Asset Pricing with Return Extrapolation*

Lawrence J. Jin and Pengfei Sui†

February 22, 2018

ABSTRACT

We present a new model of asset prices based on return extrapolation. The model is a Lucas-type general equilibrium framework, in which the agent has Epstein-Zin preferences and extrapolative beliefs. Unlike earlier return extrapolation models, our model allows for a quantitative comparison with the data on asset prices. When the agent's beliefs are calibrated to match survey expectations of investors, the model generates excess volatility and predictability of stock returns, a high equity premium, a low and stable risk-free rate, and a low correlation between stock returns and consumption growth. We compare our model with prominent rational models and document their different implications.

JEL classification: G02, G12

Keywords: Expectations, Return Extrapolation, Stock Market Movements

*We thank Nicholas Barberis, John Campbell, Stefano Cassella, Ricardo De la O, Michael Ewens, Cary Frydman, Robin Greenwood, David Hirshleifer, Philip Hoffman, Jonathan Ingersoll, Theresa Kuchler, Jiacui Li, Sean Myers, Cameron Peng, Paulo Rodrigues, Paul Sangrey, Andrei Shleifer, and seminar participants at Caltech, Maastricht University, Tilburg, University of California, Irvine, University of Southern California, the Young Economists Symposium at Yale, the Caltech Junior Faculty Behavioral Finance Conference, and the NBER Behavioral Finance meeting for helpful comments. Please send correspondence to Lawrence J. Jin, California Institute of Technology, 1200 E. California Blvd. MC 228-77, Pasadena, CA, 91125; telephone: 626-395-4558. E-mail: lawrence.jin@caltech.edu.

†Both authors' affiliation is the California Institute of Technology.

In financial economics, there is growing interest in "return extrapolation," the idea that investors' beliefs about an asset's future return are a positive function of the asset's recent past returns. Models with return extrapolation have two appealing features. First, they are consistent with survey evidence on the beliefs of real-world investors.[1] Second, they show promise in matching important asset pricing facts, such as volatility and predictability in the aggregate market, momentum and reversals in the cross-section, and bubbles (Barberis, Greenwood, Jin, and Shleifer (2015, 2017); Hong and Stein (1999)).

One limitation of existing models of return extrapolation, however, is that they can only be compared to the data in a *qualitative* way. Early models, such as Cutler, Poterba, and Summers (1990) and DeLong, Shleifer, Summers, and Waldmann (1990), highlight the conceptual importance of return extrapolation, but they are not designed to match asset pricing facts quantitatively. Barberis et al. (2015) is a dynamic consumption-based model that tries to make sense of both survey expectations and aggregate stock market prices. However, the simplifying assumptions in the model make it difficult to evaluate the model's fit with the empirical facts. For instance, their model adopts a framework with constant absolute risk aversion (CARA) preferences and a constant interest rate. Under these assumptions, many ratio-based quantities that we study in asset pricing (e.g., the price-dividend ratio) do not have well-defined distributions in the model and therefore do not have properties that match what we observe in the data.

In this paper, we propose a new model of aggregate stock market prices based on return extrapolation that overcomes this limitation. The goal of the paper is to see if the model can match important facts about the aggregate stock market when the agent's beliefs are calibrated to match survey expectations of investors, and to compare the model in a quantitative way to rational expectations models of the stock market.

We consider a Lucas economy in continuous time with a representative agent. The Lucas tree is a claim to an aggregate consumption process which follows a geometric Brownian motion. Besides the Lucas tree, there are two tradeable assets in the economy: the stock market and an instantaneous riskless asset. The stock market is a claim to an aggregate dividend process whose growth rate

[1] Among others, Vissing-Jorgensen (2004), Bacchetta, Mertens, and van Wincoop (2009), Amromin and Sharpe (2013), Greenwood and Shleifer (2014), Koijen, Schmeling, and Vrugt (2015), and Kuchler and Zafar (2016) document that many individual and institutional investors have extrapolative expectations: they believe that the stock market will continue rising in value after a sequence of high past returns, and that it will continue falling in value after a sequence of low past returns.

is positively correlated with consumption growth. The riskless asset is in zero net supply with its interest rate determined in equilibrium. The representative agent has Epstein-Zin preferences and extrapolative beliefs. She perceives that the expected growth rate of stock market prices is governed by a switching process between two regimes. If recent price growth of the stock market has been high, the agent thinks it is likely that a high-mean price growth regime is generating prices and therefore forecasts high price growth in the future. Conversely, if recent price growth has been low, the agent thinks that it is likely that a low mean-price growth regime is generating prices and therefore forecasts low price growth in the future.

We calibrate the agent's beliefs to match the survey expectations of investors studied in Greenwood and Shleifer (2014). Specifically, we set the belief-based parameters so that, for a regression of the agent's expectations about future stock market returns on past twelve-month returns, the model produces a regression coefficient and a t-statistic that match the empirical estimates from surveys. Our parameter choice also allows the agent's beliefs to match the survey evidence on the relative weight investors put on recent versus distant past returns when forming beliefs about future returns. Overall, the model generates a degree of extrapolative expectations for the agent that matches the empirical magnitude. With the agent's beliefs disciplined by survey data, the model quantitatively matches important facts about the aggregate stock market: it generates significant excess volatility and predictability of stock market returns, a high equity premium, a low and stable interest rate, as well as a low correlation between stock market returns and consumption growth.

We now explain the intuition for the model's implications, starting with excess volatility. The model generates significant excess volatility from the interaction between return extrapolation and Epstein-Zin preferences. Suppose that the stock market has had high past returns. In such a case, return extrapolation leads the agent to forecast high future returns. Under Epstein-Zin preferences, the separation between the elasticity of intertemporal substitution and risk aversion gives rise to a strong intertemporal substitution effect. Therefore, the agent's forecast of high future returns leads her to push up the current price significantly, generating excess volatility.[2]

[2]A feedback loop emerges from this mechanism. If current returns are high, that makes the agent think that future returns will also be high, which leads her to push up prices, increasing current returns further, and so on. In general, there is a danger that this feedback loop could "explode." Nonetheless, in the model, we assume the agent believes that the expected growth rate of stock market prices tends to switch over time from one regime to the other; she therefore believes her optimism will decline in the future. As a result, the cumulative impact of the feedback loop on investor expectations and asset prices is finite; the model remains stable. Models like Cutler et al. (1990) and Barberis et al. (2015) instead introduce fully rational investors in order to counteract the behavioral investors

The mechanism described above for generating excess volatility, together with a strong degree of mean reversion in the agent's expectations about stock market returns, produces the long-horizon predictability of stock market returns that we observe in the data. The agent's beliefs mean-revert, for two reasons. First, by assumption, the agent believes that the expected growth rate of stock market prices tends to switch over time from one regime to the other: the agent believes that her expectations about stock market returns will mean-revert. Second, the agent's return expectations mean-revert *faster* than what she perceives: when the agent thinks that the future price growth is high, future price growth tends to be low endogenously, causing her return expectations to decrease at a pace that exceeds her anticipation. As a result, following periods with a high price-dividend ratio—this is when the high past price growth of the stock market pushes up the agent's expectation about future returns and hence her demand for the stock market—the agent's return expectation tends to revert back to its mean, giving rise to low subsequent returns and hence the predictability of stock market returns using the price-dividend ratio.

Next, we turn to the model's implications for the equity premium. Three factors affect the long-run equity premium *perceived* by the agent. First, because the agent is risk averse, excess volatility causes her to demand a higher equity premium. Second, return extrapolation gives rise to perceived persistence of the aggregate dividend process, which, under Epstein-Zin preferences, is significantly priced, pushing up the perceived equity premium. Finally, the separation between the elasticity of intertemporal substitution and risk aversion helps to keep the equilibrium interest rate low and hence keep the equity premium high. Furthermore, the *true* long-run equity premium is significantly higher than the perceived one. In the model, the agent's beliefs mean-revert faster than what she perceives. Given this, she underestimates short-term stock market fluctuations and hence the risk associated with the stock market. In other words, if an infinitesimal rational agent, one that has the same preferences as the behavioral agent but holds rational beliefs, enters our economy, she would have demanded a higher equity premium: the model produces a true average equity premium that is substantially higher than the perceived equity premium.

Finally, the model generates low interest rate volatility and a low correlation between stock market returns and consumption growth. In the model, the agent separately forms beliefs about the dividend growth of the stock market and about aggregate consumption growth. Here, we assume

and preserve equilibrium.

that the bias in the agent's beliefs about consumption growth derives only from the bias in her beliefs about dividend growth. Given the low observed correlation between consumption growth and dividend growth, the bias in the agent's beliefs about consumption growth is small, consistent with the lack of empirical evidence that investors have extrapolative beliefs about consumption growth. The agent's approximately correct beliefs about consumption growth allow the model to generate low interest rate volatility. They also imply that the agent's beliefs about stock market returns—they co-move strongly with her beliefs about dividend growth—are not significantly affected by fluctuations in consumption growth, giving rise to the low observed correlation between stock market returns and consumption growth.

Although our model is based on return extrapolation, it yields direct implications for cash flow expectations. When the past price growth of the stock market has been high, this has a positive effect not only on the agent's beliefs about future returns, but also on her beliefs about future dividend growth; indeed, her expectations about dividend growth rise at a pace that exceeds her expectations about future returns.[3] Given this, a Campbell-Shiller decomposition using the agent's *subjective* expectations about stock market returns and dividend growth shows that changes in subjective expectations about future dividend growth explain most of the volatility of the price-dividend ratio. This model implication is consistent with the recent empirical findings of de la O and Myers (2017): they find that during periods when the price-dividend ratio of the U.S. stock market is high, investors' expectations of future dividend growth are much higher than their expectations of future stock market returns. As a result, changes in investors' subjective expectations of future dividend growth explain the majority of stock market movements. Importantly, the fact that prices in our model are mainly correlated with cash flow expectations is a consequence of the Campbell-Shiller accounting identity; this statement is about correlation, not about causality. The agent's *return* expectations determine her cash flow expectations and are the cause of price movements. Given this, our model simultaneously explains the empirical findings of de la O and Myers (2017) on cash flow expectations and the empirical findings of Greenwood and Shleifer (2014) on return expectations. At the same time, the model also explains the empirical findings of Cochrane (2008) and Cochrane (2011) that, under rational expectations, the variation of the price-dividend ratio comes primarily from discount rate variation.

[3] We provide a detailed explanation of this finding in Sections I and II.

Our model also points to some challenges: when calibrated to the survey expectations data, the model predicts a persistence of the price-dividend ratio that is significantly lower than its empirical value. In other words, to match the empirical persistence of the price-dividend ratio, investors need to form beliefs about future returns based on many years of past returns. However, the available survey evidence suggests that they focus on just the past year or two.

After presenting the model, we compare it to the standard rational expectations models of the aggregate stock market. As with the habit formation model of Campbell and Cochrane (1999), the long-run risks models of Bansal and Yaron (2004) and Bansal, Kiku, and Yaron (2012), and the rare disasters models of Barro (2006), Gabaix (2012), and Wachter (2013), our model is developed in a Lucas economy with a representative agent. This model structure allows for a direct comparison between our model and models with rational expectations. Here, we focus on the long-run risks models because these are the models most related to ours. We document some different implications.

First, our model differs from the long-run risks models in the way the agent forms expectations. In Bansal and Yaron (2004), dividend growth and consumption growth share a stochastic yet persistent component. High past stock market returns are typically caused by positive shocks to this common component, which, given its persistence, implies high dividend growth and hence high raw returns moving forward. That is, the agent in Bansal and Yaron (2004) has extrapolative beliefs about future raw returns. At the same time, precisely because dividend growth and consumption growth share a persistent component, the comovement between the agent's beliefs about stock market returns—these rationally drive returns—and her beliefs about consumption growth—these determine the interest rate in equilibrium—is high. That is, when the raw returns are high, the interest rate is also high. As a result, the agent does not hold extrapolative beliefs about *excess* returns. In our model, however, the agent extrapolates past stock market returns, but extrapolates past consumption growth much less: the comovement between her beliefs about stock market returns and her beliefs about consumption growth is low. Therefore, the agent has extrapolative beliefs about *both* raw returns and excess returns.

Furthermore, these two models yield different implications for asset prices. Our model produces an equity premium that does not vary significantly with changes in the elasticity of intertemporal substitution. On the contrary, long-run risks models cannot generate a high equity premium with a low elasticity of intertemporal substitution. To see this model difference, we first note that the

agent's beliefs in our model are much less persistent than the stochastic component of dividend and consumption growth in Bansal and Yaron (2004), allowing the equilibrium interest rate and hence the equity premium in our model to be less responsive to changes in the elasticity of intertemporal substitution. At the same time, the perceived dividend growth in our model depends more strongly on the agent's beliefs about the price growth of the stock market, pushing up the perceived equity premium; as a comparison, dividend growth in Bansal and Yaron (2004) depends much less on the stochastic growth component. Finally, the true long-run equity premium in our model is above the perceived one, allowing the equity premium to be high even when the elasticity of intertemporal substitution is low.

Our paper adds to a new wave of theories that attempt to understand the role of belief formation in driving the behavior of asset prices and the macroeconomy (Fuster, Hebert, and Laibson (2011); Gennaioli, Shleifer, and Vishny (2012); Choi and Mertens (2013); Alti and Tetlock (2014); Hirshleifer, Li, and Yu (2015); Barberis et al. (2015); Jin (2015); Ehling, Graniero, and Heyerdahl-Larsen (2015); Vanasco, Malmendier, and Pouzo (2015); Pagel (2016); Collin-Dufresne, Johannes, and Lochstoer (2016a,b); Greenwood, Hanson, and Jin (2016); Glaeser and Nathanson (2017); De-Fusco, Nathanson, and Zwick (2017); Bordalo, Gennaioli, and Shleifer (2018)). Our paper also adds to a growing literature on the source of stock price movements (Cochrane (2008); Cochrane (2011); Chen, Da, and Zhao (2013); de la O and Myers (2017)). Furthermore, it is related to theories of model uncertainty and ambiguity aversion such as Bidder and Dew-Becker (2016). These models typically assume that agents learn about the dynamic properties of the consumption process or the dividend process. Therefore, they are closely linked to the fundamental extrapolation models in the behavioral finance literature, but do not match survey evidence on return expectations. Finally, our paper speaks to the debate between Bansal et al. (2012) and Beeler and Campbell (2012) which focuses on excess predictability: the notion that, in the long-run risks literature, future consumption growth and dividend growth are excessively predicted by current variables such as the price-dividend ratio and the interest rate (see also, Collin-Dufresne et al. (2016b)). Our model does not give rise to excess predictability: return extrapolation in the model only generates *perceived* but not true persistence in consumption growth and dividend growth.

The paper proceeds as follows. In Section I, we lay out the basic elements of the model and characterize its solution. In Section II, we parameterize the model and examine its implications in

detail. Section III provides a comparative statics analysis. Section IV discusses differences between our model and rational expectations models. Section V further compares the model to a model with fundamental extrapolation, the notion that some investors hold extrapolative expectations about the future dividend growth of the stock market. Section VI concludes and suggests directions for future research. All technical details are in the Appendix.

I. The Model

In this section, we first describe the model setup and characterize its solution, and then derive equilibrium quantities that are important for understanding the implications of the model.

I.1. Model setup

Asset space.—We consider an infinite-horizon Lucas economy in continuous time with a representative agent. The Lucas tree is a claim to an aggregate consumption process. We assume it is a geometric Brownian motion

$$dC_t/C_t = g_C dt + \sigma_C d\omega_t^C, \tag{1}$$

and we denote the price of the Lucas tree at time t as P_t^C.

Besides the Lucas tree, there are two other tradeable assets in the economy; they are the main focus of our analysis. The first asset is the stock market which is a claim to an aggregate dividend process given by

$$dD_t/D_t = g_D dt + \sigma_D d\omega_t^D; \tag{2}$$

we denote the price of the stock market at time t as P_t^D.[4] Both ω_t^D and ω_t^C are standard Brownian motions. We assume that the instantaneous correlation between dD_t and dC_t is ρ: $\mathbb{E}_t[d\omega_t^D \cdot d\omega_t^C] = \rho dt$. The second asset is an instantaneous riskless asset. This asset is in zero net supply, and its

[4]Since the aggregate consumption process in the model is exogenous, the dividend payment from the stock market does not further affect consumption. As a result, we can think of the stock market as an asset in zero net supply with a shadow price determined in equilibrium. This is a common assumption adopted by many other consumption-based models such as Campbell and Cochrane (1999) and Barberis, Huang, and Santos (2001).

interest rate r_t is determined in equilibrium.

Agent's preferences.—We follow Epstein and Zin (1989, 1991) and assume that the agent has a recursive intertemporal utility

$$U_t = \left[(1 - e^{-\delta dt})C_t^{1-\psi} dt + e^{-\delta dt} \left(\mathbb{E}_t^e[\tilde{U}_{t+dt}^{1-\gamma}]\right)^{(1-\psi)/(1-\gamma)}\right]^{1/(1-\psi)}, \tag{3}$$

where δ is the subjective discount rate, $\gamma > 0$ is the coefficient of relative risk aversion, and $\psi > 0$ is the reciprocal of the elasticity of intertemporal substitution. When ψ equals γ, (3) reduces to power utility. The superscript "e" is an abbreviation for "extrapolative" expectations: the certainty equivalence in (3) is computed under the representative agent's subjective beliefs, which, as we specify later, incorporate the notion of return extrapolation.

The subjective Euler equation, or the first-order condition, is

$$\mathbb{E}_t^e \left[e^{-\delta(1-\gamma)dt/(1-\psi)} \left(\frac{\tilde{C}_{t+dt}}{C_t}\right)^{-\psi(1-\gamma)/(1-\psi)} \tilde{M}_{t+dt}^{(\psi-\gamma)/(1-\psi)} \tilde{R}_{j,t+dt} \right] = 1. \tag{4}$$

Here $\tilde{M}_{t+dt}$ is the gross return on the optimal portfolio held by the agent from time t to time $t+dt$. In a Lucas economy with a representative agent, the optimal portfolio in equilibrium is the Lucas tree itself, and therefore

$$\tilde{M}_{t+dt} = \frac{\tilde{P}_{t+dt}^C + \tilde{C}_t dt}{P_t^C} = \frac{\tilde{P}_{t+dt}^C + \tilde{C}_{t+dt} dt}{P_t^C} + o(dt). \tag{5}$$

On the other hand, $\tilde{R}_{j,t+dt}$ is the gross return on *any* tradeable asset j in the market from time t to time $t+dt$; as mentioned above, the two tradeable assets we focus on are the stock market and the riskless asset.

Agent's beliefs.—We now turn to the key part of the model: the representative agent's beliefs about stock market returns. According to surveys, real-world investors form beliefs about future stock market returns by extrapolating past returns (Vissing-Jorgensen (2004); Bacchetta et al. (2009); Amromin and Sharpe (2013); Greenwood and Shleifer (2014); Koijen et al. (2015); Kuchler and Zafar (2016)). One natural way to capture this notion of return extrapolation is through a regime-switching model. Specifically, we suppose that the agent believes that the expected growth

rate of stock market prices is governed by $(1-\theta)g_D + \theta\tilde{\mu}_{S,t}$, where $\tilde{\mu}_{S,t}$ is a latent variable which switches between a high value μ_H in a high-mean price growth regime H and a low value μ_L ($\mu_L < \mu_H$) in a low-mean price growth regime L with the following transition matrix[5]

$$\begin{array}{c} \\ \tilde{\mu}_{S,t} = \mu_H \\ \tilde{\mu}_{S,t} = \mu_L \end{array} \begin{array}{c} \begin{array}{cc} \tilde{\mu}_{S,t+dt} = \mu_H & \tilde{\mu}_{S,t+dt} = \mu_L \end{array} \\ \begin{pmatrix} 1-\chi dt & \chi dt \\ \lambda dt & 1-\lambda dt \end{pmatrix} \end{array}. \tag{6}$$

Here χ and λ are the intensities for the transitions of regime from H to L and from L to H, respectively, and the parameter θ ($0 \leq \theta \leq 1$) controls the extent to which the agent's beliefs are extrapolative: setting θ to zero makes the agent's beliefs fully rational.

