Online Appendix

A Theory of Falling Growth and Rising Rents

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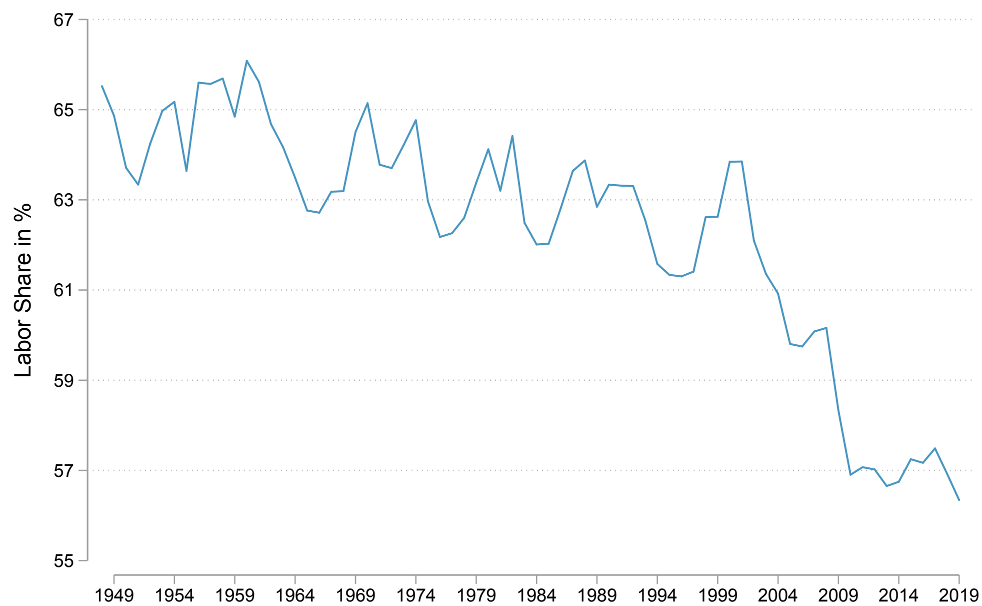
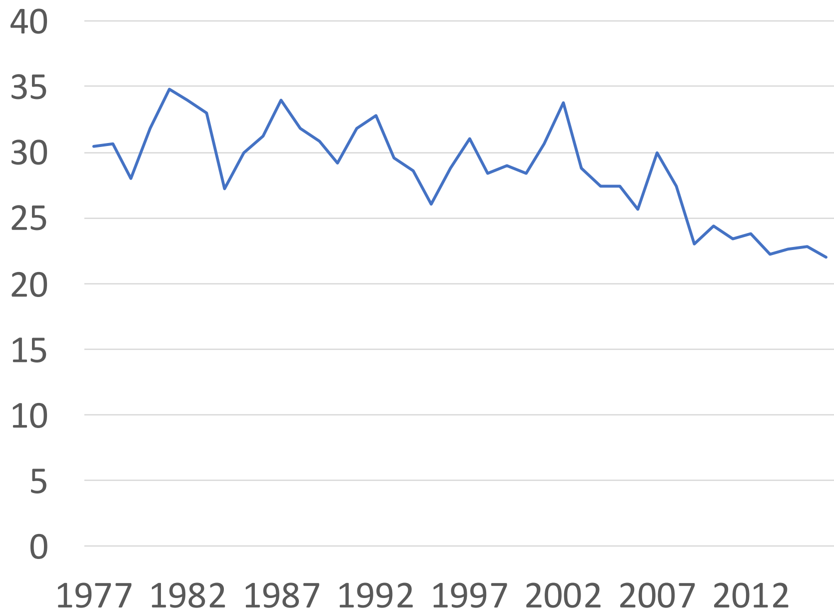
OA-A Additional figures

