Low Interest Rates, Market Power, and Productivity Growth*

Ernest Liu
Princeton University

Atif Mian
Princeton University and NBER

Amir Sufi
University of Chicago Booth School of Business and NBER

March 22, 2018

[Preliminary: Please do not share or cite]

Abstract

We build an endogenous growth model to show how low interest rates can increase market power and lower aggregate productivity growth. The model delivers both a traditional expansion in productivity growth in response to lower interest rates, and a slowdown in productivity growth from an increase in market power. When interest rates fall to low levels, the strategic competition effect dominates: the distance between a market leader and a follower increases which reduces investments in productivity by both. The model predicts that very low interest rates lead to an increase in market concentration and mark-ups, a decline in creative destruction and firm entry, a widening productivity-gap between the leader and followers within an industry, and ultimately a decline in productivity growth. We provide empirical evidence in support of these predictions.

*We thank Andy Haldane for helpful comments. We thank Michael Varley and Thomas Kroen for excellent research assistance and the Julis Rabinowitz Center For Public Policy and Finance at Princeton for financial support. Liu: ernestliu@princeton.edu; Mian: atif@princeton.edu; Sufi: amir.sufi@chicagobooth.edu.
1 Introduction

Long term interest rates in advanced economies have been on a downward trajectory since the early 1980s, reaching very low values in the aftermath of the Great Recession. Such low interest rates have sparked renewed discussion of causes and consequences. This study focuses on the supply-side implications. In particular, is it possible that low interest rates make markets less competitive thus lowering business dynamism and growth?

In traditional models, a decline in the interest rate has a neutral or positive effect on productivity growth. A lower interest rate increases the present value of future returns to productivity enhancement and therefore encourages firms to devote more resources to boosting productivity. This study builds a model rooted in the dynamic contest literature that highlights a second mechanism that can overturn the traditional result at very low interest rates. When interest rates fall below a threshold, a further decline in the interest rate boosts market concentration enough to endogenously reduce investment in productivity growth by both market leaders and followers.

Growth in the model is an outcome of a stochastic investment game between a market leader and a follower in each industry (e.g., Aghion, Harris, Howitt and Vickers (2001)). An investment in productivity enhancement in the model is a binary decision which stochastically increases the relative productivity position of the firm. The economy has a continuum of product lines, each of which is occupied by a high productivity leader and a lower productivity follower. Both the leader and the follower make investments in productivity that stochastically improve their respective positions. The heart of the model lies in the strategic incentives to invest in productivity enhancement: the leader wants to maintain the higher profits associated with being more productive and the follower understands the difficulty in overcoming the lead already enjoyed by the leader.

The state variable of the model is the productivity difference between the market leader and the follower. A main result of the model is that the leader has a (weakly) more aggressive investment policy in all states. When the productivity difference between the leader and the follower is small, both firms invest. But when the productivity difference becomes large, the discouraged follower gives up while the leader continues to invest in order to solidify her market position. We refer
to the former as the “competitive region” of the state space, and the latter as the “monopolistic region.” At the bifurcation of the state space, there is a large marginal benefit of investment by the leader: if the leader can get into the monopolistic region, the threat of losing her position becomes significantly lower as the follower is discouraged from investing to catch up.

The key question examined in the model is: what happens to relative market position and productivity growth when there is a decline in the interest rate which the firms use to discount profits? The traditional effect is present in the model: a drop in the interest rate causes both the leader and the follower to increase investment in productivity enhancement for any difference in productivity levels. All future profits from becoming a market leader are valued more highly, inducing higher investment.

However, a decline in the interest rate also induces a strategic effect which lowers total investment in productivity enhancement in the industry. This occurs because the leader boosts investment by (weakly) more than the follower when interest rates fall. This is a crucial result of the model, and so it is worth explaining why this occurs.

When interest rate declines, both leader and follower increase their investment due to the traditional effect of lower interest rate increasing present value of future profits. However, the increase in investment is always (weakly) stronger for the leader. This results in an increase in the steady state distance between the leader and the follower, i.e. industry market structure tends to become more monopolistic. The leader enjoys greater advantage and hence stronger market power over the follower. The increased steady state distance between the leader and follower at lower interest rate discourages the follower from investing, and as a result the leader also feels less of a need to invest. This is the key strategic effect that tends to reduce investment in steady state as interest rate declines in a low interest rate regime.

If interest rates become low enough, the gap between the leader and follower becomes so large that the model produces an additional effect which further reduces productivity growth: the leader is more likely to behave as a “lazy monopolist” and reduces effort to innovate because there is such a small chance that the follower will ever threaten her position. In the extreme, very low interest rates both discourage the follower from innovation, and increase the share of leaders
who behave like lazy monopolists since they feel no significant threat to their market leadership position.

Aggregate growth of the economy is determined by the steady state distribution of industries that are in the monopolistic region versus industries that are in the competitive region. Since both leader and follower invest in the competitive region, growth is higher if a greater proportion of industries are in the competitive region. However, as we show in our model, at low enough interest rates the monopolistic region always dominates. In fact, the probability that an industry belongs to the monopolistic region goes to one as interest rate goes to zero. In other words, the magnitude of the negative effect of reduced market competition “explodes” as long-term real interest rate nears zero.

This key result gives us an inverted-U relationship between long-term interest rate and productivity growth. A decline in interest rate in the “normal interest rate regime” increases growth as in traditional models. However, once interest rate enters the “low interest rate regime”, a further decline in interest rate works to lower productivity growth. The last result is driven by the strategic competition effect of low interest rate on market power.

Relative to the existing literature, the model provides an alternative interpretation for trends in long-term interest rates, market concentration, productivity growth, and the phenomenon of secular stagnation. In traditional models, equilibrium interest rates and growth are determined by the intersection of aggregate demand from the household Euler equation and a supply curve. A prominent explanation in the secular stagnation literature for the decline in long-term interest rates is a long-term inward shift in the aggregate demand curve that started in the 1980s (e.g., Summers (2014)).