Given this *perceived* regime-switching model—this is not the true model—if recent stock market price growth has been high, the agent thinks it is likely that the high-mean price growth regime is generating prices and therefore forecasts high price growth in the future. Conversely, if recent price growth has been low, the agent thinks it is likely that the low-mean price growth regime is generating prices and therefore forecasts low price growth in the future. Formally, at each point in time, the agent computes the expected value of the latent variable $\tilde{\mu}_{S,t}$ given the history of past price growth: $S_t \equiv \mathbb{E}[\tilde{\mu}_{S,t}|\mathcal{F}_t^P]$. That is, she applies optimal filtering theory (see, for instance, Lipster and Shiryaev (2001)) and obtains

$$\begin{aligned} dS_t &= (\lambda\mu_H + \chi\mu_L - (\lambda+\chi)S_t)dt + (\sigma_{P,t}^D)^{-1}\theta(\mu_H - S_t)(S_t - \mu_L)d\omega_t^e \\ &\equiv \mu_S^e(S_t)dt + \sigma_S(S_t)d\omega_t^e, \end{aligned} \tag{7}$$

where $d\omega_t^e \equiv [dP_t^D/P_t^D - (1-\theta)g_D dt - \theta S_t dt]/\sigma_{P,t}^D$ is a standard Brownian innovation term from the agent's perspective. As a result, she perceives the evolution of the stock market price P_t^D to be

$$dP_t^D/P_t^D = \mu_P^{D,e}(S_t)dt + \sigma_P^D(S_t)d\omega_t^e, \tag{8}$$

where

$$\mu_P^{D,e}(S_t) = (1-\theta)g_D + \theta S_t. \tag{9}$$

[5]The models of Barberis, Shleifer, and Vishny (1998), Veronesi (1999), and Jin (2015) also adopt a regime-switching learning structure.

The agent's expectation about price growth $\mu_P^{D,e}(S_t)$ is therefore a linear combination of a rational component g_D and a behavioral component S_t; hereafter we call S_t the sentiment variable.

In summary, the evolution of sentiment in (7) captures return extrapolation: high past price growth dP_t^D/P_t^D pushes up the perceived shock $d\omega_t^e$, which leads the agent to raise her expectation of the sentiment variable S_t, causing her expectation about future price growth $\mu_P^{D,e}(S_t)$ to rise.[6]

Although the subjective evolution of sentiment (7) is derived through optimal learning, the representative agent, it should be emphasized, does *not* hold rational expectations. With rational expectations, the agent will realize in the long run that the regime-switching model (6) is incorrect: she can look at the historical distribution of $d\omega_t^e$ and realize that it does not fit a normal distribution with a mean of 0 and a variance of dt. Instead, the agent in our behavioral model always believes that the regime-switching model is correct. In reality, it is possible that investors in the market learn over time that their mental model is incorrect. At the same time, new investors who hold extrapolative beliefs may continuously enter the market. The stable belief system in (6) is an analytically convenient way to capture these dynamics. Alternatively, if equations (6) and (7) represent the *true* data generating process, then the agent does hold rational expectations. In that case, the model becomes a fully rational model with incomplete information.[7] We discuss the predictions of such a model in Section IV.

So far we have been focusing on the agent's beliefs about stock market prices. These beliefs also have direct implications for the agent's beliefs about dividend growth. If we write the perceived dividend process as

$$dD_t/D_t = g_D^e(S_t)dt + \sigma_D d\omega_t^e, \tag{10}$$

we can connect the agent's expectation about dividend growth $g_D^e(S_t)$ to her expectation about stock market price growth $\mu_P^{D,e}(S_t)$. To formally make this connection, we first observe that all the ratio-based quantities in our model (e.g., the price-dividend ratio of the stock market) are a

[6] There are many ways to specify the evolution of S_t in order to capture return extrapolation. We derive S_t from a regime-switching model for two reasons. First, such a learning model captures base rate neglect, an important consequence of the representativeness heuristic (Tversky and Kahneman (1974)). To see this, note that the perceived regimes or states, H and L, are not part of the true states of the economy. As a result, assigning positive probability weights to these regimes reflect the bias that the investor neglects the zero base rate associated with such regimes. Second, bounding S_t by a finite range (μ_L, μ_H) reduces the analytical difficulty of solving the model.

[7] Information is incomplete in the sense that the agent does not directly observe the latent variable $\tilde{\mu}_{S,t}$.

function of the sentiment variable S_t; we can define $f(S_t) \equiv P_t^D/D_t$. We then apply Ito's lemma on both sides of this equation $f(S_t) = P_t^D/D_t$ and match terms to obtain

$$g_D^e(S_t) = \underbrace{(1-\theta)g_D + \theta S_t}_{\text{expectation of price growth}} \quad \underbrace{-(f'/f)\mu_S^e(S_t)}_{\text{expectation of sentiment evolution}} \quad \underbrace{+\sigma_D^2 - \sigma_P^D(S_t)\sigma_D - \tfrac{1}{2}(f''/f)(\sigma_S(S_t))^2}_{\text{Ito correction terms}}, \tag{11}$$

where

$$\sigma_P^D(S_t) = \frac{\sigma_D + \sqrt{\sigma_D^2 + 4\theta(\mu_H - S_t)(S_t - \mu_L)(f'/f)}}{2} > \sigma_D. \tag{12}$$

Equation (11) highlights an "expectations transmission mechanism:" it says that the agent's expectation about dividend growth equals the sum of her expectation about stock market price growth, her expectation about how the price-dividend ratio evolves with respect to changes in sentiment, and the Ito correction terms that are related to the agent's risk aversion and the volatility of dividend growth, price growth, and changes in sentiment. In this way, the agent's expectation about price growth affects her expectation about dividend growth.

With the parameter values we specify later, equation (11) suggests that the agent's expectation about dividend growth is *more* responsive to changes in sentiment than her expectation about price growth. Under Epstein-Zin preferences, the separation between the elasticity of intertemporal substitution and risk aversion gives rise to a strong intertemporal substitution effect. As a result, when the past price growth has been high, the agent's forecast of high future price growth leads her to push up the current price-dividend ratio, making it a positive function of sentiment. Furthermore, under the regime-switching model, the agent perceives sentiment to be mean-reverting: $\mu_S^e(S_t)$ in (7) is a negative function of S_t. This suggests that the agent also perceives the price-dividend ratio to be mean-reverting. Together, these two conditions—the price-dividend ratio is a positive function of sentiment and is perceived to be mean-reverting—imply that the agent anticipates that the price-dividend ratio will decline from a high value when she expects high future price growth. That is, when the agent expects high future price growth, her expectation about dividend growth rises at a pace that exceeds her expectation about future price growth.

To complete the description of the model, we need to further specify the agent's beliefs about consumption growth. To do this, first note that, with a local correlation of ρ between consumption growth and dividend growth, we can rewrite the aggregate consumption process of (1) as

$$dC_t/C_t = g_C dt + \sigma_C \left(\rho d\omega_t^D + \sqrt{1-\rho^2} d\omega_t^{\perp} \right), \tag{13}$$

where $\omega_t^{\perp}$ is a Brownian motion that is locally uncorrelated with ω_t^D, the Brownian shock on dividends. We then assume that the agent perceives the consumption process as

$$dC_t/C_t = g_C^e(S_t) dt + \sigma_C \left(\rho d\omega_t^e + \sqrt{1-\rho^2} d\omega_t^{\perp} \right). \tag{14}$$

That is, we replace the true Brownian shock on dividends $d\omega_t^D$ by the agent's perceived Brownian shock $d\omega_t^e$ and factor the difference between these two Brownian shocks into $g_C^e(S_t)$, the agent's subjective expectation about consumption growth. Conceptually, this amounts to assuming that the bias in the agent's beliefs about consumption growth comes only from the bias in her beliefs about dividend growth.[8] In doing so, we derive the agent's expectation about dividend growth as

$$g_C^e(S_t) - g_C = \rho \sigma_C \sigma_D^{-1} (g_D^e(S_t) - g_D). \tag{15}$$

Empirically, the correlation between consumption growth and dividend growth is low—ρ is positive but low—and consumption growth is much less volatile than dividend growth—σ_C is much smaller than σ_D. As a result, (15) implies that the bias in the agent's expectation about consumption growth—the difference between $g_C^e(S_t)$ and g_C—is small. This is in keeping with the lack of any evidence that investors have extrapolative beliefs about consumption growth.[9] Moreover, the agent's approximately correct beliefs about consumption growth allow the model to generate low interest rate volatility and a low correlation between consumption growth and stock market returns, both of which are consistent with the data (Campbell (2003); Hansen and Singleton

[8] For any alternative assumption, one needs to explain why the investor has incorrect beliefs about consumption above and beyond her incorrect beliefs about dividends.

[9] Consistent with the way we model the agent's expectations about consumption growth, Kuchler and Zafar (2016) find that survey expectations are "asset-specific:" respondents who become pessimistic about their employment situation after experiencing unemployment are not pessimistic about other economic outcomes, such as stock prices or interest rates. Similarly, Huang (2016) finds that investors who become optimistic about an industry's future returns after having positive prior investment experience in the industry do not invest heavily in a dissimilar industry.

(1982, 1983)).

I.2. Model solution

The subjective Euler equation in (4) shows that, when pricing the stock market, the gross return from holding the Lucas tree is also part of the pricing kernel. This observation has two implications. First, both the price-dividend ratio $f(S_t) = P_t^D/D_t$ and the wealth-consumption ratio P_t^C/C_t are functions of the sentiment variable S_t; we can define $l(S_t) \equiv P_t^C/C_t$. Second, the two functions f and l are interrelated through Euler equations, so they need to be solved simultaneously. Specifically, using the Euler equation to price the stock market—setting $\tilde{R}_{j,t+dt}$ in (4) to the gross return on the stock market—we obtain

$$0 = \left[\begin{array}{l} -\frac{(1-\gamma)}{1-\psi}\delta - \gamma g_C^e + g_D^e + [(f'/f) + \frac{\psi-\gamma}{1-\psi}(l'/l)]\mu_S^e + \frac{1}{2}[(f''/f) + \frac{\psi-\gamma}{1-\psi}(l''/l)]\sigma_S^2 \\ \\ +\frac{\gamma(\gamma+1)}{2}\sigma_C^2 + \frac{1}{2}\frac{\psi-\gamma}{1-\psi}\frac{2\psi-\gamma-1}{1-\psi}(l'/l)^2\sigma_S^2 - \frac{\gamma(\psi-\gamma)}{1-\psi}\rho\sigma_C\sigma_S(l'/l) - \gamma\rho\sigma_C\sigma_D - \gamma\rho\sigma_C\sigma_S(f'/f) \\ \\ +\frac{\psi-\gamma}{1-\psi}\sigma_D\sigma_S(l'/l) + \frac{\psi-\gamma}{1-\psi}\sigma_S^2(l'/l)(f'/f) + \sigma_D\sigma_S(f'/f) + \frac{\psi-\gamma}{1-\psi}l^{-1} + f^{-1} \end{array} \right]. \tag{16}$$

Similarly, using the Euler equation to price the Lucas tree—setting $\tilde{R}_{j,t+dt}$ in (4) to the gross return on the Lucas tree—we obtain

$$0 = \left[\begin{array}{l} -\frac{1-\gamma}{1-\psi}\delta - (\gamma-1)g_C^e + \frac{\gamma(\gamma-1)}{2}\sigma_C^2 + \frac{1-\gamma}{1-\psi}(l'/l)\mu_S^e + \frac{1-\gamma}{2(1-\psi)}(l''/l)\sigma_S^2 \\ \\ +\frac{1}{2}\frac{1-\gamma}{1-\psi}\frac{\psi-\gamma}{1-\psi}(l'/l)^2\sigma_S^2 + \frac{(1-\gamma)^2}{1-\psi}\rho\sigma_C\sigma_S(l'/l) + \frac{1-\gamma}{1-\psi}l^{-1} \end{array} \right]. \tag{17}$$

Substituting μ_S and σ_S from (7), g_D^e and σ_P^D from (11) and (12), and g_C^e from (15) into equations (16) and (17), we then obtain a system of two ordinary differential equations that jointly determines the evolutions of f and l.[10] The detailed derivation of (16) and (17) is in the Appendix.

Regarding the boundary conditions for solving the differential equations, note that, in (16) and (17), the second derivative terms are all multiplied by σ_S, and that σ_S goes to zero as S

[10] When $\theta = 0$, our model reduces to a fully rational benchmark. In this case, equations (16) and (17) lead to

$$f = \left[\delta + \psi g_C - g_D - \tfrac{\gamma(\psi+1)}{2}\sigma_C^2 + \gamma\rho\sigma_C\sigma_D\right]^{-1}, \quad l = \left[\delta + (\psi-1)g_C - \tfrac{\gamma(\psi-1)}{2}\sigma_C^2\right]^{-1}.$$

approaches either μ_H or μ_L. As a result, μ_H and μ_L are both singular points, and therefore no boundary condition is required.

Equations (16) and (17) cannot be solved analytically. We apply a projection method with Chebyshev polynomials to solve them numerically. We leave the details of the numerical procedure to the Appendix.

I.3. Important equilibrium quantities

With the model solution at hand, we derive equilibrium quantities that are important for understanding the model's implications. Specifically, we derive the dynamics of the interest rate, the objective and subjective expectations of stock market returns, and the steady-state distribution of the sentiment variable.

For the interest rate, we use the Euler equation in (4) to price the riskless asset—we set $\tilde{R}_{j,t+dt}$ to the gross return on the riskless asset $1 + r_t dt$—and obtain

$$r_t = \tfrac{1-\gamma}{1-\psi}\delta + \gamma g_C^e - \tfrac{\gamma(\gamma+1)}{2}\sigma_C^2 - \tfrac{\psi-\gamma}{1-\psi} \times \left[\begin{array}{l} (\mu_S^e - \gamma\rho\sigma_C\sigma_S)(l'/l) + \tfrac{1}{2}\sigma_S^2(l''/l) \\ \\ +\tfrac{2\psi-\gamma-1}{2(1-\psi)}\sigma_S^2(l'/l)^2 + l^{-1} \end{array} \right]. \tag{18}$$

The interest rate is linked to the agent's time preferences, her subjective expectation about consumption growth, precautionary saving, as well as how the wealth-consumption ratio P_t^C/C_t responds to changes in sentiment.[11]

To understand the risk-return tradeoff in the model, we compute, at each point in time, both the agent's expectation about future stock market returns and the (objectively measured) rational expectation about future stock market returns. From equations (8) and (9), the log excess return on the stock market from time t to time $t + dt$ is

$$\begin{aligned} r_{t+dt}^{D,e} dt &\equiv \ell\mathrm{n}(P_{t+dt}^D + D_{t+dt}dt) - \ell\mathrm{n}(P_t^D) - r_t dt \\ &= [(1-\theta)g_D + \theta S_t + f^{-1} - \tfrac{1}{2}(\sigma_P^D)^2 - r_t]dt + \sigma_P^D d\omega_t^e. \end{aligned} \tag{19}$$

[11] When $\theta = 0$, $r = \delta + \psi g_C - \frac{\gamma(\psi+1)}{2}\sigma_C^2$.

Therefore, the agent's subjective expectation about the log excess return is

$$\mathbb{E}_t^e[r_{t+dt}^{D,e}] = (1-\theta)g_D + \theta S_t + f^{-1} - \tfrac{1}{2}(\sigma_P^D)^2 - r_t, \tag{20}$$

and the subjective Sharpe ratio is $[(1-\theta)g_D + \theta S_t + f^{-1} - \frac{1}{2}(\sigma_P^D)^2 - r_t]/\sigma_P^D$.

Next, to compute the rational expectation about the stock market return, we compare (2) with (10) and obtain a relationship between the true and perceived Brownian shocks

$$d\omega_t^e = d\omega_t^D - (g_D^e(S_t) - g_D)dt/\sigma_D. \tag{21}$$

We then substitute (21) into (19) and derive

$$r_{t+dt}^{D,e}dt = [(1-\theta)g_D + \theta S_t + f^{-1} - \sigma_D^{-1}\sigma_P^D(g_D^e - g_D) - \tfrac{1}{2}(\sigma_P^D)^2 - r_t]dt + \sigma_P^D d\omega_t^D. \tag{22}$$

As a result, the rational expectation about the log excess return on the stock market is

$$\mathbb{E}_t[r_{t+dt}^{D,e}] = (1-\theta)g_D + \theta S_t + f^{-1} - \sigma_D^{-1}\sigma_P^D(g_D^e - g_D) - \tfrac{1}{2}(\sigma_P^D)^2 - r_t, \tag{23}$$

and the objectively measured Sharpe ratio of the stock market return is $[(1-\theta)g_D + \theta S_t + f^{-1} - \sigma_D^{-1}\sigma_P^D(g_D^e - g_D) - \frac{1}{2}(\sigma_P^D)^2 - r_t]/\sigma_P^D$.

All the ratio-based quantities in this model such as the agent's expectation about stock market returns and the interest rate are a function of the sentiment variable S_t. Given this, to provide a statistical assessment of the model's fit with the empirical facts, we also compute the steady-state distribution for the sentiment variable S_t as *objectively* measured by an outside econometrician. To that end, we first obtain the objective evolution of sentiment by substituting the change-of-measure equation (21) into the subjective evolution of sentiment in (7)

$$dS_t = [\mu_S^e(S_t) + \sigma_D^{-1}\sigma_S(S_t)(g_D - g_D^e(S_t))]dt + \sigma_S(S_t)d\omega_t^D. \tag{24}$$

Compared to the subjective evolution of sentiment, the objective evolution exhibits a larger degree of mean reversion: the additional term $\sigma_D^{-1}\sigma_S(S_t)(g_D - g_D^e(S_t))$ in (24) is a negative function of

sentiment.

Denote the objective steady-state distribution for sentiment as $\xi(S)$. Based on (24), we then derive $\xi(S)$ as the solution to the Kolmogorov forward equation (the Fokker-Planck equation)

$$\begin{aligned} 0 &= \tfrac{1}{2}\tfrac{d^2}{dS^2}\left(\sigma_S^2(S)\xi(S)\right) - \tfrac{d}{dS}\left([\mu_S^e(S_t) + \sigma_D^{-1}\sigma_S(S_t)(g_D - g_D^e(S_t))]\xi(S)\right) \\ &= (\sigma_S')^2\xi + \sigma_S\sigma_S''\xi + 2\sigma_S\sigma_S'\xi' + \tfrac{1}{2}\sigma_S^2\xi'' \\ &\quad - [(\mu_S^e)' + \sigma_D^{-1}\sigma_S'(g_D - g_D^e) - \sigma_D^{-1}\sigma_S(g_D^e)']\xi - [\mu_S^e + \sigma_D^{-1}\sigma_S(g_D - g_D^e)]\xi', \end{aligned} \tag{25}$$

where σ_S and g_D^e are from (7) and (11), respectively, and the expressions for σ_S', σ_S'', $(\mu_S^e)'$ and $(g_D^e)'$ are provided in the Appendix. In addition, the steady-state distribution must integrate to one.

II. Model Implications

In this section, we examine the implications of the model. We begin by setting the benchmark values for the model parameters. In particular, we calibrate the agent's beliefs to match the survey evidence documented in Greenwood and Shleifer (2014). We then look at two building blocks for the model's implications: a set of important equilibrium quantities, each as a function of sentiment; and the steady-state distribution of sentiment. Finally, we discuss the model's implications for asset prices.

II.1. Model parameterization

There are three types of parameters: asset parameters, utility parameters, and belief parameters. For the asset parameters, we set $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$. These values are consistent with those used in Campbell and Cochrane (1999), Barberis et al. (2001), Bansal and Yaron (2004), and Beeler and Campbell (2012).[12] For the utility parameters, we set γ, the coefficient of relative risk aversion, to 10. As pointed out in Bansal et al. (2012) and Bansal and Yaron (2004), the long-run risks literature—a literature that depends significantly on the parameter values of Epstein-Zin preferences for its model implications—typically assigns a

[12] The parameter values for g_C and g_D are set such that both $\ell\text{n}(C)$ and $\ell\text{n}(D)$ grow, on average, at an annual rate of 1.84%; this rate is also used in Barberis et al. (2001).

value of 10 or below for γ. Bansal and Yaron (2004), for instance, set γ to either 10 or 7.5.[13] For ψ, the reciprocal of the elasticity of intertemporal substitution, there exists a wide range of estimates in the asset pricing literature. The majority of previous papers suggests that ψ should be lower than one, but several other papers argue the opposite.[14] Given this, we set ψ to 0.9, a value that implies an elasticity of intertemporal substitution slightly above one. We explain in Section IV that our model's implications are quantitatively robust even with an elasticity of intertemporal substitution significantly lower than one. Finally, for δ, the subjective discount rate, we assign a value of 2%.

