# Structuring Mortgages for Macroeconomic Stability* 

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#### Abstract

We study mortgage design features aimed at stabilizing the macroeconomy. Using a calibrated life-cycle model with competitive risk-averse lenders, we consider an adjustablerate mortgage (ARM) with an option that during recessions allows borrowers to pay only interest on their loan and extend its maturity. We find that this option has several advantages: it stabilizes consumption growth over the business cycle, shifts defaults to expansions, and lowers the equilibrium mortgage rate by stabilizing cash flows to lenders. These advantages are magnified in a low and stable real interest rate environment where the standard ARM delivers less budget relief in a recession.


[^0]
## 1 Introduction

Events in the last decade have shown that adjustable-rate mortgages (ARMs) have advantages over fixed-rate mortgages (FRMs) in stabilizing the economy, at least when the central bank has monetary independence and can lower the short-term interest rate in a recession (Eberly and Krishnamurthy 2014). A lower short rate provides automatic budget relief for ARM borrowers and helps to support their spending. It can also provide some relief to FRM borrowers, but this requires both a decline in the long-term mortgage rate and refinancing, which may be constrained by declining house prices and tightening credit standards. Barriers to FRM refinancing in the aftermath of the Great Recession were an important concern of US policymakers and motivated the introduction of the Home Affordable Refinance Program (HARP) (Di Maggio et al. 2016). ${ }^{1}$

We argue that the stabilizing properties of plain-vanilla ARMs can be enhanced by adding an interest-only option that applies only during recessions, and that allows borrowers to extend loan maturity. This option is included in the contract ex-ante and is available to all borrowers. During a recession, any borrower who decides to take advantage of the option pays only loan interest, with principal loan repayments restarting after the recession ends, and with loan maturity extended. This proposal provides additional budget relief to distressed borrowers. It is particularly valuable in a low-interest-rate environment where the short rate is already low during economic expansion, so that the zero lower bound constrains the ability of the central bank to lower the interest rate at the onset of a recession.

We use a quantitative model to evaluate our proposal and to compare it with other mortgage designs considered in the recent literature (Eberly and Krishnamurthy 2014, Guren, Krishnamurthy, and McQuade 2017). Our model has several important features. First, the demand for loans comes from households who use them to purchase houses or to refinance existing mortgages. Their earnings are subject to both economy-wide and individual specific shocks, as in Guvenen, Ozkan, and Song (2014). Recessions are characterized by lower expected earnings growth and a higher likelihood of a large drop in earnings. The probability of a house price

[^1]drop is also higher during bad times. Therefore, recessions are riskier, and so are loans granted at such times.

Second, loans are provided by competitive lenders. Lenders are risk-averse, so they value cash flows received in recessions more than those received in expansions. This, combined with the fact that loans granted in recessions are riskier, leads to higher loan premia in bad times. We solve for equilibrium loan premia, taking account of the fact that higher mortgage rates increase defaults, dampening the direct effect of higher rates on lender profitability as in the classic literature on credit rationing (Stiglitz and Weiss 1981).

There is evidence that credit standards are looser in expansions than in recessions (for example Keys et al. 2010 and Corbae and Quintin 2015). ${ }^{2}$ With this evidence in mind, and to ensure that lenders break even on a risk-adjusted basis, a third feature of our model is that mortgage lending criteria are tighter in bad times. Specifically, the maximum loan-to-value ratio declines during recessions, which constrains borrowers' ability to refinance during bad times.

It is equally important for our model to capture what happens in the years prior to a recession. During the boom years of the mid-2000s, high levels of mortgage cash-out refinancing increased household leverage at the onset of the financial crisis (Khandani, Lo, and Merton 2013, Chen, Michaux and Roussanov 2013). A fourth feature of our model is that it allows cash-out refinancing. In each period agents can prepay their existing loan and take out a new loan with a higher principal value, subject to current loan-to-value constraints. These debt market dynamics affect the benefits to borrowers of an option to extend loan maturity, and the impact that the option has on lender cash flows.

Finally, our model abstracts from inflation dynamics. To economize on state variables, we consider a real economy in which either all mortgages are inflation-indexed, or the price level is constant. While this is an obvious limitation of our analysis, we believe our results are empirically relevant given the limited variability in realized and expected inflation over the last 15 years.

Our model delivers the following results about an interest-only option added to a standard adjustable-rate mortgage. Not all borrowers exercise the option. Some borrowers keep on

[^2]making loan principal repayments during bad times, motivated by precautionary motives and a desire to deleverage. However, because some borrowers do exercise the option, it leads to a smaller drop in consumption and a lower mortgage default rate during recessions. Those individuals who exercise the option reach the end of the recession with higher debt levels than would otherwise be the case, leading to an increase in defaults during expansions. However, the option increases ex-ante borrower welfare for the same level of risk-adjusted profits of lenders.

We also use our model to evaluate FRMs. We solve for the mortgage premia in recessions and expansions that keep the net present value of the risk-adjusted cash-flows of lenders constant over the economic cycle and equal across mortgage contracts. We find that FRMs are less effective than ARMs in stabilizing the economy during bad times. Because our model abstracts from inflation uncertainty, borrowers are actually better off with a FRM than with a plainvanilla ARM, because they benefit from stable real mortgage payments. However, the ARM with an interest-only option is preferred to either of the standard mortgage contracts.

Another contract design we consider is a FRM with an option to switch to an ARM during recessions, as proposed by Eberly and Krishnamurthy (2014) and Guren, Krishnamurthy, and McQuade (2017). In our model the switching option does have a stabilizing effect in the economy during bad times. However, borrowers switch when interest rates are low during recessions, reducing payments to risk-averse lenders at times when their consumption is already low. Lenders need to be compensated for this ex-ante, in the form of a higher mortgage rate which makes the option to switch expensive and hence relatively unattractive for borrowers.

The attractiveness of our interest-only option is even greater in a version of our model parameterized to a low-interest-rate environment. In such an environment, plain-vanilla ARMs are less effective in stabilizing the economy, but the interest-only option restores stabilization, while increasing borrower welfare and delivering the same risk-adjusted cash flows to lenders.

While we use US data to parameterize our model, the effects that we model and our proposal are relevant for other countries as well. For instance, Figure 1 plots mortgage rates for the UK over time for two different mortgage types and LTVs, $75 \%$ and $95 \%$. When the crisis hit, the official Bank of England rate declined, but mortgage premia increased and high LTV loans disappeared from the market. These dynamics are captured through our modelling of supply side. Figure 1 also shows that UK interest rates have remained low for a number of years after the crisis.

Although our work is relevant for policymakers trying to stabilize the economy, we stop short of building a general equilibrium model with endogenous income or house prices. Instead we look at the response of consumption and defaults to given income and house price shocks. These are the first-round effects that might in general equilibrium feed back into income and house prices, creating a downward spiral in the worst case. In the literature, Guren et al. (2017) go further and endogenize house prices, but they also do not endogenize income. Beraja et al. (2017) also treat both house prices and income as exogenous, and furthermore they ignore default. We think that there is value in measuring first-round effects because these will be robust to errors in modeling the general equilibrium. An alternative approach is that of Piskorski and Tchistyi (2017) who solve a general equilibrium model in closed form that shows how mortgage markets interact with housing and labor market risks. Their model is designed to provide intuition on the equilibrium channels, but is not suitable for quantitative analysis. ${ }^{3}$

We can also motivate our study as applied to contract design. For a variety of contracts, we calculate ex-ante borrower welfare while imposing a feasibility constraint that risk-adjusted lender profits must be sufficiently high. This is similar to the focus of Piskorski and Tchistyi (2010). One of our contributions is to risk-adjust lender profits in this exercise, rather than using the shortcut of assuming that lenders are risk-neutral. In addition, we consider a stochastic equilibrium in which shocks occur repeatedly and both borrowers and lenders correctly anticipate the occurrence of these shocks, rather than looking at unanticipated one-time shocks.

The paper is structured as follows. In section 2 we present the model and its parameterization. Section 3 discusses the results. Section 4 presents robustness exercises, and section 5 concludes.

## 2 The Model

In each period $t$ a new set of agents enters our economy and stays in it for $T$ periods. Therefore our economy has an overlapping generations structure. Our model economy is real and

[^3]stationary. Even though agents face expected positive real income growth during the periods in which they are in our economy, in each period a new cohort of agents is born and an old cohort drops out. The agents that are born face the same (initial) level of house prices relative to income. One possible interpretation is that there are common long-term trends in real house prices and aggregate income, which we abstract from, to focus on cyclical fluctuations. In addition, our model captures the behavior of the group of individuals who use a mortgage loan to buy a house. And even though we solve endogenously for some equilibrium prices (the loan premium), in several other dimensions our model is partial equilibrium. We make these modeling choices so as to be able to model with more realism several of the features of the mortgage contracts that are the focus of our paper.

### 2.1 Baseline model setup

### 2.1.1 Aggregate state

In each period $t$ the economy may be in either an expansion or a recession. An indicator variable $I_{t}$ equals one in an expansion, and zero otherwise. An exogenous transition probability matrix governs the evolution between these states. Persistence in the aggregate state of the economy is captured by the parameterization of this matrix.

The risk-free real interest rate is also exogenous, but stochastic and correlated with the business cycle. Let $r_{1 t}=\log \left(1+R_{1 t}\right)$ denote the $\log$ real rate, the $\log$ of the gross real return on a default-free one-period bond held from time $t$ to time $t+1$. In each period the log real rate is either high or low, with probabilities that depend on whether the economy is in an expansion or recession. We write the unconditional mean and standard deviation of $r_{1 t}$ as $\mu_{r}$ and $\sigma_{r}$, respectively.

We model house price variation in a similar fashion. The change in the log real price of housing, $\Delta p_{t}^{H}$, is either high or low with probabilities that depend on the state of the economy. We write the unconditional mean and standard deviation of $\Delta p_{t}^{H}$ as $\mu_{H}$ and $\sigma_{H}$, respectively. We set $\mu_{H}$ equal to zero, but since house price increases are more likely to occur if the economy is in an expansion, and there is persistence in the business cycle, the conditional expectation of house price changes is higher during an expansion than during a recession.

### 2.1.2 Demand for mortgage loans

The demand for mortgage loans comes from overlapping generations of agents entering the economy, as well as from existing agents refinancing their mortgages. All agents entering the economy are initially identical, with identical wealth and permanent income, but they subsequently experience idiosyncratic labor income shocks that imply cross-sectional heterogeneity increasing with age.

## Initial home purchase

At the time that agent $i$ initially enters the economy (denoted $t_{i}$ ) he or she buys a house of size $H_{i, t_{i}}$ using a downpayment financed from an initial wealth endowment $W_{i, t_{i}}$ and a mortgage loan with maturity $T$. The house size that the agent buys depends on the prevailing level of house prices $\left(P_{t_{i}}^{H}\right)$ and aggregate income $\left(Y_{t_{i}}\right)$ at time of entry.

We let $d_{i, t_{i}}$ denote the downpayment as a proportion of the house value. It is indexed by $t_{i}$ to allow for the possibility that it depends on the state of the economy. The loan amount the agent takes, $K_{i, t_{i}}$, is given by:

$$
\begin{equation*}
K_{i, t_{i}}=\left(1-d_{i, t_{i}}\right) P_{t_{i}}^{H} H_{i, t_{i}} . \tag{1}
\end{equation*}
$$

A higher proportional downpayment implies that agents use more of their previously accumulated savings to buy the house and therefore take on a loan with a lower loan-to-value (LTV) ratio $K_{i, t_{i}} /\left(P_{t_{i}}^{H} H_{i, t_{i}}\right)=\left(1-d_{i, t_{i}}\right)$.

To ensure stationarity of the model we assume that initial wealth and loan size vary in proportion to the level of aggregate income. We also assume that loan size normalized by aggregate income is invariant to the level of house prices. That is, for a given downpayment, when agents enter the economy they always take out a mortgage with the same initial loan-to-income (LTI) ratio $K_{i}=K_{i, t_{i}} / Y_{t_{i}}$. This requires that agents who enter the economy after a period of house price increases (decreases) buy a smaller (larger) house. This assumption simplifies the model solution since, in combination with the assumptions we make on preferences, it implies that we do not need to keep track of the level of house prices at the time of a home purchase.

Finally, we further simplify the model by assuming that the downpayment ratio and hence
the LTV ratio vary exogenously with the state of the economy, but that the LTI ratio $K_{i}$ does not change from expansions to recessions. The sizes of new houses purchased vary with the business cycle to make this possible. This assumption ensures that the sizes of new mortgages are constant over time in relation to aggregate income, which again economizes on state variables when we solve our model.

