The Collateralizability Premium*

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Abstract

This paper studies the implications of credit market frictions for the cross-section of stock returns. A common prediction of macroeconomic theories of credit market frictions is that the tightness of financial constraints is countercyclical. As a result, capital that can be used as collateral to relax such constraints provides insurance against aggregate shocks. We provide empirical evidence that supports the above prediction of the theory. A long-short portfolio constructed from financially constrained firms using our collateralizability measure provides an excess return of 6.48% per year. We develop a general equilibrium model with heterogeneous firms and financial constraints to quantitatively account for the effect of collateralizability on the cross-section of expected returns.

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1 Introduction

A large volume of literature in economics and finance emphasizes the importance of credit market frictions in affecting macroeconomic fluctuations.[1] Although models differ in details, a common prediction is that financial constraints exacerbate economic downturns because they are more binding in recessions. As a result, theories of financial frictions predict that assets that relax financial constraints should provide insurance against aggregate shocks. We evaluate the implication of this mechanism on the cross-section of equity returns.

From the asset pricing perspective, when financial constraints are binding, the value of collateralizable capital includes not only the dividends it generates, but also the present value of the Lagrangian multipliers of the collateral constraints it relaxes. If financial constraints are tighter in recessions, then firms that hold more collateralizable capital should require lower expected returns in equilibrium, because the collateralizability of its assets provides a hedge against the risk of being financially constrained and makes the firm less risky.

To examine the relationship between asset collateralizability and expected returns, we first construct a measure of firms' asset collateralizability. Guided by the corporate finance theory that links firms' capital structure decisions to collateral constraints, for example, Rampini and Viswanathan (2013), we measure asset collateralizability as the value-weighted average of the collateralizability of the different types of assets owned by the firm. Our measure can be interpreted as the fraction of the firm valuation that can be attributed to the collateralizability of its assets.

We sort stocks into portfolios according to our collateralizability measure. We document that the spread between the portfolio with low collateralizability measure and that with high collateralizability measure averages about 2.76% per year. Consistent with theory, the "collateralizability spread" is more significant among financially constrained firms and increases to 6.48% per year if we focus only on the subset of financially constrained firms. The difference in returns remains significant after controlling for market portfolios, and conventional factors such as size, value, momentum, and profitability. In addition, we also show that in the data, the collateralizability spread is predicted by measures of financing constraints, such as the TED spread.

To quantify the effect of asset collateralizability on the cross-section of expected returns, we develop a general equilibrium model with heterogeneous firms and financial constraints. In our model, high productivity firms require more capital and borrow from the rest of the economy. In addition, equity owners differ in their borrowing capacity because their net

[1]Quadrini (2011) and Brunnermeier et al. (2012) provide comprehensive reviews of this literature.

worth is determined by the historical returns of the firms they invest in, which are subject to idiosyncratic shocks. As a result, the heterogeneity in net worth and financing need translates into differences in asset collateralizability in equilibrium: equity owners with high need for capital and low net worth acquires more collateralizable capital to finance their debt.

We show that at the aggregate level, collateralizable capital requires lower expected returns in equilibrium, and in the cross-section, firms with high asset collateralizability earn low risk premiums. Our calibrated model matches well the conventional asset pricing moments and macroeconomic quantity dynamics and is able to account for the empirical relationship between asset collateralizability, leverage, and expected returns.

Related Literature This paper builds on the large macro literature that studies the role of credit market frictions in affecting business cycles. We refer the readers to Quadrini (2011) and Brunnermeier et al. (2012) for recent reviews of this literature. The papers that are most related to ours are those emphasize the importance of borrowing constraints and contract enforcements, such as Kiyotaki and Moore (1997), Kiyotaki and Moore (2012), Gertler and Kiyotaki (2010), He and Krishnamurthy (2014), and Brunnermeier and Sannikov (2014). A common prediction of this literature is that the tightness of borrowing constraints are counter-cyclical. We study the implication of this prediction on the cross-section of expected returns.

Our paper is also related to the corporate finance literature that emphasize the importance of asset collateralizability on firms' capital structure decisions. Albuquerque and Hopenhayn (2004) study dynamic financing with limited commitment. Rampini and Viswanathan (2010) and Rampini and Viswanathan (2013) develop a joint theory of capital structure and risk management based on asset collateralizability. Schmid (2008) considers the quantitative implications of dynamic financing with collateral constraints. Falato et al. (2013) provides empirical evidence for the link between asset collateralizability and leverage in aggregate time series and the cross section.

Our paper belongs to the broadly defined production based asset pricing literature, for which Kogan and Papanikolaou (2012) provides an excellent survey. From the methodological point of view, our general equilibrium model allows for a cross section of firms with heterogeneous productivity and is related to previous work including Gomes et al. (2003), Gârleanu et al. (2012), Ai and Kiku (2012), and Kogan et al. (2017). Compared to the above papers, our model incorporate financial frictions and our aggregation technique is novel.

Our paper is also connected to the literature that links investment to the cross section of expected returns. Zhang (2005) provides an investment-based explanation for the value premium. Chan et al. (2001), Li (2011), and Lin (2012) focus on the relationship between R&D investment and stock returns. Eisfeldt and Papanikolaou (2013) develop a model of organization capital and expected returns.

The rest of the paper is organized as follows. We summarize our empirical results on the relationship between asset collateralizability in Section 2. We describe a general equilibrium model with collateral constraints in Section 3 and analyze its asset pricing implications in 4. In Section 5, we provide a quantitative analysis of our model. Section 6 concludes.

2 Empirical Facts

2.1 Measuring collateralizability

To establish the link between asset collateralizability and expected returns, we first construct a measure of collateralizability at the firm level. Models with collateral constraints typically feature financing constraints that take the following general form:

$$B_i \leq \sum_{j=1}^{J} \zeta_j q_j K'_{i,j}, \tag{1}$$

where we assume that there are J types of capital that differ in their collateralizability. We use B_i for the total amount of borrowing for firm i, q_j for the market price of capital of type j and $K_{i,j}$ for firm $i's$ holdings of capital of type j. We suppress the time index to save notation, but use K' for the next period capital to follow the convention of one-period time to build in neoclassical models.

We assume that capital differ in their collateralizability parameter, and our purpose here is to construct a measure that summarizes the collateralizability of firm's total capital stock. We define define the collateralizability measure for firm i, ζ_i as the value-weighted average of the collateralizability parameter of all types of capital:

$$\zeta_i \equiv \sum_{j=1}^{J} \zeta_j \frac{q_j K'_{i,j}}{V_i}.$$

Note that in models of financing constraints, the value of collateralizable capital includes both the present value of the dividends it generates and that of the Lagrangian multipliers on the collateral constraints it relaxes. In Section 4 of the paper, we show that in our model, the measure ζ_i can be intuitively interpreted as weight of the Lagrangian multiplier in firms' asset valuation and it provides a sufficient statistic for the effect of collateralizability on expected returns.

To construct the collateralizability measure, ζ_i for each firm, we follow a two-step procedure. First, we use a regression based approach to estimate the callateralizability parameters ζ_j for each type of capital. Motivated by the previous work, for example, Rampini and Viswanathan (2013) Rampini and Viswanathan (2017), we broadly classify firm assets into three categories base on their collateralizability: structures, equipment, and intangible capital. Dividing both sides of inequality (1) and focusing on the subset of firms whose collateral constraints are binding, we have:

$$\frac{B_{i,t}}{V_{i,t}} = \sum_{j=1}^{J} \zeta_j \frac{q_{j,t} K_{i,j,t+1}}{V_{i,t}}.$$

Empirically, we run a panel regression of firm leverage, $\frac{B_{i,t}}{V_{i,t}}$, on the weight on each types of their capital to estimate the collateralizability parameter ζ_j for structures and equipment, respectively.[2]

Second, the firm i specific "collateralizability score", denoted as $\zeta_{i,t}$, is defined as a weighted average of collateralizability by

$$\zeta_{i,t} = \sum_{j=1}^{J} \hat{\zeta}_j \times \frac{q_{j,t} K_{i,j,t+1}}{V_{i,t}}.$$

We provide the details for the construction of the collateralizability measure in Appendix 7.1.

2.2 Collateralizability and expected returns

In this section, we provide empirical evidence on the relationship between asset collateralizability and expected returns. We follow the standard procedure and sort stocks into to portfolios based on the collateralizability measure we developed in the last section. Table 1 reports the average value-weighted portfolio returns sorted on firm's collateralizability. We make two observations. First, over the whole sample, firms with a higher collateralizability score have 0.23% higher average returns per month (i.e. 2.76 % per year) than firms with lower collateralizability. Second, focusing on the subset of financially constrained firms, where

[2] We impose the restriction that $\zeta_j = 0$ for intangible capital both because previous work typically argue that intangible capital cannot be used as collateral, and because its empirical estimate is slightly negative in unrestricted regressions.

financial constraint is measured by the size and age index (SA index hereafter) of Hadlock and Pierce (2010), we find that the collateralizability spread is two times larger than that of the whole sample: 0.54 % per month (i.e. 6.48% per year). The difference in return is economically and statistically significant with a t-statistic of 2.21.

Table 1: **Univariate Portfolio Sorting on Asset Collateralizability, Value Weighted**

This table reports the monthly excess stock returns and their statistics. At the end of June each year t, we sort all the firms into five quintiles based on collateralizability measure at the end of year $t-1$. The portfolios are reformed every June. This table reports monthly average excess returns R^e, standard errors σ, t-statistics (t). We split the whole sample into constrained and unconstrained firms, as classified by SA index.

	1	2	3	4	5	1-5
	Whole sample					
$R^e(\%)$	0.77	0.62	0.55	0.60	0.48	0.29
(t)	3.62	2.75	2.56	2.77	1.81	1.55
$\sigma(\%)$	4.61	4.93	4.67	4.69	5.74	4.05
	Financially constrained firms, SA index					
$R^e(\%)$	0.90	0.83	0.86	0.66	0.37	0.54
(t)	2.54	2.39	2.76	2.36	1.23	2.21
$\sigma(\%)$	7.77	7.58	6.80	6.09	6.49	5.28
	Financially unconstrained firms, SA index					
$R^e(\%)$	0.76	0.59	0.64	0.63	0.53	0.23
(t)	3.60	2.60	2.79	3.03	2.44	1.39
$\sigma(\%)$	4.58	4.98	4.99	4.57	4.69	3.64

In the Appendix 7.2 we provide various robustness check of the above evidence. First, we show that the return spread across collateralizability sorted portfolios is generally even stronger and statistically more significant for equal-weighted scheme, and is robust to alternative empirical measures of financial constraints. Second, we also show that the collateralizability spread remains significant after controlling for commonly used factors, for example, the Carhart (1997) four factors and the Fama and French (2015) five factors.