We now turn to the belief parameters. We set μ_H and μ_L, the mean price growth in the high and low regimes, to 15% and -15%, respectively. As we will see later in this section, the probability of the agent's price growth expectations approaching the boundaries of μ_H and μ_L is approximately zero. As a result, the model's implications are not very sensitive to the choice of μ_H and μ_L. Next, we focus on θ, the parameter that controls the extent to which the representative agent is behavioral, and χ and λ, the perceived transition intensities between the high- and low-mean price growth regimes. We calibrate these three parameters to match the survey expectations of investors studied in Greenwood and Shleifer (2014). Specifically, we set $\theta = 0.5$ and $\chi = \lambda = 0.18$ so that the agent's beliefs match survey data along two dimensions.[15] First, for a regression of the agent's expectations about future stock market returns on past twelve-month returns, our parameter choice allows the model to produce a regression coefficient and a t-statistic that match the empirical estimates from surveys. Second, our parameter choice allows the agent's beliefs to match the survey evidence on the relative weight investors put on recent versus distant past returns when forming beliefs about future returns. Below we examine these two dimensions in detail.

Empirically, Greenwood and Shleifer (2014) regress survey expectations about future stock market returns on past twelve-month cumulative raw returns across various survey expectations measures. They find that the regression coefficient is positive and statistically significant. To justify our parameter values for θ, χ, and λ, we want to run the same regression in the context of the model. One caveat, however, is that we are uncertain about what survey respondents think the definition

[13] An estimate of 10 for γ is also the maximum magnitude that Mehra and Prescott (1985) find reasonable.

[14] See Bansal et al. (2012) for a discussion of this point.

[15] Recall that $\theta = 0$ means the agent is fully rational, whereas $\theta = 1$ means that the agent is fully behavioral. Therefore, 0.5 is a natural default value for θ: it implies that the representative agent is approximately an aggregation of rational and behavioral agents with equal population weights.

of return is. Does it include the dividend yield or not? Is it a raw return or an excess return? Given this caveat, we examine *four* measures of return expectations: $\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)]$, the agent's expectation about the percentage return on the stock market, $\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)] - r_t$, the agent's expectation about the percentage return in excess of the interest rate, $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$, the agent's expectation about the price growth of the stock market, and $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)] - r_t$, the agent's expectation about the price growth in excess of the interest rate. The latter two measures are also plausible candidates because investors may not actively think about the dividend yield when answering survey questions.[16]

[Place Table 1 about here]

Table 1 reports the regression coefficient, its t-statistic, the intercept, as well as the adjusted R-squared, when regressing each of the four measures of return expectations described above on either the past twelve-month cumulative raw return or the current log price-dividend ratio, over a sample of 15 years or 50 years. Each reported value—for instance, the regression coefficient—is averaged over 100 trials, with each trial being a regression using monthly data simulated from the model. Here we make two observations. First, the magnitude of the agent's extrapolative beliefs about future stock market returns matches the empirical values suggested by Greenwood and Shleifer (2014). Regressing the agent's expectation about future price growth ($\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$) on the past twelve-month cumulative raw return for a 15-year simulated sample, the regression coefficient is 4.0% with a Newey-West adjusted t-statistic of 8.4. Running the same regression for a 50-year simulated sample, the regression coefficient is 4.0% with a t-statistic of 12.1. As a comparison, for a 5-year sample of data from the Michigan survey, the regression coefficient is 3.9% with a t-statistic of 1.68; for a 15-year sample of data from the Gallup survey, the regression coefficient, after some conversion, is 8% with a t-statistic of 8.81.

Second, by comparing the regression coefficients and the t-statistics across the four measures of return expectations, we find that including the dividend yield in the calculation of return reduces the regression coefficient by about a half, but does not significantly affect the t-statistic. Therefore, even though we model return extrapolation as extrapolating past price growth, the agent also holds

[16] Hartzmark and Solomon (2017) provide empirical evidence for the idea that investors do not take the dividend yield into account when calculating returns. Barberis et al. (2015) also take this interpretation when calibrating their model parameters to survey expectations.

extrapolative expectations about the total return. Furthermore, subtracting the interest rate from the expectation of returns only has a small impact on the regression results because of low interest rate volatility. In summary, across all four measures of return expectations, the agent extrapolates past stock market returns when forming expectations about future returns. In Section IV, we compare these regression results with those from the rational expectations models of Bansal and Yaron (2004) and Bansal et al. (2012).

Our parameter choice of θ, χ, and λ is also disciplined by matching the agent's beliefs with the survey evidence on the relative weight of recent versus distant past returns in determining investors' return expectations. Specifically, we estimate the following non-linear least squares regression

$$\text{Expectation}_t = a + b\sum_{j=1}^{n} w_j R^D_{(t-j\Delta t)\to(t-(j-1)\Delta t)} + \varepsilon_t \tag{26}$$

using model simulations, where Expectation_t is the agent's time-t expectation about stock market returns, $R^D_{(t-j\Delta t)\to(t-(j-1)\Delta t)}$ is the raw return from time $t-j\Delta t$ to $t-(j-1)\Delta t$, and $w_j \equiv e^{-\phi(j-1)\Delta t}/\sum_{l=1}^{n} e^{-\phi(l-1)\Delta t}$. In Equation (26), each past realized return is assigned a weight. The weight decreases exponentially the further back we go into the past, and the coefficient ϕ measures the speed of this exponential decline. When ϕ is high, the agent's expectation is determined primarily by recent past returns; when it is low, even distant past returns have a significant impact on the agent's current expectation.

[Place Table 2 about here]

Table 2 reports the intercept a, the regression coefficient b, the adjusted R-squared, and most importantly, the parameter ϕ. As before, we examine four expectations measures, $\mathbb{E}^e_t[(dP^D_t + D_t dt)/(P^D_t dt)]$, $\mathbb{E}^e_t[dP^D_t/(P^D_t dt)]$, $\mathbb{E}^e_t[(dP^D_t + D_t dt)/(P^D_t dt)] - r_t$, and $\mathbb{E}^e_t[dP^D_t/(P^D_t dt)] - r_t$. Each reported value is averaged over 100 trials, with each trial being a regression using simulated data with a monthly frequency over either 15 years or 50 years. We set Δt, the time interval for each past return in (26), to 1/12 (one month), and we set n, the total number of past returns on the right hand side of (26), to 600.[17]

Across the four expectations measures, the estimation of ϕ is stable: it is around 0.42. This

[17] We choose $n = 600$ because further increasing n has a minimal impact on the estimated values of the parameter ϕ and the adjusted R-squared.

value means that a monthly return three years ago is weighted about 25% as much as the most recent return; that is, the agent looks back a couple of years when forming beliefs about future returns. For comparison, Barberis et al. (2015) run the same regression (26) using survey data documented in Greenwood and Shleifer (2014); they estimate ϕ at a value of 0.44. We choose the values of θ, χ, and λ such that the model generates about the same estimate of ϕ as surveys.

The literature has not reached consensus on the value of ϕ. On the one hand, Greenwood and Shleifer (2014) and Kuchler and Zafar (2016) find that investor expectations depend only on recent returns. On the other hand, Malmendier and Nagel (2011, 2013) and Vanasco et al. (2015) suggest that distant past events may also play an important role when investors form beliefs. Reconciling this discrepancy is beyond the scope of the paper. Here, we provide two possible explanations. First, investors may simultaneously adopt two mechanisms when forming beliefs: one that focuses on recent past events such as daily stock market fluctuations, the other that focuses on infrequent but salient events such as a stock market crash. Second, the horizon over which investors form expectations may affect how far they look back into the past. For instance, the survey expectations data studied in Greenwood and Shleifer (2014) are based on questions that ask investors to forecast stock market returns over the next six to twelve months, which may prompt investors to look back only a couple of years. On the other hand, the equity holdings data studied in Malmendier and Nagel (2011) are based on equity investment decisions that may require investors to forecast equity returns over the next couple of decades; they may therefore examine equity performance over the past few decades.

[Place Table 3 about here]

We summarize the default parameter values in Table 3. In Section III, we further provide a comparative statics analysis to examine the sensitivity of the model's implications to changes in these parameter values.

II.2. Building blocks

We start with two building blocks for understanding the model's implications. First, we analyze a set of important equilibrium quantities. We then look at the steady-state distribution of sentiment.

Figure 1 plots the price-dividend ratio of the stock market f, the volatility of stock market returns σ_P^D, the rational expectation about the log excess return $\mathbb{E}[r^{D,e}]$ (the conditional equity premium), and the interest rate r, each as a function of the sentiment variable S.

[Place Figure 1 about here]

Figure 1 shows that the model generates substantial excess volatility: the volatility of dividend growth is 11%, while the volatility of stock market returns is typically above 20%. This result stems from the interaction between return extrapolation and Epstein-Zin preferences. With the coefficient of relative risk aversion γ significantly higher than the reciprocal of the elasticity of intertemporal substitution ψ—we set γ to 10 and ψ to 0.9—the intertemporal substitution effect strongly dominates the wealth effect. Given this, when the stock market has had high past price growth, the agent's forecast of high future price growth—this is a result of the agent extrapolating past price growth—leads her to push up the current price, causing the current price growth to rise. When the current price growth is higher, the agent's forecast of future price growth also becomes higher, which leads her to push up the current price, causing the current price growth to further rise, and so on. In other words, a feedback loop emerges from the interaction between beliefs and preferences, giving rise to significant excess volatility.

The mechanism described above for generating excess volatility also allows the model to generate a strong procyclical pattern for the price-dividend ratio of the stock market. With a strong intertemporal substitution effect in the model, the agent's forecast of high future price growth following high past price growth also leads her to push up the current price-dividend ratio. Figure 1 shows that the price-dividend ratio f is indeed a positive function of sentiment S.

Furthermore, the model generates a strong countercyclical pattern for the true equity premium. Suppose the stock market has had high past price growth. The agent's expectation about future price growth then increases, pushing up the stock market price relative to dividends. Given that sentiment on average tends to revert back to its mean, the price-dividend ratio also tends to mean-revert, leading to low future returns. In addition, the agent's high expectation about future price growth also makes her optimistic about future consumption growth, although to a much lesser extent. This in turn causes the agent to push up the equilibrium interest rate. Together, both a low (rational) expectation of stock market returns and a high interest rate contribute to a low equity

premium during high-sentiment periods. Of these two forces, the first one dominates: moving S_t from its mean to the top 25% percentile level causes a total decrease of 9.7% for the equity premium, out of which 9.5% comes from the decrease in the expected log return of the stock market.

[Place Figure 2 about here]

The second building block for the model's implications is the objectively measured steady-state distribution of sentiment. Figure 2 plots this steady-state distribution. Under the true probability measure, sentiment exhibits a strong degree of mean reversion, for two reasons. First, the agent believes that sentiment will naturally mean-revert: with a regime-switching model, the agent believes that the expected growth rate of stock market prices tends to switch over time from one regime to the other. Second, the agent's price growth expectations mean-revert faster than what she perceives: when the agent thinks that the future price growth is high, future price growth tends to be low endogenously, causing her price growth expectations to decrease at a pace that exceeds her anticipation. Overall, the steady-state distribution has a mean of 2.0% and a standard deviation of 2.7%: the chance of sentiment approaching the extreme values of μ_H and μ_L is close to zero.

II.3. Model implications for asset prices

The two building blocks—the quantitative relation between important equilibrium quantities and sentiment as well as the steady-state distribution of sentiment—allow us to systematically study the model's implications for asset prices. We begin with examining the long-run properties of stock market prices and returns. Table 4 reports the model's predictions for six important moments, and compares them side by side with the empirical values. In general, the model matches the facts: it generates significant excess volatility, a high equity premium, a Sharpe ratio similar to the empirical value, an interest rate that has a low level and low volatility, and a price-dividend ratio whose average level is close to the empirical one.

[Place Table 4 about here]

As explained above, the model generates significant excess volatility from the interaction between return extrapolation and Epstein-Zin preferences. This interaction is quantitatively impor-

tant. Without return extrapolation, Epstein-Zin preferences alone with *i.i.d.* dividend growth and consumption growth do not lead to any excess volatility. Without Epstein-Zin preferences—that is, setting both ψ and γ to 10 while keeping all the other parameter values unchanged—return extrapolation alone leads to average return volatility of 13.8%, which implies much less excess volatility compared to the data.

The model also generates a significant equity premium: when measured as the rational expectation of log excess returns, the true long-run equity premium is 4.9%; when measured as the rational expectation of excess returns $\mathbb{E}[(dP_t^D + D_t dt)/(P_t^D dt) - r_t]$, it is 8.6%. In order to understand this model implication, we first note that the model produces a substantial long-run perceived equity premium—this is what the agent thinks she will receive as the average equity premium. When measured as the subjective expectation of log excess returns, the perceived long-run equity premium is 1.6%; when measured as the subjective expectation of excess returns $\mathbb{E}^e[(dP_t^D + D_t dt)/(P_t^D dt) - r_t]$, it is 5.1%. Three factors affect the perceived long-run equity premium. First and most intuitively, excess volatility causes the agent to demand a higher equity premium because she is risk averse. Second, return extrapolation gives rise to *perceived* persistence of both the aggregate dividend process and, to a lesser extent, the aggregate consumption process.[18] Under Epstein-Zin preferences, this perceived persistence is significantly priced, pushing up the perceived equity premium. Finally, the separation between the elasticity of intertemporal substitution and risk aversion keeps the equilibrium interest rate low and hence helps to keep the equity premium high.

[Place Figure 3 about here]

Furthermore, with incorrect beliefs, the *true* long-run equity premium in the model can be significantly different from the perceived long-run equity premium. We find that the *true* long-run equity premium is significantly higher than the perceived one. In the model, the agent's beliefs mean-revert faster than what she perceives. Given this, she underestimates short-term stock market fluctuations and hence the risk associated with the stock market. In other words, if an infinitesimal rational agent, one that has the same preferences as the behavioral agent but holds rational beliefs, enters our economy, she would have demanded a higher equity premium: the model produces a

[18] The agent is averse to persistent shocks when $\gamma > \psi$; with our choice of parameter values, this condition is satisfied.

true average equity premium that is substantially higher than the perceived equity premium. To illustrate this point, Figure 3 plots the objective (rational) expectation and the agent's subjective expectation about price growth: for S_t less than 3.2%, the objective expectation about price growth is higher than the subjective expectation; for S_t greater than 3.2%, the opposite is true. Because the sentiment distribution has a mean of 2.0% and a standard deviation of 2.7%, Figure 3 suggests that, about 67% of the time, the rational expectation about price growth is above the subjective expectation. That is, the true price growth is more likely to be higher than the perceived price growth. As a result, the model produces a true average equity premium that is substantially higher than the perceived average equity premium.

During high-sentiment periods, the model produces a *negative* equity premium: the equity premium averaged over the top quartile of the sentiment distribution is −13.05%. In general, rational expectations models—for instance, long-run risks models and habit formation models—do not generate a negative equity premium at any time.[19] In our model, however, subjective expectations and objective expectations of stock market returns differ significantly during high- or low-sentiment periods: when the sentiment level is high, the agent expects high stock market returns moving forward, but precisely because of her incorrect beliefs, future stock market returns are low on average, generating a negative equity premium. This model implication is consistent with the recent empirical findings of Greenwood and Hanson (2013), Baron and Xiong (2015), Cassella and Gulen (2017), and Yang and Zhang (2016): these papers document that the expected excess return turns negative during high-sentiment periods.

Next, we examine the model's implications for the predictability of stock market returns. Empirically, Campbell and Shiller (1988) and Fama and French (1988) document that a regression of future log excess returns on the current log price-dividend ratio gives a negative and significant regression coefficient. Moreover, the predictive power of the price-dividend ratio is greater when future returns are calculated over longer horizons.

[Place Table 5 about here]

Table 5 reports the regression coefficient β_j and the adjusted R-squared for a regression of the

[19] Strictly speaking, a rational expectations model can generate a negative equity premium if the stock market negatively correlates with some other risk factors in the agent's portfolio and therefore serves as a diversification device.

log excess return of the stock market from time t to time $t+j$ on the current log price-dividend ratio

$$r^{D,e}_{t\to t+j} = \alpha_j + \beta_j \ell\mathrm{n}(P^D_t / D_t) + \varepsilon_{j,t} \tag{27}$$

over various time horizons j. We calculate the regression coefficients and the R-squared using 10,000 years of monthly data simulated from the model, and compare them side by side with the empirical values. Consistent with the data, β_j is negative and its magnitude increases as the time horizon j increases. A strong degree of mean reversion in sentiment, together with the feedback loop described above for generating excess volatility, allows the model to produce the long-horizon predictability of stock market returns. When the stock market has had high past price growth, the agent's expectation about future price growth increases, pushing up the current price-dividend ratio. Since the agent's expectation—the sentiment variable—tends to revert back to its mean, subsequent returns are low on average, giving rise to a negative regression coefficient in (27).

The magnitudes of the regression coefficient and the R-squared generated from the model are broadly consistent with the empirical values. One difference, however, is that in the model, the R-squared begins to decrease as the time horizon j increases beyond three years, whereas in the data, the R-squared keeps rising over longer horizons. To understand this difference, recall that we calibrate the model to the survey expectations by setting θ to 0.5 and setting λ and χ to 0.18: the agent looks back a couple of years when forming beliefs about future returns. Given this parameter choice, the mean reversion of sentiment tends to occur over the first few years. Over longer horizons, no additional mean reversion in the agent's beliefs contributes to the predictability of stock market returns.

We now further investigate the model's implications for the correlation between stock market returns and consumption growth. Empirically, Hansen and Singleton (1982, 1983) document that this correlation is low. Nonetheless, most consumption-based asset pricing models generate a high, if not perfect, correlation between stock market returns and consumption growth. By imposing rational expectations, these models treat the consumption-based pricing kernels as the only source of stock market movements; stock market movements are driven by changes in consumption. As a result, the correlation between stock market returns and consumption growth is high.[20]

[20] One exception is Barberis et al. (2001). They use "narrow framing", the notion that investors may evaluate financial risks in isolation from consumption risks, to generate a low correlation between stock market returns and

[Place Table 6 about here]

Table 6 reports both the model-implied values and the empirical values for the correlation between consumption growth and stock market returns. Consistent with the data, the model produces a low correlation: the correlation between annual log consumption growth and annual log excess returns is 0.19 in the model, and similarly it is 0.09 in the data. In comparison, the model of Campbell and Cochrane (1999) generates a correlation of 0.79, a much higher value. In our model, we assume that the bias in the agent's beliefs about consumption growth comes only from the bias in her beliefs about dividend growth. Given the low correlation observed in the data between consumption growth and dividend growth, the bias in the agent's beliefs about consumption growth in small. As a result, the agent's beliefs about stock market returns—they co-move strongly with her beliefs about dividend growth—are not significantly affected by fluctuations in consumption growth, giving rise to the low observed correlation between stock market returns and consumption growth.

Table 6 shows that the model also generates a small but negative correlation between the current consumption growth and the stock market return in the subsequent period, an implication that is consistent with the data. Recall that consumption growth and dividend growth are weakly but positively correlated. If the current consumption growth is high, dividend growth is also high on average, which leads the agent to push up the current price, increasing the current price growth and the current level of sentiment. In the subsequent period, sentiment reverts towards its mean value, giving rise to a low stock market return.

[Place Figure 4 about here]

Although our model is based on return extrapolation, it yields direct implications for cash flow expectations. The expectations transmission mechanism described by equation (11) suggests that the agent's expectation about dividend growth is more responsive to changes in sentiment than her expectation about price growth. Moreover, the total return equals the sum of the price growth and the dividend yield, and the dividend yield decreases as sentiment increases. Therefore, the

consumption growth. Specifically, they use power utility as the agent's preferences over consumption, but use prospect theory developed by Kahneman and Tversky (1979) as the agent's preferences over financial wealth.

agent's expectation about price growth is more responsive to changes in sentiment compared to her expectation about stock market returns.

