Labor Income Dynamics over the Business Cycle

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Abstract: What do labor income dynamics look like over the business cycle? How does exposure to aggregate income risk change over the life cycle and with education? To what extent do taxes, transfers and the family attenuate aggregate income risk? In this paper, we use Norwegian population panel data to answer these important questions. We first provide a detailed statistical analysis of the income process to answer these important questions. We model the income process as a first-order quantile-autoregressive process and let individuals with different education levels have a separate income process, and within each skill group and cohort we allow the conditional quantile functions to vary unrestrictively over time. We find that exposure to cyclical income risk varies substantially across skill groups and over the life-cycle; We also find that the skewness of the distribution of income decreases in response to a drop in GDP and as we condition on higher quantiles of the earnings distribution. This paper then aims to show that a structural representation of a frictional job ladder model with heterogeneous agents and aggregate shocks can explain these facts.

Keywords: Business Cycle; Labor Income Risk;

JEL codes:

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1 Introduction

What do labor income dynamics look like over the business cycle? How does exposure to aggregate income risk change over the life cycle and with education? To what extent do taxes, transfers and the family attenuate aggregate income risk? In this paper, we use Norwegian population panel data to answer these important questions. We first provide a detailed statistical analysis of the income process to answer these important questions. The mean and higher order moments of the conditional income distribution are thereby at the center of our empirical analysis. We model the income process as a first-order quantile-autoregressive process and let individuals with different education levels have a separate income process, and within each skill group and cohort we allow the conditional quantile functions to vary unrestrictively over time. In order to exlore the impact of the tax and transfers system as well as spouse's income as a mechanism of attenuation or insurance to labour income shocks, we focus on three dimensions of income: individual market income, individual disposable income, and family disposable income. Overall, our estimates are consistent with a variety of evidence on income risk over the business cycle (see e.g. Storesletten, Telmer, and Yaron, 2004; Guvenen, Ozkan, and Song, 2014; Busch, Domeij, Guvenen, and Madera, 2016; Busch and Ludwig, 2016). We find that exposure to cyclical income risk varies substantially across skill groups and over the life-cycle; our findings also suggest that the progressive nature of the Norwegian tax-transfer system as well as spouse's income play a key role in reducing aggregate income variation.

We find that the skewness of the distribution of income decreases in response to a drop in GDP and as we condition on higher quantiles of the earnings distribution. This paper then aims to show that a structural representation of a frictional job ladder model with heterogeneous agents and aggregate shocks can explain these facts. Climbing up the job ladder implies an increasing risk of experiencing large negative income shocks by falling off the job ladder. The job ladder thererefore generates an income process in which skewness decreases with income. Negative aggregate shocks further amplify this effect. The model we propose is an extension of Lise and Robin (2017) enabling us to calculate the exact dynamics of the wage distribution associated with the dynamics of worker firm matches in an economy with aggregate shocks and heterogenous agents.

The rest of the paper is organized as follows. Section 2 provides a description of the Norwegian administrative data. The empirical specification of the income process and the estimation results are presented in Section 3. Section 4 outlines the extension of Lise and Robin (2017).

2 Data Description

2.1 Norwegian Registry Data

The empirical analysis employs several registry databases maintained by Statistics Norway that we can link through unique individual identifiers. This allows us to construct a rich longitudinal data set containing individual records for the entire resident population of Norway from 1993 to 2014. The panel data set contains individual demographic information (including gender, date of birth, marital status, family size and composition) and socio-economic data (including income from various sources and schooling). The data also contains unique family identifiers that enable us to measure income at the household level by matching individuals with their spouses, and parents to their children.

Registry data are ideal for our purpose of studying income dynamics, in particular because of the long time dimension and the large number of cross-sectional observations. The coverage and reliability of Norwegian registry data are considered to be outstanding (Atkinson, Rainwater, and Smeeding, 1995). There is no attrition from the original sample because of the need to ask for permission from individuals to access their records. In Norway, these records are in the public domain. The income data also pertain to all individuals and all jobs, and not only to jobs covered by social security and measures of incomes are recorded without any top- or bottom coding. In addition, there are little reporting or recollection errors, which is a common problem with survey-based micro datasets. The income data come from individual tax records with detailed information about the different sources of income, while educational attainment is reported directly to Statistics Norway by the educational establishments.

The income variables that we consider are defined as follows. The first variable is annual *individual market income*. This includes all wage income and income from self-employment. The second variable is *individual disposable income* measured as individual market income plus cash transfers and net of taxes. The third income variable is *family disposable income*, defined as the sum of the individual disposable income of the spouses. Throughout the empirical analysis we treat cohabiting couples identical to married couples.[1] All income measures are deflated to the base year 2015 using the Consumer Price Index (CPI) provided by Statistics Norway and converted to US Dollars by using the average of the daily exchange rates in 2015.[2] The life cycle is measured by *potential labor market experience* of the household head, defined as age minus years of schooling minus 6. Accordingly, we define a *cohort* as the set of household with the same potential experience in any given year.

For our baseline specification we focus on households with a male earner as household head. To account for heterogeneity by educational attainment, we partition the sample into three mutually exclusive groups according to educational levels of the household head. Low skilled is defined as not having completed high school (15% of the sample), medium skilled includes those with a high school degree (51% of the sample), and the high skilled consists of those who have attended college (33% of the sample). To ensure fairly long records on earnings for each household, we focus on cohorts that enter the labour market between 1965 and 2005. To minimize the impact of heterogeneity in the initial transition from education to the labor market, we restrict our analysis to household heads with at least 5 years of potential experience. Similarily, to avoid problems related to retirement, we focus on households with up to 45 (low-skilled), 42 (medium-skilled) or 37 (high-skilled) years of potential experience. These restrictions correspond to the level of potential experience of the median household head at age 60 for the respective skill groups. We further drop household-year observations if the household composition has changed compared to the previous year. Furthermore, we exclude household-year observations if the household head earns less than a time-varying minimum threshold. This threshold is given by 0.1 basic amounts, which amounts to roughly 1,109 USD in 2015.[3] Similarly, we require that household heads earn at least one basic amount over all the years we observe them. Such minimum threshold conditions are fairly standard in the income dynamics literature and ensure that we select households with a reasonably strong attachment to the labor market attachment (see e.g. Meghir and Pistaferri, 2004 and Guvenen, Karahan, Ozkan, and Song, 2016 among others).

[1]Cohabitants are identified from the National Survey of Population and Housing in 1990, the National Censuses of Population and Housing in 2001, and from Statistics Norway's own datasets identifying cohabiting couples after 2004. Further, cohabiting couples are roughly identified from annual registry data as men and women living at the same address who at some point have children together or become cohabitating couples, or who get married at a later point in time.

[2]The average daily exchange rate (NOK/USD) in 2015 was 8.0739 according to the data made available by the Norwegian Central Bank.

[3]One basic amount is the so called threshold of substantial gainful activity. The nominal level of the threshold varies year-by-year according to the development of wages in the Norwegian economy.

Applying these restrictions leaves us with an unbalanced panel of 16,537,533 household-year observations. Table 1 displays some summary statistics for the three subsamples and Figure 1 shows the age profiles in the different measures of income by education levels.

variable	N	Mean	Std. Dev.	10pctile	25pctile	Median	75pctile	90pctile
age	2,542,516	39.64	9.33	27.00	32.00	40.00	47.00	52.00
experience	2,542,516	23.73	9.46	11.00	16.00	24.00	31.00	37.00
years of schooling	2,542,516	9.91	0.59	9.00	10.00	10.00	10.00	10.00
married	2,542,516	0.70	0.46	0.00	0.00	1.00	1.00	1.00
number of children	2,542,516	0.94	1.10	0.00	0.00	1.00	2.00	2.00
market income	2,542,516	49,528.89	30,038.07	19,643.93	35,537.42	46,967.71	59,958.03	77,386.88
disposable income	2,542,516	37,584.46	18,029.15	21,754.86	28,611.25	35,623.09	44,064.19	54,430.72
family disposable income	2,542,516	57,789.06	27,032.08	27,078.92	38,862.63	56,810.27	72,897.09	88,363.15

(a) Low-skilled

variable	N	Mean	Std. Dev.	10pctile	25pctile	Median	75pctile	90pctile
age	8,498,414	41.41	9.32	29.00	34.00	41.00	49.00	54.00
experience	8,498,414	22.65	9.53	10.00	15.00	23.00	30.00	36.00
years of schooling	8,498,414	12.77	0.74	12.00	13.00	13.00	13.00	13.00
married	8,498,414	0.77	0.42	0.00	1.00	1.00	1.00	1.00
number of children	8,498,414	1.01	1.11	0.00	0.00	1.00	2.00	3.00
market income	8,498,414	58,966.46	44,812.50	30,852.09	42,384.14	53,788.00	69,228.41	91,162.52
disposable income	8,498,414	42,557.42	29,143.88	25,906.05	32,141.79	39,539.04	49,190.11	61,742.23
family disposable income	8,498,414	65,511.26	35,416.60	33,429.66	47,373.23	63,660.09	80,062.27	97,135.00

(b) Medium-skilled

variable	N	Mean	Std. Dev.	10pctile	25pctile	Median	75pctile	90pctile
age	5,496,603	42.28	8.99	31.00	35.00	42.00	49.00	55.00
experience	5,496,603	19.60	8.84	8.00	12.00	19.00	27.00	32.00
years of schooling	5,496,603	16.68	1.79	14.00	15.00	17.00	18.00	19.00
married	5,496,603	0.79	0.41	0.00	1.00	1.00	1.00	1.00
number of children	5,496,603	1.08	1.13	0.00	0.00	1.00	2.00	3.00
market income	5,496,603	79,052.47	84,054.63	36,588.67	50,779.18	66,755.31	92,051.87	128,629.19
disposable income	5,496,603	52,798.77	52,045.87	28,614.21	36,621.47	46,535.05	60,732.01	80,473.67
family disposable income	5,496,603	80,332.24	58,251.38	37,526.74	55,660.39	75,578.48	97,053.20	122,520.53

(c) High-skilled

Table 1: Summary Statistics

Notes: This table shows summary statistics of key variables by educational levels. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Experience is defined as age minus years of education minus 6. The number of children refers to the number of individuals that are younger than 18 years who live in the same household. All nominal variables are deflated and converted to 2015 USD.

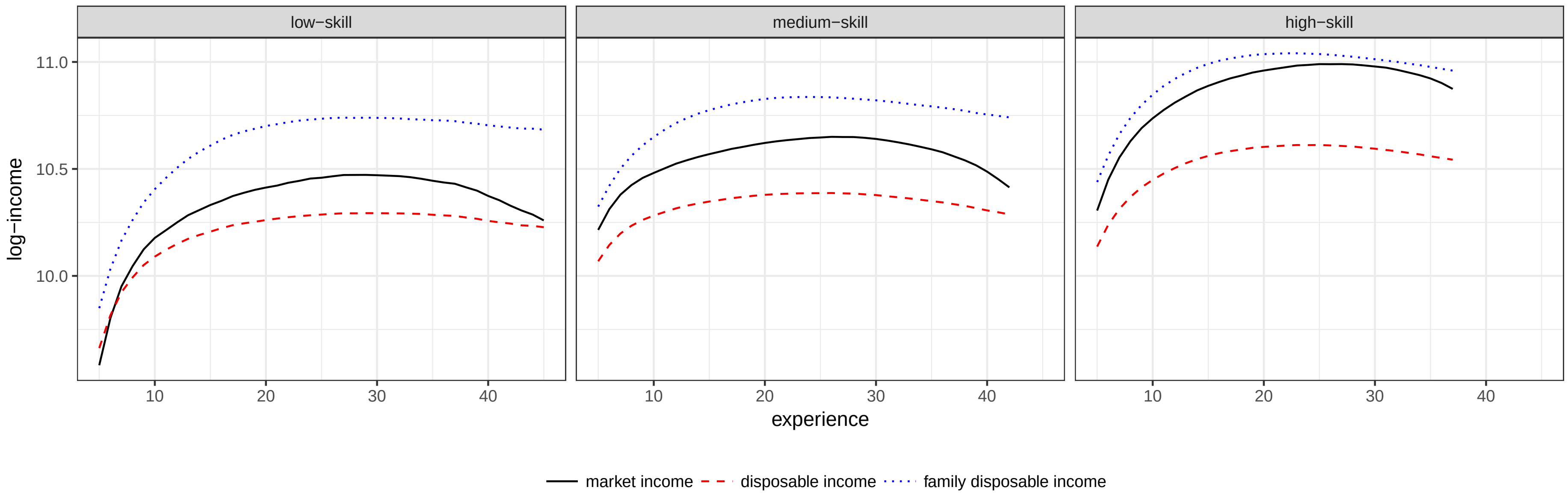


Figure 1: Age profiles in the log of income

Notes: This figure shows the age profiles in the log of income by educational levels. The age profiles are adjusted for education-specific calendar time effects. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Experience is defined as age minus years of education minus 6.

2.2 Business Cycle Conditions

For our baseline specification we use the annual real Gross Domestic Product (GDP) of mainland Norway as the business cycle indicator.[4] To capture the business cycle, we apply a Hodrick-Prescott filter to the natural logarithm of annual real GDP of mainland Norway, using a smoothing parameter of 6.25 for annual data as suggested by Ravn and Uhlig (2002). Hence the business cycle conditions are expressed in terms of the log deviation of the real GDP from its trend. For robustness checks, we also consider the unemployment rate as an alternative business cycle indicator.[5] Figure (2) shows the evolution of the business cycle indicators over time.