## Preferences

As in Campbell and Cocco (2015) we assume preference separability between housing and non-housing consumption and that house size remains fixed throughout the time during which agent $i$ is in our economy. Under these assumptions we can drop housing from the preference specification. Our agents choose non-durable consumption and manage their mortgages to maximize

$$
\begin{equation*}
\mathrm{E}_{t_{i}} \sum_{t=t_{i}}^{t_{i}+T} \beta_{i}^{t-t_{i}} \frac{C_{i t}^{1-\gamma_{i}}}{1-\gamma_{i}}+\beta_{i}^{T} b_{i} \frac{W_{i, t_{i}+T+1}^{1-\gamma_{i}}}{1-\gamma_{i}}, \tag{2}
\end{equation*}
$$

where $W_{i, t_{i}+T+1}$ denotes terminal wealth that includes both financial and housing wealth. If agents have positive outstanding debt at the terminal date, we calculate terminal wealth net of the debt outstanding. The parameter $b_{i}$ measures the relative importance of utility derived from terminal wealth. It controls the incentives of individuals to accumulate longer-term savings. These preferences give rise to a precautionary savings motive with relative prudence equal to $\gamma_{i}+1$.

## Labor income

In each period agents' labor income $\left(Y_{i t}\right)$ evolves according to the process estimated by Guvenen, Ozkan and Song (2014). Recessions are characterized by a smaller probability of a large increase in labor income and an increased probability of a large drop in labor income. As usual, we use a lower case letter to denote the natural log of the variable, so that $y_{i t} \equiv$ $\log \left(Y_{i t}\right)$. Log real labor income is the sum of a transitory $\left(\epsilon_{i t}\right)$ and a persistent $\left(z_{i t}\right)$ component. Innovations to the persistent component feature a mixture of normals:

$$
\begin{equation*}
y_{i t}=z_{i t}+\epsilon_{i t} \tag{3}
\end{equation*}
$$

$$
\begin{equation*}
z_{i t}=\rho z_{i, t-1}+\eta_{i t} \tag{4}
\end{equation*}
$$

where $\epsilon_{i t} \sim \mathcal{N}\left(0, \sigma_{\epsilon}\right)$ and:

$$
\eta_{i t}= \begin{cases}\eta_{i t}^{1} \sim \mathcal{N}\left(\mu_{1, I_{t}}, \sigma_{1}\right), & \text { with probability } p_{1}  \tag{5}\\ \eta_{i t}^{2} \sim \mathcal{N}\left(\mu_{2, I_{t}}, \sigma_{2}\right), & \text { with probability } 1-p_{1}\end{cases}
$$

where recall the subscript $I_{t}$ indicates whether period $t$ is an expansion or a recession. This setup allows us to capture important deviations of labor income growth from normality, including negative skewness and excess kurtosis, and business cycle variation in expected labor income growth through the different means of the normal distributions. The higher probability of a large drop in labor income in recessions is likely to affect borrowers' incentives to default on mortgage loans.

We model the tax code in the simplest possible way, by considering a linear taxation rule. Gross labor income and interest earned are taxed at the constant tax rate $\phi$.

### 2.1.3 Terms of mortgage loans

We study two types of mortgage contracts that differ in the interest rate risk that agents face, adjustable-rate mortgages (ARMs) and fixed-rate mortgages (FRMs). Since our model abstracts from inflation risk, the fixed-rate mortgages we model are implicitly inflation-indexed and not the nominal contracts observed in reality.

## Adjustable-rate mortgages

The interest rate on ARMs is the short-term interest rate plus a mortgage premium $\psi_{i, t_{i}}^{A R M}$ :

$$
\begin{equation*}
R_{i t}^{A R M}=R_{1 t}+\psi_{i, t_{i}}^{A R M} \tag{6}
\end{equation*}
$$

The mortgage premium compensates lenders for prepayment and for default risk. The subscripts $i$ and $t_{i}$ allow for the possibility that the premium depends on borrower characteristics and on the aggregate state of the economy at the time that the loan begins. The loan premium remains fixed over the life of the loan, but the loan rate fluctuates with the level of short rates.

The period $t$ payment due on the mortgage taken by agent $i$ is given by:

$$
\begin{equation*}
L_{i t}^{A R M}=R_{i t}^{A R M} D_{i t}+\Delta D_{i, t+1} \tag{7}
\end{equation*}
$$

where $D_{i t}$ is the principal amount outstanding on the loan at the beginning of period $t$ before any mortgage payments are made in that period and $\Delta D_{i, t+1}$ is the loan principal repayment due in period $t$. To economize on state variables we assume that in each period the principal reduction is the same that would occur in a fixed-rate loan with an exogenously specified mortgage rate. This allows us to link principal outstanding to the loan period.

## Fixed-rate mortgages

The interest rate on FRMs is fixed over the life of the loan. It is equal to the long-term bond rate at the time that the loan begins plus a mortgage premium $\psi_{i, t_{i}}^{F R M}$ :

$$
\begin{equation*}
R_{i}^{F R M}=R_{T, t_{i}}+\psi_{i, t_{i}}^{F R M} . \tag{8}
\end{equation*}
$$

To model long-term bond rates we assume that the log expectations hypothesis of the term structure holds, so that expected log returns on bonds of all maturities are equal. By specifying the expectations hypothesis in logs, we ensure that it is consistent across all holding periods and allow for long bonds to have somewhat higher simple average returns resulting from their greater return volatility.

## Refinancing, default, and prepayment options

We model three options that borrowers have in mortgage contracts: to refinance, to default, and to prepay. The option to refinance the loan, i.e. to prepay the existing loan and simultaneously take out a new one, has a monetary cost of $\theta_{R}$, but it allows agents to extract additional cash from their accumulated home equity. Agents choose how much additional equity to extract subject to the downpayment constraint prevailing at the time that the refinancing takes place. We maintain the function that maps loan amount to maturity for non-refinanced mortgages: therefore, if agents refinance to the initial loan amount the new loan has maturity $T$, but we allow agents to refinance to larger or smaller loan amounts than the initial one, with longer or shorter maturities accordingly.

Borrowers also have an option to default on the loan. In case of default they lose the house in foreclosure, are excluded from credit markets, and become renters for the remaining time
horizon. In addition default carries a utility penalty in the period that the agent defaults equal to $\lambda$ which can be interpreted as a social stigma cost (Guiso, Sapienza, and Zingales 2013). Mortgage loans are non-recourse, so lenders have no claim on labor income in the event of default. We model a lower bound on consumption which can be interpreted as arising from social security benefits or other transfers, and which ensures that borrowers' decisions are not dominated by extremely unlikely states with extremely high marginal utility of consumption.

Finally, borrowers with positive home equity have the option to sell their house, prepay their loan, and become renters for the remaining time horizon. We assume that the house rented is the same size as the one previously owned. The rental cost is equal to the user cost of housing plus a rental premium of $\varepsilon$. We follow Campbell and Cocco (2015) and define the date $t$ rental cost $R C_{i t}$ for a house of size $H_{i, t_{i}}$ as:

$$
\begin{equation*}
R C_{i t}=\left[R_{1 t}-E_{t}\left[\exp \left(\Delta p_{t+1}^{H}\right)-1\right]+\tau_{p}+m_{p}+\varepsilon\right] P_{t}^{H} H_{i, t_{i}} \tag{9}
\end{equation*}
$$

where $R_{1 t}$ is the one-period real-rate, $E_{t}\left[\exp \left(\Delta p_{t+1}^{H}-1\right]\right.$ is the expected real house price change from period $t$ to period $t+1$, and $\tau_{p}$ and $m_{p}$ are the property tax rate and the proportional housing maintenance cost respectively.

### 2.1.4 Supply of mortgage loans

Lenders originate loans at the initial date when agents enter the economy, and in later periods when there is refinancing. In periods subsequent to loan origination, lenders receive the mortgage payments, unless borrowers decide to default or to refinance. In case of default, lenders take possession of the house and sell it in the same period at current prices, net of transaction costs. In case of refinancing, lenders receive the balance outstanding on the current mortgage and write a new mortgage contract with a new principal amount.

The loan premium compensates lenders for default, prepayment, and the costs of originating and servicing loans. It depends on the type of loan and on the state of the economy at the time that the loan is originated, to capture the fact that expected default and prepayment behavior is different for loans originated under different economic conditions.

We also model the possibility that lenders use tighter lending criteria in recessions, by allowing the maximum LTV (denoted $L T V^{\max }$ ) for loans originated in bad times to be lower
than that for loans originated in good times. This makes it more difficult for agents to refinance their loans and extract home equity during recessions.

We assume a competitive market for the supply of loans and solve for the loan premia such that lenders achieve a given level of risk-adjusted net present value of loan cash-flows (denoted $\pi$ ). To capture lenders' risk aversion, or the fact that also for lenders cash-flows received in recessions are more valuable than those received in expansions, we use our model to derive an exogenous pricing kernel. More specifically, we solve our model for agents who own their houses outright (without mortgage debt) and who have large financial assets. The motivation is that these are the permanent income consumers who through lenders provide loans to mortgage borrowers. The marginal utility of these long-horizon agents is the relevant metric for pricing cash flows received in different states of the world. For this group of permanent income individuals we calculate aggregate consumption, denoted $C_{t}^{a}$, by averaging individual consumption. Finally, we calculate the implied discount rates for the different time $t$ aggregate states of the world (recession or expansion with a low or high real interest rate) using:

$$
\begin{equation*}
\mathrm{E}_{t} \beta\left(\frac{C_{t+1}^{a}}{C_{t}^{a}}\right)^{-\gamma} \tag{10}
\end{equation*}
$$

We use these risk-adjusted discount rates to calculate the present value of lenders' cashflows. We give details in the parameterization section. Naturally, the risk adjusted discount factors contribute to the debt dynamics generated by the model and play a role in the benefits to agents and costs to lenders of the option to extend loan maturity.

### 2.1.5 Equilibrium mortgage premia

Equilibrium mortgage premia in our model could in principle depend on all the state variables. But given the large number of state variables, this would make the problem intractable. Therefore, for most of the cases considered we assume that mortgage premia depend only on the loan type and on the state of the business cycle at loan origination.

The calculation of equilibrium mortgage premia requires that we solve for a fixed point. The refinancing and default choices of borrowers depend on the loan premium, so we need to iterate on candidate loan premia until lenders achieve the break-even level of profitability given borrowers' choices. When LTV ratios are high, it is possible that no fixed point exists as in

Stiglitz and Weiss (1981); this is one reason why we impose maximum LTV ratios exogenously. Furthermore, because borrowers may refinance their loans in either expansions or recessions, the cash flows of lenders and the loan premium for loans initiated in a recession depend on the loan premium for loans initiated in an expansion (and vice versa). Therefore we solve simultaneously for the recession and expansion premia such that the expected present discounted value of lenders' cash flows is equated across the two aggregate states.

### 2.2 Model timing and solution

### 2.2.1 Timing, choice and state variables

The timing of the problem is such that at the beginning of each period $t$ the state of the economy $\left(I_{t}\right)$, interest rates $\left(R_{1 t}\right)$, house prices $\left(P_{t}^{H}\right)$ and labor income $\left(Y_{i t}\right)$ are realized. We define cash-on-hand in period $t$ as the sum of wealth and realized income: $X_{i t} \equiv W_{i t}+Y_{i t}$. The remaining state variables of the problem are the level of permanent income $Z_{i t}$, the level of debt outstanding/loan period $K_{i t}$, the loan premia $\psi_{i t}$ (equivalently, the state of the economy when the agent's mortgage was originated), and whether the agent has previously moved to the rental market $I_{i t}^{R}$. We denote the time $t$ state space for agent $i$ by $\Omega_{t i} \equiv\left\{I_{t}, R_{1 t}, P_{t}^{H}, X_{i t}, Z_{i t}, K_{i t}, \psi_{i t}, I_{i t}^{R}\right\}$.

The level of debt, $K_{i t}$, and loan premia, $\psi_{i t}$, pin down home equity and the mortgage payments due (for the ARM). For the FRM loan we also need to keep track of the level of interest rates at mortgage initiation since it determines the loan rate and required mortgage payments (thus the set $\Omega_{t i}$ has one additional state variable). For both contracts, loan premia are endogenously determined at origination and remain unchanged until loan termination.

After the realization of the random variables is observed, borrowers decide whether to make the scheduled mortgage payments, refinance, default, or prepay the loan. If they refinance, borrowers need to decide the new loan amount, subject to the prevailing downpayment constraint. In addition, they decide in each period their consumption of non-durable goods. The problem is simpler for borrowers who have previously defaulted, and need only choose how much to consume and save in each period.

