Traffic in the City
The Impact of Infrastructure Improvements in the Presence of Endogenous Traffic Congestion

Treb Allen\textsuperscript{1} Costas Arkolakis\textsuperscript{2}

\textsuperscript{1}Dartmouth and NBER \textsuperscript{2}Yale and NBER

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\textsuperscript{1}An excerpt from “The Welfare Effects of Transportation Infrastructure Improvements”
Motivation

- Recent “quantitative” revolution in urban economics
  - Spearheaded by flexible theory (Ahlfeldt Redding Sturm Wolf ’15)
  - Fueled with swaths of spatial data

- The “elephant in the room”: Roads
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• Key benefit: evaluation of major transportation infrastructure
  • Trains (Heblich Redding Sturm ’20), subways (Severen ’19), dedicated bus lanes (Tsivanidis ’19)

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A New Quantitative Urban Framework with Traffic

- Quantitative urban framework:
  - Agents choose where to live, where to work, & commuting route.
  - Commute/route choice $\rightarrow$ traffic $\rightarrow$ congestion $\rightarrow$ commuting costs

Retains the old benefits...

- Analytically tractable
- "Exact hat" link to data to perform counterfactuals

...But with new benefits too:

- A gravity equation for traffic.
- Counterfactuals use (easily observed) traffic data.
- Scale matters.

Illustration: Estimate ROI of adding lane-miles to every link in Seattle road network.
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- Illustration: Estimate ROI of adding lane-miles to every link in Seattle road network.
Related literature

• Quantitative evaluations of transportation infrastructure
  • Donaldson ’12, Allen and Arkolakis ’14, Ahlfeldt et. al. ’15,
    Donaldson and Hornbeck ’16, Alder ’16, Severen ’19, Tsivanidis ’19,
    Heblich Redding Sturm ’20

• Empirical evidence on importance of congestion
  • Duranton and Turner ’11, Anderson ’14

• Optimal transportation policy computationally
  • Alder ’16, Fajgelbaum and Schaal ’20
Outline of Talk

Introduction

**A Quantitative Urban Framework with Traffic Congestion**
- Model setup
- The Routing Problem
- Traffic and Congestion
- Equilibrium
- Implications of Traffic Congestion
- Counterfactuals

The welfare impacts of improving the Seattle road network

Conclusion
Standard components

- City comprises $i \in \{1, \ldots, N\} \equiv \mathcal{N}$ locations, $\bar{L}$ agents.
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- Agents choose where to live & work, yielding *commuting gravity*:

$$L_{ij} = \left(\left(\frac{\text{amenity at home}}{u_i} \times \frac{\text{productivity at work}}{A_j} \right)^\theta\right) \times \left(\frac{\text{aggregate population}}{\bar{L}}\right) \times \left(\frac{\text{expected welfare}}{W^\theta}\right)$$
**Standard components**

- City comprises \( i \in \{1, \ldots, N\} \equiv \mathcal{N} \) locations, \( \bar{L} \) agents.

- Agents choose where to live & work, yielding *commuting gravity*:

\[
L_{ij} = \left( \frac{\text{amenity at home } u_i \times \text{productivity at work } A_j}{\tau_{ij} \text{ commuting cost}} \right)^\theta \times \left( \frac{\text{aggregate population } \bar{L}}{\text{expected welfare } W^\theta} \right)
\]

- Productivities and amenities in each location can be written as:

\[
A_i = \bar{A}_i \times \left( L_i^F \right)^\alpha \\
u_i = \bar{u}_i \times \left( L_i^R \right)^\beta
\]
Standard components

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- Productivities and amenities in each location can be written as:

$$A_i = \bar{A}_i \times (L_i^F)^\alpha$$

$$u_i = \bar{u}_i \times (L_i^R)^\beta$$

- Given elasticities $\{\alpha, \beta, \theta\}$, geography $\{\bar{A}_i, \bar{u}_i\}$, and costs $\tau_{ij}$, equilibrium is $\{L_i^F, L_i^R\}$ such that:

$$L_i^R = \sum_{j \in \mathcal{N}} L_{ij}, \quad L_i^F = \sum_{i \in \mathcal{N}} L_{ij}$$
New component: Endogenous commuting costs

