Learning about the Neighborhood*

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March 2018

Abstract

We develop a model of neighborhood choice to analyze information aggregation and learning in housing and commercial real estate markets. In the presence of pervasive informational frictions, housing prices serve as important signals to households and commercial developers about the economic strength of a neighborhood. Through this learning channel, noise from housing market supply and demand shocks can propagate from housing prices to the local economy, distorting not only migration into the neighborhood, but also supply of commercial facility. Our analysis also provides testable, nuanced implications on how the magnitudes of these noise effects vary across neighborhoods with different elasticity of housing supply and degree of complementarity of their industries.

*We are grateful to Itay Goldstein and seminar participants of 2018 AEA Meetings and Fordham University for helpful comments.

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Widespread optimism is recognized as an important driver of the U.S. housing cycle in the 2000s that led to the subsequent financial crisis and the Great Recession, e.g., Cheng, Raina and Xiong (2014) and Kaplan, Mitman, and Violante (2017). How was the optimism developed? The literature has emphasized the importance of accounting for home buyers' expectations, in particular extrapolative expectations, in understanding dramatic housing boom and bust cycles, e.g., Case and Shiller (2003), Glaeser, Gyourko, and Saiz (2008), Piazzesi and Schneider (2009), and Glaeser and Nathanson (2017). However, much of the analyses and discussions are made in the absence of a systematic framework that anchors home buyers' expectations to their information aggregation and learning process. In this paper, we help fill this gap by developing a model for analyzing information aggregation and learning in housing markets, and its spillover to other investment decisions such as development of commercial real estate.

Specifically, we develop a model to analyze how information frictions affect the learning and beliefs of households and developers about a neighborhood, which in turn drives both housing market dynamics and investment decisions in the neighborhood. The model features a continuum of households in an open neighborhood, which can be viewed as a county. Each household has a choice of whether to move into the neighborhood by purchasing a house, and has a Cobb-Douglas utility function over its consumption of its own good and its aggregate consumption of the goods produced by other households in the neighborhood. This complementarity in households' consumption motivates each household to learn about the unobservable economic strength of the neighborhood, which determines the common productivity of all households and, consequently, their desire to live in the neighborhood. Each household requires both labor, which it supplies, and commercial facility, to produce its good according to a Cobb-Douglas production function of these two inputs. Since the price of commercial facility depends on its marginal product across households in the neighborhood, competitive commercial developers must also form expectations about the economic strength of the neighborhood when determining how much commercial facility to develop.

The local housing market provides a useful platform for aggregating private information about the economic strength of the neighborhood. It is intuitive that the traded housing price reflects the net effect of demand and supply factors, in a similar spirit of the classic models of Grossman and Stiglitz (1980) and Hellwig (1980) for information aggregation in asset markets. Different from the linear equilibrium in these models, our model features

an important neighborhood selection, through which only households with private signals above a certain equilibrium cutoff choose to live in the neighborhood. This selection makes our model inherently non-linear, which posts a great challenge to households' learning and information aggregation. Nevertheless, we are able to derive the equilibrium in analytical forms, building on the cutoff equilibrium framework developed by Goldstein, Ozdenoren, and Yuan (2013) and Albagli, Hellwig, and Tsyvinski (2014, 2015) for asset markets.

There are two key features contributing to the tractability. First, despite the equilibrium housing price being a non-linear function, its information content about the neighborhood strength can nevertheless be summarized by a linear sufficient statistic, which keeps households' learning from the housing price tractable. Second, despite each household's housing demand being non-linear, the Law of Large Numbers allows us to aggregate their housing demand, and to derive a cutoff equilibrium for the housing market. In our setting, each household possesses a private signal regarding the neighborhood common productivity. By aggregating the households' housing demand, the housing price aggregates their private signals. The presence of unobservable supply shocks, however, prevents the housing price from perfectly revealing the neighborhood strength and acts as a source of informational noise in the housing price.

Our model allows us to analyze how informational frictions affect not only the housing price but also the households' neighborhood, labor and production choices, which, in turn, determine their demands for housing and commercial facility. The housing price plays a key role in affecting agents' expectations. Through this learning channel, noise in the housing price, originated from either demand or supply side of the housing market, may affect the housing price and the local economy. That is, by pushing up the housing price, a noise factor may lead to more households moving into the neighborhood, a more pronounced housing cycle, and, interestingly, greater over-supply of commercial facility and a more pronounced commercial real estate cycle in the neighborhood.

Our analysis highlights how the magnitudes of these noise effects may vary across different neighborhoods along two dimensions: 1) elasticity of housing supply in the neighborhood and 2) the degree of households' consumption complementarity. In particular, the noise effect induced by agents' learning on the housing price is hump-shaped with respect to housing supply elasticity, due to the following reason. At one end with housing supply being infinitely inelastic, the housing price is fully determined by housing demand and thus perfectly reveals

the strength of the neighborhood; at the other end with housing supply being perfectly elastic, housing price is fully determined by housing supply and is thus not affected by household expectations. At both ends, learning does not distort the housing price. As a result, the noise effect on housing price is strongest at intermediate supply elasticities. This insight reflects that supply-side characteristics of the local housing market may interfere with the informativeness of the housing price about housing demand and neighborhood strength, leading to nuanced empirical predictions.

In particular, our analysis shows that the distortionary effects induced by learning on population inflow and housing and commercial real estate cycles are most pronounced in areas with intermediate supply elasticties, rather than areas with the most inelastic housing supply. This result helps explain why areas like Las Vegas and Pheonix with relatively more elastic housing supplies had more dramatic housing cycles than New York and San Francisco, as documented in, for instance, Davidoff (2013), Glaeser (2013), and Nathanson and Zwick (2017). Our analysis also shows that these distortionary effects induced by learning tend to increase with households' consumption complementarity because greater complementarity makes learning about the neighborhood strength a more important part of household decisions. These results give rise to testable hypotheses in the cross-section when sorting areas by supply elasticity or the degree of complementarity of their industries.

Our analysis also highlights a learning externality in that when making housing choices, households do not internalize the subsequent effects on the expectations of commercial developers. To the extent that any overbuilding of offices and commercial infrastructure is difficult to reverse in the short or medium-term, the excess supply can have prolonged, overhang effects on the local economies. Gao, Sockin, and Xiong (2017), for instance, find consistent evidence that supply overhang in housing markets helped transmit the adverse impact of housing speculation to the real economy during the recent bust.

Our work features a tractable cutoff equilibrium framework, similar to that in Goldstein, Ozdenoren, and Yuan (2013) and Albagli, Hellwig, and Tsyvinski (2014, 2015). Goldstein, Ozdenoren, and Yuan (2013) investigate the feedback to the investment decisions of a single firm when managers, but not investors, learn from prices. Albagli, Hellwig, and Tsyvinski (2014, 2015) focus on the role of asymmetry in security payoffs in distorting asset prices and firm investment incentives when future shareholders learn from prices to determine their valuations. These models commonly employ risk-neutral agents, normally distributed asset

fundamentals, and position limits to deliver tractable nonlinear equilibria. In contrast, we focus on the feedback induced by learning from housing prices to household neighborhood choice and labor decisions in an equilibrium production setting with consumption complementarity and goods trading between households, and the spillover to investment decisions of commercial developers. By showing that the cutoff equilibrium framework can be conveniently adopted to analyze learning effects in this complex setting, our model substantially expands the scope of this framework.

Our model differs from Burnside, Eichenbaum, and Rebelo (2016), which offers a model of housing market booms and busts based on the epidemic spreading of optimistic or pessimistic beliefs among home buyers through their social interactions. Our learning-based mechanism is also different from Nathanson and Zwick (2017), which studies the hoarding of land by home builders in certain elastic areas as a mechanism to amplify price volatility in the recent U.S. housing cycle. Glaeser and Nathanson (2017) presents a model of biased learning in housing markets, building on current buyers not adjusting for the expectations of past buyers, and instead assuming that past prices reflect only contemporaneous demand. This incorrect inference gives rise to correlated errors in housing demand forecasts over time, which in turn generate excess volatility, momentum, and mean-reversion in housing prices. In contrast to this model, informational frictions in our model anchor on the interaction between the demand and supply sides, and feed back to both housing price and real outcomes. This key feature is also different from the amplification to price volatility induced by dispersed information and short-sale constraints featured in Favara and Song (2014).

By focusing on information aggregation and learning of symmetrically informed households with dispersed private information, our study differs in emphasis from those that analyze the presence of information asymmetry between buyers and sellers of homes, such as Garmaise and Moskowitz (2004) and Kurlat and Stroebel (2014). Neither does our model emphasize the potential asymmetry between in-town and out-of-town home buyers, which is shown to be important by Chinco and Mayer (2015).

In addition, there are extensive studies in the housing literature highlighting the roles played by both demand-side and supply-side factors in driving housing cycles. On the demand side, Himmelberg, Mayer, and Sinai (2005) focus on interest rates, Poterba, Weil, and Shiller (1991) on tax changes, Mian and Sufi (2009) on credit expansion, and DeFusco, Nathanson, and Zwick (2017) and Gao, Sockin and Xiong (2017) on investment home pur-

chases. On the supply side, Glaeser, Gyourko, Saiz (2008) emphasize supply as a key force in mitigating housing bubbles, Haughwout, Peach, Sporn and Tracy (2012) provide a detailed account of the housing supply side during the U.S. housing cycle in the 2000s, and Gyourko (2009b) systematically reviews the literature on housing supply. By introducing informational frictions, our analysis shows that supply-side and demand-side factors are not mutually independent. Supply shocks can affect housing and commercial real estate demand by acting as informational noise in learning, and influence households' and commercial developers' expectations of the strength of the neighborhood.

1 The Model

The model has two periods $t \in \{1, 2\}$. There are three types of agents in the economy: households looking to buy homes in a neighborhood or elsewhere, home builders, and commercial real estate developers. Suppose that the neighborhood is new and all households purchase houses from home builders in a centralized market at $t = 1$ after choosing whether to live in the neighborhood. Households choose their labor supply and demand for commercial facilities, such as offices and warehouses, to complete production, and consume consumption goods at $t = 2$. Our intention is to capture the decision of a generation of home owners to move into a neighborhood, and we view the two periods as representing a long period in which they live together and share amenities, as well as exchange their goods and services.

1.1 Households

We consider a pool of households, indexed by $i \in [0, 1]$, each of which can choose to live in a neighborhood or elsewhere. We can divide the unit interval into the partition $\{\mathcal{N}, \mathcal{O}\}$, with $\mathcal{N} \cap \mathcal{O} = \varnothing$ and $\mathcal{N} \cup \mathcal{O} = [0, 1]$. Let $H_i = 1$ if household i chooses to live in the neighborhood, i.e., $i \in \mathcal{N}$, and $H_i = 0$ if it chooses to live elsewhere.[1] If household i at $t = 1$ chooses to live in the neighborhood, it must purchase one house at price P. This reflects, in part, that housing is an indivisible asset and a discrete purchase, consistent with the insights of Piazzesi and Schneider (2009).

Household i in the neighborhood has a Cobb-Douglas utility function over consumption of its own good $C_i(i)$ and its consumption of the goods produced by all other households in

[1] See Van Nieuwerburgh and Weill (2010) for a systematic treatment of moving decisions by households across neighborhoods.

the neighborhood $\{C_j(i)\}_{j\in\mathcal{N}}$:

$$U\left(\{C_j(i)\}_{j\in\mathcal{N}};\mathcal{N}\right)=\left(\frac{C_i(i)}{1-\eta_c}\right)^{1-\eta_c}\left(\frac{\int_{\mathcal{N}/i}C_j(i)\,dj}{\eta_c}\right)^{\eta_c}. \tag{1}$$

The parameter $\eta_c\in(0,1)$ measures the weights of different consumption components in the utility function. A higher η_c means a stronger complementarity between household i's consumption of its own good and its consumption of the composite good produced by the other households in the neighborhood. As we will discuss later, this utility specification implies that each household cares about the strength of the neighborhood, i.e., the productivities of other households in the neighborhood. This assumption leads to strategic complementarity in each household's housing demand, as motivated by the empirical findings of Ioannides and Zabel (2003).[2]

The production function of household i is also Cobb-Douglas $e^{A_i}K_i^{\alpha}l_i^{1-\alpha}$, where l_i is the household's labor choice and A_i is its productivity. Different from the usual production function of having capital as an input, we introduce another factor K_i as commercial facility with a share of $\alpha\in(0,1)$ in the production function. We broadly interpret the commercial facility as office space, infrastructure, or other investment households can use for their productive activities in the neighborhood. As we describe later, the households buy commercial facility from commercial developers. When households are more productive in the neighborhood, the marginal productivity of commercial facility is higher, and consequently commercial develops would be able to sell more commercial facility at higher prices. Introducing commercial facility allows us to discuss how learning affects price and supply of not only residential housing but also commercial real estate and other related investment in the neighborhood.[3]

Household i's productivity A_i is comprised of a component A, common to all households in the neighborhood, and an idiosyncratic component ε_i:

$$A_i=A+\varepsilon_i,$$

where $A\sim\mathcal{N}\left(\bar{A},\tau_A^{-1}\right)$ and $\varepsilon_i\sim\mathcal{N}\left(0,\tau_\varepsilon^{-1}\right)$ are both normally distributed and independent of each other. Furthermore, we assume that $\int\varepsilon_i d\Phi(\varepsilon_i)=0$ by the Strong Law of Large

[2]There are other types of social interactions between households living in a neighborhood, which are explored, for instance, in Durlauf (2004) and Glaeser, Sacerdote, and Scheinkman (2003).

[3]One can extend our analysis to consider K to be a public good, in which case its price is the tax a local government that faces a balanced budget can raise to offset the cost of construction. Our model then has implications for how housing markets impact local government fiscal policy.

Numbers. The common productivity, A, represents the strength of the neighborhood, as a higher A implies a more productive neighborhood. As A determines the households' aggregate demand for housing, it represents the demand-side fundamental. One can view τ_ε as a measure of household diversity.

As a result of realistic informational frictions, A is not observable to households at $t = 1$ when they need to make the decision of whether to live in the neighborhood. Instead, each household observes its own productivity A_i, after examining what it can do if it chooses to live in the neighborhood. Intuitively, A_i combines the strength of the neighborhood A and the household's own attribute ε_i. Thus, A_i also serves as a noisy private signal about A at $t = 1$, as the household cannot fully separate its own attribute from the opportunity provided by the neighborhood. The parameter τ_ε governs both the diversity in the neighborhood, or dispersion in productivity, and the precision of this private signal. As $\tau_\varepsilon \to \infty$, the households' signals become infinitely precise and the informational frictions about A vanish. Households care about the strength of the neighborhood because of complementarity in their demand for consumption. Consequently, while a household may have a fairly good understanding of its own productivity when moving into a neighborhood, complementarity in consumption demand motivates it to pay attention to housing prices to learn about the average level A for the neighborhood.

