Is Inequality Increasing in *r* - *g*? Piketty's Principle of Capitalist Economics and the Dynamics of Inequality in Britain, 1210-2013[1]

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Abstract. Piketty argues that wealth inequality is sharply increasing in *r* - *g* and refers to *r* > *g* as 'the central contradiction of capitalist economics', where *r* is asset returns and *g* is real income growth. To assess whether inequality is increasing in the (*r*-*g*)-gap this paper 1) constructs unique annual data for the UK on asset returns for a balanced portfolio and several other variables over the period 1210-2013 and 2) examines whether the dynamics in the wealth-income ratio, *W*-*Y*, and capital's income share, S^K, are governed by (*r*–*g*). It is shown that *r* and *g* are robust and significant determinants of wealth and income inequality and have been the major forces behind the large inequality waves over the past eight centuries.

JEL Classification: E1, E2, O4, N1, N30, P1
Keywords: Inequality and the (*r*-*g*)-gap; dynamics of inequality; inequality in the UK, 1210-2013

1. Introduction

Central to Piketty's (2014) *Capitalism in the 21st Century* is that the growth in wealth inequality is governed by the gap between the returns to wealth, *r*, and economy-wide income growth, *g*. Piketty (2014) refers to *r* > *g* as "the central contradiction of capitalist economics" (p. 398). Assuming that real asset returns remain fairly constant, Piketty (2014) predicts that the (*r*-*g*)-gap will widen over the rest of this century because of reduced population growth and per capita income growth – a prediction that has been met with a storm of resistance from the economics profession, mostly because the (*r*-*g*)-gap does not necessarily increase in response to a reduced *g* in standard canonical Euler equations and because, in standard growth models, the inequality *r* > *g* ensures dynamic efficiency and is consistent with constant steady-state capital-income ratios (Abel *et al*., 1989). Piketty's hypothesis that wealth

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inequality is increasing in the (r–g)-gap is closely related to what is sometimes referred to Piketty's third law that inequality tends to diverge when $r > g$.[2]

Despite being central to the inequality debate, hardly any empirical work has been undertaken to investigate the relationship between inequality and the (r-g)-gap, reflecting, to some extent, the absence of continuous long data on wealth and income inequality and asset returns, not to mention the difficulties associated with the construction of a composite measure of r that includes all asset classes. Annual data on the W-Y ratio distributed on asset classes since 1970 have only recently become available for a few countries and there have been very few attempts, if any, to construct a composite r containing all asset classes.[3] In his casual inspection, Piketty (2014, Figures 10.10 and 10.11) shows that a large positive and relatively constant (r-g)-gap prevailed from Antiquity to the early 20th century in the UK and France and he argues this was responsible for a high W-Y ratio and, consequently, high wealth inequality. However, Piketty's (2014) analysis is based on very few observations, proxies r by bond rates, and fixes the interest rate at a constant level of 4.5% before the period 1700-1820.

Relying mostly on recent data, Acemoglu and Robinson (2015) and Goes (2016) are two of the few attempts to test whether inequality is increasing in the (r-g)-gap. Acemoglu and Robinson (2015) fail to find significant positive effects of the (r–g)-gap on top 1% income shares, and conclude that inequality is driven by more important factors than the (r–g)-gap, such as institutions. Almost the same conclusion is reached by Goes (2016) who finds that capital's income share, as a proxy for inequality, is *negatively* related to the (r-g)-gap for more than 75% of his country sample. However, the data used by Acemoglu and Robinson (2015) and Goes (2016) are highly problematic.[4]

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[2] Piketty (2014) does not use the term "third law of capitalist economics". Ray (2015) and Acemoglu and Robinson (2015) refer to $r > g$ as "Piketty's third law". Acemoglu and Robinson's definition of Piketty's Third Law is: "whenever $r >$ g; there will be a tendency for inequality to diverge" (2015, online Appendix p. 9).

[3] Piketty and Zucman (2014) construct long historical W-Y data for the UK (1700-2010), France (1700-2010), the US (1770-2010) and Germany (1870-2010); however, their W-Y data are not detailed sufficiently on the wealth categories that are needed to use them as weights is a composite measure of asset returns. Furthermore, their data are heavily interpolated, often with 50 year intervals or more, and their historical wealth estimates are mostly based on discounted profit flows with constant discount factors, which, as argued by Madsen (2016a), are likely to lead to biased estimates of wealth.

[4] In their tests of Piketty's (r-g)-hypothesis, Acemoglu and Robinson (2015) and Goes (2016) use, as proxies for asset returns: 1) government bond rates minus contemporaneous inflation; and 2) the returns to fixed capital based on capital's income share from the Penn World Table version 8.0. Along the same lines, Piketty (2014) uses the post-tax real bond rate in his analysis of the (r-g)-gap since the antique. Overall, the existing evidence and tests of Piketty's (r-g)-hypothesis are highly problematic because the returns cover only a fraction of assets and are often not even properly measured for individual assets. Furthermore, since land rent, which is a large source of capital income (Caselli and Feyrer, 2007), are not included as capital income share from the Penn World Table and the fact that almost all government consumption is wage costs, the coefficients of r are biased towards zero in the estimates of Acemoglu and Robinson (2015) and Goes (2016). Finally, the problem in basing returns on factor shares in national accounts, as discussed in detail below, is that a large share of asset returns are capital gains which are not included in national account factor shares although they are a significant part of permanent income of the wealthy.

This paper constructs a large *annual* dataset containing several variables and examines whether wealth and income inequality in Britain has been driven by the (*r*-*g*)-gap and, therefore, the extent to which the (*r*-*g*)-gap can be used as an analytical tool for inequality dynamics and inequality forecasting. More precisely, the paper examines the effects of *r* and *g* on the *W*-*Y* ratio and capital's income share (including income from non-reproducibles), S^W, for Britain over the past eight centuries, where *r* is measured as a weighted average of after-tax real returns on non-human assets. Capital gains on non-reproducibles are included in S^W in half of the S^W regressions because capital gains increase permanent income and, therefore, should be included as capital income. This concept is consistent with Haig-Simons classical definition of income, which states that income is what we can consume while keeping our real wealth intact (see, for discussion, Roine and Waldenström, 2015).

The data cover several economic epochs in history, which not only enables assessment of whether the nexus between the (*r*-*g*)-gap and inequality applies to all modes of production, but also the extent to which the great historical waves in the *W*-*Y* ratio and S^W are related to the (*r*-*g*)-gap. The analysis covers the late medieval period when agriculture was the dominant mode of production, the First and the Second British Industrial Revolutions where manufacturing steadily took over as the leading sector of growth, and the third phase, starting around the first oil price shock in 1973/74, in which ICT-services have become the main growth promoting sector.

The paper makes the following contributions to the literature. To enable tests of whether inequality is increasing in the (*r*-*g*)-gap, a unique annual historical dataset is constructed for Britain over the period 1210-2013 containing several variables such as returns to assets, wealth and income inequality proxies, tariff rates, saving, openness, taxes, constraints on executive, contract-intensive money, and food prices among other variables. Asset returns are estimated as the average real returns to non-residential fixed capital, agricultural land, housing, government debt, net foreign assets, gold, silver, farm buildings, and livestock, weighted by the share of each asset in total wealth. In Section 3.7 it is argued that the *W*-*Y* ratio and S^W with are good proxies for wealth and income inequality and may even be better measures of inequality than conventional measures of inequality such as Gini coefficients and top wealth and income shares; particularly when capital gains on non-reproducibles are included in S^W.

It is imperative to use ultra-long data in the analysis because the *W*-*Y* ratio and S^W move at very low frequencies that even extend beyond a century, driven by slow-moving intergenerational wealth accumulation and waves in asset prices; thus reducing the value of analyses covering even a century. Econometrically, long data increase the efficiency of the parameter estimates and reduce their bias. Davidson and McKinnon (2006), for example, show that instrument variable parameter estimates can be severely biased in small samples. Furthermore, long historical data enable one to gain insight into the dependency of wealth accumulation on the source of returns; e.g. whether the returns are predominantly derived from of dividend yields, as during the industrial epoch, or real capital gains on land, as during the pre-industrial period and post-industrial period in which service sector is the dominant mode of production and urban land is the principal component of wealth accumulation.

As another contribution, it is tested whether the *W*-*Y* ratio and S^W can be explained by *r* and *g* and saving rates. Inflation, tariffs, and real food prices are used as instruments for *r*, *g* and saving rates to deal with endogeneity and measurement errors. To further reduce the potential endogeneity bias, contract intensive money, constraints on executive, openness and taxes are controlled for in the structural regressions to allow for the effects of globalization waves on the inequality path, and for the possibility that the inequality path has been partly driven by institutions, as argued by Acemoglu and Robinson (2015).

The empirical exercise gives three principal insights. First, three large waves in *r* and the (*r*-*g*)-gap are identified over the past eight centuries and graphical evidence suggests that these waves coincide with the waves in inequality. The waves in the data are an outcome of political struggles between capitalists, the landed class and workers, and major shocks such as wars and epidemics; factors that result in inflations and deflations, fluctuations in tariff rates, and real food prices fluctuations. In the ultra-long run, however, the *W*-*Y* ratio and S^W converge towards a constant, as predicted by extant models of economic growth. Second, it is shown that a large fraction of *r* has been driven by real capital gains on non-reproducibles and that these have been a great source of the evolution of inequality since 1210. Third, it is shown that *r* and *g* are highly significant and robust determinants of the *W*-*Y* ratio and S^K, and they have the signs predicted by Piketty's theory.

The rest of the paper is organized as follows. The nexus between the (*r*-*g*)-gap and inequality to economic theory and the main criticism of Piketty's $r > g$-hypothesis are briefly discussed in Section 2, while data construction, data reliability, and graphical analyses are reviewed in Section 3. Regression results and robustness checks are presented in Sections 4 and 5, and Section 6 concludes.

2. The (*r*-*g*)-Gap, Inequality and Growth

2.1 The functional relationship between inequality and the (r-g)-gap.

According to Piketty (2014, 2015a, 2015b) inequality is increasing in *r* because it increases the wealth accumulation, thus amplifying the initial heterogeneity of the wealth distribution, and it is decreasing in *g* because it reduces the value of existing wealth relative to the new wealth generated in the economy. He, furthermore, argues that reduced growth in this century will enlarge the (*r*-*g*)-gap and further the increasing inequality that started in the early 1980s.

However, Piketty's proposition that a declining growth in this century will further inequality and that $r > g$ will lead to *increasing* inequality have been met with criticism. As stressed by Acemoglu and Robinson (2015) and Mankiw (2015), $r > g$ holds in steady state in standard growth models, and yet, the capital-output ratio, K-Y, remains constant. As a central model in macroeconomics, for example, the dynastic model poses the following steady state relationship:

$$r = \gamma g^A + \rho, \tag{1}$$

where r is real asset returns; g^A is productivity growth; γ is the inverse intertemporal elasticity of substitution; and ρ is the consumer's time preference. If, for example, $\gamma = 1$, then $r - g^A = \rho$ and it follows automatically that $r > g^A$ in steady state.

In the $r > g$ debate there has, however, been some confusion as to whether the (r-g)-gap has level or growth effects on inequality. The criticism of Mankiw (2015), among others, relates to some of the statements by Piketty (2014) that a sufficiently large (r-g)-gap has permanent growth effects on inequality. Whether a positive (r - g)-gap is a force towards divergence or convergence of wealth across the distribution of wealth in steady state is not obvious from the discussion in the literature and has caused some confusion; partly reflecting the mixed signals given by Piketty.[5] As shown in the online Appendix, almost all macro models predict a level-level relationship between the (r-g)-gap and inequality proxied mostly by the W-Y ratio and S^W.

Another concern is whether inequality is equally affected by changes to r and g and, therefore, that the (r-g)-gap is the right metric to assess the forces that are guiding inequality. As discussed in the online Appendix and the empirical section, changes to g and r may have quite different effects on inequality, partly depending on the forces driving r and g. The regression analysis below overcome these concerns by including r and g individually and by allowing for r and g to have growth as well as level effects of on inequality.

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[5] Piketty (2014), for example, suggests that the (r-g)-gap has permanent growth effects on inequality by the following statement: "If the difference $r – g$ surpasses a certain threshold, there is no equilibrium distribution: inequality of wealth will increase without limit, and the gap between the peak of the distribution and the average will grow indefinitely" (p. 258). Similarly, a higher saving rate will lead to permanently growing inequality: "If one saves more, because one's fortune is large enough to live well while consuming somewhat less of one's annual rent, then one's fortune will increase more rapidly than the economy, and inequality of wealth will tend to increase even if one contributes no income from labor" (p. 351). This reasoning gains support from Solow (2014) who states that: "This is Piketty's main point, and his new and powerful contribution to an old topic: as long as the rate of return exceeds the rate of growth, the income and wealth of the rich will grow faster than the typical income from work" (p. 1). However, in other places in his 2014 book and in later writings, Piketty suggests that the (r-g)-gap has level effects on inequality. For example, Piketty (2014) notes that "the distribution of wealth tends toward a long-run equilibrium and that the equilibrium level of inequality is an increasing function of the gap r - g" (p. 258). Furthermore, Piketty (2016) argues that the (r-g)-gap impacts on the level of inequality. For example, he remarks that "a central property of this large class of models is that for a given structure of shocks, the long-run magnitude of wealth inequality will tend to be magnified if r-g is higher" (p. 1).

2.2 Is the (r-g)-gap increasing or decreasing in g?

Piketty's (2014) prediction of increasing inequality throughout the 21st century rests on the assumption that a the (*r*-*g*)-gap will decrease in response to a reduction in *g* relative to the 20th century without any counter-balancing response in *r*. However, intertemporal utility-maximizing models predict that *r* is increasing in *g*. Eq. (1), for example, shows that a reduced growth rate also reduces *r*. If $\gamma > 1$, then reduced *g* will result in a *decreasing W-Y* ratio because *r* declines more than proportionally to the decrease in *g*. This is a well-known result and, as shown in the online Appendix, *r* responds approximately proportionally to changes in *g* in most of the oft-used growth models such as the Ramsey model of Mankiw (2015); the OLG model of Ryoji (2015); and the Solow growth model under Cobb-Douglas technology. Furthermore, Krusell and Smith (2014) show that saving rates beyond the Golden-Rule and high intertemporal substitution rates are required to generate a negative relationship between the (*r*-*g*)-gap and *g*.

The nexus between (*r*-*g*)-gap and *g* are for four reasons much less clear-cut when utility-maximizing models are combined with endogenous growth models, CES technology is assumed and capital adjustment costs are allowed for. First, the nexus between (*r*-*g*)-gap and *g* is highly likely to be negative in steady state in the Solow model under the CES technology assumption. Second, Barro and Sala-i-Martin (1992) show that *r* is independent of *g* in steady state in the most influential first-generation models of endogenous economic growth (learning-by-doing models, the AK model, and product variety models). Third, Sala-i-Martin (1992) show that *r* may even be *diminishing* in *g* when investment adjust costs are allowed for, as in all Tobin's *q* models of investment. An increase in time preference, for example, shifts the preference as well as the production schedule up in the *r*-*g* space, resulting in an increase in *r* but a decrease in *g*.

Fourth, the inclusion of land in the production function further complicates the relationship *r* and *g*. In agricultural economies as the UK up to the 19th century and most developing countries today, one would expect a positive relationship between *r* and population growth, g^n, because population growth drives returns to land up when land is non-reproducible provided that the economy is relatively closed. The link between g^A and *r* is more complex in agrarian economies because a large fraction of consumers may be trapped in a Malthusian equilibrium and the extra earnings to land-holders from the growing land productivity depends on the elasticity of demand for agricultural produce. Finally, the link between *r* and *g* weakens as the economy is financially opens up to the outside world and *r* and *g* become almost independent for a small economy under perfect capital mobility and fixed exchange rates, where fixed exchange rates have approximately prevailed in the period 1210-1973 except for the short-lived suspensions of the Gold Standard in the periods 1914-1925 and 1932-1948.

Empirically, the literature fails to find a significantly positive relationship between r and g as discussed in the online Appendix. Using the data constructed in this paper the regressions in the online Appendix fail to find a significantly positive relationship between r and g; thus giving support to the findings of the literature. In any event, the extent to which g influences r is immaterial for the results in this paper since r and g are simultaneously included in the inequality regressions.

3. Data construction

Most of the discussion in this section centers on the principal variables: S^W, the W-Y ratio, g and, particularly r, since, together with g, is the focus variable in the analysis and offers a significant advance over the proxies for r used in the existing literature. Here, S^W and the W-Y ratio are used as proxies for inequality. Asset returns, r, are here computed as a weighted average of real after-tax returns on non-human assets, where the weights are the share of each individual asset in the total portfolio. Following Piketty (2014) and Piketty and Zucman (2014), wealth, W, is defined as total non-human private wealth at market prices and, therefore, includes productive and non-productive wealth and real capital gains on non-reproducibles.[6] The main data and the broad principles behind the data construction are discussed in this section. Data sources, further details of data construction, and the construction of control variables and instruments are discussed in depth in the online Appendix.

