Appendix to Global Diversification for Long-Horizon Investors

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Appendix A. Calculating News Components

A.1 Excess Bond Returns Decomposition (3 News Components)

Define the log one-period nominal return on a nominal n-period coupon bond as

$$
\begin{aligned}
r^{\$}_{n,t+1} &= \log\left(1 + R^{\$}_{n,t+1}\right) = \log\left(P_{n-1,t+1} + C\right) - \log\left(P_{n,t}\right) \\
&= p_{n-1,t+1} - p_{n,t} + \log\left(1 + \exp\left(c - p_{n-1,t+1}\right)\right) \\
&\approx k + \rho_b p_{n-1,t+1} + \left(1 - \rho_b\right) c - p_{n,t},
\end{aligned}
\tag{1}
$$

where $\rho_b = \frac{1}{1+\exp\left(\overline{c-p}\right)}$ and $k = -\log\left(\rho_b\right) - \left(1-\rho_b\right)\log\left(\frac{1}{\rho_b} - 1\right)$. Solving forward and imposing the terminal condition that $p_{n-j,t+j}|_{j=n} = 0$, we get that

$$
p_{n,t} = \left(k + \left(1-\rho_b\right) c\right)\left(\sum_{j=0}^{n-1} \rho_b^j\right) - \sum_{j=0}^{n-1} r^{\$}_{n-j,t+1+j}\rho_b^j.
$$

Plugging this expression in to the unexpected bond return from Eq. (1), we get that

$$
\begin{aligned}
\left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[r^{\$}_{n,t+1}\right] &= \left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[\rho_b p_{n-1,t+1}\right] - \left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[p_{n,t}\right] \\
&= \left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[\rho_b p_{n-1,t+1}\right] \\
&= -\left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[\sum_{j=1}^{n-1} r^{\$}_{n-j,t+1+j}\rho_b^j\right].
\end{aligned}
\tag{2}
$$

We can write $r^{\$}_{n,t+1} = xr_{n,t+1} + r^{\$}_{f,t+1}$, where $xr_{n,t+1}$ is the excess log 1-period return on a nominal n-period coupon bond and $r^{\$}_{f,t+1}$ is the realized nominal return of the 1-period nominal bond, which is the same as the yield of the 1-period nominal bond $y^N_{1,t}$.

Decomposing the surprise bond return in Eq. (2) gives

$$
\left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[xr_{n,t+1} + r^{\$}_{f,t+1}\right] = -\left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[\sum_{j=1}^{n-1} \rho_b^j xr_{n-j,t+1+j}\right] - \left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[\sum_{j=1}^{n-1} \rho_b^j r^{\$}_{f,t+1+j}\right].
$$

The LHS can be simplified by noting that

$$
\left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[r^{\$}_{f,t+1}\right] = \left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[y^N_{1,t}\right] = 0.
$$

To simplify the RHS, we simply note that the realized nominal return of the 1-period nominal bond is the realized real return of the 1-period nominal bond plus realized inflation: $r^{\$}_{f,t+1} = r_{f,t+1} + \pi_{t+1}$. The second term on the RHS is then

$$
\left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[\sum_{j=1}^{n-1} \rho_b^j r^{\$}_{f,t+1+j}\right] = \left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[\sum_{j=1}^{n-1} \rho_b^j r_{f,t+1+j}\right] + \left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[\sum_{j=1}^{n-1} \rho_b^j \pi_{t+1+j}\right].
\tag{3}
$$

Putting together the simplified LHS and RHS, we have the following 3 news component decomposition for unexpected excess bond returns:

$$
\left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[xr_{n,t+1}\right] = N_{CF,n,t+1} - N_{RR,n,t+1} - N_{RP,n,t+1}
$$

where

$$
\begin{aligned}
N_{CF,n,t+1} = -N_{INFL,n,t+1} &\equiv -\left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[\sum_{j=1}^{n-1} \rho_b^j \pi_{t+1+j}\right], \\
N_{RR,n,t+1} &\equiv \left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[\sum_{j=1}^{n-1} \rho_b^j r_{f,t+1+j}\right], \text{ and} \\
N_{RP,n,t+1} &\equiv \left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[\sum_{j=1}^{n-1} \rho_b^j xr_{n-j,t+1+j}\right].
\end{aligned}
\tag{4}
$$

To extract the news components from the VAR, consider the vector of state variables

$$\tilde{\mathbf{z}}_{t+1} = \left[xr_{s,t+1}, xr_{n,t+1}, d_{t+1} - p_{t+1}, \pi_{t+1}, y_{1,t+1}, y^N_{10,t+1} - y^N_{1,t+1}\right]. \tag{5}$$

The main VAR equation is $\tilde{\mathbf{z}}_{t+1} = a + \mathbf{A}\tilde{\mathbf{z}}_t + \mathbf{u}_{t+1}$, which leads to $\mathbb{E}_t\left[\tilde{\mathbf{z}}_{t+j}\right] = \mathbf{A}^j\tilde{\mathbf{z}}_t$ and $\left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[\tilde{\mathbf{z}}_{t+j}\right] = \mathbf{A}^{j-1}\mathbf{u}_{t+1}$. It is then straightforward to see how the decomposition can be written in VAR notation:

$$\begin{aligned}
\left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[xr_{n,t+1}\right] =& \mathbf{e2}'\mathbf{u}_{t+1}, \\
N_{CF,n,t+1} =& -\mathbf{e4}'\left(\sum_{j=1}^{n-1}\rho_b^j\mathbf{A}^j\right)\mathbf{u}_{t+1}, \\
N_{RR,n,t+1} =& \mathbf{e5}'\left(\sum_{j=1}^{n-1}\rho_b^j\mathbf{A}^{j-1}\right)\mathbf{u}_{t+1} - \mathbf{e4}'\left(\sum_{j=1}^{n-1}\rho_b^j\mathbf{A}^j\right)\mathbf{u}_{t+1}, \text{ and} \\
N_{RP,n,t+1} =& N_{CF,n,t+1} - N_{RR,n,t+1} - \left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[xr_{n,t+1}\right].
\end{aligned}$$

We get $N_{RR,n,t+1}$ by using Eq. (3) to express real rate news in terms of nominal rate news and inflation news. Finally, we back out $N_{RP,n,t+1}$ as the residual.

A.2 Excess Stock Returns Decomposition (3 News Components)

We start with Campbell-Shiller decomposition which decompose the news on real stock return into news on growth of log real dividend and news on log real interest rate

$$\left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[r_{s,t+1}\right] = N_{CF,s,t+1} - N_{DR,s,t+1},$$

where

$$\begin{aligned}
N_{CF,s,t+1} \equiv& \left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[\sum_{j=0}^{\infty}\rho_s^j\Delta d_{t+1+j}\right] \text{ and} \\
N_{DR,s,t+1} \equiv& \left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[\sum_{j=1}^{\infty}\rho_s^j r_{s,t+1+j}\right].
\end{aligned} \tag{6}$$

We can relate the 2 news component decomposition to the 3 news component decomposition as follows. Note that the excess return could be written as $xr_{s,t+1+j} = r_{s,t+1+j} - r_{f,t+1+j}$, we have

$$\begin{aligned}
N_{DR,s,t+1} =& \left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[\sum_{j=1}^{\infty}\rho_s^j r_{s,t+1+j}\right] \\
=& \left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[\sum_{j=1}^{\infty}\rho_s^j xr_{s,t+1+j}\right] + \left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[\sum_{j=0}^{\infty}\rho_s^j r_{f,t+1+j}\right] - \left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[r_{f,t+1+j}\right].
\end{aligned}$$

Combining this with the decomposition we have

$$\begin{aligned}
\left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[xr_{s,t+1}\right] + \left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[r_{f,t+1}\right] =& N_{CF,s,t+1} - N_{DR,s,t+1} \\
=& N_{CF,s,t+1} - \left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[\sum_{j=1}^{\infty}\rho_s^j xr_{s,t+1+j}\right] - \left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[\sum_{j=0}^{\infty}\rho_s^j r_{f,t+1+j}\right] + \left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)[r
\end{aligned}$$

Thus we have

$$\left(\mathbb{E}_{t+1} - \mathbb{E}_t\right)\left[xr_{s,t+1}\right] = N_{CF,s,t+1} - N_{RR,s,t+1} - N_{RP,s,t+1}$$

where

$$
\begin{aligned}
N_{CF,s,t+1} &\equiv (\mathbb{E}_{t+1} - \mathbb{E}_t) \left[\sum_{j=0}^{\infty} \rho_s^j \Delta d_{t+1+j} \right], \\
N_{RR,s,t+1} &\equiv (\mathbb{E}_{t+1} - \mathbb{E}_t) \left[\sum_{j=0}^{\infty} \rho_s^j r_{f,t+1+j} \right], \text{ and} \\
N_{RP,s,t+1} &\equiv (\mathbb{E}_{t+1} - \mathbb{E}_t) \left[\sum_{j=1}^{\infty} \rho_s^j xr_{s,t+1+j} \right].
\end{aligned} \tag{7}
$$

With the same vector of state variables $\mathbf{z}_{t+1}$ as in Eq. (5), we write the decomposition in VAR notation:

$$
\begin{aligned}
(\mathbb{E}_{t+1} - \mathbb{E}_t) \left[xr_{s,t+1} \right] =& \mathbf{e1}' \mathbf{u}_{t+1}, \\
N_{CF,s,t+1} =& (\mathbb{E}_{t+1} - \mathbb{E}_t) \left[xr_{s,t+1} \right] + N_{RR,s,t+1} + N_{RP,s,t+1}, \\
N_{RR,s,t+1} =& \mathbf{e5}' \left(\sum_{j=1}^{\infty} \rho_s^j \mathbf{A}^{j-1} \right) \mathbf{u}_{t+1} - \mathbf{e4}' \left(\sum_{j=0}^{\infty} \rho_s^j \mathbf{A}^j \right) \mathbf{u}_{t+1}, \text{ and} \\
N_{RP,s,t+1} =& \mathbf{e1}' \left(\sum_{j=1}^{\infty} \rho_s^j \mathbf{A}^j \right) \mathbf{u}_{t+1}.
\end{aligned}
$$

Similar to the case with bonds, we get $N_{RR,n,t+1}$ by using an infinite-sum version of Eq. (3) to express real rate news in terms of nominal rate news and inflation news. Note that the first term in $N_{RR,s,t+1}$ starts from $j = 1$ instead of $j = 0$ because $(\mathbb{E}_{t+1} - \mathbb{E}_t) \left[y_{1,t}^N \right] = 0$. Finally, we back out $N_{CF,s,t+1}$ as the residual.

Appendix B. Derivation of Results in Section 3.2

We want to derive the general formula for k period portfolio return variance, where the portfolio is constructed by holding equal weight on N identical markets. The starting point is from our stylized symmetrical model of asset returns of Section 3

$$\begin{cases} r_{i,t+1} = \mu_1 + \beta s_{i,t} + u_{i,t+1} \\ s_{i,t+1} = \mu_2 + \phi s_{i,t} + u_{si,t+1} \end{cases} \tag{8}$$

and we could also write the VAR residual in terms of news terms $u_{i,t+1} = N_{CF,i,t+1} - N_{DR,i,t+1}$ and $u_{si,t+1} = \frac{1}{\lambda} N_{DR,i,t+1}$, where $\lambda = \frac{\rho\beta}{1-\rho\phi}$. The log portfolio return over k period horizon (from t to $t+k$) is [1]

$$r_{p,t+k}^{(k)} = r_0^{(k)} + \alpha_t'(r_{t+k}^{(k)} - r_0^{(k)}\boldsymbol{l}) + \frac{1}{2}\alpha_t(k)^2\sigma_t(k)^2 - \frac{1}{2}\alpha_t(k)\Sigma_t(k)\alpha_t(k) \tag{9}$$

and the variance of k period portfolio return is

$$V_t[r_{p,t+k}^{(k)}] = \frac{1}{N} V_t[r_{i,t+k}^{(k)}] + (1 - \frac{1}{N}) C_t[r_{i,t+k}^{(k)}, r_{j,t+k}^{(k)}] \tag{10}$$

where $r_{i,t+k}^{(k)} = \sum_{l=1}^{k} r_{i,t+l}$ is the k period log return of market i.

The term of interest in the expression is the cross-country covariance. Let's now derive the general expression for the covariance term. Note that the 1 period return at $t+l$ could be written as

$$r_{i,t+l} = \mu_1 + \beta s_{i,t+l-1} + u_{i,t+l}$$

$$= \mu_1 + \beta(\phi s_{i,t+l-2} + u_{si,t+l-1}) + u_{i,t+l}$$

$$\cdots$$

$$= \mu_1 + \beta\phi^{l-1} s_{i,t} + \beta \sum_{m=1}^{l-1} \phi^{m-1} u_{si,t+l-m} + u_{i,t+l} \tag{11}$$

and

$$C_t[r_{i,t+l}, r_{j,t+l}] = C_t[\beta \sum_{m=1}^{l-1} \phi^{m-1} u_{si,t+l-m} + u_{i,t+l}, \beta \sum_{m=1}^{l-1} \phi^{m-1} u_{sj,t+l-m} + u_{j,t+l}]$$

$$= C_t[\frac{\beta}{\lambda} \sum_{m=1}^{l-1} \phi^{m-1} N_{DR,i,t+l-m} + N_{CF,i,t+l} - N_{DR,i,t+l}, \frac{\beta}{\lambda} \sum_{m=1}^{l-1} \phi^{m-1} N_{DR,j,t+l-m} + N_{CF,j,t+l} - N_{DR,j,t+l}] \tag{12}$$

We make the assumption that $(for\ \forall l \geqslant 1, i \neq j)$

$$C_t[N_{CF,i,t+l}, N_{CF,j,t+l}] \equiv \sigma_{CF,CF}^{xc}$$

$$C_t[N_{CF,i,t+l}, N_{DR,j,t+l}] \equiv \sigma_{CF,DR}^{xc}$$

$$C_t[N_{DR,i,t+l}, N_{DR,j,t+l}] \equiv \sigma_{DR,DR}^{xc}$$

Thus we have

$$C_t[r_{i,t+l}, r_{j,t+l}] = [\frac{\beta^2}{\lambda^2} \frac{(1-(\phi^2)^{l-1})}{1-\phi^2} + 1]\sigma_{DR,DR}^{xc} + \sigma_{CF,CF}^{xc} - 2\sigma_{CF,DR}^{xc} \tag{13}$$

For the cross-period & cross-country covariance, we have

$$C_t[r_{i,t+l}, r_{j,t+l+p}] = C_t[\beta \sum_{m=1}^{l-1} \phi^{m-1} u_{si,t+l-m} + u_{i,t+l}, \beta \sum_{m=1}^{l+p-1} \phi^{m-1} u_{sj,t+l+p-m} + u_{j,t+l+p}]$$