We start with each household's problem at $t = 2$ and then go backwardly to describe its problem at $t = 1$. At $t = 2$, we assume that A is revealed to all agents. Furthermore, we assume that each household experiences a disutility for labor $\frac{l_i^{1+\psi}}{1+\psi}$, and that a household in the neighborhood $\mathcal{N}$ maximizes its utility at $t = 2$ by choosing labor l_i, commercial facility K_i, and its consumption demand $\{C_j(i)\}_{j\in\mathcal{N}}$:

$$\begin{aligned} U_i &= \max_{\left\{\{C_j(i)\}_{j\in\mathcal{N}}, l_i, K_i\right\}} U\left(\{C_j(i)\}_{j\in\mathcal{N}}; \mathcal{N}\right) - \frac{l_i^{1+\psi}}{1+\psi} \qquad (2) \\ &\text{such that } \; p_i C_i(i) + \int_{\mathcal{N}/i} p_j C_j(i)\, dj + RK_i = p_i e^{A_i} K_i^{\alpha} l_i^{1-\alpha}, \end{aligned}$$

where p_i is the price of the good it produces, P is the housing price in the neighborhood, and R is the unit price of commercial facility. Households behave competitively and take the prices of their goods as given.

At $t = 1$, before choosing its consumption, commercial facility usage, and labor supply, a household needs to decide whether to live in the neighborhood. In addition to their private

signals, all households and commercial developers observe a noisy public signal Q about the strength of the neighborhood A:

$$Q = A + \tau_Q^{-1/2} \varepsilon_Q,$$

where $\varepsilon_Q \sim \mathcal{N}(0,1)$ is independent of all other shocks. As τ_Q becomes arbitrarily large, A becomes common knowledge to all agents.

In addition to the utility flow U_i at $t = 2$ from final consumption, we assume that households have quasi-linear expected utility at $t = 1$, and incur a linear utility penalty equal to the housing price P if they choose to live in the neighborhood and thus have to buy a house. Given that households have Cobb-Douglas preferences over their consumption, they are effectively risk-neutral at $t = 1$, and their utility flow is then the value of their final consumption bundle less the cost of housing.[4] Households make their neighborhood choice subject to a participation constraint that their expected utility from moving into the neighborhood $E[U_i|\mathcal{I}_i] - P$ must (weakly) exceed a reservation utility, which we normalize to 0. One can interpret the reservation utility as the expected value of getting a draw of productivity from another potential neighborhood less the cost of search. Household i makes its neighborhood choice:

$$\max\{E[U_i|\mathcal{I}_i] - P, 0\} \tag{3}$$

The choice of neighborhood is made at $t = 1$ subject to each household's information set $\mathcal{I}_i = \{A_i, P, Q\}$, which includes its private productivity signal A_i, the public signal Q, and the housing price P.[5]

1.2 Commercial Developers

In addition to households, there is a continuum of risk-neutral commercial developers that develop commercial facility at $t = 1$, and sells them to households for their production at $t = 2$. The representative developer cares about R, the price of commercial facility at $t = 2$, which depend on the marginal productivity of the facility. This, in turn, depends on the

[4] For simplicity, our model does not incorporate resale of the housing after $t = 2$. As a result, we cannot simply deduct the housing price P as the housing cost from the household's budget constraint at $t = 2$. Instead, we separately treat the housing cost as a utility cost proportional to the housing price at $t = 1$. This utility cost is sufficient to capture the notion that a higher housing price implies a greater housing cost to the household without explicitly accounting for different components of the housing cost, such as initial cost of purchase, cost of mortgage loan, and resale value later.

[5] We do not include the volume of housing transactions in the information set as a result of a realistic consideration that, in practice, people observe only delayed reports of total housing transactions at highly aggregated levels, such as national or metropolitan levels.

strength of the neighborhood, and which households choose to live in the neighborhood. The housing price in the neighborhood serves as a useful signal to the developer when deciding how much commercial facilities to develop at $t = 1$.

To simplify our analysis and distinguish our mechanism from that of Rosen (1979) and Roback (1982), we decouple the supply of residential housing from the supply of commercial real estate. We assume that commercial developers can develop K units of commercial facility by incurring a convex effort cost $\frac{1}{\lambda}K^{\lambda}$, where $\lambda > 1$.

We assume that households buy commercial facilities from commercial developers when production occurs at $t = 2$, and that commercial developers must forecast this demand when choosing how much commercial facility K to develop at $t = 1$. The representative commercial developer takes the commercial facility price R as given, and chooses K to maximize its expected profit:

$$\Pi_c = \sup_K E\left[RK - \frac{1}{\lambda}K^{\lambda}\middle|\mathcal{I}^c\right] \tag{4}$$

where $\mathcal{I}^c = \{P, Q\}$ is the public information set, which includes the housing price P and the public signal Q. It then follows that the optimal choice of commercial facility sets the marginal cost, $K^{\lambda-1}$, equal to the marginal benefit, $E\left[R|\mathcal{I}^c\right]$:

$$K = E\left[R|\mathcal{I}^c\right]^{\frac{1}{\lambda-1}}.$$

The choice of commercial facility is influenced by the expectation of the commercial developer about future neighborhood productivity, which is affected by the realization of the housing price P. Market-clearing in the market for commercial facility at $t = 2$ requires that

$$\int_{\mathcal{N}} K_i di = K \int_{\mathcal{N}} di, \tag{5}$$

where $\int_{\mathcal{N}} di$ represents the population of households that live in the neighborhood.

The commercial developers' decision to develop commercial facility at $t = 1$ gives another source of amplification for informational frictions. In addition to distorting neighborhood choice of potential household entrants, informational frictions in housing markets also distort local investment in the neighborhood.

1.3 Home Builders

There is a population of home builders, indexed on a continuum $[0, 1]$, in the neighborhood. Home builders also face uncertainty about the aggregate strength of the neighborhood and

the ability of the supply side to respond to the demand. Specifically, builder i builds a single house subject to a disutility from labor:

$$e^{-\frac{1}{1+k}\omega_i} S_i,$$

where $S_i \in \{0, 1\}$ is the builder's decision to build and

$$\omega_i = \xi + e_i$$

is the builder's productivity, which is correlated across builders in the neighborhood through ξ. We assume that $\xi = k\zeta$, where $k \in (0, \infty)$ is a constant parameter, and ζ represents an unobserved, common shock to building cost in the neighborhood. From the perspective of households and builders, $\zeta \sim \mathcal{N}\left(\bar{\zeta}, \tau_\zeta^{-1}\right)$. Then $\xi = k\zeta$ can be interpreted as a supply shock with normal distribution $\xi \sim \mathcal{N}\left(\bar{\xi}, k^2\tau_\zeta^{-1}\right)$ with $\bar{\xi} = k\bar{\zeta}$. Furthermore, $e_i \sim \mathcal{N}\left(0, \tau_e^{-1}\right)$ such that $\int e_i d\Phi\left(e_i\right) = 0$ by the Strong Law of Large Numbers.

Builders in the neighborhood at $t = 1$ maximize their revenue:

$$\Pi_s\left(S_i\right) = \max_{S_i} \left(P - e^{-\frac{1}{1+k}\omega_i}\right) S_i. \tag{6}$$

Since builders are risk-neutral, it is easy to determine the builders' optimal supply curve:

$$S_i = \begin{cases} 1 & \text{if } P \geq e^{-\frac{k\zeta + e_i}{1+k}} \\ 0 & \text{if } P < e^{-\frac{k\zeta + e_i}{1+k}} \end{cases}. \tag{7}$$

The parameter k measures the supply elasticity of the neighborhood. A more elastic neighborhood has a larger supply shock, i.e., the supply shock has greater mean and variance. In the housing market equilibrium, the supply shock ξ not only affects the supply side but also the demand side, as it acts as informational noise in the price signal when the households use the price to learn about the common productivity A.

1.4 Noisy Rational Expectations Cutoff Equilibrium

Our model features a noisy rational expectations cutoff equilibrium, which requires clearing of the two real estate markets that is consistent with the optimal behavior of households, home builders and commercial developers:

- Household optimization: each household chooses H_i at $t = 1$ to solve its maximization problem in (3), and then chooses $\left\{\left\{C_j\left(i\right)\right\}_{i \in \mathcal{N}}, l_i, K_i\right\}$ at $t = 2$ to solve its maximization problem in (2).

- Commercial developer optimization: the representative developer chooses K at $t = 1$ to solve its maximization problem in (4).

- Builder optimization: each builder chooses S_i at $t = 1$ to solve his maximization problem in (6).

- At $t = 1$, the residential housing market clears:

$$\int_{-\infty}^{\infty} H_i\left(A_i, P, Q\right) d\Phi\left(\varepsilon_i\right) = \int_{-\infty}^{\infty} S_i\left(\omega_i, P, Q\right) d\Phi\left(e_i\right),$$

  where each household's housing demand $H_i\left(A_i, P, Q\right)$ depends on its productivity A_i, the housing price P, and the public signal Q, and each builder's housing supply $S_i\left(\omega_i, P, Q\right)$ depends on its productivity ω_i, the housing price P, and the public signal Q. The demand from households and supply from builders are integrated over the idiosyncratic components of their productivities $\left\{\varepsilon_i\right\}_{i\in[0,1]}$ and $\left\{e_i\right\}_{i\in[0,1]}$, respectively.

- At $t = 2$, the market for each household's good clears:

$$C_i\left(i\right) + \int_{\mathcal{N}/i} C_j\left(i\right) dj = e^{A_i} K_i^{\alpha} l_i^{1-\alpha}, \quad \forall\ i \in \mathcal{N},$$

  and the market for commercial facility clears:

$$\int_{\mathcal{N}} K_i di = K \int_{\mathcal{N}} di.$$

2 Equilibrium

In this section, we analyze a symmetric cutoff equilibrium, in which the choice of each household to live in the neighborhood is monotonic with respect to its own productivity A_i.

2.1 Choices of Households and Commercial Developers

We first analyze household choices. At $t = 2$, households need to make their production and consumption decisions, after the strength of the neighborhood A is revealed to the public, and home builders and commercial developers have also made their choices at $t = 1$. Household i has $e^{A_i} K_i^{\alpha} l_i^{1-\alpha}$ units of good i for consumption and trading with other households. It maximizes its utility function given in (2). The following proposition describes the household's consumption, labor, and commercial facility choices. Its marginal utility of goods consumption also gives the equilibrium goods price.

Proposition 1 *Households i's optimal goods consumption at $t=2$ are*

$$C_i(i) = (1-\eta_c)(1-\alpha)\, e^{A_i} K_i^{\alpha} l_i^{1-\alpha},\ C_j(i) = \frac{1}{\Phi\left(\sqrt{\tau_\varepsilon}\,(A-A^*)\right)} \eta_c (1-\alpha)\, e^{A_j} K_j^{\alpha} l_j^{1-\alpha},$$

and the price of its produced good is

$$p_i = e^{\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\eta_c(A-A_i)+\frac{1}{2}\eta_c\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\right)^2\tau_\varepsilon^{-1}} \frac{\Phi^{\eta_c}\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi^{\eta_c}\left(\sqrt{\tau_\varepsilon}\,(A-A^*)\right)}.$$

Its optimal labor and commercial facility choices are

$$\begin{aligned}
\log l_i &= \frac{1}{1-\alpha}\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\frac{\eta_c}{\psi}A + \frac{1-\eta_c}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A_i - \frac{\alpha}{1-\alpha}\frac{1}{\psi}\log R \\
&\quad + \frac{1}{1-\alpha}\frac{\eta_c}{\psi}\log\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\varepsilon}\,(A-A^*)\right)} + l_0, \\
\log K_i &= \frac{1}{1-\alpha}\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\frac{1+\psi}{\psi}\eta_c A + \frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A_i \\
&\quad - \frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\log R + \frac{1}{1-\alpha}\frac{1+\psi}{\psi}\eta_c\log\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\varepsilon}\,(A-A^*)\right)} + h_0,
\end{aligned}$$

with constants l_0 and h_0 given in the Appendix. Furthermore, the expected utility of household i at $t=1$ is given by

$$E\left[U\left(\{C_j(i)\}_{j\in\mathcal{N}};\mathcal{N}\right) - \frac{l_i^{1+\psi}}{1+\psi}\middle|\,\mathcal{I}_i\right] = (1-\alpha)\frac{\psi}{1+\psi}E\left[p_i e^{A_i}K_i^{\alpha}l_i^{1-\alpha}\middle|\,\mathcal{I}_i\right].$$

Proposition 1 shows that each household spends a fraction $1-\eta_c$ of its wealth (excluding housing wealth) on consuming its own good $C_i(i)$ and a fraction η_c on goods produced by its neighbors $\int_{\mathcal{N}/i} C_j(i)\,dj$. When $\eta_c = 1/2$, the household consumes its own good and the goods of its neighbors equally. The price of each good is determined by its output relative to that of the rest of the neighborhood. One household's good is more valuable when the rest of the neighborhood produces more, and thus each household needs to take into account the labor decisions of the other households in its neighborhood when making its own decision. The proposition demonstrates that the labor chosen by a household is determined by not only its own productivity e^{A_i} but also the aggregate productivities of other households in the neighborhood. This latter component arises from the complementarity in the household's utility function.

Proposition 1 also reveals that the optimal choice of labor for each household is log-linear with the strength of the neighborhood A, its own productivity A_i, and the logarithm of the

commercial real estate price $\log R$. The final (nonconstant) term reflects selection, in that only households with productivity above A^* enter the neighborhood. Since A is the mean of the distribution of household productivity, it shows up in this truncation. This proposition also demonstrates that household i's optimal choice of commercial facility has a similar functional form. The household's optimal labor choice and demand for commercial facility are both increasing in the strength of the neighborhood A because a higher A represents improved trading opportunities with its neighbors, while they are both decreasing in the price of commercial facility $\log R$.

We now discuss each household's decision on whether to live in the neighborhood at $t = 1$ when it still faces uncertainty about A. As a result of Cobb-Douglas utility, the household is effectively risk-neutral over its aggregate consumption, and its optimal choice reflects the difference between its expected output in the neighborhood and the cost of living in the neighborhood, which is the price P to buy a house. Then, household i's neighborhood decision is given by

$$H_i = \begin{cases} 1 & \text{if} \quad (1-\alpha)\frac{\psi}{1+\psi} E\left[p_i e^{A_i} K_i^{\alpha} l_i^{1-\alpha} \middle| \mathcal{I}_i\right] \geq P \\ 0 & \text{if} \quad (1-\alpha)\frac{\psi}{1+\psi} E\left[p_i e^{A_i} K_i^{\alpha} l_i^{1-\alpha} \middle| \mathcal{I}_i\right] < P \end{cases}.$$

This decision rule for neighborhood choice supports our conjecture to search for a cutoff strategy for each household, in which only households with productivities above a critical level A^* enter the neighborhood. This cutoff is eventually solved as a fixed point in the equilibrium.

Given each household's equilibrium cutoff A^* at $t = 1$ and optimal choices at $t = 2$, we can impose market-clearing in the market for commercial facility to arrive at its price R at $t = 2$. Commercial developers forecast this price when choosing their optimal stock of commercial facility to develop at $t = 1$. These observations are summarized by the following proposition.

