal tax on robots equal to $t_m^* = 99\%$. For higher labor supply elasticities of $\epsilon = 0.3$ or $\epsilon = 0.5$, the previous tax goes up to $t_m^* = 269\%$ and $t_m^* = 410\%$, respectively.

More generally, in order to take into account the heterogeneity in relative wage elasticities reported in Figure 1a when implementing the formula of 2, one would need data on the share of robots employed with workers of a particular type, $\frac{y_m(\theta)}{|y_m|}$. Since we do not have access to such data, we instead compute the bounds on t_m^* associated with the minimum and maximum values of $\frac{\epsilon(\theta)}{\epsilon(\theta)+1} \frac{d \ln \omega(\theta)}{d \ln |y_m|}$. For a constant labor supply elasticity equal to $\epsilon = 0.1$, this leads to a lower-bound on the tax on robots equal to 104% and 117%, respectively.

Trade Example. Our second example focuses on Chinese imports. Using the same difference-in-difference strategy as in Autor, Dorn and Hanson (2013), Chetverikov, Larsen and Palmer (2016) have estimated the effect on log wages of a \$1,000 increase in Chinese imports per worker at different percentiles of the wage distribution. Following the same approach as in the case of robots, we can transform the previous semi-elasticity into an elasticity. Figure 1b reports the local elasticity of relative wages (solid line) using $|y_m| \simeq 2.2$ as the value of Chinese imports, in thousands of US dollars, per worker for the

United States in 2007.

The average value of the relative wage elasticity, $\frac{d \ln \omega}{d \ln |y_m|}$, is of the same order of magnitude as the one implied by the estimates of Acemoglu and Restrepo (2017), around 0.5 (dashed line in Figure 1b). Compared to the robot example, however, the ratio of total labor earnings to total Chinese imports in 2007 is only 26.4, an order of magnitude smaller than the ratio to total spending on robots. Implementing again the formula of Proposition (2) in the absence of heterogeneity in labor supply and relative wage elasticities, we obtain a much smaller tariff: $t_m^* = 15\%$ for $\epsilon = 0.1$, 42% for $\epsilon = 0.3$, and 65% for $\epsilon = 0.5$.

Another key difference between the two examples is that the relative wage elasticity, $\frac{d \ln \omega(\theta)}{d \ln |y_m|}$, is much more heterogeneous across quantiles in the trade case, as can be seen in Figure 1. This leads to much broader bounds on the efficient tariffs when that heterogeneity is taken into account. For a constant labor supply elasticity $\epsilon = 0.1$, the lower-bound on the optimal tariff now is equal to 2%, whereas the upper-bound is equal to 7%.

5 Comparative Statics

To provide further intuition about the forces that shape optimal taxes, we turn to a special case of the economic environment presented in Section 2. There is one final good, indexed by f, and one intermediate good, indexed by m, which could be either robots that are produced domestically or machines that are imported from abroad. The question that we are interested in is: If robots or traded goods were to become cheaper, and exacerbate inequality further, should taxes be raised further as well?

5.1 A Parametric Example

Agents have quasi-linear preferences,

$$U(\theta) = C(\theta) - \frac{(n(\theta))^{1+1/\epsilon}}{1+1/\epsilon}, \tag{5}$$

with $C(\theta)$ the consumption of the unique final good, which we use as our numeraire, $q_f = p_f = p_f^* = 1$, and ϵ the constant labor supply elasticity. Old technology firms produce the final good, $y_f \geq 0$, using workers, $n \equiv \{n(\theta)\}$, and machines, $y_m \leq 0$, as an input. Their production set is given by

$$G(y_f, y_m, n) = y - \max_{\{\tilde{y}_m(\theta)\}} \{\int g(\tilde{y}_m(\theta), n(\theta); \theta) dF(\theta) | \int \tilde{y}_m(\theta) dF(\theta) \leq -y_m\}, \tag{6}$$

with $g(\tilde{y}_m(\theta), n(\theta); \theta)$ a Cobb-Douglas production function,

$$g(\tilde{y}_m(\theta), n(\theta); \theta) = \exp(\alpha(\theta)) \cdot (\frac{\tilde{y}_m(\theta)}{\beta(\theta)})^{\beta(\theta)} (\frac{n(\theta)}{1-\beta(\theta)})^{1-\beta(\theta)}, \tag{7}$$

where $\tilde{y}_m(\theta)$ represents the number of machines combined with workers of type θ to produce the final good, $\alpha(\theta) \equiv \frac{\alpha \ln(1-\theta)}{\beta \ln(1-\theta)-1}$, and $\beta(\theta) \equiv \frac{\beta \ln(1-\theta)}{\beta \ln(1-\theta)-1}$, with $\alpha, \beta > 0$. Like in the examples of Section 2.2, new technology firms produce machines, $y_m^* \geq 0$, using the final good, $y_f^* \leq 0$,

$$G^*(y_f^*, y_m^*) = \phi y_f^* + y_m^*, \tag{8}$$

where ϕ measures the productivity of machine producers.

Let p_m and p_m^* denote the price of robots faced by old and new technology firms. Profit maximization by new technology firms implies

$$p_m^* = 1/\phi,$$

whereas profit maximization by old technology firms implies

$$w(p_m; \theta) = (1-\theta)^{-1/\gamma(p_m)},$$

with $\gamma(p_m) \equiv 1/(\alpha - \beta \ln p_m)$. Under the restriction that $\gamma(p_m) > 0$, which we maintain throughout, wages are increasing in θ and Pareto distributed with shape parameter equal to $\gamma(p_m)$ and lower bound equal to 1. By construction, more skilled workers tend to use machines relatively more; $\beta(\theta)$ is increasing in θ. So an increase in the price of machines tends to lower their wages relatively more and decrease inequality,

$$\frac{d \ln \omega(\theta)}{d \ln p_m} = -\frac{d \ln \gamma(p_m)}{d \ln p_m} = -\beta\gamma(p_m) < 0.$$

Here, because of additive separability in production, machines directly affect inequality by affecting relative marginal products of labor, but not indirectly through further changes in relative labor supply.[13]

5.2 The Rawlsian Tax Is Decreasing

We propose to study how t_m^* varies with ϕ in the case of a government with Rawlsian preferences.

[13]This second effect is the focus of the analysis of optimal income taxes in Stiglitz (1982).

For comparative static purposes, a limitation of the formula in Proposition (2) is that it involves the entire schedule of optimal marginal tax rates $\{\tau(\theta)\}$. These are themselves endogenous objects that will respond to productivity shocks. In Appendix D.2, we demonstrate how to substitute for those by combining the first-order conditions with respect to $U(\theta)$ and $n(\theta)$. This leads to

$$\frac{t_m^*}{1+t_m^*} = \frac{\frac{\epsilon}{\epsilon+1}\frac{d\ln\omega}{d\ln|y_m|}\tau^*}{1-\frac{\epsilon}{\epsilon+1}\frac{d\ln\omega}{d\ln|y_m|}\tau^*}\frac{1-s_m}{s_m}, \tag{9}$$

where the elasticity of relative wages, $\frac{d\ln\omega}{d\ln|y_m|} \equiv -\beta\gamma(p_m)\frac{\partial\ln p_m}{\partial\ln|y_m(p_m,n)|}$, is now constant across agents; $\tau^* \equiv \frac{\epsilon+1}{\epsilon+1+\epsilon\gamma(p_m)}$ corresponds to the optimal marginal tax rate that would be imposed in the absence of a tax on machines, as in Diamond (1998), Saez (2001), and Scheuer and Werning (2017); and $s_m \equiv \frac{p_m|y_m|}{\int x(\theta)dF(\theta)+p_m|y_m|}$ measures the share of machines in gross output. After expressing the three previous statistics, $\frac{d\ln\omega}{d\ln|y_m|}$, τ^*, and s_m, as functions of t_m^* and ϕ, we can apply the Implicit Function Theorem to determine the monotonicity of the optimal tax, as we do in Appendix D.2. This leads to our final proposition.

Proposition 3. *Suppose that equations (5)-(7) hold. Then the optimal Rawlsian tax, t_m^*, decreases with the productivity of machine makers, ϕ.*

In this economy, more machines always increase inequality, $\frac{d\ln\omega}{d\ln|y_m|} > 0$, but for comparative static purposes, the relevant question is whether this effect gets exacerbated as the new technology improves. Here, one can check that $\frac{\partial}{\partial\phi}\frac{d\ln\omega}{d\ln|y_m|} < 0$ both because relative wages are becoming less responsive to the price of machines, $\frac{\partial}{\partial\phi}|\frac{d\ln\omega}{d\ln|p_m|}| < 0$, and because the demand for machines is becoming more elastic, $\frac{\partial}{\partial\phi}|\frac{\partial\ln p_m}{\partial\ln|y_m(p_m,n)|}| < 0$, due to the increase in the labor supply of high-skilled workers whose demand for machines is more elastic. One can also check that these two effects dominate the increase in the marginal tax rate, $\frac{\partial\tau^*}{\partial\phi} > 0$, in response to greater inequality. For a given share of machines s_m, this implies that the total fiscal externality associated with new machines decreases. Since the share of machines increase with improvements in the new technology, $\frac{\partial s_m}{\partial\phi} > 0$, the fiscal externality per machine a fortiori decreases and so does the tax on machines.

As this example illustrates, cheaper robots may lead to a higher share of robots in the economy, more inequality, but a lower optimal tax on robots. Likewise, more imports and more inequality, in spite of the government having extreme distributional concerns and imports causing inequality, may be optimally met with less trade protection. This does not occur because redistribution is becoming more costly as the economy gets more open.[14] Here, the elasticity of labor supply is fixed and τ^* increases with ϕ. This also does not occur because redistribution through income taxation is becoming more attractive. Everything else being equal, an increase in τ^* raises the tax on imports. Rather the decrease in the tax on imports captures a standard Pigouvian intuition: as ϕ increases, the total fiscal externality associated with imports increases, but the externality per unit of imports does not, leading to a lower value for the optimal tax.