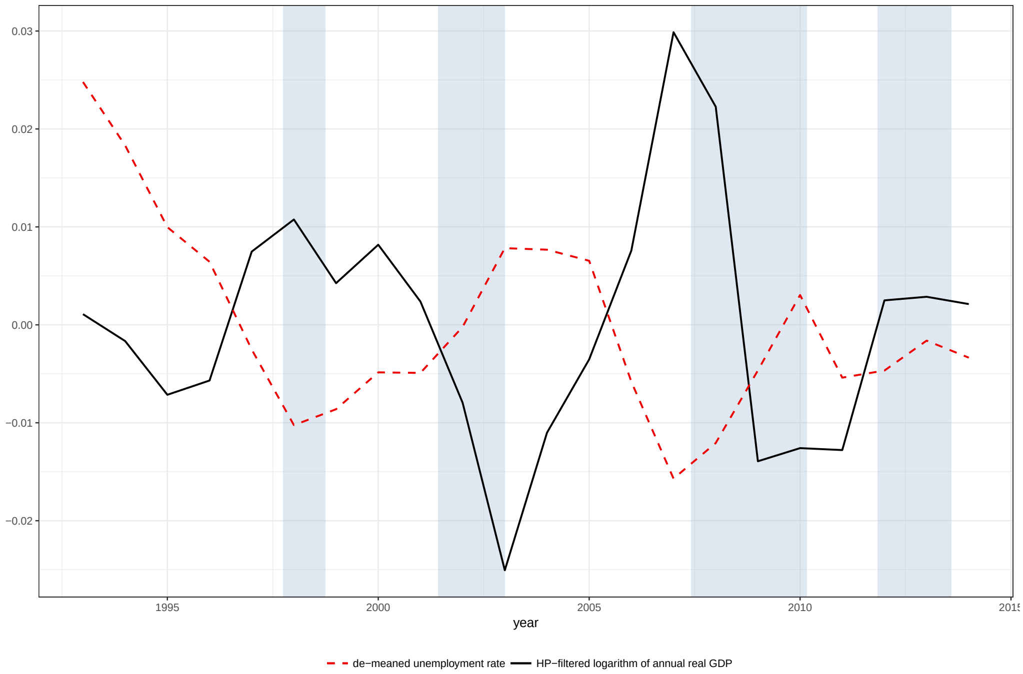


Figure 2: Business cycle indicators

Notes: This figure shows the HP-filtered natural logarithm of annual real GDP for mainland Norway and the de-meaned unemployment rate for males (age 15-74) based on Labour Force Survey (LFS) data. The grey bars indicate recessions (peak to trough) in Norway as dated by the OECD.

The Norwegian economy experienced 4 recessions over the sample period (see Aastveit, Jore, and Ravazzolo, 2016). Norway experienced a deep and long-lasting recession just before the sample period. This particular recession turned into a banking crisis that was accompanied by an unprecedented increase in the unemployment rate. The Great Recession had a relatively moderate impact on the Norwegian labour market with an unemployment rate raising by less than 2 percentage points.

[4]The GDP series relates to mainland Norway and therefore excludes offshore acitivity, namely oil and gas extraction and international shipping. The main reason for the exclusion of these sectors is the fact that their production may show large fluctuations that have very small short term effects on the Norwegian labor market (see Aastveit, Jore, and Ravazzolo, 2016).

[5]The time-series for the unemployment rate is based on data from the Labour Force Survey data and corresponds to the unemployment rate of males age 15 - 74.

3 Empirical Evidence

3.1 The Conditional Mean

Specification. We start by investigating how mean income changes over the business cycle. We decompose the cross-sectional variation in income attributable to observable household characteristics such as marital status, household size, region of residence, potential experience, cohort and calendar time. Ideally one would like to control for the effects from potential experience (h), cohort (c) and calendar time (t) in an unrestricted way. However, potential experience, year and cohort are perfectly collinear and therefore the level effects of these three factors cannot be separately identified.[6]

An extent literature has pointed out several ways to deal with this fundamental identification problem. The standard approach involves imposing one or more restrictions. Deaton (1997) for example suggests a normalisation that makes the year effects average to zero over the sample period and be orthogonal to a time trend, such that all growth is attributed to experience and cohort effects. Another widespread approach is to assume that either cohort or time effects are zero, so that secular trends appear only in time or cohort effects respectively.[7] We follow an approach suggested by Heckman and Robb (1985) instead. The general idea is to treat experience, cohort and time effects as proxy variables for underlying variables which are not themselves linearly dependent. Hence we interpret time effects as proxy variables for underlying aggregate economic conditions that in principle can be measured by business cycle indicators. Conveniently, this approach allows us to measure the average cyclical variation of log income. Hence we specify

$$\ln Y_{it} = X_{it}'\beta + \psi_h + \lambda_c + \xi \ln Z_t + \varepsilon_{it}, \tag{1}$$

where ψ_h and λ_c denote experience and cohort fixed effects, and X_{it} captures the deterministic income effects attributable to household size, marital status and region. Of particular interest in the context of our analysis is the coefficient ξ which measures the average elasticity of income with respect to the contemporaneous cyclical conditions Z_t. To allow for the possibility that the cyclicality of income changes over the life cycle, we implement an alternative specification,

$$\ln Y_{it} = X_{it}'\beta + \psi_h + \lambda_c + \sum_h \xi_h \mathbb{1}_h \ln Z_t, \tag{2}$$

where $\mathbb{1}_h$ is an indicator variable that takes on the value one if the individual has potential experience h. The series $\{\xi_h\}$ measures the average elasticity of income with respect to the contemporaneous cyclical conditions at any stage in the life cycle.

Results. We estimate the specification (1) and (2) by pooled OLS separately for each skill group and for each income measure, using log deviations of real GDP from its trend as business cycle indicator. Table 2 reports the results of these regressions.

For each skill group, we find that individual market income is more cyclical than individual disposable income, which in turn is more cyclical than family disposable income. For instance, a one-percent

[6]Even if we were to define cohort by the year of birth, we would still have to deal with a substantial problem of multicollinearity.

[7]As highlighted by Heathcote, Storesletten, and Violante (2005), the age profiles of inequality can look very different depending on whether one assumes cohort or time effects to be at work. In a recent paper, Schulhofer-Wohl (2013) proposes an estimation method for structural life cycle models that avoids the documented problems of the standard approach.

	market income (1)	market income (2)	disposable income (1)	disposable income (2)	family disposable income (1)	family disposable income (2)
ln(real GDP, det.)	2.046*** (0.033)		1.111*** (0.020)		0.876*** (0.017)	
$\mathbb{1}\{\text{exp.}=5\}\times$ln(real GDP, det.)		4.426*** (0.347)		2.056*** (0.208)		1.951*** (0.173)
$\mathbb{1}\{\text{exp.}=10\}\times$ln(real GDP, det.)		3.550*** (0.255)		1.626*** (0.153)		1.251*** (0.127)
$\mathbb{1}\{\text{exp.}=20\}\times$ln(real GDP, det.)		2.141*** (0.199)		1.168*** (0.119)		0.912*** (0.099)
$\mathbb{1}\{\text{exp.}=30\}\times$ln(real GDP, det.)		1.412*** (0.182)		0.953*** (0.109)		0.779*** (0.090)
$\mathbb{1}\{\text{exp.}=45\}\times$ln(real GDP, det.)		2.606*** (0.777)		0.615 (0.466)		0.161 (0.387)
R^2	0.10	0.10	0.19	0.19	0.58	0.58
Num. Obs.	2,542,450	2,542,450	2,542,450	2,542,450	2,542,450	2,542,450

(a) low-skilled

	market income (1)	market income (2)	disposable income (1)	disposable income (2)	family disposable income (1)	family disposable income (2)
ln(real GDP, det.)	1.474*** (0.015)		1.071*** (0.010)		0.851*** (0.009)	
$\mathbb{1}\{\text{exp.}=5\}\times$ln(real GDP, det.)		3.063*** (0.075)		1.980*** (0.094)		1.739*** (0.077)
$\mathbb{1}\{\text{exp.}=10\}\times$ln(real GDP, det.)		1.987*** (0.054)		1.275*** (0.073)		0.939*** (0.060)
$\mathbb{1}\{\text{exp.}=20\}\times$ln(real GDP, det.)		1.519*** (0.047)		1.128*** (0.059)		0.917*** (0.049)
$\mathbb{1}\{\text{exp.}=30\}\times$ln(real GDP, det.)		1.320*** (0.057)		0.988*** (0.062)		0.785*** (0.051)
$\mathbb{1}\{\text{exp.}=42\}\times$ln(real GDP, det.)		0.109 (0.124)		0.467*** (0.083)		0.375*** (0.068)
R^2	0.11	0.11	0.18	0.18	0.54	0.54
Num. Obs.	8,498,212	8,498,212	8,498,212	8,498,212	8,498,212	8,498,212

(b) medium-skilled

	market income (1)	market income (2)	disposable income (1)	disposable income (2)	family disposable income (1)	family disposable income (2)
ln(real GDP, det.)	0.866*** (0.021)		0.727*** (0.016)		0.604*** (0.013)	
$\mathbb{1}\{\text{exp.}=5\}\times$ln(real GDP, det.)		1.693*** (0.131)		0.985*** (0.097)		0.886*** (0.080)
$\mathbb{1}\{\text{exp.}=10\}\times$ln(real GDP, det.)		1.172*** (0.116)		0.870*** (0.086)		0.631*** (0.071)
$\mathbb{1}\{\text{exp.}=20\}\times$ln(real GDP, det.)		0.917*** (0.122)		0.816*** (0.090)		0.612*** (0.074)
$\mathbb{1}\{\text{exp.}=30\}\times$ln(real GDP, det.)		0.635*** (0.129)		0.633** (0.096)		0.530*** (0.079)
$\mathbb{1}\{\text{exp.}=37\}\times$ln(real GDP, det.)		0.742*** (0.148)		0.721*** (0.110)		0.603*** (0.090)
R^2	0.16	0.16	0.19	0.19	0.50	0.50
Num. Obs.	5,496,151	5,496,151	5,496,151	5,496,151	5,496,151	5,496,151

(c) high-skilled

Table 2: Aggregate variation in income

Notes: This table present coefficients of pooled OLS regressions. The results from specifications (1) and (2) are presented in column (1) and (2) respectively. The business cycle indicator is the HP-filtered natural logarithm of real GDP of mainland Norway. Other controls include potential experience fixed effects, cohort fixed effects, region fixed effects, a dummy for marital status and dummies for the number of children in the household. Standard errors are presented in parentheses. ***, **, and *, represent statistical significance at 0.1%, 1%, and 5% levels, respectively.

negative deviation of GDP from its trend implies that market income for low-skilled males is reduced by around 2.05% on average. This is in contrast to around 1.11% for individual disposable income and 0.88% for family disposable income. Another key finding is that, low-skilled households have the largest exposure to aggregate cyclical income risk, while high-skilled households are the least exposed to cyclical fluctuations. For example, high skilled households experience on average a 0.6% drop in their family disposable income in response to a one percent deviation of GDP from its trend. The corresponding number for medium skilled households is 0.85% and 0.88% for the low skilled. Taken together, these results suggest that (i) the cyclicality of income varies substantially across skill groups, and (ii) the tax-transfer system as well as spouse's income play an important role in reducing aggregate income variation.

We now shift attention to the results of specification (2) presented in columns (2). We find that income is more cyclical for workers early in their careers. For example, a one-percent negative deviation of GDP from its trend implies on average a reduction of around 4.43% in market income among low skilled males five years after leaving education. Twenty and thirty years after having completed education, this number is reduced to around 2.14% and 1.41% repectively. We find a similiar pattern for the medium- and high-skilled, and after accounting for taxes and transfers and the income of the spouse. Taken together, these results suggest that the average exposure to cyclical risk differs substantially across different points in the life cycle, with younger individuals experiencing a more cyclical income stream.

Finally, we examine whether the previous results remain economically and statistically significant if we use the demeaned unemployment rate as business cycle indicator. We also investigate whether including individual fixed effects instead of cohort fixed effects significantly changes the previous results. The prior results remain robust against these different specifications and are reported in Appendix Table 5 and Appendix Table 6 respectively.

3.2 Distributional Dynamics

Specification. We now shift our attention to the dynamics of the distribution of income. We model the conditional distribution of income using quantile-autoregressions (Koenker and Xiao, 2006). For *a given cohort* of households, log-income is assumed to evolve over time according to

$$\ln Y_{it} = Q_t(Y_{it-1}, X_{it}, u_{it}), \tag{3}$$

where $Q_t(Y_{i,t-1}, X_{it}, \tau)$ denotes the τ-th quantile of log income conditional on lagged income and observable characteristics X_{it}. The innovation u_{it} is uniformly distributed on the unit interval, and we allow the conditional quantile functions to vary with time and across cohorts in an unrestricted way. For each cohort and time period, we specify the τ-th conditional quantile function as,

$$Q_t(Y_{it-1}, X_{it}, \tau) = f(Y_{it-1}, \tau) + X_{it}'\beta(\tau). \tag{4}$$

In practice, we approximate f using a low-order orthogonal polynomial.[8]

Conditional Uncertainty and Skewness. The quantile autoregressive process (3) allows us to characterise any quantile of the conditional distribution, and therefore provides a very detailed picture

[8]Nonlinear persistence is a key feature of the income process as highlighted by Arellano, Blundell, and Bonhomme (2016). See Appendix Figure 5.

of income risk. In the following we restrict ourselves to a few characteristics of the conditional distribution. In line with the recent literature on income risk over the business cycle (e.g. Guvenen, Ozkan, and Song, 2014), we focus on two key characteristics of a distribution: dispersion and asymmetry.

Typically, the dispersion of a distribution is measured by its standard deviation. A corresponding quantile-based measure for some $\tau \in (0.5, 1)$ is

$$\sigma_t(Y_{it-1}, X_{it}, \tau) = Q_t(Y_{it-1}, X_{it}, \tau) - Q_t(Y_{it-1}, X_{it}, 1-\tau). \tag{5}$$

From the perspective of household i in period $t-1$, the measure $\sigma_t(Y_{it-1}, X_{it}, 0.75)$ represents the interquartile range of the household's possible income realisations one period ahead in t. Put differently, it measures the width of the range which holds 50% of the household's income next period and as such can be interpreted as a measure of conditional income uncertainty. We define

$$\bar{\sigma}_t(\tau) \equiv \mathbb{E}[\sigma_t(Y_{it-1}, X_{it}, \tau)], \tag{6}$$

$$\tilde{\sigma}_t(y, \tau) \equiv \mathbb{E}[\sigma_t(Y_{it-1}, X_{it}, \tau)|Y_{it-1} = y] \tag{7}$$

to be the corresponding average and conditional measures of income dispersion respectively.