Proposition 2 *Given K units of commercial facility developed by commercial developers at $t = 1$, the price of commercial facility R at $t = 2$ takes the log-linear form:*

$$\begin{aligned} \log R \;=\; & \frac{1+\psi}{\psi+\alpha} A - (1-\alpha)\frac{\psi}{\psi+\alpha}\log K + \frac{1+\psi}{\psi+\alpha}\eta_c \log \frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2} + \frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\varepsilon}\,(A-A^*)\right)} \\ & + (1-\alpha)\frac{\psi}{\psi+\alpha}\log \frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2} + \frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\varepsilon}\,(A-A^*)\right)} + r_0, \end{aligned}$$

with constant r_0 given in the Appendix. The optimal supply of commercial facility by commercial developers at $t = 1$ is given by

$$\log K = \frac{1}{\lambda - \alpha\frac{1+\psi}{\psi+\alpha}} \log E \left[e^{\frac{1+\psi}{\psi+\alpha}A} \left(\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2} + \frac{A-A^*}{\tau_\varepsilon^{-1/2}} \right)}{\Phi\left(\sqrt{\tau_\varepsilon}\,(A-A^*)\right)} \right)^{\frac{1+\psi}{\psi+\alpha}\eta_c} \right. \tag{8}$$
$$\left. \cdot \left(\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2} + \frac{A-A^*}{\tau_\varepsilon^{-1/2}} \right)}{\Phi\left(\sqrt{\tau_\varepsilon}\,(A-A^*)\right)} \right)^{\frac{\psi(1-\alpha)}{\psi+\alpha}} \middle| \mathcal{I}^c \right] + k_0,$$

with constant k_0 given in the Appendix.

Proposition 2 reveals that the commercial real estate price at $t = 2$ is increasing in the strength of the neighborhood A with the last two (nonconstant) terms reflecting selection by households into the neighborhood, and is decreasing in the supply of commercial facility K. It also demonstrates that the optimal supply of commercial facility reflects expectations over not only the strength of the neighborhood A, but also the impact of truncation from the neighborhood choice of households on the expected price of commercial facility at $t = 2$. The expectation term captures not only the expected productivity from the terms-of-trade (relative prices of household goods) in the first ratio, but also the dispersion in labor productivity in the second ratio.

2.2 Perfect-Information Benchmark

With perfect information, all households, home builders, and commercial developers observe the strength of the neighborhood A when making their respective decisions. It is straightforward to show that the optimal choice of commercial facility K simplifies to

$$\log K = \frac{\frac{1+\psi}{\psi+\alpha}}{\lambda - \alpha\frac{1+\psi}{\psi+\alpha}} A + \frac{\frac{1+\psi}{\psi+\alpha}}{\lambda - \alpha\frac{1+\psi}{\psi+\alpha}} \eta_c \left\{ \log \left[\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2} + \frac{A-A^*}{\tau_\varepsilon^{-1/2}} \right)}{\Phi\left(\sqrt{\tau_\varepsilon}\,(A-A^*)\right)} \right] \right.$$
$$\left. + (1-\alpha)\frac{\psi}{1+\psi}\frac{\frac{1+\psi}{\psi+\alpha}}{\lambda - \alpha\frac{1+\psi}{\psi+\alpha}} \log \left[\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2} + \frac{A-A^*}{\tau_\varepsilon^{-1/2}} \right)}{\Phi\left(\sqrt{\tau_\varepsilon}\,(A-A^*)\right)} \right] \right\} + k_0,$$

where k_0 is given in the Appendix and $\frac{\frac{1+\psi}{\psi+\alpha}}{\lambda - \alpha\frac{1+\psi}{\psi+\alpha}} > 0$ since $\lambda - \alpha\frac{1+\psi}{\psi+\alpha} > \lambda - 1 > 0$.

Similar to the labor choice of households from Proposition 1, the supply of commercial facility is log-linear with respect to the strength of the neighborhood A, with a correction term

for the truncation in the household population that occurs because of household selection into the neighborhood. This truncation term reflects two forces. The first is that a smaller population implies less demand for a given choice of commercial facility per household, while the second reflects that the price at which households charge each other for their goods p_i is also affected by this truncation.

We now characterize the neighborhood choice of households and the housing price. Households will sort into the neighborhood according to a cutoff equilibrium determined by the net benefit of living in the neighborhood, which trades off the opportunity of trading with other households in the neighborhood with the price of housing. Despite the inherent nonlinearity of our framework, we derive a tractable, unique cutoff equilibrium that is characterized by the solution to a fixed-point problem over the endogenous cutoff of entry in the neighborhood, A^*. This is summarized in the following proposition.

Proposition 3 *In the absence of informational frictions, there exists a unique cutoff equilibrium, in which the following hold: 1) household i follows a cutoff strategy in its neighborhood choice such that*

$$H_i = \begin{cases} 1 & \text{if } A_i \geq A^* \\ 0 & \text{if } A_i < A^* \end{cases},$$

where $A^*(A, \xi)$ solves equation (21) in the Appendix; 2) the cutoff productivity $A^*(A, \xi)$ is monotonically decreasing in ξ and increasing (hump-shaped) in A if $\eta_c < (>)\, \eta_c^*$, where η_c^* is given in the Appendix; 3) the population that enters the neighborhood is monotonically increasing in both A and ξ; 4) the housing price takes the following log-linear form:

$$\log P = \frac{1}{1+k}\left(\sqrt{\frac{\tau_\varepsilon}{\tau_e}}\left(A - A^*\right) - \xi\right); \tag{9}$$

and 5) the housing price P, and consequently the utility of the household with the cutoff productivity A^*, is increasing and convex in A.

Proposition 3 characterizes the cutoff equilibrium in the economy in the absence of informational frictions, and confirms the optimality of a cutoff strategy for households in their neighborhood choice. Households sort based on their individual productivity into the neighborhood, with the more productive, which expect more gains from living in the neighborhood, entering and participating in production at $t = 2$. This determines the supply of labor at $t = 2$, and, through this channel, the price of commercial facility at $t = 2$.

The proposition also provides comparative statics of the equilibrium cutoff household $A^*(A,\xi)$. This cutoff is decreasing in ξ, since a lower house price causes more households to enter the neighborhood for a given neighborhood strength A, and consequently a higher population enters the neighborhood as ξ increases. The cutoff, in contrast, is increasing in neighborhood strength A, since a higher A implies a higher housing price, and can also raise the price of commercial facility, depending on the supply response of commercial developers. This dominates the countervailing force that a higher A also signals more gains from trade due to complementarity in household consumption. Though the cutoff productivity increases, more households ultimately enter the neighborhood because a higher A shifts right (in the sense of first order stochastic dominance) the distribution of households more than it moves the cutoff.

Given a cutoff productivity $A^*(A,\xi)$, the housing price P positively loads on the strength of the neighborhood A, since a higher A implies stronger demand for housing, and loads negative on the supply shock ξ, reflecting that a discount is needed to ensure that a positive shift in housing supply is absorbed by a larger household population. As one would expect, the cutoff A^* enters negatively into the price since households above the cutoff sort into the neighborhood. The higher the cutoff, the fewer the households that enter the neighborhood, and the lower the housing price that is needed to clear the market with the lower housing demand. Despite its log-linear representation, the housing price is actually a generalized linear function of $\sqrt{\frac{\tau_\varepsilon}{\tau_e}}A-\xi$, since A^* is an implicit function of A and $\log P$.

As a result of endogenous selection into the neighborhood, the productivity of the neighborhood is determined by which households choose to live there. The aggregate productivity of the neighborhood $A_{\mathcal{N}}$ is given by:

$$A_{\mathcal{N}}=\log\int_{A^*}^{\infty}e^{A_j}d\Phi(\varepsilon_j)=A+\frac{1}{2}\tau_\varepsilon^{-1}+\log\Phi\left(\tau_\varepsilon^{-1/2}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right).$$

The first two terms would be what one would expect without neighborhood choice, while the third term reflects that productivity is truncated by selection. Importantly, since $A^*=A^*(A,\xi)$, it follows that A^* depends on the housing price in the neighborhood, introducing feedback from housing price to real decisions. Similar aggregation results exist for total income $\int_{\mathcal{N}}e^{A_j}p_iK^\alpha l_j^{1-\alpha}d\Phi(\varepsilon_j)$ and labor supply $\int_{\mathcal{N}}l_jd\Phi(\varepsilon_j)$ as well.

2.3 Cutoff Equilibrium with Informational Frictions

Having characterized the perfect-information benchmark equilibrium, we now turn to the equilibrium at $t = 1$ in the presence of informational frictions. With informational frictions, households and developers must now forecast the strength of the neighborhood A, and the realized price of commercial facility R, when choosing whether to live in the neighborhood, and when deciding the amount of commercial facility to develop. Each household's type A_i serves as a private signal about the strength of the neighborhood A. Since types are positively correlated with this common productivity, higher types also have more optimistic expectations about A. As such, we anticipate and conjecture that households will again follow a cutoff strategy when deciding whether to live in the neighborhood.

Due to the cutoff strategy used by households, the equilibrium housing price is a nonlinear function of A, which posts a challenge to our derivation of the learning of households and developers. It turns out that the equilibrium housing price maintains the same function form as in (9) for the perfect-information case. As a result, the information content of the publicly observed housing price can be summarized by a sufficient statistic $z(P)$ that is linear in A and the supply shock ξ:

$$z(P) = A - \sqrt{\frac{\tau_e}{\tau_\varepsilon}}\xi. \tag{10}$$

In our analysis, we shall first conjecture this linear sufficient statistic and then verify that it indeed holds in the equilibrium. This conjectured linear statistic helps to ensure tractability of the equilibrium despite that the equilibrium housing price is highly nonlinear.

By solving for the learning of households and commercial developers based on the conjectured sufficient statistic from the housing price, and by clearing the aggregate housing demand of the households with the supply from home builders, we derive the housing market equilibrium. The following proposition summarizes the housing price, each household's housing demand, and the supply of commercial facility in this equilibrium.

Proposition 4 *There exists a cutoff equilibrium in the presence of informational frictions, in which the following hold: 1) the housing price takes the log-linear form:*

$$\log P = \frac{1}{1+k}\left(\sqrt{\frac{\tau_\varepsilon}{\tau_e}}(A - A^*) - \xi\right) = \frac{1}{1+k}\sqrt{\frac{\tau_\varepsilon}{\tau_e}}(z - A^*); \tag{11}$$

2) the posterior of household i after observing housing price P, the public signal Q, and its

own productivity A_i is Gaussian with the conditional mean $\hat{A}_i$ and variance $\hat{\tau}_A$ given by

$$\begin{aligned}\hat{A}_i &= \hat{\tau}_A^{-1}\left(\tau_A\bar{A}+\tau_Q Q+\frac{\tau_\varepsilon}{\tau_e}\tau_\xi\left(\sqrt{\frac{\tau_e}{\tau_\varepsilon}}\left((1+k)\log P+\bar{\xi}\right)+A^*\right)+\tau_\varepsilon A_i\right),\\ \hat{\tau}_A &= \tau_A+\tau_Q+\frac{\tau_\varepsilon}{\tau_e}\tau_\xi+\tau_\varepsilon,\end{aligned}$$

and the posterior of commercial developers is also Gaussian with the conditional mean $\hat{A}^c$ and variance $\hat{\tau}_A^c$ given by

$$\begin{aligned}\hat{A}^c &= \hat{\tau}_A^{c-1}\left(\tau_A\bar{A}+\tau_Q Q+\frac{\tau_\varepsilon}{\tau_e}\tau_\xi\left(\sqrt{\frac{\tau_e}{\tau_\varepsilon}}\left((1+k)\log P+\bar{\xi}\right)+A^*\right)\right),\\ \hat{\tau}_A^c &= \tau_A+\tau_Q+\frac{\tau_\varepsilon}{\tau_e}\tau_\xi;\end{aligned}$$

3) household i follows the cutoff strategy in its neighborhood choice:

$$H_i=\begin{cases}1 & \text{if } A_i\geq A^*\\ 0 & \text{if } A_i<A^*\end{cases},$$

where $A^*(z,Q)$ solves equation (23) in the Appendix; 4) the supply of commercial facility takes the form:

$$\log K=\frac{1}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\log F\left(\hat{A}^c-A^*,\hat{\tau}_A^c\right)+\frac{\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}A^*+k_0,$$

where $F\left(\hat{A}^c-A^*,\hat{\tau}_A^c\right)$ is given in the Appendix, and $\log K$ is increasing in the conditional belief of commercial developers $\hat{A}^c$; and 5) the equilibrium converges to the perfect-information benchmark in Proposition 3 as $\tau_Q\nearrow\infty$.

Proposition 4 confirms that in the presence of informational frictions, each household adopts a cutoff strategy. Informational frictions make the household's equilibrium cutoff $A^*(z,Q)$ a function of $z(P)=1+k\sqrt{\frac{\tau_e}{\tau_\varepsilon}}\log P+A^*$,which is a summary statistic of the publicly observed housing price P, and the public signal Q, rather than A and ξ as in the perfect-information benchmark. This equilibrium cutoff is a key channel for informational frictions to affect the housing price, as well as commercial developers' decision to develop commercial facility.

In the presence of informational frictions, the demand-side fundamental A and the supply-side shock ξ are not directly observed by the public and, as a result, do not directly affect the housing price and other equilibrium variables. Instead, their equilibrium effects are bundled together in the housing price P through the specific form in z. Similarly, their effects on

other equilibrium variables are also bundled through z. Thus, we can examine the impact of a shock to either A or ξ by analyzing a shock to z. The equilibrium housing price in (11) directly implies that

$$\frac{\partial \log P}{\partial z} = \frac{1}{1+k}\sqrt{\frac{\tau_\varepsilon}{\tau_e}}\left(1 - \frac{\partial A^*}{\partial z}\right).$$

That is, depending on the sign of $\frac{\partial A^*}{\partial z}$, the equilibrium cutoff A^* may amplify or dampen the housing price effect of the fundamental shock z. Specifically, if $\frac{\partial A^*}{\partial z} < 0$, there is an amplification effect. This amplification effect makes housing prices more volatile, as highlighted by Albagli, Hellwig, and Tsyvinski (2015) in their analysis of the cutoff equilibrium in an asset market. This interesting feature also differentiates our cutoff equilibrium from other type of non-linear equilibrium with asymmetric information, such as the log-linear equilibrium developed by Sockin and Xiong (2015) to analyze commodity markets. In their equilibrium, prices become less sensitive to their analogue of z in the presence of informational frictions. This occurs because households, on aggregate, underreact to the fundamental shock in their private signals because of noise.

In the perfect-information benchmark, the public signal Q has no impact on neither the equilibrium cutoff A^* nor the housing price because both the demand-side fundamental $\not{A}$ and the supply-side shock ξ are publicly observable. In the presence of informational frictions, Q affects the equilibrium as it affects agents' expectations. The equilibrium housing price in (11) shows that

$$\frac{\partial \log P}{\partial Q} = -\frac{1}{1+k}\sqrt{\frac{\tau_\varepsilon}{\tau_e}}\frac{\partial A^*}{\partial Q}.$$

In other wrods, by affecting the households' expectations of A and subsequently their cutoff productivity to enter the neighborhood, the noise in the public signal Q affacts the population in the neighnorhood and the equilibrium housing price $\log P$. Subsequently, Q also affects the price of commercial facility, as well as commercial developers' optimal choice of how much commercial facility to develop.

