Optimal Bank Regulation
In the Presence of Credit and Run-Risk

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Online Appendix

Appendix A reports the proofs to propositions, lemmas and corollaries in the paper, while appendix B reports additional derivations and extensions.

A Proofs

A.1 Proof of Lemma 1

First, consider that $\xi \leq \hat{\xi} \equiv (\delta D(1+r_D) - LIQ)/I$. Then, $\theta(\xi,1) = (LIQ+\xi\cdot I)/(D(1+r_D)) \leq (LIQ+\hat{\xi}I)/(D(1+r_D)) = \delta$. Also, $\hat{\lambda}(\xi)$ in (13) can be written as $[\theta(\xi,1)(1+r_D)(1+r_I) - \xi(1+\bar{r}_D) - \xi\cdot X/(\omega D)]/((1+r_D)(1+r_I) - \xi(1+\bar{r}_D))$, which is smaller than $\theta(\xi,1)$ as long as $\theta(\xi,1) < 1 + X/[\omega D(1+\bar{r}_D)((1+r_D)(1+r_I) - \xi(1+\bar{r}_D))]$, which is always true. So, $\hat{\lambda}(\xi) < \delta$ as well, and only the full run region is possible. Next, define ξ^{ld} as the solution to $\hat{\lambda}(\xi^{ld}) = \delta$; $\hat{\hat{\xi}}$ as the solution to $\theta(\hat{\hat{\xi}},1) = 1$, yielding $\hat{\hat{\xi}} = (D(1+r_D) - LIQ)/I$; and ξ^{ud} as the solution to $\hat{\lambda}(\xi^{ud}) = 1$, yielding $\xi^{ud} = \hat{\hat{\xi}}/(1 - X/(\omega I(1+r_I))) > \hat{\hat{\xi}}$. Moreover, $\hat{\lambda}(\hat{\hat{\xi}}) > \delta$ and $\partial\hat{\lambda}(\xi)/\partial\xi > 0$, so $\xi^{ld} < \hat{\hat{\xi}} < \xi^{ud}$. Finally, $\xi^{ud} < \overline{\xi}$, because $X < \overline{X} \equiv \omega I(1+r_I)(1 - \hat{\hat{\xi}}/\overline{\xi})$. Using these observations, it is easy to establish the non-empty regions for the remaining $\xi \in (\hat{\xi}, \overline{\xi}]$ in the Lemma.

A.2 Proof of Proposition 1

The proof follows the steps in Goldstein and Pauzner (2005) but includes additional derivations and arguments to tackle the perverse state monotonicity as well as the monitoring incentives and limited liability of the bank.

An equilibrium with threshold x^* exists only if $\Delta(x^*, x^*) = 0$ given by (19). Consider a potential threshold x'. We will show that x' exists and it satisfies (19) at exactly one point, $x' = x^*$.

By the existence of ξ^{ld} and ξ^{ud} defined in Lemma 1, $\Delta(x',x')$ is negative for $x' \leq \xi^{ld} - \varepsilon$ and positive for $x' \geq \xi^{ud} + \varepsilon$. Thus, in order to establish that a threshold equilibrium exists, it suffices to show that $\Delta(x',x')$ is continuous in $x' \in [\xi^{ld} - \varepsilon, \xi^{ud} + \varepsilon]$. It is convenient to write the utility differential $\Delta(x',x')$ as $\Delta(\hat{x}+\Delta x, \hat{x}+\Delta x)$ for some $\hat{x}$ such that Δx is the change in both the signal that

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the marginal saver receives and the threshold strategy. Then,

$$\begin{aligned}\Delta(\hat{x}+\Delta x,\hat{x}+\Delta x) &= \frac{1}{2\varepsilon}\int_{\hat{x}+\Delta x-\varepsilon}^{\hat{x}+\Delta x+\varepsilon}\nu(\xi,\lambda(\xi,\hat{x}+\Delta x))d\xi \\ &= \frac{1}{2\varepsilon}\int_{\hat{x}-\varepsilon}^{\hat{x}+\varepsilon}\nu(\xi+\Delta x,\lambda(\xi+\Delta x,\hat{x}+\Delta x))d\xi \\ &= \frac{1}{2\varepsilon}\int_{\hat{x}-\varepsilon}^{\hat{x}+\varepsilon}\nu(\xi+\Delta x,\lambda(\xi,\hat{x}))d\xi, \end{aligned} \tag{A.1}$$

because $\lambda(\xi+\Delta x,\hat{x}+\Delta x)=\lambda(\xi,\hat{x})$ from (15). In other words, the marginal saver's belief about how many other savers withdraw is unchanged when her private signal and the threshold strategy change by the same amount. Yet, she expects ξ to be higher for $\Delta x>0$ and lower for $\Delta x<0$, which is reflected in the calculation of $\nu(\xi+\Delta x,\lambda(\xi,\hat{x}))$. Thus, we need to show that, for a given distribution of $\lambda's$, the integral in (A.1) is continuous in Δx.

The integrand $\nu(\xi+\Delta x,\lambda(\xi,\hat{x}))$ in (A.1) is a piecewise function such that each sub-function is computed over a distribution of λ unaffected by Δx, but the interval for each sub-function depends on Δx. The thresholds $\hat{\lambda}_{\Delta x}$ and $\hat{\theta}_{\Delta x}$, which show the level of withdrawals above which the bank does not monitor and there is a full run (Lemma 1), are functions of certain levels of the liquidation value, which belong in the posterior distribution of ξ and are denoted by $\xi_{\hat{\lambda}_{\Delta x}}$ and $\xi_{\hat{\theta}_{\Delta x}}$, respectively.

Given that the distribution of λs does not change with Δx, we can compute $\xi_{\hat{\lambda}_{\Delta x}}$ as a function of Δx by equating the portion of savers withdrawing at this liquidation value for signal $\hat{x}$, which is $\lambda(\xi_{\hat{\lambda}_{\Delta x}},\hat{x})$ given by (15), to the threshold $\hat{\lambda}_{\Delta x}(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x)$, which moves with Δx and is given by (13):

$$\begin{aligned}&\lambda(\xi_{\hat{\lambda}_{\Delta x}},\hat{x})=\hat{\lambda}_{\Delta x}(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x)\\ &\Rightarrow \delta+(1-\delta)\frac{\hat{x}-\xi_{\hat{\lambda}_{\Delta x}}+\varepsilon}{2\varepsilon}=\frac{\left[LIQ+\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)I\right](1+r_I)-\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)(D(1+\bar{r}_D)+X/\omega)}{D\left[(1+r_D)(1+r_I)-\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)(1+\bar{r}_D)\right]}.\end{aligned} \tag{A.2}$$

Similarly, we can compute $\xi_{\hat{\theta}_{\Delta x}}$ as a function of Δx by equating the portion of savers withdrawing at this liquidation value for signal $\hat{x}$, which is $\lambda(\xi_{\hat{\theta}_{\Delta x}},\hat{x})$ given by (15), to the threshold $\hat{\theta}_{\Delta x}(\xi_{\hat{\theta}_{\Delta x}}+\Delta x)\equiv\theta(\xi_{\hat{\theta}_{\Delta x}}+\Delta x,1)$, which moves with Δx and is given by (10):

$$\begin{aligned}&\lambda(\xi_{\hat{\theta}_{\Delta x}},\hat{x})=\hat{\theta}_{\Delta x}(\xi_{\hat{\theta}_{\Delta x}}+\Delta x)\\ &\Rightarrow \delta+(1-\delta)\frac{\hat{x}-\xi_{\hat{\theta}_{\Delta x}}+\varepsilon}{2\varepsilon}=\frac{LIQ+\left(\xi_{\hat{\theta}_{\Delta x}}+\Delta x\right)I}{D(1+r_D)}.\end{aligned} \tag{A.3}$$

To ease notation, we will denote $\hat{\lambda}_{\Delta x}(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x)$ and $\hat{\theta}_{\Delta x}(\xi_{\hat{\theta}_{\Delta x}}+\Delta x)$, that is the value if the threshold at fundamentals $\xi_{\hat{\lambda}_{\Delta x}}+\Delta x$, by $\hat{\lambda}_{\Delta x}$ and $\hat{\theta}_{\Delta x}$, respectively.

Because the number of savers withdrawing decreases as fundamentals improve for given strategy threshold–see equation (15)–and $\hat{\lambda}_{\Delta x} < \hat{\theta}_{\Delta x}$ from Lemma 1, we get that $\xi_{\hat{\theta}_{\Delta x}} < \xi_{\hat{\lambda}_{\Delta x}}$. Thus, using (17), (A.1) can be written as:

$$\Delta(\hat{x}+\Delta x,\hat{x}+\Delta x) = -\frac{1}{2\varepsilon}\int_{\hat{x}-\varepsilon}^{\xi_{\hat{\theta}_{\Delta x}}} \frac{LIQ+(\xi+\Delta x)I}{\lambda(\xi,\hat{x})}d\xi - \frac{1}{2\varepsilon}\int_{\xi_{\hat{\theta}_{\Delta x}}}^{\xi_{\hat{\lambda}_{\Delta x}}} D(1+r_D)d\xi$$
$$+\frac{1}{2\varepsilon}\int_{\xi_{\hat{\lambda}_{\Delta x}}}^{\hat{x}+\varepsilon}\left\{\omega D(1+\bar{r}_D) - D(1+r_D)\right\}d\xi. \tag{A.4}$$

All the integrands ν in (A.4) are bounded and continuous in Δx, the thresholds $\xi_{\hat{\theta}_{\Delta x}}$ and $\xi_{\hat{\lambda}_{\Delta x}}$ change continuously with Δx from (A.3) and (A.2), and the only discontinuity in ν across regions occurs at one discrete point, $\xi_{\hat{\lambda}_{\Delta x}}$. Hence, $\Delta(\hat{x}+\Delta x,\hat{x}+\Delta x)$ is continuous and a threshold equilibrium exists.

We will now establish that the threshold equilibrium is unique. By implicitly differentiating (A.2) and (A.3), we get:

$$\frac{d\xi_{\hat{\lambda}_{\Delta x}}}{d\Delta x} = -\frac{2\varepsilon\Gamma_{\xi_{\hat{\lambda}_{\Delta x}}}}{1-\delta+2\varepsilon\Gamma_{\xi_{\hat{\lambda}_{\Delta x}}}} < 0 \tag{A.5}$$

because

$$\Gamma_{\xi_{\hat{\lambda}_{\Delta x}}} \equiv \frac{I(1+r_I)-(D(1+\bar{r}_D)+X/\omega)}{D\left[(1+r_D)(1+r_I)-\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)(1+\bar{r}_D)\right]} + \frac{\hat{\lambda}_{\Delta x}(1+\bar{r}_D)}{(1+r_D)(1+r_I)-\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)(1+\bar{r}_D)} > 0,$$

and

$$\frac{d\xi_{\hat{\theta}_{\Delta x}}}{d\Delta x} = -\frac{2\varepsilon I}{(1-\delta)D(1+r_D)+2\varepsilon I} < 0. \tag{A.6}$$

(A.5) and (A.6) tell us that, as fundamentals become better ($\Delta x > 0$), the region where the banker monitors and the region that a run does not occur become bigger.

The derivative of (A.4) with respect to Δx is:

$$d\frac{\Delta(\hat{x}+\Delta x,\hat{x}+\Delta x)}{d\Delta x} = -\frac{1}{2\varepsilon}\int_{\hat{x}-\varepsilon}^{\xi_{\hat{\theta}_{\Delta x}}} \frac{I}{\lambda(\xi,\hat{x})}d\xi - \frac{1}{2\varepsilon}\frac{d\xi_{\hat{\lambda}_{\Delta x}}}{d\Delta x}\omega D(1+\bar{r}_D), \tag{A.7}$$

because $(LIQ+(\xi_{\hat{\theta}_{\Delta x}}+\Delta x)I)/\lambda(\xi_{\hat{\theta}_{\Delta}},\hat{x}) = D(1+r_D)$ from (A.3).

The first term (A.7) is negative and captures the perverse incentives of making withdrawals more profitable when fundamentals are stronger given that the run is underway. The second term in (A.7) is positive and represents the payoff change from decreasing the threshold $\xi_{\hat{\lambda}_{\Delta x}}$ where the banker ceases to monitor. As a result, we cannot unambiguously sign the derivative for any signal x. However, as we discussed in the paper, it suffices to evaluate (A.7) at a *candidate* threshold $\hat{x}$, which as we established above, that exists. If the derivative is positive at candidate threshold, we can conclude that (A.4) does not cross zero from above and, given continuity, the threshold is unique.

Adding and subtracting $1/(2\varepsilon)\int_{\hat{x}-\varepsilon}^{\xi_{\hat{\theta}_{\Delta x}}}\xi_{\hat{\lambda}_{\Delta x}}I/\lambda(\xi,\hat{x})d\xi$ to $\Delta(\hat{x}+\Delta x,\hat{x}+\Delta x)=0$ in (A.4) we get that:

$$-\frac{1}{2\varepsilon}\int_{\hat{x}-\varepsilon}^{\xi_{\hat{\theta}_{\Delta x}}}\frac{I}{\lambda(\xi,\hat{x})}d\xi=\frac{1}{\xi_{\hat{\lambda}_{\Delta x}}+\Delta x}\frac{1}{2\varepsilon}\left[\int_{\hat{x}-\varepsilon}^{\xi_{\hat{\theta}_{\Delta x}}}\frac{LIQ+\left(\xi-\xi_{\hat{\lambda}_{\Delta x}}\right)I}{\lambda(\xi,\hat{x})}d\xi+\int_{\xi_{\hat{\theta}_{\Delta x}}}^{\xi_{\hat{\lambda}_{\Delta x}}}D(1+r_D)d\xi\right]$$

$$-\frac{1}{\xi_{\hat{\lambda}_{\Delta x}}+\Delta x}\frac{1}{2\varepsilon}\int_{\xi_{\hat{\lambda}_{\Delta x}}}^{\hat{x}+\varepsilon}\{\omega D(1+\bar{r}_D)-D(1+r_D)\}\,d\xi. \tag{A.8}$$

Substituting (A.8) in (A.7) we get:

$$d\frac{\Delta(\hat{x}+\Delta x,\hat{x}+\Delta x)}{d\Delta x}=-\frac{1}{\xi_{\hat{\lambda}_{\Delta x}}+\Delta x}\frac{1}{2\varepsilon}\omega D(1+\bar{r}_D)\left[\frac{d\xi_{\hat{\lambda}_{\Delta x}}}{d\Delta x}\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)+\left(\hat{x}+\varepsilon-\xi_{\hat{\lambda}_{\Delta}}\right)\right]$$

$$+\frac{1}{\xi_{\hat{\lambda}_{\Delta x}}+\Delta x}\frac{1}{2\varepsilon}\left[\int_{\hat{x}-\varepsilon}^{\xi_{\hat{\theta}_{\Delta x}}}\frac{LIQ+\left(\xi-\xi_{\hat{\lambda}_{\Delta x}}\right)I}{\lambda(\xi,\hat{x})}d\xi+\int_{\xi_{\hat{\theta}_{\Delta x}}}^{\xi_{\hat{\lambda}_{\Delta x}}}D(1+r_D)d\xi\right]. \tag{A.9}$$

Using (17), (A.2) and (A.5), the bracketed terms in the first line in (A.9), can be written as:

$$\frac{d\xi_{\hat{\lambda}_{\Delta x}}}{d\Delta x}\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)+\left(\hat{x}+\varepsilon-\xi_{\hat{\lambda}_{\Delta}}\right)=$$

$$-\frac{2\varepsilon}{1-\delta+2\varepsilon\Gamma_{\xi_{\hat{\lambda}_{\Delta x}}}}\left[\Gamma_{\xi_{\hat{\lambda}_{\Delta x}}}\cdot\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)-\left(\hat{\lambda}_{\Delta x}-\delta\right)+\left(\hat{\lambda}_{\Delta x}-\delta\right)\left(\delta-2\varepsilon\Gamma_{\xi_{\hat{\lambda}_{\Delta x}}}\right)\right]. \tag{A.10}$$

Consider the terms in A.10 separately and use the definition of $\hat{\lambda}_{\Delta x}$:

$$\Gamma_{\xi_{\hat{\lambda}_{\Delta x}}}\cdot\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)-\left(\hat{\lambda}_{\Delta x}-\delta\right)=$$

$$\frac{(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x)I(1+r_I)-(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x)(D(1+\bar{r}_D)+X/\omega)+(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x)\hat{\lambda}_{\Delta x}D(1+\bar{r}_D))}{D\left[(1+r_D)(1+r_I)-\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)(1+\bar{r}_D)\right]}$$

$$\frac{-\hat{\lambda}_{\Delta x}D(1+r_D)(1+r_I)+\left(\hat{\lambda}_{\Delta x}-\delta\right)\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)D(1+\bar{r}_D)+\delta D(1+r_D)(1+r_I)}{D\left[(1+r_D)(1+r_I)-\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)(1+\bar{r}_D)\right]}$$

$$=\frac{\left(\hat{\lambda}_{\Delta x}-\delta\right)\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)D(1+\bar{r}_D)+(\delta D(1+r_D)-LIQ)(1+r_I)}{D\left[(1+r_D)(1+r_I)-\left(\xi_{\hat{\lambda}_{\Delta x}}+\Delta x\right)(1+\bar{r}_D)\right]}, \tag{A.11}$$

which is positive from Lemma 1. Hence, there exists small enough noise such that A.10 is negative (bracketed terms positive) and the first line in A.9 is positive.

Now, consider the bracketed terms in the second line A.9, which can be written, by substituting the definition of $\lambda(\xi,\hat{x})$ from (15), as:

$$\int_{\hat{x}-\varepsilon}^{\xi_{\hat{\theta}_{\Delta x}}} \frac{LIQ+\left(\xi-\xi_{\hat{\lambda}_{\Delta x}}\right)I}{\frac{1+\delta}{2}+\frac{\hat{x}-\xi}{2\varepsilon}}d\xi+\int_{\xi_{\hat{\theta}_{\Delta x}}}^{\xi_{\hat{\lambda}_{\Delta x}}} D(1+r_D)d\xi. \tag{A.12}$$

The first term in (A.9) can be made very close to zero for small enough noise, and hence the second line in (A.9) is positive as well.

This concludes the argument to establish uniqueness of a threshold equilibrium $x' = x^*$ for small noise. See section 2.4 in the paper for a simpler version of this proof in the case of limiting noise, $\varepsilon \to 0$.

To conclude the proof, we need to show that the threshold equilibrium is indeed an equilibrium, i.e., $\Delta(x_i,x^*)$ in (18) is positive for all $x_i > x^*$ and negative for all $x_i < x^*$. A higher (lower) signal indicates not only that the fundamental state is better (worse), but also that fewer (more) patient savers withdraw. Both forces result in lower (higher) incentive to withdraw under global strategic complementarities and state monotonicity. But this is less obvious under one-sided strategic complementarities and perverse state monotonicity: In the run region, the incentive to withdraw increases the fewer savers withdraw and the higher the fundamental state is, which complicates the argument. Goldstein and Pauzner (2005) show that the single-crossing property is sufficient to show that the candidate threshold is indeed an equilibrium in a model without global strategic complementarities but with state monotonicity. We show below that single-crossing is sufficient even under perverse state monotonicity.

