Consumption-led Growth*

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Abstract

What is the relationship between trade and current account openness and growth? Can a catching-up economy borrow like Argentina or Spain and grow like China? To address these questions, we develop a model of endogenous converge growth, which we study under various policy regimes regarding trade and capital account openness. In the model, entrepreneurs adopt heterogenous projects based on their profitability. Trade openness has two effects on the relative profitability of tradable projects. First, the foreign competition effect unambiguously discourages tradable innovation. Second, the relative market size effect may favor or discourage tradable innovation. We show that balanced trade ensures that the two effects exactly offset each other, while trade deficits unambiguously favor non-tradable innovation. The increase in domestic consumption associated with international borrowing results in a relative market size effect that reinforces the foreign competition effect to discourage tradable innovation, as well as the aggregate innovation rate and the pace of productivity convergence. We further show that net exports relative to domestic absorption is a sufficient statistic for the feedback effect from aggregate allocation into sectoral productivity growth, and we find empirical support for the predictions of the model in the panel of sectoral productivity growth rates in OECD countries. A sudden stop in capital flows during the transition phase results simultaneously in a recession due to a fall in local demand and a sharp rebound in tradable productivity growth, provided the labor market can adjust flexibly via a sharp decline in the wage rate.

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1 Introduction

What is the relationship between trade and current account openness and growth? Can a catching-up economy borrow like Argentina or Spain and grow like China? Or is there some basic inconsistency between these two outcomes? Furthermore, is there something special about the Chinese openness and its growth experience, and if so, is it about trade flows, current account surpluses, or FDI flows? To address these questions, we develop a model of endogenous converge growth, which we study under various policy regimes regarding trade and capital account openness. A catching-up economy borrows to smooth consumption in anticipation of the future higher productivity levels. We study, in particular, the feedback effect from such current account deficits during the transition into the convergence growth trajectory of the economy, referring to it as *consumption-led growth*.[1]

In the model, agents select into entrepreneurial activity when expected profits are high relative to wages, as in Lucas (1978), and then select among heterogenous business projects based on their profitability. The economic profitability of the projects does not always correspond to their physical productivity, and we show that this gap is systematically related to the aggregate trade imbalance of the country. The projects become obsolete over time, and in steady state the inflow of new ideas is offset by the exit of the existing ones.

The model features a neoclassical convergence force by which backward economies grow faster as the aggregate and lagging sectors exhibit a faster catch-up productivity growth. Over and above this convergence force, trade and current account openness have a systematic effect on both the overall pace and the composition of productivity growth in the economy, by affecting the relative profitability of tradable and non-tradable projects, as well as the level of expected profits.[2]

Openness to international trade and financial flows affects relative profitability of tradable production via two channels. First, there is a relative market size effect, both due to a change in domestic consumption and due to foreign market access for tradable products. The direction of this effect can go both ways, in favor or against tradable innovation. The second effect operates via foreign competition, which reduces the tradable price index, and unambiguously discourages tradable innovation. In additionally, both international trade and international borrowing lead to an increase in domestic wages, due to terms-of-trade and home-market-

[1] A natural analogy in the policy debate is to export-led growth on one hand and import substitution policies on the other. We emphasize, however, that consumption-led growth is not a policy-induced outcome, it is rather a pattern of market equilibrium growth outcomes in the period of large current account deficits associated with borrowing along the transition path. In contrast, export-led-growth and import-substitution policies are introduced to steer away from the market allocation with a particular policy objective, and often rely on trade policy or capital controls, which we also study in the context of our model.

[2] Of course, tradability of a good is a modeling abstraction, and more generally we think of products as either exportable (e.g., competitive on the world market) or non-exportable.

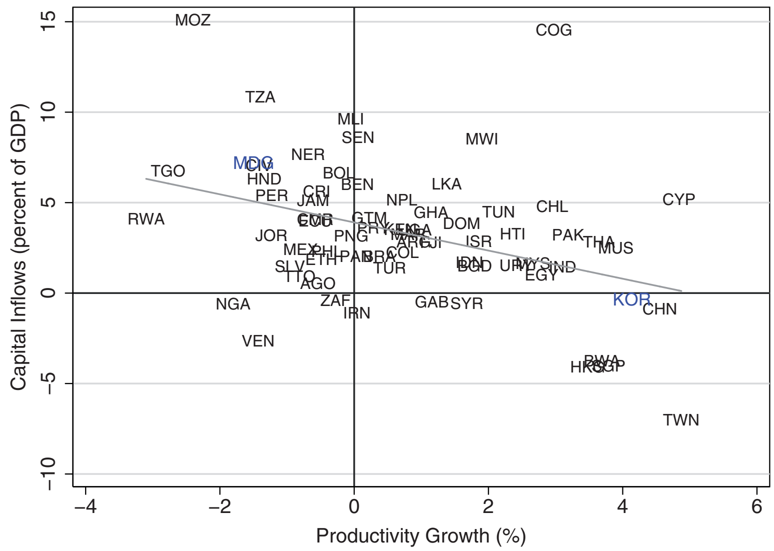

Figure 1: The capital flows allocation puzzle

Note: Reproduced from Gourinchas and Jeanne (2013). Average productivity growth and average capital inflows between 1980 and 2000. 68 non-OECD countries.

size effects respectively, which further reduces the relative profitability of the tradable innovation. We show that balanced trade ensures that the two effects exactly offset each other, and thus does not alter the tradability composition of projects adopted by entrepreneurs and the convergence trajectory of the economy.

At the same time, trade deficits unambiguously favor non-tradable innovation. External borrowing leads to a higher domestic consumption, which is disproportionately spent on the locally-produced goods. The induced market size effect reinforces the foreign competition effect to discourage tradable innovation. We show that trade deficits tilt productivity evolution away from the tradable sector, resulting in a delayed productivity convergence in this sector. In contrast, the non-tradable technological convergences comes faster under transitory current account deficits.

Furthermore, we show that external borrowing reduces expected profits relative to wages in the economy, and thus discourages entrepreneurial activity, the aggregate innovation rate and the pace of the overall productivity convergence. Trade surpluses, whether market or policy-driven, have the opposite effect, and result in a higher rate of innovation with a tilt towards the tradable sector and away from the non-tradable sector. This prediction of the model provides an economic rationale for the "allocation puzzle" in international capital flows documented by Gourinchas and Jeanne (2013, see Figure 1). Namely, they documented that capital inflows correlated negatively with aggregate productivity growth in the cross-section of developing countries, in contrast with the prediction of neoclassical growth theory, which emphasizes international borrowing for capital deepening.

We further show that the ratio of net exports to domestic absorption is a sufficient statistic for the feedback effect from aggregate economic outcomes into sectoral productivity growth. In other words, once the trade deficit is controlled for, other macroeconomic outcomes such as the aggregate unit labor costs and the real exchange rate have no residual predictive ability for the patterns of sectoral productivity growth. In addition, the nature of trade deficits — whether they are equilibrium outcomes or policy induced — is not essential for this result. Based on this result, we derive a reduced-form relationship between sectoral productivity growth and the degree of sectoral tradability interacted with the economy-wide trade balance-to-absorption ratio, additionally controlling for the initial productivity level in the sector. We test for this relationship in the panel of sectoral productivity growth rates across OECD countries. Consistent with the predictions of the theory, we find a sizable feedback effect from current account surpluses into the increased relative pace of productivity growth in the more tradable sectors, and vice versa, after controlling for country and industry fixed effects.

We use the model to study the efficiency properties of the convergence growth trajectories, as well as consider a number of extensions and applications of the baseline model. In particular, we study the consequences of a sudden stop in the financial flows during the borrowing phase of the transition. A sudden stop results in a reversal of the trade deficit, which is associated with an abrupt drop in local demand, a recession in the non-tradable sector, and a fast productivity growth take-off in the tradable sector, provided the labor market can adjust flexibly via a sharp decline in the wage rate.

Our interest in the consumption-led growth is motivated by the recent experience in the Euro Zone, where large current account imbalances between Southern and Northern Europe in the 2000s were associated with large shifts in the employment allocation towards the non-tradable sectors in the South [see figure 1 here]. This was followed by a steep and long-lasting recession in the aftermath of the 2008-09 global financial crisis, hitting Southern Europe particularly hard. The question we are interested in is whether the growth experience in the Southern Europe was affected by the current account deficits, and whether the steepness of the recession was in part due to the growth patterns in the early and mid 2000s.

Related literature The neoclassical analysis of growth in an economy open to financial flows was carried out by Barro, Mankiw, and Sala-i-Martin (1995): a country on a convergence trajectory borrows from the rest of the world to build-up its capital stock, which results in a faster neoclassical convergence. Our baseline model abstract from capital formation (see Section 6) and focuses on the capital inflows that finance consumption rather than investment, arguably a relevant description of the current account deficits in the Southern Europe in 2000s (see Blanchard and Giavazzi 2002).

The alternative mechanisms of the relationship between economy openness and growth were studied in Rivera-Batiz and Romer (1991), Ventura (1997), Acemoglu and Ventura (2002), Parente and Prescott (2002) , Aoki, Benigno, and Kiyotaki (2009). The empirical studies of the relationship between trade openness and growth include Frankel and Romer (1999), Ben-David (1993), Dollar and Kraay (2003). The patterns of financial flows and growth are studied in Gourinchas and Jeanne (2006, 2013), Alfaro, Kalemli-Özcan, and Volosovych (2008) and Gopinath, Kalemli-Özcan, Karabarbounis, and Villegas-Sanchez (2017).

Our model of endogenous convergence growth shares some common features with the models of learning-by-doing and the Dutch disease: see Corden and Neary (1982), Krugman (1987), Young (1991) and the large literature that followed (see recently Korinek and Serven 2010, Benigno and Fornaro 2012, 2014). Our environment is, however, different from an environment with technological externalities and increasing returns, and is instead characterized by decreasing returns from innovation. Alternative arguments for export-led growth are provided in Rajan and Subramanian (2005) and other papers (see also Rodrik 2008). Optimal development policies in an economy with financial frictions are studied in Itskhoki and Moll (2014). See also Song, Storesletten, and Zilibotti (2011).

Our model is built on the ideas from the endogenous growth literature (see Aghion and Howitt book), as well as the literature on firm dynamics, trade and growth (in particular, Kortum 1997, Eaton and Kortum 2001, 2002, Klette and Kortum 2004). The efficiency properties of growth models are studied in Matsuyama (1995). The models of endogenous growth and trade include: Grossman and Helpman (1993), Atkeson and Burstein (2010), Alvarez, Buera, and Lucas (2017), Perla, Tonetti, and Waugh (2015), Buera and Oberfield (2017).

[TO BE COMPLETED]

2 Model Setup

Consider a real small open economy in continuous time, which faces an exogenous world interest rate r^* and a world price of an international traded good basket $P_F^* \equiv 1$, which is used as a numeraire. The rest of the world is a continuum of small developed countries with wage and technology given by $W^* = A_T^* = A_N^* = A^*$, resulting in prices $P_F^* = P_N^* = P^* = 1$. The small open economy under consideration is on a catch-up growth trajectory, with initial productivity levels $A_T(0), A_N(0) < A^*$. We study the endogenous convergence paths of the tradable and non-tradable productivities under open and closed trade balance and current account.

2.1 Environment

We start by describing the economic environment conditional on the path of technology, which we endogenize in the following subsection.

Households The economy is populated by a representative household that maximizes present value of utility over consumption and labor:

$$\int_0^\infty \beta^t u(C_t, L_t)\mathrm{d}t, \qquad \text{where} \qquad u(C,L) = \frac{1}{1+\sigma}C^{1-\sigma} - \frac{\psi}{1+\varphi}L^{1+\varphi},$$

subject to a flow budget constraint

$$\dot{B}_t = r^* B_t + (W_t L_t + \Pi_t - P_t C_t), \tag{1}$$

where B_t is an instantaneous real bond paying out in units of the world tradable good, Π_t are aggregate dividends (profits), and W_t is the wage rate. The optimal labor supply satisfies:

$$\psi C_t^\sigma L_t^\varphi = \frac{W_t}{P_t}. \tag{2}$$

The relative risk aversion is $\sigma \geq 0$ and the inverse Frisch elasticity is $\varphi \in [0, \infty]$, and we consider the various limiting cases. In particular, in the limit $\varphi = \infty$, labor supply is exogenously given by $\bar{L} = 1$. In what follows we normalize $\psi = 1$ for simplicity.

Demand The final consumption good C is a CES aggregator across a continuum of industries, a fraction $\gamma \in [0, 1]$ of which are internationally tradable and the remaining $(1 - \gamma)$ are non-tradable.[3] Specifically, we have:

$$C = \left[\gamma C_T^{\frac{\eta-1}{\eta}} + (1-\gamma) C_N^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}} \qquad \text{and} \qquad C_T = \left[\kappa^{\frac{1}{\rho}} C_F^{\frac{\rho-1}{\rho}} + (1-\kappa)^{\frac{1}{\rho}} C_H^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho-1}{\rho}},$$

where $\eta \in [0, \rho)$ is the elasticity of substitution between tradables and non-tradables, $\rho > 1$ is the elasticity of substitution between home and foreign varieties of tradables, and $\kappa \in [0, 1]$ captures the expenditure share on foreign-produced goods, with the limit $\kappa \to 1$ corresponding to the case without home bias (this structure is borrowed from Galí and Monacelli 2005). In what follows, our baseline case is Cobb-Douglas with $\eta = 1$, with the expenditure share on tradable varieties equal to γ.

The home-produced tradable and non-tradable bundles are CES aggregators across individual varieties (or industries):

$$C_H = \left[\frac{1}{\gamma}\int_0^{\Lambda_T} C_H(i)^{\frac{\rho-1}{\rho}} \mathrm{d}i\right]^{\frac{\rho}{\rho-1}} \qquad \text{and} \qquad C_N = \left[\frac{1}{1-\gamma}\int_0^{\Lambda_N} C_N(i)^{\frac{\rho-1}{\rho}} \mathrm{d}i\right]^{\frac{\rho}{\rho-1}},$$

where Λ_T and Λ_N are the measures of home tradable and non-tradable varieties. Note that the elasticity of substitution, ρ, is the same across all varieties, both domestically- and internationally-produced, an assumption we make for tractability.

The resulting price index is given by:

$$P = \left[\gamma P_T^{1-\eta} + (1-\gamma) P_N^{1-\eta}\right]^{\frac{1}{1-\eta}}, \quad \text{where} \quad P_T = \left[\kappa P_F^{1-\rho} + (1-\kappa) P_H^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{3}$$

is the price aggregator between home- and foreign-produced tradable varieties. We denote with P_H and P_N the *average* prices of home-produced tradables and non-tradables respectively:[4]

$$P_H = \left[\frac{1}{\gamma}\int_0^{\Lambda_T} P_H(i)^{1-\rho} \mathrm{d}i\right]^{\frac{1}{1-\rho}} \quad \text{and} \quad P_N = \left[\frac{1}{1-\gamma}\int_0^{\Lambda_N} P_N(i)^{1-\rho} \mathrm{d}i\right]^{\frac{1}{1-\rho}}. \tag{4}$$

[3] In general, γ controls the expenditure share on tradables, and exactly equals it in the Cobb-Douglas case ($\eta = 1$). In the other special case of a single-tier utility ($\eta = \rho$), γ correspond to the fraction of tradable industries by count. See discussion in Appendix A.1.

[4] As we discuss below, we allow $\Lambda_T > \gamma$ and $\Lambda_N > 1 - \gamma$, and therefore P_H and P_N are *average* prices per unit of expenditure in the tradable and non-tradable sectors respectively (not per count of industries), while the *ideal* price indexes are given by γP_T and $(1-\gamma)P_N$ respectively. We adopt this formulation with average prices for symmetry between individual tradable and non-tradable sectors, and since it is not the conventional case, we provide an explicit derivation of the demand and price indexes in Appendix A.1.

Further, the domestic demand for the home-produced varieties is given by:

$$C_H(i) = (1-\kappa)\left(\frac{P_H(i)}{P_T}\right)^{-\rho}\left(\frac{P_T}{P}\right)^{-\eta} C \quad \text{and} \quad C_N(i) = \left(\frac{P_N(i)}{P_N}\right)^{-\rho}\left(\frac{P_N}{P}\right)^{-\eta} C. \tag{5}$$

and the total expenditure Y equals

$$\begin{aligned} Y \equiv PC &= \gamma(P_F C_F + P_H C_H) + (1-\gamma)P_N C_N \\ &= \gamma P_F C_F + \int_0^{\Lambda_T} P_H(i)C_H(i)\mathrm{d}i + \int_0^{\Lambda_N} P_N(i)C_N(i)\mathrm{d}i, \end{aligned}$$

where γ is the share of the tradable sectors and $P_H C_H$ is the average expenditure per tradable sector, and similarly for foreign tradable and home non-tradables. Finally, the total expenditure on the foreign-produced tradables, or *aggregate imports*, equals:

$$X^* = \gamma P_F C_F = \gamma\kappa\left(\frac{P_F}{P_T}\right)^{1-\rho}\left(\frac{P_T}{P}\right)^{1-\eta} Y. \tag{6}$$

In the presence of iceberg trade costs, $\tau \geq 1$, the price of the foreign tradable good in the home market is given by:[5]

$$P_F = \tau P_F^* = \tau.$$

Our baseline case assumes zero trade costs ($\tau = 1$), so that the home bias in consumption is fully driven by $\kappa < 1$ in preferences. This case is convenient as the counterfactual closed economy with $\kappa = 0$ and the open economy with $\kappa > 0$ with balanced trade have the same price level and welfare in the long run, as we explain below.

Net exports Lastly, the foreign-market demand for the domestically produced tradable varieties is given by:

$$C_H^*(i) = \kappa\left(\frac{\tau P_H(i)}{P_T^*}\right)^{-\rho}\left(\frac{P_T^*}{P^*}\right)^{-\eta} C^*, \tag{7}$$

where $P^* = P_T^* = 1$ and $Y^* = C^* = (A^*)^{\frac{1+\varphi}{\sigma+\varphi}}$ is the aggregate foreign expenditure (consumption). Therefore, the aggregate foreign expenditure on the home-produced tradables, or *aggregate exports* of home, equals:

$$X = \gamma P_H^* C_H^* = \gamma\kappa(\tau P_H)^{1-\rho} Y^*. \tag{8}$$

[5] The $\tau \geq 1$ iceberg trade cost is specific to the small open economy in question. The rest of the world is fully integrated and faces no trade costs ($\tau^* = 1$) on internal trade flows, which is why the assumption that $P^* = P_T^* = 1$ is internally consistent.

The home net exports is hence given by:

$$NX = X - X^* = \gamma\kappa\tau^{1-\rho}\Big[P_H^{1-\rho}Y^* - P_T^{\rho-\eta}P^{\eta-1}Y\Big], \tag{9}$$

which can be expressed in terms of relative demand Y/Y^*, terms of trade $\mathcal{S} \equiv P_F/P_H = 1/P_H$ and real exchange rate $\mathcal{Q} \equiv P^*/P = 1/P$.[6]

Technology and revenues The home tradable varieties $i \in [0, \Lambda_T]$ have access to technologies $\{Z_T(i)\}$ and a linear production function $Y_T(i) = Z_T(i)L$, where L is the industry labor input.[7] The output of the tradable variety is split between the home- and foreign-market sales: $Y_T(i) = C_H(i) + C_H^*(i)$. If access to the technology in sector i is non-excludable (i.e., all home agents have access to it), then pricing is competitive and equal to the marginal cost:

$$P_H(i) = \frac{W}{Z_T(i)}.$$

Otherwise, if the technology is privately owned, the optimal markup price is given by:

$$P_H(i) = \frac{\rho}{\rho-1}\frac{W}{Z_T(i)},$$

for both home and foreign markets. The non-tradable varieties $i \in [0, \Lambda_N]$ are symmetric with available technologies $\{Z_N(i)\}$, but non-tradables can be marketed only at home and do not directly compete with foreign varieties.

Furthermore, a home non-tradable firm that sets price $P_N(i)$ generates revenues:

$$R_N(i) = P_N(i)C_N(i) = \left(\frac{P_N(i)}{P_N}\right)^{1-\rho} R_N, \qquad \text{where} \qquad R_N = P_N C_N = \left(\frac{P_N}{P}\right)^{1-\eta} Y$$

is a non-tradable demand (or revenue) shifter. In contrast, a home tradable firm that sets producer price $P_H(i)$ for sales at home and abroad generates total revenues:

$$R_T(i) = P_H(i)C_H(i) + \tau P_H(i)C_H^*(i) = \left(\frac{P_H(i)}{P_H}\right)^{1-\rho} R_T,$$

where

$$R_T = (1-\kappa)\left(\frac{P_H}{P_T}\right)^{1-\rho}\left(\frac{P_T}{P}\right)^{1-\eta} Y + \kappa\left(\tau P_H\right)^{1-\rho} Y^*.$$

is a revenue shifter for tradable products. Relative to autarky (with $\kappa = 0$), the revenues of

[6]In particular, we can rewrite (9) as $\frac{NX}{Y} = \gamma\kappa\tau^{1-\rho}\Big[\mathcal{S}^{\rho-1}\frac{Y^*}{Y} - \mathcal{Q}^{1-\eta}\left[\kappa\tau^{1-\rho} + (1-\kappa)\mathcal{S}^{\rho-1}\right]^{\frac{\rho-\eta}{1-\rho}}\Big]$.

[7]In Section XX, we generalize the technology to be CRS in labor and capital…

a tradable variety are lower in the home market due to competition from foreign tradables, as $(1-\kappa)(P_H/P_T)^{1-\rho} < 1$, but are higher because of the additional access to the foreign market Y^*. Trade balance (9) imposes a tight relationship between these two effects, as we will see below.

Next, we look at the profits from a privately owned technology in a tradable versus non-tradable sector:

$$\Pi_N(i) = \frac{1}{\rho}R_N(i) = \frac{1}{\rho}\left(\frac{\rho}{\rho-1}\frac{W}{P_N Z_N(i)}\right)^{1-\rho} R_N, \tag{10}$$

$$\Pi_T(i) = \frac{1}{\rho}R_T(i) = \frac{1}{\rho}\left(\frac{\rho}{\rho-1}\frac{W}{P_H Z_T(i)}\right)^{1-\rho} R_T, \tag{11}$$

where $\rho/(\rho-1)$ is the optimal markup over the marginal cost and hence $1/\rho$ is the share of profits in revenues.