The complementarity between households reinforces the effects of informational frictions. Without complementarity, a stronger neighborhood, i.e., higher A, is bad news for households, because a higher A raises not only the housing price, but also the price of commercial facility. With complementarity, however, a stronger neighborhood could be good news for households, because it means that other households in the neighborhood are more productive, and thus a better opportunity for trade. In the presence of informational frictions, complementarity gives each household a stronger incentive to learn about A and thus strengthens

the potential distortionary effects from such learning.

Supply elasticity also plays an important and nuanced role in the distortionary effects of learning. It is instructive to consider two polar cases for supply elasticity. When supply is infinitely inelastic (i.e., $k \to 0$), housing prices are only determined by the strength of the neighborhood A, and prices are fully revealing to households and commercial developers. As a result, there is not any distortion from the learning when supply is infinitely inelastic. On the other hand, when supply is infinitely elastic (i.e., $k \to \infty$), prices converge to $\log P = -\zeta$, which is driven only by the supply shock.[6] In this case, prices contain no information about demand, and therefore no information about the strength of the neighborhood. Consequently, the learning from housing price and the potential distortion of such learning both dissipate as supply elasticity approaches infinity. These two polar cases demonstrate that the distortion caused by learning on housing price has a humped shape with respect to supply elasticity.

3 Model Implications

We now investigate several implications of our model regarding how informational frictions affect the dynamics of housing and commercial real estate markets. We provide comparative statistics to illustrate how several key aspects of the neighborhood and its real estate markets vary across two dimensions: 1) supply elasticity k, and 2) the degree of consumption complementarity in household utility η_c. Supply elasticity is a natural candidate for classifying the cross-section of housing markets. It has been emphasized in the literature, in work including Malpezzi and Wachter (2005) and Glaeser, Gyourko, and Saiz (2008), to help explain certain features of housing cycles, such as housing price volatility. Similarly, the degree of complementarity captures the agglomeration and spillover effects that lead to coordination among firms and industries that locate in one area, such as the financial industry in New York City, the technology sector in San Francisco, the Research Triangle in North Carolina, and the oil industry in Houston. As emphasized, for instance, by Dougal, Parsons, and Titman (2015), employers and/or workers can benefit from locating in close proximity to competitors, either from knowledge spillovers or from the implicit insurance in labor markets.

While we have analytical expressions for most equilibrium outcomes, the key equilibrium

[6] It is straightforward to see from equation (23) that A^* remains finite a.s. as $k \to \infty$, allowing us to take the limit.

cutoff A^* needs to be numerically solved from the fixed-point condition in equation (23). We therefore analyze the equilibrium properties of A^* and other variables through a series of numerical illustrations. The benchmark parameters we choose for the numerical examples are provided in Table 1. For the share of commercial facility in households' production, we treat it as being similar to capital, and select the typical estimate of $\alpha = 0.33$. For the Frisch elasticity of labor supply, we choose $\psi = 2.5$, which is within the typical range found in the literature. We set τ_ζ to be four-fold larger than τ_A to ensure that with perfect information, the log housing price variance is monotonically declining in supply elasticity, as is observed empirically. We set $\lambda = 1.1$ to have commercial facility be in elastic supply, and avoid having convexity in its production function. We choose for the neighborhood fundamentals $A = \zeta = -0.5$, though the qualitative patterns we highlight hold more generically for a wide range of shock values. In addition, we set the public signal Q to 0.

τ_A	0.50	τ_ζ	2.00	τ_ϵ	0.20
τ_Q	1.00	η_c	0.50	α	0.33
ψ	2.50	k	0.50	λ	1.10
$\overline{A}$	0	$\overline{\zeta}$	0		

Table 1: Benchmark Parameters for Numerical Illustrations

3.1 Equilibrium Cutoff

Our model features an equilibrium cutoff productivity for the marginal household to enter the neighborhood, which hinges on the households' learning process about the neighborhood. This, in turn, determines the population flow into the neighborhood, and the dynamics of both housing and commercial real estate markets. As a consequence, the equilibrium productivity cutoff serves as a channel for informational frictions to impact the local economy.

Figure 1 illustrates how the cutoff responds to random shocks. We focus on two types of shocks, a noise shock Q and a fundamental shock z. The first row considers a random shock to the public signal Q, by computing the partial derivative of A^* with respect to Q across different values of supply elasticity k in the left panel and degree of complementarity η_c in the right panel. Q has no impact on the equilibrium in the perfect-information benchmark. In the presence of informational frictions, however, the shock affects households' expectations about A, as they use the public signal to infer the value of A. By making households more optimistic about A, a positive shock to Q raises each household's utility, and this lowers the

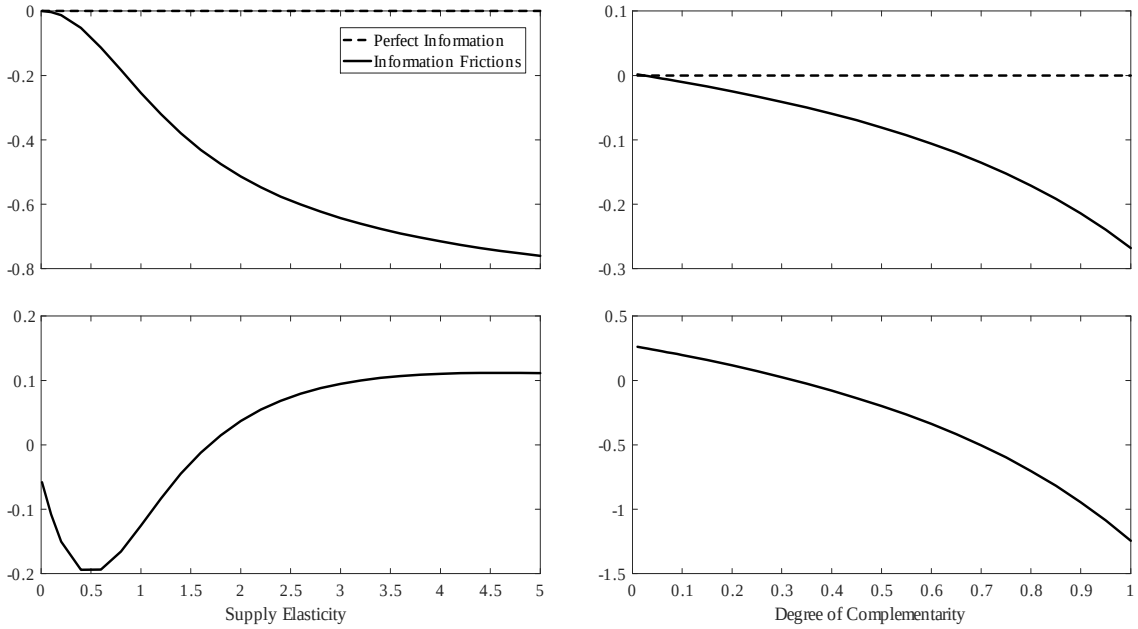

Figure 1: The Response of the equilibrium cutoff productivity to a noise shock Q (the first row) and a fundamental shock z (the second row) across housing supply elasticity (left) and degree of complementarity (right).

cutoff productivity of the marginal household that enters the neighborhood. This induces a greater population flow to the neighborhood.

Interestingly, this learning effect is stronger when supply elasticity is greater (the upper-left panel of Figure 1), or when the households' consumption elasticity is greater (the upper-right panel of Figure 1). The former result results from the fact that greater supply elasticity makes the housing price more dependent on supply-side factors, and therefore less informative of the neighborhood productivity. Consequently, households place a greater weight on the public signal Q in their learning about A, and this amplifies the effect of the noise shock to Q. The latter result is driven by the greater role that household learning plays as consumption complementarity increases, as a higher complementarity makes each household more concerned about the neighborhood's productivity.

The second row of Figure 1 considers a fundamental shock to z. As we discussed earlier, this shock can be a demand-side shock to A or a supply-side shock to ξ, which are bundled together in z according to (10). Interestingly, the left panel shows that $\frac{\partial A^*}{\partial z}$ has a U-shape with respect to supply elasticity and is particularly negative when supply elasticity is in an intermediate value around 0.5. It turns positive when supply elasticity rises roughly

above 1.8. This U-shape originates from the monotonic learning effect of the housing price. As households use housing price as a key source of information in their learning of the neighborhood strength A and this learning effect has a particularly strong effect when supply elasticity has an intermediate value, making the equilibrium cutoff particularly sensitive to the z shock. The negative value of the effect implies that in response to the better neighborhood fundamental, households reduce their cutoff, resulting in more households in the neighborhood, despite the higher housing price. The right panel further shows that $\frac{\partial A^*}{\partial z}$ decreases monotonically with the degree of complementarity. Specifically, $\frac{\partial A^*}{\partial z}$ is positive when complementarity is low and becomes more negative as complementarity rises. This pattern confirms our earlier discussion that the learning effect from housing price strengthens with complementarity.

Relating the model to empirical predictions, the noise shock to Q represents a non-fundamental shift in housing demand. One may broadly interpret this non-fundamental demand shock, in practice, as originating from different sources. For instance, it can be noise in public information, as featured in Morris and Shin (2002) and Hellwig (2005), housing market optimism, as in Ferreira and Gyourko (2011), Gao, Sockin and Xiong (2017), and Kaplan, Mitman, and Violante (2017), or credit expansion from the banking sector, as in Mian and Sufi (2009). Our analysis illustrates a mechanism for these non-fundamental shocks to induce greater population flow into the neighborhood through the households' learning channel. One may be able to test this effect across different regions with properly designed measures of these non-fundamental shocks. Housing supply elasticity can be measured, for instance, as in Saiz (2010), while complementarity in an area can be measured by its industry complementarity, as suggested by Dougal, Parsons, and Titman (2015). One can consequently directly test the cross-sectional implication of our model that non-fundamental shocks, such as the noise shock, have a greater impact in inducing stronger population inflow to areas with greater industry complementarity and intermediate supply elasticities.

3.2 Housing Cycle

We now examine the reaction of the housing market to the noise shock Q and a fundamental shock. For the sake of clarity, we explicitly consider a negative shock to housing supply, rather than the generic z shock, which can be either a demand-side shock to A or a supply shock.

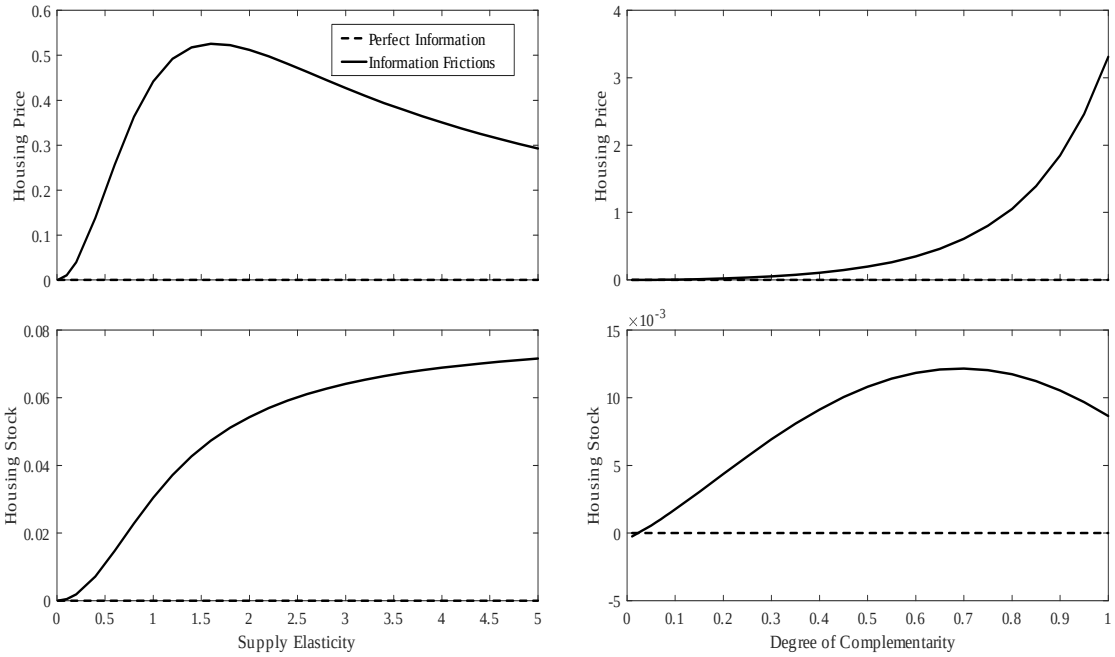

Figure 2: Housing responses to a noise shock to the public signal Q across supply elasticity (left) and degree of complementarity (right).

Figure 2 illustrates the impacts of the noise shock to Q on the housing price and housing stock in the neighborhood, by computing their partial derivatives with respect to Q across different values of supply elasticity k in the two left panels, and across different values of the degree of consumption complementarity η_c in the two right panels. As we discussed earlier, in the absence of informational frictions, this shock has no effect on the housing market. In the presence of informational frictions, the noise shock raises both housing price and housing stock relative to the perfect-information benchmark, because the shock boosts the agents' expectations about the neighborhood's productivity. Interestingly, the upper-left panel shows that this effect on housing price is hump-shaped with respect to supply elasticity, and peaks at an intermediate value. This results from the non-monotonicity of the distortionary effect of learning that we discussed earlier. When housing supply is infinitely inelastic, the noise shock has a muted effect on households' expectations because the price is fully revealing. When housing supply is infinitely elastic, the housing price is fully determined by supply shock and is immune from the households' learning of A. As a result, the price distortion caused by household learning is strongest when supply elasticity is in an intermediate range.