First, consider that $x_i < x^*$. Then we can decompose the intervals $[x_i-\varepsilon, x_i+\varepsilon]$ and $[x^*-\varepsilon, x^*+\varepsilon]$ into a common part $c = [x_i-\varepsilon, x_i+\varepsilon] \cap [x^*-\varepsilon, x^*+\varepsilon]$ and two disjoint parts $d^i = [x_i-\varepsilon, x_i+\varepsilon] \backslash c$ and $d^* = [x^*-\varepsilon, x^*+\varepsilon] \backslash c$. Thus, (18) and (19) can be written as:

$$\Delta(x_i,x^*) = \Delta^i_{\xi\in c} + \Delta^i_{\xi\in d^i}, \tag{A.13}$$

$$\Delta(x^*,x^*) = \Delta^*_{\xi\in c} + \Delta^*_{\xi\in d^i}. \tag{A.14}$$

All savers have the same belief about the (deterministic) number of withdrawals for threshold strategy x^*, which are given by $\lambda(\xi,x^*)$ for the level of fundamentals ξ. What changes with the signals is the posterior belief about ξ and, hence, the possible realizations of λ. From (15), $\lambda(\xi,x^*)$ is always one over d^i, thus $\Delta^i_{\xi\in d^i} = \int_{\xi\in d^i} \nu(\xi,1)d\xi = -\int_{\xi\in d^i}(LIQ+\xi\cdot I)d\xi < 0$. As a result, it suffices to show that $\Delta^i_{\xi\in c} < 0$. We will use the facts that A.14 is zero, and that ν changes sign ("crosses zero") only once and it is positive for higher values of ξ and negative for lower values of ξ in the interval $[x^*-\varepsilon, x^*+\varepsilon]$. Hence, $\Delta^*_{\xi\in d^*} > 0$ and $\Delta^*_{\xi\in c} < 0$, since the fundamentals are higher over d^* than c. The fact that v may be increasing in ξ in the lower segment of c does not matter, because v is still negative in that segment. If $\Delta^i_{\xi\in c} \leq \Delta^*_{\xi\in c}$, then we get the desired result. First, consider the case that all $\xi \in c$ are below the monitoring threshold $\xi_{\hat{\lambda}}$ given by equating (15) and (13), i.e., $\lambda(\xi_{\hat{\lambda}},x^*) = \hat{\lambda}(\xi_{\hat{\lambda}})$. Then, it is obvious that v is negative over c and $\Delta^i_{\xi\in c} = \Delta^*_{\xi\in c} < 0$. Second, consider the case that ξ_{λ^*} lies within c. Because a saver that receives signal x_i still believes that the number of withdrawals at each ξ is given by $\lambda(\xi, x^*)$, the (perceived) monitoring threshold, $\xi_{\hat{\lambda}}$, is not affected by the signal. Hence, $\Delta^i_{\xi\in c} = \Delta^*_{\xi\in c} < 0$ in this case as well, which concludes the argument. Essentially, observing a signal x_i below x^* shifts probability from positive values of ν to negative values of ν (recall that noise is uniformly distributed) and, thus, $\Delta(x_i, x^*) < \Delta(x^*, x^*)$. Note that the argument holds trivially if the interval c is empty. The proof for $x_i > x^*$ is similar, which verifies that x^* is indeed a threshold equilibrium.

A.3 Proof of Corollary 1

Totally differentiating (19), we get that $\partial\xi^*/\partial z = -(\partial GG^*/\partial z)/(\partial GG^*/\partial\xi^*)$, where z can be any of I, LIQ, D, r_I, r_D, or $\bar{r}_D$. Recall that $\partial GG^*/\partial\xi > 0$ from (22). Then, $\partial\xi^*/\partial I < 0$, because $\partial GG^*/\partial I = \omega D(1+\bar{r}_D)\partial\lambda^*/\partial I - \int_{\theta^*}^1 \xi^*/\lambda d\lambda = \omega D(1+\bar{r}_D)[\partial\hat{\lambda}(\xi^*)/\partial I - (\lambda^* - \delta)/I] + (\theta^* - \delta)D(1+r_D)/I + \int_{\theta^*}^1 LIQ/(\lambda I)d\lambda > 0$, from $\partial\lambda^*/\partial I - (\lambda^* - \delta)/I = [(\delta D(1+r_D) - LIQ)(1+r_I) + (1-\delta)\xi^* D(1+\bar{r}_D) + \xi^* X/\omega]/[I \cdot D((1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)] > 0$ from Lemma 1.

Moreover, $\partial\xi^*/\partial D > 0$ because $\partial GG^*/\partial D = \omega D(1+\bar{r}_D)[\partial\lambda^*/\partial D + (\lambda^* - \delta)/D] - (\theta^* - \delta)(1+r_D) < 0$, from $\partial\lambda^*/\partial D = -\xi^*(1+\bar{r}_D)/[D((1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)] - \lambda^*/D < 0$.

The partial effect of the loan rate and the early deposit rate are, respectively, negative and positive, because $\partial GG^*/\partial r_I = \omega D(1+\bar{r}_D)\partial\lambda^*/\partial r_I = \omega D(1+\bar{r}_D)[(1+r_D)(1+\bar{r}_D)\xi^*(1-\theta^*) + \xi^* X/\omega]/[((1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)] > 0$ and $\partial GG^*/\partial r_D = \omega D(1+\bar{r}_D)\partial\lambda^*/\partial r_D - (\theta^* - \delta)D = -\omega D(1+\bar{r}_D)\lambda^*(1+r_I)/[((1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)] - (\theta^* - \delta)D < 0$.

However, the sign of $\partial\xi^*/\partial LIQ < 0$ is ambiguous, because $\partial GG^*/\partial LIQ = \omega D(1+\bar{r}_D)\partial\lambda^*/\partial LIQ - \int_{\theta^*}^1 1/\lambda d\lambda = \omega D(1+\bar{r}_D)\cdot[\partial\lambda^*/\partial LIQ - (\lambda^* - \delta)/LIQ] + (\theta^* - \delta)D(1+r_D)/LIQ + \int_{\theta^*}^1 \xi I/(\lambda LIQ)d\lambda$, and we cannot unambiguously sign $\partial\lambda^*/\partial LIQ - (\lambda^* - \delta)/LIQ = [(\delta D(1+r_D) - \xi^* I)(1+r_I) + (1-\delta)\xi^* D(1+\bar{r}_D) + \xi^* X/\omega]/[I \cdot D((1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)]$.

Finally, the sign of $\partial\xi^*/\partial\bar{r}_D$ is also ambiguous, because $\partial GG^*/\partial\bar{r}_D = \omega D(1+\bar{r}_D)\partial\lambda^*/\partial\bar{r}_D + \omega D(\lambda^* - \delta) = \omega D[(\lambda^* - \delta)(1+r_D)(1+r_I) - (1-\delta)\xi^*(1+\bar{r}_D)]/[(1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)]$, which cannot be unambiguously signed.

A.4 Proof of Corollary 3

Given that $c(0) = 0$ and $c'(\cdot) > 0$, $c''(\cdot) = 0$ implies that $c(x) = a_c \cdot x$, with $a_c > 0$. Then, $\partial\mathbb{U}^*_E/\partial\xi^* = -[c'(I)I - c(I)]/\Delta_\xi = 0$. Moreover, the surplus to E, $(1-q)c''(I)I$, is zero. Similarly, given that $V(0) = 0$ and $V'(\cdot) > 0$, $V''(\cdot) = 0$ implies that $V(x) = a_v \cdot x$, with $a_v > 0$. Then, $\partial\mathbb{U}^*_S/\partial\xi^* = -[V(D(1+r_D)) - V'(D(1+r_D))D(1+r_D)]/\Delta_\xi = 0$. Moreover, the surplus from the transaction services of deposits, $(1-q)V''(D(1+r_D))D(1+r_D)^2$, is zero. Finally, the surplus in terms of period 1 utility, $U''(e_R - D)D$, is zero for $U''(\cdot) = 0$ as well as for $LIQ_S > 0$, because then $\mathbb{U}^*_S = \mathbb{U}^\alpha_S$ given that $V(D(1+r_D)) - V'(D(1+r_D))D(1+r_D) = 0$ from condition (ii). In turn, $LIQ_S > 0$ if savers endowment is higher than some threshold $\bar{e}_S$ at which (3) holds with equality.

A.5 Proof of Proposition 2

First, set $X = 0$ to make the determination of the run threshold in (20) scale invariant. Dividing it by the balance sheet size, $E + D$ (or $I + LIQ$), (20) becomes:

$$GG_{BS} = \int_{\delta}^{\hat{\lambda}} \omega(1-k)(1+\bar{r}_D)d\lambda - \int_{\delta}^{\theta^*} (1-k)(1+r_D) - \int_{\theta^*}^{1} \frac{\xi^*(1-\ell)+\ell}{\lambda} d\lambda = 0, \tag{A.15}$$

where

$$\hat{\lambda}_{BS} = \frac{(\xi^*(1-\ell)+\ell)(1+r_I) - \xi^*((1-k)(1+\bar{r}_D))}{(1-k)[(1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)]}. \tag{A.16}$$

Thus, k affects the payoff differential in a partial run as well as the range that monitoring occurs, $\hat{\lambda} - \delta$, via its effect on bank profitability. Totally differentiating (A.15) with respect to k, while keeping ℓ, r_I, r_D and $\bar{r}_D$ constant we get:

$$\frac{\partial GG_{BS}}{\partial k} = \underbrace{\frac{\partial \hat{\lambda}}{\partial k}\omega(1-k)(1+\bar{r}_D)}_{\text{More monitoring}} - \underbrace{(\hat{\lambda}-\delta)[\omega(1+\bar{r}_D) - (1+r_D)]}_{\substack{\text{Lower payoff} \\ \text{given monitoring}}} + \underbrace{(\theta^* - \hat{\lambda})(1+r_D)}_{\substack{\text{'Higher' payoff} \\ \text{absent monitoring}}}, \tag{A.17}$$

where $\partial\hat{\lambda}/\partial k > 0$. Hence, the trade-off from setting a higher requirement $k \geq \bar{k}$ is that monitoring becomes more probable, but the payoff to depositors is smaller given monitoring. Combining the two effects, we get that

$$\frac{\partial GG_{BS}}{\partial k} = \left[\frac{\xi^*(1+\bar{r}_D)}{(1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)} + \delta\right]\omega(1+\bar{r}_D) + (\theta^* - \delta)(1+r_D) > 0, \tag{A.18}$$

which implies that $\partial\xi^*/\partial k = -(\partial GG^*_{BS}/\partial k)/(\partial GG^*_{BS}/\partial\xi^*) < 0$, i.e., higher k reduces the run probability q.

Finally, note that $CR = k(1-\ell)$. So tightening leverage is equivalent to setting a higher capital requirement, all else being equal, and, thus, higher CR reduces the run probability q.

A.6 Proof of Proposition 3

The proof uses material described in proof A.5. Totally differentiating (A.15) with respect to ℓ, while keeping k, r_I, r_D and $\bar{r}_D$ constant, we get:

$$\frac{\partial GG_{BS}}{\partial \ell} = \underbrace{\frac{\partial \hat{\lambda}}{\partial \ell}\omega(1-k)(1+\bar{r}_D)}_{\text{More monitoring}} - \underbrace{\int_{\theta^*}^{1} \frac{1-\xi^*}{\lambda} d\lambda}_{\substack{\text{Higher payoff} \\ \text{in full run}}}, \tag{A.19}$$

where $\partial\hat{\lambda}/\partial\ell > 0$. Hence, the trade-off from setting a higher requirement $\ell \geq \bar{\ell}$ is that monitoring becomes more probable, but the incentives to join a full run increase. Combining the two effects, we get that

$$\frac{\partial GG_{BS}}{\partial \ell} = (1-\xi^*)\left[\frac{\omega(1+\bar{r}_D)(1+r_I)}{(1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)} + \log\theta^*\right], \tag{A.20}$$

which is definitely positive if $\log\theta^* > -1$ given that $\omega(1+\bar{r}_D) > 1+r_D$. In turn, this is satisfied under sufficient conditions $\delta > e^{-1}$ given that $\theta^* > \delta$ or $\ell > \hat{\ell} \equiv \left[e^{-1}\cdot(1-k)(1+r_D)-\xi^*\right]/(1-\xi^*)$, which is true for high enough ξ^* in the private equilibrium.

Finally, note that $LCR = ((1-\underline{\xi})\ell+\underline{\xi})/(k(1+r_D))$ and $NSFR = (k+(1-\delta)(1-k))/(1-\ell)$. So increasing ℓ is equivalent to increasing LCR or $NSFR$, all else being equal, and, thus, higher LCR or $NSFR$ reduce the run probability q.

A.7 Proof of Proposition 4

We show that three tools are needed to replicate the planner's allocations. We start with case i), which considers corrective taxes, and then turn to cases ii) and iii), which consider combinations of taxation and regulatory-ratio tools.

Our conjecture is that three tools are needed. For example, consider τ_I, τ_D and τ_{LIQ}. As shown in section B.6, condition (B.27) is sufficient and necessary to replicate the planner's allocations. This condition takes the following matrix form under the three aforementioned Pigouvian taxation tools

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & -1 & 0 \end{bmatrix}\begin{bmatrix} \tau_I \\ \tau_D \\ \tau_{LIQ} \end{bmatrix} = \begin{bmatrix} AAM_{WD} \\ CSM_{WD} \\ SIM_{WD} \end{bmatrix}, \tag{A.21}$$

which reduces to

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} \tau_I \\ \tau_D \\ \tau_{LIQ} \end{bmatrix} = \begin{bmatrix} -(CSM_{WD}+SIM_{WD}) \\ CSM_{WD} \\ -(AAM_{WD}+CSM_{WD}+SIM_{WD}) \end{bmatrix}. \tag{A.22}$$

Since, generically, $AAM_{WD} \neq 0$, $CSM_{WD} \neq 0$, $SIM_{WD} \neq 0$, $CSM_{WD}+SIM_{WD} = -AAM_{WD} + (w_S\cdot\partial\mathbb{U}_S^*/\partial\xi^* + w_E\cdot\partial\mathbb{U}_E^*/\partial\xi^*)\cdot\partial\xi^*/\partial LIQ \neq 0$ and $AAM_{WD}+CSM_{WD}+SIM_{WD} = (w_S\cdot\partial\mathbb{U}_S^*/\partial\xi^* + w_E\cdot\partial\mathbb{U}_E^*/\partial\xi^*)\cdot\partial\xi^*/\partial LIQ \neq 0$, (A.22) tells us that three tools are needed to replicate the planner's allocations.

Alternatively, we could consider a combination of τ_I, τ_E, and τ_{LIQ}. Following the same methodology, we get $\tau_I = -SIM_{WD} \neq 0$, $\tau_E = -CSM_{WD} \neq 0$, and $\tau_{LIQ} = -(AAM_{WD}+SIM_{WD}) = CSM_{WD} - (w_S\cdot\partial\mathbb{U}_S^*/\partial\xi^* + w_E\cdot\partial\mathbb{U}_E^*/\partial\xi^*)\cdot\partial\xi^*/\partial LIQ \neq 0$. Other possible combinations would also yield the same result, i.e., that three tools are needed to replicate the planner's allocations.

Turning to combinations of taxation and regulatory-ratio tools, consider the following mix: A capital requirement $\overline{CR}$, a liquidity requirement $\bar{\ell}$ and a subsidy on deposit-taking $-\tau_D$. Condition

(B.27) becomes

$$\begin{bmatrix} \overline{CR} & 1 & 0 \\ 1 & 0 & 1 \\ -\overline{CR} & -\overline{\ell} & -1 \end{bmatrix} \begin{bmatrix} \lambda_{CR} \\ \lambda_{\ell} \\ \tau_D \end{bmatrix} = \begin{bmatrix} AAM_{WD} \\ CSM_{WD} \\ SIM_{WD} \end{bmatrix}, \tag{A.23}$$

which reduces to

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{CR} \\ \lambda_{\ell} \\ \tau_D \end{bmatrix} = \frac{1}{1-\overline{CR}(1-\overline{\ell})} \begin{bmatrix} \overline{\ell}\cdot AAM_{WD} + CSM_{WD} + SIM_{WD} \\ AAM_{WD} - \overline{CR}\cdot (AAM_{WD} + CSM_{WD} + SIM_{WD}) \\ -\overline{\ell}\cdot AAM_{WD} - (1-\overline{\ell})\cdot \overline{CR}\cdot CSM_{WD} - SIM_{WD} \end{bmatrix}. \tag{A.24}$$

Thus, λ_{CR} and λ_{ℓ} are positive, i.e., the capital and liquidity requirement are binding, if $CR < AAM_{WD}/(AAM_{WD} + CSM_{WD} + SIM_{WD}) < 1-\ell$.

Finally, consider a regulatory mix consisting for a capital requirement $\overline{CR}$, a lending subsidy $-\tau_I$, and a deposit-taking subsidy $-\tau_D$. Condition (B.27) becomes

$$\begin{bmatrix} \overline{CR} & 1 & 0 \\ 1 & 0 & 1 \\ -\overline{CR} & -1 & -1 \end{bmatrix} \begin{bmatrix} \lambda_{CR} \\ \tau_I \\ \tau_D \end{bmatrix} = \begin{bmatrix} AAM_{WD} \\ CSM_{WD} \\ SIM_{WD} \end{bmatrix}, \tag{A.25}$$

which reduces to

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \lambda_{CR} \\ \tau_I \\ \tau_D \end{bmatrix} = \begin{bmatrix} AAM_{WD} + CSM_{WD} + SIM_{WD} \\ AAM_{WD} - \overline{CR}\cdot (AAM_{WD} + CSM_{WD} + SIM_{WD}) \\ -(AAM_{WD} + SIM_{WD}) \end{bmatrix}. \tag{A.26}$$

Thus, $\lambda_{CR} > 0$, i.e., capital requirements are binding because $AAM_{WD} + CSM_{WD} + SIM_{WD} = (w_S \partial \mathbb{U}_S^*/\partial \xi^* + w_E \partial \mathbb{U}_E^*/\partial \xi^*)\cdot \partial \xi^*/\partial LIQ > 0$ under the assumptions in Proposition 3. Moreover, a lending subsidy requires $AAM_{WD} < \overline{CR}\cdot (AAM_{WD} + CSM_{WD} + SIM_{WD})$, which is trivially guaranteed for high enough w_E making $AAM_{WD} < 0$. Otherwise, a tax on lending would be needed, which is equivalent to a liquidity requirement examined above.

B Extensions and Additional Derivations

B.1 Intermediation Margins in Private Equilibrium

The first-order conditions (24) together with the four constraints in $\mathcal{Y}$ can be combined to characterize the private equilibrium as follows. If (24) gives an interior $r_D > 0$, then it is used to determine r_D as a function of all other variables in $\mathcal{C}$; otherwise, set $r_D = 0$. Then, use (2), (7), (9), and (20) to express (implicitly) $\bar{r}_D$, r_I, E, and ξ^* in terms of I, LIQ and D. The next step is to express the shadow values on the four constraints $\mathcal{Y}$ in terms of I, LIQ, and D. The shadow value of funds is determined by the first-condition with respect to E,

$$\psi_{BS} = W'(e_B + D - I - LIQ), \tag{B.1}$$

where we have substituted $E = I + LIQ - D$.