Figure 4 plots the agent's expectation about stock market returns, $\mathbb{E}^e[(dP_t^D + D_t dt)/(P_t^D dt)]$, the agent's expectation about price growth, $\mathbb{E}^e[dP_t^D/(P_t^D dt)]$, and the agent's expectation about dividend growth, $\mathbb{E}^e[dD_t/(D_t dt)]$, each as a function of the sentiment variable S. Quantitatively, Figure 4 suggests that a one-standard deviation (2.7%) increase in sentiment from its mean (2.0%) pushes up the agent's expectation about stock market returns from 7.44% to 8.22%—a small increase of 0.78%—while it pushes up the agent's expectation about dividend growth from -0.04% to 6.48%—a much larger increase of 6.52%. Also, recall from Figure 1 that a one-standard deviation increase in sentiment from its mean pushes up the price-dividend ratio of the stock market from 19.16 to 21.75. These results together imply that the price-dividend ratio is mainly correlated with the agent's expectation about dividend growth.

To further understand stock market movements, we follow the procedure in Campbell and Shiller (1988) to decompose, in the context of the model, the log price-dividend ratio of the stock market:

$$\ell\mathrm{n}(P_t^D/D_t) \approx \sum_{j=0}^{\infty} \xi^j \left(\Delta d_{(t+j\Delta t)\to(t+(j+1)\Delta t)} - r^D_{(t+j\Delta t)\to(t+(j+1)\Delta t)}\right) - (\ell\mathrm{n}(\xi) + (1-\xi)\zeta)/(1-\xi), \tag{28}$$

where ζ is the in-sample average of the annual log dividend-price ratio, $\xi = e^{-\zeta}/(\Delta t + e^{-\zeta})$, $\Delta d_{(t+j\Delta t)\to(t+(j+1)\Delta t)}$ is the log dividend growth from time $t+j\Delta t$ to $t+(j+1)\Delta t$, and $r^D_{(t+j\Delta t)\to(t+(j+1)\Delta t)}$ is the log gross return over the same period. Equation (28) says that the movement of price-dividend ratio comes from either the movement of future dividend growth—this is called "cash flow news"—or the movement of future returns—this is called "discount rate news." The standard approach that empirically addresses the relative importance of these two components is to look at future *realized*

dividend growth and returns, and compute

$$1 \approx \underbrace{\frac{\mathbb{C}\text{ov}\left(\sum_{j=0}^{\infty}\xi^{j}\Delta d_{(t+j\Delta t)\to(t+(j+1)\Delta t)},\ell\text{n}(P_t^D/D_t)\right)}{\mathbb{V}\text{ar}\left(\ell\text{n}(P_t^D/D_t)\right)}}_{CF_{objective}} + \underbrace{\frac{\mathbb{C}\text{ov}\left(-\sum_{j=0}^{\infty}\xi^{j}r^{D}_{(t+j\Delta t)\to(t+(j+1)\Delta t)},\ell\text{n}(P_t^D/D_t)\right)}{\mathbb{V}\text{ar}\left(\ell\text{n}(P_t^D/D_t)\right)}}_{DR_{objective}}. \tag{29}$$

The first term on the right hand side of (29), $CF_{objective}$, is the contribution of changes in cash flow news to stock market movements, and the second term, $DR_{objective}$, is the contribution of changes in discount rate news to stock market movements. By using future realized dividend growth and returns, this approach effectively imposes rational expectations. Most empirical studies that have conducted a Campbell-Shiller decomposition take this approach.

However, in a model with incorrect beliefs, we can further study the relation between the agent's *subjective* expectations and stock market movements by taking the subjective expectations on both sides of (28) and computing

$$1 \approx \underbrace{\frac{\mathbb{C}\text{ov}\left(\mathbb{E}_t^e[\sum_{j=0}^{\infty}\xi^{j}\Delta d_{(t+j\Delta t)\to(t+(j+1)\Delta t)}],\ell\text{n}(P_t^D/D_t)\right)}{\mathbb{V}\text{ar}\left(\ell\text{n}(P_t^D/D_t)\right)}}_{CF_{subjective}} + \underbrace{\frac{\mathbb{C}\text{ov}\left(-\mathbb{E}_t^e[\sum_{j=0}^{\infty}\xi^{j}r^{D}_{(t+j\Delta t)\to(t+(j+1)\Delta t)}],\ell\text{n}(P_t^D/D_t)\right)}{\mathbb{V}\text{ar}\left(\ell\text{n}(P_t^D/D_t)\right)}}_{DR_{subjective}}. \tag{30}$$

The first term on the right hand side of (30), $CF_{subjective}$, is the contribution of changes in subjective expectations about cash flow news to stock market movements, and the second term, $DR_{subjective}$, is the contribution of changes in subjective expectations about discount rate news to stock market movements.

[Place Table 7 about here]

Table 7 reports the four coefficients, $CF_{objective}$, $DR_{objective}$, $CF_{subjective}$, $DR_{subjective}$, as well as their corresponding adjusted R-squared. These coefficients and R-squared are calculated using

10,000 years of monthly data simulated from the model. By using future realized dividend growth and stock market returns and therefore imposing rational expectations, we obtain $DR_{objective} = 0.98$ with a R-squared of 0.209 and $CF_{objective} = 0.02$ with a R-squared of 1.2×10^{-4}. This result replicates the empirical finding of the volatility test literature that the variation of the price-dividend ratio comes primarily from discount rate variation (see Cochrane (2008) and Cochrane (2011)).

On the other hand, by relaxing the rational expectations assumption and using the agent's subjective expectations about dividend growth and returns, we obtain $DR_{subjective} = -0.08$ with a R-squared of 0.982 and $CF_{subjective} = 1.08$ with a R-squared of 0.984. This result unveils a very different picture and highlights the importance of expectations data: changes in the agent's subjective expectations about future cash flow news explain the majority of stock market movements. Empirically, de la O and Myers (2017) find that $DR_{subjective} = -0.09$ and $CF_{subjective} = 1.09$. These values match the theoretical values from our model.

Importantly, the fact that prices in our model are mainly correlated with cash flow expectations is a consequence of the Campbell-Shiller accounting identity; this statement is about correlation, not about causality. The agent's *return* expectations determine her cash flow expectations and are the cause of price movements. Given this, our model simultaneously explains the empirical findings of de la O and Myers (2017) on cash flow expectations and the empirical findings of Greenwood and Shleifer (2014) on return expectations. We provide additional discussion about the relation between return expectations and cash flow expectations in the Appendix.

[Place Table 8 about here]

The model also points to some challenges: when calibrated to the survey expectations data, the model predicts a persistence of the price-dividend ratio that is significantly lower than its empirical value. Table 8 presents the empirical values and theoretical values for the autocorrelations of asset prices. Empirically, price-dividend ratios are highly persistent at short lags. Nonetheless, the model produces a persistence for the price-dividend ratio that is much *lower* than the empirical value: the autocorrelation of $\ell n(P^D/D)$ with a lag of three years is 0.5 in the data, but it is essentially zero in the model. In the model, the persistence of the price-dividend ratio is driven by the persistence of the agent's beliefs. The available survey evidence suggests that investors focus on just the past

year or two when forming beliefs about future returns. Therefore, when calibrated to surveys, the agent's beliefs tend to mean-revert in a couple of years. However, to match the empirical persistence of the price-dividend ratio, the agent's beliefs need to be much more persistent: the agent needs to form beliefs about future returns based on many years of past returns.

We leave a careful reconciliation of the survey expectations about stock market returns and the observed persistence of the price-dividend ratio to future research. One possibility, however, is to develop a framework that allows for the interaction between financial frictions and the agent's beliefs. Lopez-Salido, Stein, and Zakrajsek (2016) show that various types of financial frictions are empirically more persistent than investor sentiment. As a result, investor beliefs can affect asset prices through their interaction with financial frictions, making their impact more persistent.

We conclude this section by discussing the role of rational arbitrageurs. The model has a representative agent whose beliefs are biased. One natural question to ask is: if we introduce rational arbitrageurs, to what extent can they counteract the mispricing caused by the behavioral agent and therefore attenuate the significance of the model implications? Developing such a two-agent model is beyond the scope of the paper. However, three observations suggest that our model implications will remain intact after taking rational arbitrageurs into account.

First, in an economy with both rational and behavioral agents who have recursive preferences, the behavioral agents may eventually dominate the market: there is a positive probability that they take up most of the wealth in the economy in the long run. This is a key finding in Borovicka (2016). It suggests that our model's implications can be the limiting implications of a model with both rational and behavioral agents in the initial period. Second, in an economy with heterogeneous beliefs, asset prices are jointly determined by agents' beliefs weighted by their risk tolerances. A positive fundamental shock causes optimists to gain a larger fraction of wealth and increases their risk tolerance relative to pessimists, which in turn gives optimists a greater weight in driving asset prices, pushing asset prices further up. As a result, heterogeneity in investor beliefs can be an additional source of excess volatility; it can further amplify—rather than attenuate—our model implications.[21] Lastly, as pointed out by Barberis et al. (2015), extrapolative expectations are persistent in a dynamic model, which means that the behavioral agents who extrapolate past returns have persistently high demand for the stock market following high stock market returns. The

[21] See Xiong (2013) for more discussion of this amplification mechanism.

persistence of this demand prevents near-future stock market returns from becoming too low, which reduces the incentive of rational agents to counteract mispricing. In other words, the persistence of extrapolative beliefs limits the impact of rational arbitrageurs on asset prices.

III. Comparative Statics

In this section, we examine the sensitivity of the model's implications to changes in parameter values. We focus on parameters that either have dispersed estimates in the literature or cannot be directly observed from the data. Specifically, for the utility parameters, we study how changes in γ, the coefficient of relative risk aversion, and ψ, the reciprocal of the elasticity of intertemporal substitution, affect the equity premium and the volatility of stock market returns. For the belief parameters, we look at how changes in θ affect the equity premium, the volatility of stock market returns, the price-dividend ratio, and the average interest rate. We also examine how changes in θ, χ, and λ affect return predictability and the persistence of the price-dividend ratio.

III.1. Utility parameters

Figure 5 plots the long-run average of the equity premium and the volatility of stock market returns, each as a function of γ or ψ. The coefficient of relative risk aversion is positively related to the equity premium but negatively related to the volatility of returns. Lower risk aversion naturally leads the agent to require a lower equity premium for risk compensation; reducing γ from 10 to 5, the model still explains 75% of the observed equity premium. At the same time, lower risk aversion strengthens the feedback loop described earlier since it increases the agent's demand for risky assets. Therefore, it leads to higher return volatility.

[Place Figure 5 about here]

Within the examined range, changes in the elasticity of intertemporal substitution do not significantly affect the average equity premium and the average volatility of returns. As a result, our model implications are quantitatively robust to changes in ψ. We provide a detailed explanation of this observation in the next section when we compare our model with the model of Bansal and Yaron (2004).

III.2. Belief parameters

Belief Parameters.—Figure 6a plots the long-run average of the equity premium, the volatility of stock market returns, the price-dividend ratio, and the interest rate, each as a function of θ, the parameter that measures the extent to which the agent is behavioral. Setting θ to zero gives the agent fully rational beliefs. In this case, the equity premium is 0.23%; the return volatility equals the fundamental volatility of 11%; the price-dividend ratio stays constant at 135; and the interest rate stays at 2.35%. With $\theta = 0$, the model fails to match the long-run properties of the stock market. Increasing θ from zero allows the feedback loop described above to emerge, which generates excess volatility, pushes up the perceived equity premium, and significantly reduces the price-dividend ratio. At the same time, a higher θ—and the agent's more extrapolative beliefs about stock market returns—leads the agent to perceive a higher persistence in dividend growth, which, under Epstein-Zin preferences, is significantly priced, causing the perceived equity premium to further rise. Finally, a higher θ leads to a true equity premium that is significantly higher than the perceived one.

[Place Figure 6a about here]

Overall, a 1% increase in θ leads to a 0.09% increase in the equity premium and a 0.29% increase in the volatility of returns. On the other hand, the effect of θ on the interest rate is small. A higher θ increases the extrapolation bias in the agent's beliefs about stock market returns, but does not significantly affect her beliefs about consumption growth, which determine the equilibrium interest rate.

Figure 6b examines how changes in θ, χ, and λ affect the predictability of stock market returns and the persistence of the price-dividend ratio. Here the predictability of returns is measured by the slope coefficient in a regression of the next year's log excess return on the current log price-dividend ratio; the persistence of the price-dividend ratio is measured by the one-year autocorrelation of log price-dividend ratios.

[Place Figure 6b about here]

Figure 6b shows that higher values of θ, χ, and λ lead to stronger predictability of returns and a lower persistence of the price-dividend ratio. Higher values of χ and λ—that is, higher

perceived transition intensities between the high- and low-mean price growth regimes—suggest that the agent focuses on a shorter history of past return realizations when forming beliefs about future returns. A higher value of θ has a similar implication: it suggests that the agent exhibits a stronger extrapolation bias, which means that the agent deviates more from a rational agent whose beliefs depend on both recent and distant past returns. In other words, with a higher θ, the agent relies more heavily on recent past returns when forming beliefs about future returns. Therefore, higher values of θ, χ, and λ all lead to a stronger degree of mean reversion in sentiment, which in turn gives rise to stronger predictability of returns and a lower persistence of the price-dividend ratio.

The comparative statics results in Figure 6b are consistent with recent empirical findings. At the aggregate level, Cassella and Gulen (2017) find that, during periods when investors' expectations about future returns depend on both recent and distant past returns, the price-dividend ratio does not strongly predict the next year's return. Conversely, during periods when investors' expectations depend primarily on recent past returns, the price-dividend ratio strongly predicts the next year's return. In our model, higher values of θ, χ, and λ lead to a higher ϕ, and therefore the agent's expectations about future returns depend more heavily on recent past returns. In the meantime, they also lead to stronger return predictability. Overall, the model implies that, when the agent forms beliefs based on a short history of past returns, the predictability of returns is strong. To give an example, increasing θ from 0.05 to 0.5 changes ϕ from 0.37 to 0.43. At the same time, it changes β_1, the slope coefficient for a regression of the next year's log excess return on the current log price-dividend ratio, from -0.32 to -0.72; the corresponding R-squared increases from 0.001 to 0.13. In the cross-section, Da, Huang, and Jin (2017) show that stocks associated with a larger extrapolation bias—beliefs of forecasters on these stocks depend more strongly on recent past returns—exhibit stronger return reversals. Applying our model to individual stocks gives the same prediction.

IV. Comparison with Rational Expectations Models

In this section, we provide a quantitative comparison between our model and several models with rational expectations. First, we look at a rational expectations model that is most analogous

to our behavioral model, one in which the regime-switching process characterized in Section I represents the *true* data generating process. We then examine differences between our model and the rational expectations models of Bansal and Yaron (2004) and Bansal et al. (2012): we focus on the long-run risks models because these are the models most related to ours.

IV.1. The true regime-switching model

The rational expectations model most analogous to our model is the one that assumes the regime-switching process characterized in equations (6) and (7) is the true data generating process. In this case, the true evolution of the stock market price is

$$dP_t^D/P_t^D = [(1-\theta)g_D + \theta\tilde{\mu}_{S,t}]dt + \sigma_P^D(S_t)d\omega_t^P, \tag{31}$$

where ω_t^P is a standard Brownian motion. As with the behavioral model, the agent in this model does not directly observe the latent variable $\tilde{\mu}_{S,t}$. Instead, she uses past stock market prices to form a Bayesian estimate of $\tilde{\mu}_{S,t}$: $S_t = \mathbb{E}[\tilde{\mu}_{S,t}|\mathcal{F}_t^P]$. That is, the perceived evolution of stock market price in (8) is fully rational. We further assume that the perceived dividend process (10) and the perceived consumption process (14) are also rational.

By construction, this rational expectations model produces the same equilibrium prices as our behavioral model: the solutions to the differential equations of (16) and (17) also apply to this model. Nonetheless, the two models have statistical properties that are significantly different. One difference, for instance, lies in the models' implications for the predictability of stock market returns.

[Place Table 9 about here]

Table 9 reports the regression coefficient β_j and the adjusted R-squared for a regression of the log excess return of the stock market from time t to time $t+j$ on the current log price-dividend ratio $\ell\text{n}(P_t^D/D_t)$ over various time horizons j (one to seven years), now using the true regime-switching model. Table 9 shows that the model fails to produce the predictability of stock market returns documented in Campbell and Shiller (1988) and Fama and French (1988): both the regression coefficients and the R-squared are close to zero. In contrast, Table 5 shows that the behavioral

model produces the observed predictability of stock market returns.

With rational expectations, the agent's beliefs about stock market returns are on average correct. Therefore, following high past price growth, the agent properly anticipates high future price growth, which pushes down the dividend yield in equilibrium, leading to flat returns in subsequent periods. As a result, future returns do not vary significantly with the current price-dividend ratio, giving rise to the lack of return predictability in the true regime-switching model.

IV.2. The long-run risks models

Table 10 reports the regression coefficients and t-statistics when regressing four measures of rational expectations of return—raw return or excess return, with or without dividend yield—either on the past twelve-month return or on the current log price-dividend ratio. The regressions are based on simulated data from Bansal and Yaron (2004). Interestingly, their model generates, to some extent, extrapolative expectations about future raw returns: a regression of the agent's expectations—this is also the rational expectation—about the next twelve-month total return on past twelve-month total return yields a coefficient of 2.5% with a t-statistic of 2.4 for a 15-year simulated sample; the regression coefficient is 3.0% with a t-statistic of 3.8 for a 50-year simulated sample.

[Place Table 10 about here]

In Bansal and Yaron (2004), dividend growth and consumption growth share a stochastic yet persistent component. High past stock market returns are typically caused by positive shocks to this common component, which, given its persistence, implies high dividend growth and hence high raw returns moving forward. That is, the agent in Bansal and Yaron (2004) has extrapolative beliefs about future raw returns. At the same time, precisely because dividend growth and consumption growth share a persistent component, high dividend growth tends to coincide with high consumption growth, which implies a high interest rate in equilibrium. That is, when raw returns are high, the interest rate is also high. As a result, the agent does not have extrapolative beliefs about future *excess* returns: as shown in Table 10, regressing the agent's expectation about future excess returns on past returns, the regression coefficient and the t-statistic are both close to zero.

These regression results highlight one fundamental difference between our model and the model of Bansal and Yaron (2004). In our model, the agent extrapolates past returns on the stock market, but extrapolates past consumption growth much less. The comovement between the agent's beliefs about returns and her beliefs about consumption growth is low. On the contrary, in Bansal and Yaron (2004), the comovement between the agent's beliefs about stock market returns—these rationally drive returns—and her beliefs about consumption growth—these determine the interest rate in equilibrium—is high. Therefore, the agent in our model has extrapolative beliefs about *both* raw returns and excess returns, whereas the agent in Bansal and Yaron (2004) has extrapolative beliefs only about raw returns.

Furthermore, as shown in Figure 5, our model produces an equity premium and return volatility that do not vary significantly with respect to changes in the elasticity of intertemporal substitution. On the contrary, the long-run risks models cannot generate a high equity premium with a low elasticity of intertemporal substitution. For instance, setting the elasticity of intertemporal substitution to 0.5, our model generates an equity premium of 7.1% (measured as the rational expectation of excess returns), while the model of Bansal and Yaron (2004) produces an equity premium between 1% and 2%. Given this contrast, our model does not face the challenge of defending an elasticity of intertemporal substitution that is greater than one.

To understand this model difference, first note that sentiment, the state variable in our model, is much less persistent than the stochastic component of dividend and consumption growth in Bansal and Yaron (2004), allowing the equilibrium interest rate and hence the equity premium in our model to be less responsive to changes in the elasticity of intertemporal substitution. At the same time, the perceived dividend growth in our model depends more strongly on the state variable of sentiment, pushing up the perceived equity premium; as a comparison, dividend growth in Bansal and Yaron (2004) depends much less on the stochastic growth component.[22] Finally, the true average equity premium in our model is above the perceived one, allowing the equity premium to be high even when the elasticity of intertemporal substitution is low.