We set up the problem recursively and define two distinct value functions: $V$ is the value of repaying the loan or refinancing and $V^{R}$ is the value of moving to the rental market (either through default or through mortgage prepayment). If the agent has a loan outstanding the

Bellman equation is given by:

$$
\begin{equation*}
V_{i t}\left(\Omega_{t i}\right)=\max \left\{U\left(C_{i t}\right)+\beta E_{t} \max \left[V_{i, t+1}(\cdot), V_{i, t+1}^{R}(\cdot)\right]\right\}, \tag{11}
\end{equation*}
$$

where $V^{R}$ denotes the value obtained from moving to the rental market. The Bellman equation for an agent in the rental market is given by:

$$
\begin{equation*}
\left.V_{i t}^{R}\left(X_{i, t}, Z_{i t}, I_{t}, R_{1 t}, P_{t}^{H}\right)=\max \left\{U\left(C_{i t}\right)+\beta E_{t} V_{i, t+1}^{R}(\cdot)\right]\right\} \tag{12}
\end{equation*}
$$

In periods when the agent does not move to the rental market and does not refinance his or her loan, cash-on-hand evolves according to:

$$
\begin{align*}
X_{i, t+1}= & {\left[X_{i t}-C_{i t}-L_{i t}^{\text {LoanType }}-P C_{t}+\phi R_{i t}^{\text {LoanType }} D_{i, t-1}\right]\left(1+(1-\phi) R_{1 t}\right) }  \tag{13}\\
& +(1-\phi) Y_{i, t+1},
\end{align*}
$$

where LoanType $\in\{F R M, A R M\}$. Cash-on-hand in period $t+1$ is equal to cash-on-hand in period $t$, minus consumption $\left(C_{i t}\right)$, mortgage payments $\left(L_{i t}^{\text {LoanType }}\right.$ ), property maintenance and tax costs $\left(P C_{t}\right)$, plus the interest tax shield, the interest on savings and realized labor income (net of income taxes). ${ }^{4}$

If the agent decides to tap into home equity through loan refinancing, he or she must choose a new loan amount $K_{i t}^{\prime}$, prepay the outstanding amount of the old loan $\left(D_{i t}\right)$, and pay a refinancing cost of $\theta_{R}$. In such a situation cash-on-hand evolves according to:

$$
\begin{align*}
X_{i, t+1}= & {\left[X_{i t}-C_{i t}-\left(1+R_{i t}^{\text {LoanType }}\right) D_{i t}+\phi R_{i t}^{\text {LoanType }} D_{i, t-1}+K_{i t}^{\prime}-P C_{t}-\theta_{R}\right] } \\
& \left(1+(1-\phi) R_{1 t}\right)+(1-\phi) Y_{i, t+1} . \tag{14}
\end{align*}
$$

The choice of the new loan amount $\left(K_{i t}^{\prime}\right)$ is subject to a LTV constraint such that $K_{i t}^{\prime} \leq$ $L T V_{t}^{\max } P_{t}^{H} H_{i, t_{i}}$, where we allow $L T V^{\max }$ to depend on the aggregate business cycle $I_{t}$.

If the agent has positive home equity, he or she can decide to move to the rental market. Such a decision happens at the beginning of the period. In such a case the agent receives the

[^4]net proceeds from selling the house (net of transaction costs $\theta_{c}$ ) minus the outstanding loan amount (which is prepaid). The law of motion for cash-on-hand is:
\[

$$
\begin{align*}
X_{i, t+1}= & {\left[X_{i t}-C_{i t}+\left(1-\theta_{c}\right) P_{t}^{H} H_{i, t_{i}}-\left(1+R_{i t}^{\text {LoanType }}\right) D_{i t}\right.} \\
& \left.+\phi R_{i t}^{\text {LoanType }} D_{i, t-1}-R C_{i t}\right]\left(1+(1-\phi) R_{1 t}\right)+(1-\phi) Y_{i, t+1}, \tag{15}
\end{align*}
$$
\]

i.e. agents receive the net proceeds from selling the house but need to start paying the rental cost $R C_{i t}$. Finally, cash-on-hand for agents already in the rental market or for agents who default is given by:

$$
\begin{equation*}
X_{i, t+1}=\left[X_{i t}-C_{i t}-R C_{i t}\right]\left(1+(1-\phi) R_{1 t}\right)+(1-\phi) Y_{i, t+1} \tag{16}
\end{equation*}
$$

### 2.2.2 Numerical solution

We solve the agents' problem by backwards induction for given values for the loan premium and for given maximum LTV. We give details on the numerical solution methodology in the appendix. We use the optimal policy functions, four hundred different paths for the aggregate variables (recession/expansion, house prices and interest rates), and the realizations of individual earnings to generate simulated data, over a forty-year period. In each period a new set of agents enters the economy and stays in it for twenty years. We discard the first twenty periods as burn-in and report the statistics for the last twenty periods of our simulated economy. This ensures that in each period a new set of agents enters our economy at the same time that a set of agents drops out from our sample. For each aggregate state and at each point in time there are 550 agents in our data (i.e. 25 agents for each age cohort). We use these data to calculate the net present value of lenders' cash flows. We iterate simultaneously on the recession and expansion loan premium until we find the fixed point at which lenders' achieve the target profitability, given agents' optimal choices. We generate simulated data for the different experiments that we carry out, but the realizations for the random variables are the same throughout so that different experiments are comparable.

### 2.3 Alternative mortgage structures

We study two alternatives to the standard mortgages described so far. First, we augment ARM loans with an option that gives borrowers the choice to pay only interest and to extend loan maturity during recessions. We allow all borrowers, including those with negative home equity, to take advantage of this option if they wish to do so. There is no monetary cost of exercising the option. If borrowers extend maturity, debt service temporarily comprises only interest, with principal repayments restarting the following period and loan maturity extended by one period. For multi-year recessions, borrowers choose whether to exercise the option in each of the recession years.

In this extended model there is an additional choice variable in recession years, the agents' decision of whether to extend maturity. The model solution does not require an additional state variable. Lender cash flows for the periods in which borrowers exercise the option include only interest so that

$$
\begin{equation*}
C F_{i t}^{\text {lender }}=D_{i t} R_{i t}^{A R M} \tag{17}
\end{equation*}
$$

In this way the option to extend loan maturity provides cash-flow relief to borrowers during bad times.

A second alternative mortgage structure we study combines the FRM loan with an option that allows borrowers to costlessly switch during recessions to an ARM loan with the same level of principal outstanding. Since interest rates are more likely to be low during bad times, the switch to an ARM allows borrowers to take advantage of low rates to reduce their required mortgage payments.

Both these changes to mortgage structure have an impact on lenders' cash flows that we take into account by solving for the level of loan premia so that the net present value of risk-adjusted cash flows is the same as in the base model.

### 2.4 Parameterization

We use several data sources and estimates from papers in the literature to parameterize our model. Table 1 summarizes our parameter choices.

### 2.4.1 Aggregate state variables

To parameterize the transition probability matrix between expansions and recessions we use NBER business cycle dates. The conditional transition probabilities reported in panel A of Table 1 capture the persistence in the aggregate state of the economy. Expansions are more persistent than recessions: the probability that an expansion continues from one quarter to the next is 0.82 , while the probability that a recession continues is only 0.37 .

Panel B of Table 1 summarizes our parameter choices for real interest rates. We calculate the expected real interest rate using quarterly data on 1-year nominal Treasury bond yields and on expected inflation from the Michigan survey from 1977Q4 to 2014Q3. Over the whole sample period the real interest rate was on average higher in recessions than in expansions: $1.59 \%$ compared to $2.44 \%$, respectively. However, this was driven mainly by the recessions of the early 1980s. If one focuses on the period after 1985 the average real interest rate was on average higher in expansions than in recessions: $1.12 \%$ compared to $0.04 \%$, respectively. The unconditional mean over this period was $1 \%$ and the standard deviation was $2.5 \%$.

In our model the real interest rate can either be low or high. We set the unconditional probabilities of low and high rates to be equal, so for a mean of $1 \%$ and a standard deviation of $2.5 \%$ the two possible values for the real rate are $-1.5 \%$ and $3.6 \%$. We adjust the conditional probabilities of low and high rates to match the post- 1985 means during expansion and recession, which implies a 0.48 probability of a low rate in an expansion, and a 0.62 probability in a recession. This real interest rate process inherits the persistence of the business cycle variable.

Panel C of Table 1 reports our parameter choices for house prices. We match the unconditional mean and standard deviation of log house price changes from Campbell and Cocco (2015). To parameterize the relation between house price changes and the aggregate state of the economy we use Case-Shiller house price data. In our model house prices can either increase or decrease by $16.2 \%$ each period. We calculate the conditional probabilities of house price declines in expansion and recession to match the average house price increase of $1 \%$ in an expansion and decline of $3 \%$ in a recession observed in the S\&P/Case-Shiller 20-city composite Home Price Index data. During an expansion, the probability of a house price increase is 0.52 , whereas this probability is only 0.39 in a recession.

### 2.4.2 Preference parameters and the labor income process

Panel D of Table 1 reports preference parameters. We set the subjective time discount factor to 0.98 , the coefficient of relative risk aversion to 2 , and the bequest parameter $b$ so that agents in our model accumulate financial savings at a rate similar to that observed in the data. More precisely we target a terminal value for financial wealth that roughly matches the average level of $\$ 20,400$ observed for individuals aged between 35 and 44 in the 2013 wave of the Survey of Consumer Finances (SCF).

We take earnings process parameters from Guvenen et. al (2014), and report them in panel E of Table 1. We assume a flat income tax rate of $20 \%$, with mortgage interest tax deductible at this rate.

### 2.4.3 Mortgage and housing parameters

Panel F of Table 1 reports the parameters we use to model the mortgage and housing markets. In the base model the initial loan-to-income (LTI) ratio is constant at 3.5. The maximum LTV is 0.9 for loans initiated in expansions and 0.8 for loans initiated in recessions. These constraints restrict agents' ability to refinance their loans. We also set downpayments at the time that agents enter the economy equal to the minimum values of 0.1 and 0.2 implied by the maximum LTVs. Mortgages have an initial maturity of 20 years.

We solve endogenously for mortgage premia but we need a base value for lender profitability $\pi$ that compensates lenders for their unmodeled costs. We have used monthly data on ARM effective rates from the Federal Housing Finance Agency, covering the period 1986-2008. From these effective rates we subtract the one-year bond yield and calculate the average mortgage premium during NBER recession months to be $3 \%$. We use this value to determine $\pi$ and solve all other loan premia to generate the same profitability.

We assume that as long as a mortgage is outstanding, lenders must pay a $0.25 \%$ per year servicing cost. This parameter helps to determine the profitability of mortgages that remain outstanding relative to those that prepay or default. There are no charges to borrowers for prepaying mortgages, but refinancing incurs a $\$ 1000$ fixed cost. There is a $6 \%$ commission paid on house value when a mortgage is prepaid and the house is sold, and an equivalent $6 \%$ loss to lenders in the event of default.

Beyond the financial implications of default, we assume that default creates disutility for borrowers through a "stigma" effect. In the base case we set the value for the stigma parameter $\lambda$ to 0.1 so that the average mortgage default rates generated by the model match those in the data. A high value for stigma reduces the incentives for individuals to default for strategic reasons, so that when default happens it is more likely to occur when borrowers have low income. As shown by Guiso, Sapienza, and Zingales (2013) the importance of moral and strategic considerations for the default decision differs across individuals, so we plan to solve our model for alternative values for stigma.

Finally, our model has three housing parameters: property taxes at $1.5 \%$ of value per year, maintenance expenses at $2.5 \%$ of value per year (both taken from Campbell and Cocco 2015), and a rental premium of $1 \%$.

### 2.4.4 Pricing kernel

In order to discount lenders' cashflows taking into account differences in marginal utility between different states of the world we solve the model for agents with the same preferences as our mortgage borrowers, but unlike them, these agents own a house outright without mortgage debt and have accumulated financial savings in excess of $\$ 100,000$. The idea is that these are the agents who would ultimately be providing loans (through financial intermediaries) to those buying a house. We use these agents' consumption choices to calculate their aggregate consumption and implied discount rates for the four different states of the world at time $t$ (recession or expansion with a low or high real interest rate) using equation (10) above.

Panel A of Table 2 reports the resulting risk-adjusted discount rates in the third column. These discount rates imply an unrealistically high unconditional average riskfree rate, shown at the bottom of the panel. Therefore we retain their dispersion but adjust their mean so that the implied average risk-free rate matches the average risk-free rate we assumed in Table 1. These discount rates that we use to discount lender cash-flows are shown in the last column of Table 2. They imply a higher cost of capital for lenders during recessions relative to expansions.

### 2.4.5 Low-interest-rate environment

The advantages of ARMs over FRMs in stabilizing the economy may be reduced in a low-interest-rate environment. If the short-term interest rate is already low at the start of a recession, the ability of the central bank to lower it further may be limited. In order to explore this issue, we solve our model for a low-interest-rate environment, with the short rate parameterized to the post-2000 period. The mean real interest rate is lower and equal to -0.01 , less variable with a standard deviation of 0.0196, and there are no significant differences in its level between expansions and recessions. Therefore, in this environment the risk-free rate is uncorrelated with the business cycle unlike the assumption of procyclicality that we made in our base case.

When we solve our model for this low-interest-rate environment we recalculate the riskadjusted discount rates. We report their values in Panel B of Table 2. However, we do not take into account any effects that the low-interest-rate environment may have on house prices and on initial loan values. It would be straightforward to let initial LTIs be higher in a low-interest-rate environment.

## 3 Model Results

In section 3.1 we describe the results for a baseline ARM contract. In section 3.2 we evaluate the impact of augmenting the ARM with an option to extend maturity during a recession. In section 3.3 we compare this with a permanent option to extend maturity regardless of business cycle conditions. In section 3.4 we solve our model for the FRM contract, and in section 3.5 we evaluate the effects of allowing FRM borrowers to switch costlessly to an ARM in recessions. In section 3.6 we study a low-interest-rate environment. Section 3.7 briefly summarizes the comparison of all these mortgage contracts.