The collateralizability spread among financially constrained firms is qualitatively consistent with theoretical models of financial constraints. The fact that the spread is quantitatively small, but still present, among financially unconstrained firms is also consistent with theory: in forward looking dynamic models, collateralizability adds to asset value even for currently unconstrained firms because of the possibility of its relaxing financial constraints in the future. The expectation of being financially constrained in the future will factor into the current asset valuation and affect asset returns. In the next Section, we develop a general equilibrium model to formalize the above intuition and to quantitatively account for the (negative) collateralizability premium.

3 A general equilibrium model

This section describes the ingredients of our quantitative theory of the collateralizability spread. The aggregate aspect of the model is intended to follow standard macro models with collateral constraints such as Kiyotaki and Moore (1997) and Gertler and Kiyotaki (2010). We allow for heterogeneity in the collateralizability of assets as in Rampini and Viswanathan (2013). The key additional elements in the construction of our theory is idiosyncratic productivity shocks and firm entry and exit. These features allow us to generate a quantitatively plausible firm dynamics in order to study the implication of financial constraints for the cross section of equity returns.

3.1 Households

Time is infinite and discrete. The representative household consists of a continuum of workers and a continuum of entrepreneurs. Workers and entrepreneurs receive their incomes every period and submit them to the planner of the household, who makes decisions for consumption for all members of the household. Entrepreneurs and workers make their financial decisions separately.[3]

The household ranks her utility according to the following recursive preference as in Epstein and Zin (1989):

$$U_t = \left\{ (1-\beta) C_t^{1-\frac{1}{\psi}} + \beta (E_t[U_{t+1}^{1-\gamma}])^{\frac{1-\frac{1}{\psi}}{1-\gamma}} \right\}^{\frac{1}{1-\frac{1}{\psi}}},$$

where β is the time discount rate, ψ is the intertemporal elasticity of substitution, and γ is the relative risk aversion. As we will show later in the paper, together with the endogenous equilibrium long run risk, the recursive preference in our model generates a volatile pricing kernel and a significant equity premium as in Bansal and Yaron (2004).

In every period t, the household purchases $B_t(i)$ amount of risk-free bond from entrepreneur i, from which she expects to receive $B_t(i) R_{t+1}^f$ in the next period, where R_{t+1}^f denotes the risk-free interest rate from period t to $t+1$. In addition, she receives capital income $\Pi_t(i)$ from entrepreneur i and labor income $W_t L_t(j)$ from worker j. Without loss

[3] As Gertler and Kiyotaki (2010), we make the assumption that household members make joint decisions on their consumption to avoid the need to keep the distribution of entrepreneur income as the state variable.

of generality, we assume that all workers are endowed with the same number of hours per period, and suppress the dependence of $L_t(j)$ on j. The household budget constraint at time t can therefore be written as:

$$C_t + \int B_t(i)\,di = W_t L_t + R_t^f \int B_{t-1}(i)\,di + \int \Pi_t(i)\,di.$$

Let M_{t+1} denote the the stochastic discount factor implied by household consumption. Under recursive utility, $M_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{U_{t+1}}{E_t[U_{t+1}^{1-\gamma}]^{\frac{1}{1-\gamma}}}\right)^{\frac{1}{\psi}-\gamma}$, the optimality of the intertemporal saving decisions implies that the risk-free interest rate must satisfy:

$$E_t[M_{t+1}]R_{t+1}^f = 1.$$

3.2 Entrepreneurs

Entrepreneurs are agents who operate a productive idea. An entrepreneur who starts at time 0 draws an idea with initial productivity $\bar{z}$ and begins operation with initial net worth N_0. Under our convention, N_0 is also the total net worth of all entrepreneurs at time 0 because the total measure of all entrepreneurs is normalized to one.

Let $N_{i,t}$ denote the net worth of an entrepreneur i at the end of time t, and let $B_{i,t}$ denote the total amount of risk-free bond the entrepreneur issues to the household, the time-t budget constraint for the entrepreneur is

$$q_{K,t}K_{i,t+1} + q_{H,t}H_{i,t+1} = N_{i,t} + B_{i,t}.^{4} \tag{2}$$

Here we assume that there are two types of capital, K and H, that differ in their collateralizability and use $q_{K,t}$ and $q_{H,t}$ for their prices at time t. $K_{i,t+1}$ and $H_{i,t+1}$ is the amount of capital that entrepreneur i purchases at the end of period t, which can be used for production in period $t+1$. We assume that at the end of period t, the entrepreneur has an opportunity to default on his lending contract and abscond with all of the H-type capital and a $1-\zeta$ fraction of the $K-$type capital. Because lenders can retrieve ζ fraction of the type-K capital

[4] We assume that the entrepreneur have access to only risk-free borrowing contracts and do not allow for state-contigents debt.

upon default, borrowing is limited by

$$B_{i,t} \leq \zeta q_{K,t} K_{i,t+1}. \tag{3}$$

The type-K capital can therefore be interpreted as collateralizable capital and type-H capital is non-collateralizable.

In period $t+1$, the productivity of entrepreneur i evolves according to

$$z_{i,t+1} = z_{i,t} e^{\mu + \sigma \varepsilon_{i,t+1}}, \tag{4}$$

where $\varepsilon_{i,t+1}$ is a Gaussian shock i.i.d. across agents and over time. We use $\pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}, H_{i,t+1}\right)$ to denote the equilibrium profit that entrepreneur i makes in period $t+1$, where $\bar{A}_{t+1}$ is the aggregate productivity in period $t+1$.

In each period after production, the entrepreneur experiences a liquidation shock with probability λ, upon which he loses his idea and needs to liquidate his net worth back to the household.[5] If the liquidation shock happens, the entrepreneur restarts with a draw of a new idea with initial productivity $\bar{z}$ and an initial net worth $\bar{z}\chi N_t$ in period $t+1$, where N_t is the total (average) net worth of the economy in period t, and χ is a parameter that determines the ratio of the initial net worth of entrepreneurs relative to that of the economy-wide average. Conditioning on not receiving a liquidation shock, the net worth of entrepreneur i at the end of period $t+1$ is determined by:

$$\begin{aligned} N_{i,t+1} &= \pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}, H_{i,t+1}\right) + (1-\delta)\, q_{K,t+1} K_{i,t+1} \\ &\quad + (1-\delta)\, q_{H,t+1} H_{i,t+1} - R_{f,t+1} B_{i,t}. \end{aligned} \tag{5}$$

The interpretation is that the entrepreneur receives $\pi\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}, H_{i,t+1}\right)$ from production. His capital holdings depreciate at rate δ, and he needs to pay back the debt borrowed last period, $R_{f,t+1} B_{i,t}$.

Because whenever liquidity shock happens, entrepreneurs submit their net worth to the household who chooses consumption collectively for all members, they value their net worth using the same pricing kernel as the household. Let $V_t^i\left(N_{i,t}\right)$ denote the value function of entrepreneur i, it must satisfy the following Bellman equation:

$$V_t^i\left(N_{i,t}\right) = \max_{K_{i,t+1}, H_{i,t+1}, N_{i,t+1}} E_t\left[M_{t+1}\{(1-\lambda) N_{i,t+1} + \lambda V_{t+1}^i\left(N_{i,t+1}\right)\}\right], \tag{6}$$

[5] This assumption effectively make entrepreneurs less patient than the household and prevents them from saving out of the financial constraint.

where the law of motion of $N_{i,t+1}$ is given by (5).

We use variables without an i subscript to denote economy-wide aggregate quantities, the aggregate net worth in the entrepreneur sector satisfies

$$N_{t+1} = (1-\lambda)\left[\begin{array}{c} \pi\left(\bar{A}_{t+1}, K_{t+1}, H_{t+1}\right) + (1-\delta)\, q_{K,t+1} K_{t+1} \\ + (1-\delta)\, q_{H,t+1} H_{t+1} - R_{f,t+1} B_t \end{array}\right] + \lambda \bar{z} \chi N_t, \tag{7}$$

where $\pi\left(\bar{A}_{t+1}, K_{t+1}, H_{t+1}\right)$ is the aggregate profit of all entrepreneurs.

3.3 Production

3.3.1 Final output

Let $z(i,t)$ denote the idiosyncratic productivity for firm i at time t. We assume that output is produced through the following production technology:

$$\Pi\left(A_t, z_{i,t}, K_{i,t}, H_{i,t}\right) = \max_{N_{i,t}} A(t)\left[z(i,t)^{1-\nu}\left(K_{i,t}+H_{i,t}\right)^{\nu}\right]^{\alpha} L_{i,t}^{1-\alpha} - W(t)\, L_{i,t}, \tag{8}$$

where $W(t)$ is the equilibrium wage rate, and $L_{i,t}$ is the amount of labor hired by entrepreneur i at time t. In our formulation, α is capital share, and ν is the span of control parameter as in Atkeson and Kehoe (2005).

Note that collateralizable and non-collateralizable capitals are perfect substitutes in production. This assumption is made for tractability.