The shadow value on the deposit supply schedule can be obtained from (24) with respect to $\bar{r}_D$, which yields

$$\psi_{DS} = -\left(\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D} + \psi_{GG}\frac{\partial GG}{\partial \bar{r}_D}\right)\frac{\partial DS}{\partial \bar{r}_D}^{-1}. \tag{B.2}$$

The choice of $\bar{r}_D$ matters for the banker via the effect on profits and on the run dynamics. The shadow value determined in (B.2) captures the sum of these effects as the deposit rate moves along the deposit supply schedule. Because the three variables of interest—I, LIQ, and D—affect the loan demand directly as well as indirectly via $\bar{r}_D$, their overall effect on the deposit supply will be scaled by the shadow value ψ_{DS} in their respective first-order conditions.

The shadow value on the loan demand schedule can be obtained from (24) with respect to r_I, which yields

$$\psi_{LD} = -\left(\frac{\partial \mathbb{U}_B}{\partial r_I} + \psi_{GG}\frac{\partial GG}{\partial r_I}\right)\frac{\partial LD}{\partial r_I}^{-1}. \tag{B.3}$$

Similar to (B.2), condition (B.3) says that the shadow value on the loan demand is measured by how a change in the loan rate along the loan demand schedule affects banker's utility.

Equivalently, combining (24) for $\mathcal{C} = \xi^*$, (B.2) and (B.3), the shadow value on the global game constraint is given by

$$\psi_{GG} = -\frac{d\mathbb{U}_B}{d\xi^*}\cdot\frac{dGG}{d\xi^*}^{-1}, \tag{B.4}$$

where $d\mathbb{U}_B/d\xi^*$ is the total effect of the run threshold ξ^* on banker's utility, which captures the partial direct effect—$\partial\mathbb{U}_B/\partial\xi^*$ in (B.70)—and the partial indirect effects via the deposit and loan rate, in (B.70) and (B.70), respectively:

$$\frac{d\mathbb{U}_B}{d\xi^*} = \left(\frac{\partial \mathbb{U}_B}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial \bar{r}_D}\frac{\partial DS}{\partial \bar{r}_D}^{-1}\frac{\partial DS}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\frac{\partial LD}{\partial \xi^*}\right). \tag{B.5}$$

Similarly, $dGG/d\xi^*$ is the total effect of the run threshold ξ^* on the utility differential determining the run behavior, which captures the partial direct effect—$\partial GG/\partial\xi^*$ in (22)—and the partial indirect effects via the deposit and loan rate, in (B.94) and (B.102), respectively:

$$\frac{dGG}{d\xi^*} = \frac{\partial GG}{\partial \xi^*} - \frac{\partial GG}{\partial \bar{r}_D}\frac{\partial DS}{\partial \bar{r}_D}^{-1}\frac{\partial DS}{\partial \xi^*} - \frac{\partial GG}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\frac{\partial LD}{\partial \xi^*}. \tag{B.6}$$

Overall, ψ_{GG} measures the effect of a change in the run threshold, which is consistent with optimal run behavior, i.e., along the global game constraint, on B's welfare.

Combining (24) with respect to LIQ and I and substituting in (B.2), (B.3), and (B.4), we obtain the asset allocation margin in the private equilibrium (AAM_{PE}):

$$\overbrace{\frac{\partial \mathbb{U}_B}{\partial LIQ}-\frac{\partial \mathbb{U}_B}{\partial I}-\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D}\frac{\partial DS}{\partial \bar{r}_D}^{-1}\left(\frac{\partial DS}{\partial LIQ}-\frac{\partial DS}{\partial I}\right)-\frac{\partial \mathbb{U}_B}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\left(\frac{\partial LD}{\partial LIQ}-\frac{\partial LD}{\partial I}\right)}^{\text{Effect of asset mix on } \mathbb{U}_B \text{ via bank profits: } d\mathbb{U}_B/dLIQ - d\mathbb{U}_B/dI}$$

$$+\underbrace{\left(\frac{\partial \mathbb{U}_B}{\partial \xi^*}-\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D}\frac{\partial DS}{\partial \bar{r}_D}^{-1}\frac{\partial DS}{\partial \xi^*}-\frac{\partial \mathbb{U}_B}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\frac{\partial LD}{\partial \xi^*}\right)}_{\text{Effect on } \mathbb{U}_B \text{ via } \xi^*\text{: } d\mathbb{U}_B/d\xi^*}\cdot\underbrace{\left(\frac{\partial \xi^*}{\partial LIQ}-\frac{d\xi^*}{dI}\right)}_{\text{Effect of asset mix on } \xi^*}=0, \tag{B.7}$$

where $d\xi^*/dLIQ$ and $d\xi^*/dI$ are obtained from total differentiation of (20), hence

$$\frac{d\xi^*}{dLIQ}-\frac{d\xi^*}{dI}=-\left[\frac{\partial GG}{\partial LIQ}-\frac{\partial GG}{\partial I}-\frac{\partial GG}{\partial \bar{r}_D}\frac{\partial DS}{\partial \bar{r}_D}^{-1}\left(\frac{\partial DS}{\partial LIQ}-\frac{\partial DS}{\partial I}\right)-\frac{\partial GG}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\left(\frac{\partial LD}{\partial LIQ}-\frac{\partial LD}{\partial I}\right)\right]\cdot\frac{dGG}{d\xi^*}^{-1}. \tag{B.8}$$

The asset allocation margin in (B.7) captures the decision to substitute a unit of loans with a unit of liquid assets. The banker in the private equilibrium weighs the effect of the change in the asset mix on the bank profitability (the first line) and on the run threshold, which determines run-risk, because both affect her welfare (the second line). The asset mix matters for bank profits because of portfolio effects (first two terms in first line), but also because of the way it influences the profit margin via the loan rate and deposit rates (remaining terms in first line). The latter (general equilibrium) effect via rates captures how the asset mix matters for the loan rate or deposit rate that entrepreneurs and depositors are willing to accept. Similarly, the asset mix changes the payoffs governing the run dynamics directly and indirectly via the loan and deposit rates (captured by (B.8)), which in turn affect the run threshold influencing B's welfare directly and indirectly via the loan and deposit rates.

Similarly, combining (24) with respect to E and D and substituting in (B.2), (B.3), and (B.4), we obtain the capital structure margin in the private equilibrium (CSM_{PE}):

$$\overbrace{\frac{\partial \mathbb{U}_B}{\partial E}-\frac{\partial \mathbb{U}_B}{\partial D}+\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D}\frac{\partial DS}{\partial \bar{r}_D}^{-1}\frac{\partial DS}{\partial D}+\frac{\partial \mathbb{U}_B}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\frac{\partial LD}{\partial D}}^{\text{Effect of liabilities mix on } \mathbb{U}_B \text{ via bank profits: } d\mathbb{U}_B/dE - d\mathbb{U}_B/dD}$$

$$-\underbrace{\left(\frac{\partial \mathbb{U}_B}{\partial \xi^*}-\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D}\frac{\partial DS}{\partial \bar{r}_D}^{-1}\frac{\partial DS}{\partial \xi^*}-\frac{\partial \mathbb{U}_B}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\frac{\partial LD}{\partial \xi^*}\right)}_{\text{Effect on } \mathbb{U}_B \text{ via } \xi^*\text{: } d\mathbb{U}_B/d\xi^*}\cdot\underbrace{\frac{d\xi^*}{dD}}_{\substack{\text{Effect of liabilities}\\ \text{mix on } \xi^*}}=0, \tag{B.9}$$

where

$$\frac{d\xi^*}{dD}=-\left[\frac{\partial GG}{\partial D}-\frac{\partial GG}{\partial \bar{r}_D}\frac{\partial DS}{\partial \bar{r}_D}^{-1}\frac{\partial DS}{\partial D}-\frac{\partial GG}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\frac{\partial LD}{\partial D}\right]\cdot\frac{dGG}{d\xi^*}^{-1}. \tag{B.10}$$

Finally, combining (24) with respect to I and D and substituting in (B.2), (B.3), and (B.4), we obtain the margin for the scale of intermediation in the private equilibrium (SIM_{PE}):

$$\overbrace{\frac{\partial \mathbb{U}_B}{\partial I}+\frac{\partial \mathbb{U}_B}{\partial D}-\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D}\frac{\partial DS}{\partial \bar{r}_D}^{-1}\left(\frac{\partial DS}{\partial I}+\frac{\partial DS}{\partial D}\right)-\frac{\partial \mathbb{U}_B}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\left(\frac{\partial LD}{\partial I}+\frac{\partial LD}{\partial D}\right)}^{\text{Effect of intermediation scale on } \mathbb{U}_B \text{ via bank profits: } d\mathbb{U}_B/dI+d\mathbb{U}_B/dD}$$

$$+\underbrace{\left(\frac{\partial \mathbb{U}_B}{\partial \xi^*}-\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D}\frac{\partial DS}{\partial \bar{r}_D}^{-1}\frac{\partial DS}{\partial \xi^*}-\frac{\partial \mathbb{U}_B}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\frac{\partial LD}{\partial \xi^*}\right)}_{\text{Effect on } \mathbb{U}_B \text{ via } \xi^*:\ d\mathbb{U}_B/d\xi^*}\cdot\underbrace{\left(\frac{d\xi^*}{dI}+\frac{d\xi^*}{dD}\right)}_{\text{Effect of intermediation scale on } \xi^*}=0, \qquad \text{(B.11)}$$

where

$$\frac{d\xi^*}{dI}+\frac{d\xi^*}{dD}=-\left[\frac{\partial GG}{\partial I}+\frac{\partial GG}{\partial D}-\frac{\partial GG}{\partial \bar{r}_D}\frac{\partial DS}{\partial \bar{r}_D}^{-1}\left(\frac{\partial DS}{\partial I}+\frac{\partial DS}{\partial D}\right)-\frac{\partial GG}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\left(\frac{\partial LD}{\partial I}+\frac{\partial LD}{\partial D}\right)\right]\cdot\frac{dGG}{d\xi^*}^{-1}. \qquad \text{(B.12)}$$

Note that, expanding the first two terms in (B.11), we get that

$$\frac{\partial \mathbb{U}_B}{\partial I}+\frac{\partial \mathbb{U}_B}{\partial D}=\omega\Big\{[(1-q)-\delta(1+r_D)log(\bar{\xi}/\xi^*)/\Delta_\xi](1+r^I)-(1-\delta)(1+\bar{r}_D)\Big\}, \qquad \text{(B.13)}$$

where q is the run probability. Hence, the third margin capturing the scale of intermediation can be proxied by the intermediation spread between the loan rate, r_I, and the late deposit rate, $\bar{r}_D$.

The three intermediation margins pin down the three free variables I, LIQ, and D. The remaining variables, E, ξ^*, r_I, and $\bar{r}_D$, are implicitly functions of the three free variables via constraints (9), (20), (7) and (2), which are always binding in equilibrium . Hence, there are three degrees of freedom and the private equilibrium is characterized by (B.7), (B.9) and (B.11).

B.2 Intermediation Margins in Social Planner's Equilibrium

As for the private equilibrium, we can use the first-order conditions (34) in the planning problem together with the four constraints in $\mathcal{Y}$ to characterize the planning allocations. In particular, we use the first-order condition with respect to r_D to determine its value—which is zero as in the PE— and (2), (7), (9), and (20) to express (implicitly) $\bar{r}_D$, r_I, E, and ξ^* in terms of I, LIQ, and D. As discussed, we consider a planner that respects the deposit supply and loan demand schedule, because we want to focus on regulation to affect bank's behavior. See section B.9 for a more powerful planner, who can levy distortionary taxes to affect the private deposit supply and loan demand schedules.

Up to this point everything is analogous to the characterization of PE allocations in section B.2. But, the planner also cares about the direct effect on S and E welfare as captured in the social welfare function (29). This influences the functional form of the Lagrange multipliers on constraints $\mathcal{Y}$, which are denoted by $\zeta_{\mathcal{Y}}$ instead of $\psi_{\mathcal{Y}}$.

The functional forms of ζ_{BS}, ζ_{DS}, and ζ_{LD} are the same as for ψ_{BS}, ψ_{DS}, and ψ_{LD} given by (B.1), (B.2) and (B.3), with the exception that the latter two are functions of ζ_{GG} instead of ψ_{GG}. The reason is that E, $\bar{r}_D$, and r_I do not appear directly in (29). Note, this does not mean that the equilibrium values of these Lagrange multipliers are the same in the private and planning solutions. But, the multiplier ζ_{GG} on constraint (20) will have a different functional form compared to (B.4), because ξ^* appears in the indirect utilities:

$$\begin{aligned}\zeta_{GG} &= -\left(\frac{\partial \mathbb{U}_B}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial r_3^D}\frac{\partial DS}{\partial r_3^D}^{-1}\frac{\partial DS}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial r^I}\frac{\partial LD}{\partial r^I}^{-1}\frac{\partial LD}{\partial \xi^*} + w_S\frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E\frac{\partial \mathbb{U}_E^*}{\partial \xi^*}\right)\frac{dGG}{d\xi^*}^{-1} \\ &= \psi_{GG} - \left(w_S\frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E\frac{\partial \mathbb{U}_E^*}{\partial \xi^*}\right)\frac{dGG}{d\xi^*}^{-1} \\ &= \psi_{GG}\left[1 + \left(w_S\frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E\frac{\partial \mathbb{U}_E^*}{\partial \xi^*}\right)\frac{d\mathbb{U}_B}{d\xi^*}\right], \end{aligned} \tag{B.14}$$

where the terms in red are the additional terms in the planner's problem. Because $\partial \mathbb{U}_S^*/\partial \xi^* = -[V(D) - V'(D)D]\Delta_\xi < 0$, $\partial \mathbb{U}_E^*/\partial \xi^* = -[c'(I)I - c(I)]\Delta_\xi < 0$, and, from (B.5), $d\mathbb{U}_B/d\xi^* < 0$,[1] internalizing the run externalities makes the Lagrange multiplier on the constraint (20) higher (when evaluated at the PE allocations). We can now derive the wedges between the private and social intermediation margins.

Combining (34) with respect to *LIQ* and *I* together with (25), we can derive the following wedge in the Asset Allocation Margin (also reported in (35)):

$$AAM_{WD} = \underbrace{\left(w_S\frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E\frac{\partial \mathbb{U}_E^*}{\partial \xi^*}\right)\cdot\left(\frac{\partial \xi^*}{\partial LIQ} - \frac{\partial \xi^*}{\partial I}\right)}_{\text{Run externality from asset allocation}} - \underbrace{w_E(1-q)c''(I)I}_{\text{Surplus to E from additional } I} \tag{B.15}$$

As discussed, the first term in (B.15) captures how a shift in the asset allocation from loans to liquid asset holdings affects savers' and entrepreneurs' welfare via its effect on the run probability. Both savers and entrepreneurs are worse-off when run-risk goes up, i.e., $\partial \mathbb{U}_S^*/\partial \xi^* < 0$ and $\partial \mathbb{U}_E^*/\partial \xi^* < 0$. Thus, if run-risk decreases when the asset allocation shifts towards liquid assets, then the planner would want a more liquid asset mix. The second term in (B.15) captures the surplus created for the entrepreneur from an additional unit of investment. This term is negative (including the minus sign) if the planner puts weight on *E* and if *E* extracts some surplus to start with, i.e., $c'' > 0$, which is true for a strictly convex function.

Examining the run externalities in more detail the term $\partial \xi^*/\partial LIQ - \partial \xi^*/\partial I$ captures exactly the effects of substituting a unit of liquid assets with a unit of loans on the probability of a run and is given by (B.8). If it is negative, correcting the run externalities requires a more liquid asset mix. The first and third terms inside the bracket are unambiguously positive, respectively. The first term captures the direct effect of shifting the asset allocation towards more liquid assets on the incentive to run. Combining (B.83) and (B.82) we get that $\partial GG/\partial LIQ - \partial GG/\partial I = \partial GG/\partial \ell$, which

[1] $\partial \mathbb{U}_B/\partial \xi^* < 0$ from (B.70); $\partial \mathbb{U}_B/\partial \bar{r}_D < 0$ from (B.73); $\partial DS/\partial \bar{r}_D > 0$ from (B.97); $\partial DS/\partial DS/\xi^* < 0$ from (B.94); $\partial \mathbb{U}_B/\partial r_I > 0$ from (B.71); $\partial LD/\partial r_I < 0$ from (B.103); $\partial LD/\partial \xi^*$ from (B.102) can be positive or negative, so the effect through the loan demand cannot be unambiguously determined. However, in our examples, the first two terms dominate, and overall, a higher run threshold reduces banker's utility all else being equal.

is positive as we prove in Proposition 3 (recall that $\ell \equiv LIQ/(I+LIQ)$ is the share of liquid assets in the asset portfolio). Intuitively, a more liquid asset portfolio (directly) decreases the incentives to run. The third term captures the indirect effect via the loan rate. In particular, $\partial GG/\partial r_I > 0$ from (B.79) and (B.87), $\partial LD/\partial r_I = -\omega \int_{\xi^*}^{\overline{\xi}} (1-y) I d\xi/\Delta_\xi < 0$ from (B.103), and, combining (B.98) and (B.99), we get that $\partial LD/\partial LIQ - \partial LD/\partial I = \omega[A-(1+r_I)]log(\overline{\xi}/\xi^*)/\Delta_\xi[LIQ+I-\delta D(1+r_D)]/I^2 - (1-q)c''(I) > 0$. Thus, the third term in (B.8)—including the minus sign—is positive. Intuitively, a higher loan rate reduces the incentives to run, because it increases bank profits and, hence, the region where the banker decides to monitor. In turn, a more liquid asset portfolio increases the loan demand, because of convex investment costs and fewer loans being recalled, pushing up the loan rate entrepreneurs are willing to pay.

Finally, the second term captures the indirect effect via the late deposit rate. If we could show that this is always positive, then the whole expression would be negative given that $dGG/d\xi^*$ needs to be positive to have $\psi_{GG} > 0$ in the private equilibrium. In particular, combining (B.91) and (B.90) we get that $\partial DS/\partial LIQ - \partial DS/\partial I = q[1-(\xi^* + \underline{\xi})/2-]/D > 0$, while $\partial DS/\partial \bar{r}_D = \omega(1-q)(1-\delta) > 0$ from (B.97). Intuitively, a more liquid asset portfolio increases the demand for deposits because it increases the probability of being repaid in a run, θ, and, thus, pushes down the rate depositors demand. However, the effect of the deposit rate on the incentives to run, $\partial GG/\partial \bar{r}_D$, could be unambiguously determined:

$$\frac{\partial GG}{\partial \bar{r}_D} = \underbrace{(\hat{\lambda}-\delta)\omega D}_{\substack{\text{Higher payoff} \\ \text{given monitoring}}} + \underbrace{\frac{\partial \hat{\lambda}}{\partial \bar{r}_D}\omega D(1+\bar{r}_D)}_{\substack{\text{Lower chance} \\ \text{of monitoring}}} = \omega D \frac{(\lambda-\delta)(1+r_D)(1+r_I)-(1-\delta)\xi^*(1+\bar{r}_D)}{(1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)}. \tag{B.16}$$

In other words, a higher deposit rate increases the payoff from waiting given monitoring, which reduces the incentives to run, but also reduces the chances that monitoring takes place, i.e., $\partial\hat{\lambda}/\partial\bar{r}_D < 0$ from (B.81), which increases the incentives to run. We haven't been able to sign the overall effect analytically, but, in all the examples we have studied, we find that $\partial GG/\partial \bar{r}_D > 0$, or in other words a higher deposit rate reduces the run probability, all else being equal.[2] This is an important and novel channel in our model (see the discussion at the end of section 2.4).