Figure OA-1: U.S. labor share



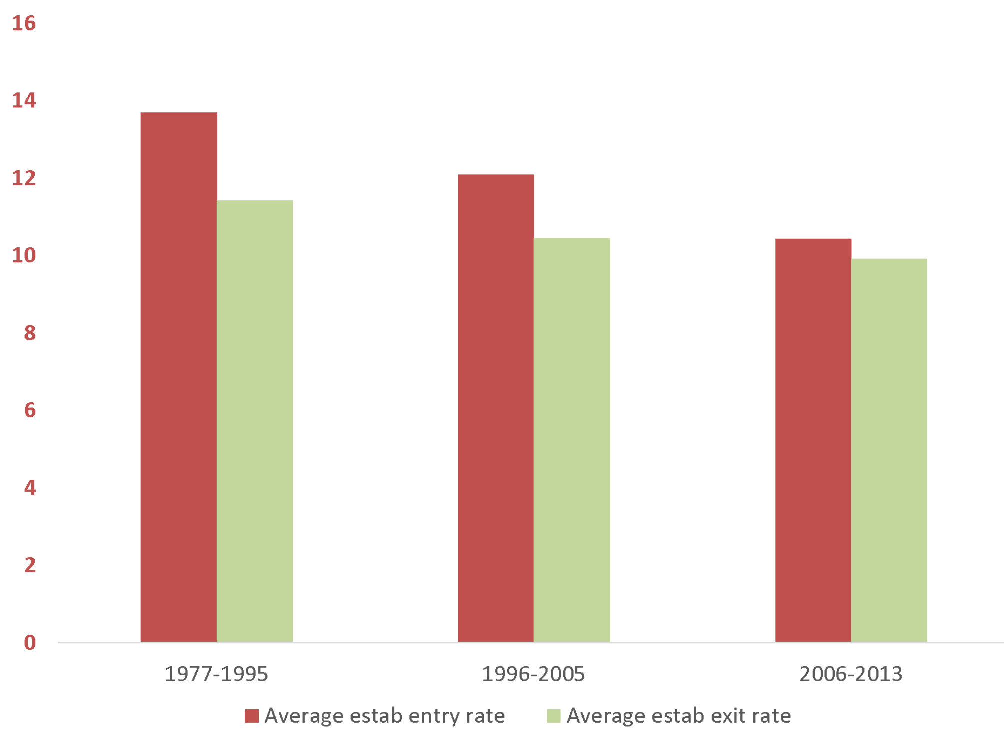
Source: BLS. Aggregate labor compensation of all employed persons as a share of aggregate output for the nonfarm business sector.

Figure OA-2: Falling job reallocation rate 1977–2016



Source: U.S. Census Bureau Business Dynamic Statistics. Job reallocation rate across establishments in U.S. nonfarm business sector.

Figure OA-3: Entry and exit rates of establishments



Source: U.S. Census Bureau Business Dynamic Statistics. Average entry and exit rates at the establishment level in U.S. nonfarm business sector.

OA-B Productivity growth by IT intensity

In Figure 5, we plot the 5-year moving average of MFP growth for two groups of sectors: IT producing and non IT producing. Column "Baseline" of Table OA-1 summarizes the classification of the sectors underlying Figure 5. Industries called "IT producing" are computer and electronic, computer system design and publishing industries as in the classification by Fernald (2015). We calculate yearly productivity growth rate by adding R&D and IP contribution to BLS MFP and then converting the sum to labor augmenting form. The other not IT producing industries are ranked based on the average value of their IT capital relative to value added over the years 1987 to 2016 and then split into two categories: IT intensive and non-IT intensive such that the share of total value added in the two groups is roughly the same. We consider 3 digit sectors spanning the entire non-farm businesses excluding finance and aggregate the MFP growth rates using value-added weights. In Figure 6, we used the same classification to look at the unweighted average of the labor share across IT producing, IT intensive and non IT intensive groups.

In this Appendix, we show that the results from Figures 5 and 6 are robust to considering alternative measures of IT intensity to classify the not IT producing industries into an IT intensive and a non-IT intensive group. Note that we do not change the group of IT producing industries in any of these alternatives as this definition is not based on the measure of IT intensity that we consider here. The list of industries within each group is reported in Table OA-1.

1. In our first alternative (Alt. 1), we use the capital share of computer and software of each sector taken from the BLS and plot the resulting MFP growth rates and labor income shares in Figures OA-4.

2. In our second alternative (Alt. 2), we use gross investment in computer and communication equipment over value added as a measure of IT intensity. The results can be seen in Figures OA-5.

3. Our third alternative (Alt. 3) builds on the idea that the Integrated Industry-Level Production Account suffer from a potential drawback described in Haltiwanger (2015). Indeed, the BEA uses a top down approach to measure capital flow, by looking at how much capital goods are produced (and adding up import - export) by specific industries and then break it down to other industry by looking at specific occupations that work in this industry (for example, a computer scientist is likely to use a computer). While this critique mostly applies to the 1997 version of the industry account, we nevertheless consider an alternative measure that is not based on the BEA data. More precisely, we build on the work of Autor et al. (1998) and measure IT intensity in non-IT producing sectors using their measure of computer use taken from the Current Population Survey (CPS) in 1993. This measure considers the fraction of worker who directly use a computer keyboard at work in each industry.[1] The results can be found in Figures OA-6.

4. Finally, our fourth alternative (Alt. 4) does the same as our baseline, but considers the measure of IT intensity only up until 1995 (instead of over the whole period). The results are plotted in Figures OA-7.

[1]We match the industry classification used in Autor et al. (1998) with the one we use in Table OA-1 by hand. The number of industries is larger in the case of the BEA and we restrict to the set of sectors from Autor et al. (1998).