It is convenient to write the profit function explicitly by maximizing out labor in equation (8) and using the labor market clearing, $\int L_{i,t} di = 1$, to get

$$L_{i,t} = \frac{z(i,t)^{1-\nu}\left(K_{i,t}+H_{i,t}\right)^{\nu}}{\int z(i,t)^{1-\nu}\left(K_{i,t}+H_{i,t}\right)^{\nu} di}, \tag{9}$$

and

$$\Pi\left(\bar{A}_t, z_{i,t}, K_{i,t}, H_{i,t}\right) = \alpha \bar{A}(t)\, z(i,t)^{1-\nu}\left(K_{i,t}+H_{i,t}\right)^{1-\nu}\left[\int z(i,t)^{1-\nu}\left(K_{i,t}+H_{i,t}\right)^{\nu} di\right]^{\alpha-1}. \tag{10}$$

Let $y_{i,t} = \bar{A}(t)\left[z(i,t)^{1-\nu}\left(K_{i,t}+H_{i,t}\right)^{\nu}\right]^{\alpha} L_{i,t}^{1-\alpha}$ be the output of firm i at time t, the total output of the economy therefore satisfies

$$Y_t = \int y_{i,t} di = A(t) \left[\int z(i,t)^{1-\nu} (K_{i,t} + H_{i,t})^{\nu} di \right]^{\alpha}. \tag{11}$$

3.3.2 Capital goods

We assume that capital goods are produced from a constant-return-to-scale and convext adjustment cost function $G(I, K+H)$, that is, one unit of investment costs $G(I, K+H)$ units of consumption goods. Therefore, the aggregate resource constraint is

$$C_t + I_t + G(I_t, K_t + H_t) = Y_t,$$

where Y_t is the total amount of output in period t. Without loss of generality, we assume that $G(I_t, K_t + H_t) = g\left(\frac{I_t}{K_t + H_t}\right)(K_t + H_t)$ for some covex function g.

We assume that ϕ fraction of new investment goods can be used for type-K capital and the rest, $1-\phi$ fraction can be used for type-H capital. This is another simplifying assumption. Because at the aggregate level, the ratio of type K capital and type H capital is always $\frac{\phi}{1-\phi}$, the total capital stock of the economy can be summarized as a single state variable. The aggregate capital stocks of the economy must satisfy:

$$\begin{aligned} K_{t+1} &= (1-\delta) K_t + \phi I_t \\ H_{t+1} &= (1-\delta) H_t + (1-\phi) I_t. \end{aligned}$$

4 Equilibrium Asset Pricing

4.1 Aggregation

Our economy is one with both aggregate and idiosyncratic productivity shocks. In general, we need to use the joint distribution of capital and networth as an infinite-dimensional state variable in order to characterize the equilibrium recursively. In this section, we present an aggregation result and show that the aggregate quantities and prices of our model can be characterized without any reference to distributions. Given the aggregate quantities and prices, quantities and shadow prices at the individual firm level can be constructed from equilibrium conditions.

Distribution of idiosyncratic productivity In our model, the law of motion of idiosyncratic productivity shocks, $z_{i,t+1} = z_{i,t}e^{\mu+\sigma\varepsilon_{i,t+1}}$, is time invariant, which implies that the cross-section distribution of $z_{i,t}$ enventually converge to a stationary distribution.[6] At the macro level, the heterogeneity of idiosyncratic productivity can be conveniently summarized by a simple statistic: $Z(t) = \int z(i,t)\, di$. It is useful to compute this integral explicitly.

Given the law of motion of $z_{i,t}$, we have:

$$Z(t+1) = (1-\lambda)\int z(i,t)\, e^{\varepsilon(i,t+1)} di + \lambda\bar{z}$$

The interpretation is that only a fraction $(1-\lambda)$ survive to the next period, and λ fraction of the entrepreneur redraw of new productivity of $\bar{z}$. Note that $\varepsilon(i,t+1)$ is independent of $z_{i,t}$; therefore we can integrate out $\varepsilon(i,t+1)$ first and write the above as:

$$\begin{aligned} Z(t+1) &= (1-\lambda)\int z(i,t)\, E\left[e^{\varepsilon(i,t+1)}\right] di \ + \lambda\bar{z} \\ &= (1-\lambda)\, Z(t)\, e^{\mu+\frac{1}{2}\sigma^2} + \lambda\bar{z}, \end{aligned}$$

where the last line uses the property of the log-normal distribution. It is easy to see that if we choose the normalization $\bar{z} = \frac{1}{\lambda}\left[1-(1-\lambda)\, e^{\mu+\frac{1}{2}\sigma^2}\right]$, and start the economy at $Z(0) = 1$, then $Z(t) = 1$ for all t. This will be the assumption we maintain for the rest of the paper.

Firm profit We assume that $\varepsilon(i,t+1)$ is observed at the end of period t when the entrepreneurs plan for the next period capital. As we show in the appendix, this implies that entrepreneur will choose $K_{i,t+t}+H_{i,t+1}$ to be proportional to $z_{i,t+1}$. Because $\int z(i,t+1)\, di = 1$, we must have

$$K_{i,t+t} + H_{i,t+1} = z_{i,t+1}\left(K_{t+1} + H_{t+1}\right),$$

where $K_{t+1} + H_{t+1}$ are aggregate quantities.

The assumption that capitals are chosen after $z_{i,t+1}$ is observed implies that total output does not depend on the joint distribution of idiosyncratic productivity and capital and allows us to write $Y_t = \bar{A}(t)\left(K_{t+1} + H_{t+1}\right)^{\alpha}\int z(i,t)\, di = \bar{A}(t)\left(K_{t+1} + H_{t+1}\right)^{\alpha}$. It also implies that the profit at the firm level is proportional to productivity:

$$\Pi\left(\bar{A}_t, z_{i,t}, K_{i,t}, H_{i,t}\right) = \alpha\bar{A}(t)\, z(i,t)\left(K_t + H_t\right)^{\alpha},$$

[6] In fact, the stationary distribution of $z_{i,t}$ is a double-sided Pareto distribution. Our model is therefore consistent with the empirical evidence of the power law distribution of firm size.

and the marginal production of capital are equalized across firms:

$$\frac{\partial}{\partial K_{i,t}}\Pi\left(\bar{A}_t, z_{i,t}, K_{i,t}, H_{i,t}\right) = \frac{\partial}{\partial H_{i,t}}\Pi\left(\bar{A}_t, z_{i,t}, K_{i,t}, H_{i,t}\right) = \alpha \bar{A}\left(t\right)\left(K_t + H_t\right)^{\alpha-1}. \tag{12}$$

Intertemporal optimality Having simplified the profit functions, we can derive the optimality conditions for the entrepreneur's maximization problem, (6). Note that given equilibrium prices, the objective function and the constraints are linear in net worth; therefore, the value function must be linear. We write $V_t^i\left(N_{i,t}\right) = \mu_t^i N_{i,t}$, where μ_t^i can be interpreted as the marginal value of net worth for entrepreneur i. Also, let η_t^i be the Lagrangian multiplier of the collateral constraint (3), the first order condition with respect to $B_{i,t}$ implies

$$\mu_t^i = E_t\left[\tilde{M}_{t+1}^i\right] R_{t+1}^f + \eta_t^i, \tag{13}$$

where we use the notation:

$$\tilde{M}_{t+1}^i = M_{t+1}[\left(1-\lambda\right)\mu_{t+1}^i + \lambda]. \tag{14}$$

The interpretation is the one unit of net worth allows the entrepreneur to reduce one unit of borrowing, the present value of which is $E_t\left[\tilde{M}_{t+1}^i\right] R_{t+1}^f$, and relaxes the collateral constraint, the benefit of which is measured by η_t^i.

Similarly, the first order condition for $K_{i,t+1}$ is

$$\mu_t^i = E_t\left[\tilde{M}_{t+1}^i \frac{\Pi_K\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}, H_{i,t+1}\right) + \left(1-\delta\right) q_{K,t+1}}{q_{K,t}}\right] + \zeta\eta_t^i. \tag{15}$$

An additional unit of type-K capital allows the entrepreneur to purchase $\frac{1}{q_{K,t}}$ unit of capital, which pays a profit of $\Pi_K\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}, H_{i,t+1}\right)$ in the next period before it depreciates at rate δ. In addition, a fraction ζ of type-K capital can be used as collateral and to relax the borrowing constraint.

Finally, optimality with respect to the choice of type-H capital implies

$$\mu_t^i = E_t\left[\tilde{M}_{t+1}^i \frac{\Pi_H\left(\bar{A}_{t+1}, z_{i,t+1}, K_{i,t+1}, H_{i,t+1}\right) + \left(1-\delta_K\right) q_{H,t+1}}{q_{H,t}}\right]. \tag{16}$$

Recursive construction of the equilibrium Note that in our model, firms differ in their net worth, which by (5), depends on the entire history of idiosyncratic productivity shocks, and the need for capital, which depends on the realization of the next-period productivity shock. Therefore in general, the marginal benefit of net worth, μ_t^i and the tightness of the collateral constraint, η_t^i depend on firm history. Below we show that despite the heterogeneity in net worth and capital holdings across firms, our model permits an equilibrium in which μ_t^i and η_t^i are equalized across firms, and aggregate quantities can be determined independent of the distribution of net worth and capital.

Note that the assumption that type-K and type-H capital are perfect substitutes and the assumption that the idiosyncratic shock $z_{i,t+1}$ is observed before the decisions on $K_{i,t+1}$ and $H_{i,t+1}$ are made imply that the marginal product of both types of capital are equalized within and across firms, as shown in (12). As a result, equations (13)-(16) permits solutions where μ_t^i and η_t^i are not firm-specific. Intuitively, because the marginal product of capital depends only on the sum of $K_{i,t+1} + H_{i,t+1}$ and not their composition, entrepreneurs will choose the total amount of capital to equalize its marginal product across firms — this is possible as $z_{i,t+1}$ is observed in period t. Depending on his borrowing need, an entrepreneur can then determine the amount of $K_{i,t+1}$ to satisfy the collateral constraint. Because capital can be purchased on a competitive market, entrepreneurs will choose $K_{i,t+1}$ to equalize its price and its marginal benefit, which include the marginal product of capital and the Lagrangian multiplier η_t^i. Because both the price and the marginal product of capital is equalized across firms, so must the tightness of the collateral constraint.

