DEFERRED ACCEPTANCE AUCTIONS WITHOUT OPTIMIZATION

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OUTLINE

1. DESCRIBE CLASS OF DESCENDING CLOCK AUCTIONS
2. DA OPTIMIZATION ⇔ SUBSTITUTES ⇔ MATROID CONSTRAINTS
3. GREEDY ALGORITHMS FOR KNAPSACK PROBLEMS
4. OBVIOUS STRATEGY-PROOFNESS FOR SINGLE-MINDED BIDDERS, GROUP STRATEGY-PROOFNESS
5. BUDGET CONSTRAINTS AND COST-SHARING (MOULIN)
6. WINNER PRIVACY CHARACTERIZATION
7. COST: COMPETITIVE EQUILIBRIUM CHARACTERIZATION
8. COST MINIMIZATION: EXACT AND APPROXIMATE
9. COMPUTATIONS WITH THE ACTUAL AUCTION

DESCRIBE CLASS OF DESCENDING CLOCK AUCTIONS (INDEXED BY A FUNCTION p)

- PERIODS: $t = 1,2,\ldots$
- ACTIVE BIDDERS $A_t \subseteq N$
- HISTORY: $A^t = (A_1, \ldots, A_t)$ where $A_t \subseteq A_{t-1} \subseteq \cdots \subseteq A_1 = N$
- CLOCK PRICE RULE: $p: H \to \mathbb{R}_+^N$ such that $p_t(A^t) \leq p_{t-1}(A^{t-1})$ for all $t \geq 2, A^t$.
- EACH ACTIVE BIDDER accepts or rejects its price if that price has changed.
- AUCTION ENDS at T when $p_T(A^T) = p_{T-1}(A^{T-1})$.
- WINNERS are the bidders in A^T.

OPTIMIZATION & SUBSTITUTES

- It is a familiar result that, when goods are substitutes for the buyer, there is a clock auction that implements the Vickrey outcome.
- Let $\alpha(v)$ be an "optimizing" allocation rule, that is, one that for some F and some non-decreasing $(\gamma_i)_{i \in N}$ satisfies

$$\alpha(v) \in \underset{A \subseteq N}{\operatorname{argmax}}\, F(A) - \sum_{i \in A} \gamma_i(v_i)$$

Theorem

Suppose that for any finite set $V = \times_{i \in N} V_i \subset \mathbb{R}_+^N$, α restricted to V is implemented by a clock auction. Then on any such set V with "no ties," α has the substitutes property.

MATROID

Let X be a finite set and $\mathcal{R} \subseteq 2^X$.

The elements of $\mathcal{R}$ are called the *independent* sets.

$(X, \mathcal{R})$ is a *matroid* if

1. Downward comprehensive: $S' \subset S \in \mathcal{R} \Rightarrow S' \in \mathcal{R}$
2. Non-empty: $\emptyset \in \mathcal{R}$
3. Augmentation: $[S, S' \in \mathcal{R}\ and\ |S| > |S'|] \Rightarrow \exists n \in S - S'such\ that\ S' \cup \{n\} \in \mathcal{R}$

MATROIDS AND SUBSTITUTES

Informally, ignore ties and think of $\alpha(w; v)$ as a demand function when the price vector v varies:

"Theorem"
Let $\alpha(w, v) = \arg\max_{S \in \mathcal{R}} \sum_{n \in S} w_n - v_n$. The elements of X are (gross) substitutes for all $v \in \mathbb{R}_+^X$ if and only if $\mathcal{R}$ is a matroid.

MATROID INTUITION

1. A discrete descending clock auction "greedily" rejects the "worst" option myopically: it does not look carefully at how that rejection affects the remaining feasible sets of options.

2. With substitutes, "greedy" rejection works for this reason: If it is optimal to reject the worst offer from some set of offers, then it remains optimal to reject that offer when other offers are improved.

3. The key to optimality of a greedy algorithm is that the constraints have the *augmentation property*, which characterizes matroids.

   Augmentation: $[S, S' \in \mathcal{R} \; and \; |S| > |S'|] \Rightarrow \exists n \in S - S' such \; that \; S' \cup \{n\} \in \mathcal{R}$

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KNAPSACK PROBLEM AND GREEDY ALGORITHM

$$\max_S \sum_{n \in S} v_n \ \ subject\ to\ \sum_{n \in S} s_n \leq K$$

- Approximate optimization by a greedy algorithm *without a matroid*. Order the items so that v_n / s_n decreases with n and pack in that order if there is room. Otherwise, set the item aside and continue.
- If the first item set aside is item m, then loss compared to optimization is no more than $\left(K - \sum_{j=1}^{m-1} s_j\right) v_m$.
- Potentially relevant for repacking TV stations (in each DMA)