3.1 Asset returns

Asset returns on a private portfolio are constructed annually as the weighted average of the real returns to government bonds (or rent charges), stocks, agricultural land, housing, and corporate capital stock, where the weights are their respective shares in total private wealth computed as a chain index:

$$r_t = \alpha_t^B r_t^B + \alpha_t^K r_t^K + \alpha_t^T r_t^T + \alpha_t^H r_t^H + \alpha_t^{G\&S} r_t^{G\&S} + \alpha_t^{LS} r_t^{LS} + \alpha_t^{FB} r_t^{FB} + \alpha_t^{FA} r_t^{FA}, \qquad (2)$$

where α^X = asset X's share in total wealth and r^X is asset X's real expected after-tax returns; B = government bonds; K is non-agricultural non-residential fixed capital; T = agricultural land; H =

[6] Empirically, Piketty's (2014) analysis and, the analysis here, deviates from standard analyses in two respects. First, Piketty values wealth at market prices, which conflates the distinction between changes in quantities and relative prices, which have implications for the dynamics of inequality and the mapping between Piketty's framework and the predictions of growth models. Second, Piketty (2014) uses the terms 'wealth' (W) and 'capital' (K) synonymously, which has caused some confusion in the literature because the accumulation of distinctive types of wealth has very different implications for the dynamics and measurement of inequality as well as the feedback effects of the W-Y ratio on r. In almost all growth models capital is defined as fixed non-residential reproducible capital, K, and an increasing K, *ceteris paribus*, is associated with a reduced r unless the elasticity of substitution between K and L exceeds one. The letters K and W are used here to distinguished wealth, W, and fixed non-residential capital stock, K.

housing (value of residential structures plus urban land); *G&S* = gold and silver; *LS* = livestock; *FB* = farm buildings; and *FA* = foreign assets.

The after tax returns for each individual asset are computed as:

$$r_t^B = (1-\tau_t^Y) r_t^{N,B} - \pi_t^{CPI}, \tag{3}$$

$$r_t^K = (1-\tau_t^Y)\left(\frac{Div}{P^S}\right)_t + g_t^S - \pi_t^{CPI}, \tag{4}$$

$$r_t^T = (1-\tau_t^T)(1-\tau_t^Y)\left(\frac{Rent^T}{P^T}\right)_t + g_t^T - \pi_t^{CPI}, \tag{5}$$

$$r_t^H = [\,\theta_t(1-\tau_t^H) + (1-\theta_t)(1-\tau_t^Y)]\left(\frac{Rent^H}{P^H}\right)_t + g_t^H - \pi_t^{CPI}, \tag{6}$$

$$r_t^{G\&S} = g_t^{G\&S} - \pi_t^e, \tag{7}$$

$$r_t^{FB} = r_t^{LS} = (1-\tau_t^Y) r_t^{RC} - \pi_t^{CPI}, \tag{8}$$

$$r_t^{FA} = (1-\tau_t^Y)\left(\frac{Div^{FA}}{Fass}\right)_t + g_t^S - \pi_t^{CPI}, \tag{9}$$

where τ^Y = income tax rate; r^B is the real after-tax bond rate on government debt; $r^{N,B}$ = nominal bond rate on government debt; π^{CPI} = consumer price inflation rate; *Div* = dividends per stock; P^S = stock prices; g^S = capital gains on stocks; τ^T = agricultural land taxes; $Rent^T$ = land rent per hectare; P^T = price per hectare of agricultural land; g^T = capital gains on agricultural land; τ^H = property taxes on houses; θ = share of housing wealth that is owner occupied; $Rent^H$ = unit housing rent; P^H = unit house prices; g^H is capital gains on housing; $g^{G\&S}$ = capital gains on gold and silver; r^{RC} = interest charges; Div^{FA} = dividends on foreign assets; *Fass* = value of net foreign assets; g^{FA} = capital gains on foreign assets.

Returns to each individual asset are in most cases self-explanatory; however, the returns to foreign assets, bonds and fixed capital require some discussion. Returns to foreign assets is the sum of after-tax dividend yields on foreign assets plus capital gains on domestic stocks, where the dividend yield is estimated as the net income from abroad as a percentage of the net foreign asset position. Capital gains on domestic stocks are used for capital gains on foreign assets because the data on the growth in the net foreign asset position are not of sufficiently good quality to be used; even the most recent data.[7]

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[7] I initially computed capital gains from net foreign assets using the following model:

$$g_t^F = g_t^{Fass} - \left(\frac{CA}{Fass}\right)_{t-1},$$

where *CA* is current account on the balance of payments, and g^{Fass} is the growth in the value of the foreign asset position. While this approach is theoretically correct, the generated capital gains are implausibly high during the Napoleonic War and, particularly, after 1969. Using this method capital gains apparently exceeded 600% each year over the period 1974-

Rent charges are used for bond returns, r^B, before 1775 and government bond rates thereafter as bonds gained a stronger foothold as the most important nominal debt instruments (Clark, 1988). Although bond rates are repeatedly used as proxies for asset returns in the literature, (e.g. Piketty, 2014), they are particularly biased measures of returns on balanced portfolios back in history when the usury laws were enforced. Interest rates on credit lasting less than one year, for example, often exceeded 50% in the medieval period (Homer and Sylla, 2005). It can, therefore, be highly misleading to use interest rates from the medieval period; particularly since data on rent charges are available. Rent charges are perpetual fixed nominal obligations that give the buyer the right to receive a specified payment each year forever. Since they were secured by land or houses and other property they were considered as safe loans with low default probabilities and were not affected by medieval usury laws because they were regarded as a sale in advance of rent from property and not as a loan (Clark, 1988). Rent charges was a popular way of raising capital until the 19th century and were sold at least as early as the late 12th century (Clark, 1988). As argued by Clark (1988) rent charges are probably the closest one can come to the long historical cost of capital.

The returns to non-residential non-agricultural fixed capital are estimated as real after tax stock returns (see, Eq. (4)) because corporate earnings data are unavailable before 1900 and, more importantly, because stock returns adequately account for differential tax treatments of dividends and capital gains that are the result of additional fixed capital investment or share-buy-backs. Furthermore, in contrast to corporate earnings national account data, stock returns implicitly account for negative capital gains resulting from creative destruction in which new technologies replace older ones. The data on dividends and stock prices are available back to 1695. Being probably the best early measure of the cost of capital, real rental charges are used as proxies for returns to fixed capital before 1695.

The share of each asset in total wealth, α^X, is based on the wealth estimates of Madsen (2016a). The guiding principle behind the construction of the wealth data is that 1) commercial fixed capital is valued by the volume of fixed capital stock multiplied by Tobin's q; 2) urban and agricultural land are valued at market prices and are estimated as the market value of the average unit of asset times its quantity; and 3) the value of farm structures, gold and silver and livestock are valued at acquisition costs.[8]

1976, in spite of falling global asset prices, and the annual capital losses exceeded 22% each year over the ten years up to 2013; a period during which the global capital markets experienced large capital gains.

[8] An alternative method, predominantly used to generate wealth before WWII, is created from earnings records that are converted to wealth by multiplying by a fixed number; the so-called 'years of purchase' that is typically set to 20. In other words, this method assumes that earnings are perpetual discounted with a constant discount rate regardless of whether earnings are atypical and discount rates change over time and, as such, is only a very rough approximation (see, for a critical assessment, Madsen, 2016a).

The advantage of the approach used here is that it is transparent, is based on annually changing weights (chain index), includes capital gains on each individual asset, and allows of movements of the relative prices of capital/wealth and consumption (see, on the last point, Caselli and Fayrer, 2007). The method is superior to existing methods because it covers the main assets in a balanced portfolio and allows for capital gains on all assets. An alternative method, which is used by Acemoglu and Robinson (2015) and Goes (2016), is to recover real returns from the equation $r^{SK} = S^K \cdot Y/W - \delta$, where δ is depreciation; however, this method requires data for W that are only sparsely available, mostly do not include important assets such as land, livestock, foreign assets, gold and silver, and requires weighted averages of depreciation rates that vary substantially for asset classes and over time as the production structure changes.[9] Problematic for this method is also that taxes cannot be adequately allowed for because they are specific to each asset and type of return (dividend yield versus capital gain) and that capital gains on non-reproducible assets are not included in S^K. Thus, r^{SK} is likely to be a severely biased proxy for the real post-tax returns on a balanced portfolio.

The downside of the estimates undertaken here is that returns on individual assets may underestimate those of wealthy individuals (Piketty, 2014) and returns to subsoil wealth are unaccounted for. Data on the value of and the returns to mines are not available and even current methods do not provide wealth estimates for mines but estimate subsoil wealth by discounting rents by a constant discount rate (see, for discussion, Caselli and Fayrer, 2007). Finally, although some of the data are interpolated on decennial frequencies this should not affect the results since the estimates are undertaken in 10-year intervals.

3.2 The W-Y ratio

The *W-Y* ratio is estimated as the ratio of private wealth and net national income. Private wealth is computed as:

$$W_t = q_t P_t^K K_t^{Fix} + P_t^K K_t^{Agr} + P_t^R K_t^{Res} + P_t^K K_t^{Dur} + P_t^{Agr} T_t^{Agr} + P_t^{Urb} T_t^{Urb}$$
$$+ P_t^L S_t^{Live} + P_t^{G\&S} G\&S_t + NFA_t + P_t^I S_t^{Int} + Debt_t^G - P_t^K K_t^G,$$

[9] Goes (2016) uses the post corporate-tax sovereign bond rate minus contemporary inflation. It is not clear, however, why the corporate tax rate as opposed to the direct tax rate, is used to compute the post-tax interest rate. In the robustness section he estimates returns as capital's share times the *Y-K* ratio from the Penn World Tables, where *K* is fixed capital. There are several troubles with his approach; most importantly that S^K includes labor income of self-employed, wealth in the denominator of the *Y-W* ratio based on fixed capital, and returns are gross returns; i.e. is inclusive capital depreciation.

where K^{Fix} is the stock of fixed non-residential and non-agricultural capital; P^K is the price of fixed capital (non-residential investment price deflator); q is Tobin's q; K^{Res} is the stock of residential structures; P^R is the price of residential stock (residential investment deflator); K^{Dur} is the stock of durable goods; K^{Agr} is agricultural fixed capital stock; P^{Agr} is the price of agricultural land; T^{Agr} is the agricultural land area (arable plus pastoral land); T^{Urb} is the size of the urban land; P^{Urb} is the price of urban land; S^{Live} is the livestock; P^L is the price of livestock; $G\&S$ is the quantity of monetary and non-monetary gold and silver; $P^{G\&S}$ is the price of gold and silver; NFA is the value of net foreign assets; S^{Int} is intangible stock; P^I is the price of intellectual knowledge; K^G is government fixed capital; and $Debt^G$ is net government debt.

3.3 Saving

Private saving is estimated as follows:

$$S_t^P = I_t^{Tot} + I_t^{LS} + CA_t + I_t^{G\&S}, \tag{10}$$

where S^P is gross private saving; I^{Tot} is total nominal gross fixed capital formation at acquisition costs; I^{LS} is nominal investment in breeding and working livestock at acquisition costs; CA is the current account balance at current prices; and $I^{G\&S}$ is the nominal investment in monetary gold and silver. The saving given by Eq. (10) extends conventional saving estimates by including investment in livestock, gold and silver to make saving consistent with the wealth estimates. Furthermore, investment in livestock, (monetary) gold and silver is part of saving because they are generated from income and are not yet consumed.

Total fixed capital formation is available from Feinstein (1988) after 1760. Before then, I^{Tot} is computed as:

$$I_t^{Tot} = I_t^{Ind} + I_t^{Ships} + I_t^{Dwel} + I_t^{Agr} + I_t^{Land}, \tag{11}$$

where I^{Ind} is industrial investment (mining and manufacturing); I^{Ships} is investment in ships; I^{Dwel} is investment in residential structures; I^{Agr} is investment in agricultural buildings (barns/stalls/byres); and I^{Land} is investment in land improvement, enclosure and fencing. Investment in sector X, I^X, where X = *Ind*, *Ships*, *Dwel*, *Agr* and *Land*, is recovered from the equation, $I_t^X = K_t^X - (1 - \delta^X)K_{t-1}^X$, where δ^X is the depreciation rate for sector X's fixed capital stock.

3.4 Share of income going to capital

Two measures of capital's income share are constructed as proxies of inequality; a conventional and one in which capital gains on non-reproducibles are included:

$$S_t^W = 1 - \Psi_{1855} \frac{w_t^{BL}[\Phi_t^A \theta_t^{RA} + (1-\Phi_t^A)(0.5+0.5\theta_t^{RBC})]Pop_t}{Y_t^N}, \tag{12}$$

and

$$S_t^{WCG} = S_t^W + \frac{[(g_t^T - \pi_t^{CPI}) + (g_t^H - \pi_t^{CPI})]}{Y_t^N}, \tag{13}$$

where S^W is capital's income share of net national income excluding capital gains on non-reproducibles wealth (conventional case); S^{WCG} is capital's income share including capital gains on non-reproducible wealth; *Pop* is population; w^{BL} is *annual* wages of unskilled labor; θ^{RA} is the ratio of daily wages of agricultural workers and unskilled labor; θ^{RBC} is the ratio of daily of skilled and unskilled labor; Φ^A is the employment share of agriculture in total employment; Y^N is nominal net national income; g^H is capital gains on housing (underlying land plus structure); g^T is capital gains on agricultural land; π^{CPI} is consumer price inflation in decimal points; and Ψ_{1855} is a conversion factor to realign the level of S^W to capital's share derived from national accounts in 1855, when national account data become available. After 1855 S^W is estimated as one minus the share of compensation to employees in net national income, where labor earnings of the self-employed are imputed into compensation to employees (earnings from labor of self-employed labor and working family members are counted as profits in national accounts). Nominal net national income is used in the denominator to ensure that S^W is based on earnings after capital depreciation. Both measures of capital shares are used in the regressions below.

The first measure is capital's share of net national income, S^W, is estimated as one minus the share of income going to labor. Compensation to employees is computed as employment times annual earnings per worker, where workers' average earnings are annual earnings of unskilled labor adjusted by the share of different types of labor and their hourly wages relative to that of unskilled labor (annul income are only available for unskilled labor). The annual wage data are constructed by Humphries and Weisdorf (2016) and offer a marked improvement over previous wage data, which are based on daily wages, because they incorporate variations in the annual number of working days that appear to have changed substantially over time.

Capital gains on urban and agricultural land are included in the second measure of capital's share, S^{WCG}, because they add directly to wealth inequality and, indirectly to income inequality when the assets are sold, borrowed against, or passed on to the next generation in the form of gifts or bequests. While capital gains on non-reproducibles are not included in national account estimates of S^W, capital gains on fixed capital, in the form of capital gains on stocks, are contained in S^K as corporate earnings in steady state, noting that real stock prices only increase in the long run because of retained earnings, which are included in S^W.[10] It is crucial to include real capital gains on non-reproducibles in any measure of income inequality and, particularly, wealth inequality, because they have historically been significant sources of capital income.

3.5 Income

The real and nominal GDP data are from Clark (2010) (real national income) (over the period 1210-1270; from Broadberry *et al*. (2015) over the period 1270-1820 and spliced to Bank of England's GDP data after 1820. The data constructed by Broadberry *et al*. (2015) constitutes the most thorough and probably the most credible reconstruction of long historical GDP data undertaken thus far.

3.6 W-Y ratio and factor income shares as measures of inequality

There are considerable difficulties associated with the measurement of income and, particularly, wealth inequality and common for all available measures of inequality, is that none of them are perfect. Although the classical error-in-variables bias does not normally apply to dependent variables, the parameter estimates will be biased if the measurement errors between the dependent and independent variables are correlated, and significant measurement errors will reduce the efficiency of the parameter estimates.

Thus far, tests of the nexus between inequality and the (*r*-*g*)-gap have almost entirely, if not always, been based on top income shares and capital's income share even though the (*r*-*g*)-gap applies to wealth inequality; probably reflecting that the existing wealth inequality data are generally of low quality, cover relatively short time periods, have breaks, and contain a lot of missing observations, suggesting that alternative measures of wealth inequality are called for (see for a thorough discussion of the existing wealth inequality data, Roine and Waldenström, 2015).[11] Similarly, there are several breaks in the top income inequality data and one needs to splice various top deciles to get a continuous series. The only relatively continuous long data available are top 0.05% income share-tax units covering the period 1908-1994. Thus, while data on top 10% or top 1% income shares can be

[10] Real stock prices do not increase in the long run because of capital gains but solely because of retained earnings – earnings that are not paid out as dividends but retained within the company for further expansion of the capital stock or for share buybacks (see, for an exposition, Madsen and Davis, 2006). Share prices deflated by consumer prices of the firm that pays out all earnings tend to decrease over time as a result of creative destruction and investment-specific technological progress.

[11] For UK the wealth inequality data have been constructed by Atkinson and Harrison (1978) and extended by Atkinson (see Roine and Waldenström, 2015) by adjusting tax data of the deceased by the inverse mortality rates for different age, sex and social groups. The data are available over the years 1800, 1810, 1911, 1923-1930, 1936, 1938, 1950-2005 except 1963 and 2004 (Roine and Waldenström, 2015).

constructed over the period 1908-2013, they need to be spliced from data of many different top income deciles (see Roine and Waldenström, 2015).