We formalize the above observation by providing a recursive characterization of the equilibrium. We make one final assumption that the aggregate productivity is given by $\bar{A}_t = A_t \left(K_t + H_t\right)^{1-\alpha}$, where $\{A_t\}_{t=0}^{\infty}$ is a Markov process. This assumption generates endogenous growth, which combined with the recursive preference, enhances the volatility of the pricing kernel, as in long-run risks models.[7]

Let lower case variables denote aggregate quantities normalized by current-period capital stock. The equilibrium objects are: consumption, $c\left(A, n\right)$, investment, $i\left(A, n\right)$, the marginal value of net worth, $\mu\left(A, n\right)$, the Lagrangian muliplier on the collateral constraint, $\eta\left(A, n\right)$, the price of type-K capital, $q_K\left(A, n\right)$, the price of type-H capital, $q_H\left(A, n\right)$, and the risk-free interest rate, $R_f\left(A, n\right)$ as functions of state variables. Given these equilibrium functionals, we can define

$$\Gamma\left(A, n\right) = \frac{K'}{K} = (1-\delta) + i\left(A, n\right)$$

as the growth rate of the capital stock, and constuct the law of motion of the endogenous

[7]See Bansal and Yaron (2004) and Kung and Schmid (2015).

state variable n from equation (7):

$$n' = (1-\lambda)\left[\alpha A' + \phi q_K(A',n') + (1-\phi)\, q_H(A',n') - \zeta\phi q_K(A,n)\frac{R_f(A,n)}{\Gamma(A,n)}\right] + \lambda\chi\frac{n}{\Gamma(A,n)}.$$

With the law of motion of the state variables, we can construct the normalized utility of the household as the fixed point of:

$$u(A,n) = \left\{(1-\beta)c(A,n)_t^{1-\frac{1}{\psi}} + \beta\Gamma(A,n)^{1-\frac{1}{\psi}}\left(E[u(A',n')^{1-\gamma}]\right)^{\frac{1-\frac{1}{\psi}}{1-\gamma}}\right\}^{\frac{1}{1-\frac{1}{\psi}}}.$$

The stochastic discount factors can then be written as:

$$\begin{aligned} M' &= \beta\left[\frac{c(A',n')\,\Gamma(A,n)}{c(A,n)}\right]^{-\frac{1}{\psi}}\left[\frac{u(A',n')}{E\left[u(A',n')^{1-\gamma}\right]^{\frac{1}{1-\gamma}}}\right]^{\frac{1}{\psi}-\gamma}, \\ \tilde{M}' &= M'[(1-\lambda)\,\mu(A',n') + \lambda]. \end{aligned}$$

Proposition 4.1. *(Recursive equilibrium)*

The equilibrium functionals, $c(A,n)$, $i(A,n)$, $\mu(A,n)$, $\eta(A,n)$, $q_K(A,n)$, $q_H(A,n)$, and $R_f(A,n)$ are the solution to the following set of functional equations:

$$E[M'|A]\,R_f(A,n) = 1$$

$$\mu(A,n) = E\left[\tilde{M}'\middle| A\right] R_f(A,n) + \eta(\theta,n)$$

$$\mu(A,n) = E\left[\tilde{M}'\frac{\alpha A' + (1-\delta)\,q_K(A',n')}{q_K(A,n)}\middle| A\right] + \zeta\eta(A,n)$$

$$\mu(A,n) = E\left[\tilde{M}'\frac{\alpha A' + (1-\delta)\,q_H(A',n')}{q_H(A,n)}\middle| A\right]$$

$$n = (1-\zeta)\,q_K(A,n) + q_H(A,n)$$

$$G'(i(A,n)) = \phi q_K(A,n) + (1-\phi)\,q_H(A,n)$$

$$c(\theta,n) + i(\theta,n) + g(i(\theta,n)) = \theta$$

The above proposition allows us to solve the aggregate quantities of the economy first, and use the law of motion of idiosyncratic productivity and firm-level budget constraint (2) and (3) to construct the cross-section of net worth and capital holdings.

4.2 The cross-section of expected returns

Collateralizability spread at the aggregate level Our models allow for two types of capital, type-K capital is collateralizable, and type-H capital is not. The difference in the returns of the claim to one unit of type-K capital and the claim to one unit of type-H capital can be interpreted as the (negative) collateralizability premium at the aggregate level. Note that one unit of type j capital costs $q_{j,t}$ in period t and it pays off $\Pi_{j,t+1} + (1-\delta)\, q_{j,t+1}$ in the next period, for $j = K, H$. Therefore, the returns on the claims to the two types of capital are given by:

$$R_{j,t+1} = \frac{\alpha A_{t+1} + (1-\delta)\, q_{j,t+1}}{q_{j,t}}, \quad j = K, H.$$

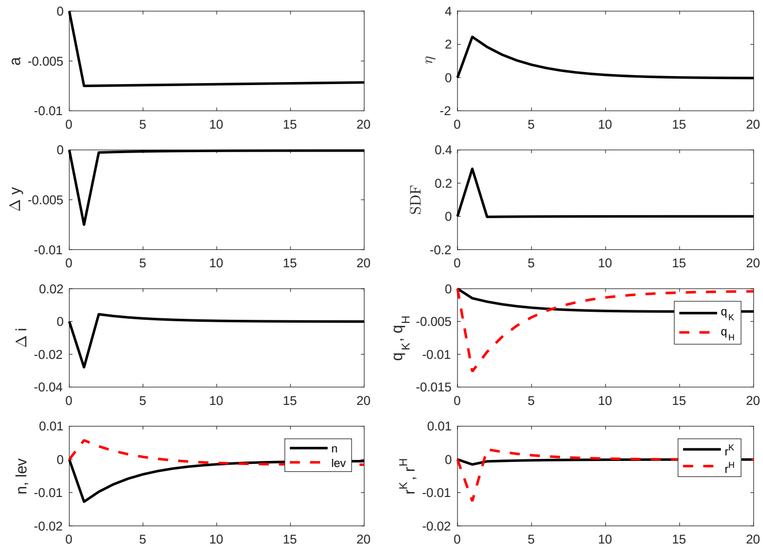
Of course, risk premiums are determined by the covariances of the payoffs with respect to the stochastic discount factor. Given that the marginal product of capital component of the payoff are identical for both types of capital, the key to understand the collateralizability premium is the cyclical properties of the price of capital, $q_{j,t+1}$.

We can iterate equations (15) and (16) forward to obtain expression for $q_{K,t}$ and $q_{H,t}$ as present value of future cash flows. Clearly, the present value of $q_{K,t}$ will contain the Lagrangain multipliers $\left\{\eta^i_{t+j}\right\}_{j=0}^{\infty}$, and the present value of $q_{H,t}$ does not. Because the Lagrangian multipliers are counter-cyclical and act as a hedge, $q_{K,t}$ will be less sensitive to productivity shocks. These asset pricing implications of our model are best illustrated with impulse response functions.

In Figure 1, we plot the percentage deviations of quantities (left column) and prices (right column) from the steady state in response to a one-standard deviation negative shock to the aggregate productivity.

We make two observations. First, a negative productivity shock lowers output and investment (second and third panel on the left column) as in standard macro models. In addition, as shown in the bottom panel on the left, entrepreneur net worth drops sharply and leverage rise immediately. Second, upon a negative productivity shock, because the entrepreneur net worth drops sharply, so does the price of type-H capital. However, the decrease in the price of the collateralizable capital is much smaller by comparison. This is because the Lagrangian multiplier on the collateral constraint, η increases upon impact and offsets the effect of negative productivity shock on the price of type K capital. As a result, the return of type-H capital responds much less to negative productivity shocks than that of the type-H capital. Collateralizable capital is less risky than non-collateralizable capital in our model.

Figure 1: Impulse Responses to TFP shock



This figure plots the log-deviations from the steady state for quantities (left panel) and prices (right panel) with respect to a one-standard deviation negative shock to aggregate productivity. One period is a month. All parameters are calibrated as in Table 2.

Collateralizability spread at the firm level In our model, equity claims to firms can be freely traded among entrepreneurs. The return on an entrepreneur's net worth is $\frac{N_{i,t+1}}{N_{i,t}}$. Using (2) and (5), we can write this return as

$$\begin{aligned} & \frac{\alpha A_{t+1}\left(K_{i,t+1}+H_{i,t+1}\right)+\left(1-\delta\right)q_{K,t+1}K_{i,t+1}+\left(1-\delta\right)q_{H,t+1}H_{i,t+1}-R_{f,t+1}B_{i,t}}{q_{K,t}K_{i,t+1}+q_{H,t}H_{i,t+1}-B_{i,t}} \\ = & \frac{V_{i,t}}{N_{i,t}}\left\{\frac{q_{K,t}K_{i,t+1}}{V_{it}}R_{k,t+1}+\frac{q_{H,t}H_{i,t+1}}{V_{i,t}}R_{H,t+1}-R_{f,t+1}\right\}+R_{f,t+1}, \end{aligned}$$

where we define $V_{i,t}=q_{K,t}K_{i,t+1}+q_{H,t}H_{i,t+1}$ to be the total value of firm i's asset at time t. The above expression has intuitive interpretations. The term $\frac{q_{K,t}K_{i,t+1}}{V_{it}}R_{k,t+1}+\frac{q_{H,t}H_{i,t+1}}{V_{i,t}}R_{H,t+1}$ is the weighted average return on firm i's asset, and $\frac{V_{i,t}}{N_{i,t}}$ is the leverage ratio. That is, the return on equity is the leverage-adjusted weighted average return on assets.

To understand the collateralizability premium at the firm level, note that the return on a firm's asset is the value weighted return of the different types of capital owned by the firm. Because type-H capital provides a higher expected return then type-K capital, firms with more collateralizable capital earns lower risk premium. In our model, the above relationship between asset collateralizability and expected return is best summarized by the collateralizability measure we constructed in Section 2 of the paper. To see this, letting j

index the type of capital, and using the fact that μ_t^i and η_t^i are identical across firms, equations (15) and (16) can be summarized as:

$$\mu_t q_{j,t} K_{j,t+1} = E_t \left[\tilde{M}_{t+1}^i \left\{ \Pi_{j,t+1} + (1-\delta)\, q_{j,t+1} \right\} K_{j,t+1} \right] + \zeta_j \eta_t q_{j,t} K_{j,t+1} \tag{17}$$

Let $V_t = \sum_{j=1}^{J} q_{j,t} K_{j,t+1}$ be the total value of the firm's asset. Dividing the above equation by V_t and summing over all j, we have:

$$\mu_t = \frac{\sum_{j=1}^{J} E_t \left[\tilde{M}_{t+1}^i \left\{ \Pi_{j,t+1} + (1-\delta)\, q_{j,t+1} \right\} K_{j,t+1} \right]}{V_t} + \eta_t \sum_{j=1}^{J} \zeta_j \frac{q_{j,t} K_{j,t+1}}{V_t}. \tag{18}$$

Note that μ_t is the shadow value of entrepreneur net worth. Equation (18) decompose μ_t into two parts. Because the term $E_t \left[\tilde{M}_{t+1}^i \left\{ \Pi_{j,t+1} + (1-\delta)\, q_{j,t+1} \right\} K_{j,t+1} \right]$ can be interpreted as the present value of the cash flow generated by type j capital, the first component is the fraction of firm value that comes from dividend cash flow. The second component is the Lagrangian multiplier on the collateral constraint multiplied by our measure of asset collateralizability.