Table 11 repeats the regression analyses of Table 10 using the model of Bansal et al. (2012). Compared to the original model of Bansal and Yaron (2004), Bansal et al. (2012) introduces ad-

[22] Bansal and Yaron (2004) set their "leverage parameter" ϕ to 3.5. In comparison, with return extrapolation, our model effectively sets ϕ to 14.5, a much higher value.

ditional time variation in the long-run risks that further reduces the model's ability to generate extrapolative expectations: when regressing the agent's expectation of raw returns on past returns, the coefficient now becomes insignificant.

[Place Table 11 about here]

We complete our discussion in this section by making two remarks. First, the persistence introduced by the long-run risks models on consumption growth and dividend growth leads to excess predictability, the notion that future consumption growth and dividend growth are excessively predicted by current variables such as the price-dividend ratio and the interest rate (see Beeler and Campbell (2012) and Collin-Dufresne et al. (2016b) for a detailed discussion). Our model does not give rise to excess predictability: return extrapolation in the model only generates *perceived* but not true long-run risks, and therefore the true consumption growth and dividend growth remain unpredictable.

Second, the rational expectations models and our model generate different implications regarding the role of cash flow expectations in understanding stock market movements. In models with rational expectations, the stock market price is mainly correlated with subjective discount rate (return) expectations; it is not significantly correlated with cash flow expectations. In our model, however, the stock market price is mainly correlated with the agent's subjective expectations of future cash flow growth.

V. Fundamental Extrapolation

A literature in behavioral finance focuses on fundamental extrapolation, the notion that some investors hold extrapolative expectations about *fundamentals* such as dividend growth or GDP growth (Barberis et al. (1998); Fuster et al. (2011); Choi and Mertens (2013); Alti and Tetlock (2014); Hirshleifer et al. (2015)). In this section, we provide a quantitative comparison between our model and a model with fundamental extrapolation. To facilitate the comparison, we keep the two models almost identical. The only difference is that, in the model with fundamental extrapolation, sentiment is constructed from past dividend growth, whereas in the model with return extrapolation, sentiment is constructed from past price growth. Below we first briefly describe the key

assumptions in this fundamental extrapolation model. We then discuss the model's implications.

V.1. Model setup

With fundamental extrapolation, the agent believes that the expected growth rate of *dividends*—instead of the expected growth rate of stock market prices in the case of return extrapolation—is governed by $(1-\theta)g_D + \theta\tilde{\mu}_{S,t}$, where $\tilde{\mu}_{S,t}$ is a latent variable that follows a regime-switching process described in Section I. The agent does not directly observe the latent variable $\tilde{\mu}_{S,t}$. Instead, she computes its expected value given the history of past dividend growth: $S_t \equiv \mathbb{E}[\tilde{\mu}_{S,t}|\mathcal{F}_t^D]$. She then applies optimal filtering theory and derives

$$\begin{aligned} dS_t =&(\lambda\mu_H + \chi\mu_L - (\lambda+\chi)S_t)dt + \sigma_D^{-1}\theta(\mu_H - S_t)(S_t - \mu_L)d\omega_t^e \\ \equiv&\mu_S^e(S_t)dt + \sigma_S(S_t)d\omega_t^e, \end{aligned} \tag{32}$$

where $d\omega_t^e \equiv [dD_t/D_t - (1-\theta)g_D dt - \theta S_t dt]/\sigma_D$ is a standard Brownian innovation term from the agent's perspective. That is, she perceives the evolution of dividend as

$$dD_t/D_t = g_D^e(S_t)dt + \sigma_D d\omega_t^e, \tag{33}$$

where

$$g_D^e(S_t) = (1-\theta)g_D + \theta S_t. \tag{34}$$

In other words, the agent's expectation about dividend growth $g_D^e(S_t)$ is a linear combination of a rational component g_D and a sentiment component S_t constructed from past dividend growth. On the other hand, the perceived evolution of the stock market price can be derived as

$$dP_t^D/P_t^D = \mu_P^{D,e}(S_t)dt + \sigma_P^D(S_t)d\omega_t^e, \tag{35}$$

where

$$\begin{aligned} \sigma_P^D(S) =&\sigma_D + (f'/f)\sigma_D^{-1}\theta(\mu_H - S)(S - \mu_L), \\ \mu_P^{D,e}(S) =&(f'/f)\mu_S^e + \tfrac{1}{2}(f''/f)\sigma_S^2 + (1-\theta)g_D + \theta S - \sigma_D^2 + \sigma_D\sigma_P^D(S). \end{aligned} \tag{36}$$

As before, f is defined as the price-dividend ratio of the stock market.

As with the return extrapolation model, equations (16) and (17) determine f and l, the price-dividend ratio and the wealth-consumption ratio, except that μ_S, σ_S, g_D^e, $\mu_P^{D,e}$, and σ_P^D are now from (32), (34), and (36).

V.2. Model implications

We first examine the model's implications for investor expectations. Table 12 reports the regression coefficient, its t-statistic, the intercept, as well as the adjusted R-squared, when regressing the four measures of return expectations on either the past twelve-month cumulative raw returns or the current log price-dividend ratio. With fundamental extrapolation, the regression coefficient on past returns and the t-statistic are both close to zero.

[Place Table 12 about here]

Suppose past dividend growth has been high. On the one hand, it results in high past returns. On the other hand, fundamental extrapolation leads the agent to expect high dividend growth moving forward, but *not* high returns: following high past dividend growth, the stock market price increases to the extent that the agent's expectation of future returns does not change significantly. A fundamental extrapolation model with a representative agent therefore faces a challenge in explaining survey expectations about returns. In contrast, our return extrapolation model is constructed to explain these survey expectations.[23]

[Place Table 13 about here]

Table 13 analyzes the model's fit with the long-run properties of the stock market. Using the same parameters that allow the return extrapolation model to well explain the important moments of the stock market, the fundamental extrapolation model generates lower excess volatility and a much lower equity premium. Quantitatively, fundamental extrapolation generates 74.4% of excess volatility and 37.9% of the equity premium that return extrapolation produces.

[23] A fundamental extrapolation model with *heterogeneous* agents—for instance, one with both an agent who extrapolates past dividend growth and an agent who is fully rational—can potentially generate extrapolative expectations of returns for the behavioral agent in the model. See the model of Ehling et al. (2015) as an example.

This quantitative comparison highlights the importance of the feedback loop described above in matching asset pricing facts. With return extrapolation, the feedback loop emerges because extrapolative beliefs are applied to the stock market return, a variable that is *endogenously* determined in equilibrium. With fundamental extrapolation, however, the feedback loop is absent because extrapolative beliefs are applied to dividend growth, a variable that is exogenous in the model: high past dividend growth makes the agent optimistic about future dividend growth and therefore pushes up the current price, but the higher price does not further affect the agent's beliefs about future dividend growth.

This feedback loop also points to a methodological contribution of the paper. Equation (32) shows that, in a fundamental extrapolation model, sentiment S, the state variable that drives asset prices dynamics, can be exogenously specified without solving the equilibrium; this greatly simplifies the model. On the other hand, with return extrapolation, sentiment S determines—and is endogenously determined by—asset prices. As a result, such a model requires solving beliefs and asset prices simultaneously, and therefore imposes a greater modeling challenge. Our numerical approach to solving a system of differential equations provides a solution to this challenge.

VI. Conclusion

We build a new return extrapolation model that can be brought to the data quantitatively. With the agent's beliefs calibrated to fit the extrapolative expectations data documented in surveys, the model matches the long-run properties of stock market prices: it generates a high average equity premium, significant excess volatility, a low average interest rate, low interest rate volatility, and a price-dividend ratio whose average level is similar to the empirical one. The model also matches the dynamic behavior of stock market prices: it produces the long-horizon predictability of stock market returns, and generates the observed low correlation between stock market returns and consumption growth. We compare our model to the long-run risks models and find that our model's quantitative implications are more robust to changes in the elasticity of intertemporal substitution.

Our analysis has left several important issues for future work. First, when calibrated with the survey expectations data, the model predicts a persistence of the price-dividend ratio that is significantly lower than its empirical value. To reconcile the survey expectations about stock market

returns with the observed persistence of the price-dividend ratio, we need a deeper understanding about how investors form beliefs. Second, the extrapolation framework is closely linked to theories of model uncertainty. A careful investigation of this connection may produce useful insights to both literatures. Finally, our representative-agent model neglects an important channel that affects asset prices: the time-varying fraction of wealth held by behavioral agents. Explicitly incorporating rational agents into our framework may lead to additional implications.

Appendices

A. *Derivation of the Differential Equations (16) and (17)*

For the subjective Euler equation (4), setting $\tilde{R}_{j,t+dt}$, the return on the tradeable asset, to the gross return on the stock market, the equation becomes

$$\mathbb{E}_t^e\left[e^{-\delta(1-\gamma)dt/(1-\psi)}\left(\frac{\tilde{C}_{t+dt}}{C_t}\right)^{-\psi(1-\gamma)/(1-\psi)}\left(\frac{\tilde{P}^C_{t+dt}+\tilde{C}_t dt}{P^C_t}\right)^{(\psi-\gamma)/(1-\psi)}\frac{\tilde{P}^D_{t+dt}+\tilde{D}_{t+dt}dt}{P^D_t}\right]=1. \tag{A.1}$$

Using Taylor expansion, (A.1) becomes

$$\begin{aligned}&\mathbb{E}_t^e\left[e^{-\delta(1-\gamma)dt/(1-\psi)}\left(\tilde{C}_{t+dt}\right)^{-\psi(1-\gamma)/(1-\psi)}(\tilde{P}^C_{t+dt})^{(\psi-\gamma)/(1-\psi)}\tilde{P}^D_{t+dt}\left(1+\tfrac{\psi-\gamma}{1-\psi}\tfrac{\tilde{C}_{t+dt}}{\tilde{P}^C_{t+dt}}dt+\tfrac{\tilde{D}_{t+dt}}{\tilde{P}^D_{t+dt}}dt\right)\right]\\&=C_t^{-\psi(1-\gamma)/(1-\psi)}(P^C_t)^{(\psi-\gamma)/(1-\psi)}P^D_t.\end{aligned} \tag{A.2}$$

Rearranging terms gives

$$0=\mathbb{E}_t^e\left[\begin{array}{l}d(\Theta C^{(\psi-\gamma)/(1-\psi)}l^{(\psi-\gamma)/(1-\psi)}Df)+\frac{\psi-\gamma}{1-\psi}\Theta C^{(\psi-\gamma)/(1-\psi)}l^{(2\psi-\gamma-1)/(1-\psi)}Dfdt\\ \\ +\Theta C^{(\psi-\gamma)/(1-\psi)}l^{(\psi-\gamma)/(1-\psi)}Ddt\end{array}\right], \tag{A.3}$$

where $\Theta(C,t)\equiv e^{-\delta(1-\gamma)t/(1-\psi)}C^{-\psi(1-\gamma)/(1-\psi)}$. By Ito's lemma, (A.3) leads to

$$0=\mathbb{E}_t^e\left[\begin{array}{l}-\frac{\delta(1-\gamma)}{1-\psi}dt-\gamma(dC/C)+(dD/D)+(df/f)+\frac{\psi-\gamma}{1-\psi}(dl/l)+\frac{\gamma(\gamma+1)}{2}(dC/C)^2\\ \\ +\frac{1}{2}\frac{\psi-\gamma}{1-\psi}\frac{2\psi-\gamma-1}{1-\psi}(dl/l)^2-\frac{\gamma(\psi-\gamma)}{1-\psi}(dC/C)(dl/l)-\gamma(dC/C)(dD/D)-\gamma(dC/C)(df/f)\\ \\ +\frac{\psi-\gamma}{1-\psi}(dl/l)(dD/D)+\frac{\psi-\gamma}{1-\psi}(dl/l)(df/f)+(df/f)(dD/D)+\frac{\psi-\gamma}{1-\psi}l^{-1}dt+f^{-1}dt\end{array}\right]. \tag{A.4}$$

Using (7), (10), and (14) to further simplify (A.4) gives (16).

Setting $\tilde{R}_{j,t+dt}$ in (4) to the gross return on the Lucas tree, the subjective Euler equation (4)

becomes

$$\mathbb{E}_t^e\left[e^{-\delta(1-\gamma)dt/(1-\psi)}\left(\frac{\tilde{C}_{t+dt}}{C_t}\right)^{-\psi(1-\gamma)/(1-\psi)}\left(\frac{\tilde{P}^C_{t+dt}+\tilde{C}_t dt}{P^C_t}\right)^{(1-\gamma)/(1-\psi)}\right]=1. \tag{A.5}$$

Rearranging terms yields

$$0=\mathbb{E}_t^e\left[d(\Theta C^{(1-\gamma)/(1-\psi)}l^{(1-\gamma)/(1-\psi)})+\tfrac{1-\gamma}{1-\psi}\Theta C^{(1-\gamma)/(1-\psi)}l^{(\psi-\gamma)/(1-\psi)}dt\right]. \tag{A.6}$$

By Ito's lemma, (A.6) leads to

$$0=\mathbb{E}_t^e\left[\begin{array}{l}-\frac{1-\gamma}{1-\psi}\delta dt-(\gamma-1)(dC/C)+\frac{\gamma(\gamma-1)}{2}(dC/C)^2+\frac{1-\gamma}{1-\psi}(dl/l)\\ \\ +\frac{1}{2}\frac{1-\gamma}{1-\psi}\frac{\psi-\gamma}{1-\psi}(dl/l)^2+\frac{(1-\gamma)^2}{1-\psi}(dC/C)(dl/l)+\frac{1-\gamma}{1-\psi}l^{-1}dt\end{array}\right]. \tag{A.7}$$

Using (7) and (14) to further simplify (A.7) gives (17). ■

B. Steady-State Distribution for Sentiment

Below we provide all the terms necessary for solving the Kolmogorov forward equation (25). From the expression of σ_S in (7)

$$\begin{aligned}\sigma_S' =&\frac{\theta\sigma_P^D(\mu_H+\mu_L-2S)-\theta(\mu_H-S)(S-\mu_L)(\sigma_P^D)'}{(\sigma_P^D)^2},\\ \sigma_S'' =&\frac{\theta(\mu_H-S)(S-\mu_L)\{2[(\sigma_P^D)']^2-\sigma_P^D(\sigma_P^D)''\}}{(\sigma_P^D)^3}-2\theta\frac{\sigma_P^D(\sigma_P^D)'(\mu_H+\mu_L-2S)+(\sigma_P^D)^2}{(\sigma_P^D)^3}.\end{aligned} \tag{B.1}$$

For the expression of μ_S^e in (7) and the expression of g_D^e in (11)

$$\begin{aligned}(\mu_S^e)' =&-(\lambda+\chi),\\ (g_D^e)' =&\theta-\sigma_D(\sigma_P^D)'-\mu'_S(f'/f)-\mu_S[f''/f-(f')^2/f^2]\\ &-\sigma_S\sigma'_S(f''/f)-\tfrac{1}{2}\sigma_S^2[f'''/f-f'f''/f^2],\end{aligned} \tag{B.2}$$

where σ_P^D is from (11), and $(\sigma_P^D)'$ and $(\sigma_P^D)''$ are

$$
\begin{aligned}
(\sigma_P^D)' =& \frac{\theta(\mu_H+\mu_L-2S)(f'/f)+\theta(\mu_H-S)(S-\mu_L)[f''/f-(f')^2/f^2]}{\sqrt{\sigma_D^2+4\theta(\mu_H-S)(S-\mu_L)(f'/f)}},\\
(\sigma_P^D)'' =& -\frac{2\{\theta(\mu_H+\mu_L-2S)(f'/f)+\theta(\mu_H-S)(S-\mu_L)[f''/f-(f')^2/f^2]\}^2}{[\sigma_D^2+4\theta(\mu_H-S)(S-\mu_L)(f'/f)]^{3/2}}\\
&+\frac{-2\theta f'/f+2\theta(\mu_H+\mu_L-2S)[f''/f-(f')^2/f^2]}{\sqrt{\sigma_D^2+4\theta(\mu_H-S)(S-\mu_L)(f'/f)}}\\
&+\frac{\theta(\mu_H-S)(S-\mu_L)[f'''/f-3(f'f'')/f^2+2(f')^3/f^3]}{\sqrt{\sigma_D^2+4\theta(\mu_H-S)(S-\mu_L)(f'/f)}}.
\end{aligned}
\tag{B.3}
$$

■

C. *Numerical Procedure for Solving the Equilibrium*

We use a projection method with Chebyshev polynomials to jointly solve the two differential equations (16) and (17). The value of the sentiment variable S ranges from μ_L to μ_H, whereas the domain for Chebyshev polynomials is [–1, 1]. Therefore, we transform S to a new state variable z

$$
z \equiv aS+b, \qquad \text{where} \quad a=\frac{2}{\mu_H-\mu_L},\ b=-\frac{\mu_H+\mu_L}{\mu_H-\mu_L}, \tag{C.1}
$$

and we define $h(z)\equiv f(S(z))$ and $j(z)\equiv l(S(z))$. Equations (16) and (17) can be rewritten as

$$
0=\left[\begin{array}{l}
-\frac{(1-\gamma)}{1-\psi}\delta-\gamma g_C^e+g_D^e+[(h'/h)+\frac{\psi-\gamma}{1-\psi}(j'/j)]a\mu_S^e+\frac{1}{2}[(h''/h)+\frac{\psi-\gamma}{1-\psi}(j''/j)]a^2\sigma_S^2\\
+\frac{\gamma(\gamma+1)}{2}\sigma_C^2+\frac{1}{2}\frac{\psi-\gamma}{1-\psi}\frac{2\psi-\gamma-1}{1-\psi}(aj'/j)^2\sigma_S^2-\frac{\gamma(\psi-\gamma)}{1-\psi}\rho\sigma_C\sigma_S(aj'/j)-\gamma\rho\sigma_C\sigma_D-\gamma\rho\sigma_C\sigma_S(ah'/h)\\
+\frac{\psi-\gamma}{1-\psi}\sigma_D\sigma_S(aj'/j)+\frac{\psi-\gamma}{1-\psi}\sigma_S^2a^2(j'/j)(h'/h)+\sigma_D\sigma_S(ah'/h)+\frac{\psi-\gamma}{1-\psi}j^{-1}+h^{-1}
\end{array}\right]
\tag{C.2}
$$

and

$$
0=\left[\begin{array}{l}
-\frac{1-\gamma}{1-\psi}\delta-(\gamma-1)g_C^e+\frac{\gamma(\gamma-1)}{2}\sigma_C^2+\frac{1-\gamma}{1-\psi}(aj'/j)\mu_S^e+\frac{1-\gamma}{2(1-\psi)}(a^2j''/j)\sigma_S^2\\
+\frac{1}{2}\frac{1-\gamma}{1-\psi}\frac{\psi-\gamma}{1-\psi}(aj'/j)^2\sigma_S^2+\frac{(1-\gamma)^2}{1-\psi}\rho\sigma_C\sigma_S(aj'/j)+\frac{1-\gamma}{1-\psi}j^{-1}
\end{array}\right].
\tag{C.3}
$$

We approximate h and j by

$$\hat{h}(z) = \sum_{r=0}^{n} a_r T_r(z), \qquad \hat{l}(z) = \sum_{r=0}^{m} b_r T_r(z), \tag{C.4}$$

where $T_r(z)$ is the r^{th} degree Chebyshev polynomial of the first kind.[24] The projection method chooses the coefficients $\{a_r\}_{r=0}^{n}$ and $\{b_r\}_{r=0}^{m}$ so that the differential equations are *approximately* satisfied. One criterion for a sufficient approximation is to minimize the weighted sum of squared errors

$$\begin{aligned}
&\sum_{i=1}^{N} \frac{1}{\sqrt{1-z_i^2}} \left[\begin{array}{l}
-\frac{(1-\gamma)}{1-\psi}\delta - \gamma g_C^e + g_D^e + [(\hat{h}'/\hat{h}) + \frac{\psi-\gamma}{1-\psi}(\hat{j}'/\hat{j})] a\mu_S^e + \frac{1}{2}[(\hat{h}''/\hat{h}) + \frac{\psi-\gamma}{1-\psi}(\hat{j}''/\hat{j})] a^2\sigma_S^2 \\
+\frac{\gamma(\gamma+1)}{2}\sigma_C^2 + \frac{1}{2}\frac{\psi-\gamma}{1-\psi}\frac{2\psi-\gamma-1}{1-\psi}(a\hat{j}'/\hat{j})^2\sigma_S^2 - \frac{\gamma(\psi-\gamma)}{1-\psi}\rho\sigma_C\sigma_S(a\hat{j}'/\hat{j}) - \gamma\rho\sigma_C\sigma_D \\
-\gamma\rho\sigma_C\sigma_S(a\hat{h}'/\hat{h}) + \frac{\psi-\gamma}{1-\psi}\sigma_D\sigma_S(a\hat{j}'/\hat{j}) + \frac{\psi-\gamma}{1-\psi}\sigma_S^2 a^2(\hat{j}'/\hat{j})(\hat{h}'/\hat{h}) + \sigma_D\sigma_S(a\hat{h}'/\hat{h}) \\
+\frac{\psi-\gamma}{1-\psi}\hat{j}^{-1} + \hat{h}^{-1}
\end{array} \right]^2_{z=z_i} \\
&+\sum_{i=1}^{N} \frac{1}{\sqrt{1-z_i^2}} \left[\begin{array}{l}
-\frac{1-\gamma}{1-\psi}\delta - (\gamma-1)g_C^e + \frac{\gamma(\gamma-1)}{2}\sigma_C^2 + \frac{1-\gamma}{1-\psi}(a\hat{j}'/\hat{j})\mu_S^e + \frac{1-\gamma}{2(1-\psi)}(a^2\hat{j}''/\hat{j})\sigma_S^2 \\
+\frac{1}{2}\frac{1-\gamma}{1-\psi}\frac{\psi-\gamma}{1-\psi}(a\hat{j}'/\hat{j})^2\sigma_S^2 + \frac{(1-\gamma)^2}{1-\psi}\rho\sigma_C\sigma_S(a\hat{j}'/\hat{j}) + \frac{1-\gamma}{1-\psi}\hat{j}^{-1}
\end{array} \right]^2_{z=z_i},
\end{aligned} \tag{C.5}$$

where $\{z_i\}_{i=0}^{N}$ are the N zeros of $T_N(z)$. By the Chebyshev interpolation theorem, if N is sufficiently larger than n and m, and if the sum of weighted square in (C.5) is sufficiently small, the approximated functions $\hat{h}(z)$ and $\hat{l}(z)$ are sufficiently close to the true solutions.