6 Conclusion

Our paper has focused on two broad sets of issues. The first one is related to the evaluation of new technologies. In spite of tax instruments being limited and the government having concerns for redistribution, we have shown that productivity shocks can be evaluated using a simple envelope argument, like in first best environments. Our finding stands in sharp contrast with the existing results of a large literature concerned with distortions and welfare. Our envelope result implies that firms developing new technologies should not be taxed or subsidized. It also implies that the welfare gains from trade or the welfare gains from the introduction of robots can still be computed by integrating below the demand curves for foreign goods and robots, respectively. Distributional concerns and various distortions may affect how much we trade or how much we use robots, but not the welfare implications from changes in the demand for those.

The second set of issues is related to the management of new technologies. We have asked: should we tax or subsidize firms using new technologies? And to the extent that taxes should be imposed, what are the sufficient statistics that can guide the optimal taxation of new technology firms in practice? In second best environments—where income taxation is available, but taxes on specific factors are not—we have shown that there is a case for taxing new technology firms, if taxes on old technology firms are unavailable. We have derived multiple optimal tax formulas and demonstrated how these can be implemented with existing reduced-form evidence to compute optimal taxes on robots or trade.

Finally, we provided an illustrative comparative static showing that more robots or more trade may go hand in hand with more inequality and lower taxes, despite robots or trade being responsible for the rise in inequality, and governments having extreme preferences for redistribution.

[14] This is the point emphasized by Itskhoki (2008) and Antras, de Gortari and Itskhoki (2017) in an economy where entrepreneurs can decide whether to export or not. This makes labor supply decisions more elastic in an open economy, which may reduce redistribution at the optimum.

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A Section 2

A.1 Competitive Equilibrium with Taxes

We first provide a formal definition of a competitive equilibrium with taxes.

Demand. Households maximize utility taking prices p and the income tax schedule T as given. Since preferences are weakly separable, the demand of any household θ is given by the solution to the following two-step problem

$$c(\theta) \in \operatorname{argmax}_{\tilde{c}(\theta)} \{v(\tilde{c}(\theta)) | p \cdot \tilde{c}(\theta) \leq w(\theta)n(\theta) - T(w(\theta)n(\theta))\}, \tag{A.1}$$

$$n(\theta) \in \operatorname{argmax}_{\tilde{n}(\theta)} \{u(C(p, w(\theta)\tilde{n}(\theta) - T(w(\theta)\tilde{n}(\theta))), \tilde{n}(\theta))\}, \tag{A.2}$$

where $C(p, r(\theta)) \equiv \max_{\tilde{c}(\theta)} \{v(\tilde{c}(\theta)) | p \cdot \tilde{c}(\theta) \leq r(\theta)\}$ denotes the indirect utility function associated with the lower stage given prices p and post-tax earnings $r(\theta) = w(\theta)n(\theta) - T(w(\theta)n(\theta))$.

Supply. Firms maximize profits taking prices p and p^* as given,

$$y, n \in \operatorname{argmax}_{\tilde{y}, \tilde{n}} \{p \cdot \tilde{y} - \int w(\theta)\tilde{n}(\theta) dF(\theta) | G(\tilde{y}, \tilde{n}) \leq 0\}, \tag{A.3}$$

$$y^* \in \operatorname{argmax}_{\tilde{y}^*} \{p^* \cdot \tilde{y}^* | G^*(\tilde{y}^*; \phi) \leq 0\}. \tag{A.4}$$

Market Clearing. Demand equals supply for all goods,

$$\int c_i(\theta) dF(\theta) = y_i + y_i^*, \text{ for all } i. \tag{A.5}$$

Non-Arbitrage. Prices faced by old and new technology firms satisfy

$$p_i = (1 + t_i^*) p_i^*, \text{ for all } i. \tag{A.6}$$

Equilibrium. A competitive equilibrium with taxes (T, t^*) corresponds to an allocation—, $c \equiv \{c(\theta)\}$, $n \equiv \{n(\theta)\}$, $y \equiv \{y_i\}$, and $y^* \equiv \{y_i^*\}$—and a system of prices and wages—$p \equiv \{p_i\}$, $p^* \equiv \{p_i^*\}$, and $w \equiv \{w(\theta)\}$—such that:

i. households maximize their utility, condition (A.1) and (A.2);

ii. firms maximize their profits, conditions (A.3) and (A.4);

iii. good markets clear, condition (A.5);

iv. prices satisfy the non-arbitrage condition (A.6);.

A.2 Planning Problem

The government problem is

$$\max_{T,t^*,c,n,y,y^*,p,p^*,w} \bar{W}(c,n) \tag{A.7}$$

subject to (A.1), (A.2), (A.3), (A.4), (A.5), and (A.6).

B Section 3

B.1 Envelope Result

Let us define the feasible set $\mathcal{Z}$ such that

$$\mathcal{Z} = \{(c,n,y)|\exists(T,p,w) \text{ such that } (c,n,y) \text{ satisfy (A.1), (A.2), and (A.3)}\}$$

The government problem (A.7) can then be rearranged as

$$\max_{(c,n,y)\in\mathcal{Z}} \bar{W}(c,n) \tag{B.1a}$$

$$G^*(\int c(\theta)dF(\theta) - y; \phi) \leq 0. \tag{B.1b}$$

For any solution to (B.1), there exists (T, p, w) such that (A.1), (A.2), and (A.3) hold. One can then construct y^*such that (A.5) holds,

$$y_i^* = \int c_i(\theta)dF(\theta) - y_i \text{ for all } i;$$

p^*such that (A.4) holds,

$$p_i^* = G^*_{y_i^*}(y^*)/G^*_{y_1^*}(y^*) \text{ for all } i;$$

and t^* such that (A.6) holds,

$$t_i^* = p_i/p_i^* - 1 \text{ for all } i.$$

B.2 Valuation of New Technologies

Starting from equation (4), the equivalent variation, $EV(\phi_0, \phi_1)$, associated with a productivity shock from ϕ_0 to ϕ_1 can be computed as the unique solution to the differential equation

$$\frac{dEV(\phi,\phi_1)}{d\phi} = \frac{\partial G^*(y^*(\phi, EV(\phi,\phi_1)); \phi)}{\partial \phi}, \text{ with initial condition } EV(\phi_1,\phi_1) = 0, \tag{B.2}$$

evaluated at $\phi = \phi_0$. Likewise, the compensating variation, $CV(\phi_0, \phi_1)$, can be computed as the unique solution to

$$\frac{dCV(\phi_0, \phi)}{d\phi} = \frac{\partial G^*(y^*(\phi, -CV(\phi_0, \phi)); \phi)}{\partial \phi}, \text{ with initial condition } CV(\phi_0, \phi_0) = 0, \tag{B.3}$$

evaluated at $\phi = \phi_1$. In equations (B.2) and (B.3), $y^*(\phi, EV(\phi, \phi_1))$ and $y^*(\phi, -CV(\phi_0, \phi))$ are the counterparts to the compensated Hicksian demand functions in standard consumer theory, evaluated at the final and the initial utility level, respectively.

C Section (4)

C.1 Lemma (1)

Consider a variation around the initial competitive equilibrium with taxes (t^*, T). Formally, the new tax schedule is given by

$$t_i^*(\epsilon) = t_i^* + \epsilon \bar{t}_i^*,$$
$$T(x; \epsilon) = T(x) + \epsilon \bar{T}(x) \text{ for all } x \geq 0,$$

for some arbitrary vector new technology taxes, $\bar{t}^*$, income tax schedule, $\bar{T}$, and $\epsilon \in \mathbb{R}$.

Let $\{c(\theta, \epsilon)\}$, $\{n(\theta, \epsilon)\}$, $\{y_i(\epsilon)\}$, and $\{y_i^*(\epsilon)\}$ denote the associated equilibrium allocation; let $\{p_i(\epsilon)\}$, $\{p_i^*(\epsilon)\}$ and $\{w(\theta, \epsilon)\}$ denote the associated prices and wages; and let $\{U(\theta, \epsilon)\}$ denote the associated utility levels. They satisfy

$$U(\theta, \epsilon) = \max_{\tilde{n}(\theta)} u(C(\{p_i(\epsilon)\}, R(w(\theta, \epsilon)\tilde{n}(\theta), \epsilon), \tilde{n}(\theta))$$
$$w(\theta, \epsilon) R_x(w(\theta, \epsilon) n(\theta, \epsilon), \epsilon) = -\frac{u_n(C(\{p_i(\epsilon)\}, R(w(\theta, \epsilon)n(\theta, \epsilon), \epsilon), n(\theta, \epsilon))}{u_C(C(\{p_i(\epsilon)\}, R(w(\theta, \epsilon)n(\theta, \epsilon), \epsilon), n(\theta, \epsilon))C_R(\{p_i(\epsilon)\}, R(w(\theta, \epsilon)n(\theta, \epsilon), \epsilon)}$$

$$\begin{aligned}
c(\theta, \epsilon) &= c(\{p_i(\epsilon)\}, C(\{p_i(\epsilon)\}, R(w(\theta, \epsilon)n(\theta, \epsilon), \epsilon)) \\
y_i(\epsilon) &= y_i(\{p_i(\epsilon)\}, \{n(\theta, \epsilon)\}) \\
y_i^*(\epsilon) &= \int c(\theta, \epsilon) dF(\theta) - y_i(\epsilon) \\
p_i^*(\epsilon) &= G^*_{y_i}(\{y_j^*(\epsilon)\}) / G^*_{y_1}(\{y_j^*(\epsilon)\}) \\
p_i(\epsilon) &= (1 + t_i^*(\epsilon)) p_i^*(\epsilon) \\
w(\theta, \epsilon) &= w(\{p_i(\epsilon)\}, \{n(\theta', \epsilon)\}; \theta) \\
G^*(y^*(\epsilon)) &= 0
\end{aligned}$$

where $R(x; \epsilon) \equiv x - T(x, \epsilon)$ denotes the retention function associated with the income tax schedule and $y(p,n)$ denotes the solution to (A.3) given labor supply n.

