Online Appendix for An Exploration of Trend-Cycle Decomposition Methodologies in Simulated Data

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1 Estimation of the Morley, Nelson, and Zivot (2003) Model with U.S. Real GDP per Capita

Estimating the model for the full sample from 1947:1 to 2019:1 using the natural logarithm of U.S. real GDP per capita gives

$$\Delta y_t = \underset{(0.036}{0.211} + \underset{(0.077)}{1.330}\Delta y_{t-1} - \underset{(0.058)}{0.776}\Delta y_{t-2} + \epsilon_t - \underset{(0.085)}{1.099}\epsilon_{t-1} + \underset{(0.062)}{0.663}\epsilon_{t-2}. \tag{1}$$

These parameter estimates are relatively close to the estimates in the main paper, and the implied long-run impulse response coefficient from this estimation is also quite similar to the estimate in the main paper:

$$A_0 = \phi(1)^{-1}\theta(1) = (1 - 1.099 + 0.663)/(1 - 1.340 + 0.776) = 1.290. \tag{2}$$

References

Baxter, M. and R. G. King (1999). Measuring business cycles: approximate band-pass filters for economic time series. *Review of Economics and Statistics 81*(4), 575–593.

Clark, P. K. (1987). The cyclical component of us economic activity. *Quarterly Journal of Economics 102*(4), 797–814.

Hamilton, J. D. (2018). Why you should never use the Hodrick-Prescott filter. *Review of Economics and Statistics 100*(5), 831–843.

Hodrick, R. J. and E. C. Prescott (1997). Postwar US business cycles: an empirical investigation. *Journal of Money, Credit, and Banking*, 1–16.

Morley, J. C., C. R. Nelson, and E. Zivot (2003). Why are the Beveridge-Nelson and unobserved-components decompositions of gdp so different? *Review of Economics and Statistics 85*(2), 235–243.

Table 1: Statistics on U.S. Real GDP per Capita

The table presents two sets of statistics on data from the natural logarithm of U.S. real GDP per capita. The sample period is 1947:1 to 2019:1. In Panel A the columns labeled "H", "HP", and "BK" refer to the respective methods of decomposing a time series into trend and cycle of Hamilton (2018), Hodrick and Prescott (1997), and Baxter and King (1999), and the statistics are the standard deviations of the cyclical components and the standard deviations of the changes in the trends. Panel B reports the square roots of the Variance Ratios.

Panel A: Standard Deviations

	H	HP	BK
Standard Deviation of Cycle	3.348	1.562	1.504
Standard Deviation of ΔTrend	1.034	0.233	0.246

Panel B: Square Roots of Variance Ratios

k	10	20	30	40	50	60	70	80
$\sqrt{V_k}$	1.328	1.277	1.248	1.209	1.140	1.112	1.106	1.056

Table 2: Simulated Statistics from a Random Walk

The table presents sample means of statistics from 5,000 simulations of length 578 of a random walk with drift calibrated such that the drift and the standard deviation coincide, respectively, with the sample mean and sample standard deviation of the rate of growth of GDP. In Panel A the columns labeled "H", "HP", and "BK" refer to the respective methods of decomposing a time series into trend and cycle of Hamilton (2018), Hodrick and Prescott (1997), and Baxter and King (1999). The column labeled "In Sim" is the sample mean of the realized values of the row statistic in the simulations. Panel B presents the slope coefficient and R^2 in the regression of the simulated cycle minus the HP or BK cycle on a constant and the H cycle minus the HP or BK cycle. Panel C presents the square roots of the Variance Ratios.

Panel A: Standard Deviations, Correlations, and RMSEs

	H	HP	BK	In Sim
Standard Deviation of Cycle	2.590	1.197	1.075	2.616
Standard Deviation of ΔTrend	0.912	0.187	0.193	0.933
Correlation of Cycles	0.990	0.655	0.627	
Correlation of ΔTrends	0.980	0.151	0.156	
RMSE of Cycles	0.441	2.063	2.071	

Panel B: Regression Diagnostics

	HP	BK
Slope Coefficient	1.000	1.001
R^2	0.968	0.971

Panel C: Square Roots of Variance Ratios

k	10	20	30	40	50	60	70	80
$\sqrt{V_k}$	1.000	1.000	1.000	0.999	0.998	0.998	0.998	0.998

Table 3: Simulated Statistics from an ARIMA model

The table presents sample means of statistics from 5,000 simulations of length 578 from the ARIMA(2,1,2) model of Morley et al. (2003) estimated with GDP data for the full sample. The model is

$$\Delta y_t = 0.320 + 1.271\Delta y_{t-1} - 0.682\Delta y_{t-2} + \epsilon_t - 0.979\epsilon_{t-1} + 0.540\epsilon_{t-2},$$

and ϵ_t is distributed $N(0, 0.7236)$. In Panel A the columns labeled "H", "HP", and "BK" refer to the respective methods of decomposing a time series into trend and cycle of Hamilton (2018), Hodrick and Prescott (1997), and Baxter and King (1999). The column labeled "In Sim" is the sample mean of the realized values of the row statistic in the simulations. Panel B presents the slope coefficient and R^2 in the regression of the simulated cycle minus the HP or BK cycle on a constant and the H cycle minus the HP or BK cycle. Panel C presents the square roots of the Variance Ratios.