The upper-right panel of Figure 2 shows that the effect of the noise shock on the housing

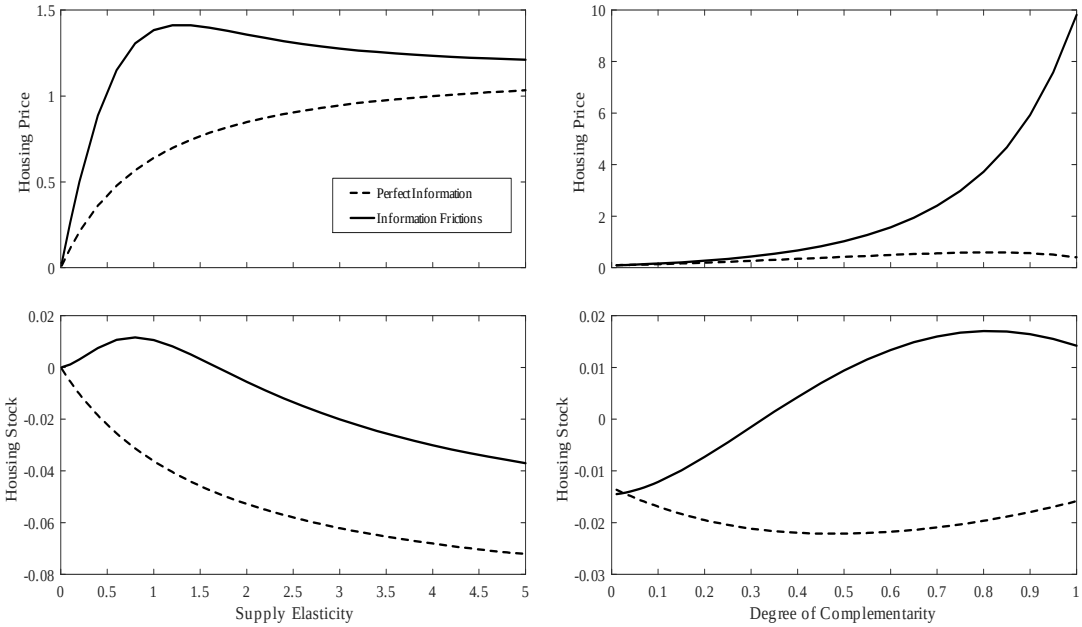

Figure 3: Housing responses to a negative supply shock across supply elasticity (left) and degree of complementarity (right).

price is increasing with the complementarity. As complementarity rises, each household cares more about trading goods with other households. This makes households' expectations about the neighborhood's productivity a greater determinant of the equilibrium housing price, since the housing price is equal to the utility of the marginal household with the cutoff productivity. This, in turn, causes the noise shock to have a greater effect on the housing price as complementarity increases. That the growth in housing stock is hump-shaped reflects that near perfect complementarity, almost all households are already entering the neighborhood and the marginal effect of the increase in equilibrium cutoff on neighborhood population diminishes.

We now consider a negative shock to the building cost shock ζ. Figure 3 displays the responses of the housing price and housing stock to this shock across different values of supply elasticity k in the two left panels, and across different degrees of consumption complementarity η_c in the two right panels. In the perfect-information benchmark, the housing price increases with a negative supply shock, and the price increase rises with supply elasticity. In contrast, the housing stock falls with the negative supply shock since the higher housing price discourages more households from entering, and the supply drop is greater when supply elasticity is larger.

In the presence of informational frictions, the negative supply shock is, in part, interpreted by households as a positive demand shock when they observe a higher housing price. This learning effect, in turn, pushes up the housing price and housing stock, relative to the perfect-information benchmark, as shown in the left panels of Figure 3. Across supply elasticity, these distortions are hump-shaped because the impact of learning from the housing price is most pronounced at intermediate supply elasticities, and, consequently, the response of the housing price and housing stock also peak at an intermediate range. As consumption complementarity increases, the learning effect from the negative supply shock is amplified, since households put more weight on the neighborhood's strength when determining whether to enter the neighborhood. This is shown in the upper-right panel of Figure 3. Similar to the noise demand shock, the impact on the housing stock is hump-shaped, since most households are already entering the neighborhood as η_c nears perfect complementarity.

While our model is static and cannot deliver a boom-and-bust housing cycle across periods, one may intuitively interpret the deviation of housing price induced by the noise demand shock and the supply shock from its value in the perfect-information benchmark in Figures 2 and 3 as a non-fundamental driven price boom, which would eventually reverse. Then, we have a testable implication for housing cycle—non-fundamental shocks, such as the noise demand shock and the supply shock, can lead to a more pronounced housing cycle in areas with greater industry complementarity and intermediate housing supply elasticities. This implication helps to explain why during the recent U.S. housing cycle, Las Vegas and Phoenix experienced more pronounced housing price boom and bust cycles than San Francisco and New York, which have more inelastic housing supply.

3.3 Commercial Real Estate Cycle

By affecting agents' expectations, informational frictions not only distort the housing price and housing stock but also other investment decisions related to the neighborhood. The commercial real estate market featured in our model allows us to analyze such effects. We first analyze in Figure 4 how the price and stock of commercial facilities react to the noise shock Q across different values of supply elasticity k in the three left panels, and across the degree of consumption complementarity η_c in the three right panels. While households acquire commercial facility only at $t = 2$ at the price R, we can also compute the shadow price of commercial facility at $t = 1$ as the commercial developers' marginal development cost

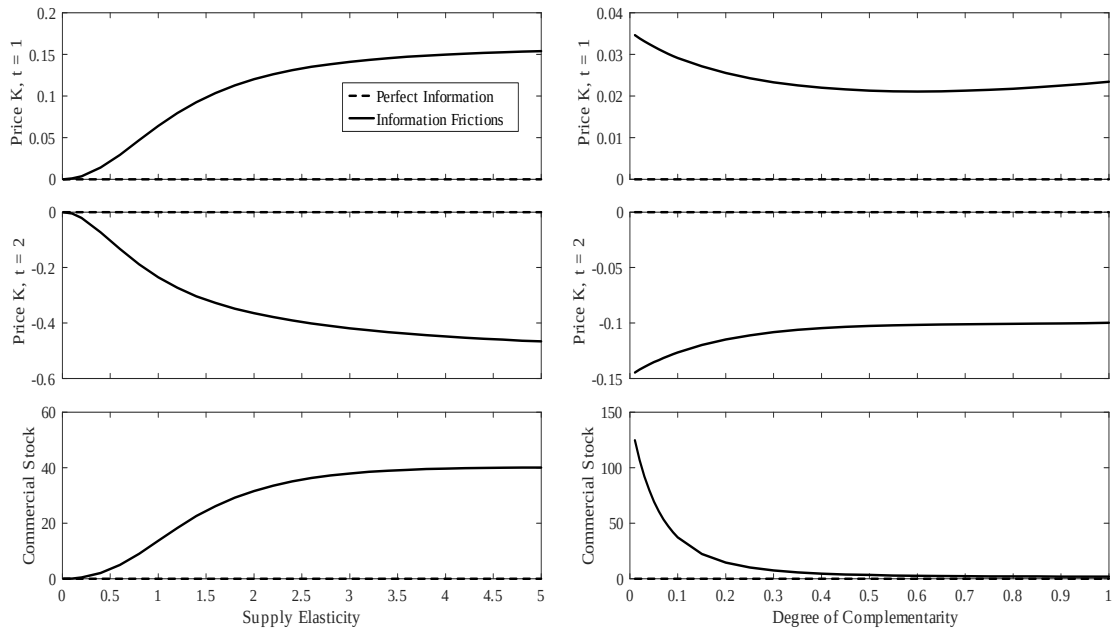

Figure 4: Neighborhood responses to a noise shock to the public signal Q across supply elasticity (left) and degree of complementarity (right).

$K^{\lambda-1}$ when they develop the facility. This shadow price reflects the developers' expectations about the price that will prevail at $t = 2$. We depict this shadow price of the commercial facility at $t = 1$ in the first row of Figure 4, its market price R at $t = 2$ in the second row, and the stock of commercial facility K built by the developers at $t = 1$ in the third row.

As we discussed before, in the perfect-information benchmark, the noise shock Q has no impact on agents' expectations, and consequently no impact on the price and stock of commercial facility. In the presence of informational frictions, however, the noise shock boosts agents' expectations about A. As a result, it pushes up both the shadow price and supply of commercial facility at $t = 1$, relative to the perfect-information benchmark. When the households come to buy the commercial facility at $t = 2$, the market price is determined by their realized productivity, and thus falls to reflect that A had been overestimated at $t = 1$. As a result, the noise shock causes a boom in the market for the commercial facility at $t = 1$, in terms of both price and supply, and a bust at $t = 2$ when the price reverses.

Interestingly, the magnitude of this boom-and-bust cycle, measured by the deviation of the price response at either $t = 1$ or $t = 2$ from the perfect-information benchmark, is monotonically increasing with supply elasticity. As we discussed before, as supply elasticity rises, the housing price is driven more by supply side factors, and is thus less informative

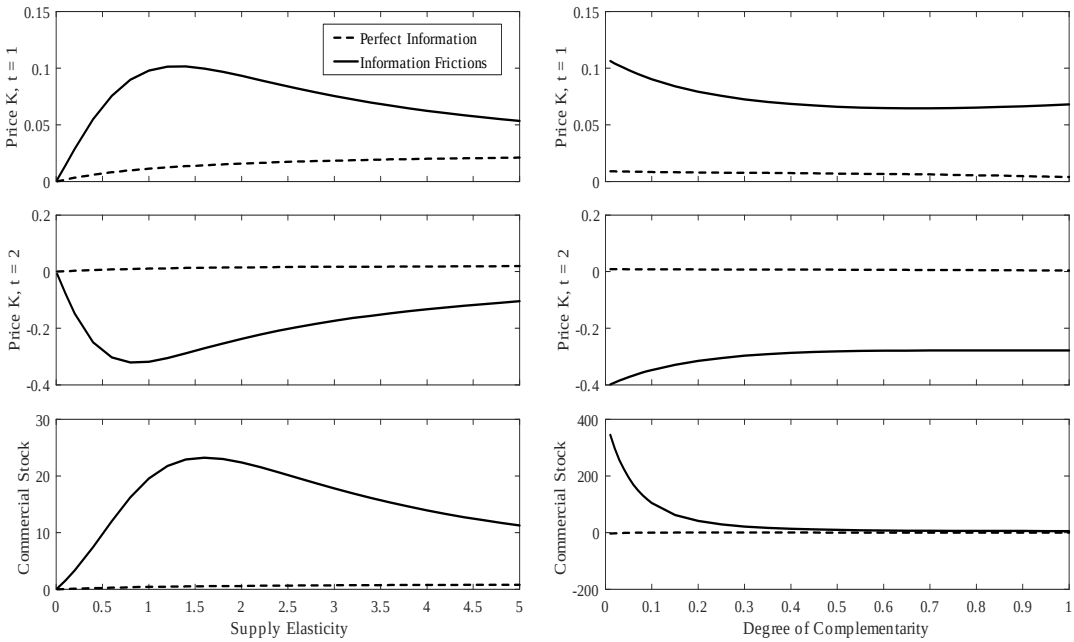

Figure 5: Neighborhood responses to a negative supply shock across supply elasticity (left) and degree of complementarity (right).

about the neighborhood productivity A. Consequently, the public signal Q gets a greater weight in the agents' learning process about A, giving the noise shock to Q a larger impact on the market for commercial facility. With respect to consumption complementarity, the demand shock has the largest impact on commercial real estate at lower levels of complementarity. This occurs because the price of commercial facilities at $t = 2$ is less sensitive to the neighborhood strength A when there is more coordination among households in their production decisions, and because the average marginal product of commercial facility is lower the more households that enter the neighborhood. In contrast to the housing market, the largest boom and bust in the shadow price and the subsequent market price R occurs at low levels of complementarity.

We now analyze how the commercial real estate market reacts to a negative supply shock ζ to the housing market in Figure 5, which shows the responses of the commercial facility's shadow price at $t = 1$ in the first row, its market price at $t = 2$ in the second row, and its supply at $t = 1$ in the third row, across housing supply elasticity in the three left panels and across households' consumption complementarity in the three right panels.

In the perfect-information benchmark, the negative supply shock only impacts the housing price, and, through this channel, the cutoff productivity of the households that enter the

neighborhood. As is apparent, this direct effect has only a modest impact on the commercial real estate market. In the presence of informational frictions, its impact on the commercial real estate market is substantially larger. This occurs because the negative supply shock is partially interpreted by the agents as a positive shock to the neighborhood productivity when they learn from the housing price about the neighborhood productivity A. Consequently, it distorts the agents' expectations about A upwardly, leading to overoptimism about the local economy. This results in both a higher shadow price and more supply of commercial facilities at $t = 1$, and a greater price reversal at $t = 2$. Interestingly, the magnitudes of these effects are all hump-shaped with respect to housing supply elasticity, as a result of the hump-shaped distortion to agents' expectations caused by their learning from the housing price. Similar to the noise demand shock, the negative supply shock distorts the commercial real estate market by leading to overoptimism about A, and it is most pronounced at low levels of consumption complementarity.

Our analysis shows that non-fundamental shocks to the housing market lead to not only a housing cycle, but also to a boom and bust in the market for commercial facilities. This is consistent with Gyourko (2009a), who highlights that the recent U.S. housing cycle was accompanied by a similar boom and bust in commercial real estate. Though also characterized by a dramatic run-up and collapse in prices, this second boom and bust, and its relation to local economic outcomes, have received less attention. It is difficult to simply attribute this commercial real estate boom to the subprime credit expansion that had played an important role for the housing boom, as the credit expansion was mainly targeting households. One may attribute it to widespread optimism, and our model provides a coherent explanation for the shared optimism in both housing and commercial real estate markets. Specifically, our model shows that non-fundamental shocks may lead to joint cycles in housing and commercial real estate markets, especially in areas with intermediate values of housing supply elasticity.

4 Conclusion

In this paper, we introduce a model of information aggregation in housing and commercial real estate markets, and examine its implications for not only housing prices, but also economic outcomes such as neighborhood choice and the supply of commercial real estate. We provide empirical predictions for the expected response of neighborhoods to noise from both

demand and supply sides across supply elasticity and the degree of consumption complementarity, and offer a rationale for the synchronized boom and bust cycles observed in the U.S. housing and commercial real estate markets during the 2000s.

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Appendix Proofs of Propositions

A.1 Proof of Proposition 1

The first order conditions of household i's optimization problem in (2) respect to $C_i(i)$ and $C_j(i)$ at an interior point are:

$$C_i(i) \quad : \quad \frac{1-\eta_c}{C_i(i)} U\left(\{C_k(i)\}_{k\in\mathcal{N}};\mathcal{N}\right) = \theta_i p_i, \tag{12}$$

$$C_j(i) \quad : \quad \frac{\eta_c}{\int_{\mathcal{N}/i} C_j dj} U\left(\{C_k(i)\}_{k\in\mathcal{N}};\mathcal{N}\right) = \theta_i p_j, \tag{13}$$

where θ_i is the Lagrange multiplier for the budget constraint. Rewriting (13) as

$$\frac{\eta_c C_j}{\int_{\mathcal{N}/i} C_j dj} U\left(\{C_k(i)\}_{k\in\mathcal{N}};\mathcal{N}\right) = \theta_i p_j C_j$$

and integrating over $\mathcal{N}$, we arrive at

$$\eta_c U\left(\{C_k(i)\}_{k\in\mathcal{N}};\mathcal{N}\right) = \theta_i \int_{\mathcal{N}/i} p_j C_j dj.$$

Dividing equations (12) by this expression leads to $\frac{\eta_c}{1-\eta_c} = \frac{\int_{\mathcal{N}/i} p_j C_j(i) dj}{p_i C_i(i)}$, which in a symmetric equilibrium implies $p_j C_j(i) = \frac{1}{\Phi(\sqrt{\tau_\varepsilon}(A-A^*))} \frac{\eta_c}{1-\eta_c} p_i C_i(i)$. By substituting this equation back to the household's budget constraint in (2), we obtain:

$$C_i(i) = (1-\eta_c)(1-\alpha) e^{A_i} K_i^\alpha l_i^{1-\alpha}.$$

The market-clearing for the household's good requires that $C_i(i) + \int_{\mathcal{N}/i} C_i(j) dj = (1-\alpha) e^{A_i} K_i^\alpha l_i^{1-\alpha}$, which implies that $C_i(j) = \frac{1}{\Phi(\sqrt{\tau_\varepsilon}(A-A^*))} \eta_c (1-\alpha) e^{A_i} K_i^\alpha l_i^{1-\alpha}$.