- Commuting costs $\tau_{ij}$ are *endogenous*, depend on:
  - Agents’ routing problem: What is the optimal path through the infrastructure network (taking traffic as given)?
  - Traffic congestion: How do agents’ route choice, choice of where to live and work affect use of each link in the infrastructure network?
New component: Endogenous commuting costs

- Commuting costs $\tau_{ij}$ are *endogenous*, depend on:
  - Agents’ routing problem: What is the optimal path through the infrastructure network (taking traffic as given)?
  - Traffic congestion: How do agents’ route choice, choice of where to live and work affect use of each link in the infrastructure network?
  - Feedback loop: traffic congestion affects route choice & choice of where to live and work.
Infrastructure network

- $N$ locations arrayed on a weighted network.

Let $t_{kl} \geq 1$ be the iceberg commuting cost incurred by traveling from $k$ to $l$ on the infrastructure network, where:

$$t_{kl} = \bar{t}_{kl} \times (\Xi_{kl}) \lambda \quad (1)$$

- $\bar{t}_{kl} \geq 1$ is the (first nature) quality of the infrastructure connection.
- If $\bar{t}_{kl} < \infty$, we say that $k$ and $l$ are a link.
- $\Xi_{kl}$ is the traffic on link $k$ to $l$.
- $\lambda$ is strength of traffic congestion ($\lambda = 0$ in a standard model).
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Example of infrastructure network
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  Model setup
  The Routing Problem
  Traffic and Congestion
  Equilibrium
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The welfare impacts of improving the Seattle road network

Conclusion
The routing choice problem

- A route from $i$ to $j$ of length $K$ is a sequence of locations beginning with $i$ and ending with $j$:

$$r = \{i, r_1, r_2, \ldots, r_{K-1}, j\}$$
The routing choice problem

• A route from \( i \) to \( j \) of length \( K \) is a sequence of locations beginning with \( i \) and ending with \( j \):

\[
 r = \{i, r_1, r_2, \ldots, r_{K-1}, j\}
\]

• Let \( \mathcal{R}_{ij} \) be the set of all possible routes from \( i \) to \( j \). Then a route \( r \in \mathcal{R}_{ij} \) incurs an iceberg cost of:

\[
\tau_{ij, r} = \prod_{l=1}^{K} t_{r_{l-1}, r_l}
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The routing choice problem

- A *route* from $i$ to $j$ of length $K$ is a sequence of locations beginning with $i$ and ending with $j$:

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- Let $\mathcal{R}_{ij}$ be the set of all possible routes from $i$ to $j$. Then a route $r \in \mathcal{R}_{ij}$ incurs an iceberg cost of:

$$\tau_{ij,r} = \prod_{l=1}^{K} t_{r_{l-1},r_l}$$

- Assume agents choose where to live, where to work, & route to maximize:

$$V_{ij,r}(\nu) = \left( A_j u_i / \prod_{l=1}^{K} t_{r_{l-1},r_l} \right) \times \varepsilon_{ij,r}(\nu).$$

with Frechet distributed idiosyncratic shock $\varepsilon_{ij,r}(\nu)$. 
Endogenous commuting costs

- Solving the maximization problem and summing across all possible routes from $i$ to $j$ yields commuting gravity equation from above:

$$L_{ij} = \left( \frac{u_i \times A_j}{\tau_{ij}} \right)^\theta \times \left( \frac{\bar{L}}{W^\theta} \right)$$

where:

$$\tau_{ij} \equiv \left( \sum_{r \in \mathcal{R}_{ij}} \left( \prod_{l=1}^{K} t_{r_{l-1}, r_l} \right)^{-\theta} \right)^{-\frac{1}{\theta}}$$

is the *endogenous* commuting cost.
An analytical solution

• Define the *weighted adjacency matrix* \( A \equiv \begin{bmatrix} a_{ij} \equiv t_{ij}^{-\theta} \end{bmatrix} \).