The relative profits in the two sectors are hence given by:

$$\frac{\Pi_T(i)}{\Pi_N(i)} = \chi\cdot\left(\frac{Z_T(i)}{Z_N(i)}\right)^{\rho-1}, \qquad \text{where} \qquad \chi \equiv \left(\frac{P_H}{P_N}\right)^{\rho-1}\frac{R_T}{R_N}. \tag{12}$$

The relative profits are higher in the tradable sector, the higher is relative idiosyncratic productivity $Z_T(i)/Z_N(i)$, the higher is the relative price of domestic competitors P_H/P_N (i.e., less local competition), and the higher is the relative revenue shifter R_T/R_N. The relative revenue shifter, in turn, depends on both the relative size of the foreign and home markets (the Y^*/Y term, adjusted for price levels) and on the tightness of the competition from foreign tradable firms (P_T/P_H):

$$\frac{R_T}{R_N} = \left(\frac{P_T}{P_N}\right)^{1-\eta}\cdot\underbrace{\left(\frac{P_T}{P_H}\right)^{\rho-1}\cdot\left[(1-\kappa)+\kappa\cdot\frac{\tau^{1-\rho}\,Y^*}{P_T^{\rho-\eta}P^{\eta-1}Y}\right]}_{\gtreqless 1 \text{ iff } NX \gtreqless 0 \text{ by Lemma 1}}. \tag{13}$$

In addition, when $\eta \neq 1$, this ratio depends on P_T/P_N, as households endogenously shift expenditure across sectors. We now show that the last two terms cancel out under balanced trade:

Lemma 1 (i) *Under balanced trade ($NX=0$), the relative revenue shifter of tradable and non-tradable industries $R_T/R_N = \big(P_T/P_N\big)^{1-\eta}$, and the relative profit shifter $\chi = \big(\frac{P_T}{P_H}\big)^{1-\eta}\big(\frac{P_H}{P_N}\big)^{\rho-\eta}$.* (ii) *In the Cobb-Douglas case ($\eta=1$), this further simplifies to $R_T/R_N = 1$ and $\chi = (P_H/P_N)^{\rho-1}$, with $R_T/R_N > 1$ iff $NX > 0$, and vice versa.*

Proof: From (9), trade balance $NX = 0$ implies $P_H^{1-\rho}Y^* = P_T^{\rho-\eta}P^{\eta-1}Y$. Intuitively, a productivity differential reflected in relative income Y/Y^* requires an adjustment in the terms of trade (i.e., prices of home tradables P_H, which determine P_T and P) to ensure trade balance.[8] Substituting this relationship into the expression for R_T, we have:

$$R_T = \left[(1-\kappa)P_H^{1-\rho} + \kappa\tau^{1-\rho}\right] P_T^{\rho-\eta} P^{\eta-1} Y = \left(\frac{P_T}{P}\right)^{1-\eta} Y,$$

where the second equality obtains from the definition of P_T in (3) given that $P_F = \tau$. Dividing by R_N yields the result for R_T/R_N, and the results for χ follow from its definition in (12).

Now in the general case with $NX \neq 0$, the expression in (13) can be simplified using (9):

$$\frac{R_T}{R_N} = \left(\frac{P_T}{P_N}\right)^{1-\eta} \left[1 + \kappa \left(\frac{\tau}{P_T}\right)^{1-\rho} \frac{NX}{X^*}\right] = \left(\frac{P_T}{P_N}\right)^{1-\eta} \left[1 + \left(\frac{P_T}{P}\right)^{\eta-1} \frac{NX}{\gamma Y}\right], \tag{14}$$

where $X^* > 0$ is aggregate imports, and using the CES demand (6) property that

$$\gamma\kappa \left(\frac{\tau}{P_T}\right)^{1-\rho} = \left(\frac{P_T}{P}\right)^{\eta-1} \frac{X^*}{Y}.$$

Hence, when $\eta = 1$, $\frac{R_T}{R_N} - 1$ has the same sign as NX. ■

Lastly, we define aggregate (average) sectoral productivity levels as:

$$A_T = \left[\frac{1}{\gamma}\int_0^{\Lambda_T} Z_T(i)^{\rho-1} \mathrm{d}i\right]^{\frac{1}{\rho-1}} \quad \text{and} \quad A_N = \left[\frac{1}{1-\gamma}\int_0^{\Lambda_N} Z_N(i)^{\rho-1} \mathrm{d}i\right]^{\frac{1}{\rho-1}}, \tag{15}$$

as well as the aggregate productivity level in parallel with the aggregate price index in (3):

$$A = \left[\gamma A_T^{\eta-1} + (1-\gamma)A_N^{\eta-1}\right]^{\frac{1}{\eta-1}}, \tag{16}$$

which in the Cobb-Douglas limit simplifies to $A = A_T^{\gamma} A_N^{1-\gamma}$.

As explained below, at each instant only a measure zero of industries have privately owned technology, while almost all industries have non-excludable technology and competitive pricing. As a result, the aggregate profits are zero $\Pi = 0$ and the average price levels for the home-produced tradable and non-tradable baskets are given by:

$$P_H = \frac{W}{A_T} \qquad \text{and} \qquad P_N = \frac{W}{A_N}. \tag{17}$$

[8] Formally, in this small open economy, the terms of trade equal $\mathcal{S} = P_H$ and the real exchange rate is $\mathcal{Q} = P$. Note that the tradable price level P_T is determined by $\mathcal{S}$, while the overall price level P, and hence the real exchange rate, also depend on the relative price of non-tradables (as can be seen from (3)).

Substituting P_H into the expression for tradable price index in (3), we have:

$$P_T = \left[\kappa\tau^{1-\rho} + (1-\kappa)(W/A_T)^{1-\rho}\right]^{1/(1-\rho)}.$$

Finally, we combine this with the definition of the price index in (3) to obtain:

$$P = W\Big/\left[\left[\kappa\big(W/(\tau A_T)\big)^{\rho-1} + (1-\kappa)\right]^{\frac{\eta-1}{\rho-1}}\gamma A_T^{\eta-1} + (1-\gamma)A_N^{\eta-1}\right]^{\frac{1}{\eta-1}}, \tag{18}$$

which also defines an equilibrium relationship between real wage $w \equiv W/P$, country productivity (A_T, A_N) and unit labor cost W/A_T. Note that in the counterfactual autarky economy with $\kappa = 0$, expression (18) simplifies to $w = A$, and real wage always equals aggregate productivity. This is not generally the case in an open economy.

Market clearing and static equilibrium In equilibrium, labor and product markets must clear. Both of these equilibrium requirements can be summarized by the following aggregate condition (see Appendix A.2):

$$Y + NX = WL. \tag{19}$$

That is, total home expenditure $Y = PC$ plus net exports NX equals total home income WL.

Furthermore, under balanced trade $NX = 0$, market clearing requires $Y = PC = WL$, which together with labor supply condition (2) implies $C = (W/P)^{\frac{1+\varphi}{\sigma+\varphi}}$. Substituting this solution into $Y = PC$, and further into the definition of NX (9), trade balance pins down the equilibrium value of the wage rate W, as well as for the other variables (C, L, Y, P).

Outside the case of financial autarky, with $NX \neq 0$ in general, the equilibrium allocation (W, P, C, L, Y) depends on the value of NX, and is continuous in it. In Appendix A.3, we provide a log-linearized solution for static equilibrium as a function of the endogenous trade surplus, NX. The intertemporal budget constraint requires that

$$B(0) + \int_0^\infty e^{-r^*t} NX(t)\mathrm{d}t = 0. \tag{20}$$

2.2 Technology adoption and evolution

The aggregate productivity state at time t is characterize by a quadruplet $\mathbf{S}(t)$ with $\mathbf{S} = (\Lambda_T, \Lambda_N, A_T, A_N)$, where Λ_J is the set of available varieties in sector $J \in \{T, N\}$ and A_J is the corresponding level of aggregate technology, which obtains from idiosyncratic industry productivities $\mathbf{Z}_J = \{Z_J(i)\}_{i\in[0,\Lambda_J]}$ according to (15). We now describe the endogenous dynamics of the aggregate productivity state $\mathbf{S}$, its steady state, as well as the cross-sectional distribution of productivities $\mathbf{Z} = (\mathbf{Z}_T, \mathbf{Z}_N)$.

Over a short period of time $\mathrm{d}t$, a mass $\lambda \mathrm{d}t$ of entrepreneurs receive new ideas. For now we treat λ as a parameter, and we make it an endogenous equilibrium outcome in Section 4.3. Each entrepreneur has n ideas, which correspond to n potential new varieties, a fraction γ of which are tradable and the remaining $(1-\gamma)n$ are non-tradable.[9] Entrepreneurs observe the tradability $J(\ell) \in \{T, N\}$ for each idea $\ell \in \{1..n\}$, as well as its productivity $Z_{J(\ell)}(\ell)$. Each entrepreneur can select only one idea $\hat{\ell}$ to implement as a new variety. The technology for the new variety is proprietary for one period, and becomes common knowledge next period. (A period in the continuous-time model is an instant.) The entrepreneur can, thus, receive an instantaneous profit $\Pi_{J(\ell)}(\ell)$, and chooses the project for implementation according to:

$$\hat{\ell} = \arg \max_{\ell \in \{1..n\}} \Pi_{J(\ell)}(\ell). \tag{21}$$

We denote $\hat{J} = J(\hat{\ell})$, $\hat{Z} = Z_{\hat{J}}(\hat{\ell})$ and $\hat{\Pi} = \Pi_{\hat{J}}(\hat{\ell})$. We also denote

$$\hat{Z}_J = \max_{\ell \,:\, J(\ell) = J} Z_{J(\ell)}(\ell) \tag{22}$$

the best idea productivity-wise among the entrepreneur's ideas within each sector $J \in \{T, N\}$. Furthermore, we use the notation $\hat{\Pi}_J$ for the profit of the best project in sector J, which is calculated according to (10)–(11) given best productivity options $\hat{Z}_J$.

Lastly, we assume that the productivity of all new ideas are drawn randomly from a Frechet distribution:

$$Z_{J(\ell)}(\ell) \sim iid\, Frechet(z, \theta), \tag{23}$$

where $z > 0$ is the location (mean) parameter and $\theta > \max\{\rho - 1, 1\}$ is the shape (dispersion) parameter. That is, the cdf of the productivity draws is given by $F(Z) = \exp\big\{ - z \cdot Z^{-\theta} \big\}$. With this distributional assumption, we have the following result:[10]

[9] In the model, nz is a sufficient statistic (where z is the Frechet parameter in (23)), with all outcomes isomorphic under various combinations of n and z. An interesting limit is $n \to \infty$ and $z \to 0$ such that $nz = const$, in which case entrepreneurs have a large number of ideas with most of them unproductive. In this limit, the Frechet distribution for $\hat{Z}_J$ in (22) arises endogenously independently of the primitive distributional assumption for individual productivity draws.

[10] See Appendix A.4 for a discussion of the useful Frechet properties that imply the results in Lemmas 2–3. In particular, maximum over n iid $Frechet(z, \theta)$ is $Frechet(nz, \theta)$, and linear and power transformation of Frechet are also Frechet. Lastly, a conditional distribution of $Frechet(z_1, \theta)$ conditional on being bigger than another independent $Frechet(z_2, \theta)$ is $Frechet(z_1 + z_2, \theta)$.

Lemma 2 *With the selection criterion* (21) *and under the distributional assumption* (23)*, the probability that an entrepreneur adopts a tradable project, i.e. that* $\hat{\Pi} = \hat{\Pi}_T > \hat{\Pi}_N$*, equals:*

$$\pi_T \equiv \mathbb{P}\{\hat{\Pi}_T > \hat{\Pi}_N\} = \frac{\gamma \cdot \chi^{\frac{\theta}{\rho-1}}}{\gamma \cdot \chi^{\frac{\theta}{\rho-1}} + (1-\gamma)}, \tag{24}$$

where χ *is the relative profit shifter defined in* (12)*.*

The formal proof is contained in Appendix A.4, where we show that both $\hat{\Pi}_T$ and $\hat{\Pi}_N$ are distributed Frechet with shape parameter $\frac{\theta}{\rho-1}$ and with the ratio of the location parameters given by $\frac{\gamma}{1-\gamma} \cdot \chi^{\frac{\theta}{\rho-1}}$. Then, expression (24) for the adoption probability π_T follows from the Eaton and Kortum (2002) algebra. Intuitively, the probability of adopting a tradable project depends on the relative frequency of tradable ideas $\frac{\gamma}{1-\gamma}$ and the relative profitability of the tradable projects χ. The probability of tradable-project adoption can be higher or lower than γ depending on whether χ is higher or lower than 1.

We further assume that existing technologies die (or become obsolete) at rate $\delta \geq \lambda$, and thus the dynamics of the available set of varieties is given by:

$$\dot{\Lambda}_T = \lambda\pi_T - \delta\Lambda_T \qquad \text{and} \qquad \dot{\Lambda}_N = \lambda(1-\pi_T) - \delta\Lambda_N, \tag{25}$$

where π_T evolves endogenously over time. Our baseline parametric assumption is $\lambda = \delta$, which as we will see below allows the small open economy to converge with the world technology frontier. Alternatively, if $\lambda < \delta$, the country will stay strictly within the world technological frontier.[11]

Lastly, we characterize the evolution of the aggregate productivities in each sector:

Lemma 3 *The aggregate productivity dynamics is given by:*

$$\frac{\dot{A}_T}{A_T} = \frac{1}{\rho-1}\left[\lambda\left(\frac{\pi_T}{\gamma}\right)^{\nu}\left(\frac{A_T}{A^*}\right)^{1-\rho} - \delta\right], \tag{26}$$

$$\frac{\dot{A}_N}{A_N} = \frac{1}{\rho-1}\left[\lambda\left(\frac{1-\pi_T}{1-\gamma}\right)^{\nu}\left(\frac{A_N}{A^*}\right)^{1-\rho} - \delta\right], \tag{27}$$

where $\nu \equiv 1 - \frac{\rho-1}{\theta} \in (0,1)$*,* $A^* \equiv (nz)^{\frac{1}{\theta}}\Gamma(\nu)^{\frac{1}{\rho-1}}$ *is the world technological frontier, and* $\Gamma(\nu) \equiv \int_0^\infty x^{\nu-1}e^{-x}\mathrm{d}x$ *is the Gamma-function.*

[11] Indeed, we interpret δ as the global rate at which any technology in the world economy becomes obsolete, while λ is the country-specific entrepreneurial capacity. Only countries with $\lambda = \delta$ can catch up with the world frontier, while countries with $\lambda < \delta$ permanently lag behind by a constant factor, as we discuss below.

We again provide the formal proof in Appendix A.4, and offer here an intuitive discussion. From the definition of the average productivity in (15), it follows that over a short interval $\mathrm{d}t$:

$$\frac{\mathrm{d}A_T}{A_T} = \frac{A_T^{1-\rho}}{\rho-1} \cdot \left[\lambda \pi_T \frac{1}{\gamma} \mathbb{E}\Big\{ \hat{Z}_T^{\rho-1} \big| \hat{\Pi}_T > \hat{\Pi}_N \Big\} - \delta A_T^{\rho-1} \right] \cdot \mathrm{d}t.$$

Indeed, the new tradable projects arrive at rate $\lambda\pi_T$ and have the conditional distribution $\hat{Z}_T\big|_{\hat{\Pi}_T > \hat{\Pi}_N}$, while the existing project die randomly from the existing stock with mean A_T at rate δ. The imperfect substitutability between technologies (with elasticity $\rho > 1$) implies that the impact of these arrivals and departures is amplified by the power $\rho - 1$ and discounted by the existing level of $A_T^{\rho-1}$. Lastly, using the properties of the Frechet distribution, the conditional expectation of the productivity of the new technology arrivals is $\mathbb{E}\big\{ \hat{Z}_T^{\rho-1} \big| \hat{\Pi}_T > \hat{\Pi}_N \big\} = \big(\frac{\gamma}{\pi_T}\big)^{\frac{\rho-1}{\theta}} (A^*)^{\rho-1}$, with A^* as defined in Lemma 3. From here, taking the limit $\mathrm{d}t \to 0$ results in (26), and similar steps for the non-tradable sector result in (27).

We note here three implications of Lemma 3. First, under imperfect substitutability $\rho > 1$, the productivity dynamics is subject to decreasing returns, as the productivity levels increase towards A^*. Second, the effect on productivity of the death of projects is proportional to A_T, rather than A^*, which is the second convergence force in the model. Lastly, a greater probability of adopting a tradable project ($\pi_T > \gamma$, which recall from (24) follows from $\chi > 1$) accelerates the convergence in the tradable sector, and decelerates it in the non-tradable sector. This effect, however, is less than proportional, as $\nu < 1$, reflecting the fact that increasing sectoral adoption on extensive margin lowers the productivity of the marginally adopted projects in the sector.

Equations (24)–(27) characterize the evolution of the aggregate productivity state of the economy in both sectors. This completes the description of the model economy. The key feature of this economy is that the cross sectional allocation at time t has a feedback effect on the trajectory of the technology evolution, via the relative profit shifter $\chi(t)$. We now study how various international trade regimes in goods and assets affect the equilibrium catch-up growth trajectories in this small open economy.

3 Closed Economy Benchmark

We consider here a counterfactual closed economy with $\kappa = 0$. In this economy, tradable and non-tradable sectors are symmetric, and the transition dynamics is still characterized by the dynamic system (25)–(27) with π_T given by (24), which satisfies:

$$\chi = \left(\frac{\pi_T}{1-\pi_T} \frac{1-\gamma}{\gamma} \right)^{1-\nu} = \left(\frac{P_T}{P_N} \right)^{\rho-1} \left(\frac{P_T}{P_N} \right)^{1-\eta} = \left(\frac{A_N}{A_T} \right)^{\rho-\eta}, \tag{28}$$

where the second equality comes from (12) and the third equality follows from the fact that $\kappa = 0$ implies $P_T = P_H = W/A_T$ and $P_N = W/A_N$ (by (3) and (17)). We solve for:

$$\pi_T = \frac{\gamma \left(A_N/A_T\right)^{\theta \frac{\rho-\eta}{\rho-1}}}{\gamma \left(A_N/A_T\right)^{\theta \frac{\rho-\eta}{\rho-1}} + (1-\gamma)}, \tag{29}$$

such that $\pi_T = \gamma$ iff $A_T = A_N$, and π_T is a decreasing function of A_T/A_N, under our parameter restriction $\eta < \rho$.[12] A productivity advantage in a given sector reduces the relative price of this sector, driving up the competition for new projects and lowering their profits. This discourages entry in this sector.

We can now characterize the equilibrium productivity dynamics in the closed economy:

Proposition 1 (i) *Starting from a symmetric initial state $A_T(0) = A_N(0)$, the equilibrium project selection probability $\pi_T(t) = \gamma$ for all $t \geq 0$, and the productivity path in both sectors $J \in \{T, N\}$ is given by:*

$$A_J(t) = \left[e^{-\delta t} A_J(0)^{\rho-1} + \left(1 - e^{-\delta t}\right)\bar{A}^{\rho-1}\right]^{\frac{1}{\rho-1}}, \qquad \textit{where} \qquad \bar{A} \equiv A^* \cdot \left(\frac{\lambda}{\delta}\right)^{\frac{1}{\rho-1}} \tag{30}$$

is the steady state level of productivity in both sectors. (ii) *When $A_N(0) > A_T(0)$, the equilibrium project selection satisfies* (29) *and features $\pi_T(t) > \gamma$ and $A_N(t) > A_T(t)$ for all $t \geq 0$, and vice versa. Independently of the initial conditions, the steady state productivity levels are $\bar{A}$ in both sectors, and steady state project choice is $\bar{\pi}_T = \gamma$.*

Proof: From (29), $A_T(0) = A_N(0)$ implies $\pi_T(0) = \gamma$, which in turn implies $\dot{A}_T(0) = \dot{A}_N(0)$ from the productivity dynamics system (26)–(27). By induction, this implies $A_T(t) = A_N(t)$, and hence $\pi_T(t) = \gamma$, for all $t \geq 0$. Under these circumstances, the law of motion for productivity in each sector is given by an autonomous ODE:

$$\frac{\dot{A}_J(t)}{A_J(t)} = \frac{\delta}{\rho - 1}\left[\left(\frac{A_J(t)}{\bar{A}}\right)^{1-\rho} - 1\right], \tag{31}$$

which the solution given by (30).

According to (29), $A_N(t) > A_T(t)$ implies $\pi_T(t) > \gamma$, which from (26)–(27) implies $\dot{A}_T(t)/A_T(t) > \dot{A}_N(t)/A_N(t)$, and an asymptotic convergence of the two sectoral productivities. Indeed, the convergence cannot happen in finite time. If it did happen at some finite $s < \infty$, then $A_T(s) = A_N(s)$ and $\pi_T(s) = \gamma$ imply $\dot{A}_T(s) = \dot{A}_N(s)$, constituting a contradiction that s is the instance of convergence (by rolling time backwards). Therefore, we must

[12] Note that with $\eta = \rho$, we have $\pi_T = \gamma$ independently of the productivity levels. For $\eta > \rho$, the project selection favors the sector with an existing productivity advantage.

have $A_N(t) > A_T(t)$ and $\pi_T(t) > \gamma$ for all $t \geq 0$. The law of motion for the productivity vector is an autonomous ODE system given by:

$$\frac{\dot{A}_T(t)}{A_T(t)} = \frac{\delta}{\rho - 1}\left[\frac{\Big(A_T(t)/\bar{A}\Big)^{1-\rho}}{\Big[\gamma + (1-\gamma)(A_T(t)/A_N(t))^{\theta\frac{\rho-\eta}{\rho-1}}\Big]^{1-\nu}} - 1\right], \tag{32}$$

with a symmetric equation for $\dot{A}_N(t)$, which implies that $\bar{A}$ is the steady state level of productivity in both sectors. We illustrate this equilibrium transition dynamic in Figure 2. ■

We make three remarks regarding Proposition 1. First, the rate of productivity convergence in (31) is shaped only by δ and ρ, while the steady state productivity level is additionally shaped by λ. Second, the small open economy converges to the world technological frontier A^* in both sectors if $\lambda = \delta$, and lags behind it even in the long run if $\lambda < \delta$. Recall that δ is the world-wide death rate of existing technologies, while λ is the country-specific capacity for entrepreneurship. Lastly, note that the steady state measures of home-produced varieties in the two sectors are given by:

$$\bar{\Lambda}_T = \gamma\frac{\lambda}{\delta} \qquad \text{and} \qquad \bar{\Lambda}_N = (1-\gamma)\frac{\lambda}{\delta},$$

so that $\bar{\Lambda}_T = \gamma$ and $\bar{\Lambda}_N = 1-\gamma$ (with $\bar{\Lambda} = \bar{\Lambda}_T + \bar{\Lambda}_N = 1$) if $\lambda = \delta$, and strictly less otherwise.[13]

Efficiency of the transition

We close this section with a discussion of the efficiency property of the equilibrium path of productivity. In closed economy, the path of the aggregate productivity level A is a *sufficient statistic* for aggregate allocation and welfare. Indeed, normalizing the price level $P = 1$, we have $W = A$, as well as:

$$C = w^{\frac{1+\varphi}{\sigma+\varphi}}, \qquad L = w^{\frac{1-\sigma}{\sigma+\varphi}} \qquad \text{where} \qquad w \equiv \frac{W}{P} = A \tag{33}$$

is the real wage, and the flow utility is given by $u(C,L) = \frac{\sigma+\varphi}{(1-\sigma)(1+\varphi)} A^{\frac{(1-\sigma)(1+\varphi)}{\sigma+\varphi}}$. The derivation is in Appendix A.5, where we further prove the following efficiency result for the equilibrium

[13]Furthermore, if the initial productivities at $t = 0$ satisfy

$$A_T(t) = A^* \cdot \left(\frac{\Lambda_T(t)}{\gamma}\right)^{\frac{1}{\rho-1}} = A_N(t) = A^* \cdot \left(\frac{\Lambda_N(t)}{1-\gamma}\right)^{\frac{1}{\rho-1}},$$

then these relationships hold for all $t \geq 0$, otherwise they hold only asymptotically as $t \to \infty$.

path of project choice $\pi_T(t)$:[14]

Proposition 2 (i) *Suppose the initial productivity levels are the same, $A_T(0) = A_N(0)$. Then, sectoral productivity dynamics (31) with $A_T(t) = A_N(t)$ for all $t \geq 0$, which obtains under the project choice rule $\pi_T(t) \equiv \gamma$, maximizes the aggregate productivity level $A(t)$ at each point in time $t \geq 0$ and for any $\eta \in [0, \rho]$, and hence is also welfare maximizing in the closed economy.* (ii) *Next, suppose, for concreteness, $A_N(t) > A_T(t)$ at some $t \geq 0$. Then the optimal project choice $\pi_T^*(t) \in \big(\gamma, \pi_T(t)\big)$, and hence the laissez-faire dynamics with $\pi_T(t)$ defined by (28) is suboptimal in this case.*[15]

Intuitively, the tradable and non-tradable sectors are symmetric in the closed economy with aggregate productivity given by a concave aggregator of sectoral productivities. Setting $\pi_T = \gamma$ results in a symmetric evolution of productivities throughout the economy, which maximizes both the path of the aggregate productivity and welfare in the closed economy, starting from a symmetric initial state. We set up the closed economy to be symmetric and efficient with the purpose of illuminating the effects from economy openness, which we study next.