In response to lower long term interest rates driven by an inward shift in aggregate demand, traditional models with no frictions predict either no change in growth or a positive effect.\footnote{e.g. The supply-side relationship between growth and interest rate is flat in Solow and Ramsey style growth models that assume exogenous growth. The relationship is downward sloping in endogenous growth models such as Aghion and Howitt (1992) and Klette and Kortum (2004), i.e. the supply-side response to a fall in interest rate is to boost growth all else equal. The downward sloping relationship can become â“verticalâ” in the limit as in Grossman and Helpman (1991) and Romer (1990)} To generate a lower level of GDP in response to a shift inward in aggregate demand, business cycle
models typically rely on frictions associated with the zero lower bound (e.g., Krugman (1998); Eggertsson and Krugman (2012); Guerrieri and Lorenzoni (2017)). If such frictions are persistent, they can eventually cause a decline in GDP growth Benigno and Fornaro (n.d.) as well as the level of GDP (Eggertsson, Mehrotra and Robbins (2017)).

The model presented here proposes an explanation for secular stagnation that is distinct from the mechanisms discussed in the existing literature. As in the existing literature, the model assumes that the decline in long-term interest rates over the past 35 years has been driven by an inward shift in aggregate demand. However, unlike previous models, the model presented here illuminates an endogenous response of firms that generates higher market concentration, lower productivity growth, and hence lower GDP growth. The model generates these outcomes without reliance on frictions such as the zero lower bound on nominal interest rates. When interest rates are already low, a further decline in interest rates driven by an inward shift in the aggregate demand makes it more likely that firms enter the monopolistic region of the state space. In such a circumstance, firms endogenously respond to lower interest rates by reducing investments in productivity for the reasons outlined above.\(^2\)

We provide a number of empirical results from the U.S. and other advanced OECD countries in support of the mechanism highlighted by our model. First, the model predicts that a decline in real interest rates driven by an inward shift in the aggregate demand curve will be associated with a rise in industry concentration, markups and profit share, and a decline in business dynamism. The relationship between operating profits as a share of GDP and long-term (10 year) interest rate since 1985 is indeed strongly negative. Moreover, a fall in interest rate is also strongly associated with a fall in business creation and business destruction. The rise in profit share and concentration, and fall in business entry and exit is already discussed and analyzed in papers such as De Loecker and Eeckhout (2017), Barkai (2018), Autor, Dorn, Katz, Patterson and Van Reenen (2017), Gutiérrez and Philippon (2016, 2017), Grullon, Larkin and Michaely (2016), Davis and Haltiwanger (2014), Decker, Haltiwanger, Jarmin and Miranda (2016), Haltiwanger (2015), Hathaway and Litan (2015), and Andrews, Criscuolo and Gal (2016).

\(^2\)There has been work in the past that links interest rates to the level of productivity (e.g. Caballero, Hoshi and Kashyap (2008); and Gopinath, Kalemi-Ozcan, Karabarbounis and Villegas-Sanchez (2017)). This study focuses on the relationship between interest rates and the growth rate of productivity.
Second, our model predicts that as long-term interest rate declines, the productivity gap between industry leaders relative to followers increases, and it is this increase in productivity gap that lowers aggregate productivity growth. These are quite unique predictions of our framework. As we discuss in the empirical section, new firm-level data from the OECD confirms both of these predictions. Berlingieri and Criscuolo (2017) and Andrews et al. (2016) show that the productivity gap between 90th versus 10th percentile firms within narrowly defined industries has been increasing since 2000 across a range of OECD countries. Furthermore, Andrews et al. (2016) show that the productivity gap between market leaders and followers has risen the most in the industries where productivity growth has slowed down the most. These results provide reasonably strong support for the mechanism underlying our theoretical framework.

Third, our model has precise predictions regarding the relative valuation of industry leaders versus followers as well as the relative P/E ratio of industry leaders versus followers. A fall in long-term interest rate increases the expected profit flow for industry leader relative to the follower, and makes industry leader’s position more persistent. Thus both relative valuation as well as relative P/E ratio for leader versus follower in an industry should increase as interest rate declines - especially at very low long term rates. Using data from compustat on publicly listed firms, we find strong support for this hypothesis. Lower interest rates are associated with a rise in the fraction of total stock market value of the top five firms in each industry. Further, the price to earnings ratio of the top five firms in each industry relative to the price to earnings ratio of remaining is also negatively correlated with long term interest rates. This suggests that the profits of market leaders have become more persistent as interest rates have fallen, consistent with our model’s predictions.

Finally, the ultimate prediction of the model is a positive correlation between long term interest rates and aggregate productivity growth. This prediction is confirmed in data covering the United States and OECD countries from 1985 and 2016. As long term interest rates have fallen, advanced economies of the world have seen a rise in market concentration and a decline in productivity growth, as predicted by the model.

3Relatedly Gutiérrez and Philippon (2016, 2017), Lee, Shin and Stulz (2016) show sharp decline of investment relative to operating surplus and that investment gap is especially pronounced in concentrated industries.

4While our focus is on steady state relationship between long-term interest rate and productivity growth, Cette,
Our paper contributes to the endogenous growth literature, and the literature on dynamic contest games in particular. Our framework on competition within an industry differs from the seminal work of Aghion et al. (2001) in that innovation happens step-by-step. The follower can thus not leap-frog the leader instantly. Our model is thus more relevant for the “normal” process of innovation where innovation is gradual and step-by-step. In periods or industries that are subject to major technological revolutions, a large exogenous boost to technological progress - driven by leap-frogging technologies - can overcome the negative market competition effect even at low interest rates.

2 A Theory of Interest Rate, Market Power and Growth

2.1 Model Setup

The economy has a continuum of markets indexed by \( \nu \in [0, 1] \), each with two forward-looking and risk-neutral firms engaging in dynamic competition. Time is continuous. For expositional simplicity, we drop the market subscript \( \nu \) and discuss the game in a generic market.