[1] The formula for portfolio return below is derived in the appendix of Campbell and Viceira (2002) "Strategic Asset Allocation: Portfolio Choice for Long-Term Investors"

$$= C_t[u_{i,t+l} + \beta u_{si,t+l-1} + \beta\phi u_{si,t+l-2} + \cdots + \beta\phi^{l-2} u_{si,t+1}, \beta\phi^{p-1} u_{sj,t+l} + \beta\phi^{p} u_{sj,t+l-1} + \beta\phi^{p+1} u_{sj,t+l-2} + \cdots + \beta\phi^{l+p-2} u_{sj,t+1}]$$

$$= \beta\phi^{p-1} C_t[u_{i,t+l}, u_{sj,t+l}] + \beta^2\phi^{p} C_t[u_{si,t+l-1}, u_{sj,t+l-1}] + \beta^2\phi^{p+2} C_t[u_{si,t+l-2}, u_{sj,t+l-2}] + \cdots + \beta^2\phi^{p+2(l-2)} C_t[u_{si,t+1}, u_{sj,t+1}]$$

$$= \frac{\beta\phi^{p-1}}{\lambda}(\sigma^{xc}_{CF,DR} - \sigma^{xc}_{DR,DR}) + \frac{\beta^2\phi^{p}}{\lambda^2}\frac{1-(\phi^2)^{l-1}}{1-\phi^2}\sigma^{xc}_{DR,DR} \tag{14}$$

with $p \geqslant 1$. Using the results above, we could get the k period cross-country return covariance

$$C_t[r^{(k)}_{i,t+k}, r^{(k)}_{j,t+k}] = \sum_{l=1}^{k} C_t[r_{i,t+l}, r_{j,t+l}] + 2\sum_{l=1}^{k-1}\sum_{p=1}^{k-l} C_t[r_{i,t+l}, r_{j,t+l+p}]$$

$$= \sum_{l=1}^{k}\left([\frac{\beta^2}{\lambda^2}\frac{(1-(\phi^2)^{l-1})}{1-\phi^2} + 1]\sigma^{xc}_{DR,DR} + \sigma^{xc}_{CF,CF} - 2\sigma^{xc}_{CF,DR}\right) + 2\sum_{l=1}^{k-1}\sum_{p=1}^{k-l}\left(\frac{\beta\phi^{p-1}}{\lambda}(\sigma^{xc}_{CF,DR} - \sigma^{xc}_{DR,DR}) + \frac{\beta^2\phi^{p}}{\lambda^2}\frac{1-(\phi^2)^{l-1}}{1-\phi^2}\sigma^{xc}_{DR,DR}\right)$$

$$= \left([\frac{\beta^2}{\lambda^2}\frac{(k - \frac{1-(\phi^2)^k}{1-\phi^2})}{1-\phi^2} + k]\sigma^{xc}_{DR,DR} + k\sigma^{xc}_{CF,CF} - 2k\sigma^{xc}_{CF,DR}\right)$$

$$+2\sum_{l=1}^{k-1}\left(\frac{\beta}{\lambda(1-\phi)}(1-\phi^{k-l})(\sigma^{xc}_{CF,DR} - \sigma^{xc}_{DR,DR}) + \frac{\beta^2}{\lambda^2}\frac{1-(\phi^2)^{l-1}}{1-\phi^2}\frac{\phi(1-\phi^{k-l})}{1-\phi}\sigma^{xc}_{DR,DR}\right)$$

$$= \left([\frac{\beta^2}{\lambda^2}\frac{(k - \frac{1-(\phi^2)^k}{1-\phi^2})}{1-\phi^2} + k]\sigma^{xc}_{DR,DR} + k\sigma^{xc}_{CF,CF} - 2k\sigma^{xc}_{CF,DR}\right)$$

$$+2\left(\frac{\beta}{\lambda(1-\phi)}(k-1-\phi\frac{1-\phi^{k-1}}{1-\phi})(\sigma^{xc}_{CF,DR} - \sigma^{xc}_{DR,DR}) + \frac{\beta^2\phi}{\lambda^2(1-\phi^2)(1-\phi)}(k-1+\frac{(\phi^{k-1}-1)(\phi-\phi^{k-1})}{1-\phi} - \frac{1-(\phi^2)^{k-1}}{1-\phi^2})\sigma^{xc}_{DR,DR}\right)$$

$$= k\sigma^{xc}_{CF,CF} + 2k\left(\frac{\beta}{\lambda(1-\phi)}(\frac{k-1}{k} - \frac{\phi}{k}\frac{1-\phi^{k-1}}{1-\phi}) - 1\right)\sigma^{xc}_{CF,DR}$$

$$+\left(\frac{\beta^2}{\lambda^2}\frac{(k - \frac{1-(\phi^2)^k}{1-\phi^2})}{1-\phi^2} + 2\frac{\beta^2\phi}{\lambda^2(1-\phi^2)(1-\phi)}(k-1+\frac{(\phi^{k-1}-1)(\phi-\phi^{k-1})}{1-\phi} - \frac{1-(\phi^2)^{k-1}}{1-\phi^2}) - 2\frac{\beta}{\lambda(1-\phi)}(k-1-\phi\frac{1-\phi^{k-1}}{1-\phi}) + k\right)\sigma^{xc}_{DR,} \tag{15}$$

We further simplify the coefficient on $\sigma^{xc}_{DR,DR}$ as

$$\frac{\beta^2}{\lambda^2}\frac{(k - \frac{1-(\phi^2)^k}{1-\phi^2})}{1-\phi^2} + 2\frac{\beta^2\phi}{\lambda^2(1-\phi^2)(1-\phi)}(k-1+\frac{(\phi^{k-1}-1)(\phi-\phi^{k-1})}{1-\phi} - \frac{1-(\phi^2)^{k-1}}{1-\phi^2}) - 2\frac{\beta}{\lambda(1-\phi)}(k-1-\phi\frac{1-\phi^{k-1}}{1-\phi}) + k$$

$$= k\left(\frac{\beta^2}{\lambda^2}\frac{(1 - \frac{1-(\phi^2)^k}{k(1-\phi^2)})}{1-\phi^2} + 2\frac{\beta^2\phi}{\lambda^2(1-\phi^2)(1-\phi)}(\frac{k-1}{k}+\frac{(\phi^{k-1}-1)(\phi-\phi^{k-1})}{k(1-\phi)} - \frac{1-(\phi^2)^{k-1}}{k(1-\phi^2)}) - 2\frac{\beta}{\lambda(1-\phi)}(\frac{k-1}{k}-\phi\frac{1-\phi^{k-1}}{k(1-\phi)}) + 1\right)$$

$$= k\left\{\frac{\beta^2}{\lambda^2(1-\phi)(1+\phi)}\left(1 - \frac{1-(\phi^2)^k}{k(1-\phi)(1+\phi)} + 2\frac{\phi}{(1-\phi)}(\frac{k-1}{k} + \frac{(\phi^{k-1}-1)(\phi-\phi^{k-1})}{k(1-\phi)} - \frac{1-(\phi^2)^{k-1}}{k(1-\phi)(1+\phi)})\right) - 2\frac{\beta}{\lambda(1-\phi)}(\frac{k-1}{k}-\phi\frac{1-\phi^{k-1}}{k(1-\phi)}) + 1\right\}$$

$$= k\left\{\left(\frac{\beta}{\lambda(1-\phi)}\right)^2\left(\frac{1-\phi}{1+\phi} - \frac{1-(\phi^2)^k}{k(1+\phi)(1+\phi)} + 2\frac{\phi}{(1+\phi)}\left(\frac{k-1}{k} + \frac{(\phi^{k-1}-1)(\phi-\phi^{k-1})}{k(1-\phi)} - \frac{1-(\phi^2)^{k-1}}{k(1-\phi)(1+\phi)}\right) - \left(\frac{k-1}{k} - \phi\frac{1-\phi^{k-1}}{k(1-\phi)}\right)^2\right) + \left(\frac{\beta}{\lambda(1-\phi)}\right)^2\right.$$

$$= k\left\{\left(\frac{\beta}{\lambda(1-\phi)}\right)^2\left(\frac{1-\phi}{1+\phi} - \frac{1-(\phi^2)^k}{k(1+\phi)(1+\phi)} + 2\frac{\phi}{(1+\phi)}\left(\frac{k-1}{k} + \frac{(\phi^{k-1}-1)(\phi-\phi^{k-1})}{k(1-\phi)} - \frac{1-(\phi^2)^{k-1}}{k(1-\phi)(1+\phi)}\right) - \left(\frac{k-1}{k} - \phi\frac{1-\phi^{k-1}}{k(1-\phi)}\right)^2\right) + \left(\left(\frac{\beta}{\lambda(1-\phi)}\right)\right.\right.$$

If we define $a(k;\beta,\phi,\lambda) \equiv 1 - \left(\frac{\beta}{\lambda(1-\phi)}\right)\left(\frac{k-1}{k} - \phi\frac{1-\phi^{k-1}}{k(1-\phi)}\right)$ then equation (11) could be written as

$$\frac{1}{k}C_t[r^{(k)}_{i,t+k}, r^{(k)}_{j,t+k}] = \sigma^{xc}_{CF,CF} + \left[a(k;\beta,\phi,\lambda)^2 + b(k;\beta,\phi,\lambda)\right]\sigma^{xc}_{DR,DR} - 2a(k;\beta,\phi,\lambda)\sigma^{xc}_{CF,DR} \tag{16}$$

where

$$b(k;\beta,\phi,\lambda) \equiv \left(\frac{\beta}{\lambda(1-\phi)}\right)^2\left(\frac{1-\phi}{1+\phi} - \frac{1-(\phi^2)^k}{k(1+\phi)(1+\phi)} + 2\frac{\phi}{(1+\phi)}\left(\frac{k-1}{k} + \frac{(\phi^{k-1}-1)(\phi-\phi^{k-1})}{k(1-\phi)} - \frac{1-(\phi^2)^{k-1}}{k(1-\phi)(1+\phi)}\right) - \left(\frac{k-1}{k} - \phi\frac{1-\phi^{k-1}}{k(1-\phi)}\right)^2\right) \tag{17}$$

we could show that $lim_{k\to+\infty} b(k;\beta,\phi,\lambda)=0$.

Finally we have the asymptotic result

$$lim_{k\to+\infty}\ \frac{C_t[r_{i,t+k}^{(k)},r_{j,t+k}^{(k)}]}{k}=\sigma_{CF,CF}^{xc}+2(\frac{\beta}{\lambda(1-\phi)}-1)\sigma_{CF,DR}^{xc}+(\frac{\beta^2}{\lambda^2(1-\phi^2)}+\frac{2\beta^2\phi}{\lambda^2(1-\phi^2)(1-\phi)}-\frac{2\beta}{\lambda(1-\phi)}+1)\sigma_{DR,DR}^{xc} \quad (18)$$

Now we derive the range of the coefficients for variance-covariance terms in Eq (12), note that $\lambda=\frac{\rho\beta}{1-\rho\phi}$

$$\frac{\beta}{\lambda(1-\phi)}-1=\frac{1-\rho\phi}{\rho}\frac{1}{(1-\phi)}-1>\frac{1}{\rho}-1>0$$

and

$$\begin{aligned}&\frac{\beta^2}{\lambda^2(1-\phi^2)}+\frac{2\beta^2\phi}{\lambda^2(1-\phi^2)(1-\phi)}-\frac{2\beta}{\lambda(1-\phi)}+1\\&=\left(\frac{\beta}{\lambda(1-\phi)}\right)^2-\frac{2\beta}{\lambda(1-\phi)}+1\\&=\left(\frac{\beta}{\lambda(1-\phi)}-1\right)^2\\&=\left(\frac{1-\rho\phi}{\rho-\rho\phi}-1\right)^2\end{aligned}$$

we know that ρ and ϕ are close to but smaller than 1, and if we assume that $\rho>\frac{1}{2-\phi}$, we have $\left(\frac{1-\rho\phi}{\rho-\rho\phi}-1\right)^2<1$. Thus we could have

$$0<\frac{\beta^2}{\lambda^2(1-\phi^2)}+\frac{2\beta^2\phi}{\lambda^2(1-\phi^2)(1-\phi)}-\frac{2\beta}{\lambda(1-\phi)}+1<1$$

under the assumption.

Numerical Calibration:

We try to use the formula to explain the positive gap between the portfolio variance of the benchmark case and the case in which integration is purely driven by increased DR news correlation. In our benchmark case, we set $\sigma_{CF,CF}^{xc}=\sigma_{CF,DR}^{xc}=\sigma_{DR,DR}^{xc}=0$, therefore

$$lim_{k\to+\infty}\ \sqrt{V_t[r_{p,t+k}^{(k)}]/k}=lim_{k\to+\infty}\ \sqrt{\frac{1}{N}V_t[r_{i,t+k}^{(k)}]/k} \quad (19)$$

. And for the integrated case purely driven by increased DR news correlation, we have

$$lim_{k\to+\infty}\ \sqrt{V_t[r_{p,t+k}^{(k)}]/k}=lim_{k\to+\infty}\ \sqrt{\frac{1}{N}V_t[r_{i,t+k}^{(k)}]/k+(1-\frac{1}{N})(\frac{\beta^2}{\lambda^2(1-\phi^2)}+\frac{2\beta^2\phi}{\lambda^2(1-\phi^2)(1-\phi)}-\frac{2\beta}{\lambda(1-\phi)}+1)\sigma_{DR,DR}^{xc}} \quad (20)$$

and we have

$$\frac{\beta^2}{\lambda^2(1-\phi^2)}+\frac{2\beta^2\phi}{\lambda^2(1-\phi^2)(1-\phi)}-\frac{2\beta}{\lambda(1-\phi)}+1=0.0175 \quad (21)$$

therefore explains the positive gap between the two variance plot in our 2 country symmetrical experiment.

The coefficient of the term $\sigma_{DR,DR}^{xc}$ in Eq (11) standardized by k

$$\frac{1}{k}\left(\frac{\beta^2}{\lambda^2}\frac{(k-\frac{1-(\phi^2)^k}{1-\phi^2})}{1-\phi^2}+2\frac{\beta^2\phi}{\lambda^2(1-\phi^2)(1-\phi)}(k-1+\frac{(\phi^{k-1}-1)(\phi-\phi^{k-1})}{1-\phi}-\frac{1-(\phi^2)^{k-1}}{1-\phi^2})-2\frac{\beta}{\lambda(1-\phi)}(k-1-\phi\frac{1-\phi^{k-1}}{1-\phi})+k\right) \quad (22)$$

is a function of investment horizon k, and the coefficient annualized by k should converge to the value in Eq (15). The coefficient as a function of k is plotted in Figure 3.

In the next step, we calibrate the variance under the two cases (integration purely driven by increased cross country CF-CF/ DR-DR correlation). Under the limit case where $k \to +\infty$ we have

$$(\frac{\beta^2}{\lambda^2(1-\phi^2)} + \frac{2\beta^2\phi}{\lambda^2(1-\phi)^2} - \frac{2\beta}{\lambda(1-\phi)} + 1)\sigma_{DR,DR}^{xc} = 0.000010$$

where $\sigma_{DR,DR}^{xc} = \rho_{DR,DR}^{xc}\sigma_{DR}\sigma_{DR}$ and cross country DR correlation $\rho_{DR,DR}^{xc} = 0.25$. Similarly we get

$$\sigma_{CF,CF}^{xc} = \rho_{CF,CF}^{xc}\sigma_{CF}\sigma_{CF} = 0.0012$$

where $\rho_{CF,CF}^{xc} = 0.335$. In the calibration, we see that when integration purely driven by increased cross country CF-CF correlation, the impact on portfolio variance is permanent. When the integration is purely driven by increased cross country DR-DR correlation, the impact on portfolio variance is temporary, and dies out at long horizons. This matches with our intuition perfectly, and we see from the calibration that $(\frac{\beta^2}{\lambda^2(1-\phi^2)} + \frac{2\beta^2\phi}{\lambda^2(1-\phi)^2} - \frac{2\beta}{\lambda(1-\phi)} + 1)\sigma_{DR,DR}^{xc} \ll \sigma_{CF,CF}^{xc}$.

Lemma: Assuming
(1) $0.5 < \rho < 1$ and $0.5 < \phi < 1$ (trivially satisfied for time preference factor ρ and persistence of state variable ϕ).
(2) $\rho > \frac{2\phi^2+3\phi+1}{\phi^2+3\phi+2}$

We can conclude that the coefficient $\frac{1}{k}\left[a(k;\beta,\phi,\lambda)^2 + b(k;\beta,\phi,\lambda)\right]$ is positive and decreasing in k (these are sufficient but not necessary conditions). The impact of covariance term $\sigma_{DR,DR}^{xc}$ on per-period portfolio variance decreases as investment horizon k increases.