The degree of asymmetry of a distribution is measured by its skewness. For some $\tau \in (0.5, 1)$, a quantile-based measure of conditional skewness is

$$\gamma_t(Y_{it-1}, X_{it}, \tau) = \frac{Q_t(Y_{it-1}, X_{it}, \tau) + Q_t(Y_{it-1}, X_{it}, 1-\tau) - 2Q_t(Y_{it-1}, X_{it}, 0.5)}{Q_t(Y_{it-1}, X_{it}, \tau) - Q_t(Y_{it-1}, X_{it}, 1-\tau)}, \tag{8}$$

and similarly we define

$$\bar{\gamma}_t(\tau) \equiv \mathbb{E}[\gamma_t(Y_{it-1}, X_{it}, \tau)], \tag{9}$$

$$\tilde{\gamma}_t(y, \tau) \equiv \mathbb{E}[\gamma_t(Y_{it-1}, X_{it}, \tau)|Y_{it-1} = y], \tag{10}$$

to be the corresponding average and conditional measures of skewness. For the interpretation of some of the empirical results discussed further below, it is useful to note the link between the conditional measure of dispersion and skewness. Rearranging (8) yields,

$$\frac{Q_t(Y_{it-1}, X_{it}, \tau) - Q_t(Y_{it-1}, X_{it}, 0.5)}{\sigma_t(Y_{it-1}, X_{it}, \tau)} = \frac{\gamma_t(Y_{it-1}, X_{it}, \tau) + 1}{2}, \tag{11}$$

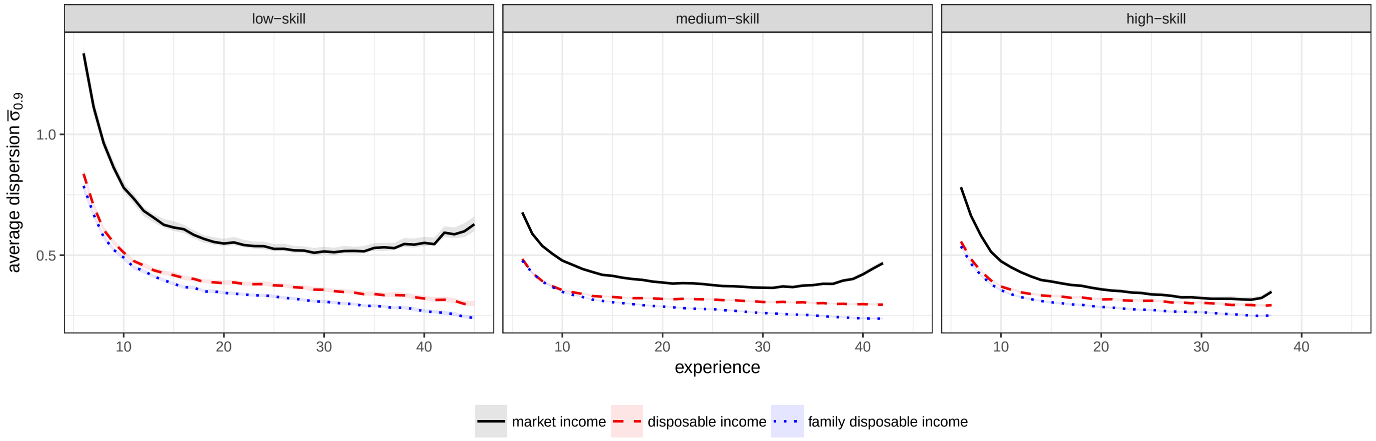
which implies that less than half of the overall dispersion is explained by the length of the upper tail if the distribution is skewed to the left ($\gamma_t(Y_{it-1}, X_{it}, \tau) < 0$).

Results. We estimate specification (4) for different values of $\tau \in (0, 1)$ separately by year, cohort, and skill group. This procedure gives us measures of average and conditional income uncertainty and skewness that vary across cohorts, time and experience. Of course, and as already pointed out in Section 3.1, not all three effects can then be identified without additional restrictions.

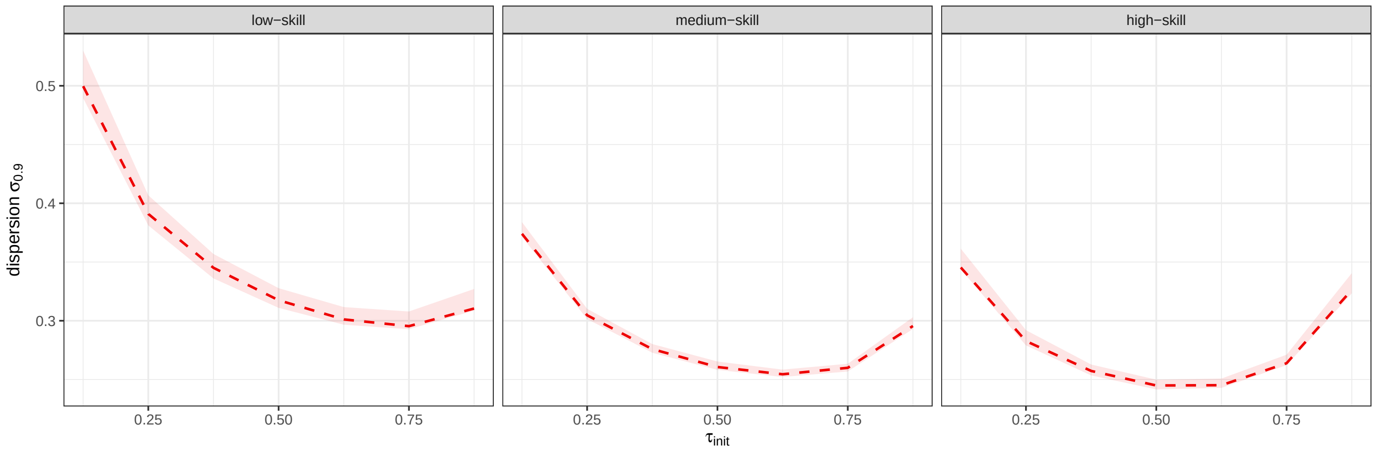
As before, we interpret time effects as proxy variables for underlying aggregate economic conditions that are measured by business cycle indicators (Heckman and Robb, 1985). Specifically, we calculate correlations between the various measures of conditional uncertainty and skewness, and the business cycle indicator, whilst controlling for experience and cohort fixed effects. To fix ideas, consider the following regression framework:

$$m_{ct} = \alpha + \psi_h + \lambda_c + \xi \ln Z_t + \varepsilon_{ct}, \tag{12}$$

where ψ_h and λ_c denote experience and cohort fixed effects respectively. The dependent variable m_{ct} corresponds to one of the cohort and time specific estimates of the dispersion and skewness measures in (6) - (10). The coefficient ξ then captures the cyclicality of this measure, while the coefficients on the experience dummies trace out the average life cycle profiles.



(a) Average dispersion over the life cycle



(b) Conditional dispersion in disposable income

Figure 3: Dispersion

Notes: This figure shows (a) the life cycle profile of the average dispersion (6) with $\tau = 0.9$, (b) dispersion in disposable income (7) with $\tau = 0.9$ and evaluated at a value of lagged income that corresponds to the τ_{init} percentile of the distribution of Y_{it-1}conditional on observable characteristics and 20 years of potential experience. The estimates are net of cohort and business cycle effects based on model (12). Shaded areas represent 95% point-wise confidence intervals based on a block bootstrap procedure of the quantile auto-regressions (4) with 100 replications. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.

Figure (3a) shows the estimated life cycle profiles of average income dispersion (6) with $\tau = 0.9$. We find that the magnitude of income dispersion varies systematically with potential experience and across skill groups. Households are exposed to a substantial amount of income uncertainty early in their careers. We also observe that at almost any stage in the life-cycle the low skilled are exposed to more income uncertainty than the medium skilled, who in turn face more uncertainty than the high skilled. A comparison across the different measures of income reveals that the tax- and transfer system leads to a remarkable compression of incomes. A pooling of resources at the household level leads to a further reduction in dispersion. These results suggest the tax- and transfer system and the family provide a significant amount of insurance against income shocks.

We also observe that exposure to income uncertainty varies systematically in the cross-section. As shown in Figure (3b), conditional dispersion (7) roughly follows a U-shape. Households at the top and bottom of the cross-sectional income distribution in $t-1$ face a larger dispersion of their income realisations in t compared to those in the middle of the distribution. Appendix Figures (8) and (9) show that samiliar patterns hold for other income measures and at different points in the life cycle. Taken together, these results suggest that income uncertainty varies systematically over the life cycle, within the cross-section and across skill groups.

In Table (3) we report the cyclicality of the average and conditional income dispersion. Columns (a) show the estimated cyclicality of the average income dispersion measures, while columns (b) report the income dispersion measures for those currently at the bottom, middle and top of the income distribution. Overall, the effects we find are quantitatively small. This is consistent with the empirical evidence reported in Guvenen, Ozkan, and Song (2014) and Busch, Domeij, Guvenen, and Madera (2016), who investigate the dispersion in income growth rates over the business cycle for the US, Germany and Sweden. To investigate the reason behind these small changes, we take a closer look at the movements in the tails of the conditional distribution. Our estimates of the conditional quantile functions reveal that the 90-50 and 50-10 income ratios largely move together over the business cycle. We do however find some differences across income measures, skill groups and across the income distribution. For instance, a one-percent increase in GDP implies that on average the 90-10 ratio in market income for the low-skilled decreases by around 2.8 percentage points. For a low-skilled male with 20 years of experience this implies that the 90th percentile is now around 47% larger than the 10th percentile, compared to around 50% if GDP has not changed. A closer inspection of the tails reveals that this is driven by the shrinkage of the bottom tail that outweighs the expansion of the upper tail. Our results in column (b) suggest that across all skill groups this effect is especially strong for those who are currently at the bottom of the income distribution, but the effect largely disappears when we take taxes and transfers and the income of the spouse into account.

τ_{init}	market income (a)	(b) 0.1	0.5	0.9	disposable income (a)	(b) 0.1	0.5	0.9	family disposable income (a)	(b) 0.1	0.5	0.9
ln(real GDP, det.)	−2.763	−5.832	−2.387	−0.703	0.061	0.197	0.029	−0.005	0.011	0.051	−0.004	0.014
	(0.105)	(0.288)	(0.103)	(0.107)	(0.050)	(0.122)	(0.052)	(0.068)	(0.044)	(0.109)	(0.042)	(0.059)

(a) low-skilled

τ_{init}	market income (a) -	(b) 0.1	0.5	0.9	disposable income (a)	(b) 0.1	0.5	0.9	family disposable income (a)	(b) 0.1	0.5	0.9
ln(real GDP, det.)	−0.772	−2.539	−0.482	0.204	0.178	0.228	0.176	0.147	0.115	0.288	0.072	0.074
	(0.031)	(0.088)	(0.030)	(0.035)	(0.019)	0.048)	(0.018)	(0.031)	(0.016)	(0.035)	(0.017)	(0.024)

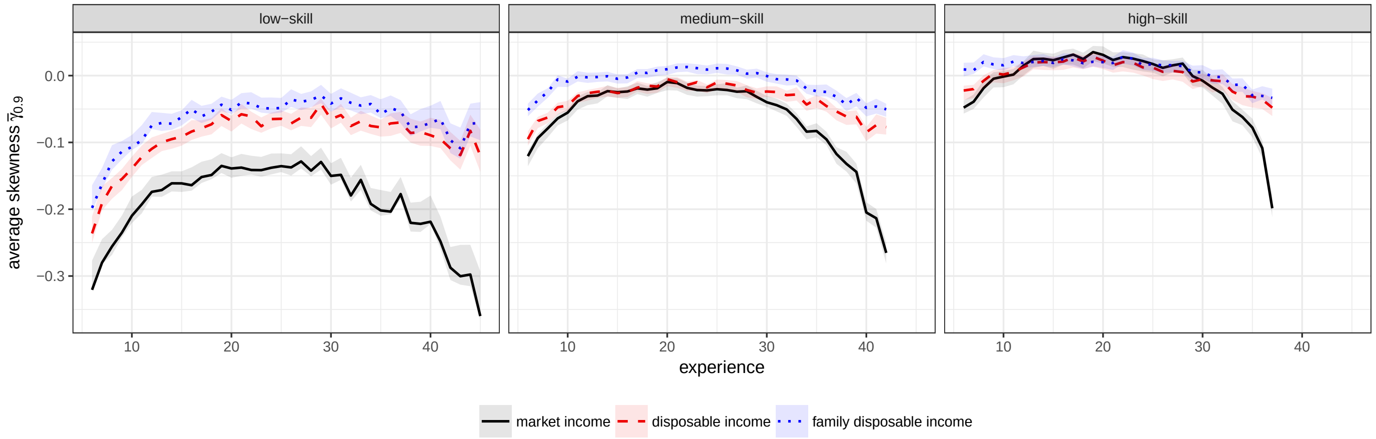
(b) medium-skilled

τ_{init}	market income (a)	(b) 0.1	0.5	0.9	disposable income (a)	(b) 0.1	0.5	0.9	family disposable income (a)	(b) 0.1	0.5	0.9
ln(real GDP, det.)	0.001	−0.704	0.086	0.427	0.530	0.772	0.486	0.413	0.377	0.550	0.317	0.378
	(0.032)	(0.098)	(0.025)	(0.045)	(0.020)	(0.058)	(0.022)	(0.043)	(0.021)	(0.041)	(0.017)	(0.030)

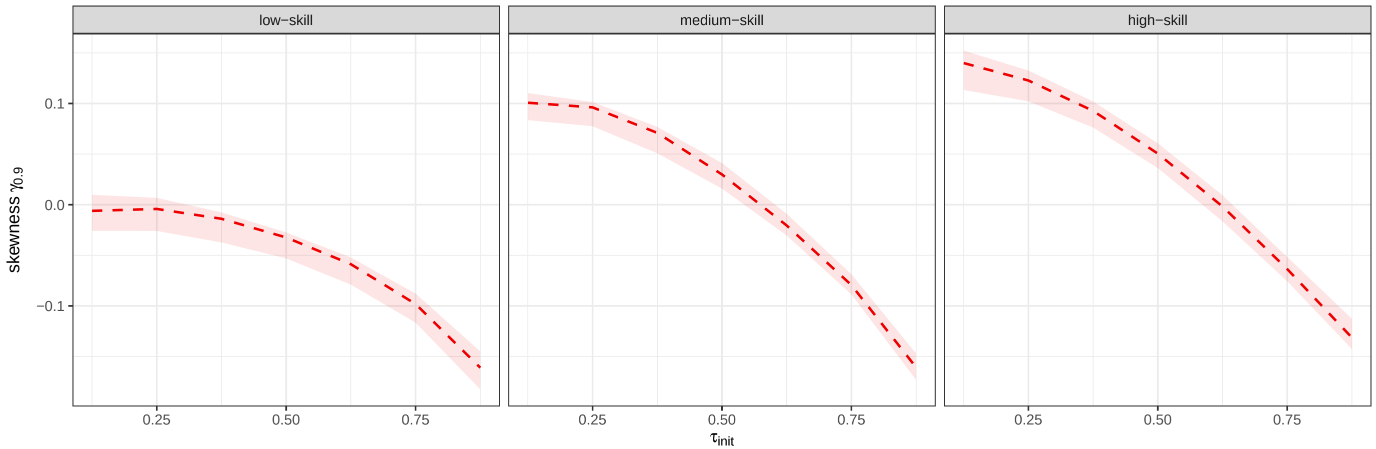
(c) high-skilled

Table 3: Cyclicality of dispersion

Notes: This table presents the results of model (12). The business cycle indicator is the HP-filtered natural logarithm of real GDP of mainland Norway. The dependent variables in columns (a) are the cohort and time specific measures of average dispersion as defined in (6) with $\tau = 0.9$. The dependent variables in columns (b) are the cohort and time specific measures of dispersion as defined in (7) with $\tau = 0.9$ and evaluated at a value of lagged income that corresponds to the τ_{init} percentile of the distribution of Y_{it-1}conditional on observable characteristics. Standard errors are presented in parentheses and are based on a block bootstrap procedure of the quantile auto-regressions (4) with 100 replications. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college.