Let \( z_1(t), z_2(t) \in \mathbb{Z}_{>0} \) denote the productivity levels of the two market participants, and let \( s(t) = |z_1(t) - z_2(t)| \in \mathbb{Z}_{\geq 0} \) be the state variable that captures their productivity gap. When \( s = 0 \), the two participants are said to be neck-to-neck; when \( s > 0 \), one of the firm is a temporary leader (L) while the other is a follower (C). The leader earns flow profit \( \pi_s \) and the follower earns \( \pi_{-s} \) given productivity gap \( s \).

**Assumption 1.** The flow profits \( \pi_s, -\pi_s, \) and \( (\pi_s + \pi_{-s}) \) are non-negative, bounded, weakly concave and weakly increasing in \( s \), with limits

\[
\lim_{s \to \infty} \pi_s = \bar{\pi},
\]

\[
\lim_{s \to \infty} \pi_{-s} = 0.
\]

Because higher states are associated with higher joint profits and more unequal profits between the leader and the follower, we interpret state \( s \) to be more competitive than state \( s' \) if \( s < s' \) and

Fernald and Mojon (2016) show in two-variable VAR that a negative shock to long-term interest rate leads to a fall in productivity growth, consistent with our framework.
more concentrated if \( s > s' \).

Each firm can invest in order to improve its productivity, which evolves in step-increments. Investment in state \( s \) is a binary decision \( \eta_s \in \{0, \eta\} \). Should the firm chooses to invest \( (\eta_s = \eta) \), it pays a cost \( c\eta \) in exchange for a Poisson rate \( \eta \) with which the firm’s productivity improves by one. Specifically, given investment decisions \( \{\eta_s, \eta_{-s}\} \) over interval \( \Delta \) at time \( t \), the leader and the follower pay cost \( c \cdot \eta_s \Delta \) and \( c \cdot \eta_{-s} \Delta \) respectively, and the state \( s \) transitions according to

\[
\begin{aligned}
s (t + \Delta) &= \begin{cases} 
    s (t) + 1 & \text{with probability } \Delta \cdot \eta_s \\
    s (t) - 1 & \text{with probability } \Delta \cdot (\kappa + \eta_{-s}) \\
    s (t) & \text{otherwise},
\end{cases}
\end{aligned}
\]

where \( \kappa \) is the exogenous Poisson rate of technology diffusion, enabling the follower to close productivity gap by one step. The key assumption embodied in the investment technology is that catching up is a gradual process: the productivity gap has to be closed step-by-step, and the follower cannot “leapfrog” the leader by overtaking leadership with one successful innovation.

Interest rate is modeled as the rate at which firms discount future payoffs. Each firm’s value \( v_s (t) \) in state \( s \) at time \( t \) can be expressed as the expected present-discount-value of future profits net of investment costs:

\[
v_s (t) = \mathbb{E} \left[ \int_0^\infty e^{-r\tau} \{ \pi (t + \tau) - c (t + \tau) \} | s \right].
\]

We look for a stationary symmetric Markov-perfect equilibrium such that the value functions and investment decisions depend on the state but not the time index. The stationary HJB equation for the two firms are respectively

\[
rv_s = \pi_s + (\kappa + \eta_{-s}) (v_{s-1} - v_s) + \max \{0, \eta (v_{s+1} - v_s - c)\}
\]

\[
rv_{-s} = \pi_{-s} + \eta_s (v_{-(s+1)} - v_{-s}) + \kappa (v_{-(s-1)} - v_{-s}) + \max \{0, \eta (v_{s+1} - v_s - c)\}.
\]
Definition 1. A stationary symmetric Markov-perfect equilibrium is a collection of value functions and investment decisions \( \{ \eta_s, \eta_{-s}, v_s, v_{-s} \}_{s=0}^{\infty} \) that satisfy the infinite system of equations in (1) and (2).

Given the investment decisions \( \{ \eta_s, \eta_{-s} \}_{s=0}^{\infty} \), an equilibrium induces an ergodic Markov process over state transitions and emits a unique stationary distribution over the state space. Let \( M_s \) denote the expected time for a firm currently in state \( s \) to reach state zero. It is a measure of dynamic monopoly power, as it captures the expected duration in which the firm stays as a leader.

Definition 2. A steady-state of the economy \( \{ \mu_s \}_{s=0}^{\infty} \) induced by equilibrium investments \( \{ \eta_s, \eta_{-s} \}_{s=0}^{\infty} \) is a distribution over the state space such that

\[
2\mu_0\eta_0 = (\mu_1 + \kappa) \mu_1
\]

\[
\mu_s\eta_s = (\eta_{-(s+1)} + \kappa) \mu_{s+1}
\]

for all \( s > 0 \), and

\[
\sum_{s=0}^{\infty} \mu_s = 1.
\]

In a steady-state, the aggregate investment is \( I \equiv \sum_{s=0}^{\infty} \mu_s (\eta_s + \eta_{-s}) \) and the productivity growth rate of the economy is

\[
g \equiv \lim_{\Delta \to 0} \left( \mathbb{E} \left[ \frac{z_1(t + \Delta) + z_2(t + \Delta)}{2} \right] - \mathbb{E} \left[ \frac{z_1(t) + z_2(t)}{2} \right] \right).
\]

Let \( M \equiv \sum_{s=1}^{\infty} \mu_s M_s \) denote the average duration of market leadership.

In the definition, the aggregate productivity growth rate is measured as the average productivity growth across individual firms. This is without loss of generality because in a steady-state, leaders and followers in all markets have the same expected growth rate of productivity:

\[
\lim_{\Delta \to 0} \left( \mathbb{E} [z_1(t + \Delta)] - \mathbb{E} [z_1(t)] \right) = \lim_{\Delta \to 0} \left( \mathbb{E} [z_2(t + \Delta)] - \mathbb{E} [z_2(t)] \right).
\]

We impose the following assumption on the investment technology \( \eta \).