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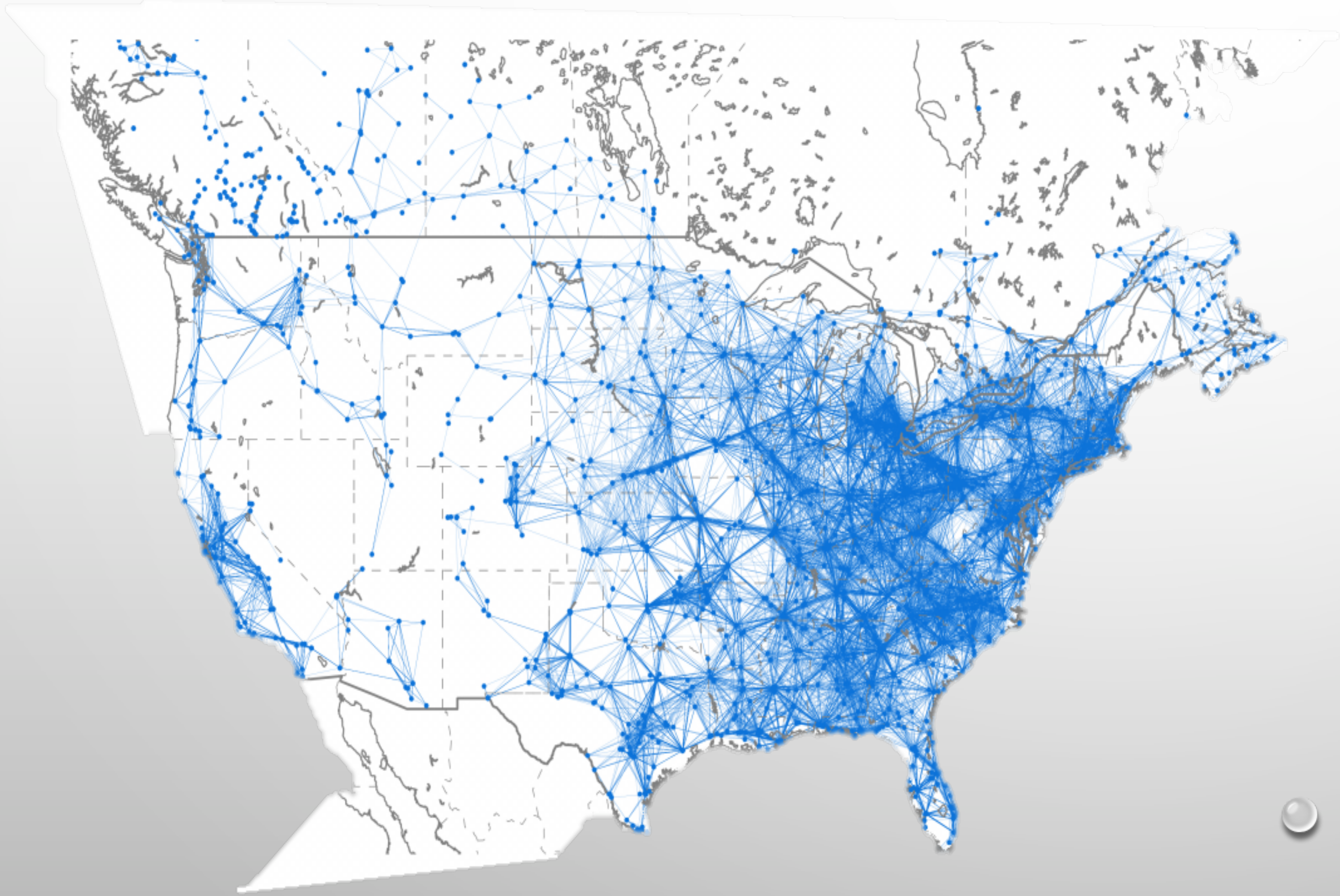
COMPLEXITY OF CONSTRAINTS



A set of stations is feasible if therre is a way to assign channels to stations without interference.

About 130,000 co-channel constraints shown in the graph.

Graph coloring is an NP-complete problem.

COMPUTABILITY OF VICKREY PRICES

- Let $S \in \mathcal{F}$ mean that S is a feasible set of broadcasters.
- Then, the Vickrey price for a station i that goes off air is

$$p_i = \left(\max_{S \in \mathcal{F}} \sum\nolimits_{j \in S} v_j \right) - \left(\max_{\substack{S \in \mathcal{F} \\ S \ni i}} \sum\nolimits_{j \in S} v_j \right)$$

- With 2000 stations, a 1% computation error in one of the maximizations leads to a pricing error of $\approx 20\times$average station value.
- *Conclusion: Vickrey prices are not computable in practice.*

BIDDER EXPERIENCE

Dear Mr. Broadcaster:

We have heard your concerns about the complexity of the spectrum reallocation process. You may even be unsure about whether to participate or how much to bid. To make things as easy as possible for you, we have adopted a Nobel-prize winning auction procedure called the "Vickrey auction."

In this auction, all you need to do is to tell us what your broadcast rights are worth to you. We'll figure out whether you are a winner and, if so, how much to pay to buy your rights. The rules will ensure that it is in your interest to report truthfully. That is the magic of the Vickrey auction!