Given our restriction on W, social welfare as a function of ϵ can be expressed as

$$\tilde{W}(\epsilon) = \int \lambda(z)\tilde{U}(z,\epsilon)dz$$

where $\tilde{U}(z,\epsilon)$ denote the utility level associated with the quantile z of the earning distribution, i.e.,

$$z = \int_{U(\theta,\epsilon)\leq\tilde{U}(z,\epsilon)} dF(\theta)$$

Reynold's Theorem implies

$$\frac{d\tilde{U}(z,\epsilon)}{d\epsilon} = \int \frac{dU(\theta,\epsilon)}{d\epsilon} f(\theta|U(\theta,\epsilon) = \tilde{U}(z,\epsilon))d\theta.$$

By the Envelope Theorem,

$$\frac{dU(\theta,\epsilon)}{d\epsilon} = u_C(\theta,\epsilon)C_R(\theta,\epsilon)[R_x(w(\theta,\epsilon)n(\theta,\epsilon),\epsilon)n(\theta,\epsilon)w_\epsilon(\theta,\epsilon) - c(\theta,\epsilon)\cdot p_\epsilon(\epsilon) + R_\epsilon(w(\theta,\epsilon)n(\theta,\epsilon),\epsilon)]$$

Note that all households θ with the same utility, $U(\theta,\epsilon) = \tilde{U}(z,\epsilon)$, must also have the same wage, the same labor supply, the same vector of consumption, and the same marginal utility of income,

$$\begin{aligned}
w(\theta,\epsilon) &= \bar{w}(z,\epsilon) \text{ for all } \theta \text{ such that } U(\theta,\epsilon) = \tilde{U}(z,\epsilon);\\
n(\theta,\epsilon) &= \bar{n}(z,\epsilon) \text{ for all such } \theta \text{ that } U(\theta,\epsilon) = \tilde{U}(z,\epsilon);\\
c(\theta,\epsilon) &= \bar{c}(z,\epsilon) \text{ for all such } \theta \text{ that } U(\theta,\epsilon) = \tilde{U}(z,\epsilon);\\
u_C(\theta,\epsilon)C_R(\theta,\epsilon) &= \zeta(z,\epsilon) \text{ for all } \theta \text{ such that } U(\theta,\epsilon) = \tilde{U}(z,\epsilon).
\end{aligned}$$

Let us define the average wage response at quantile z as

$$\bar{w}_\epsilon(z,\epsilon) = \int w_\epsilon(\theta,\epsilon)f(\theta|U(\theta,\epsilon) = \tilde{U}(z,\epsilon))d\theta$$

We therefore get

$$\frac{d\tilde{U}(z,\epsilon)}{d\epsilon} = \int \zeta(z,\epsilon)[(1-\tau(z))\bar{n}(z,\epsilon)\bar{w}_\epsilon(z,\epsilon) - \bar{c}(z,\epsilon)\cdot p_\epsilon(\epsilon) + R_\epsilon(\bar{x}(z),\epsilon)]dz$$

where $\bar{x}(z) = \bar{w}(z,\epsilon)\bar{n}(z,\epsilon)$ denotes earnings at quantile z and $\tau(z) = 1 - R_x(\bar{x}(z),\epsilon)$. In turn, we have

$$\frac{d\tilde{W}(\epsilon)}{d\epsilon} = \int \lambda(z)\zeta(z,\epsilon)[(1-\tau(z))\bar{n}(z,\epsilon)\bar{w}_\epsilon(z,\epsilon) - \bar{c}(z,\epsilon)\cdot p_\epsilon(\epsilon) + R_\epsilon(\bar{x}(z),\epsilon)]dz. \tag{C.1}$$

Now consider the change in the cost of resources used by the new technology,

$$\begin{aligned}\frac{dG^*(y^*(\epsilon))}{d\epsilon} =& \nabla G^*(y^*(\epsilon)) \cdot y^*_\epsilon(\epsilon)\\
=& G^*_{y_1}(y^*(\epsilon))\{[p^*(\epsilon) - p(\epsilon)] \cdot y^*_\epsilon(\epsilon) + p \cdot y^*_\epsilon(\epsilon)\}\\
=& G^*_{y_1}(y^*(\epsilon))\{[p^*(\epsilon) - p(\epsilon)] \cdot y^*_\epsilon(\epsilon) + p(\epsilon) \cdot \int c_\epsilon(\theta,\epsilon)dF(\theta) - p(\epsilon) \cdot y_\epsilon(\epsilon)\}\\
=& G^*_{y_1}(y^*(\epsilon))\{[p^*(\epsilon) - p(\epsilon)] \cdot y^*_\epsilon(\epsilon) + p(\epsilon) \cdot \int c_\epsilon(\theta,\epsilon)dF(\theta) - \int w(\theta,\epsilon)n_\epsilon(\theta,\epsilon)dF(\theta)\}\\
=& G^*_{y_1}(y^*(\epsilon))\{[p^*(\epsilon) - p(\epsilon)] \cdot y^*_\epsilon(\epsilon)\\
&+ \int (R_\epsilon(w(\theta,\epsilon)n(\theta,\epsilon),\epsilon) + (1-\tau(\theta))n(\theta,\epsilon)\, w_\epsilon(\theta,\epsilon) + c(\theta,\epsilon) \cdot p_\epsilon(\epsilon) - \tau(\theta)w(\theta,\epsilon)n_\epsilon(\theta,\epsilon))\, dF(\theta)\end{aligned}$$

Let us define the average labor supply response at quantile z as

$$\bar{n}_\epsilon(z,\epsilon) = \int n_\epsilon(\theta,\epsilon)f(\theta|U(\theta,\epsilon) = \tilde{U}(z,\epsilon))d\theta.$$

Using this notation, we can rearrange the previous expression as

$$\begin{aligned}\frac{dG^*(y^*(\epsilon))}{d\epsilon} =& G^*_{y_1}(y^*(\epsilon))\{[p^*(\epsilon) - p(\epsilon)] \cdot y^*_\epsilon(\epsilon)\\
&+ \int [R_\epsilon(\bar{x}(z),\epsilon) + (1-\tau(z))\bar{n}(z,\epsilon)\bar{w}_\epsilon(z,\epsilon) - \bar{c}(z,\epsilon) \cdot p_\epsilon(\epsilon)]dz\\
&- \int \tau(z,\epsilon)w(z,\epsilon)n_\epsilon(z,\epsilon)]dz\}\end{aligned} \tag{C.2}$$

Let $\gamma > 0$ represents the marginal value of resources in the hand of the government. A necessary condition for a feasible variation not to improve welfare is that

$$\left.\frac{d\tilde{W}(\epsilon)}{d\epsilon}\right|_{\epsilon=0} - \gamma \left.\frac{dG^*(y^*(\epsilon))}{d\epsilon}\right|_{\epsilon=0} = 0.$$

By equations C.1 and (C.2),

$$\begin{aligned}&(p - p^*) \cdot \delta y^* - \int \tau(z)w(z)\delta n(z)\, dz\\
&\qquad = \int \tilde{\lambda}(z)((1-\tau(z))n(z)\, \delta w(z) - c(z) \cdot \delta p - \delta T(x(z)))\, dz.\end{aligned}$$

.

C.2 Proposition 2

All variations consist of a change in t^* and an associated change in p^*, potentially accompanied by a change in the income tax schedule T, such that $dy_1, dy^*_i \neq 0$ and $dy^*_j = 0$ for all j.

Variation $\delta T = 0$ From Lemma 1, if $\delta T = 0$, then

$$(p - p^*) \cdot \delta y^* - \int \tau(z) w(z) \delta n(z)\, dz = \int (\tilde{\lambda}(z) - 1)((1 - \tau(z)) n(z)\, \delta w(z) - c(z) \cdot \delta p)\, dz$$

Expressing $\delta n(z)$ as a function of the change in the wage and the labor supply elasticity, we get for a variation such that $dy_1, dy_i^* \neq 0$ and $dy_j^* = 0$ for all j,

$$p_i - p_i^* = t_i^* p_i^* = \int \left((\tilde{\lambda}(z) - 1)(1 - \tau(z)) + \tau(z)\, \varepsilon^u(z) \right) n(z) \frac{dw(z)}{dy_i^*}|_{\delta T = 0}\, dz - \int (\tilde{\lambda}(z) - 1) c(z) \cdot \frac{dp}{dy}|_{\delta T = 0}\, dz$$

Variation $\delta n = 0$ Define

$$\hat{R}(x, \epsilon) = C(R(x, \epsilon), q(\epsilon))$$

We have

$$V(\hat{R}, n) = u(\hat{R}, n)$$

$$MRS(\hat{R}, n) = -\frac{u_n(\hat{R}, n)}{u_C(\hat{R}, n)}$$

FOC for labor:

$$MRS(\hat{R}, n) = w \hat{R}_x$$

Differentiating leads to:

$$MRS_{\hat{R}} \hat{R}_\epsilon + MRS_{\hat{R}} \hat{R}_x n w_\epsilon + MRS_n n_\epsilon = \hat{R}_{xx} w x_\epsilon + \hat{R}_{x\epsilon} w + \hat{R}_x w_\epsilon$$

In order to get $n_\epsilon = 0$, we need $\hat{R}_\epsilon$ to solve:

$$MRS_{\hat{R}} \hat{R}_\epsilon + MRS_{\hat{R}} \hat{R}_x n w_\epsilon = \hat{R}_{xx} w x_\epsilon + \hat{R}_{x\epsilon} w + \hat{R}_x w_\epsilon$$

ie

$$\hat{R}_{x\epsilon} - \frac{MRS_{\hat{R}}}{w} \hat{R}_\epsilon = -\frac{w_\epsilon}{w} \hat{R}_x [1 + \frac{\hat{R}_{xx} w n}{\hat{R}_x}] + \frac{MRS_{\hat{R}}}{w} \hat{R}_x n w_\epsilon$$

Let

$$\rho(v) = \frac{MRS_{\hat{R}}}{w}(v)$$
$$g(v) = -\frac{w_\epsilon}{w} \hat{R}_x [1 + \frac{\hat{R}_{xx} w n}{\hat{R}_x}] + \rho \hat{R}_x n w_\epsilon$$

The previous ODE becomes:

$$\hat{R}_{x\epsilon} - \rho \hat{R}_\epsilon = g$$

Solving we get:

$$\hat{R}_\epsilon(x(\theta),\epsilon) = \int_0^\theta \exp[\int_z^\theta \rho(v)dx(v)]g(z)dx(z) + \exp[\int_0^\theta \rho(v)dx(v)]\hat{R}_\epsilon(x(0),\epsilon)$$