• Define \( B \equiv (I - A)^{-1} \) and \( b_{ij} \equiv [B]_{ij} \).
An analytical solution

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- If $\rho(A) < 1$ then:
  \[ \tau_{ij} = cb_{ij}^{-\frac{1}{\theta}} \]  
  (2)
- Mapping from infrastructure network to commuting costs (!)
An analytical solution

• Define the weighted adjacency matrix $\mathbf{A} \equiv \begin{bmatrix} a_{ij} & \equiv t_{ij}^{-\theta} \end{bmatrix}$.

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• Notes:

• As $\theta \to \infty$, $\tau_{ij}$ converge to commuting cost for least cost route (generalization of Dijkstra algorithm).
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- Mapping from infrastructure network to commuting costs (!)

- **Notes:**
  - As $\theta \to \infty$, $\tau_{ij}$ converge to commuting cost for least cost route (generalization of Dijkstra algorithm).
  - Analogy to path integral formulation of quantum mechanics: “space of all possible paths of the system in between the initial and final states, including those that are absurd by classical standards”
Outline of Talk

Introduction

A Quantitative Urban Framework with Traffic Congestion
  Model setup
  The Routing Problem
  Traffic and Congestion
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The welfare impacts of improving the Seattle road network

Conclusion
From routing to traffic

• Equation (2) yields the commuting cost taking traffic congestion as given. But what is the equilibrium traffic?

\[ \pi_{kl}^{ij} = (\tau_{ij} \times t_{kl} \times \tau_{lj}) / \theta \]

Intuition: More out of the way links are used less.
From routing to traffic

- Equation (2) yields the commuting cost taking traffic congestion as given. But what is the equilibrium traffic?

- First step: calculate the intensity with which a particular link is used on the way from \( i \) to \( j \):

\[
\pi_{ij}^{kl} = \left( \frac{\tau_{ij}}{\tau_{ik} \times t_{kl} \times \tau_{lj}} \right)^\theta
\]

- Intuition: More out of the way links are used less.
Link intensity: traveling from $i = 1$ to $j = 25$
From routing to traffic

• Equation (2) yields the commuting cost taking traffic congestion as given. But what is the equilibrium traffic?

• First step: calculate the intensity with which a particular link is used on the way from $i$ to $j$:

$$
\pi_{ij}^{kl} = \left( \frac{\tau_{ij}}{\tau_{ik} \times t_{kl} \times \tau_{lj}} \right)^\theta
$$

• Intuition: More out of the way links are used less.

• Second step: Sum over all origins and destinations to get traffic:

$$
\Xi_{kl} = \sum_{i,j \in N} L_{ij} \pi_{ij}^{kl}
$$
A gravity equation for traffic

• Standard *commuting gravity equation*:

\[
L_{ij} = \tau_{ij}^{-\theta} \times \frac{L_i^R}{RMA_i} \times \frac{L_j^F}{FMA_j} \times \frac{\bar{L}}{W^\theta},
\]

where

• Residential market access: \( RMA_i = \sum_j \tau_{ij}^{-\theta} \times \frac{L_j^F}{FMA_j} \)

• Firm market access: \( FMA_j = \sum_i \tau_{ij}^{-\theta} \times \frac{L_i^R}{RMA_i} \).
A gravity equation for traffic

- **Standard commuting gravity equation:**

  \[ L_{ij} = \tau_{ij}^{-\theta} \times \frac{L_i^R}{RMA_i} \times \frac{L_j^F}{FMA_j} \times \frac{\bar{L}}{W^\theta}, \]

  where

  - Residential market access: \( RMA_i = \sum_j \tau_{ij}^{-\theta} \times \frac{L_j^F}{FMA_j} \)
  - Firm market access: \( FMA_j = \sum_i \tau_{ij}^{-\theta} \times \frac{L_i^R}{RMA_i} \)

- **New traffic gravity equation:**

  \[ \Xi_{kl} = t_{kl}^{-\theta} \times FMA_k \times RMA_l \times \frac{\bar{L}}{W^\theta} \] (3)

  - **Intuition:** Greater \( FMA_k \), more traffic flowing in. Greater \( RMA_l \), more traffic flowing out.
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The welfare impacts of improving the Seattle road network

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The effect of traffic congestion

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• Traffic gravity equation (3) shows how market access affects equilibrium traffic flows...
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To summarize:

Traffic gravity equation (3) shows how market access affects equilibrium traffic flows...