These issues, coupled with the short length of the data call for alternative measures of wealth and income inequality that cover a long time span and are reliable indicators of inequality. However, are S^W, S^{WCG}, and the *W-Y* ratio good proxies of inequality? The share of income going to capital, S^W/S^{WCG}, is likely to be a good indicator of personal income and wealth distributions well into the 20th century when there was a strong and clear divide between owners of land and fixed capital and workers. The working and the landless classes were representative of almost the entire wage-earning population and this class derived little income from wealth (Prados de la Escosura, 2008). The almost pure class system coupled with a low variation of earnings of the working class relative to that of the capitalist/landed class, resulted in an almost perfect correlation between S^W/S^{WCG} and income inequality as shown formally in the online Appendix. Even urban property, as the most basic asset of the broad population today, was owned predominantly by the landed class (Justman and Grandstein, 1999). Furthermore, the homeownership ratio was low at approximately 20% from 1798 to 1939, increased to approximately 30% from 1939-1961 and it was first after 1970 that the homeownership ratio climb above 50%, suggesting a very unequal distribution of housing wealth throughout history, amplified by a highly unequal distribution of housing wealth among house owners (Bethan and Dorling, 2004; Gyourko *et al*., 2013; the homeownership data, which are not available before 1798, are listed in the online Appendix).

As an indicator of inequality S^W/S^{WCG} may have weakened recently as the share of high income earners among employees has increased (Piketty, 2015a, 2015b). However, several recent studies suggest that the positive correlation between the personal and the functional income distribution has remained highly significant in the post-WWII period as high income earning employees have experienced large real capital gains on their assets and have large pension funds that are invested in assets (Roine and Waldenström, 2012). Furthermore, Gennaioli *et al*. (2014) show that financial income is a function of intermediated wealth, which, in conjunction with financial sector employees being a large fraction of high income earners, gives an indirect, but strong, link between employees' inequality and the *W-Y* ratio and S^W. For the UK, Bell and Van Reenen (2014) find that bankers' bonuses account for two-thirds of the increase in the top 1% share over the period 1999-2008; thus confirming a strong link between profits and high-income employees. Presumably, this also holds for earnings of employees in real estate in which fees are in a fixed constant proportion to property values.

Importantly, capital gains on non-reproducibles and, in most instances, stocks are not included in conventional income inequality proxies such as Gini's and top income shares although they are part of permanent income and, therefore, are influential for income and wealth inequality. Based on data for Sweden, Roine and Waldenström (2012) find that the top 1% income share in total income was, on average, 40% higher over the period 1990-2008 when realized capital gains are counted as income. For the US over the period 1916-1998 Piketty and Saez (2003) find that 18% of the income of the top 1% share, on average, is attributed to capital gains. For the UK real capital gains share of total real returns has fluctuated between 15% and 30% over the period 1210-2013 and has been above 20% over the past two centuries if the real capital losses on government bonds are omitted from the estimates (see online Data Appendix for sources). Since capital gains/losses fluctuate substantially at frequencies of 50-100 years, the omission of capital gains from conventional income inequality measures will often give a misleading picture of levels and, particularly, movements in income and wealth inequality. Capital gains on non-reproducibles, furthermore, amplify the income inequality between employees as a large fraction of income of high-income earners in the finance sector derive from bonuses that are often linked to capital gains and earnings of real estate agents are often in a fixed constant proportion to property values.

Is S^W also a useful proxy for wealth inequality? If wealth is down to zero at the end of the lifespan, as the permanent income and the life-cycle hypotheses assume, and the population age-distribution is constant, the answer is approximately yes under the maintained hypothesis of certainty under these hypotheses because wealth mirrors permanent income over the lifespan. This result holds in dynastic models where consumption is spread over future generations as well as in economies with low social mobility and in which the inheritance is a large fraction of income. As shown by Piketty (2014) inheritance is a large fraction of income and tends towards approximately 20% in the long run. Examining the period 1270-2010, Clark (2014) finds a low social mobility in England over the period 1170-1800 with only a slight increase after 1800. The intergenerational correlation of underlying social status jumps from 0.83 before 1800 to 0.73 thereafter, suggesting that the stubbornly low social mobility that prevailed in the mediaeval England has almost persisted to the present day. The low social mobility implies a persistent wealth distribution and that the relative income position of an individual will be echoed in the individual's relative wealth position, particularly when capital gains on non-reproducible assets are allowed for in the estimates of factor shares.

Turning to the *W*-*Y* ratio, there are several reasons why the *W*-*Y* ratio is a good proxy for wealth and income inequality and why the *W*-*Y* ratio serves as a fundamental complement/substitute for other inequality measures; particularly wealth inequality. First, since inequality in wealth holdings is higher than that of income, the overall income inequality will increase in response to an increase in the *W*-*Y* ratio provided that the relative dispersions of *W* and *Y* are preserved (Krussel and Smith, 2015).

Second, the housing wealth of the wealthy in the post-WWII period has increased proportionally more than that of the underprivileged class because the wealthy live in the more sort-after areas and in wealthier cities where urban land supply elasticities are significantly lower than that of the poorer neighborhoods and cities (Bethan and Dorling, 2004; Gyourko *et al*., 2013). Building on a model with overlapping generations of heterogeneous altruistic households, Borri and Reichlin (2015) show that Baumol's cost disease in the building industry is partly responsible for the increasing housing share in wealth, wealth-to-income ratios and wealth inequality, especially since housing takes a sizable share of intergenerational bequests. The same argument applies to the agricultural based economies in which large landholders have the most fertile land, tend to implement land-improving new technologies at a faster rate, and have scale advantages over small farmers, implying that inequality is increasing in the W-Y ratio.

Third, the W-Y ratio impacts directly on income inequality through the share of income going to capital, S^W, following Piketty's (2014) first law of capitalist economics, *viz*, $S^W = r \times W/Y$. Provided that returns to fixed capital are not significantly affected by the W-Y ratio, it follows that the W-Y ratio is positively related to inequality as the share of income derived from assets is a steeply positive function of income as argued above (see also Piketty, 2015a). A potential problem associated with this reasoning, as often pointed out in the literature including Piketty (2015a, 2015b), is that S^W is not positively related to the W-Y ratio if an investment-induced decline in the marginal productivity of capital dominates the positive effects of the capital-induced increase in the W-Y ratio. This is the case if the elasticity of substitution between labor and capital is below one, which seems to be the consensus in the literature (Jones, 2003; Ray, 2015), but recently challenged by Karabarbounis and Neiman (2014) and Piketty (2015a, 2015b).

Importantly, however, this line of reasoning only applies to reproducible factors of production that are subject to diminishing returns and for which real capital gains are zero in the long run. Reproducibles, such as livestock and fixed capital, however, are only a relatively small fraction of W. Increases in the W-Y ratio induced by non-reproducible wealth, such as agricultural and urban land, come almost entirely from real capital gains since the supply of arable land is quite inelastic. Similarly, the returns of precious metals and foreign assets are independent of the quantity of these assets held by the public. Using the estimates in this paper, the value of fixed capital has, on average, only been 19% of total wealth over the period 1210-2013, and 17% in the period 1750-2013. Thus, in response to W-Y expansions that preserve the distribution of wealth on asset types, capital's income share will only increase if the elasticity of substitution between capital and labor is extremely low, suggesting that capital's income share is, on average, increasing in the W-Y ratio.[12]

Furthermore, the increasing globalization, starting during the early industrialization, has partly counterbalanced diminishing returns to fixed capital through foreign direct investment (especially outsourcing), overseas portfolio investments and immigration flows. The key point here is that investors channel their investment into assets with the highest returns adjusted for risk (correlation with investor's consumption, covariance with other assets, exchange rate risk, confiscation risk *etc*.). Finally, Jones (2003) presents a model in which the elasticity of substitution is small in the short run but approaches one in the long run because the long-run elasticity of substitution is governed by the distribution of ideas; the ease with which new ideas appropriate for a different input mix can be discovered.

Table 1 displays the correlation between S^W and the W-Y gap and the often used proxies for inequality (top 0.5% income, the Gini coefficient and top 10% wealth share). Capital gains on non-reproducibles are not included in S^W because they are not accounted for in the considered inequality measures and, therefore, should not be included in capital's share. The sample period covers the maximum period at which almost continuous data for top 0.5%, the income Gini, and top 10% wealth are available and the numbers in parentheses are *t*-ratios. Considering first the S^W column, the correlation coefficients are statistically and economically highly significant, suggesting large commonalities between S^W and conventional inequality measures. These results are consistent with the findings of a strong positive relationship between personal and functional income distributions for other countries (see Giovannoni, 2010, for the OECD countries in the Post-WWII period; Jacobson and Occhino, 2012, for the US; Adler and Schmid, 2013, for Germany; and Bengtsson and Waldenström, 2017, for OECD countries over the last century). The correlation between the W-Y ratio

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[12] For the CES production function, $Y = \left[\alpha K^{(\sigma-1)/\sigma} + (1-\alpha)L^{(\sigma-1)/\sigma}\right]^{(\sigma-1)/\sigma}$, the rate of returns on fixed capital is $r^K = \alpha(K/Y)^{-1/\sigma}$, where σ is the elasticity of substitution. This implies that capital's income share is given by $S^K = r^K * K/Y = \alpha(K/Y)^{(\sigma-1)/\sigma}$, where S^K is the fixed capital's income share. The share of income going to capital, S^W, is given by $S^W = S^K + S^T = r^K * K/Y + r^T * T/Y = \alpha\left(\varphi\frac{W}{Y}\right)^{\frac{\sigma-1}{\sigma}} + r^T * (1-\varphi)\, W/Y$, where φ is the share of fixed capital in total wealth; S^K is the share of income going to fixed non-residential capital; and T is the value of land (land and fixed capital are assumed here to be the only assets). Thus, the change in the share of income going to wealth holders in response to a change in wealth that keeps φ constant, is given by: $\frac{\partial S^W}{\partial (W/Y)} = \alpha\varphi\frac{\sigma-1}{\sigma}\left(\varphi\frac{W}{Y}\right)^{-\frac{1}{\sigma}} + r^T * (1-\varphi)$. Using the approximate average values for the UK over the period 1210-2013, $\alpha = 0.3$, $\varphi = 0.2$, and $W/Y = 10$ (so $K/Y = 2$), and setting $\sigma = 0.5$, then S^W is a positive function of the W-Y ratio if $\frac{\partial S^W}{\partial (W/Y)} = -0.015 + r^T * 0.8 > 0$ or $r^T > 0.0187$, where 0.0187 is approximately a third of the returns to land estimated here. Here it is assumed that the returns to land are fixed and independent of investment in fixed capital stock; however, it is likely that the returns to land are an increasing function of investment in land improvement and structures. For the given parameter values this result holds for any value of the elasticity of substitution $\sigma > 0.006$ for W-Y = 5.

and S^W is also highly significant (note the long data period), which is an important result because S^W is used as an alternative proxy for wealth inequality in the regressions below.

Table 1. Correlation coefficients between inequality measures

Dep Var.	S^W	***W/Y***	**Period**
Top 0.5% Income	0.94(17.7)***	0.60(4.10)***	1918-2013
Income Gini	0.55(-5.09)***	0.36(-1.97)**	1948-2013
W/Y	0.57(15.6)***	1.00	1221-2013
Top 10% Wealth	0.53(4.76)***	0.64(7.09)***	1933-2003

Notes. The numbers in parentheses are *t*-statistics and are based on heteroscedastic and serial correlation consistent standard errors. The data are measured in 10-year overlapping averages following most of the estimates below. See the online Data Appendix for data sources. The wealth inequality data are interpolated between the following years: 1931-1936 (linearly interpolated), 1936-1938 (linearly interpolated), 1938-1950 (interpolated using top 1% income shares); and 1962-1964 (linearly interpolated). *** = significant at the 1% level; ** = significant at the 5% level.

Turning to the *W-Y* column in Table 1, the top 0.5% income share and the income Gini are positive and statistically significantly correlated with the *W-Y* ratio; however, not nearly as significant as their correlation with S^W. A statistically much more significant relationship is found between the *W-Y* ratio and the top 10% wealth share and this is likely to have been much higher had the wealth inequality data not recently been flat although there are strong reasons to believe that wealth inequality has been increasing in the UK since 2000 (Alvaredo *et al*., 2016). These results are highly indicative of the *W-Y* ratio being a strong proxy for wealth inequality; however, less so for income inequality – as intended in the regression analysis below.

Furthermore, the highly significantly positive relationship between *W-Y* and S^W implies that the potential counter-response of *r* to a change in the *W-Y* ratio is not sufficiently large to counteract the change in the *W-Y* ratio, which may own to one or more of the following: 1) that the elasticity of substitution between labor and fixed capital exceeds one (unlikely); 2) that changes in the *W-Y* ratio have been dominated by real capital gains on non-reproducibles and less so by changes in fixed capital for which diminishing returns applies; 3) that diminishing returns to fixed capital have been countered by a highly elastic supply of labor in a globalized economy or, more likely, that, at least well into the 19th century, there has been a highly elastic supply of rural labor readily available to operate the machinery in the urban areas; 4) that overseas direct investment has been a large fraction of total fixed investment and diminishing returns have been overcome by the sheer size of the foreign markets; and/or 5) that the positive relationship between S^W and the *W-Y* ratio is partly driven by feedback effects from S^W to the *W-Y* ratio through saving.

Overall, S^W/S^{WCG} and the *W-Y* ratio are likely to serve as good proxies for inequality because they 1) are significantly correlated with conventional inequality measures; 2) are available on an annual basis over long periods; 3) allow for real capital gains on non-reproducibles; 4) capture the fact that high-income earners derive a significant part, if not the majority, of their income from wealth or income that originates from profits or capital gains; and 5) *W-Y* affects inequality through Piketty's first law. Furthermore, the *W-Y* ratio includes assets that are rarely accounted for at the individual level, such as gold and silver, and perhaps most importantly, since S^W and the *W-Y* ratio are used as dependent variables, their measurement errors are unlikely to bias the parameter estimates of the key variables because their measurement errors are unlikely to be correlated with the regressors and the regression error term.

Finally, the model of Kesten (1973) gives an explicit mapping between r, s^W, and g and the *W-Y* ratio (see, also Acemoglu and Robinson's, 2015, online Appendix). The law of motion of individual i's assets, W_i, is given by:

$$W_{i,t+1} = (s^W + r + \varepsilon_{it})W_{it} + Z_{it},$$

where s^W is the marginal propensity to save out of wealth; Z is stochastic labor income; and $r + \varepsilon_{it}$ is the stochastic rate of asset return (see, for a complete derivation, the appendix of Acemoglu and Robinson, 2015, online Appendix). Individual i is assumed to be infinity lived.

Dividing this model by Y_{it} and normalizing the *W-Y* ratio to one, w, yields:

$$w_{i,t+1} = \varrho w_{it} + z_{it},$$

where $\varrho = \frac{r + s^W + \varepsilon_{it}}{1+g}$. Provided that $E\{\varrho\} < 1$, this model predicts that the distribution of W converges to a stationary distribution with a Pareto tail:

$$\mathcal{B}(\mathrm{W}) = 1 - \Pr\left[W_{i,t+1} \leq W\right] = \Gamma^{-\nu},$$

where ν is the Pareto exponent, $\nu \geq 1$, and inequality *decreases* with ν. If the entire distribution is Pareto, then the top k percentile's wealth share is $(k/100)^{(1-\nu)/\nu}$. For example, if $\nu = 2$ it follows that the fraction of wealth belonging to the top 1% is 10%.

Assuming that ϱ is log normally distributed with a mean of $[\ln(1-\bar{z}) - \sigma^2/2]$ and a variance of σ^2, Acemoglu and Robinson (2015, online Appendix) show that:[13]

$$\nu \cong 1 - \frac{\ln(r - g + s^W)}{\sigma^2/2},$$

[13] A similar relationship can be derived even if γ is not log normally distributed (see, for an exposition, Acemoglu and Robinson, 2015, online Appendix).

from which it follows that ν is decreasing in $r - g + s^W$ and increasing in σ^2. Wealth inequality is a positive function of ($r - g + s^W$) as wealth capitalizes at a faster rate and is less likely to be overtaken by growth. This analysis suggests that $(W/Y)_t$ is governed by r, g, s^W, $\sigma^{W/Y}$ and $(W/Y)_{t-1}$ – a model specification that guides the empirics presented below.