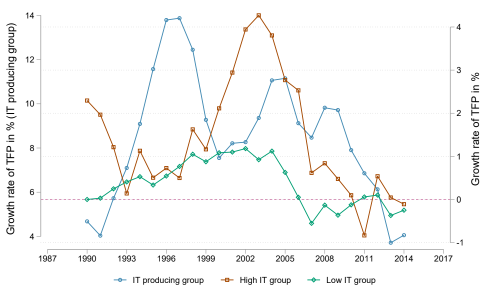
Table OA-1: List of sector by IT intensity

NAICS (2012)	Sector	Baseline	Alt. 1	Alt. 2	Alt. 3	Alt. 4
721	Accommodation	0	0	0		0
561	Administrative & Support Services	1	1	1		1
481	Air Transportation	1	0	0	1	1
621	Ambulatory Health Care Services	0	0	0		0
713	Amusements, Gambling	0	0	1		0
315,316	Apparel & Leather	0	0	0	0	0
515,517	Broadcasting & telecommunications	1	1	1	1	1
325	Chemical Products	1	1	0	1	1
334	Computer & Electronic Products	2	2	2	2	2
5415	Computer Systems Design	2	2	2	2	2
23	Construction	0	0	0	0	0
518,519	Data processing, internet publishing	1	1	1		1
61	Educational Services	0	0	0		0
335	Electrical Equipment	1	1	0	1	1
332	Fabricated Metal Products	0	0	0	0	0
311,312	Food and Beverage & Tobacco Products	0	0	0	0	0
722	Food Services & Drinking Places	0	0	0		0
337	Furniture & Related Products	0	0	0	0	0
622,623	Hospitals & Nursing & Residential Care Facilities	0	0	0		0
5411	Legal Services	1	1	1		0
333	Machinery	1	1	0	1	1
55	Management of Enterprises	0	1	1		0
339	Miscellaneous Manufacturing	0	0	1	1	1
5412-5414,5416-5419	Miscellaneous Professional, Scientific, & Tech Services	0	1	1		1
512	Motion picture & sound recording industries	0	0	0	1	0
336	Transportation Equipment	0	0	1	0	0
327	Nonmetallic Mineral Products	0	0	0	0	1
81	Other Services, except Government	1	0	0		0
487,488,492	Other Transportation and Support Activities	0	0	1	1	0
336	Transportation Equipment	0	1	1	1	1
322	Paper Products	0	0	0	0	0
711,712	Performing Arts	0	0	0		0
324	Petroleum & Coal Products	0	1	0	1	0
486	Pipeline Transportation	1	0	1	0	1
326	Plastics & Rubber Products	0	0	0	1	0
331	Primary Metal Products	0	0	0	0	0
323	Printing & Related Support Activities	0	0	1		0
511	Publishing industries (includes software)	2	2	2	2	2
482	Rail Transportation	0	0	0	0	0
44,45	Retail Trade	1	0	1	0	1
624	Social Assistance	0	0	0		0
313,314	Textile Mills & Textile Product Mills	0	0	0	0	0
485	Transit & Ground Passenger Transportation	1	0	0	0	1
484	Truck Transportation	1	0	0	0	0
493	Warehousing & Storage	0	0	0	0	0
562	Waste Management & Remediation Services	1	0	0		1
483	Water Transportation	1	0	0	1	1
42	Wholesale Trade	1	1	1	1	1
321	Wood Products	0	0	0	0	0

Notes: Classification of sectors between IT producing (2), IT intensive (1) and non IT intensive (0) across 5 different specifications as described in Appendix OA-B.

Figure OA-4: Productivity growth and labor share by IT intensity - Alternative 1

(a) TFP growth



(b) Labor share

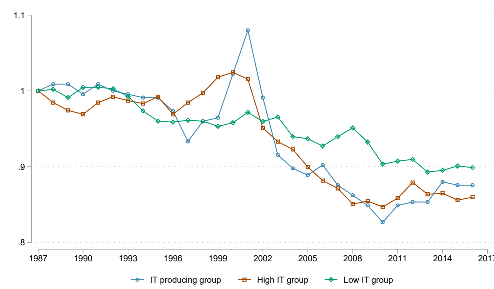
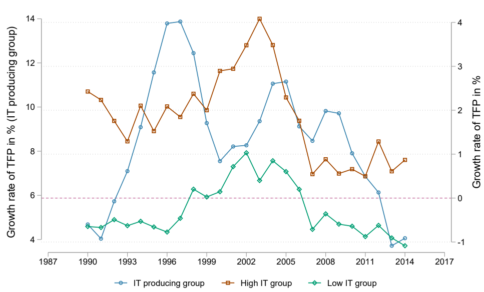