More generally, the laissez-faire dynamics is inefficient, as is the case with an asymmetric initial condition. This is because the private and the social benefits of innovation are not perfectly aligned. The planner would adopt a less aggressive project selection policy, closing down the productivity gap between sectors more gradually. The reason is that in the laissez-faire markets, the entrepreneurs are driven by current profits and do not internalize the negative spillover they have on the returns from future innovation in a given sector. Thus, current relative profits give too strong of an incentive for tilting the innovation efforts towards a lagging sector.[16]

[14] In this analysis we think of $\pi_T = \mathbb{P}\{(1+\varsigma)\hat{\Pi}_T > \hat{\Pi}_N\}$ as a control, where $\varsigma \in [-1, \infty)$ is a policy tool, which allows to span any $\pi_T \in [0, 1]$. The productivity dynamics in (26)–(27) as a function of π_T still applies.

[15] We can further show that $\pi_T^*(t)$ decreases over time towards γ in the long run, and also that $\pi_T^*(s) < \bar{\pi}_T(s)$, where $\pi_T(t) \equiv \bar{\pi}_T(s)$ for all $t \in [0, s]$ is the project choice rule that maximizes aggregate productivity at $t = s$, $A(s)$, as we explain in Appendix A.5.

[16] The planner can improve the allocation by giving patents with exclusivity rights to the innovators, with the optimal duration of patents changing along the convergence path. In particular, we can show that indefinite patents are not optimal in this model.

4 Catch-up Growth in a Small Open Economy

4.1 Closed capital account

First, we briefly consider the case in which our baseline small open economy with $\kappa > 0$ runs balanced trade each period, that is $NX(t) \equiv 0$ and hence $B(t) \equiv 0$. By Lemma 1, this implies

$$\chi = \left(\frac{P_T}{P_N}\right)^{1-\eta}\left(\frac{P_H}{P_N}\right)^{1-\rho} = \left[(1-\kappa) + \kappa\left(\tau\frac{A_T}{W}\right)^{1-\rho}\right]^{\frac{1-\eta}{1-\rho}} \cdot \left(\frac{A_N}{A_T}\right)^{\rho-\eta},$$

where we used the definition of the tradable price index in (3) and the expression for sectoral prices from (17). The difference with χ in the counterfactual closed economy is the square bracket in front, which is absent when either $\kappa = 0$, or $\eta = 1$. Indeed, static balanced trade changes the price of tradables by offering access to foreign varieties. This lowers the tradable price index $P_T < P_H$ iff $\tau < P_H = W/A_T$, i.e. when foreign tradables are cheaper than domestic tradables after adjusting for iceberg trade costs.[17] Outside the Cobb-Douglas case, when $\eta \neq 1$, this difference between P_T and P_H results in a reallocation of expenditure across sectors. This changes the relative profitability in the two sectors, affecting the incentive to adopt tradable versus non-tradable projects. In the Cobb-Douglas case, the expenditure shares are fixed, and this difference does not affect the equilibrium project selection.

We summarize the implications of this discussion in:

Proposition 3 (i) *In the Cobb-Douglas case ($\eta = 1$), the productivity dynamics in an open economy under closed capital account ($NX(t) \equiv 0$) is equivalent to the productivity dynamics in the closed economy, as described in Proposition 1. In particular, starting from $A_T(0) = A_N(0)$, the project choice is $\pi_T(t) = \gamma$ for all $t \geq 0$.* (ii) *With $\eta \neq 1$, $P_T(t) < P_H(t)$ leads to $\pi_T(t) < \gamma$ when $\eta < 1$ and to $\pi_T(t) > \gamma$ when $\eta > 1$, starting in a symmetric state $A_T(t) = A_N(t)$.*[18]

Despite the similarity in the laissez-faire productivity dynamics, the allocations in the open economy under balanced trade differ from those in the closed economy. In particular, from labor and product market clearing (2) and (19), we have:

$$C = w^{\frac{1+\varphi}{\sigma+\varphi}} \qquad \text{and} \qquad L = w^{\frac{1-\sigma}{\sigma+\varphi}}, \tag{34}$$

[17] Note that welfare cannot be directly compared across economies with different preferences parametrized by $\kappa \in [0, 1]$. Our counterfactual closed economy features $\kappa = 0$ (instead of $\tau = \infty$). Nonetheless, with $\tau = 1$, the closed economy and the open economy with balanced trade feature the same long-run productivity and welfare, which allows for a direct comparison across models in this case.

[18] $P_T(0) < P_H(0)$ is indeed the likely outcome, which obtains when τ is not very large. We provide an explicit condition for $P_T(0) < P_H(0)$ expressed in terms of primitives in Appendix A.2.

as before (see (33)), but now the real wage $w = W/P$ does not simply reflect the home productivity level A, but also depends on the terms of trade effects from international trade. We provide here a simple log-linear expression for the real wage, which obtains in the special case of log-Cobb-Douglas utility ($\sigma = \eta = 1$, i.e. the Cole and Obstfeld (1991) case):

$$w \equiv \frac{W}{P} = A \left(\tau^{1-2\rho} \frac{A^*}{A_T} \right)^{\frac{\kappa\gamma}{1+(2-\kappa)(\rho-1)}}, \tag{35}$$

and the general non-linear expression is provided in Appendix A.2.[19] We adopt the log-Cobb-Douglas case as a point of approximation not only for its tractability, but because it is an empirically reasonable point of approximation, with the plausible values of σ and η likely being in the neighborhood of 1. The economic effects are smooth and continuous in the values of parameters in this neighborhood.[20]

Note the expression in bracket in (35), which is exactly the adjustment for the terms of trade effect. When home lags behind in terms of tradable productivity $A_T < A^*$, the terms of trade have a beneficial effect on the real wage, provided that τ is not too high and $\rho < \infty$. As a result, both real wages and consumption increase not only in A_T and A_N, but also in A^*. In other words, an open economy benefits from international trade along the transition (by raising its consumption level), even when it has no access to international borrowing for consumption smoothing. We summarize this in:

Proposition 4 (i) *Both nominal and real wages are homogenous of degree one in* (A_T, A_N, A^*)*, necessarily increasing in* A_T *and* A^**, and decreasing in* τ*, and increasing in* A_N *iff* $\sigma > 1$. (ii) *When* $\sigma = \eta = 1$*,* $\tau \approx 1$ *and* $A_T(t), A_N(t) < A^*$*, the real wage and consumption satisfy:*

$$\min\{A_T(t), A_N(t)\} < w(t) < A^* \qquad \textit{and} \qquad C(t) = w(t) > A(t),$$

while the nominal wage $A_T(t) < W(t) < A^*$*.* (iii) *Laissez-faire productivity dynamics is suboptimal; in particular, starting from* $A_T(0) = A_N(0)$*, the welfare maximizing project choice is* $\pi_T(t) < \gamma$*, resulting in* $A_T(t) < A_N(t)$*, for all* t*.*

[19] In addition to $\sigma = \eta = 1$, this expression also uses a Cobb-Douglas approximation to the tradable price index P_T, which is accurate in the limits of full or no home bias (both $\kappa \to 0$ and $\kappa \to 1$), as well as for any κ when $\rho \gg 1$ and $\tau = 1$ (in this case, $W \to A_T$ as $\rho \to \infty$, and the approximation is exact when $W = \tau A_T$).

[20] Note that the elasticity of employment in terms of real wage is proportional to $1 - \sigma$, with income and substitution effects cancelling out when $\sigma = 1$. While the exact $\sigma = 1$ case is, of course, a simplification, and the elasticity is likely to be positive or negative, yet it is likely to be small (a point of approximation we adopt here).

Proof: Rewrite the trade balance equation (9) using $\sigma = \eta = 1$ as:

$$\frac{W}{A^*} = \left[(1-\kappa)\left(\frac{W}{\tau A_T}\right)^{1-\rho} + \kappa\right]\left(\frac{\tau W}{A_T}\right)^{1-\rho} \qquad \Rightarrow \qquad W \approx \left[\frac{A_T^{(2-\kappa)(\rho-1)} A^*}{\tau^{\kappa(\rho-1)}}\right]^{\frac{1}{1+(2-\kappa)(\rho-1)}},$$

where we used the definition of P_T (3), the expression for P_H (17), and the facts that when $\sigma = 1$, $Y^* = A^*$ and $Y = PC = WC/w = W$ (see (34)). This allows to solve for the nominal wage as a homogenous of degree one function in (A^*, A_T) and decreasing in τ. In Appendix A.2 we show that this result generalizes, as stated in the lemma part (i). The real wage follows from the definition of the price index (18) evaluated at $\eta \approx 1$:

$$w = \frac{W}{P} = A\left[1-\kappa+\kappa\left(\frac{W}{\tau A_T}\right)^{\rho-1}\right]^{\frac{\gamma}{\rho-1}} \approx A\left(\frac{W}{\tau A_T}\right)^{\kappa\gamma} = A\left[\frac{1}{\tau^{1+2(\rho-1)}}\frac{A^*}{A_T}\right]^{\frac{\kappa\gamma}{1+(2-\kappa)(\rho-1)}}.$$

The approximations in both expressions are exact as $\kappa \to 0$, $\kappa \to 1$, or $W/(\tau A_T) \to 1$, where the latter endogenous condition is satisfied for any $\kappa \in (0,1)$ when $\tau = 1$ and $\rho \to \infty$. These expressions imply the statements in part (ii) of the lemma.

The last part of the proposition follows by the same logic as the proof of Proposition 2 in Appendix A.5, after noting that the objective is no longer proportional to

$$\log A(t) = \gamma \log A_T(t) + (1-\gamma)\log A_N(t),$$

but is instead proportional to

$$\log w(t) = \gamma\frac{1-\kappa+(2-\kappa)(\rho-1)}{1+(2-\kappa)(\rho-1)}\log A_T(t) + (1-\gamma)\log A_N(t) - \Xi(t),$$

where $\Xi(t)$ is the term that does not depend on policy (but depends on A^* and τ). Here we specialized the expression to the special case with $\sigma = \eta = 1$, but the result is more general. In order to maximize $w(s)$ at any $s > 0$, the optimal control $\pi_T(t)$ is constant for $t \in [0, s]$, and equals $\pi_T(t) \equiv \hat{\pi}_T(s)$, which satisfies:

$$\left(\frac{\hat{\pi}_T(s)}{1-\hat{\pi}_T(s)}\frac{1-\gamma}{\gamma}\right)^{1-\nu} = \underbrace{\frac{1-\kappa+(2-\kappa)(\rho-1)}{1+(2-\kappa)(\rho-1)}}_{<1}\left(\frac{A_T(s)}{A_N(s)}\right)^{1-\rho},$$

from which it follows that $\hat{\pi}_T(s) < \gamma$ if $A_T(0)$ is not too small relative to $A_N(0)$, and in particular $A_T(0) \geq A_N(0)$ is sufficient, but not necessary. ■

To summarize the results of this section, the productivity dynamics in an economy with balanced international trade is the same as in the closed economy, when elasticity between tradables and non-tradables $\eta = 1$ (our baseline case). Access to international trade increases wages, consumption and welfare along the transition path, due to the positive terms-of-trade externality when trading with a more developed rest-of-the-world. The same terms-of-trade externality makes the symmetric laissez-faire productivity dynamics suboptimal, as the welfare maximizing government would tilt project choice away from the tradable sector to take a greater advantage of the terms-of-trade externality. Indeed, in an economy with balanced trade, the benefits of tradable productivity A_T improvement are less than proportional, as they induce a deterioration in the terms of trade, while the benefits from non-tradable productivity A_N improvement are still fully captured domestically (see (35)).[21]

4.2 Financial openness

We now study the productivity effects of capital account openness, when a country can borrow for consumption smoothing during the transition growth phase. In this section, we specialize the analysis to the Cobb-Douglas case ($\eta = 1$), and the results hold by continuity in the neighborhood of $\eta = 1$, which is the relevant range empirically.[22]

The result of Lemmas 1–3 apply, and the productivity dynamics still satisfies (26)–(27) with π_T pinned down by:

$$\left(\frac{\pi_T}{1-\pi_T}\frac{1-\gamma}{\gamma}\right)^{1-\nu} = \chi = \left(\frac{A_N}{A_T}\right)^{\rho-1}\left[1+\frac{NX}{\gamma Y}\right], \tag{36}$$

as follows from (12), (14) and (24). Note that $NX/(\gamma Y)$ is the ratio of net exports to the tradable expenditure (absorption) in the economy. Therefore, $\pi_T < \gamma$ whenever $\chi < 1$. Furthermore, trade deficit $NX < 0$ directly implies $\chi < (A_N/A_T)^{\rho-1}$, and vice versa. This allows us to prove the following important intermediate result:

Lemma 4 *Assume $\eta = 1$. Then $NX(t) < 0$ and $A_T(t) \geq A_N(t)$ imply $\dot{A}_T(t) < \dot{A}_N(t)$. Conversely, $NX(t) > 0$ and $A_T(t) \leq A_N(t)$ imply $\dot{A}_T(t) > \dot{A}_N(t)$.*

[21] A conjecture to be verified: if the terms-of-trade effect is internalized (or there is an optimal tariff in place), the symmetric laissez-faire productivity dynamics is again optimal, as in the closed economy (see Bagwell and Staiger 2004).

[22] In fact, $\eta < 1$ case reinforces the results, as $P_T < P_H$ in this case further tilts the project adoption towards the non-tradable sector.

Proof: Combining equations (26)–(27) from Lemma 3, one can obtain:

$$\frac{\rho-1}{\delta}\big[\dot{A}_T-\dot{A}_N\big]=\bar{A}^{\rho-1}\left[\left(\frac{\pi_T}{\gamma}\right)^{\nu}A_T^{2-\rho}-\left(\frac{1-\pi_T}{1-\gamma}\right)^{\nu}A_N^{2-\rho}\right]-(A_T-A_N)$$
$$=\frac{\bar{A}^{\rho-1}}{A_N^{\rho-2}}\left(\frac{1-\pi_T}{1-\gamma}\right)^{\nu}\Bigg[\underbrace{\chi^{\frac{\nu}{1-\nu}}\left(\frac{A_N}{A_T}\right)^{\rho-2}-1}_{<0}\Bigg]-(A_T-A_N)<0,$$

where the inequality inside the square bracket follows from (36) and the condition of the lemma that $NX<0$ and $A_T\geq A_N$. Indeed, under these circumstances:

$$\chi^{\frac{\nu}{1-\nu}}\left(\frac{A_N}{A_T}\right)^{\rho-2}=\left(\frac{A_N}{A_T}\right)^{\theta-1}\left[1+\frac{NX}{\gamma Y}\right]^{\frac{\nu}{1-\nu}}<1,$$

where our parameter restriction $\theta>\max\{\rho-1,1\}$ is a sufficient condition. ■

Lemma 4 implies our two main results, characterizing the steady state and transition dynamics in the open economy:

Proposition 5 *A steady state with a negative net foreign asset position and a trade surplus, $\overline{NX}=-r^*\bar{B}>0$, is associated with tradable productivity exceeding non-tradable productivity:*

$$\bar{A}_T>\bar{A}=A^*\left(\lambda/\delta\right)^{1/(\rho-1)}>\bar{A}_N,$$

and vice versa.

Proof: Setting $\dot{A}_T=\dot{A}_N=0$ in (26)–(27) yields:

$$\bar{A}_T=\bar{A}\left(\frac{\bar{\pi}_T}{\gamma}\right)^{\frac{\nu}{\rho-1}}\qquad\text{and}\qquad\bar{A}_N=\bar{A}\left(\frac{1-\bar{\pi}_T}{1-\gamma}\right)^{\frac{\nu}{\rho-1}}.$$

With this, (36) implies:

$$\frac{\bar{\pi}_T}{1-\bar{\pi}_T}\frac{1-\gamma}{\gamma}=1+\frac{\overline{NX}}{\gamma\bar{Y}}>1\qquad\text{iff}\qquad\overline{NX}=\bar{X}-\bar{X}^*>0,$$

and hence $\bar{\pi}_T>\gamma$, yielding the result of the proposition. ■

Proposition 6 *Starting from a symmetric initial state $A_T(0)=A_N(0)<\bar{A}$ and assuming that $NX(0)<0$, there exist two cutoffs t_1 and t_2, such that $0<t_1<t_2<\infty$, and:*[23]

[23]Furthermore, one can show that $\dot{A}_N>\dot{A}_T$ on some open interval to the right of $t=0$ and the sign changes for all $t\in(t_3,\infty)$ for some $t_3\in(0,t_1)$. The growth rates of sectoral productivity $\dot{A}_N/A_N$ and $\dot{A}_T/A_T$ exhibit a similar property.

- *$NX(t) < 0$ for $t \in [0, t_1)$ and $NX(t) > 0$ for $t > t_1$, and*
- *$A_T(t) < A_N(t)$ for $t \in (0, t_2)$ and $A_T(t) > A_N(t)$ for $t > t_2$.*

At $t = t_2$, $A_T(t) = A_N(t)$, and the aggregate productivity in the open economy $A(t)$ is strictly lower than that obtained under the closed economy dynamics (characterized in Proposition 1).

Proof: We show in appendix that $NX(0) < 0$ is a sufficient condition for the existence of a unique $t_1 > 0$ such that $NX(t_1) = 0$.[24] Then by Lemma 4, we know that $A_N(t) > A_T(t)$ for all $t \in (0, t_1]$, and by continuity also on an open interval of $t > t_1$. Similarly, as $t \to \infty$, $NX(t) \to \overline{NX} > 0$ to ensure intertemporal budget constraint (20), and by Proposition 5, $A_J(t) \to \bar{A}_J$ such that $\bar{A}_T > \bar{A}_N$. Since $NX(t) > 0$ for all $t > t_1$, there exists a unique $t_2 \in (t_1, \infty)$ such that $A_T(t_2) = A_N(t_2) = A(t_2)$, where aggregate productivity is defined in (16). By Proposition 2, since $\pi_T(t) \neq \gamma$ for all t in the open economy, $A(t_2)$ must be strictly lower than that obtained at t_2 under the closed economy (or closed capital account) dynamics with $\pi_T \equiv \gamma$. ■

These two proposition characterize the transition productivity dynamics under international financial openness in a catching-up economy, and Figure 2 provides an illustration. Trade deficits in the early part of the transition tilt the relative profitability towards the non-tradable projects, by reducing R_T/R_N (see (13)). This happens by means of two effects: the relative market size effect $\tau^{1-\rho}Y^*/(P_T^{\rho-1}Y)$ and the relative competition effect $(P_T/P_H)^{\rho-1}$, which on net favor the non-tradable sector when $NX < 0$ (see (14)), resulting in $\pi_T(t) < \gamma$ and $A_N(t) > A_T(t)$ during the early transition. Eventually, trade deficits must turn into trade surpluses, and $\pi_T(t) > \gamma$. A catching-up economy that borrows to smooth consumption must run a long-run trade surplus to satisfy the intertemporal budget constraint. This implies a long-run reversal in relative productivities, with $A_T(t) > A_N(t)$ eventually, and $\bar{A}_T > \bar{A}_N$ in the long-run.[25]

Note that Propositions 5–6 rely only on the transition dynamics of the net exports in the economy, and so apply in general, independently of the parameter values and the specific characterization of the equilibrium allocation. Indeed, these propositions reflect the conventional neoclassical forces in an economy with an endogenous productivity dynamics, and this

[24] Monotonic increase in $NX(t)$ over time t is a sufficient condition for $NX(0) < 0$, which is an implication of consumption smoothing, provided that $A_T(t)$ monotonically increases over time. There is, however, a possibility of an inverted-U pattern of $NX(t)$ over time, if $A_T(t)$ falls initially (a pathalogical case, which we can try to rule out by an appropriate choice of parameters). Even with a possible non-monotonicity in $NX(t)$ over time, a sufficient requirement for the result is that $NX(0) < 0$, which is quite natural for a catching-up economy (we check this condition numerically).

[25] If there is a force to reduce NFA in the long-run, as in Schmitt-Grohé and Uribe (2003), then in the long-run $\overline{NX} = -r^*\bar{B} = 0$ and $\bar{A}_T = \bar{A}_N$, as under closed capital account. However, since the time pattern of trade deficits and surpluses during the transition is similar in this case, our results still characterize accurately the convergence dynamics in this case, up to the very long-run.