3.7 Reliability of the data

Although returns data offer a marked improvement over previous attempts to measure returns on a balanced portfolio, the quality of the data is likely to deteriorate as we go back in time, particularly because some of the data on returns are mostly available on decennial frequencies before the late 1600s and, importantly, their geographical coverage is not of today's standard.[14] Countering this problem is the fact that the economy back in time was much simpler and easier to measure than now because consumption back then consisted of a fewer and simpler goods in which the quality hardly changed over time in contrast to the past century or two in which it takes several years before the ever increasing range and quality of new products are incorporated into price indices and quality improvements are extremely difficult to measure; particularly for investment goods (see, e.g., Gordon, 1990). The measurement problems associated with investment deflators since the start of industrialization has biased the measurement of fixed investment and hence the value of capital. Similarly, housing wealth is substantially harder to measure today than in the past because the variety of houses and the value of urban land has increased substantially more than agricultural land over the past century. Before circa 1870, when agricultural land was the dominant asset in total wealth, the massive problems associated with price measurement of the 20th and 21st centuries, would have been much less of a concern. The quality of the data across assets and time is discussed in depth in the online Data Appendix.

3.8 Summary statistics

Table 2 provides summary statistics of the most important variables. Most noticeable is the high standard deviations of r and g, reflecting the significant shocks of the past such as the Black Death and occasional prolonged crop failures, that often resulted in reduced growth and accelerating prices. The

[14] Very few, if any, composite measures of returns have been constructed and most researchers, including Piketty (2014) and Acemoglu and Robinson (2015), rely on real bond returns or returns to fixed capital. Real bond returns are well-known to be well below the returns to stocks and most other non-fixed income assets and to be sensitive to unexpected inflation while the returns to land, precious metal, livestock and fixed capital, at least in theory, are hedged against inflation. Thus, while the returns estimated here contain measure errors, they offer big improvements over previous asset returns proxies; particularly those of Piketty (2014, Ch. 10), where asset returns over the past 2000 years are based on a few observations for bond rates.

high, usually unexpected, price volatility translated into large swings in capital-gains adjusted income shares, S^{WCG}, that occasionally go well beyond the limits of the income-only factor shares, $0 < S^W < 1$.

Table 2. Summary statistics.

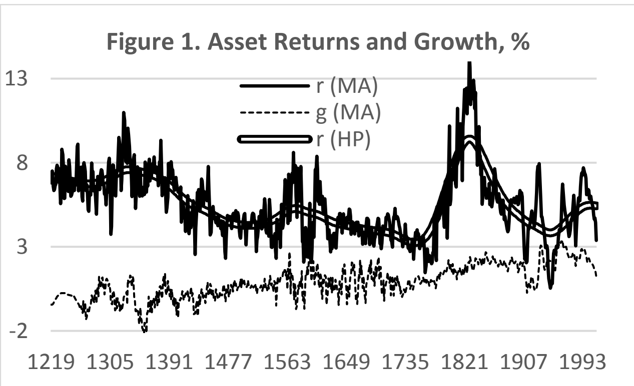
	Mean	SD		Min	Max		Mean	SD	Min	Max
S^W	0.545	0.074		0.304	0.699	r	0.052	0.104	-0.326	0.575
S^{WCG}	0.588	0.402		-0.885	4.054	g	0.007	0.057	-0.292	0.239
W/Y	5.437	1.152		3.307	9.465	s^W	0.024	0.024	-0.063	0.173

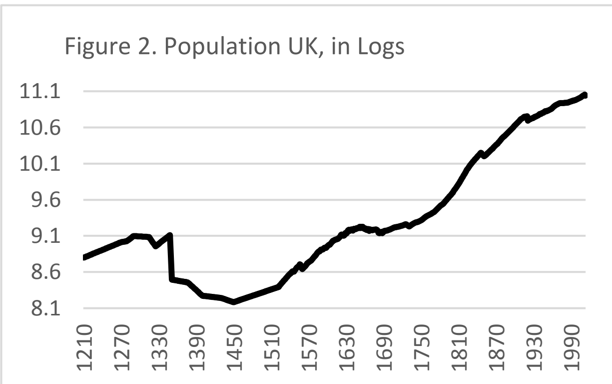
Notes. SD = standard deviation. The data are measured annually and in decimal points. Data period is 1211-2013 (N = 803).

3.8 Graphical analysis

3.8.1 Evolution in asset returns

Figure 1 shows the trends in r and g. Three trends in r can be identified in the data: 1) An ultra-long-run slow moving trend; 2) three long-term waves typically at 100-150 years' length; and 3) fluctuations below 100 years of durations influenced by business-cycles, wars, crop failures and asset market bubbles. The trend in real asset returns is surprisingly constant in the ultra-long run, fluctuating around the sample average of 5.2%. Regressing r on a time-trend, yields a coefficient of -0.001(-0.44), where the figure in parenthesis is the t-value and N = 804. The absence of a significant ultra-long-run trend in r may come as a surprise as most historical evidence points toward prohibitively high interest rates in the medieval Europe (see, e.g., Marshall, 1890, pp. 680-681; Homer and Sylla, 2005); however, as discussed above, medieval interest rates can be highly misleading proxies for asset returns since non-interest debt instruments, at least in England from the late 12th century were predominantly used (Clark, 1988).





Notes. The trend-line in Figure 1 is computed from the HP-filter with a smoothing factor of 100,000. The other lines in Figure 1 are computed as 25-years centered moving averages of the actual data. Here g is the annual growth in total real GDP and r is the annual real after-tax returns to assets.

Three and a half long-term waves in *r* can be identified during the approximate periods 1225-1349, 1560-1660, 1780-1870, and a half cycle starting in 1960. The first wave during the 13^{th} and the first half on the 14^{th} centuries was not unique to medieval Britain but was widespread across West Europe and it appears that a high time-preference was the underlying cause (see, e.g., Marshall, 1890, pp. 680-681; Clark, 1988; Homer and Sylla, 2005) or simply that usury laws and moral values discouraged lending and, consequently, pushed interest rates up by excess demand for loans. The high asset returns were reduced abruptly by the reduced supply of agricultural labor and reduced demand for agricultural products following the Black Death, and *r* didn't start to increase again until the population started to pick up two centuries later (for population growth, see Figure 2). Returns, however, the marked reduction in population in the first years of the Black Death only resulted in a modest reduction in *r* because many farmers likely switched from labor intensive arable cultivation to less labor intensive pasture (Clark, 2001).[15]

The second wave in *r* during the approximate period 1560-1660 was a result of high returns to the key assets such as fixed capital and land (agricultural and urban). The increasing the returns to agricultural land was likely a result of the three-fold increase in real food prices over the period 1570-1709 and significant technological progress in agriculture (Madsen *et al*. 2010). Until 1580 there were few innovations in British agriculture in the extensive (more land is brought under cultivation) as well as the intensive margins (higher yield per hectare). Increasing crop rotation, technological innovations, better draining methods brought about by innovations, and the increasing spread of agricultural technical manuals all contributed to the increasing agricultural productivity (Madsen *et al*. 2010). Furthermore, the period 1500-1650 marks the scientific and educational revolutions that increased the knowledge stock per worker and led to increasing returns to fixed capital, as predicted by the Schumpeterian theory of asset returns (see, Madsen, 2016; Madsen *et al*., 2017).

A reversal of the high asset returns occurred over the next century, 1650-1750, as a result of institutional improvements and tariff rate reductions. In their seminal paper, North and Weingast, (1989) argue that the Glorious Revolution of 1688, as probably the most important milestone in British institutional history, resulted in lower interest rates on public debt which also ensured a positive externality for the parallel development of a market for private debt and underpinned the institutional

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[15] Land cultivation was not only highly labor intensive during the harvest but also during the periods of plowing, weeding, fertilizing the soil etc. In other words, provided labor and crop land were complements, while labor and pastoral land were substitutes, the Plague-induced labor shortage gave landowners an incentive to substitute animal production for corn production. This scenario is consistent with the time-profile of real wages: farmworkers experienced a two-fold increase in real wages over the period 1347-1350 and a 33% reduction over the period 1350-1352 (see online Data Appendix for sources). Using an alternative dataset to the one used here, Clark (2001) finds that real wages of agricultural workers did not increase at the onset of the Black Death and immediately thereafter.

structure for pooling savings and for intermediation between borrowers and lenders. The reduction in macro tariff rates from approximately 50% to 20% (see Figure 7 below) was also influential for the reduction in r through reduced prices on imports of corn.

The third, and the most pronounced wave in r occurred in the first half of the 19th century and was in particular a result of extraordinarily high returns to agricultural land, housing, and fixed capital. The fundamental forces behind this increase were 1) large productivity advances in agriculture that increased the returns to land; 2) the marked technological progress in manufacturing that increased the returns to fixed capital since the capital deepening was a slow process often delayed by difficulties in getting access to credit; 3) forbidden unions made it difficult for workers to gain a stake in the profits; and 4) the imposition of the Corn Laws over the period 1815-1846 resulted in increasing real prices of agricultural products that already started up from a high level.

The Corn Laws imposed import barriers and steep tariff rates on imports of corn were not designed to safeguard farmers from distress but rather to preserve the extraordinarily high profits that prevailed during the Napoleonic war-years (Fletcher, 1961). The real returns to land, were on average, no less than 17.5% over the period 1815-1846 during which the Corn Laws were in place. The introduction of the Corn Laws is a good example of how influential institutions can be for inequality, as advocated by Acemoglu and Robinson (2015) and Piketty (2014, 2015a, 2015b). Almost coinciding with the high land returns, stock returns were extraordinarily high during the period 1801-1874 which was probably related to the high population growth rates that kept real wages flat despite solid productivity advances that reduced real wages below workers' marginal productivity and enhanced profits.

It is unclear the extent to which the increasing income growth, g, fueled by high population and economy-wide productivity growth rates, exerted upward pressure on asset returns during the first half of the 19th century as predicted by some models of asset returns since it is not possible to empirically verify a positive relationship between r and g for UK (see online Appendix). It cannot be ruled out, however, that the strong population pressure in the urban areas increased the marginal productivity of capital or at least prevented it from declining in response to the capital deepening during this period.

The Trade Union Act of 1871, which legislated trade unions, and the Great Agricultural Depression over the period 1873-1896 marked the end of the high asset returns enjoyed during most of the 19th century. WWI signals a further blow to the high asset returns experienced during the 19th century and asset returns decline to an average low of 3.1% during the period 1915-1982; a return that even underperforms the average low of 3.4% that prevailed during the period 1350-1560. Falling real prices of agricultural produce, starting in the early 1920s, and the often negative corporate profits during the Great Depression forced asset returns down during the 1930s.

The three inflationary episodes, 1915-1920, 1940-1952 and 1973-1982 markedly reduced the real returns to bonds because the inflations were probably unexpected (or bond-holders did not fully understand the implications of inflation on bond returns) and, at the same time, government bonds increased their weight substantially in the overall portfolio. The share of government debt in total wealth increased from 7% in 1913 to approximately 40% during WWI and fluctuated around this level up until the late 1950s, and first reached the pre-WWI level in 1977. Since the increasing inflation was only fractionally passed on to higher nominal interest rates and the top-income tax rates increased to historical heights during WWII and remained high up to the early 1980s, the resulting negative post-tax real bond returns were major contributors to the low returns during this period (Roine *et al*., 2009; Piketty, 2014).

The increasing union power following the Trade Union Act of 1871, which in terms of unionization and strike activity, came to a halt in 1982, marks a turning point for asset returns. Reduced union power, disinflation, real house price inflation and reduced top income taxes have all contributed to the upswing in asset returns since 1982 (Roine *et al*., 2009; Piketty, 2014).

3.8.2 The income growth path

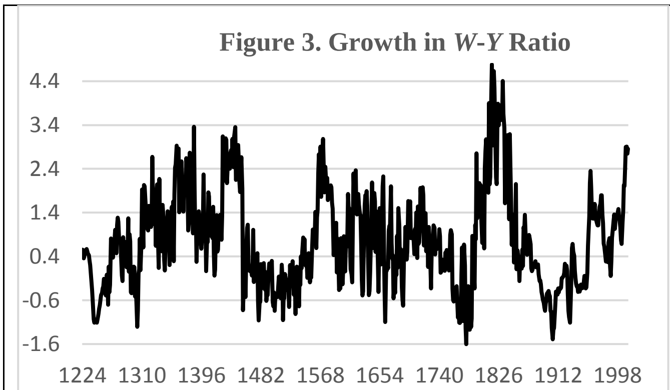
As seen from Figure 1, the trend growth rate, *g*, was close to zero from 1210 until the mid-16th century because population growth rates, apart for the marked negative population shock 1348-1351 (Figure 1), were close to zero and slow productivity growth rates driven predominantly by modest advances in technology and education (Madsen, 2016b). The growth rates picked up substantially during the later 16th century. The spread of the movable printer, which reduced book prices substantially during the 16th and the 17th centuries, was pivotal for the educational expansion in the UK and the dissemination of knowledge which, coupled with the scientific revolution in the 17th century and increasing innovative activity, lead to productivity advances not experienced earlier in recorded British history (Galor, 2005; Madsen *et al*., 2010; Madsen, 2016b).

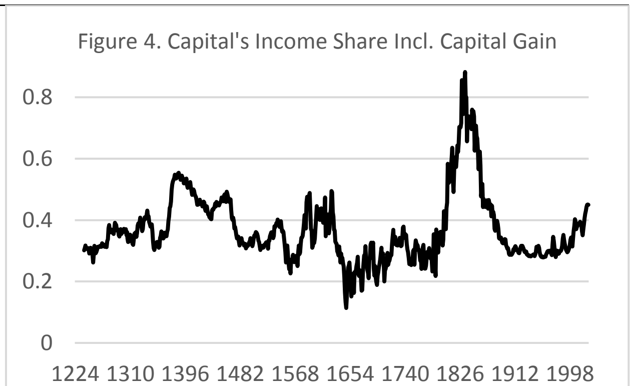
The high population growth rates during the 16th and the 17th centuries additionally increased *g* directly; however, the total effect on *g* was reduced by the population growth drag introduced by land as a semi-fixed factor of production (Madsen *et al*., 2010; Madsen, 2016b). Assuming that the output elasticity of land was 0.5 and capital's income share a third, the net effect of population growth on *g* was only 0.25%.

The productivity advances during the first phase of the British Industrial Revolution were associated with increasing population pressure (Figure 2) because the quantity effect remained strong relative to the quality effect in the fertility decision (Galor, 2005; Madsen *et al*., 2017). The fertility transition first occurred later during the British Industrial Revolution as parents substituted quality for quantity in response to increasing returns to education (Galor, 2005) and increased opportunity costs of having children triggered by a reduced gender wage gap (Madsen *et al*., 2017). The fertility transition coupled with slow productivity advances over the period 1910-1928, reduced g substantially. Subsequently, the educational and scientific advances over the period 1929-2007 resulted in strong productivity advances that kept g high despite low population growth rates (Madsen *et al*., 2010).

3.8.3 W-Y ratio and capital's income share

The growth in the W-Y ratio and capital's income share, S^{WCG}, are displayed in Figures 3 and 4. The growth in the W-Y ratio is considered more relevant than its level because the lagged value of the dependent variable in the W-Y regressions below is approximately 0.8, pointing towards highly persistent effects to shocks in the W-Y ratio. The periods 1370-1470, 1550-1650, and 1800-1870 are associated with grow in the W-Y ratio and the level of S^{WCG} above their ultra-long trends and tend to coincide with the waves in r and $-g$ and the (r-g)-gap. The increasing time-preference associated with the Black Death and the marked negative population growth rates (see Figure 2) are likely to have been influential for the 1370-1470 wave in the W-Y ratio and S^{WCG}. The marked technological progress during the period 1550-1650, as discussed above, increased returns to fixed capital and land. Although we would expect capital deepening to reduce returns to required returns relatively quickly in modern economies in which fixed reproducible capital is the dominant source of wealth, the relatively inelastic supply of land and credit to finance reproducible capital are likely to have prevented returns to wealth to be driven down by supply forces.





Notes. The data are 25-year centered moving averages. Capital's income share, S^{WCG}, is computed using Eq. (13)

The marked decline in S^{WCG} and the growth in the W-Y ratio, over the period from 1833 to 1904 are exceptional and deserves some discussion. Half of the decline is a gravitation towards the ultra-long-run trend and, as discussed above, capital's extraordinarily high share over the period 1800-1840 was predominantly a result of extraordinarily high land rents driven up by a lucrative market for agricultural produce during the Anglo-French wars 1794-1815 (including the Napoleonic war) and the high prices of agricultural produce kept artificially high by the Corn Laws. Land rents were deemed to fall as soon as the Corn Laws were repealed. Capital's income shares and, to some extent, the *W-Y* ratio were driven down by a sequence of events such as the repeal of the Corn Laws in 1846, the legalization of unions in 1871, the Great Agricultural Depression over the period 1873-1896 (see, e.g., Fletcher, 1961), the increase in union membership rates from 9% in 1905 to 25% in 1922 and the almost three-fold increase in consumer prices over the period 1913-1920 all contributed to the sharp reduction in capital's income share and the *W-Y* ratio.

Furthermore, real stock prices lost half of their value over the period 1813-1921, where the lion's share of the loss was concentrated over the high inflation period 1914-1918. The *increasing* inflation during the period 1914-1918 may have reduced real stock returns because of inflation illusion among shareholders. As discussed in more detail below, Modigliani and Cohn (1979) hypothesize that share-holders erroneously discount expected returns by the nominal interest rate and fail to factor into to their valuation that inflation, on a one-to-one basis, erodes the real value of firms' debt. Under the maintained hypothesis of inflation illusion, the *increasing* inflation from 0% in 1914 to 25% in 1918 may have been influential for the negative stock returns during the period.