Turning to the capital structure margin, we combine (34) with respect to E and D together with (26) to get the following wedge (also reported in (36)):

$$CSM_{WD} = \underbrace{-\left(w_S \frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E \frac{\partial \mathbb{U}_E^*}{\partial \xi^*}\right)\frac{\partial \xi^*}{\partial D}}_{\text{Run externality from liabilities mix}} + \underbrace{w_R\left[U''(e_R-D)D+(1-q)V''(D(1+r_D)D(1+r_D)^2\right]}_{\text{Surplus to S from additional } D} \tag{B.17}$$

Similar to the wedge in the asset allocation margin, the wedge in the capital structure margin features two components. The first term captures how a shift in the liabilities mix from deposits to equity affects savers' and entrepreneurs' welfare via its effect on the run probability. Both savers and

[2] Recall that $\partial\xi^*/\partial\bar{r}_D = -(\partial GG/\partial\bar{r}_D)/(\partial GG/\partial\xi^*)$.

entrepreneurs are worse-off when run-risk goes up. Thus, if run-risk decreases when the liabilities mix shifts towards equity, then the planner would want lower leverage.[3] The second term captures the surplus created for savers from an additional unit of deposits and is negative if the planner puts weigh on S and if S extracts some surplus to start with, which is true for strictly concave functions U and V. In others words, shifting the capital structure away from deposits entails a welfare cost because of the lower surplus from deposit services to savers.

With respect to the run externalities, the term $\partial\xi^*/\partial D$ can be decomposed into a direct effect on the incentives to run and two indirect effects via the deposit and loan rate depicted in (B.10). Similar to above, the direct effect and the indirect effect via the loan rate can be unambiguously signed, but the indirect effect via the deposit rate cannot. In particular, expanding (B.84), we get $\partial GG/\partial D = -\omega(1+\bar{r}_D)[\delta(1+r_D)(1+r_I)+(1-\delta)\xi^*(1+\bar{r}_D)] - (1+r_D)(\theta^*-\delta) < 0$, while $\partial DS/\partial D < 0$ from (B.92), and $\partial LD/\partial D < 0$ from (B.100). But, as mentioned, we find that $\partial GG/\partial \bar{r}_D > 0$, which means that the indirect effect via the deposit rate operates in the opposite direction compared with both the direct effect and indirect effect via the loan rate. Nevertheless, the indirect effect via the deposit rate does not dominate the other two effects and $\partial\xi^*/\partial D > 0$, which means that run-risk increases when deposits go up, all else being equal.

Lastly, combining first-order condition (34) with respect to I and D together with (27), we obtain the following wedge in the scale of intermediation margin (also reported in (37):

$$SIM_{WD} = \overbrace{\left(w_S\frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E\frac{\partial \mathbb{U}_E^*}{\partial \xi^*}\right)\cdot\left(\frac{\partial\xi^*}{\partial I}+\frac{\partial\xi^*}{\partial D}\right)}^{\text{Run externality from intermediation scale}} \underbrace{-w_S[U''(e_S-D)D+(1-q)V''(D(1+r_D))D(1+r_D)^2]+w_E(1-q)c''(I)I}_{\text{Surplus to } S \text{ and } E \text{ from higher intermediation scale}}. \tag{B.18}$$

Similar to the other two wedges, (B.18) has a component that captures the externality from the intermediation scale on the run probability and a component that captures the surplus created for S and E. The latter is unambiguously positive as more intermediation, i.e., more deposits channeled to lending, increases the surplus to both savers and entrepreneurs. The impact of the intermediation scale of the run probability captured by $\partial\xi^*/\partial I + \partial\xi^*/\partial D$ (see (B.12) for the detailed expression) comprises of a direct effect as well as indirect effects via the deposit and loan rate, similar to the other two wedges. All three components are hard to sign analytically without knowing the ratio of lending to deposits, I/D, in the private equilibrium.

Overall, the planner balances the run externalities and the additional surpluses to savers and entrepreneurs when deciding how the asset allocation, the capital structure, and the scale of intermediation should differ from the private equilibrium.

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[3] Note that the equity capital choice does not directly enter the global game constraint GG. Thus, the effect of shifting the capital structure from deposit to equity is written as $\partial\xi^*/\partial E - \partial\xi^*/\partial D = -\partial\xi^*/\partial D$.

B.3 Disciplining Role of Runs

This section explores what is the role of the run in disciplining the banker. In particular, we compute the private equilibrium in which the banker does not internalize the effect of her actions on the run probability, i.e., $\psi_{GG} = 0$ in (24). Table B.1 reports the results. The banker chooses allocations that result in higher run-risk and all agents are worse-off, while capital and liquidity are much lower than in the socially optimal outcomes even when E is favored. As expected, there is a much bigger scope for regulation if B neglected her impact on run-risk and all agents could be made better-off.

	PE	PE	SP for weights (w_E, w_S)		
	$\psi_{GG} > 0$	$\psi_{GG} = 0$	(0.00,0.20)	(0.10,0.10)	(0.20,0.00)
I	0.862	0.827	0.785	0.873	0.906
LIQ	0.052	0.000	0.221	0.060	0.000
D	0.875	0.803	0.962	0.894	0.867
E	0.038	0.024	0.044	0.039	0.038
r_I	3.097	3.119	3.198	3.089	3.042
$\bar{r}_D$	0.717	0.581	0.804	0.767	0.758
q	0.407	0.425	0.386	0.403	0.408
ℓ	0.057	0.000	0.219	0.065	0.000
k	0.042	0.028	0.044	0.042	0.042
$r_I - (1-\delta)\bar{r}_D$	2.739	2.828	2.796	2.705	2.663
$I + LIQ$	0.914	0.827	1.006	0.933	0.906
$I - E$	0.824	0.803	0.741	0.834	0.867
$E(Div)$	0.745	0.703	0.755	0.747	0.743
$\Delta\mathbb{U}_E$	-	-1.11%	-1.66%	0.33%	1.19%
$\Delta\mathbb{U}_S$	-	-2.10%	3.63%	0.71%	-0.30%
$\Delta\mathbb{U}_B$	-	-0.89%	-0.44%	-0.05%	-0.09%

Table B.1: Private equilibrium allocations when the banker does and does not internalize the effect of her action on run-risk versus Socially optimal solutions. The welfare changes are computed over the level of welfare in the private equilibrium where the banker internalizes run-risk, which is normalized to one for each agent.

B.4 Incomplete Deposit Contracts and Lack of Commitment

This section studies the case of incomplete deposit contracts. First, we show how the wedges in the intermediation margins between the private and social solutions change. Second, we report how the numerical solution for the private equilibrium under incomplete contracts compares to the private and social outcomes discussed in section 3.2.

Under incomplete deposit contracts, the banker only internalizes the effect of terms specified in the deposit contract on the deposit supply. At minimum, these terms include the amount of deposits, D, and the deposit rates, r_D and $\bar{r}_D$. As a result, the banker would be tempted to deviate when choosing the rest of the balance sheet after she has entered into a deposit contract and received the deposits. The banker does understand that taking more risk increases the cost of raising deposits and would ideally want to promise depositors that she will behave prudently. But, after the deposit contract has been signed, the banker has an incentive to deviate towards lending more, holding fewer liquid assets, and raising less equity.

Depositors have rational expectations and ex ante require that the banker offers higher deposit rates to compensate for the anticipated risk-taking due to the lack of commitment. As a result, the deposit supply schedule has the same functional form to the benchmark environment. The difference in the private equilibrium comes from the fact that the banker will not include the effect of I, LIQ and ξ^* on DS in the respective first-order conditions, i.e., the respective (24) will not include the terms multiplied by ψ_{DS}.

The Lagrange multipliers ψ_{DS} and ψ_{LD} will have the same functional form derived in (B.2) and (B.3), respectively. However, the functional form of ψ_{GG} will be different from the one in (B.4):

$$\widehat{\psi}_{GG} = -\frac{\frac{\partial \mathbb{U}_B}{\partial \xi^*} - \frac{\partial \mathbb{U}_B}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\frac{\partial LD}{\partial \xi^*}}{\frac{\partial GG}{\partial \xi^*} - \frac{\partial GG}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\frac{\partial LD}{\partial \xi^*}}. \tag{B.19}$$

So the difference between the multipliers in the social equilibrium, ζ_{GG} given by (B.14), and private equilibrium, $\widehat{\psi}_{GG}$ given by (B.19), does not only come from the presence of run externalities, but also from externalities arising from contract incompleteness:

$$\zeta_{GG} - \widehat{\psi}_{GG} = \underbrace{-\left(w_S\frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E\frac{\partial \mathbb{U}_E^*}{\partial \xi^*}\right)\frac{dGG}{d\xi^*}^{-1}}_{\text{Run externality}} + \underbrace{\left(\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D} + \widehat{\psi}_{GG}\frac{\partial GG}{\partial \bar{r}_D}\right)\frac{\partial DS}{\partial \bar{r}_D}^{-1}\frac{\partial DS}{\partial \xi^*}\frac{dGG}{d\xi^*}^{-1}}_{\text{Incomplete contract externality}}, \tag{B.20}$$

where $dGG/d\xi^*$ is given as before by (22). Note that, under complete contracts, the difference between ζ_{GG} and ψ_{GG}, given by (B.4), is only due to the run externality, i.e., the first term in (B.20).

The three expressions in (B.21), (B.22), and (B.23) below report the wedges in the three intermediation margins, separating the externalities stemming from incomplete contracts.

The wedge is the asset allocation margin is:

$$\widehat{AAM}_{WD} = \underbrace{\left(w_S \frac{\partial \mathbb{U}_S^*}{\partial \xi^*} + w_E \frac{\partial \mathbb{U}_E^*}{\partial \xi^*}\right) \cdot \left(\frac{\partial \xi^*}{\partial LIQ} - \frac{\partial \xi^*}{\partial I}\right)}_{\text{Run externality from asset allocation}} - \underbrace{w_E (1-q) c''(I) I}_{\text{Surplus to E from additional } I}$$

$$\underbrace{+ \left(\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D} + \widehat{\psi}_{GG} \frac{\partial GG}{\partial \bar{r}_D}\right) \frac{\partial DS}{\partial \bar{r}_D}^{-1} \frac{\partial DS}{\partial \xi^*} \cdot \left[\frac{\partial GG}{\partial LIQ} - \frac{\partial GG}{\partial I} - \frac{\partial GG}{\partial r_I} \frac{\partial LD}{\partial r_I}^{-1} \left(\frac{\partial LD}{\partial LIQ} - \frac{\partial LD}{\partial I}\right)\right] \frac{dGG}{d\xi^*}^{-1}}_{\text{(Indirect) Incomplete contract externality}}$$

$$\underbrace{- \left(\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D} + \zeta_{GG} \frac{\partial GG}{\partial \bar{r}_D}\right) \frac{\partial DS}{\partial \bar{r}_D}^{-1} \left(\frac{\partial DS}{\partial LIQ} - \frac{\partial DS}{\partial I}\right)}_{\text{(Direct) Incomplete contract externality}}. \tag{B.21}$$

The terms in the first line are the same ones in (B.15) under complete deposit contracts. The second line captures the effect of the asset allocation on the run threshold, which in turn was not part of the deposit contract that the banker would internalize. The last line captures the direct effect of incomplete contracts on the asset allocation margin, i.e., the planner internalizes how the asset mix affects the deposit supply schedule.

Examining the direct effect first, note that $-(\partial \mathbb{U}_B/\partial \bar{r}_D + \zeta_{GG} \cdot \partial GG/\partial \bar{r}_D) \cdot (\partial DS/\partial \bar{r}_D)^{-1}$ is the planner's Lagrange multiplier on the deposit supply schedule, which we expect to be positive as long as the planner wants to encourage the supply of deposits by offering a higher deposit rate. Also, we have shown that $(\partial DS/\partial I - \partial DS/\partial LIQ) < 0$, which means that having a less liquid asset mix will adversely affect the supply of deposits. The planner internalizes this, but the banker may have an incentive to deviate and take more asset risk after the deposit contract has been signed. The overall direct effect is negative, which means the planner (and a banker that can commit) would like to implement a more liquid asset allocation.

Turning to the indirect externality, $-(\partial \mathbb{U}_B/\partial \bar{r}_D + \widehat{\psi}_{GG} \cdot \partial GG/\partial \bar{r}_D) \cdot (\partial DS/\partial \bar{r}_D)^{-1}$ is the multiplier on DS in the private problem, which we expect to be positive for the same reasons as above. The overall effect from all the terms is negative (see the benchmark model where we sign the other terms). This means that the planner (and a banker that can commit) would, similarly to the direct effect, also want a more liquid asset allocation. Hence, the incomplete contract externality results in more asset risk in private allocations.

Similarly, the wedges for the capital structure becomes:

$$\widehat{CSM}_{WD} = \underbrace{-\left(w_S\frac{\partial \mathbb{U}_S^*}{\partial \xi^*}+w_E\frac{\partial \mathbb{U}_E^*}{\partial \xi^*}\right)\frac{\partial \xi^*}{\partial D}}_{\text{Run externality from liabilities mix}}+\underbrace{w_R\left[U''(e_R-D)D+(1-q)V''(D(1+r_D)D(1+r_D)^2\right]}_{\text{Surplus to S from additional } D}$$

$$\underbrace{-\left(\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D}+\widehat{\Psi}_{GG}\frac{\partial GG}{\partial \bar{r}_D}\right)\frac{\partial DS}{\partial \bar{r}_D}^{-1}\frac{\partial DS}{\partial \xi^*}\cdot\left[\frac{\partial GG}{\partial D}-\frac{\partial GG}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\frac{\partial LD}{\partial D}-\frac{\partial GG}{\partial \bar{r}_D}\frac{\partial DS}{\partial \bar{r}_D}^{-1}\frac{\partial DS}{\partial D}\right]\frac{dGG}{d\xi^*}^{-1}}_{\text{(Indirect) Incomplete contract externality}}. \tag{B.22}$$

Note that in (B.22), the additional terms stem only for the indirect incomplete contract externality through the run threshold because the banker internalizes the effect of D on the deposit supply schedule.

Finally, the wedge in the scale of intermediation margin becomes:

$$\widehat{SIM}_{WD} = \overbrace{\left(w_S\frac{\partial \mathbb{U}_S^*}{\partial \xi^*}+w_E\frac{\partial \mathbb{U}_E^*}{\partial \xi^*}\right)\cdot\left(\frac{\partial \xi^*}{\partial I}+\frac{\partial \xi^*}{\partial D}\right)}^{\text{Run externality from intermediation scale}}$$
$$\underbrace{-w_S[U''(e_S-D)D+(1-q)V''(D(1+r_D))D(1+r_D)^2]+w_E(1-q)c''(I)I}_{\text{Surplus to } S \text{ and } E \text{ from higher intermediation scale}}$$

$$\underbrace{+\left(\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D}+\widehat{\Psi}_{GG}\frac{\partial GG}{\partial \bar{r}_D}\right)\frac{\partial DS}{\partial \bar{r}_D}^{-1}\frac{\partial DS}{\partial \xi^*}\cdot \cdot\left[\frac{\partial GG}{\partial I}+\frac{\partial GG}{\partial D}-\frac{\partial GG}{\partial r_I}\frac{\partial LD}{\partial r_I}^{-1}\left(\frac{\partial LD}{\partial I}+\frac{\partial LD}{\partial D}\right)-\frac{\partial GG}{\partial \bar{r}_D}\frac{\partial DS}{\partial \bar{r}_D}^{-1}\frac{\partial DS}{\partial D}\right]\frac{dGG}{d\xi^*}^{-1}}_{\text{(Indirect) Incomplete contract externality}}$$

$$\underbrace{-\left(\frac{\partial \mathbb{U}_B}{\partial \bar{r}_D}+\zeta_{GG}\frac{\partial GG}{\partial \bar{r}_D}\right)\frac{\partial DS}{\partial \bar{r}_D}^{-1}\frac{\partial DS}{\partial I}}_{\text{(Direct) Incomplete contract externality}}. \tag{B.23}$$

Along the same lines, the direct incomplete contract externality in (B.23) is only due to the choice of I.

Overall, the externalities from incomplete contracts are an additional source of divergence between private and social intermediation margins. Table B.2 compares the private equilibrium under incomplete deposit contracts to both the private equilibrium under complete contracts and the socially optimal allocations. Comparing the private equilibria under complete and incomplete contracts, we can see that the banker has an incentive to choose a less liquid asset portfolio and more leveraged capital structure. The inability to commit, results in higher run-risk, and the banker needs to cut deposit-taking in order to sustain a not-too-low profit margin. All agents are worse-off. As a result, the planner cannot only fix the run externalities, but also the inefficiencies arising from incomplete deposit contracts. In other words, the planner can “enable" the banker to commit, which can be beneficial for all agents including the banker.

	PE	PE	SP for weights (w_E, w_S)		
	Complete	*Incomplete*	(0.00,0.20)	(0.10,0.10)	(0.20,0.00)
I	0.862	0.825	0.785	0.873	0.906
LIQ	0.052	0.000	0.221	0.060	0.000
D	0.875	0.797	0.962	0.894	0.867
E	0.038	0.028	0.044	0.039	0.038
r_I	3.097	3.121	3.198	3.089	3.042
$\bar{r}_D$	0.717	0.550	0.804	0.767	0.758
q	0.407	0.424	0.386	0.403	0.408
ℓ	0.057	0.000	0.219	0.065	0.000
k	0.042	0.035	0.044	0.042	0.042
$r_I - (1-\delta)\bar{r}_D$	2.739	2.846	2.796	2.705	2.663
$I + LIQ$	0.914	0.825	1.006	0.933	0.906
$I - E$	0.824	0.797	0.741	0.834	0.867
$E(Div)$	0.745	0.714	0.755	0.747	0.743
$\Delta\mathbb{U}_E$	-	-1.14%	-1.66%	0.33%	1.19%
$\Delta\mathbb{U}_S$	-	-2.12%	3.63%	0.71%	-0.30%
$\Delta\mathbb{U}_B$	-	-0.85%	-0.44%	-0.05%	-0.09%

Table B.2: Private equilibrium allocations under complete and incomplete deposit contracts versus Socially optimal solutions. The welfare changes are computed over the level of welfare in the private equilibrium under complete contracts, which is normalized to one for each agent.