Assumption 2. \( 2c\eta > \bar{\pi} \) and \( \eta > \kappa \).
Because firms are risk-neutral, we can alternatively interpret the binary investment decision as a continuous one, i.e. firms can choose bounded investment \( \eta \in [0, \eta] \) with marginal cost \( c \). The first part of the assumption ensures that the upperbound on investment is “not too tight”, in the sense that it is unprofitable for both firms to undertake the maximum investment simultaneously and permanently given the maximum joint flow payoffs. The second part of the assumption states that when the follower does not invest, the leader’s endogenous investment can be large enough to overcome the productivity diffusion such that his market power is expected to increase over time.

### 2.2 Steady-state Equilibrium

In the model, firms not only invest for gains in the flow profits in higher states; more importantly, they invest in order to enhance the respective market positions, the advancement of which enables the firms to reach for even higher profits in the future. For the follower, closing the productivity gap by one step enables him further closes the gaps in the future and eventually catches up with the leader. For the leader, widening the productivity gap brings higher profits, the option value to further increase the lead in the future, as well as the improved expected duration of market leadership, because it would now take the follower additional steps to catch up.

To understand the equilibrium, we first describe the structure of value functions given any sequence of (potentially non-equilibrium) investment decisions \( \{\eta_s, \eta_{-s}\}_{s=0}^{\infty} \). The fact that firms are forward-looking implies that value function in each state can be written as a weighted average of flow payoffs in all ergodic states induced by the investments, i.e.,

\[
v_s = \sum_{s' = -\infty}^{\infty} \lambda_{s'|s} \left( \frac{\pi_{s'} - c\eta_{s'}}{r} \right), \quad \text{where} \quad \sum_{s' = -\infty}^{\infty} \lambda_{s'|s} = 1 \quad \text{for all} \ s.
\]

The term \( \left( \frac{\pi_{s'} - c\eta_{s'}}{r} \right) \) represents permanent-value in state \( s' \), or in other words, the present-discount value of the flow payoffs in state \( s' \) if a firm stays in state \( s' \) permanently. In equilibrium, the value in state \( s \) can be written as a weighted average of the permanent-value across all ergodic states. The weight \( \lambda_{s'|s} \) can be interpreted as the present-discounted value of time that
the firm is going to be in state $s'$, given that the current state is $s$. The weights $\{\lambda_{s'|s}\}_{s'=s}^{\infty}$ is a measure conditional on the current state. When the current state $s$ is high, the firm is expected to spend more time in higher states, and the conditional distribution $\{\lambda_{s'|s+1}\}_{s'=s}^{\infty}$ first-order-stochastically dominates $\{\lambda_{s'|s}\}_{s'=s}^{-\infty}$ for all $s$.

Lastly, the conditional distribution $\{\lambda_{s'|s}\}_{s'=s}^{\infty}$ depends on the equilibrium investment decisions as well as the interest rate. Holding investment decisions fixed, $|\lambda_{s'|s+1} - \lambda_{s'|s}|$ is lower when the interest rate $r$ is low. This is because under low $r$, the value functions place higher weights on future payoffs, and the temporary difference in value due to the different starting states becomes less important. Specifically, holding investment decisions constant,

$$rv_s \rightarrow \bar{v} \quad \text{for all } s \in \mathbb{Z}.$$ 

We now analyze the equilibrium structure.

**Lemma 1.** In an equilibrium, value functions are monotone in the state: $v_{s+1} \geq v_s$ and $v_{-(s+1)} \leq v_s$ for all $s \geq 0$.

**Proposition 1.** Consider a generic equilibrium. If $\eta_0 = \eta$, then investment decisions follow “cut-off” rules: there exists integers $(n, k) \in \mathbb{Z}^2_{\geq 0}$ such that

$$\eta_s = \begin{cases} 
\eta & \text{for } s \leq n \\
0 & \text{for } s > n,
\end{cases} \quad \eta_{-s} = \begin{cases} 
\eta & \text{for } s < k \\
0 & \text{for } s > k,
\end{cases}$$

and $\eta_{-k} \in [0, \eta]$. That is, leader strictly invests in states 1 through $n$, and strictly does not invest after state $n$; the follower strictly invests in states 1 through $(k - 1)$, weakly invests in state $k$, and does not invest after state $k$.

For certain parameter range, there exists a trivial equilibrium in which neither firm invests in state 0 (i.e., $\eta_0 = 0$). State 0 is therefore an absorbing state, and aggregate investment is zero in the steady-state. This equilibrium arises when the flow profit is state 0 is high relative to $\pi_1$, and firms in the neck-to-neck state do not have sufficient incentive to invest, despite that this state has the least joint profits. We do not focus on this uninteresting equilibrium in our analysis.
The proposition establishes that in a non-trivial equilibrium, investments are weakly decreasing in the productivity gap $s$. This is a manifestation of two effects. First, when the follower is too far behind, the firm value is low and the marginal value of catching up by one step is not worth the investment cost. This is also known as the “discouragement effect” in the dynamic contest literature (Konrad (2012)). Second, the leader’s strategy is monotone due to a “lazy monopolist” effect: when the leader is far ahead of the follower, he ceases investment because the marginal gain in value brought by advancing market position is no longer worth the investment costs. The proposition also illustrates in a generic equilibrium, the leader is never indifferent between the two actions while the follower is indifferent in at most one state, $k$, which is the last state in which he weakly invests. For expositional purposes, in what follows we assume the follower strictly invests in this state, with $\eta_{-k} = \eta$.