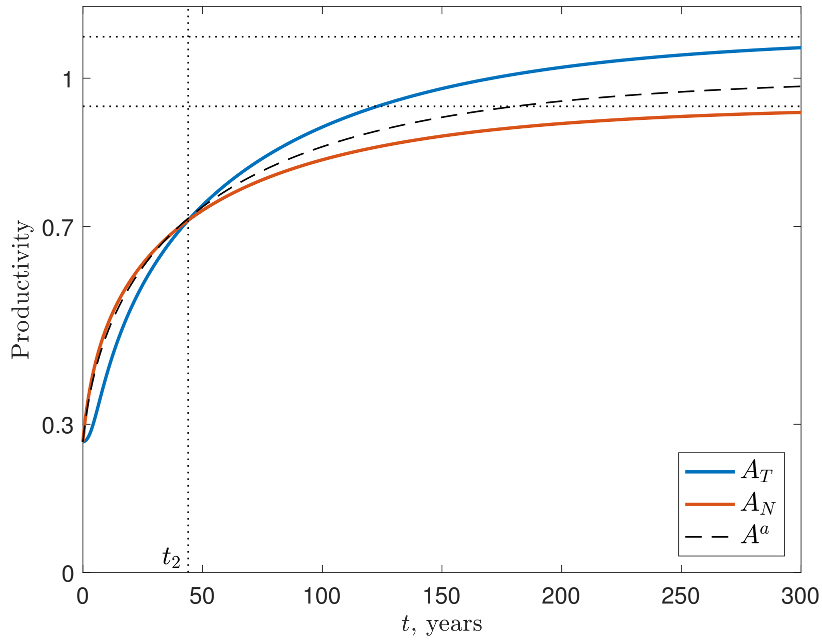

Figure 2: Productivity convergence in closed and open economies

is why we view them as capturing a robust relationship between current account openness and productivity evolution, likely to be present in a variety of environments and empirical contexts, akin to the convergence force in the neoclassical growth model. In Section 6, we upgrade the baseline mechanism with a number of additional ingredients, which may prove more or less important in various empirical contexts. Nonetheless, the baseline mechanism captured in this section always persists.

Equilibrium allocation dynamics

We now characterize the equilibrium allocation along the convergence path. In doing so, we take the technologies $(A_T(t), A_N(t))$ as given at each point in time, with their evolution characterized by Propositions 5–6. Furthermore, we assume $r^* = -\log \beta$, and the economy smoothes consumption $C(t)$ according to the intertemporal optimaility.[26] Given $C(t)$ and the technology vector, we solve for $NX(t)$ and the rest of the equilibrium allocation, in particular the real wage $w(t)$. In the text, we adopt our conventional approximation with $\eta = 1$ and $\rho \gg 1$, and we provide the exact expressions in Appendix A.2.

Combining (2) with (19), we show that market clearing under open capital account results

[26] Since r^* is in terms of foreign tradables, and the domestic price level $P(t)$ decreases along the transition, the domestic consumption level $C(t)$ increases over time with $\dot{C}(t)$ pinned down by intertemporal optimality, and the level of consumption determined by the intertemporal budget constraint (20).

in the following consumption and employment allocation (*cf* (34)):

$$C = w^{\frac{1+\varphi}{\sigma+\varphi}}(1+nx)^{-\frac{\varphi}{\sigma+\varphi}}, \quad L = w^{\frac{1-\sigma}{\sigma+\varphi}}(1+nx)^{\frac{\sigma}{\sigma+\varphi}} \quad \text{where} \quad w = A\left(\frac{W}{\tau A_T}\right)^{\kappa\gamma}, \tag{37}$$

and $nx \equiv NX/Y$ is trade surplus scaled by domestic absorption $Y = PC$. Substituting prices into the definition of net exports in (9), we have:[27]

$$nx = \frac{\gamma\kappa}{\left(\frac{W}{\tau A_T}\right)^{\rho-\kappa\gamma}}\left[\tau^{1-2\rho}\frac{A^{*\frac{1+\varphi}{\sigma+\varphi}}}{C}\frac{A}{A_T} - \left(\frac{W}{\tau A_T}\right)^{(1-\kappa\gamma)+(2-\kappa)(\rho-1)}\right]. \tag{38}$$

Equations (37) and (38) allow to solve for equilibrium allocation (w, W, NX, L) as a function of productivities (A_T, A_N, A^*) and endogenous consumption level C. We prove in the appendix:

Proposition 7 *Along an equilibrium path with consumption smoothing:* (i) *net exports $NX(t)$ and real wage $w(t)$ increase with aggregate productivity $A(t)$, as well as with individual sectoral productivities $A_T(t)$ and $A_N(t)$, but less than proportionately for wages so that $w(t)/A(t)$ decreases.* (ii) *nominal wage $W(t)$ falls with $A_N(t)$, as well as with $A_T(t)$ when $\rho \approx 1$, while it increases with $A_T(t)$ and $A(t)$ when ρ is sufficiently large.*[28]

The implication of Proposition 7 is that the real wage $w(t)$ jumps up discontinuously upon opening to financial flows, which results in higher consumption level on impact by means of trade deficits. Thereafter, real wage increases with productivity, but less than proportionately. Recall from Proposition 4 that opening to trade, under closed capital account, already leads to a discontinuous increase in wages and consumption, and this effect is further reinforced by trade deficits along the early transition phase. From (37) we immediately observe that employment increases with real wage $w(t)$ and decreases with consumption level $C(t)$, with both elasticities proportional to the Frish elasticity $1/\varphi$, and therefore a large φ results in a flat path of employment, arguably consistent with the data. The model also implies that nominal wage rate $W(t)$ and price level $P(t)$ decrease with non-tradable productivity $A_N(t)$ and increase with tradable productivity $A_T(t)$, provided that ρ is sufficiently large, which we

[27] These equations hold by virtue of static equilibrium conditions (market clearing), and the only dynamic link comes from the equilibrium path of consumption C, which in particular could be $C = \hat{C}$ under perfect consumption smoothing. In addition, we have $NX = nxPC = nxWC/w$, so that:

$$NX = \gamma\kappa\tau^{2(1-\rho)}\left(\frac{W}{\tau A_T}\right)^{-(\rho-1)}\left[A^{*\frac{1+\varphi}{\sigma+\varphi}} - \tau^{1+2(\rho-1)}\frac{A_T}{A}\left(\frac{W}{\tau A_T}\right)^{(1-\kappa\gamma)+(2-\kappa)(\rho-1)}C\right].$$

[28] Additionally, we show that nominal price level $P(t)$ falls with $A_N(t)$ and $A(t)$, and also with $A_T(t)$ if $\rho \approx 1$ or $\gamma \approx 1$, while it may increase with A_T if ρ is large and γ is small.

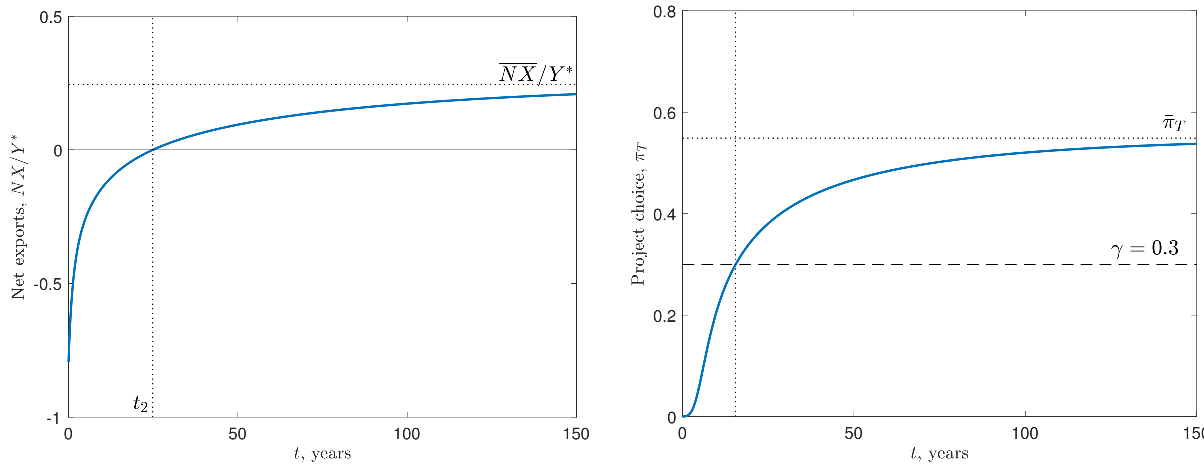

Figure 3: Dynamics of the equilibrium allocation

take to be the empirically relevant case.[29] We illustrate the equilibrium path of these variables in Figure 3 [TO BE COMPLETED]

4.3 Endogenous innovation rate

In the baseline model, the inflow rate of aggregate innovation λ is exogenous, and only the choice of projects between tradable and non-tradable is endogenous. Therefore, international openness can affect only the path of sectoral productivities, while leaving the path of aggregate quantity of projects adopted unchanged. We now consider an extension in which the innovation rate λ is also determined endogenously by profit-maximizing entrepreneurs.

Specifically, we adopt an occupational choice formulation as in Lucas (1978). At each time t, agents decide whether to be workers and earn nominal wage $W(t)$ per unit of labor, or to become entrepreneurs and earn an expected profit $\mathbb{E}\hat{\Pi}(t)$. Being an entrepreneur is associated with an effort cost ϕ (in units of labor), which is distributed according to a cdf $\Phi(\phi)$ in the cross-section of agents. Therefore, only agents for whom $\mathbb{E}\hat{\Pi}(t) \geq \phi W(t)$ will select into being entrepreneurs. Therefore, the innovation rate is determined by:

$$\lambda = \Phi\left(\frac{\mathbb{E}\hat{\Pi}(t)}{W(t)}\right),$$

and hence is increasing in the ratio of expected profits to nominal wages, $\mathbb{E}\hat{\Pi}(t)/W(t)$.[30]

[29] A sufficient condition during the trade deficit phase ($NX = X - X^* \leq 0$) and assuming not too high Frish elasticity ($1/\varphi \leq 1$) is $\rho \geq 1 + \gamma Y/X^*$. If imports are 15% of GDP and tradable expenditure is 30% of income, both conservative, $\rho \geq 3$ is sufficient. In practice, the results holds for considerably lower values of ρ.

[30] We assume that the mass of entrepreneurs at each point in time is negligible relative to total labor force, and therefore we do not need to subtract the number (measure) of entrepreneurs from the total labor supply in the

Lastly, note that the expected profit from innovation is:

$$\mathbb{E}\hat{\Pi} \equiv \mathbb{E} \max_{\ell \in \{1..n\}} \Pi_{J(\ell)}(\ell) = \left(\frac{\rho A_N}{\rho - 1}\right)^{1-\rho} \frac{R_N}{\rho}\, \mathbb{E} \max\left\{\chi \hat{Z}_T^{\rho-1}, \hat{Z}_N^{\rho-1}\right\},$$

where we used the definition of Π_T, Π_N and χ in (10)–(12) and the $\hat{Z}_J$ notation from (22).

Using the properties of the Frechet distribution summarized in Appendix A.4, we have the following characterization:[31]

$$\frac{\mathbb{E}\hat{\Pi}}{W} = \varrho \left(\frac{A^*}{A} \cdot \frac{A}{\hat{A}_\theta}\right)^{\rho-1} \left[\frac{\gamma \chi^{\frac{\theta}{\rho-1}} + (1-\gamma)}{\gamma (A_N/A_T)^\theta + (1-\gamma)}\right]^{\frac{\rho-1}{\theta}} \frac{C}{w}, \tag{39}$$

where $\varrho \equiv \frac{1}{\rho}\left(\frac{\rho}{\rho-1}\right)^{1-\rho}$ is a constant profitability term and $\hat{A}_\theta \equiv \left[\gamma A_T^{-\theta} + (1-\gamma) A_N^{-\theta}\right]^{-1/\theta}$ is a CES-average productivity with Frechet θ used as a curvature parameter. Note that $\hat{A}_\theta = A$ when $A_T = A_N$, and outside this case $\hat{A}_\theta < A$. Finally, recall from (36) that:

$$\chi^{\frac{\theta}{\rho-1}} = (A_N/A_T)^\theta \left[1 + NX/(\gamma Y)\right]^{\frac{\theta}{\rho-1}}.$$

This characterization allows us to prove the following result:

Proposition 8 (i) *The aggregate innovation rate λ increases in the productivity gap to the frontier A^*/A and in the gap between sectoral productivity levels, which results in $A/\hat{A}_\theta > 1$.* (ii) *With balanced trade $NX = 0$, $\mathbb{E}\hat{\Pi}/W = \varrho \left(A^*/\hat{A}_\theta\right)^{\rho-1} w^{\frac{1-\sigma}{\sigma+\varphi}}$, and hence the aggregate innovation rate increases with the productivity (welfare) gain associated with trade iff $\sigma < 1$.* (iii) *Assume $\sigma = 1$, then $A_N \geq A_T$ is a sufficient condition for the aggregate innovation rate to increase in NX around $NX = 0$, with the slope of the effect increasing in A_N/A_T.*

Proof: (i) and (ii) follow directly from (39), combined with (34) and the fact that $\chi = (A_N/A_T)^{\rho-1}$

economy, L. Formally, this can be achieved by placing a near complete mass on very high entrepreneurial effort cost $\phi \approx \infty$, i.e. $\lim_{\phi\to\infty}[1 - \Phi(\phi)] = 1$. Also note that λ is the extensive margin of entrepreneurial activity, while the intensive margin reflected in n and Z remains unchanged in this formulation.

[31] **Proof:** From definition (22) and the distributional assumption (23), we have for $\mathcal{Z} \equiv \max\left\{\chi \hat{Z}_T^{\rho-1}, \hat{Z}_N^{\rho-1}\right\}$:

$$\mathcal{Z} \sim Frechet\left(\left[\gamma \chi^{\frac{\theta}{\rho-1}} + (1-\gamma)\right] nz, \frac{\theta}{\rho-1}\right) \quad \Rightarrow \quad \mathbb{E}\mathcal{Z} = \left[\gamma \chi^{\frac{\theta}{\rho-1}} + (1-\gamma)\right]^{\frac{\rho-1}{\theta}} A^{*\rho-1},$$

where A^* is the world productivity frontier defined in Lemma 3. Since with $\eta = 1$, $R_N = P_N C_N = PC$, hence $R_N/W = C/w$, and we obtain the expression in the text after rearranging the productivity terms using $\hat{A}_\theta$.

when $NX = 0$. Next, for (iii), substitute (37) into (39) and rearrange to obtain:

$$\frac{\mathbb{E}\hat{\Pi}}{W} = \varrho\left(\frac{A^*}{\hat{A}_\theta}\right)^{\rho-1} \frac{\left(1+\gamma\left(\hat{A}_\theta/A_T\right)^\theta\left[\left(1+NX/(\gamma Y)\right)^{\theta/(\rho-1)}-1\right]\right)^{\frac{\rho-1}{\theta}}}{(1+NX/Y)^{\frac{\varphi}{1+\varphi}}}.$$

First order approximation around $NX/Y = 0$ yields:

$$\frac{\mathbb{E}\hat{\Pi}}{W} \approx \varrho\left(\frac{A^*}{\hat{A}_\theta}\right)^{\rho-1}\left[1+\left(\left(\hat{A}_\theta/A_T\right)^\theta-\tfrac{\varphi}{1+\varphi}\right)\frac{NX}{Y}\right],$$

which is increasing in NX provided $\hat{A}_\theta \geq A_T$ (or equivalent, $A_N \geq A_T$), since $\varphi/(1+\varphi) < 1$. A weaker necessary condition for λ to increase in NX is $(A_T/A_N)^\theta < 1 + \frac{1}{(1-\gamma)\varphi}$. ■

This extension of the model again exhibits a neoclassical convergence force, by which growth rates are higher the further away is the economy from the frontier, $A/A^* < 1$, as innovation is more profitable early on in the transition, given $\rho > 1$. Note that this can be consistent with the economy never reaching the frontier A^*, with the state state productivity level given by $\bar{A} = A^*(\bar{\lambda}/\delta)^{1/(\rho-1)} < A^*$, as in Proposition 1, but with an endogenous long-run rate of innovation $\bar{\lambda} < \delta$, which is defined by the following fixed point condition:

$$\bar{\lambda} = \Phi\left(\varrho(A^*/\bar{A})^{\rho-1}\right) = \Phi\left(\varrho\bar{\lambda}/\delta\right).$$

A country can differ in $\bar{\lambda}$ due to a country-specific high cost of innovation reflected in a shifted out distribution $\Phi(\cdot)$.

More interestingly, the country innovates faster when there is an asymmetry in the productivity levels across sectors, $A_T \neq A_N$ resulting in $\hat{A}_\theta < A$. This is because lower sectoral productivity relative to the average economy-wide productivity leaves more room for a profitable innovation in that sector, raising the overall expected profit, $\mathbb{E}\hat{\Pi}/W$.

When a country runs balanced trade, the only feedback into the innovation decision relative to a closed economy is from the increased aggregate productivity, arising from the gains from trade. This effect is captured by the term $C/w = w^{\frac{1-\sigma}{\sigma+\varphi}}$, and it is only present when labor is supplied with a positive Frish elasticity, $1/\varphi > 0$. The effect on innovation may be positive with $\sigma < 1$, or negative with $\sigma > 1$, and it is nil when $\sigma = 1$ so that income and substitution effects of trade cancel out.

Lastly, and most importantly given our interest, the model exhibits a robust link between current account imbalances and the aggregate rate of innovation. Starting around $A_T \approx A_N$, the model exhibits a force by which the aggregate innovation rate λ is increasing with trade surpluses and decreasing with trade deficits.[32] In general, there are two effects of $NX \neq 0$ on the expected profit from innovation, $\mathbb{E}\hat{\Pi}/W$, and hence on the equilibrium innovation rate λ. The first effect is negative and operates via the size of the market relative to the real wage, C/w, which is decreasing in NX (see (37)). This effect is stronger the larger is φ, i.e. the less elastic is the labor supply, resulting in a steeper decrease of C/w in NX. The second effect is positive, and arises due to R_T/R_N and χ being increasing in NX, as the access to the large foreign market increases profitability and encourages innovation. This effect is stronger the less developed is the tradable sector relative to non-tradable, i.e. the lower is A_T/A_N, as otherwise the profitability of the tradable innovation is suppressed. We prove in the proposition that this latter effect always dominates around $NX/Y \approx 0$, provided that A_T is not much larger than A_N.

This result rationalizes why countries, such as China, that run trade surpluses maybe also experience periods of unusually fast growth, while countries that run trade deficits, such as Spain and Argentina, may in contrast experience periods of stagnating productivity growth. Note that this relationship does not depend on whether the trade surpluses and deficits are policy induced or are an outcome of decentralized equilibrium forces. Furthermore, from Proposition 6, we know that A_N/A_T increases during early transition with $NX < 0$, and now from Proposition 8 we see that this dynamics effect can lead to a further slow down of aggregate innovation, i.e. a fall in λ, acting as a dynamic amplification force in the model with endogenous productivity dynamics. Intuitively, a relatively low A_T encourages tradable innovation, and makes it particularly sensitive to stimulus coming from trade surpluses. In other words, the highest tradable productivity growth can be achieved when the initial tradable productivity is low and the country runs a trade surplus. In contrast, running a trade deficit under these circumstances is particularly discouraging for tradable innovation.

5 Empirical Implications

We now explore the implications of the model for the dynamics of productivity and unit labor costs, and provide some suggestive empirical evidence supporting the model mechanism.

5.1 Productivity growth

The main implication of our analysis is that trade deficits tilt the path of relative productivity towards the non-tradable sector, while trade surpluses do the reverse. This implication is at the core of our modeling framework and depends exclusively on the presence of endogenous

[32] The latter comparative statics requires that the trade deficits are not too large, as with very large negative NX the positive effect on innovation from high C/w may start to dominate.

project selection block in the model. Indeed, the results here come from Lemma 3 (equations (26)–(27)), which characterizes the dynamics of productivity, where endogenous feedback is summarizes by the project selection probability π_T. In the absence of this feedback, with $\pi_T \equiv \gamma$, the model features an exogenous productivity catch-up trajectory. Furthermore, Lemmas 1–2 show that π_T monotonically increases in the ratio of aggregate revenues in the tradable and non-tradable sectors R_T/R_N, which in turn increases in the net exports of the country NX (as summarized in (36)):

$$\frac{\pi_T}{1-\pi_T} = \frac{\gamma}{1-\gamma}\left(\frac{A_N}{A_T}\right)^{\theta}\left[1+\frac{NX}{\gamma Y}\right]^{\frac{\theta}{1-\rho}}.$$

This implies that the relative growth rate in the tradable sector, $\frac{\dot{A}_T}{A_T} - \frac{\dot{A}_N}{A_N}$, shifts out with trade surpluses of a country and shifts down with trade deficits.

To illustrate this effect, we provide here an approximate expression for the relative productivity growth rates, which is exact around any symmetric productivity level $A_T = A_N = A_0$ and balanced trade $NX = 0$:

$$\begin{aligned}\frac{\dot{A}_T(t)}{A_T(t)} - \frac{\dot{A}_N(t)}{A_N(t)} &= g_0\left[-(\rho-1)\log\frac{A_T(t)}{A_N(t)} + \frac{\nu\big(\pi_T(t)-\gamma\big)}{\gamma(1-\gamma)}\right] \\ &= g_0\left[-(\rho-1)\,(1+\mu)\log\frac{A_T(t)}{A_N(t)} + \frac{\mu}{\gamma}\frac{NX(t)}{Y_0}\right],\end{aligned} \tag{40}$$

where $\mu \equiv \frac{1}{2\gamma(1-\gamma)}\frac{\nu}{1-\nu}$, $g_0 \equiv \frac{\delta}{\rho-1}\left(\frac{\bar{A}}{A_0}\right)^{\rho-1}$ and Y_0 is domestic absorption at the point of approximation (see Appendix A.3). We further show that the baseline growth rates in both sectors are given by $G_0 \equiv \frac{\delta}{\rho-1}\left[\left(\frac{\bar{A}}{A_0}\right)^{\rho-1} - 1\right] > 0$ iff $A_0 < \bar{A}$. Therefore, there are two forces in the model. The first is a *neoclassical convergence force*, which pushes both sectoral productivities towards their steady state levels (at rate G_0) and introduces a mean reversion force if one sector advances ahead (the first term in (40)). The second is the endogenous feedback term proportional to $(\pi_T - \gamma)$, which emerges when $NX \neq 0$, and tilts the productivity growth away from the tradable sector whenever a country runs a trade deficit.

We emphasize that this prediction of the model does not depend on the rest of the equilibrium system, in particular on whether the international trade and consumption-savings decision of the country are optimal or shaped by distortionary policies. Therefore, we expect the relationship in (40) to hold in the cross-section of countries, in particular including both countries like China that save along the catch-up phase and countries like Argentina and Spain that borrow instead.

