Aging and Deflation:

A *Politico-Economic* Perspective*

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VERY PRELIMINARY

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Abstract

How does the grayer society affect the political decision making regarding inflation rate in the long-run? Is deflation preferred as the society gets older? In order to answer to these questions, we first compute the optimal inflation rates in the stationary population for young and old respectively, by using a New Keynesian model with the overlapping generations. Then, we explore how the weighted average of optimal inflation rates by population has developed with the on-going societal aging in Japan. Old prefers lower inflation rates as aging deepens. The fluctuations in the weighted average of optimal inflation rates are, however, very small compared to those found in the data. Since our focus is on the long-run optimal inflation rate by the Ramsey planner, we deliberately abstract heterogeneous impacts from surprise inflation via nominal asset holdings as empirically examined in Doepke and Schneider (2006). The result in this paper implies that if there exists any political bias toward deflation in the grayer society, it should be due not to the optimal inflation rates in the long-run but to nominal contracts in financial transactions.

Keywords: Optimal inflation rates; Societal aging;
JEL codes: E52; J11

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1 Introduction

Japan has been experiencing long-lasting deflation or low inflation rates for more than two decades. Some claim that this is due to the failure of monetary policy with insufficient policy reaction by the Bank of Japan, based on the idea that "Inflation is always and everywhere a monetary phenomenon," by Friedman (1963). On the other hand, because of its long-lasting nature of deflation or low inflation rates, some point out that chronic deflation or disinflation should have its root in the structural issues. An interesting observation in Japan about the relationship between the structural factor and nominal development is that deflation or disinflation started around the mid-1990s when the working age population also started decreasing. Is it a causal relationship or a mere coincidence? In this paper, we explore new insight on the relationship between deflation and aging from a *politico-economic* perspective.

Since aging and low inflation rates are not phenomena intrinsic only to Japan and now observed among developed economies, several researchers start investigating the possible causal relationship between inflation dynamics and demographic change. Carvalho and Ferrero (2014) and Fujita and Fujiwara (2016) discuss how societal aging can lead to the declining natural rate of interest. Carvalho and Ferrero (2014) focus on the demand channel, or *consumption-saving heterogeneity*. Longer longevity induces higher saving rates for self-insurance. Such a saving-for-retirement motive can account for roughly 30% to 50% of the decline in real interest rates in Japan. The decline in fertility rate, however, does not have large impacts. On the other hand, Fujita and Fujiwara (2016) quantify the impact of the supply channel, or *skill (productivity) heterogeneity*. The changes in the demographic structure induce significant low-frequency movements in *per-capita* consumption growth and the real interest rate through the changes in the composition of skilled (old) and unskilled (young) workers. This mechanism can accounts for roughly 40% of the declines in the real interest rate observed between the 1980s and 2000s in Japan. The key is the declining fertility (labor participation) rate.

Several other studies investigate whether preference for inflation or deflation may depend on age. Doepke and Schneider (2006) empirically investigate the redistribution effect of inflation. Since old own more nominal financial assets, they are more vulnerable to unanticipated inflation rates. On the other hand, surprise inflation can be beneficial for young because they are borrowers with nominal contracts. This implicitly implies that as the society gets grayer, the social preference goes toward less inflation. Bullard et al. (2012) construct an overlapping generations model with two assets, capital and money. If old agents have more influence on the political decision making, relatively low inflation is chosen because lower inflation reduces the opportunity cost of holding and money becomes relatively more attractive and capital accumulation is thus reduced. This raises interest rate, which is preferred by old since they rely more on capital income than labor income.

In this paper, we also inquire into whether there are possible structural relationship between demographic change and inflation dynamics, but with different angles from previous studies. They are the optimal inflation rate in the spirit of Schmitt-Grohe and Uribe (2010) and the political economy in the spirit of Bassetto (2008), who studies the inter-generational conflicts in tax policy in overlapping generations.

As comprehensively analyzed in Schmitt-Grohe and Uribe (2010), specifically for Calvo (1983) contract in Ascari (2004) and for Rotemberg (1982) adjustment cost in Bilbiie et al. (2014), inflation rates are not neutral even in the (stochastic) steady states. Inflation rates affect markups in the long-run through such nominal rigidities as considered in Calvo (1983) and Rotemberg (1982) and have impacts on real variables. As a result, optimal inflation rates can be positive or negative depending on the deep parameters. Since the structural parameters differentiate the behavior of old and young, the optimal inflation rates are likely to be different between these two agents.

We first compute the optimal inflation rates in the stationary population for young and old, respectively, and how they change with different demographic settings. We then investigate whether the historical inflation dynamics are consistent with weighted average of optimal inflation rates between young and old with time-varying demographic factors. In Japan, as shown in Figure 1,[1] (a) the life-expectancy becomes longer, (b) the population growth rates becomes lower, (c) as a result, the young/old population ratio has been increasing, and (d) young's asset holdings has been decreasing. For this purpose, we employ the overlapping generations model with nominal rigidities by Fujiwara and Teranishi (2008), where there are two consumers: young and old. They are different in life-expectancy and productivity. Such a model enable us to investigate how optimal inflation rates change by different demographic structures illustrated in Figure 1.

The demographic structure can potentially have some implications for optimal inflation rates for young and old. According to our simulation results, old prefers lower inflation rates as aging deepens. The fluctuations in the weighted average of optimal inflation rates are, however, very small compared to those found in the data. Since our focus is the long-run optimal inflation rate by the Ramsey planner, we deliberately abstract heterogeneous impacts from surprise inflation via nominal asset holdings. The

[1] The top left panel is the asset holding of young divided by total asset holding. The top right panel is life expectancy at the age 65. The bottom left panel is the number of old divided by number of young. Young and old are respectively defined as the population of age 20 to 65 and the one of age over 65. Population growth rate is plotted in the bottom right panel expressed as the percentile change.