Figure OA-5: Productivity growth and labor share by IT intensity - Alternative 2

(a) TFP growth



(b) Labor share

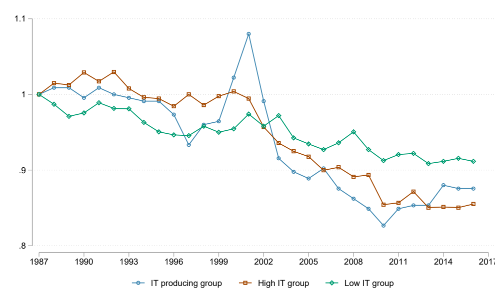


Figure OA-6: Productivity growth and labor share by IT intensity - Alternative 3

(a) TFP growth (b) Labor share

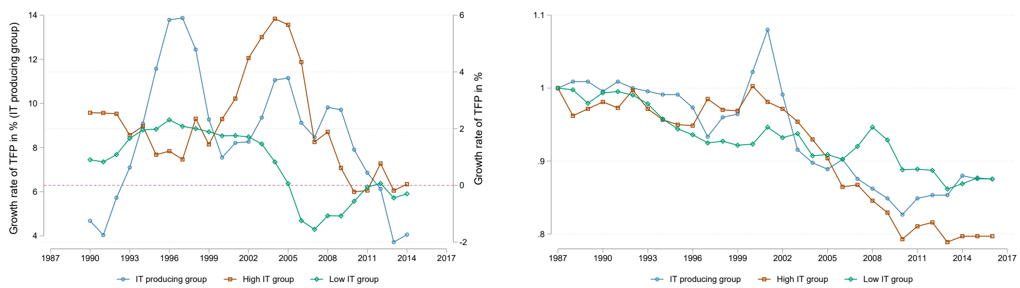
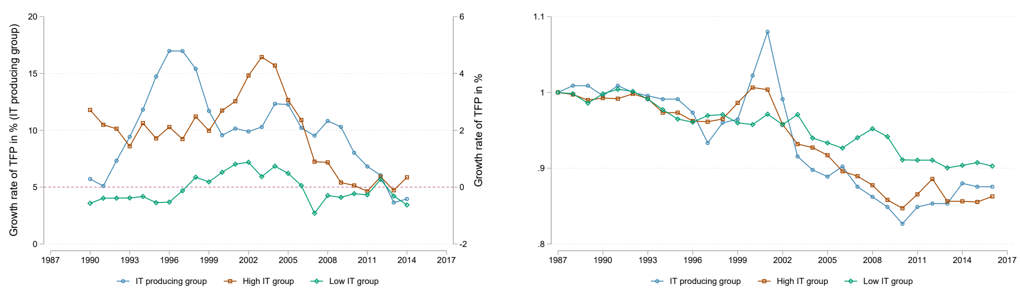


Figure OA-7: Productivity growth and labor share by IT intensity - Alternative 4

(a) TFP growth (b) Labor share



OA-C Additional Calibration Results

Table OA-2: Effect of 20% decline in ψ_o on untargeted moments

	Date	Model
1. 2006–18 productivity growth rate (ppt)	1.10	1.62
% growth slowdown explained	**27.8%**	
2. Between change in labor share (%)	-12.5	-2.2
3. change in aggregate labor share (%)	-8.1	-0.4
4. within change in labor share (%)	4.5	1.8
5. change in intangible share (ppt)	1.5	-0.1
6. change in concentration (ppt)	4.8	8.7

Table OA-3: Effect of 50% decline in ψ_o on untargeted moments

	Date	Model
1. 2006–18 productivity growth rate (ppt)	1.10	1.17
% growth slowdown explained	**90.4%**	
2. Between change in labor share (%)	-12.5	-12.1
3. change in aggregate labor share (%)	-8.1	-5.1
4. within change in labor share (%)	4.5	7.0
5. change in intangible share (ppt)	1.5	1.3
6. change in concentration (ppt)	4.8	41.0

Source: 1: BLS MFP series. 2–5: Autor et al. (2019), BLS, KLEMS. 6: Corrado, Haskel, Jona-Lasinio and Iommi (2012). We lower ψ_o by 35.4% to match the decline in the relative price of IT products from 1996–2005. The data change in the intangible share is from 1995 to 2006–2010. The change in concentration and labor share moments (within, between and aggregate) is from 1987–1992 to 1997–2012.

Cobb-Douglas

Table OA-4: Cobb-Douglas: Parameter Values

Calibrated		
Definition	Parameter	Value
1. share of H-type firms	ϕ	0.010
2. quality step	γ	1.273
3. discount factor	β	0.956
4. initial overhead cost	ψ_o^0	0.026
5. R&D costs	ψ_r	1.006
6. productivity gap	Δ	1.182
Assigned		
Definition	Parameter	Value
7. CES	σ	1
8. CRRA	θ	1

Table OA-5: Cobb-Douglas: model fit

Targeted	Target	Model
1. top 10% concentration 1987–1992	64.0	56.1
2. productivity growth 1949–1995	1.82	1.82
3. aggregate markup	1.27	1.27
4. real interest rate	6.1	6.5
5. intangible share	10.4	12.0
6. labor share and size relation	-1.1	-1.1

Source: 1 and 6: Autor et al. (2019). 2: BLS MFP series. 3: Hall (2018). 4: Farhi and Gourio (2018). 5: Corrado et al. (2012).

Table OA-6: Cobb-Douglas: effect of a decline in ψ_o on untargeted moments

	Data	Model
1. 2006–18 productivity growth rate (ppt)	1.10	1.07
% of growth slowdown explained	**103.9%**	
2. change in aggregate labor share (%)	-8.1	-0.3
3. within change in labor share (%)	4.5	5.1
4. between change in labor share (%)	-12.5	-5.4
5. change in concentration (ppt)	4.8	28.7
6. change in intangible share (ppt)	1.5	-1.4

Source: 1: BLS MFP series. 2–5: Autor et al. (2019), BLS KLEMS. 6: Corrado, Haskel, Jona-Lasinio and Iommi (2012). We lower ψ_o by 35.4% to match the decline in the relative price of IT products from 1996–2005. The data change in the intangible share is from 1995 to 2006–2010. The change in concentration and labor share moments (within, between and aggregate) is from 1987–1992 to 1997–2012.