Let us find $\hat{R}_\epsilon(0,\epsilon)$ such that resource constraint holds. Using Lemma 1, we know that:

$$0 = (p^* - q)\int c_\epsilon dF(\theta) + (p - p^*)y_\epsilon + \int (R_\epsilon + (1-\tau)nw_\epsilon + cq_\epsilon - \tau w n_\epsilon)\, dF(\theta)$$

which can be rearranged (using Roy's identity) as

$$0 = (p^* - q)\int c_\epsilon dF(\theta) + (p - p^*)y_\epsilon + \int \left(\hat{R}_\epsilon + \hat{R}_x n w_\epsilon - C_R \tau w n_\epsilon\right)\frac{1}{C_R} dF(\theta)$$

With $n_\epsilon = 0$, this leads to

$$\begin{aligned} 0 &= (p - p^*)y_\epsilon + (p^* - q)\int c_\epsilon dF(\theta) \\ &+ \int \left(\int_0^\theta \exp[\int_z^\theta \rho(v)dx(v)]g(z)dx(z) + \exp[\int_0^\theta \rho(v)dx(v)]\hat{R}_\epsilon(x(0),\epsilon) + \hat{R}_x n w_\epsilon\right)\frac{1}{C_R} dF(\theta) \end{aligned}$$

In turn,

$$R_\epsilon(0,\epsilon) = \frac{(p^* - p)\cdot y_\epsilon - (p^* - q)\int c_\epsilon dF(\theta) - \int \{[\int_0^\theta \exp[\int_z^\theta \rho(v)dx(v)]g(z)dx(z)] + \hat{R}_x n w_\epsilon\}\frac{1}{C_R} dF(\theta)}{\int \exp[\int_0^\theta \rho(v)dx(v)]\frac{1}{C_R} dF(\theta)}$$

At the optimum we need:

$$\int [\hat{R}_\epsilon + \hat{R}_x w_\epsilon n] d\tilde{\Lambda}(\theta) = 0$$

where $\tilde{\Lambda}$ corresponds to the distribution of Pareto weights adjusted by marginal utility of aggregate consumption

$$\tilde{\lambda}(\theta) = \lambda(\theta) u_C$$

Let $\tilde{F}$ corresponds to the distribution of Pareto weights adjusted by marginal value of aggregate consumption:

$$\tilde{f}(\theta) = \frac{1}{C_R} f(\theta)$$

Substituting for $\hat{R}_\epsilon$, this leads to

$$\int [\int_0^\theta \exp[\int_z^\theta \rho(v)dx(v)]g(z)dx(z)]d\tilde{\Lambda}(\theta)$$
$$+\hat{R}_\epsilon(x(0),\epsilon)\int \exp[\int_0^\theta \rho(v)dx(v)]d\tilde{\Lambda}(\theta)$$
$$+\int \hat{R}_x w_\epsilon n d\tilde{\Lambda}(\theta) = 0$$

$$\int [\int_0^\theta \exp[\int_z^\theta \rho(v)dx(v)]g(z)dx(z)]d\tilde{\Lambda}(\theta)$$
$$+\hat{R}_\epsilon(x(0),\epsilon)\{\int \exp[\int_0^\theta \rho(v)dx(v)]d\tilde{F}(\theta)\}\frac{\int \exp[\int_0^\theta \rho(v)dx(v)]d\tilde{\Lambda}(\theta)}{\int \exp[\int_0^\theta \rho(v)dx(v)]d\tilde{F}(\theta)}$$
$$+\int \hat{R}_x w_\epsilon n d\tilde{\Lambda}(\theta) = 0$$

$$\int [\int_0^\theta \exp[\int_z^\theta \rho(v)dx(v)]g(z)dx(z)]d\tilde{\Lambda}(\theta)$$
$$+\{(p^*-p)\cdot y_\epsilon - (p^*-q)\int c_\epsilon dF(\theta) - \int [\int_0^\theta \exp[\int_z^\theta \rho(v)dx(v)]g(z)dx(z) + \hat{R}_x w_\epsilon n]d\tilde{F}(\theta)\}$$
$$\times\frac{\int \exp[\int_0^\theta \rho(v)dx(v)]d\tilde{\Lambda}(\theta)}{\int \exp[\int_0^\theta \rho(v)dx(v)]d\tilde{F}(\theta)}$$
$$+\int \hat{R}_x w_\epsilon n d\tilde{\Lambda}(\theta) = 0$$

$$(p^*-p)\cdot y_\epsilon - (p^*-q)\int c_\epsilon dF(\theta) = -\frac{\int \exp[\int_0^\theta \rho(v)dx(v)]d\tilde{F}(\theta)}{\int \exp[\int_0^\theta \rho(v)dx(v)]d\tilde{\Lambda}(\theta)}\int [\int_0^\theta \exp[\int_z^\theta \rho(v)dx(v)]g(z)dx(z)]d\tilde{\Lambda}(\theta)$$
$$+\int [\int_0^\theta \exp[\int_z^\theta \rho(v)dx(v)]g(z)dx(z)]d\tilde{F}(\theta)$$
$$+\int [\hat{R}_x w_\epsilon n]d\tilde{F}(\theta) - \frac{\int \exp[\int_0^\theta \rho(v)dx(v)]d\tilde{F}(\theta)}{\int \exp[\int_0^\theta \rho(v)dx(v)]d\tilde{\Lambda}(\theta)}\int \hat{R}_x w_\epsilon n d\tilde{\Lambda}(\theta)$$

Let $\bar{\Lambda}(\theta) = \frac{\int \exp[\int_0^\theta \rho(v)dx(v)]d\tilde{F}(\theta)}{\int \exp[\int_0^\theta \rho(v)dx(v)]d\tilde{\Lambda}(\theta)}\tilde{\Lambda}(\theta)$. Then

$$
\begin{aligned}
(p^* - p) \cdot y_\epsilon - (p^* - q) \int c_\epsilon dF(\theta) &= \int \{ \int_0^\theta \exp[\int_z^\theta \rho(v) dx(v)] g(z) dx(z) \} d(\tilde{F}(\theta) - \bar{\Lambda}(\theta)) \\
&+ \int \hat{R}_x w_\epsilon n d(\tilde{F}(\theta) - \bar{\Lambda}(\theta)) \\
&= \int \{ \int_0^\theta \exp[- \int_0^z \rho(v) dx(v)] g(z) dx(z) \} \exp[\int_0^\theta \rho(v) dx(v)] d(\tilde{F}(\theta) - \bar{\Lambda}(\theta)) \\
&+ \int \hat{R}_x w_\epsilon n d(\tilde{F}(\theta) - \bar{\Lambda}(\theta))
\end{aligned}
$$

Let

$$
\xi(\theta) = \int_0^\theta \exp[\int_0^z \rho(v) dx(v)] d(\tilde{F}(z) - \bar{\Lambda}(z))
$$

Integrating by parts,

$$
(p^* - p) \cdot y_\epsilon - (p^* - q) \int c_\epsilon dF(\theta) = - \int \xi(\theta) \exp[- \int_0^\theta \rho(v) dx(v)] g(\theta) x'(\theta) d\theta + \int \hat{R}_x w_\epsilon n d(\tilde{F}(\theta) - \bar{\Lambda}(\theta)) \tag{C.3}
$$

Let us use the fact

$$
\begin{aligned}
\xi(\bar{\theta}) &= \int_0^{\bar{\theta}} \exp[\int_0^\theta \rho(v) dx(v)] d(\tilde{F}(\theta) - \bar{\Lambda}(\theta)) \\
&= \int_0^{\bar{\theta}} \exp[\int_0^\theta \rho(v) dx(v)] d\tilde{F}(\theta) \\
&- \frac{\int \exp[\int_0^\theta \rho(v) dx(v)] d\tilde{F}(\theta)}{\int \exp[\int_0^\theta \rho(v) dx(v)] d\tilde{\Lambda}(\theta)} \int_0^{\bar{\theta}} \exp[\int_0^\theta \rho(v) dx(v)] d\tilde{\Lambda}(\theta) \\
&= 0
\end{aligned}
$$

This leads to:

$$
\begin{aligned}
\xi(\theta) &= \int_0^\theta \exp[\int_0^z \rho(v) dx(v)] d(\tilde{F}(z) - \bar{\Lambda}(z)) \\
&- \int_0^{\bar{\theta}} \exp[\int_0^z \rho(v) dx(v)] d(\tilde{F}(z) - \bar{\Lambda}(z)) \\
&= - \int_\theta^{\bar{\theta}} \exp[\int_0^z \rho(v) dx(v)] d(\tilde{F}(z) - \bar{\Lambda}(z)) \\
&= - \exp[\int_0^\theta \rho(v) dx(v)] \int_\theta^{\bar{\theta}} \exp[\int_\theta^z \rho(v) dx(v)] d(\tilde{F}(z) - \bar{\Lambda}(z))
\end{aligned}
$$

Now let

$$
\psi(\theta) = \int_\theta^{\bar{\theta}} \exp[\int_\theta^z \rho(v) dx(v)] d(\tilde{F}(z) - \bar{\Lambda}(z))
$$

We can therefore rearrange C.3 as

$$
\begin{aligned}
(p^* - p) \cdot y_\epsilon - (p^* - q) \int c_\epsilon dF(\theta) &= \int \psi(\theta) g(\theta) x'(\theta) d\theta + \int \hat{R}_x w_\epsilon n d(\tilde{F}(\theta) - \bar{\Lambda}(\theta)) \\
&= \int \psi(\theta) g(\theta) x'(\theta) d\theta \\
&+ \int \exp[-\int_0^\theta \rho(v) dx(v)] \hat{R}_x w_\epsilon n \exp[\int_0^\theta \rho(v) dx(v)] d(\tilde{F}(\theta) - \bar{\Lambda}(\theta))
\end{aligned}
$$

So integrating the second term by parts again,

$$
\begin{aligned}
(p^* - p) \cdot y_\epsilon - (p^* - q) \int c_\epsilon dF(\theta) &= \int \psi(\theta) g(\theta) x'(\theta) d\theta \\
&+ \int \psi(\theta) \{\hat{R}_{xx} x'(\theta) w_\epsilon n + \hat{R}_x w_{\epsilon\theta} n + \hat{R}_x w_\epsilon n_\theta\} d\theta \\
&- \int \psi(\theta) \rho(\theta) x'(\theta) \hat{R}_x w_\epsilon n d\theta
\end{aligned}
$$