### 3.1 Baseline ARM

Table 3 reports model results for ARMs in the base case, without an option to extend loan maturity. The top of the table shows key results for the economy as a whole, while the lower parts of the table report results for subgroups of agents who default on their mortgages, refinance them, or make required mortgage payments. (Prepayments are relatively rare and are omitted
from the table for simplicity.)
The first column of the table reports unconditional results and the second and third columns report results conditional on recession and expansion, respectively. The first row of the table shows that the $3.0 \%$ premium for loans originated in recessions is significantly higher than the $1.3 \%$ premium for loans originated in expansions, which are safer because of the cyclical variation in idiosyncratic income risk. The second row of the table reports the average log consumption growth that results from mortgage borrowers' optimal choices, and for comparison the third row reports the exogenous average income growth. Due to borrowers' leverage and fluctuations in collateral value, the difference in consumption growth between recessions and expansions is larger than the difference in income growth. When a recession hits and income drops, levered agents are forced to cut consumption proportionally more than income to meet their mortgage payments.

In order to provide insights into the determinants of default, refinancing, and regular mortgage payment, the remainder of the table reports summary statistics for these outcomes. We start by showing their incidence. Default is relatively rare but more common in a recession $(1.9 \%)$ than in an expansion ( $1.2 \%$ ). Refinancing is rare in a recession ( $1.4 \%$ ) but common in an expansion (11.5\%). Regular mortgage payments are made by $95.3 \%$ of agents in recessions, and $86.3 \%$ in expansions. The remaining agents, $1.4 \%$ in a recession and $1.0 \%$ in an expansion, prepay their mortgages and move to rental housing.

The remainder of the table reports summary statistics for the three groups of defaulters, refinancers, and mortgage payers. Defaulters and refinancers have significantly lower average labor income and have recently experienced declines in labor income. Defaulters have much higher LTI ratios and ratios of mortgage payments to income, and their LTV ratios are above one, while refinancers have low LTVs before refinancing that remain relatively moderate after refinancing. Related to this, defaulters have experienced declining house prices while refinancers have experienced recent increases in their house prices.

There are several reasons why mortgage refinancing is procyclical in our model. Borrowers refinance when they have positive home equity after an increase in house prices. This is more likely to be the case in an expansion than in a recession. Furthermore, expected income growth is higher and income risk lower in expansions than in recessions: agents with precautionary savings motives respond to this by levering more aggressively. Finally, mortgage rates tend
to be lower in expansions due to the lower mortgage premium on loans initiated at such times (which more than offsets the effect of a higher short-term interest rate). Borrowers who refinance extract home equity ("cash out"): in expansions, average LTVs increase at refinancing from 0.65 to 0.80 .

Procyclical cash-out refinancing increases household leverage at the time that a recession hits. The recession then causes larger declines in consumption, more defaults, and greater losses given default for lenders. This type of borrower behavior has been previously studied by Khandani, Lo, and Merton (2013), who call it the "ratchet effect" on leverage, and Chen, Michaux, and Roussanov (2013).

We have modelled a lower maximum LTV for loans originated in recessions than for loans originated in expansions. This was motivated by the observation that in reality one does observe a tightening of lending criteria during bad times. But we have also tried to solve our model for the same value for the LTV constraint in recessions and in expansions, equal to 0.90 . We could not find a fixed point for the mortgage premia that delivered the same net present value of the risk-adjusted profits of lenders in recessions and in expansions. The economic reason is simple. When we increased the mortgage premium for loans originated in recessions, the probability of default and the losses to lenders increased, counteracting the positive effects of the higher loan premium on lenders' cash flows. This result is in the spirit of Stigliz and Weiss (1981).

### 3.2 ARM with an interest-only option in recessions

We evaluate the effects of combining the ARM with an option that allows agents to pay only interest and extend loan maturity in recessions. This option cannot be replicated through mortgage refinancing for two reasons. First, the option to extend maturity is available to all borrowers, including those with low or negative home equity. In other words, we assume that the leverage constraint does not apply in the case of maturity extension. Second, unlike for mortgage refinancing for which there is a loan origination cost, we assume that there is no monetary cost associated with the exercise of the option to extend maturity.

Table 4 reports results for the ARM with an interest-only option during recessions, using the same format as Table 3. The first row shows that equilibrium mortgage premia are smaller than in the base case, $2.7 \%$ rather than $3.0 \%$ for loans originated in recessions and $1.2 \%$ rather
than $1.3 \%$ for loans originated in expansions. There are two main reasons for these declines: first, a lower probability of default during bad times when discount rates are high; and second, a lower probability of loan termination through mortgage refinancing, which contributes to a longer expected duration of loans allowing lenders to earn the mortgage premium for more periods. These effects are visible in the lower portion of Table 4, and they imply that lower mortgage premia are required to deliver the same net present value of risk-adjusted cash flows to lenders.

The second row of Table 4 reports average log consumption growth. Comparing to the second row of Table 3 we see that now there is a smaller decline in consumption during recessions of $1.0 \%$ compared to a previous decline of $1.6 \%$. Average log consumption growth in expansions is lower than before at $5.5 \%$ rather than $5.9 \%$. The option to extend loan maturity allows agents to defer payments in recessions and in this way to better smooth consumption over the business cycle. The unconditional mean of consumption growth declines from $4.2 \%$ in the base case to $4.0 \%$ due to a reduced precautionary savings motive. This implies that agents' consumption is higher when they enter the economy and they are able to better smooth consumption over the life cycle.

In order to obtain a summary measure of the benefits to mortgage borrowers, we compare their utility with and without the option to extend maturity and calculate consumptionequivalent variations. The welfare benefits of the interest-only option are equivalent to $1.3 \%$ of annual consumption. This is reported in the first row of Table 5. We will return to the remaining rows of this table once we describe other types of mortgage contracts.

The lower part of Table 4 reports summary statistics for four different mortgage decisions: default, refinance, make mortgage payments, and extend loan maturity. The statistics on incidence show that the option to extend maturity reduces the default rate in recessions, from $1.9 \%$ in the base case to $0.5 \%$. However, some of the agents who exercise the option to extend maturity end up defaulting when the recession ends and they need to start making principal repayments once again. Therefore the default rate in expansions increases from $1.2 \%$ to $1.6 \%$. Overall (across expansions and recessions) there is no significant change in default rates relative to the base case. The option to extend loan maturity shifts defaults from recessions to expansions, which may have benefits in stabilizing the macroeconomy, rather than reducing the unconditional average default rate.

Comparing the characteristics of defaulters to those in Table 3 for the baseline ARM, we see that default now takes place at higher LTI and particularly LTV ratios. The increases in these ratios are significantly larger in recessions than in expansions. The increases in LTI and LTV for agents who default in expansions are in part explained by the fact that those who exercise the option to extend maturity reach the end of a recession with higher leverage than they would have had if the option to extend was not available and they had made the scheduled loan principal repayments.

Table 4 also reports the characteristics of the borrowers who decide to extend maturity. Borrowers use the option to extend loan maturity on $63.9 \%$ of the occasions when it is available. Recall that the option is only available in recessions, hence the " $n / a$ " in the expansion column. The average LTV ratio for extended loans is 1.0 , so many of the borrowers who extend maturity would not be able to refinance the loan due to the leverage constraint. Borrowers who decide to extend maturity have lower than average income, but still higher than those who decide to default. They have recently experienced moderate earnings declines.

To further characterize the consumption stabilization provided by the interest-only option in recessions, we calculate the average changes in consumption during recessions for different movements of the risk-free rate. The motivation is simple: monetary authorities may be able to provide cash-flow relief to ARM borrowers by reducing interest rates during recessions.

The results are shown in Table 7. The first row reports the results for the base case ARM. The first column reports the average log consumption change when interest rates are low in the period before the recession and they stay low during the recession, the second column when they are high before the recession and they are reduced in the recession period, and so on. The results confirm the notion that a decline in interest rates during recessions helps ARM borrowers and stimulates their consumption, with a model-implied aggregate increase of $1.4 \%$. However, this requires that monetary authorities do have the possibility of reducing rates when the recession hits. If rates are already low and stay low in a recession, the consumption of ARM borrowers declines by $0.6 \%$. In the second row of Table 7 we report similar statistics for ARMs combined with an interest-only option to extend maturity during recessions. The policy option has a sizeable effect on aggregate consumption, reducing the consumption decline when rates are already low at the onset of a recession to only $0.1 \%$. We will return to the remaining rows of this table when we discuss FRMs and the low-interest-rate environment.

### 3.2.1 Decisions after option exercise

The option to extend maturity can be used by agents to obtain cash-flow relief during bad times. But it can also be used strategically. Individuals in a situation of negative home equity can use the option to postpone loan principal repayments and bet on future house price increases. If price increases fail to materialize and house prices decline further, agents will then default in the following period. The deferral of principal repayments means that individuals reach the later period with larger outstanding loan balances than if the option to defer was not available, which increases further the incentives to default and the losses to lenders in case of default.

To illustrate more clearly the strategic use of the option to extend maturity, in Table 6 we report the loan decisions of agents in the period after they have exercised the option. The table answers the question, conditional on having exercised the option to extend the loan maturity at time $t$, what do agents do at time $t+1$ ? Do they go back to making their regular mortgage payments, do they use the option to extend the loan again (in case of a recession), do they default, refinance, or prepay the loan? The table reports the percentage of agents who makes each of these decisions conditional on the business cycle variable at $t+1$ and whether house prices decreased or increased between times $t$ and $t+1$ (Panel A), or whether agents' income decreased or increased between times $t$ and $t+1$ (Panels B).

This table has several interesting results. First, agents who have used the policy at $t$ are very likely to use it again at $t+1$ if the economy is still in a recession. The probability of this happening varies between 0.80 and 0.90 . Second, the majority of the individuals who have used the policy at $t$ go back to making the regular mortgage payments at time $t+1$ in case the economy moves into an expansion. There are however significant quantitative differences depending on the realization of house price changes.

Panel A shows that if the economy goes back to an expansion at $t+1$ around $9.3 \%$ of borrowers decide to default if house prices decrease compared to a $1.4 \%$ default rate if house prices increase. Panel B reports probabilities conditional on the realized income change. If the economy goes back to an expansion at time $t+1,6.3 \%$ of agents decide to default if their income declines compared to $4.3 \%$ if their income increases. These default sensitivities reflect the strategic and cash-flow default motives of agents in our model, which also affect their exercise of the maturity extension option.

### 3.3 ARM with a permanent interest-only option

Before discussing results for FRMs and a low-interest-rate environment it is useful to consider the implications of giving agents the option to extend maturity both in expansions and in recessions. Panel A of Table 8 reports results for this case. If the option to extend maturity is always available, the default rate in recessions jumps to $1.5 \%$ compared to $0.5 \%$ when the option to extend maturity is only available in recessions. The reason is straightforward: the availability of maturity extension in expansions leads borrowers to increase their leverage. When a recession arrives, more leveraged agents are more likely to default. However, the recession default rate is still lower than the $1.9 \%$ recession default rate of the the plain-vanilla ARM. The unconditional average loan premium for the permanent interest-only option is $1.4 \%$, which is lower than the unconditional average loan premium of $1.5 \%$ from the ARM contract with the option only in recessions. Loans with a permanent interest-only option are outstanding for longer periods and lenders benefit from the additional interest they receive.

The permanent option to extend mortgage maturity allows agents to better smooth consumption across the life-cycle. The unconditional average consumption growth rate with a permanent option is $3.8 \%$, compared to $4.0 \%$ when the option is only available in recessions, and $4.2 \%$ for the standard ARM contract. Agents increase their consumption early on in the life-cycle, which translates into fairly large welfare gains: $4.9 \%$ in consumption equivalent units, compared to the plain vanilla ARM.

This large welfare gain is partially due to the reduction in the refinancing costs incurred by agents. In our model, agents with positive home equity can access cash, but need to pay a fixed cost of refinancing. They want to do so more often in expansions. We interpret these refinancing costs as sunk costs arising from property valuation and the writing of contracts for the new mortgage, costs that do not exist when maturity is extended. When agents are given a permanent interest-only option, they increase leverage by exercising the option instead of by refinancing their loans: refinancing rates (not shown in the table) decrease from $9 \%$ to $5 \%$ unconditionally.

To investigate further the role of refinancing costs, we have solved the model allowing agents to always extend the maturity of their loans, but assuming that in expansions they incur a monetary cost equal to the refinancing cost (the option is still free in recessions). The results are reported in Panel B of Table 8. This scenario is almost equivalent to the scenario
where maturity extension is only available in recessions: default rates, consumption changes and welfare gains are almost the same as the ones reported in the previous section and in Tables 4 and 5. Loan premia in expansions increase slightly due to the fact that agents who pay the cost of exercising the option in expansions are riskier and the increase in leverage makes them more likely to default.