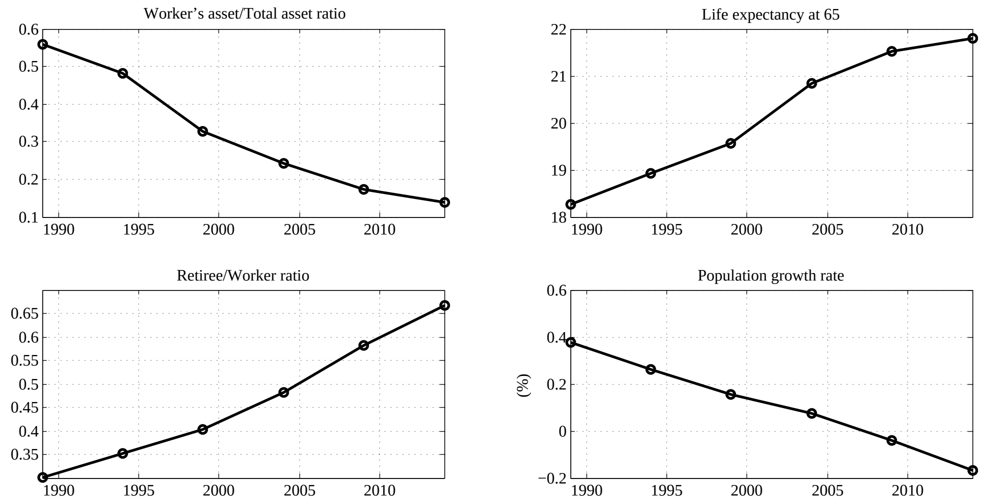

Figure 1: Japanese Data

result in this paper implies that if there exists any political bias toward deflation in the grayer society, it should be due not to the optimal inflation rates in the long-run but to nominal contracts in financial transactions. Careful analysis on the household's balance sheet, in particular, how components of assets and liabilities are constrained by nominal contracts, or which assets are owned by each agent, is needed to understand the asymmetric impacts of surprise inflation to young and old.

The remainder of the paper is structured as follows. Section 2 describes the model used in this paper. In Section 3, we show how optimal inflation rates are different between young and old and how they change by different demographic structures. Section 4 examines whether the demographic factors can explain Japan's deflation or low inflation rates qualitatively as well as quantitatively. Section 5 concludes.

2 The Model

In order to investigate the effects of societal aging on the optimal inflation rate, we use the overlapping generation (OLG) model analyzed in Fujiwara and Teranishi (2008), that extend the analytical framework in Gertler (1999) to incorporate nominal rigidities and monetary policy. Unlike the standard overlapping generation model, the transition from young to old follows a Markov process with the latter being the absorbing state.[2] Consequently, without resort to numerical analysis with the large number of, say 50x4=200, generations, the analysis over the quarterly frequency, where monetary policy is considered to be effective, becomes possible in a tractable as well as analytical framework.

[2]The overlapping generation model by Gertler (1999) can be considered as the generalized Blanchard-Yaari model *a la* Blanchard (1985) and Yaari (1965).

There are five agents in this model economy: two types of consumers: young and old; firms; a capital producer; the central bank. The problems which each agent except for the central banks faces are as follows. The central bank sets inflation rate in order to maximizes welfare.[3]

2.1 Consumers

Each young faces a constant probability ω to become old, while each old remains in the population with the survival probability γ.[4] Each type of agent is different in labor productivity. Both supply unit of labor but old's relative labor productivity is assumed to be ξ. Setting $\xi = 0$ corresponds to the economy where old agents never work. In the benchmark model shown below, ξ is set to be zero. The model of variable labor supply by old and results from it are shown in Appendix B.

Let us first discuss the optimization problem of old, which is assumed to be the absorbing state.

2.1.1 Old

At time t, an old, denoted by superscript o, who were born at period j and became old at period k, maximizes welfare:

$$V^o_{j,k,t} := \left\{ \left[C^o_{j,k,t}\right]^\rho + \beta\gamma \left(V^o_{j,k,t+1}\right)^\rho \right\}^{\frac{1}{\rho}},$$

subject to the budget constraint:

$$\frac{A^o_{j,k,t}}{P_t} = \frac{R_{t-1}}{\gamma} \frac{A^o_{j,k,t-1}}{P_t} - C^o_{j,k,t} + \frac{W_t}{P_t}\xi + D^o_{j,k,t}.$$

C_t, A_t, P_t, and R_t denote consumption, financial assets, aggregate price, and nominal interest, respectively. Old does not supply labor. D_t is the sum of the transfer (or tax) from the government and profits rebated from the intermediate goods producers and the capital producer by the ownership of these firms.[5] β and ρ define the common subjective discount factor for both old and young and the inverse of the inter temporal elasticity of substitution, respectively. The next period welfare is discounted by $\beta\gamma$ since old must take the survival rate into account. The rate of return from holding financial assets is divided by γ because of the perfect annuity market among old. Bequests are distributed equally among surviving old.

[3]The details of the derivation are shown in Appendix B.
[4]To be precise, since old also works, they should be termed old workers.
[5]Final goods as well as capital producers yield zero profit.

2.1.2 Young

A young, which is denoted by the subscript y, who were born at period j maximizes the life time utility:

$$V_{j,t}^{y} := \left\{ \left[C_{j,t}^{y} \right]^{\rho} + \beta \left[\omega V_{j,t+1}^{y} + (1-\omega) V_{j,t+1}^{o} \right]^{\rho} \right\}^{\frac{1}{\rho}}.$$

subject to the budget constraint

$$\frac{A_{j,t}^{y}}{P_t} = R_t \frac{A_{j,t-1}^{y}}{P_t} + \frac{W_t}{P_t} - C_{j,t}^{y} + D_{j,t}^{y}.$$

Since each young becomes old with probability ω, the next period value is weighted value of old and young. In contrast to old, young supplies unit of labor and obtain nominal wage W_t.