Lastly, we compare the implication for productivity growth (40) with that for the employment allocation across sectors, which we show is also related to the relative revenues in the two sectors:[33]

$$\frac{L_T}{L_N} = \frac{\gamma}{1-\gamma}\frac{R_T}{R_N} = \frac{\gamma}{1-\gamma}\left[1 + \frac{NX}{\gamma Y}\right],$$

where the second equality substitutes expression (14) for R_T/R_N from the proof of Lemma 1 under the Cobb-Douglas assumption $\eta = 1$. We see that in the closed economies or when countries run a balanced trade account, Cobb-Douglas preferences ensures that a fraction γ of labor input always goes to the tradable sector. Trade deficits, just like with relative growth rates, shift labor allocation away from the tradable sector. This effect is, however, more general and is also present in an exogenous growth version of the model with $\pi_T \equiv \gamma$. Therefore, the distinctive feature of our model is its prediction for the relationship between trade deficits and relative sectoral productivity growth, which we test in the data.[34]

Empirical analysis We test the implication of the theory, summarized in (40), in the panel of sector-country productivity growth rates using the KLEMS database for OECD countries. Specifically, we estimate the following empirical specification:

$$\Delta \log A_{ks} = d_k + d_s + b \cdot \log A^0_{ks} + c \cdot \Lambda_s \cdot \frac{TB_k}{Y_k} + \varepsilon_{ks}, \tag{41}$$

where $\Delta \log A_{ks}$ is a measure of productivity growth in sector s country k, d_k and d_s are country and sector fixed effects, A^0_{ks} is the initial productivity level, and the coefficient b captures the neoclassical convergence force. Our main focus however is on the interaction term of a sectoral-level *home share* Λ_s (a measure of *non*-tradability, namely sectoral share of domestic absorption in output, averaged over time and median across countries) and a country-level measure of trade surpluses over GDP $\frac{TB_k}{Y_k}$, also averaged over time period. The theory predicts $b < 0$ and $c < 0$.[35] That is, sectors with initially high productivity level have slower produc-

[33] Indeed, since at every instant almost all firms are price takers, we have $WL_N(i) = P_N(i)C_N(i)$ and $WL_T(i) = P_H(i)C_H(i) + P^*_H(i)C^*_H(i)$. Aggregating across $i \in [0, \Lambda_J]$, and using the demand schedules and definitions of the price indexes, we arrive at the result in the text.

[34] The model also allows to split the dynamics of revenues and expenditure on tradables and non-tradables into the price and quantity movements. Under Cobb-Douglas ($\eta = 1$), the aggregate expenditure (and revenues) in the non-tradable sector are constant as a share of total expenditure: $(1-\gamma)R_N = (1-\gamma)P_N C_N = PC$, and therefore $C_N/C \propto (P_N/P)^{-1}$. Using the definitions of the price indexes, we have:

$$\frac{P_N}{P} = \left(\frac{A_T}{A_N}\right)^{\gamma}\left[1 - \kappa + \kappa\left(\frac{W}{\tau A_T}\right)^{\rho-1}\right]^{\frac{\gamma}{\rho-1}},$$

and therefore P_N/P is high when A_T/A_N is high or when W/A_T is high (see below on ULC, where we show that W/A_T is highest early on in the transition).

[35] Specifically, assume that in each sector s there are some tradable and non-tradable varieties (e.g., drawn at random from γ tradable and $1-\gamma$ non-tradable sectors in the model). Sectors with a larger fraction of non-

Table 1: Trade balance and sectoral productivity growth

Dep. var: $\Delta \log A_{ks}$	VA/L (1)	RVA/L (2)	KLEMS (3)	VA/L (4)	RVA/L (5)
$\Lambda_s \times \frac{TB_k}{Y_k}$	-0.36^{***} (0.10)	-0.41^{**} (0.15)	0.07 (0.20)	-0.20 (0.14)	-0.00 (0.14)
$\log A^0_{ks}$	-4.75^{**} (1.76)	-4.43^{***} (0.98)	-0.74 (0.72)	-2.17^{**} (0.73)	-3.40^{***} (0.56)
R^2	0.68	0.57	0.33	0.54	0.59
Observations	532	530	399	399	399
Sample	VA	RVA	KLEMS	KLEMS	KLEMS
$\log A^0_{ks}$	VA/L	RVA/L	VA/L	VA/L	RVA/L

Note: Results from estimation of equation (41), see text for details. Robust standard errors in brackets.

tivity growth, and non-tradable sectors grow relatively slower in periods of trade surpluses (and vice versa).

We test the theory using a 6-year period 2001–2007, taking a long difference for the productivity growth measures and 2000 as the base year for initial productivity levels. The sectoral tradability and country trade surplus measures are time averages over this period. The sample contains 17 OECD countries[36] and 33 sectors in the KLEMS database (roughly at the 3-digit level of industry aggregation).

We use three different proxies for productivity: value added per worker (labor productivity, VA/L), value added per worker deflated by sectoral price index (RVA/L), and the KLEMS-estimated measure of sectoral productivity. The latter measure is an index, and therefore we cannot control for its initial level, and use the value-added productivity measures instead. The caveat with the labor productivity measures is that with decreasing marginal products they may reflect shifts along the labor demand curves rather than changes in productivity, which may bias downwards our results even if the sectoral productivity mechanism is at play.[37] Note that if productivity gains are mostly in terms of cost reduction, we should use the deflated measures of value added, while if they are mostly in terms of quality improvement, we should use the non-deflated value added. In the data, all three measures of productivity growth are

tradable varieties have a larger Λ_s. In the extreme, if all varieties in a sector weere non-tradable, $\Lambda_s = 1$. However, such sectors are rare in the data: the interquartile range for Λ_s is $[0.607, 0.975]$. Therefore, for empirical implementation we adopt this more continuous interpretation of the baseline model. Our results are similar when instead of taking the median sectoral tradability measure across countries we use the country-specific or the US sectoral tradability measures.

[36] Austria, Belgium, Czech Republic, Germany, Denmark, Spain, Finland, France, UK, Italy, Luxembourg, Latvia, Netherlands, Slovak Republic, Slovenia, Sweden, USA.

[37] If sectors expanding in productivity draw in additional labor, this may offset part of the measured productivity gains.

positively correlated, with the weakest correlation of 0.49 between VA/L and KLEMS and the strongest correlation of 0.93 between KLEMS and RVA/L (the correlation between VA/L and RVA/L is 0.67). Importantly, we have VA/L measures for virtually all countries and sectors, while KLEMS measures are missing for almost 30% of country-sector observations.

We report the estimation results in Table 1. Using the value-added measures of productivity (VA/L and RVA/L), we find a strong convergence effect from initial productivity level ($b < 0$) and a sizable interaction term $c < 0$: indeed, non-tradable sectors grow slower in periods of trade surpluses. The magnitude of this effect is as follows: a 6% trade deficit in GDP is associated with a 1% slower productivity growth rate for sectors at the 75th percentile of tradability relative to sectors at the 25th percentile of tradability ($-1\% \approx -0.4 \cdot 0.4 \cdot 6\%$, where 0.4 is approximately the interquartile range for our sectoral tradability measure). This interaction effect becomes insignificant when we use the KLEMS measure of sectoral productivity, however, as we show in columns 4 and 5, this is largely due to the loss of country-sample observations in the KLEMS sample (in particular, we loose countries which have important variation in their trade balance ratios in our sample). We conclude that we find tentative empirical support for the sectoral productivity growth mechanism emphasized by our model.

5.2 Unit labor costs

We now consider the dynamics of the *unit labor costs* (ULC), which we define as the ratio of real wage rate to aggregate productivity, $w(t)/A(t)$. From (37), we know that $w(t)/A(t)$ is monotonically related with an alternative tradable measure of ULC, namely the ratio of nominal wage rate to tradable productivity, $W(t)/A_T(t)$.[38] For expositional simplicity, the expressions below are reproduced for the special Cole-Obstfeld case with $\sigma = \eta = 1$ and $\tau = 1$, but the results apply more generally (see Appendix A.2). In the closed economy (superscript a for autarky):

$$w^a(t) = C^a(t) = A(t),$$

and therefore unit labor costs always equal unity, and do not change along the transition path.

From Propositions 4, we further know that $w(t)$ jumps up on impact of opening to balanced trade flows, and thus for a given productivity level this implies an instantaneous increase in

[38] Given our real model with foreign price of tradables chosen as numeraire, nominal wage rate is measured in units of foreign tradables, and thus is an equally meaningful benchmark for the definition of ULC. In practice, ULC are sometimes defined as $\frac{W(t)}{A(t)} = \frac{W(t)}{A_T(t)}\left(\frac{A_T(t)}{A_N(t)}\right)^{1-\gamma}$, which suggests lower ULC when $A_N(t) > A_T(t)$, not necessarily an appealing feature.

the unit labor costs. We have from (34)–(35) that with balanced trade (superscript b):[39]

$$w^b(t) = C^b(t) = A(t)\left(\frac{A^*}{A_T(t)}\right)^{\frac{\kappa\gamma}{1+(2-\kappa)(\rho-1)}} > A(t) \quad \text{when} \quad A_T(t) < A^*.$$

An increase in ULC is often interpreted as a loss of competitiveness, however, here it is a reflection of an improvement in the terms of trade and the resulting gains from trade (associated with an increase in $C(t)$ under balanced trade). Therefore, one must be cautious when interpreting the data on $w(t)/A(t)$, as trade openness has a systematic effect of increasing it without necessarily implying any deterioration in country's competitiveness.

Lastly, consider the case of a switch from balanced trade to an open financial account. Upon financial liberalization, consumption level jumps further above the balanced-trade level of consumption, which exceeds the autarky level. In Appendix A.2, we show (see Lemma 5) that $w(0)$ increases on impact together with $C(0)$ such that

$$w^b(0) < w(0) < C(0).$$

Financial liberalization allows to increase consumption on impact, but this is associated with an increase in both real and nominal wages, and a decline in international competitiveness. Lastly, from Proposition 7 we know that $w(t)/A(t)$ decreases with productivity gains, and therefore ULC are high on impact and decreasing over time.[40] A country that borrows along the equilibrium path has high ULC in the short-to-medium run, and it becomes a low-ULC country in the long-run, when due to trade surplus $NX > 0$, ULC may fall below 1. In case if borrowing is possible, but constrained, the initial jump in ULC can be smoothed out translating into an early period of increasing ULC associated with gradual increase in borrowing, until eventually ULC start to come down.

This analysis suggests that high measured ULC is a natural implication of openness to both trade and international financial flows. ULC are highest on impact of capital account liberalization, and then gradually decline as the country becomes more productive. Furthermore, an increase in measured ULC does not necessary reflect a drop in competitiveness, as it may arise due to an improvement in the terms of trade. Lastly, the dynamics of ULC does not depend on whether project adoption is exogenous or endogenous. Nonetheless, the endogenous feed-

[39]More generally, note from (35) that $w(t)/A(t)$ increases as τ falls, so ULC increase with any incremental reduction in variable trade costs. The condition for $w(t)/A(t) > 1$ is $\tau < (A^*/A_T)^{1/(2\rho-1)}$. However, note that our result that $w(t)/A(t) \equiv 1$ in autarky under $\kappa = 0$ is only directly comparable with the open economy model if $\tau = 1$. ULC are, in general, higher in more open relative to more closed economies (holding preference parameter κ constant).

[40]When ULC are measured as $W(t)/A(t)$, a fast increase in $A_N(t)/A_T(t)$ may lead to a further increase in ULC in the short run, which may be an empirically-relevant case.

back of openness into productivity dynamics may alter the time path of ULC, in particular if non-tradable productivity growth faster than tradable productivity in the short run.

6 Extensions and Applications

6.1 Rollover crisis

We consider here a scenario of an unexpected exogenous rollover crisis along the convergence path, which we discussed in Section 4.2. The rollover crisis happens at some date $s \in (0, t_1)$, when a country still runs a trade deficit (recall Proposition 6) and has a net foreign asset position

$$B(s) = \int_0^s e^{r^*(s-t)} NX(t) \mathrm{d}t < 0.$$

Further, the technology state at this date is $\big(A_N(s), A_T(s)\big)$, such that $A_N(s) > A_T(s)$.

We further assume that debt takes a form of a long-term fixed income contract, depreciating at rate δ, so that a coupon payment each period is equal to $(r + \delta)B(t)$. Since absent a rollover crisis, there is no aggregate uncertainty, this debt structure is inconsequential for the transition dynamics, as the country issues (or buys back) additional debt to exactly accommodate the path of trade surpluses and deficits, $\{NX(t)\}_{t \geq 0}$, that is $\dot{B}(t) = r^* B(t) + NX(t)$.

Upon a rollover crisis, we assume that a country cannot issue any additional debt, and hence needs to depreciate its outstanding debt at a constant rate δ, as it gradually matures. That is, $\dot{B}(t) = -\delta B(t)$, which from the flow budget constraint implies $NX(t) = -(r+\delta)B(t) > 0$. We rewrite this as follows:

$$nx(t) = -(r + \delta)b(t),$$

where $nx(t) \equiv NX(t)/Y(t)$ and $b(t) \equiv B(t)/Y(t)$, such that

$$\dot{b}(t) = \frac{\dot{B}(t)}{B(t)} - \frac{\dot{Y}(t)}{Y(t)} = -\big(\delta + g_y(t)\big),$$

starting from an initial condition $b(s)$ and where $g_y(t)$ is an exogenous growth rate of domestic absorption $Y(t)$.[41]

We now characterize the dynamics of productivity and equilibrium allocation under two alternative scenarios regarding the flexibility of the labor market.

[41]Note that the path of trade surpluses $NX(t)$ is exogenous, while the path of normalized trade surpluses $nx(t)$ also depends on the feedback into absorption $Y(t)$. However, since in the characterization of equilibrium, $nx(t)$ plays a more prominent role, we choose to focus on this variable here.

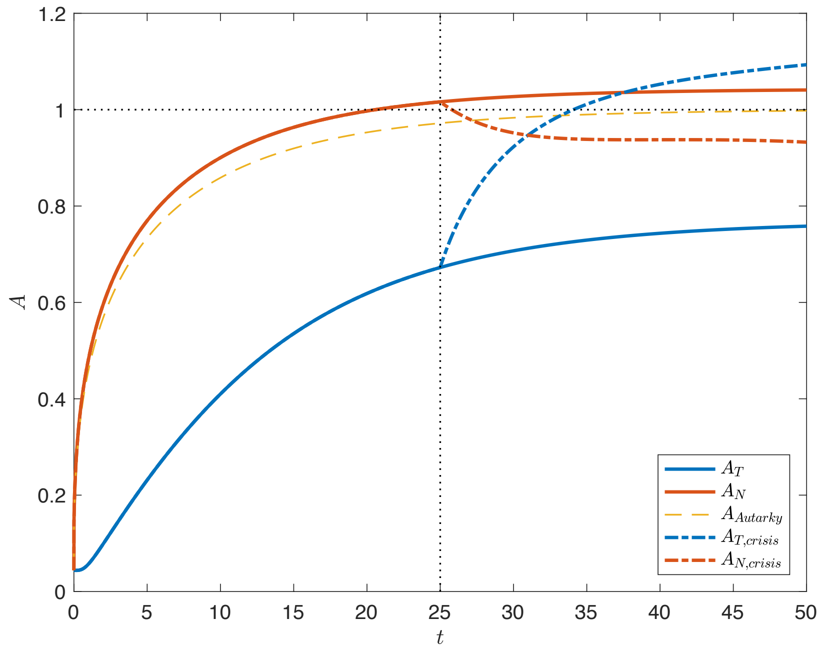

Figure 4: Productivity evolution before and after a sudden stop

Flexible labor market We first start with the analysis of the rollover crisis with a flexible labor market, so that the equilibrium characterization of Section 4 applies. In particular, the productivity dynamics is still characterized by (26)–(27), with $\pi_T(t)$ given by (36):

$$\frac{\pi_T(t)}{1-\pi_T(t)} = \frac{\gamma}{1-\gamma}\left(\frac{A_N(t)}{A_T(t)}\right)^{\theta}\left[1+nx(t)/\gamma\right]^{\frac{\theta}{\rho-1}}.$$

A discontinuous switch from a trade deficit $nx(t) < 0$ to a trade surplus $nx(t) > 0$ at $t = s$ implies a discontinuous jump up in $\pi_T(t)$ at $t = s$. That is, there is a discontinuous switch towards the adoption of tradable as opposed to non-tradable projects. This results in a discontinuous increase in the relative productivity growth rate in the tradable sector, $\dot{A}_T(t)/A_T(t) - \dot{A}_N(t)/A_N(t)$, which necessarily turns positive for $t \geq s$ (since $A_N(s) > A_T(s)$, see Lemma 4). This result suggests a fast growth rebound in the tradable sector immediately following a rollover crisis, an empirical outcome documented in a number of sudden stop episodes. In fact, the larger the imbalance, the stronger is the tradable productivity rebound.

Next, we can use equilibrium conditions (37)-(38), to solve for the equilibrium allocation $(w, W, C, L, Y)(t)$ as a function of $nx(t)$ and productivity $(A_T(t), A_N(t))$. This fully characterizes the equilibrium dynamics of the economy in a sudden stop. We show that a sudden increase in nx results in a collapse in consumption C and wages (both real w and nominal W), an increase in employment L. As a result, the country both gains international competitiveness, as unit labor costs fall, and the size of the domestic market shrink (Y/Y^*), with both effects favoring tradable innovation and discouraging non-tradable innovation. We illustrate these effects in Figure 4.

Interestingly, despite the increase in net exports, the GDP of the country may collapse due to a fall in the size of the domestic consumption and a reallocation of labor away from non-tradable towards the underdeveloped tradable sector. This recession emerges without sticky wages and exchange rate pegs, and due to a structural imbalance in the productivity evolution, which is suboptimal ex post.[42] We next study the amplification effects that arise with inflexible labor markets.

Downward wage rigidity A switch from $NX < 0$ to $NX > 0$ has a discontinuous effect on $\dot{A}_T$ and $\dot{A}_N$, with a recession driven by a collapse in $P_N(t)$. Sticky wages limit the reallocation towards the tradable sector, both in levels, but also limit the extent of switch in π_T and $\dot{A}_T$.

Not only the depth of the recession, but also no growth effects upon sudden stop

China case: external demand collapse after a period of export-led growth.

6.2 Misallocation (and capital flows)

CHINA: what type of misallocation makes $NX > 0$ policy welfare improving? Start with a labor misallocation wedge towards N. Maybe labor transition from rural to urban, as a separate goal. Or productivity transfer from abroad, which is somehow more likely in the tradable sector. Or maybe a "big push" theory, where building the base of the domestic demand is more difficult than employing workers at low wage.

6.3 Physical capital and financial frictions (collateral constraints)

Convergence in capital, but divergence in tradable productivity in the short run...

A model in which capital is needed to start new projects, but not as much on intensive margin, then the capital flows will be disproportionately to the sector with higher $\dot{\Lambda}_J$, which can be non-tradable in early transition.

7 Optimal Policy [TO BE COMPLETED]

[42] **Additionally**: (1) Compare two setups: the same path of $\{NX(t)\}_{t\in[0,T)}$, but in one case $\pi_T \equiv \gamma$ (exogenous growth) and in the other $\pi_T < \gamma$ (endogenous feedback to trade deficits) — is the size of the recession different? How does the difference depend on sticky wages?

(2) We can also study the counterfactual of smoothing out the trade deficit adjustment (like in Spain and Greece vs Argentina) — the trade off is the slower productivity recovery.

A Appendix

A.1 Preferences and demand

The preference structure is given by a nested CES utility over tradables (home and foreign) and non-tradables:

$$
\begin{aligned}
C &= \left[\gamma C_T^{\frac{\eta-1}{\eta}} + (1-\gamma) C_N^{\frac{\eta-1}{\eta}}\right]^{\frac{\eta}{\eta-1}}, \qquad \eta \geq 0 \\
C_T &= \left[\kappa^{\frac{1}{\rho}} C_F^{\frac{\rho-1}{\rho}} + (1-\kappa)^{\frac{1}{\rho}} C_H^{\frac{\rho-1}{\rho}}\right]^{\frac{\rho}{\rho-1}}, \qquad \rho > 1, \\
C_H &= \left[\frac{1}{\gamma}\int_0^{\Lambda_T} C_H(i)^{\frac{\rho-1}{\rho}} \mathrm{d}i\right]^{\frac{\rho}{\rho-1}}, \\
C_N &= \left[\frac{1}{\gamma}\int_0^{\Lambda_N} C_N(i)^{\frac{\rho-1}{\rho}} \mathrm{d}i\right]^{\frac{\rho}{\rho-1}},
\end{aligned}
$$

with the prices given by $\{P_H(i)\}_{i\in[0,\Lambda_T]}$, $\{P_N(i)\}_{i\in[0,\Lambda_N]}$ and P_F for the foreign-produced tradable basket. Under these circumstances, the expenditure minimization:

$$
\min_{\substack{C_F,\\ \{C_H(i)\},\{C_N(i)\}}} Y = \gamma P_F C_F + \int_0^{\Lambda_T} P_H(i) C_H(i)\mathrm{d}i + \int_0^{\Lambda_N} P_N(i) C_N(i)\mathrm{d}i
$$

results in the following demand schedules:

$$
\begin{aligned}
C_H(i) &= \left(\frac{P_H(i)}{P_H}\right)^{-\rho} C_H \qquad \text{and} \qquad C_N(i) = \left(\frac{P_N(i)}{P_N}\right)^{-\rho} C_N, \\
C_F &= \kappa\left(\frac{P_F}{P_T}\right)^{-\rho} C_T \qquad \text{and} \qquad C_H = (1-\kappa)\left(\frac{P_H}{P_T}\right)^{-\rho} C_T, \\
C_T &= \left(\frac{P_T}{P}\right)^{-\eta} C \qquad \text{and} \qquad C_N = \left(\frac{P_N}{P}\right)^{-\eta} C, \\
Y &= PC,
\end{aligned}
$$

with the auxillary price indexes defined by:

$$
\begin{aligned}
P &= \left[\gamma P_T^{1-\eta} + (1-\gamma) P_N^{1-\eta}\right]^{\frac{1}{1-\eta}} \qquad \text{and} \qquad P_T = \left[\kappa P_F^{1-\rho} + (1-\kappa) P_H^{1-\rho}\right]^{\frac{1}{1-\rho}}, \\
P_H &= \left[\frac{1}{\gamma}\int_0^{\Lambda_T} P_H(i)^{1-\rho}\mathrm{d}i\right]^{\frac{1}{1-\rho}} \qquad \text{and} \qquad P_N = \left[\frac{1}{1-\gamma}\int_0^{\Lambda_N} P_N(i)^{1-\rho}\mathrm{d}i\right]^{\frac{1}{1-\rho}}.
\end{aligned}
$$

Note that

$$\int_0^{\Lambda_T} P_H(i)C_H(i)\mathrm{d}i = \gamma P_H C_H \qquad \text{and} \qquad \int_0^{\Lambda_N} P_N(i)C_N(i)\mathrm{d}i = (1-\gamma)P_N C_N,$$

so that

$$Y = PC = \gamma(P_F C_F + P_H C_H) + (1-\gamma)P_N C_N.$$

Therefore, P_F, P_H and P_N are average price indexes (per unit of sectoral expenditure), while γP_F, γP_H and $(1-\gamma)P_N$ are ideal price indexes for the corresponding baskets of varieties. Similarly, $P_H C_H$ is the "average" spending on a typical home-produced tradable variety, while $\gamma P_H C_H$ is the total spending on all tradable varieties, which account for a fraction γ of all sectors (tradable and non-tradable). In the limiting case of Cobb-Douglas upper-tier aggregator ($\eta \to 1$), γ equals exactly the share of all tradable sectors in the total expenditure.