In our model, μ_t and η_t are common across all firms. All types of capital generate the same marginal product in the future. As a result, expected returns differ only because the composition of asset collateralizability, which is completely summarized by the asset collateralizability measure, $\sum_{j=1}^{J} \zeta_j \frac{q_{j,t} K_{j,t+1}}{V_t}$. As show we show in the next section, this parallel between our model and our empirical procedure allows our model to match very well the quantitative features of the collateralizability spread in the data.

5 Quantitative Analysis

In this section, we examine whether our model can quantitatively account for the collateralizability premium in the data. We calibrate the model parameters to match moments of macroeconomic quantities and asset prices at the aggregate level and study its implications on the cross-section of expected returns. We show that our model can quantitative replicate the main features of firm characteristics, and produce a collateralizability premium comparable to that in the data. In addition, we also documents that aggregate measures of financing constraint, such as the TED spread, can predict the collateralizability spread in the data, and quantitatively replicate this predictive regression inside the model.

5.1 Calibration

We calibrate our model at monthly frequency, and list the parameters and the corresponding macroeconomic moments that we used in our calibration procedure in Table 2. We group our parameters into four blocks. In the first block, we list the parameters which can be determined by the previous literature. In particular, we set the relative risk aversion $\gamma = 10$ and intertemporal elasticity of substitution $\psi = 2$, in line with the long-run risks literature, for instance, Bansal and Yaron (2004). The capital share parameter, α, is set to be 0.30, as in the standard RBC literature. The span of control parameter ν is set to be 0.85, consistent with Atkeson and Kehoe (2005).

Table 2: **Calibration**

We calibrate the model at monthly frequency. This table reports the parameter values and the corresponding moments (annalized) we used in the calibration procedure.

Parameter	Symbol	Value	Target/Source	Moments (Annual)
Relative risk aversion	γ	10	Bansal and Yaron (2004)	-
IES	ψ	2	Bansal and Yaron (2004)	-
Capital share	α	0.3	RBC Literature	-
Span of control parameter	ν	0.85	Atkeson and Kehoe (2005)	-
Mean productivity growth rate	$E(\tilde{A})$	0.005	Mean GDP growth rate	2%
Time discount factor	β	0.998	Average risk-free rate	1%
Share of type-K investment	ϕ	0.50	Average capital ratio, K/H	1
Capital depreciation rate	δ	0.008	Annual capital depreciation	10%
Death rate of entrepreneurs	λ	0.01	Corporate survival rate	90%
Collateralizability parameter	ζ	0.985	Corporate debt to asset ratio	0.55
Transfer to entering entrepreneurs	χ	0.77	Average C/I ratio	4
Persistence of TFP shock	ρ^A	0.989	Autocorrelation of GDP growth	0.49
Vol. of TFP shock	σ^A	0.015	Volatility of GDP growth	3.05%
Invest. adj. cost parameter	τ	23	Vol. of investment growth	10%
Mean idio. productivity growth	μ_Z	0.003	Mean idio. productivity growth	4%
Vol. of idio. productivity growth	σ_Z	0.057	Vol. of idio. productivity growth	20%

The second block of parameters are determined by matching a set of first moments of quantities and prices to their empirical counterparts. We set the average economy-wide productivity growth rate $E(\tilde{a})$ to match a mean growth rate of U.S. economy of 2% per year. The time discount factor β is set to match the average real risk free rate of 1% per year. The share of K-type capital investment, ϕ is set to be 0.5 to maintain a unit average capital ratio of K and H. The capital depreciation rate is set to match a 10% annual capital depreciation rate in the data. Note that, in the current calibration, we maintain the symmetry in the share parameter ϕ and the depreciation rate δ for both types of capital. By doing so, we can single out the implications of collateralizablity on the cross-sectional return spread. The death rate of entrepreneurs is calibrated to be 0.01, targeting an average corporate duration of 10 years. We calibrate the remaining two parameters related to financial frictions, namely, the collateralizability parameter, ζ, and the transfer to entering entrepreneurs, χ, by jointly matching two moments. They include a non-financial corporate sector leverage ratio, defined as the debt to asset ratio, of 0.55, the average consumption to investment ratio $E(C/I) = 4$. Such targeted leverage ratio is broadly in line with the median ratio of U.S. non-financial firms in COMPUSTAT.

The third block of parameters are not directly related to the steady states of the economy, instead they are determined by the second moments in the data. The persistence parameter ρ and the standard deviation σ^A is chosen to match the first-order autocorrelation and the volatility of the aggregate output growth. The elasticity parameters of the adjustment cost functions, τ are set to allow the model to achieve a reasonable high volatility of investment, in line with the data.

The last block of the parameters are related to the firm idiosyncratic productivity shocks. We calibrate them to match the mean and volatility of the idiosyncratic productivity growth of the cross-section of U.S. non-financial firms in the Compustat database.

Computation Method Based on our calibrated parameters, the collateral constraint is binding at the steady state. Therefore, following the prior macroeconomic literature, for instance, Gertler and Kiyotaki (2010), we assume the constraint is binding over the narrow region around the steady state, and the local approximation solution method is a good approximation. We therefore solve the model using a second-order local approximation around the stochastic steady state, computed using the `dynare++` package.

5.2 Simulation

In this section, we report the model simulated moments in the aggregate and the cross-section, and compare them with the data. We simulate the model at the monthly frequency and aggregate the data to form annual observations. Each simulation has a length of 160 years. We drop the first half of each simulation to avoid dependence on initial values, and repeat the process 5,000 times. At the cross-sectional level, each simulation contains 2,500 firms.

5.2.1 Aggregate moments

In this section, we focus on model's quantitative performance at the aggregate level, and document that our model can match a wide set of conventional moments in macroeconomic quantities and asset prices. More importantly, it delivers a sizable collateralizability spread at the aggregate level.

Table 3 reports the key moments of macroeconomic quantities (top panel) and those of asset returns (bottom panel) respectively, and compare them with data counterpart whenever available. The top panel shows that the model simulated data is broadly consistent with the basic features of the aggregate macro-economy in terms of volatilities, correlation and persistence of output, consumption and investment. Our current model maintains the success of neoclassical growth model in accounting for the dynamics of macroeconomic quantities.

Table 3: **Model Simulations and Quantitative Results**

This table presents the moments from the model simulation. The market return R_M corresponds to the return on entrepreneurs' net worth and embodies an endogenous financial leverage. R_K^L, R_H^L denotes the levered capital returns, by the average financial leverage in the economy. We simulate the economy at monthly frequency, then aggregate the monthly observations to annual frequency. The moments reported are based on the annual observations. Number in parenthesis are standard errors of the calculated moments.

Moments	Data	Model
$\sigma(\Delta y)$	3.05 (0.60)	2.95
$\sigma(\Delta c)$	2.53 (0.56)	2.77
$\sigma(\Delta i)$	10.30 (2.36)	4.77
$corr(\Delta c, \Delta i)$	0.39(0.29)	0.68
$AC1(\Delta y)$	0.49(0.15)	0.49
$E[R_M - R_f]$	5.71 (2.25)	6.17
$\sigma(R_M - R_f)$	20.89 (2.21)	2.92
$E[R_f]$	1.10 (0.16)	0.82
$\sigma(R_f)$	0.97 (0.31)	1.05
$E[R_H^L - R_f]$		12.74
$E[R_K^L - R_f]$		1.26
$E[R_H^L - R_K^L]$		11.48

Focusing on the asset pricing moments (bottom panel), we make two observations. First, our model is reasonably successful in generating asset pricing moments at the aggregate level. In particular, it replicates a low and smooth risk free rate, with mean 0.82% and volatility 1.05%. The equity premium in this economy is 6.17%, comparable to 5.7% in the data. Second and more importantly, our model is also able to generate a sizable return spread between collateralizable and non-collateralizable capital. The aggregate collateralizability spread is amount to 12.74%. [8]

5.2.2 Cross-sectional moments

In the section, we simulate the cross-section of firms, in which the heterogeneity is driven by idiosyncratic productivity shocks and firm entry and exit. We document that the model can generate a quantitatively plausible firm dynamics, in particular, the model simulation can replicate some key features about the relationship between collateralizability and firm characteristics. Furthermore, the model is able to deliver the collateralizability spread quantitatively when we replicate the standard portfolio sorting procedure.

Collateralizability and Firm Characteristics In Table 4, we document how firm differences in their collateralizability are related to firm characteristics, both in the data (Panel A) and in the model (Panel B). We report the time-series average of the mean firm characteristics in each quintile portfolio.

We make several observations from the data (Panel A). First, firms with higher collateralizability are expected to have higher debt capacity, and in turn, higher financial leverage. This feature is robustness to various measures of financial leverage in the literature. Second, firms' book-to-market ratio and size are increasing with collateralizability. Third, across three measures of financial constraint, i.e. SA index, WW index, and whether a firm pays dividend or not, we observe that firms with more collateralizable capital are less likely to be financially constrained, in line with our model prediction.

Turning attention to the model (Panel B), we observe the model performs reasonably well in quantitatively replicating these patterns. In particular, the model can generate a similar magnitude of dispersion in collateralizability in quintiles, which is critical to generate a comparable collateralizability spread in the model with its data counterpart.

Collateralizability spread Table 5 demonstrates model's ability to generate quantitatively comparable return spread across collateralizability sorted portfolios. Panel A reports

[8] In the model, the market return is defined as the return on the net worth of entrepreneurs, and it endogenously embodies a financial leverage due to the entrepreneurs' levered position. However, the returns on capital are unlevered. For consistency, we lever them up by the average leverage ratio in the economy, and denote levered capital returns as R_K^L and R_H^L.