... Through congestion (equation 1), traffic flows affects travel costs, which through routing choice (equation 2) affects commuting costs...

... And commuting costs affect market access through (standard) equilibrium channels.
The effect of traffic congestion

- To summarize:
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- A massive fixed point problem!
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• To summarize:

  • Traffic gravity equation (3) shows how market access affects equilibrium traffic flows...
  
  • ... Through congestion (equation 1), traffic flows affects travel costs, which through routing choice (equation 2) affects commuting costs...
  
  • ... And commuting costs affect market access through (standard) equilibrium channels.

• A massive fixed point problem!

  • ...but it turns out to not be too bad at all.
Equilibrium

- Eqm. conditions $L_i^R = \sum_j L_{ij}$, $L_i^F = \sum_j L_{ji}$ in a standard model are:

\[
(l_i^R)^{1-\theta \beta} = \chi \sum_j \tau_{ij}^{-\theta} \bar{u}_i^\theta \bar{A}_j^\theta (l_j^F)^{\theta \alpha}
\]

\[
(l_i^F)^{1-\theta \alpha} = \chi \sum_j \tau_{ji}^{-\theta} \bar{u}_j^\theta \bar{A}_i^\theta (l_j^R)^{\theta \beta}
\]

where $\chi \equiv \bar{L}_\alpha + \beta W$, $l_i^R \equiv L_i^R / \bar{L}$ and $l_i^F \equiv L_i^F / \bar{L}$ are labor shares.
Equilibrium

- Eqm. conditions $L_i^R = \sum_j L_{ij}$, $L_i^F = \sum_j L_{ji}$ in a standard model are:

  $$(l_i^R)^{1-\theta \beta} = \chi \sum_j \tau_{ij}^{-\theta} \tilde{u}_i^\theta \tilde{A}_j^\theta (l_j^F)^{\theta \alpha}$$

  $$(l_i^F)^{1-\theta \alpha} = \chi \sum_j \tau_{ij}^{-\theta} \tilde{u}_j^\theta \tilde{A}_i^\theta (l_j^R)^{\theta \beta}$$

- With traffic congestion, they become:

  $$(l_i^R)^{1-\theta \beta} (l_i^F)^{\theta \lambda (1-\alpha \theta) \frac{1+\theta \lambda}{1+\theta \lambda}} = \chi \tilde{A}_i^\theta \tilde{u}_i^\theta (l_i^F)^{\theta (\alpha+\lambda) \frac{1+\theta \lambda}{1+\theta \lambda}} + \chi \frac{\theta \lambda}{1+\theta \lambda} \sum_j (\tilde{t}_{ij} \tilde{L}^\lambda)^{-\theta \frac{1}{1+\theta \lambda}} \tilde{A}_i^\theta \frac{\theta \lambda}{1+\theta \lambda} \tilde{u}_i^\theta \tilde{u}_j^\theta (l_j^R)^{1-\theta \beta \frac{1+\theta \lambda}{1+\theta \lambda}}$$

  $$(l_i^F)^{\theta \lambda (1-\beta \theta) \frac{1+\theta \lambda}{1+\theta \lambda}} (l_i^R)^{1-\theta \alpha} = \chi \tilde{A}_i^\theta \tilde{u}_i^\theta (l_i^R)^{\theta (\beta+\lambda) \frac{1+\theta \lambda}{1+\theta \lambda}} + \chi \frac{\theta \lambda}{1+\theta \lambda} \sum_j (\tilde{t}_{ji} \tilde{L}^\lambda)^{-\theta \frac{1}{1+\theta \lambda}} \tilde{A}_i^\theta \frac{\theta \lambda}{1+\theta \lambda} \tilde{A}_j^\theta (l_j^F)^{\theta (\beta+\lambda) \frac{1+\theta \lambda}{1+\theta \lambda}}$$

where $\chi \equiv \frac{\bar{L}^{\alpha+\beta}}{W}$, $l_i^R \equiv L_i^R / \bar{L}$ and $l_i^F \equiv L_i^F / \bar{L}$ are labor shares.
Equilibrium