Proof: $f(k) \equiv \frac{1}{k}\left[a(k;\beta,\phi,\lambda)^2 + b(k;\beta,\phi,\lambda)\right]$

$$= \frac{1}{k}\left(\frac{\beta^2}{\lambda^2}\frac{(k-\frac{1-(\phi^2)^k}{1-\phi^2})}{1-\phi^2} + 2\frac{\beta^2\phi}{\lambda^2(1-\phi^2)(1-\phi)}(k-1+\frac{(\phi^{k-1}-1)(\phi-\phi^{k-1})}{1-\phi} - \frac{1-(\phi^2)^{k-1}}{1-\phi^2}) - 2\frac{\beta}{\lambda(1-\phi)}(k-1-\phi\frac{1-\phi^{k-1}}{1-\phi}) + k\right)$$

$$= \left(\frac{\beta^2}{\lambda^2}\frac{1}{1-\phi^2}(1-\frac{1-(\phi^2)^k}{k(1-\phi^2)}) + 2\frac{\beta^2\phi}{\lambda^2(1-\phi^2)(1-\phi)}(1-\frac{1}{k}+\frac{(\phi^{k-1}-1)(\phi-\phi^{k-1})}{k(1-\phi)} - \frac{1-(\phi^2)^{k-1}}{k(1-\phi^2)}) - 2\frac{\beta}{\lambda(1-\phi)}(1-\frac{1}{k}-\frac{\phi}{k}\frac{1-\phi^{k-1}}{1-\phi}) + 1\right)$$

$$= Const + \frac{1}{k}\left(-\frac{\beta^2}{\lambda^2}\frac{(1-\phi^k)(1+\phi^k)}{(1-\phi^2)^2} + 2\frac{\beta^2\phi}{\lambda^2(1-\phi^2)(1-\phi)}\frac{-1+\phi^2+(\phi^{k-1}-1)(\phi-\phi^{k-1})(1+\phi)-1+\phi^{2(k-1)}}{(1-\phi^2)} + 2\frac{\beta}{\lambda(1-\phi)}\frac{1-\phi^k}{1-\phi}\right)$$

$$= Const + \frac{1}{k}\left(-\frac{\beta^2}{\lambda^2}\frac{(1-\phi^k)(1+\phi^k)}{(1-\phi^2)^2} + 2\frac{\beta^2\phi}{\lambda^2(1-\phi)}\frac{(2+\phi-\phi^{k+1})(\phi^k-1)}{(1-\phi^2)^2} + 2\frac{\beta}{\lambda(1-\phi)}\frac{1-\phi^k}{1-\phi}\right)$$

$$= Const + \frac{1}{k}\frac{\beta}{\lambda}\frac{1-\phi^k}{(1-\phi)^2}\left(\frac{\beta}{\lambda}\frac{\phi^k(2\phi^2+\phi-1)-2\phi^2-3\phi-1}{(1+\phi)^2(1-\phi)} + 2\right)$$

where

$$Const = \frac{\beta^2}{\lambda^2}\frac{1}{1-\phi^2} + 2\frac{\beta^2\phi}{\lambda^2(1-\phi^2)(1-\phi)} - 2\frac{\beta}{\lambda(1-\phi)} + 1$$

$$= \frac{\beta^2(1-\phi) + 2\beta^2\phi - 2\beta\lambda(1-\phi^2) + \lambda^2(1-\phi^2)(1-\phi)}{\lambda^2(1-\phi^2)(1-\phi)}$$

$$= \frac{(\beta-\lambda(1-\phi))^2}{\lambda^2(1-\phi)^2} > 0$$

Note that ρ and ϕ are close to but smaller than 1, and $\frac{\beta}{\lambda} = \frac{1-\rho\phi}{\rho}$. We want to find sufficient conditions so that $f(k)$ is decreasing in k. Since $f(k) = g(k)h(k)$ and $f'(k) = g\prime(k)h(k) + g(k)h'(k)$, $f'(k) < 0 \iff g(k)h'(k) < -g'(k)h(k)$. Since $g(k) > 0$, it will be sufficient if we could show that $g'(k) < 0$, $h'(k) < 0$ and $h(k) > 0$.

We first show that $g(k) \equiv \frac{1}{k}\frac{\beta}{\lambda}\frac{1-\phi^k}{(1-\phi)^2}$ decrease in k for $\phi \in (0,1)$. Take the first order derivative we get $g'(k) = \frac{\beta}{\lambda}\frac{1}{(1-\phi)^2}\frac{\phi^k(1-k\ ln\phi)-1}{k^2}$. To show $g'(k) < 0$, we need to show that $m(\phi) = \phi^k(1-k\ ln\phi) - 1 < 0$ for $\phi \in (0,1)$ and $\forall k$. This could be easily proved since $m'(\phi) = -k^2\phi^{k-1}ln(\phi) > 0$ for $\phi \in (0,1)$ and $m(1) = 0$. Thus $g(k)$ is positive and decrease in k. Then we want to know the property of $h(k) = \frac{\beta}{\lambda}\frac{\phi^k(2\phi^2+\phi-1)-2\phi^2-3\phi-1}{(1+\phi)^2(1-\phi)} + 2$. We also notice given that $2\phi^2+\phi-1 > 0$ (which hold as long as $\phi > 0.5$), $h(k)$ is decreasing in k. Thus it would be sufficient to prove the lemma if we know $h(k) > 0$ for $\forall k$. Since $h(k)$ is decreasing in k, we only need $\lim_{k\to\infty} h(k) = -\frac{\beta}{\lambda}\frac{2\phi^2+3\phi+1}{(1+\phi)^2(1-\phi)} + 2 = -\frac{1-\rho\phi}{\rho(1-\phi)}\frac{2\phi^2+3\phi+1}{(1+\phi)^2} + 2 > 0$ to hold. This is equivalent to $\rho > \frac{2\phi^2+3\phi+1}{\phi^2+3\phi+2}$. Under this condition, we know both $g(k)$ and $h(k)$ are positive and decreasing, therefore $f(k) = g(k)h(k)$ is positive and decreasing in k.

Appendix C. Symmetrical Model for Asset Returns

We introduce a two-state-variable symmetrical model for stocks, which includes excess stock return and dividend price ratio as state variables. In particular, the dynamics of the variables are given by:

$$xr_{s,t+1} = \mu_1 + \beta(d_t - p_t) + u_{xr,t+1} \tag{23}$$

$$d_{t+1} - p_{t+1} = \mu_2 + \phi(d_t - p_t) + u_{dp,t+1} \tag{24}$$

We denote $u_t = [u_{xr,t}, u_{dp,t}]'$ and assume the VAR shocks are covariance stationary $E(u_t) = \mathbf{0}$, $E(u_t u_s) = \begin{cases} \Sigma^{wc} & (t = s) \\ \mathbf{0} & (t \neq s) \end{cases}$.The superscript wc stands for within-country, and we use xc to represent cross-country in later part of the paper.

C.1 Connect VAR shocks to structural shocks

We decompose stock excess returns into two structural shocks: cash flow news and discount rate news. In the symmetrical model (VAR) with two state variables, there's actually a one-to-one mapping from the structural shocks to VAR shocks. Recall from the decomposition

$$N_{RR,t+1} \equiv (E_{t+1} - E_t)\left[\sum_{j=0}^{\infty} \rho_s^j r_{f,t+1+j}\right] = (E_{t+1} - E_t)\left[\sum_{j=0}^{\infty} \rho_s^j \left(y_{1,t+j}^N - \pi_{t+1+j}\right)\right] = 0$$

This is because the short nominal rate and inflation are assume to be zero in our symmetrical model.

$$N_{RP,t+1} \equiv (E_{t+1} - E_t)\left[\sum_{j=0}^{\infty} \rho_s^j xr_{s,t+1+j}\right] = \frac{\rho_s \beta}{1 - \rho_s \phi} u_{dp,t+1}$$

Therefore we have the discount rate news

$$N_{DR,t+1} = N_{RR,t+1} + N_{RP,t+1} = \frac{\rho_s \beta}{1 - \rho_s \phi} u_{dp,t+1}$$

and the cash flow news is calculated from the identity

$$N_{CF,t+1} = (E_{t+1} - E_t)\left[xr_{s,t+1}\right] + N_{DR,t+1} = u_{xr,t+1} + \frac{\rho_s \beta}{1 - \rho_s \phi} u_{dp,t+1}$$

To summarize, we have

$$\begin{bmatrix} N_{CF,t+1} \\ N_{DR,t+1} \end{bmatrix} = \begin{bmatrix} 1 & \frac{\rho_s \beta}{1-\rho_s \phi} \\ 0 & \frac{\rho_s \beta}{1-\rho_s \phi} \end{bmatrix} \begin{bmatrix} u_{xr,t+1} \\ u_{dp,t+1} \end{bmatrix} \tag{25}$$

which connects the VAR shocks to structural shocks. Or in matrix notation $\varepsilon_{t+1} = P u_{t+1}$, where ε_{t+1} is the structural shock, u_{t+1} the VAR shocks and P the transformation matrix.

C.2 From single country to a world with N identical countries

To further explore the benefit of international diversification, we design an experiment in a world with N clones (N-replica world composed of N identical countries, and we use the US data to get empirical results). To explain the experiment in detail, we first introduce some notations. Let $\Sigma^{wc} \equiv Var(u_{t+1})$ be the within country VAR covariance matrix, and $\Sigma^{xc} \equiv Cov(u_{i,t+1}, u_{j,t+1})$ $(i \neq j)$ is defined as the cross-country VAR covariance matrix (between country i and j). Since all covariance matrix Σ could be decomposed into volatility component $G \equiv diag(\Sigma)^{1/2}$ and correlation component $(\Gamma \equiv diag(\Sigma)^{-1/2} \Sigma diag(\Sigma)^{-1/2})$, we have the following decomposition for within-country and cross-country VAR covariance matrix

$$\Sigma^{wc} \equiv G_\Sigma \Gamma_\Sigma^{wc} G_\Sigma' \tag{26}$$

$$\Sigma^{xc} \equiv G_\Sigma \Gamma_\Sigma^{xc} G_\Sigma' \tag{27}$$

By using this notation we have implicitly assumed all countries are identical, i.e. $\Sigma_i^{wc} = \Sigma_j^{wc}$ and $\Sigma_{ij}^{xc} = \Sigma_{lm}^{xc}$ $(i \neq j, l \neq m)$, which also implies $G_{\Sigma,i} = G_{\Sigma,j}$, $\Gamma_{\Sigma,i}^{wc} = \Gamma_{\Sigma,j}^{wc}$, $\Gamma_{\Sigma,ij}^{xc} = \Gamma_{\Sigma,lm}^{xc}$.

Then the covariance matrix for the global VAR shock in the N-replica economy is

$$\Sigma_{glo} = \begin{bmatrix} \Sigma^{wc} & \Sigma^{xc} & \dots & \Sigma^{xc} \\ \Sigma^{xc} & \Sigma^{wc} & \dots & \Sigma^{xc} \\ \vdots & \vdots & \dots & \vdots \\ \Sigma^{xc} & \Sigma^{xc} & \dots & \Sigma^{wc} \end{bmatrix}$$

with Σ^{wc} as diagonal blocks and Σ^{xc} as off diagonal blocks . Later we use Σ_{glo} international portfolio allocation analysis.

C.3 Connect the VAR covariance matrix to structural covariance matrix in a world with N identical countries

Let $\Omega^{wc} \equiv Var(\varepsilon_{t+1})$ be the within country structural covariance matrix, and $\Omega^{xc} \equiv Cov(\varepsilon_{i,t+1}, \varepsilon_{j,t+1})$ $(i \neq j)$ is defined as the cross-country structural covariance matrix (between country i and j). Analogous to the decomposition above, we have

$$\Omega^{xc} \equiv G_\Omega \Gamma_\Omega^{xc} G_\Omega' \tag{28}$$

$$\Omega^{wc} \equiv G_\Omega \Gamma_\Omega^{wc} G_\Omega' \tag{29}$$

From the relation $\varepsilon_{t+1} = Pu_{t+1}$, we can take cross-country covariance $Cov(\varepsilon_{i,t+1}, \varepsilon_{j,t+1}) = PCov(u_{i,t+1}, u_{j,t+1})P'$ and get an identity $\Omega^{xc} = P\Sigma^{xc}P'$. Of course, $\Omega^{wc} = P\Sigma^{wc}P'$ also holds.

The identity could be rewritten as

$$G_\Omega \Gamma_\Omega^{xc} G_\Omega' = PG_\Sigma \Gamma_\Sigma^{xc} G_\Sigma' P' \tag{30}$$

Applying the vec operator to both sides and using the trick that $vec(ABC) = (C' \otimes A) \cdot vec(B)$ (see Hamilton 1994 Proposition 10.4) we have

$$(G_\Omega \otimes G_\Omega) \cdot vec\,(\Gamma_\Omega^{xc}) = ((PG_\Sigma) \otimes (PG_\Sigma)) \cdot vec\,(\Gamma_\Sigma^{xc}) \tag{31}$$

Now we've got a mapping from cross-country structural shock correlation matrix to cross-country VAR shock correlation matrix. If $((PG_\Sigma) \otimes (DG_\Sigma))$ is nonsingular, we could rewrite the relationship as

$$vec\,(\Gamma_\Sigma^{xc}) = ((PG_\Sigma) \otimes (PG_\Sigma))^{-1} (G_\Omega \otimes G_\Omega) \cdot vec\,(\Gamma_\Omega^{xc}) \tag{32}$$

And similarly, we have

$$(G_\Omega \otimes G_\Omega) \cdot vec\,(\Gamma_\Omega^{wc}) = ((PG_\Sigma) \otimes (PG_\Sigma)) \cdot vec\,(\Gamma_\Sigma^{wc}) \tag{33}$$

We could also analogously define the covariance matrix for the global structural shock

$$\Omega_{glo} = \begin{bmatrix} \Omega^{wc} & \Omega^{xc} & \dots & \Omega^{xc} \\ \Omega^{xc} & \Omega^{wc} & \dots & \Omega^{xc} \\ \vdots & \vdots & \dots & \vdots \\ \Omega^{xc} & \Omega^{xc} & \dots & \Omega^{wc} \end{bmatrix}$$

And equations (33) and (34) give us the connection between Ω_{glo} and Σ_{glo}.

C.4 Illustrative example using the symmetrical model

From the analysis above, we know there's a connection between the global structural shocks and global VAR shocks. And we could design some experiments using this connection to study the effect of international integration on portfolio allocation. Empirically, we follow the steps below:

1. Estimate a single country symmetrical model using the US historical data. From this we could get a estimate for the covariance matrix Σ^{wc} (or equivalently G_Σ and Γ_Σ^{wc}). P matrix could also be calculated from the reduced form VAR coefficients.
2. Using the identity $\Omega^{wc} = P\Sigma^{wc}P'$, we have an estimate of Ω^{wc}(or equivalently G_Ω and Γ_Ω^{wc}).
3. Manually set values for the cross-country structural shock correlation matrix Γ_Ω^{xc}. From equation (?) we will be able to get the implied cross-country VAR shock correlation matrix Γ_Σ^{xc}.
4. Construct the implied global VAR covariance matrix Σ_{glo}, based on our input Γ_Ω^{xc} in step 3. Given Σ_{glo}, we could study the implications of international integration on global portfolio allocation.

Specifically, we assign 3 set of values toΓ_Ω^{xc} in step 3 above, each corresponds a scenario below :

1st Scenario: $\Gamma_\Omega^{xc} = \mathbf{0}$

This is a benchmark case without international integration, where all cross-country structural shocks are uncorrelated.

2nd Scenario: $\Gamma_{\Omega}^{xc} = \begin{bmatrix} \Gamma_{\Omega,11}^{xc} & 0 \\ 0 & 0 \end{bmatrix}$

where $\Gamma_{\Omega,11}^{xc}$ denote the cross-country CF news correlation.

This is a case with international integration, and the integration is purely driven by increased CF news correlation:

3rd Scenario: $\Gamma_{\Omega}^{xc} = \begin{bmatrix} 0 & 0 \\ 0 & \Gamma_{\Omega,22}^{xc} \end{bmatrix}$

where $\Gamma_{\Omega,22}^{xc}$ denote the cross-country DR news correlation.

This is a case with international integration, and the integration is purely driven by increased DR news correlation.

C.5 Implied Correlation Structure of VAR in Section 3.3

	First Scenario		Second Scenario		Third Scenario	
Corr	$u_{xr,s}$	u_{dp}	$u_{xr,s}$	u_{dp}	$u_{xr,s}$	u_{dp}
$u_{xr,s}$	0	0	0.070	0	0.070	-0.087
u_{dp}	0	0	0	0	-0.087	0.109

C.6 From 2 state variables (symmetrical model) to 6 state variables (general model)

It's very easy to incorporate the symmetrical model in a more general framework. Recall that our general model for a single country is a VAR with 6 state variables

$$\tilde{\mathbf{z}}_{t+1} = a + \mathbf{A}\tilde{\mathbf{z}}_t + \mathbf{u}_{t+1}$$

where $\tilde{\mathbf{z}}_{t+1} = [xr_{s,t+1}, xr_{n,t+1}, d_{t+1} - p_{t+1}, \pi_{t+1}, y_{1,t+1}^N, y_{10,t+1}^N - y_{1,t+1}^N]$. Out symmetrical model is a special case of the general model with

$$a = \begin{bmatrix} \mu_1 \\ 0 \\ \mu_2 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & \beta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \phi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

and

$$\mathbf{u}_{t+1} = \begin{bmatrix} u_{xr,t+1} \\ 0 \\ u_{dp,t+1} \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Appendix D. Data Description

We consider a number of time series from 7 major OECD countries, which accounts for 62% of total world market shares by end of 2014.The full sample period is 1986:01 to 2016:12, yielding 372 monthly observations. We split the full sample to two sub-periods, with the sub-period 1 from 1986:01 to 1999:12 and the sub-period 2 from 2000:01 to 2016:12. Returns are in U.S. dollar currency-hedged terms in excess of the three-month U.S. Treasury bill rate.