Capital's share and the growth in the *W-Y* ratio, remained low up to the 1970s as a result of inflationary spells, increasing marginal tax rates and increased unionization, noting that the temporary increase in the *W-Y* ratio during the approximate period 1916-1960 was caused by the markedly increasing government debt incurred during the world wars. The weakening of unions, as signified by a fall in the union membership rate from its historical peak of 46% in 1979 to 22% in 2013, has eased the wage pressure and increased the profit rate in the corporate sector.

Consistent with most growth theories, Piketty's first two laws and Kaldor's (1957) stylized facts, S^{WCG}, S^W and the *W-Y* ratio have tended toward constants in the long run, noting that the growth in the *W-Y* ratio has tended towards zero in the long-run.[16] Piketty's first two laws yield the prediction that $S^W = rW/Y = rs^N/g$, where s^N is the net saving rate. Piketty (2014) and Piketty and Zucman (2014) argue that S^K and *W-Y* both gravitate towards steady states determined by r, s^N and g and that

[16] Dickey-Fuller tests suggest that $S^{W,}$ S^{WCG} and the *W-Y* ratio are mean-reverting. The Dickey-Fuller test with constant terms included yield the values of -3.88 (S^W), -30.5 (S^{WCG}), -7.11(*W-Y*), indicating that the null hypothesis of unit roots is strongly rejected at conventional levels in all cases, where annual data are used over the period 1211-2013. The critical value for the Dickey-Fuller test with a constant included is -3.43 at the 1% level when the number of observations exceeds 500.

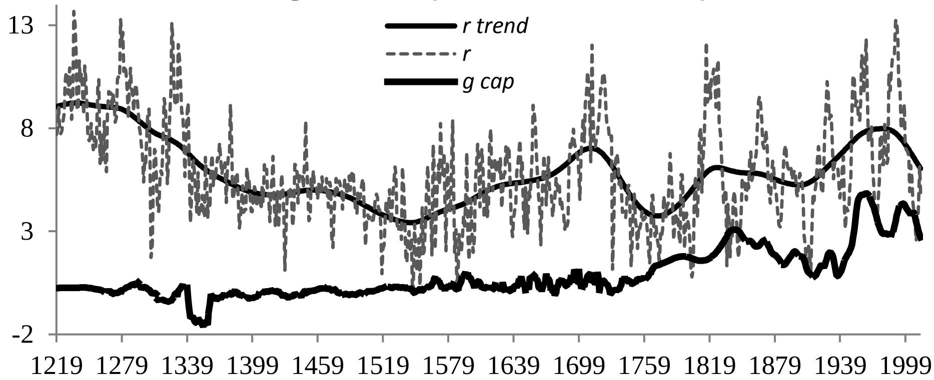
deviations from these benchmark long-run are only temporary. Groth and Madsen (2016) show that S^W may be driven away from its equilibrium by adverse supply shocks that lead to dampened internal oscillations and hump-shaped impulse-response functions around the steady state, because of a high degree of real-wage rigidity combined with a low elasticity of factor substitution. The wage shocks immediately after WWI and WWII and in the 1970s, for example, pushed real wages above their long-run equilibrium and, consequently, increased labor's income share. These shocks had two immediate effects; substitution towards more capital-intensive production methods and reductions in Tobin's *q*, which in turn led to reduced capital investment. The resulting excess unemployment reduced wage growth and capital's income share gradually recovered.

3.9 Dynamic efficiency

Several economists have challenged Piketty's (2014) claim that $r > g$ leads to an 'endless inegalitarian spiral' from the perspective that: 1) an increasing wealth concentration, which is driven by an increasing *K*-*Y* ratio, where *K* is non-residential fixed capital, will eventually drive *r* below *g*; and 2) the condition $r > g$, modified to apply to fixed capital, is a natural steady-state condition as long as the saving rate does not exceed a level at which the capital stock is pushed beyond the Golden Rule level (see, e.g., Abel *et al*., 1989). Dynamic efficiency requires that $r^K > g^K$, where r^K is the real returns to fixed capital and g^K is the growth in the volume of fixed capital. If $r^K < g^K$ we are in the problematic situation of dynamic inefficiency in the sense that capital is over-accumulated and all generations can be better off by reducing their saving and that firms can increase stockholder value by paying earnings out rather than retaining and reinvesting them.

As long as the economy is dynamically efficient, then wealth accumulation leading to an increasing *W*-*Y* ratio is compatible with Piketty's hypothesis that the *W*-*Y* ratio is an increasing function of the (*r*-*g*)-gap. If, on the other hand, the economy is dynamic inefficient we are in a situation where further wealth accumulation may reduce the share of income going to capital. Furthermore, an increase in the *W*-*Y* ratio, which is not driven by fixed capital accumulation, will not drive a dynamic efficient economy into dynamic inefficiency since dynamic efficiency applies to the *K*-*Y* ratio and not the *W*-*Y* ratio and, therefore, requires $r^K > g^K$ and *not* $r > g$. For the UK, *K* is approximately 20% of *W* and this number has been relatively constant over time and that r^K has historically often been quite different from *r*. Thus, the *W*-*Y* ratio can be increasing in the (*r*-*g*)-gap without the economy necessarily being overcapitalized with reproducible capital. A feature of fixed capital is that if r^K exceeds its steady state level determined by the Euler equation, it follows that capital accumulation will automatically reduce r^K because of diminishing returns to capital. Returns to bonds, agricultural land and housing, by contrast, can remain below *g* without the economy staying overcapitalized.

Figure 5. Dynamic Efficiency



Notes. Returns, r^K, are 25-year centered moving averages of pre-tax returns on non-residential fixed capital and g^K is a 25-year centered moving average of the growth in real fixed capital including agricultural buildings and fixed capital. The r^K-trend double-lined line is computed from the HP-filter with a smoothing factor of 100,000.

The historical evidence suggests that the British economy has been dynamically efficient. The time-profiles of r^K and g^K are displayed in Figure 5. Focusing on the long-run trends in r^K and g^K, the British economy has, by a large margin, been dynamically efficient at least since 1210. The ($r^K - g^K$)-gap has narrowed slightly after the onset of the First British Industrial Revolution in 1760 and has, on average, been 3.6% over the period 1760-2013 and 4.0% over the period 1900-2013. The narrowing of the ($r^K - g^K$)-gap after 1760 has been a result of an increase in g^K and not a reduction in r^K, where the historically high growth in non-residential fixed capital stock has not resulted in a decline in r^K because g^K has been predominantly driven by a high population growth and technological progress and these factors have out weighted the r^K–reducing effects of a potentially increasing *K*-*Y* ratio through a higher saving rate.

The finding of dynamic efficiency is consistent with the results of Abel *et al*. (1989), who show that the English economy was dynamically efficient every year, between 1960 and 1984, by a large margin. Furthermore, the ($r^K - g^K$)-gap has not been significantly negatively related to the *W*-*Y* ratio because *K* is a low fraction of *W* and because investors allocate their portfolio where the returns are highest subject to the riskiness of the assets (the *t*-value is -0.45 from regressing (*W*/*Y*) on [$r^K - g^K$]). Between 1210 and the end of the 15th century the *W*-*Y* ratio was well above its long-run trend and the ($r^K - g^K$)-gap was above and not below its ultra-long trend, as would have been the case if a high and increasing *W*-*Y* is incompatible with dynamic efficiency (compare Figures 3 and 6). Focusing on the post-1760 period, the *W*-*Y* ratio peaked in the first half of the 19th century without having any noticeable impact on the ($r^K - g^K$)-gap. Thus, there is not necessarily a contradiction between Piketty's (*r*-*g*)-hypothesis and dynamic efficiency.

4. Regression analysis

4.1 Model specification

Two baseline regressions are used to test Piketty's (*r*-*g*)-hypothesis and, more broadly, the extent to which wealth and income inequalities are determined by *r, g,* the saving propensity and the persistence of inequality to shocks. The models are stochastically specified as:

$$ln(W/Y)_t = \varrho_0 + \varrho_1 ln(W/Y)_{t-1} + \varrho_2 r_t + \varrho_3 g_t + \varrho_4 s_t^W + \vartheta X_t + \varepsilon_{1,t}, \tag{14}$$

$$S_t^{WCG} = \zeta_0 + \zeta_1 S_{t-1}^{WCG} + \zeta_2 r_t + \zeta_3 g_t + X_t \xi' + \varepsilon_{2,t}, \tag{15}$$

where *W* is total private nominal wealth; *Y* is nominal net national income (GNP minus depreciation and plus net foreign income); S^{WCG} is the share of income going to wealth including capital gains on non-reproducible capital (S^K is used in some regressions); *r* is the real post-tax return to wealth; *g* is the growth in real net national income; s^W is the gross propensity to save out of wealth (private nominal saving divided by private nominal wealth); *X* is a vector of control variables; and ε is a stochastic error term. The coefficients are expected to have the signs as follows: *r* positive; *g* negative; and s^W positive. The models are estimated over the period 1222-2013 in 10-year intervals to smooth out erratic movements in the data to allow for some of the dynamic adjustment towards steady state and to reduce business-cycle influences on the estimates.

Lagged dependent variables are included in the models 1) to capture the effects of omitted variables that are serial correlated; 2) to allow for slow adjustment towards long-run equilibrium; 3) to capture mean-reverting dynamics and persistent effects in the dependent variable that may be endogenous to inequality; 4) because it follows the predictions of the Kesten (1973) model of inequality as presented in the end of Section 3.5; 5) to reduce the bias of the coefficients of the focus variables, which is introduced by confounding variables that push S^W and the *W*-*Y* ratio out of its steady-state trajectory, such as crises, epidemics, social unrest, war, *etc.*; 6) because it is not clear from the literature whether *r* and *g* have temporary or permanent effects on the growth in the *W*-*Y* ratio and S^W (permanent growth effects of *r* and *g* follow from the limited case where ϱ_1 and τ_1 are one); and 7) bequests creates wealth persistence because most wealth stems from inheritance (Piketty, 2011, and 2014, p. 268). Piketty (2011) shows, for a sufficiently large (*r* – *g*)-gap, that in steady state the bequest-income ratio is approximately equal to (*W*/*Y*)/*Gen*, where *Gen* is the average length of a generation (the average age at which parents have children). Since *Gen* is relatively constant, it follows that bequests create persistence in the *W*-*Y* ratio.

The saving propensity out of wealth, s^W, is included in Eq. (14) because it is a crucial determinant of wealth accumulation (see, e.g., Piketty, 2014; Piketty and Zucman, 2014) and in the representative-agent framework, each family needs to reinvest the fraction g/r of its capital income to keep the W-Y ratio constant. Furthermore, s^W is a key determinant of wealth inequality in the models of Kesten (1973) and Jones (2015). Another reason for including s^W in the regressions is that the r and g terms may capture the effects of shocks to saving that impact simultaneously on r, g and wealth inequality (Krussel and Smith, 2014). Krussel and Smith (2014), for example, argue that increasing length of unemployment will lead to reduced wealth inequality because poor consumers self-insure against unemployment by accumulating additional wealth. This effect will captured by the inclusion of s^W in the regression; however, in contrast to the models discussed here the effects of s^W on inequality will be negative.

4.1 Identification

In the IV regressions, r, and g are treated as endogenous, because of potential feedback effects from the dependent variables, confounding factors and errors-in-variable biases. Real food price inflation, consumer price inflation, and tariffs are used as instruments for r and g. From the onset it is important to stress that although instruments are reasonably exogenous, the exclusion restrictions may not be fully satisfied for all instruments as discussed further below.[17]

High real food price inflation is likely to be associated with high r and low g through a combination of high earnings and low output growth in agriculture, particularly before the pre-industrial growth regime in which the wealth associated with agricultural production (agricultural land, buildings and livestock) was the dominant source of national wealth (Madsen, 2016a). The industrialization and the expansion of the modern state gradually changed this picture and industrial and residential capital evolved to become the largest sources of private wealth in Britain (Madsen, 2016a). However, real food price inflation has recently regained some of its influence on total returns, along with a markedly increasing share of housing in total private wealth. Housing wealth is linked to real food prices through the price of land. The higher are real prices of food, the higher are the earnings in the agricultural sector, which in turn will tend to be capitalized in agricultural land prices. Farmland

[17] Educational attainment and life expectancy at birth were also considered as instruments in an early draft of the paper following the predictions of the model of Becker and Mulligan (1997) in which the time preference is shown to be a negative function of life expectancy and educational attainment. Educational attainment (from Madsen, 2016b) was, however, insignificant in all the first-stage model specifications covering the period 1270-2013 and the coefficients of life expectancy covering the period 1510-2013 (also from Madsen, 2016b) is negative and has a t-value of only about 3, depending on model specification, estimation period *etc*. Education and life expectancy were consequently omitted from the first-stage regressions.

prices will influence urban land prices through ripple effects starting from urban land expansion at the fringes of cities (Madsen, 2009). Food prices are likely to be exogenous because they have, to a large extent, been determined by weather conditions, crop disease, population pressure, tariffs and world food prices (O'Rourke and Williamson, 2002); forces that are quite independent of the *W-Y* ratio. Furthermore, the exclusion restriction is likely to be satisfied in that food price shocks have macroeconomic effects on the *W-Y* ratio through population growth (starvation and poor health) and asset returns (land rent).

Inflation is influential for real returns when inflation is unanticipated as it lowers the real value of contractual nominal payments, such as housing rent, agricultural land rent, and interests in government bonds. Furthermore, Modigliani and Cohn (1979) argue that inflation reduces real returns to equity because 1) growing inflation, *ceteris paribus*, reduces accounting earnings as it increases nominal interest payments on debt, and investors fail to account for the fact that inflation erodes the real value of the firm's debt; and 2) investors tend to discount future earnings by nominal interest rates. Furthermore, the level and the change in inflation can be taxing for owners of fixed-interest rate government debt; particularly during the approximate period 1940-1980 when marginal income tax rates were well above their long-run trend (Roine *et al*., 2009). As shown in Figure 6 the major inflationary periods, 1549-1575, 1799-1813, 1915-1920, 1940-1952, and 1968-1982, have been associated with low or negative real after-tax returns (compare Figures 1 and 6). Conversely, the two major deflationary periods, 1318-1339 and 1814-1823, were associated with high real returns.

Figure 6. Inflation, %

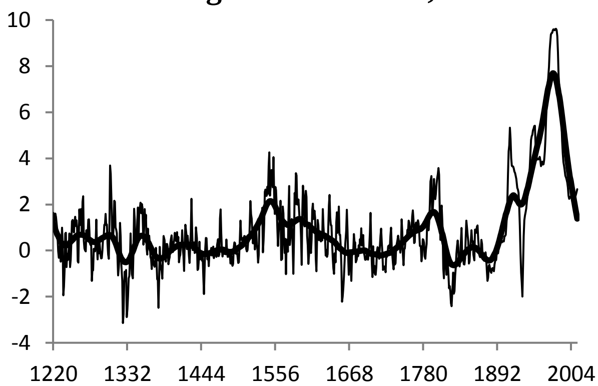
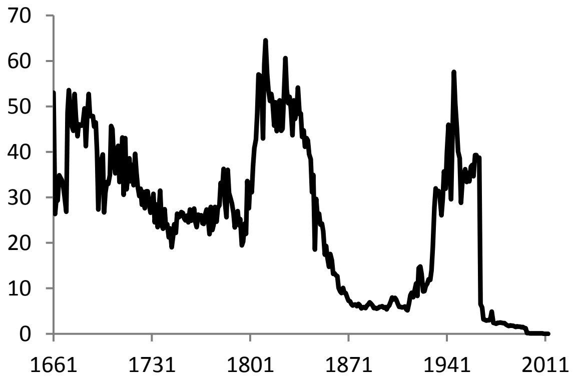


Figure 7. Tariff Rate



Notes. The trend inflation is estimated from the Hodrick-Prescott filter with a smoothing factor of 10,000. The thin line in Figure 6 is a 19-year centered moving average of the rate of inflation.

Macro tariff rates, calculated as the ratio of import duties and imports, are used as instrument for (*r-g*) because they have played a major role in the history of the political economy of rent seeking and, at the same time, they reduce income through various channels. It is, for example, well-known that tariffs on agricultural products have often been used to protect the vested interests of the landed class. Probably the best example of successful rent seeking through tariffs and trade barriers in British history is the marked increase in tariff rates following the introduction of the Corn Laws in 1815, as discussed above. The effects of the Corn Laws on tariff rates, first repealed in 1846, are highly visible in Figure 7, where tariff rates are measured as custom revenue divided by nominal imports. The macro tariff rates were a whopping 50% during this period, noting that macro tariff rates underestimate the effective tariff rates because of substitution away from products most affected by the tariffs. Similarly, capitalizing on the widening trade deficit after the wars between France and the UK in the period 1689-1713, protectionists engineered the imposition of high tariffs on a range of imports from France (Nye, 2007).