2.2 Firms

Final goods, Y_t, is produced by the final goods firm in a competitive market. Differentiated intermediary goods $Y_{i,t}$ are aggregated by the the production technology:

$$Y_t = \left[\int_0^1 (Y_{i,t})^{\frac{\kappa-1}{\kappa}} di \right]^{\frac{\kappa}{\kappa-1}}.$$

The parameter κ denotes the elasticity of substitution among differentiated intermediate goods. Given the aggregate price level P_t and the price of each intermediary goods $P_{i,t}$, profit maximization by final goods firm yields the demand for each intermediate goods:

$$Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\kappa} Y_t. \tag{1}$$

Firm i in monopolistically competitive market uses non-differentiated labor $L_{i,t}$ and capital $K_{i,t-1}$ in order to produce differentiated intermediary goods$Y_{i,t}$. The production function of the intermediate goods is given by

$$Y_{i,t} = (\exp(Z_t) L_{i,t})^{1-\alpha} K_{i,t-1}^{\alpha}, \tag{2}$$

where α is capital share. The exogenous variable Z_t represents aggregate technology shock, which are common for all intermediary firms. Labor is supplied by consumers with nominal wage rate W_t. Capital is rent to intermediary firms at real rate R_t^K from capital producers. The real cost minimization problem is given by

$$\min \left\{ \frac{W_t}{P_t} L_{i,t} + r_t^K K_{i,t} \right\}$$

subject to the production function (2). This gives the optimal factor price conditions:

$$\frac{W_t}{P_t} = (1-\alpha)\, \psi_t \left(\exp\left(Z_t\right)\right)^{1-\alpha} \left(L_{i,t}\right)^{-\alpha} K_{i,t-1}^{\alpha},$$

$$r_t^K = \alpha \psi_t \left(\exp\left(Z_t\right) L_{i,t}\right)^{1-\alpha} K_{i,t-1}^{\alpha-1},$$

where ψ_t denotes real marginal cost.

Since each intermediary firm is in a monopolistically competitive market, it can choose the optimal price to maximize the profit with the Rotemberg (1982) price adjustment cost with cost parameter ϕ. Instantaneous real profit $\Pi_{i,t}^I$ is given by

$$\Pi_{i,t}^I = (1+\tau) \frac{P_{i,t}}{P_t} Y_{i,t} - \psi_t Y_{i,t} - \frac{\phi}{2} \left(\frac{P_{i,t}}{P_{i,t-1}} - 1 \right)^2 Y_t.$$

We assume that the intermediaries are owned by consumers, therefore, the profit is rebated equally to all consumers. Let $m_{0,t}$ denote the pricing kernel. Then, the profit maximization problem by price setting becomes

$$\max E_t \sum_{t=0}^{\infty} \left(m_{0,t}\right) \left[(1+\tau) \frac{P_{i,t}}{P_t} Y_{i,t} - \psi_t Y_{i,t} - \frac{\phi}{2} \left(\frac{P_{i,t}}{P_{i,t-1}} - 1 \right)^2 Y_t \right]$$

subject to the demand for intermediary goods in (1). Since there are heterogeneous agents, it is not trivial how to define the pricing kernel. In this paper, following Ghironi (2008) and Fujiwara and Teranishi (2008), we only conduct perfect foresight simulations, and therefore all assets yield the same rate of return among different agents both *ex ante* and *ex post*. In other words, the profit is discounted by the *risk free rate*. This assumption only matters at the initial period when the target level of inflation is altered.

In order to eliminate the distortion from monopolistic competition, production subsidy $\tau = \frac{1}{\kappa - 1}$ is assumed. This subsidy is financed by lump sum tax to households. The optimality condition is given by

$$m_{0,t}\left[(1+\tau)(-\kappa+1)(P_{i,t})^{-\kappa}\left(\frac{1}{P_t}\right)^{-\kappa+1}Y_t+\kappa\psi_t(P_{i,t})^{-\kappa-1}\left(\frac{1}{P_t}\right)^{-\kappa}Y_t-\phi\left(\frac{P_{i,t}}{P_{i,t-1}}-1\right)\frac{1}{P_{i,t-1}}Y_t\right]$$
$$+m_{0,t+1}E_t\phi\left(\frac{P_{i,t+1}}{P_{i,t}}-1\right)Y_{t+1}\frac{P_{i,t+1}}{P_{i,t}^2}=0.$$

In a symmetric equilibrium where $P_{i,t}=P_t$, this condition collapses to the following New Keynesian Phillips Curve:

$$\kappa(\psi_t-1)Y_t-\phi(\pi_t-1)Y_t\pi_t+E_t\frac{m_{0,t+1}}{m_{0,t}}\phi(\pi_{t+1}-1)\pi_{t+1}Y_{t+1}=0,$$

where $\pi_t:=\frac{P_t}{P_{t-1}}$.

2.3 Capital Producer

A capital producer maximizes the profit:

$$\sum_{t=0}^{\infty}m_{0,t}\left(\Pi_t^K\right),$$

where

$$\Pi_t^K:=\frac{A_t}{P_t}-R_{t-1}\frac{A_{t-1}}{P_t}+r_{t-1}^K K_{t-1}-I_t,$$

subject to the capital producing technology:

$$K_t=(1-\delta)K_{t-1}+\left[1-S\left(\frac{I_t}{I_{t-1}}\right)\right]I_t.$$

A capital producer issues financial assets A_t with nominal rate of return R_t. Such funding from households and the receipts from the renting the capital to the intermediate goods producer are allocated to the repayment of borrowing from households and investment I_t. $S(\cdot)$ denotes the investment growth adjustment cost used in Christiano et al. (2005):

$$S(x_t):=s\left(\frac{x_t^2}{2}-x_t+\frac{1}{2}\right).$$

This capital producer can be also interpreted as a financial intermediary.

2.4 Aggregate Conditions

The financial market clears with

$$q_t K_t = \frac{A_t}{P_t},$$

and

$$A_t = A_t^y + A_t^o,$$

where $A_t^y = \sum_{j=0}^{\infty} A_{j,t}^y$ and $A_t^o = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} A_{j,k,t}^o$. q_t denotes Tobin's Q, which is given by the Lagrange multiplier on the constraint in the capital producer's profit maximization problem.

The good market clears as

$$Y_t = C_t + I_t + \frac{\phi}{2} (\pi_t - 1)^2 Y_t,$$

where $C_t = C_t^y + C_t^o$, $C_t^y = \sum_{j=0}^{\infty} C_{j,t}^y$ and $C_t^o = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} C_{j,k,t}^o$.

