Foreign Reserve Management

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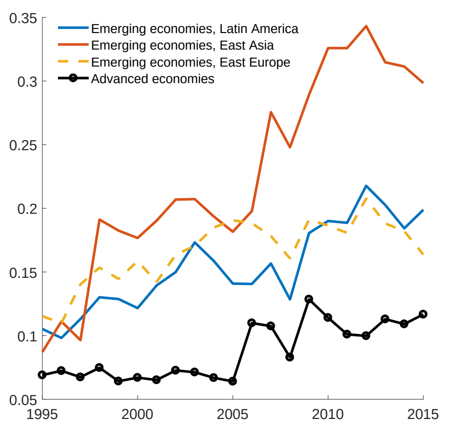
Motivation

Over the past 20 years massive increase in foreign reserves holdings by Central Banks around the world



Reserves-GDP

Motivation (ctd)

Why do countries hold foreign reserves? Two motives

1. *Precautionary role*: reserves used as a buffer for bad shocks
2. *Exchange rate management*: reserves used to achieve a policy for nominal exchange rates

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This paper: Given exchange rates and monetary policy objectives, How should a Central Bank manage its reserve portfolio?

Foreign Reserve management without uncertainty

CB has a monetary policy objective: $\{i, e_t, e_{t+1}\}$

Suppose that $(1+i)\frac{e_t}{e_{t+1}} > (1+i^\star)$ (needs limited arbitrage)

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$$u'(c_t) = \beta \left[(1+i)\frac{e_t}{e_{t+1}} \right] u'(c_{t+1})$$

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Policy has two costs

- Current consumption is too low
- Resource loss from capital inflows: Foreigners exploit interest differential and *earn a profit*

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- **Multiple** consumption profiles consistent with *the same nominal rate and exchange rate*
- CB can implement any of them by managing its foreign reserves portfolio
    - Tilts consumption towards the future, as before
    - But can also *change consumption across states*

With uncertainty (continued)

- Thus CB has more options with uncertainty

For example:

- A negative covariance between the appreciation and future consumption boosts current consumption *for the same targets*:

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Trade-off: consumption smoothing vs resource losses

Resolving the trade-off

When potential capital inflows are small – resource losses are small

- Optimal to focus on consumption smoothing
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When potential capital inflows are large – resources losses are large

- Optimal to focus on minimizing resource losses
- Purchase relatively safe foreign portfolio

Framework

- Two-period model, $t \in \{1, 2\}$
  - Small open economy (central bank + households)
  - International Financial Market
  - Foreign Intermediaries
- Uncertainty realized at $t = 2$
  - $s \in S \equiv \{s_2, ..., s_N\}, \pi(s)$
- One (tradable) good, law of one price, foreign price normalized to 1

Asset markets: complete but segmented

International financial markets (IFM)

- Full set of Arrow-Debreu securities in foreign currency:
    - Security s: 1 unit of foreign currency in state s, 0 otherwise
    - Price $q(s)$ in terms of foreign currency at $t = 1$

Domestic financial market

- Full set of Arrow-Debreu securities in domestic currency
    - Security s: 1 unit of domestic currency in state s, 0 otherwise
    - Price $p(s)$ in terms of domestic currency at $t = 1$

Intermediaries

- Trade securities with SOE & IFM

Households

- Endowment: $(y_1, \{y_2(s)\})$, transfers: $(\{T_2(s)\})$

$$\max_{c_1,\{c_2(s),a(s),f(s)\}} \left\{ u(c_1) + \beta \sum_{s\in S} \pi(s) u(c_2(s)) \right\}$$

subject to:

$$y_1 = c_1 + \sum_{s\in S} \left[q(s)f(s) + p(s)\frac{a(s)}{e_1} \right]$$

$$y_2(s) + T_2(s) + f(s) + \frac{a(s)}{e_2(s)} = c_2(s) \quad \forall s \in S$$

$$f(s) \geq 0, \quad \forall s \in S$$

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$f(s), a(s)$: holdings of foreign and domestic security s

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Foreign Intermediaries

- Endowed with capital $\bar{w}$

$$\max_{\{d_1^\star, d_2^\star(s), a^\star(s), f^\star(s)\}} d_1^\star + \sum_{s \in S} \pi(s) \Lambda(s) d_2^\star(s)$$

subject to:

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Consider $\Lambda(s) = \frac{q(s)}{\pi(s)}$ (same SDF as IFM)

Central Bank

- CB has an objective for the nominal interest rate and exchange rates that we take as given: $(i, e_1, \{e_2(s)\})$

$$1 + i = \left(\sum_{s \in S} p(s) \right)^{-1} \tag{NIRC}$$

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- Given objectives, CB chooses policies to maximize welfare

Characterizing equilibria: Arbitrage returns

- **Arbitrage return** for security s:

$$\kappa(s) \equiv \frac{\frac{e_1}{e_2(s)p(s)}}{\frac{1}{q(s)}} - 1$$

$\kappa(s) > 0 \Rightarrow$ domestic security paying in state s yields higher return

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- Intermediaries: invest all available funds in highest return security
- Denote by $\bar{\kappa} \equiv \max_s\{\kappa(s)\}$

Characterizing equilibria: Resource constraint

$$y_1 - c_1 + \sum_{s \in S} q(s)[y_2(s) - c_2(s)] - \sum_{s \in S} a^*(s) \frac{p(s)}{e_1} \kappa(s) = 0$$

- Profits for intermediaries are losses for the SOE

$$\sum_{s \in S} a^*(s) \frac{p(s)}{e_1} \kappa(s) = \bar{\kappa} \bar{w}$$

Central bank objective and $\Delta(i)$

Given CB objective $(i, e_1, \{e_2(s)\})$, consider the return differential between risk-free home and foreign bonds

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We study two regimes $(i, e_1, \{e_2(s)\})$

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Capital Inflow Regime

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Which assets $\{F(s)\}$ should CB buy?