These examples show that tariff rates may not necessarily only be increased in response to low returns to land, but that they are a function of a complex political process related to increasing (Corn Laws), decreasing (tariffs during the Great Depression and WWII), or unaltered (entrance to the Common Market in 1973) wealth positions of the landed class. Thus, tariff rates are likely to be independent of rent. Furthermore, the exclusion restriction is likely to be satisfied since tariffs impact on the *W-Y* ratio through land rent because the tariff-induced changes to returns result in increasing wealth accumulation through saving and capital gains.

4.2 W-Y regression results

The results of regressing Eq. (14) are shown in Table 3, where the IV estimator is used in the regression in columns (4)-(6) and the OLS estimator in the other regressions in the table. Consider first the regressions in columns (1)-(3). In the regression in column (1) in which s^W is excluded, r and g are both highly significant and have their expected signs. In the estimates in column (2) the coefficients of r and g remain significant and of the expected signs when the propensity to save out of wealth, s^W, is added to the model, and s^W is significantly positive, as predicted by a couple of the models presented in the online Appendix (the models of Kesten, 1973, and Jones, 2015, and, implicitly in the models of Mankiw, 2015, and Solow). The statistical significance coefficients remain high and the magnitudes of the coefficients are close to those of the baseline regression in column (2) when non-overlapping data are used in the regressions (column (3)). Common for all the regressions in columns (1)-(3) (and subsequent regressions) the coefficients of the lagged dependent variable are all highly significant and approximately 0.7, indicating shifts in g and r have persistent; but not permanent growth effects on the *W-Y* ratio and, therefore, wealth inequality.

Table 3. Parameter estimates of Eq. (14).

	1	2	3	4	5	6	7	8	9	10
Dep. Var.	$ln(W/Y)_t$	$ln(W/Y)_t$	$ln(W/Y)_t$	$ln(W/Y)_t$	$ln(W/Y)_t$	$ln(W/Y)_t$	$ln(W/Y)_t$	$ln(W/Y)_t$	$ln(W/Y)_t$	$ln(W/Y)_t$
	OLS	OLS	OLS/Ex Overlap	IV	IV	IV	OLS/Post-1800	OLS/Pre-1600	OLS/Post-1600	Growth decomp.
$ln(W/Y)_{t-1}$	0.78*** (29.7)	0.89*** (24.2)	0.84*** (9.09)	0.75*** (27.9)	0.73*** (18.7)		0.37*** (3.95)	0.98*** (22.6)	0.58*** (9.77)	0.86*** (24.1)
r_t	1.85*** (7.01)	2.47*** (8.08)	2.69*** (3.15)	0.92*** (2.52)	0.79** (2.07)	4.41*** (5.44)	2.54*** (7.36)	3.23*** (6.06)	2.34*** (6.67)	2.38*** (7.94)
g_t	-5.81*** (9.30)	-7.35*** (10.3)	-7.07*** (4.26)	-3.37*** (4.25)	-2.61*** (2.23)	-12.1*** (8.79)	-4.32*** (5.29)	-7.88*** (7.11)	-9.23*** (12.8)	
g_t^A										-8.63*** (11.3)
g_t^n										-4.27*** (6.02)
s_t^W		2.53*** (4.64)	1.64 (1.18)		-0.56 (0.79)		0.65 (0.75)	12.8*** (4.51)	-1.14* (1.89)	2.34*** (4.31)
R^2	0.65	0.66	0.62	NA	NA	NA	0.37	0.58	0.72	0.67
# Obs	792	792	79	792	792	792	214	378	414	792
Est. Period	1222-2013	1222-2013	1222-2013	1222-2013	1222-2013	1222-2013	1800-2013	1222-1600	1600-2013	1222-2013

Notes. The numbers in parentheses are absolute *t*-statistics based on standard errors that are robust to serial correlation and heteroscedasticity. g^A = per capita real income growth; g^n = population growth. Constant terms are included in the regressions but not reported. The data are measured in 10-year overlapping intervals except the regression in column (3) in which the data are measured in 10-year non-overlapping intervals. OLS/Ex Overlap, which is in ten-year non-overlapping observations. The following instruments are used in the IV regression in column (4): Inflation, real food price inflation, and macro tariff rates. *: Significant at 10%; **: Significant at 5%; and ***: Significant at 1%.

Instruments are used in the regressions in columns (4)-(6). The *F*-tests for excluded instruments all exceed the value of 158 for the endogenous variables, suggesting that the potential bias of the coefficients of the regressors is well below 1%. Turning to the second-stage regressions in Table 3, the coefficients of *g* and *r* are statistically highly significant and have absolute log-run elasticities that are lower than those of the OLS regressions which may, as far as *r* is concerned, be associated with a negative feedback effect from the *W*-*Y* ratio to *r* for reproducibles.

The model is regressed over the periods 1600-2013, 1800-2013, and 1222-1600 in columns (7)-(9). The coefficients of *r* and *g* are consistently highly significant and are remarkably stable in magnitude over time, suggesting that the nexus between the *W*-*Y* ratio and *r* and *g* is independent of the mode of production and that the relationship between these variables is robust. The coefficient of s^W, however, is only significant before 1600, indicating saving has not been an important of wealth accumulation in the post-1600 period. The insignificance of s^W in the post-1600 period is also consistent with the overall results in Table 3 that s^W in not a robust determinant of the *W*-*Y* ratio. An alternative interpretation of this result is that s^W was an important determinant of the *W*-*Y* ratio in the pre-industrial era because almost all wealth accumulation was through saving and not capital gain while the primary source of wealth accumulation the post-1600 has been capital gain.

Growth, *g*, is decomposed into population growth, g^n, and per capita income growth, g^A, in the last column in Table 3. The parameter estimates of *r* and s^W are close to those of the baseline regression in column (2). The coefficients of g^n and g^A are both highly significantly; however, the coefficient of g^A is twice as high as that of g^n, which is inconsistent with theories predicting that g^n and g^A have the same effects on the W-Y ratio and inequality. However, if g^n dilutes wealth concentration through inheritance it will affect the W-Y ratio with a time-lag. If g^n is lagged 10 years, the coefficients of g^n and g^A become even more significant than those in in the regression in the last column in Table 3 and the coefficients of approximately the same magnitude, suggesting that g^n and g^A are equally influential for inequality in the long run (the regression is not shown).

The coefficients of r and g are economically highly significant in all the regressions in Table 3. Referring to the baseline regression in column (2) a permanent one percentage point increase in r is associated with an approximately 2.5% increase in the W-Y ratio in the short run and 12.5% in the long run. Consequently, the buoyant movements in r over history have been influential for the evolution in the W-Y ratio. The absolute effect of an increase in g is twice as big as that of an increase in r, implying that the (r-2g)-gap, as opposed to the (r-g)-gap, is a more precise determinant of the W-Y ratio and hence wealth inequality. Assuming that r and g are independent the 2 percentage point increase in g from the stationary economy in the pre-1600 period to the post-1820 period, has contributed to an approximately 50% increase in the W-Y ratio. Thus, overall r and g have been influential for the W-Y-path over the past eight centuries.

Are the results in Table 3 consistent with the predictions of economic models? Most economic models pose a positive and often complex non-linear relationship between the W-Y ratio and r, g and s^W, where the coefficients of r, g and s^W depend on parameters that are model specific. The only restriction imposed in some of the models summarized in the online Appendix is that the coefficients of s^W, r and -g are the same, restrictions that are easily rejected even at the 1% level, in the regressions in Table 3. In absolute terms, the coefficient of g is too large relative to that of r, which, in turn, is too large relative to that of s^W. These results may be an outcome of measurement errors, omitted variables and that the models are not sufficiently general. Furthermore, the stronger impact of g than r on the W-Y ratio in absolute terms is consistent with the Solow model that the K-Y ratio is determined by g but not r and that the coefficient of g is higher than one because depreciation is omitted from the g-term.

4.3 Capital's share regressions

The results of regressing Eq. (15), where capital's income share is the dependent variable, are shown in Table 4. The coefficients of r and g are both highly significant and have the expected signs in the baseline regressions in the first two columns. These results remain intact in the non-overlapping 10-year interval regression in column (3). Comparing the S^W and the S^{WCG} regression results in columns (1) and (2) there are distinct differences, however. First, the coefficient of the lagged dependent variable is 0.84 in the S^W regression, suggesting that shocks to r and g have highly persistent effects on factor shares. Comparatively, the lagged coefficient of S^{WCG} is driven down to a low at 0.41 by low persistence in capital gains.

Second, the coefficients of r and g are statistically significantly higher in the S^{WCG} regression than in the S^{W} regression; however, the statistical significance in the S^{W} regression is likely driven down by the highly significant coefficient of the lagged variable. It is well-known that lags of highly persistent dependent variables, such as S^{W}, remove a lot of the statistical significance of the other regressors even if they are relevant and their statistical significance increases substantially when the lag of S^{K} is omitted (results are not shown). Third, the absolute and the relative values of the coefficients of r and g are much closer in magnitude in the S^{WCG} regressions than the S^{K} regressions. Although the restriction $\zeta_2 = -\zeta_3$ is rejected at conventional significance levels, it has to be noted that this rejection is partly driven by the high statistical significance of r and g (see the Wald tests of the restriction $\zeta_2 = -\zeta_3$ in Table 4). Furthermore, the restriction $\zeta_2 = -\zeta_3$ cannot be rejected below the 8% level when the lagged dependent variable is omitted from the regression (last column).

Table 4. Parameter estimates of Eq. (15).

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
Dep. Var.	S_t^{W}	S_t^{WCG}	S_t^{WCG}	S_t^{W}	S_t^{WCG}	S_t^{WCG}	S_t^{WCG}	S_t^{WCG}
	OLS	OLS	OLS/Ex OL	IV	IV	OLS	OLS	OLS
S_{t-1}^{W}/ S_{t-1}^{WCG}	0.84(19.1)***	0.41(9.60)***	0.36(3.79)***	0.52(10.3)***	0.20(6.28)***	0.13(3.04)***	0.35(7.28)***	0.37(6.59)***
r_t	0.42(5.04)***	3.76(21.4)***	4.22(10.2)***	1.48(8.59)***	3.85(16.3)***	2.66(19.8)***	4.30(15.8)***	4.27(11.8)***
g_t	-1.41(3.15)***	-2.97(8.12)***	-3.26(3.44)***	-6.80(8.90)***	-8.66(15.2)***	-0.74(2.05)**	-2.53(5.68)***	-3.23(4.76)***
$\chi^2(1)$	5.16***	4.33**	1.09	57.6***	63.6***	25.5***	19.8***	3.24*
R^2	0.88	0.70	0.72	NA	NA	0.64	0.70	0.68
# Obs	792	792	79	792	792	379	414	314
Est. Period	1222-2013	1222-2013	1222-2013	1222-2013	1222-2013	1222-1600	1600-2013	1800-2013

Notes. See notes to Table 3. S^{W} = capital's income share (including land rent) excluding capital gains on non-reproducibles; S^{WCG} = capital's income share including capital gain on renewables. Constant terms are included in the regressions but not shown. Returns are measured in 10-year moving averages and the growth in Y is measured in 10-year differences. The variables are in decimal points and growth, g, and returns, r, are annualized. OLS/Ex OL = ten-year *non*-overlapping observations. $\chi^2(1)$ = Wald test of the restriction $\zeta_2 = -\zeta_3$ (coefficients of r and g), distributed as $\chi^2(1)$ under the null hypothesis that the restriction is satisfied.

The coefficients of r and g remain statistically and economically highly significant in the IV regressions in columns (4) and (5), indicating that the significance of r and g in the OLS regressions is not driven by feedback effects from S^{W} or S^{WCG} to r and g. The long-run coefficients of r and g are, in absolute terms, significantly higher in the IV than the OLS regressions, suggesting the absence of positive feedback effects from the W-Y ratio to r and g in the OLS estimates. For r this means that factor shares that are driven by changes in the W-Y ratio are not countered by potentially adverse responses in r as often stressed in critique of Piketty's second law.

The results of regressing the S^{WCG} model over the periods1222-1600, 1600-2013, and 1800-2013 are shown in columns (6)-(8) in Table 4. The regressions covering the period 1600-2013 (column

(7)) give results that are close to the regression results covering the entire period (column (2)) in terms of statistical significance and magnitude of the long-run coefficients. Conversely, the coefficients of the lagged dependent variable are substantially lower in pre-1600 regressions, than that of the post-1600 regression. This may reflect the large swings in factor income shares that are driven by large fluctuations in output prices and quantities combined with sticky nominal wages, as characteristic for the predominantly agricultural society in the pre-1600 period; thus automatically reducing the persistence in factor shares (these swings are not visible in Figure 3 because the fluctuations are smoothed out).[18] The coefficients of r and g remain statistically highly significant in the pre-1600 period; however, the magnitude of their long-run coefficients are well below their post-1600 counterparts, predominantly due to the low persistence in factor shares.

Common for the regressions is that the economic significance of the coefficients of r and g are generally high. Referring to the baseline regression in column (2) in Table 4, a one percentage point increase in r is associated with a 3.8% increase in capital's share, S^{WCG}, in the short run and 6.4% in the long run. Considering the 5 percentage point reduction in the trend in r over the period 1830-1950, to take an extreme example, resulted in a reduction in capital's share of almost 32%, suggesting that the declining r was a massive force behind the reduced inequality over this period. A one percentage point increase in g is associated with a 3.0% increase in capital's share, S^{WCG}, in the short run and 5.0% in the long run. The shift in the growth trend from approximately 0.7% before 1800 to 2.1% thereafter, resulted in a 7% long-run reduction in S^{WCG}, suggesting that the historically larger shifts in r have been a greater source of inequality fluctuations than g. A similar conclusion is reached when the effects of a one standard deviation change in r and g is considered. A one standard deviation increase in r (SD^r = 10.40) results in a 67.0% (10.4*3.8/(1-0.41)) increase in capital's share. The corresponding number is -29.3 (5.76*3.0/(1-0.41)) for a one standard deviation increase in g (SD^g = 5.76).

In terms of theory consistence, the negative coefficient of g is inconsistent with the post-Keynesian models (see, e.g., Kaldor, 1955; Pasinetti, 1962) in which an increasing demand, g, is associated with increasing mark-ups and, therefore, an increasing share of income going to capital. The results are only consistent with standard growth models to the extent that g dilutes capital per effective units of labor. To the extent that the restriction $\zeta_2 = -\zeta_3$ cannot be rejected, as is the case for the model in column (3) in Table 4, the results are consistent with the Ramsey model of Mankiw (2015) in which the ratio of each capitalist's consumption over that of each worker, c^k/c^w, as an approximation of S^W, is proportional to the (r-g)-gap. The finding that the coefficient of r is significantly smaller than

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[18] Regressing consumer price inflation on its lagged value using annual data over the period 1222-1600, yields a coefficient of lagged inflation of 0.24(5.03), where the value in parenthesis is the t-statistic. For wage inflation the corresponding estimate is 0.64(17.0), indicating a much higher degree of wage than price stickiness.

that of g in absolute values suggests that the restriction $\zeta_2 = -\zeta_3$ in Piketty's analysis is too stringent. As shown by De Nardi *et al*. (2015) the restriction, $\zeta_2 = -\zeta_3$, may not be met for various reasons depending on the setup of the model describing inequality. De Nardi *et al*. (2015), for example, suggest that $\zeta_2 < -\zeta_3$ because a lower rate of population growth is associated with a higher ratio of deaths to births and, consequently, a higher average bequest size. Since bequests are luxury goods, population growth will have a more significant impact on wealth concentration at the right tail of the wealth distribution than will an increase in r.

5 Extensions and robustness checks

The gain more insight into the drivers and sources of wealth and income inequality the models in this section are modified in the following capacities: 1) W limited to the value of fixed non-residential capital measured at market values or at acquisition costs; 2) the model specification follows the predictions of the Solow model; and 3) returns, r, are decomposed into dividend yields and real capital gains. Furthermore, the models are extended with variables that are likely to influence factor shares and the W-Y ratio and, at the same time, might be correlated with the explanatory variables. This extension is to ensure that the results thus far have not been driven by the exclusion of relevant determinants of the outcome variables. As will become apparent, the regressions below give important insight into inequality dynamics and the fundamental drivers of inequality.

5.1 Non-residential fixed capital

Wealth has thus far been measured as the sum of reproducible and non-reproducible capital; however, economic growth theories and some of the debate surrounding Piketty's laws apply to non-residential reproducible capital. To cater for this, the W-Y ratio is first measured as non-residential fixed capital at market values, $W^{K.M}$, divided by net national income in the regression in the first column in Table 5. The economic and statistical significance of the coefficients of g and r are substantially lower than that of the baseline regressions, reinforcing that r and g are much more important drivers of the W-Y ratio than the $W^{K.M}$-Y ratio. This result suggests that the strength of Piketty's (r-g)-hypothesis hinges on the inclusion of non-reproducibles in wealth and, therefore, needs to be discussed in this context.