Table 4: **Collateralizability and Firm Characteristics**

This table shows the mean of firm characteristics of the collateralizability sorted portoflios. Financial debt (FD) is defined as long-term debt (DLTT), plus debt in current liability (DLC). Book equity (BE) is stockholder's book equity (SEQ), plus balance sheet deferred taxes and investment tax credit (TXDITC) if available, minus the book value of preferred stock (PSTK/PSTKRV/PSTKL depends on availability). Market equity (ME) is defined as the price of stock times share outstanding (SHROUT). Book leverage denominated by book asset is defined as FD/AT, book leverage denominated by book equity is defined as FD/(BE+FD), market leverage is FD/(FD+ME), book to market ratio (BM) is BE/ME. Asset turnover is defined as sales (SALES) to book assets (AT), return on assets is net income (NI) to total asset (AT). Investment rate is defined as physical investment (CAPX) denominated by total asset (AT). Firm age is the years since a firm has record in COMPUSTAT. WW and SA indices are financial constraint measures, they are from Whited and Wu (2006) and Hadlock and Pierce (2010), respectively. The firms are more financially constrained when the WW and SA index are assigned with higher values. Prob(Dividend) is the probability of a firm paying dividend in a quintile.

Panel A: Data

	1	2	3	4	5
Collateralizability	0.070	0.118	0.160	0.216	0.733
FD/AT	0.150	0.194	0.211	0.227	0.234
FD/(FD+BE)	0.202	0.255	0.279	0.297	0.305
FD/(AT-BE+ME)	0.113	0.158	0.178	0.194	0.200
BM	0.644	0.750	0.813	0.834	0.842
Size	4.669	5.219	5.391	5.506	5.538
SA	-2.608	-2.938	-3.053	-3.083	-3.069
WW	-0.211	-0.273	-0.276	-0.198	-0.307
Prob(Dividend)	0.333	0.444	0.513	0.539	0.515

Panel B: Model

	1	2	3	4	5
Collateralizability	0.116	0.259	0.375	0.504	0.697
Book Leverage	0.141	0.315	0.454	0.611	0.844
Market Leverage	0.154	0.328	0.455	0.585	0.759
BM	1.122	1.068	1.005	0.899	0.491
Size	0.480	0.594	0.768	1.092	3.073

Table 5: **Collateralizability Spread, Data and Model Comparison**

This table reports the monthly excess stock returns and their statistics. The table reports monthly average excess returns R^e, standard errors σ, t-statistics (t) across portfolios. Panel A reports the 5 quintile portfolios sorted on colalteralizability on constrained firms, classified by the SA index. Panel B reports 5 quintile portfolios sorted on simulated data.

Panel A: Data, constrained firms

Value-weighted Portfolios

	1	2	3	4	5	5-1
$R^e(\%)$	0.90	0.83	0.86	0.66	0.37	0.54
(t)	2.54	2.39	2.76	2.36	1.23	2.21
$\sigma(\%)$	7.77	7.58	6.80	6.09	6.49	5.28

Equal-weighted Portfolios

	1	2	3	4	5	5-1
$R^e(\%)$	1.60	1.33	1.21	1.03	0.89	0.71
(t)	4.26	4.35	4.25	3.80	3.19	3.04
$\sigma(\%)$	8.17	6.66	6.18	5.90	6.09	5.05

Panel B: Model

Value-weighted Portfolios

	1	2	3	4	5	5-1
$R^e(\%)$	0.78	0.73	0.63	0.50	0.35	0.44
(t)	8.87	9.15	9.40	9.78	10.66	7.59
$\sigma(\%)$	2.09	1.91	1.60	1.23	0.78	1.39

Equal-weighted Portfolios

	1	2	3	4	5	5-1
$R^e(\%)$	0.78	0.73	0.63	0.48	0.34	0.43
(t)	8.89	9.14	9.41	9.74	10.80	7.59
$\sigma(\%)$	2.11	1.91	1.59	1.18	0.76	1.35

the portfolio returns in the data, while Panel B reports the model counterparts. We observe that the model can generate a return spread of low minus high collateralizability portfolios of 0.44% per month, comparable to 0.54% per month in the data under the value-weighted scheme. The similar comparison applies for equal weighted portfolio returns.

5.3 Conditional collateralizability spread

In this section, we test an additional model implication on the conditional collateralizability spread. Our model predicts that when the collateral constraint is more binding, the conditional collateralizability spread increases. This is due to the time varying risk premium channel. When constraint binds more, the financial constraint mechanism becomes stronger, and, in turn, the covariance of realized return spread and the SDF becomes larger in magnitude, and leads to a higher conditional spread.

In order to formally test this prediction, we follow Frazzini and Pedersen (2014), and use the TED spread (i.e. the interest rate spread between LIBOR rate and the Tbill rate) as the proxy for the aggregate financing constraint measure. When TED spread is high, the aggregate financing condition in the economy becomes tighter. In the model, we map the TED spread to the interest rate spread between the shadow interest rate among entrepreneurs and that of the household risk-free loan. With this aggregate financing constraint measure, we run predictive regression of the interest rate spread on the next period's collateralizability spread both in the data and model.

Table 6 reports regression results of our tests. We make two observations from the table. First, from the model simulation (Panel B), the positive and significant predictive coefficient confirms our intuition. Second, from the data, in particular, we document the predictive coefficients for the sub-sample of financially constrained firms are positive and significant at least at 10% level. The fact that the positive predictability is quantitatively weaker, but still present, among the financially unconstrained firms is also consistent with our theory, since in the model the entrepreneurs are forward looking and will factor the probability of being financially constrained in the future in the current asset valuation.

In summary, the empirical tests of the conditional spread in this section provides direct evidence that further supports our key model mechanism on the key role of financial frictions in generating the collateralizability spread.

Table 6: **Predictive Regression of the Collateralizability Spread**

This table shows one-month ahead predictive regression of aggregate financing constraint measure, proxied by the TED spread, on collateralizability spread. TED spread is defined as the interest rate spread between LIBOR rate and the Tbill rate. Both are obtained St. Louis Fed. The column "Measure" indicates what we use to classify firms into unconstrained and constrained groups. We use SA, WW and dividend paying dummy to group firms into unconstrained and constrained groups. Column "uncons." indicates we use L-H portfolio sorted on unconstrained firms. Column "cons." indicates we use L-H portfolio sorted on constrained firms. Panel B reports the predictive regression from the simulated data. Numbers in brackets are standard errors adjust for serial correlation.

	Measure	cons.	uncons.
Panel A: Data			
β_{TED}	SA	0.190**	0.059
$s.e.$		[0.085]	[0.068]
β_{TED}	WW	0.142*	0.065
$s.e.$		[0.086]	[0.066]
β_{TED}	Dividend	0.186*	0.064
$s.e.$		[0.095]	[0.077]
Panel B: Model			
β_{TED}	Model	0.25***	
$s.e.$		[0.011]	

standard errors in brackets
* p<0.10, ** p<0.05, *** p<0.01

6 Conclusion

In this study, we present a general equilibrium asset pricing model with collateral constraint and two types of assets which differ in their collateralizability. Our model predicts that the collateralizable asset provides an insurance against aggregate shocks and therefore is expected to earn lower expected return, because it relaxes the countercyclical collateral constraint in bad times. We measure non-collateralizabile capital based on structure, equipment and intangible capital, and document the empirical evidence which is consistent with the predictions of our model. In particular, we find in the data that stock of more constrained firms with a larger share of non-collateralizable capital earn on average 6.3% higher return annually than those of firms with a lower share. When calibrate our model to standard statistics of the dynamics of macroeconomic quantities, we show that the credit market friction channel is quantitatively important at determining the cross-section of asset returns.

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7 Appendix

7.1 Empirical details on collateralizability measure

This section provides empirical details on the construction of the firm specific collateralizability measure, and it is complementary to the methodology of the measurement laid down in Section 2.

In the empirical implementation, we first find empirical proxies for the share of each type of capital, and run the empirical counterpart of the leverage regression as in equation (2.1), and then calculate the firm specific "collateralizability score".

Our major sources of information are from (1) the firm level balance sheet data in CRSP/Compustat Merged Database Annual Industrial Annual Files; (2) Monthly stock returns in the CRSP dataset; and (3) Industry level capital stock data is from the BEA table: fixed assets by industry. We adopt the standard screen process for the CRSP/Compustat Merged Database. Since 1975, FASB standardize the accounting rule of R&D expenditures, therefore the sample starts from 1975. We exclude utility (SIC code between 4900 and 4999) and financial firms (SIC between 6000 and 6999). Additionally, we keep common stocks that are traded on NYSE, AMEX and NASDAQ. Firms with negative book value of assets, book equity or sales are excluded. Delisting bias is corrected following Shumway (1997).

In order to obtain a long sample with broader coverage[9], we use BEA narrowly defined industry level non-residential fixed asset (structure, equipment and intellectual) to back out industry level structure and equipment capital shares. The BEA industry classification is of 63 industries. We match the BEA industry level measure of structure and equipment capital shares to COMPUSTAT firm level data using NAICS code, assuming that, for a given year, firms in the same industry have the same structure and equipment capital shares. Since fixed assets in BEA tables have a comparable notion as the PPENT in COMPUSTAT, we multiply BEA structure and equipment share with industry level PPENT to book asset (AT) ratio, so that this measure has the same denominator as in the book leverage regression. Therefore, the final measure of structure and equipment share for industry j at the end of year t are defined as

[9]COMPUSTAT only splits the composites of physical capital (PPENT) between 1969 and 1997. However, even for years between 1969 and 1997, only 40% observations have non-missing record for PPENB, PPENME and PPENLI.

$$
\begin{aligned}
StructShare_{j,t} &= \frac{\text{Structure}^{BEA}_{j,t}}{\text{Fixed Asset}^{BEA}_{j,t}}\frac{PPENT_{j,t}}{AT_{j,t}} \\
EquipShare_{j,t} &= \frac{\text{Equipment}^{BEA}_{j,t}}{\text{Fixed Asset}^{BEA}_{j,t}}\frac{PPENT_{j,t}}{AT_{j,t}},
\end{aligned}
$$

where $\text{Structure}^{BEA}_{j,t}$ and $\text{Equipment}^{BEA}_{j,t}$ are value of structure and equipment capital from BEA table, respectively. $\text{Fixed Asset}^{BEA}_{j,t}$ is the total value of non-residential fixed asset.