- What are the trade-offs involved?
- What are the key goals that should be pursued?

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  Details
- Each $\{\kappa(s)\}$ corresponds to a reserve policy

Interlude II: From arbitrage gaps to reserves, $\kappa(s) \rightarrow F(s)$

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- CB has to buy $F(s)$ to deliver consumption goods in that state

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- Intervention large so that capital inflows cannot undo it

Optimal portfolio management

The CB has two goals:

1. Minimize intertemporal distortions
2. Minimize resource losses generated by interventions

- Goals not necessarily aligned
- Degree of financial openness controls the trade-off

Optimal policy depends on financial openness

- We study next two cases

Financially closed economy

Optimal policy. Assume $\bar{w} = 0$ and $q(s) = \beta^{\star}\pi(s)$

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- Key idea: promise high $\kappa(s)$ (more consumption) when e_2 appreciates (i.e., when nominal bond pays more).

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NIRC binds from below:

$$\frac{1+i^{\star}}{1+i} \geq \mathbb{E}\left(\frac{e_1}{e_2(s)}\right)\mathbb{E}\left(\frac{1}{1+\kappa(s)}\right) + \mathrm{Cov}\left(\frac{e_1}{e_2(s)}, \frac{1}{1+\kappa(s)}\right)$$

- To reduce average *intertemporal* distortion $\sim \mathbb{E}(\kappa)$, increase *intratemporal* distortions.

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Prescriptions for reserve policy

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Assume that output volatility is low, we have that

- Buy assets thay pay when the currency appreciates

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$$\min_{\{\kappa(s)\}_{s\in S}} \left\{\max_s\{\kappa(s)\}\right\}$$

$$s.t. \qquad 0 \leq 1 + i - \sum_s \frac{q(s)e_1}{(1+\kappa(s))e_2(s)} \qquad \text{(NIRC)}$$

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- Optimal policy calls for equal gaps $\kappa(s) = \kappa \ \forall s$
  - only allocation in which intermediaries demand risk free bonds
- Some leeway about actual CB portfolio
  - e.g. purchase foreign risk-free bonds

Conclusion

- Developed a framework to analyze the reserve management problem for a CB with nominal objectives
- Uncovered a trade-off that is key for the portfolio choice
- Optimal portfolio hinges on *the potential size of capital flows*
    - Relatively small flows:
        - Invest in foreign assets that pay when the currency appreciates
    - Relatively large flows:
        - Make sure intermediaries demand domestic risk free bonds
- Agenda
    - Implementation with specific assets (e.g. bonds and equity)
    - Closed economy implications

CB must open positive "gaps"

For some $s, \kappa(s) > 0$

Under $\kappa(s) \leq 0$

$$\sum_{s \in S} p(s) = \sum_{s \in S} q(s) \frac{e_1}{e_2(s)(1+\kappa(s))} \geq \sum_{s \in S} q(s) \frac{e_1}{e_2(s)} = \frac{1+\Delta(i)}{1+i}$$

Since $\Delta(i) > 0$,

$$\left[\sum_{s \in S} p(s) \right]^{-1} < (1+i)$$

Interest rate is too low relative to NIRC.

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A key condition: arbitrage return on risk-free bond

Investor with one unit of the consumption good

- Invest it in domestic risk free bond:

Cost today: 1 Benefit tomorrow: $\left\{\left(\frac{e_1}{\sum_s p(s)}\right)\frac{1}{e_2(s)}\right\}$

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- If $\sum_s q(s)\left(\left(\frac{e_1}{\sum_s p(s)}\right)\frac{1}{e_2(s)}\right) \neq 1 \Rightarrow$

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- If $\sum_s q(s) \left(\left(\frac{e_1}{\sum_s p(s)} \right) \frac{1}{e_2(s)} \right) \neq 1 \Rightarrow$
  - Note $\Delta(i) > 0 \iff \sum_{s \in S} q(s)(e_1(1+i)\frac{1}{e_2(s)}) > 1$

A key condition: arbitrage return on risk-free bond

Investor with one unit of the consumption good

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Cost today: 1 Benefit tomorrow: $\left\{ \left(\frac{e_1}{\sum_s p(s)} \right) \frac{1}{e_2(s)} \right\}$

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Characterizing equilibria: Balance of Payment

- Trade deficits and net foreign assets:

$$\underbrace{c_1 - y_1}_{\text{trade deficit}} = \underbrace{\frac{\sum_s p(s)a^\star(s)}{e_1}}_{\text{foreign liabilities}} - \underbrace{\sum_s q(s)\left[f(s) + F(s)\right]}_{\text{foreign assets}}$$