A natural question to ask is what happens if there is also a cost of exercising the option to extend maturity in recessions. Panel C of table 8 reports the results. In this scenario, the option becomes unattractive to borrowers. The probability of using the option in recession (not shown in the table) decreases from $63 \%$ to $8 \%$. The agents who benefited the most from the free interest-only option were cash-constrained agents who now have to pay a cost to use the policy. The postponing of principal repayment does not give them a sufficient incentive to incur the cost. Therefore in this scenario both default rates and consumption changes are similar to the benchmark ARM with no interest-only option, and the welfare gains of the costly option are close to zero. Finally, loan premia slightly increase compared to the contract with the free option as again the pool of borrowers who use the option and increase their leverage is now riskier.

### 3.4 Baseline FRM

We now solve our model for FRMs. Recall that since our model has zero inflation these loans should be interpreted as inflation-indexed FRMs and not the commonly observed nominal FRMs. We find the equilibrium mortgage premia so that the net present value of the riskadjusted profits of lenders is the same in recessions and expansions and equal to that of ARMs. We report these premia at the top of Table 9 , which has the same structure as Tables 3 and 4. To facilitate the comparison to ARMs, we report premia relative to the short rate. In our model there is a one-to-one mapping between short and long rates. The equilibrium loan premia are $3.5 \%$ in recessions and $1.1 \%$ in expansions. Recall that the corresponding premia for ARMs were $3.0 \%$ and $1.3 \%$, so that the premium for FRMs is higher in recessions and lower in expansions. The term structure tends to be upward sloping in recessions so that the FRM premium relative to the short rate tends to be higher at such times.

FRMs lead to larger consumption declines in recessions than ARMs: $1.9 \%$ compared to
$1.6 \%$. This occurs because interest rates tend to be low in recessions, but FRM borrowers with insufficient home equity to refinance are unable to take advantage of low interest rates and are forced to cut consumption by more. Details are provided in the third row of Table 7, where we report the aggregate consumption change of FRM borrowers in recessions conditional on the interest rate movement in the period. FRM consumption declines are greater than ARM consumption declines whenever interest rates are low during the recession (whether or not they were already low at the start of the recession).

FRMs generate higher default rates in recessions of $2.1 \%$, compared to $1.9 \%$ for ARMs. FRM borrowers in a situation of negative equity and unable to refinance cannot take advantage of the decline in interest rates that tends to occur during bad times. Looking at the characteristics of the FRM borrowers who decide to default, they tend to do so for slightly smaller declines in earnings and lower levels of mortgage payments to income than ARM borrowers. This reflects the fact that FRM borrowers are relatively less likely to default for cash-flow reasons than ARM borrowers (Campbell and Cocco 2015).

Average refinancing rates for FRM loans are greater than was the case for ARMs. To more easily compare the properties of different loans we assume that refinancing takes place to the same type of loan, i.e. FRM borrowers refinance to a FRM. FRM borrowers refinance to extract home equity, but also to take advantage of low interest rates. The short-term rates at the time of FRM refinancing are on average $0.8 \%$ in recessions and $1.0 \%$ in expansions. The corresponding values for ARMs in Table 3 were $1.4 \%$ and $2.0 \%$, respectively.

Despite the macroeconomic disadvantages of FRMs, in our model agents prefer them to plain-vanilla ARMs: the welfare gain of a FRM is equivalent to $0.4 \%$ of annual consumption (second row of Table 5). A fixed real rate eliminates the cash-flow risk of ARMs that agents in our model dislike. However, ARMs with an option to extend maturity generate even larger welfare benefits than FRMs, while also assisting macroeconomic stabilization.

### 3.5 FRM with an option to switch to ARM in a recession

Guren, Krishnamurthy, and McQuade (2017) emphasize the benefits for macroeconomic stability of switching borrowers from a FRM to an ARM when a recession hits. The switch allows agents to benefit from lower rates and mortgage payments. We investigate in the context of
our model the benefits of giving borrowers the option to switch to an ARM during bad times. We assume that such a switch is costless and that all borrowers, including those with negative home equity, are allowed to switch. When they do so, they switch to a plain vanilla ARM with the same principal outstanding.

Before we discuss our results it is important to note some differences between our setting and that of Guren et al (2017). We treat the switch as a borrower option and incorporate lenders' risk aversion in our analysis. In addition, we evaluate the properties of the option in a stochastic equilibrium in which shocks occur repeatedly in the manner that borrowers and lenders anticipate, rather than looking at its properties in a single scenario designed to match the housing downturn of the late 2000s. On the other hand, we do not solve for equilibrium in the housing market, so that we are only able to capture the first-round effects of mortgage design.

We solve for mortgage premia so that the net present value of the risk-adjusted cash-flows of lenders is the same as when the option to switch is not available. The results in Table 10 show that this leads to an increase in loan premia to $4.0 \%$ in recessions and $1.4 \%$ in expansions, compared to plain vanilla FRM premia of $3.5 \%$ and $1.1 \%$ respectively. FRM borrowers switch to an ARM during recessions when interest rates are at their lowest level. This imposes losses on lenders at times when their marginal utility is high. Lenders therefore need to be compensated ex-ante with a higher loan premium to be willing to give borrowers the option to switch.

The switching option does have a stabilizing effect in the economy during bad times. As Table 10 shows there is, in recessions, a smaller drop in consumption and a lower default rate compared to a plain-vanilla FRM. The average default rate in recessions is also lower than that generated by a plain-vanilla ARM. However, the ARM with an interest-only option has an even lower consumption decline during recessions. This is true regardless of the movements in interest rates that occur during the recession, as shown by Table 7.

Roughly $20 \%$ of borrowers decide to exercise the option to switch from a FRM to an ARM when it is available. Those who do so have average LTVs of 0.90 , so these are agents who are unable to refinance due to the LTV constraint. The exercise of the option allows them to lower mortgage payments to income from 0.27 to 0.15 , and in this way it provides cash-flow relief. The immediate cash-flow relief is however lower than the relief provided by the ARM with the maturity extension option (which allows agents to lower the ratio of mortgage payments to
income from 0.25 to 0.09 ).
The increase in loan premia required by lenders makes the option to switch to an ARM unattractively expensive for borrowers. The welfare gain of a FRM with an option to switch is $-0.2 \%$ relative to a plain-vanilla FRM. Relative to the plain-vanilla ARM, the FRM with the switching option delivers a welfare gain of $0.2 \%$ as reported in Table 5.

### 3.6 A low-interest-rate environment

Plain-vanilla ARMs do not help to stabilize the economy when the real interest rate is low before the recession and constrained by the zero lower bound during the recession. To quantify this and to investigate the extent to which the option to extend maturity helps, we solve our model for the low-interest-rate environment discussed in the parameterization section. When doing so we keep the loan amount and the initial LTI at 3.5, similar to the base case. We re-calculate the risk-adjusted discount rates that correspond to this interest rate scenario (reported in Panel B of Table 2), and solve for the mortgage premia such that the expected present discounted value of lender cash-flows remains unchanged.

The results for plain-vanilla ARMs are shown in Table 11. As expected, due to lower average short rates, equilibrium loan premia are lower. However, the drop in consumption in bad times almost doubles to $-3.0 \%$, compared to $-1.6 \%$ in the base case. Interest rates are no more likely to fall in recessions than in expansions, and therefore do not provide cash-flow relief to borrowers, who are forced to cut consumption by more. The unconditional average consumption growth rate is smaller than in the base case, as the lower required mortgage payments of the low-interest-rate environment allow borrowers to consume more non-durable goods earlier on, and better smooth lifetime consumption. Agents also have a lower incentive to save (due to the lower returns on their savings), which further reduces average consumption growth.

In spite of the lower mortgage payments, default rates in the low-interest-rate environment are almost identical to those of the base case. The main reason is that lower average short rates imply a lower user cost of housing and rental prices, and so a lower cost of default. The characteristics of the borrowers who choose to default are in line with those in the base case. On the other hand, in a low-interest-rate environment borrowers are more aggressive in their refinancing behavior, particularly so in expansions: they refinance at slightly higher initial

LTVs and end up with even higher LTVs.

### 3.6.1 Interest-only option in a low-interest-rate environment

In a low-interest-rate environment the option to extend maturity is still effective at stabilizing the economy in bad times. The drop in consumption is $-2.2 \%$, smaller than the drop of $-3.0 \%$ when the option is not available (second row of Table 12). As in the base case, there is a large drop in default rates during recessions, but an increase in expansions, so that the overall average default rate remains unchanged.

In a low-interest-rate environment borrowers are more likely to exercise the option to extend, doing so on $70 \%$ of the occasions on which it is available. The vast majority of those who exercise the option would not have been able to refinance: the average LTV is 1.02 . However, there are some agents who substitute maturity extension for refinancing during recessions: the annual refinancing rate in a recession is $1.4 \%$ when the extension option is not available, but drops to $0.6 \%$ in the presence of the option.

Borrowers benefit more from the option to extend in the low-interest-rate environment than in the base case: the welfare gains are $1.4 \%$ of annual consumption compared to a previous gain of $1.3 \%$ (last row of Table 5). The welfare gains are higher than in the base case partly due to the higher cash-flow relief the option to extend maturity provides agents in relation to the base case. In the base case, when using the option, agents make an interest payment of $8.5 \%$ of income. In the low-interest-rate scenario, the payment is $1.3 \%$ of income. This compares with counterfactual payments if the option was not exercised of $25.0 \%$ and $17.2 \%$ respectively. Thus agents obtain much greater relief from their mortgage obligations in the low-interest-rate environment.

### 3.7 Summary comparison of mortgage contracts

We summarize our comparison of mortgage contracts in Figures 2 and 3. Figure 2 shows unconditional average loan premia and default rates on the left, and the cyclicality of loan premia and default rates (the difference between the levels of the variables in recessions and expansions) on the right. The blue bars refer to ARMs, the red bars to FRMs, and the green bars to ARMs in a low-interest-rate environment. The top part of the figure shows the lower
and less cyclical loan premia implied by a recession-contingent interest-only option for an ARM, and the higher and more cyclical loan premia implied by a recession-contingent option to switch a FRM to an ARM. The bottom part of the figure shows that average default rates are little affected by mortgage contract design, but defaults are much less procyclical when borrowers have ARMs with interest-only options. FRMs with switching options are less effective at stabilizing default rates over the business cycle.

Figure 3 shows unconditional average consumption growth at the top left and the cyclicality of consumption growth (the difference between consumption growth in expansions and recessions) at the top right. Average consumption growth is lower in a low-interest-rate environment, but relatively little affected by mortgage contract design. However, consumption growth is less procyclical when borrowers have ARMs with interest-only options. The stabilizing effects of such options persist even in a low-interest-rate environment where plain-vanilla ARMs do not perform well.

The bottom of Figure 3 reports consumption-equivalent welfare gains for alternative mortgage contracts, using a plain-vanilla ARM as the base case. The interest-only option generates substantial welfare gains in both the baseline environment and the low-interest-rate environment. While borrowers do prefer FRMs to plain-vanilla ARMs, they prefer ARMs with an interest-only option to all the alternative contracts we have considered.

## 4 Robustness

Table 13 presents a series of robustness exercises in which we vary key features of our environment and repeat the comparison between a plain-vanilla ARM and an ARM with a maturity extension option during recessions. For reference, Panel A of the table repeats the benchmark comparison that we have already discussed.

### 4.1 Inflation

We examine the effects on our analysis of a deterministic $2 \%$ inflation rate. Positive inflation generates a downward tilt in real mortgage payments, which are higher in the early years of a mortgage than in the later years. We solve for mortgage premia that make the present values of
lenders' cash-flows the same as in the case of zero inflation, in both recessions and expansions. Mortgage premia increase relative to the base case, and more so for loans initiated in recessions. Higher real mortgage payments in the early years increase the likelihood of loan termination through refinancing: the unconditional probability increases from $9.3 \%$ for the case of zero inflation to $9.6 \%$ for the $2 \%$ inflation scenario. On the other hand, the unconditional incidence of default deceases, although only slightly, from $1.3 \%$ for zero inflation to $1.2 \%$ for positive inflation. In the latter case, real mortgage payments are higher early on, when lenders are protected against the risk of default through the down payment. If and when house prices decline, real outstanding debt is lower which explains the reduction in default rates. Higher initial real mortgage payments are also the reason for the slightly higher unconditional consumption growth in the positive inflation economy.

We now analyze the option to extend loan maturity. When the option is exercised, borrowers make only the real interest payments due in that period, but must repay the remaining real loan balance from the following period onwards. In other words, the option involves zero real amortization but negative nominal amortization of the loan balance: when the option is exercised the nominal balance is increased by $2 \%$, with a corresponding increase in the remaining nominal mortgage payments, to compensate lenders for the effects of inflation on their cash flows. The results in Panel B of Table 13 show that in this scenario the maturity extension option is effective in stabilizing the economy: it leads to a shift in defaults from recessions to expansions and a much smaller drop in consumption during bad times. Borrowers benefit from the option, which yields a welfare gain of $1.3 \%$.