Special cases and interpretation of the utility In the special case with $\eta = \rho$, we have a single-tier preferences, which we write as:

$$\eta = \rho: \qquad C = \left[\gamma \kappa^{\frac{1}{\rho}} C_F^{\frac{\rho-1}{\rho}} + (1-\kappa)^{\frac{1}{\rho}} \int_0^{\Lambda_T} C_H(i)^{\frac{\rho-1}{\rho}}\mathrm{d}i + \int_0^{\Lambda_N} C_N(i)^{\frac{\rho-1}{\rho}}\mathrm{d}i\right]^{\frac{\rho}{\rho-1}}.$$

In the limit of full home bias $\kappa = 0$, this expression further simplifies to:

$$\eta = \rho,\ \kappa = 0: \qquad C = \left[\int_0^{\Lambda_T} C_H(i)^{\frac{\rho-1}{\rho}}\mathrm{d}i + \int_0^{\Lambda_N} C_N(i)^{\frac{\rho-1}{\rho}}\mathrm{d}i\right]^{\frac{\rho}{\rho-1}}.$$

If we also have $\Lambda_T = \gamma$ and $\Lambda_N = 1-\gamma$, then we further simplify to:

$$\eta = \rho,\ \kappa = 0,\ \Lambda_T = \gamma,\ \Lambda_N = 1-\gamma: \qquad C = \left[\int_0^1 C(i)^{\frac{\rho-1}{\rho}}\mathrm{d}i\right]^{\frac{\rho}{\rho-1}},$$

with the interpretation that the first $i \in [0,\gamma]$ are the tradable sectors and the remaining $i \in (\gamma, 1]$ are the non-tradable sectors. In this case, γ has the interpretation of the fraction of tradable sectors (by count). More generally, we interpret γ as the parameter that controls average expenditure on tradable sectors, and equals it exactly in the case of the Cobb-Douglas upper-tier utility ($\eta = 1$).

If we additionally restrict $\Lambda_T \le \gamma$ and $\Lambda_N \le 1-\gamma$, then we can define $\tilde{\Lambda}_T = \frac{1}{\gamma}\Lambda_T \in [0,1]$ and $\tilde{\Lambda}_N = \frac{1}{1-\gamma}\Lambda_N \in [0,1]$ and interpret them as the shares of tradable and non-tradable industries respectively with a technology draw, while the remaining industries (varieties) lack technology and are not produced. This is the most intuitive interpretation of our setup, and it applies for a general case with $\eta \neq \rho$ and $\kappa > 0$. It nonetheless requires $\Lambda_T \leq \gamma$ and $\Lambda_N \leq 1-\gamma$, a restriction that we do not impose in our model.

In the general case, γ controls the expenditure share on tradables, and Λ_T and Λ_N are the available varieties in the two sectors respectively. Then our price indexes and productivities in (4) and (15), as well as expenditures $P_H C_H$ and $P_N C_N$, have the interpretation of average prices, productivities and expenditures per unit of expenditure on tradable and non-tradable sectors respectively.

A.2 Market clearing and static general equilibrium

We characterize the static general equilibrium vector (C, L, Y, W, P, χ) as a parametric function of $(NX; A_T, A_N)$. First, we combine (2) with (19) to solve for:

$$C = w^{\frac{1+\varphi}{\sigma+\varphi}}(1+nx)^{-\frac{\varphi}{\sigma+\varphi}} \qquad \text{and} \qquad L = w^{\frac{1-\sigma}{\sigma+\varphi}}(1+nx)^{\frac{\sigma}{\sigma+\varphi}},$$

as a function of the real wage w (from (18)) and net export-expenditure ratio nx (from (9)):

$$\begin{aligned}
w \equiv \frac{W}{P} &= \left[\gamma A_T^{\eta-1}\omega_T^{\eta-1} + (1-\gamma)A_N^{\eta-1}\right]^{\frac{1}{\eta-1}} \\
&= A\left[1+\gamma\left(\frac{A_T}{A}\right)^{\eta-1}\left(\omega_T^{\eta-1}-1\right)\right]^{\frac{1}{\eta-1}}, \qquad (42)\\
nx \equiv \frac{NX}{Y} &= \gamma\kappa\tau^{1-\rho}\left[\left(\frac{W}{A_T}\right)^{1-\rho}\frac{Y^*}{Y} - \left(\frac{W}{A_T\omega_T}\right)^{\rho-\eta}\left(\frac{W}{w}\right)^{\eta-1}\right] \\
&= \gamma\kappa\left(\frac{\tau W}{A_T}\right)^{1-\rho}\frac{1}{W}\left[(1+nx)^{\frac{\varphi}{\sigma+\varphi}}\left[(A^*)^{\frac{1+\varphi}{\sigma+\varphi}}w^{\frac{\sigma-1}{\sigma+\varphi}}\right] - \frac{W^{2\rho-1}w^{1-\eta}}{A_T^{2\rho-\eta-1}\omega_T^{\rho-\eta}}\right], \qquad (43)
\end{aligned}$$

where A is the aggregate productivity defined in (16) and ω_T is the relative cost of tradables:

$$\omega_T \equiv \frac{P_H}{P_T} = \frac{W}{P_T A_T} = \left[(1-\kappa)+\kappa\left(\frac{W}{\tau A_T}\right)^{\rho-1}\right]^{\frac{1}{\rho-1}},$$

and where we used the definition (3) of P_T and the fact (4) that $P_H = W/A_T$, as well as the definition of expenditure $Y = PC$, and we used the fact that $Y^* = (A^*)^{\frac{1+\varphi}{\sigma+\varphi}}$.

Additionally, we are interested in characterizing the equilibrium value of χ from (12):

$$\chi = \left(\frac{A_N}{A_T}\right)^{\rho-\eta}\cdot\underbrace{\left(\frac{P_T}{P_H}\right)^{\rho-\eta}\cdot\left[(1-\kappa)+\kappa\cdot\frac{\tau^{1-\rho}Y^*}{P_T^{\rho-\eta}P^\eta C}\right]}_{=\omega_T^{\eta-1} \text{ iff } NX=0 \text{ by Lemma 1}}.$$

Note that (43) characterizes W, which is needed to recover w from (42), both as a function of nx. With this we can recover the full equilibrium vector as a function of nx, as well as productivities A_T and A_N.

The two sources of CES non-linearity in the solution (in w and ω_T) can be approximated as follows:

$$\omega_T \approx \left(\frac{W}{\tau A_T}\right)^{\kappa} \qquad \text{and} \qquad w \approx A\,\omega_T^{\gamma} \approx A\left(\frac{W}{\tau A_T}\right)^{\kappa\gamma}, \tag{44}$$

where the first approximation is exact in the Cobb-Douglas limit ($\eta \to 1$) and the second approximation is exact when $W/(\tau A_T) \to 1$ (or when $\kappa \approx 0$ or 1). Note that we assume $\rho \gg 1$, so do not consider the approximation around $\rho = 1$. With large ρ, the approximation that $W \approx A_T$ (when $\tau \approx 1$) is in fact quite reasonable, as we see below.

With these approximations, the system simplifies as follows:

$$nx = \frac{\gamma\kappa\left(\frac{\tau W}{A_T}\right)^{1-\rho}\frac{A^{1-\eta}}{W}}{\left(\frac{W}{\tau A_T}\right)^{\kappa[(\rho-\eta)-\gamma(1-\eta)]}}\left[(1+nx)^{\frac{\varphi}{\sigma+\varphi}}(A^*)^{\frac{1+\varphi}{\sigma+\varphi}}A^{\frac{\sigma-1}{\sigma+\varphi}-(1-\eta)}\left(\frac{W}{\tau A_T}\right)^{\kappa\gamma\frac{\sigma-1}{\sigma+\varphi}+\kappa[(\rho-\eta)-\gamma(1-\eta)]} - \frac{W^{2\rho-1}}{A_T^{2\rho-\eta-1}}\right],$$

In the Cobb-Douglas case ($\eta = 1$), this simplifies to:

$$nx\frac{W\left(\frac{W}{A_T}\right)^{(\rho-1)-\kappa\gamma\frac{\sigma-1}{\sigma+\varphi}}}{\gamma\kappa\tau^{(1-\rho)(1-\kappa)}} = \left[\frac{(1+nx)^{\frac{\varphi}{\sigma+\varphi}}}{\tau^{\kappa\gamma\frac{\sigma-1}{\sigma+\varphi}+\kappa(\rho-1)}}(A^*)^{\frac{1+\varphi}{\sigma+\varphi}}A^{\frac{\sigma-1}{\sigma+\varphi}} - W\left(\frac{W}{A_T}\right)^{2(\rho-1)-\kappa\gamma\frac{\sigma-1}{\sigma+\varphi}-\kappa(\rho-1)}\right],$$

Balanced trade $NX = 0$ and hence $nx = 0$, simplifies the characterization of W in (43) to:

$$W = A_T^{\frac{2\rho-\eta-1}{2\rho-1}}\,\omega_T^{\frac{\rho-\eta}{2\rho-1}}\left[(A^*)^{\frac{1+\varphi}{\sigma+\varphi}}\big(Af(\omega_T)\big)^{\frac{\sigma-1}{\sigma+\varphi}-(1-\eta)}\right]^{\frac{1}{2\rho-1}},$$

where $f(\omega_T) = \left[1+\gamma\left(\frac{A_T}{A}\right)^{\eta-1}\left(\omega_T^{\eta-1}-1\right)\right]^{1/(\eta-1)}$ and ω_T is an increasing function of W/A_T with an elasticity approximately given by $\kappa \in [0,1]$, so that $f(\omega_T)$ is an increasing function of W/A_T with an elasticity approximately $\gamma\kappa$.

In the special case of Cobb-Douglas ($\eta = 1$) and no home bias ($\kappa = 1$) we obtain a closed-form solution (using (44), which is exact in this case):

$$W = \tau^{-\frac{\rho-1+\gamma\frac{\sigma-1}{\sigma+\varphi}}{\rho-\gamma\frac{\sigma-1}{\sigma+\varphi}}}\left[A_T^{\rho-1}A_N^{(1-\gamma)\frac{\sigma-1}{\sigma+\varphi}}(A^*)^{\frac{1+\varphi}{\sigma+\varphi}}\right]^{\frac{1}{\rho-\gamma\frac{\sigma-1}{\sigma+\varphi}}}$$

Outside this case, we have a log-differential solution:

$$\hat{w} =,$$
$$\hat{W} = .$$

CONDITION FOR $P_T < P_H$:

$$1 < \frac{W}{\tau A_T} = \frac{1}{\tau}\omega_T^{\frac{\rho-\eta}{2\rho-1}}\left[A_T^{-\eta}(A^*)^{\frac{1+\varphi}{\sigma+\varphi}}\left(Af(\omega_T)\right)^{\frac{\sigma-1}{\sigma+\varphi}-(1-\eta)}\right]^{\frac{1}{2\rho-1}},$$

WHICH ESSENTIALLY REQUIRES THAT τ IS NOT VERY HIGH, SINCE $W > A_T$

THE APPROXIMATE SOLUTION $\eta = 1$ and ω_T approximation:

$$W = \left[\frac{A_T^{\rho-1+(1-\kappa)(\rho-1)+(1-\kappa)\gamma\frac{\sigma-1}{\sigma+\varphi}} A_N^{(1-\gamma)\frac{\sigma-1}{\sigma+\varphi}} A^{*\frac{1+\varphi}{\sigma+\varphi}}}{\tau^{\kappa\gamma\frac{\sigma-1}{\sigma+\varphi}+\kappa(\rho-1)}}\right]^{\frac{1}{\rho+(1-\kappa)(\rho-1)-\kappa\gamma\frac{\sigma-1}{\sigma+\varphi}}}$$

and

$$C = w^{\frac{1+\varphi}{\sigma+\varphi}}, \quad \text{where} \quad w \equiv \frac{W}{P} = A\left(\frac{W}{\tau A_T}\right)^{\kappa\gamma} \quad \text{and} \quad \frac{W}{\tau A_T} = \left[\frac{A^{\frac{\sigma-1}{\sigma+\varphi}} A^{*\frac{1+\varphi}{\sigma+\varphi}}}{A_T\tau^{2\rho-1}}\right]^{\frac{1}{\rho+(1-\kappa)(\rho-1)-\kappa\gamma\frac{\sigma-1}{\sigma+\varphi}}}$$

Results:

1. W is homogenous degree one in (A_T, A_N, A^*), increasing in all of them when $\sigma > 1$, and additionally W decreases in τ.

   Furthermore, if $\sigma \geq 1$, $\tau = 1$ and $A_T(t), A_N(t) < A^*$, it is sufficient for

   $$\min\{A_T(t), A_N(t)\} < W(t) < A^*.$$

   And if $\sigma = 1$, then $A_T(t) < W(t) < A^*$.

2. When $\sigma = 1$, W does not depend on A_N, and it is decreasing in A_N iff $\sigma < 1$.

   When $\sigma = \eta = 1$:

   $$W = \left[\frac{A_T^{(2-\kappa)(\rho-1)}A^*}{\tau^{\kappa(\rho-1)}}\right]^{\frac{1}{1+(2-\kappa)(\rho-1)}}$$

3. As $\rho \to \infty$ (tradable goods become perfect substitutes internationally), $W = A_T\tau^{-\frac{\kappa}{2-\kappa}}$, and does not depend on A_N and A^*.

4. In the limit of the closed economy $\kappa \to 0$,

$$W = \left[A_T^{2(\rho-1)} A^{\frac{\sigma-1}{\sigma+\varphi}} A^{*\frac{1+\varphi}{\sigma+\varphi}}\right]^{\frac{1}{2\rho-1}}$$

5. Real wage:

$$w = \left[\frac{A^{1+(2-\kappa)(\rho-1)} A^{*\kappa\gamma\frac{1+\varphi}{\sigma+\varphi}}}{A_T^{\kappa\gamma}\tau^{\kappa\gamma(2\rho-1)}}\right]^{\frac{1}{1+(2-\kappa)(\rho-1)-\kappa\gamma\frac{\sigma-1}{\sigma+\varphi}}}$$

Real wages is between $A(t)$ and A^* if $\tau = 1$ and $\sigma = 1$, otherwise the condition is:

$$\frac{(A^*/A_T)^{\frac{1+\varphi}{\sigma+\varphi}}}{\tau^{2\rho-1}} > (A_T/A)^{\frac{\sigma-1}{\sigma+\varphi}}$$

The Cobb-Douglas case ($\eta = 1$) Full system:

$$\begin{aligned}
w &= A\omega_T^{\gamma} \qquad \text{with} \qquad \omega_T = \left[(1-\kappa) + \kappa\left(\frac{W}{\tau A_T}\right)^{\rho-1}\right]^{\frac{1}{\rho-1}}, \\
nx &= \gamma\kappa\left(\frac{\tau W}{A_T}\right)^{1-\rho}\frac{1}{W}\left[(1+nx)^{\frac{\varphi}{\sigma+\varphi}}\left[(A^*)^{\frac{1+\varphi}{\sigma+\varphi}} w^{\frac{\sigma-1}{\sigma+\varphi}}\right] - \frac{W^{2\rho-1}}{A_T^{2(\rho-1)}\omega_T^{\rho-1}}\right], \\
C &= w^{\frac{1+\varphi}{\sigma+\varphi}}(1+nx)^{-\frac{\varphi}{\sigma+\varphi}} = \bar{C}
\end{aligned}$$

and

$$\int_0^{\infty} \beta^t NX_t \mathrm{d}t = 0, \qquad \text{where} \qquad NX = nx \cdot Y = \frac{nx \cdot WC}{w},$$

Above we used $P = W/w$ and $Y = PC = WC/w$, and we also have $L = w^{\frac{1-\sigma}{\sigma+\varphi}}(1+nx)^{\frac{\sigma}{\sigma+\varphi}}$. From this, we can further recover the equilibrium values of sectoral prices (P_T, P_N) and sectoral allocations (C_T, C_N, L_T, L_N). Constant consumption at some level $\bar{C}$ is the result of the perfect consumption smoothing under $\beta R^* = 1$.

From the three-equation system above we can solve for (W, w, nx) as a function of $(A, A_T, \bar{C})$ and (A^*, τ) and other parameters, while the last integral condition pins down the equilibrium level of $\bar{C}$. The first line implies a monotonic parametric relationship between $W/(\tau A_T)$ and w/A, such that:

$$w = Ah\left(\frac{W}{\tau A_T}\right) \qquad \text{and} \qquad W = \tau A_T H\left(\frac{w}{A}\right), \tag{45}$$

where function $h(x) \equiv \left[1-\kappa+\kappa x^{\rho-1}\right]^{\gamma/(\rho-1)}$ and has $h'(\cdot) > 0$, $h''(\cdot) < 0$ and $H(\cdot) = h^{-1}(\cdot)$, and with the property that both $x/h(x)$ and $x/h(x)^{1/\gamma}$ increase in x. Furthermore, the third equation implies that w is increasing in nx and in $\bar{C}$:

$$w = (1+nx)^{\frac{\varphi}{1+\varphi}} \bar{C}^{\frac{\sigma+\varphi}{1+\varphi}}. \tag{46}$$

This leaves us with the last equation, which determines nx:

$$nx = \frac{\gamma\kappa}{\tau^{2(\rho-1)}} \left(\frac{W}{\tau A_T}\right)^{1-\rho} \frac{1}{W} \left[(1+nx)^{\frac{\varphi}{\sigma+\varphi}} A^{*\frac{1+\varphi}{\sigma+\varphi}} A^{\frac{\sigma-1}{\sigma+\varphi}} \left(\frac{w}{A}\right)^{\frac{\sigma-1}{\sigma+\varphi}} - \frac{A_T \tau^{2\rho-1} \left(\frac{W}{\tau A_T}\right)^{2\rho-1}}{(1-\kappa) + \kappa \left(\frac{W}{\tau A_T}\right)^{\rho-1}} \right].$$

Using (46), we can rewrite this condition as:

$$nx = \gamma\kappa \frac{h\left(\frac{W}{\tau A_T}\right)}{\left(\frac{W}{\tau A_T}\right)^{\rho}} \left[\frac{1}{\tau^{2\rho-1}} \frac{A}{A_T} \frac{A^{*\frac{1+\varphi}{\sigma+\varphi}}}{\bar{C}} - \frac{\left(\frac{W}{\tau A_T}\right)^{2\rho-1}}{\left[(1-\kappa) + \kappa \left(\frac{W}{\tau A_T}\right)^{\rho-1}\right] h\left(\frac{W}{\tau A_T}\right)} \right] \tag{47}$$

Note that (46) is an increasing relationship between nx and w, while (47) (in light of (45)) defines a decreasing relationship, with $\bar{C}$ shifting the two curves.

We also use the expression for the level of net exports:

$$NX = \frac{W\bar{C}}{w} nx = \frac{\tau^{-2(\rho-1)}\gamma\kappa}{\left(\frac{W}{\tau A_T}\right)^{\rho-1}} \left[A^{*\frac{1+\varphi}{\sigma+\varphi}} - \frac{\tau^{2\rho-1} \bar{C} \frac{A_T}{A} \left(\frac{W}{\tau A_T}\right)^{2\rho-1}}{\left[(1-\kappa) + \kappa \left(\frac{W}{\tau A_T}\right)^{\rho-1}\right] h\left(\frac{W}{\tau A_T}\right)} \right]. \tag{48}$$

Equations (45)–(48) allow to characterize the response of endogenous variables (w, W, nx, NX) to productivity (A_T, A_N, A^*), as well as to the endogenous level of consumption $\bar{C}$. Some of the results are particularly easy to see when we use approximation $h(x) \approx x^{\gamma\kappa}$, but they are true exactly as well. We can now provide a:

Proof of Lemma ?? and Proposition 7 Expand:

$$\begin{aligned}
\hat{w} &= \hat{A} + \frac{X^*}{Y}(\hat{W} - \hat{\tau} - \hat{A}_T), \\
\hat{w} &= \frac{\varphi}{1+\varphi}\widehat{nx} + \frac{\sigma+\varphi}{1+\varphi}\hat{C}, \\
\widehat{nx} &= \frac{X}{GDP}\left(\hat{A} - \hat{A}_T + \frac{1+\varphi}{\sigma+\varphi}\hat{A}^* - \hat{C} - (2\rho-1)\hat{\tau}\right) \\
&\quad - \frac{X^*}{GDP}\left[\frac{X}{X^*}\left(\rho - \frac{X^*}{Y}\right) + (\rho-1)\left(1 - \frac{X^*}{\gamma Y}\right)\right](\hat{W} - \hat{\tau} - \hat{A}_T),
\end{aligned}$$

where $\widehat{nx} \equiv \mathrm{d}\log(1+nx)$, $GDP \equiv Y + NX = WL$, and since $nx = \frac{NX}{Y} = \frac{X-X^*}{Y}$ and

$$\frac{xh'(x)}{h(x)} = \frac{\gamma\kappa x^{\rho-1}}{1-\kappa+\kappa x^{\rho-1}}\Big|_{x=\frac{W}{\tau A_T}} = \gamma\kappa\left(\frac{\tau}{P_T}\right)^{1-\rho} = \gamma\frac{P_F C_F}{P_T C_T} = \frac{X^*}{PC}.$$

Additionally we have:

$$\begin{aligned}\widehat{NX} &= \frac{\mathrm{d}NX}{GDP} = \widehat{nx} + \frac{X-X^*}{GDP}\hat{Y}\\ &= \frac{X}{GDP}\frac{1+\varphi}{\sigma+\varphi}\hat{A}^* - \frac{X^*}{GDP}(\hat{C}+\hat{A}_T-\hat{A}) - \left[\frac{X^*}{GDP} + \frac{X}{GDP}2(\rho-1)\right]\hat{\tau}\\ &- \frac{X^*}{GDP}\left[(\rho-1)\left(1+\frac{X}{X^*}-\frac{X^*}{\gamma Y}\right) + \left(1-\frac{X^*}{Y}\right)\right](\hat{W}-\hat{\tau}-\hat{A}_T),\end{aligned}$$

since $Y = PC = WC/w$. Note that with our approximation in the text $X^*/Y \approx \gamma\kappa$.