The profits are distributed by the amount of asset holdings for each agent:

$$D_t^o = \sum_{j=0}^{\infty} \sum_{k=0}^{\infty} D_{j,k,t}^o = \frac{A_{t-1}^o}{A_{t-1}^y + A_{t-1}^o} D_t,$$

and

$$D_t^y = \sum_{j=0}^{\infty} D_{j,t}^y = \frac{A_{t-1}^y}{A_{t-1}^y + A_{t-1}^o} D_t,$$

where

$$D_t := \Pi_t^I + \Pi_t^K - \tau Y_t = \left[1 - \psi_t - \frac{\phi}{2} (\pi - 1)^2\right] Y_t + \frac{A_t}{P_t} - R_{t-1} \frac{A_{t-1}}{P_t} + r_{t-1}^K K_{t-1} - I_t,$$

under a symmetric equilibrium.

2.5 Equilibrium Conditions

2.5.1 Population

Let N_t^r and N_t^w be the population of old and young at period t. Then, the population dynamics for young and old are, respectively, given by

$$N_{t+1}^y = b N_t^y + \omega N_t^y,$$

and

$$N_{t+1}^o = \gamma N_t^o + (1 - \omega) N_t^y,$$

where b denotes the birth rate. Since $N^y_{t+1} = (b+\omega) N^y_t$, the growth rate of young population is given by $b+\omega-1$. Given these population laws of motion, the ratio of the number of old over that of young , denoted by Γ_t, evolves as

$$\Gamma_{t+1} = \frac{N^o_{t+1}}{N^y_{t+1}} = \frac{\gamma N^o_t + (1-\omega) N^y_t}{b N^y_t + \omega N^y_t} = \frac{\gamma}{b+\omega}\Gamma_t + \frac{1-\omega}{b+\omega}.$$

At the stationary population, the ratio of the number of old over that of young remain constant:

$$\Gamma = \frac{1-\omega}{b+\omega-\gamma}.$$

2.5.2 Equilibrium Conditions in a Monopolistically Competitive Market

As the details are shown in Appendix A, from the first order necessary conditions of above optimization problems, we have the equilibrium conditions under a monopolistically competitive market:

$$Y_t = [\exp(z_t)]^{1-\alpha} K^\alpha_t,$$

$$\frac{W_t}{P_t} = (1-\alpha)\psi_t [\exp(z_t)]^{1-\alpha} K^\alpha_t,$$

$$r^K_t = \alpha\psi_t [\exp(z_t)]^{1-\alpha} K^{\alpha-1}_t,$$

$$D_t = \left[1-\psi_t-\frac{\phi}{2}(\pi_t-1)^2\right] Y_t + \frac{A^y_t + A^o_t}{P_t} - R_{t-1}\frac{A^y_{t-1}+A^o_{t-1}}{P_t} + r^K_{t-1}K_{t-1} - I_t, \quad (3)$$

$$\kappa(\psi_t-1) Y_t - \phi(\pi_t-1)\pi_t Y_t + m_{t,t+1}\phi(\pi_{t+1}-1)\pi_{t+1}Y_{t+1} = 0, \quad (4)$$

$$K_t = (1-\delta) K_{t-1} + \left[1 - S\left(\frac{I_t}{I_{t-1}}\right)\right] I_t,$$

$$q_t\left[1 - S\left(\frac{I_t}{I_{t-1}}\right) - S'\left(\frac{I_t}{I_{t-1}}\right)\frac{I_t}{I_{t-1}}\right] + \frac{\pi_{t+1}}{R_t} q_{t+1} S'\left(\frac{I_{t+1}}{I_t}\right)\left(\frac{I_{t+1}}{I_t}\right)^2 = 1,$$

$$q_t = \frac{\pi_{t+1}}{R_t}\left[q_{t+1}(1-\delta) + r^K_{t+1}\right],$$

$$q_t K_t = \frac{A^y_t + A^o_t}{P_t},$$

$$\frac{A^o_t}{P_t} = R_{t-1}\frac{A^o_{t-1}}{P_t} - C^o_t + \frac{A^o_{t-1}}{A^y_{t-1}+A^o_{t-1}} D_t + (1-\omega)\left(R_t\frac{A^y_{t-1}}{P_t} + \frac{W_t}{P_t} - C^y_t + \frac{1}{A^y_{t-1}+A^o_{t-1}} D_t\right),$$

$$C^o_t = \epsilon_t\theta_t\left(R_{t-1}\frac{A^o_{t-1}}{P_t} + \Theta^o_t\right), \quad (5)$$

$$\left(\frac{\epsilon_t\theta_t}{1-\epsilon_t\theta_t}\right)^{\rho-1} = \beta\gamma^{1-\rho}\left(\frac{R_t}{\gamma\pi_{t+1}}\right)^\rho (\epsilon_{t+1}\theta_{t+1})^{\rho-1},$$

$$C_t^y = \theta_t \left(R_{t-1} \frac{A_{t-1}^y}{P_t} + \Theta_t^y \right), \tag{6}$$

$$\left(\frac{\theta_t}{1-\theta_t} \right)^{\rho-1} = \beta \left(\frac{R_t \Phi_{t+1}}{\pi_{t+1}} \right)^{\rho} (\theta_{t+1})^{\rho-1},$$

$$\Theta_t^o = \frac{A_{t-1}^o}{A_{t-1}^y + A_{t-1}^o} D_t + \frac{\gamma \pi_{t+1}}{(1+n) R_t} \Theta_{t+1}^o, \tag{7}$$

$$\Theta_t^y = w_t + \frac{A_{t-1}^y}{A_{t-1}^y + A_{t-1}^o} D_t + \omega \frac{\pi_{t+1}}{(1+n) R_t \Phi_{t+1}} \Theta_{t+1}^y + (1-\omega) \epsilon_{t+1}^{\frac{\rho-1}{\rho}} \frac{\pi_{t+1}}{(1+n) R_t \Phi_{t+1}} \Theta_{t+1}^o,$$

$$Y_t = C_t^o + C_t^y + \left[1 - S\left(\frac{I_t}{I_{t-1}} \right) \right] I_t + \frac{\phi}{2} (\pi_t - 1)^2 Y_t,$$

$$L_t = 1 + \xi \Gamma_t,$$

where for simplicity of analysis, we define an auxiliary variable:

$$\Phi_t := \omega + (1-\omega) \epsilon_t^{\frac{\rho-1}{\rho}} \xi^{\nu-1}.$$

The newly defined variables θ_t and $\epsilon_t \theta_t$ denote the marginal propensity to consume for young and old, respectively. These equations together with monetary policy, which maximizes welfare, determines the equilibrium.