The computations that we do will be very hard ones, and we cannot guarantee that they will be exactly correct. Also, federal law forbids us to share the information that you would need to check them.

….

IMPORTANT STRATEGIC PROPERTIES OF THE DESCENDING CLOCK AUCTION

Theorem

For any single-minded bidder, the descending clock auction is obviously strategy-proof.

Theorem

For any set of single-minded bidders, the descending clock auction is group strategy-proof.

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ACCOMMODATING BUDGET CONSTRAINTS

- Vickrey auctions cannot accommodate budget constraints.
- In a descending clock auction, if the procurement budget is exceeded, modify p and continue to reduce prices.
  - Cancel the procurement if the final result is unacceptable.

Theorem

This budget-constrained modification of the original descending clock auction is still a descending clock auction.

- Special case: Moulin's model of cost sharing for a group procurement.

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COMMUNICATION PROTOCOLS

DEFINITION: A *communications protocol* (Γ, σ, t) consists of a

1. finite extensive form mechanism $\Gamma = (N, S)$ coupled with …
2. a profile of strategies $\sigma_n: V_n \to S_n$ for each player $n \in N$
3. a labelling of the terminal nodes $t(s) \subseteq N$ (identifying the "winners")

Any communications protocol "*implements*" an allocation function $\alpha: \times_{n \in N} V_n \to 2^N$, where $\alpha(v) = t(\sigma(v))$.

WINNER PRIVACY AND INCENTIVES

DEFINITION: A protocol satisfies *unconditional winner privacy* (*UWP*) if

$$n \in \alpha(v_n, v_{-n}) \cap \alpha(v'_n, v_{-n}) \Rightarrow t(v_n, v_{-n}) = t(v'_n, v_{-n}).$$

DEFINITION: A protocol is *ex post incentive compatible (EPIC)* if, for all value profiles v the selected strategies form a full information Nash equilibrium.

Theorem

1. Every DA clock auction with truthful bidding satisfies *UWP* and *EPIC*
2. If α can be implemented by any protocol that satisfies UWP and EPIC, then it can be implemented by a descending clock auction with truthful bidding.

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"COMPETITIVE EQUILIBRIUM"

DEFINITION: Given v, the pair (S, p), with $S \subseteq N$ (the "winners") and $p \in \mathbb{R}_+^N$, is aN α-competitive equilibrium of the auction setting if

SUPPLY CONDITION: $p_n > v_n \Rightarrow n \in S$ and $p_n < v_n \Rightarrow n \notin S$

DEMAND CONDITION: $S = \alpha(p)$

Theorem

1. Let $S = \alpha(v)$ be the set of winners from a descending clock auction and let p_S be the clock auction prices. Then $\left(S, (p_S, v_{-S})\right)$ is a maximal price α-competitive equilibrium.
2. Moreover, the prices (p_S, v_{-S}) are full-information Nash equilibrium bids in the related first-price auction with the same winners selection rule.

In contrast, for the Vickrey auction, prices can be uncompetitively high.

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MYERSON-STYLE EXPECTED COST MINIMIZATION

$\min_{X,P} E[\sum_{n\in N} P_n(v)]$ subject to

(IC) $\ldots$for all n, v_n, v_n'

(PC) … for all n, v_n

(PF) $supp\big(X(v)\big) \subseteq \mathcal{F}$ for all v

$\Rightarrow$

$$\min_{supp(X(\cdot))\in\mathcal{F}} E\left[\sum_{n\in N} C_n(v_n)\, X_n(v)\right]$$

$x_n(\cdot)$ nondecreasing for all n

where $C_n(v_n) = v_n - \frac{F_n(v_n)}{f_n(v_n)}$ is the virtual cost function for bidder n.

Theorem

If $(N, \mathcal{F})$ is a matroid, then the "descending clock auction" that sets price $C_n^{-1}((1-t)\bar{v})$ for bidder n at time t is an expected cost minimizing auction.

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EFFICIENCY AND COST

Descending auction sets prices by:

$$p_n(t) = q(t) \times (Population_n^{0.5} \times Links_n^{0.5})$$

$Population_n$ connects price to virtual cost

$Links_n$ connects to "knapsack size"

Milgrom-Segal paper reports simulations results for a sub-problem, including only stations within 2 links of NYC, so that Vickrey outcomes are computable.

- SIMULATION RESULTS
  - Vickrey mean computation time: 90 CPU days
  - DA computation time: 1.5 CPU hours
  - Vickrey mean efficiency ratio: 100%
  - DA mean efficiency ratio: 95%
  - Mean cost DA/Vickrey ratio: 76%

SUPPLY REDUCTION?

- SIMULATION RESULTS IN THE WORKS.
- THRESHOLD PRICES FOR ~3000 STATIONS ARE COMPUTABLE, BUT REQUIRE RUNNING THE AUCTION 3000 TIMES.
- INTENTION IS TO COMPARE THRESHOLD PRICES FOR LOSERS AND ACTUAL BIDS FOR WINNERS TO ESTIMATED STATION VALUES.

SUMMARY

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THANK YOU!