(a) Average skewness over the life cycle



(b) Conditional skewness in disposable income

Figure 4: Skewness

Notes: This figure shows (a) the life cycle profile of the average skewness (9) with $\tau = 0.9$, (b) skewness in disposable income (10) with $\tau = 0.9$ and evaluated at a value of lagged income that corresponds to the τ_{init} percentile of the distribution of Y_{it-1}conditional on observable characteristics and 20 years of potential experience. The estimates are net of cohort and business cyle effects based on model (12). Shaded areas represent 95% point-wise confidence intervals based on a block bootstrap procedure of the quantile auto-regressions (4) with 100 replications. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.

We now shift attention to the asymmetry of the conditional distribution. Figure (4a) shows the life cycle profile of the average skewness measure, defined in (9) and evaluated at $\tau = 0.9$. First, note that the differences in the skewness profiles across the different measures of income and skill groups. At any experience level, the income distributions tend to be more left-skewed for the low-skilled than for the medium- and high-skilled. Consider for example low-skilled households 10 years after completing education. Skewness in market income is around -0.21 compared to -0.06 and 0.0 for the medium- and high-skilled. According to equation (11), this implies that the upper to lower tail ratio is around 40/60 for the low-skilled, compared to 50/50 for the high-skilled. We also find a pronounced variation in skewness with potential experience, with average skewness roughly following an inverted U-shape over the life cycle. A comparison across income measures reveals that taxes and transfers as well as spousal income tend to reduce skewness risk, especially for the low-skilled and towards the end of the life cycle. Consistent with the empirical evidence in Guvenen, Karahan, Ozkan, and Song (2016) and Arellano, Blundell, and Bonhomme (2016), we also find substantial variation in exposure to skewness risk in the cross-section. Figure (4b) plots estimates for the conditional skewness (8) for households in the middle of the life cycle. Households who are at the top of the income distribution today are exposed to more skewness risk regarding their income realisations of tomorrow. Put differently, the likelihood of large downward movements is higher for those at the top compared to those at the bottom of today's cross-sectional distribution. Appendix Figures (10) and (11) show that roughly the same pattern can be observed for other income measures and at different points in the life cycle. Taken together, these results suggest that conditional skewness varies systematically over the life cycle, within the cross-section and across skill groups. We also find that the tax and transfer system and the pooling of income at the household level appears to decrease skweness risk especially for the low-skilled and towards the end of the life cycle.

Finally, we shift attention to the cyclical properties of the average and conditional skewness measures are reported in Table (4) columns (a) and (b) respectively. Consistent with the empirical evidence in Guvenen, Ozkan, and Song (2014) and Busch and Ludwig (2016), we find that skewness is strongly procyclical across all income measures, skill groups and across the income distribution. For instance, a two-percent drop in GDP implies that the average skewness of market income for medium-skilled households decreases by around 0.065. As shown in Figure (4a), the average skewness in market income for medium-skilled households with 20 years of potential experience is around 0, implying an upper to lower tail ratio of 50/50. A two-percent drop in GDP then corresponds to an upper-to-lower tail ratio of 46/54. Taxes and transfers and the income of the spouse attenuate this effect. For individual disposable income the upper to lower tail ratio changes from 50/50 to 48/52, while the corresponding ratio for family disposable income becomes 49/51.

	market income				disposable income				family disposable income			
	(a)	(b)			(a)	(b)			(a)	(b)		
τ_{init}		0.1	0.5	0.9		0.1	0.5	0.9		0.1	0.5	0.9
ln(real GDP, det.)	4.129	3.867	5.305	2.691	1.471	1.072	2.085	1.536	0.673	0.051	1.282	1.614
	(0.102)	(0.173)	(0.136)	(0.164)	(0.088)	(0.130)	(0.102)	(0.150)	(0.125)	(0.109)	(0.111)	(0.135)

(a) low-skilled

	market income				disposable income				family disposable income			
	(a)	(b)			(a)	(b)			(a)	(b)		
τ_{init}	-	0.1	0.5	0.9		0.1	0.5	0.9		0.1	0.5	0.9
ln(real GDP, det.)	3.266	4.135	3.943	1.873	1.579	1.345	2.001	1.608	1.216	0.806	1.405	1.838
	(0.065)	(0.121)	(0.079)	(0.095)	(0.050)	0.108)	(0.067)	(0.089)	(0.050)	(0.075)	(0.059)	(0.079)

(b) medium-skilled

	market income				disposable income				family disposable income			
	(a)	(b)			(a)	(b)			(a)	(b)		
τ_{init}		0.1	0.5	0.9		0.1	0.5	0.9		0.1	0.5	0.9
ln(real GDP, det.)	3.012	3.128	3.669	1.786	2.151	1.778	2.690	1.597	1.886	1.374	2.166	2.191
	(0.068)	(0.132)	(0.095)	(0.109)	(0.064)	(0.114)	(0.086)	(0.111)	(0.056)	(0.106)	(0.061)	(0.105)

(c) high-skilled

Table 4: Cyclicality of skewness

Notes: This table presents the results of model (12). The business cycle indicator is the HP-filtered natural logarithm of real GDP of mainland Norway. The dependent variables in columns (a) are the cohort and time specific measures of average skewness as defined in (9) with $\tau = 0.9$. The dependent variables in columns (b) are the cohort and time specific measures of skewness as defined in (10) with $\tau = 0.9$ and evaluated at a value of lagged income that corresponds to the τ_{init} percentile of the distribution of Y_{it-1}conditional on observable characteristics. Standard errors are presented in parentheses and are based on a block bootstrap procedure of the quantile auto-regressions (4) with 100 replications. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college.

4 A Search Theoretic Framework

This section develops a tractable, search-theoretic model that is consistent with the empirical evidence presented above. The model extends Lise and Robin (2017) who develop a random search model for the allocations of heterogeneous workers to heterogeneous firms in a frictional labor market with aggregate productivity shocks. By making slightly different assumptions on the employment contracts, our extension delivers the same allocations as in Lise and Robin (2017), but in addition allows us to solve for individual wages in any given match and at any aggregate state of the economy.

4.1 The Environment

Aggregate Shocks. Time is discrete and indexed by $t \in \mathbb{N}$. The aggregate state of the economy follows a Markov chain $\{z_t\}$ with transition probabilities $\pi(z, z') \geq 0$ and $\sum_{z'} \pi(z, z') = 1$.

Agents. The economy consists of a large number of risk-neutral, finitely-lived workers. Workers are characterized by their type $x = (\alpha, h)$. The first component is the worker's time-invariant ability $\alpha \in \{\alpha_1, ..., \alpha_N\}$. The second component is the worker's age $h \in \{1, ..., H\}$. The number of workers of type x, $\ell(x)$, is exogenous and constant over time. Retiring workers are replaced by unemployed labour market entrants. There are also J different types of jobs in the economy and each job is characterized by its permanent productivity factor $y \in \{y_1 < \cdots < y_J\}$. A worker matched with a type-y job receives a wage in exchange for producing value added $f(x, y, z)$.

The number of matched and unmatched workers and jobs changes over time. Let $u_t(x)$ denote the measure of unemployed workers of type x at the beginning of period t. Similarily, let the measure of matches between type-y jobs and workers of type x be given by $m_t(x, y)$, with $u_t(x) + \sum_y m_t(x, y) = \ell(x)$. Firms also recruit workers in every period by posting job-vacancies that last for one period. Let $v_t(y)$ denote the measure of type-y vacancies posted in period t, and define for expositional purposes

$$\Omega_t \equiv \{z_t, u_t(x), m_t(x, y), v_t(y)\},$$

which summarizes the aggregate conditions at time t.

4.1.1 Separations and Meetings

In each period, workers lose jobs and new matches are formed. Let $S(x, y, \Omega_t)$ denote the surplus of a match involving a worker of type x and a firm y under aggregate conditions Ω_t. Whenever the match surplus turns negative, the worker is laid-off endogenously. In addition, we assume that a fraction δ of otherwise profitable matches separates due to exogenous reasons. Workers and firms engage in a random, sequential search process in a single labour market. A firm's recruitment activities for a type-y job are associated with costs $c(y) > 0$, $c' > 0$ and $c'' > 0$. Total recruitment effort in the economy in period t depends on aggregate conditions, and is defined as

$$V(\Omega_t) \equiv \sum_y v_t(y)$$

Workers search for jobs regardless of employment, but the efficiency of their search technology may differ across employment states. Together they produce aggregate search effort in period t defined as

$$L(\Omega_t) \equiv \sum_x u_t(x) + s \sum_{x,y} m_t(x, y),$$

where $s > 0$ is a parameter that governs the search efficiency of employed workers relative to unemployed workers. Both sides are brought together by an exogenous meeting function. The number of meetings between workers and firms in period t is given by

$$M(\Omega_t) = \mu L(\Omega_t)^\gamma V(\Omega_t)^{1-\gamma}. \tag{13}$$

The probability at which an unemployed worker meets a vacancy is defined as $\lambda(\Omega_t) \equiv M(\Omega_t)/L(\Omega_t)$, while the probability for an employed worker is $s\lambda(\Omega_t)$. The probability for any vacancy to contact a worker is defined as $q(\Omega_t) \equiv M(\Omega_t)/V(\Omega_t)$.

4.2 Employment Contracts

An employment contract between a worker x and a firm with job y is a sharing rule $(\omega, 1 - \omega)$ that specifies how the match surplus is split between the two parties. With linear preferences the match surplus $S(x, y, \Omega)$ only depends on the characteristics of the match, the aggregate conditions and not how it is split between the two parties. A contract can be renegotiated by mutual consent only, and is set through the sequential auction framework of Postel-Vinay and Robin (2002). Matches with a negative surplus do not exist, since both parties would be better off by dissolving the match. Thus at any point in time only matches with a positive match surplus are viable.

Let $W(x, y, \omega, \Omega)$ denote the present value of a being matched with job y for a worker of type x under a contracted sharing rule ω and aggregate conditions Ω. Similiarily, let $B(x, \Omega)$ denote the value of the worker's outside option, i.e. the present value of unemployment. Being matched with a job y under a sharing rule $(\omega, 1-\omega)$ implies that the worker enjoys a surplus given by

$$W(x, y, \omega, \Omega) - B(x, \Omega) = \omega S(x, y, \Omega), \tag{14}$$

The firm's surplus on the is equal to the firm's present value of profits,

$$\Pi(x, y, \omega, \Omega) = (1-\omega) S(x, y, \Omega), \tag{15}$$

where the free entry condition ensures that the present value of job y being vacant is nil. A match between an unemployed worker x and a firm y is formed whenever the surplus is non-negative. We assume that firms have full monopsony power with respect to unemployed workers, which implies that $\omega = 0$ and firms retain the entire match surplus.

Poaching. When a worker x currently employed in job y is offered a type-y' job, both the incumbent and the poaching firm engage in Bertrand competition about the worker's services. The result of this bargaining game is that the worker ends up in job y' whenever the match-surplus is greater than the surplus of the current match. Let $\mathcal{M}_1(x, y, \Omega) = \{y' \mid S(x, y', \Omega) > S(x, y, \Omega)\}$ denote the set of jobs y' which would induce the worker to change jobs. At the new job y' the worker receives a share

$$\omega_1(x, y', y, \Omega) = \frac{S(x, y, \Omega)}{S(x, y', \Omega)}, \tag{16}$$

where y becomes the negotiation benchmark. If the poaching firm offers the worker a job y' for which the surplus net of training costs is less than the current surplus, the worker will stay in job y. The worker might nevertheless be able to use this offer to re-negotiate his current contract and increase his current share. This will be the case whenever the $y' \in \mathcal{M}_0(x, y, \Omega) = \{y' \mid \omega S(x, y, \Omega) \leq S(x, y', \Omega) \leq S(x, y, \Omega)\}$ and the worker's new share of the match surplus is given by

$$\omega_0(x, y, y', \Omega) = \frac{S(x, y', \Omega)}{S(x, y, \Omega)}, \tag{17}$$

with y' as the negotiation benchmark. Any offer from jobs $y' \notin \{\mathcal{M}_1(a, x, y, \Omega) \cup \mathcal{M}_0(a, x, y, \Omega)\}$ do not lead to an increase in the worker's share or induce hom to move jobs. Therefore they will be discarded by the worker.

4.3 Value Functions and the Match Surplus

The next step in solving the model is to characterize the worker' s and the firm's surplus which have been kept implicit so far.

Value of Unemployment. Consider an unemployed worker of type x who currently receives income $b(x, z)$. Next period, unless he reaches the retirement age, he will be matched with a job y' with probability $\lambda(\Omega')v(y')/V(\Omega')$. Thus, the Bellman equation reads

$$B(x,\Omega) = b(x,z) + \mathbb{1}\{h < H\}\beta\mathbb{E}_{\Omega'}\Bigg\{\Big(1 - \lambda(\Omega')\sum_{y'\in\mathcal{B}}\frac{v(y')}{V(\Omega')}\Big)B(x',\Omega') \\ + \lambda(\Omega')\sum_{y'\in\mathcal{Y}}\frac{v(y')}{V(\Omega')}\max\{W(x',y',0,\Omega'),B(x',\Omega')\}\;\Big|\;\Omega\Bigg\},$$

where $\mathcal{Y} = \{y' \mid S(x',y',\Omega') \geq 0\}$. Note that the expectation is taking over future aggregate conditions Ω' which includes the distributions of matched and unmatched workers as well as the distribution of job-openings. However, a firm is assumed to have full monopsony power vis-a-vis unemployed workers implying that the firm retains the entire match-surplus. It then follows from (14) that $W(x',y',0,\Omega') = B(x',\Omega')$, and the Bellman equation simplifies to

$$B(x,z) = b(x,z) + \mathbb{1}\{h < H\}\beta\mathbb{E}_{z'}\left\{B(x',z')\;\Big|\;z\right\}, \tag{18}$$

where it is worth to note that the dimensionality of the state space is considerably reduced.