Discussion: Surprise Inflation Our model abstracts the effects of surprise inflation on different consumers through nominal asset holdings analyzed in Doepke and Schneider (2006). The *solved out consumption functions* in (5) and (6) are expressed as the multiple of the marginal propensity to consume and the wealth, which include initial nominal assets divided by the price level at t. Jump in the price level or inflation seems to affect the wealth of young and old differently.

If (3) and (7), which determine the profit and the financial wealth for old respectively, are substituted in (5), the solved out consumption function for old collapses to

$$\begin{aligned} C_t^o \quad = \quad & \epsilon_t \theta_t \{ \frac{A_{t-1}^o}{A_{t-1}^y + A_{t-1}^o} \left\{ \left[1 - \psi_t - \frac{\phi}{2} (\pi_t - 1)^2 \right] Y_t + \frac{A_t^y + A_t^o}{P_t} + r_{t-1}^K K_{t-1} - I_t \right\} \\ & + \frac{\gamma \pi_{t+1}}{(1+n) R_t} \Theta_{t+1}^o \}. \end{aligned}$$

Initial real asset position, which is nominal asset position divided by the price level:

A^o_{t-1}/P_t, disappears from the wealth. Thus, surprise inflation does not alter the initial real asset position. This irrelevance result stems from our assumption that the profits are shared by the same asset ratio for good producers as well as the capital producer (financial intermediary) and all financial assets are idential.

2.5.3 Aggregate Value

We can obtain aggregate value for young and old as indirect utility as

$$V^y_t = (\theta_t)^{-\frac{1}{\rho}} C^y_t, \tag{8}$$

and

$$V^o_t = (\epsilon_t \theta_t)^{-\frac{1}{\rho}} C^o_t. \tag{9}$$

These are the targets by the central bank to maximize. To be precise, we suppose a situation that there are two political parties: young party and old party. Young (old) party represents the young consumers and insists that the central banks should conduct monetary policy to maximize V^y_t (V^o_t) at time t. Note that there is no heterogeneity within each agent.

2.5.4 Monetary Policy

The central bank is equipped with a commitment technology, aiming to maximize welfare defined in (8) or (9). Welfare is evaluated at the beginning of transition from the initial state to the one with the new steady state inflation rate. Initial state is given by that under zero inflation steady state. As shown in Bilbiie et al. (2014), difference in the initial state variables can lead to incorrect welfare evaluation. The same initial state variables are assumed when comparing welfare.[6]

2.5.5 Calibration

The parameter calibration is shown in Table 1. The model is simulated at the quarterly frequency. The discount factor β and capital depreciation δ are set at $1.04^{-\frac{1}{4}}$ and $1.01^{-\frac{1}{4}} - 1$, respectively. Under our benchmark calibration, we set the parameters for demographic dynamics ω and γ so that on average, each works for 45 years and lives as old for 15 years. They are set to $\frac{45\times4-1}{45\times4} = 0.9944$ and $\frac{15\times4-1}{15\times4} = 0.9833$. Population growth rate is set to zero, which implies $b = 1 - \omega = 0.0055$. Other parameters are

[6] The optimal inflation rate found in this way depends on the initial state variables. However, even if we set initial state as steady state of $\pm 5\%$ inflation rate, the change is small and our main message still holds.

set to conventional values following Fujiwara and Teranishi (2008). Capital share α and elasticity of substitution of intermediate goods κ are set to $\frac{1}{3}$ and 10, respectively. For the parameter of Rotemberg (1982) cost ϕ, we use 50 so that the New Keynesian Philips Curve of our model matches with the one implied by Calvo (1983) type price setting where quarter fraction of firms change prices in each period on average. Invest adjustment cost is $s = 2.48$, which is taken from Christiano et al. (2005). Elasticity of intertemporal substitution σ is set to 0.5 which is consistent with Yogo (2004). Also, for benchmark case, ξ is set to zero.

Table 1: Benchmark Parameter values

	Parameters	Values
ω	transition probability to old	0.9944
γ	survival rate	0.9833
b	birth rate	$1-\omega = 0.0055$
β	discount factor	$1.04^{-\frac{1}{4}}$
σ	IES	0.5
ρ	Curvature	$\frac{\sigma-1}{\sigma} = -1$
α	capital share	$\frac{1}{3}$
κ	elasticity of substitution	10
ϕ	Rotemberg cost parameter	50
δ	capital depreciation rate	$1.01^{-\frac{1}{4}} - 1$
s	investment adjustment cost parameter	2.48

3 Optimal Inflation Rate

As explained in the previous section, the optimal inflation rate is computed as ones that maximize welfare of young and old. In this section, we explore how optimal inflation rates change by different demographic structures. We only consider optimal monetary policy under stationary population, since our interest is on how optimal inflation changes according to the change in demographics. We address it by changing such parameters as γ and b, that define the demographic structure.

Let us first explain as to why inflation in the stationary population can have real implications. As discussed in Bilbiie et al. (2014), the New Keynesian Phillips curve in (4) implies that fall in inflation rates raises the marginal cost when interest rates are low, while rise in inflation rates raises the marginal cost when interest rates are high. To highlight this relationship, consider the New Keynesian Phillips curve in steady state detrended by the total population:

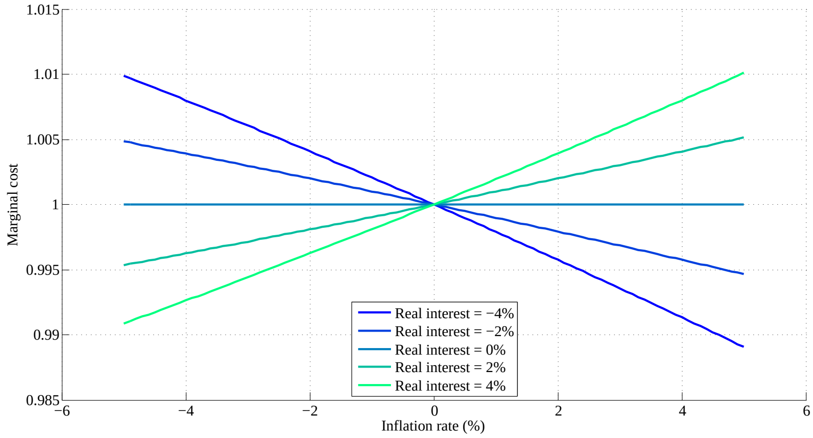

Figure 2: Steady state relationship with in CONK

$$\kappa\left(\psi-1\right)-\phi\left(\pi-1\right)\pi+\frac{\pi}{R}\phi\left(\pi-1\right)\pi\left(1+n\right)=0$$
$$\Leftrightarrow \psi=1+\frac{1}{\kappa}\left(1-\frac{\pi}{R}\left(1+n\right)\right)\phi\left(\pi-1\right)\pi=\frac{\kappa+\phi\pi\left[-\frac{\pi^2}{R}\left(1+n\right)+\frac{\pi}{R}\left(1+n\right)+\pi-1\right]}{\kappa},$$

where n denotes the population growth rate. Take the derivative of the RHS with respect to π gives

$$\phi\frac{\left[-3\frac{\pi^2}{R}\left(1+n\right)+2\frac{\pi}{R}\left(1+n\right)+1\right]}{\kappa}\Bigg|_{\pi=1}=\phi\frac{\left[-\frac{1}{R}\left(1+n\right)+1\right]}{\kappa},$$

which is positive when $R>1+n$. Namely, in steady state, marginal cost rises as inflation increases if and only if nominal interest is larger than the population growth rate. Figure 2 shows the relationship between the inflation rate and the marginal cost can be upward or downward sloping depending on the level of the steady state real interest rate.

Changes in the marginal cost can potential affect young and old differently. Higher marginal cost raises real wage and interest rates. Increase in wage will welfare-enhancing for workers since wage is their main source of income. On the other hand, low marginal cost can have positive effect on old because low marginal cost increases firm's profit. Since old has two source of income, i.e. return from financial assets and profits divided to old, this channel becomes more important depending on the amount of asset holdings. Thus, inflation rates in the steady state or stationary population matters for relative welfare between young and old.

In order to understand how demographic changes illustrated in Figure 1 affect optimal inflation rates for both young and old, respectively, we examine how changes in (a) the life expectancy, (b) the population growth rate, (c) the relative population, and (d) the relative asset holdings between young and old.

3.1 Life Expectancy

Figure 3 shows how the optimal inflation rates vary depending on the parameter values of γ. The vertical axis shows the optimal annualized inflation rate and the horizontal axis shows the life expectancy for old define by γ.

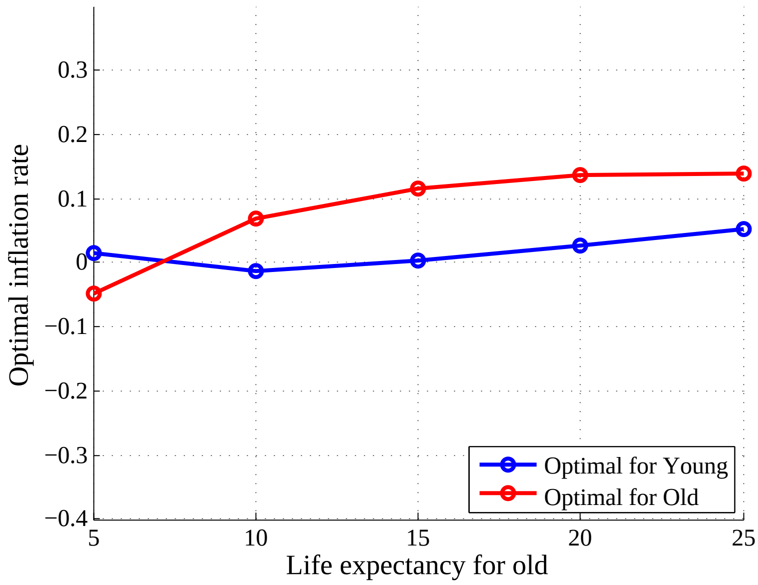

Figure 3: Optimal inflation rate by life expectancy

Young prefer lower inflation rate with longer longevity. This is because interest rate is lowered due to higher motive toward the saving-for-retirement in our overlapping generations economy. The lower the interest rate (and the higher marginal cost) gets, the higher the wage becomes. Such changes in macroeconomic variables are preferred by young because they can enjoy higher wage, which at the same time increases the marginal propensity to consume under our calibration of intertemporal elasticity of substitution being smaller than unity. On the other hand, old wants inflation rates to be higher because profits becomes larger. Note that if life expectancy is very long, specifically more than 35, the optimal inflation rate for old declines gradually because asset accumulation is large and return from the asset becomes more important source of income.

When the duration of old is short, however, young prefers higher inflation rate than old. In this case, since interest rate is high due to the relatively small saving for retirement, rise in inflation leads to a rise in marginal cost, which contrasts to the

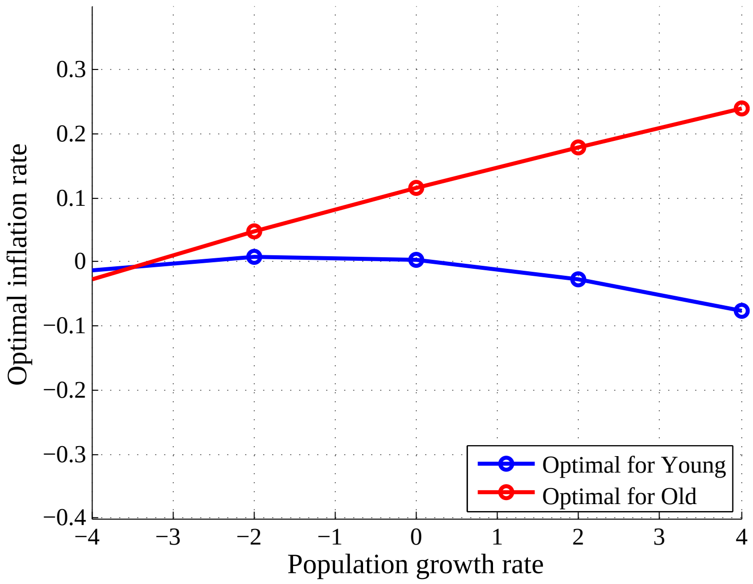

Figure 4: Optimal inflation rate by population growth

previous case. Indeed, real interest rate is greater than one and less than one when life expectancy is five years and ten years respectively. The rise in marginal cost contributes to raise wage and real rate of return. Thus, higher inflation rates are preferred by young while old can enjoy more consumption from lower inflation rate following the exactly opposite logic.