Table OA-7: Cobb-Douglas, decline in ψ_o: Initial vs. new steady state

	Initial	New
1. creative destruction rate ($z^\star$)	7.53	4.43
2. % of H-type products ($S^\star$)	56.1	84.8
3. % of H-type sales ($\widetilde{S}^\star$)	56.1	84.8
4. markup of H-type firms	1.37	1.30
5. markup of L-type firms	1.16	1.10
6. aggregate markup	1.26	1.27
7. R&D/PY (%)	7.6	4.5
8. overhead/PY (%)	4.4	6.1
9. rent/PY (%)	9.0	10.7
10. real interest rate (%)	6.5	5.7

OA-D Solving for the transition dynamics

This section lays out the numerical method used to compute the transition dynamics in Section 5.2. Let n_t be the number of product a firm holds and let h_t be the share of these products where the firm faces a high productivity second-best producer. We define $m_t \equiv n_t h_t$. The dynamic problem of a firm of type $j = H, L$ in equation (21) of the main text, can be expressed (after dividing the objective by Q_0) as

$$\begin{aligned}
\max_{\{n_s,m_s\}_{s=1}^{\infty}} \quad & \{\pi_j(n_0,m_0) - (n_1 - n_0(1-z_1))\psi_r\} \frac{Y_0}{Q_0} \\
+ \quad & \frac{\gamma^{z_1}}{1+r_1} \{\pi_j(n_1,m_1) - (n_2 - n_1(1-z_2))\psi_r\} \frac{Y_1}{Q_1} \\
+ \quad & \frac{\gamma^{z_1}}{1+r_1}\frac{\gamma^{z_2}}{1+r_2} \{\pi_j(n_2,m_2) - (n_3 - n_2(1-z_3))\psi_r\} \frac{Y_2}{Q_2} \\
\vdots \\
+ \quad & \prod_{\tau=1}^{t} \frac{\gamma^{z_\tau}}{1+r_\tau} \{\pi_j(n_t,m_t) - (n_{t+1} - n_t(1-z_{t+1}))\psi_r\} \frac{Y_t}{Q_t} \\
\vdots
\end{aligned} \tag{OA-1}$$

for a given $m_0 \equiv n_0 h_0 = n_0 S_0$, and subject to

$$m_t = m_{t-1}(1-z_t) + S_{t-1}(n_t - (1-z_t)n_{t-1}), \quad t = 1, 2, \ldots \tag{OA-2}$$

$$n_t \geq n_{t-1}(1-z_t), \quad t = 1, 2, \ldots \tag{OA-3}$$

where

$$\pi_H(n_t,m_t) = m_t\left(1 - \frac{1}{\gamma}\right) + (n_t - m_t)\left(1 - \frac{1}{\Delta\gamma}\right) - \psi_o \frac{1}{2} n_t^2, \tag{OA-4}$$

and

$$\pi_L(n_t,m_t) = m_t\left(1 - \frac{\Delta}{\gamma}\right) + (n_t - m_t)\left(1 - \frac{1}{\gamma}\right) - \psi_o \frac{1}{2} n_t^2. \tag{OA-5}$$

The j index only shows up through this profit function because we start the transition dynamics from an initial steady state where $h_{0,j} = S_0$ for $j = H, L$. As a result, only the profit functions differ between the high and low type firms.

We can iterate (OA-2) backward to express m_t as a function of all past n choices as

$$\begin{aligned} m_t &= (m_0 - S_0 n_0)\prod_{b=1}^{t}(1-z_b) + S_{t-1}n_t + \sum_{a=1}^{t-1}(S_{a-1}-S_a)n_a\prod_{b=1}^{t-a}(1-z_{a+b}) \\ &= S_{t-1}n_t + \sum_{a=1}^{t-1}(S_{a-1}-S_a)n_a\prod_{b=1}^{t-a}(1-z_{a+b}) \quad \forall t = 1,2,\ldots \end{aligned} \tag{OA-6}$$

The second equality follows from $h_{j,0} = S_0$. We denote the function for m_t by $m_t(\{n_s\}_{s=1}^{t})$.

We can then derive the derivative of m_{t+k} with respect to n_t (we suppress the j subscript since the expression is the same for the two types)

$$\frac{\partial m_{t+k}(\{n_s\}_{s=1}^{t+k})}{\partial n_t} = \begin{cases} 0 & \text{if } k < 0 \\ S_{t-1} & \text{if } k = 0 \\ (S_{t-1}-S_t)\prod_{b=1}^{k}(1-z_{t+b}) & \text{if } k > 0. \end{cases} \tag{OA-7}$$

This is the effect of increasing the number of products in period t by one unit on the number of products facing a high type second-best firm in period $t+k$ (while holding the number of product in all other periods constant). Adding a product in t adds $(1-z_{t+1})$ products in $t+1$. x_{t+1} therefore needs to drop by $(1-z_{t+1})$ to keep n_{t+1} constant. All other $x_\tau, \tau > t+2$ are then kept unchanged.