We now study the comparative statics with respect to $\bar{C}$ and productivity:

$$\hat{w} - \hat{A} = \frac{-\hat{A} + \frac{\sigma+\varphi\frac{Y-X^*}{GDP}}{1+\varphi}\hat{C} + \frac{X}{GDP}\left(\frac{\varphi}{1+\varphi}(\hat{A}-\hat{A}_T) + \frac{\varphi}{\sigma+\varphi}\hat{A}^*\right)}{1 + \frac{\varphi}{1+\varphi}\frac{Y}{GDP}\left[\frac{X}{X^*}\left(\rho - \frac{X^*}{Y}\right) + (\rho-1)\left(1-\frac{X^*}{\gamma Y}\right)\right]},$$

and

$$\begin{aligned}\widehat{NX} &= \frac{X}{GDP}\frac{1+\varphi}{\sigma+\varphi}\hat{A}^* - \frac{X^*}{GDP}(\hat{C}+\hat{A}_T-\hat{A})\\ &+ \Psi\left[\hat{A} - \frac{\sigma+\varphi\frac{Y-X^*}{GDP}}{1+\varphi}\hat{C} - \frac{X}{GDP}\left(\frac{\varphi}{1+\varphi}(\hat{A}-\hat{A}_T) + \frac{\varphi}{\sigma+\varphi}\hat{A}^*\right)\right],\end{aligned}$$

where $\Psi \equiv \frac{\frac{Y}{GDP}\left[(\rho-1)\left(1+\frac{X}{X^*}-\frac{X^*}{\gamma Y}\right)+\left(1-\frac{X^*}{Y}\right)\right]}{1+\frac{\varphi}{1+\varphi}\frac{Y}{GDP}\left[\frac{X}{X^*}\left(\rho-\frac{X^*}{Y}\right)+(\rho-1)\left(1-\frac{X^*}{\gamma Y}\right)\right]}$ such that $\frac{\varphi}{1+\varphi}\Psi \in (0,1)$.

Lastly, employment from (2) is simply proportional to $w^{1/\varphi}$ given $\bar{C}$, and in addition is declining in $\bar{C}^{\sigma/\varphi}$.

From this we can confirm (using the facts that $X < GDP = Y + X - X^*$ and $Y > \gamma Y > X^*$):

1. w and W increase and NX decreases in $\bar{C}$.
2. w, W and NX increase in A^* (holding $\bar{C}$ constant).
3. w and NX increase and w/A and W/A_T decrease in A (holding A_N/A_T constant).
4. w and NX increase and w/A decreases in A_T.
5. w and NX increase, and w/A deceases in A_N.

6. W decreases in A_N. W increases in A_T iff $\frac{\varphi}{1+\varphi}\frac{X}{GDP}\left[1+\frac{\rho-1}{\gamma}\frac{X^*}{X}\frac{1+\frac{X}{X^*}-\frac{X^*}{\gamma Y}}{1-\frac{X^*}{\gamma Y}}\right]>1$.
   W increases in A iff $\frac{X^*}{Y}\left[1+\frac{\varphi}{1+\varphi}\frac{Y}{GDP}\left[\frac{X}{X^*}\left(\rho-\frac{X^*}{Y}\right)+(\rho-1)\left(1-\frac{X^*}{\gamma Y}\right)\right]\right]>1$.
7. P necessarily falls in A_N and A, and may increase in A_T if ρ is sufficiently large
8. Employment L increases with w (for a given $\bar{C}$), but falls in $\bar{C}$.

Comparative statics in productivity:

$$\hat{w}-\hat{A}=\frac{-\hat{A}+\frac{X}{GDP}\frac{\varphi}{1+\varphi}(1-\gamma)(\hat{A}_N-\hat{A}_T)}{1+\frac{\varphi}{1+\varphi}\frac{Y}{GDP}\left[\frac{X}{X^*}\left(\rho-\frac{X^*}{Y}\right)+(\rho-1)\left(1-\frac{X^*}{\gamma Y}\right)\right]},$$
$$\widehat{NX}=\Psi\hat{A}-\left[1-\frac{X}{X^*}\frac{\varphi\Psi}{1+\varphi}\right]\frac{X^*}{GDP}(1-\gamma)(\hat{A}_T-\hat{A}_N),$$

Since $\hat{w}-\hat{A}=\frac{X^*}{Y}\left(\hat{W}-\hat{\tau}-\hat{A}_T\right)$, we have:

$$\hat{W}=\hat{A}_T-\frac{Y}{X^*}\frac{\hat{A}+\frac{X}{GDP}\frac{\varphi}{1+\varphi}(1-\gamma)(\hat{A}_T-\hat{A}_N)}{1+\frac{\varphi}{1+\varphi}\frac{Y}{GDP}\left[\frac{X}{X^*}\left(\rho-\frac{X^*}{Y}\right)+(\rho-1)\left(1-\frac{X^*}{\gamma Y}\right)\right]}$$

and therefore W is increasing in $\hat{A}=\hat{A}_T=\hat{A}_N$ when:

$$(\rho-1)\frac{\varphi}{1+\varphi}\frac{X^*}{GDP}\left(1-\frac{X^*}{\gamma Y}+\frac{X}{X^*}\right)>\left[1-\frac{\varphi}{1+\varphi}\frac{X}{GDP}\right]\left(1-\frac{X^*}{Y}\right).$$

One can verify that this inequality fails when $\rho\to 1$, while it holds for ρ sufficiently large.

For A_T:

$$\hat{w}=\frac{\frac{X}{X^*}\left(\rho-\frac{X^*}{\gamma Y}\right)+(\rho-1)\left(1-\frac{X^*}{\gamma Y}\right)}{1+\frac{\varphi}{1+\varphi}\frac{Y}{GDP}\left[\frac{X}{X^*}\left(\rho-\frac{X^*}{Y}\right)+(\rho-1)\left(1-\frac{X^*}{\gamma Y}\right)\right]}\frac{\varphi}{1+\varphi}\frac{Y}{GDP}\gamma\hat{A}_T,$$
$$\hat{W}=\frac{1+\frac{\varphi}{1+\varphi}\frac{Y}{GDP}\left[\frac{X}{X^*}\left(\rho-\frac{X^*}{Y}\right)+(\rho-1)\left(1-\frac{X^*}{\gamma Y}\right)\right]-\frac{Y}{X^*}\gamma-\frac{Y}{X^*}\frac{X}{GDP}\frac{\varphi}{1+\varphi}(1-\gamma)}{1+\frac{\varphi}{1+\varphi}\frac{Y}{GDP}\left[\frac{X}{X^*}\left(\rho-\frac{X^*}{Y}\right)+(\rho-1)\left(1-\frac{X^*}{\gamma Y}\right)\right]}\hat{A}_T,$$
$$\widehat{NX}=\left[\Psi\gamma+\frac{X\frac{\varphi\Psi}{1+\varphi}-X^*}{GDP}(1-\gamma)\right]\hat{A}_T,$$

One can directly verify that the last expression in square brackets is always positive as:

$$\Psi\left(\gamma+(1-\gamma)\frac{\varphi}{1+\varphi}\frac{X}{GDP}\right)>(1-\gamma)\frac{X^*}{GDP},$$

because after substituting in Ψ and simplifying, it is equivalent to:

$$(\rho-1)\left(1-\frac{X^*}{\gamma Y}+\frac{X}{X^*}\right)\left[1+\frac{\varphi}{1+\varphi}\frac{1-\gamma}{\gamma}\frac{X-X^*}{GDP}\right]+\left(1-\frac{X^*}{\gamma Y}\right)>0,$$

as both terms are positive. Indeed, $\gamma Y>X^*$ and $\gamma GDP+(1-\gamma)(X-X^*)=\gamma Y-X^*+X>0$. Finally, W increases in A_T iff:

$$(\rho-1)\frac{\varphi}{1+\varphi}\frac{X^*}{GDP}\left(1-\frac{X^*}{\gamma Y}+\frac{X}{X^*}\right)>\gamma\left[1-\frac{\varphi}{1+\varphi}\frac{X}{GDP}\right]\left(1-\frac{X^*}{\gamma Y}\right),$$

which is naturally a weaker condition than for W to increase in A, but this condition also holds for sufficiently large ρ and is violated for $\rho\to 1$.

For A_N:

$$\begin{aligned}
\frac{X^*}{Y}\hat{W}=\hat{w}-\hat{A}&=-\frac{1-\frac{X}{GDP}\frac{\varphi}{1+\varphi}}{1+\frac{\varphi}{1+\varphi}\frac{Y}{GDP}\left[\frac{X}{X^*}\left(\rho-\frac{X^*}{Y}\right)+(\rho-1)\left(1-\frac{X^*}{\gamma Y}\right)\right]}(1-\gamma)\hat{A}_N,\\
\hat{w}&=\frac{\frac{X}{X^*}\rho+(\rho-1)\left(1-\frac{X^*}{\gamma Y}\right)}{1+\frac{\varphi}{1+\varphi}\frac{Y}{GDP}\left[\frac{X}{X^*}\left(\rho-\frac{X^*}{Y}\right)+(\rho-1)\left(1-\frac{X^*}{\gamma Y}\right)\right]}\frac{\varphi}{1+\varphi}\frac{Y}{GDP}(1-\gamma)\hat{A}_N,\\
\widehat{NX}&=\left[\Psi\left(1-\frac{\varphi}{1+\varphi}\frac{X}{GDP}\right)+\frac{X^*}{GDP}\right](1-\gamma)\hat{A}_N.
\end{aligned}$$

Comparative statics for P and L

$$P=\frac{W}{w}=\frac{W/(\tau A_T)}{w/A}\frac{\tau A_T}{A}.$$

Therefore,

$$\begin{aligned}
\hat{P}&=\left(\frac{Y}{X^*}-1\right)(\hat{w}-\hat{A})+\left(\hat{A}_T-\hat{A}\right)\\
&=\frac{\left[1+(\rho-1)\frac{\varphi}{1+\varphi}\frac{Y}{GDP}\left(1-\frac{X^*}{\gamma Y}+\frac{X}{X^*}\right)\right](1-\gamma)\left(\hat{A}_T-\hat{A}_N\right)-\frac{Y}{X^*}\left(1-\frac{X^*}{Y}\right)\hat{A}}{1+\frac{\varphi}{1+\varphi}\frac{Y}{GDP}\left[\frac{X}{X^*}\left(1-\frac{X^*}{Y}\right)+(\rho-1)\left(1-\frac{X^*}{\gamma Y}+\frac{X}{X^*}\right)\right]},
\end{aligned}$$

and we have P declining in A and A_N and increasing in A_T iff:

$$1+\frac{(\rho-1)(1-\gamma)\frac{\varphi}{1+\varphi}\frac{X^*}{GDP}-\gamma}{(\rho-1)(1-\gamma)\frac{\varphi}{1+\varphi}\frac{X}{GDP}}\left(1-\frac{X^*}{\gamma Y}\right)>0,$$

which holds for ρ sufficiently large and fails for $\rho\to 1$.

Lastly, from (37), we have $L=w^{1/\varphi}\bar{C}^{-\sigma/\varphi}$, and the comparative statics for L is the same as for w, apart from L decreasing in $\bar{C}$, which can be verified directly. As $\varphi\to\infty$, L is given by an exogenous constant. ■

Lemma 5 *ULC with financial openness:*

$$w^a(0)=C^a(0)<C^b(0)=w^b(0)<w(0)<\hat{C}.$$

Proof: Denote with $x\equiv W/(\tau A_T)$ and with $z\equiv C/A^{\frac{1+\varphi}{\sigma+\varphi}}$. Then we can write:

$$\frac{h(x)^{\frac{1+\varphi}{\varphi}}}{z^{\frac{\sigma+\varphi}{\varphi}}}-1=\frac{\gamma\kappa}{x^{\rho-\kappa\gamma}}\left[\tau^{1-2\rho}\frac{A^{*\frac{1+\varphi}{\sigma+\varphi}}A^{\frac{\sigma-1}{\sigma+\varphi}}}{A_T}\frac{1}{z}-x^{(1-\kappa\gamma)+(2-\kappa)(\rho-1)}\right].\tag{49}$$

Under balanced trade $NX=0$ and the LHS of (49) is zero, and hence we can solve for:

$$z=h(x)^{\frac{1+\varphi}{\sigma+\varphi}},$$
$$x^{(1-\kappa\gamma)+(2-\kappa)(\rho-1)}h(x)^{\frac{1+\varphi}{\sigma+\varphi}}=\tau^{1-2\rho}\frac{A^{*\frac{1+\varphi}{\sigma+\varphi}}A^{\frac{\sigma-1}{\sigma+\varphi}}}{A_T},$$

which characterizes $w^b(t)$ and $C^b(t)$ as a function of productivity and trade costs.

We next expand (49):

$$\left[\frac{1+\varphi}{\varphi}\frac{h(x)^{\frac{1+\varphi}{\varphi}}}{z^{\frac{\sigma+\varphi}{\varphi}}}\frac{X^*}{Y}+(\rho-\kappa\gamma)\left(\frac{h(x)^{\frac{1+\varphi}{\varphi}}}{z^{\frac{\sigma+\varphi}{\varphi}}}-1\right)+\gamma\kappa[(1-\kappa\gamma)+(2-\kappa)(\rho-1)]x^{(1-\kappa)(\rho-1)}\right]\hat{x}$$
$$=\left[\frac{\sigma+\varphi}{\varphi}\frac{h(x)^{\frac{1+\varphi}{\varphi}}}{z^{\frac{\sigma+\varphi}{\varphi}}}-\frac{\gamma\kappa}{x^{\rho-\kappa\gamma}}\tau^{1-2\rho}\frac{A^{*\frac{1+\varphi}{\sigma+\varphi}}A^{\frac{\sigma-1}{\sigma+\varphi}}}{A_T}\frac{1}{z}\right]\hat{z}.$$

AROUND $NX=0$:

$$\hat{x}=\frac{\frac{\sigma+\varphi}{\varphi}-\gamma\kappa x^{(1-\kappa)(\rho-1)}}{\frac{1+\varphi}{\varphi}\frac{X^*}{Y}+\gamma\kappa[(1-\kappa\gamma)+(2-\kappa)(\rho-1)]x^{(1-\kappa)(\rho-1)}}\hat{z}.$$

CONCLUSION: x increases in z under a mild regularity condition. Since $C(0)=\hat{C}>C^b(0)$, this means that $w(0)>w^b(0)$. Furthermore, we know that $w(0)<\hat{C}$ from (37) since

$nx(0) < 0$. ■

A.3 Log-linearized solution for static equilibrium

Productivity dynamics Start by log-linearizing (26)–(27) and (36):

$$1+\frac{\rho-1}{\delta}\frac{\dot{A}_T}{A_T}=\frac{\left(\frac{\bar{A}}{A_T}\right)^{\rho-1}\left(\frac{A_N}{A_T}\right)^{\theta-(\rho-1)}\left[1+\kappa\left(\frac{\tau}{P_T}\right)^{1-\rho}\frac{NX}{X^*}\right]^{\frac{\theta}{\rho-1}-1}}{\left[1-\gamma+\gamma\left(\frac{A_N}{A_T}\right)^{\theta}\left[1+\kappa\left(\frac{\tau}{P_T}\right)^{1-\rho}\frac{NX}{X^*}\right]^{\frac{\theta}{\rho-1}}\right]^{\nu}},$$

$$1+\frac{\rho-1}{\delta}\frac{\dot{A}_N}{A_N}=\frac{\left(\frac{\bar{A}}{A_N}\right)^{\rho-1}}{\left[1-\gamma+\gamma\left(\frac{A_N}{A_T}\right)^{\theta}\left[1+\kappa\left(\frac{\tau}{P_T}\right)^{1-\rho}\frac{NX}{X^*}\right]^{\frac{\theta}{\rho-1}}\right]^{\nu}},$$

and therefore:

$$\frac{1+\frac{\rho-1}{\delta}\frac{\dot{A}_T}{A_T}}{1+\frac{\rho-1}{\delta}\frac{\dot{A}_N}{A_N}}=\left(\frac{A_N}{A_T}\right)^{\theta}\left[1+\kappa\left(\frac{\tau}{P_T}\right)^{1-\rho}\frac{NX}{X^*}\right]^{\frac{\theta}{\rho-1}-1}$$

where we used:

$$\frac{\pi_T}{\gamma}=\frac{\chi^{\frac{\theta}{\rho-1}}}{1-\gamma+\gamma\chi^{\frac{\theta}{\rho-1}}}=\frac{\left(\frac{A_N}{A_T}\right)^{\theta}\left[1+\kappa\left(\frac{\tau}{P_T}\right)^{1-\rho}\frac{NX}{X^*}\right]^{\frac{\theta}{\rho-1}}}{1-\gamma+\gamma\left(\frac{A_N}{A_T}\right)^{\theta}\left[1+\kappa\left(\frac{\tau}{P_T}\right)^{1-\rho}\frac{NX}{X^*}\right]^{\frac{\theta}{\rho-1}}},$$

$$\frac{1-\pi_T}{1-\gamma}=\frac{1}{1-\gamma+\gamma\chi^{\frac{\theta}{\rho-1}}}=\frac{1}{1-\gamma+\gamma\left(\frac{A_N}{A_T}\right)^{\theta}\left[1+\kappa\left(\frac{\tau}{P_T}\right)^{1-\rho}\frac{NX}{X^*}\right]^{\frac{\theta}{\rho-1}}}$$

Approximation around $A_T(0)=A_N(0)=A(0)<\bar{A}$ and $NX=0$ (implying $\chi=1$ and $\pi_T=\gamma$), and $X^*/Y=\gamma\kappa$ and $P_T=\tau=1$:

$$\begin{aligned}\dot{a}_T&=G_0+g_0\left[-(\rho-1)a_T+\frac{\nu}{\gamma}(\pi_T-\gamma)\right],\\ \dot{a}_N&=G_0+g_0\left[-(\rho-1)a_N-\frac{\nu}{1-\gamma}(\pi_T-\gamma)\right],\\ 2(1-\nu)(\pi_T-\gamma)=\chi-1&=(\rho-1)(a_N-a_T)+\frac{1}{\gamma}\widehat{nx}\end{aligned}$$

where $a_T\equiv\log A_T$, $g_0\equiv\frac{\delta}{\rho-1}\left(\frac{\bar{A}}{A(0)}\right)^{\rho-1}$ and $G_0\equiv g_0-\frac{\delta}{\rho-1}$ is the growth rate under balanced trade $nx = 0$ (and $\pi_T = \gamma$), and $\widehat{nx} = NX/Y(0)$. One interesting observation for aggregate productivity:

$$\dot{a} = \gamma \dot{a}_T + (1-\gamma)\dot{a}_N = g_0 - G_0(\rho - 1)a,$$

and it the effects of tilting wash out to first order (of course, this is only true when approximating around in a symmetric state $A_T(0) = A_N(0)$). Further, combine the equations to obtain:

$$\begin{aligned}\dot{a}_T - \dot{a}_N &= G_0\left[-(\rho-1)(a_T - a_N) + \frac{\nu}{\gamma(1-\gamma)}(\pi_T - \gamma)\right] \\ &= G_0\left[-(\rho-1)\left(1 + \frac{1}{2\gamma(1-\gamma)}\frac{\nu}{1-\nu}\right)(a_T - a_N) + \frac{1}{2\gamma^2(1-\gamma)}\frac{\nu}{1-\nu}\widehat{nx}\right],\end{aligned}$$

where the first terms if the "convergence" terms and the second term is the endogenous feedback from trade deficit. This equation can be taken to the data.

A.4 Properties of the Frechet distribution

Consider $x \sim Frechet(T, \theta)$ with cdf $F(x) = e^{-Tx^{-\theta}}$. List of properties:

1. The mean is:

$$\mathbb{E}x = T^{1/\theta}\,\Gamma\left(1 - \frac{1}{\theta}\right)$$

   Proof: $\mathbb{E}x = \int_0^\infty x \mathrm{d}\left(e^{-Tx^{-\theta}}\right) = -\int_0^\infty x e^{-Tx^{-\theta}} \mathrm{d}\big(\overbrace{Tx^{-\theta}}^{\equiv z}\big) = T^{1/\theta}\int_0^\infty z^{-1/\theta}e^{-z}\mathrm{d}z = T^{1/\theta}\Gamma(1 - 1/\theta)$, where Gamma-function is $\Gamma(\alpha) = \int_0^\infty x^{\alpha-1}e^{-x}\mathrm{d}x$. ■

2. If $x \sim Frechet(T, \theta)$, then $ax \sim Frechet(Ta^\theta, \theta)$ and $x^\alpha \sim Frechet(T, \theta/\alpha)$

   Proof: $\mathbb{P}\{ax < z\} = \mathbb{P}\{x < z/a\} = e^{-Ta^\theta z^{-\theta}}$ and $\mathbb{P}\{x^\alpha < z\} = e^{-Tz^{-\theta/\alpha}}$. ■

3. If $x_1, \ldots, x_n$ are iid Frechet with the same shape parameter θ, but different means $T_1, \ldots, T_n$, then

$$\max\{x_1, \ldots, x_n\} \sim Frechet(T_1 + \ldots + T_n, \theta).$$

   Proof: $\mathbb{P}\big\{\max\{x_1, x_2, \ldots, x_n\} < z\big\} = \prod_1^n \mathbb{P}\{x_i < z\} = e^{-(T_1+\ldots+T_n)e^{-\theta}}$. ■

   This can be generalized to correlated draws...

4. In the same case, $\mathbb{P}\big\{x_j \geq \max\{x_1, x_2, \ldots, x_n\}\big\} = \frac{T_j}{T_1+\ldots+T_n}$.[43]

   Proof: $\mathbb{P}\big\{x_j \geq \max\{x_1, x_2, \ldots, x_n\}\big\} = \int_0^\infty \prod_{i\neq j} F_i(x_j)\mathrm{d}F_j(x_j) = \frac{T_j}{T_1+\ldots+T_n} \underbrace{\int_0^\infty \mathrm{d}F_{\max}(z)}_{=1}$,

   where the substitution of variables is again $z = T_j x_j^{-\theta}$ and $F_{\max}(z) = \mathbb{P}\big\{\max\{x_1, \ldots, x_n\} < z\big\}$.