Note that:

$$
\begin{aligned}
x'(\theta)\{-\frac{w_\epsilon}{w}\} + w_{\epsilon\theta} n + w_\epsilon n_\theta &= [w_\theta n + w n_\theta][-\frac{w_\epsilon}{w}] + w_{\epsilon\theta} n + w_\epsilon n_\theta \\
&= w_\theta n[-\frac{w_\epsilon}{w}] + w_{\epsilon\theta} n \\
&= wn[\frac{w w_{\epsilon\theta} - w_\theta w_\epsilon}{w^2}] \\
&= wn \frac{d}{d\epsilon} \frac{d \ln w}{d\theta} \\
&= (wn)\omega_\epsilon
\end{aligned}
$$

This leads to

$$
(p^* - p) \cdot y_\epsilon - (p^* - q) \int c_\epsilon dF(\theta) = \int \psi(\theta) \hat{R}_x w n \omega_\epsilon d\theta.
$$

For $p = q$, we get

$$
(p - p^*) \cdot y^*_\epsilon = \int \psi(\theta) \hat{R}_x w n \omega_\epsilon d\theta.
$$

Variation dU=0 From Lemma 1, we know that if $\delta U(z) = 0$ for all z. Then

$$
(p - p^*) \cdot \delta y^* = \int \tau(z) w(z) \delta n(z)\, dz \tag{C.4}
$$

Let us now compute $\delta n(z)$. We have

$$
R_\epsilon(y(z,\epsilon),\epsilon) + R_y(y(z,\epsilon),\epsilon) w_\epsilon(z,\epsilon) n(z,\epsilon) = 0
$$

$$MRS(R(y(z,\epsilon),\epsilon),n(z,\epsilon)) = R_y(y(z,\epsilon),\epsilon)w(z,\epsilon)$$

$$y(z,\epsilon) = n(z,\epsilon)w(z,\epsilon)$$

Differentiating

$$\left(R_{\epsilon y} + R_{yy}w_\epsilon n\right) y_z + R_y w_{\epsilon z} n + R_y w_\epsilon n_z = 0$$

$$\left(R_{\epsilon y} + R_{yy}w_\epsilon n\right) y_\epsilon + R_{\epsilon\epsilon} + R_{y\epsilon}w_\epsilon n + R_y w_{\epsilon\epsilon} n + R_y w_\epsilon n_e = 0$$

$$MRS_c R_y y_z + MRS_n n_z = R_{yy} w y_z + R_y w_z$$

$$MRS_c(R_y y_\epsilon + R_\epsilon) + MRS_n n_\epsilon = R_{yy} w y_\epsilon + R_{y\epsilon} w + R_y w_\epsilon$$

$$y_z = n_z w + n w_z$$

$$y_\epsilon = n_\epsilon w + n w_\epsilon$$

But using that $R_\epsilon + R_y w_\epsilon n = 0$ the forth equation is equivalent to

$$MRS_c R_y w n_\epsilon + MRS_n n_\epsilon = R_{yy} w y_\epsilon + R_{y\epsilon} w + R_y w_\epsilon$$

Knowns: $w(z,\epsilon)$ hence w_ϵ, $w_{\epsilon z}$, $w_{\epsilon\epsilon}$ and R_y, R_{yy} and h''
Unknowns: n_z, n_ϵ, y_z,y_ϵ,$R_{y\epsilon}$,$R_{\epsilon\epsilon}$.
Substituting out y_z and y_ϵ, we get a system of 4x4,

$$\left(R_{\epsilon y} + R_{yy}w_\epsilon n\right)(n_z w + n w_z) + R_y w_{\epsilon z} n + R_y w_\epsilon n_z = 0$$

$$\left(R_{\epsilon y} + R_{yy}w_\epsilon n\right)(n_\epsilon w + n w_\epsilon) + R_{\epsilon\epsilon} + R_{y\epsilon}w_\epsilon n + R_y w_{\epsilon\epsilon} n + R_y w_\epsilon n_\epsilon = 0$$

$$MRS_c R_y (n_z w + n w_z) + MRS_n n_z = R_{yy} w (n_z w + n w_z) + R_y w_z$$

$$MRS_c R_y n_\epsilon w + MRS_n n_\epsilon = R_{yy} w (n_\epsilon w + n w_\epsilon) + R_{y\epsilon} w + R_y w_\epsilon$$

Solving the third equation and rearranging the forth gives,

$$n_z = \frac{R_{yy} w n + R_y - MRS_c R_y n}{MRS_c R_y w + MRS_n - R_{yy} w^2} w_z$$

$$\left(MRS_c R_y w + MRS_n - R_{yy} w^2\right) n_\epsilon = (R_{yy} w n + R_y) w_\epsilon + R_{y\epsilon} w$$

Substituting n_z into the first equation

$$\left(R_{\epsilon y}+R_{yy}w_\epsilon n\right)\left(\frac{R_{yy}wn+R_y-MRS_cR_yn}{MRS_cR_yw+MRS_n-R_{yy}w^2}w_zw+nw_z\right)$$
$$+R_yw_{\epsilon z}n+R_yw_\epsilon\left(\frac{R_{yy}wn+R_y-MRS_cR_yn}{MRS_cR_yw+MRS_n-R_{yy}w^2}w_z\right)=0$$

Multiplying through by $MRS_cR_yw+MRS_n-R_{yy}w^2$ gives

$$\left(R_{\epsilon y}+R_{yy}w_\epsilon n\right)\left(\left(R_{yy}wn+R_y-MRS_cR_yn\right)w_zw+\left(MRS_cR_yw+MRS_n-R_{yy}w^2\right)nw_z\right)$$
$$+\left(MRS_cR_yw+MRS_n-R_{yy}w^2\right)R_yw_{\epsilon z}n+R_yw_\epsilon\left(R_{yy}wn+R_y-MRS_cR_yn\right)w_z=0$$

Cancelling

$$\left(R_{\epsilon y}+R_{yy}w_\epsilon n\right)\left(R_yw+MRS_nn\right)w_z$$
$$+\left(MRS_cR_yw+MRS_n-R_{yy}w^2\right)R_yw_{\epsilon z}n+R_yw_\epsilon\left(R_{yy}wn+R_y-MRS_cR_yn\right)w_z=0$$

and solving (using $MRS=R_yw$)

$$\begin{aligned}\Rightarrow R_{\epsilon y}=&-R_{yy}w_\epsilon n-\frac{MRS_cR_yw+MRS_n-R_{yy}w^2}{R_yw+MRS_nn}R_y\frac{w_{\epsilon z}}{w_z}n\\&-R_yw_\epsilon\frac{R_{yy}wn+R_y-MRS_cR_yn}{R_yw+MRS_nn}\\=&-R_{yy}nw_\epsilon-\frac{MRS_cR_yw+MRS_n-R_{yy}w^2}{1+\frac{MRS_n}{MRS}n}\frac{w_{\epsilon z}}{w_zw}n-\frac{R_{yy}wn+R_y-MRS_cR_yn}{1+\frac{MRS_n}{MRS}n}\frac{w_\epsilon}{w}\end{aligned}$$

Substituting into the forth equation gives

$$\begin{aligned}\left(MRS_cR_yw+MRS_n-R_{yy}w^2\right)n_\epsilon=&(R_{yy}wn+R_y)w_\epsilon\\&-R_{yy}wnw_\epsilon-\frac{MRS_cR_yw+MRS_n-R_{yy}w^2}{1+\frac{MRS_n}{MRS}n}\frac{w_{\epsilon z}}{w_z}n-\frac{R_{yy}wn+R_y-MRS_cR_yn}{1+\frac{MRS_n}{MRS}n}w_\epsilon\\=&\frac{\frac{MRS_n}{MRS}n}{1+\frac{MRS_n}{MRS}n}(R_{yy}wn+R_y-MRS_cR_yn)w_\epsilon-R_{yy}wnw_\epsilon-\frac{MRS_cR_yw+MRS_n-R_{yy}w^2}{1+\frac{MRS_n}{MRS}n}\frac{w_{\epsilon z}}{w_z}n\\&+MRS_cR_ynw_\epsilon\end{aligned}$$

Also note that

$$\frac{1}{\frac{w_z}{w}}\frac{\partial}{\partial\epsilon}\left(\frac{w_z}{w}\right)=\frac{1}{\frac{w_z}{w}}\left(\frac{w_{z\epsilon}}{w}-\frac{w_zw_\epsilon}{w^2}\right)=\frac{w_{z\epsilon}}{w_z}-\frac{w_\epsilon}{w}$$

So that

$$
\begin{aligned}
\left(MRS_c R_y w + MRS_n - R_{yy} w^2\right) n_\epsilon = & \frac{\frac{MRS_n}{MRS} n}{1 + \frac{MRS_n}{MRS} n} (R_{yy} w n + R_y - MRS_c R_y n) w_\epsilon \\
& - R_{yy} w n w_\epsilon - \frac{MRS_c R_y w + MRS_n - R_{yy} w^2}{1 + \frac{MRS_n}{MRS} n} \left(\frac{1}{\frac{w_z}{w}} \frac{\partial}{\partial \epsilon} \left(\frac{w_z}{w}\right) + \frac{w_\epsilon}{w} \right) n
\end{aligned}
$$

$$
\begin{aligned}
\left(MRS_c R_y w + MRS_n - R_{yy} w^2\right) n_\epsilon = & \frac{\frac{MRS_n}{MRS} n}{1 + \frac{MRS_n}{MRS} n} (R_y - MRS_c R_y n) w_\epsilon \\
& - \frac{MRS_c R_y w + MRS_n - R_{yy} w^2}{1 + \frac{MRS_n}{MRS} n} n \frac{1}{\frac{w_z}{w}} \frac{\partial}{\partial \epsilon} \left(\frac{w_z}{w}\right) \\
& - \frac{MRS_c R_y w + MRS_n}{1 + \frac{MRS_n}{MRS} n} R_y \frac{w_\epsilon}{MRS} n
\end{aligned}
$$

$$
\begin{aligned}
\left(MRS_c R_y w + MRS_n - R_{yy} w^2\right) n_\epsilon = & -\frac{\frac{MRS_n}{MRS} n}{1 + \frac{MRS_n}{MRS} n} MRS_c R_y n w_\epsilon - \frac{1}{1 + \frac{MRS_n}{MRS} n} MRS_c R_y n w_\epsilon \\
& - \frac{MRS_c R_y w + MRS_n - R_{yy} w^2}{1 + \frac{MRS_n}{MRS} n} n \frac{1}{\frac{w_z}{w}} \frac{\partial}{\partial \epsilon} \left(\frac{w_z}{w}\right) \\
& + MRS_c R_y n w_e
\end{aligned}
$$