### 4.2 Higher average house price growth

In Panel C of Table 13 we evaluate the robustness of our results to a higher value of $1 \%$ rather than $0 \%$ for average log real house price growth. Unsurprisingly, positive house price growth increases the incidence of loan termination through refinancing and prepayment (as households tap into their home equity) and reduces default probabilities both in expansions and in recessions. However the effects of house price growth on default probabilities are smaller than one might have expected. The reason is that higher expected real house price growth reduces the user cost of housing, making the default option relatively more attractive.

The required loan premia are almost unchanged relative to the benchmark calibration.
The option to extend loan maturity is effective in stabilizing the economy with higher log real house price growth. It leads to a reduction in recession default rates from $1.7 \%$ to $0.8 \%$ and a smaller drop in consumption during bad times, of $-1.1 \%$ instead of $-1.7 \%$ when the option is not available. As before the option is welfare improving.

### 4.3 Zero rental premium

In the benchmark calibration we set the rental premium to $1 \%$, which could reflect the compensation required by property owners for moral-hazard costs associated with renting (for example, a rental property may incur higher maintenance costs compared to an owner-occupied unit). In Panel D of Table 13 we evaluate the effectiveness of the maturity extension option in an economy with a zero rental premium. The reduced rental premium reduces the cost of mortgage default, so borrowers default more often. This reduces lenders' profits compared to the economy in which the rental premium is zero.

We might try to increase loan premia so as to achieve the same level of profitability as in the benchmark economy, but that tends to increase default rates further. A second possibility would be to increase the stigma cost associated with default to counteract the effects of the lower rental premium. A third alternative is to let the target level of lender profits be different in the two economies, but adjust loan premia so that profitability is the same for all scenarios in the economy with a zero rental premium. This is the route that we follow. More precisely, we set the loan premium in a recession at $3.0 \%$, as in the benchmark economy. We calculate the corresponding present value of lenders' cash flows, and we use this value to calculate loan premia for the remaining cases (the plain-vanilla ARM in expansions and ARMs with maturity extension options in both recessions and expansions). The results are shown in Panel D of Table 13. As before, the maturity extension option is effective in stabilizing the economy in recessions.

### 4.4 Foreclosure discount

The final robustness exercise that we carry out is to assume that in the case of default there is a foreclosure discount of $26 \%$ in the value of property sold by lenders, as estimated by Campbell,

Giglio, and Pathak (2011). This naturally reduces lender profitability relative to the benchmark economy. We follow the same approach as for the case of a zero rental premium, fixing the loan premium in a recession to $3.0 \%$, calculating the corresponding present value of lenders' cash flows, using that as a target, and solving for the loan premia across alternative scenarios so as to achieve the same level of profitability.

Panel E of Table 13 shows the results. The option to extend maturity leads to a reduction in recession default rates from $1.8 \%$ to $0.5 \%$ and a smaller reduction in consumption in bad times of $1.0 \%$, compared to a reduction of $1.6 \%$ when the option to extend maturity is not available. One important point is that in this robustness exercise we have assumed that the foreclosure discount is the same in booms and in recessions. If we had assumed that it was larger in recessions, the maturity extension option would become an even more attractive policy.

## 5 Conclusion

We have used a quantitative dynamic model of borrower behavior to evaluate changes to the design of mortgage contracts aimed at increasing macroeconomic stability. In our model the demand for loans comes from borrowers who purchase a house using a mortgage that is a given multiple of income. After the initial period, borrowers decide in each period how much to consume and save, and whether to refinance to a new mortgage, to default, or to prepay their mortgage and move to rental housing. Mortgage loans are supplied by risk-averse lenders subject to LTV constraints. We have solved for a stochastic equilibrium where agents anticipate the occurrence of individual and aggregate shocks, but these shocks (to income, interest rates, and house prices) are exogenous in our model.

We have analyzed several changes to mortgage contract design. The two most important are a plain-vanilla ARM contract combined with an option to pay only interest and to extend the loan maturity in recessions, and a FRM contract combined with an option to switch to an ARM during recessions. The former has several advantages. Relative to a standard ARM, it stabilizes consumption growth over the business cycle, it shifts defaults to expansions, and it has a lower premium because cash flows to lenders are more stable and less cyclical. On the other hand, relative to a standard FRM, a FRM with an option to switch to an ARM modestly stabilizes consumption growth over the business cycle and modestly reduces defaults
in recessions, but it commands a higher premium because lenders lose payments in recessions.
We have proposed the introduction of a recession-contingent interest-only option in a plainvanilla ARM contract. However, we have not taken into account the evidence that borrowers are often slow to exercise options. Andersen et al. (2018) show this in an environment where FRMs can be refinanced regardless of income, credit score, or home equity. It has also been documented in the US for prequalified refinancing offers by Johnson, Meier, and Toubia (2015) and Keys, Pope, and Pope (2016). ${ }^{5}$ Earlier work on this sluggishness includes Schwartz (2006) and Campbell (2006). In our context the interest-only option is used by borrowers to lower their mortgage payments at times of financial distress, so inertia is likely to be a smaller issue here. However, in future work we plan to investigate the quantitative effects of including an inertia parameter (a cost of option exercise) in our model.

[^5]
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## A Numerical Solution

We take the loan premium and the form of mortgage contact as given and solve the agent's problem using backward induction. In the last period $(t=T)$ the agent simply decides whether or not to default on the outstanding debt. In case of no default the agent derives utility from terminal wealth (including housing wealth). We then iterate backwards. For every period $t$ prior to $T$, and for each combination of values for the state variables, we optimize over the choice variables using a discrete grid search. We use an equally spaced grid for cash-on-hand $\left(X_{t}\right)$ and for the log level of permanent income $\left(z_{t}\right)$. In order to improve accuracy and efficiency of the solution we let the boundaries for the grid $z_{t}$ to increase from period $t$ to $T$.

We use a single debt grid for all the loan types under analysis. This has the advantage of keeping the possible loan amounts fixed across all our experiments (i.e. agents face the same choices when deciding to refinance). The loan premia for each experiment are endogenously determined and interest payments are computed as the product of the interest rate of the contract times the principal outstanding. The density functions of the random variables were approximated using Gaussian quadrature. For points that do not lie on the state space grid, we evaluate the value function using a cubic spline interpolation for the cash-on-hand grid and a linear interpolation for the permanent income grid. We calculate the value function associated with all possible values for the control variables and optimize over these. In this way we obtain the optimal choices of consumption, refinancing to tap into home equity, mortgage prepayment, default, and when available whether to use a mortgage modification option. In the case of the option to switch from a FRM to an ARM we first solve the ARM problem and then the FRM problem. When solving the FRM problem at each combination of state variables we check the maximum between the FRM value function and the previously computed ARM value function and store the maximum among these and the corresponding choice of keeping the FRM contract or switching to an ARM.

We use the optimal choices of the agents to calculate the cash-flows of lenders. If their NPV per loan, conditional on the business cycle, is higher (lower) than the target profitability we decrease (increase) the loan premium and solve the agent's problem again. We repeat the process until we find a fixed point in which the present value of the risk-adjusted cash-flows of lenders is the same for each loan type under analysis.

Figure 1: Mortgage rates in the U.K.
This figure plots mortgage rates in the U.K. for two different types of mortgage and LTVs


-     - 2-year variable rate LTV = 95\% —— 2-year fixed rate LTV = 95\%
——Official Bank Rate ——2-year fixed rate LTV $=75 \%$
-     - 2-year variable rate LTV $=75 \%$

Figure 2: Loan premia and default rates across scenarios

This figure plots in a bar chart the loan premia (top two figures) and default rates (bottom two figures) for the scenarios under analysis. The left subplots show the unconditional averages and the subplots on the right show the cyclicality (the difference between the averages conditional on being in a recession or an expansion).


Figure 3: Consumption growth and welfare gains
This figure plots in a bar chart the consumption growth (top two figures) and welfare gains (bottom figure) for the scenarios under analysis. The left subplots show the unconditional averages. The right subplots show the cyclicality (the difference between the averages conditional on being in a recession or an expansion).

Average consumption growth


Cyclicality of consumption growth


## Welfare gains



Table 1: Baseline parameters.
This table reports the baseline model parameters.

| Description | Parameter | Value |
| :---: | :---: | :---: |
| Panel A: Business cycle transition probabilities |  |  |
| P (recession \| recession) |  | 0.37 |
| P (recession \| expansion) |  | 0.18 |
| Panel B: Real interest rate |  |  |
| Mean log real rate | $\mu_{r}$ | 0.01 |
| St. dev. of real rate | $\sigma_{r}$ | 0.025 |
| High value log real risk-free |  | 0.035 |
| Low value log real risk-free |  | -0.015 |
| P (high rate \| recession) |  | 0.38 |
| P (high rate \| expansion) |  | 0.52 |
| Panel C: House prices |  |  |
| Mean log house price change | $\mu_{H}$ | 0 |
| St dev log house price change | $\sigma_{e}$ | 0.162 |
| High log house price growth |  | 0.162 |
| Low log house price growth |  | -0.162 |
| P (increase in house prices \| recession) |  | 0.39 |
| P (increase in house prices \| expansion) |  | 0.52 |
| Panel D: Time and preference parameters |  |  |
| Subjective discount factor | $\beta$ | 0.98 |
| Risk aversion | $\gamma$ | 2 |
| Number of periods | $T$ | 20 |
| Utility of terminal wealth | $b$ | 10 |
| Panel E: Labor income process |  |  |
| Log permanent income AR(1) coefficient | $\rho$ | 0.979 |
| Prob. aggregate/idiosyncratic shock | $p_{1}$ | 0.49 |
| Mean log earnings growth expansion (1) | $\mu_{1 E}$ | 0.119 |
| Mean log earnings growth expansion (2) | $\mu_{2 E}$ | -0.026 |
| Mean log earnings growth recession (1) | $\mu_{1 R}$ | -0.102 |
| Mean log earnings growth recession (2) | $\mu_{2 R}$ | 0.094 |
| St. dev permanent income shock (1) | $\sigma_{1}$ | 0.325 |
| St. dev permanent income shock (2) | $\sigma_{2}$ | 0.001 |
| St. dev. temporary shock | $\sigma_{\epsilon}$ | 0.186 |
| Tax rate | $\phi$ | 20\% |

Table 1 - continued
Panel F: Loan and rental market parameters

| Initial loan to income | $l t i$ |  |  |
| :--- | :---: | :---: | :---: |
| Initial loan to value expansion (recession) | $l t v$ | $0.9(0.8)$ |  |
| Loan premium (ARM, recession) | $\psi^{A R M}$ | 0.03 |  |
| Servicing costs (as \% of loan outstanding) |  | 0.0025 |  |
| Loan maturity | $\tau$ | 20 years |  |
| Default utility penalty | $\lambda$ | 0.1 |  |
| Prepayment cost | $\theta_{P}$ | 0 |  |
| Refinancing cost | $\theta_{R}$ | $\$ 1000$ |  |
| House sale commission | $\theta_{c}$ | 0.06 |  |
| Property taxes | $\tau_{p}$ | 0.015 |  |
| Maintenance expenses | $m_{p}$ | 0.025 |  |
| Rental premium | $\varepsilon$ | 0.01 |  |

Table 2: Real risk-free rate and risk adjusted discount factors
In this table we report the risk-free rate, the risk-adjusted discount factors and mean preserving spread on the risk-adjusted discount factors. Panel A reports the values for the benchmark parameterization and panel B the values for the low interest rate environment parameterization. The first column of this table reports the values of the risk-free rate. The second columns reports the implied risk-adjusted discount factors for an agent with power utility a coefficient of relative risk aversion $\gamma$ equal to 2 , a rate of time preference $\beta$ equal to 0.98 and a bequest motive intensity $b$ equal to 10 and the income process specified in section 2.1.2. We use these preference parameters and the model predicted growth rates of average consumption to calculate the risk-adjusted discount rates. The last column reports a mean-preserving spread of the risk-adjusted discount factors.