5. $\mathbb{P}\big\{x_1 < z | x_1 \geq \max\{x_1, x_2, \ldots, x_n\}\big\} = \mathbb{P}\big\{\max\{x_1, x_2, \ldots, x_n\} < z\big\} = F_{\max}(z)$, which is the cdf of $Frechet(T_1 + \ldots + T_n, \theta)$.

   Proof: $\mathbb{P}\big\{x_1 < z | x_1 \geq \max\{x_1, x_2, \ldots, x_n\}\big\} = \frac{\mathbb{P}\{x_1 < z \cup x_1 \geq \max\{x_1, x_2, \ldots, x_n\}\}}{\mathbb{P}\big\{x_1 \geq \max\{x_1, x_2, \ldots, x_n\}\big\}}$ $= \frac{\mathbb{P}\{\max\{x_1, x_2, \ldots, x_n\} < z \cup x_1 \geq \max\{x_1, x_2, \ldots, x_n\}\}}{\mathbb{P}\big\{x_1 \geq \max\{x_1, x_2, \ldots, x_n\}\big\}} = \frac{\mathbb{P}\{\max\{x_1, x_2, \ldots, x_n\} < z\}\mathbb{P}\{x_1 \geq \max\{x_1, x_2, \ldots, x_n\}\}}{\mathbb{P}\big\{x_1 \geq \max\{x_1, x_2, \ldots, x_n\}\big\}} =$ $\mathbb{P}\{\max\{x_1, x_2, \ldots, x_n\} < z\}$ and the only part that requires confirmation of the independence of $\mathbf{1}\{\max\{x_1, x_2, \ldots, x_n\} < z\}$ and $\mathbf{1}\{x_1 \geq \max\{x_1, x_2, \ldots, x_n\}\}$. It is easy to prove the stronger claim that $\mathbb{P}\big\{x_1 \geq \max\{x_1, \ldots, x_n\}\big| \max\{x_1, \ldots, x_n\} < z\}$ does not depend on z and equals the unconditional probability $\mathbb{P}\big\{x_1 \geq \max\{x_1, x_2, \ldots, x_n\}\big\} = \frac{T_1}{T_1+\ldots+T_n}$. Indeed: $\mathbb{P}\big\{x_1 \geq \max\{x_1, \ldots, x_n\} \cup \max\{x_1, \ldots, x_n\} < z\big\} = \int_0^z \prod_{i\neq 1} F_i(x_1)\mathrm{d}F_1(x_1) = \frac{T_1}{T_1+\ldots+T_n} \int_0^z \mathrm{d}F_{\max}(z) = \frac{T_1}{T_1+\ldots+T_n} \mathbb{P}\big\{\max\{x_1, \ldots, x_n\} < z\big\}$.

A.5 Dynamics and efficiency in the closed economy

Proof of Proposition 2: Consider the general productivity dynamics system (26)–(27), starting from some initial condition $\big(A_T(0), A_N(0)\big)$, in which we treat $\pi_T(t)$ as a control. Indeed, this dynamic system obtains directly from the entrepreneur's choice of project characterized by $\max\Big\{(1+\varsigma)\hat{\Pi}_T, \hat{\Pi}_N\Big\}$, where $\varsigma(t) \in [-1, \infty)$ is the control (e.g., subsidy), which allows to trace all possible values $\pi_T(t) \in [0, 1]$ at any point in time t.

Consider now the problem of choosing $\{\pi_T(t)\}_{t\in[0,s]}$ to maximize aggregate productivity $A(s)$, as defined in (16), at some finite time $s > 0$. The Hamiltonian for this optimization

[43] Note: In 4 and 5 we use the following result:

$$\int_0^z F_2(x_1)\mathrm{d}F_1(x_1) = -\int_0^z e^{-(T_1+T_2)x_1^{-\theta}} \mathrm{d}(T_1 x_1^{-\theta}) = \frac{T_1}{T_1+T_2} \int_{(T_1+T_2)z^{-\theta}}^\infty e^{-y}\mathrm{d}y$$
$$= -\frac{T_1}{T_1+T_2} e^{-y}\Big|_{(T_1+T_2)z^{-\theta}}^\infty = \frac{T_1}{T_1+T_2} e^{-(T_1+T_2)z^{-\theta}} = \frac{T_1}{T_1+T_2} F_{\max}(z),$$

or equivalently:

$$\int_0^z F_2(x_1)\mathrm{d}F_1(x_1) = \frac{T_1}{T_1+T_2} \int_0^z e^{-(T_1+T_2)x_1^{-\theta}} \mathrm{d}(-(T_1+T_2)x_1^{-\theta})$$
$$= \frac{T_1}{T_1+T_2} \int_0^z \mathrm{d}\left(e^{-(T_1+T_2)x_1^{-\theta}}\right) = \frac{T_1}{T_1+T_2} F_{\max}(z).$$

problem is:

$$\mathcal{H}(t) = \frac{\mu_T(t)}{\rho-1}\left[\lambda\left(\frac{\pi_T(t)}{\gamma}\right)^{\nu} A^{*\rho-1}A_T(t)^{2-\rho} - \delta A_T(t)\right] + \frac{\mu_N(t)}{\rho-1}\left[\lambda\left(\frac{1-\pi_T(t)}{1-\gamma}\right)^{\nu} A^{*\rho-1}A_N(t)^{2-\rho} - \delta A_N(t)\right],$$

where μ_T and μ_N are the co-state variables. The Pontryagin's maximum principle requires that the following optimality conditions hold for all $t \in (0, s)$:

$$\begin{aligned}
\dot{\mu}_T &= -\frac{\partial \mathcal{H}}{\partial A_T} = \frac{\mu_T}{\rho-1}\left[(\rho-2)\lambda\left(\frac{\pi_T}{\gamma}\right)^{\nu} A^{*\rho-1}A_T^{1-\rho} + \delta\right],\\
\dot{\mu}_N &= -\frac{\partial \mathcal{H}}{\partial A_N} = \frac{\mu_N}{\rho-1}\left[(\rho-2)\lambda\left(\frac{1-\pi_T}{1-\gamma}\right)^{\nu} A^{*\rho-1}A_N^{1-\rho} + \delta\right],\\
0 &= \quad \frac{\partial \mathcal{H}}{\partial \pi_T} = \frac{\nu\lambda A^{*\rho-1}}{\rho-1}\left[\frac{\mu_T A_T^{2-\rho}}{\pi_T^{1-\nu}\gamma^{\nu}} - \frac{\mu_N A_N^{2-\rho}}{(1-\pi_T)^{1-\nu}(1-\gamma)^{\nu}}\right],
\end{aligned}$$

and optimality at the right end (for $t = s$):

$$\begin{aligned}
\mu_T(s) &= \frac{\partial A(s)}{\partial A_T(s)} = \gamma\left(\frac{A_T(s)}{A(s)}\right)^{\eta-2},\\
\mu_N(s) &= \frac{\partial A(s)}{\partial A_N(s)} = (1-\gamma)\left(\frac{A_N(s)}{A(s)}\right)^{\eta-2}.
\end{aligned}$$

We simplify the set of dynamic optimality conditions as:

$$\begin{aligned}
\frac{\dot{\mu}_T}{\mu_T} &= (\rho-2)B\left(\frac{\pi_T}{\gamma}\right)^{\nu} A_T^{1-\rho} + D,\\
\frac{\dot{\mu}_N}{\mu_N} &= (\rho-2)B\left(\frac{1-\pi_T}{1-\gamma}\right)^{\nu} A_N^{1-\rho} + D,\\
\frac{\dot{A}_T}{A_T} &= B\left(\frac{\pi_T}{\gamma}\right)^{\nu} A_T^{1-\rho} - D,\\
\frac{\dot{A}_N}{A_N} &= B\left(\frac{1-\pi_T}{1-\gamma}\right)^{\nu} A_N^{1-\rho} - D,
\end{aligned}$$

where $B \equiv \lambda A^{*\rho-1}/(\rho-1)$ and $D \equiv \delta/(\rho-1)$, and the static optimality condition for π_T as:

$$\left(\frac{\pi_T}{1-\pi_T}\frac{1-\gamma}{\gamma}\right)^{1-\nu} = \frac{1-\gamma}{\gamma}\frac{\mu_T A_T^{2-\rho}}{\mu_N A_N^{2-\rho}}. \tag{50}$$

This implies the following optimal evolution of π_T:

$$(1-\nu)\frac{\mathrm{d}}{\mathrm{d}t}\log\frac{\pi_T}{1-\pi_T} = \left(\frac{\dot{\mu}_T}{\mu_T} + (2-\rho)\frac{\dot{A}_T}{A_T}\right) - \left(\frac{\dot{\mu}_N}{\mu_N} + (2-\rho)\frac{\dot{A}_N}{A_N}\right) = 0.$$

That is, $\pi_T(t) = const$ for all $t \in (0,s)$ maximizes $A(s)$, and therefore we can have either $\pi_T \equiv \gamma$, or $\pi_T > \gamma$, or $\pi_T < \gamma$ at every point of the transition that maximizes terminal productivity $A(s)$.

To establish the optimal level of π_T, we consider the optimality at the right end, which we write as:

$$\frac{\bar{\mu}_T \bar{A}_T^{2-\eta}}{\bar{\mu}_N \bar{A}_N^{2-\eta}} = \frac{\gamma}{1-\gamma}. \tag{51}$$

Since μ_J and A_J are non-jump variables, while π_T is a constant on $t \in (0,s)$, we can consider the limit $t \to s$, and combining with the optimality for π_T (50), we have:

$$\left(\frac{\pi_T}{1-\pi_T}\frac{1-\gamma}{\gamma}\right)^{1-\nu} = \left(\frac{\bar{A}_N}{\bar{A}_T}\right)^{\rho-\eta}. \tag{52}$$

Recall that $\eta < \rho$.[44] It follows that

$$\operatorname{sign}\{\pi_T - \gamma\} = \operatorname{sign}\{(\rho-\eta)(\bar{A}_N - \bar{A}_T)\} = \operatorname{sign}\big\{(\rho-\eta)\big(A_N(0) - A_T(0)\big)\big\},$$

where the last claim is established by rolling the productivity evolution backwards.[45] This implies that $A(s)$-maximizing level of $\pi_T > \gamma$ iff $A_T(0) < A_N(0)$, and vice versa.

Lastly, we consider two special case when the optimal $\pi_T \equiv \gamma$. First, independently of the parameters η and ρ, this is the case when the initial condition is symmetric, $A_T(0) = A_N(0)$. Indeed, in this case $\pi_T = \gamma$ implies $A_T \equiv A_N$ for all t, including at $t = s$, which establishes the optimality of $\pi_T \equiv \gamma$ from (52). Second, independently of the initial condition $(A_T(0), A_N(0))$, $\pi_T \equiv \gamma$ is optimal if $\eta = \rho$, since in this case the dynamic optimality (50) is consistent with the terminal optimality (51) for $\pi_T = \gamma$. ■

Welfare maximization in the closed economy Substituting (33) into the household objective, we obtain the social welfare function in the closed economy, which only depends on

[44]This is sufficient for the second order condition to hold.

[45]For concreteness, consider the case with $\bar{A}_T > \bar{A}_N$. This implies $\pi_T < \gamma$, which favors the non-tradable sector during the transition ($\dot{A}_T < \dot{A}_N$). But this implies then that $A_T(0) - A_N(0) > \bar{A}_T - \bar{A}_N > 0$, and hence the initial and the terminal productivity differentials are of the same sign.

the path of aggregate productivity:

$$\mathbb{W}_0 \equiv \max \frac{\varphi+\sigma}{(1-\sigma)(1+\varphi)} \int_0^\infty \beta^t A(t)^{(1-\sigma)\frac{1+\varphi}{\sigma+\varphi}} \mathrm{d}t$$

Interesting special cases: 1) $\sigma = 0$ implies $\max \frac{\varphi}{1+\varphi} \int_0^\infty \beta^t A(t)\mathrm{d}t$ and 2) $\sigma = 1$ implies $\max \int_0^\infty \beta^t \log A(t)\mathrm{d}t + \frac{1}{\log\beta}\frac{1}{1+\varphi}$. The general Hamiltonian for optimal policy $\{\pi_T(t)\}$ is given by:

$$\begin{aligned}\mathcal{H}(t) = & \frac{\varphi+\sigma}{(1-\sigma)(1+\varphi)} A(t)^{(1-\sigma)\frac{1+\varphi}{\sigma+\varphi}} \\ & + \frac{\mu_T(t)}{\rho-1}\left[\lambda\left(\frac{\pi_T(t)}{\gamma}\right)^\nu A^{*\rho-1} A_T(t)^{2-\rho} - \delta A_T(t)\right] \\ & + \frac{\mu_N(t)}{\rho-1}\left[\lambda\left(\frac{1-\pi_T(t)}{1-\gamma}\right)^\nu A^{*\rho-1} A_N(t)^{2-\rho} - \delta A_N(t)\right].\end{aligned}$$

We denote with $\{\pi_T^*(t)\}_{t\geq 0}$ the path of optimal policy maximizing $\mathbb{W}_0$, and recall that $\pi_T(t) = \bar{\pi}_T(s)$ for all $t \in [0, s]$ is the optimal policy that maximizes $A(s)$ for some $s > 0$. We have the following result:

Proposition 9 (i) *If $A_T(0) = A_N(0)$, then $\pi_T^*(t) = \bar{\pi}_T(t) = \gamma$ for all $t \geq 0$.* (ii) *If $A_T(0) < A_N(0)$, then $\gamma < \pi_T^*(t) < \bar{\pi}_T(t)$ and $\frac{\partial}{\partial t}\pi_T^*(t) < 0$ for all $t \geq 0$, and $\lim_{t\to\infty}\pi_T^*(t) = \lim_{t\to\infty}\bar{\pi}_T(t) = \gamma$. The case with $A_T(0) > A_N(0)$ has an analogical characterization.*

Proof: We show that $\pi_T^*(t)$ is a forward-looking weighted average of $\{\bar{\pi}_T(s)\}_{s\geq t}\ldots$ ■

…

The optimality conditions are:

$$\dot{\mu}_T - \vartheta\mu_T = -\frac{\partial\mathcal{H}}{\partial A_T} = -\gamma\left(\frac{A_T}{A}\right)^{\eta-2} A^{\zeta-1} + \mu_T\frac{\delta}{\rho-1}\left[(\rho-2)\left(\frac{\bar{A}}{A_T}\right)^{\rho-1}\left(\frac{\pi_T}{\gamma}\right)^\nu + 1\right],$$

$$\dot{\mu}_N - \vartheta\mu_N = -\frac{\partial\mathcal{H}}{\partial A_N} = -(1-\gamma)\left(\frac{A_N}{A}\right)^{\eta-2} A^{\zeta-1} + \mu_N\frac{\delta}{\rho-1}\left[(\rho-2)\left(\frac{\bar{A}}{A_N}\right)^{\rho-1}\left(\frac{1-\pi_T}{1-\gamma}\right)^\nu + 1\right]$$

and

$$\left(\frac{\pi_T}{1-\pi_T}\frac{1-\gamma}{\gamma}\right)^{1-\nu} = \frac{1-\gamma}{\gamma}\frac{\mu_T}{\mu_N}\left(\frac{A_N}{A_T}\right)^{\rho-2},$$

where

$$\zeta \equiv (1-\sigma)\frac{1+\varphi}{\sigma+\varphi}.$$

Next, we do the following substitution of variables:

$$\xi_T \equiv \frac{\mu_T A_T}{\gamma} \qquad \text{and} \qquad \xi_N \equiv \frac{\mu_N A_N}{1-\gamma} \qquad \Rightarrow \qquad \frac{\dot{\xi}_J}{\xi_J} = \frac{\dot{\mu}_J}{\mu_J} + \frac{\dot{A}_J}{A_J},$$

and rewrite the optimality as:

$$\left(\frac{\pi_T}{1-\pi_T}\frac{1-\gamma}{\gamma}\right)^{1-\nu} = \frac{\xi_T}{\xi_N}\left(\frac{A_N}{A_T}\right)^{\rho-1}$$

and

$$\begin{aligned}
\frac{\dot{\xi}_T}{\xi_T} &= \left[\vartheta + \delta\left(\frac{\bar{A}}{A_T}\right)^{\rho-1}\left(\frac{\pi_T}{\gamma}\right)^{\nu}\right] - \left(\frac{A_T}{A}\right)^{\eta-1}\frac{A^{\zeta}}{\xi_T}, \\
\frac{\dot{\xi}_N}{\xi_N} &= \left[\vartheta + \delta\left(\frac{\bar{A}}{A_N}\right)^{\rho-1}\left(\frac{1-\pi_T}{1-\gamma}\right)^{\nu}\right] - \left(\frac{A_N}{A}\right)^{\eta-1}\frac{A^{\zeta}}{\xi_N},
\end{aligned}$$

where we substituted in:

$$\begin{aligned}
\frac{\dot{A}_T}{A_T} &= \frac{\delta}{\rho-1}\left[\left(\frac{\bar{A}}{A_T}\right)^{\rho-1}\left(\frac{\pi_T}{\gamma}\right)^{\nu} - 1\right], \\
\frac{\dot{A}_N}{A_N} &= \frac{\delta}{\rho-1}\left[\left(\frac{\bar{A}}{A_N}\right)^{\rho-1}\left(\frac{1-\pi_T}{1-\gamma}\right)^{\nu} - 1\right].
\end{aligned}$$

The last five equations characterize planner's optimality, while the last four equations characterize general evolution of state and co-state variables for an arbitrary path of π_T.

We rewrite for both $J \in \{T, N\}$:

$$\dot{\xi}_J(t) = -a_J(t) + b_J(t)\xi_T(t),$$

$$\text{where} \qquad a_J(t) \equiv \left(\frac{A_J(t)}{A(t)}\right)^{\eta-1} A(t)^{\zeta} \qquad \text{and} \qquad b_J(t) \equiv \vartheta + \delta\left(\frac{\bar{A}}{A_J(t)}\right)^{\rho-1}\left(\frac{\pi_J(t)}{\gamma_J}\right)^{\nu},$$

and we denoted $\gamma_N = 1-\gamma_T = 1-\gamma$ and $\pi_N = 1-\pi_T$. The forward solution to this equation is given by:[46]

$$\xi_J(t) = \int_t^{\infty} e^{-\int_t^s b_J(z)\mathrm{d}z} a_J(s)\mathrm{d}s + \lim_{s\to\infty} e^{-\int_t^s b_J(z)\mathrm{d}z}\xi_J(s),$$

where the last term converges to zero by optimality. Note that $b_J(t) \geq \vartheta$ plays the role of a

[46]Note that the full differential is

$$\mathrm{d}[c(t)\xi(t)] = -a(t)c(t)\mathrm{d}t, \qquad \text{where} \qquad \frac{\dot{c}(t)}{c(t)} = -b(t)$$

so that

$$c(s)\xi(s)\Big|_{s=t}^{\infty} = -\int_t^{\infty} a(s)c(s)\mathrm{d}s \qquad \text{and} \qquad \log c(s)\Big|_{s=t}^{\infty} = -\int_t^{\infty} b(s)\mathrm{d}s \quad \Rightarrow \quad \frac{c(s)}{c(t)} = e^{-\int_t^s b(z)\mathrm{d}z}.$$

Dividing the first expression through by $c(t)$ and assuming transversality, we have the result.

discount factor, which reflects both discount rate ϑ and the spillover into the returns to future J-sector innovations. In turn, $a_J(t)$ reflects the flow benefit from J-sector innovation. Note that:

$$\frac{a_T(t)}{a_N(t)} = \left(\frac{A_N(t)}{A_T(t)}\right)^{1-\eta} = \frac{R_T(t)}{R_N(t)},$$

and if

$$\frac{\xi_T(t)}{\xi_N(t)} \equiv \frac{a_T(t)}{a_N(t)}$$

then the planner's allocation coincides with the laissez-faire. Using the forward solution for $\xi_J(t)$ we can prove:

Lemma 6 *In the limit of perfect impatience, $\vartheta \to \infty$, we have $\xi_T(t)/\xi_N(t) \to a_T(t)/a_N(t)$ for all t, and the laissez-faire allocation corresponds to the planner's allocation.*

More generally, planner's allocation features:

$$\left(\frac{\pi_T}{1-\pi_T}\frac{1-\gamma}{\gamma}\right)^{1-\nu} = \frac{\xi_T}{\xi_N}\left(\frac{A_N}{A_T}\right)^{\rho-1} \qquad \Rightarrow \qquad \pi_T = \frac{\gamma}{\gamma + (1-\gamma)\left(\frac{\xi_T}{\xi_N}\right)^{\frac{\theta}{\rho-1}}\left(\frac{A_T}{A_N}\right)^{\theta}}.$$

and therefore we have

$$b_T(t) = \vartheta + \delta\frac{\left(\bar{A}/A_T\right)^{\rho-1}}{\left(\gamma + (1-\gamma)\left(\frac{\xi_T}{\xi_N}\right)^{\frac{\theta}{\rho-1}}\left(\frac{A_T}{A_N}\right)^{\theta}\right)^{\nu}} \quad \text{and} \quad b_N(t) = \vartheta + \delta\frac{\left(\bar{A}/A_N\right)^{\rho-1}}{\left(\gamma\left(\frac{\xi_N}{\xi_T}\right)^{\frac{\theta}{\rho-1}}\left(\frac{A_N}{A_T}\right)^{\theta} + 1-\gamma\right)^{\nu}}.$$

Instead of making $\pi_T/(1-\pi_T)$ proportional to R_T/R_N, the planner makes it proportional to ξ_T/ξ_N, which has two peculiar properties:

1. ξ_J aggregates future a_J, where $a_T/a_N = R_T/R_N$;
2. the discount rate is $b_J \geq \vartheta + \delta$, and it reflect impatience ϑ, death rate of projects δ, as well as the externality on the future innovation rate $\delta\left[\left(\frac{\bar{A}}{A_J}\right)^{\rho-1}\left(\frac{\pi_J}{\gamma_J}\right)^{\nu} - 1\right] \geq 0$.

Patents can help with the implementation, but indefinite patents are not an optimal arrangement for two reasons:

1. they would involve discounting at rate $\delta + \vartheta < b_J$, so a greater loading on the future than needed (hence finite horizons will be optimal);
2. they aggregate future profitability, not future revenue shifters, and the gap between the two is the sectoral price index to $\rho - 1$.

Nature of inefficiency: From the discussion above it is clear that the nature of inefficiency of the laissez-faire allocation is that it does not take into account the spillover from today's sectoral innovation into the return to this sector's innovation in the future. More innovation today means lower return to innovation in the future. Without patents, the laissez-faire allocation will result in too much of an innovational tilt towards the lagging behind sector, while the planner would partially smooth this tilt out. If there were such a market, future innovators would pay current innovators to encourage them to innovate less in the lagging sector.

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