Panel A: Standard Deviations, Correlations, and RMSEs

	H	HP	BK	In Sim
Standard Deviation of Cycle	3.304	1.535	1.474	3.334
Standard Deviation of ΔTrend	1.096	0.234	0.243	1.076
Correlation of Cycles	0.988	0.679	0.671	
Correlation of ΔTrends	0.963	0.158	0.167	
RMSE of Cycles	0.601	2.577	2.612	

Panel B: Regression Diagnostics

	HP	BK
Slope Coefficient	0.991	0.992
R^2	0.960	0.962

Panel C: Square Roots of Variance Ratios

k	10	20	30	40	50	60	70	80
$\sqrt{V_k}$	1.271	1.267	1.263	1.260	1.258	1.257	1.256	1.256

Table 4: Simulated Statistics from a Model with Constant Unconditional Mean Change in Trend

The table presents sample means of statistics from 5,000 simulations of length 578 in which the simulated time series is the sum of a stochastic trend and a stochastic cycle,

$$y_t = g_t + c_t$$

The trend is modeled as an ARIMA(1,1,0). The change in the trend is

$$\Delta g_t = 0.07786 + 0.900 \Delta g_{t-1} + \nu_t,$$

and the innovations are drawn from a $N(0, 0.002)$. The cyclical component is modeled as an ARIMA(2,0,0)

$$c_t = 1.25 c_{t-1} - 0.45 c_{t-2} + \epsilon_t,$$

and the innovations are drawn from a $N(0, 0.6385)$. The innovations in the trend and cycle are uncorrelated. In Panel A the columns labeled "H", "HP", and "BK" refer to the respective methods of decomposing a time series into trend and cycle of Hamilton (2018), Hodrick and Prescott (1997), and Baxter and King (1999). The column labeled "In Sim" is the sample mean of the realized values of the row statistic in the simulations. Panel B presents the slope coefficient and R^2 in the regression of the simulated cycle minus the HP or BK cycle on a constant and the H cycle minus the HP or BK cycle. Panel C presents the square roots of the Variance Ratios.

Panel A: Standard Deviations, Correlations, and RMSEs

	H	HP	BK	In Sim
Standard Deviation of Cycle	2.444	1.432	1.367	1.755
Standard Deviation of ΔTrend	0.790	0.105	0.118	0.100
Correlation of Cycles	0.739	0.885	0.853	
Correlation of ΔTrends	0.046	0.570	0.515	
RMSE of Cycles	1.653	0.840	0.943	

Panel B: Regression Diagnostics

	HP	BK
Slope Coefficient	0.086	0.151
R^2	0.034	0.076

Panel C: Square Roots of Variance Ratios

k	10	20	30	40	50	60	70	80
$\sqrt{V_k}$	0.911	0.708	0.638	0.604	0.582	0.567	0.557	0.549

Table 5: Simulated Statistics from the Clark (1987) Unobserved Components Model

The table presents sample means of statistics from 5,000 simulations of length 578 generate from the Clark (1987) model estimated on the full sample. The simulated time series is the sum of a stochastic trend and a stochastic cycle,

$$y_t = g_t + c_t.$$

The change in the trend has a conditional mean,

$$\Delta g_t = d_{t-1} + 0.545 w_t,$$

and the conditional mean is a random walk with a relatively small standard deviation of its innovation,

$$d_t = d_{t-1} + 0.021 u_t.$$

The cyclical component is modeled as an ARIMA(2,0,0),

$$c_t = 1.510 c_{t-1} - 0.565 c_{t-2} + 0.603 v_t,$$

and the three innovations, u_t, w_t, and v_t, are standard normal random variables. The three innovations are uncorrelated. In Panel A the columns labeled "H", "HP", and "BK" refer to the respective methods of decomposing a time series into trend and cycle of Hamilton (2018), Hodrick and Prescott (1997), and Baxter and King (1999). The column labeled "In Sim" is the sample mean of the realized values of the row statistic in the simulations. Panel B presents the slope coefficient and R^2 in the regression of the simulated cycle minus the HP or BK cycle on a constant and the H cycle minus the HP or BK cycle. Panel C presents the square roots of the Variance Ratios.