Panel A: Benchmark environment

| Aggregate state |  | Risk-free |  | Risk-adjusted |
| :--- | :---: | :---: | :---: | :---: |

Panel B: Low interest rate environment

| Aggregate state | Risk-free | Risk-adjusted | Risk-adjusted mean preserving spread |
| :---: | :---: | :---: | :---: |
| Recession and low interest rate | -0.029 | 0.073 | 0.012 |
| Recession and high interest rate | 0.010 | 0.103 | 0.042 |
| Expansion and low interest rate | -0.029 | 0.027 | -0.034 |
| Expansion and high interest rate | 0.010 | 0.055 | -0.006 |
| Implied risk-free | -0.010 | 0.052 | -0.010 |

Table 3: Baseline ARM
This table reports the baseline model results for a plain-vanilla ARM contract. The data are obtained by simulating the model with the baseline parameters reported in Table 1. The first column reports unconditional moments and the second (third) column reports moments conditional on the economy being in a recession (expansion). The first three rows report the loan premia, average log consumption growth and log income growth. The remaining rows report key model moments conditional on agents' decisions (default, mortgage refinance and make mortgage payments).

|  | Unconditional | Recession | Expansion |
| :---: | :---: | :---: | :---: |
| Loan premia | 0.016 | 0.030 | 0.012 |
| Average log cons. growth | 0.042 | -0.016 | 0.059 |
| Log change in income | 0.027 | -0.009 | 0.038 |
| Incidence |  |  |  |
| Default | 0.013 | 0.019 | 0.012 |
| Refinance | 0.093 | 0.014 | 0.115 |
| Pay | 0.883 | 0.953 | 0.863 |
| Income |  |  |  |
| Default | 0.619 | 0.604 | 0.626 |
| Refinance | 0.970 | 1.094 | 0.966 |
| Pay | 2.052 | 1.894 | 2.102 |
| Log change in income |  |  |  |
| Default | -0.138 | -0.210 | -0.104 |
| Refinance | -0.047 | -0.166 | -0.043 |
| Pay | 0.035 | -0.004 | 0.048 |
| Risk-free interest rate |  |  |  |
| Default | 0.014 | 0.009 | 0.016 |
| Refinance | 0.020 | 0.014 | 0.020 |
| Pay | 0.009 | 0.004 | 0.010 |
| Loan to income |  |  |  |
| Default | 7.692 | 7.965 | 7.567 |
| Refinance | 5.071 | 4.842 | 5.079 |
| Pay | 2.964 | 3.202 | 2.889 |

Table 3-continued

| Mortgage payment to income |  |  |  |
| :---: | :---: | :---: | :---: |
| Default | 0.515 | 0.502 | 0.521 |
| Refinance | 0.368 | 0.399 | 0.367 |
| Pay | 0.236 | 0.237 | 0.236 |
| Loan to value |  |  |  |
| Default | 1.388 | 1.393 | 1.386 |
| Before refinancing | 0.647 | 0.435 | 0.654 |
| After refinancing | 0.790 | 0.524 | 0.799 |
| Pay | 0.801 | 0.826 | 0.793 |
| Delta HP |  |  |  |
| Default | -0.155 | -0.155 | -0.155 |
| Refinance | 0.101 | 0.068 | 0.102 |
| Pay | -0.009 | -0.032 | -0.001 |

Table 4: ARM contract with option to extend maturity in recessions
This table reports the model results for an ARM contract where agents have the option to extend the maturity of the contract. The loan premium is endogenously determined to keep lenders' net present value of cash-flows per loan equal to the baseline ARM with no option to extend the maturity. The first column reports unconditional moments and second (third) column reports moments conditional on the economy being in a recession (expansion). The first three rows report the loan premia, average log consumption growth and log income growth. The remaining rows report key model moments conditional on agents' decisions (default, mortgage refinance and make mortgage payments).

|  | Unconditional | Recession | Expansion |
| :---: | :---: | :---: | :---: |
| Loan premia | 0.015 | 0.026 | 0.012 |
| Average log cons. growth | 0.040 | -0.010 | 0.055 |
| Log change in income | 0.027 | -0.009 | 0.038 |
| Incidence |  |  |  |
| Default | 0.013 | 0.005 | 0.016 |
| Refinance | 0.085 | 0.006 | 0.108 |
| Pay | 0.748 | 0.340 | 0.865 |
| Extend | 0.142 | 0.639 | n/a |
| Income |  |  |  |
| Default | 0.649 | 0.698 | 0.645 |
| Refinance | 0.979 | 1.194 | 0.975 |
| Pay | 2.193 | 3.188 | 2.082 |
| Extend | 1.128 | 1.128 | n/a |
| Log change in income |  |  |  |
| Default | -0.089 | -0.144 | -0.084 |
| Refinance | -0.042 | -0.200 | -0.039 |
| Pay | 0.044 | 0.019 | 0.047 |
| Extend | -0.024 | -0.024 | $\mathrm{n} / \mathrm{a}$ |
| Risk-free interest rate |  |  |  |
| Default | 0.015 | 0.017 | 0.015 |
| Refinance | 0.020 | 0.015 | 0.020 |
| Pay | 0.009 | 0.000 | 0.010 |
| Extend | 0.006 | 0.006 | n/a |

Table 4 - continued

| Loan to income |  |  |  |
| :---: | :---: | :---: | :---: |
| Default | 7.828 | 9.151 | 7.718 |
| Refinance | 5.169 | 5.154 | 5.169 |
| Pay | 2.890 | 1.408 | 3.057 |
| Extend | 4.653 | 4.653 | n/a |
| Mortgage payment to income |  |  |  |
| Default | 0.517 | 0.603 | 0.509 |
| Refinance | 0.362 | 0.403 | 0.361 |
| Pay | 0.243 | 0.243 | 0.243 |
| If no extension | 0.250 | 0.250 | n/a |
| With extension | 0.085 | 0.085 | n/a |
| Loan to value |  |  |  |
| Default | 1.524 | 1.508 | 1.525 |
| Before refinancing | 0.663 | 0.410 | 0.667 |
| After refinancing | 0.799 | 0.505 | 0.804 |
| Pay | 0.836 | 0.721 | 0.849 |
| Extend | 0.996 | 0.996 | n/a |
| Delta HP |  |  |  |
| Default | -0.138 | -0.136 | -0.138 |
| Refinance | 0.105 | 0.080 | 0.106 |
| Pay | -0.002 | -0.014 | 0.000 |
| Extend | -0.044 | -0.044 | n/a |

Table 5: Welfare gains
This table reports the welfare gains of the several mortgage types relative to the ARM baseline with no option to extend maturity. The welfare benefits of each option are calculated under the form of consumption equivalent variations, so that the reported value measures the percentage increase (decrease) in consumption required to make the agent as well off in the alternative under analysis as in the ARM baseline. The first row reports the welfare gain of the ARM with option to extend maturity. The second row reports the welfare gain of the FRM. The third row the welfare gain of an FRM contract with an option to switch to an ARM. The last row reports the welfare gain of an ARM with option to extend maturity in a low interest rate environment in comparison to the plain vanilla ARM also in a low interest rate environment (LIRE).

| ARM with option to extend maturity | $1.31 \%$ |
| :--- | :--- |
| FRM | $0.44 \%$ |
| FRM with option to switch to an ARM | $0.23 \%$ |
| ARM with option to extend maturity (LIRE) | $1.40 \%$ |

Table 6: Decisions in the period after the maturity extension policy is used.
This table describes the agents' decisions in the period after the maturity extension policy has been used. The decisions available depend on whether the economy is in expansion (first two columns) or in recession (last two columns). Panel A reports the percentage of agents who at $t+1$ conditional on having used the policy at $t$ go back to make mortgage payments, use the policy again, default, prepayment or refinance, depending on whether house prices decrease or increase. Panel B reports the same but conditional on an increase (decrease) in income.

|  | Recession at t+1 |  | Expansion at t+1 |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Panel A: House Price Changes |  |  |  |
|  | $\Delta P_{t+1}^{H}<\overline{0}$ | $\Delta P_{t+1}^{H}>0$ | $\Delta P_{t+1}^{H}<0$ | $\Delta P_{t+1}^{H}>0$ |
| Make mortgage payment | 6.94 | 16.03 | 83.98 | 78.79 |
| Use policy again | 90.43 | 80.15 | n/a | n/a |
| Default | 1.76 | 0.32 | 9.28 | 1.44 |
| Prepayment | 0.52 | 2.06 | 0.45 | 1.62 |
| Refinance | 0.35 | 1.45 | 6.29 | 18.15 |
| Panel B: Income Changes |  |  |  |  |
|  | $\Delta Y_{t+1}<0$ | $\Delta Y_{t+1}>0$ | $\Delta Y_{t+1}<0$ | $\Delta Y_{t+1}>0$ |
| Make mortgage payment | 7.81 | 12.86 | 78.49 | 83.7 |
| Use policy again | 88.06 | 84.97 | n/a | n/a |
| Default | 1.45 | 0.97 | 6.27 | 4.33 |
| Prepayment | 1.57 | 0.72 | 1.36 | 0.79 |
| Refinance | 1.11 | 0.48 | 13.88 | 11.18 |

Table 7: Changes in consumption conditional on interest rate changes
This table reports average log consumption changes conditional on the economy moving to a recession and conditional on the movement of the risk-free real interest rate. The first column reports the results for the change in consumption when interest rates are low and remain low. The second (third) column the results for when there is a decrease (increase) in interest rates. The last column the results when interest rates are kept high. The first row of the table has the results for the plain-vanilla ARM, the second row the results for an ARM contract with option to extend maturity, the third row the results for a standard FRM contract and the last row for a FRM contract with the option to convert to an ARM contract in recessions.

|  | $\Delta \log C \mid$ recession |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Low to Low | High to Low | Low to High | High to High |
| Baseline case - ARM | -0.006 | 0.014 | -0.055 | -0.037 |
| Maturity extension - ARM | -0.001 | 0.018 | -0.047 | -0.029 |
| Baseline - FRM | -0.015 | 0.002 | -0.047 | -0.030 |
| FRM with option to convert to ARM | -0.008 | 0.011 | -0.052 | -0.034 |

Table 8: Alternative maturity extension policies
This table reports the key results (default rates, consumption growth, loan premia and welfare gains) of allowing agents to always extend the maturity of their mortgage contracts and the results of having to pay a cost to use the option. Panel A reports the results of always allowing the agents to extend the maturity of their contracts. Panel B reports the same results in model where it is costly to extend the maturity in expansions. Finally, Panel C reports the results of a model where maturity extension is only possible in recessions at a cost. The cost is set to be identical to the refinancing cost.

| Panel A: ARM |  |  |  | with free permanent option to extend maturity |
| :--- | :---: | :---: | :---: | :---: |
|  | $\frac{\text { Unconditional }}{}$ | Recession |  |  |
| Default rate | 0.014 |  | $\frac{\text { Expansion }}{0.015}$ | 0.014 |
| Loan premia | 0.038 |  | 0.022 | 0.012 |
| Consumption changes | 0.038 | -0.015 | 0.053 |  |
| Welfare gains | $4.94 \%$ |  |  |  |

Panel B: ARM with permanent option to extend maturity, costly in expansions

| Default rate | 0.013 | 0.006 | 0.015 |
| :--- | :--- | :--- | :--- |
| Loan premia | 0.015 | 0.025 | 0.013 |
| Consumption changes | 0.040 | -0.011 | 0.055 |
| Welfare gains | $1.45 \%$ |  |  |


| Panel C: ARM with costly option to extend maturity in recessions |  |  |  |
| :--- | :---: | :---: | :---: |
| Default rate | 0.013 | 0.014 | 0.013 |
| Loan premia | 0.017 | 0.031 | 0.013 |
| Consumption changes | 0.042 | -0.015 | 0.058 |
| Welfare gains | $0.02 \%$ |  |  |

Table 9: Baseline FRM
This table reports the model results for the baseline FRM contract. The structure of the table is the same as Table 3. The loan premia reported in the first row of the table are endogenously determined using the pricing kernel described in section 2.1.4 such that lenders achieve the same average NPV per loan.

|  | Unconditional | Recession | Expansion |
| :---: | :---: | :---: | :---: |
| Loan premia | 0.016 | 0.035 | 0.011 |
| Average log cons. growth | 0.041 | -0.019 | 0.058 |
| Log change in income | 0.027 | -0.009 | 0.038 |
| Incidence |  |  |  |
| Default | 0.013 | 0.021 | 0.011 |
| Refinance | 0.097 | 0.015 | 0.121 |
| Pay | 0.878 | 0.950 | 0.858 |
| Income |  |  |  |
| Default | 0.627 | 0.618 | 0.631 |
| Refinance | 1.053 | 1.093 | 1.052 |
| Pay | 2.046 | 1.897 | 2.092 |
| Log change in income |  |  |  |
| Default | -0.134 | -0.193 | -0.102 |
| Refinance | -0.038 | -0.166 | -0.034 |
| Pay | 0.035 | -0.004 | 0.047 |
| Risk-free interest rate |  |  |  |
| Default | -0.001 | -0.004 | 0.001 |
| Refinance | 0.010 | 0.008 | 0.010 |
| Pay | 0.010 | 0.004 | 0.012 |
| Loan to income |  |  |  |
| Default | 7.670 | 7.871 | 7.562 |
| Refinance | 5.045 | 4.855 | 5.051 |
| Pay | 2.998 | 3.224 | 2.926 |
| Mortgage payment to income |  |  |  |
| Default | 0.478 | 0.495 | 0.469 |
| Refinance | 0.315 | 0.358 | 0.314 |
| Pay | 0.240 | 0.255 | 0.235 |
| Loan to value |  |  |  |
| Default | 1.399 | 1.400 | 1.399 |
| Before refinancing | 0.659 | 0.445 | 0.666 |
| After refinancing | 0.793 | 0.530 | 0.802 |
| Pay | 0.810 | 0.833 | 0.803 |
| Delta HP |  |  |  |
| Default | -0.153 | -0.154 | -0.153 |
| Refinance | 0.105 | 0.071 | 0.106 |
| Pay | -0.009 | -0.032 | -0.002 |

Table 10: FRM contract with option to switch to an ARM contract in recession This table reports the results for a FRM mortgage contract with the option to switch to an ARM in recessions. The remining rows report statistics conditional on the decisions to default, refinance and make mortgage payments and the switch to an ARM contract.