B.5 Loan Market and Price-Taking Behavior

This section presents the private equilibrium outcomes when the banker acts as a price-taker in the loan market, i.e., she takes r_I as given and does not internalize the effect of the other choices in $\mathcal{C}$ on the loan demand schedule (7). Technically, this means that the first-order conditions (24) in the private equilibrium should not include the terms multiplied by ψ_{LD}. This would introduce an additional reason why the privately and socially optimal allocations diverge on top of the run externalities and surplus considerations present in the three wedges in (35), (36), and (37). Indeed, one can derive expressions similar to (B.21), (B.22), and (B.23) where, instead of the terms for the incomplete deposit contract externalities, there would be terms capturing the externalities from pricing taking behavior in the loan market.

Table B.3 compares the private equilibrium when the banker is a price-taker in the loan market with both the private equilibrium when the banker internalizes the loan demand schedule and the socially optimal allocations. Comparing the two private equilibria, the banker extends more loans to entrepreneurs, which reduces the loan rate and the profit margin, when she does not internalize how her choice affects the loan demand by entrepreneurs. Naturally, the banker is worse-off compared with the private equilibrium where she fully internalizes her actions. Entrepreneurs and savers are better-off—the former because they get more loans and the latter because the banker raises more deposits to fund lending, which pushes up the deposit rate. It may seem that social welfare is higher in the private equilibrium with price-taking behavior compared with the socially optimal outcomes. This is not true. Consider, for example, that the social welfare weights are $w_E = w_S = 0.1$. The difference between social welfare in the planner's solution and the private equilibrium with risk-taking behavior is 1.06%, i.e., the planner does better. The planner still cares about the banker and, thus, she chooses allocations that favor the banker, who did not internalize how her actions affected loan demand. The concerns about the banker's welfare diminish as more weight is placed on S and E, which can be seen from the last column where the social welfare weights are $w_E = w_S = 0.5$. Social welfare is higher by 0.05% in the planner solution compared with the private equilibrium with price-taking behavior.

	PE	PE	SP for weights (w_E, w_S)			
		Price-taking	(0.00,0.20)	(0.10,0.10)	(0.20,0.00)	(0.50,0.50)
I	0.862	0.976	0.785	0.873	0.906	0.944
LIQ	0.052	0.000	0.221	0.060	0.000	0.056
D	0.875	0.933	0.962	0.894	0.867	0.957
E	0.038	0.043	0.044	0.039	0.038	0.043
r_I	3.097	2.966	3.198	3.089	3.042	3.015
$\bar{r}_D$	0.717	0.981	0.804	0.767	0.758	0.996
q	0.407	0.400	0.386	0.403	0.408	0.395
ℓ	0.057	0.000	0.219	0.065	0.000	0.056
k	0.042	0.044	0.044	0.042	0.042	0.043
$r_I - (1-\delta)\bar{r}_D$	2.739	2.475	2.796	2.705	2.663	2.516
$I + LIQ$	0.914	0.976	1.006	0.933	0.906	1.000
$I - E$	0.824	0.933	0.741	0.834	0.867	0.901
$E(Div)$	0.745	0.740	0.755	0.747	0.743	0.739
$\Delta\mathbb{U}_E$	-	3.57%	-1.66%	0.33%	1.19%	2.57%
$\Delta\mathbb{U}_S$	-	2.33%	3.63%	0.71%	-0.30%	3.37%
$\Delta\mathbb{U}_B$	-	-1.60%	-0.44%	-0.05%	-0.09%	-1.57%

Table B.3: Private equilibrium allocations with and without price-taking banker's behavior versus Socially optimal solutions. The welfare changes are computed over the level of welfare in the private equilibrium where the banker internalizes the loan demand schedule, which is normalized to one for each agent.

B.6 Tools-Augmented Planner

We consider a tools-augmented planner who is endowed with the set of tools $\mathcal{T} \in \mathbb{T}$ and wants to replicate the social planner's allocations $\mathcal{C}_{sp}$, as a private equilibrium. The tools-augmented planner's problem is akin to a Ramsey planner's problem in the public finance literature.

We will consider two types of tools. First, restrictions on regulatory ratios denoted by $\mathcal{T}_R$. Second, Pigouvian taxes imposed directly on $B's$ payoffs and denoted by $\mathcal{T}_P$. For each $\mathcal{T}_R$, there is a regulatory constraint $RC(\mathcal{T}_R,\mathcal{C}) \geq 0$, which ties the tool with the endogenous variables $\mathcal{C}$, while for each $\mathcal{T}_P$, there is an additional term in B's utility, $\mathbb{U}_B(\mathcal{T}_P,\mathcal{C})$. It is important to note that the regulatory constraints, RC, are defined as inequalities, i.e., the planner can tighten them but not loosen them while there are no restrictions on Pigouvian taxes which can be positive or negative. Let $\psi_{\mathcal{T}_R}$ be the multipliers that the banker in the private equilibrium assigns to constraint $RC(\mathcal{T}_R,\mathcal{C}) \geq 0$.

Under regulation, the optimization margins change to:

$$AAM_{\mathbb{T}} : AAM_{PE} + \sum_{\mathcal{T}} \left\{ \psi_{\mathcal{T}_R} \left[\frac{\partial RC(\mathcal{T}_R,\mathcal{C})}{\partial LIQ} - \frac{\partial RC(\mathcal{T}_R,\mathcal{C})}{\partial I} \right] + \frac{\partial \mathbb{U}_B(\mathcal{T}_P,\mathcal{C})}{\partial LIQ} - \frac{\partial \mathbb{U}_B(\mathcal{T}_P,\mathcal{C})}{\partial I} \right\} = 0, \tag{B.24}$$

$$CSM_{\mathbb{T}} : CSM_{PE} + \sum_{\mathcal{T}} \left\{ \psi_{\mathcal{T}_R} \left[\frac{\partial RC(\mathcal{T}_R,\mathcal{C})}{\partial E} - \frac{\partial RC(\mathcal{T}_R,\mathcal{C})}{\partial D} \right] + \frac{\partial \mathbb{U}_B(\mathcal{T}_P,\mathcal{C})}{\partial E} - \frac{\partial \mathbb{U}_B(\mathcal{T}_P,\mathcal{C})}{\partial D} \right\} = 0, \tag{B.25}$$

$$SIM_{\mathbb{T}} : SIM_{PE} + \sum_{\mathcal{T}} \left\{ \psi_{\mathcal{T}_R} \left[\frac{\partial RC(\mathcal{T}_R,\mathcal{C})}{\partial I} + \frac{\partial RC(\mathcal{T}_R,\mathcal{C})}{\partial D} \right] + \frac{\partial \mathbb{U}_B(\mathcal{T}_P,\mathcal{C})}{\partial I} + \frac{\partial \mathbb{U}_B(\mathcal{T}_P,\mathcal{C})}{\partial D} \right\} = 0. \tag{B.26}$$

We will show that in order to implement the equilibrium allocations of the social planner, denoted by $\mathcal{C}_{sp}$, it is not necessary to solve the full problem of the tools-augmented planner. Instead, it suffices that there are tools, $\mathcal{T} = \{\mathcal{T}_R, \mathcal{T}_P\}$, that first satisfy the regulatory constraints $RC(\mathcal{T}_R,\mathcal{C}_{sp}) = 0$ at the planner's allocations, and, second, the intermediation margins in the associated equilibrium are the same as the intermediation margins of the planner. Essentially, this means that the additional terms in (B.24), (B.25), and (B.26) need to equal the wedges derived in (35), (36), and (37). In matrix form, this can be written as:

$$\begin{bmatrix} \Delta_{\mathcal{T}_R} \\ \Delta_{\mathcal{T}_P} \end{bmatrix} \cdot \begin{bmatrix} \Psi_{\mathcal{T}_R} & \bar{\mathcal{T}}_{\mathrm{P}} \end{bmatrix} = WD_{sp}, \tag{B.27}$$

where $\Psi_{\mathcal{T}_R}$ is the $\mathcal{T}_R x1$ vector of the multiplier on the $\mathcal{T}_R$ regulatory constraints, WD_{sp} is the $3x1$ vector of the wedges in the three intermediation margins evaluated at the planner's equilibrium values, $\Delta_{\mathcal{T}_R} \equiv \Delta RC(\mathcal{T}_R,\mathcal{C}_{sp})$ is the $3x\mathcal{T}_R$ matrix of the partial derivatives of the relevant variables for each intermediation margin on the $\mathcal{T}_R$ regulatory constraints, $\bar{\mathcal{T}}_P$ is the $\mathcal{T}_P x1$ vector of Pigouvian tools, and $\Delta_{\mathcal{T}_P}$ is the $3x\mathcal{T}_P$ matrix of the coefficient on the tools $\mathcal{T}_P$ in the partial derivatives of the utility terms, i.e., $\Delta_{\mathcal{T}_P} \cdot \bar{\mathcal{T}}_P \equiv \Delta \mathbb{U}_B(\mathcal{T},\mathcal{C}_{sp})$.

Given that there are at most three distorted wedges in intermediation margins, only three independent tools are needed, i.e, $\#\mathcal{T}_R + \#\mathcal{T}_P = 3$. First, consider that there are three tools $\mathcal{T}_P$ and none

$\mathcal{T}_R$. As long as $\Delta_{\mathcal{T}_P}$ is invertible, three Pigouvian tools $\mathcal{T}_P$ are sufficient to implement the planner's solution, i.e., (B.27) has a solution even if regulatory-ratio tools $\mathcal{T}_R$ are not considered. Alternatively, the planner's allocations can be implemented with just three regulatory-ratio tools, if first, the matrix $\Delta_{\mathcal{T}_R}$ is invertible and, second, all elements in $\Psi_{\mathcal{T}_R}$ are positive. But, three regulatory-ratio tools (capital or liquidity) may not be linearly independent, because choosing two of them may replicate the value of the third. For example, a capital and a liquidity tool can be jointly binding, but two liquidity tools cannot. Additionally, some of resulting multipliers $\psi_{\mathcal{T}_R}$ may be negative, because the planner may want to encourage instead of restrict activity (recall that the regulatory constraints RC are inequalities). Indeed, this is the case we study in section 4.3. For these reasons, we combine a capital and a liquidity tool with a (Pigouvian) subsidy on deposit interest expenses to implement the planner's allocations when savers are favored, while a capital tool is combined with (Pigouvian) deposit and lending subsidies when entrepreneurs are favored. When both regulatory-ratio and Pigouvian tools are used, it suffices that the matrix $\Delta_{\mathcal{T}} \equiv [\Delta_{\mathcal{T}_R}\ \ \Delta_{\mathcal{T}_\mathrm{R}}]'$ is invertible in order to implement the planner's allocations.

We now show that (B.27) is a necessary and sufficient condition such that the social planner's solution described in section 3.1 can be decentralized as a private equilibrium by using regulatory tools $\mathcal{T} = \{\mathcal{T}_R, \mathcal{T}_P\} \in \mathbb{T}$. The tools-augmented planner not only chooses optimally allocations $\mathcal{C}$, but also the level of tools $\mathcal{T} \in \mathbb{T}$ and the multipliers $\psi_{\mathcal{T}_R}$, which are the shadow values that the banker assigns to constraints $RC(\mathcal{T}_R, \mathcal{C}) \geq 0$ in the new equilibrium. Her problem is:

$$\max_{\mathcal{C},\mathcal{T},\psi_{\mathcal{T}_R}} \quad \mathbb{U}_{\mathrm{sp}}^{\mathbb{T}} \quad s.t.\ \ \mathcal{Y}(\mathcal{C}) = 0,\ \ RC(\mathcal{T}_R, \mathcal{C}) \geq 0,\ \ AAM_{\mathcal{T}}(\mathcal{T}_R, \mathcal{C}, \Psi_{\mathcal{T}}) = 0,\ \mathrm{CSM}_{\mathcal{T}}(\mathcal{T}_\mathrm{R}, \mathcal{C}, \Psi_{\mathcal{T}}) = 0,$$
$$SIM_{\mathcal{T}}(\mathcal{T}_R, \mathcal{C}, \Psi_{\mathcal{T}}) = 0. \tag{B.28}$$

Note that the additional utility terms $\mathbb{U}_B(\mathcal{T}_P, \mathcal{C})$ due to Pigouvian taxation tools do not appear in the utility that the tools-augmented planner maximizes, because she engages in lump-sum transfers of equal size.

The first-order condition with respect to $\mathcal{C}$ (similar to first-order condition (34)) are:

$$\frac{\partial \mathbb{U}_B}{\partial \mathcal{C}} + w_S \frac{\partial \mathbb{U}_S^*}{\partial \mathcal{C}} + w_E \frac{\partial \mathbb{U}_E^*}{\partial \mathcal{C}} + \sum_{\mathcal{Y}} \zeta_{\mathcal{Y}} \frac{\partial \mathcal{Y}}{\partial \mathcal{C}}$$
$$+ \sum_{\mathcal{T}_R} \zeta_{\mathcal{T}_R} \frac{\partial RC}{\partial \mathcal{C}} + \zeta_{AAM} \frac{\partial AAM_{\mathcal{T}}}{\partial \mathcal{C}} + \zeta_{CSM} \frac{\partial CSM_{\mathcal{T}}}{\partial \mathcal{C}} + \zeta_{SIM} \frac{\partial SIM_{\mathcal{T}}}{\partial \mathcal{C}} = 0, \tag{B.29}$$

where $\zeta_{\mathcal{T}_R}$, ζ_{AAM}, ζ_{CSM} and ζ_{SIM} are the multipliers the tool-augmented planner assigns to regulatory constraints and the three regulation-distorted intermediation margins.

The first-order conditions with respect to the level of tools $\mathcal{T}_R$ and $\mathcal{T}_P$, respectively, are:

$$\zeta_{\mathcal{T}_R} \frac{\partial RC}{\partial \mathcal{T}_R} + \zeta_{AAM} \frac{\partial AAM_{\mathcal{T}}}{\partial \mathcal{T}_R} + \zeta_{CSM} \frac{\partial CSM_{\mathcal{T}}}{\partial \mathcal{T}_R} + \zeta_{SIM} \frac{\partial SIM_{\mathcal{T}}}{\partial \mathcal{T}_R} = 0, \tag{B.30}$$

and

$$\zeta_{AAM} \frac{\partial AAM_{\mathcal{T}}}{\partial \mathcal{T}_P} + \zeta_{CSM} \frac{\partial CSM_{\mathcal{T}}}{\partial \mathcal{T}_P} + \zeta_{SIM} \frac{\partial SIM_{\mathcal{T}}}{\partial \mathcal{T}_P} = 0, \tag{B.31}$$

Finally, choosing optimally the multipliers $\psi_{\mathcal{T}_R}$ yields:

$$\zeta_{AAM} \frac{\partial AAM_{\mathcal{T}}}{\partial \psi_{\mathcal{T}_R}} + \zeta_{CSM} \frac{\partial CSM_{\mathcal{T}}}{\partial \psi_{\mathcal{T}_R}} + \zeta_{SIM} \frac{\partial SIM_{\mathcal{T}}}{\partial \psi_{\mathcal{T}_R}} = 0. \tag{B.32}$$

The solutions of the social and tools-augmented planners coincide if the optimality conditions (34) and (B.29) coincide, i.e., if $\zeta_{AAM} = \zeta_{CSM} = \zeta_{SIM} = 0$ and $\zeta_{\mathcal{T}_R} = 0$ for all tools $\mathcal{T}_R$.

To prove sufficiency, note that augmenting (B.32) and (B.31) and representing them in combat form yields $\Delta_{\mathcal{T}}{}' \cdot [\zeta_{AAM} \;\; \zeta_{\text{CSM}} \;\; \zeta_{\text{SIM}}]' = 0$. Given that $\Delta_{\mathcal{T}}$ should be invertible for (B.27) to yield a solution, its transpose is also invertible, and the only solution is $\zeta_{AAM} = \zeta_{CSM} = \zeta_{SIM} = 0$. Thus, all $\zeta_{\mathcal{T}_R} = 0$ in (B.30) are also zero, and (34) and (B.29) coincide.

To prove necessity, suppose that (B.27) does not hold, i.e., $\Delta_{\mathcal{T}}$ is not invertible and some or all multipliers $\zeta_{\mathcal{T}_R}$, ζ_{AAM}, ζ_{CSM} and ζ_{SIM} do not need to be zero. Using conditions (B.29), we can derive intermediation margins $AAM_{TAP} = AAM_{SP} + AAM_{TAP,WD}$, $CSM_{TAP} = CSM_{SP} + CSM_{TAP,WD}$ and $SIM_{TAP} = SIM_{SP} + SIM_{TAP,WD}$ for the tool-augmented planner, where the wedges are linear combination of the multipliers $\zeta_{\mathcal{T}_R}$, ζ_{AAM}, ζ_{CSM} and ζ_{SIM}. The social planner's and tools-augmented planner's solutions coincide if wedges $AAM_{TAP,WD}$, $CSM_{TAP,WD}$, and $SIM_{TAP,WD}$ are all zero, which in principle is possible by varying $\zeta_{\mathcal{T}_R}$, ζ_{AAM}, ζ_{CSM} and ζ_{SIM}. However, equations (B.30), (B.31), and (B.32) remove as many degrees of freedom and are, thus, generically satisfied when all multipliers are zero—a contradiction.

B.7 Perfectly Elastic Demand Curve

This section studies the special case of a perfectly elastic demand curve, which obtains for $c_I = 0$. Then, from (7), $1 + r_I = A$, and, from (8), $\mathbb{U}_E^* = 0$. There are two implications of abstracting from a downward sloping demand curve. First, the planner does not consider the welfare of entrepreneurs as they make zero profits. Second, the banker cannot manipulate her profit margin by adjusting the volume of lending to affect the loan rate and all of the adjustment in the intermediation spread is happening via the deposit rate.

Table B.4 reports the results from implementing single and combined regulatory tools, which are consistent with the results in the baseline model when the planner places more weight on savers. Tightening the leverage requirement forces the banker to raise more equity and results in some substitution away from liquid assets toward loans. The difference with the case of a downward sloping loan demand curve is that here the banker can extend more lending without pushing the loan rate down and, hence, eroding her profit margin. As in the general case, funding more loans with equity reduces the need for deposits, which also pushes down the deposit rate improving the intermediation margin and enabling the banker to raise more equity. Expected dividends are higher, but the banker is worse-off, because she had to contribute more equity than what was optimal for her in the private equilibrium.

The results for liquidity regulation are similar to the baselines ones. The only difference is that the reduction in lending does not boost loan rates, and hence the banker cannot increase deposit-taking as much as she would be able to under a downward sloping demand curve. Contrary to the baseline case, the deposit rate falls because the lower run probability dominates the effect from the somewhat higher deposit demand. In the baseline case, lower lending could support a higher spread and, thus, the banker could increase deposit-taking a lot, undoing the effect of lower run-risk on deposit rates and pushing them to levels above their private equilibrium ones.

Comparing column "ℓ" and "$k\&\ell$", we see that leverage and liquidity regulations can be combined to improve welfare. The margin effect of adding a leverage requirement on top of the liquidity requirement is small in this example but goes in the direction of the baseline results. Finally, a subsidy on deposits, $\tau_D = -3.61\%$, is needed to fully implement the planner's allocations, as is the case in the baseline model when the planner puts higher weight on savers.