D.1 Currency-hedged Return

Before further explaining our data in details, we first introduce the concept of currency hedged excess return. Consider a home investor from US buying assets in a foreign country (for example in Japan), we are interested in his excess returns from this investment denominated in home currency. We use a superscript $*$ to denote a foreign variable. S_t denotes the spot foreign exchange rate, and an increase in S_t means home currency is weakening relative to foreign currency. To conduct this trade, the investor at time t has to exchange 1 US dollar into $\frac{1}{S_t}$ Japanese yen and invest in Japanese capital market, then converts the money back to USD at time $t+1$ when the investment is liquidated. Thus the (unhedged) 1-period return in Japanese market (measured in dollars) is

$$1 + R_{JPN,t+1} \equiv (1 + R^*_{JPN,t+1})\frac{S_{t+1}}{S_t}$$

where $R^*_{JPN,t+1}$ is return in Japanese asset denominated in Japanese yen (local return).

However, due to the uncertainty in future exchange rate S_{t+1}, the investor will want to lock down the future exchange rate using a currency forward at forward rate F_t. So the currency hedged return of a US investor investing in Japan is defined as

$$1 + R^h_{JPN,t+1} \equiv (1 + R^*_{JPN,t+1})\frac{F_t}{S_t}$$

Recall from the covered interest rate parity (CIP), we also have

$$1 + i_{US,t+1} = (1 + i^*_{JPN,t+1})\frac{F_t}{S_t}$$

where $i_{US,t+1}$ is the nominal interest rate for the US, while $i_{JPN,t+1}$ is the nominal interest rate for Japan. The intuition for this equation is that the investor should not have arbitrage opportunities, or alternatively, should be indifferent to invest locally or abroad if the currency risk of investing in foreign country is hedged. This equation holds pretty well unless there's counter-party risk or barriers to financial integration (transaction costs, taxes, capital controls, et cetera).

Combining the two equations above, we know that the excess currency hedged return of a US investor investing in Japan is

$$\frac{1 + R^h_{JPN,t+1}}{1 + i_{US,t+1}} = \frac{1 + R^*_{JPN,t+1}}{1 + i^*_{JPN,t+1}}$$

or in log terms

$$r^h_{JPN,t+1} - r_{f,US,t+1} = r^*_{JPN,t+1} - r^*_{f,JPN,t+1}$$

where $r_{f,US,t+1} = \ln(1 + i_{US,t+1})$ and $r_{f,JPN,t+1} = \ln(1 + i^*_{JPN,t+1})$ are the risk free rates in US and Japan. Thus, we have shown that the excess currency-hedged return of US investors investing in Japan is the same as the excess return of Japanese investors investing in home country (local excess return).

D.2 Main Variables

Now we introduce our main variables briefly.

Returns, Dividend Yield and Inflation

The international portfolio we consider are constructed from country level index in equity and bonds. The country level stock returns are measured as dollar returns on MSCI net total return indices, which reinvest dividends after the deduction of withholding taxes. We use Merill Lynch total return indices (7yr-10yr) to get bond returns. The dividend yield is measured as the log of MSCI dividend yield (MSDY), which is calculated using the trailing 12-month cash earnings per share figure. All the data on stock and bond returns as well as dividend yields are from Datastream. Table 2.A reports sample correlations of monthly bond and stock returns for the period January 1986 to December 2016. Returns are in U.S. Dollar currency-hedged terms in excess of the three-month U.S. Treasury bill rate. Table 2.B and 2.C further look at the correlations in the two sub-samples we are studying.

For the inflation, we get data from both Datastream and Global Financial Data (GFD). We first get annualized inflation rates from Datastream. But for France and UK, the data does not go back far enough because data comes from newer HICP that started in 1990's; thus, we compute inflation manually using CPI for France and RPI for UK from GSD.

Foreign Exchange Rates

We get spot currency levels and one-month forward currency levels from Datastream. The currency levels are all in terms of 1 US dollar except for British Pound (GBP), so we invert GBP to get correct reference frame. The (unhedged) currency returns are calculated as $\ln(\frac{S_{t+1}}{S_t})$ for spot currency levels for 1 USD, and the currency-hedged returns are calculated as $\ln\frac{F_t}{S_t}$ for forward and spot currency levels for 1 USD. Note that French and German data switch to Euros at the beginning of 1999.

Short Term and Long Term Nominal Interest Rate

We use 1 month T-bill rate for US short term nominal interest rate, and for other countries we use different rates on short term financial instruments including 1 month Euribor rates, bank loan rates or overnight money market interest rates. The data are from GFD and central bank websites. Long term nominal interest rate are represented using 10 year yields. The US series is from CRSP Fixed Term Indices and other countries from GFD.

D.3 Data Source

Variable	Source	Description	Download Information
Equity Index	Datastream	MSCI net returns in USD using MSNR (net dividends reinvested); sheet also contains MSCI price indices in USD using MSPI (no dividends reinvested) and MSCI return indices in USD using MSRI (gross dividends reinvested); get returns with simple division of levels; can also get local returns as opposed to USD returns. Take simple USD returns from MSNR and takes LN of gross returns.	MSAUSTL, MSCNDAL, MSFRNCL, MSGERML, MSJPANL, MSUTDKL, MSUSAML with fields MSNR, MSPI, or MSRI
Dividend yields	Datastream	Dividend yields; take LN	MSAUSTL, MSCNDAL, MSFRNCL, MSGERML, MSJPANL, MSUTDKL, MSUSAML with field MSDY
Bond Index	Datastream	Merrill Lynch total return indices; get simple returns with simple division of levels; numbers are already in USD. We take only 7y-10y sector TR and takes LN of gross returns	Datastream tickers: MLAD1T3, MLAD3T5, MLAD5T7, MLAD710, MLCD1T3, MLCD3T5, MLCD5T7, MLCD710, MLFF1T3, MLFF3T5, MLFF5T7, MLFF710, MLDM1T3, MLDM3T5, MLDM5T7, MLDM710, MLJP1T3, MLJP3T5, MLJP5T7, MLJP710, MLUK1T3, MLUK3T5, MLUK5T7, MLUK710, MLUS1T3, MLUS3T5, MLUS5T7, MLUS710
Inflation	Datastream and Global Financial Data(GFD)	Get annualized inflation rates from Datastream and take monthly differences to account for seasonality; for France and UK, data does not go back far enough because data comes from newer HICP that started in 1990's; thus, use GFD to get older CPI for France and RPI for UK and manually compute inflation. We take LN of 1 + monthly difference.	Datastream tickers: AUCPANNL, BDCPANNL, CNCPANNL, FRCPANNL, JPCPANNL, UKCPANNL, USCPANNL; GFD tickers: CPAUSM, CPCANM, CPFRAM (this is French CPI), CPHFRAM (this is French HICP), CPDEUM, CPJPNM, CPGBRM (this is UK RPI), CPHGBRM (this is UK HICP), CPUSAM

FX Data (spot and forward currency level)	Datastream	Currency returns calculated as LN(SPOT(t+1)/SPOT(t)) for SPOT currency levels for 1 USD; hedged currency returns calculated as LN(FWD(t)/SPOT(t)) for FWD and SPOT currency levels for 1 USD; note that French and German data switch to Euros at the beginning of 1999	Get spot currency levels with BBAUDSP, BBCADSP, BBFRFSP, BBDEMSP, BBJPYSP, BBGBPSP, BBEURSP - these are all in terms of 1 USD except for GBP, so need to invert GBP to get correct reference frame; get 1m forward currency levels with BBAUD1F, BBCAD1F, BBFRF1F, BBDEM1F, BBJPY1F, BBGBP1F, BBEUR1F - these are all in terms of 1 USD except for GBP, so need to invert GBP to get correct reference frame
Short Term Interest Rate	GFD and websites	Short nominal rates; Australia: target FF rates; Canada: bank rates, which are discount rates or +25bp over target FF rates; France/Germany: 1 month Euribor rates; Japan: basic discount rates/basic loan rates; UK: bank rates, which are discount rates; US: 12*RF where RF is the 1 month T-bill rate; take LN of (1+SR) as defined above and divides by 12 to get monthly figure	Australia: GFD (from Global Currency Hedging paper) until 200605, then from `http://www.rba.gov.au/statistics/cash-rate.html`; Canada: `http://www.bankofcanada.ca/rates/interest-rates/canadian-interest-rates/`; France: GFD (from Global Currency Hedging paper) until 200412, then from `http://www.global-rates.com/interest-rates/euribor/2010.aspx`; Germany: GFD (from Global Currency Hedging paper) until 200412, then from `http://www.global-rates.com/interest-rates/euribor/2010.aspx`; Japan: `http://www.boj.or.jp/en/statistics/boj/other/discount/index.htm/`; UK: `http://www.bankofengland.co.uk/mfsd/iadb/Repo.asp?Travel=NIxRPx`; US: from Ken French's website
Long Term Interest Rate	GFD and CRSP	Long nominal rates; essentially CMT at 5y and 10y points; takes LN of 1 + LR using the 10y point and divides by 12 to get monthly figure	For non-US, use GFD and the following symbols: IGAUS5D, IGCANB5D, IGFRA5D, IGDEU5D, IGJPN5D, IGGBR5D; IGAUS10D, IGCAN10D, IGFRA10D, IGDEU10D, IGJPN10D, IGGBR10D for US, use CRSP Fixed Term Indices (Daily Series of Yield to Maturity) and the data for 2014 comes from, taking the yield at the end of each month `http://www.treasury.gov/resource-center/data-chart-center/interest-rates/Pages/TextView.aspx?data=yieldYear&year=2014`
Market Capitalization	World Bank	Market capitalization of each country	"Market capitalization of listed companies (current US$)" on world bank website `http://data.worldbank.org/indicator/CM.MKT.LCAP.CD/countries`
Credit Spread	GFD and Datastream	Investmet grade corporate bond index of each country in excess of the government bond index. For US, we use Moody's Baa-Aaa as credit spread.	For Corporate Bonds: Australia use GFD series "INAUSW" before 2005, use series "Non-financial corporate BBB-rated bonds – Yield – 7 year target tenor" from Reserve Bank of Australia starting from 200501. Canada use GFD series "INCANLTW" until 2006, then switch to "S&P CANADA IG CORP BOND IDX" (from datastream) starting from 200605. France use GFD series "INFRAM". Germany use GFD series "INDEUD". Japan use GFD series "INJPNW". UK use GFD series "INGBRW". Government bonds are from GFD with the following symbols: IGAUS5D, IGAUS10D, IGFRA3D, IGDEU5D, IGJPN5D, IGGBR10D. For US, we use Moody's Baa-Aaa as credit spread.

Real GDP	GFD	Real GDP in domestic currency	From GFD, tickers as follows: GDPCCAN (Canada Real GDP in 2007 Dollars) GDPCDEU (Germany Real GDP in 2010 Euros) GDPCAUS (Australia Real GDP in 2007-2008 Dollars), GDPCGBR (Great Britain Real GDP in 2008 Pounds), GDPCFRA (France Real GDP in 2010 Euros), GDPCJPN (Japan Real GDP in 2010 Yen), GDPCUSA (United States Real GDP in 2009 Dollars)
Real Industrial Production	GFD	Industrial Production Index in each country	From GFD, tickers as follows: NDAUTM, NDCANM, NDDEUM, NDFRAM, NDGBRM, NDJPNM, USINDPROM
Real Consumption	GFD	Private Final Consumption Expenditure in each country. We adjusted for inflation to get real variables (if the original variable is nominal).	From GFD, tickers as follows: GDPPCRAUSQ, GDPPCCANQ, GDPPCFRAQ, GDPPCDEUQ, GDPPCRJPNQ, GDPPCGBRQ, GDPPCUSAQ
Real Corporate Earnings	Datastream	Corporate profit, income or surplus aggretate to country level. We adjust for inflation to get real variables.	From Datastream, tickers as follows: USPROFTSB, AUPROFTSB, CNPROFTSB, BDPROFTSB, JPNETPRFB, UKPROFTSB, FRNFCGOSB
Real Dividend	Datastream	Use country level dividend yield and stock price index and multiply to get level of dividend ($D_t = \frac{D_t}{P_t} \times P_t$). And then real by nominal dividend growth adjusted for inflation.	We use MSCI price index (MSPI) and dividend yield (MSDY). Tickers are as follows: MSAUSTL, MSCNDAL, MSFRNCL, MSGERML, MSJPANL, MSUTDKL, MSUSAML with fields MSPI and MSDY.

D.4 Correlation Summary Statistics

Table D.4 - Correlations (Jan. 1986 - Dec. 2016)															
		Bonds							Stocks						
		AUS	CAN	FRA	GER	JPN	UKI	USA	AUS	CAN	FRA	GER	JPN	UKI	USA
Bonds	AUS	1.00													
	CAN	0.55	1.00												
	FRA	0.46	0.52	1.00											
	GER	0.49	0.58	0.86	1.00										
	JPN	0.22	0.33	0.30	0.39	1.00									
	UKI	0.53	0.44	0.57	0.59	0.27	1.00								
	USA	0.55	0.71	0.60	0.64	0.31	0.39	1.00							
Stocks	AUS	0.21	-0.04	-0.06	-0.11	-0.11	0.13	-0.16	1.00						
	CAN	0.07	0.10	-0.07	-0.11	-0.04	0.03	-0.09	0.63	1.00					
	FRA	-0.03	-0.02	0.09	-0.02	0.02	0.03	-0.14	0.57	0.63	1.00				
	GER	-0.03	-0.05	-0.04	-0.10	-0.05	-0.05	-0.19	0.56	0.60	0.84	1.00			
	JPN	-0.10	0.00	-0.03	-0.08	0.00	-0.02	-0.16	0.44	0.46	0.51	0.46	1.00		
	UKI	0.12	0.07	0.03	-0.03	0.01	0.15	-0.06	0.66	0.68	0.73	0.68	0.45	1.00	
	USA	0.04	0.08	-0.02	-0.11	0.00	0.03	-0.05	0.63	0.78	0.71	0.69	0.49	0.79	1.00

Table 2.B - Correlations (Jan. 1986 - Dec. 1999)															
		Bonds							Stocks						
		AUS	CAN	FRA	GER	JPN	UKI	USA	AUS	CAN	FRA	GER	JPN	UKI	USA
Bonds	AUS	1.00													
	CAN	0.44	1.00												
	FRA	0.31	0.39	1.00											
	GER	0.31	0.46	0.78	1.00										
	JPN	0.18	0.34	0.30	0.43	1.00									
	UKI	0.44	0.29	0.45	0.46	0.24	1.00								
	USA	0.40	0.64	0.48	0.51	0.31	0.17	1.00							
Stocks	AUS	0.44	0.01	0.01	-0.01	-0.12	0.29	-0.10	1.00						
	CAN	0.39	0.30	0.08	0.06	0.04	0.21	0.08	0.64	1.00					
	FRA	0.18	0.12	0.40	0.31	0.09	0.22	0.08	0.48	0.55	1.00				
	GER	0.25	0.13	0.24	0.23	-0.02	0.12	0.06	0.51	0.54	0.76	1.00			
	JPN	0.08	0.17	0.13	0.12	0.14	0.15	0.00	0.34	0.39	0.42	0.32	1.00		
	UKI	0.37	0.19	0.20	0.17	0.04	0.33	0.09	0.64	0.66	0.62	0.58	0.37	1.00	
	USA	0.35	0.34	0.19	0.12	0.05	0.22	0.24	0.58	0.78	0.59	0.55	0.36	0.74	1.00

Table 2.C - Correlations (Jan. 2000 - Dec. 2016)															
		Bonds							Stocks						
		AUS	CAN	FRA	GER	JPN	UKI	USA	AUS	CAN	FRA	GER	JPN	UKI	USA
Bonds	AUS	1.00													
	CAN	0.73	1.00												
	FRA	0.66	0.70	1.00											
	GER	0.71	0.73	0.94	1.00										
	JPN	0.35	0.33	0.36	0.39	1.00									
	UKI	0.72	0.76	0.78	0.84	0.37	1.00								
	USA	0.74	0.83	0.72	0.76	0.36	0.76	1.00							
Stocks	AUS	-0.21	-0.12	-0.17	-0.24	-0.08	-0.22	-0.24	1.00						
	CAN	-0.29	-0.13	-0.21	-0.26	-0.17	-0.22	-0.24	0.66	1.00					
	FRA	-0.31	-0.20	-0.25	-0.34	-0.13	-0.27	-0.36	0.71	0.72	1.00				
	GER	-0.34	-0.24	-0.29	-0.36	-0.12	-0.28	-0.38	0.66	0.65	0.92	1.00			
	JPN	-0.34	-0.24	-0.20	-0.28	-0.31	-0.31	-0.33	0.61	0.55	0.62	0.59	1.00		
	UKI	-0.22	-0.09	-0.16	-0.24	-0.06	-0.15	-0.22	0.71	0.72	0.86	0.79	0.56	1.00	
	USA	-0.30	-0.21	-0.23	-0.30	-0.11	-0.25	-0.30	0.73	0.78	0.83	0.81	0.62	0.84	1.00

This table reports sample correlations of monthly bond and stock returns for the whole sample (January 1986 to December 2016), early sample (January 1986 to December 1999) and late sample (January 2000 to December 2016). Returns are in U.S. Dollar currency-hedged terms in excess of the three-month U.S. Treasury bill rate.