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(l_i^R)^{1-\theta\beta} = \chi \sum_j \tau_{ij}^{\theta} \bar{u}_i^{\theta} \bar{A}_j^{\theta} (l_j^F)^{\theta\alpha}
$$

$$
(l_i^F)^{1-\theta\alpha} = \chi \sum_j \tau_{ji}^{\theta} \bar{u}_j^{\theta} \bar{A}_i^{\theta} (l_j^R)^{\theta\beta}
$$

- With traffic congestion, they become:

$$
(l_i^R)^{1-\theta\beta} (l_i^F)_{\frac{\theta\lambda(1-\alpha\theta)}{1+\theta\lambda}} = \chi \bar{A}_i^{\theta} \bar{u}_i^{\theta} (l_i^F)_{\frac{\theta(\alpha+\lambda)}{1+\theta\lambda}} + \chi_{\frac{\theta\lambda}{1+\theta\lambda}} \sum_j (\bar{t}_{ij}\bar{L})^{-\frac{\theta}{1+\theta\lambda}} \bar{A}_i^{\theta} \bar{u}_i^{\theta} \bar{u}_j^{\theta} \bar{A}_j^{-\frac{\theta}{1+\theta\lambda}} (l_j^R)^{\frac{1-\beta\theta}{1+\theta\lambda}}
$$

$$
(l_i^F)_{\frac{\theta\lambda(1-\beta\theta)}{1+\theta\lambda}} (l_i^R)^{1-\theta\alpha} = \chi \bar{A}_i^{\theta} \bar{u}_i^{\theta} (l_i^R)_{\frac{\theta(\beta+\lambda)}{1+\theta\lambda}} + \chi_{\frac{\theta\lambda}{1+\theta\lambda}} \sum_j (\bar{t}_{ji}\bar{L})^{-\frac{\theta}{1+\theta\lambda}} \bar{A}_i^{\theta} u_i^{\theta} \bar{A}_j^{\theta} \bar{A}_j^{-\frac{\theta}{1+\theta\lambda}} (l_j^F)^{\frac{1-\alpha\theta}{1+\theta\lambda}}
$$

where $\chi \equiv \frac{\bar{L}^{\alpha+\beta}}{W}$, $l_i^R \equiv L_i^R/\bar{L}$ and $l_i^F \equiv L_i^F/\bar{L}$ are labor shares.

- Same number of equations & unknowns, new structure!
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Introduction

A Quantitative Urban Framework with Traffic Congestion
  Model setup
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  Traffic and Congestion
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The welfare impacts of improving the Seattle road network

Conclusion
Traffic congestion acts like a standard dispersion force...
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... but unlike a standard dispersion force, scale matters
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A Quantitative Urban Framework with Traffic Congestion
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The welfare impacts of improving the Seattle road network

Conclusion
Counterfactuals

- Standard model: write system in changes using observed data:

\[
(\hat{l}_R^{1-\theta_\beta}) = \hat{\chi} \sum_j \left( \frac{L_{ij}}{L_{Ri}} \right)^{\hat{r}_{ij} - \theta} \left( \hat{l}_F^i \right)^{\theta_\alpha}
\]

\[
(\hat{l}_F^{1-\theta_\alpha}) = \hat{\chi} \sum_j \left( \frac{L_{ji}}{L_{Fi}} \right)^{\hat{r}_{ji} - \theta} \left( \hat{l}_R^j \right)^{\theta_\beta}
\]
Counterfactuals

- Standard model: write system in changes using observed data:

\[
(\hat{l}_R^i)^{1-\theta \beta} = \hat{\chi} \sum_j \left( \frac{L_{ij}}{L_i^R} \right)^\theta \hat{l}_{ij} (\hat{l}_j^F)^{\theta \alpha}
\]

\[
(\hat{l}_F^i)^{1-\theta \alpha} = \hat{\chi} \sum_j \left( \frac{L_{ji}}{L_i^F} \right)^\theta \hat{l}_{ji} (\hat{l}_j^R)^{\theta \beta}
\]