Cancelling gives

$$
n_\epsilon = -\frac{1}{1 + \frac{MRS_n}{MRS} n} n \frac{1}{\frac{w_z}{w}} \frac{\partial}{\partial \epsilon} \left(\frac{w_z}{w}\right).
$$

Substituting into C.4, we obtain

$$
p_i - p_i^* = t_i^* p_i^* = \int \tau(\theta) w(z) n(z) \frac{\epsilon(z)}{\epsilon(z) + 1} \frac{1}{\omega(z)} \frac{d\omega(z)}{dy_i^*} |_{\delta U = 0} \, dz.
$$

D Section 5

D.1 Preliminary Results

Planning Problem Using the revelation principle, we rearrange the planning problem as follows

$$
\max_{U,n,p,q} \int U(\theta) d\Lambda(\theta) \tag{D.1a}
$$

subject to

$$U'(\theta) = -u_n(C(n(\theta), U(\theta)), n(\theta))n(\theta)\omega(p, n; \theta), \tag{D.1b}$$

$$G^*(c(q, n, U) - y(p, n); \phi) \leq 0. \tag{D.1c}$$

where we use the following notation. On the demand side, we let $c(q, C(\theta))$ denote the solution to (A.1) given consumer prices q and aggregate consumption $C(\theta)$; we let $C(n(\theta), U(\theta))$ denote the aggregate consumption required to achieve utility $U(\theta)$ given labor supply $n(\theta)$, that is the solution to $u(C, n(\theta)) = U(\theta)$; and we let $c(q, n, U) = \int c(q, C(n(\theta), U(\theta)))dF(\theta)$ denote the total demand for goods conditional on prices, q, labor supply, $n \equiv \{n(\theta)\}$, and utility levels, $U \equiv \{U(\theta)\}$. Likewise, on the supply side, we let $y(p, n)$ denote the solution to (A.3) given producer prices p and labor demand n, and we let $w(p, n; \theta)$ denote the associated equilibrium wage for each household θ,

$$w(p, n; \theta) = G_{n(\theta)}(y(p, n), n) \times \frac{p \cdot y(p, n)}{\int n(\theta')G_{n(\theta')}(y(p, n), n)dF(\theta')},$$

obtained from the first-order condition with respect to $n(\theta)$, with $G_{n(\theta)} \equiv \partial G/\partial n(\theta)$.

First-order Conditions The Lagrangian associated with the planner's problem (D.1) is given by

$$\mathcal{L} = \int U(\theta)d\Lambda(\theta) + \int \mu(\theta) \left(U'(\theta) + u_n(C(n(\theta), U(\theta)), n(\theta))n(\theta)\omega(\{p_i\}, \{n(\theta)\})\right) d\theta \\ - \gamma G^*(\{c_j(\{q_i\}, \{n(\theta)\}, \{U(\theta)\}) - y_j(\{p_i\}, \{n(\theta)\})\}; \phi).$$

Integrating by parts, we get

$$\mathcal{L} = \int U(\theta)d\Lambda(\theta) - \int \mu'(\theta)U(\theta)d\theta + U(\bar{\theta})\mu(\bar{\theta}) - U(\underline{\theta})\mu(\underline{\theta}) \\ + \int \mu(\theta)u_n(C(n(\theta), U(\theta)), n(\theta))n(\theta)\omega(\{p_i\}, \{n(\theta)\})d\theta \\ - \gamma G^*(\{c_j(\{q_i\}, \{n(\theta)\}, \{U(\theta)\}) - y_j(\{p_i\}, \{n(\theta)\})\}; \phi).$$

Since $U(\bar{\theta})$ and $U(\underline{\theta})$ are free we must have

$$\mu(\underline{\theta}) = \mu(\bar{\theta}) = 0.$$

With Respect to $U(\theta)$ The first-order condition with respect to $U(\theta)$ leads to

$$\lambda(\theta) - \mu'(\theta) + \mu(\theta)u_{nC}(\theta)C_{U(\theta)}(\theta)n(\theta)\omega(\theta) - \gamma\nabla_{y^*}G^* \cdot c_{U(\theta)}f(\theta) = 0,$$

where λ denotes the density associated with Λ. From condition (A.4) and the normalization $p_1^* = 1$, we know that $p_i^* = G_{y_i}^*/G_{y_1}^*$ for all i. From the fact that $D_q c \cdot (p^* - q) = 0$ at the optimum, as argued in the main text, and the normalization $q_1 = 1$, we also know that $p_i^* = q_i$ for all i. Using these two observations, we get

$$\mu'(\theta) - \mu(\theta)u_{nC}(\theta)C_{U(\theta)}(\theta)n(\theta)\omega(\theta) = \lambda(\theta) - \gamma G_{y_1}^* \zeta(\theta) f(\theta),$$

with $\zeta(\theta) \equiv \sum p_j^* \frac{dc_j(\{q_i\}, C(n(\theta), U(\theta)))}{dU(\theta)}$. Let $\tilde{u}(C, x, \theta) \equiv u(C, x/w(\theta))$, $\tilde{u}_\theta \equiv \frac{\partial \tilde{u}}{\partial \theta}$, and $\tilde{u}_{\theta C} \equiv \frac{\partial^2 \tilde{u}}{\partial \theta \partial C}$. By definition, we have

$$\tilde{u}_\theta(\theta) = -u_n(\theta)n(\theta)\omega(\theta)$$

and

$$\tilde{u}_{\theta C}(\theta) = -u_{nC}(\theta)n(\theta)\omega(\theta)$$

Using the previous notation and using the fact that $C_{U(\theta)} = 1/u_C(\theta) = 1/\tilde{u}_C(\theta)$, we can rearrange the above first-order condition as

$$\mu'(\theta)\tilde{u}_C(\theta) + \mu(\theta)\tilde{u}_{\theta C}(\theta) = \tilde{u}_C(\theta)(\lambda(\theta) - \gamma G_{y_1}^* \zeta(\theta) f(\theta)). \tag{D.2}$$

Let $\tilde{\mu}(\theta) \equiv \mu(\theta)\tilde{u}_C(\theta)$. By definition, we also have

$$\tilde{\mu}'(\theta) = \mu'(\theta)\tilde{u}_C(\theta) + \mu(\theta)\tilde{u}_C'(\theta)$$

with

$$\tilde{u}_C'(\theta) = \tilde{u}_{\theta C}(\theta) + \tilde{u}_{CC}(\theta)C'(\theta) + \tilde{u}_{Cx}(\theta)x'(\theta),$$

which can be rearranged as

$$\tilde{u}_C'(\theta) = \tilde{u}_{\theta C}(\theta) + x'(\theta)[\tilde{u}_{CC}(\theta)\frac{dC}{dx} + \tilde{u}_{Cx}(\theta)]. \tag{D.3}$$

From agent θ's budget constraint, $e(q, C) = R(x)$, we know that

$$\frac{dC}{dx} = \frac{R'(x)}{e_C},$$

and from the first-order condition associated with (A.2), we know that

$$\frac{R'(x)}{e_C} = -\frac{\tilde{u}_x}{\tilde{u}_C}.$$

Combining the three previous equations, we obtain

$$\tilde{u}_C'(\theta) = \tilde{u}_{\theta C}(\theta) + \tilde{u}_C(\theta)x'(\theta)\rho(\theta), \tag{D.4}$$

where $\rho(\theta) \equiv \frac{\partial(\tilde{u}_x/\tilde{u}_C)}{\partial C} = \frac{\tilde{u}_{Cx}}{\tilde{u}_C} - \tilde{u}_{CC}\frac{\tilde{u}_x}{\tilde{u}_C^2}$ denotes the partial derivative, with respect to aggregate consumption, of the marginal rate of substitution between earnings and consumption. In turn, equations (D.2), (D.3), and (D.4) imply

$$\tilde{\mu}'(\theta) - \tilde{\mu}(\theta)\frac{dx}{d\theta}\rho(\theta) = \tilde{u}_C[\lambda(\theta) - \gamma G_{y_1}^* \zeta(\theta)f(\theta)].$$

Solving forward and using the fact that $\tilde{\mu}(\bar{\theta}) = 0$, we get

$$\begin{aligned}\tilde{\mu}(\theta) = &- \int_\theta^{\bar{\theta}} \exp[-\int_\theta^z \rho(v)dx(v)]\tilde{u}_C(z)d\Lambda(z) \\ &+ \gamma \int_\theta^{\bar{\theta}} \exp[-\int_\theta^z \rho(v)dx(v)]\tilde{u}_C(z)G_{y_1}^*\zeta(z)dF(z).\end{aligned}$$

Since $\tilde{\mu}(\underline{\theta}) = 0$, we must also have

$$\gamma = \frac{\int_{\underline{\theta}}^{\bar{\theta}} \exp[-\int_\theta^z \rho(v)dx(v)]\tilde{u}_C(z)d\Lambda(z)}{\int_{\underline{\theta}}^{\bar{\theta}} \exp[-\int_\theta^z \rho(v)dx(v)]\tilde{u}_C(z)G_{y_1}^*\zeta(z)dF(z)},$$

which implies

$$\begin{aligned}\frac{\tilde{\mu}(\theta)}{\gamma G_{y_1}^*} = &\int_\theta^{\bar{\theta}} \exp[-\int_\theta^z \rho(v)dx(v)]\tilde{u}_C(z)\zeta(z)dF(z) \\ &- \frac{\int_\theta^{\bar{\theta}} \exp[-\int_\theta^z \rho(v)dx(v)]\tilde{u}_C(z)d\Lambda(z)}{\int_{\underline{\theta}}^{\bar{\theta}} \exp[-\int_{\underline{\theta}}^z \rho(v)dx(v)]\tilde{u}_C(z)d\Lambda(z)} \cdot \int_{\underline{\theta}}^{\bar{\theta}} \exp[-\int_{\underline{\theta}}^z \rho(v)dx(v)]\tilde{u}_C(z)\zeta(z)dF(z).\end{aligned}$$