What is the effect on m_{t+k}? Adding a product in t adds $S_{t-1}(1-z_{t+1})$ products with a high-type follower in $t+1$ while lowering x_{t+1} by $(1-z_{t+1})$ in $t+1$ reduces high type follower by $S_t(1-z_{t+1})$. The net effect on m_{t+1} is $(S_{t-1}-S_t)(1-z_{t+1})$. This change decays at the rate of creative destruction such that the $\prod_{b=1}^{k}(1-z_{t+b})$ term shows up. Hence what matters for m_{t+k}, $k > 0$ is the change in the composition of the pool the additional product is drawn from, i.e., the difference between S_t and S_{t-1}. If $S_t = S_{t-1}$ an increase in n_t has no effect on m_{t+k}, $k > 0$. If $S_t > S_{t-1}$, the change shrinks the number of products with high-type followers. Vice versa for $S_t < S_{t-1}$.

Substituting (OA-7) into (OA-4) and (OA-5) and taking derivatives with respect to n yields

$$\frac{\partial \pi_{t+k,H}(n_{t+k}, m_{t+k}(\{n_s\}_{s=1}^{t+k}))}{\partial n_t} = \begin{cases} 0 & \text{if } k < 0 \\ S_{t-1}\frac{1-\Delta}{\Delta\gamma} + 1 - \frac{1}{\Delta\gamma} - \psi_o n_t & \text{if } k = 0 \\ \frac{1-\Delta}{\Delta\gamma}(S_{t-1} - S_t)\prod_{b=1}^{k}(1 - z_{t+b}) & \text{if } k > 0 \end{cases} \tag{OA-8}$$

and

$$\frac{\partial \pi_{t+k,L}(n_{t+k}, m_{t+k}(\{n_s\}_{s=1}^{t+k}))}{\partial n_t} = \begin{cases} 0 & \text{if } k < 0 \\ S_{t-1}\frac{1-\Delta}{\gamma} + 1 - \frac{1}{\gamma} - \psi_o n_t & \text{if } k = 0 \\ \frac{1-\Delta}{\gamma}(S_{t-1} - S_t)\prod_{b=1}^{k}(1 - z_{t+b}) & \text{if } k > 0. \end{cases} \tag{OA-9}$$

It is useful to rewrite the objective function in (OA-1) before taking first-order conditions. First, we use the Euler equation to express the discount factors as

$$\prod_{t=a}^{b} \frac{\gamma^{z_t}}{1 + r_t} = \beta^{b-a+1}\frac{y_{a-1}c_{a-1}}{y_b c_b},$$

where $y_t \equiv Y_t/Q_t$ and c_t denotes consumption share of output C_t/Y_t. This consumption share can be expressed as

$$\begin{aligned} c_t \equiv \frac{C_t}{Y_t} &= 1 - \frac{O_t}{Y_t} - \frac{Z_t}{Y_t} \\ &= 1 - \left(\phi n_{tH}^2 + (1-\phi) n_{tL}^2\right)\frac{\psi_o J}{2} - \psi_r z_{t+1} \\ &= 1 - \left(\frac{S_t^2}{\phi} + \frac{(1-S_t)^2}{1-\phi}\right)\frac{\psi_o}{2J} - \psi_r z_{t+1} = c(S_t, z_{t+1}). \end{aligned} \tag{OA-10}$$

Substituting this expression into the objective function (OA-1), dividing by

y_0 and rearranging allows us to express the problem of a firm of type $j = H, L$ as

$$\max_{\{n_{t,j}\}_{t=1}^{\infty}} \quad \pi_j(n_{0,j}, m_{0,j}) + n_{0,j}(1-z_1)\psi_r \tag{OA-11}$$
$$+ \quad \sum_{t=1}^{\infty} \beta^t \frac{c_0}{c_t} \left\{ \pi_j(n_{tj}, m_{t,j}(\{n_{s,j}\}_{s=1}^{t})) + \psi_r n_t \left[(1-z_{t+1}) - \frac{c_t}{c_{t-1}\beta} \right] \right\}$$

subject to

$$n_{t,j} \geq n_{t-1}(1-z_t), \quad t = 1, 2, \ldots \tag{OA-12}$$

The first-order conditions of the above objective function with respect to $n_{t,j}, t = 1, 2, \ldots$ are

$$\frac{\partial \pi_j(n_t, m_{t,j}(n_{t,j}))}{\partial n_t} + \Lambda_{t,j} - \Lambda_{t+1,j}(1-z_{t+1}) \tag{OA-13}$$
$$= \quad \psi_r \left[\frac{c_t}{c_{t-1}\beta} - (1-z_{t+1}) \right] + f_j \frac{1-\Delta}{\Delta\gamma}(S_t - S_{t-1}) \sum_{a=t+1}^{\infty} \beta^{a-t} \frac{c_t}{c_a} \prod_{b=1}^{a-t} (1-z_{t+b})$$

and

$$\Lambda_{t,j} \geq 0, \quad n_{t,j} \geq n_{t-1,j}(1-z_t), \quad \Lambda_{t,j}(n_{t,j} - n_{t-1,j}(1-z_t)) = 0,$$

where $f_j = \Delta$ if $j = L$ and $f_j = 1$ otherwise. Λ_{tj} denotes the Lagrangean multiplier on the inequality constraint (OA-12). We will solve two such "representative" firm problem, one for the H type and one for the L type.