The first order condition in equation (12) also gives the price of the good produced by household i. Since the household's budget constraint in (2) is entirely in nominal terms, the price system is only identified up to θ_i, the Lagrange multiplier. We therefore normalize θ_i to 1. It follows that:

$$p_i = \frac{1-\eta_c}{C_i(i)} U\left(\{C_j(i)\}_{j\in\mathcal{N}};\mathcal{N}\right) = \left(e^{A_i} l_i^{1-\alpha} K_i^\alpha\right)^{-\eta_c} \left(\frac{1}{\Phi\left(\sqrt{\tau_\varepsilon}(A-A^*)\right)} \int_{\mathcal{N}/i} e^{A_j} l_j^{1-\alpha} K_j^\alpha dj\right)^{\eta_c}. \tag{14}$$

Furthermore, given equation (1), it follows since $C_i(i) = (1-\eta_c)(1-\alpha) e^{A_i} K_i^\alpha l_i^{1-\alpha}$ and $C_j(i) = \frac{1}{\Phi(\sqrt{\tau_\varepsilon}(A-A^*))} \eta_c (1-\alpha) e^{A_j} K_j^\alpha l_j^{1-\alpha}$ that:

$$\begin{aligned} U\left(\{C_k(i)\}_{k\in\mathcal{N}};\mathcal{N}\right) &= (1-\alpha)\left(e^{A_i} l_i^{1-\alpha} K_i^\alpha\right)^{1-\eta_c} \left(\frac{1}{\Phi\left(\sqrt{\tau_\varepsilon}(A-A^*)\right)} \int_{\mathcal{N}/i} e^{A_j} K_j^\alpha l_j^{1-\alpha} dj\right)^{\eta_c} \\ &= (1-\alpha) p_i e^{A_i} K_i^\alpha l_i^{1-\alpha}, \end{aligned}$$

from substituting with the household's budget constraint at $t=2$.

The first-order conditions for household i's choice of l_i at an interior point is:

$$l_i^\psi = (1-\alpha)\theta_i p_i e^{A_i} \left(\frac{K_i}{l_i}\right)^\alpha. \tag{15}$$

from equation (12). Substituting $\theta_i = 1$ and p_i with equation (14), it follows that:

$$\log l_i = \frac{1}{\psi+\alpha+(1-\alpha)\eta_c} \log(1-\alpha) + \frac{1}{\psi+\alpha+(1-\alpha)\eta_c} \log\left(\left(e^{A_i} K_i^\alpha\right)^{(1-\eta_c)} \left(\frac{\int_{\mathcal{N}/i} e^{A_j} K_j^\alpha l_j^{1-\alpha} dj}{\Phi\left(\sqrt{\tau_\varepsilon}(A-A^*)\right)}\right)^{\eta_c}\right). \tag{16}$$

The optimal labor choice of household i, consequently, represents a fixed point problem over the optimal labor strategies of other households in the neighborhood.

Noting that $K_i = \left(\frac{\alpha p_i e^{A_i} l_i^{1-\alpha}}{R}\right)^{\frac{1}{1-\alpha}}$, we can substitute in the price function p_i to arrive at:

$$\begin{aligned}\log K_i &= \frac{1}{1-(1-\eta_c)\,\alpha}\log\left(\left(e^{A_i}l_i^{1-\alpha}\right)^{1-\eta_c}\left(\frac{1}{\Phi\left(\sqrt{\tau_\varepsilon}\,(A-A^*)\right)}\int_{\mathcal{N}/i} e^{A_j}l_j^{1-\alpha}K_j^{\alpha}dj\right)^{\eta_c}\right)\\ &\quad -\frac{1}{1-(1-\eta_c)\,\alpha}\log R+\frac{1}{1-(1-\eta_c)\,\alpha}\log\alpha, \qquad (17)\end{aligned}$$

which is a fixed-point problem for the optimal choice of commercial facility.

Given the optimal labor supply of household i l_i and optimal demand for commercial land K_i jointly satisfy the functional fixed-point equations (16) and (17), let us conjecture for i for which $A_i \geq A^*$, so that $i \in \mathcal{N}$ is in the neighborhood, that:

$$\log l_i = l_0 + l_A A + l_s A_i + l_R \log R + l_\Phi \log \frac{\Phi\left(\left(1+(\alpha h_s+(1-\alpha)\,l_s)\right)\tau_\varepsilon^{-1/2}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\varepsilon}\,(A-A^*)\right)},$$

where R is the rental rate of commercial land, and that capital satisfies:

$$\log K_i = h_0 + h_A A + h_s A_i + h_R \log R + h_\Phi \log \frac{\Phi\left(\left(1+(\alpha h_s+(1-\alpha)\,l_s)\right)\tau_\varepsilon^{-1/2}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\varepsilon}\,(A-A^*)\right)},$$

Substituting these conjectures into the fixed-point recursion for labor, equation (16), we arrive, by the method of undetermined coefficients, at the coefficient restrictions:

$$\begin{aligned}cons &: (\psi+\alpha)\,l_0 = \log(1-\alpha)+\alpha h_0+\frac{1}{2}\eta_c\left(1+\alpha h_s+(1-\alpha)\,l_s\right)^2\tau_\varepsilon^{-1},\\ A &: (\psi+\alpha)\,l_A = \alpha h_A+\left(1+\alpha h_s+(1-\alpha)\,l_s\right)\eta_c,\\ A_i &: (\psi+\alpha+(1-\alpha)\,\eta_c)\,l_s = (1-\eta_c)\left(1+\alpha h_s\right),\\ \log R &: (\psi+\alpha)\,l_R = \alpha h_R,\\ \Phi &: (\psi+\alpha)\,l_\Phi = \eta_c+\alpha h_\Phi.\end{aligned}$$

Similarly, substituting these conjectures into the fixed-point recursion for commercial land, equation (17), we arrive at the coefficient restrictions:

$$\begin{aligned}cons &: (1-\alpha)\,h_0 = (1-\alpha)\,l_0+\frac{1}{2}\eta_c\left(1+\alpha h_s+(1-\alpha)\,l_s\right)^2\tau_\varepsilon^{-1}+\log\alpha,\\ A &: (1-\alpha)\,h_A = (1-\alpha)\,l_A+\eta_c\left(1+\alpha h_s+(1-\alpha)\,l_s\right),\\ A_i &: (1-(1-\eta)\,\alpha)\,h_s = (1-\eta_c)\left(1+(1-\alpha)\,l_s\right),\\ \log R &: (1-\alpha)\,h_R = (1-\alpha)\,l_R-1,\\ \Phi &: (1-\alpha)\,h_\Phi = (1-\alpha)\,l_\Phi+\eta_c.\end{aligned}$$

We consequently have ten linear equations and ten coefficients, from which follows that:

$$\begin{aligned}
l_0 &= \frac{1}{2}\frac{1}{1-\alpha}\frac{\eta_c}{\psi}\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\right)^2\tau_\varepsilon^{-1}+\frac{\alpha}{1-\alpha}\frac{1}{\psi}\log\alpha+\frac{1}{\psi}\log(1-\alpha),\\
l_A &= \frac{1}{1-\alpha}\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\frac{\eta_c}{\psi},\\
l_s &= \frac{1-\eta_c}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c},\\
l_R &= -\frac{\alpha}{1-\alpha}\frac{1}{\psi},\\
l_\Phi &= \frac{1}{1-\alpha}\frac{\eta_c}{\psi},
\end{aligned}$$

and

$$\begin{aligned}
h_0 &= \frac{1}{2}\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\eta_c\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\right)^2\tau_\varepsilon^{-1}+\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\log\alpha+\frac{1}{\psi}\log(1-\alpha),\\
h_A &= \frac{1}{1-\alpha}\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\frac{1+\psi}{\psi}\eta_c,\\
h_s &= \frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c},\\
h_R &= -\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi},\\
h_\Phi &= \frac{1}{1-\alpha}\frac{1+\psi}{\psi}\eta_c,
\end{aligned}$$

which confirms the conjectures.

Consequently, we find that, for $A_i \geq A^*$:

$$\begin{aligned}
\log l_i &= \frac{1}{2}\frac{1}{1-\alpha}\frac{\eta_c}{\psi}\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\right)^2\tau_\varepsilon^{-1}+\frac{\alpha}{1-\alpha}\frac{1}{\psi}\log\alpha+\frac{1}{\psi}\log(1-\alpha)\\
&\quad+\frac{1-\eta_c}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A_i+\frac{1}{1-\alpha}\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\frac{\eta_c}{\psi}A\\
&\quad-\frac{\alpha}{1-\alpha}\frac{1}{\psi}\log R+\frac{1}{1-\alpha}\frac{\eta_c}{\psi}\log\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\varepsilon}(A-A^*)\right)},
\end{aligned}$$

and:

$$\begin{aligned}
\log K_i &= \frac{1}{2}\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\eta_c\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\right)^2\tau_\varepsilon^{-1}+\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\log\alpha+\frac{1}{\psi}\log(1-\alpha)\\
&\quad+\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A_i+\frac{1}{1-\alpha}\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\frac{1+\psi}{\psi}\eta_c A\\
&\quad-\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\log R+\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\eta_c\log\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\varepsilon}(A-A^*)\right)}.
\end{aligned}$$

Substituting this functional form for the labor supply and commercial labor demand of household i into equation (14), the price of household $i's$ good then reduces to:

$$p_i = e^{\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\eta_c(A-A_i)+\frac{1}{2}\eta_c\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\right)^2\tau_\varepsilon^{-1}}\left(\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\varepsilon}\left(A-A^*\right)\right)}\right)^{\eta_c}.$$

Finally, that $U\left(\left\{C_k\left(i\right)\right\}_{k\in\mathcal{N}};\mathcal{N}\right)=\left(1-\alpha\right)p_i e^{A_i}K_i^{\alpha}l_i^{1-\alpha}$, implies:

$$E\left[U\left(\left\{C_j\left(i\right)\right\}_{j\in\mathcal{N}};\mathcal{N}\right)-\frac{l_i^{1+\psi}}{1+\psi}\middle|\mathcal{I}_i\right]=\left(1-\alpha\right)\frac{\psi}{1+\psi}E\left[p_i e^{A_i}K_i^{\alpha}l_i^{1-\alpha}\middle|\mathcal{I}_i\right].$$

A.2 Proof of Proposition 2

Substituting the optimal demand for commercial land K_i into the market-clearing condition for the commercial facility in (5) reveals that the price R is given by:

$$\begin{aligned}\log R &= \frac{1+\psi}{\psi+\alpha}A-\left(1-\alpha\right)\frac{\psi}{\psi+\alpha}\log K+\frac{1+\psi}{\psi+\alpha}\eta_c\log\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\varepsilon}\left(A-A^*\right)\right)}\\ &\quad+\left(1-\alpha\right)\frac{\psi}{\psi+\alpha}\log\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\varepsilon}\left(A-A^*\right)\right)}+r_0,\end{aligned}$$

where K is the total amount of commercial facility developed by commercial developers at $t=1$, and

$$r_0=\log\alpha+\frac{1-\alpha}{\psi+\alpha}\log\left(1-\alpha\right)+\frac{1}{2}\left(\frac{1+\psi}{\psi+\alpha}\eta_c+\left(1-\alpha\right)\frac{\psi}{\psi+\alpha}\left(1-\eta_c\right)^2\right)\left(\frac{1+\psi}{\left(1-\alpha\right)\psi+\left(1+\alpha\psi\right)\eta_c}\right)^2\tau_\varepsilon^{-1}.$$

Since market-clearing in the market for commercial facility imposes that $K\int_{i\in\mathcal{N}}di=\int_{i\in\mathcal{N}}K_i di$, it follows from equation (4) that the optimal choice of how much commercial facility commercial developers create is given by equation (8) with constant k_0 is given by

$$k_0=\frac{\log\alpha+\frac{1-\alpha}{\psi+\alpha}\log\left(1-\alpha\right)+\frac{1}{2}\left(\frac{1+\psi}{\psi+\alpha}\eta_c+\left(1-\alpha\right)\frac{\psi}{\psi+\alpha}\left(1-\eta_c\right)^2\right)\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\right)^2\tau_\varepsilon^{-1}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}.$$

A.3 Proof of Proposition 3

When all households and builders observe A directly, there are no longer information frictions in the economy. By substituting for prices, the optimal labor and commercial facility choices of household i, the realized commercial facility price R, and commercial facility demand

K_i from Proposition 2, the utility of household i at $t = 1$ from choosing to live in the neighborhood is

$$E\left[U_i|\mathcal{I}_i\right] = (1-\alpha)\frac{\psi}{1+\psi}e^{u_0+u_A A+\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A_i}\cdot\left(\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\varepsilon}\left(A-A^*\right)\right)}\right)^{u_\Phi}$$
$$\times\left(\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\varepsilon}\left(A-A^*\right)\right)}\right)^{(1-\lambda)\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}},$$

where

$$u_0 = \frac{1}{2}\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\left(\lambda\eta_c\frac{1-\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}-\frac{\alpha\left(\lambda-1\right)}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\frac{\psi\left(1-\alpha\right)}{\psi+\alpha}\left(1-\eta_c\right)^2\right)\left(\frac{1+\psi}{\left(1-\alpha\right)\psi+\left(1+\alpha\psi\right)\eta_c}\right)^2\tau_\varepsilon^{-1}$$
$$+\frac{1}{\psi}\frac{1-\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\left(\frac{\alpha}{1-\alpha}\left(1+\psi\right)\log\alpha+\lambda\log\left(1-\alpha\right)\right),$$
$$u_A = \frac{1}{1-\alpha}\frac{1+\psi}{\psi}\left(\frac{1+\psi}{\left(1-\alpha\right)\psi+\left(1+\alpha\psi\right)\eta_c}\eta_c-\left(\lambda-1\right)\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\right),$$
$$u_\Phi = \frac{\lambda\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\eta_c>0.$$

Since the household with the critical productivity A^* must be indifferent to its neighborhood choice at the cutoff, it follows that $U_i - P = 0$, which implies:

$$e^{\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A_i}\left(\frac{\Phi\left(\frac{(1+\psi)\tau_\varepsilon^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\varepsilon}\left(A-A^*\right)\right)}\right)^{u_\Phi}\left(\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)\tau_\varepsilon^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\sqrt{\tau_\varepsilon}\left(A-A^*\right)\right)}\right)^{\frac{(1-\lambda)\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}}$$
$$= \frac{1+\psi}{\psi\left(1-\alpha\right)}e^{-u_0-u_A A}P, \quad \text{with} \quad A_i = A^* \tag{18}$$

which implies the benefit of living with more productive households is offset by the higher cost of living in the neighborhood.