Appendix E. VAR Model Estimation

Table E1. Pooled VAR(1) Model Estimates

Panel A

Model estimates	Coefficients on lagged variables (1)	(2)	(3)	(4)	(5)	(6)	Rsq
(1) log stock excess returns	0.081	0.110	0.012	0.002	-0.776	1.305	0.015
	(2.249)	(1.151)	(2.187)	(0.005)	(-0.737)	(0.632)	
(2) log bond excess returns	-0.050	0.059	0.003	-0.227	0.458	2.232	0.042
	(-4.786)	(1.939)	(1.766)	(-1.800)	(1.433)	(3.432)	
(3) log dividend yield	-0.078	-0.141	0.978	0.142	-0.281	-3.879	0.963
	(-2.057)	(-1.390)	(161.895)	(0.328)	(-0.254)	(-1.776)	
(4) log inflation	0.004	-0.008	0.000	0.164	0.267	-0.014	0.085
	(2.580)	(-1.674)	(0.035)	(6.606)	(5.809)	(-0.145)	
(5) log short rate	0.000	-0.002	0.000	0.004	1.003	0.068	0.981
	(1.282)	(-4.188)	(-1.280)	(2.217)	(237.262)	(7.051)	
(6) log yield spread	0.000	0.001	0.000	-0.002	-0.011	0.910	0.863
	(1.952)	(1.070)	(-0.289)	(-0.841)	(-2.004)	(74.947)	

Panel B

Within-country Residual Correlation Matrix (1986.01-2016.12)
averaged over 7 countries
average annualized volatility*100 in diagonal

	(1)	(2)	(3)	(4)	(5)	(6)
(1) log stock excess returns	17.702	0.062	-0.897	0.024	-0.018	-0.031
(2) log bond excess returns	0.062	5.829	-0.055	-0.076	-0.183	-0.461
(3) log dividend yield	-0.897	-0.055	19.684	0.025	0.033	0.023
(4) log inflation	0.024	-0.076	0.025	1.115	0.055	0.013
(5) log short rate	-0.018	-0.183	0.033	0.055	0.102	-0.711
(6) log yield spread	-0.031	-0.461	0.023	0.013	-0.711	0.119

Cross-country Residual Correlation Matrix (1986.01-2016.12)
averaged over 7 countries
diagonal terms are average cross-country correlation of the same state variable

	(1)	(2)	(3)	(4)	(5)	(6)
(1) log stock excess returns	0.610	-0.050	-0.571	0.006	0.003	0.030
(2) log bond excess returns	0.000	0.458	0.002	-0.072	-0.051	-0.288
(3) log dividend yield	-0.546	0.044	0.531	0.017	0.010	-0.039
(4) log inflation	0.013	-0.036	0.014	0.186	0.032	0.001
(5) log short rate	0.007	-0.045	0.009	0.049	0.128	-0.062
(6) log yield spread	-0.010	-0.257	0.000	0.015	-0.087	0.259

Panel C

Within-country Residual Correlation Matrix (1986.01-1999.12) averaged over 7 countries diagonal terms are annualized average volatility*100						
	(1)	(2)	(3)	(4)	(5)	(6)
(1) log stock excess returns	19.213	0.293	-0.926	-0.026	-0.088	-0.109
(2) log bond excess returns	0.293	6.743	-0.290	-0.071	-0.209	-0.400
(3) log dividend yield	-0.926	-0.290	20.863	0.058	0.083	0.115
(4) log inflation	-0.026	-0.071	0.058	1.058	0.041	0.021
(5) log short rate	-0.088	-0.209	0.083	0.041	0.136	-0.721
(6) log yield spread	-0.109	-0.400	0.115	0.021	-0.721	0.153

Cross-country Residual Correlation Matrix (1986.01-1999.12) averaged over 7 countries diagonal terms are average cross-country correlation of the same state variable						
	(1)	(2)	(3)	(4)	(5)	(6)
(1) log stock excess returns	0.538	0.080	-0.527	-0.060	-0.047	-0.012
(2) log bond excess returns	0.183	0.370	-0.177	-0.060	-0.072	-0.213
(3) log dividend yield	-0.508	-0.084	0.509	0.069	0.045	0.015
(4) log inflation	-0.016	-0.020	0.027	0.093	0.006	0.009
(5) log short rate	-0.035	-0.054	0.030	0.034	0.097	-0.032
(6) log yield spread	-0.074	-0.196	0.078	0.033	-0.050	0.188

Panel D

Within-country Residual Correlation Matrix (2000.01-2016.12) averaged over 7 countries diagonal terms are annualized average volatility*100						
	(1)	(2)	(3)	(4)	(5)	(6)
(1) log stock excess returns	16.244	-0.239	-0.871	0.071	0.129	0.095
(2) log bond excess returns	-0.239	4.863	0.247	-0.086	-0.125	-0.643
(3) log dividend yield	-0.871	0.247	18.416	-0.008	-0.080	-0.120
(4) log inflation	0.071	-0.086	-0.008	1.135	0.091	0.017
(5) log short rate	0.129	-0.125	-0.080	0.091	0.053	-0.625
(6) log yield spread	0.095	-0.643	-0.120	0.017	-0.625	0.074

Cross-country Residual Correlation Matrix (2000.01-2016.12) averaged over 7 countries diagonal terms are average cross-country correlation of the same state variable						
	(1)	(2)	(3)	(4)	(5)	(6)
(1) log stock excess returns	0.700	-0.220	-0.633	0.068	0.110	0.083
(2) log bond excess returns	-0.225	0.605	0.216	-0.101	-0.008	-0.442
(3) log dividend yield	-0.600	0.198	0.573	-0.030	-0.061	-0.104
(4) log inflation	0.046	-0.070	-0.006	0.249	0.057	0.014
(5) log short rate	0.115	-0.035	-0.040	0.107	0.271	-0.171
(6) log yield spread	0.100	-0.439	-0.140	0.004	-0.206	0.486

Table E2. VAR(1) Model Estimates [Australia]

Panel A. Model estimates

	Coefficients on lagged variables							
		(1)	(2)	(3)	(4)	(5)	(6)	Rsq
(1) log stock excess returns	0.080	0.024	-0.273	0.023	0.118	-0.774	0.714	0.018
	(1.081)	(0.566)	(-1.540)	(1.049)	(0.105)	(-0.553)	(0.298)	
(2) log bond excess returns	0.027	-0.041	0.110	0.009	-0.389	0.698	2.597	0.047
	(1.322)	(-2.144)	(1.889)	(1.433)	(-0.862)	(1.643)	(2.586)	
(3) log dividend yield	-0.164	-0.051	0.291	0.950	1.044	-0.158	-4.370	0.923
	(-2.058)	(-0.948)	(1.486)	(40.451)	(0.774)	(-0.103)	(-1.519)	
(4) log inflation	-0.001	0.001	-0.003	0.000	0.737	0.117	0.001	0.709
	(-0.558)	(0.751)	(-0.974)	(-0.598)	(10.216)	(2.553)	(0.021)	
(5) log short rate	0.000	0.000	0.002	0.000	0.044	0.985	0.179	0.956
	(0.305)	(0.605)	(0.880)	(0.474)	(2.262)	(44.080)	(3.591)	
(6) log yield spread	-0.001	0.000	-0.003	0.000	-0.043	0.004	0.786	0.702
	(-0.797)	(0.255)	(-1.821)	(-1.052)	(-2.238)	(0.180)	(15.170)	

Panel B. Residual correlation matrix

	(1)	(2)	(3)	(4)	(5)	(6)
(1) log stock excess returns	17.152	0.210	-0.918	0.001	-0.041	-0.032
(2) log bond excess returns	0.210	6.349	-0.177	-0.058	-0.061	-0.288
(3) log dividend yield	-0.918	-0.177	18.997	0.004	0.027	0.040
(4) log inflation	0.001	-0.058	0.004	0.437	0.091	-0.068
(5) log short rate	-0.041	-0.061	0.027	0.091	0.215	-0.933
(6) log yield spread	-0.032	-0.288	0.040	-0.068	-0.933	0.229

Table E3. VAR(1) Model Estimates [Canada]

Panel A. Model estimates

	Coefficients on lagged variables							
		(1)	(2)	(3)	(4)	(5)	(6)	Rsq
(1) log stock excess returns	0.033	0.116	0.155	0.008	0.509	-0.983	1.908	0.029
	(0.843)	(1.915)	(1.238)	(0.757)	(0.787)	(-0.902)	(0.755)	
(2) log bond excess returns	0.007	-0.078	0.044	0.002	-0.065	0.429	2.555	0.057
	(0.654)	(-2.810)	(0.652)	(0.741)	(-0.194)	(0.969)	(2.210)	
(3) log dividend yield	-0.076	-0.128	-0.217	0.978	-0.518	-0.450	-5.245	0.970
	(-1.529)	(-2.016)	(-1.648)	(73.081)	(-0.655)	(-0.378)	(-1.867)	
(4) log inflation	0.000	0.008	-0.008	0.000	0.109	0.247	-0.129	0.077
	(0.110)	(1.726)	(-0.774)	(-0.228)	(1.632)	(2.717)	(-0.675)	
(5) log short rate	0.000	0.000	-0.004	0.000	0.000	1.000	0.029	0.989
	(-1.635)	(-0.172)	(-3.091)	(-1.499)	(-0.025)	(136.467)	(1.451)	
(6) log yield spread	0.000	0.001	0.002	0.000	0.000	-0.006	0.952	0.929
	(1.156)	(2.073)	(2.259)	(0.893)	(-0.014)	(-0.849)	(49.350)	

Panel B. Residual correlation matrix

	(1)	(2)	(3)	(4)	(5)	(6)
(1) log stock excess returns	15.048	0.119	-0.911	0.090	-0.016	-0.041
(2) log bond excess returns	0.119	5.837	-0.113	0.009	-0.309	-0.367
(3) log dividend yield	-0.911	-0.113	16.963	-0.045	0.035	0.036
(4) log inflation	0.090	0.009	-0.045	1.170	0.027	-0.014
(5) log short rate	-0.016	-0.309	0.035	0.027	0.095	-0.724
(6) log yield spread	-0.041	-0.367	0.036	-0.014	-0.724	0.099

Table E4. VAR(1) Model Estimates [France]

Panel A. Model estimates

	Coefficients on lagged variables							
		(1)	(2)	(3)	(4)	(5)	(6)	Rsq
(1) log stock excess returns	0.024	0.100	0.467	0.007	1.062	-0.310	3.542	0.034
	(0.559)	(1.440)	(2.129)	(0.625)	(1.009)	(-0.170)	(0.753)	
(2) log bond excess returns	0.027	-0.030	0.079	0.008	-0.650	0.568	2.187	0.063
	(2.457)	(-2.003)	(1.338)	(2.601)	(-2.693)	(1.290)	(1.976)	
(3) log dividend yield	-0.100	-0.081	-0.581	0.968	-0.688	-0.781	-6.171	0.937
	(-1.893)	(-1.118)	(-2.523)	(66.628)	(-0.575)	(-0.414)	(-1.222)	
(4) log inflation	0.000	0.005	-0.002	0.000	-0.028	0.264	0.192	0.053
	(0.087)	(2.072)	(-0.187)	(-0.124)	(-0.502)	(3.149)	(0.973)	
(5) log short rate	-0.001	0.000	-0.003	0.000	-0.002	1.009	0.054	0.993
	(-3.076)	(-0.381)	(-3.523)	(-2.961)	(-0.406)	(144.631)	(1.756)	
(6) log yield spread	0.000	0.000	0.002	0.000	0.008	-0.018	0.927	0.922
	(1.282)	(1.786)	(1.701)	(0.866)	(1.586)	(-2.023)	(28.193)	

Panel B. Residual correlation matrix

	(1)	(2)	(3)	(4)	(5)	(6)
(1) log stock excess returns	19.104	0.090	-0.858	-0.029	-0.011	-0.072
(2) log bond excess returns	0.090	5.076	-0.022	-0.146	-0.150	-0.500
(3) log dividend yield	-0.858	-0.022	21.924	0.127	-0.015	0.060
(4) log inflation	-0.029	-0.146	0.127	0.912	0.095	0.033
(5) log short rate	-0.011	-0.150	-0.015	0.095	0.080	-0.747
(6) log yield spread	-0.072	-0.500	0.060	0.033	-0.747	0.097

Table E5. VAR(1) Model Estimates [Germany]

Panel A. Model estimates

	Coefficients on lagged variables							
		(1)	(2)	(3)	(4)	(5)	(6)	Rsq
(1) log stock excess returns	0.085	0.097	0.020	0.022	-0.548	-2.237	3.821	0.024
	(1.544)	(1.503)	(0.082)	(1.497)	(-0.579)	(-1.380)	(0.802)	
(2) log bond excess returns	0.007	-0.040	0.055	0.001	-0.350	-0.185	1.017	0.052
	(0.662)	(-3.110)	(1.014)	(0.445)	(-1.559)	(-0.484)	(1.077)	
(3) log dividend yield	-0.170	-0.112	-0.069	0.951	0.328	1.066	-7.180	0.924
	(-2.900)	(-1.675)	(-0.269)	(59.279)	(0.321)	(0.647)	(-1.494)	
(4) log inflation	0.003	0.005	-0.010	0.000	-0.125	0.236	-0.416	0.051
	(1.202)	(1.912)	(-0.968)	(0.620)	(-2.217)	(2.026)	(-1.874)	
(5) log short rate	0.000	0.000	-0.003	0.000	0.002	1.003	0.028	0.993
	(-1.749)	(1.028)	(-4.439)	(-1.613)	(0.603)	(272.038)	(1.903)	
(6) log yield spread	0.000	0.000	0.002	0.000	0.002	-0.004	0.969	0.939
	(1.423)	(1.877)	(2.609)	(1.277)	(0.735)	(-0.657)	(58.023)	

Panel B. Residual correlation matrix

	(1)	(2)	(3)	(4)	(5)	(6)
(1) log stock excess returns	21.496	-0.095	-0.875	0.085	0.083	-0.034
(2) log bond excess returns	-0.095	4.789	0.081	-0.127	-0.310	-0.529
(3) log dividend yield	-0.875	0.081	23.371	-0.046	-0.042	0.017
(4) log inflation	0.085	-0.127	-0.046	1.140	0.033	0.096
(5) log short rate	0.083	-0.310	-0.042	0.033	0.054	-0.581
(6) log yield spread	-0.034	-0.529	0.017	0.096	-0.581	0.069

Table E6. VAR(1) Model Estimates [Japan]

Panel A. Model estimates

	Coefficients on lagged variables							
		(1)	(2)	(3)	(4)	(5)	(6)	Rsq
(1) log stock excess returns	0.048	0.119	0.211	0.011	-0.535	-0.291	4.355	0.022
	(1.128)	(1.795)	(0.885)	(1.077)	(-0.815)	(-0.095)	(0.646)	
(2) log bond excess returns	0.035	-0.034	0.136	0.009	-0.018	0.888	7.955	0.095
	(3.297)	(-2.619)	(2.095)	(3.311)	(-0.112)	(1.172)	(4.093)	
(3) log dividend yield	-0.096	-0.128	-0.225	0.975	0.631	-2.666	-13.637	0.981
	(-1.609)	(-1.592)	(-0.783)	(65.865)	(0.846)	(-0.722)	(-1.537)	
(4) log inflation	0.000	0.002	0.001	0.000	0.181	0.374	-0.138	0.051
	(-0.083)	(0.383)	(0.034)	(-0.098)	(4.731)	(1.692)	(-0.279)	
(5) log short rate	0.000	0.000	-0.001	0.000	0.002	0.984	-0.003	0.992
	(-2.430)	(-0.166)	(-2.350)	(-2.439)	(1.027)	(121.339)	(-0.253)	
(6) log yield spread	0.000	0.000	-0.001	0.000	-0.002	0.004	0.924	0.930
	(-2.229)	(2.376)	(-0.921)	(-2.424)	(-1.194)	(0.421)	(41.469)	