In the following we define

$$d_t \equiv \sum_{a=t+1}^{\infty} \beta^{a-t} \frac{c_t}{c_a} \prod_{b=1}^{a-t} (1-z_{t+b}). \tag{OA-14}$$

One can show that

$$\begin{aligned} d_{t-1} &= \sum_{a=t}^{\infty} \beta^{a-t+1} \frac{c_{t-1}}{c_a} \prod_{b=1}^{a-t+1} (1 - z_{t-1+b}) \\ &= \beta(1-z_t) \frac{c_{t-1}}{c_t} \sum_{a=t}^{\infty} \beta^{a-t} \frac{c_t}{c_a} \prod_{b=1}^{a-t} (1 - z_{t+b}) \\ &= \beta(1-z_t) \frac{c_{t-1}}{c_t} (1 + d_t). \qquad \text{(OA-15)} \end{aligned}$$

Replacing $n_{t,H} = \frac{S_t}{\phi J}$ and $n_{t,L} = \frac{(1-S_t)}{(1-\phi)J}$ in (OA-8) and (OA-9) allows us to write

$$\frac{\partial \pi_H(n_{t,H}, m_{t,H}(n_{t,H}))}{\partial n_{t,H}} = S_{t-1} \frac{1-\Delta}{\gamma \Delta} + 1 - \frac{1}{\Delta \gamma} - \psi_o \frac{S_t}{\phi J} \equiv \frac{\partial \pi_H}{\partial n_{t,H}}(S_t) \qquad \text{(OA-16)}$$

and

$$\frac{\partial \pi_L(n_{t,L}, m_{t,L}(n_{t,L}))}{\partial n_{t,L}} = S_{t-1} \frac{1-\Delta}{\gamma} + 1 - \frac{1}{\gamma} - \psi_o \frac{(1-S_t)}{(1-\phi)J} \equiv \frac{\partial \pi_L}{\partial n_{t,L}}(S_t). \qquad \text{(OA-17)}$$

Substituting (OA-14) and (OA-15) into (OA-13) yields the following set of equations for each period $t = 1, 2, \ldots$

$$\begin{aligned} &\frac{\partial \pi_H}{\partial n_{t,H}}(S_t) + \Lambda_{t,H} - \Lambda_{t+1,H}(1 - z_{t+1}) \\ &= \psi_r \left[\frac{c_t}{c_{t-1}\beta} - (1 - z_{t+1}) \right] + \frac{1-\Delta}{\Delta\gamma}(S_t - S_{t-1}) d_t, \qquad \text{(OA-18)} \\ &\frac{\partial \pi_L}{\partial n_{t,L}}(S_t) + \Lambda_{t,L} - \Lambda_{t+1,L}(1 - z_{t+1}) \\ &= \psi_r \left[\frac{c_t}{c_{t-1}\beta} - (1 - z_{t+1}) \right] + \frac{1-\Delta}{\gamma}(S_t - S_{t-1}) d_t, \qquad \text{(OA-19)} \\ d_t &= d_{t-1} \frac{1}{\beta(1-z_t)} \frac{c_t}{c_{t-1}} - 1, \qquad \text{(OA-20)} \\ h_{t,H} &= (h_{t-1,H} - S_{t-1}) \frac{S_{t-1}}{S_t} (1 - z_t) + S_{t-1}, \qquad \text{(OA-21)} \\ h_{t,L} &= (h_{t-1,L} - S_{t-1}) \frac{1 - S_{t-1}}{1 - S_t} (1 - z_t) + S_{t-1}, \qquad \text{(OA-22)} \end{aligned}$$

and

$$\Lambda_{t,j} \geq 0,\; n_{t,j} \geq n_{t-1,j}(1-z_t),\; \Lambda_{t,j}(n_{t,j} - n_{t-1,j}(1-z_t)) = 0,\; j = H, L. \tag{OA-23}$$

OA-D.1 Forward iteration algorithm

Given $(d_{t-1}, z_t, S_{t-1}, h_{t-1,H}, h_{t-1,L})$ and the Λ_js, equations (OA-18) to (OA-22) solves for $(d_t, z_{t+1}, S_t, h_{tH}, h_{tL})$. First, we guess that both types of firms have interior solution, i.e., $\Lambda_{tj}, \Lambda_{t+1,j} = 0$ for $j = L, H$. Then, we can multiply (OA-18) by Δ and subtract (OA-19) to eliminate the d_t term on the RHS. This yields

$$\Delta - 1 - \Delta\psi_o \frac{S_t}{\phi J} + \psi_o \frac{1-S_t}{(1-\phi)J} = (\Delta - 1)\psi_r \left[\frac{c_t}{c_{t-1}\beta} - (1 - z_{t+1})\right]. \tag{OA-24}$$