For the numerical results in the main text, we set $m = 40$, $n = 40$, $N = 400$. We then apply the Levenberg-Marquardt algorithm and obtain a minimized sum of squared errors less than 10^{-11}. The small size of the total error indicates convergence of the numerical solution. The solution is also insensitive to the choice of n, m, or N. Together, these findings suggest that the numerical solutions are a sufficient approximation for the true h and j functions.

The same numerical procedure is applied to solving the Kolmogorov forward equation (25). ■

[24] See Mason and Handscomb (2003) for a detailed discussion of the properties of Chebyshev polynomials.

D. *Additional Discussion about Return Expectations and Cash Flow Expectations*

The direct implication of return extrapolation is that the agent's subjective expectation about the future stock market return

$$\mathbb{E}_t^e[P_{t+dt}^D] = 1 + \mathbb{E}_t^e[r_{t+dt}^D]dt = \mathbb{E}_t^e\left[\frac{P_{t+dt}^D}{P_t^D}\right] + \frac{D_t dt}{P_t^D} \tag{D.1}$$

is a positive function of the stock market's recent past returns. Rearranging terms gives

$$\frac{P_t^D}{D_t} = \frac{1}{\mathbb{E}_t^e[r_{t+dt}^D] - \mathbb{E}_t^e[dP_t^D/(P_t^D dt)]}. \tag{D.2}$$

That is, the current price-dividend ratio is determined by the agent's subjective expectation about the future stock market return $\mathbb{E}_t^e[r_{t+dt}^D]$ and the agent's subjective expectation about future price growth $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$. Equation (D.2) does not suggest an explicit role for the agent's expectation about dividend growth in determining the price-dividend ratio.

However, two conditions allow us to link the price-dividend ratio of the stock market to the agent's expectation about dividend growth. First, the law of iterated expectations must hold so that we can iterate forward the Euler equation (4) with the stock market as the tradeable asset. Second, the transversality condition must hold so that the economy permits no bubbles.[25] These two conditions allow us to obtain

$$\frac{P_t^D}{D_t} = \mathbb{E}_t^e\left[\int_t^\infty e^{-\delta(1-\gamma)(s-t)/(1-\psi)}\left(\frac{\tilde{C}_s}{C_t}\right)^{-\psi(1-\gamma)/(1-\psi)} \tilde{M}_{t\to s}^{(\psi-\gamma)/(1-\psi)}\left(\frac{\tilde{D}_s}{D_t}\right) ds\right], \tag{D.3}$$

where $\tilde{M}_{t\to s}$ denotes the continuously compounded gross return for holding the Lucas tree from time t to time s ($> t$). Equation (D.3) says that the current price-dividend ratio of the stock market equals the agent's subjective expectation of the sum of discounted future dividend growths.

For an infinitely-lived agent, (D.3) further implies that the agent is aware of the fact that both her expectation about future price growth and her expectation about future returns are linked to her expectation about future dividend growth. The specific relationship between these expectations are discussed in Sections I and II. ■

[25] The transversality condition holds in this economy as the stock market price is bounded by a finite range.

REFERENCES

Alti, Aydogan, and Paul Tetlock, 2014, Biased beliefs, asset prices, and investment: A structural approach, *Journal of Finance* 69, 325–361.

Amromin, Gene, and Steven A. Sharpe, 2013, Expectation of risk and return among household investors: Are their sharpe ratios countercyclical?, Working paper, Federal Reserve Board, Washington DC.

Bacchetta, Philippe, Elmar Mertens, and Eric van Wincoop, 2009, Predictability in financial markets: What does survey expectations tell us?, *Journal of International Money and Finance* 28, 406–426.

Bansal, Ravi, Dana Kiku, and Amir Yaron, 2012, An empirical evaluation of the long-run risks model for asset prices, *Critical Finance Review* 183–221.

Bansal, Ravi, and Amir Yaron, 2004, Risks for the long run: A potential resolution of asset pricing puzzles, *Journal of Finance* 59, 1481–1509.

Barberis, Nicholas, Robin Greenwood, Lawrence Jin, and Andrei Shleifer, 2015, X-capm: An extrapolative capital asset pricing model, *Journal of Financial Economics* 115, 1–24.

Barberis, Nicholas, Robin Greenwood, Lawrence Jin, and Andrei Shleifer, 2017, Extrapolation and bubbles, *Journal of Financial Economics* forthcoming.

Barberis, Nicholas, Ming Huang, and Tano Santos, 2001, Prospect theory and asset prices, *Quarterly Journal of Economics* 116, 1–53.

Barberis, Nicholas, Andrei Shleifer, and Robert Vishny, 1998, A model of investor sentiment, *Journal of Financial Economics* 49, 307–343.

Baron, Matthew, and Wei Xiong, 2015, Credit expansion and neglected crash risk, Working paper, Princeton University.

Barro, Robert, 2006, Rare disasters and asset markets in the twentieth century, *Quarterly Journal of Economics* 121, 823–866.

Beeler, Jason, and John Y. Campbell, 2012, The long-run risks model and aggregate asset prices: An empirical assessment, *Critical Finance Review* 141–182.

Bidder, Rhys, and Ian Dew-Becker, 2016, Long-run risk is the worst-case scenario, *American Economic Review* 106, 2494–2527.

Bordalo, Pedro, Nicola Gennaioli, and Andrei Shleifer, 2018, Diagnostic expectations and credit cycles, *Journal of Finance* forthcoming.

Borovicka, Jaroslav, 2016, Survival and long-run dynamics with heterogeneous beliefs under recursive preferences, Working paper, New York University.

Campbell, John Y., 2003, Consumption-based asset pricing, in George M. Constantinides, Milton Harris, and Rene M. Stulz, eds., *Handbook of the Economics of Finance*, volume 1B, chapter 13, 801–885 (Elsevier: Amsterdam).

Campbell, John Y., and John Cochrane, 1999, By force of habit: A consumption-based explanation of aggregate stock market behavior, *Journal of Political Economy* 107, 205–251.

Campbell, John Y., and Robert Shiller, 1988, The dividend-price ratio and expectations of future dividends and discount factors, *Review of Financial Studies* 1, 195–228.

Cassella, Stefano, and Huseyin Gulen, 2017, Extrapolation bias and the predictability of stock returns by price-scaled variables, *Review of Financial Studies* forthcoming.

Chen, Long, Zhi Da, and Xinlei Zhao, 2013, What drives stock market movements?, *Review of Financial Studies* 16, 841–876.

Choi, James, and Thomas Mertens, 2013, Extrapolative expectations and the equity premium, Working paper, New York University and Yale University.

Cochrane, John H., 2008, The dog that did not bark: A defense of return predictability, *Review of Financial Studies* 21, 1533–1575.

Cochrane, John H., 2011, Presidential address: Discount rates, *Journal of Finance* 66, 1047–1108.

Collin-Dufresne, Pierre, Michael Johannes, and Lars A. Lochstoer, 2016a, Asset pricing when 'this time is different', Working paper, Ecole Polytechnique Federale de Lausanne and Columbia Business School.

Collin-Dufresne, Pierre, Michael Johannes, and Lars A. Lochstoer, 2016b, Parameter learning in general equilibrium: The asset pricing implications, *American Economic Review* 106, 664–698.

Cutler, David M., James M. Poterba, and Lawrence H. Summers, 1990, Speculative dynamics and the role of feedback traders, *American Economic Review Papers and Proceedings* 80, 63–68.

Da, Zhi, Xing Huang, and Lawrence Jin, 2017, Extrapolative beliefs in the cross-section: What can we learn from the crowds?, Working paper, University of Notre Dame, Washington University at St. Louis, and California Institute of Technology.

de la O, Ricardo, and Sean Myers, 2017, Subjective cash flows and discount rates, Working paper, Stanford University.

DeFusco, Anthony A., Charles G. Nathanson, and Eric Zwick, 2017, Speculative dynamics of prices and volume, Working paper, Northwestern University and University of Chicago.

DeLong, Bradford, Andrei Shleifer, Lawrence Summers, and Robert Waldmann, 1990, Positive feedback investment strategies and destablizing rational speculation, *The Journal of Finance* 45, 379–395.

Ehling, Paul, Alessandro Graniero, and Christian Heyerdahl-Larsen, 2015, Asset prices and portfolio choice with learning from experience, Working paper, BI Norwegian Business School and London Business School.

Epstein, Larry, and Stanley Zin, 1989, Substitution, risk aversion, and the temporal behavior of consumption and asset returns: A theoretical framework, *Econometrica* 57, 937–969.

Epstein, Larry, and Stanley Zin, 1991, Substitution, risk aversion, and the temporal behavior of consumption and asset returns: An empirical analysis, *Journal of Political Economy* 99, 263–286.

Fama, Eugene, and Kenneth French, 1988, Dividend yields and expected stock returns, *Journal of Financial Economics* 22, 3–25.

Fuster, Andreas, Benjamin Hebert, and David Laibson, 2011, Natural expectations, macroeconomic dynamics, and asset pricing, *NBER Macroeconomics Annual* 26, 1–48.

Gabaix, Xavier, 2012, Variable rare disasters: An exactly solved framework for ten puzzles in macro–finance, *Quarterly Journal of Economics* 127, 645–700.

Gennaioli, Nicola, Andrei Shleifer, and Robert Vishny, 2012, Neglected risks, financial innovation, and financial fragility, *Journal of Financial Economics* 104, 452–468.

Glaeser, Edward L., and Charles G. Nathanson, 2017, An extrapolative model of house price dynamics, *Journal of Financial Economics* 126, 147–170.

Greenwood, Robin, Sam Hanson, and Lawrence Jin, 2016, A model of credit market sentiment, Working paper, California Institute of Technology and Harvard University.

Greenwood, Robin, and Samuel G. Hanson, 2013, Issuer quality and corporate bond returns, *Review of Financial Studies* 26, 1483–1525.

Greenwood, Robin, and Andrei Shleifer, 2014, Expectations of returns and expected returns, *Review of Financial Studies* 27, 714–746.

Hansen, Lars, and Kenneth Singleton, 1982, Generalized instrumental variables estimation of nonlinear rational expectations models, *Econometrica* 50, 1269–1286.

Hansen, Lars, and Kenneth Singleton, 1983, Stochastic consumption, risk aversion, and the temporal behavior of asset returns, *Journal of Political Economy* 91, 249–265.

Hartzmark, Samuel M., and David H. Solomon, 2017, Reconsidering returns, Working paper, University of Chicago and Boston College.

Hirshleifer, David, Jun Li, and Jianfeng Yu, 2015, Asset pricing in production economies with extrapolative expectations, *Journal of Monetary Economics* 76, 87–106.

Hong, Harrison, and Jeremy C. Stein, 1999, A unified theory of underreaction, momentum trading, and overreaction in asset markets, *Journal of Finance* 54, 2143–2184.

Huang, Xing, 2016, Mark twain's cat: Investment experience, categorical thinking, and stock selection, Working paper, Michigan State University.

Jin, Lawrence J., 2015, A speculative asset pricing model of financial instability, Working paper, California Institute of Technology.

Kahneman, Daniel, and Amos Tversky, 1979, Prospect theory: An analysis of decision under risk, *Econometrica* 47, 263–292.

Koijen, Ralph S. J., Maik Schmeling, and Evert B. Vrugt, 2015, Survey expectations of returns and asset pricing puzzles, Working paper, London Business School.

Kuchler, Theresa, and Basit Zafar, 2016, Personal experiences and expectations about aggregate outcomes, Working paper, New York University and Federal Reserve Bank of New York.

Lipster, Robert, and Albert Shiryaev, 2001, *Statistics of Random Processes I, General Theory*, second edition (Springer-Verlag, New York).

Lopez-Salido, David, Jeremy C. Stein, and Egon Zakrajsek, 2016, Credit-market sentiment and the business cycle .

Malmendier, Ulrike, and Stefan Nagel, 2011, Depression-babies: Do macroeconomic experiences affect risk-taking?, *Quarterly Journal of Economics* 126, 373–416.

Malmendier, Ulrike, and Stefan Nagel, 2013, Learning from inflation experiences, *Quarterly Journal of Economics* forthcoming.

Mason, John C., and David C. Handscomb, 2003, *Chebyshev Polynomials* (Chapman & Hall/CRC).

Mehra, Rajnish, and Edward C. Prescott, 1985, The equity premium: A puzzle, *Journal of Monetary Economics* 15, 145–161.

Pagel, Michaela, 2016, Expectations-based reference-dependent preferences and asset pricing, *Journal of European Economic Association* 14, 468–514.

Tversky, Amos, and Daniel Kahneman, 1974, Judgment under uncertainty: Heuristics and biases, *Science* 185, 1124–1131.

Vanasco, Victoria, Ulrike Malmendier, and Demian Pouzo, 2015, A theory of experience effects, Working paper, Stanford University and University of California, Berkeley.

Veronesi, Pietro, 1999, Stock market overreaction to bad news in good times: A rational expectations equilibrium model, *Review of Financial Studies* 12, 975–1007.

Vissing-Jorgensen, Annette, 2004, Perspectives on behavioral finance: Does "irrationality" disappear with wealth? Evidence from expectations and actions, *NBER Macroeconomics Annual 2003* 18, 139–194.

Wachter, Jessica A., 2013, Can time-varying risk of rare disasters explain aggregate stock market volatility?, *Journal of Finance* 68, 987–1035.

Xiong, Wei, 2013, Bubbles, crises, and heterogeneous beliefs, in Jean-Pierre Fouque, and Joe Langsam, eds., *Handbook for Systemic Risk*, 663–713 (Cambridge University Press).

Yang, David C., and Fan Zhang, 2016, The household equity share and expected market returns, Working paper, University of California, Irvine.

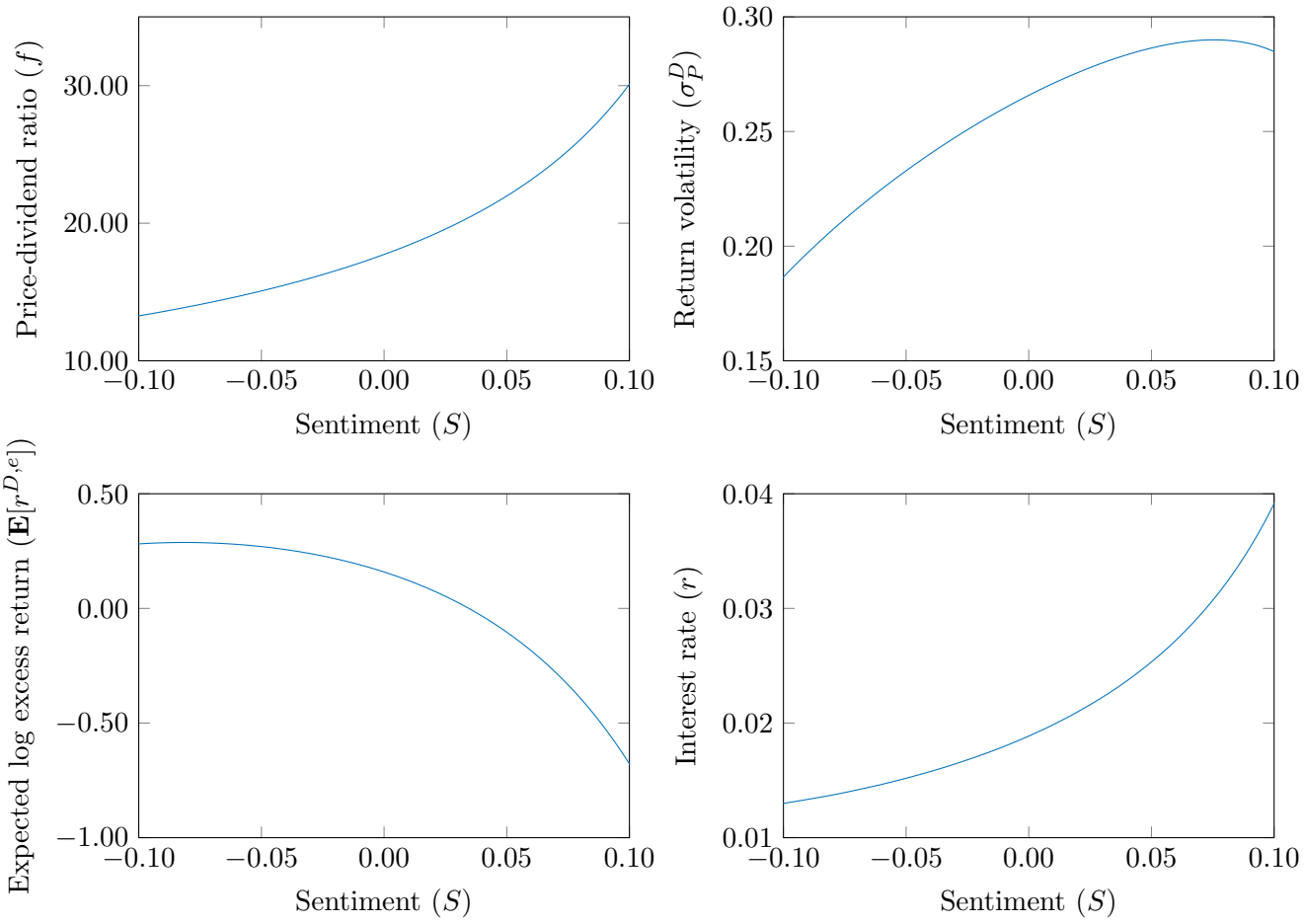

Figure 1. Important Equilibrium Quantities Each as a Function of Sentiment. The figure plots the price-dividend ratio of the stock market f, the volatility of stock market returns σ_P^D, the rational expectation about the log excess return $\mathbb{E}[r^{D,e}]$ (the conditional equity premium), and the interest rate r, each as a function of the sentiment variable S. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

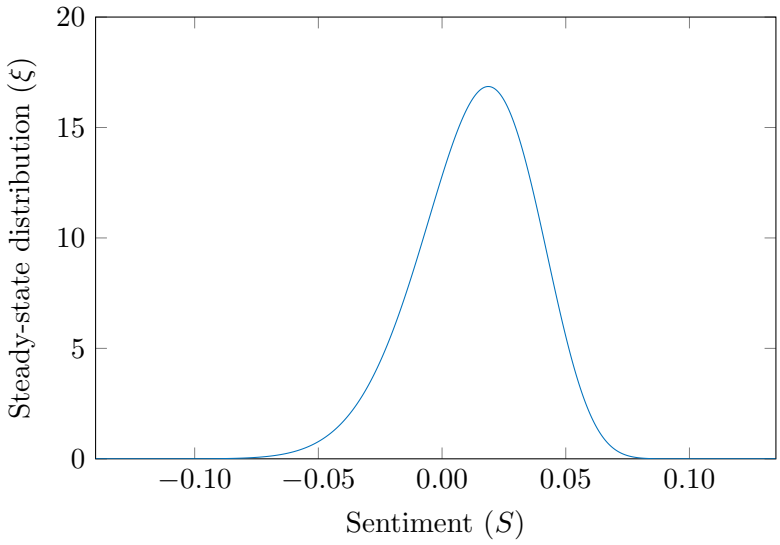

Figure 2. Objectively Measured Steady-State Distribution of Sentiment. The figure plots the objective steady-state distribution of sentiment ξ as a function of the sentiment variable S. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = –15\%$.