Fixing the critical value A^* and price P, we see that the LHS of equation (18) is increasing in monotonically in A_i, since $\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\left(1-\eta_c\right)>0$. This confirms the optimality of the cutoff strategy that households with $A_i \geq A^*$ enter the neighborhood, and households with $A_i < A^*$ choose to live somewhere else. Since $A_i = A+\varepsilon_i$, it then follows that a fraction $\Phi\left(-\sqrt{\tau_\varepsilon}\left(A^*-A\right)\right)$ enter the neighborhood, and a fraction $\Phi\left(\sqrt{\tau_\varepsilon}\left(A^*-A\right)\right)$ choose to live somewhere else. As one can see, it is the integral over the idiosyncratic productivity shocks of households ε_i that determines the fraction of households in the neighborhood.

From the optimal supply of housing by builder i in the neighborhood (7), there exists a critical value ω^* :

$$\omega^* = -(1+k)\log P, \tag{19}$$

such that builders with productivity $\omega_i \geq \omega^*$ build houses. Thus, a fraction $\Phi\left(-\sqrt{\tau_e}\left(\omega^* - \xi\right)\right)$ build houses in the neighborhood. Imposing market-clearing, it must be the case that

$$\Phi\left(-\sqrt{\tau_\varepsilon}\left(A^* - A\right)\right) = \Phi\left(-\sqrt{\tau_e}\left(\omega^* - \xi\right)\right).$$

Since the CDF of the normal distribution is montonically increasing, we can invert the above market-clearing conditions, and impose equation (19) to arrive at

$$\log P = \frac{1}{1+k}\left(\sqrt{\frac{\tau_\varepsilon}{\tau_e}}\left(A - A^*\right) - \xi\right). \tag{20}$$

By substituting for P in equation (18), we obtain an equation to determine the equilibrium cutoff $A^* = A^*\left(A, \xi\right)$:

$$\begin{aligned} & e^{\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{\sqrt{\tau_\varepsilon/\tau_e}}{1+k}\right)A^*}\left(\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)\tau_\varepsilon^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}\right)^{\frac{(1-\lambda)\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}} \frac{\Phi\left(\frac{(1+\psi)\tau_\varepsilon^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)^{u_\Phi}}{\Phi\left(\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)^{u_\Phi}} \\ = & \frac{1+\psi}{\psi\left(1-\alpha\right)} e^{\left(\frac{1}{1+k}\sqrt{\frac{\tau_\varepsilon}{\tau_e}}-u_A\right)A-\frac{1}{1+k}\xi-u_0}. \end{aligned} \tag{21}$$

Taking the derivative of the log of the LHS of equation (21) with respect to A^* gives

$$\begin{aligned} & \frac{d\log LHS}{dA^*} \\ = & u_\Phi \frac{1}{\tau_\varepsilon^{-1/2}}\left(\frac{\phi\left(\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)} - \frac{\phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}\right) + \frac{\left(1+\psi\right)\left(1-\eta_c\right)}{\left(1-\alpha\right)\psi+\left(1+\alpha\psi\right)\eta_c} \\ & + \frac{1}{1+k}\sqrt{\frac{\tau_\varepsilon}{\tau_e}} - \frac{1}{\tau_\varepsilon^{-1/2}}\frac{\left(\lambda-1\right)\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\left(\frac{\phi\left(\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)} - \frac{\phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}\right). \end{aligned}$$

The term in parentheses are nonnegative by the properties of the normal CDF. The last term is nonpositive, since $\lambda > 1$, and attains its minimum at $A^* \to \infty$, from which follows, substituting for u_Φ, that

$$\text{As } A^* \to \infty, \ \frac{d\log LHS}{dA^*} \to \frac{1}{1+k}\sqrt{\frac{\tau_\varepsilon}{\tau_e}} + \frac{\lambda\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}} > 0.$$

Consequently, since $\frac{d\log LHS}{dA^*} > 0$ when the last term attains its (nonpositive) minimum, it follows that $\frac{d\log LHS}{dA^*} > 0$. Therefore, $\log LHS$, and consequently LHS, is monotonically

increasing in A^*. Since the RHS of equation (21) is independent of A^*, it follows that the LHS and RHS of equation (21) intersect at most once. Therefore, the can be, at most, one cutoff equilibrium. Furthermore, since the LHS of equation (21) tends to 0 as $A^* \to -\infty$, and the RHS is nonnegative, it follows that a cutoff equilibrium always exists. Therefore, there exists a unique cutoff equilibrium in this economy.

It is straightforward to apply the Implicit Function Theorem to (21) to obtain

$$\begin{aligned}\frac{dA^*}{dA} &= \frac{\frac{1}{1+k}\sqrt{\frac{\tau_\varepsilon}{\tau_e}} - \frac{d\log LHS}{dA} - u_A}{\frac{d\log LHS}{dA^*}} \\ \frac{dA^*}{d\xi} &= -\frac{1}{1+k}\frac{1}{\frac{d\log LHS}{dA^*}} < 0,\end{aligned}$$

where

$$\begin{aligned}\frac{d\log LHS}{dA} &= -u_\Phi \frac{1}{\tau_\varepsilon^{-1/2}}\left(\frac{\phi\left(\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)} - \frac{\phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2} + \frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2} + \frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}\right) \\ &\quad + \frac{1}{\tau_\varepsilon^{-1/2}}(\lambda-1)\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\left(\frac{\phi\left(\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)} - \frac{\phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2} + \frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2} + \frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)}\right).\end{aligned}$$

Note that the nonpositive term in $\frac{d\log LHS}{dA}$ achieves its minimum at $A \to -\infty$, at which

$$\frac{d\log LHS}{dA} \to ((\lambda-1)\alpha(1-\eta_c) - \lambda\eta_c)\frac{\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}.$$

Then, as $A \to -\infty$, the numerator of $\frac{dA^*}{dA}$ converges to

$$\begin{aligned}\frac{1}{1+k}\sqrt{\frac{\tau_\varepsilon}{\tau_e}} - \frac{d\log LHS}{dA} - u_A \;\to\; {}_{A\to-\infty} &- \frac{(1+\psi)\left(\frac{((\lambda-1)\alpha(1-\eta_c)-\lambda\eta_c)\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}} + \frac{1}{1-\alpha}\frac{1+\psi}{\psi}\eta_c\right)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c} \\ &+ \frac{1}{1-\alpha}\frac{1+\psi}{\psi}\frac{(\lambda-1)\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}} + \frac{1}{1+k}\sqrt{\frac{\tau_\varepsilon}{\tau_e}},\end{aligned}$$

which is positive. Consequently $\frac{dA^*}{dA}\big|_{A^*=-\infty} > 0$. In contrast, as $A^* \to \infty$, one has that

$$\begin{aligned}&\frac{1}{1+k}\sqrt{\frac{\tau_\varepsilon}{\tau_e}} - \frac{d\log LHS}{dA} - u_A \\ \to\; {}_{A\to\infty} &\frac{1}{1+k}\sqrt{\frac{\tau_\varepsilon}{\tau_e}} - \frac{1}{1-\alpha}\frac{1+\psi}{\psi}\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\eta_c - (\lambda-1)\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\right),\end{aligned}$$

which is negative if

$$\eta_c > \eta_c^* = (1-\alpha)\frac{\psi}{1+\alpha\psi}\frac{\frac{\psi}{1+\psi}\frac{1-\alpha}{1+k}\sqrt{\frac{\tau_\varepsilon}{\tau_e}} + (\lambda-1)\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}}{\frac{1+\psi}{1+\alpha\psi} - \frac{\psi}{1+\psi}\frac{1-\alpha}{1+k}\sqrt{\frac{\tau_\varepsilon}{\tau_e}} - (\lambda-1)\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}}.$$

We can rewrite equation (21) as:

$$e^{-\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{1}{1+k}\sqrt{\frac{\tau_\varepsilon}{\tau_e}}\right)s}\left(\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)\tau_\varepsilon^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{s}{\tau_\varepsilon^{-1/2}}\right)}{\Phi\left(\frac{s}{\tau_\varepsilon^{-1/2}}\right)}\right)^{\frac{(1-\lambda)\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}}\frac{\Phi\left(\frac{(1+\psi)\tau_\varepsilon^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{s}{\tau_\varepsilon^{-1/2}}\right)^{u_\Phi}}{\Phi\left(\frac{s}{\tau_\varepsilon^{-1/2}}\right)^{u_\Phi}}$$

$$= \frac{1+\psi}{\psi(1-\alpha)}e^{-\lambda\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\frac{1-\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}A-\frac{1}{1+k}\xi-u_0},$$

where $s = A - A^*$ determines the population that enter the neighborhood. It is straightforward to show that

$$\frac{d\log LHS}{ds} = -\frac{d\log LHS}{dA^*} < 0.$$

Consequently, we have

$$\begin{aligned}\frac{ds}{d\xi} &= -\frac{\frac{1}{1+k}}{\frac{d\log LHS}{ds}} > 0,\\ \frac{ds}{dA} &= -\frac{\lambda\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\frac{1-\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}}{\frac{d\log LHS}{ds}} > 0.\end{aligned}$$

Thus, the population that enters, $\Phi\left(\sqrt{\tau_\varepsilon}s\right)$, is increasing in A and ξ. Furthermore, it follows from (20) that

$$\frac{d\log P}{dA} = \frac{1}{1+k}\sqrt{\frac{\tau_\varepsilon}{\tau_e}}\frac{ds}{dA} > 0,$$

and therefore the log housing price is increasing in A.

Finally, we recognize that:

$$\frac{d^2P}{dA^2} = \left(\frac{ds}{dA}\right)^2 P + \frac{d^2s}{dA^2}P = \left(\frac{ds}{dA}\right)^2 P + \frac{\lambda\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\frac{1-\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}}{\left(\frac{d\log LHS}{ds}\right)^2}\frac{ds}{dA}\frac{d^2\log LHS}{ds^2}P,$$

where $\lambda\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\frac{1-\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\frac{ds}{dA} > 0$ by the above arguments. It follows that from calculating $\frac{d^2\log LHS}{ds^2}$ that:

$$\lim_{s\to-\infty}\frac{d^2\log LHS}{ds^2} = (\lambda(\alpha-\eta_c)-\alpha)\frac{\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\frac{1}{\tau_\varepsilon^{-1}},$$

and therefore, as $P \to \infty$, from the expression for $\frac{d^2P}{dA^2}$ one has that $\frac{d^2P}{dA^2} \to \infty$. Furthermore, as $s \to -\infty$,

$$\frac{d \log LHS}{ds} \to -\left(\frac{1}{1+k} \sqrt{\frac{\tau_\varepsilon}{\tau_e}} + \frac{\lambda \frac{1+\psi}{\psi+\alpha}}{\lambda - \alpha \frac{1+\psi}{\psi+\alpha}} \right),$$

and

$$\lim_{s \to \infty} \frac{d^2 \log LHS}{ds^2} = 0,$$

and $P \to 0$ at an exponential rate. Consequently, as $s \to -\infty$, $\frac{d^2P}{dA^2} \to 0$. Since $\frac{d^2P}{dA^2}$ is continuous, it follows that $\frac{d^2P}{dA^2} \geq 0$. Consequently, P is convex in A. Since, in equilibrium, the housing price is equal to the utility of the household with the cutoff productivity, it follows that this utility is also convex and increasing in A.

A.4 Proof of Proposition 4

Given our assumption about the sufficient statistic in housing price, each household's posterior about A is Gaussian $A \,|\mathcal{I}_i \sim \mathcal{N}\left(\hat{A}_i, \hat{\tau}_A^{-1}\right)$ with conditional mean and variance of

$$\begin{aligned}
\hat{A}_i &= \bar{A} + \tau_A^{-1} \left[\begin{array}{ccc} 1 & 1 & 1 \end{array} \right] \left[\begin{array}{ccc} \tau_A^{-1} + \tau_Q^{-1} & \tau_A^{-1} & \tau_A^{-1} \\ \tau_A^{-1} & \tau_A^{-1} + z_\xi^{-2} \tau_\xi^{-1} & \tau_A^{-1} \\ \tau_A^{-1} & \tau_A^{-1} & \tau_A^{-1} + \tau_\varepsilon^{-1} \end{array} \right]^{-1} \left[\begin{array}{c} Q - \bar{A} \\ z(P) - \bar{A} \\ A_i - \bar{A} \end{array} \right] \\
&= \hat{\tau}_A^{-1} \left(\tau_A \bar{A} + \tau_Q Q + z_\xi^2 \tau_\xi z(P) + \tau_\varepsilon A_i \right), \\
\hat{\tau}_A &= \tau_A + \tau_Q + z_\xi^2 \tau_\xi + \tau_\varepsilon.
\end{aligned}$$

Note that the conditional estimate of $\hat{A}_i$ of household i is increasing in its own productivity A_i. Similarly, the posterior for commercial developers about A is Gaussian $A \,|\mathcal{I}^c \sim \mathcal{N}\left(\hat{A}^c, \hat{\tau}_A^{c-1}\right)$, where

$$\begin{aligned}
\hat{A}^c &= \bar{A} + \tau_A^{-1} \left[\begin{array}{cc} 1 & 1 \end{array} \right] \left[\begin{array}{cc} \tau_A^{-1} + \tau_Q^{-1} & \tau_A^{-1} \\ \tau_A^{-1} & \tau_A^{-1} + z_\xi^{-2} \tau_\xi^{-1} \end{array} \right]^{-1} \left[\begin{array}{c} Q - \bar{A} \\ z(P) - \bar{A} \end{array} \right] \\
&= \hat{\tau}_A^{c-1} \left(\tau_A \bar{A} + \tau_Q Q + z_\xi^2 \tau_\xi z(P) \right), \\
\hat{\tau}_A^c &= \tau_A + \tau_Q + z_\xi^2 \tau_\xi.
\end{aligned}$$

This completes our characterization of learning by households and commercial developers.