Value of Employment. Next consider a worker who is currently earning a wage w. The match will separate netx period whenever the match surplus turns negative or when an exogenous shock with probability δ destroys the otherwise profitable match. If neither is the case, the worker eventually receives an outside offer from a type-y' job with probability $s\lambda(\Omega')\frac{v(y')}{V(\Omega')}$. If $y' \in \mathcal{M}_1(x',y,\Omega')$, he or she will move to firm y' receiving a share ω_1 of the surplus according to (16). If the worker meets a firm of type $y' \in \mathcal{M}_0(x',y,\Omega')$, the worker will use this offer to receive a share $\omega_0 > \omega$ of the surplus according to (17). Thus the worker's Bellman equation reads,

$$W(x,y,\omega,\Omega) = w + \mathbb{1}\{h < H\}\mathbb{E}_{\Omega'}\Bigg\{\left(\mathbb{1}\left\{S(x',y,\Omega') < 0\right\} + \delta\mathbb{1}\left\{S(x',y,\Omega') \geq 0\right\}\right)B(x',z') \\ + (1-\delta)\mathbb{1}\left\{S(x',y,\Omega') \geq 0\right\}\Bigg[\Big(1 - s\lambda(\Omega')\sum_{y'\in\mathcal{M}_1\cup\mathcal{M}_0}\frac{v(y')}{V(\Omega')}\Big)W(x',y,\omega,\Omega') \\ + s\lambda(\Omega')\Big(\sum_{y'\in\mathcal{M}_1}\frac{v(y')}{V(\Omega')}W(x',y',\omega_1,\Omega') + \sum_{y'\in\mathcal{M}_0}\frac{v(y')}{V(\Omega')}W(x',y,\omega_0,\Omega')\Big)\Bigg]\;\Big|\;\Omega\Bigg\}, \tag{19}$$

where $\omega_1 = \frac{S(x',y,\Omega')}{S(x',y',\Omega')}$ and $\omega_0 = \frac{S(x',y',\Omega')}{S(x',y,\Omega')}$.

Firm Value. Consider the present value of profits of a job y filled by a worker of type x. Current profits are given by the difference in match output and the wage paid to the worker. The job might dissapear tomorrow due to endogenous separation as a consequence of the aggregate state, for exogenous reasons with probability δ or due to the worker ageing that potentially might drive the match surplus negative. The current job also dissappears when the worker receives and accepts an offer from another firm $y' \in \mathcal{M}_1(a',x,y,\Omega')$. In these cases the continuation value is equal to the value of a vacant job, which is equal to 0 due to free entry. If however the worker receives an outside offer from a firm $y' \in \mathcal{M}_0(a',x,y,\Omega')$, the worker stays with the firm under a re-negotiated share of the match surplus $\omega_0 > \omega$ and with probability $1 - s\lambda(\Omega')\sum_{y'\in\mathcal{M}_1\cup\mathcal{M}_0}\frac{v(y')}{V(\Omega')}$ the worker receives no offer that would lead to a credible threat to leave the match. Thus the firm's present value of profits are

$$\Pi(x,y,\omega,\Omega) = f(x,y,z) - w + \mathbb{1}\{h<H\}\beta\mathbb{E}_{\Omega'}\Bigg\{(1-\delta)\mathbf{1}\left\{S(x',y,\Omega')\geq 0\right\} \tag{20}$$

$$\times\Bigg(s\lambda(\Omega')\sum_{y'\in\mathcal{M}_0}\frac{v(y')}{V(\Omega')}\Pi(x',y,\omega_0,\Omega') + \Bigg(1 - s\lambda(\Omega')\sum_{y'\in\mathcal{M}_1\cup\mathcal{M}_0}\frac{v(y')}{V(\Omega')}\Bigg)\Pi(x',y,\omega,\Omega')\Bigg)\ \Big|\ \Omega\Bigg\},$$

with $\omega_0 \equiv \frac{S(x',y',\Omega')}{S(x',y,\Omega')}$

The Match Surplus. The match surplus of a match (x,y,ω) under aggregate conditions Ω is defined as

$$S(x,y,\Omega) := W(x,y,\omega,\Omega) - B(x,\Omega) + \Pi(x,y,\omega,\Omega).$$

Plugging in the expressions for the value functions (18), (19), and (20) yields a simple recursive expression for the total match-surplus (see Appendix 6):

$$S(x,y,z) = f(x,y,z) - b(x,z) + \mathbb{1}\{h<H\}\beta(1-\delta)\mathbb{E}_{z'}\left\{\max\left\{S(x',y,z'),0\right\}\ \Big|\ z\right\}.$$

The surplus depends on aggregate conditions Ω only through the aggregate state z, but not on the distribution of vacancies and matched and unmatched workers. The size of the surplus of the match next period only changes through changes in the worker's age and the aggregate productivity level z. Outside-offers on the other hand do not change the size of the surplus, only how it is split between the two parties. Therefore the expectation about next period's surplus can be taken without taking into account the law of motion for the time-varying distributions $\{v_t(y), m_t(a,x,y), u_t(a,x)\}$. This is the key result in Lise and Robin (2017).

4.4 Vacancy Creation

In each period t, firms engage in recruitment activities to hire workers for a job y by posting job-openings $v_t(y)$. Firms choose $v_t(y)$ to maximize the return from recruiting,

$$\max_{v_t(y)}\left\{-c\left(v_t(y)\right) + v_t(y)\frac{M(\Omega_t)}{V(\Omega_t)}J(y,\Omega_t)\right\}$$

where $c(.)$ are recruitment costs and $J(y,\Omega_t)$ are the expected profits from filling a vacancy upon contacting a worker:

$$J(y,\Omega_t) = \sum_x \frac{\lambda(\Omega_t)u_t(x)}{M(\Omega_t)}\max\{S(x,y,z_t),0\}$$
$$+\sum_{x,y'}\frac{s\lambda(\Omega_t)m_t(x,y')}{M(\Omega_t)}\max\{S(x,y,z_t)-S(x,y',z_t),0\}. \tag{21}$$

Optimality requires that the marginal costs of creating a vacancy equals marginal profits,

$$c'(v_t(y)) = \frac{M(\Omega_t)}{V(\Omega_t)}J(y,\Omega_t).$$

With recruitment costs given by

$$c(v_t(y)) = c_0 \frac{v_t(y)^{1+c_1}}{1+c_1},$$

with $c_0 > 0$ and $c_1 > 0$, it follows that

$$v_t(y) = \left(\frac{M(\Omega_t) J(y, \Omega_t)}{V(\Omega_t) c_0} \right)^{\frac{1}{c_1}}.$$

With $\frac{M(\Omega_t)}{V(\Omega_t)} = \frac{\mu}{\theta_t^\gamma}$ and $\theta_t = V_t / L_t$, it follows that

$$v_t(y) = \left(\frac{\mu J(y, \Omega_t)}{\theta_t^\gamma c_0} \right)^{\frac{1}{c_1}}$$

Summing over y and dividing by $L(\Omega_t)$ yields

$$\theta_t = \frac{1}{L(\Omega_t)} \sum_y \left(\frac{\mu J(y, \Omega_t)}{\theta_t^\gamma c_0} \right)^{\frac{1}{c_1}}$$

and re-arranging

$$\theta_t = \left(\frac{1}{L(\Omega_t)} \sum_y \left(\frac{\mu J(y, \Omega_t)}{c_0} \right)^{\frac{1}{c_1}} \right)^{\frac{c_1}{c_1+\gamma}}$$

4.5 Employment Dynamics

Having established that the match surplus is sufficient to characterize the mobility decisions of workers, we proceed by characterizing the dynamics of the distributions of matches and unmatched workers as time passes within period t. The within-period timing of events is as follows: At the beginning of every period the aggregate shock is realised, workers age and eventually retire. Existing matches then eventually separate following the resolution of uncertainty. After that, firms decide about their recruitment activities and new matches between workers and jobs are formed. Existing and newly formed matches produce output, workers collect their wages and firms collect their profits.

All jobs with a match-surplus $S(x, y, z_t) < 0$ break up by mutual agreement between the firm and the worker. Simultaneously another exogenous fraction δ of matches with a non-negative surplus is destroyed due to exogenous reasons. Thus after the separation stage (at time $t+$), the measure of workers of type x who are unemployed is given by,

$$u_{t+}(x) = u_t(x) + \sum_y \left[\mathbb{1}\{S(x, y, z_t) < 0\} + \delta \mathbb{1}\{S(x, y, z_t) \geq 0\} \right] m_t(x, y). \tag{22}$$

The measure of matches between workers of type x and job y under contract ω is then given by

$$g_{t+}(x, y, \omega) = (1 - \delta) \mathbb{1}\{S(x, y, z_t) > 0\} g_t(x, y, \omega),$$

with $\int g_{t+}(x, y, \omega) \mathrm{d}\omega = m_{t+}(x, y)$.

After separations have taken place, workers and firms then meet according to the aggregate meeting function (13). An unemployed worker of type x meets a job y with probability $\lambda(\Omega_t) \frac{v_t(y)}{V(\Omega_t)}$ and both parties decide to form a match whenever the match-surplus is positive. Thus, the measure of workers of type x that remain unemployed at the end of period t is given by

$$u_{t+1}(x) = u_{t+}(x) \left[1 - \sum_y \lambda(\Omega_t) \frac{v_t(y)}{V(\Omega_t)} \mathbf{1}\{S(x,y,z_t) > 0\} \right].$$

To determine $g_{t+1}(x,y,\omega)$, consider the inflow and outflow due to job-to-job and unemployment-to-employment transitions first. Whenever a worker x currently employed in a job-type y' with surplus $S(x,y',z_t) < S(x,y,z_t)$ is contacted by a type-y job, he will move to the type$-y$ job. If $\frac{S(x,y',z_t)}{S(x,y,z_t)} = \omega$, then this job-to-job transition implies a flow of a worker x into a job of type y with share ω. We denote by $\mathcal{G}^+_{j2j}(x,y',\Omega) = \{y' \mid S(x,y',z_t) < S(x,y,z_t)\}$ the set of all type y' jobs that would induce the worker x to move to job y. Similiarily, we denote by $\mathcal{G}^-_{j2j}(x,y',\Omega) = \{y' \mid S(x,y',\Omega) > S(x,y,\Omega)\}$ the set of type y' jobs that would induce the worker x to leave the job y upon meeting firm y'. Also, unemployed workers of type x contacted by a type y firm move into employment if $S(x,y,z_t) \geq 0$. This unemployment-to-employment flow represents an inflow to $g_{t+}(x,y,0)$.

Second, consider the inflow and outflow due to internal promotion of workers induced by Bertrand competition between firm y with firm y'. If a worker currently receiving a share $\omega' < \omega$ is contacted by a type-y' job with $\omega' S(x,y,z_t) < S(x,y',z_t) < S(x,y,z_t)$ and $\omega = \frac{S(x,y',z_t)}{S(x,y,z_t)}$, then Bertrand competition delivers a share $\omega > \omega'$ for worker x at firm y. Let $\mathcal{G}_B = \{y' \mid S(x,y',z_t) < S(x,y,z_t)\}$ denote the set of firms that would potentially deliver an increasing share of the surplus upon contacting. Thus the measure of workers x that are employed in type-y jobs with a surplus share ω at the end of period t is given by

$$\begin{aligned}
g_{t+1}(x,y,\omega) &= g_{t+}(x,y,\omega) + u_{t+}(x)\lambda(\Omega_t)\frac{v_t(y)}{V(\Omega_t)}\mathbb{1}\{\omega = 0\}\mathbb{1}\{S(x,y,z_t) > 0\} \\
&+ s\lambda(\Omega_t)\Bigg(\sum_{y' \in \mathcal{G}^+_{j2j}} \frac{v_t(y')}{V(\Omega_t)} m_{t+}(x,y')\mathbb{1}\left\{\omega = \frac{S(x,y',z_t)}{S(x,y,z_t)}\right\} - \sum_{y' \in \mathcal{G}^-_{j2j}} \frac{v_t(y')}{V(\Omega_t)} m_{t+}(x,y') \\
&+ \int_{\omega' < \omega} \sum_{y' \in \mathcal{G}_B} g_{t+}(x,y,\omega')\frac{v_t(y')}{V(\Omega_t)}\mathbb{1}\left\{\omega = \frac{S(x,y',z_t)}{S(x,y,z_t)}\right\} \\
&- \sum_{y' \in \mathcal{G}_B} g_{t+}(x,y,\omega)\frac{v_t(y')}{V(\Omega_t)}\mathbb{1}\left\{\omega < \frac{S(x,y',z_t)}{S(x,y,z_t)}\right\}\Bigg).
\end{aligned}$$

4.6 Wage Dynamics

A worker x in job y under contract $(\omega, 1-\omega)$ receives a wage that depends on the aggregate conditions and that is given by (see Appendix)

$$\begin{aligned}
w(x,y,\omega,\Omega) &= \omega f(x,y,z) + (1-\omega)b(x,z) - \beta\mathbb{E}_{\Omega'}\Bigg\{(1-\delta)\mathbb{1}\left\{S(x',y,z') \geq 0\right\} \\
&\times s\lambda(\Omega')\Bigg(\sum_{y' \in \mathcal{M}_0} \frac{v(y')}{V(\Omega')}\big(S(x',y',z') - \omega S(x',y,z')\big) + (1-\omega)\sum_{y' \in \mathcal{M}_1} \frac{v(y')}{V(\Omega')} S(x',y,z').
\end{aligned}$$

5 Calibration

- calibrate the model to match the evidence on earnings dynamics reported above.