- With traffic congestion:

\[
(\hat{l}_R^i)^{-\theta \beta + 1} (\hat{l}_F^i) \frac{\theta \lambda (1-\alpha \theta)}{1+\theta \lambda} = \hat{\chi} \left( \frac{L_i^F}{L_i^R + \sum_j \Xi_{ij}} \right) (\hat{l}_i^F)^{\theta (\alpha + \lambda)} \frac{\theta \lambda}{1+\theta \lambda} \sum_j \left( \frac{\Xi_{ij}}{L_i^F + \sum_j \Xi_{ij}} \right) \hat{t}_{ij} (\hat{l}_j^R)^{\frac{\theta}{1+\theta \lambda}} (\hat{l}_j^F)^{\frac{1-\beta \theta}{1+\theta \lambda}}
\]

\[
(\hat{l}_R^i)^{\frac{\theta \lambda (1-\beta \theta)}{1+\theta \lambda}} (\hat{l}_F^i)^{-\theta \alpha + 1} = \hat{\chi} \left( \frac{L_i^R}{L_i^R + \sum_j \Xi_{ji}} \right) (\hat{l}_i^R)^{\theta (\beta + \lambda)} \frac{\theta \lambda}{1+\theta \lambda} \sum_j \left( \frac{\Xi_{ji}}{L_i^R + \sum_j \Xi_{ji}} \right) \hat{t}_{ji} (\hat{l}_j^F)^{\frac{\theta}{1+\theta \lambda}} (\hat{l}_j^F)^{\frac{1-\alpha \theta}{1+\theta \lambda}}
\]
Counterfactuals

• Standard model: write system in changes using observed data:

\[
(\hat{\gamma}_i^R)^{1-\theta \beta} = \hat{\chi} \sum_j \left( \frac{L_{ij}}{L_i^R} \right) \hat{\tau}_{ij}^{-\theta} (\hat{\gamma}_j^F)^{\theta \alpha}
\]

\[
(\hat{\gamma}_i^F)^{1-\theta \alpha} = \hat{\chi} \sum_j \left( \frac{L_{ji}}{L_i^F} \right) \hat{\tau}_{ji}^{-\theta} (\hat{\gamma}_j^R)^{\theta \beta}
\]

• With traffic congestion:

\[
(\hat{\gamma}_i^R)^{-\theta \beta + 1} \left( \frac{\theta \lambda (1-\alpha \theta)}{1+\theta \lambda} \right) = \hat{\chi} \left( \frac{L_i^F}{L_i^R + \sum_j \Xi_{ij}} \right) \left( \frac{\theta (\lambda + \lambda)}{1+\theta \lambda} \right) + \hat{\chi} \left( \frac{\theta \lambda}{1+\theta \lambda} \right) \sum_j \left( \frac{\Xi_{ij}}{L_i^F + \sum_j \Xi_{ij}} \right) \hat{\tau}_{ij}^{-\theta} \left( \frac{\theta}{1+\theta \lambda} \right) \left( \hat{\gamma}_j^R \right)^{1-\beta \theta}
\]

\[
(\hat{\gamma}_i^R)^{\theta \lambda (1-\beta \theta)} \left( \frac{L_i^R}{L_i^R + \sum_j \Xi_{ji}} \right) \left( \frac{\theta \lambda}{1+\theta \lambda} \right) = \hat{\chi} \left( \frac{L_i^R}{L_i^R + \sum_j \Xi_{ji}} \right) \left( \frac{\theta (\beta + \lambda)}{1+\theta \lambda} \right) + \hat{\chi} \left( \frac{\theta \lambda}{1+\theta \lambda} \right) \sum_j \left( \frac{\Xi_{ji}}{L_i^R + \sum_j \Xi_{ji}} \right) \hat{\tau}_{ji}^{-\theta} \left( \frac{\theta}{1+\theta \lambda} \right) \left( \hat{\gamma}_j^F \right)^{1-\alpha \theta}
\]

• Same close marriage between theory and data, but now using traffic data!
Outline of Talk

Introduction

A Quantitative Urban Framework with Traffic Congestion

The welfare impacts of improving the Seattle road network
   Empirical Context & Data
   Estimation
   The welfare impacts of improving the Seattle Road Network

Conclusion
Why Seattle?