The regression in column (2) in Table 5 is closer in spirit of the Solow model than the models considered thus far. Here, I regress the steady state condition of the Solow model, $\ln(K/Y) = \ln(W^{K.A}/Y) = \ln[s^Y/(g + \delta)]$, augmented with r^K, where δ is the depreciation rate for non-residential fixed capital; $W^{K.A}$ is measured as the value of fixed non-residential capital at acquisition costs; r^K is the real returns to stocks following Eq. (3); and s^Y is the propensity to save out of income where saving is measured as private gross saving. The coefficients of s^Y and r^K are positive and significant at the 1% level, while the coefficient of $(g + \delta)$ is insignificant. The restriction that the long run coefficient of $\ln[s^Y/(g + \delta)]$ equals one, as predicted by the Solow model, is rejected at conventional significance levels [$\chi(2)$ = 123].

The regression results of the Solow model remain unaffected when r^K is excluded from the regression (the results are not shown). The significantly positive coefficient of r^K suggests that an increasing $W^{K.A}$-Y ratio is *not* associated with lower returns to non-residential fixed capital because of a negative feedback effect from $W^{K.A}$-Y to r^K as is often argued in the literature (see, e.g. Piketty, 2015a; Ray, 2015). This result suggest that foreign investment and effective foreign and domestic labor supply may have countered diminishing returns to fixed reproducible capital or that the elasticity of substitution between labor and capital exceeds one as found by Karabarbounis and Neiman (2014). In fact, the historical evidence suggests that migration flows and cross-border capital movements have been high in Britain for centuries (Madsen, 2016a). Furthermore, the positive coefficient of r^K indicates that the positive relationship between r and the W-Y ratio is neither driven by non-reproducibles nor by short-term real capital gains.

Table 5. Robustness checks and extensions

	1	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
Dep. Var.	$ln(W^{FC.M}/Y)_t$	$ln(W^{FC.A}/Y)_t$	$ln(W/Y)_t$	$ln(W/Y)_t$	$ln(W/Y)_t$	S_t^{WCG}	S_t^{WCG}	S_t^{WCG}	S_t^{W}
	OLS. Fixed capital	OLS Fixed capital	OLS Div Y. /Cap. G.	OLS All controls	OLS Sign. controls	OLS Div Y. /Cap. G.	OLS Incl. all controls	OLS Incl. sig. controls	OLS Incl. sig. controls
(Dep Var)$_{t-1}$	0.90*** (15.2)	0.87*** (28.2)	0.78*** (23.5)	0.94*** (25.2)	0.91*** (26.3)	0.45*** (11.0)	0.17*** (5.13)	0.17*** (4.65)	0.56*** (14.1)
r_t				2.60*** (7.48)	2.43*** (8.40)		3.44*** (23.9)	3.40*** (23.0)	0.29*** (4.41)
g_t	-0.02 (1.62)		-7.87*** (12.5)	-8.95*** (10.8)	-8.82*** (9.92)	-3.62*** (9.75)	-0.75*** (2.65)	-0.91*** (3.53)	-0.47** (2.20)
s_t^W	1.47*** (4.44)		0.78* (1.67)	0.96 (1.27)					
r_t^K	0.59*** (4.02)	0.66*** (2.64)							
$\ln(g_t + \delta_t)$		0.02 (1.27)							
$\ln s_t^Y$		0.19*** (6.21)					0.41*** (5.48)	0.35*** (5.28)	0.35*** (5.28)
r_t^{CG}			4.76*** (11.6)			5.40*** (22.4)			
r_t^{DY}			1.44*** (3.28)			1.78*** (6.89)			
$\ln(W/Y)_t^{GD.}$				-0.06*** (2.93)	-0.04** (2.07)		0.03* (1.89)	0.03* (1.89)	
(M2-H0)/M2)$_t$				0.22*** (2.36)	0.25*** (5.09)		-0.06 (1.25)		-0.12*** (8.41)
Exec$_t$				0.01 (0.94)			0.01 (1.27)		
$(IM/Y^N)_t$				-0.26 (0.86)			-1.03*** (5.95)	-1.09*** (13.5)	
Tax$_t$				0.12 (1.09)			-0.14*** (2.62)	-0.15*** (3.66)	-0.14*** (5.83)
σ^2				0.03 (1.37)			0.04*** (6.62)	0.04*** (4.51)	0.07*** (4.57)

R^2	0.86	0.65	0.73	0.67	0.65	0.75	0.82	0.82	0.94
# Obs	792	792	792	792	792	792	792	792	792
Est. Period	1222-2013	1222-2013	1222-2013	1222-2013	1222-2013	1222-2013	1222-2013	1222-2013	1222-2013

Notes. See notes to Table 3. Dividend yield, r^{DY}, is weighted real returns on all wealth minus real capital gain on agricultural and urban land, r^{CG}; $W^{FC.M}$ = value of fixed capital stock at market prices; $W^{FC.A}$ = value of fixed capital stock at acquisition costs; *Exec* = constraints on executive; (IM/Y^N) = nominal imports divided by nominal GNP; $(W/Y)^{GD}$ is the government debt-income ratio; s^Y is the propensity to save out of income; r^K is the after-tax real returns to fixed capital; ($M2$-$H0$)/$M2$) is contract intensive money where $H0$ is coins and notes and $M2$ is broad money; *Tax* = top-income tax; and σ^2 = the standard deviation of ln(W/Y) (Column (4)) or S^{WCF} (Column (6)), measured as the annual variation in the data within each 10-year interval. Column 1: The dependent variable is the ratio of non-residential fixed capital in market values and nominal GNI. Column 2: The dependent variable is the ratio of real non-residential fixed capital in acquisition costs. Column 3: r is decomposed into dividend yield, r^{DY}, and real capital gain on non-reproducibles, r^{CG}. Column 4: All controls are included in the regression. Column 5: r is decomposed into dividend yield, r^{DY}, and real capital gain on non-reproducibles, r^{CG}. Column 6: Control variables are included in the regression. Column 7: Only the significant control variables (at 10%) from the regression in column (6) is included in the regression.

5.2 Decomposing r into capital gains and dividend yields

To gain deeper insight into the principal sources of wealth accumulation, asset returns are decomposed into dividend yields, r^{DY}, and real capital gains on agricultural and urban land, r^{CG}, in the regressions in columns (3) and (6) in Table 5. Dividend yield is a real magnitude because implicit price deflators are included in the numerator (dividends) as well as the denominator (asset value) and, therefore, it needs not be transformed to a real value by deducting consumer price inflation. Real capital gains on fixed assets (stocks) are not included in r^{CG} because capital gains on stocks in steady state are driven by retained earnings and share buy-backs, otherwise Tobin's q would be permanently driven above its steady state equilibrium.

For both the *W*-*Y* regression and S^{WCG} regressions, the coefficients of r^{DY} and r^{CG} are both highly significant; however, the coefficient of r^{CG} is more than three times as large as that of r^{DY}, showing that capital gains are much more important sources of wealth accumulation than dividend yields. These results suggest that the marginal propensity to consume out of dividend yield exceed that of capital gains, probably because 1) credit constraints prevent landholders from borrowing against capital gains on land; 2) capital gains are not expected to be permanent and, therefore, not expected to increase permanent income; 3) landholders have imperfect knowledge of the real asset appreciation; and/or 4) habit persistence in consumption among landholders results in increasing saving in response to real capital gains on assets since dividend yields are a more stable source of income than capital gains. Consequently, the higher the share of land in total wealth, the larger is the potential for an increasing *W*-*Y* ratio and wealth inequality as experienced up to 1870s and since the 1980s. Conversely, the gradual erosion of land's share in total wealth as industrialization unfolded, contributed to a reduction in *W*-*Y* ratio during the approximate period 1875-1975. The declining share of non-reproducibles in total wealth from the 1870s to the 1970s was associated with declining wealth inequality, whereas the declining land wealth share has been reversed along with the expanding housing wealth since the early 1980s.

These results have implications for the measurement and the dynamics of wealth inequality. The finding that capital gains on non-reproducibles are much more important sources of variations in wealth inequality than dividend yields suggests that under modes of production in which non-reproducibles are a high share of total wealth, such as in the pre-industrial period (agricultural land) and more recently (urban property), that capital gains on land are much more influential for inequality than rental income in general. For measurement of wealth and income inequality, it implies that conventional measures of inequality insufficiently capture increases in inequality in periods of secular increases in land values; a conclusion also reached by Roine and Waldenström (2012). The aforementioned flat wealth inequality Gini coefficient since 2000 is a clear example of how standard wealth inequality measures fail to capture potentially large secular changes in inequality. Over the periods of sustained increasing inequality, such as the periods 1741-1871 and 1978-2013, real capital gains have been 40.0% higher than dividend yield, suggesting that the distortion in standard measures of inequality is even higher in periods of increasing inequality.

5.3 Inclusion of control variables

Government debt, institutional quality, import propensity, the variance of the dependent variable, top income taxes, and the propensity to save out of income are included as controls in the regressions in columns (4)-(5 and (7)-(9) because there are good reasons why they may have influenced inequality and weakened the significance of *r* and *g* in the regressions. Common for these control variables is that various authors have stressed their importance for the inequality path.

The government debt-income ratio, $(W/Y)^{GD}$, may affect inequality because after-tax real returns on government debt were often rendered low or negative by unexpected inflation during wars. Although tax-payers may increase their saving in response to higher government debt to counter their future tax liabilities, this effect is captured by the saving terms s^W or s^Y in the regressions. The variance of the *W*-*Y* ratio, σ^2, is positively related to inequality following the predictions of the models of Kesten (1973) and Jones (2015) and standard portfolio models in which the required returns are positive functions of the riskiness of the asset. The propensity to import, Im/Y^N, where Y^N is nominal income is, before the second half of the 20^{th} century, likely to be negatively related to inequality because import of overseas agricultural products tended to reduce the prices on basic food and, conversely, reduce the income of the landed class.

Top income tax rates are included as control variables following Piketty’s (2014) argument that high income earners bargain harder for income shares when income tax rates are low because they have more to gain from incremental earnings at lower tax rates. Based on calibrations of their model Hubmer *et al*. (2016) show the marked decrease in tax progressivity since the late 1970s has by far been the most powerful force for increasing wealth inequality in the US, because a reduced tax progression spreads out the distribution of after-tax resources available for consumption and saving and increases the returns to saving; thus leading to higher wealth accumulation. Empirically, Roine *et al*. (2009) find that top tax rates significantly reduce income inequality. Tax rates are measured as the sum of income tax rates of the top 1% decile (statutory rates) and land tax rates because they jointly constitute taxation of high income earners. Land taxes were the dominant form of income taxes before 1843 when income taxes were permanently introduced (low income tax rates were temporarily in place in the period 1798-1816), and land tax rates were gradually reduced and phased out after 1843.

Institutions have often been stressed as essential determinants of inequality. As staunch advocates of the importance of institutions on economic development and inequality, Acemoglu and Robinson (2015) argue that changes in income shares are predominantly determined by institutions rather than *r* and *g*. The Corn Laws, the prohibition of unions before the Trade Union Act in 1871, and the rent reducing consequences of Glorious Revolution of 1688, as argued by North and Weingast (1989), are examples of institutional influences on asset returns.

Constraints on the executive and contract intensive money are used as indicators of the quality of institutions. Following the Polity IV, constraints on executive, *Exec*, is constructed as institutional constraints on the decision-making powers of governments, and it is operationalized by a seven-point scale, where the minimum value of 1 implies "executive authority" and the maximum value of 7 signifies "executive parity or subordination". Changes in the index over the period from 1210 up to 1832, when it reached the maximum value and has stayed there since, mostly reflect the changing power relationship between the Crown and the Parliament. Significant institutional changes constitute the Magna Carta of 1215, when everybody, including the king, became subject to the law, the Civil War of 1642-1649, when Parliamentary forces defeated Charles I, and the Glorious Revolution of 1688-1689, which gave Parliament supremacy over the King (see, North and Weingast, 1989, for discussion of the history of institutions in Britain).

Introduced by Clague *et al*. (1999), contract-intensive money, as another dimension of institutions, is proxied by $(M2\text{-}H0)/M2$, where $M2$ is broad money supply and $H0$ is the monetary base. Clague *et al*. (1999) argue that the better a government enforces contracts the higher the level of contract-intensive money. In economies with excellent third-party contract enforcement, credit and monetary deposits will be the preferred store of money and medium of exchange over cash money, because they are safe, efficient, in most cases pay interest, and facilitate the tracking of credit history and, thereby better enable banks screening borrowers. Conversely, if contracts are not enforced by the government, 1) the safety of money in financial institutions is not guaranteed; 2) repayment of loans cannot be taken for granted; and 3) lenders do not have the any security rights to mortgage assets if a borrower defaults (Clague *et al*., 1999). In these cases, cash will be the preferred medium of exchange over credit. Contract-intensive money affects the wealth accumulation process by simultaneously impacting r and inequality through the same channels as constraints on executive, *Exec*.

Considering the *W*-*Y* regression in column (4), all the control variables are insignificant except for two of the covariates; a result that stands firm when all the insignificant covariates are deleted (column (5)). The government debt-income ratio, $(W/Y)^{GD}$, is significantly negative, which is consistent with the fact that returns on government debt have been driven down because of the inflationary implications of debt finance. The coefficient of contract intensive money is significantly positive, suggesting that an improvement of contractual arrangements is associated with increasing wealth inequality – a result that makes sense from the perspective that asset accumulation is eased by better credit facilities and finance arrangements. Finally, the coefficients of r and g remain highly significant; thus giving further support to the thesis that r, g and s^W are the principal drivers of wealth inequality proxied by the *W*-*Y* ratio.

Turning to the S^{WCG}-regressions in columns (7)-(8) the coefficients of the controls are statistically mostly highly significant determinants of S^{WCG} and S^W, where the insignificant controls in the regressions in column (7) are deleted in the regression in column (8). The coefficients of import propensity are negative and highly significant in the S^{WCG} regression, suggesting that openness benefits labor income shares because British imports have traditionally competed with agricultural production and, as discussed above, it has long been in the interest of the landed class to keep profits up through restrictions on imports. The coefficient of σ^2 is positive and significant in accordance with expectations.

The coefficients of top income taxes are negative in the S^{WCG} and S^W regressions; thus giving support to Piketty's (2014) postulate that high income earners have less incentive to bargain for higher income shares when taxes are high because of reduced expected gains from bargaining over income shares. The coefficients of s^Y are highly significantly positive, which is inconsistent with the predictions of the post-Keynesian models of Kaldor (1955) and Pasinetti (1962) in which increasing saving rates curb consumption and, consequently, reduce price-markups over marginal costs through reduced aggregate demand. However, the result is consistent with Piketty's (2014) prediction that a saving-induced increase in the *W*-*Y* ratio increases capital's share of income because there is no or very little counter-balancing reduction in r. Finally, contract-intensive money, $(M2\text{-}H0)/M2$ is significantly negative in the S^W regression, suggesting that improvements in contractional arrangements are associated with reduced income inequality as argued by Acemoglu and Robinson (2015).

Overall, the regressions in Table 5 show that the baseline results are robust to inclusion of variables that have been highlighted in the literature as being important for inequality. The coefficients of *r* and *g* remain highly significant and of the expected sign in all regressions. Moreover, the high *R*-squared in the regressions indicate that the models explain a large share of the variation in the *W-Y* ratio, S^{WCG} and S^{W}. This is particularly true in the S^{WCG} regressions in which *R*-squared reaches the value of 0.82, which, combined with a very low coefficient of the lagged dependent variable, suggesting that the model is very good at tracking capital-gains augmented capital shares despite the large variations in capital gains on land as well as stocks. Finally, *r* and *g* are the key-drivers in the S^{WCG} regressions. Comparing Tables 4 and 5, *R*-squared improves only slightly by the inclusion of the control variables and *R*-squared drops to 0.38 if *r* and *g* are excluded from the S^{WCG} regressions in Table 5 (the results are not shown), reinforcing the importance of *r* and *g* as central determinants of the evolution of inequality.

6. Concluding remarks

Piketty's hypothesis that inequality is increasing in the (*r*–*g*)-gap, is highly controversial and yet little, if any, empirical research has examined the validity of the law and the extent to which *r* and *g* are significant determinants of wealth and income inequality. Constructing annual data for Britain over the period 1210-2013, this paper is a first attempt to overcome this lacuna in the literature by examining the dynamics of the *W-Y*-ratio and capital's income share, S^{WCG} (where S^{WCG} is capital's income share including capital gains on non-reproducibles), and their relationship to *r* and *g*. It is shown that *r* and *g* have highly significant and quite persistent effects on wealth and income inequality as proxied by the *W-Y* ratio and S^{W}/S^{WCG}. The results of the paper give five principal insights into the dynamics of inequality, the influence of *r* and *g* on wealth and income inequality and the importance of distinguishing between reproducibles and non-reproducibles.