	PE	k	ℓ	$k\&\ell$	$k,\ell\&\tau_D$
I	0.550	0.558	0.503	0.503	0.510
LIQ	0.109	0.103	0.193	0.193	0.195
D	0.627	0.625	0.663	0.663	0.672
E	0.031	0.036	0.033	0.033	0.034
r_I	2.300	2.300	2.300	2.300	2.300
$\bar{r}_D$	0.754	0.734	0.690	0.689	0.721
q	0.510	0.506	0.504	0.503	0.501
ℓ	0.165	0.156	0.277	0.277	0.277
k	0.047	0.055	0.047	0.048	0.048
$r_I - (1-\delta)\bar{r}_D$	1.923	1.933	1.955	1.955	1.939
$I + LIQ$	0.658	0.661	0.695	0.696	0.706
$I - E$	0.519	0.522	0.470	0.470	0.477
$E(Div)$	0.244	0.256	0.248	0.249	0.249
$\Delta\mathbb{U}_E$	-	0.00%	0.00%	0.00%	0.00%
$\Delta\mathbb{U}_S$	-	-0.05%	0.962%	0.965%	1.23%
$\Delta\mathbb{U}_B$	-	-0.05%	-0.06%	-0.06%	-0.08%
$\Delta\mathbb{S}_{sp}$	-	-0.01%	0.90%	0.90%	1.16%

Table B.4: Equilibrium allocation under perfectly elastic loan demand. The welfare changes are computed over the level of welfare in the private equilibrium, which is normalized to one for each agent.

B.8 Direct Lending

This section derives the conditions for direct lending to entrepreneurs by savers and computes the equilibrium outcomes for the parametrization in section 3.2.

Direct lending requires the individual savers to be able to monitor the entrepreneur. Denote by X_S the monitoring cost to an individual saver, which we assume can be higher or equal to the monitoring cost of the banker, i.e., $X_S \geq X$. At $t = 1$, an individual saver can lend to the entrepreneur, I_{dl}, at interest rate r_{dl}. In the intermediate period, she would liquidate all of her loans if she turns out to be impatient. Otherwise, the saver waits until the final period and receives the percentage repayment on the loans she made. The saver's utility under direct lending is given by

$$\mathbb{U}_S^{dl} = U(e_R - I_{dl}) + \beta\delta \int_{\underline{\xi}}^{\overline{\xi}} \xi \cdot I_{dl} \frac{d\xi}{\Delta_\xi} + \beta^2(1-\delta) \sum_s (\omega I_{dl}(1+r_{dl}) - X_S) \frac{d\xi}{\Delta_\xi}. \tag{B.33}$$

The entrepreneur will choose I_{dl} to maximize her utlity $\mathbb{U}_E^{dl} = (1-\delta)[\omega I_{dl}(1+r_{dl}) - c(I_{dl})]$; with probability δ, an individual entrepreneur has her project liquidated and receives zero utility, while, with probability $1-\delta$, the saver does not liquidate the project, and the entrepreneur incurs the effort cost and defaults in the bad state. E's optimizing behavior yields the following loan demand schedule:

$$1 + r_{dl} = A - c'(I_{dl})/\omega. \tag{B.34}$$

Because each individual saver is sufficiently small, she takes the loan rate as given and, thus, the loan supply schedule is:

$$1 + r_{dl} = \frac{1}{\omega\beta^2(1-\delta)} \left[U'(e_R - I_{dl}) - \beta\delta E(\xi) \right]. \tag{B.35}$$

The intersection of the loan demand and loan supply schedule in (B.34) and (B.35) yields the equilibrium loan rate and loan amount. Finally, S's and E's levels of welfare in equilibrium are given by $\mathbb{U}_S^{dl*} = U(e_R - I_{dl}) + U'(e_R - I_{dl})I_{dl} - \beta^2(1-\delta)X_S$ and $\mathbb{U}_E^{dl*} = (1-\delta)\left(c'(I_{dl})I_{dl} - c(I_d l)\right)$. Table B.5 reports the equilibrium in the loan market together with how S's and E's levels of welfare compare across three cases: the bank intermediation private equilibrium reported in section 3.2, the direct lending equilibrium, and the autarkic outcome when S uses only the storage technology. Savers are better-off under bank intermediation as they enjoy the transaction services of deposits, and they do not need to pay the monitoring cost. However, for $X = X_S$, savers are better-off lending directly to E compared with autarky. There is a level of the monitoring cost X_S that this stops being true ($X_S/X > 1.46$ in our example). Direct lending is higher than bank lending because the bank strategically curtails credit extension to secure higher loan rates and increase her profit margins. Note that the monitoring cost does not affect the marginal choice of I_{dl} because it is not incurred per unit of loans extended but rather applies to the whole portfolio.

B.9 Additional Distortionary Tools

This section extends the analysis in section 3 by allowing the planner to use tools to distort the deposit supply and loan demand schedules of savers and entrepreneurs. We consider generic tools, τ_{DS} for the deposit supply schedule and τ_{LD} for the loan demand schedule, and discuss how they

	Loan rate	Loan amount	$\%\Delta\mathbb{U}_S$	$\%\Delta\mathbb{U}_E$
Intermediation	3.097	0.862	-	-
Direct lending	1.272	0.928	-3.20%	0.38%
Autarky	-	-	-4.34%	-7.61%

Table B.5: Equilibrium allocation under bank intermediation, direct lending, and autarky with storage. The welfare changes are computed over the level of welfare in the private equilibrium with bank intermediation, which is normalized to one for each agent.

can be implemented in practice.

The deposit supply schedule (2) that the planner faces becomes:

$$U'(e_S - D - LIQ_S) = \left[\beta\delta + \beta^2(1-\delta)\right](1+r_D)\int_{\underline{\xi}}^{\xi^*} \theta(\xi,1)\frac{d\xi}{\Delta_\xi} + \left[\beta\delta(1+r_D) + \beta^2(1-\delta)\omega(1+\bar{r}_D) + V'\left(D(1+r_D)\right)(1+r_D)\right](1-q) + \tau_{DS}. \tag{B.36}$$

The planner can distort the willingness of savers to hold deposits at given deposit rates by varying the level of the distortionary tool τ_{DS}. In other words, the planner can set τ_{DS}, which implies that (B.36) stops being a constraint in her optimization problem defined in Definition 2 and, thus, $\zeta_{DS} = 0$ in (34). The intervention can be implemented, for example, either as a tax on the supply of deposits at $t = 1$ or as a tax on the interest income accruing to late depositors at $t = 3$ when the bank is solvent. In the first case, the tax can be computed as $-\tau_{DS}/U'(e_R - D - LIQ_S)$, while in the second, as $-\tau_{DS}/(\beta^2(1-\delta)\omega(1+\bar{r}_D)(1-q))$. If $\tau_{DS} < 0$, then a tax is levied, while $\tau_{DS} > 0$ implies a subsidy. We assume that the planner rebates the tax proceeds back to the same agents in the same period in a lump-sum fashion in order to neutralize any income effects.

Similarly, the loan demand schedule (7) becomes:

$$\int_{\xi^*}^{\bar{\xi}} \{\omega[A - (1+r^I)](1 - y(\xi,\delta)) - c'(I)\}\frac{d\xi}{\Delta_\xi} + \tau_{LD} = 0. \tag{B.37}$$

The planner can distort the willingness of entrepreneurs to borrow by varying the level of the distortionary tool τ_{LD}, such that, if $\tau_{LD} \neq 0$, then $\zeta_{LD} = 0$ in (34). The intervention can be implemented with a tax on loan repayment in the good state of the world, which can be computed as $-\tau_{LD}/(\int_{\xi^*}^{\bar{\xi}}(1-y(\xi,\delta))(1+r_I)d\xi/\Delta_\xi)$. If $\tau_{LD} < 0$, then a tax is levied, while $\tau_{LD} > 0$ implies a subsidy.

Given that the planner may choose to distort the deposit supply and loan demand schedules, we cannot use the indirect utilities (4) and (8), which imply the social welfare function (29). Instead, the planner maximizes the more elaborate social welfare function (28), which considers the direct utilities (1) and (5).

Table B.6 reports the planning equilibria under two sets of weights and three configurations: the benchmark one where the planner respects the deposit supply and loan demand schedule, a second where she distorts the deposit supply, and a third where she distorts the loan demand. When the planner can distort the deposit supply schedule, she can convince savers to supply deposits even if this is not optimal for them. For example, a distortionary subsidy on deposits makes savers want to supply more deposits and accept lower deposit rates, which is beneficial for the banker and entrepreneurs: lower deposit rates increase the profit margin of the banker, who is then willing to extend more loans at a lower loan rate. The planner goes all the way down to extracting all the surplus from depositors and pushing them to their participation constraint, i.e., their utility in autarky.[4] The opposite is true when the planner distorts the loan demand schedule, i.e., $\tau_{LD} \neq 0$. A distortionary subsidy on investment/borrowing to E allows the planner to increase lending without having to attract E by offering a lower loan rate. Higher lending requires more deposits, which pushes deposit rates up and enhances transaction services. Yet, the planner can compensate the banker with a higher loan rate, which provides incentives for injecting more equity. The planner goes all the way to extracting all the surplus from entrepreneurs and pushing them to their participation constraint.

The higher the weight on entrepreneurs is, the higher is their utility under a positive τ_{DS}, which distorts the deposit supply schedule urging savers to supply more deposits for lower deposit rates. Similarly, the higher the weight on savers is, the higher is their utility under a positive τ_{LD}, which distorts the loan demand schedule urging entrepreneurs to borrow more for higher loan rates. In all cases, the planner still cares about the banker and can transfer some of the surplus back to her. As the weight on other agents becomes higher, this transfer will become smaller. Note that this was not possible in our benchmark analysis because both the banker and the planner had to respect the deposit supply and loan demand schedules. Hence, the banker was always losing from the planner's interventions.

B.10 Negative Interest Rates and Run-Proof Banking

In our benchmark results in section 3.2, we assumed that the bank cannot set (charge) negative interest rates for early withdrawals, i.e., $r_D \geq 0$. In this section, we relax this assumption. On the one hand, a negative r_D may allow the bank to eliminate all run-risk, i.e., the bank is run-proof despite issuing demandable deposits. On the other hand, a negative r_D would reduce the utility from transaction services and the bank would need to offer a higher late deposit rate, $\bar{r}_D$; otherwise, savers may choose to self-insure by holding the liquid asset and stop using the bank.

There are two subtle assumptions we have made that offer the best chance for negative rates to eliminate run-risk without hurting welfare. The first assumption is that $V(0) = 0$, i.e., if the bank sets $r_D = -100\%$, then savers get zero utility. This is important because, if $V(0) \to -\infty$, savers would become explosively worse-off as r_D became more and more negative, and they would require an explosively high $\bar{r}_D$ to supply deposits. As a result, intermediation would be impossible under

[4] Given that the planner distorts the deposit supply schedule (2), savers' welfare in equilibrium is not given by (4) but rather (1). Hence, there is no guarantee that savers are strictly better-off under bank intermediation compared to autarky (see footnote 4).

	PE	SP for $(w_E, w_S) = (0.05, 0.15)$			SP for $(w_E, w_S) = (0.15, 0.05)$		
		No tools	$\tau_{DS} \neq 0$	$\tau_{LD} \neq 0$	No tools	$\tau_{DS} \neq 0$	$\tau_{LD} \neq 0$
I	0.862	0.841	1.190	1.002	0.899	1.211	0.991
LIQ	0.052	0.119	0.000	0.000	0.012	0.000	0.000
D	0.875	0.919	1.164	0.956	0.873	1.184	0.946
E	0.038	0.041	0.026	0.045	0.038	0.027	0.045
r_I	3.097	3.131	2.706	3.312	3.051	2.678	3.316
$\bar{r}_D$	0.717	0.778	0.668	1.047	0.761	0.689	1.008
q	0.407	0.398	0.422	0.392	0.407	0.420	0.393
ℓ	0.057	0.124	0.000	0.000	0.013	0.000	0.000
k	0.042	0.043	0.022	0.045	0.042	0.022	0.046
$r_I - (1-\delta)\bar{r}_D$	3.239	3.242	2.872	3.288	3.170	2.833	3.312
$I + LIQ$	0.914	0.960	1.190	1.002	0.911	1.211	0.991
$I - E$	0.824	0.800	1.164	0.956	0.861	1.184	0.946
$E(Div)$	0.745	0.750	0.858	0.864	0.743	0.859	0.864
$\Delta\mathbb{U}_E$	0.076	-0.44%	11.87%	-7.61%	1.02%	12.97%	-7.61%
$\Delta\mathbb{U}_S$	1.050	1.74%	-4.34%	3.35%	-0.10%	-4.34%	2.87%
$\Delta\mathbb{U}_B$	0.327	-0.13%	14.13%	10.11%	-0.08%	14.02%	10.16%
τ_{DS}	-	-	0.164	-	-	0.173	-
τ_{LD}	-	-	-	0.122	-	-	0.120

Table B.6: Privately versus Socially Optimal Solutions when additional distortionary tools are available. The welfare changes are computed over the level of welfare in the private equilibrium, which is normalized to one for each agent.

very negative r_D. The second assumption is that the liquid asset cannot offer the transaction services of deposits (or if it does, its services are inferior to the ones offered by deposits). If deposits and the liquid assets were perfect substitute for transactions, then savers would very quickly switch to holding the liquid assets once r_D became sufficiently negative. Instead, in our environment, savers would still be willing to hold deposits along with the liquid asset, even under considerably negative r_D. The reason is that savers' utility under bank intermediation is strictly higher than in autarky for $r_D > -100\%$ (see the discussion at the end of section 2.1).

We derive below the conditions under which run-proof banking is possible as well as the corresponding private and social equilibrium allocation for our benchmark parametrization. Our analysis has focused on the externalities induced by the banker's behavior when there is positive run-risk in equilibrium. Thus, the natural candidate for comparison is run-proof banking, which is possible given the two aforementioned assumptions.[5]

Given Lemma 1, banking is run-proof if $\hat{\lambda}(\underline{\xi})$—given by combining (13) and (10) for the lowest

[5]This does not mean that negative interest rates are only compatible with run-proof banking. Although quantitatively the results in section 3.2 will differ if we allow for negative r_D, the wedges derived in section 3.1 will maintain their functional form and, hence, the sources of divergence between the privately and socially optimal solutions will be the same.

realization of the liquidation value $\xi = \underline{\xi}$ and the highest number of withdrawals $\lambda = 1$—is not lower than one. Hence, liquid asset holdings should satisfy:

$$\hat{\lambda}(\underline{\xi}) = \frac{\theta(\underline{\xi},1) - \frac{\underline{\xi}(1+\bar{r}_D)}{(1+r_D)(1+r_I)} - \frac{\underline{\xi}\cdot X/\omega}{D(1+r_D)(1+r_I)}}{1 - \frac{\underline{\xi}(1+\bar{r}_D)}{(1+r_D)(1+r_I)}} \geq 1 \Rightarrow \theta(\underline{\xi},1) \geq 1 + \frac{\underline{\xi}\cdot X/\omega}{D(1+r_D)(1+r_I)}$$

$$\Rightarrow LIQ \geq D(1+r_D) - \underline{\xi}\left(I - \frac{X}{\omega(1+r_I)}\right). \tag{B.38}$$

Beyond that level, it is inefficient to hold liquid assets, so (B.38) holds with equality.

If $LIQ \geq \delta D(1+r_D)$ or

$$(1-\delta)D(1+r_D) \geq \underline{\xi}\left(I - \frac{X}{\omega(1+r_I)}\right), \tag{B.39}$$

then the liquid asset holdings of the bank are higher than the predictable early withdrawals and the bank will transfer the excess liquidity, $\widehat{LIQ} = (1-\delta)D(1+r_D) - \underline{\xi}\,(I - X/(\omega(1+r_I)))$ in the last period. Otherwise, the bank will need to recall a fraction $\hat{y}(\xi) = [\underline{\xi}(I - X/(\omega(1+r_I))) - (1-\delta)D(1+r_D)]/(\xi \cdot I)$ of loans to pay early withdrawals. Note that, if $\underline{\xi}$ is very small, then (B.39) is satisfied for a larger range of negative r_D; at the limit, as $\underline{\xi} \to 0$, (B.39) is satisfied for all $r_D > -100\%$, i.e., the bank does not need to recall any loans. For the rest of this section, we assume that $\underline{\xi} \to 0$ in order to simplify the algebra.

In the absence of run-risk, the loan demand schedule can be written as

$$1 + r_I = A - c'(I)/\omega. \tag{B.40}$$

The deposit supply schedule is more elaborate, because the excess liquidity $\widehat{LIQ}$ will be distributed pro-rata to the $(1-\delta)$ patient savers when the bank defaults in the bad state of the world.[6] Thus, the percentage payment, $\mathcal{R}$, to patient savers in the bad state is $\mathcal{R} = \widehat{LIQ}/((1-\delta)D(1+\bar{r}_D))$ and the total payoff from holding deposits in the bad state is $\mathcal{R}\cdot D(1+\bar{r}_D)$. Individual savers take the percentage payment in the bad state as given when choosing the amount of deposits. The deposit

[6] In the bad state, the bank defaults if $\widehat{LIQ} < (1-\delta)D(1+\bar{r}_D)$, which is true because $\bar{r}_D > r_D$.

supply schedule is, thus, given by the following first-order condition with respect to D:

$$\overbrace{-U'(e_S - D - LIQ_S)}^{\text{Consumption cost of depositing}} + \overbrace{\beta\delta(1+r_D)}^{\text{Marginal payoff to impatient } S} + \overbrace{\beta^2(1-\delta)\omega(1+\bar{r}_D)}^{\text{Marginal payoff to patient } S \text{ in good state}} + \overbrace{\beta^2(1-\delta)(1-\omega)\mathcal{R}\cdot(1+\bar{r}_D)}^{\text{Marginal payoff to patient } S \text{ in bad state}}$$
$$+ \underbrace{V'(D(1+r_D))(1+r_D)}_{\text{Marginal payoff from transaction services}} = 0$$

$$\Rightarrow \omega(1-\delta)(\bar{r}_D - r_D) = \frac{U'(e_S - D - LIQ_S) - V'(D(1+r_D))(1+r_D) - \left(\beta\delta + \beta^2(1-\delta)\right)(1+r_D)}{\beta^2}, \tag{B.41}$$

where we have substituted the definition of $\mathcal{R}$.