Panel B. Residual correlation matrix

	(1)	(2)	(3)	(4)	(5)	(6)
(1) log stock excess returns	19.861	0.002	-0.860	0.041	-0.043	0.001
(2) log bond excess returns	0.002	4.927	-0.015	0.016	-0.187	-0.742
(3) log dividend yield	-0.860	-0.015	22.954	0.004	0.066	0.005
(4) log inflation	0.041	0.016	0.004	1.475	0.032	-0.016
(5) log short rate	-0.043	-0.187	0.066	0.032	0.038	-0.407
(6) log yield spread	0.001	-0.742	0.005	-0.016	-0.407	0.059

Table E7. VAR(1) Model Estimates [United Kingdom]

Panel A. Model estimates

	Coefficients on lagged variables							
		(1)	(2)	(3)	(4)	(5)	(6)	Rsq
(1) log stock excess returns	0.093	0.029	0.248	0.026	0.031	-0.930	0.189	0.034
	(1.833)	(0.518)	(2.419)	(1.917)	(0.048)	(-0.627)	(0.059)	
(2) log bond excess returns	0.023	-0.069	-0.010	0.006	0.034	-0.006	1.086	0.031
	(1.249)	(-1.452)	(-0.128)	(1.273)	(0.119)	(-0.010)	(0.843)	
(3) log dividend yield	-0.083	-0.025	-0.264	0.975	-0.147	0.287	-1.345	0.952
	(-1.487)	(-0.413)	(-2.374)	(63.898)	(-0.220)	(0.183)	(-0.404)	
(4) log inflation	0.003	0.004	-0.012	0.000	0.133	0.243	0.013	0.070
	(0.584)	(0.976)	(-0.891)	(0.336)	(2.046)	(2.197)	(0.051)	
(5) log short rate	-0.001	0.000	-0.002	0.000	0.005	1.010	0.042	0.995
	(-2.122)	(0.928)	(-3.303)	(-2.033)	(1.492)	(135.818)	(2.432)	
(6) log yield spread	0.001	0.000	0.001	0.000	-0.006	-0.017	0.942	0.954
	(1.949)	(0.069)	(1.735)	(1.683)	(-1.190)	(-2.033)	(48.304)	

Panel B. Residual correlation matrix

	(1)	(2)	(3)	(4)	(5)	(6)
(1) log stock excess returns	15.385	0.153	-0.907	-0.011	-0.053	-0.099
(2) log bond excess returns	0.153	7.345	-0.144	-0.102	-0.304	-0.424
(3) log dividend yield	-0.907	-0.144	17.152	0.066	0.058	0.091
(4) log inflation	-0.011	-0.102	0.066	1.412	0.117	0.033
(5) log short rate	-0.053	-0.304	0.058	0.117	0.075	-0.578
(6) log yield spread	-0.099	-0.424	0.091	0.033	-0.578	0.096

Table E8. VAR(1) Model Estimates [United States]

Panel A. Model estimates

	Coefficients on lagged variables							
		(1)	(2)	(3)	(4)	(5)	(6)	Rsq
(1) log stock excess returns	0.111	0.062	0.030	0.023	0.212	-3.259	-6.290	0.024
	(2.633)	(0.876)	(0.219)	(2.505)	(0.290)	(-1.799)	(-1.693)	
(2) log bond excess returns	-0.015	-0.079	0.034	-0.003	-0.831	1.422	3.670	0.075
	(-0.979)	(-3.138)	(0.584)	(-0.854)	(-2.323)	(2.263)	(2.760)	
(3) log dividend yield	-0.092	-0.044	-0.001	0.979	0.369	1.421	4.942	0.981
	(-2.117)	(-0.643)	(-0.006)	(101.402)	(0.556)	(0.749)	(1.350)	
(4) log inflation	0.002	0.009	-0.012	0.000	0.448	0.158	-0.052	0.263
	(0.593)	(1.635)	(-1.394)	(0.346)	(5.998)	(1.506)	(-0.246)	
(5) log short rate	-0.001	0.001	0.000	0.000	0.006	1.029	0.154	0.964
	(-3.505)	(0.742)	(-0.321)	(-2.639)	(1.063)	(99.190)	(5.568)	
(6) log yield spread	0.001	0.000	0.000	0.000	0.003	-0.050	0.803	0.777
	(3.487)	(0.412)	(-0.092)	(2.650)	(0.342)	(-3.578)	(25.018)	

Panel B. Residual correlation matrix

	(1)	(2)	(3)	(4)	(5)	(6)
(1) log stock excess returns	15.127	-0.034	-0.959	-0.024	0.031	-0.024
(2) log bond excess returns	-0.034	6.147	0.015	-0.133	0.013	-0.452
(3) log dividend yield	-0.959	0.015	15.323	0.038	0.005	0.002
(4) log inflation	-0.024	-0.133	0.038	0.971	-0.025	0.076
(5) log short rate	0.031	0.013	0.005	-0.025	0.133	-0.883
(6) log yield spread	-0.024	-0.452	0.002	0.076	-0.883	0.160

Appendix F. Fisher Transformation and Correlation Contribution

F.1 Fisher Transformation

We use Fisher transformation to test the hypothesis that cross-country correlations of the news components of excess stock returns are different between 1986-1999 subperiod and the 2000-2016 subperiod. Define $z = \frac{1}{2}\ln\left(\frac{1+r}{1-r}\right)$. If (X, Y) is bivariate normal, and if (X_i, Y_i) used to form r are independent, then $z \sim \mathcal{N}\left(\frac{1}{2}\ln\left(\frac{1+\rho}{1-\rho}\right), \frac{1}{N-3}\right)$, where N is the sample size. For two samples of data, the early subperiod (1) and the late subperiod (2), define $z_1 = \frac{1}{2}\ln\left(\frac{1+r_1}{1-r_1}\right)$ and $z_2 = \frac{1}{2}\ln\left(\frac{1+r_2}{1-r_2}\right)$. The difference is $z_1 - z_2 \sim \mathcal{N}\left(\frac{1}{2}\ln\left(\frac{1+\rho_1}{1-\rho_1}\right) - \frac{1}{2}\ln\left(\frac{1+\rho_2}{1-\rho_2}\right), \frac{1}{N_1-3} + \frac{1}{N_2-3}\right)$. p-values can then be obtained in the normal way.

F.2 Correlation Contribution

For stocks, we can decompose the excess return news $\tilde{xs}_{t+1} = (\mathbb{E}_{t+1} - \mathbb{E}_t)[xr_{s,t+1}] = N_{CF,t+1} - N_{RR,t+1} - N_{RP,t+1}$. For bonds we can decompose its excess return news as $\tilde{xr}_{t+1} = (\mathbb{E}_{t+1} - \mathbb{E}_t)[xr_{n,t+1}] = N_{CF,n,t+1} - N_{RR,n,t+1} - N_{RP,n,t+1}$. (an increase in $N_{CF,n,t+1}$ for bonds is interpreted as negative inflation news).

The reported "Component Contributions" in Tables 4-6 look at how much of the average covariance in excess returns is being explained by covariances of news components. E.g., in Table 4, the stocks cash flow/stocks real rate across countries component contribution is calculated as $\frac{1}{N(N-1)/2}\sum_i \sum_{j \neq i} \frac{Cov(N_{CF,i}, N_{RR,j})}{Cov(\tilde{xs}_i, \tilde{xs}_j)}$. For a given (i,j) pair, the denominator $Cov(\tilde{xs}_i, \tilde{xs}_j) = Cov(N_{CF,i} - N_{RR,i} - N_{RP,i}, N_{CF,j} - N_{RR,j} - N_{RP,j})$ can be broken into 9 covariances of news components. Therefore, the 9 terms in the "Component Contributions" table always sum up to 1.

Appendix G. Semidefinite Programming Method

We do a constrained minimization problem to estimate the covariance matrices which satisfy two constraints: A). volatility matrix and within-country correlation are the same across two sample period. B). covariance matrix is positive semi-definite. First we decompose a covariance matrix into volatility matrix and correlation matrix

$$\Sigma = D\Gamma D = \begin{pmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_m \end{pmatrix} \begin{pmatrix} 1 & \cdots & \rho_{1m} \\ \vdots & \ddots & \vdots \\ \rho_{1m} & \cdots & 1 \end{pmatrix} \begin{pmatrix} \sigma_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \sigma_m \end{pmatrix}$$

Where the σ_i and ρ_{ij} $(i, j = 1, ..., m)$ are the coefficients to be estimated. Suppose $\widehat{\Sigma_1}$ and $\widehat{\Sigma_2}$ are the sample covariance matrices for early period and late period (known), then we need to estimate two covariance matrix $\Sigma_1 = D_1\Gamma_1 D_1$ and $\Sigma_2 = D_2\Gamma_2 D_2$ with the constraint $D_1 = D_2 = D$ and $\Gamma_1^{within} = \Gamma_2^{within}$. We use the minimum distance estimation, and this is a well defined constrained optimization problem

$$\min_{\Sigma_1, \Sigma_2} \left\{ \| \widehat{\Sigma_1} - \Sigma_1 \|_2 + \| \widehat{\Sigma_2} - \Sigma_2 \|_2 \right\}$$

$$\Longleftrightarrow \min_{D, \Gamma_1, \Gamma_2} \left\{ \| \widehat{\Sigma_1} - D\Gamma_1 D \|_2 + \| \widehat{\Sigma_2} - D\Gamma_2 D \|_2 \right\}$$

$$s.t.\ \Gamma_i \succcurlyeq 0\ (i = 1, 2)$$

$$\Gamma_2^{within} = \Gamma_1^{within}$$

where $\| \, . \, \|_2$ represents the norm in L^2 space ($\| A - B \|_2 = \sum_{i,j}(a_{ij} - b_{ij})^2$), the notation $\Gamma \succcurlyeq 0$ means the matrix Γ is positive semi-definite, and Γ^{within} denotes the within-country correlation. To solve the Semidefinite programming (SDP) problem, we use the MATLAB package CVX by Stephen Boyd. `http://cvxr.com/cvx/doc/sdp.html`

Appendix H. VAR Model with Stochastic Volatility

Estimating VAR with Stochastic Volatility

We follow the methodology in Campbell, Giglio, Polk and Turley (CGPT 2017) in estimating VAR with stochastic volatility. Our VAR includes 8 state variables: stock excess returns, bond excess returns, dividend yield, inflation, short rate, yield spread, credit spread and EVAR. This adds two additional variables to our baseline VAR (credit spread and EVAR). The credit spread is constructed following the methodology in Kang and Pflueger (2013). It's constructed as the log yields of investment grade corporate bond index subtracted by log yields of nominal government bond[2]. For U.S. credit spread, we use Moody's Baa log yield minus Aaa log yield. Figure 1 plots the country level credit spread in our sample. As argued in CGPT 2017, shocks to credit spread to some degree reflect news about aggregate default probabilities, which in turn should reflect news about the market's future cash flows and volatility.

We use daily MSCI price index (MSPI) denominated in USD to constructed monthly realized variance (RVAR). The daily return is constructed by taking the daily difference of the price index $r_{t+1} = ln(\frac{P_{t+1}}{P_t})$. The monthly realized variance is the sum of daily squared return. In estimation of the VAR, we use a two stage method (as in CGPT 2017). In the first stage, we construct period $t+1$ expected market variance ($EVAR_t$) based on information available at period t (i.e. all state variables at period t: $\boldsymbol{x_t}$). Following CGPT, we fit the regression using weighted Least Squares (WLS). Specifically, we weight each observation ($RVAR_{t+1}, \boldsymbol{x_t}$) by previous period's realized variance $RVAR_t^{-1}$. And we use a shrinkage factor as indicated in CGPT to ensure the ratio of weights across observations is not too extreme. In the second stage, we estimate a VAR with the first stage fitted value EVAR as a state variable. The second stage VAR is also estimated using WLS except that now the weight becomes $EVAR_t^{-1}$. We continue to apply the shrinkage factor in the second stage estimation. The results of the first stage regressions and second stage VAR estimations for 7 countries are reported in Tables H.1 to H.7.

Simulating Symmetrical Model with Stochastic Volatility

To understand the impact of stochastic volatility on portfolio risk, we add volatility shock into our stylized symmetrical model of asset returns of Section 3 and simulate the symmetrical model with stochastic volatility. The new model has the following data generating process

$$r_{t+1} = \mu_r + \beta s_t + \sigma_t u_{r,t+1}$$

$$s_{t+1} = \mu_s + \phi s_t + \sigma_t u_{s,t+1}$$

$$\sigma_{t+1} = (1-\psi) + \psi \sigma_t + v_{\sigma,t+1}$$

The only difference from our previous symmetrical model is that here we added add a volatility, which follows a AR(1) process with persistence ψ. Now the innovations to other variables (s_t and r_t) become heteroskedastic. In the simulation, we assume a symmetrical model for 7 countries, and the shocks to the 7 country VAR follow a multivariate normal process. In the simulation, we set $\phi = 0.9857$ and $\beta = 0.0123$, which are estimated from US data. For the volatility persistence, we compared two values in simulation: $\psi = 0.9$ and $\psi = 0.99$.

As a robustness check, we first reproduced the results in Figure 4 Panel B by simulating the 7 country symmetrical model of 2 state variables (excess stock return, dividend price ratio) over a horizon of 800 periods. We simulate 20000 paths. Then we simulate our symmetrical model with stochastic volatility specified above. We set the within-country correlation of volatility news and excess stock return news $corr(v_{\sigma,i}, u_{r,i})$ to be -0.625 and the within-country correlation of volatility news and dividend yield news $corr(v_{\sigma,i}, u_{r,i})$ to be 0.595. The numbers come from our VAR estimation results in Appendix Table H7.

We focus on two exercises in the simulation. In the first exercise, the volatility news are not correlated across countries (i.e. $corr(v_{\sigma,i}, v_{\sigma,j}) = 0$ for $\forall i \neq j$). Compare this with the symmetrical model of 2 state variables (Figure 4 Panel B), we could see the impact of stochastic volatility on portfolio risk. In the second exercise, we make volatility news correlated across countries ($corr(v_{\sigma,i}, v_{\sigma,j}) = 0.3$ for $\forall i \neq j$) and everything else the same as in the first exercise. This exercise studies how volatility integration impacts portfolio risk. In both exercises, we tried two specifications for the volatility persistence ($\psi = 0.9$ and $\psi = 0.99$). We see that when volatility is more persistent, the impact on portfolio risk is greater.

[2]We selected government bonds with appropriate maturity so that the duration of it roughly match the duration of corporate bond indexes.

Figure H.1: International credit spreads. This figure shows the monthly credit spreads for Australia, Canada, France, Germany, Japan, the UK, and the US. It's constructed as the log yields of investment grade corporate bond index subtracted by log yields of duration matched nominal government bond. For U.S. credit spread, we use Moody's Baa log yield minus Aaa log yield.