6 Conclusion

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Appendix

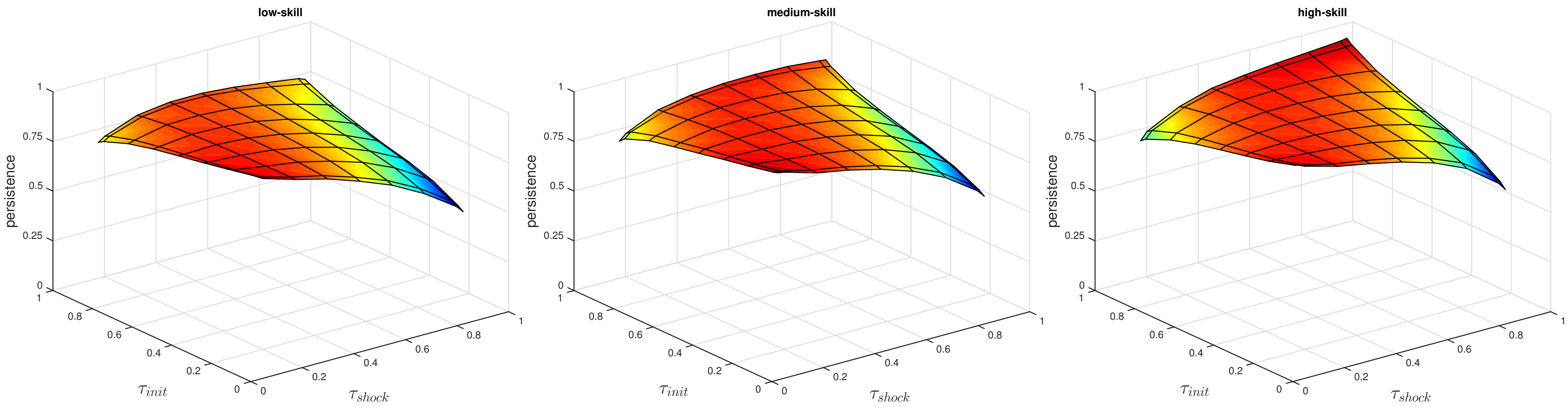


Figure 5: Persistence in family disposable income

Notes: This figure shows the average persistence in family disposable income at 20 years of potential experience. Cohort and business cycle effects are taken out based on model (12). Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.

	market income		disposable income		family disposable income	
unemp. rate, de-meaned	-0.039^{***} (0.000)		-0.013^{***} (0.000)		-0.011^{***} (0.000)	
$\mathbb{1}\{\text{exp.}=5\}\times$unemp. rate		-0.097^{***} (0.005)		-0.041^{***} (0.003)		-0.041^{***} (0.002)
$\mathbb{1}\{\text{exp.}=10\}\times$unemp. rate		-0.056^{***} (0.003)		-0.017^{***} (0.002)		-0.014^{***} (0.001)
$\mathbb{1}\{\text{exp.}=20\}\times$unemp. rate		-0.032^{***} (0.002)		-0.008^{***} (0.001)		-0.007^{***} (0.001)
$\mathbb{1}\{\text{exp.}=30\}\times$unemp. rate		-0.034^{***} (0.003)		-0.023^{***} (0.002)		-0.019^{***} (0.002)
$\mathbb{1}\{\text{exp.}=45\}\times$unemp. rate		-0.041^{***} (0.003)		-0.023^{*} (0.011)		-0.014 (0.009)
R^2	0.10	0.10	0.19	0.19	0.58	0.58
Num. Obs.	2,542,450	2,542,450	2,542,450	2,542,450	2,542,450	2,542,450

(a) low-skilled

	market income		disposable income		family disposable income	
unemp. rate, de-meaned	-0.025^{***} (0.000)		-0.011^{***} (0.000)		-0.009^{***} (0.000)	
$\mathbb{1}\{\text{exp.}=5\}\times$unemp. rate		-0.067^{***} (0.002)		-0.039^{***} (0.001)		-0.035^{***} (0.001)
$\mathbb{1}\{\text{exp.}=10\}\times$unemp. rate		-0.034^{***} (0.001)		-0.012^{***} (0.001)		-0.008^{***} (0.001)
$\mathbb{1}\{\text{exp.}=20\}\times$unemp. rate		-0.020^{***} (0.001)		-0.005^{***} (0.001)		-0.004^{***} (0.001)
$\mathbb{1}\{\text{exp.}=30\}\times$unemp. rate		-0.024^{***} (0.002)		-0.019^{***} (0.001)		-0.016^{***} (0.001)
$\mathbb{1}\{\text{exp.}=42\}\times$unemp. rate		0.003 (0.003)		-0.013^{***} (0.002)		-0.011^{***} (0.002)
R^2	0.11	0.11	0.18	0.18	0.54	0.54
Num. Obs.	8,498,212	8,498,212	8,498,212	8,498,212	8,498,212	8,498,212

(b) medium-skilled

	market income		disposable income		family disposable income	
unemp. rate, de-meaned	-0.011^{***} (0.002)		-0.004^{***} (0.000)		-0.002^{***} (0.000)	
$\mathbb{1}\{\text{exp.}=5\}\times$unemp. rate		-0.026^{***} (0.002)		-0.017^{***} (0.001)		-0.013^{***} (0.001)
$\mathbb{1}\{\text{exp.}=10\}\times$unemp. rate		-0.018^{***} (0.002)		-0.008^{***} (0.001)		-0.005^{***} (0.001)
$\mathbb{1}\{\text{exp.}=20\}\times$unemp. rate		-0.003 (0.002)		0.003^{**} (0.001)		0.004^{***} (0.001)
$\mathbb{1}\{\text{exp.}=30\}\times$unemp. rate		-0.016^{***} (0.002)		-0.017^{***} (0.002)		-0.014^{***} (0.001)
$\mathbb{1}\{\text{exp.}=35\}\times$unemp. rate		-0.014^{***} (0.003)		-0.011 (0.002)		-0.010^{***} (0.002)
R^2	0.16	0.16	0.19	0.19	0.50	0.50
Num. Obs.	5,496,151	5,496,151	5,496,151	5,496,151	5,496,151	5,496,151

(c) high-skilled

Table 5: Aggregate cyclical income risk: Unemployment rate

Notes: This table present coefficients of pooled OLS regressions. The results from specifications (1) and (2) are presented in column (1) and (2) respectively. The business cycle indicator is the demeaned unemployment rate. Other controls include potential experience fixed effects, cohort fixed effects, region fixed effects and marital status and dummies for the number of children in the household. Standard errors are presented in parentheses. ***, **, and *, represent statistical significance at 0.1%, 1%, and 5% levels, respectively.

	market income		disposable income		family disposable income	
ln(real GDP, det.)	2.229***		1.201***		0.933***	
	(0.027)		(0.015)		(0.012)	
$\mathbb{1}\{\text{exp.}=5\}\times\ln(\text{real GDP, det.})$		4.764***		2.179***		2.021***
		(0.297)		(0.167)		(0.136)
$\mathbb{1}\{\text{exp.}=10\}\times\ln(\text{real GDP, det.})$		3.821***		1.761***		1.302***
		(0.206)		(0.112)		(0.094)
$\mathbb{1}\{\text{exp.}=20\}\times\ln(\text{real GDP, det.})$		2.340***		1.252***		0.950***
		(0.157)		(0.088)		(0.072)
$\mathbb{1}\{\text{exp.}=30\}\times\ln(\text{real GDP, det.})$		1.541***		0.983***		0.802***
		(0.143)		(0.080)		(0.066)
$\mathbb{1}\{\text{exp.}=45\}\times\ln(\text{real GDP, det.})$		2.692***		0.349		−0.039
		(0.616)		(0.346)		(0.282)
R^2	0.49	0.49	0.59	0.59	0.80	0.80
Household FE?	yes	yes	yes	yes	yes	yes
Num. Obs.	2,542,450	2,542,450	2,542,450	2,542,450	2,542,450	2,542,450

(a) low-skilled

	market income		disposable income		family disposable income	
ln(real GDP, det.)	1.546***		1.106***		0.867***	
	(0.011)		(0.007)		(0.006)	
$\mathbb{1}\{\text{exp.}=5\}\times\ln(\text{real GDP, det.})$		3.022***		1.911***		1.648***
		(0.107)		(0.068)		(0.055)
$\mathbb{1}\{\text{exp.}=10\}\times\ln(\text{real GDP, det.})$		2.052***		1.320***		0.961***
		(0.081)		(0.051)		(0.041)
$\mathbb{1}\{\text{exp.}=20\}\times\ln(\text{real GDP, det.})$		1.555***		1.138***		0.912***
		(0.065)		(0.042)		(0.033)
$\mathbb{1}\{\text{exp.}=30\}\times\ln(\text{real GDP, det.})$		1.382***		1.019***		0.808***
		(0.069)		(0.044)		(0.035)
$\mathbb{1}\{\text{exp.}=42\}\times\ln(\text{real GDP, det.})$		0.153		0.530***		0.398***
		(0.092)		(0.058)		(0.047)
R^2	0.55	0.55	0.63	0.63	0.80	0.80
Household FE?	yes	yes	yes	yes	yes	yes
Num. Obs.	8,498,212	8,498,212	8,498,212	8,498,212	8,498,212	8,498,212

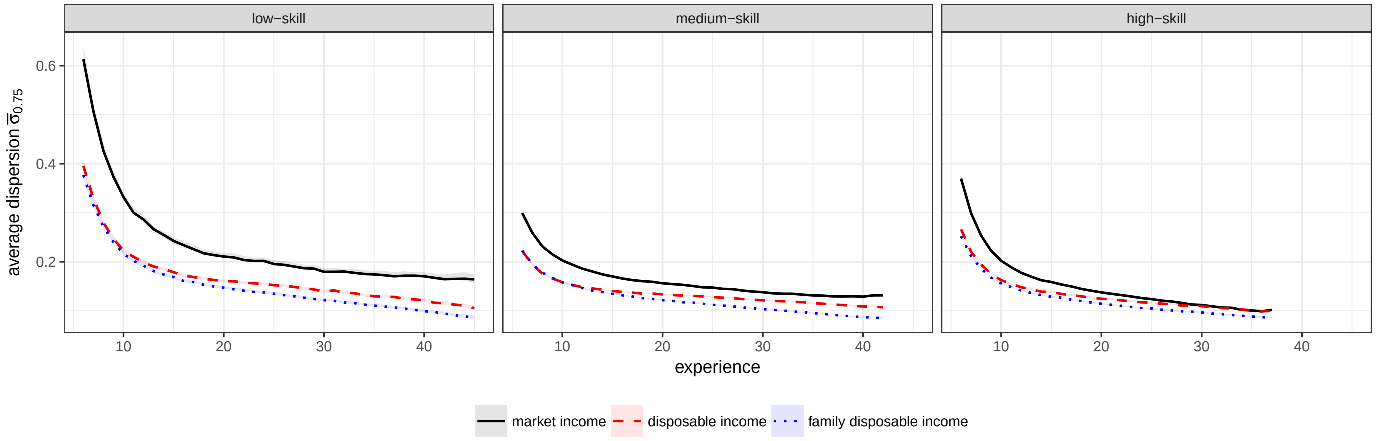
(b) medium-skilled

	market income		disposable income		family disposable income	
ln(real GDP, det.)	0.987***		0.809***		0.660***	
	(0.014)		(0.010)		(0.008)	
$\mathbb{1}\{\text{exp.}=5\}\times\ln(\text{real GDP, det.})$		1.894***		1.128***		0.949***
		(0.090)		(0.065)		(0.053)
$\mathbb{1}\{\text{exp.}=10\}\times\ln(\text{real GDP, det.})$		1.326***		0.987***		0.704***
		(0.078)		(0.056)		(0.046)
$\mathbb{1}\{\text{exp.}=20\}\times\ln(\text{real GDP, det.})$		0.943***		0.849***		0.632***
		(0.082)		(0.059)		(0.048)
$\mathbb{1}\{\text{exp.}=30\}\times\ln(\text{real GDP, det.})$		0.684***		0.687***		0.580***
		(0.086)		(0.062)		(0.051)
$\mathbb{1}\{\text{exp.}=35\}\times\ln(\text{real GDP, det.})$		0.537***		0.467***		0.443***
		(0.094)		(0.068)		(0.056)
R^2	0.66	0.66	0.69	0.69	0.81	0.81
Household FE?	yes	yes	yes	yes	yes	yes
Num. Obs.	5,496,151	5,496,151	5,496,151	5,496,151	5,496,151	5,496,151

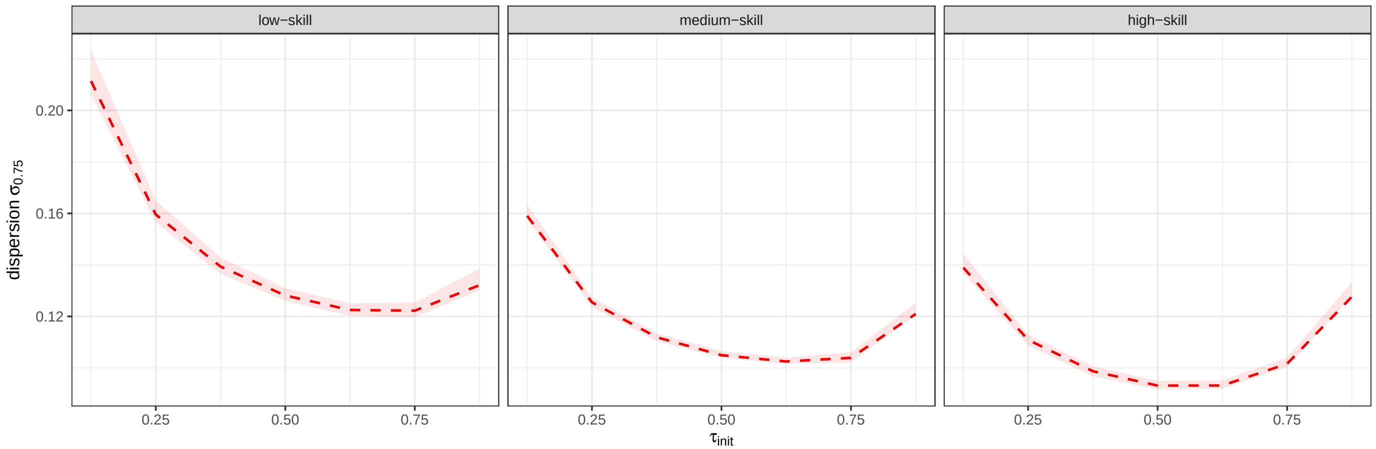
(c) high-skilled

Table 6: Aggregate cyclical income risk: Household fixed effects

Notes: This table present coefficients of fixed effects regressions. The results from specifications (1) and (2) are presented in column (1) and (2) respectively. The business cycle indicator is the HP-filtered natural logarithm of real GDP of mainland Norway. Other controls include potential experience fixed effects, region fixed effects, marital status and dummies for the number of children in the household. Standard errors are presented in parentheses. ***, **, and *, represent statistical significance at 0.1%, 1%, and 5% levels, respectively.