**Proposition 2.** The leader weakly invests more than the follower in any state: $n \geq k$.

The proposition shows that the leader invests in more state than the follower does, a phenomenon we refer to as *leader dominance*. To aid the understanding of this result, we depict the value functions in the following figure. The black curve represents the value function of the leader, while the purple curve represents that of the follower. The two verticle lines respectively represent $k$ and $n$, the last states in which the follower and the leader weakly invests.
The intuition for leader dominance is as follows. In states that the follower does not invest, the productivity gap closes at a slow rate $\kappa$. Conversely, in states that the follower does invest, the gap closes at a fast rate $(\eta + \kappa)$. Firms are motivated to invest because of the high future flow payoffs in high states after consecutive successful investments. The leader is motivate to invest in all states $s \leq n$ in order to reach state $n + 1$, so that he can enjoy the flow payoff $\pi_n$ without having to pay the investment costs in the state. The state $(n + 1)$ is especially attractive if the follower does not invest in that state, because the leader can enjoy the payoff for a longer expected duration before the state stochastically transitions down to $n$, after which he has to incur investment cost again. The follower, on the other hand, is also motivated by future payoffs. He incurs investment costs in exchange for the possibility of closing the gap and catching up with the leader, and for the possibility of eventually becoming the leader himself in the future so that he can enjoy the high flow payoffs. In other words, investment decisions for both forward-looking firms are motivated by the high flow profits in the high states, and the incentive to reach these states is stronger for the
leader because the leader is closer to those high-payoff states.

Another way to describe the intuition is to consider the contradiction brought by \( n < k \). Suppose the leader stops investing before the follower does. In this case, the high flow payoff \( \pi_{n+1} \) is transient for the leader as market leadership is very much fleeting due to the high rate of downward state transition, implying the payoffs for being a leader is low. Because firms are forward-looking and their value functions depend on future payoffs, the low value of being a leader “trickles down” to affect the value function in all states. Therefore, low payoffs of being a leader in turn diminishes the incentive for the follower to invest, generating a contradiction to the presumption that follower invests more than the leader does.

The structure of an equilibrium can therefore be represented by the following diagram. States are represented by circles, going from state 0 on the very left and state \((n + 1)\) on the very right. The coloring of a circle represents investment decisions: states in which the firm weakly invests are represented by dark circles, while white ones represent those in which the firm does not invest. The top row represents leader’s investment decisions while the bottom row represent the follower’s investment decisions.

The fact that the leader invests in more states than the follower does enables us to partition the ergodic state space into two regions: one in which both firms invest and another in which the follower does not. In the first region, the state transitions up with Poisson rate \( \eta \) and transitions down with rate \((\eta + \kappa)\). In expectation, the state \( s \) decreases over time in this region, and the market structure tends to move towards being more competitive. For this reason, we refer to this as the competitive region. Note that this label is not a reflection of the static profits, which can be very much concentrated on the leader. Rather, the label reflects the fact that joint profits
tend to decrease dynamically. In the second region, the state transitions up with Poisson rate \( \eta \) and transitions down with rate \( \kappa \). Relative to the first region, the downward transition rate is much lower while the upward transition rate is the same; thus the market structure tends to move towards being more concentrated. For this region, we refer to this as the monopolistic region.

The aggregate productivity growth rate in the economy is a weighted average of the productivity growth rate in each market and it depends on both the state-dependent investment decisions as well as the distribution of market structure. In a steady-state, the distribution of market structure itself is a function of the investment decisions \( \{\eta_s, \eta_{-s}\}_{s=0}^{\infty} \). Let \( \mu_C \equiv \sum_{s=1}^{k} \mu_s \) be the fraction of markets in the competitive region in a steady-state and let \( \mu_M \equiv \sum_{s=k+1}^{\infty} \mu_s \) be the fraction of markets in the monopolistic region.

**Proposition 3.** In a stationary distribution \( \{\mu_s\}_{s=0}^{\infty} \) induced by cut-off strategies \((n,k)\), the aggregate productivity growth rate is

\[
g = \mu_C \cdot (\eta + \kappa) + \mu_M \cdot \kappa.
\]

Steady-state growth rate is increasing in \( k \) and decreasing in \( n \).

The proposition shows that steady-state growth is increasing in the fraction of markets in the competitive region and decreasing in the fraction of markets in the monopolistic region. In the competitive region, both firms are investing and, as a result, productivity improvements are rapid, state transition rate is high, dynamic competition is fierce, and market power tends to decrease over time. On the other hand, only the leader is investing in the monopolistic region, and the
rate of state transition and productivity growth is low. The fraction of markets in the competitive region, hence the steady-state growth rate, is increasing in $k$ and decreasing in $n$.

**Proposition 4.** Let $\{e\}$ be the set of equilibria. Let $v^e_s$ be the value function of state $s$ in equilibrium $e$, and let $(n^e, k^e)$ denote the investment strategies in the equilibrium. Generically, all equilibria can be Pareto-ranked. That is, there exists an equilibrium $(\star)$ such that $v^\star_s \geq v^e_s$ for all $s$ and all equilibria $e$. In this Pareto-optimal equilibrium, $n^\star \leq n^e$ and $k^\star \geq k^e$ for all $e$, and steady-state growth rate is maximized.

This proposition shows the existence, for any interest rate $r$, of a Pareto-optimal equilibrium in which firm value is maximized across all states and features the highest steady-state growth rate among all equilibria. In what follows, we refer to it as equilibrium $(\star)$ and its investment cutoffs as $(n^\star, k^\star)$, while we keep using $(n, k)$ to represent any generic equilibrium.

Now consider the effects of a decline in the interest rate. Conventional intuitions suggest that ceteris paribus, when firms discount future profits at a lower rate, the incentive to invest should increase because the cost of investment is lower relative to future benefits. This intuition does hold in our model, as manifested in the following proposition.

**Proposition 5.** The cutoff states $n^\star$ and $k^\star$, are both decreasing in $r$. Moreover, in any non-trivial equilibrium,

\[
\lim_{r \to 0} k = \lim_{r \to 0} (n - k) = \infty.
\]

The proposition suggests that as interest rate declines, both the leader and the follower weakly raise investment in all states in equilibrium $(\star)$. Moreover, as interest rate converges to zero, firms become infinitely patient and they sustain investment despite being arbitrarily far behind or ahead.

Our main theoretical contribution concerns the behavior of aggregate variables as $r \to 0$.

**Theorem.** As $r \to 0$, the fraction of markets in the competitive region vanishes:

\[
\lim_{r \to 0} \mu^C = 0,
\]

and the monopoly region becomes absorbing:

\[
\lim_{r \to 0} \mu^M = 1.
\]
In the limit as $r \to 0$, the aggregate variables in the economy behave as follows.