We now turn to the optimal decision of commercial developers. Since the posterior for $A - A^*$ of households is conditionally Gaussian, it follows that the expectations in the expression of K in Proposition 2 is a function of the two conditional moments, $\hat{A}^c - A^*$ and

$\hat{\tau}_A^c$. Let

$$F\left(\hat{A}^c - A^*, \hat{\tau}_A^c\right) = E\left[\left(\frac{e^{(A-A^*)}\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2} + \frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)^{\eta_c}}{\Phi\left(\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)^{\eta_c+\frac{\psi(1-\alpha)}{1+\psi}}\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2} + \frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)^{-\frac{\psi(1-\alpha)}{1+\psi}}}\right)^{\frac{1+\psi}{\psi+\alpha}} \middle| \mathcal{I}^c\right].$$

Define $z = \frac{A-A^*}{\tau_\varepsilon^{-1/2}}$ and the function $f(z)$:

$$f(z) = e^{\tau_\varepsilon^{-1/2}z}\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2} + z\right)^{\eta_c}}{\Phi(z)^{\eta_c}}\left(\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2} + z\right)}{\Phi(z)}\right)^{\frac{\psi}{1+\psi}(1-\alpha)},$$

which is the term inside the bracket in the expectation. Then, it follows that

$$\begin{aligned}\frac{1}{f(z)}\frac{df(z)}{dz} &= \tau_\varepsilon^{-1/2} + \eta_c\left(\frac{\phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2} + z\right)}{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2} + z\right)} - \frac{\phi(z)}{\Phi(z)}\right) \\ &\quad + \frac{\psi}{1+\psi}(1-\alpha)\left(\frac{\phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2} + z\right)}{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2} + z\right)} - \frac{\phi(z)}{\Phi(z)}\right).\end{aligned}$$

Notice that $\frac{\phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+z\right)}{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+z\right)} - \frac{\phi(z)}{\Phi(z)}$ achieves its minimum as $z \to -\infty$. Applying L'Hospital's Rule, it follows that the minimum of $\frac{1}{f(z)}\frac{df(z)}{dz}$ is given by

$$\begin{aligned}\lim_{z\to-\infty}\frac{1}{f(z)}\frac{df(z)}{dz} &= \tau_\varepsilon^{-1/2} + \lim_{z\to-\infty}\eta_c\left(\frac{\frac{d}{dz}\phi\left(\frac{1+\psi}{\psi+\alpha+(1-\alpha)\eta_c}\tau_\varepsilon^{-1/2} + z\right)}{\phi\left(\frac{1+\psi}{\psi+\alpha+(1-\alpha)\eta_c}\tau_\varepsilon^{-1/2} + z\right)} - \frac{\frac{d}{dz}\phi(z)}{\phi(z)}\right) \\ &\quad + \frac{\psi}{1+\psi}(1-\alpha)\left(\frac{\frac{d}{dz}\phi\left(\frac{(1+\psi)(1-\eta_c)}{\psi+\alpha+(1-\alpha)\eta_c}\tau_\varepsilon^{-1/2} + z\right)}{\phi\left(\frac{(1+\psi)(1-\eta_c)}{\psi+\alpha+(1-\alpha)\eta_c}\tau_\varepsilon^{-1/2} + z\right)} - \frac{\frac{d}{dz}\phi(z)}{\phi(z)}\right) \\ &= \alpha\frac{1+\psi}{\psi+\alpha+(1-\alpha)\eta_c}(1-\eta_c)\tau_\varepsilon^{-1/2} \\ &> 0\end{aligned}$$

from which follows that $\frac{1}{f(z)}\frac{df(z)}{dz} \geq 0$ for all z, and therefore $\frac{df(z)}{dz} \geq 0$, since $f(z) \geq 0$. Consequently, since $f(z)^{\frac{1+\psi}{\psi+\alpha}}$ is a monotonic transformation of $f(z)$, it follows that $\frac{dF}{dx}(x, \hat{\tau}_A) \geq 0$ since this holds for all realizations of $A - A^*$. This establishes that the optimal choice of commercial facility is increasing with $\hat{A}^c$, since $f(z)$ is increasing for each realization of z.

The optimal choice of K then takes the following form:

$$\log K = \frac{1}{\lambda - \alpha\frac{1+\psi}{\psi+\alpha}}\log F\left(\hat{A}^c - A^*, \hat{\tau}_A^c\right) + \frac{\frac{1+\psi}{\psi+\alpha}}{\lambda - \alpha\frac{1+\psi}{\psi+\alpha}}A^* + k_0.$$

By substituting the expressions for K_i and l_i into the utility of household i given in Proposition 1, we obtain

$$\begin{aligned}
& E\left[U_i|\mathcal{I}_i\right] \\
= \; & (1-\alpha)\frac{\psi}{1+\psi}e^{\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A_i+\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\left(\log F\left(\hat{A}^c-A^*,\hat{\tau}_A^c\right)+\frac{1+\psi}{\psi+\alpha}A^*\right)+\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\eta_c-\alpha\frac{1+\psi}{\psi+\alpha}\right)A^*+u_0} \\
& \cdot E\left[\left(\frac{e^{\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\left(\frac{(1+\psi)\eta_c}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}-\alpha\frac{1+\psi}{\psi+\alpha}\right)(A-A^*)}\Phi\left(\frac{(1+\psi)\tau_\varepsilon^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)^{\eta_c}}{\Phi\left(\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)^{\eta_c-\alpha}\Phi\left(\frac{(1+\psi)(1-\eta_c)\tau_\varepsilon^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)^{\alpha}}\right)^{\frac{1+\psi}{\psi+\alpha}}\middle|\mathcal{I}_i\right],
\end{aligned}$$

where u_0 is given in the proof of Proposition 3.

Since the posterior for $A-A^*$ of household i is conditionally Gaussian, it follows that the expectations in the expressions above are functions of the first two conditional moments $\hat{A}_i-A^*$ and $\hat{\tau}_A$. Let

$$G\left(\hat{A}_i-A^*,\hat{\tau}_A\right)=E\left[\left(\frac{e^{\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\left(\frac{(1+\psi)\eta_c}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}-\alpha\frac{1+\psi}{\psi+\alpha}\right)(A-A^*)}\Phi\left(\frac{(1+\psi)\tau_\varepsilon^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)^{\eta_c}}{\Phi\left(\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)^{\eta_c-\alpha}\Phi\left(\frac{(1+\psi)(1-\eta_c)\tau_\varepsilon^{-1/2}}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}+\frac{A-A^*}{\tau_\varepsilon^{-1/2}}\right)^{\alpha}}\right)^{\frac{1+\psi}{\psi+\alpha}}\middle|\mathcal{I}_i\right]$$

Define $z=\frac{A-A^*}{\tau_\varepsilon^{-1/2}}$, and the function $g(z)$

$$\begin{aligned}
g(z) \; = \; & e^{\frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\left(\frac{(1+\psi)\eta_c}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}-\alpha\frac{1+\psi}{\psi+\alpha}\right)\tau_\varepsilon^{-1/2}z}\frac{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+z\right)^{\eta_c}}{\Phi(z)^{\eta_c}} \\
& \cdot\frac{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+z\right)^{-\alpha}}{\Phi(z)^{-\alpha}},
\end{aligned}$$

as the term inside the bracket. Then, it follows that:

$$\begin{aligned}
\frac{1}{g(z)}\frac{dg(z)}{dz} \; = \; & \frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\eta_c-\alpha\frac{1+\psi}{\psi+\alpha}\right)\tau_\varepsilon^{-1/2} \\
& -\eta_c\left(\frac{\phi(z)}{\Phi(z)}-\frac{\phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+z\right)}{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+z\right)}\right) \\
& +\alpha\left(\frac{\phi(z)}{\Phi(z)}-\frac{\phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+z\right)}{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+z\right)}\right).
\end{aligned}$$

Note that $\frac{\phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+z\right)}{\Phi\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+z\right)}-\frac{\phi(z)}{\Phi(z)}$ achieves its minimum as $z\to-\infty$. Applying

L'Hospital's Rule, it follows that the minimum of $\frac{1}{g(z)}\frac{dg(z)}{dz}$ is given by

$$\begin{aligned}\lim_{z\to-\infty}\frac{1}{g(z)}\frac{dg(z)}{dz} &= \frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\eta_c-\alpha\frac{1+\psi}{\psi+\alpha}\right)\tau_\varepsilon^{-1/2}\\ &\quad+\eta_c\lim_{z\to-\infty}\left(\frac{\frac{d}{dz}\phi\left(\frac{1+\psi}{\psi+\alpha+(1-\alpha)\eta_c}\tau_\varepsilon^{-1/2}+z\right)}{\phi\left(\frac{1+\psi}{\psi+\alpha+(1-\alpha)\eta_c}\tau_\varepsilon^{-1/2}+z\right)}-\frac{\frac{d}{dz}\phi(z)}{\phi(z)}\right)\\ &\quad+\alpha\lim_{z\to-\infty}\left(\frac{\frac{d}{dz}\phi(z)}{\phi(z)}-\frac{\frac{d}{dz}\phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+z\right)}{\Phi\left(\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}+z\right)}\right)\\ &= \frac{1}{1-\alpha}\frac{\psi+\alpha}{\psi}\left(\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\eta_c-\alpha\frac{1+\psi}{\psi+\alpha}\right)\tau_\varepsilon^{-1/2}\\ &\quad+\left(\left(1-\eta_c\right)\alpha-\eta_c\right)\frac{1+\psi}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}\tau_\varepsilon^{-1/2}.\end{aligned}$$

With some manipulation, the above expression is collapsed to

$$\lim_{z\to-\infty}\frac{1}{g(z)}\frac{dg(z)}{dz}=0,$$

and it follows that $\frac{1}{g(z)}\frac{dg(z)}{dz}\geq 0$, and therefore $\frac{dg(z)}{dz}\geq 0$, since $g(z)\geq 0$. Consequently, since $g(z)^{\frac{1+\psi}{\psi+\alpha}}$ is a monotonic transformation of $g(z)$, it follows that $\frac{dG}{dx}(x,\hat{\tau}_A)\geq 0$, since this holds for all realizations of $A-A^*$.

Since the household with the critical productivity A^* must be indifferent to its neighborhood choice at the cutoff, it follows that $U_i-P=0$, which implies

$$\begin{aligned}&e^{\frac{(1+\psi)(1-\eta_c)}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}A_i+\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}\left(\log F\left(\hat{A}^c-A^*,\hat{\tau}_A^c\right)+\frac{1+\psi}{\psi+\alpha}A^*\right)+\frac{1}{1-\alpha}\frac{1+\psi}{\psi}\left(\frac{(1+\psi)\eta_c}{(1-\alpha)\psi+(1+\alpha\psi)\eta_c}-\alpha\frac{1+\psi}{\psi+\alpha}\right)A^*+u_0}\\ &\cdot G\left(\hat{A}_i-A^*,\hat{\tau}_A\right)=\frac{1+\psi}{\psi(1-\alpha)}P,\ A_i=A^*\end{aligned}\tag{22}$$

which does not depend on the unobserved A or the supply shock ξ. As such, $A^*=A^*(\log P,Q)$. Furthermore, since $\hat{A}_i^*$ is increasing in A_i and $G\left(\hat{A}_i^*-A^*,\tau_A\right)$ is (weakly) increasing in $\hat{A}_i$, it follows that the LHS of equation (22) is (weakly) monotonically increasing in A_i, confirming the cutoff strategy assumed for households is optimal. Those with the RHS being nonnegative enter the neighborhood, and those with it being negative choose to live elsewhere.

It then follows from market-clearing that

$$\Phi\left(-\sqrt{\tau_\varepsilon}\left(A^*-A\right)\right)=\Phi\left(-\sqrt{\tau_e}\left(\omega^*-\xi\right)\right).$$

Since the CDF of the normal distribution is montonically increasing, we can invert the above market-clearing condition, and impose equation (19) to arrive at

$$\log P=\frac{1}{1+k}\left(\sqrt{\frac{\tau_\varepsilon}{\tau_e}}\left(A-A^*\right)-\xi\right),$$

from which follows that

$$z(P) = \sqrt{\frac{\tau_e}{\tau_\varepsilon}}(1+k)\log P + A^* = A - \sqrt{\frac{\tau_e}{\tau_\varepsilon}}\xi,$$

and therefore $z_\xi = \sqrt{\frac{\tau_\varepsilon}{\tau_e}}$. This confirms our conjecture for the sufficient statistic in housing price and that learning by households is indeed a linear updating rule.

As a consequence, the conditional estimate of household i is

$$\begin{aligned}
\hat{A}_i &= \hat{\tau}_A^{-1}\left(\tau_A\bar{A} + \tau_Q Q + \frac{\tau_\varepsilon}{\tau_e}\tau_\xi\left(\sqrt{\frac{\tau_e}{\tau_\varepsilon}}\left((1+k)\log P + \bar{\xi}\right) + A^*\right) + \tau_\varepsilon A_i\right), \\
\hat{\tau}_A &= \tau_A + \tau_Q + \frac{\tau_\varepsilon}{\tau_e}\tau_\xi + \tau_\varepsilon,
\end{aligned}$$

and the conditional estimate of commercial developers is

$$\begin{aligned}
\hat{A}^c &= \hat{\tau}_A^{c-1}\left(\tau_A\bar{A} + \tau_Q Q + \frac{\tau_\varepsilon}{\tau_e}\tau_\xi\left(\sqrt{\frac{\tau_e}{\tau_\varepsilon}}\left((1+k)\log P + \bar{\xi}\right) + A^*\right)\right), \\
\hat{\tau}_A^c &= \tau_A + \tau_Q + \frac{\tau_\varepsilon}{\tau_e}\tau_\xi.
\end{aligned}$$

Substituting for prices, and simplifying A^* terms, we can express equation (22) as

$$e^{\left(\frac{\lambda\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}+\frac{\sqrt{\tau_\varepsilon/\tau_e}}{1+k}\right)A^*} G\left(\hat{A}^* - A^*, \hat{\tau}_A\right) F\left(\hat{A}^c - A^*, \hat{\tau}_A^c\right)^{\frac{\alpha\frac{1+\psi}{\psi+\alpha}}{\lambda-\alpha\frac{1+\psi}{\psi+\alpha}}} = \frac{1+\psi}{\psi(1-\alpha)} e^{\frac{1}{1+k}\sqrt{\frac{\tau_\varepsilon}{\tau_e}}z - u_0}, \tag{23}$$

where

$$\begin{aligned}
\hat{A}_i^* &= \hat{\tau}_A^{-1}\left(\tau_A\bar{A} + \tau_Q Q + \frac{\tau_\varepsilon}{\tau_e}\tau_\xi\left(\sqrt{\frac{\tau_e}{\tau_\varepsilon}}\left((1+k)\log P + \bar{\xi}\right) + A^*\right) + \tau_\varepsilon A^*\right), \\
\hat{A}^c &= \hat{\tau}_A^{c-1}\left(\tau_A\bar{A} + \tau_Q Q + \frac{\tau_\varepsilon}{\tau_e}\tau_\xi\left(\sqrt{\frac{\tau_e}{\tau_\varepsilon}}\left((1+k)\log P + \bar{\xi}\right) + A^*\right)\right).
\end{aligned}$$

Notice that the LHS of equation (23) is continuous in A^*. As $A^* \to -\infty$, the LHS of equation (23) converges to

$$\lim_{A^*\to-\infty} LHS = 0.$$

Furthermore, by L'Hospital's Rule and the Sandwich Theorem, one also has that

$$\lim_{A^*\to\infty} LHS = \infty.$$

Since the RHS is independent of A^*, it follows that the LHS and RHS intersect once. Therefore, a cutoff equilibrium in the economy with informational frictions exists.

Finally, notice that, as $\tau_Q \nearrow \infty$, that $\hat{A}^c$ and $\hat{A}^i$ converge to A a.s., since $\hat{\tau}_A^c,\ \hat{\tau}_A^i \nearrow \infty$. Taking the limit along a sequence of τ_Q, it is straightforward to verify that equation (22)

converges to equation (21), and therefore A^* converges to its perfect-information benchmark value. Taking similar limits for the expressions for capital and labor supply verify that they also converge to their perfect-information benchmark values, and therefore the noisy rational expectations cutoff equilibrium converges to the perfect-information benchmark economy as $\tau_Q \nearrow \infty$.