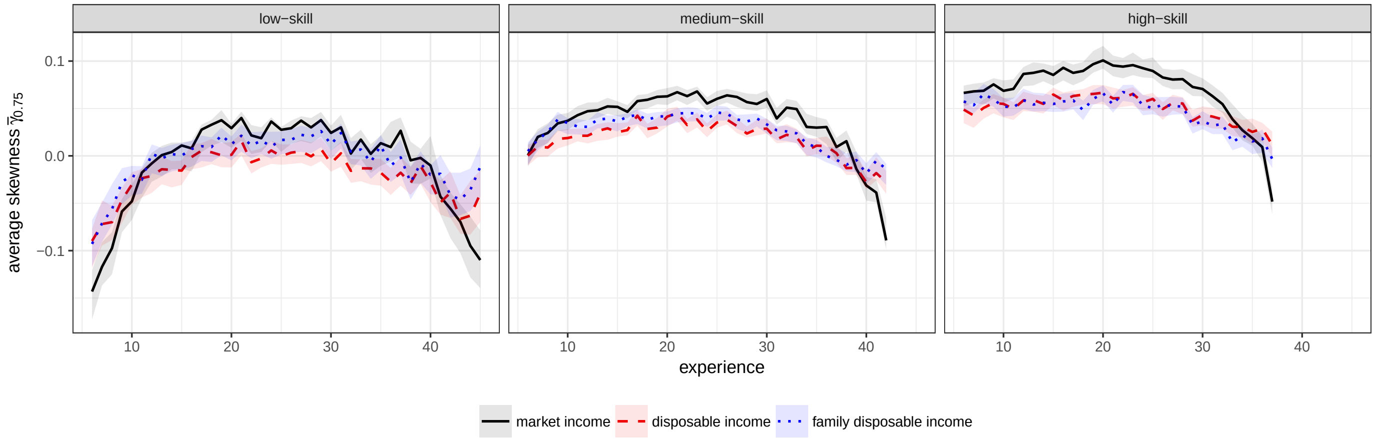
(a) Average dispersion over the life cycle



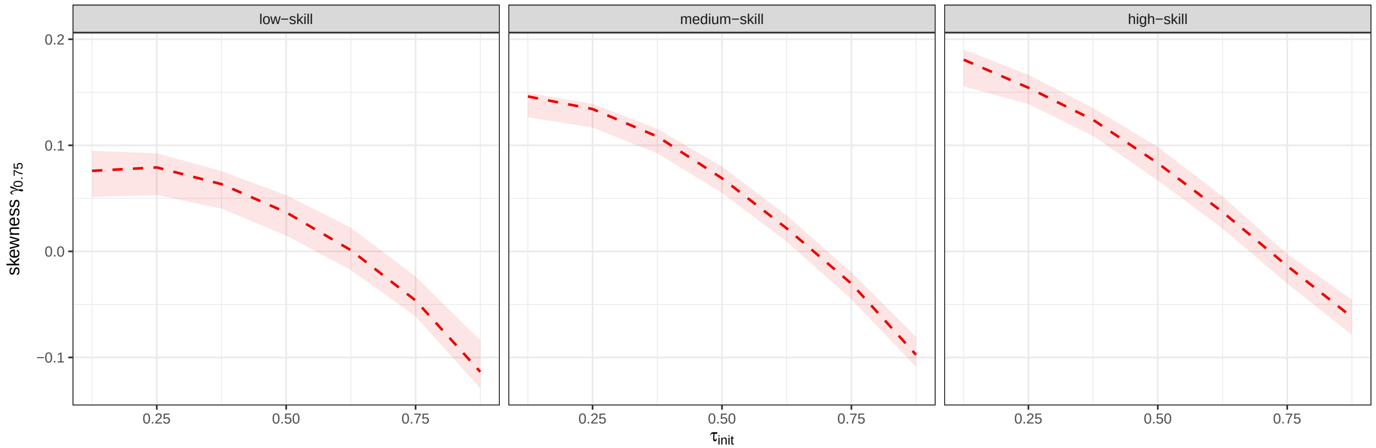
(b) Conditional dispersion in disposable income

Figure 6: Dispersion

Notes: This figure shows (a) the life cycle profile of the average dispersion (6) with $\tau = 0.75$, (b) dispersion in disposable income (7) with $\tau = 0.75$ and evaluated at a value of lagged income that corresponds to the τ_{init} percentile of the distribution of Y_{it-1}conditional on observable characteristics and 20 years of potential experience. The estimates are net of cohort and business cyle effects based on model (12). Shaded areas represent 95% point-wise confidence intervals based on a block bootstrap procedure of the quantile auto-regressions (4) with 100 replications. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.



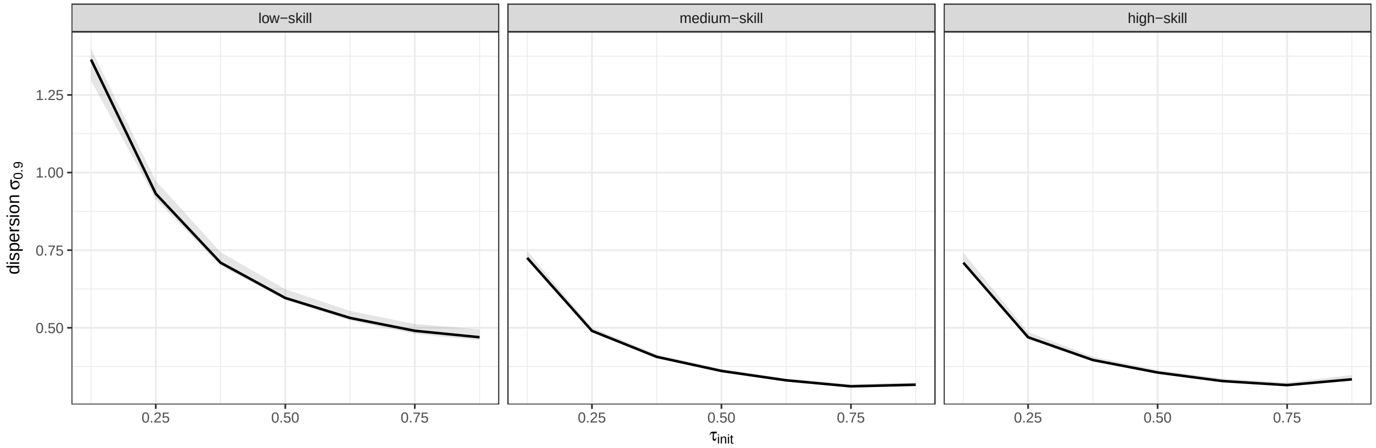
(a) Average skewness over the life cycle



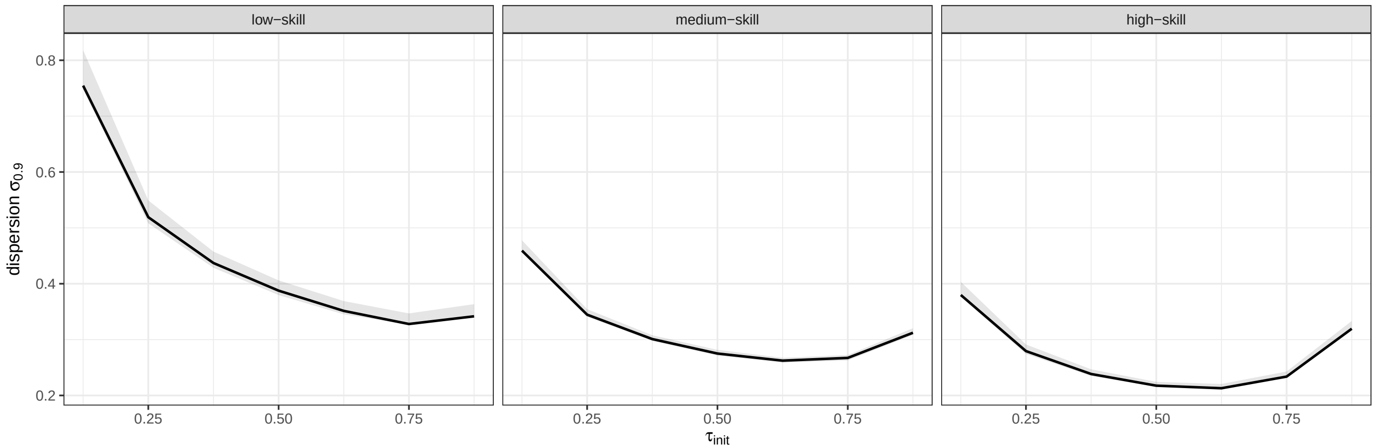
(b) Conditional skewness in disposable income

Figure 7: Skewness

Notes: This figure shows (a) the life cycle profile of the average skewness (9) with $\tau = 0.75$, (b) skewness in disposable income (10) with $\tau = 0.75$ and evaluated at a value of lagged income that corresponds to the τ_{init} percentile of the distribution of Y_{it-1}conditional on observable characteristics and 20 years of potential experience. The estimates are net of cohort and business cyle effects based on model (12). Shaded areas represent 95% point-wise confidence intervals based on a block bootstrap procedure of the quantile auto-regressions (4) with 100 replications. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.



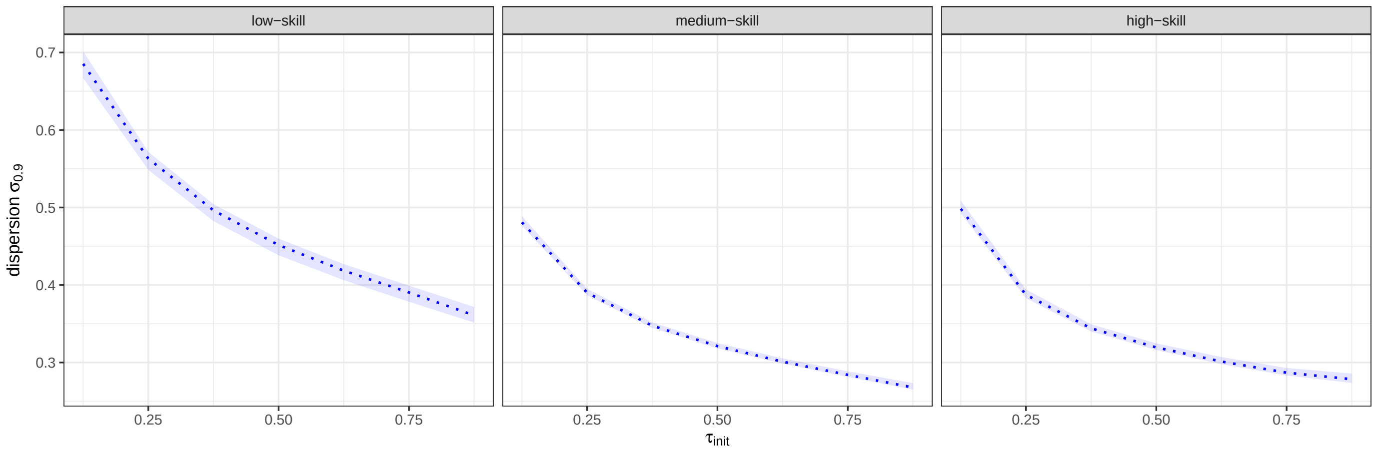
(a) 10 years of potential experience



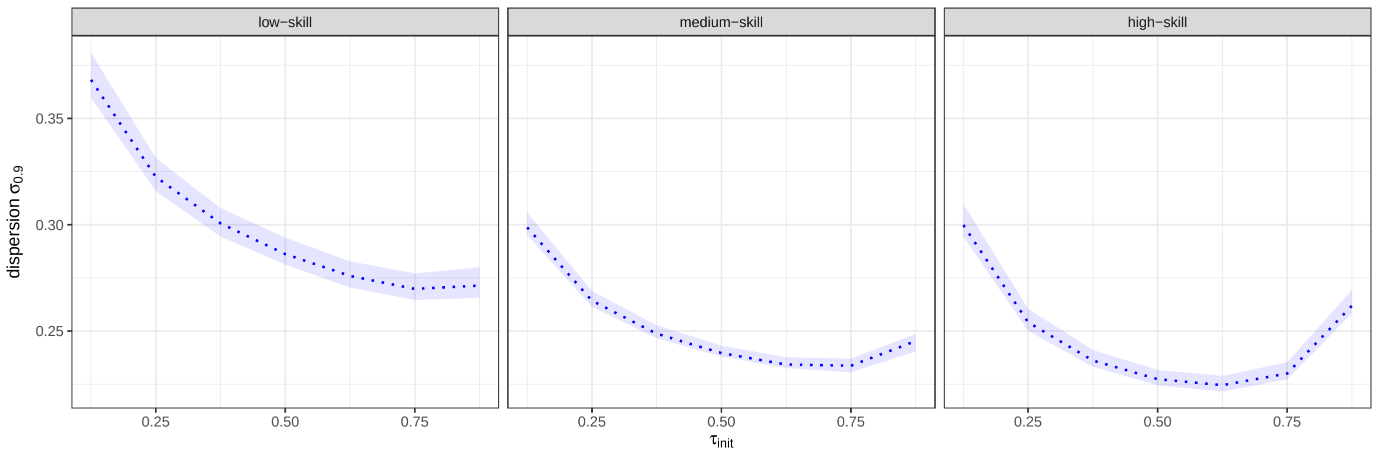
(b) 30 years of potential experience

Figure 8: Conditional dispersion in market income

Notes: This figure shows dispersion in market income (7) with $\tau = 0.9$ and evaluated at a value of lagged income that corresponds to the τ_{init} percentile of the distribution of Y_{it-1}conditional on observable characteristics, and 10 and 30 years of potential experience. The estimates are net of cohort and business cyle effects based on model (12). Shaded areas represent 95% point-wise confidence intervals based on a block bootstrap procedure of the quantile auto-regressions (4) with 100 replications. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.