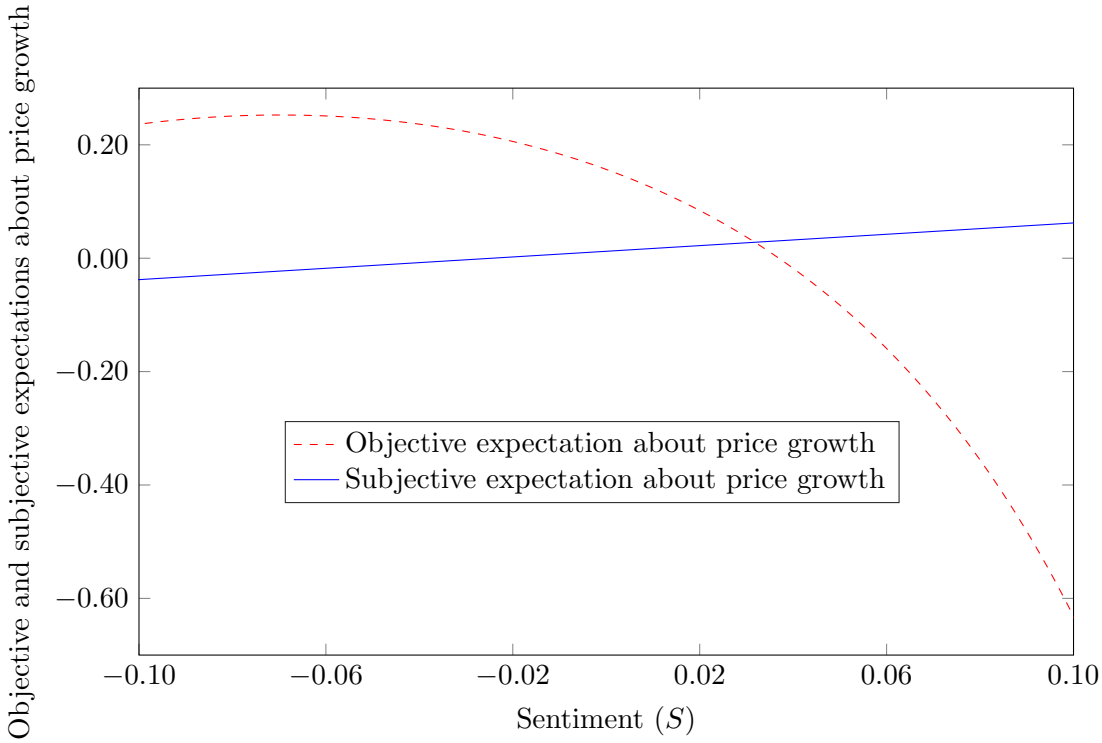

Figure 3. Objective and Subjective Expectations about Price Growth. The dashed line plots the objective (rational) expectation about price growth, $\mathbb{E}_t[(dP_t^D)/(P_t^D dt)]$, as a function of the sentiment variable S_t. The solid line plots the agent's subjective expectation about price growth, $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$, as a function of the sentiment variable S_t. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

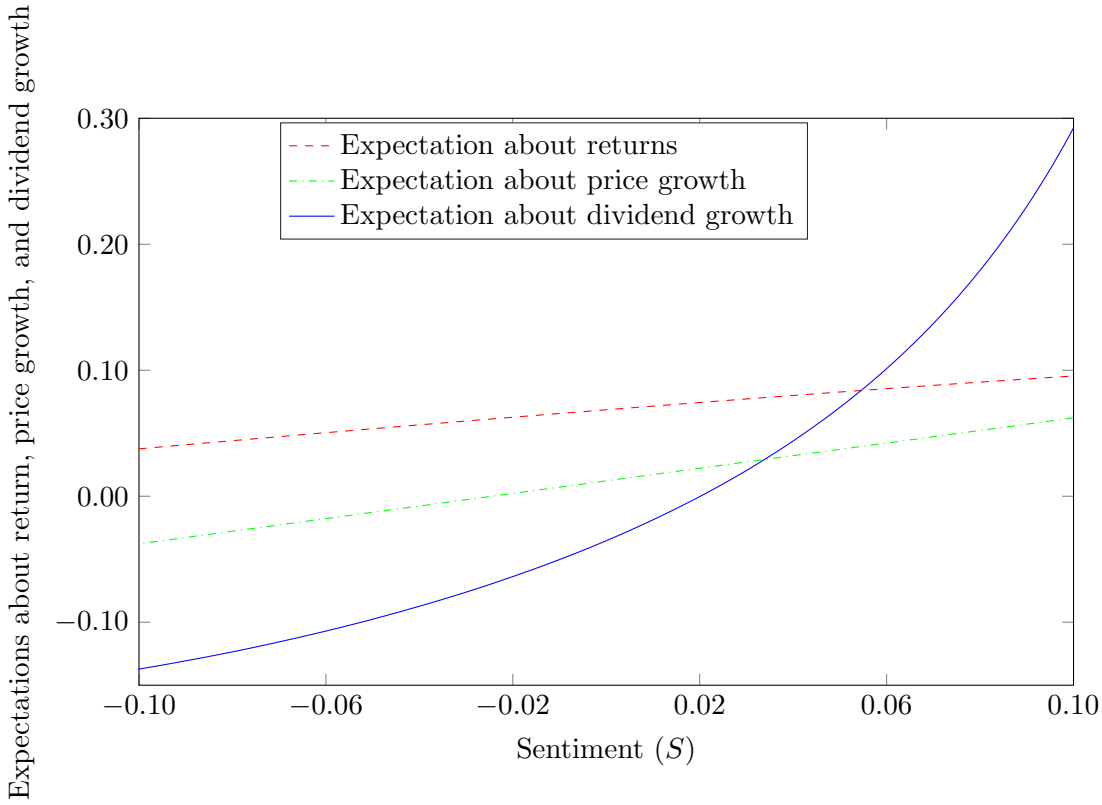

Figure 4. Agent's Expectations about Stock Market Returns, Price Growth, and Dividend Growth. The dashed line plots the agent's expectation about stock market returns, $\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)] = (1-\theta)g_D + \theta S_t + 1/f$, as a function of the sentiment variable S_t. The dotted-dashed line plots the agent's expectation about price growth, $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)] = (1-\theta)g_D + \theta S_t$, as a function of the sentiment variable S_t. The solid line plots the agent's expectation about dividend growth, $\mathbb{E}_t^e[dD_t/(D_t dt)]$, as a function of the sentiment variable S_t. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = –15\%$.

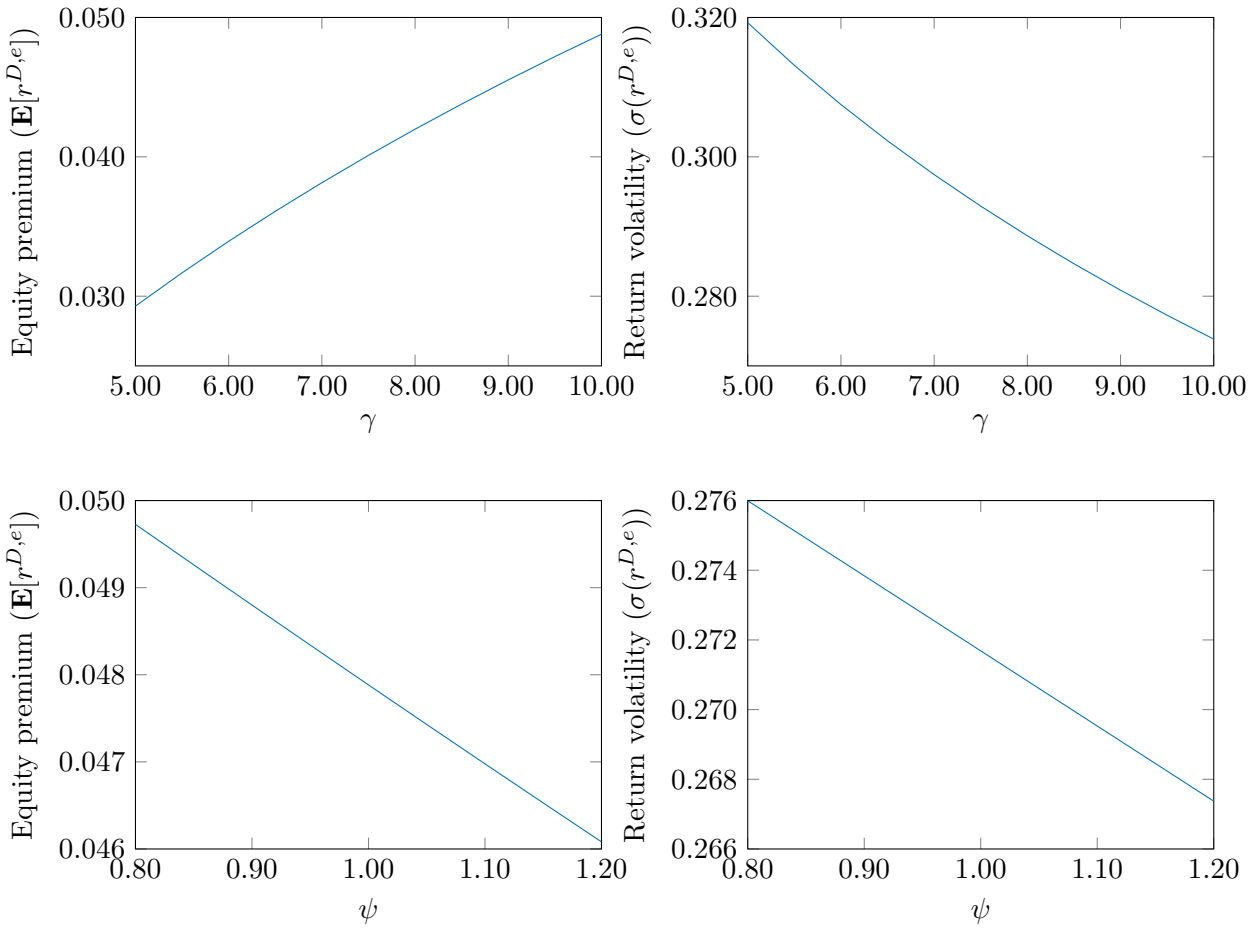

Figure 5. Comparative Statics: Utility Parameters. The upper panel plots the average equity premium $\mathbb{E}[r^{D,e}]$ and the average volatility of stock market returns $\sigma(r^{D,e})$, each as a function of γ, the coefficient of relative risk aversion. The lower panel plots the average equity premium $\mathbb{E}[r^{D,e}]$ and the average volatility of stock market returns $\sigma(r^{D,e})$, each as a function of ψ, the reciprocal of the elasticity of intertemporal substitution. The default values for γ and ψ are 10 and 0.9, respectively. The other parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

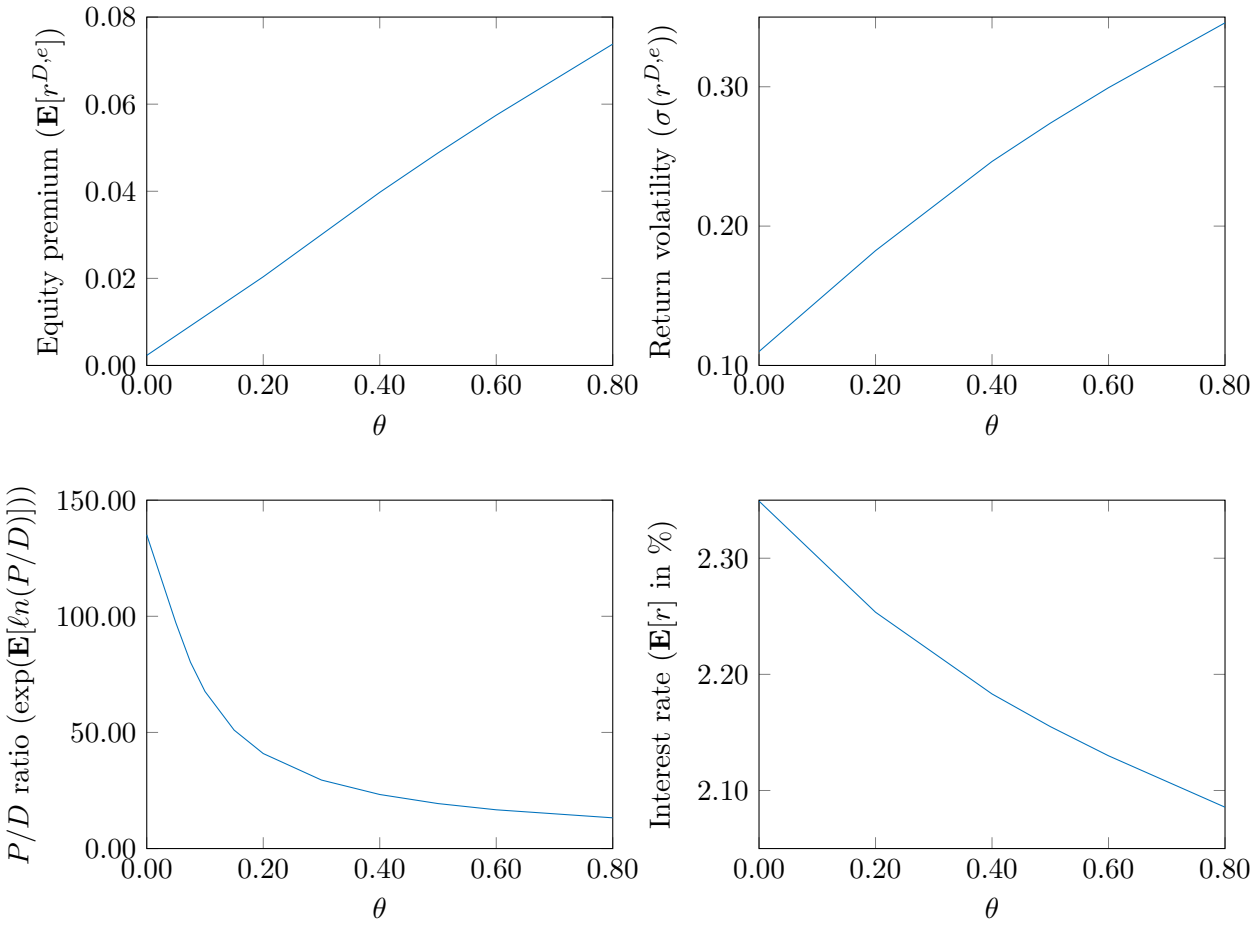

Figure 6a. Comparative Statics: Belief Parameters (I). The figure plots the average equity premium $\mathbb{E}[r^{D,e}]$, the average volatility of stock market returns $\sigma(r^{D,e})$, the average price-dividend ratio $\exp(\mathbb{E}[\ell \text{n}(P/D)])$, and the average interest rate $\mathbb{E}[r]$ (in percentage), each as a function of θ, the parameter that controls the extent to which the agent is behavioral. The other parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

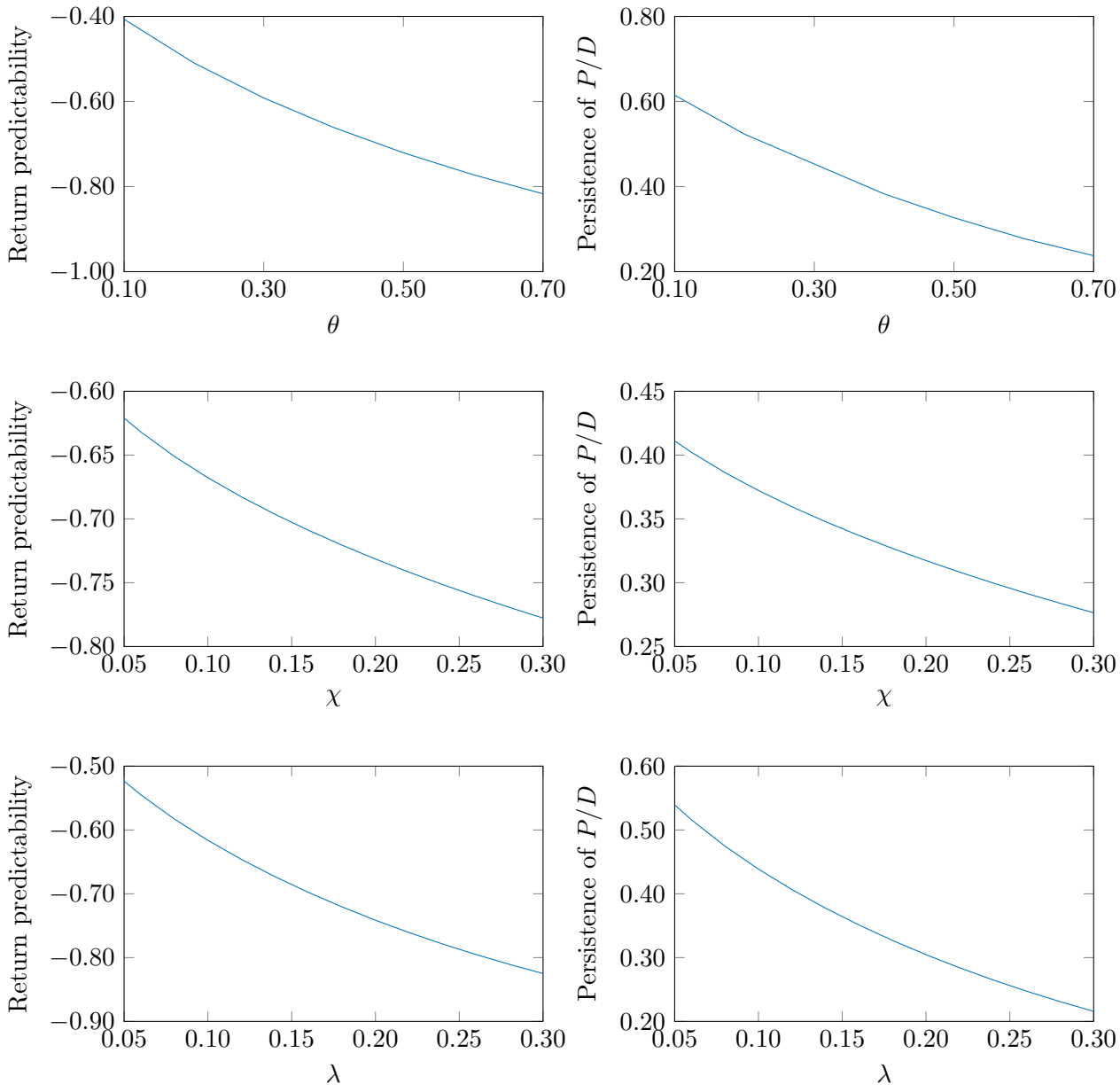

Figure 6b. Comparative Statics: Belief Parameters (II). The figure plots the predictability of stock market returns and the persistence of the price-dividend ratio, each as a function of θ, χ, or λ. The predictability of returns is measured by the slope coefficient for a regression of the next year's log excess return on the current log price-dividend ratio. The persistence of the price-dividend ratio is measured by the one-year autocorrelation of log price-dividend ratios. The default values for θ, χ, and λ are 0.5, 0.18, and -0.18, respectively. The other parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

Table 1. Investor Expectations.