1. Average flow profits for leaders and followers converge to the limit:

$$
\lim_{r \to 0} \sum_{s=0}^{\infty} \mu_s \pi_s = \pi, \quad \lim_{r \to 0} \sum_{s=0}^{\infty} \mu_s \pi_{-s} = 0.
$$

2. Market leadership becomes permanently persistent:

$$
\lim_{r \to 0} M = \infty.
$$

3. Productivity gap between leaders and followers diverges:

$$
\lim_{r \to 0} \sum_{s=0}^{\infty} \mu_s s = \infty.
$$

4. Relative market valuation of leaders and followers diverges:

$$
\lim_{r \to 0} \frac{\sum_{s=0}^{\infty} \mu_s v_s}{\sum_{s=0}^{\infty} \mu_s v_{-s}} = \infty.
$$

5. Aggregate investment declines and productivity growth slows down:

$$
\lim_{r \to 0} I = c\kappa, \quad \lim_{r \to 0} g = \kappa.
$$

The theorem states that as $r \to 0$, all markets are in the monopolistic region and leaders almost surely stay permanently as leaders. In this region, leaders invest only to counteract the exogenous diffusion of productivity, and as a result, aggregate investment and productivity growth rate decline and are both fully determined by the exogenous rate $\kappa$, there lower bound c.f. Proposition 3. The rest of the statements then follows.

To understand this result, note that

$$
\mu^C \to \mu_0 \frac{2\eta + \kappa}{\kappa}, \quad \mu^M \to \mu_{n+1} \frac{\eta}{\eta - \kappa}, \quad \text{and} \quad \mu_0 = \left(\frac{\eta + \kappa}{\eta}\right)^k \left(\frac{\kappa}{\eta}\right)^{n-k} \mu_n.
$$

The fact that $\mu^C \to 0$ implies that $\left(\frac{\eta}{\eta + \kappa}\right)^{n-k}$ converges to zero at a faster rate than $\left(\frac{\eta}{\eta + \kappa}\right)^k$, or equivalently,

$$
\lim_{r \to 0} \left(\frac{n-k}{k}\right) > \frac{\log \left(\frac{\eta}{\eta + \kappa}\right)}{\log \left(\frac{\eta + \kappa}{\eta}\right)}.
$$
Proposition 5 shows that in any equilibrium, both $k$ and $(n - k)$ diverges as $r \to 0$. Why is $(\frac{n - k}{k})$ asymptotically bounded below? The intuition is threefolds.

First, as the interest rate converges to zero, the value of being in state 0 diverges at a rate slower than $r^{-1}$ (i.e., $rv_0 \to 0$). This is because holding $(n - k)$ constant, $v_0$ is decreasing in the $k$, the size of the competitive region. In this region, the state tends to transition downwards: for every step of productivity improvement made by the leader, the follower is expected to advance by more than one step on average. As a result, starting from state 0, it is exponentially more difficult to get out of the region and as $k$ increases. Because both firms invest at the maximum rate in this region, the joint flow payoffs are negative, and $v_0$ is therefore low.

Second, the leader stops investing in state $n + 1$ despite low $r$ implies that $rv_{n+1} \to \pi - c\kappa/\eta$ converges to a positive constant, and therefore $v_0$ vanishes relative to $v_n$. This observation in turn implies that the decline in value as the leader falls from the monopolistic region into the competitive region is steep, as manifested by the “cliff” around $s = k$ in the leader’s value function in Figure 1. In fact, one can show that the slope of the value function for the leader at the cliff, $(v_{k+1} - v_k)$, diverges at the same rate as $v_{n+1}$, which is on the same order as $(\frac{\eta + \kappa}{\eta})^k$:

$$v_{k+1} - v_k = \Theta(v_{n+1}) = \Theta\left(\left(\frac{\eta + \kappa}{\eta}\right)^k\right).$$

The third step is to realize that the slope of leader’s value function around state $n$ converges to a constant, $v_{n+1} - v_n = \Theta(c)$. Since the slope of value functions at state $n$ is increasing in the slope at state $k$, which diverges and is on the same order as $\left(\frac{\eta + \kappa}{\eta}\right)^k$, implies that $(n - k)$ must be sufficiently large to neutralize the impact of $\left(\frac{\eta + \kappa}{\eta}\right)^k$ on $(v_{n+1} - v_n)$.

2.3 Model Predictions

Our main theorem makes a number of predictions about the economy as interest rate drops to zero. We plot various endogenous outcomes of our model against the interest rate by solving the model detailed above numerically.\(^5\) We start with the implied supply curve of our model, i.e. the

\(^5\)Because of the discrete nature of the action space in our model, equilibrium variables move discontinuous with the interest rate. To smooth out the discontinuity, we generate these figures by numerically solving a continuous-action
relationship between endogenous productivity growth rate $g$ and long-term interest rate $r$.

Figure 2 plots how growth rate varies with interest rate. Interest rates can be split into two regions, a “normal interest rate regime” region and a “low interest rate regime” region. When interest rates are in the normal interest rate regime, we have the traditional downward sloping supply-curve: a reduction in interest rate increases growth rate. However, once interest rate enters the low interest rate regime, the negative market competition effect highlighted by our theorem becomes the dominant force and pushes growth rate down as interest rate declines. Supply-curve is thus upward sloping in the low interest rate regime. Overall the supply-curve is non-monotonic with an inverted-U shape.

The top-left panel of Figure 3 shows the model-implied relationship between profit share, markup, and industry concentration. Profit share is measured as $\sum_{s=0}^{\infty} \mu_s \pi_s$ where total output is normalized to one. Markup and profit are identical if we make an additional Bertrand limit-pricing assumption, and industry concentration is measured as the probability an industry is in the monopolistic region with at most one firm investing. A fall in interest rates monotonically increases profit share, markup and industry concentration as a decline in interest rate increases investment incentive more strongly for an industry leader relative to the follower.