Substituting in $c(S_t, z_{t+1})$ from (OA-10) yields

$$\begin{aligned}
& \frac{c_{t-1}\beta}{(\Delta-1)\psi_r}\left[\Delta - 1 - \Delta\psi_o \frac{S_t}{\phi J} + \psi_o \frac{1-S_t}{(1-\phi)J}\right] \\
= & \left[1 - \left(\frac{S_t^2}{\phi} + \frac{(1-S_t)^2}{1-\phi}\right)\frac{\psi_o}{2J} - c_{t-1}\beta + (c_{t-1}\beta - \psi_r) z_{t+1}\right] \\
z_{t+1} = & \frac{\frac{c_{t-1}\beta}{(\Delta-1)\psi_r}\left[\Delta - 1 - \Delta\psi_o \frac{S_t}{\phi J} + \psi_o \frac{1-S_t}{(1-\phi)J}\right] - \left[1 - \left(\frac{S_t^2}{\phi} + \frac{(1-S_t)^2}{1-\phi}\right)\frac{\psi_o}{2J} - c_{t-1}\beta\right]}{c_{t-1}\beta - \psi_r} \\
\equiv & z(S_t, S_{t-1}, z_t)
\end{aligned}$$

We can substitute this into (OA-18) to get an equation with S_t as the only unknown

$$\begin{aligned}
\frac{\partial \pi_H}{\partial n_{t,H}}(S_t) = & \frac{\Delta \frac{\partial \pi_H}{\partial n_{t,H}}(S_t) - \frac{\partial \pi_L}{\partial n_{t,L}}(S_t)}{\Delta - 1} \\
& + \frac{1-\Delta}{\Delta\gamma}(S_t - S_{t-1})\left(\frac{d_{t-1}}{\beta(1-z_t)}\frac{c(S_t, z(S_t, S_{t-1}, z_t))}{c_{t-1}} - 1\right).
\end{aligned}$$

This is a quadratic function in S_t which built-in solvers easily minimize. We choose the solution that is between the old and new steady state S.

We then check if the interiority assumption is satisfied. We look for solutions where z_t is always positive. This means at least one firm innovates. After a drop in ψ_o the H types firms have higher returns to innovate. Hence positive z_t means the high type has positive R&D. We only need to worry about the case when the low type does not innovate. We first guess that the low type does not have zero innovation in consecutive periods. Then $\Lambda_{t+1,L} = 0$ if $\Lambda_{tL} > 0, n_{tL} = (1-z_t)n_{t-1,L}$. This implies $S_t = 1-(1-z_t)(1-S_{t-1})$. Substituting this into (OA-18) yields

$$S_{t-1}\frac{1-\Delta}{\gamma\Delta}+1-\frac{1}{\Delta\gamma}-\psi_o\frac{1-(1-z_t)(1-S_{t-1})}{\phi J} = \psi_r\left[\frac{c_t}{c_{t-1}\beta}-(1-z_{t+1})\right]+\frac{1-\Delta}{\Delta\gamma}z_t(1-S_{t-1})d_t.$$

This equation solves for z_{t+1}. We can then solve for $t+1$ and confirm that the interiority condition for $n_{t+1,L}$ is satisfied.

We initiate the algorithm with a guess for (z_1, d_0) and set $S_0 = h_{0,H} = h_{0,L} = S^\star_{old}$. The algorithm has an outer loop and an inner loop. The inner loop holds d_0 fixed and iterates on z_1. It iterates on (OA-18) to (OA-22) until S_t is close to the new steady state $S^\star_{new}$. Then it uses bisection to update the guess of z_1. Namely, it increases z_1 if the last value of z is lower than the new steady state $z^\star_{new}$ and reduce z_1 otherwise.

The inner loop yields a path of (z_{t+1}, S_t) that converges to $(z^\star_{new}, S^\star_{new})$ holding fixed d_0. The path implies a path for d_t that may not converge to the steady state value of $d^\star_{new} \equiv \frac{\beta(1-z^\star_{new})}{1-\beta(1-z^\star_{new})}$. The outer loop uses bisection to update d_0 until d_t also converges. Namely, it reduces d_0 if the inner loop overshoots and increases d_0 otherwise.

We stop the algorithm when (d_t, z_{t+1}, S_t) all approximately converge to the new steady state. Suppose this happens after T periods. Then we set (d_t, z_{t+1}, S_t) for $t > T$ to their new steady state values and iterate forward until $(h_{t,H}, h_{t,L})$ converges to the new steady state. We do not keep on iterating on (d_t, z_{t+1}, S_t) until $(h_{t,H}, h_{t,L})$ converges because (OA-20) is not stable outside of its fixed point. Because machine precision does not allow the algorithm to reach the exact fixed point, d_t eventually explodes as we iterate forward.

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