- The traffic in Seattle is bad.
  - Second highest commute times in the U.S.
  - No major public transportation system.

- The data in Seattle is good.
  - For roughly 1,500 miles of roads, we observe traffic, length, location, number of lanes, speed limit (HPMS).
  - Residential, workplace populations in each census block group (LODES).
  - Note: HPMS & LODES available throughout U.S.

- The geography is interesting.
  - Water & bridges result in natural bottlenecks in road network.
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  - Note: HPMS & LODES available throughout U.S.

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  - Water & bridges result in natural bottlenecks in road network.
The Seattle Road Network

Traffic (AADT)
- ≤4696
- ≤5453
- ≤10030
- ≤37680
- ≤204800

Node Population
- ≤2168
- ≤4260
- ≤7061
- ≤11470
- ≤18695
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Estimation overview

- To evaluate welfare impacts, only need to know four elasticities:
  1. Preference heterogeneity $\theta$.
  2. Productivity spillover $\alpha$.
  3. Amenity spillover $\beta$.
  4. Traffic congestion parameter $\lambda$. 

We set $\theta = 4$ (Monte et al. '18).

We set $\alpha = 0$ (Roca & Puga '17).

We set $\beta = -0.3$ (C-D share of housing).

We estimate this.
To evaluate welfare impacts, only need to know four elasticities:

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1. Preference heterogeneity $\theta$. $\leftarrow$ We set $\theta = 4$ (Monte et al. ’18).

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4. Traffic congestion parameter $\lambda$. $\leftarrow$ We estimate this.
Estimation of Traffic Congestion

- Assume:
  1. Link costs increasing in travel time:

\[
\ln t_{kl} = \delta_0 \ln \text{time}_{kl}
\]
Estimation of Traffic Congestion

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\]

  2. Speed is decreasing in traffic (AADT) per lane-mile:

\[
speed_{kl} = -\delta_1 \ln \left( \frac{\Xi_{kl}}{\text{lanes}_{kl}} \right) + \delta_{kl},
\]
Estimation of Traffic Congestion

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\[
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\]

• Then:
  1. Consistent with theory, we have:

\[
\ln t_{kl} = \delta_0 \ln \text{distance}_{kl} - \delta_0 \delta_1 \text{lanes}_{kl} - \delta_0 \delta_{kl} + \delta_0 \delta_1 \ln \Xi_{kl}
\]

\[
\equiv \ln \bar{t}_{kl} + \lambda \equiv \lambda
\]

• Note: Constructing more lane-miles reduces \( \bar{t}_{kl} \).
Estimation of Traffic Congestion

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\equiv \ln \bar{t}_{kl} + \lambda
\]

• Note: Constructing more lane-miles reduces \( \bar{t}_{kl} \).

2. Simple estimating equation:

\[
speed_{kl} = -\delta_1 \ln \left( \frac{\Xi_{kl}}{\text{lanes}_{kl}} \right) + X_{kl} \beta + \varepsilon_{kl} \\
\equiv \delta_{kl}
\]
Estimation of Traffic Congestion (ctd.)

- Estimating equation from last slide:

\[
speed_{kl} = -\delta_1 \ln \left( \frac{\Xi_{kl}}{\text{lanes}_{kl}} \right) + X_{kl} \beta + \varepsilon_{kl} \equiv \delta_{kl}
\]
Estimation of Traffic Congestion (ctd.)

- Estimating equation from last slide:

\[ \text{speed}_{kl} = -\delta_1 \ln \left( \frac{\Xi_{kl}}{\text{lanes}_{kl}} \right) + \mathbf{x}_{kl} \beta + \varepsilon_{kl} \equiv \delta_{kl} \]

- Need an IV for traffic uncorrelated with free flow rate of speed.
Estimation of Traffic Congestion (ctd.)

- Estimating equation from last slide:
  
  \[ speed_{kl} = -\delta_1 \ln \left( \frac{\Xi_{kl}}{lanes_{kl}} \right) + X_{kl} \beta + \varepsilon_{kl} \equiv \delta_{kl} \]

- Need an IV for traffic uncorrelated with free flow rate of speed.
- **Solution:** Number of turns (conditional on number of intersections).