On the other hand, when the life expectancy becomes longer than 10 years, young's optimal inflation rate is increased. This is because their income composition becomes closer to the one of old. Young need to save more and increase welfare when they become old in (9).

3.2 Population Growth

Figure 4 compares the optimal inflation rates by different population growth rate. Horizontal axis is now annual population growth rate controlled by b.

High (low) population growth rate increases (reduces) interest rates and wage since it increases the capital-labor ratio. This changes how the long-run inflation rate affects marginal costs as illustrated in Figure 2. Namely, a positive population growth likely leads to $R < 1 + n$. Young prefer low inflation to achieve high wage. Old prefers high inflation to achieve low marginal cost and to increase profits. On the other hand, when population growth rate is relatively low, relation flips since it is likely $R > 1 + n$.

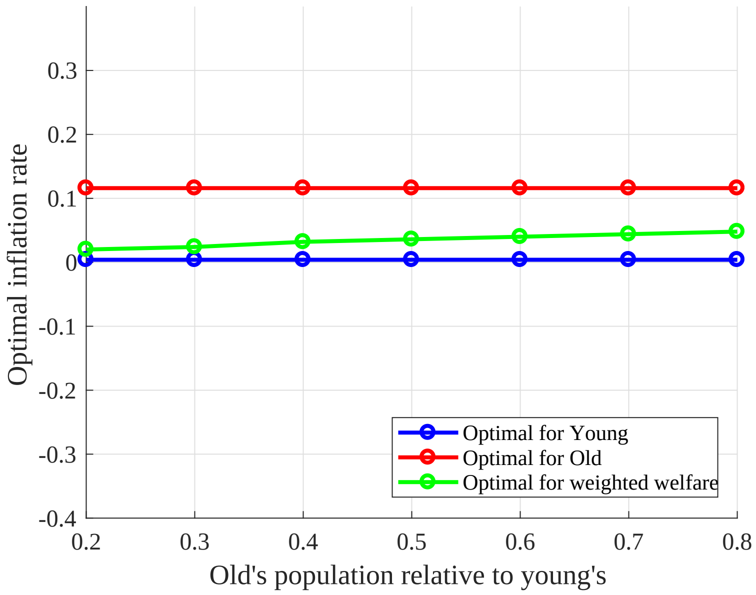

Figure 5: Optimal inflation rate by population ratio

3.3 Population Ratio

Figure 5 demonstrates how initial population ratio, N_0^y/N_0^o, affects the optimal inflation rates.

Changes in the composition of the population itself does not alter the optimal inflation rates for each young and old. The changes in the composition of the population affect only the weighted average of the optimal inflation rates.

3.4 Asset Ratio

Figure 6 illustrates how initial asset distribution affects the optimal inflation rates. In the following figure, we change the holding ratio at time zero, $A_0^y/(A_0^o + A_0^y)$, from 0.3 to 0.9. As young hold more fraction of asset, interest rate falls because they have smaller marginal propensity to consume. As implied by Figure 2, low rate of return implies that young prefers lower inflation rates. Note that our model abstracts the effects of surprise inflation on different consumers through nominal asset positions.

4 Aging and Optimal Inflation Rate

We calibrate the model to replicate the Japanese economy period by period and evaluate the optimal inflation rates in such a model.

In order to set initial state of the economy, we use the data of National Survey of Family Income and Expenditure issued by Statistics Bureau, Ministry of Internal Affairs and Communications as shown in Figure 1. We extract the data of estimated Value

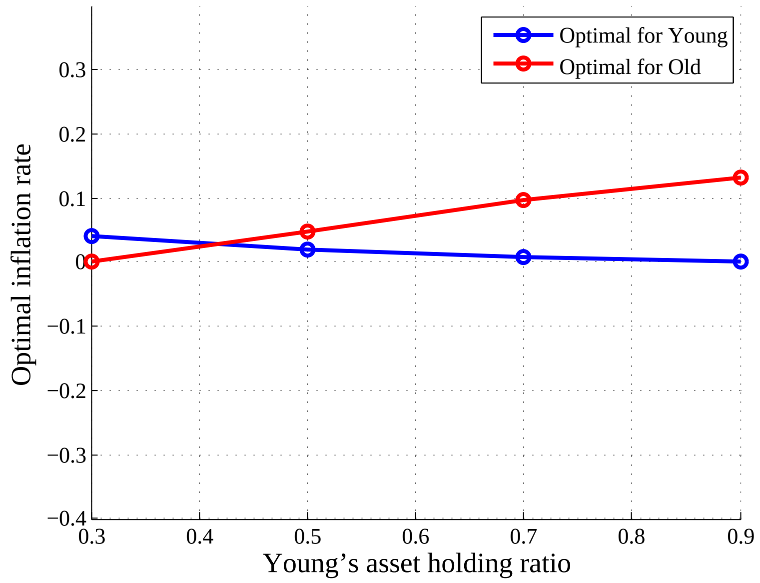

Figure 6: Optimal inflation rate by asset ratio

of Assets per Household by Age Group of Household Head from total households whenever available and from two-or-more-person households otherwise. Among the data, we compute the ratio of financial assets (savings less liabilities) between group over 60 and less than 60. We replicate the initial asset ratio, $A_0^y/(A_0^o + A_0^y)$, as observed in the data. Other initial state variables are set to those at the zero inflation steady state.