Panel A: Standard Deviations, Correlations, and RMSEs

	H	HP	BK	In Sim
Standard Deviation of Cycle	3.598	1.647	1.562	2.726
Standard Deviation of ΔTrend	1.327	0.272	0.284	0.581
Correlation of Cycles	0.474	0.644	0.664	
Correlation of ΔTrends	0.049	0.325	0.308	
RMSE of Cycles	3.362	2.141	2.109	

Panel B: Regression Diagnostics

	HP	BK
Slope Coefficient	0.050	0.056
R^2	0.015	0.017

Panel C: Square Roots of Variance Ratios

k	10	20	30	40	50	60	70	80
$\sqrt{V_k}$	1.443	1.445	1.499	1.584	1.675	1.766	1.852	1.932

Table 6: Simulated Statistics from a Model with a Changing Unconditional Mean Change in Trend

The table presents sample means of statistics from 5,000 simulations of length 578 in which the simulated time series is the sum of a stochastic trend and a stochastic cycle,

$$y_t = g_t + c_t.$$

The trend is modeled as an ARIMA(1,1,0) with three different intercepts, $\mu_1 = 0.1$ for the first third of the simulated data, $\mu_2 = 0.07786$ for the second third, and $\mu_3 = 0.0554$ for the final third. The change in the trend is

$$\Delta g_t = \mu_i + 0.900\Delta g_{t-1} + \nu_t,$$

and the innovations are drawn from a $N(0, 0.002)$. The cyclical component is modeled as an ARIMA(2,0,0)

$$c_t = 1.25c_{t-1} - 0.45c_{t-2} + \epsilon_t,$$

and the innovations are drawn from a $N(0, 0.6385)$. The innovations in the trend and cycle are uncorrelated. In Panel A the columns labeled "H", "HP", and "BK" refer to the respective methods of decomposing a time series into trend and cycle of Hamilton (2018), Hodrick and Prescott (1997), and Baxter and King (1999). The column labeled "In Sim" is the sample mean of the realized values of the row statistic in the simulations. Panel B presents the slope coefficient and R^2 in the regression of the simulated cycle minus the HP or BK cycle on a constant and the H cycle minus the HP or BK cycle. Panel C presents the square roots of the Variance Ratios.

Panel A: Standard Deviations, Correlations, and RMSEs

	H	HP	BK	In Sim
Standard Deviation of Cycle	2.494	1.433	1.369	1.755
Standard Deviation of ΔTrend	0.805	0.209	0.213	0.207
Correlation of Cycles	0.724	0.884	0.852	
Correlation of ΔTrends	0.212	0.890	0.863	
RMSE of Cycles	1.726	0.842	0.948	

Panel B: Regression Diagnostics

	HP	BK
Slope Coefficient	0.077	0.136
R^2	0.034	0.070

Panel C: Square Roots of Variance Ratios

k	10	20	30	40	50	60	70	80
$\sqrt{V_k}$	1.080	1.103	1.221	1.346	1.465	1.576	1.680	1.776

Table 7: Simulated Statistics from a Model with a Slowly Changing Unconditional Mean Change in Trend

The table presents sample means of statistics from 5,000 simulations of length 578 in which the simulated time series is the sum of a stochastic trend and a stochastic cycle,

$$y_t = g_t + c_t$$

The trend is modeled as an ARIMA(1,1,0) with a changing unconditional mean. The change in the trend is

$$\Delta g_t = 0.07786 + 0.04428(145 - t)/289 + 0.900\Delta g_{t-1} + \nu_t,$$

where t ranges from 1 to 289, and the innovations are drawn from a $N(0, 0.002)$. The cyclical component is modeled as an ARIMA(2,0,0)

$$c_t = 1.25c_{t-1} - 0.45c_{t-2} + \epsilon_t,$$

and the innovations are drawn from a $N(0, 0.6385)$. The innovations in the trend and cycle are uncorrelated. In Panel A the columns labeled "H", "HP", and "BK" refer to the respective methods of decomposing a time series into trend and cycle of Hamilton (2018), Hodrick and Prescott (1997), and Baxter and King (1999). The column labeled "In Sim" is the sample mean of the realized values of the row statistic in the simulations. Panel B presents the slope coefficient and R^2 in the regression of the simulated cycle minus the HP or BK cycle on a constant and the H cycle minus the HP or BK cycle. Panel C presents the square roots of the Variance Ratios.

Panel A: Standard Deviations, Correlations, and RMSEs

	H	HP	BK	In Sim
Standard Deviation of Cycle	2.436	1.431	1.365	1.754
Standard Deviation of ΔTrend	0.792	0.162	0.168	0.158
Correlation of Cycles	0.739	0.884	0.853	
Correlation of ΔTrends	0.145	0.821	0.778	
RMSE of Cycles	1.648	0.840	0.944	

Panel B: Regression Diagnostics

	HP	BK
Slope Coefficient	0.051	0.152
R^2	0.034	0.076

Panel C: Square Roots of Variance Ratios

k	10	20	30	40	50	60	70	80
$\sqrt{V_k}$	0.991	0.912	0.953	1.013	1.075	1.135	1.192	1.245