|  | Unconditional | Recession | Expansion |
| :---: | :---: | :---: | :---: |
| Loan premia | 0.012 | 0.040 | 0.014 |
| Average log cons. growth | 0.041 | -0.016 | 0.058 |
| Log change in income | 0.027 | -0.009 | 0.038 |
| Incidence |  |  |  |
| Default | 0.013 | 0.017 | 0.012 |
| Refinance | 0.094 | 0.012 | 0.118 |
| Pay | 0.835 | 0.749 | 0.860 |
| Switch to ARM | 0.046 | 0.207 | 0.000 |
| Income |  |  |  |
| Default | 0.622 | 0.605 | 0.630 |
| Refinance | 1.009 | 1.156 | 1.005 |
| Pay | 2.093 | 2.067 | 2.099 |
| Switch to ARM | 1.249 | 1.249 | n/a |
| Log change in income |  |  |  |
| Default | -0.134 | -0.209 | -0.102 |
| Refinance | -0.046 | -0.161 | -0.042 |
| Pay | 0.038 | -0.004 | 0.048 |
| Switch to ARM | -0.008 | -0.008 | n/a |
| Risk-free interest rate |  |  |  |
| Default | 0.009 | 0.006 | 0.011 |
| Refinance | 0.015 | 0.017 | 0.015 |
| Pay | 0.011 | 0.009 | 0.011 |
| Switch to ARM | -0.015 | -0.015 | $\mathrm{n} / \mathrm{a}$ |
| Loan to income |  |  |  |
| Default | 7.692 | 8.009 | 7.560 |
| Refinance | 5.067 | 4.585 | 5.081 |
| Pay | 2.914 | 2.976 | 2.898 |
| Switch to ARM | 4.148 | 4.148 | n/a |

Table 10-continued

| Mortgage payment to income |  |  |  |
| :---: | :---: | :---: | :---: |
| Default | 0.500 | 0.482 | 0.507 |
| Refinance | 0.349 | 0.374 | 0.349 |
| Pay | 0.242 | 0.248 | 0.241 |
| If no conversion to ARM | 0.265 | 0.265 | n/a |
| With conversion to ARM | 0.148 | 0.148 | $\mathrm{n} / \mathrm{a}$ |
| Loan to value |  |  |  |
| Default | 1.396 | 1.405 | 1.391 |
| Before refinancing | 0.651 | 0.412 | 0.658 |
| After refinancing | 0.791 | 0.501 | 0.800 |
| Pay | 0.799 | 0.809 | 0.797 |
| Switch to ARM | 0.903 | 0.903 | n/a |
| Delta HP |  |  |  |
| Default | -0.155 | -0.155 | -0.155 |
| Refinance | 0.103 | 0.062 | 0.104 |
| Pay | -0.008 | -0.032 | -0.001 |
| Switch to ARM | -0.033 | -0.033 | n/a |

Table 11: ARM in a low interest rate environment
This table reports the results for an ARM contract in a low interest rate environment. The setup is the same as the one for the ARM contract, but the risk-free rate is parameterized to the post year 2000 period. Details of the calibration are given in section 2.4.5. The table follows the same structure as Table 3 .

|  | Unconditional | Recession | Expansion |
| :---: | :---: | :---: | :---: |
| Loan premia | 0.015 | 0.028 | 0.012 |
| Average log cons. growth | 0.035 | -0.030 | 0.054 |
| Log change in income | 0.027 | -0.009 | 0.038 |
| Incidence |  |  |  |
| Default | 0.013 | 0.019 | 0.011 |
| Refinance | 0.105 | 0.014 | 0.131 |
| Pay | 0.868 | 0.951 | 0.845 |
| Income |  |  |  |
| Default | 0.621 | 0.606 | 0.629 |
| Refinance | 0.981 | 1.060 | 0.978 |
| Pay | 2.077 | 1.905 | 2.132 |
| Log change in income |  |  |  |
| Default | -0.153 | -0.225 | -0.118 |
| Refinance | -0.041 | -0.170 | -0.037 |
| Pay | 0.036 | -0.004 | 0.049 |
| Risk-free interest rate |  |  |  |
| Default | -0.006 | -0.007 | -0.006 |
| Refinance | -0.005 | -0.002 | -0.005 |
| Pay | -0.010 | -0.010 | -0.010 |
| Loan to income |  |  |  |
| Default | 7.656 | 7.918 | 7.529 |
| Refinance | 5.058 | 5.051 | 5.058 |
|  | 3.038 | 3.293 | 2.957 |
| Mortgage payment to income |  |  |  |
| Default | 0.347 | 0.361 | 0.340 |
| Refinance | 0.229 | 0.305 | 0.227 |
| Pay | 0.173 | 0.184 | 0.170 |
| Loan to value |  |  |  |
| Default | 1.426 | 1.429 | 1.425 |
| Before refinancing | 0.680 | 0.488 | 0.686 |
| After refinancing | 0.812 | 0.580 | 0.819 |
| Pay | 0.845 | 0.868 | 0.838 |
| Delta HP |  |  |  |
| Default | -0.155 | -0.154 | -0.155 |
| Refinance | 0.112 | 0.081 | 0.113 |
| Pay | -0.012 | -0.033 | -0.005 |

Table 12: ARM contract with the option to extend maturity in a low interest rate environment This table reports the results for an ARM contract with the option to extend maturity in recessions, in a low interest rate environment. Details of the calibration are given in section 2.4.5. The table follows the same structure as Table 4.

|  | Unconditional | Recession | Expansion |
| :---: | :---: | :---: | :---: |
| Loan premia | 0.015 | 0.027 | 0.012 |
| Average log cons. growth | 0.034 | -0.022 | 0.050 |
| Log change in income | 0.027 | -0.009 | 0.038 |
| Incidence |  |  |  |
| Default | 0.013 | 0.003 | 0.016 |
| Refinance | 0.097 | 0.006 | 0.123 |
| Pay | 0.720 | 0.275 | 0.848 |
| Extend | 0.157 | 0.704 | n/a |
| Income |  |  |  |
| Default | 0.651 | 0.822 | 0.641 |
| Refinance | 0.989 | 1.129 | 0.987 |
| Pay | 2.223 | 3.543 | 2.100 |
| Extend | 1.173 | 1.173 | n/a |
| Log change in income |  |  |  |
| Default | -0.092 | -0.108 | -0.091 |
| Refinance | -0.033 | -0.210 | -0.030 |
| Pay | 0.046 | 0.022 | 0.048 |
| Extend | -0.021 | -0.021 | n/a |
| Risk-free interest rate |  |  |  |
| Default | -0.007 | -0.006 | -0.007 |
| Refinance | -0.005 | -0.001 | -0.005 |
| Pay | -0.010 | -0.013 | -0.010 |
| Extend | -0.009 | -0.009 | n/a |

Table 12 - continued

| Loan to income |  |  |  |
| :---: | :---: | :---: | :---: |
| Default | 7.852 | 8.449 | 7.816 |
| Refinance | 5.159 | 5.552 | 5.153 |
| Pay | 2.987 | 1.240 | 3.150 |
| Extend | 4.610 | 4.610 | n/a |
| Mortgage payment to income |  |  |  |
| Default | 0.337 | 0.366 | 0.336 |
| Refinance | 0.219 | 0.310 | 0.218 |
| Pay | 0.181 | 0.242 | 0.175 |
| If no extension | 0.172 | 0.172 | n/a |
| With extension | 0.013 | 0.013 | $\mathrm{n} / \mathrm{a}$ |
| Loan to value |  |  |  |
| Default | 1.580 | 1.472 | 1.587 |
| Before refinancing | 0.696 | 0.458 | 0.699 |
| After refinancing | 0.820 | 0.552 | 0.824 |
| Pay | 0.891 | 0.771 | 0.902 |
| Extend | 1.021 | 1.021 | n/a |
| Delta HP |  |  |  |
| Default | -0.134 | -0.124 | -0.134 |
| Refinance | 0.116 | 0.095 | 0.116 |
| Pay | -0.005 | -0.017 | -0.004 |
| Extend | -0.040 | -0.040 | n/a |

Table 13: Robustness to parameterization
This table reports the main model results under alternative parameterizations. Panel A reports the results for the baseline parameterization described in section 2.4. Panel B reports the model results assuming a constant inflation rate of $2 \%$. In panel C we parameterize the $\log$ house price growth to be $1 \%$. In panel $D$ we set the rental premium to zero. Finally in panel E we set the foreclosure loss to $26 \%$.

| Unconditional |  | Recession | Expansion |
| :---: | :---: | :---: | :---: |
| Panel A: Benchmark calibration |  |  |  |
| Baseline ARM |  |  |  |
| Default rate | 0.013 | 0.019 | 0.012 |
| Loan premia | 0.016 | 0.030 | 0.013 |
| Consumption changes | 0.042 | -0.016 | 0.059 |
| Maturity extension ARM |  |  |  |
| Default rate | 0.013 | 0.005 | 0.016 |
| Loan premia | 0.015 | 0.027 | 0.012 |
| Consumption changes | 0.040 | -0.010 | 0.055 |
| Welfare gains | 1.31\% |  |  |
| Panel B: Constant inflation rate |  |  |  |
| Baseline ARM |  |  |  |
| Default rate | 0.012 | 0.018 | 0.011 |
| Loan premia | 0.017 | 0.033 | 0.013 |
| Consumption changes | 0.043 | -0.016 | 0.060 |
| Maturity extension ARM |  |  |  |
| Default rate | 0.013 | 0.004 | 0.015 |
| Loan premia | 0.016 | 0.032 | 0.012 |
| Consumption changes | 0.042 | -0.008 | 0.056 |
| Welfare gains | 1.29\% |  |  |
| Panel C: Positive house price growth |  |  |  |
| Baseline ARM |  |  |  |
| Default rate | 0.012 | 0.017 | 0.010 |
| Loan premia | 0.016 | 0.030 | 0.012 |
| Consumption changes | 0.041 | -0.017 | 0.058 |
| Maturity extension ARM |  |  |  |
| Default rate | 0.012 | 0.005 | 0.014 |
| Loan premia | 0.015 | 0.028 | 0.011 |
| Consumption changes | 0.040 | -0.011 | 0.055 |
| Welfare gains | 1.14\% |  |  |

Table 13-continued
$\underline{\text { Panel D: No rental premium }}$

| Baseline ARM |  |  |  |
| :---: | :---: | :---: | :---: |
| Default rate | 0.018 | 0.025 | 0.016 |
| Loan premia | 0.017 | 0.030 | 0.013 |
| Consumption changes | 0.039 | -0.017 | 0.055 |
| Maturity extension ARM |  |  |  |
| Default rate | 0.017 | 0.006 | 0.020 |
| Loan premia | 0.016 | 0.028 | 0.013 |
| Consumption changes | 0.038 | -0.012 | 0.053 |
| Welfare gains | 1.05\% |  |  |
| Panel E: Higher foreclosure loss |  |  |  |
| Baseline ARM |  |  |  |
| Default rate | 0.013 | 0.018 | 0.012 |
| Loan premia | 0.016 | 0.030 | 0.012 |
| Consumption changes | 0.042 | -0.016 | 0.059 |
| Maturity extension ARM |  |  |  |
| Default rate | 0.013 | 0.005 | 0.016 |
| Loan premia | 0.015 | 0.026 | 0.012 |
| Consumption changes | 0.041 | -0.010 | 0.055 |
| Welfare gains | 1.25\% |  |  |


[^0]:    *We would like to thank Joel Shapiro and participants at the Tenth Macro Finance Society Workshop (Boston College), the Second London Financial Intermediation Workshop (Bank of England), and at the European Central Bank seminar series for comments.
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    ${ }^{\ddagger}$ Department of Finance, London Business School.
    ${ }^{\S}$ Department of Finance, London Business School and CEPR.

[^1]:    ${ }^{1}$ A separate program, the Home Affordable Modification Mortgage (HAMP), facilitated loan modifications so as to make them more affordable and prevent foreclosures. The modifications included interest rate reduction, principal reduction, and term extension (Agarwal et al. 2017).

[^2]:    ${ }^{2}$ There is an ongoing debate about the importance of subprime lending during the credit boom of the mid2000s (Adelino, Schoar and Severino 2016).

[^3]:    ${ }^{3}$ Favilukis, Ludvigson, and Van Nieuwerburgh (2017) solve a quantitative general equilibrium model of the housing market with aggregate business cycle risk and a realistic wealth distribution. In their model the boom in house prices can be explained by a relaxation of financing constraints and a decline in the housing risk premium. Their focus is not on mortgage contract design.

[^4]:    ${ }^{4}$ Property maintenance and tax costs are a proportion of the house value, i.e., $P C_{t} \equiv\left(m_{p}+\tau_{p}(1-\phi)\right) H_{i, t_{i}} P_{t}^{H}$. We give further details in the parameterization section.

[^5]:    ${ }^{5}$ See also Agarwal, Rosen, and Yao (2016) and Agarwal et al. (2015).