The top-right panel of Figure 3 plots business dynamism against the interest rate. In the model, this corresponds to plotting $\frac{1}{M}$ against $r$, where $M$ is the average duration of market leadership. $M$ thus proxies the strength of business dynamism, i.e. how often do firms interchange their position as industry leader versus follower. The model predicts a monotonic negative effect of lower interest rate on business dynamics that progressively becomes stronger as interest rate approaches zero. At zero the industry leader become permanently entrenched in its position and the probability that follower will overtake the leader approaches zero.

The lower-left panel of Figure 3 plots the average productivity gap between the leader and follower, i.e. $\sum_{s=0}^{\infty} \mu_s s_s$ as a function of $r$. The average productivity gap increases as interest rate decline and in fact increases at an ever faster rate ultimately approaching infinity as $r$ goes to zero.

Version of our model. Specifically, we maintain the assumption that investment decisions lie in the range $[0, \eta]$, but the cost of investment is convex (rather than linear). These figures also illustrate that the main predictions of our model are robust to using a continuous action space.
The productivity gap is driven by the endogenous investment responses of leader and follower in the steady state dynamic contest game.

The lower-right panel of figure 3 plots the relative market valuation of leader versus follower, and the relative P/E ratio of leader versus follower against \( r \). The model predicts that the market valuation of leaders relative to that of followers diverges as interest rate declines to zero:

\[
\lim_{r \to 0} \frac{\sum_{s=0}^{\infty} H_s v_s}{\sum_{s=0}^{\infty} \mu_s v_{-s}} = \infty.
\]

Equation (3) further shows that the divergence of relative market valuations is a manifestation of two effects. First, low interest rate affects the steady states distribution of flow payoffs: leaders enjoy higher relative flow profits due high productivity gaps (changes in \( \{ \mu_s \}_{s=0}^{\infty} \)), and the endogenous investment costs are also affected (changes in \( \{ \eta_s, \eta_{-s} \}_{s=0}^{\infty} \)). Second and more interestingly, the relative valuations also reflect changes in the persistence of market power as encoded in the conditional measures \( \{ \lambda_{s'|s} \}_{s,s'=-\infty}^{\infty} \). Under high rates, market power is transient, and relative market value could be moderate despite leaders enjoy high flow profits. As market power becomes more persistent under low rates, the relative valuation diverges because the inequality in flow payoffs gets magnified.

The results summarized in figures 2 and 3 illustrate how our model parsimoniously connects a single “factor”, shifts in long-term interest rate driven by demand-side forces, with market structure and the nature of competition, and ultimately growth. In the following section, we compare the analytical predictions of the model with empirical data.

3 Implications for Broader Economy and Secular Stagnation

3.1 Overview

The model’s emphasis on the supply-side response to lower interest rates offers distinct insights into the relationship between aggregate demand shocks and growth, and a novel interpretation of the source of secular stagnation. The top panel of Figure 4 presents a visual representation of
many macroeconomic models in the growth-real interest rate space. In many models, the growth rate is determined by the supply side of the economy. In the absence of frictions, the growth rate is constant, which is represented by the horizontal line in the figure. The real interest rate is pinned down by an upward sloping demand curve, which is determined by the Euler equation of the household sector. Alternatively, as the middle panel shows, supply-side may be downward sloping as a reduction in interest rate increases incentives to invest and increase the growth rate.

Let us suppose that changes in the real interest rate are primarily determined by shifts in the aggregate demand curve. Such shifts could be due to short-run shocks such as a preference shock (Krugman (1998)) or a tightened borrowing constraint (Eggertsson and Krugman (2012)). But such a shift could also be due to long run structural changes in the economy, as highlighted by Summers (2014).

In traditional models, an inward shift in the aggregate demand curve leads to a lower interest rate with no effect on growth. In order to generate a lower GDP level in response to the inward shift, a common assumption in the literature is the existence of a zero lower bound on nominal interest rates, which together with nominal rigidities can lead to a real interest rate that is “too high,” thus reducing the level of GDP. In the extreme, the real interest rates may remain persistently “too high” and reduce even GDP growth considerably. The term “secular stagnation” has been used to describe such a situation.

The model presented here offers a different view. As in the traditional models, let us assume that a shift in the Euler equation is the primary determinant of changes in real interest rates and growth. As shown in the bottom panel of Figure 4, the model presented here generates an upward sloping supply curve in the growth-interest rate space for low levels of interest rates. If the aggregate demand curve shifts inward substantially, then the model produces a lower growth rate with no need to rely on frictions related to the zero lower bound on nominal interest rates.

The intuition is straight-forward from the discussion above. A large enough shift inward in the aggregate demand curve makes it more likely that the supply side of the economy enters into the monopolistic region of the productivity investment space. followers become discouraged from innovating, and leaders may eventually become lazy due to the reduced threat to their position.
This endogenous response of firm productivity investment to lower interest rates leads to a lower growth rate. The model generates secular stagnation as an endogenous supply side response to inward shifts in aggregate demand.

The rest of this section tests the empirical predictions from the model.

3.2 Model predictions in the data

This subsection examines the model predictions with data from the United States and the rest of the OECD countries. The figures plot a variety of outcomes against the nominal 10-year U.S. Treasury rate. The analysis uses the nominal rate given that inflation expectations have been well-anchored during the time period we analyze. Estimating real interest rate introduces possible noise given that their are multiple approaches to estimating the real rate and each approach relies on some structural assumptions.

Country-level total factor productivity data comes from the multi-factor productivity database from the OECD. Nominal interest rates are from the IMF except for the United States, where the interest rate data is from FRED. Our measure of profits is gross operating surplus as a share of GDP and it is also from the OECD.

The establishment entry and exit rates time series for the United States comes from the US Census’ Business Dynamics Statistics database. We use Computat to construct market concentration ratios and relative price-earnings ratios. The data on country-average productivity divergences is from Berlingieri and Criscuolo (2017).

Fact 1: Rising market concentration  Figure 5 plots the profit share of GDP against the 10-year nominal U.S. Treasury rate and shows that there is a strong negative correlation, as predicted by the model.  