- **Intuition:** Intersections uniformly costly, turns annoying.
Table: Estimating the strength of traffic congestion

<table>
<thead>
<tr>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
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<tr>
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<td>OLS</td>
<td>IV: 1st stage</td>
<td>IV</td>
<td>IV: 1st stage</td>
<td>IV</td>
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<td>AADT per Lane</td>
<td>-0.048***</td>
<td>0.118**</td>
<td>0.488*</td>
<td>0.488*</td>
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<tr>
<td></td>
<td>(0.007)</td>
<td>(0.048)</td>
<td>(0.278)</td>
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<tr>
<td>Turns along Route</td>
<td>-0.252***</td>
<td>-0.091**</td>
<td>-0.091**</td>
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<tr>
<td></td>
<td>(0.049)</td>
<td>(0.039)</td>
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<td>F-statistic</td>
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<td>5.336</td>
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<td>R-squared</td>
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<td>0.721</td>
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<td>0.875</td>
<td>-2.757</td>
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<td>Observations</td>
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<tr>
<td>Start-location FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
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<tr>
<td>End-location FE</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>No. of Intersections</td>
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<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Bilateral Route Quality</td>
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<td>No</td>
<td>No</td>
<td>Yes</td>
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</tr>
</tbody>
</table>

- Implies $\lambda = 0.11$. 
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Conclusion
Welfare elasticities \( \left( \frac{\partial \ln W}{\partial \ln \bar{t}_{kl}} \right) \) of improving each link

\[
\leq -0.0025 \\
\leq 0 \\
\leq 0.0025 \\
\leq 0.005 \\
\leq 0.0075 \\
\leq 0.01 \\
\leq 0.055
\]

- \( \sim 10\% \) of links are welfare reducing (Braess paradox in action!)
Calculating the Returns on Investment

- Calculate the annual return on investment for an additional lane-mile on every segment.
Calculating the Returns on Investment

• Calculate the annual return on investment for an additional lane-mile on every segment.

• Benefits:

\[ \frac{\partial \ln W}{\partial \ln \text{lanes}_{kl}} = \delta_0 \delta_1 \times \frac{\partial \ln W}{\partial \ln t_{kl}} \]
Calculating the Returns on Investment

• Calculate the annual return on investment for an additional lane-mile on every segment.

• Benefits:
  \[
  \frac{\partial \ln W}{\partial \ln \text{lanes}_{kl}} = \delta_0 \delta_1 \times \frac{\partial \ln W}{\partial \ln \bar{t}_{kl}}
  \]

• Costs: Latest estimates from Federal Highway Administration (FHWA) by road type & location. Assume 10 year linear depreciation.
Estimated Annualized cost of an Additional Lane-mile

Cost of Adding One Additional Lane-Mile ($m)

- ≤20.52
- ≤25.37
- ≤26.94
- ≤31.78
- ≤46.69
Return on Investment of Infrastructure Investment

- Huge heterogeneity in ROI: Mean: 17%, Median: 8%, SD: 37%.
Return on Investment of Infrastructure Investment

- Huge heterogeneity in ROI: Mean: 17%, Median: 8%, SD: 37%.
Seattle City Council won’t back second Montlake Bridge

A 10-year-old rendering of what a second Montlake Bridge could look like — via Madison Park Blogger

The state has the funds to build it but the Seattle City Council won’t — yet — back a resolution supporting a second bascule bridge connecting through the transit chokepoint between Montlake and light rail at Husky Stadium.
Conclusion

• To bolster the quantitative revolution, introduce new urban framework with traffic congestion:
  • Same analytical tractability, close marriage between theory and data.
  • New implications for welfare impacts of road construction.
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- To bolster the quantitative revolution, introduce new urban framework with traffic congestion:
  - Same analytical tractability, close marriage between theory and data.
  - New implications for welfare impacts of road construction.

- Future work could leverage wide-spread availability of traffic data to better design infrastructure networks in locations where commuting data is scarce (e.g. in developing countries).