	$\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)]$				$\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$			
$R^D_{t-12\to t}$	0.022	0.023			0.040	0.040		
	(8.2)	(11.7)			(8.4)	(12.1)		
$\ell\mathrm{n}(P/D)$			0.068	0.069			0.120	0.121
			(29.5)	(39.2)			(36.8)	(48.9)
Constant	0.07	0.07	–0.13	–0.13	0.02	0.02	–0.33	–0.34
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.57	0.54	0.98	0.98	0.58	0.55	0.99	0.99

	$\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)] - r_t$				$\mathbb{E}_t^e[dP_t^D/(P_t^D dt)] - r_t$			
$R^D_{t-12\to t}$	0.013	0.013			0.030	0.030		
	(6.6)	(9.0)			(7.8)	(11.1)		
$\ell\mathrm{n}(P/D)$			0.039	0.039			0.091	0.091
			(12.4)	(16.6)			(22.3)	(28.7)
Constant	0.05	0.05	–0.06	–0.06	–0.002	–0.002	–0.27	–0.27
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.51	0.47	0.92	0.91	0.56	0.53	0.97	0.97

The table reports the regression coefficient and the t-statistic (in parenthesis), the intercept, as well as the adjusted R-squared, for regressing the agent's expectation about future stock market returns either on the past twelve-month cumulative raw return $R^D_{t-12\to t}$ or on the current log price-dividend ratio $\ell\mathrm{n}(P_t/D_t)$, over a sample of 15 years or 50 years. In the top panel, the expectations measure for the first four columns is $\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)]$, and the expectations measure for the last four columns is $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$. In the bottom panel, the expectations measure for the first four columns is $\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)] - r_t$, and the expectations measure for the last four columns is $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)] - r_t$. Each reported value is averaged over 100 trials, and each trial represents a regression using monthly data simulated from the model. The t-statistics are calculated using a Newey-West estimator with twelve-month lags. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$.

Table 2. Determinants of Investor Expectations.

	$\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)]$		$\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$	
ϕ	0.432	0.417	0.428	0.420
a	0.064	0.064	0.004	0.004
b	1.15	1.18	2.04	2.07
Sample size	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.98	0.98	0.99	0.98
	$\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)] - r_t$		$\mathbb{E}_t^e[dP_t^D/(P_t^D dt)] - r_t$	
ϕ	0.414	0.408	0.418	0.420
a	0.047	0.047	–0.013	-0.010
b	0.70	0.67	1.58	1.55
Sample size	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.92	0.90	0.97	0.96

The table reports the parameter ϕ, the intercept a, the regression coefficient b, and the adjusted R-squared, for running the non-linear least squares regression

$$\text{Expectation}_t = a + b \sum_{j=1}^{n} w_j R^D_{(t-j\Delta t)\to(t-(j-1)\Delta t)} + \varepsilon_t,$$

over a sample of 15 years or 50 years, where $w_j \equiv e^{-\phi(j-1)\Delta t} / \sum_{l=1}^{n} e^{-\phi(l-1)\Delta t}$, $\Delta t = 1/12$, and $n = 600$. In the top panel, the expectations measure for the first four columns is $\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)]$, and the expectations measure for the last four columns is $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$. In the bottom panel, the expectations measure for the first four columns is $\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)] - r_t$, and the expectations measure for the last four columns is $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)] - r_t$. Each reported value is averaged over 100 trials, and each trial represents a regression using monthly data simulated from the model. The t-statistics are calculated using a Newey-West estimator with twelve-month lags. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = –15\%$.

Table 3. Parameter Values.

Parameter	Variable	Value
Asset parameters:		
Expected consumption growth	g_C	1.91%
Expected dividend growth	g_D	2.45%
Standard deviation of consumption growth	σ_C	3.8%
Standard deviation of dividend growth	σ_D	11%
Correlation between dD and dC	ρ	0.2
Utility parameters:		
Relative risk aversion	γ	10
Reciprocal of EIS	ψ	0.9
Subjective discount rate	δ	0.02
Belief parameters:		
Degree of extrapolation	θ	0.5
Perceived transition intensity from H to L	χ	0.18
Perceived transition intensity from L to H	λ	0.18
Upper bound of sentiment	μ_H	0.15
Lower bound of sentiment	μ_L	–0.15

Table 4. Basic Moments.

Statistic	Theoretical value	Empirical value
Equity premium ($\mathbb{E}[r^{D,e}]$)	4.88%	3.90%
Return volatility ($\sigma(r^{D,e})$)	27.4%	18.0%
Sharpe ratio ($\mathbb{E}[r^{D,e}]/\sigma(r^{D,e})$)	0.20	0.22
Interest rate ($\mathbb{E}[r]$)	2.16%	2.92%
Interest rate volatility ($\sigma(r)$)	0.33%	2.89%
Price-dividend ratio ($\exp(\mathbb{E}[\ell\text{n}(P/D)])$)	19.4	21.1

The table reports six important moments about stock market prices and returns: the long-run average of the equity premium (the rational expectation of log excess return, $\mathbb{E}[r^{D,e}]$), the average volatility of stock market returns (the volatility of log excess return, $\sigma(r^{D,e})$), the Sharpe ratio ($\mathbb{E}[r^{D,e}]/\sigma(r^{D,e})$), the average interest rate ($\mathbb{E}[r]$), interest rate volatility ($\sigma(r)$), and the average price-dividend ratio of the stock market ($\exp(\mathbb{E}[\ell\text{n}(P/D)])$). The theoretical values for these moments are computed over the objectively measured steady-state distribution of sentiment S. The model parameters are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$. For the empirical values, five out of six are from Campbell and Cochrane (1999); the empirical value for interest rate volatility is not reported in Campbell and Cochrane (1999), so we report the value from Beeler and Campbell (2012).

Table 5. Return Predictability Regressions.

Horizon (years)	Theoretical value		Empirical value	
	10× coefficient	Adjusted R-squared	10× coefficient	Adjusted R-squared
1	–7.2	0.13	–1.3	0.04
2	–9.5	0.16	–2.8	0.08
3	–10.1	0.15	–3.5	0.09
5	–10.6	0.13	–6.0	0.18
7	–11.0	0.12	–7.5	0.23

The table reports the regression coefficient β_j and the adjusted R-squared for a regression of the log excess return of stock market from time t to time $t+j$ on the current log price-dividend ratio $\ell\text{n}(P_t^D/D_t)$

$$r_{t\to t+j}^{D,e} = \alpha_j + \beta_j \ell\text{n}(P_t^D/D_t) + \varepsilon_{j,t},$$

where j = 1, 2, 3, 5, and 7 (years). The theoretical values are calculated using 10,000 years of monthly data simulated from the model. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$. The empirical values are from Campbell and Cochrane (1999).

Table 6. Correlation between Consumption Growth and Stock Returns.

Correlation	Theoretical value			Empirical value
	monthly	quarterly	annual	annual
$\mathbb{C}\text{orr}(r^{D,e}_{t\to t+1}, \ell\text{n}(C_{t-1}/C_{t-2}))$	–0.01	–0.02	–0.02	–0.05
$\mathbb{C}\text{orr}(r^{D,e}_{t\to t+1}, \ell\text{n}(C_{t}/C_{t-1}))$	–0.01	–0.03	–0.06	–0.08
$\mathbb{C}\text{orr}(r^{D,e}_{t\to t+1}, \ell\text{n}(C_{t+1}/C_{t}))$	0.20	0.20	0.19	0.09
$\mathbb{C}\text{orr}(r^{D,e}_{t\to t+1}, \ell\text{n}(C_{t+2}/C_{t+1}))$	0.00	–0.00	0.01	0.49
$\mathbb{C}\text{orr}(r^{D,e}_{t\to t+1}, \ell\text{n}(C_{t+3}/C_{t+2}))$	–0.00	–0.00	0.01	0.05

The table reports correlations between log consumption growth and log excess returns of the stock market. The log consumption growth and log excess returns are computed at either a monthly, a quarterly, or an annual horizon. Correlations are either contemporaneous or with a lead-lag structure; for instance, at a monthly frequency, $\mathbb{C}\text{orr}(r^{D,e}_{t\to t+1}, \ell\text{n}(C_{t+2}/C_{t+1}))$ is the correlation between the current monthly log excess return and the log consumption growth in the subsequent month. The theoretical values are calculated using 10,000 years of monthly data simulated from the model. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = –15\%$. The empirical values are from Campbell and Cochrane (1999).

Table 7. Campbell-Shiller Decomposition.

Coefficient	Realized dividend growth and returns	
	Value	Adjusted R-squared
$DR_{objective}$	0.98	0.209
$CF_{objective}$	0.02	1.2×10^{-4}

Coefficient	Subjective expectations about dividend growth and returns	
	Value	Adjusted R-squared
$DR_{subjective}$	-0.08	0.982
$CF_{subjective}$	1.08	0.984

The table reports the four coefficients, $CF_{objective}$, $DR_{objective}$, $CF_{subjective}$, and $DR_{objective}$, defined in equations (29) and (30), as well as their corresponding adjusted R-squared. These coefficients and R-squared are calculated using 10,000 years of monthly data simulated from the model. At each point in time, for a given level of sentiment, subjective expectations about dividend growth and returns are calculated as the average values of 100 trials. Each trial is 50 years of monthly simulated data under the agent's expectations with the given initial level of sentiment. For realized dividend growth and returns, both $\sum_{j=0}^{\infty} \xi^j \Delta d_{(t+j\Delta t)\to(t+(j+1)\Delta t)}$ and $\sum_{j=0}^{\infty} \xi^j r^D_{(t+j\Delta t)\to(t+(j+1)\Delta t)}$ are approximated using 50 years of monthly simulated data. From the simulated data, $\xi = 0.9957$. The other parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = –15\%$.

Table 8. Autocorrelations of Log Excess Returns and Log Price-dividend Ratios.

Lag (years)	Theoretical value			Empirical value		
	$\ell n(P^D/D)$	$r^{D,e}$	$\Sigma_{i=1}^{j}\rho(r_t^{D,e}, r_{t-i}^{D,e})$	$\ell n(P^D/D)$	$r^{D,e}$	$\Sigma_{i=1}^{j}\rho(r_t^{D,e}, r_{t-i}^{D,e})$
1	0.33	–0.28	–0.28	0.78	0.05	0.05
2	0.11	–0.09	–0.37	0.57	–0.21	–0.16
3	0.05	–0.02	–0.39	0.50	–0.08	–0.09
5	0.00	–0.01	–0.40	0.32	–0.14	–0.28
7	–0.02	–0.01	–0.41	0.29	0.11	–0.15

The table reports, over various lags j, the autocorrelations of log price-dividend ratios and log excess returns, as well as the partial sum of the autocorrelations of log excess returns. The operator $\rho(x, y)$ computes the sample correlation between variable x and variable y. The theoretical values are calculated using 10,000 years of monthly data simulated from the model; for each month, we compound the next 12 months of log excess returns to obtain an annual log excess return. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$. The empirical values are from Campbell and Cochrane (1999).

Table 9. Return Predictability Regressions in the True Regime-Switching Model.

Horizon (years)	Theoretical value		Empirical value	
	10× coefficient	10^3×Adjusted R-squared	10× coefficient	Adjusted R-squared
1	0.2	0.16	–1.3	0.04
2	0.4	0.31	–2.8	0.08
3	0.5	0.32	–3.5	0.09
5	0.8	0.54	–6.0	0.18
7	0.7	0.34	–7.5	0.23

The table reports the regression coefficient β_j and the adjusted R-squared for a regression of the log excess return of stock market from time t to time $t+j$ on the current log price-dividend ratio $\ell\text{n}(P_t^D/D_t)$

$$r_{t\to t+j}^{D,e} = \alpha_j + \beta_j \ell\text{n}(P_t^D/D_t) + \varepsilon_{j,t},$$

where $j = 1$, 2, 3, 5, and 7 (years). The theoretical values are calculated using 10,000 years of monthly data simulated from the true regime-switching model described in Section IV. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$. The empirical values are from Campbell and Cochrane (1999).

Table 10. Investor Expectations in Bansal and Yaron (2004).

	Expectation of return				Expectation of return w/o divd.			
$R^D_{t-12\to t}$	0.025	0.030			0.025	0.031		
	(2.4)	(3.8)			(2.5)	(4.0)		
ℓn(P/D)			0.068	0.067			0.069	0.067
			(5.6)	(7.6)			(5.8)	(7.6)
Constant	1.06	1.05	0.88	0.88	1.06	1.05	0.88	0.89
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.140	0.142	0.500	0.451	0.136	0.145	0.529	0.451

	Expectation of excess return				Expectation of excess return w/o divd.			
$R^D_{t-12\to t}$	–0.001	0.000			0.000	0.001		
	(–0.1)	(0.1)			(–0.1)	(0.2)		
ℓn(P/D)			–0.008	–0.012			–0.010	–0.011
			(–0.8)	(–1.3)			(–0.8)	(–1.2)
Constant	0.06	0.06	0.08	0.10	0.06	0.06	0.09	0.09
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.026	0.008	0.087	0.054	0.026	0.008	0.093	0.053

The table reports the regression coefficient and the t-statistic (in parenthesis), the intercept, as well as the adjusted R-squared, for regressing four measures of rational expectation of return—raw return or excess return, with or without dividend yield—either on the past twelve-month cumulative raw return or on the current log price-dividend ratio, over a sample of 15 years or 50 years. The conditional expectation of subsequent returns, the dependent variable in each regression, is computed by averaging realized returns across simulations over a twelve-month horizon for a given state of the economy. Each reported value is the estimator median over 1,000 trials, and each trial represents a regression using monthly data simulated from Bansal and Yaron (2004). The t-statistics are calculated using a Newey-West estimator with twelve-month lags. The parameters take their default values from Tables II and IV of Bansal and Yaron (2004).

Table 11. Investor Expectations in Bansal et al. (2012).

	Expectation of return				Expectation of return w/o divd.			
$R^D_{t-12\to t}$	0.006	0.010			0.006	0.011		
	(0.7)	(1.3)			(0.7)	(1.3)		
$\ell\mathrm{n}(P/D)$			–0.006	–0.041			–0.006	–0.038
			(–0.3)	(–3.1)			(–0.2)	(–2.9)
Constant	1.08	1.08	1.11	1.22	1.08	1.08	1.11	1.21
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.031	0.021	0.129	0.147	0.033	0.020	0.139	0.136

	Expectation of excess return				Expectation of excess return w/o divd.			
$R^D_{t-12\to t}$	–0.003	0.000			–0.004	–0.001		
	(–0.3)	(0.1)			(–0.5)	(–0.1)		
$\ell\mathrm{n}(P/D)$			–0.071	–0.095			–0.036	–0.054
			(–5.2)	(–10.7)			(–3.4)	(–5.2)
Constant	0.09	0.08	0.30	0.37	0.08	0.08	0.19	0.24
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.021	0.010	0.432	0.599	0.026	0.008	0.226	0.254

The table reports the regression coefficient and the t-statistic (in parenthesis), the intercept, as well as the adjusted R-squared, for regressing four measures of rational expectation of return—raw return or excess return, with or without dividend yield—either on the past twelve-month cumulative raw return or on the current log price-dividend ratio, over a sample of 15 years or 50 years. The conditional expectation of subsequent returns, the dependent variable in each regression, is computed by averaging realized returns across simulations over a twelve-month horizon for a given state of the economy. Each reported value is the estimator median over 1,000 trials, and each trial represents a regression using monthly data simulated from Bansal et al. (2012). The t-statistics are calculated using a Newey-West estimator with twelve-month lags. The parameters take their default values from Table 1 of Bansal et al. (2012).

Table 12. Investor Expectations in the Fundamental Extrapolation Model.

	$\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)]$				$\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$			
$R^D_{t-12\to t}$	0.004	0.003			0.010	0.010		
	(0.21)	(0.63)			(1.15)	(2.04)		
$\ell n(P/D)$			0.017	0.019			0.040	0.042
			(0.88)	(1.62)			(2.08)	(3.52)
Constant	0.05	0.05	–0.01	–0.02	0.03	0.03	–0.12	–0.13
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.14	0.04	0.24	0.10	0.17	0.08	0.31	0.22

	$\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)] - r_t$				$\mathbb{E}_t^e[dP_t^D/(P_t^D dt)] - r_t$			
$R^D_{t-12\to t}$	–0.004	–0.004			0.003	0.002		
	(–0.88)	(–1.13)			(0.07)	(0.42)		
$\ell n(P/D)$			–0.009	–0.007			0.014	0.016
			(–0.49)	(–0.54)			(0.71)	(1.35)
Constant	0.03	0.03	0.06	0.05	0.006	0.006	–0.05	–0.05
Sample size	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.	15 yr.	50 yr.
R^2	0.15	0.05	0.22	0.06	0.14	0.04	0.23	0.08

The table reports the regression coefficient and the t-statistic (in parenthesis), the intercept, as well as the adjusted R-squared, for regressing the agent's expectation about future stock market returns either on the past twelve-month cumulative raw return $R^D_{t-12\to t}$ or on the current log price-dividend ratio $\ell n(P_t/D_t)$, over a sample of 15 years or 50 years. In the top panel, the expectations measure for the first four columns is $\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)]$, and the expectations measure for the last four columns is $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)]$. In the bottom panel, the expectations measure for the first four columns is $\mathbb{E}_t^e[(dP_t^D + D_t dt)/(P_t^D dt)] - r_t$, and the expectations measure for the last four columns is $\mathbb{E}_t^e[dP_t^D/(P_t^D dt)] - r_t$. Each reported value is averaged over 100 trials, and each trial represents a regression using monthly data simulated from the fundamental extrapolation model described in Section V. The t-statistics are calculated using a Newey-West estimator with twelve-month lags. The parameter values are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = –15\%$.

Table 13. Basic Moments in the Fundamental Extrapolation Model.

Statistic	Theoretical value	Empirical value
Equity premium ($\mathbb{E}[r^{D,e}]$)	1.85%	3.90%
Return volatility ($\sigma(r^{D,e})$)	20.4%	18.0%
Sharpe ratio ($\mathbb{E}[r^{D,e}]/\sigma(r^{D,e})$)	0.08	0.22
Interest rate ($\mathbb{E}[r]$)	2.27%	2.92%
Interest rate volatility ($\sigma(r)$)	0.23%	2.89%
Price-dividend ratio ($\exp(\mathbb{E}[\ell n(P/D)])$)	44.4	21.1

The table reports six important moments about stock market prices and returns: the long-run average of the equity premium (the rational expectation of log excess return, $\mathbb{E}[r^{D,e}]$), the average volatility of stock market returns (the volatility of log excess return, $\sigma(r^{D,e})$), the Sharpe ratio ($\mathbb{E}[r^{D,e}]/\sigma(r^{D,e})$), the average interest rate ($\mathbb{E}[r]$), interest rate volatility ($\sigma(r)$), and the average price-dividend ratio of the stock market ($\exp(\mathbb{E}[\ell n(P/D)])$). The theoretical values for these moments are computed over the objectively measured steady-state distribution of sentiment S in the fundamental extrapolation model described in Section V. The model parameters are: $g_C = 1.91\%$, $g_D = 2.45\%$, $\sigma_C = 3.8\%$, $\sigma_D = 11\%$, $\rho = 0.2$, $\gamma = 10$, $\psi = 0.9$, $\delta = 2\%$, $\theta = 0.5$, $\chi = 0.18$, $\lambda = 0.18$, $\mu_H = 15\%$, and $\mu_L = -15\%$. For the empirical values, five out of six are from Campbell and Cochrane (1999); the empirical value for interest rate volatility is not reported in Campbell and Cochrane (1999), so we report the value from Beeler and Campbell (2012).