The survival rate for old, γ, is calibrated to match the life expectancy at age 65 from *Life Tables* published by Ministry of Health, Labor and Welfare. The birth rate, b, is set so that the population growth rate in the model, $b + \omega - 1$ match the data.

4.1 Simulation Result

Figure 7 shows the optimal inflation rates for young and old as well as the weighted average of the optimal inflation rates by population.

The optimal inflation rate is different from zero and the weighted average of the optimal inflation rates has been increasing. It is close to that of the young at the beginning and less so toward the end of the period. Declining population growth puts more weight on old.

Old prefers lower inflation rates as aging deepens, but the weighted average of inflation rates are increasing. This is because young prefers lower inflation rates than old with longer life expectancy as depicted in Figure 3.

Figure 8 compares those optimal inflation rates to the GDP deflator. According to our simulation results, the fluctuations in the weighted average of optimal inflation rates are very small compared to those found in the data. Since our focus is the

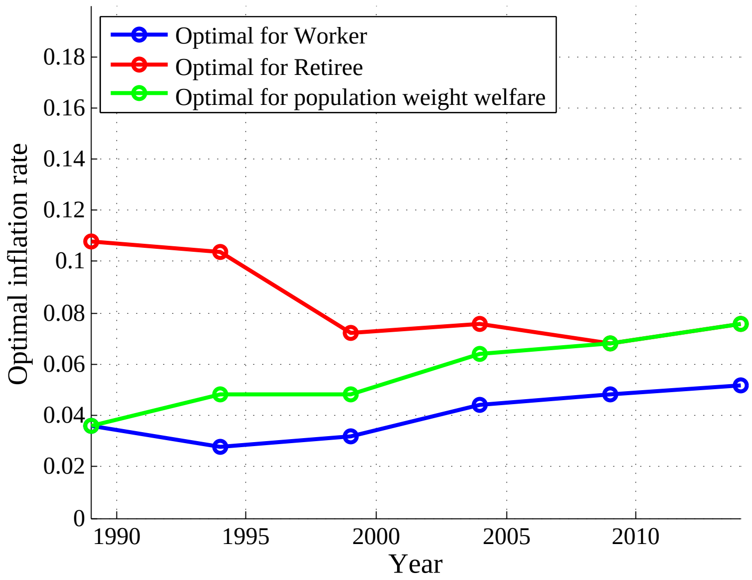

Figure 7: Optimal inflation rate

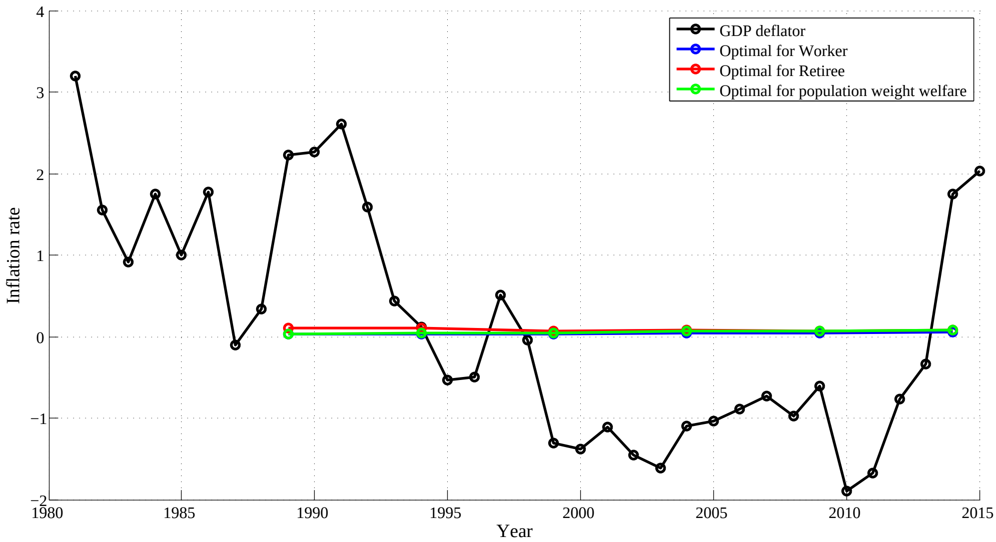

Figure 8: GDP deflator and optimal inflation rate

long-run optimal inflation rate by the Ramsey planner, we deliberately abstract heterogeneous impacts from surprise inflation via nominal asset holdings. The result in this paper implies that if there exists any political bias toward deflation in the grayer society, it should be due not to the optimal inflation rates in the long-run but to nominal contracts in financial transactions.

In our model, all assets are treated equally and all profits are shared by young and old by its nominal asset position. As a result, initial real asset positions, which is nominal asset position divided by the price level, disappear from the wealth in the solved out consumption functions. In reality, young and old are different in compositions in assets and liabilities. Also, the speed with which each nominal asset adjusts to the developments in inflation rates is different. Careful analysis on the household's balance sheet, in particular, how components of assets and liabilities are constrained by nominal contracts, or which assets are owned by each agent, is needed to understand the asymmetric impacts of surprise inflation to young and old.

5 Conclusion

In this paper, we show that the demographic structure can potentially have significant implications for optimal inflation rates for young and old. The fluctuations in the weighted average of optimal inflation rates following the aging in Japan are, however, very small compared to those found in the data. This result implies that if there exists any political bias toward deflation in the grayer society, it should be due not to the optimal inflation rates in the long-run but to nominal contracts in financial transactions. We deliberately abstract such channel.

In the balance sheet of young households, the largest asset is often the housing stock while the largest debt tends to be the mortgage loan. The house price can adjust more quickly to inflation fluctuations that the mortgage loan with nominal contracts. Under such circumstances, even net nominal asset position is positive, young households are likely to prefer higher inflation so that they can reduce the real debt, while the real asset remains to be rather constant. In order to gauge the tension surrounding optimal inflation rates between young and old, the detailed analysis of the households' balance sheet, in particular, how components of assets and liabilities are constrained by nominal contracts, or which assets are owned by each agent, is needed. Gauging the asymmetric impacts of surprise inflation rates in the general equilibrium framework is a challenging task. This agenda is left for our future study.

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Appendix

A Derivation of the Model

TBA

B Extension: Variable labor supply

TBA