With Respect to $n(\theta)$ The first-order condition with respect to $n(\theta)$ is given by

$$\begin{aligned}\gamma\phi[y_{f,n(\theta)}(p_m, n) - h'(n(\theta)) + \frac{1}{\phi}y_{m,n(\theta)}(p_m, n\}]f(\theta) \\ = \mu(\theta)[h''(n(\theta))n(\theta) + h'(n(\theta))]\omega(p_m;\theta).\end{aligned} \tag{D.5}$$

Since old technology firms choose their labor demand to maximize profits and agents choose their labor supply to maximize utility, we also know that

$$\begin{aligned}y_{f,n(\theta)}(p_m, n) + p_m y_{m,n(\theta)}(p_m, n) = w(\theta), \\ h'(n(\theta)) = w(\theta)(1 - \tau(\theta)).\end{aligned}$$

Thus, using the fact that $p_m^* = 1/\phi$, we can rearrange equation (D.5) into

$$\gamma\phi[w(\theta)\tau(\theta) + (p_m^* - p_m)y_{m,n(\theta)}]f(\theta) = \mu(\theta)h'(n(\theta))[\frac{\epsilon(\theta)+1}{\epsilon(\theta)}]\omega(p_m;\theta), \tag{D.6}$$

where $\epsilon(\theta) \equiv \frac{d\ln(n(\theta))}{d\ln h'(n(\theta)))} \geq 0$ denotes the labor supply elasticity and $y_{m,n(\theta)} \equiv \frac{dy_m(p_m,n)}{dn(\theta)}$.

With Respect to p_m The first-order condition with respect to p_m is given by

$$\gamma\phi(p_m^* - p_m)y_{m,p_m} = \int \mu(\theta)h'(n(\theta))n(\theta)\omega_{p_m}(p_m;\theta)d\theta, \tag{D.7}$$

with $y_{m,p_m} \equiv \frac{dy_m(p_m,n)}{dp_m}$. Using equation D.6 to substitute for $\mu(\theta)h'(n(\theta))/\gamma$ in equation (D.7) and noting that $d\ln y_m(p_m, n(\theta);\theta)/d\ln n(\theta) = 1$,[15] we obtain

$$p_m - p_m^* = \frac{\int \frac{\epsilon(\theta)}{\epsilon(\theta)+1} \cdot \frac{d\ln\omega(\theta)}{d\ln|y_m|} \cdot \tau(\theta) \cdot x(\theta)dF(\theta)}{|y_m|[1 - \int \frac{\epsilon(\theta)}{\epsilon(\theta)+1} \cdot \frac{d\ln\omega(\theta)}{d\ln|y_m|} \cdot \frac{y_m(\theta)}{|y_m|}dF(\theta)]}, \tag{D.8}$$

where $\frac{d\ln\omega(\theta)}{d\ln|y_m|}$ corresponds to the local elasticity of relative wages with respect to the total number of machines used by old technology firms, holding workers' labor supply fixed,

$$\frac{d\ln\omega(\theta)}{d\ln|y_m|} = \frac{\partial\ln p_m}{\partial\ln|y_m(p_m,n)|}\frac{d\ln\omega(p_m;\theta)}{d\ln p_m}.$$

Since households do not consume machines, the previous price gap can be implemented equivalently with a negative tax on old technology firms, a positive tax on new technology firms, or a combination of both. For expositional purposes, we focus in the rest of this section on the case where the tax on old technology firms has been set to zero and refer to $t_m^* = p_m/p_m^* - 1$ as the tax on machines. Given this normalization, equation (D.8) leads to

$$t_m^* = \frac{\int \frac{\epsilon(\theta)}{\epsilon(\theta)+1}\frac{d\ln\omega(\theta)}{d\ln|y_m|}\tau(\theta)x(\theta)dF(\theta)}{p_m^*|y_m|[1 - \int \frac{\epsilon(\theta)}{\epsilon(\theta)+1}\frac{d\ln\omega(\theta)}{d\ln|y_m|}\frac{y_m(\theta)}{|y_m|}dF(\theta)]}. \tag{D.9}$$

D.2 Section 5.2

Equation ((9)). Under the assumption that upper-level preferences are quasilinear, we have

$$\frac{\mu(\theta)}{\gamma\phi} = \Lambda(\theta) - F(\theta).$$

[15] Recall that $y_m(p_m, n(\theta);\theta)$ is implicitly defined as the solution to $p_m = dg(y_m(\theta), n(\theta);\theta)/dy_m(\theta)$. Since $g(\cdot,\cdot;\theta)$ is homogeneous of degree one, this is equivalent to $p_m = dg(y_m(\theta)/n(\theta), 1;\theta)/dy_m(\theta)$. Differentiating, we therefore get $d\ln y_m(p_m, n(\theta);\theta)/d\ln n(\theta) = 1$.

Together with equation (D.6), we therefore get

$$[w(\theta)\tau(\theta)+(p_m^*-p_m)y_{m,n(\theta)}]=\frac{[\Lambda(\theta)-F(\theta)]}{f(\theta)}h'(n(\theta))[\frac{\epsilon(\theta)+1}{\epsilon(\theta)}]\omega(p_m;\theta).$$

Using again the fact that $d\ln y_m(p_m,n(\theta);\theta)/d\ln n(\theta)=1$ and $h'(n(\theta))=w(\theta)(1-\tau(\theta))$, from the first-order condition of the agent's utility maximization problem, this leads to

$$\tau(\theta)=\tau^*(\theta)-\frac{p_m-p_m^*}{p_m}\frac{p_m y_m(\theta)}{x(\theta)}(1-\tau^*(\theta)), \tag{D.10}$$

with

$$\tau^*(\theta)\equiv\frac{1}{1+\frac{\epsilon(\theta)}{\epsilon(\theta)+1}\cdot\frac{f(\theta)}{(\Lambda(\theta)-F(\theta))\omega(p_m;\theta)}}.$$

Combining the previous expression with equation D.9, we obtain

$$\frac{t_m^*}{1+t_m^*}=\frac{\int\frac{\epsilon(\theta)}{\epsilon(\theta)+1}\cdot\frac{d\ln\omega(\theta)}{d\ln|y_m|}\cdot\tau^*(\theta)\cdot x(\theta)dF(\theta)}{p_m|y_m|[1-\int\frac{\epsilon(\theta)}{\epsilon(\theta)+1}\cdot\frac{d\ln\omega(\theta)}{d\ln|y_m|}\cdot\tau^*(\theta)\cdot\frac{y_m(\theta)}{|y_m|}dF(\theta)]} \tag{D.11}$$

In the parametric example of Section 5.2, we have assumed

$$\epsilon(\theta)=\epsilon \text{ for all } \theta, \tag{D.12}$$

$$\Lambda(\theta)=1 \text{ for all } \theta, \tag{D.13}$$

$$f(\theta)=1 \text{ for all } \theta, \tag{D.14}$$

$$F(\theta)=\theta \text{ for all } \theta. \tag{D.15}$$

We therefore immediately get

$$\tau^*(\theta,p_m)=\frac{1}{1+\frac{\epsilon}{\epsilon+1}\cdot\frac{1}{(1-\theta)\omega(p_m;\theta)}}. \tag{D.16}$$

In Section 5.2, we have also established that

$$w(p_m;\theta)=(1-\theta)^{-1/\gamma(p_m)},$$

which implies

$$\omega(p_m;\theta)=\frac{1}{\gamma(p_m)}\cdot\frac{1}{1-\theta}.$$

Substituting into equation (D.16), we therefore get

$$\tau^*(\theta)=\frac{1}{1+\frac{\epsilon}{\epsilon+1}\gamma(p_m)}\equiv\tau^*. \tag{D.17}$$

In Section 5.2, we have also established that

$$\frac{d\ln\omega(p_m;\theta)}{d\ln p_m} = -\beta\gamma(p_m),$$

which implies

$$\frac{d\ln\omega(\theta)}{d\ln|y_m|} = -\beta\gamma(p_m)\frac{d\ln p_m}{d\ln|y_m(p_m,n)|} \equiv \frac{d\ln\omega}{d\ln|y_m|}. \tag{D.18}$$

Combining equations (D.11), ((D.12)), (D.17) and (D.18), we obtain

$$\frac{t_m^*}{1+t_m^*} = \frac{\frac{\epsilon}{\epsilon+1}\frac{d\ln\omega}{d\ln|y_m|}\tau^*}{1-\frac{\epsilon}{\epsilon+1}\frac{d\ln\omega}{d\ln|y_m|}\tau^*(\theta,p_m)}\frac{\int x(\theta)dF(\theta)}{p_m|y_m|}, \tag{D.19}$$

Letting $s_m \equiv \frac{p_m|y_m|}{\int x(\theta)dF(\theta)+p_m|y_m|}$, this leads to equation ((9)).

Proposition 3. From equation ((9)), we know that

$$\frac{t_m^*}{1+t_m^*} = \frac{\frac{\epsilon}{\epsilon+1}\frac{d\ln\omega}{d\ln|y_m|}\tau^*}{1-\frac{\epsilon}{\epsilon+1}\frac{d\ln\omega}{d\ln|y_m|}\tau^*}\frac{1-s_m}{s_m}$$

with

$$\begin{aligned}
\frac{d\ln\omega}{d\ln|y_m|} &= -\beta\gamma(p_m)\frac{d\ln p_m}{d\ln|y_m(p_m,n)|},\\
\tau^* &= \frac{1}{1+\frac{\epsilon}{\epsilon+1}\gamma(p_m)},\\
s_m &= \frac{p_m|y_m|}{\int x(\theta)dF(\theta)+p_m|y_m|},
\end{aligned}$$

This expression can be rearranged as

$$\frac{t_m^*}{1+t_m^*} = \frac{\Phi}{\rho-\Phi}\frac{1-s_m}{s_m} \tag{D.20}$$

with

$$\begin{aligned}
\Phi &= -\frac{\epsilon\beta\gamma(p_m)}{(\epsilon+1)+\epsilon\gamma(p_m)},\\
\rho &= \frac{\partial\ln|y_m(p_m,n)|}{\partial\ln p_m}.
\end{aligned} \tag{D.21}$$

We first demonstrate that Φ, s_m, and ρ can be expressed as functions of t_m^* and ϕ.