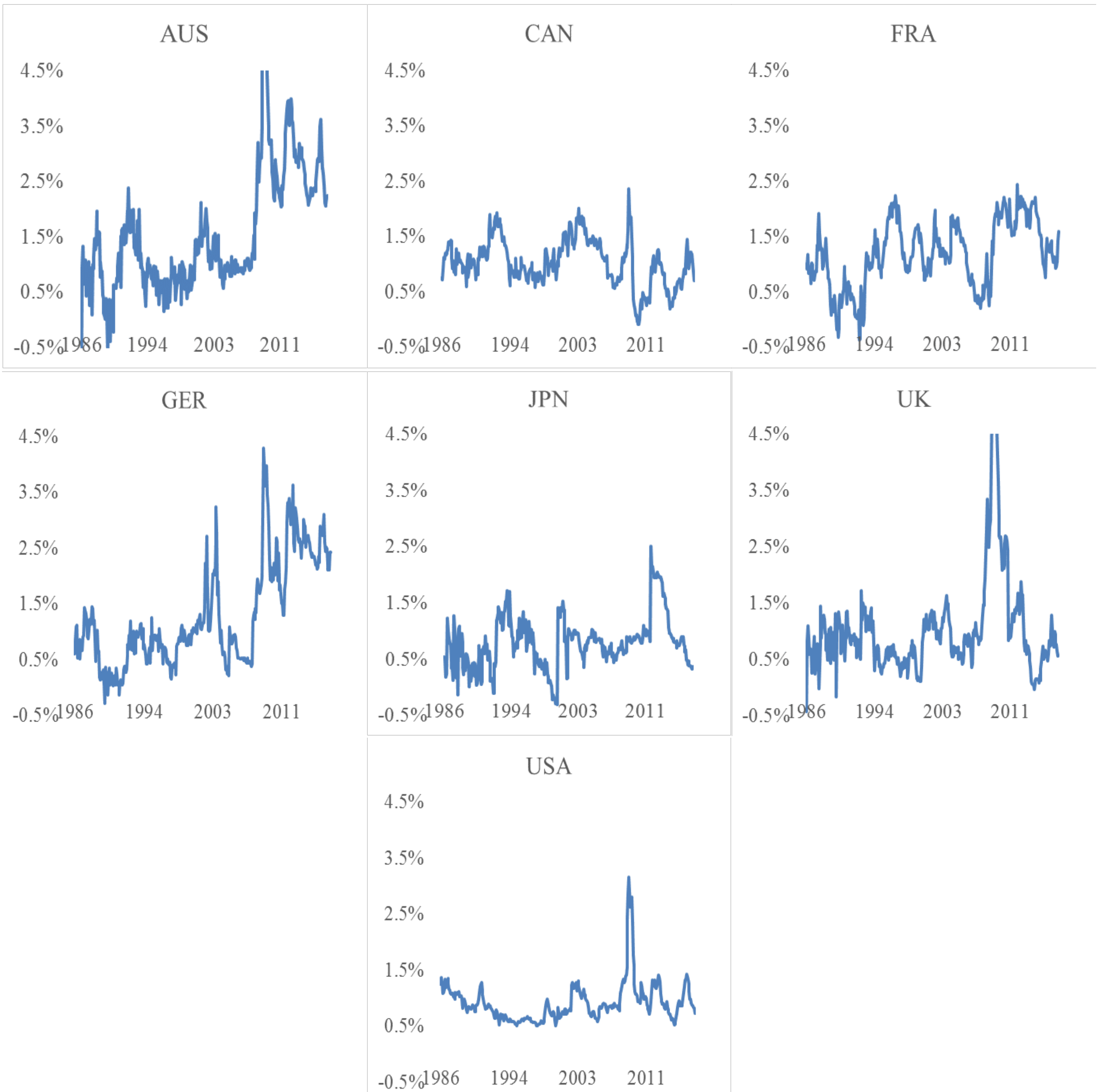

Figure H.2: International realized variance (RVAR) and expected variance (EVAR). This figure shows the monthly realized variance (RVAR) and expected variance (EVAR) for Australia, Canada, France, Germany, Japan, the UK, and the US. The monthly realized variance is constructed from daily MSCI price index (MSPI) denominated in USD.

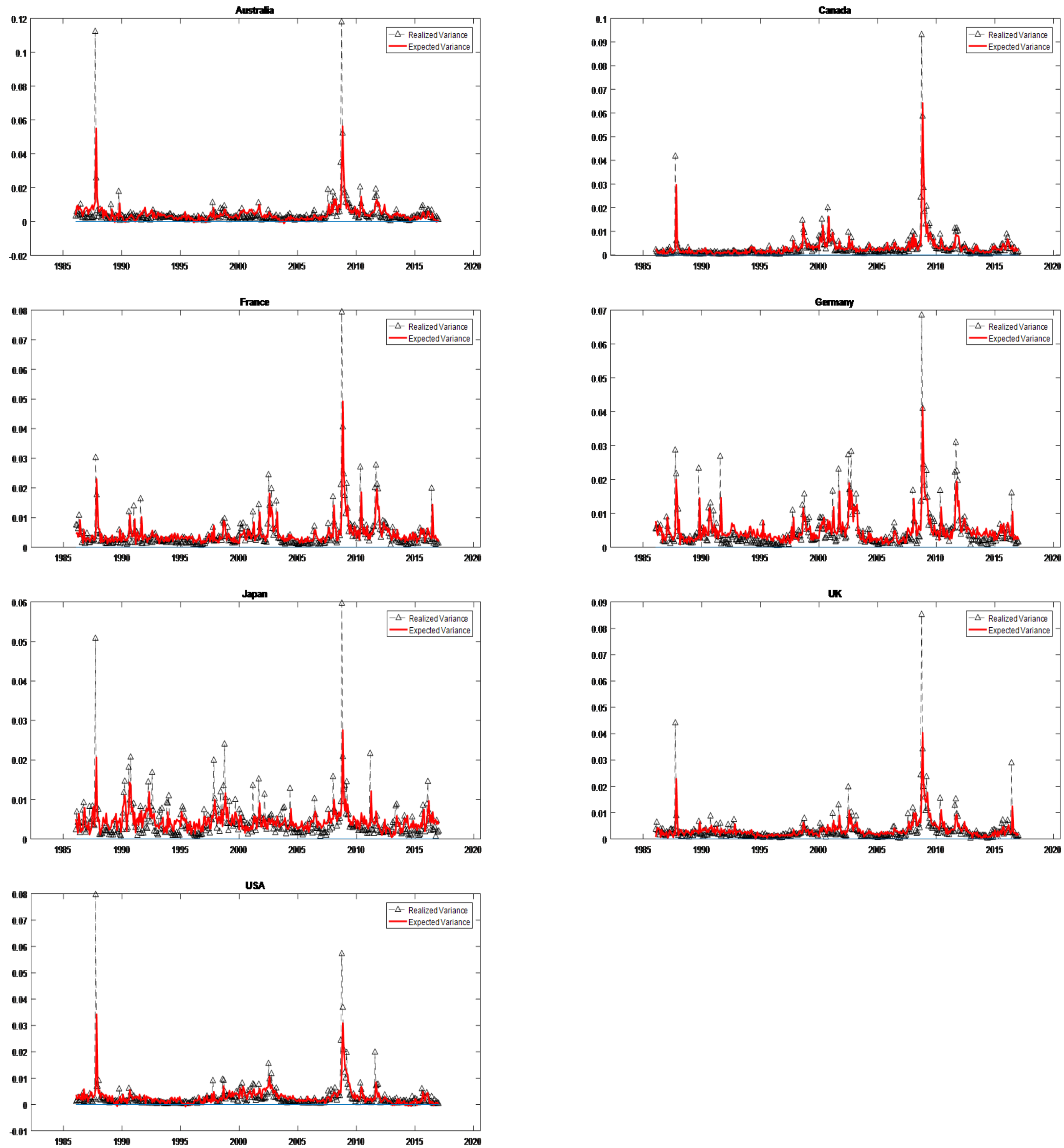

Figure H.3: Cross country correlation of heteroscedastic VAR news (stocks). This figure plots the three year 3-year moving average of average cross-country correlations of shocks to stock excess returns, cash flow news, real rate news, and risk premium news, both including the October 1987 observation and excluding it. The news components are extracted from heteroscedastic VAR.

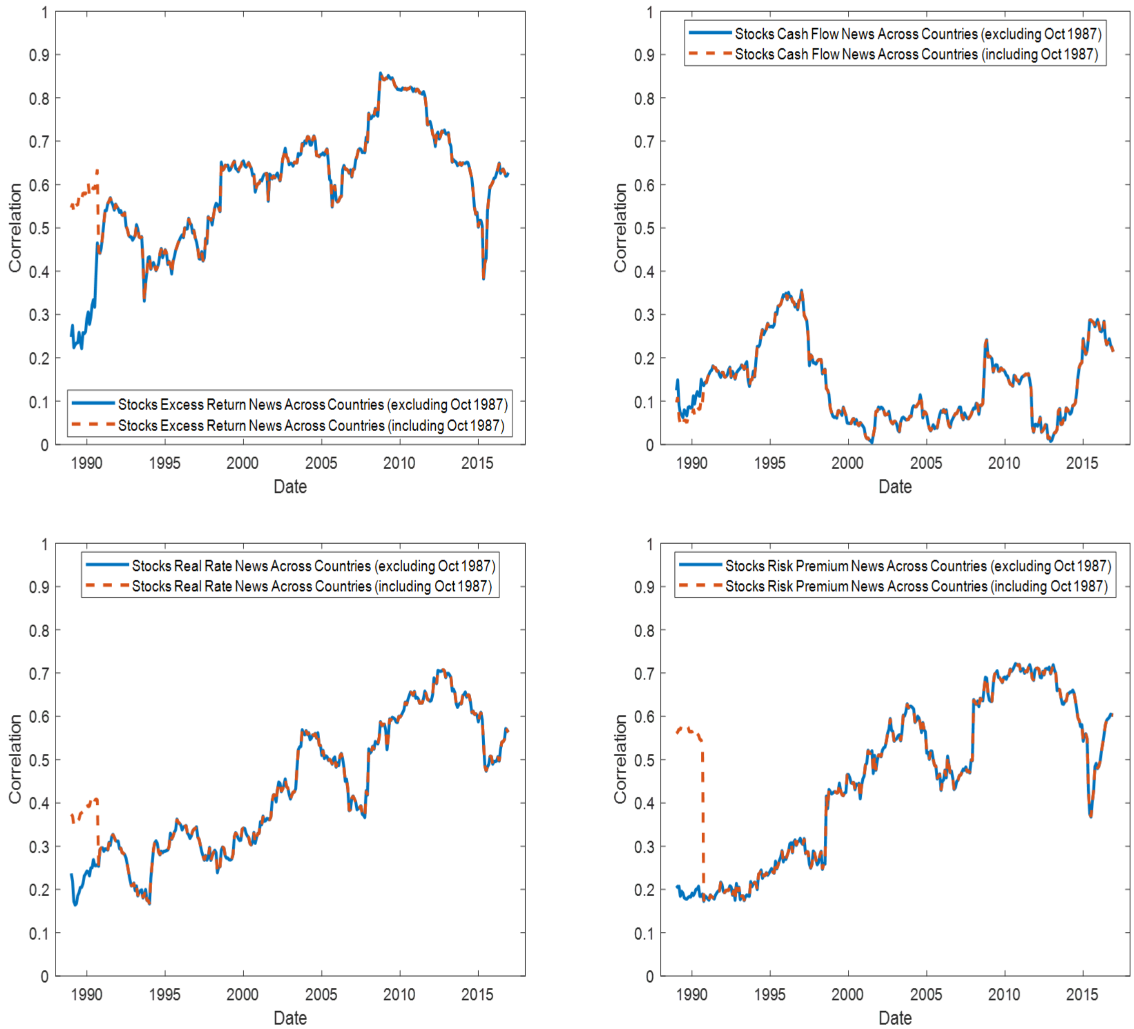

Table H1. Estimates of VAR(1) Model with Stochastic Volatility (Australia)

Panel A: Forecasting Monthly Realized Variance (RVAR)

Intercept	log stock excess returns	log bond excess returns	log dividend yield	log inflation	log short rate	log yield spread	log credit spread	RVAR	Rsq
-0.036	-0.016	0.043	-0.010	0.015	0.662	-0.074	0.244	0.407	0.242
-1.232	-2.167	1.732	-1.273	0.155	1.217	-0.203	1.717	2.447	

Panel B: VAR Estimates

Second Stage	Coefficients on lagged variables									
	Intercept	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	Rsq
(1) log stock excess returns	0.106	-0.017	-0.159	0.028	0.048	-1.443	-0.209	-0.219	-0.612	0.027
	(0.714)	(-0.242)	(-1.059)	(0.709)	(0.044)	(-0.485)	(-0.086)	(-0.272)	(-0.629)	
(2) log bond excess returns	0.034	-0.025	0.092	0.011	-0.279	0.461	2.747	-0.078	0.345	0.053
	(0.938)	(-0.829)	(1.600)	(1.120)	(-0.630)	(0.581)	(2.634)	(-0.336)	(0.938)	
(3) log dividend yield	-0.174	-0.002	0.158	0.948	1.439	0.019	-3.339	0.056	0.660	0.923
	(-1.102)	(-0.031)	(0.950)	(22.667)	(1.136)	(0.006)	(-1.156)	(0.064)	(0.539)	
(4) log inflation	-0.004	0.000	-0.001	-0.001	0.729	0.172	0.002	0.023	-0.023	0.710
	(-1.884)	(-0.038)	(-0.434)	(-1.800)	(9.289)	(2.843)	(0.038)	(1.990)	(-0.855)	
(5) log short rate	0.001	0.000	0.002	0.000	0.049	0.961	0.164	-0.010	0.001	0.956
	(0.954)	(0.383)	(1.033)	(0.989)	(2.498)	(34.186)	(3.151)	(-1.295)	(0.152)	
(6) log yield spread	-0.002	0.000	-0.004	-0.001	-0.049	0.035	0.801	0.012	-0.007	0.705
	(-1.289)	(-0.189)	(-1.845)	(-1.383)	(-2.513)	(1.144)	(15.210)	(1.422)	(-0.693)	
(7) log credit spread	0.021	0.001	-0.017	0.005	-0.060	-0.385	-0.385	0.798	0.263	0.913
	(2.892)	(0.154)	(-1.579)	(2.592)	(-0.657)	(-2.767)	(-2.129)	(15.158)	(2.256)	
(8) EVAR	-0.026	0.000	0.000	-0.007	0.002	0.547	0.068	0.178	0.483	0.443
	(-2.133)	(0.043)	(-0.012)	(-2.177)	(0.038)	(2.246)	(0.486)	(2.468)	(2.825)	

Panel C1: Residual correlation matrix (scaled)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) log stock excess returns	17.078	0.217	-0.917	0.006	-0.051	-0.024	-0.135	-0.558
(2) log bond excess returns	0.217	6.329	-0.184	-0.056	-0.060	-0.290	0.260	0.120
(3) log dividend yield	-0.917	-0.184	18.941	0.001	0.035	0.035	0.124	0.492
(4) log inflation	0.006	-0.056	0.001	0.436	0.099	-0.078	0.002	-0.090
(5) log short rate	-0.051	-0.060	0.035	0.099	0.213	-0.932	-0.113	0.104
(6) log yield spread	-0.024	-0.290	0.035	-0.078	-0.932	0.228	0.013	-0.146
(7) log credit spread	-0.135	0.260	0.124	0.002	-0.113	0.013	1.128	0.443
(8) EVAR	-0.558	0.120	0.492	-0.090	0.104	-0.146	0.443	1.306

Panel C2: Residual correlation matrix (unscaled)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) log stock excess returns	21.500	0.217	-0.922	0.028	-0.048	-0.029	-0.155	-0.578
(2) log bond excess returns	0.217	7.599	-0.172	-0.056	-0.049	-0.302	0.285	0.080
(3) log dividend yield	-0.922	-0.172	23.812	-0.025	0.028	0.039	0.151	0.518
(4) log inflation	0.028	-0.056	-0.025	0.519	0.102	-0.080	-0.044	-0.161
(5) log short rate	-0.048	-0.049	0.028	0.102	0.252	-0.932	-0.097	0.080
(6) log yield spread	-0.029	-0.302	0.039	-0.080	-0.932	0.271	-0.008	-0.108
(7) log credit spread	-0.155	0.285	0.151	-0.044	-0.097	-0.008	1.441	0.477
(8) EVAR	-0.578	0.080	0.518	-0.161	0.080	-0.108	0.477	1.916

Table H2. Estimates of VAR(1) Model with Stochastic Volatility (Canada)

Panel A: Forecasting Monthly Realized Variance (RVAR)

Intercept	log stock excess returns	log bond excess returns	log dividend yield	log inflation	log short rate	log yield spread	log credit spread	RVAR	Rsq
-0.003	-0.010	-0.011	-0.001	-0.037	-0.065	-0.158	-0.020	0.656	0.418
-1.089	-1.162	-0.797	-1.678	-0.764	-0.456	-0.644	-0.443	4.796	