First, the regressions indicate that *r* and *g* have been the principal drivers of inequality proxied by the *W-Y* ratio and S^{WCG} over the past eight centuries. Although some controls, such as institutions, taxes, openness, and savings are also significant determinants of S^{WCG}, *r* and *g* explain the lion's share of the variance in S^{WCG} regression and, jointly with saving, all of the explained variance in the *W-Y* regressions. These results imply that an increasing wealth concentration, as signified by high *W-Y*-ratios, historically has *not* been associated with a lower *r* introduced by diminishing returns to wealth combined with feedback effects from the *W-Y* ratio on *r*. This result remained intact if wealth is limited to non-residential reproducible capital stock, which has been a relatively stable 20% fraction of total wealth, indicating that diminishing returns to capital have been countered by migration flows, foreign direct investment and expanding population, and though the Malthusian mechanism. Furthermore, the analysis shows that the W-Y ratio has periodically been trending upwards without reducing the gap between returns to reproducibles and growth in capital stock, and, therefore, compromising dynamic efficiency.

Second, returns to non-reproducible wealth impact distinctly differently on inequality than do returns on reproducibles in that non-reproducibles, unlike reproducibles, are not subject to diminishing returns. Furthermore, unlike reproducibles, the increasing demand for land combined with inelastic land supply has often resulted in waves of expanding real values of non-reproducibles. The W-Y and the S^{WCG} elasticiticities of real capital gains are found to be significantly higher than those of dividend yields, suggesting that wealth and income inequality dynamics depend heavily on the source of returns. Since capital gains on assets in steady state are approximately limited to non-reproducibles, it follows that wealth accumulation is faster in economies in which land is a large component of wealth, as in the pre-industrial period and today, where the expansion of the service sector in the city areas has resulted in urban property being the largest component of wealth.

Third, the estimates give qualified support for Piketty's (r-g)-hypothesis. While support is found for Piketty's thesis that r and g are the principal drivers of wealth and income inequality, the results suggest that 1) Piketty's law needs to be extended to allow for disparate impacts of $-g$ and r on inequality; 2) that capital gains on non-reproducibles are more influential for inequality than are dividend yields; and 3) that s^W is a robust determinant of wealth inequality. Although these results do not overturn Piketty's reasoning that an increasing (r-g)-gap over the rest of the 21st century will result in increasing income and wealth inequality, they suggest that Piketty's law needs to be extended along these dimensions. Related to this is the finding that the impact of r and g on the W-Y ratio is much stronger for total wealth than for non-residential fixed capital wealth. This suggests that a broader wealth concept than the one used in standard growth models, such as the wealth concept advocated by Piketty (2014) and Piketty and Zucman (2014), is needed to analyze inequality dynamics at the macro level.

Fourth, the (r-g)-gap and inequality, W/Y and S^{WCG}, have been fluctuating around a constant trend in the ultra-long run and independent of the mode of production and level of development; thus giving little support for Piketty's (2014) conjecture of a "hyperconcentration of wealth in traditional agrarian societies and to a large extent in all societies prior to World War I" (p. 249). However, the finding that capital gains on non-reproducibles are more influential for inequality than dividend yields does suggest that mode of production matters, in that industrialized economies are more equal than economies dominated by agriculture and services. Around the ultra-long run trend three large waves and an emerging wave in r and the (r-g)-gap are identified over the past eight centuries and the waves are outcomes of political struggles between capitalists, the landed class and workers, and major shocks such as wars, epidemics and so on.

Fifth, the regressions show that shocks to the W-Y ratio are relatively persistent but not permanent and the results consistently indicated that the trend in r has been in excess of g in the UK over the past eight centuries without permanently increasing inequality measured by the W-Y ratio or factor shares. What the results show is that r, g and, to some extent, saving propensities are consistently significant determinants of inequality and, therefore, that extensions of the Piketty model should be the centerpiece of the macroeconomics of inequality.

References

Abel, A. B., Mankiw, N. G., Summers, L. H. and Zeckhauser, R. J. (1989). Assessing Dynamic Efficiency: Theory and Evidence. *Review of Economic Studies*, 56(1), 1-19.

Acemoglu, D. and Robinson, J. (2015). The Rise and Decline of General Laws of Capitalism. *Journal of Economic Perspectives*, 29(1), 3-28.

Adler, M., and Schmid, K. D. (2013). Factor Shares and Income Inequality: Empirical Evidence from Germany 2002–2008. *Journal of Applied Social Science Studies*, 133(2), 121-132.

Alvaredo, F., Atkinson A. B. and Morelli, S. (2016). The Challenge of Measuring UK Wealth Inequality in the 2000s. *Fiscal Studies*, 37(1), 13-33.

Atkinson, A.B., Harrison, A. (1978). *The Distribution of Personal Wealth in Britain*. Cambridge, Cambridge University Press.

Baker, D., DeLong, J. B. and Krugman, P. (2005). Asset Returns and Economic Growth. *Brookings Papers on Economic Activity*, 1, 289-315.

Barro, R.J., Sala-I-Martin, X. (1992). Public Finance in Models of Economic Growth. *The Review of Economic Studies*, 59(4), 645-661

Becker, G. S. and Mulligan C. B. (1997). The Endogenous Determination of Time Preference. *Quarterly Journal of Economics*, 112(3), 729-758.

Bell, B. and Van Reenen, J. (2014). Bankers and their Bonuses. *The Economic Journal*, 124(574), F1-21.

Bengtsson, E. and Waldenström, D. (2017). Capital Shares and Income Inequality: Evidence for the Long Run. Working Paper 2017-1, Uppsala University.

Bethan, T. and Dorling, D. (2004). Know Your Place: Housing, Wealth and Inequality in Great Britain 1980–2003 and Beyond. Investigation Report, Shelter. http://england. shelter.org.uk. http://england.shelter.org.uk/__data/assets/pdf_fsile/0014/40091/Know_your_place.pdf

Blanchard, O. (2016). Three Remarks on the US Treasury Yield Curve. https://piie.com/blogs/realtime-economic-issues-watch/three-remarks-us-treasury-yield-curve.

Borri, N. and Reichlin, P. (2015). The Housing Cost Disease. CEPR Discussion Paper 10756.

Bosworth, B. P. (2014). Interest Rates and Economic Growth: Are they Related? Center for Retirement Research, Working Paper 2014-8, Boston College.

Broadberry, S., Campbell, B. M. S., Klein, A., Overton, M. and van Leeuwen, B., 2015, *British Economic Growth 1270-1870*, Cambridge University Press: Cambridge.

Caselli, F. and Feyrer, J. (2007). The Marginal Product of Capital. *The Quarterly Journal of Economics*, 122(2), 535-568.

Clark, G. (1988). The Cost of Capital and Medieval Agricultural Technique. *Explorations in Economic History*, 25, 265-294.

Clark, G. (2001). Microbes and Markets: Was the Black Death an Economic Revolution? Working Paper, http://www.econ.ucdavis.edu/faculty/gclark/research.html.

Clark, G. (2010). The Macroeconomic Aggregates for England. 1209-2008, *Research in Economic History*, 27, 51-140.

Clark, G. (2014). *The Son Also Rises: Surnames and the History of Social Mobility*. Princeton, NJ: Princeton University Press.

Clague, C., Keefer, P., Knack, S. and Olson, M. (1999). Contract-Intensive Money: Contract Enforcement, Property Rights, and Economic Performance. *Journal of Economic Growth*, 4, 185-211.

Davidson, R, and MacKinnon, J. G. (2006). The Case against JIVE. *Journal of Applied Econometrics*, 21(6), 827-833.

De Nardi, M., Fella, G. and Yang, F. (2015). Piketty's Book and Macro Models of Wealth Inequality. NBER Working Paper 21730.

Feinstein, C. H. (1988). National Statistics, 1760-1920: Sources and Methods of Estimation for Domestic Reproducible Fixed Assets, Stocks and Works in Progress, Overseas Assets, and Land. In Feinstein, C. H. and Pollard, S. (eds.), *Studies in Capital Formation in the United Kingdom, 1750-1920,* Oxford: Oxford University Press, 257-471.

Fletcher, T. W. (1961). The Great Depression of English Agriculture, 1873-1896. *Economic History Review*, 13, 417-432.

Galor, O. (2005). From Stagnation to Growth: Unified Growth Theory. In Aghion, P. and Durlauf, S. (eds.), *Handbook of Economic Growth*, Vol. 1, Amsterdam: Elsevier, 171-293.

Gennaioli, N., Shleifer, A. and Vishny, R. (2014). Finance and the Preservation of Wealth. *The Quarterly Journal of Economics*, 129 (3), 1221-1254

Giovannoni, O. (2010) Functional Distribution of Income, Inequality and the Incidence of Poverty: Stylized Facts and the Role of Macroeconomic Policy. UTIP Working Paper 58, LBJ School of Public Affairs, The University of Texas at Austin.

Goes, C. (2016). Testing Piketty's Hypothesis on the Drivers of Income Inequality: Evidence from Panel VAR Heteregenous Dynamics. IMF Working Papers 16/160, International Monetary Fund.

Gordon, R. J. (1990). *The Measurement of Durable Goods Prices*. Chicago: University of Chicago Press.

Groth, C. and Madsen, J. B. (2016). Medium-Term Fluctuations and the "Great Ratios" of Economic Growth. *Journal of Macroeconomics*, 49, 149-176.

Gyourko, J., Mayer, C. and Sinai, T. (2013). Superstar Cities. *American Economic Journal: Economic Policy*, 5(4), 167-99.

Hamilton, J., Harris, E., Hatzius, J. and West, K. (2016). The Equilibrium Real Funds Rate: Past, Present, and Future. *IMF Economic Review*, 64, 660-707.

Havranek, T., Horvath, R., Irsova, Z. and Rusnak, M. (2015). Cross-Country Heterogeneity in Intertemporal Substitution. *Journal of International Economics*, 96(1), 100-118.

Herlihi, D. (1997). *The Black Death and the Transformation of the West*. Cambridge, MA: Harvard University Press.

Homer, S. and Sylla, R. (2005). *A History of Interest Rates*, 4th Edition, Hoboken, NJ: John Wiley & Sons, Inc.

Hubmer, J., Krusell, P. and Smith, A. A. (2016). The Historical Evolution of Wealth Distribution: A Quantitative-Theoretic Explanation. NBER Working Papers 23111.

Humphries, J. And Weisdorf, J. (2016). Unreal Wages? A New Empirical Foundation for the Study of Living Standards and Economic Growth in England, 1260-1860,"CAGE Online Working Paper Series 310, Competitive Advantage in the Global Economy (CAGE).

Jacobson, M., and Occhino, F. (2012). Labor's Declining Share of Income and Rising Inequality. Economic Commentary 2012-13, Federal Reserve Bank of Cleveland.

Jones, C. I. (2003). Growth, Capital Shares, and a New Perspective on Production Functions. Paper presented at the Conference on Technology, Productivity and Public Policy, 7-8 November, Center for the Study of Innovation and Productivity, Federal Reserve Bank of San Francisco. Available at: http://www.frbsf.org/economic-research/files/jones_alpha100.pdf

Jones, C. I. (2015). Simple Models of Pareto Income and Wealth Inequality, Online Appendix to Jones, C.I. (2015), Pareto and Piketty: The Macroeconomics of Top Income and Wealth Inequality. *Journal of Economic Perspectives*, 29(1), 29-46, Available at: https://www.aeaweb.org/jep/app/2901/29010029_app.pdf.

Justman, M. and Gradstein, M. (1999). The Industrial Revolution, Political Transition, and the Subsequent Decline in Inequality in 19th-Century Britain. *Explorations in Economic History*, 36(2), 109-127.

Kaldor, N. (1955). Alternative Theories of Distribution. *Review of Economic Studies*. 23(2), 83-100.

Kaldor, N. (1957). A Model of Economic Growth. *The Economic Journal*, 67(268), 591-624.

Karabarbounis, L. and Neiman, B. (2014). The Global Decline of the Labor Share. *The Quarterly Journal of Economics*, 129(1), 61-103.

Kesten, H. (1973). Random Difference Equations and Renewal Theory for Product to Random Matrices. *Acta Mathematica*, 131, 207-248.

Krussel, P. and Smith, A. A. Jr. (2014). Is Piketty's "Second Law of Capitalism" Fundamental? Mimeo, Yale.

Krussel, P. and Smith, A. A. Jr. (2015). Is Piketty's "Second Law of Capitalism" Fundamental? *Journal of Political Economy*, 123(4), 725-48.

Lopez-Bernardo, J., Lopez-Martinez, F. and Stockhammer, E. (2017). A Post-Keynesian Response to Piketty's 'Fundamental Contradiction of Capitalism', *Review of Political Economy* 28(2), 190-204.

Madsen, J. B. (2009). Taxes and the Fundamental Value of Houses. *Regional Science and Urban Economics,* 39(3), 365-376.

Madsen, J. B. (2016a). Wealth and Inequality in Eight Centuries of British Capitalism. Department of Economics, Research Discussion Paper 23-16, Monash University.

Madsen, J. B. (2016b). Human Accomplishment and the British Industrial Revolution: The Role of Great Scientists and Education in British Growth since 1270. Department of Economics, Research Discussion Paper 1-16, Monash University.

Madsen, J. B., Ang, J. B., and Banerjee, R., (2010). Four Centuries of British Economic Growth: The Roles of Technology and Population. *Journal of Economic Growth*, 15(4), 263-290.

Madsen, J. B. and Davis,E. P. (2006). Equity Prices, Productivity Growth and the New Economy. *The Economic Journal*, 116(513), 791-811.

Madsen, J. B., Minniti, A. and Venturini, F. (2015). Assessing Piketty's Laws of Capitalism. Department of Economics, Research Discussion Paper 34/15, Monash University.

Madsen, J. B., Minniti, A. and Venturini, F. (2017). A Schumpeterian Perspective on Piketty's Laws of Capitalism. Mimeo, Monash University.

Mankiw, N. G. (2015). Yes, $r > g$. So What? *American Economic Review: Papers & Proceedings*, 105(5), 43-47.

Marshall, A. (1890). *Principles of Economics*, London: Macmillan and Co.

Modigliani, F. and Cohn, R. (1979). Inflation, Rational Valuation and the Market. *Financial Analysts Journal,* 35(2), 24-44**.**

North, D. C. & Weingast, B. R. (1989). Constitutions and Commitment: The Evolution of Institutions Governing Public Choice in Seventeenth-Century England. *The Journal of Economic History*, 49(04), 803-832.

Nye, J. V. C. (2007). *War, Wine, and Taxes: The Political Economy of Anglo-French Trade, 1689-1900*. Princeton, NJ: Princeton University Press.

O'Rourke, K. H. and Williamson, J. G. (2002). After Columbus: Explaining Europe's Overseas Trade Boom, 1500-1800. *Journal of Economic History*, 62(2), 417-4456.

Pasinetti, L. (1962). Rate of Profit and Income Distribution in Relation to the Rate of Economic Growth. *Review of Economic Studies*, XXIX (4), October, 267-279.

Piketty, T. (2011). On the Long-Run Evolution of Inheritance: France 1820—2050. *The Quarterly Journal of Economics*, 126(3), 1071-1131.

Piketty, T. (2014). *Capital in the Twenty-First Century*. Cambridge, MA: Belknap Press.

Piketty, T. (2015a). About Capital in the Twenty-First Century. *American Economic Review: Papers and Proceedings*, 105(5), 48-53.

Piketty, T. (2015b). Putting Distribution Back at the Center of Economics: Reflections on Capital in the Twenty-First Century. *Journal of Economic Perspectives*. 29(1), 67-88.

Piketty, T. (2016). Capital, Predistribution, and Redistribution. CrookedTimber.Org, 16 January, 2016. Available at: http://crookedtimber.org/2016/01/04/capital-predistribution-and-redistribution/.

Piketty, T. and Saez, E. (2003). Income Inequality in the United States, 1913-1998. *Quarterly Journal of Economics*, 118(1), 1-39.

Piketty, T. and Zucman, G. (2014). Capital is Back: Wealth-Income Ratios in Rich Countries 1700–2010. *The Quarterly Journal of Economics*, 129(3), 1255-1310.

Prados de la Escosura, L. (2008). Inequality, Poverty, and the Kuznets Curve in Spain, 1850-2000. *European Review of Economic History*, 12(3), 287-324.

Ray, D. (2015). Nit-Piketty: A Comment on Thomas Piketty's Capital in the Twenty First Century. *CESifo Forum*, 16(1), 19-25.

Roine, J., Vlachos, J. and Waldenström, D. (2009). The Long-Run Determinants of Inequality: What Can we Learn from Top Income Data? *Journal of Public Economics*, 93(7-8), 974-988.

Roine, J. and Waldenström, D. (2012). On the Role of Capital Gains in Swedish Income Inequality, *Review of Income and Wealth*, 58(3), 569-587.

Roine, J. and Waldenström, D. (2015). Long-Run Trends in the Distribution of Income and Wealth. In Atkinson, A. B. and Bourguignon, F. (eds.), *Handbook of Income Distribution*, vol. 2, Amsterdam: Elsevier, North-Holland, 469-592.

Ryoji, H. (2015). Wealth Inequality, or *r-g*, in the Economic Growth Model, RIETI Discussion Paper Series 15-E-117, Chiba University.

Solow, R. M. (2014). Thomas Piketty is Right. *The New Republic*, 1-12, April 23. Available at: http://newrepublic.com/article/117429/capital-twenty-_rst-century-thomas-piketty-reviewed.