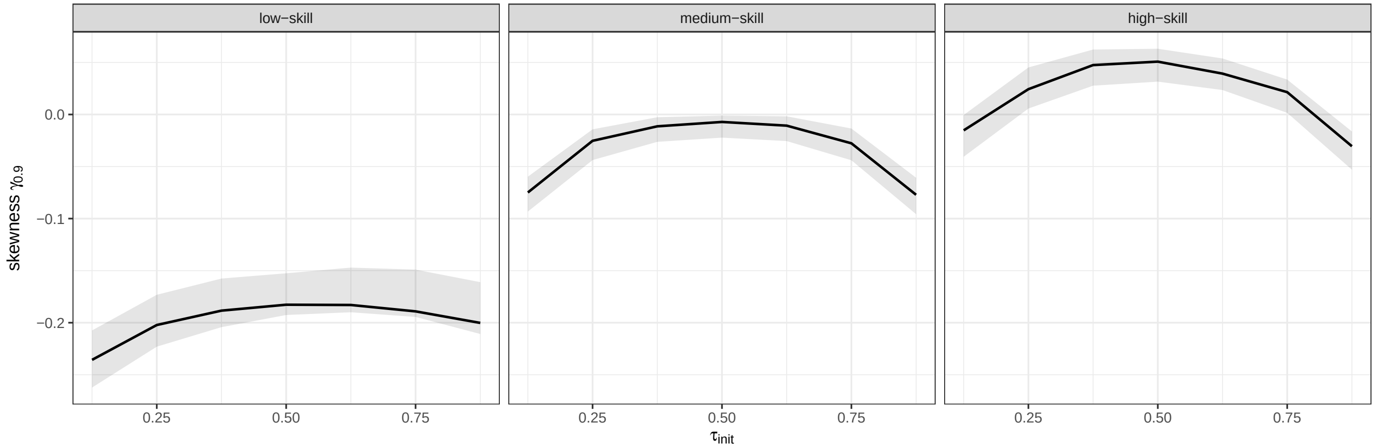
(a) 10 years of potential experience



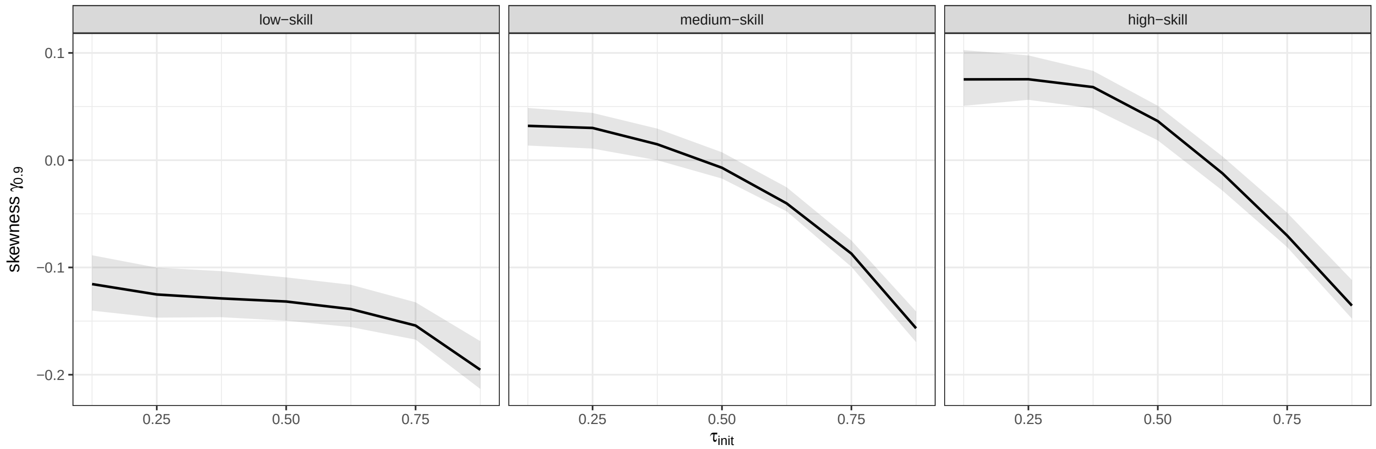
(b) 30 years of potential experience

Figure 9: Conditional dispersion in family disposable income

Notes: This figure shows dispersion in family disposable income (7) with $\tau = 0.9$ and evaluated at a value of lagged income that corresponds to the τ_{init} percentile of the distribution of Y_{it-1}conditional on observable characteristics, and 10 and 30 years of potential experience. The estimates are net of cohort and business cyle effects based on model (12). Shaded areas represent 95% point-wise confidence intervals based on a block bootstrap procedure of the quantile auto-regressions (4) with 100 replications. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.



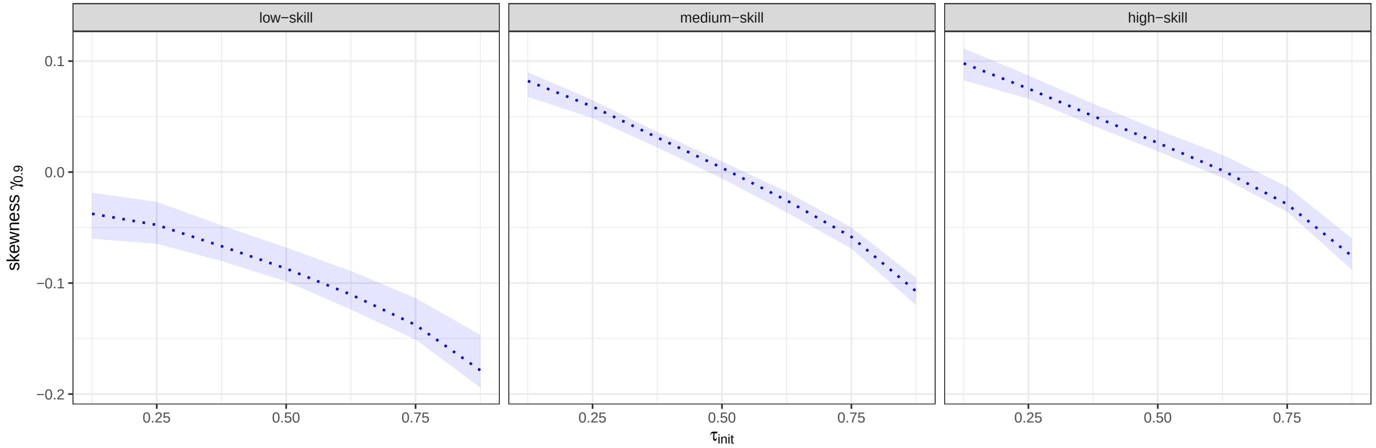
(a) 10 years of potential experience



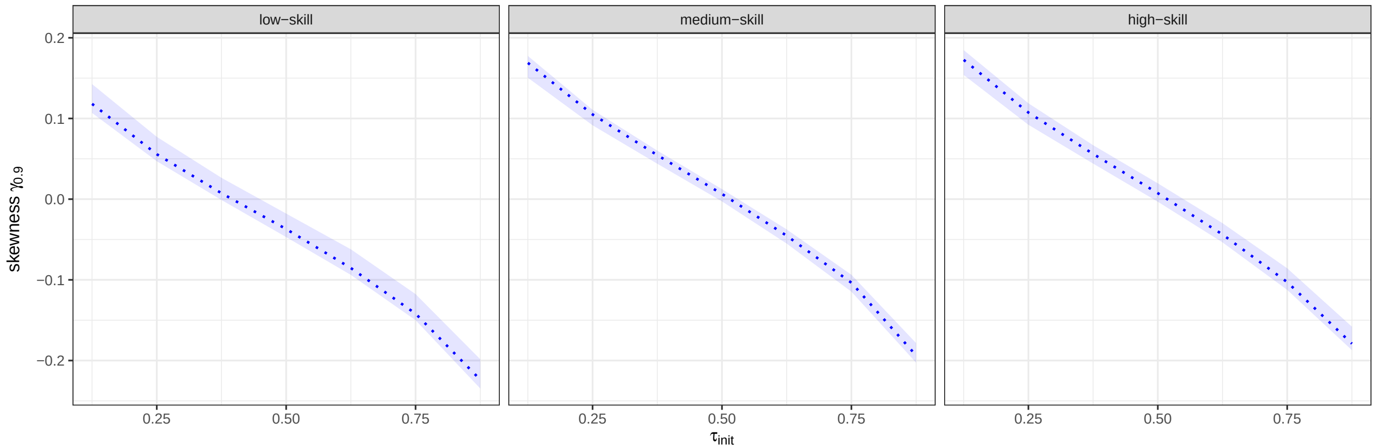
(b) 30 years of potential experience

Figure 10: Conditional skewness in market income

Notes: This figure shows skewness in market income (10) with $\tau = 0.9$ and evaluated at a value of lagged income that corresponds to the τ_{init} percentile of the distribution of Y_{it-1}conditional on observable characteristics, and 10 and 30 years of potential experience. The estimates are net of cohort and business cyle effects based on model (12). Shaded areas represent 95% point-wise confidence intervals based on a block bootstrap procedure of the quantile auto-regressions (4) with 100 replications. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.



(a) 10 years of potential experience



(b) 30 years of potential experience

Figure 11: Conditional skewness in family disposable income

Notes: This figure shows skewness in family disposable income (10) with $\tau = 0.9$ and evaluated at a value of lagged income that corresponds to the τ_{init} percentile of the distribution of Y_{it-1}conditional on observable characteristics, and 10 and 30 years of potential experience. The estimates are net of cohort and business cyle effects based on model (12). Shaded areas represent 95% point-wise confidence intervals based on a block bootstrap procedure of the quantile auto-regressions (4) with 100 replications. Low skilled is defined as not having completed high school, medium skilled includes individuals with a high school degree, and high skilled consists of individuals who have attended college. Potential labour market experience is defined as age minus years of education minus 6.

Deriving the match surplus

The match surplus of a match (x, y, ω) under aggregate conditions Ω is defined as

$$S(x, y, \Omega) := W(x, y, \omega, \Omega) - B(x, \Omega) + \Pi(x, y, \omega, \Omega).$$

Plugging in the expressions for the value functions (18), (19), and (20) yields after re-arranging,

$$\begin{aligned}
S(x, y, \Omega) = f(x, y, z) - b(x, z) + \mathbb{1}\{h < H\}\beta\mathbb{E}_{\Omega'}\Bigg\{&(1-\delta)\mathbb{1}\{S(x', y, \Omega') \geq 0\} \\
\times \Bigg[\Bigg(1 - s\lambda(\Omega') &\sum_{y' \in \mathcal{M}_1 \cup \mathcal{M}_0} \frac{v(y')}{V(\Omega')}\Bigg)\big(W(x', y, \omega, \Omega') + \Pi(x', y, \omega, \Omega') - B(x', z')\big) \\
&+ s\lambda(\Omega')\Bigg(\sum_{y' \in \mathcal{M}_1} \frac{v(y')}{V(\Omega')}\big(W(x', y', \omega_1, \Omega') - B(x', z')\big) \\
&+ \sum_{y' \in \mathcal{M}_0} \frac{v(y')}{V(\Omega')}\big(W(x', y, \omega_0, \Omega') + \Pi(x', y, \omega_0, \Omega') - B(x', z')\big)\Bigg) \;\Bigg|\; \Omega\Bigg\}. \quad (23)
\end{aligned}$$

Note that $W(x', y, \omega, \Omega') + \Pi(x', y, \omega, \Omega') - B(x', z') = S(x', y, \Omega')$ equals the surplus of the match. Further note that the size of the surplus does not depend on how it is split between the parties, so $W(x', y, \omega_0, \Omega') + \Pi(x', y, \omega_0, \Omega') - B(x', z') = S(x', y, \Omega')$. In addition, it follows from (16), that the worker receives the whole surplus of the current match when he moves to a firm y', so that $W(x', y', \omega_1, \Omega') - B(x', z') = S(x', y, \Omega')$. Therefore equation (23) simplifies to

$$S(x, y, \Omega) = f(x, y, z) - b(x, z) + \mathbb{1}\{h < H\}\beta(1-\delta)\mathbb{E}_{\Omega'}\Big\{\max\big\{S(x', y, \Omega'), 0\big\} \;\Big|\; \Omega\Big\},$$

Since the match surplus depends on aggregate conditions only through the aggregate state z, we can directly write the match surplus as,

$$S(x, y, z) = f(x, y, z) - b(x, z) + \mathbb{1}\{h < H\}\beta(1-\delta)\mathbb{E}_{z'}\Big\{\max\big\{S(x', y, z'), 0\big\} \;\Big|\; z\Big\}.$$

Deriving the wage equation

Using $W(x, y, \omega, \Omega) - B(x, \Omega) = \omega S(x, y, z)$, we can write the worker surplus in a match (x, y) under sharing rule $(\omega, 1-\omega)$ and aggregate conditions Ω as

$$\begin{aligned}
\omega S(x, y, z) = w - b(x, z) + \beta\mathbb{E}_{\Omega'}\Bigg\{&(1-\delta)\mathbb{1}\big\{S(x', y, z') \geq 0\big\} \\
&\times \Bigg[\Bigg(1 - s\lambda(\Omega') \sum_{y' \in \mathcal{M}_1 \cup \mathcal{M}_0} \frac{v(y')}{V(\Omega')}\Bigg)\omega S(x', y, z') \\
&+ s\lambda(\Omega')\Bigg(\sum_{y' \in \mathcal{M}_1} \frac{v(y')}{V(\Omega')} S(x', y, z') + \sum_{y' \in \mathcal{M}_0} \frac{v(y')}{V(\Omega')} S(x', y', z')\Bigg)\Bigg] \;\Bigg|\; \Omega\Bigg\}. \quad (24)
\end{aligned}$$

Similiarily, we can write the firm's surplus from match (x, y) as

$$(1-\omega)S(x,y,z) = f(x,y,z) - w + \beta\mathbb{E}_{\Omega'}\Bigg\{(1-\delta)\mathbb{1}\left\{S(x',y,z')\geq 0\right\}$$

$$\times\Bigg[\Bigg(1 - s\lambda(\Omega')\sum_{y'\in\mathcal{M}_1\cup\mathcal{M}_0}\frac{v(y')}{V(\Omega')}\Bigg)(1-\omega)\,S(x',y,z') \tag{25}$$

$$+s\lambda(\Omega')\sum_{y'\in\mathcal{M}_0}\frac{v(y')}{V(\Omega')}\left(S(x',y,z') - S(x',y',z')\right)\Bigg|\,\Omega\Bigg\}. \tag{26}$$

Multiplying (26) with ω and (24) with $(1-\omega)$ and subtracting from each other yields the equation for the wage of worker x in job y under contract $(\omega, 1-\omega)$ when the current aggregate conditions are Ω :

$$w(x,y,\omega,\Omega) = \omega f(x,y,z) + (1-\omega)b(x,z) - \beta\mathbb{E}_{\Omega'}\Bigg\{(1-\delta)\mathbb{1}\left\{S(x',y,z')\geq 0\right\}$$

$$\times\, s\lambda(\Omega')\Bigg(\sum_{y'\in\mathcal{M}_0}\frac{v(y')}{V(\Omega')}\big(S(x',y',z') - \omega S(x',y,z')\big) + (1-\omega)\sum_{y'\in\mathcal{M}_1}\frac{v(y')}{V(\Omega')}S(x',y,z')$$