Using the fact that $p_m = (1 + t_m^*)/\phi$, we can immediately rearrange equation (D.21) as

$$\Phi = -\frac{\epsilon\beta\gamma((1+t_m^*)/\phi)}{(\epsilon+1)+\epsilon\gamma((1+t_m^*)/\phi)} \equiv \Phi(t_m^*, \phi). \tag{D.22}$$

To express s_m and ρ as a function of t_m^* and ϕ, we further need to solve for the optimal labor supply of each agent, $n(\theta)$, which itself depends on the marginal income tax rates, $\tau(\theta)$. Together with equations (D.12)-(D.15), equation (D.10) implies

$$\tau(\theta) = \frac{\epsilon + 1 - \frac{t_m^*}{1+t_m^*}\frac{p_m y_m(\theta)}{x(\theta)}\gamma(p_m)}{\epsilon + 1 + \epsilon\gamma(p_m)}.$$

From the first-order condition of the old technology firms, we know that

$$\frac{p_m y_m(\theta)}{x(\theta)} = -\beta(\theta)\ln(1-\theta), \tag{D.23}$$

which leads to

$$\tau(\theta) = \frac{\epsilon + 1 + \frac{t_m^*}{1+t_m^*}\beta\gamma(p_m)\ln(1-\theta)}{\epsilon + 1 + \epsilon\gamma(p_m)}. \tag{D.24}$$

The optimal labor supply is given by the agent's first-order condition

$$n(\theta) = ((1-\tau(\theta))w(\theta))^{\epsilon}. \tag{D.25}$$

Combining equations (D.24) and (D.25) with the fact that $w(p_m;\theta) = (1-\theta)^{-1/\gamma(p_m)}$, we get

$$n(\theta) = (\frac{\gamma(p_m)}{\epsilon+1+\epsilon\gamma(p_m)})^{\epsilon}(\epsilon - \frac{t_m^*}{1+t_m^*}\beta\ln(1-\theta))^{\epsilon}(1-\theta)^{-\epsilon/\gamma(p_m)},$$

and in turn,

$$\int w(\theta)n(\theta)d\theta = (\frac{\gamma(p_m)}{\epsilon+1+\epsilon\gamma(p_m)})^{\epsilon}\int(\epsilon - \frac{t_m^*}{1+t_m^*}\beta\ln(1-\theta))^{\epsilon}\theta^{-\frac{1+\epsilon}{\gamma(p_m)}}d\theta$$

Using equation (D.23), we further get

$$p_m y_m(p_m, n(\theta);\theta) = -\beta\ln(1-\theta)(\frac{\gamma(p_m)}{\epsilon+1+\epsilon\gamma(p_m)})^{\epsilon}(\epsilon - \frac{t_m^*}{1+t_m^*}\beta\ln(1-\theta))^{\epsilon}(1-\theta)^{-\frac{1+\epsilon}{\gamma(p_m)}}, \tag{D.26}$$

and in turn,

$$p_m|y(p_m,n)| = -\beta(\frac{\gamma(p_m)}{\epsilon+1+\epsilon\gamma(p_m)})^{\epsilon}\int\ln(1-\theta)(\epsilon - \frac{t_m^*}{1+t_m^*}\beta\ln(1-\theta))^{\epsilon}(1-\theta)^{-\frac{1+\epsilon}{\gamma(p_m)}}d\theta. \tag{D.27}$$

The aggregate share of robots is therefore given by

$$s_m = \frac{\int \beta \ln(1-\theta)(\epsilon - \frac{t_m^*}{1+t_m^*}\beta \ln(1-\theta))^{\epsilon}(1-\theta)^{-\frac{1+\epsilon}{\gamma((1+t_m^*)/\phi)}} d\theta}{\int (\beta \ln(1-\theta) - 1)(\epsilon - \frac{t_m^*}{1+t_m^*}\beta \ln(1-\theta))^{\epsilon}(1-\theta)^{-\frac{1+\epsilon}{\gamma((1+t_m^*)/\phi)}} d\theta} \equiv s_m(t_m^*, \phi), \tag{D.28}$$

where we have again used $p_m = (1+t_m^*)/\phi$. The elasticity ρ can be computed in a similar manner. From equation (D.23) and the fact that $w(p_m;\theta) = (1-\theta)^{-1/\gamma(p_m)}$, we get

$$p_m y(p_m, n(\theta); \theta) = -\beta \ln(1-\theta) n(\theta)(1-\theta)^{-1/\gamma(p_m)}.$$

Using the previous expression with the definition of $\rho \equiv \frac{\partial \ln |y_m(p_m,n)|}{\partial \ln p_m}$, we get

$$\rho = \int \frac{y(p_m, n(\theta); \theta)}{|y_m(p_m, n)|} \frac{d \ln w(p_m;\theta)}{d \ln p_m} d\theta - 1.$$

Combining the previous expressions with equations (D.26), (D.27), and using the fact that $\frac{d \ln w(p_m;\theta)}{d \ln p_m} = -\beta \ln(1-\theta)$ and $p_m = (1+t_m^*)/\phi$, we get

$$\rho = \frac{\int (\beta \ln(1-\theta) - 1) \ln(1-\theta)(\epsilon - \frac{t_m^*}{1+t_m^*}\beta \ln(1-\theta))^{\epsilon}(1-\theta)^{-\frac{1+\epsilon}{\gamma((1+t_m^*)/\phi)}} d\theta}{\int \ln(1-\theta)(\epsilon - \frac{t_m^*}{1+t_m^*}\beta \ln(1-\theta))^{\epsilon}(1-\theta)^{-\frac{1+\epsilon}{\gamma((1+t_m^*)/\phi)}} d\theta} \equiv \rho(t_m^*, \phi). \tag{D.29}$$

At this point, we have established that the three statistics in equation (D.20) can be expressed as $\Phi(t_m^*, \phi)$, $\rho(t_m^*, \phi)$, and $s_m(t_m^*, \phi)$. We can therefore rearrange equation (D.20) as

$$H(t_m^*, \Phi(t_m^*, \phi), \rho(t_m^*, \phi), s_m(t_m^*, \phi)) = 0,$$

with

$$H(t_m^*, \Phi, \rho, s_r) \equiv \frac{\Phi}{\rho - \Phi} \cdot \frac{1 - s_m}{s_m} - \frac{t_m^*}{1+t_m^*}.$$

By the Implicit Function Theorem, we have

$$\frac{dt_m^*}{d\phi} = -\frac{dH/d\phi}{dH/dt_m^*}. \tag{D.30}$$

Since the tax on robots is chosen to maximize welfare, the second derivative of the government's value function, expressed as a function of t_m^* only, must be negative. Noting that H corresponds to its first derivative—which is equal to zero at the optimal tax—we therefore obtain

$$dH/dt_m^* < 0. \tag{D.31}$$

Since $\gamma(\cdot)$ is a strictly increasing function, equation (D.22) implies

$$\frac{\partial \Phi(t_m^*, \phi)}{\partial \phi} > 0. \tag{D.32}$$

To establish the monotonicity of s_m and ρ with respect to ϕ, it is convenient to introduce the following function:

$$d(t_m^*, \phi, \zeta; \theta) = (\epsilon - \beta \frac{t_m^*}{1+t_m^*} \ln(1-\theta))^{\epsilon} (1-\theta)^{-\frac{1+\epsilon}{\gamma((1+t_m^*)/\phi)}} (\ln(1-\theta))^{-\zeta}.$$

By construction, d is log-supermodular in (ϕ, ζ, θ). Since log-supermodularity is preserved by integration, the following function,

$$D(\phi, \zeta) = \int d(t_m^*, \phi, \zeta; \theta) d\theta,$$

is also log-supermodular. It follows that

$$\frac{D(\phi, \zeta = 0)}{D(\phi, \zeta = -1)} = \frac{\int (\epsilon - \beta \frac{t_m^*}{1+t_m^*} \ln(1-\theta))^{\epsilon} (1-\theta)^{-\frac{1+\epsilon}{\gamma((1+t_m^*)/\phi)}} d\theta}{\int (\ln(1-\theta))(\epsilon - \beta \frac{t_r^*}{1+t_r^*} \ln(1-\theta))^{\epsilon} (1-\theta)^{-\frac{1+\epsilon}{\gamma((1+t_m^*)/\phi)}} d\theta} \text{ is increasing in } \phi,$$
$$\frac{D(\phi, \zeta = -2)}{D(\phi, \zeta = -1)} = \frac{\int (\ln(1-\theta))^2 (\epsilon - \beta \frac{t_m^*}{1+t_m^*} \ln(1-\theta))^{\epsilon} (1-\theta)^{-\frac{1+\epsilon}{\gamma((1+t_m^*)/\phi)}} d\theta}{\int (\ln(1-\theta))(\epsilon - \beta \frac{t_r^*}{1+t_m^*} \ln(1-\theta))^{\epsilon} (1-\theta)^{-\frac{1+\epsilon}{\gamma((1+t_m^*)/\phi)}} d\theta} \text{ is decreasing in } \phi.$$

Noting that

$$s_m = \frac{1}{1 - \frac{1}{\beta} \frac{D(\phi, \zeta=0)}{D(\phi, \zeta=-1)}},$$
$$\rho = \beta \frac{D(\phi, \zeta = -2)}{D(\phi, \zeta = -1)} - 1,$$

we obtain that

$$\frac{\partial s_m(t_m^*, \phi)}{\partial \phi} > 0, \tag{D.33}$$
$$\frac{\partial \rho(t_m^*, \phi)}{\partial \phi} < 0. \tag{D.34}$$

Since $\frac{\partial H}{\partial \Phi} < 0$, $\frac{\partial H}{\partial s_m} < 0$, and $\frac{\partial H}{\partial \rho} > 0$, inequalities (D.32)-(D.34) imply

$$\frac{dH}{d\phi} = \frac{\partial H}{\partial \Phi} \frac{\partial \Phi}{\partial \phi} + \frac{\partial H}{\partial s_m} \frac{\partial s_m}{\partial \phi} + \frac{\partial H}{\partial \rho} \frac{\partial \rho}{\partial \phi} < 0.$$

Combining this observation with equation (D.30) and (D.31), we conclude that $dt_m^*/d\phi > 0$.