6The fact that the profit share has been rising while interest rates have been falling is the central empirical point made in Barkai (2018).
Fact 2: Reduced business dynamism  We use business creation and destruction as a proxy for followers successfully innovating and leaders losing their position. In particular, we use establishment entry and exit information for the United States from 1985 to 2014. Figure 6 shows that lower interest rates are associated with both a decline in the entry and exit rates of establishments in the United States. Our interpretation of this decline in business dynamism is that it reflects higher market power of industry leaders and the reduced incentives to enter new markets by followers.

Overall, we know that investment as a share of GDP has also gradually declined, consistent with our model’s predictions. Furthermore, in related papers, Jones and Philippon (2016) show that increase in industry concentration is associated with lower firm investment. Gutiérrez and Philippon (2016) show that investment is not correlated with market valuation and profitability after 2000s.

Fact 3: Widening productivity gap  Berlingieri and Criscuolo (2017) provide evidence in support of this prediction. The bottom two panels of Figure 7 show their findings. The bottom left panel uses data from the OECD multi-prod data set, and it shows the gap between the labor productivity of market leaders and followers. Leaders are defined to be firms in the 90th percentile of the labor productivity distribution for a given 2-digit industry, and followers are defined to be firms in the 10th percentile. The gap between leaders and followers has been increasing steadily from 2000 to 2014 as long term interest rates have fallen.

The bottom right panel extracts the year fixed effects from a country by industry level panel containing this same gap. As it shows, there is a steady increase in the productivity gap between market leaders and followers across countries and industries. A similar result is in Andrews et al. (2016), who show a widening gap in labor productivity of frontier versus laggard firms in both manufacturing and services for OECD firms.

Andrews et al. (2016) also show cross-sectional evidence supporting the predictions of the model. They show that industries in which the gap between leader and follower multi-factor productivity is rising the most are the same industries where sector-aggregate multi-factor productivity is falling the most. This empirical finding is consistent with the model. Low interest rates lead to a large gap between the leader and follower in productivity investment; as a result,
total investment in productivity of the industry falls.

**Fact 4: Stock market values**  Data on stock market valuation can also be used to assess the model. The left panel of Figure 8 plots the average share of market value captured by the five largest firms in each 2-digit industry in the U.S. against the 10-year nominal U.S. treasury rate. As predicted by our model, the correlation is strongly negative. To isolate the effects of market power persistence, on the right panel we plot the relative P/E ratios between market leaders and followers (5 largest firms v.s. the rest) against the nominal treasury rate. The model prediction is again supported in the data.

**Fact 5: Interest rates and productivity growth**  This prediction is borne out in figure 9. The figure use data covering the OECD countries. The left panel shows the log total factor productivity level against the 10-year nominal U.S. Treasury rate. The right panel shows the scatter plot of total factor productivity growth against the interest rate. There is a clear negative relationship.

In summary, lower interest rates are associated with higher market concentration, a decline in business dynamism, a widening of the gap between the productivity of leaders and followers, and a slowdown in productivity growth. This is true both in the model and in the data.

4 Conclusion

The focus of this paper is on understanding how the supply-side of the economy responds to a reduction in long-term interest rates driven by demand-side forces. The existing literature in growth either assumes no supply-side response to declining interest rates, or a positive response driven by increased incentive to invest in the face of higher discounted present value of future profits. Our point of departure from this literature lies in explicitly modeling competition within an industry and analyzing how lower interest rates effect the nature of competition. We build on the dynamic contests literature to show that in a fairly general set up and without relying on any financial or other forms of frictions, the effect of lower interest rates on growth in a low interest rate regime can be negative.
A reduction in long-term interest rates tends to make market structure less competitive within an industry. The reason is that while both leader and follower within an industry increase their investment in response to a reduction in interest rates, the increase in investment is always stronger for the leader. Thus in equilibrium the gap between the leader and follower increases as interest rate declines, making an industry less competitive and more concentrated. This negative effect of lower interest rates on industry competition tends to lower growth and overwhelms the traditional positive effect of lower interest rates on growth when interest rates are sufficiently low. There is thus a hump-shaped inverted-U supply-side relationship between growth and interest rate in our framework.

Our model delivers a distinctive upward sloping supply-curve in a low interest rate regime. We think that this insight is useful in understanding the slowdown in productivity growth in recent decades and the broader discussion regarding “secular stagnation”. The slowdown in productivity growth is global as it shows up in almost all advanced economies. The slowdown started well before the Great Recession, suggesting that cyclical forces related to the crisis are unlikely to be the trigger. And the slowdown in productivity is highly persistent, lasting well-over a decade. Thus explanations that rely on traditional Keynesian forces such as price stickiness or zero lower bound are less likely to be sufficient.

This paper introduces the possibility of low interest rates as the common global “factor” that drives the slowdown in productivity growth. The mechanism that our theory postulates delivers a number of important predictions that are supported by empirical evidence. A reduction in long term interest rate increases market concentration and market power in our model. A fall in interest rate also makes industry leadership and monopoly power more persistent. There is strong support for these predictions in the data, both in aggregate time series as well as in firm-level panel data sets.

For example, productivity gap has indeed expanded between leaders and followers, and industries where this gap has widened the most tend to have slower aggregate growth. Similarly, market value of leaders within an industry has risen relative to followers as long-term interest rate declined. There is additional evidence that the market leader position becomes more persistent with lower interest rates as the relative P/E ratio of leaders versus followers also rises. All of these
are quite unique predictions of our model that are confirmed in data.

A natural implication of our analysis is that we need to understand more carefully the forces behind the reduction in long-term interest rate in the first place. This is a fruitful area of research going forward.
References


Figure 2: Model prediction: growth and interest rate
Figure 3: Other model predictions
Figure 4: Supply and demand relationships in growth models
Figure 5: Aggregate profit share and interest rate
Figure 6: Business Dynamism
Figure 7: Widening productivity gap between leaders and followers
Figure 8: Value of stock market as a fraction of GDP
Figure 9: Productivity growth and interest rates