A negative r_D makes self-insuring through holding the liquid asset more appealing to savers. As a result, LIQ_S can be positive, in contrast to the benchmark equilibrium we have studied, and (3) will hold with equality. We assume that this is the case and verify our conjecture in equilibrium, under the same parametrization used for the benchmark equilibrium. Hence,

$$U'(e_S - D - LIQ_S) = \beta\delta + \beta^2(1-\delta). \tag{B.42}$$

Using the balance sheet constraint (9) and (B.38), we can express equity in terms of the lending, deposit and early deposit rate choices:

$$E = I + Dr_D. \tag{B.43}$$

Moreover, using (B.38), (B.40), (B.41), (B.42), and (B.43), the utility of the banker can be re-written as:

$$\begin{aligned}\mathbb{U}_B^{nr} &= W(e_B - E) + \omega\left[(1+r_I)I + \widehat{LIQ} - (1-\delta)D(1+\bar{r}_D)\right] - X \\ &= W(e_B - I - Dr_D) + \omega\Big[A\cdot I - c'(I)I/\omega + \beta^{-2}V'(D(1+r_D))(1+r_D) \\ &\qquad + \left(\beta^{-1}\delta + (1-\delta)\right)r_D\Big] - X.\end{aligned} \tag{B.44}$$

The private equilibrium is characterized by the choice of I, D, and r_D that result in the highest $\mathbb{U}_B^{nr}$ in (B.44), i.e., by the following first-order condition:

$$-W'(e_B - I - Dr_D) + \omega A - c''(I)I - c'(I) = 0, \tag{B.45}$$

$$-W'(e_B - I - Dr_D)r_D + \omega\beta^{-2}V''(D(1+r_D))(1+r_D)^2 = 0, \tag{B.46}$$

and

$$-W'(e_B - I - Dr_D)D + \omega\beta^{-2}V''(D(1+r_D))D(1+r_D) + \omega\left(\beta^{-1}\delta + (1-\delta)\right) = 0. \tag{B.47}$$

Table B.7 reports the equilibrium outcomes such that the bank is run-proof. In order to satisfy (B.38), the banker has an incentive to set a negative deposit rate for early withdrawals; otherwise, she would need to hold as many liquid asset as the amount of deposits and could not use any to extend loans. Eliminating the run reduces the risk premium savers demand to hold deposits, but it also reduces the transaction services to savers and hence the convenience yield that the bank extracts. In particular, there is a trade-off between r_D and $\bar{r}_D$. Given that deposits are safer, savers are willing to accept a negative late deposit rate $\bar{r}_D$ in exchange for a less negative r_D. A more negative r_D allows the banker to channel more deposits to loans and still eliminate all run-risk. But, savers would demand higher compensation in terms of $\bar{r}_D$, which reduces banking profits. The banker balances these two effects and offers deposit rates that are making savers indifferent between supplying deposits and self-insuring ($LIQ_S > 0$ in the run-proof equilibrium). Overall, lending and intermediation are lower in the run-proof private equilibrium, while the liquid asset holdings are substantially higher compared to our benchmark PE where we restrict $r_D \geq 0$. Still, the level deposits are comparable across the two private equilibria. For our parametrization, all agents are worse-off in the run-proof equilibrium. Nevertheless, this does not always need to be always the case. For example, if $\underline{\xi}$ was high enough, then the bank could be run-proof without the need to hold a lot of liquid assets, which hurt lending, or charge negative deposit rates for early withdrawals, which diminish transaction services.

Substituting (B.40), (B.41), and (B.42) in (5) and (1), and setting $q = 0$, we get the following indirect utility functions:

$$\mathbb{U}_E^{nr,*} = c'(I)I - c(I) \tag{B.48}$$

and

$$\mathbb{U}_S^{nr,*} = \mathbb{U}_S^{\alpha} + V(D(1+r_D)) - V'(D(1+r_D))D(1+r_D). \tag{B.49}$$

The run-proof socially optimal equilibrium allocation maximize a social welfare function $\mathbb{U}_{sp}^{nr} = \mathbb{U}_B^{nr} + w_E\mathbb{U}_E^{nr,*} + w_S\mathbb{U}_S^{nr,*}$ given (B.44), (B.48), and (B.49). As a result, the first-order conditions characterizing the planner's solution are

$$-W'(e_B - I - Dr_D) + \omega A - c''(I)I - c'(I) + w_E\partial\mathbb{U}_E^{nr,*}/\partial I = 0, \tag{B.50}$$

$$-W'(e_B - I - Dr_D)r_D + \omega\beta^{-2}V''(D(1+r_D))(1+r_D)^2 + w_S\partial\mathbb{U}_S^{nr,*}/\partial D = 0, \tag{B.51}$$

and

$$-W'(e_B - I - Dr_D)D + \omega\beta^{-2}V''(D(1+r_D))D(1+r_D) + \omega\left(\beta^{-1}\delta + (1-\delta)\right) + w_S\partial\mathbb{U}_S^{nr,*}/\partial r_D = 0, \tag{B.52}$$

	PE	PE	Run-proof SP for weights (w_E, w_S)		
	Restrict $r_D \geq 0$	*Run-proof*	(0.05,0.15)	(0.10,0.10)	(0.15,0.05)
I	0.853	0.114	0.104	0.107	0.111
LIQ	0.061	0.683	0.693	0.689	0.686
D	0.876	0.739	0.739	0.739	0.739
E	0.039	0.058	0.058	0.058	0.058
r_I	3.108	3.495	3.496	3.496	3.495
r_D	0.000	-0.076	-0.062	-0.067	-0.071
$\bar{r}_D$	0.726	-0.334	-0.344	-0.340	-0.337
q	0.408	0.000	0.000	0.000	0.000
ℓ	0.067	0.857	0.869	0.865	0.861
k	0.042	0.073	0.073	0.073	0.073
$r_I - (1-\delta)\bar{r}_D$	2.382	3.829	3.839	3.836	3.833
$I + LIQ$	0.915	0.797	0.797	0.797	0.797
$I - E$	0.815	0.056	0.046	0.049	0.053
LIQ_S	0.000	0.081	0.080	0.081	0.081
$\Delta\mathbb{U}_E$	-	-7.33%	-7.33%	-7.33%	-7.33%
$\Delta\mathbb{U}_S$	-	-1.26%	-1.19%	-1.21%	-1.23%
$\Delta\mathbb{U}_B$	-	-40.94%	-43.81%	-42.85%	-41.90%

Table B.7: Private equilibrium and socially optimal run-proof allocations. The welfare changes are computed over the level of welfare in the private equilibrium where deposit rates are restricted to be positive and banking is not run-proof. All equilibria are for $\underline{\xi} = 0$.

where $\partial\mathbb{U}_E^{nr,*}/\partial I = c''(I)I > 0$, $\partial\mathbb{U}_S^{nr,*}/\partial D = -V''\left(D(1+r_D)\right)D(1+r_D)^2 > 0$, and $\partial\mathbb{U}_S^{nr,*}/\partial r_D = -V''\left(D(1+r_D)\right)D^2(1+r_D) > 0$.

When the planner puts more weight on the saver, she choose a less negative r_D and, hence, needs higher LIQ to implement the run-proof equilibrium, which pushes lending down compared with the private equilibrium or cases where less weight is put on S. Both the banker and entrepreneurs are worse-off compared with the private equilibrium—the welfare loss for the entrepreneur is small because lending and investment are already low in the private run-proof equilibrium, and the higher investment is, the bigger the surplus to E due to the convex effort cost. Overall, all agents are worse-off, even for the planner's allocations, compared with the benchmark private equilibrium indicating that run-proof banking is not optimal for the example we present.

B.11 Outside Equity

This section extends the baseline model such that the bank has an alternative source of funding apart from the equity contributed by the banker and the deposits offered by savers. In particular, we consider a separate group of agents, who we call *outside investors* and who may choose to buy equity at a certain price from the bank at $t = 1$ in exchange for a share of the dividends at $t = 3$. These investors do not have a preference for early consumption nor do they value the transaction services of deposits contrary to savers. We assume that their preferences are the same as for bankers, but contrary to them, investors do not have the ability to monitor entrepreneurs and, hence, manage a bank themselves. We will refer to the equity contributed by the banker and investors as *inside equity* and *outside equity*, respectively.[7]

Denote by P the price of one share of outside equity and by O the number of shares issued and distributed to outside investors. Inside equity is equally divided into E shares, i.e., the banker first injects inside equity, normalizing the price of each (inside equity) share to one, and then decides how many shares to issue to outside investors and at what price. Thus, the total number of shares is $E+O$ and the total equity capital $E+P\cdot O$. It is convenient to denote bank dividends as $Div(\xi,\delta)=\omega[(1-y(\xi,\delta))\cdot I\cdot(1+r_I)-(1-\delta)\cdot D\cdot(1+\bar{r}_D)]$ and dividends per share as $DPS(\xi,\delta)=Div(\xi,\delta)/(E+O)$. Then, investors choose how much of their period 1 endowment, e_O, to invest in equity, in order to maximize

$$\mathbb{U}_O = W(e_O - P\cdot O) + \int_{\xi^*}^{\bar{\xi}} O\cdot DPS(\xi,\delta)\frac{d\xi}{\Delta_\xi}, \tag{B.53}$$

which, taking the dividends per share as given, yields the following outside equity supply (*ES*) schedule:

$$-P\cdot W'(e_O - P\cdot O) + \int_{\xi^*}^{\bar{\xi}} DPS(\xi,\delta)\frac{d\xi}{\Delta_\xi} \leq 0, \tag{B.54}$$

holding with equality for $O>0$. Finally, substituting (B.54) in (B.53) we get the following indirect utility for outside investors:

$$\mathbb{U}_O^* = W(e_O - P\cdot O) + P\cdot O\cdot W'(e_O - P\cdot O). \tag{B.55}$$

On top of the previous choices in $\mathcal{C}$, the banker will also choose the level of outside equity, O, and the price, P, that are consistent with the equity supply schedule (B.54). Hence, B's choice set becomes $\widetilde{\mathcal{C}}=\mathcal{C}\cup\{O,P\}$. Because B will receive only a fraction $E/(E+O)$ of the dividends, her utility becomes:

$$\begin{aligned}\widetilde{\mathbb{U}}_B &= W(e_B - E) + \int_{\xi^*}^{\bar{\xi}} E\cdot DPS(\xi,\delta)\frac{d\xi}{\Delta_\xi} \\ &= W(e_B - E) + \int_{\xi^*}^{\bar{\xi}} \left\{\frac{E}{E+O}\cdot\omega\cdot[(1-y(\xi,\delta))\cdot I\cdot(1+r_I)-(1-\delta)\cdot D\cdot(1+\bar{r}_D)]-X\right\}\frac{d\xi}{\Delta_\xi}.\end{aligned} \tag{B.56}$$

[7] We have assumed a different investor base for outside equity to keep the extension simple. Note that outside equity and deposit markets can be endogenously segmented, i.e., there is no need to exogenously restrict outside investors or depositors to supply deposits or equity, respectively. The decision to abstain from these markets would be consistent with equilibrium equity prices and deposit rates. Intuitively, savers would require a lower price to purchase equity, because equity is less useful for early consumption, because it is worthless in a run and secondary market trading can be frictional, and because it does not provide transaction services. Moreover, an all-equity funding structure would not be possible even if the bank preferred it due to the disciplinary role of runnable debt, which addresses the critique raised by Jacklin (1987). Similarly, outside investors would require a higher deposit rate to supply deposits, since they do not price their transaction services. See, also, Allen, Carletti, and Marquez (2015) for a model of segmented bank equity and deposit markets.

Note that the banker bears the full cost of monitoring.

The functional form of the deposit supply and loan demand schedules, (2) and (7) respectively, are unaffected by the introduction of outside equity. However, the monitoring threshold $\hat{\lambda}$, given by (13) for certain ξ, will change to $\hat{\hat{\lambda}}$, because the banker will monitor if her share of, rather than the total, dividends is higher than the monitoring cost:

$$\frac{E}{E+O}\cdot\omega\cdot[(1-y(\xi,\hat{\hat{\lambda}}))\cdot I\cdot(1+r_I)-(1-\hat{\hat{\lambda}})\cdot D\cdot(1+\bar{r}_D)]-X\geq 0$$

$$\Rightarrow\hat{\hat{\lambda}}(\xi)=\frac{(\xi\cdot I+LIQ)(1+r_I)-\xi\left(D(1+\bar{r}_D)+\frac{E+O}{E}\frac{X}{\omega}\right)}{D[(1+r_D)(1+r_I)-\xi(1+\bar{r}_D)}. \tag{B.57}$$

In other words, outside equity reduces the threshold for withdrawals under which the banker has incentives to monitor, i.e., $\partial\hat{\hat{\lambda}}/\partial O<0$. The functional form of the global game constraint (20) does not change but λ^* is replaced with $\tilde{\lambda}^*\equiv\hat{\hat{\lambda}}(\xi^*)$ in the limits of integration:

$$\widetilde{GG}=\int_{\delta}^{\tilde{\lambda}^*}[\omega D(1+\bar{r}_D)-D(1+r_D)]\,d\lambda-\int_{\tilde{\lambda}^*}^{\theta^*}D(1+r_D)d\lambda-\int_{\theta^*}^{1}\frac{LIQ+\xi^*I}{\lambda}d\lambda=0. \tag{B.58}$$

Finally, the balance sheet incorporates the funds form raising outside equity:

$$\widetilde{BS}:\quad I+LIQ=D+E+P\cdot O. \tag{B.59}$$

B chooses variables in $\widetilde{\mathcal{C}}$ to maximize $\widetilde{\mathbb{U}}_B$ in (B.56) subject to $\widetilde{\mathcal{Y}}=\{\widetilde{BS},\widetilde{GG},DS,LD,ES\}$ given by (B.59), (B.58), (2), (7), and (B.54). The private equilibrium is characterized by the following first-order conditions:

$$\frac{\partial\widetilde{\mathbb{U}}_B}{\partial\widetilde{\mathcal{C}}}+\sum_{\widetilde{\mathcal{Y}}}\psi_{\widetilde{\mathcal{Y}}}\frac{\partial\widetilde{\mathcal{Y}}}{\partial\widetilde{\mathcal{C}}}=0, \tag{B.60}$$

where ψ_{ES} is the Lagrange multiplier on the equity supply schedule and $\partial ES/\partial\widetilde{\mathcal{C}}$ are the partial derivatives reported in (B.106)-(B.115) in section B.12.[8] Thus, the banker also internalizes how her choices affect the supply of outside equity by investors. At the same time, issuing outside equity has a direct effect on the disutility from injecting inside equity as can be seen by $\partial\widetilde{\mathbb{U}}_B/\partial E=\partial\mathbb{U}_B/\partial E+O/(E+O)\int_{\xi^*}^{\bar{\xi}}DPS(\xi,\delta)d\xi/\Delta_\xi$, where $\partial\mathbb{U}_B/\partial E<0$ is given by (B.69). Hence, if the issuance of outside equity is positive, the disutility for the banker from injecting equity is lower because it increases her share of dividends. We now turn to the choice of O and P, which are new

[8]The partial derivatives $\partial DS/\partial C$ and $\partial LD/\partial C$ are given by (B.90)-(B.97) and (B.98)-(B.105), respectively, while all $\partial DS/\partial O$, $\partial DS/\partial P$, $\partial LD/\partial O$, and $\partial DS/\partial P$ are zero. For choices $c'=\{I,LIQ,D,r_I,r_D,\bar{r}_D\}$, $\partial\widetilde{\mathbb{U}}_B/\partial c'=E/(E+O)\cdot\partial\mathbb{U}_B/\partial c'$, where $\partial\mathbb{U}_B/\partial c'$ are given by (B.66), (B.67), (B.68), (B.71), (B.72) and (B.73), while $\partial\widetilde{\mathbb{U}}_B/\partial\xi^*=\partial\mathbb{U}_B/\partial\xi^*+O\cdot DPS(\xi^*,\delta)$ using (B.70). The partial derivatives $\partial\widetilde{GG}/\partial C$ have the same functional form given by (B.82)-(B.89) with the exception that $\partial\hat{\lambda}/\partial C$ is replaced by $\partial\hat{\hat{\lambda}}/\partial C$. In turn, the latter partial derivatives have the same function form with the former given by (B.74)-(B.76) and (B.79)-(B.81) with the exception of the partial derivatives with respect to E and ξ^*, which need to account for outside equity. That is, $\partial\hat{\hat{\lambda}}/\partial E=\xi\cdot O/E^2\cdot X/\omega/\{D[(1+r_D)(1+r_I)-\xi(1+\bar{r}_D)]\}$ and $\partial\hat{\hat{\lambda}}/\partial\xi^*=\partial\hat{\lambda}/\partial\xi^*-O/E\cdot X/\omega/\{D[(1+r_D)(1+r_I)-\xi(1+\bar{r}_D)]\}$ using (B.78). The remaining partial derivatives are discussed in the main text.

to our benchmark analysis.

The first-order conditions (B.60) with respect to O and P are:

$$\frac{\partial \widetilde{\mathbb{U}}_B}{\partial O} + \psi_{BS} \cdot P + \psi_{GG} \frac{\partial \widetilde{GG}}{\partial O} + \psi_{ES} \frac{\partial ES}{\partial O} = 0 \tag{B.61}$$

and

$$\psi_{BS} \cdot O + \psi_{ES} \frac{\partial ES}{\partial P} = 0. \tag{B.62}$$

Substituting (B.62) in (B.61) we get:

$$\underbrace{-\left(\frac{\partial \widetilde{\mathbb{U}}_B}{\partial O} + \psi_{GG} \frac{\partial \widetilde{GG}}{\partial O}\right)}_{\substack{\text{Cost of issuing} \\ \text{outside equity}}} = \underbrace{\psi_{BS}\left(P - O \cdot \frac{\partial ES}{\partial P}^{-1} \frac{\partial ES}{\partial O}\right)}_{\substack{\text{Benefit of issuing} \\ \text{outside equity}}}, \tag{B.63}$$

where $\partial \widetilde{\mathbb{U}}_B / \partial O = -E/(E+O) \int_{\xi^*}^{\bar{\xi}} DPS(\xi, \delta) d\xi / \Delta_\xi < 0$, $\partial ES/\partial O < 0$ from (B.110), $\partial ES/\partial P < 0$ from (B.115), and $\partial \widetilde{GG}/\partial O = \omega D(1+\bar{r}_D)\partial \hat{\hat{\lambda}}/\partial O < 0$, because $\partial \hat{\hat{\lambda}}/\partial O = -\xi/E \cdot X/\omega/\{D[(1+r_D)(1+r_I) - \xi(1+\bar{r}_D)]\} < 0$.

The cost of issuing outside equity consists of two components. First, outside equity reduces the share of dividends accruing to the banker, and second, it makes monitoring less likely, which adversely affects the probability of a run, all else being equal. Note that this does not mean that issuing outside equity increases the run probability in equilibrium, since other variables will adjust and the bank may operate with more capital and liquidity reducing the run probability, as we show in the numerical results below. The benefit of issuing outside equity stems from raising additional funds given the shadow value of funding ψ_{BS}. The banker does not take P as given, but internalizes how her choice of O affects P via ES and, hence, accurately captures the marginal funding benefit from issuing outside equity.

The social planner faces the same choices $\widetilde{C}$ and constraints $\widetilde{Y}$ as the banker in the private equilibrium but wants to maximize the social welfare function $\widetilde{\mathbb{U}}^*_{sp} = \widetilde{\mathbb{U}}_B + w_S \mathbb{U}^*_S + w_E \mathbb{U}^*_E + w_S \widetilde{\mathbb{U}}^*_O$, where the utilities are defined in (B.56), (4), (8), (B.55), and $w_O \geq 0$ is the weight assigned to outside investors.