Comparable to theoretically motivation regression equation (2.1), we run the following regression,

$$
\frac{B_{i,t}}{Asset_{i,t}} = c+\zeta_{str}StructShare_{j,t}+\zeta_{equ}EquipShare_{j,t}+\gamma X_{i,t}+\sum_j Industry_j+\sum_t Year_t+\varepsilon_{i,t},
$$

where j denotes the industry index and i denotes the firm index. $X_{i,t}$ are typical controls in the capital structure regressions, including size, book-to-market ratio, profitability, marginal tax rate, earnings volatility and bond rating. B_{it} is the total debt defined as long term debt (DLTT) plus short term debt (DLC). For robustness, we try both book leverage and market leverage regressions. If the dependent variable is book leverage, then $Asset_{it}$ is book asset (AT), if the dependent variable is market leverage, then $Asset_{it}$ is book asset plus market value of equity less book equity (AT - BE + PRCC_F*CSHO). The regression results are shown in Table 7 and Table 8 for book and market leverage ratio regressions, respectively. As we can see in both of the regressions, for financial constrained firms, there is significant asymmetry in collateralizability of structure versus equipment. In particular, structure enjoys higher collateralizability and can support more debt. The evidence here is in line with the that of Campello and Giambona (2013).

We interpret $\zeta_{str}StructShare_{jt} + \zeta_{equ}EquipShare_{jt}$ as the contribution of structure and equipment capital to financial leverage, we interpret the product of this term with book value of assets as the measure of collateralizable capital. [10]The collateralizability of firm i in year t is defined as

$$
\zeta_{i,t} = \frac{(\zeta_{str}StructShare_{j,t} + \zeta_{equ}EquipShare_{j,t})AT_{i,t}}{PPENT_{i,t} + Intangible_{i,t}}
$$

[10]We also use market value of asset as an alternative. If we construct collateralizability in that way, the empirical collteralizability spread based on this sorting measure is even stronger.

Table 7: **Capital Structure Regressions (Book Leverage)**

This table reports regression results for book leverage regression. *Struct Share* and *Equip Share* are constructed using BEA and Compustat data, as defined in Section **??**. *Size* is the market capitalization of a firm, *BM* is book-to-market ratio, Profitability is defined as OIBDP/AT. *Marginal Tax Rate* is following Graham (2000), from John Graham's website. *Earnings Volatility* is computed using 4-year windows of consecutive firm observations of *Profitability*. *RatingDummy* is a dummy variable that takes a value of 1 if the firm has either a bond rating (splticrm) or a commercial paper rating (spsticrm), and 0 otherwise.

	(1) Full	(2) Non-Dividend	(3) Dividend	(4) SA1	(5) SA2	(6) WW1	(7) WW2
Struct Share	0.215***	0.168***	0.281***	0.196***	0.262***	0.173***	0.291***
	(11.41)	(7.27)	(9.93)	(9.80)	(7.00)	(8.38)	(9.74)
Equp Share	0.0863***	0.0945***	0.0695***	0.100***	0.0750**	0.110***	0.0549*
	(4.68)	(4.00)	(2.66)	(5.01)	(2.28)	(5.38)	(1.83)
Size	-0.0207***	-0.0121***	-0.0289***	-0.0265***	-0.0471***	-0.0232***	-0.0325***
	(-20.20)	(-8.82)	(-19.26)	(-19.85)	(-20.24)	(-17.09)	(-15.92)
BM	0.00763***	0.0118***	0.000882	0.000310	-0.0368***	0.00530*	-0.00976***
	(3.60)	(3.21)	(0.33)	(0.11)	(-8.93)	(1.84)	(-3.02)
Profitability	-0.0920***	-0.235***	-0.0310***	-0.299***	-0.0000518	-0.329***	-0.0251**
	(-9.37)	(-8.32)	(-3.05)	(-13.18)	(-0.01)	(-12.57)	(-2.48)
Marg Tax Rate	0.0896***	0.0918***	0.0837***	0.111***	0.0146	0.129***	0.0338**
	(7.21)	(4.14)	(5.75)	(6.55)	(0.88)	(6.82)	(2.21)
Earning Vol	-0.308***	-0.404***	-0.259***	-0.291***	-0.171***	-0.282***	-0.235***
	(-14.07)	(-7.14)	(-11.58)	(-5.97)	(-8.15)	(-6.00)	(-9.95)
Rating Dummy	0.166***	0.134***	0.202***	0.157***	0.290***	0.145***	0.255***
	(38.60)	(25.05)	(30.89)	(35.30)	(10.98)	(31.78)	(26.63)
Constant	0.274***	0.216***	0.329***	0.334***	0.366***	0.341***	0.298***
	(11.38)	(6.97)	(8.32)	(10.10)	(7.99)	(10.26)	(10.14)
Observations	60811	31292	29450	41747	19064	35353	25150
r2	0.205	0.207	0.225	0.264	0.162	0.263	0.181

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Table 8: **Capital Structure Regressions (Market Leverage)**

This table reports regression results for book leverage regression. *Struct Share* and *Equip Share* are constructed using BEA and Compustat data, as defined in Section 7.1. *Size* is the market capitalization of a firm, *BM* is book-to-market ratio, Profitability is defined as OIBDP/AT. *Marginal Tax Rate* is following Graham (2000), from John Graham's website. *Earnings Volatility* is computed using 4-year windows of consecutive firm observations of *Profitability*. *RatingDummy* is a dummy variable that takes a value of 1 if the firm has either a bond rating (splticrm) or a commercial paper rating (spsticrm), and 0 otherwise. Standard errors are clustered at firm year level.

	(1) Full	(2) Non-Dividend	(3) Dividend	(4) SA1	(5) SA2	(6) WW1	(7) WW2
Struct Share	0.220***	0.177***	0.279***	0.211***	0.231***	0.185***	0.283***
	(12.03)	(7.88)	(10.30)	(10.68)	(6.83)	(9.01)	(10.09)
Equp Share	0.0734***	0.0810***	0.0577**	0.0919***	0.0605**	0.103***	0.0383
	(4.38)	(3.76)	(2.48)	(5.14)	(2.05)	(5.60)	(1.42)
Size	-0.0218***	-0.0144***	-0.0294***	-0.0308***	-0.0376***	-0.0271***	-0.0299***
	(-21.26)	(-11.18)	(-19.78)	(-22.59)	(-19.83)	(-18.50)	(-17.46)
BM	0.0761***	0.0858***	0.0651***	0.0594***	0.0528***	0.0649***	0.0680***
	(17.92)	(15.38)	(12.24)	(12.04)	(14.10)	(10.00)	(19.82)
Profitability	-0.0932***	-0.240***	-0.0276***	-0.315***	0.000491	-0.345***	-0.0228***
	(-11.05)	(-8.79)	(-3.57)	(-13.51)	(0.08)	(-11.92)	(-3.07)
Marg Tax Rate	0.114***	0.104***	0.110***	0.116***	0.0502***	0.132***	0.0644***
	(10.64)	(5.10)	(9.05)	(7.49)	(3.83)	(7.66)	(5.07)
Earning Vol	-0.326***	-0.405***	-0.274***	-0.336***	-0.152***	-0.298***	-0.234***
	(-15.23)	(-7.10)	(-12.97)	(-7.13)	(-9.61)	(-6.07)	(-11.72)
Rating Dummy	0.139***	0.111***	0.171***	0.135***	0.244***	0.123***	0.219***
	(35.87)	(23.86)	(28.71)	(33.67)	(9.93)	(29.76)	(23.89)
Constant	0.186***	0.101***	0.193***	0.238***	0.244***	0.235***	0.168***
	(9.09)	(4.22)	(5.45)	(9.16)	(6.32)	(9.48)	(6.33)
Observations	60811	31292	29450	41747	19064	35353	25150
r2	0.344	0.347	0.365	0.414	0.279	0.413	0.320

t statistics in parentheses
* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

where the nominator $(\zeta_{str} StructShare_{i,t} + \zeta_{equ} EquipShare_{i,t}) AT_{i,t}$ is the collateralizable capital, $Intangible_{it}$ is the intangible capital of firm i in year t. In recent literature, e.g. Eisfeldt and Papanikolaou (2013), Peters and Taylor (2016) etc, addressing the importance of taking into account of intangible capital. In the above collateralizability measure, we implicit assume the collateralizability parameter for intangible capital is equal to zero. There is empirical evidence that intangible capital can hardly be used as collateral, only 3% of total loan value is written on intangibles like patents or brands (Falato et al. (2013)). In our benchmark collateralizability measure we take into account the intangible capital measure, but our results remain qualitatively very similar if we exclude intangible capital in collateralizability measure, and only exploiting the asymmetric collateralizability between structure and equipment within tangible assets.

The details about measuring firm specific intangible capital will be provided in Appendix 7.3.

7.2 More evidence on collateralizability and expected return

In this section, we provide more evidence and robustness check for the relationship between collateralizability and expected return. This section is complementary to the evidence on collateralizability spread provided in Section 2.

As a robustness check, we try alternative measures of financial constraints, including the Whited and Wu measure (Whited and Wu (2006)), and the criteria whether a firm pays positive dividend or not in a given year. Table 9 shows that the return spreads among financially constrained firms are all economically large and statistically significant, and this phenomenon is robust to alternative financial constraint measures. The second panel of Table 9 shows that the dispersion in returns across collateralizability sorted portfolios is generally even stronger and statistically more significant for equal-weighted scheme.

The cross-sectional return spread across collateralizability sorted portfolios is distinct from Carhart four factors (Carhart, 1997), and Fama and French (2015) five factors. Table 10 reports the alphas for the whole sample and the subsamples based on financial constraint measure. The low collateralizability minus high collateralizability portfolio has an alpha of 0.53% and 0.50% per month when we use (Carhart, 1997) model and Fama French five factor model respectively. Furthermore, the alphas for the subset of financially constrained firms, classified by the SA index, display a higher magnitude and stronger statistical significance that those of the whole sample. The GRS tests reject that the alphas are jointly zero for both (Carhart, 1997) four factor model and Fama French five factor model, with p-values

Table 9: **Univariate Portfolio Sorting (Different Financial Constraints Measure)**

This table reports the portfolio sort results on constrained and unconstrained firms, classified by SA, dividend payment dummy and WW index. Column labeled with "uncons". implies that the result in this column is from unconstrained sample. Column labeled with cons. implies that the result in this column is from constrained sample.