Table B.8 reports the private and social equilibrium outcomes when the bank issues outside equity and the planner assigns zero weight to outside investors, such that we can have a more straightforward comparison to the case that there is no outside equity funding. We, first, compare the private equilibrium outcomes. Issuing outside equity allows the banker to expand the balance sheet, but reduces her share of profits. To compensate for this, the banker decreases lending to improve the profit margin and also injects more inside equity. Relying more on equity financing allows the banker to channel more deposits into the liquid asset resulting in a smaller scale of intermediation. The higher liquidity and capital ratios outweigh the negative effect of outside equity on monitoring incentives and result in lower run-risk compared to the PE without issuance of outside equity. Deposits become safer and the bank can attract deposits offering lower deposit rates, which further improves the profit margin. Although it is not reported in the table, the banker enjoys higher utility in the PE where she issues outside equity compared with the benchmark equilibrium. Comparing the privately and socially optimal outcomes when the bank issues outside equity, we derive the same conclusion as in the benchmark case. The planner chooses more liquidity and capital to favor S resulting in less intermediation, but also lower run-risk. On the contrary, the planner cuts the liquid asset holdings to support more lending and favor E resulting in higher run-risk compared with the private equilibrium.[9] Finally, comparing the socially optimal outcomes with and without the issuance of outside equity, we find that the planner can implement lower run-risk and achieve higher social welfare when outside equity funding is allowed.

We should note that these observations do not rely on the fact that the planner assigns zero weight on outside investors. If $w_O > 0$, the planner internalizes how the issuance of outside equity matters for outside investors' welfare via (B.55). Hence, the first-order conditions with respect to O and P—given by (B.61) and (B.61) in PE and SP for $w_O = 0$—incorporate additional terms:

$$\frac{\partial \widetilde{\mathbb{U}}_B}{\partial O} + \Psi_{BS} \cdot P + \Psi_{GG} \frac{\partial \widetilde{GG}}{\partial O} + \Psi_{ES} \frac{\partial ES}{\partial O} + w_O \frac{\partial \mathbb{U}_O^*}{\partial O} = 0 \tag{B.64}$$

and

$$\Psi_{BS} \cdot O + \Psi_{ES} \frac{\partial ES}{\partial P} + w_O \frac{\partial \mathbb{U}_O^*}{\partial P} = 0, \tag{B.65}$$

where $\partial \mathbb{U}_O^* / \partial O = -P^2 \cdot O \cdot W''(e_O - P \cdot O) > 0$ and $\partial \mathbb{U}_O^* / \partial P = -P \cdot O^2 \cdot W''(e_O - P \cdot O) > 0$.

Table B.9 reports the privately and socially optimal outcomes when the planner assigns $w_O > 0$. In sum, the planner offers a better price P to outside investors compared with the case that $w_O = 0$, whose welfare improves. But, the rest of the findings remain unchanged.

[9] One inconsequential difference is that for $(w_E, w_S) = (0.1, 0.1)$ the planner chooses lower lending compared with the private equilibrium in the presence of outside equity funding, which results in lower utility for E. The reason is that issuing outside equity can help savers further, and, as we show, social welfare is higher for this set of weights. If, instead, we set $(w_E, w_S) = (0.12, 0.08)$, we find that lending as well as liquidity and capital are higher resulting in higher welfare for both E and S compared with the private equilibrium (as is the case in absence of outside equity funding for $(w_E, w_S) = (0.1, 0.1)$).

	PE		SP for weights (w_E, w_S)			
			(0.1,0.1)		(0.2,0.0)	
	No OE	*OE*	*No OE*	*OE*	*No OE*	*OE*
I	0.862	0.834	0.873	0.829	0.906	0.908
LIQ_1	0.052	0.131	0.060	0.168	0.000	0.020
D	0.875	0.908	0.894	0.937	0.867	0.875
E	0.038	0.041	0.039	0.043	0.038	0.039
O	0.000	0.002	0.000	0.002	0.000	0.001
$E/(E+O)$	1.000	0.962	1.000	0.959	1.000	0.965
r_I	3.097	3.142	3.089	3.154	3.042	3.047
$\bar{r}_D$	0.717	0.697	0.767	0.748	0.758	0.727
P	-	9.902	-	9.378	-	10.502
q	0.407	0.394	0.403	0.386	0.408	0.401
ℓ	0.057	0.136	0.065	0.169	0.000	0.022
k	0.042	0.059	0.042	0.060	0.042	0.058
$r_I - (1-\delta)\bar{r}_D$	2.739	2.794	2.705	2.780	2.663	2.683
$I + LIQ$	0.914	0.965	0.933	0.997	0.906	0.928
$I - E - P \cdot O$	0.824	0.777	0.834	0.769	0.867	0.855
$E(Div)$	0.745	0.789	0.747	0.796	0.743	0.779
$\Delta\mathbb{U}_E$	-	-	0.33%	-0.62%	1.19%	1.93%
$\Delta\mathbb{U}_S$	-	-	0.71%	2.55%	-0.30%	-1.34%
$\Delta\mathbb{U}_B$	-	-	-0.05%	0.59%	-0.09%	-0.19%
$\Delta\mathbb{U}_O$	-	-	-	0.36%	-	-0.05%
$\Delta\mathbb{U}_{sp}$			0.05%	0.06%	0.14%	0.19%

Table B.8: Privately versus Socially optimal solutions when the bank issues outside equity (OE) and $w_O = 0$. The welfare changes are computed over the levels of welfare in the respective private equilibrium, which are normalized to one. We have set $e_O = 0.09$ such that outside investors are willing to buy equity at the price offered by the bank, which is true as long as $e_O > 0.062$.

	PE	SP for weights (w_E, w_S, w_O)			
		(0.1,0.1,0.0)	(0.2,0.0,0.0)	(0.1,0.1,0.1)	(0.2,0.0,0.1)
I	0.834	0.829	0.908	0.824	0.907
LIQ_1	0.131	0.168	0.020	0.179	0.023
D	0.908	0.937	0.875	0.942	0.876
E	0.041	0.043	0.039	0.043	0.039
O	0.002	0.002	0.001	0.002	0.001
$E/(E+O)$	0.962	0.959	0.965	0.956	0.963
r_I	3.142	3.154	3.047	3.161	3.048
$\bar{r}_D$	0.697	0.748	0.727	0.749	0.726
P	9.902	9.378	10.502	9.200	10.392
q	0.394	0.386	0.401	0.385	0.401
ℓ	0.136	0.169	0.022	0.178	0.024
k	0.059	0.060	0.058	0.061	0.058
$r_I - (1-\delta)\bar{r}_D$	2.794	2.780	2.683	2.787	2.685
$I + LIQ$	0.965	0.997	0.928	1.003	0.930
$I - E - P \cdot O$	0.777	0.769	0.855	0.763	0.853
$E(Div)$	0.789	0.796	0.779	0.800	0.781
$\Delta\mathbb{U}_E$	-	-0.06%	1.93%	-0.17%	1.91%
$\Delta\mathbb{U}_S$	-	1.22%	-1.34%	1.41%	-1.30%
$\Delta\mathbb{U}_B$	-	-0.06%	-0.19%	-0.07%	-0.19%
$\Delta\mathbb{U}_O$	-	0.05%	-0.05%	0.10%	-0.02%

Table B.9: Privately versus Socially optimal solutions when the bank issues outside equity. The welfare changes are computed over the levels of welfare in the private equilibrium, which are normalized to one. We have set $e_O = 0.09$ such that outside investors are willing to buy equity at the price offered by the bank, which is true as long as $e_O > 0.062$.

B.12 Derivatives

This section reports the partial derivatives of the banker's utility $\mathbb{U}_B$ in (14), the monitoring threshold $\hat{\lambda}$ in (13), the global game constraint GG in (20), the deposit supply schedule DS in (2), the loan demand schedule LD in (7), and the (outside) equity supply schedule ES in (B.54), with respect to the choice variables in $\mathbb{C}$. When it is unambiguous, we also report the sign of the derivatives.

Partial derivatives $\partial\mathbb{U}_B/\partial\mathcal{C}$.

$$\frac{\partial\mathbb{U}_B}{\partial I} = \omega(1-q)(1+r_I) > 0. \tag{B.66}$$

$$\frac{\partial\mathbb{U}_B}{\partial LIQ} = \omega(1+r_I)\log\left(\bar{\xi}/\xi^*\right)/\Delta_\xi > 0. \tag{B.67}$$

$$\frac{\partial\mathbb{U}_B}{\partial D} = -\omega\left[\delta(1+r_D)(1+r_I)\log\left(\bar{\xi}/\xi^*\right)/\Delta_\xi + (1-\delta)(1-q)(1+\bar{r}_D)\right] < 0. \tag{B.68}$$

$$\frac{\partial\mathbb{U}_B}{\partial E} = -W'(e_B - E) < 0. \tag{B.69}$$

$$\frac{\partial\mathbb{U}_B}{\partial\xi^*} = -\left[\omega\left((\xi^* I - \delta D(1+r_D) + LIQ)/\xi^*(1+r_I) - (1-\delta)D(1+\bar{r}_D)\right) - X\right]/\Delta_\xi < 0. \tag{B.70}$$

$$\frac{\partial\mathbb{U}_B}{\partial r_I} = \omega\left[(1-q)I - (\delta D(1+r_D) - LIQ)\log\left(\bar{\xi}/\xi^*\right)/\Delta_\xi\right] > 0. \tag{B.71}$$

$$\frac{\partial\mathbb{U}_B}{\partial r_D} = -\omega\delta D(1+r_I)\log\left(\bar{\xi}/\xi^*\right)/\Delta_\xi < 0. \tag{B.72}$$

$$\frac{\partial\mathbb{U}_B}{\partial\bar{r}_D} = -\omega(1-\delta)(1-q)D < 0. \tag{B.73}$$

Partial derivatives $\partial\hat{\lambda}/\partial\mathcal{C}$.

$$\frac{\partial\hat{\lambda}}{\partial I} = \xi^*(1+r_I)/\left[D\left((1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)\right)\right] > 0. \tag{B.74}$$

$$\frac{\partial\hat{\lambda}}{\partial LIQ} = (1+r_I)/\left[D\left((1+r_D)(1+r_I) - \xi^*(1+\bar{r}_D)\right)\right] > 0. \tag{B.75}$$

$$\frac{\partial\hat{\lambda}}{\partial D} = -\xi^*(1+\bar{r}_D)/\left[D\left((1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)\right)\right] - \hat{\lambda}/D < 0. \tag{B.76}$$

$$\frac{\partial\hat{\lambda}}{\partial E} = 0. \tag{B.77}$$

$$\begin{aligned}\frac{\partial\hat{\lambda}}{\partial\xi^*} &= \left[I(1+r_I)-(D(1+\bar{r}_D)+X/\omega)\right]/\left[D\left((1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)\right)\right] \\ &\quad + \hat{\lambda}(1+\bar{r}_D)/\left[(1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)\right] > 0.\end{aligned} \tag{B.78}$$

$$\begin{aligned}\frac{\partial\hat{\lambda}}{\partial r_I} &= (\xi^* I + LIQ)/\left[D\left((1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)\right)\right] \\ &\quad - \hat{\lambda}(1+r_D)/\left[(1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)\right] > 0.\end{aligned} \tag{B.79}$$

$$\frac{\partial\hat{\lambda}}{\partial r_D} = -\hat{\lambda}(1+r_I)/\left[(1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)\right] < 0. \tag{B.80}$$

$$\frac{\partial\hat{\lambda}}{\partial\bar{r}_D} = -(1-\hat{\lambda})\xi^*/\left[(1+r_D)(1+r_I)-\xi^*(1+\bar{r}_D)\right] < 0. \tag{B.81}$$

Partial derivatives $\partial GG/\partial\mathcal{C}$ (see proof of Corollary 1).

$$\frac{\partial GG}{\partial I} = \omega D(1+\bar{r}_D)\frac{\partial\hat{\lambda}}{\partial I} - \int_{\xi^*}^{1}\frac{\xi^*}{\lambda}d\lambda > 0. \tag{B.82}$$

$$\frac{\partial GG}{\partial LIQ} = \omega D(1+\bar{r}_D)\frac{\partial\hat{\lambda}}{\partial LIQ} - \int_{\xi^*}^{1}\frac{1}{\lambda}d\lambda \gtreqless 0. \tag{B.83}$$

$$\frac{\partial GG}{\partial D} = \omega D(1+\bar{r}_D)\left[\frac{\partial\hat{\lambda}}{\partial D} + (\hat{\lambda}-\delta)/D\right] - (\theta^*-\delta)(1+r_D) < 0. \tag{B.84}$$

$$\frac{\partial GG}{\partial E} = 0. \tag{B.85}$$

$$\frac{\partial GG}{\partial\xi^*} = \omega D(1+\bar{r}_D)\frac{\partial\hat{\lambda}}{\partial\xi^*} - \int_{\xi^*}^{1}\frac{I}{\lambda}d\lambda > 0. \tag{B.86}$$

$$\frac{\partial GG}{\partial r_I} = \omega D(1+\bar{r}_D)\frac{\partial\hat{\lambda}}{\partial r_I} > 0. \tag{B.87}$$

$$\frac{\partial GG}{\partial r_D} = \omega D(1+\bar{r}_D)\frac{\partial\hat{\lambda}}{\partial r_D} - D(\theta^* - \delta) < 0. \tag{B.88}$$

$$\frac{\partial GG}{\partial \bar{r}_D} = \omega D(1+\bar{r}_D)\frac{\partial\hat{\lambda}}{\partial \bar{r}_D} + \omega D(\hat{\lambda} - \delta) \gtrless 0. \tag{B.89}$$

Partial derivatives $\partial DS/\partial\mathcal{C}$.

$$\frac{\partial DS}{\partial I} = [\beta\delta + \beta^2(1-\beta)] \cdot q \cdot \frac{\xi^* + \underline{\xi}}{2} \cdot \frac{1}{D} > 0. \tag{B.90}$$

$$\frac{\partial DS}{\partial LIQ} = [\beta\delta + \beta^2(1-\beta)] \cdot q \cdot \frac{1}{D} > 0. \tag{B.91}$$

$$\begin{aligned}\frac{\partial DS}{\partial D} = {} & U''(e_S - D) - [\beta\delta + \beta^2(1-\beta)] \cdot q \cdot \left(LIQ + I \cdot \frac{\xi^* + \underline{\xi}}{2}\right)\frac{1}{D^2} \\ & + (1-q)V''(D(1+r_D))(1+r_D)^2 < 0.\end{aligned} \tag{B.92}$$

$$\frac{\partial DS}{\partial E} = 0. \tag{B.93}$$

$$\begin{aligned}\frac{\partial DS}{\partial \xi^*} = {} & \left\{[\beta\delta + \beta^2(1-\beta)]\frac{LIQ + \xi^* I}{D} - \delta\beta(1+r_D) \right. \\ & \left. -(1-\delta)\beta^2\omega(1+r_D) - V'(D(1+r_D))(1+r_D)\right\}\Delta_{\xi}^{-1} < 0.\end{aligned} \tag{B.94}$$

$$\frac{\partial DS}{\partial r_I} = 0. \tag{B.95}$$

$$\frac{\partial DS}{\partial r_D} = (1-q)\left[\beta\delta + V'(D(1+r_D)) + V''(D(1+r_D))D(1+r_D)\right] > 0. \tag{B.96}$$

$$\frac{\partial DS}{\partial \bar{r}_D} = \omega \cdot \beta^2(1-\delta)(1-q) > 0. \tag{B.97}$$

Partial derivatives $\partial LD/\partial \mathcal{C}$.

$$\frac{\partial LD}{\partial I} = \omega(A-(1+r_I))\frac{\delta D(1+r_D)-LIQ}{I^2}\frac{\log(\bar{\xi}/\xi^*)}{\Delta_\xi} - (1-q)c''(I) \lesseqgtr 0. \tag{B.98}$$

$$\frac{\partial LD}{\partial LIQ} = \omega(A-(1+r_I))\frac{1}{I}\frac{\log(\bar{\xi}/\xi^*)}{\Delta_\xi} > 0. \tag{B.99}$$

$$\frac{\partial LD}{\partial D} = -\omega(A-(1+r))\frac{\delta(1+r_D)}{I}\frac{\log(\bar{\xi}/\xi^*)}{\Delta_\xi} < 0. \tag{B.100}$$

$$\frac{\partial LD}{\partial E} = 0. \tag{B.101}$$

$$\frac{\partial LD}{\partial \xi^*} = -\left[\omega(A-(1+r_I))\frac{\xi^* I - \delta D(1+r_D)+LIQ}{\xi^* I} - c'(I)\right]\Delta_\xi^{-1} \gtreqless 0. \tag{B.102}$$

$$\frac{\partial LD}{\partial r_I} = -\omega\left[(1-q) - \frac{\delta D(1+r_D)}{I}\frac{\log(\bar{\xi}/\xi^*)}{\Delta_\xi}\right] < 0. \tag{B.103}$$

$$\frac{\partial LD}{\partial r_D} = -\omega(A-(1+r))\frac{\delta D}{I}\frac{\log(\bar{\xi}/\xi^*)}{\Delta_\xi} < 0. \tag{B.104}$$

$$\frac{\partial LD}{\partial \bar{r}_D} = 0. \tag{B.105}$$

Partial derivatives $\partial ES/\partial \tilde{\mathcal{C}}$ for extension in section B.11 .

$$\frac{\partial ES}{\partial I} = \frac{1}{E+O}\omega(1-q)(1+r_I) > 0. \tag{B.106}$$

$$\frac{\partial ES}{\partial LIQ} = \frac{1}{E+O}\omega(1+r_I)\log(\bar{\xi}/\xi^*)/\Delta_\xi > 0. \tag{B.107}$$

$$\frac{\partial ES}{\partial D} = -\frac{1}{E+O}\omega\left[\delta(1+r_D)(1+r_I)\log(\bar{\xi}/\xi^*)/\Delta_\xi + (1-\delta)(1-q)(1+\bar{r}_D)\right] < 0. \tag{B.108}$$

$$\frac{\partial ES}{\partial E} = -\frac{1}{E+O}\int_{\xi^*}^{\overline{\xi}} DPS(\xi,\delta)\frac{d\xi}{\Delta_\xi} < 0. \tag{B.109}$$

$$\frac{\partial ES}{\partial O} = P^2 \cdot W''(e_O - P\cdot O) - \frac{1}{E+O}\int_{\xi^*}^{\overline{\xi}} DPS(\xi,\delta)\frac{d\xi}{\Delta_\xi} < 0. \tag{B.110}$$

$$\frac{\partial ES}{\partial \xi^*} = -DPS(\xi^*,\delta)/\Delta_\xi < 0. \tag{B.111}$$

$$\frac{\partial ES}{\partial r_I} = \frac{1}{E+O}\omega\left[(1-q)I - (\delta D(1+r_D) - LIQ)\log(\overline{\xi}/\xi^*)/\Delta_\xi\right] > 0. \tag{B.112}$$

$$\frac{\partial ES}{\partial r_D} = -\frac{1}{E+O}\omega\delta D(1+r_I)\log(\overline{\xi}/\xi^*)/\Delta_\xi < 0. \tag{B.113}$$

$$\frac{\partial ES}{\partial \bar{r}_D} = -\frac{1}{E+O}\omega(1-\delta)(1-q)D < 0. \tag{B.114}$$

$$\frac{\partial ES}{\partial P} = P\cdot O\cdot W''(e_O - P\cdot O) - W'(e_O - P\cdot O) < 0. \tag{B.115}$$