	SA index		Dividend		WW Index	
	uncons.	cons.	uncons.	cons.	uncons.	cons.
			Value Weighted Portfolios			
(1)-(5)	0.23	0.54	0.09	0.40	0.19	0.43
t-stat	1.39	2.21	0.53	2.22	1.23	1.94
			Equal Weighted Portfolios			
(1)-(5)	0.37	0.70	0.39	0.65	0.42	0.70
t-stat	2.87	3.04	3.02	3.32	4.23	3.37

0.001 and 0.003, respectively.

Univariate portfolio sorts show us that on average, portfolios of firms with more non-collateralizable capital have higher expected returns, without controlling any other firm heterogeneity. In order to address this issue at firm level, we perform Fama-Macbeth regressions in the following form:

$$R_{i,t+1} = \alpha^i + \beta_1 log(ME_{it}) + \beta_2 log(BM_{it}) + \beta_3 Mom_{it} + \beta_4 ROA_{it} + \beta_5 Collateralizability_{it} + BLEV_{it} + \varepsilon_{it}$$

where $R_{i,t+1}$ is individual stocks' cumulative returns from July of year t to June of each year $t+1$, $log(ME_{it})$ is the nature log of firms' market capitalization at the end of June of each year t, $log(BM_{it})$ is the firms' book to market ratio at the end of June of each year t, Mom_{it} is the prior six month returns with a one-month gap between holding period and current month, ROA_{it} is income before extraordinary (IB) divide by total asset (AT) at June of year t. BLEV is financial debt (FD) divided by total asset. To avoid using future information, all the balance sheet variables measured at June of year t is using the value of fiscal year $t-1$, which is already known by investors.

Table 11 reports the results of Fama-Macbeth regression. The model indicates a significantly negative slope on collateralizability. This evidence supports our theory that if a firm has lower colateralizabity, it demands higher returns.

7.3 Measuring intangible capital

In this section, we provide details on the construction of firm specific intangible capital, used in our empirical measure of collateralizability, as in Appendix 7.1.

Table 10: **Asset Pricing Tests (sort on collateralizability)**

This table shows asset pricing test for five value weighted portfolios sorted on collateralizability measure. In Panel A, we regress the five quintile portfolios on Carhart four factor model, in Panel B we regress five portfolios on Fama French four factor model. To take in to account serial correlation, the *t*-statistics in parentheses are computed via Newey-West estimator allowing for three lags. All values reported here are in monthly frequency. We report the asset pricing test on the whole sample, we also split the sample into constrained firms and unconstrained firms using SA index.

Panel A: Carhart Four-Factor Model

	1	2	3	4	5	1-5
			Whole sample			
α	0.31	0.08	0.05	0.03	-0.20	0.50
(t)	3.29	0.95	0.75	0.41	-1.58	2.97
			Financially constrained firms, SA index			
α	0.28	0.20	0.23	-0.03	-0.40	0.68
(t)	1.92	1.51	2.05	-0.31	-2.65	3.05
			Financially unconstrained firms, SA index			
α	0.30	0.10	0.15	0.08	-0.12	0.42
(t)	3.49	1.19	2.24	1.03	-1.11	2.55

Panel B: Fama-French Five-Factor Model

	1	2	3	4	5	1-5
			Whole sample			
α	0.25	0.10	-0.01	-0.07	-0.28	0.53
(t)	2.75	1.20	-0.09	-1.08	-1.64	2.53
			Financially constrained firms, SA index			
α	0.51	0.38	0.28	-0.07	-0.56	1.07
(t)	3.12	2.86	2.42	-0.81	-3.08	4.08
			Financially unconstrained firms, SA index			
α	0.23	-0.01	0.12	-0.05	-0.26	0.49
(t)	2.62	-0.12	1.69	-0.56	-2.28	2.73

Table 11: Return Spread: Predictive Regression

$R_{i,t}$ is individual stocks' cumulative return from July of year t to June of year $t+1$, $log(ME)$ is the nature log of firms' market capitalization at the end of June of year t, $log(BM)$ is the firms' book to market ratio at the end of June of year t, Mom is the prior six month returns with a one-month gap between holding period and current month, ROA is income before extraordinary (IB) divide by total asset (AT) at June of year t. Book leverage is financial debt (FD) divided by total asset. All the balance sheet variables measured at June of year t is using the value of fiscal year $t-1$. Values in parenthesis are t-statistics estimated by Newey-West estimator allowing for three lags. Column labeled with SA, WW and Dividend refer to the constrained firms measured by the corresponding measures.

$$R_{i,t+1} = \alpha^i + \beta_1 log(ME_{it}) + \beta_2 log(BM_{it}) + \beta_3 Mom_{it} + \beta_4 ROA_{it} + \beta_5 Collateralizability_{it} + \beta_6 Lev_{it} \varepsilon_{it}.$$

	Cons.(SA)	Cons.(WW)	Cons.(Div)	All
log(ME)	-0.0903***	-0.0813***	-0.0311***	-0.0213***
	(-4.88)	(-6.01)	(-4.16)	(-3.99)
log(BM)	0.0291**	0.0360***	0.0463***	0.0340***
	(2.69)	(3.60)	(4.67)	(2.88)
Momentum	0.0334***	0.0313*	0.0564**	0.0531**
	(2.79)	(1.81)	(2.72)	(2.70)
ROA	-0.00880	-0.0340	-0.0414	-0.0322
	(-0.20)	(-0.74)	(-0.94)	(-0.63)
Collateralizability	-0.0522***	-0.0891***	-0.0797***	-0.0178*
	(-4.05)	(-3.19)	(-3.74)	(-1.89)
Book Leverage	-0.0788	-0.0339	-0.0224	0.0224
	(-1.41)	(-0.70)	(-0.42)	(0.53)
Constant	-0.301	1.248	0.477	-0.233
	(-0.16)	(0.60)	(0.23)	(-0.19)
Obs	32978	38122	45846	88535
R2	0.0570	0.0577	0.0475	0.0517

t statistics in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

The total amount of intangible capital of a firm is defined as

$$\text{Intangible capital} = \text{INTAN} + \text{R\&D capital} + \text{Organizational capital}$$

7.3.1 Externally Acquired Intangible Capital

The externally acquired capital is defined as item *INTAN* in COMPUSTAT. It reflects the amount of intangible capital a firm purchases in a given fiscal year. Firms typically capitalize this type of asset on the balance sheet as part of intangible assets. For an average firm, *INTAN* is only 19% of total intangible capital, meanwhile the median value is just 3%, consistent with Peters and Taylor (2016). I set INTAN to zero when missing.

Internally created intangible capital has two components, R&D and organizational capital, we discuss the methods to recover them from firms' balance sheets in the following sections.

7.3.2 R&D capital

Internally created R&D capital does not appear on firm's balance sheet, one can estimate it by accumulating past expenditures. R&D expenditure is from COMPUSTAT item *XRD*, "Research and Development Expense", it represents the amount of expenditures on research and development of a firm in a given fiscal year. Following Falato et al. (2013) and Peters and Taylor (2016), we recover R&D capital using perpetual inventory method[11]:

$$RD_{t+1} = (1 - \delta_{RD})RD_t + XRD_t$$

where δ_{RD} is the depreciation rate of R&D capital, consistent with Peters and Taylor (2016), the depreciation rates are following Li and Hall (2016), which is also published on BEA R&D satellite account. For unclassified industries, the depreciation rates are set to 15%. Our results are not sensitive to the choice depreciation rates.

However, this is not enough to identify the stock of capital, the initial value for R&D capital is still undefined. We use the first non-missing R&D expenditure, XRD, as the first R&D investment, then the initial stock of R&D capital is specified as,

$$RD_0 = \frac{XRD_1}{g_{RD} + \delta_{RD}} \tag{19}$$

where g_{RD} is the average annual growth rate of firm level R&D expenditure, which is 29.1%

[11]It is also used by BEA R&D satellite account.

in my sample. The sample starts from 1975, since the accounting treatment of R&D expense reporting was standardized in 1975, the amount of XRD reported by firms may not be comparable to each other before this standard is adopted, therefore previous R&D expenditures are not taken into account.

7.3.3 Organizational Capital

Another internally created component is organizational capital, it is constructed by accumulating a fraction of past SGA expense, COMPUSTAT item *XSGA*, "Selling, General and Administrative Expense". It includes lots of items, e.g. marketing expense, employee benefit, etc. It indirectly reflects reputation or human capital of a firm. Additionally it includes R&D expenses unless it's included in cost of goods sold by the company, therefore we need to exclude the R&D part from XSGA.

Peters and Taylor (2016) document that XSGA (Selling, General and Administrative Expense) includes R&D expense unless the company record R&D expense as cost of goods sold (Compustat item *COGS*), and Compustat adds R&D to *XSGA* in 90 out of 100 cases. Additionally *XSGA* do not incorporate the in process R&D expense (Compustat item *RDIP*), *RDIP* is coded as negative numbers. To exclude R&D capital from organizational capital, following Peters and Taylor (2016), I define $SGA \equiv XSGA - XRD - RDIP$, where the absolute value of $RDIP$ is basically added to SGA. Additionally, following Peters and Taylor (2016), we add a filter: when XRD exceeds XSGA but is less than COGS, or when XSGA is missing, we keep XSGA with no further adjustment. we replace missing XSGA with zero. Following Hulten (2008), Eisfeldt and Papanikolaou (2014) and Peters and Taylor (2016), we count only 30% of SGA expense as investment in organizational capital, the rest 70% is treated as operating costs.

Using the same procedure, the organizational capital is constructed as,

$$
\begin{aligned}
SGA_t &= 0.3(XSGA_t - XRD_t - RDIP_t) \\
OG_{t+1} &= (1 - \delta_{OG})OG_t + SGA_t
\end{aligned}
$$

where δ_{SGA} is set at 20%, consistent with Falato, Kadyrzhanova, Sim, Falato, and Sim (2013) and Peters and Taylor (2016), g_{RD} is the average annual growth rate of firm level XSGA. I set initial level of organizational capital as

$$
OG_0 = \frac{SGA_1}{g_{OG} + \delta_{OG}}
$$

where $g_{OG} = 18.9\%$ in the sample.