Panel B: VAR Estimates

Second Stage	Coefficients on lagged variables									
	Intercept	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	Rsq
(1) log stock excess returns	0.033	0.066	0.153	0.008	0.408	-1.537	1.107	0.344	-0.540	0.032
	(0.925)	(1.045)	(1.237)	(0.770)	(0.647)	(-1.301)	(0.425)	(0.639)	(-0.715)	
(2) log bond excess returns	0.006	-0.087	0.073	0.002	0.050	0.683	3.032	-0.287	0.079	0.061
	(0.614)	(-3.544)	(1.269)	(0.549)	(0.155)	(1.462)	(2.716)	(-1.254)	(0.233)	
(3) log dividend yield	-0.079	-0.085	-0.217	0.976	-0.392	0.249	-4.224	-0.834	0.326	0.970
	(-1.689)	(-1.210)	(-1.614)	(76.533)	(-0.520)	(0.197)	(-1.523)	(-1.378)	(0.387)	
(4) log inflation	0.001	0.007	-0.006	0.000	0.086	0.235	-0.169	-0.017	-0.026	0.080
	(0.349)	(1.409)	(-0.632)	(-0.112)	(1.352)	(2.573)	(-0.911)	(-0.392)	(-0.437)	
(5) log short rate	0.000	0.000	-0.003	0.000	-0.001	1.002	0.029	-0.007	-0.002	0.990
	(-1.857)	(-0.388)	(-3.089)	(-1.900)	(-0.228)	(139.333)	(1.628)	(-1.630)	(-0.597)	
(6) log yield spread	0.000	0.001	0.002	0.000	0.000	-0.012	0.945	0.012	0.001	0.931
	(1.136)	(2.681)	(1.678)	(1.356)	(0.048)	(-1.625)	(53.560)	(2.807)	(0.336)	
(7) log credit spread	-0.001	-0.009	0.011	0.000	0.022	0.075	0.051	0.915	0.002	0.885
	(-1.171)	(-3.739)	(2.509)	(-1.588)	(1.132)	(1.959)	(0.636)	(49.452)	(0.032)	
(8) EVAR	-0.003	0.000	-0.003	-0.001	-0.007	-0.071	-0.191	-0.018	0.656	0.406
	(-1.576)	(0.066)	(-0.277)	(-2.178)	(-0.208)	(-0.734)	(-1.126)	(-0.542)	(4.880)	

Panel C1: Residual correlation matrix (scaled)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) log stock excess returns	15.022	0.124	-0.911	0.088	-0.016	-0.045	-0.158	-0.510
(2) log bond excess returns	0.124	5.823	-0.119	0.009	-0.319	-0.362	0.396	-0.056
(3) log dividend yield	-0.911	-0.119	16.927	-0.045	0.030	0.047	0.135	0.396
(4) log inflation	0.088	0.009	-0.045	1.168	0.022	-0.008	-0.066	-0.172
(5) log short rate	-0.016	-0.319	0.030	0.022	0.094	-0.719	-0.120	-0.138
(6) log yield spread	-0.045	-0.362	0.047	-0.008	-0.719	0.097	-0.246	0.129
(7) log credit spread	-0.158	0.396	0.135	-0.066	-0.120	-0.246	0.490	0.074
(8) EVAR	-0.510	-0.056	0.396	-0.172	-0.138	0.129	0.074	1.247

Panel C2: Residual correlation matrix (unscaled)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) log stock excess returns	17.015	0.115	-0.908	0.118	0.013	-0.064	-0.144	-0.481
(2) log bond excess returns	0.115	6.559	-0.112	-0.015	-0.302	-0.392	0.454	-0.059
(3) log dividend yield	-0.908	-0.112	19.209	-0.073	0.011	0.061	0.123	0.347
(4) log inflation	0.118	-0.015	-0.073	1.301	0.056	-0.019	-0.107	-0.212
(5) log short rate	0.013	-0.302	0.011	0.056	0.102	-0.711	-0.129	-0.182
(6) log yield spread	-0.064	-0.392	0.061	-0.019	-0.711	0.108	-0.273	0.170
(7) log credit spread	-0.144	0.454	0.123	-0.107	-0.129	-0.273	0.596	0.014
(8) EVAR	-0.481	-0.059	0.347	-0.212	-0.182	0.170	0.014	1.699

Table H3. Estimates of VAR(1) Model with Stochastic Volatility (France)

Panel A: Forecasting Monthly Realized Variance (RVAR)

Intercept	log stock excess returns	log bond excess returns	log dividend yield	log inflation	log short rate	log yield spread	log credit spread	RVAR	Rsq
0.006	-0.016	0.003	0.001	-0.048	-0.043	0.123	-0.023	0.558	0.376
1.911	-3.481	0.178	1.434	-0.631	-0.280	0.271	-0.380	5.624	

Panel B: VAR Estimates

Second Stage	Coefficients on lagged variables									
	Intercept	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	Rsq
(1) log stock excess returns	0.032	0.050	0.477	0.009	0.721	-0.238	2.166	0.289	-0.759	0.037
	(0.642)	(0.656)	(2.026)	(0.726)	(0.696)	(-0.134)	(0.380)	(0.338)	(-0.698)	
(2) log bond excess returns	0.022	-0.025	0.076	0.007	-0.584	0.580	1.386	0.171	0.179	0.067
	(1.708)	(-1.341)	(1.220)	(2.199)	(-2.424)	(1.259)	(0.808)	(0.634)	(0.533)	
(3) log dividend yield	-0.104	-0.020	-0.568	0.967	-0.109	-0.940	-2.378	-0.775	1.003	0.937
	(-1.797)	(-0.251)	(-2.282)	(65.397)	(-0.095)	(-0.505)	(-0.387)	(-0.825)	(0.768)	
(4) log inflation	0.002	0.004	0.003	0.000	-0.033	0.224	0.332	-0.048	-0.044	0.062
	(0.992)	(1.394)	(0.274)	(0.552)	(-0.564)	(2.590)	(1.364)	(-1.182)	(-0.905)	
(5) log short rate	0.000	0.000	-0.001	0.000	-0.005	0.997	0.178	-0.030	-0.010	0.994
	(1.949)	(-1.245)	(-1.499)	(0.761)	(-1.445)	(189.147)	(3.686)	(-4.641)	(-2.354)	
(6) log yield spread	-0.001	0.001	0.000	0.000	0.010	-0.005	0.810	0.029	0.009	0.930
	(-2.696)	(2.013)	(-0.034)	(-2.095)	(2.408)	(-0.646)	(15.245)	(4.072)	(1.575)	
(7) log credit spread	0.003	-0.001	-0.017	0.001	0.029	-0.094	-0.151	0.947	0.026	0.907
	(1.792)	(-0.253)	(-2.065)	(1.415)	(0.890)	(-1.759)	(-0.624)	(26.491)	(0.590)	
(8) EVAR	0.005	-0.001	-0.008	0.001	-0.009	-0.048	0.045	-0.019	0.569	0.349
	(2.351)	(-0.276)	(-0.602)	(1.679)	(-0.179)	(-0.462)	(0.153)	(-0.463)	(5.550)	

Panel C1: Residual correlation matrix (scaled)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) log stock excess returns	19.079	0.093	-0.858	-0.033	-0.017	-0.074	-0.079	-0.612
(2) log bond excess returns	0.093	5.065	-0.025	-0.140	-0.138	-0.545	0.046	0.044
(3) log dividend yield	-0.858	-0.025	21.879	0.132	-0.017	0.064	0.094	0.532
(4) log inflation	-0.033	-0.140	0.132	0.908	0.069	0.060	0.022	-0.013
(5) log short rate	-0.017	-0.138	-0.017	0.069	0.073	-0.715	-0.294	-0.033
(6) log yield spread	-0.074	-0.545	0.064	0.060	-0.715	0.092	0.252	-0.007
(7) log credit spread	-0.079	0.046	0.094	0.022	-0.294	0.252	0.630	0.128
(8) EVAR	-0.612	0.044	0.532	-0.013	-0.033	-0.007	0.128	1.124

Panel C2: Residual correlation matrix (unscaled)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) log stock excess returns	22.277	0.049	-0.870	-0.027	-0.001	-0.056	-0.086	-0.579
(2) log bond excess returns	0.049	5.879	0.009	-0.165	-0.130	-0.555	-0.021	0.022
(3) log dividend yield	-0.870	0.009	25.504	0.114	-0.024	0.044	0.095	0.505
(4) log inflation	-0.027	-0.165	0.114	1.037	0.092	0.054	0.033	-0.007
(5) log short rate	-0.001	-0.130	-0.024	0.092	0.083	-0.713	-0.295	-0.065
(6) log yield spread	-0.056	-0.555	0.044	0.054	-0.713	0.106	0.286	0.024
(7) log credit spread	-0.086	-0.021	0.095	0.033	-0.295	0.286	0.732	0.177
(8) EVAR	-0.579	0.022	0.505	-0.007	-0.065	0.024	0.177	1.479

Table H4. Estimates of VAR(1) Model with Stochastic Volatility (Germany)

Panel A: Forecasting Monthly Realized Variance (RVAR)

Intercept	log stock excess returns	log bond excess returns	log dividend yield	log inflation	log short rate	log yield spread	log credit spread	RVAR	Rsq
-0.009	-0.019	-0.006	-0.002	0.036	0.666	0.214	0.179	0.456	0.388
-1.389	-3.680	-0.394	-1.461	0.448	2.106	0.565	2.244	7.912	

Panel B: VAR Estimates

Second Stage	Coefficients on lagged variables									
	Intercept	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	Rsq
(1) log stock excess returns	0.280	0.092	0.157	0.059	-0.750	-12.280	-4.784	-2.532	0.830	0.050
	(3.435)	(1.240)	(0.656)	(3.110)	(-0.832)	(-3.440)	(-1.051)	(-3.153)	(0.502)	
(2) log bond excess returns	-0.014	-0.042	0.064	-0.003	-0.274	0.751	2.029	0.225	-0.010	0.057
	(-0.824)	(-2.398)	(1.068)	(-0.724)	(-1.230)	(0.888)	(1.930)	(1.142)	(-0.026)	
(3) log dividend yield	-0.287	-0.094	-0.166	0.930	0.547	7.074	-1.003	1.518	-0.364	0.925
	(-3.341)	(-1.099)	(-0.647)	(46.235)	(0.583)	(1.878)	(-0.213)	(1.773)	(-0.197)	
(4) log inflation	0.004	0.000	-0.007	0.001	-0.141	0.336	-0.449	0.030	-0.171	0.068
	(0.845)	(0.058)	(-0.696)	(0.545)	(-2.331)	(1.826)	(-1.960)	(0.688)	(-2.523)	
(5) log short rate	0.001	0.000	-0.003	0.000	0.000	0.966	-0.005	-0.009	-0.006	0.994
	(2.483)	(-0.655)	(-4.340)	(1.933)	(0.101)	(93.908)	(-0.424)	(-3.389)	(-1.524)	
(6) log yield spread	-0.001	0.001	0.001	0.000	0.003	0.029	0.996	0.007	0.007	0.942
	(-1.920)	(2.953)	(1.737)	(-1.506)	(0.943)	(2.212)	(59.915)	(2.394)	(1.367)	
(7) log credit spread	0.007	-0.007	-0.015	0.001	-0.045	-0.422	-0.613	0.884	0.044	0.929
	(1.940)	(-2.527)	(-1.356)	(1.371)	(-1.165)	(-2.288)	(-3.082)	(19.430)	(0.729)	
(8) EVAR	-0.012	-0.002	-0.008	-0.003	0.005	0.806	0.178	0.200	0.426	0.421
	(-3.017	(-0.651	(-0.738	(-3.186	0.108	4.109	0.741	4.112	5.813	

Panel C1: Residual correlation matrix (scaled)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) log stock excess returns	21.207	-0.083	-0.875	0.080	0.035	0.001	-0.287	-0.686
(2) log bond excess returns	-0.083	4.775	0.073	-0.119	-0.298	-0.569	0.384	0.113
(3) log dividend yield	-0.875	0.073	23.262	-0.041	-0.010	-0.006	0.255	0.601
(4) log inflation	0.080	-0.119	-0.041	1.130	-0.001	0.124	-0.079	-0.081
(5) log short rate	0.035	-0.298	-0.010	-0.001	0.051	-0.547	-0.132	-0.049
(6) log yield spread	0.001	-0.569	-0.006	0.124	-0.547	0.067	-0.253	-0.040
(7) log credit spread	-0.287	0.384	0.255	-0.079	-0.132	-0.253	0.881	0.472
(8) EVAR	-0.686	0.113	0.601	-0.081	-0.049	-0.040	0.472	1.043

Panel C2: Residual correlation matrix (unscaled)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) log stock excess returns	26.729	-0.124	-0.891	0.068	0.096	-0.016	-0.320	-0.653
(2) log bond excess returns	-0.124	5.851	0.113	-0.125	-0.314	-0.550	0.388	0.112
(3) log dividend yield	-0.891	0.113	29.213	-0.029	-0.075	0.014	0.280	0.580
(4) log inflation	0.068	-0.125	-0.029	1.339	0.009	0.118	-0.076	-0.090
(5) log short rate	0.096	-0.314	-0.075	0.009	0.063	-0.555	-0.153	-0.096
(6) log yield spread	-0.016	-0.550	0.014	0.118	-0.555	0.081	-0.241	-0.002
(7) log credit spread	-0.320	0.388	0.280	-0.076	-0.153	-0.241	1.106	0.496
(8) EVAR	-0.653	0.112	0.580	-0.090	-0.096	-0.002	0.496	1.377

Table H5. Estimates of VAR(1) Model with Stochastic Volatility (Japan)

Panel A: Forecasting Monthly Realized Variance (RVAR)

Intercept	log stock excess returns	log bond excess returns	log dividend yield	log inflation	log short rate	log yield spread	log credit spread	RVAR	Rsq
0.011	-0.018	-0.028	0.002	0.058	0.260	0.746	-0.170	0.325	0.183
2.328	-3.316	-0.809	1.523	0.894	1.412	0.881	-2.926	5.167	

Panel B: VAR Estimates

Second Stage	Coefficients on lagged variables									
	Intercept	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	Rsq
(1) log stock excess returns	0.046	0.129	0.149	0.012	-0.572	0.434	5.300	0.603	0.040	0.024
	(0.765)	(1.717)	(0.695)	(0.965)	(-0.899)	(0.136)	(0.725)	(0.673)	(0.019)	
(2) log bond excess returns	0.038	-0.035	0.141	0.010	-0.045	0.752	7.478	-0.113	0.009	0.095
	(2.354)	(-1.713)	(2.229)	(2.707)	(-0.291)	(0.980)	(3.369)	(-0.552)	(0.018)	
(3) log dividend yield	-0.071	-0.216	-0.278	0.976	1.013	-1.855	-14.450	-0.492	-3.145	0.981
	(-1.019)	(-2.180)	(-1.022)	(65.218)	(1.427)	(-0.435)	(-1.605)	(-0.494)	(-1.112)	
(4) log inflation	0.004	-0.003	0.001	0.001	0.178	0.449	0.245	-0.059	-0.224	0.062
	(1.141)	(-0.539)	(0.081)	(0.891)	(3.638)	(2.043)	(0.490)	(-1.009)	(-1.491)	
(5) log short rate	0.000	0.000	-0.001	0.000	0.002	0.984	-0.006	0.000	0.001	0.992
	(-1.765)	(-0.127)	(-2.335)	(-1.945)	(0.780)	(116.578)	(-0.533)	(0.323)	(0.158)	
(6) log yield spread	0.000	0.000	-0.001	0.000	-0.002	0.007	0.929	0.002	-0.002	0.930
	(-1.547)	(1.342)	(-1.088)	(-1.877)	(-0.897)	(0.688)	(34.118)	(0.635)	(-0.326)	
(7) log credit spread	0.008	-0.004	-0.026	0.002	0.018	0.027	0.836	0.847	-0.020	0.776
	(3.019)	(-1.741)	(-2.682)	(2.794)	(0.824)	(0.244)	(2.537)	(23.371)	(-0.317)	
(8) EVAR	0.008	-0.001	-0.001	0.001	0.016	0.255	0.140	-0.149	0.286	0.254
	(3.565)	(-0.273)	(-0.074)	(1.718)	(0.707)	(2.472)	(0.490)	(-5.290)	(3.617)	

Panel C1: Residual correlation matrix (scaled)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) log stock excess returns	19.846	0.003	-0.863	0.042	-0.044	-0.002	-0.047	-0.690
(2) log bond excess returns	0.003	4.927	-0.015	0.015	-0.186	-0.742	0.265	-0.232
(3) log dividend yield	-0.863	-0.015	22.935	-0.004	0.068	0.005	0.024	0.617
(4) log inflation	0.042	0.015	-0.004	1.466	0.035	-0.017	-0.031	0.119
(5) log short rate	-0.044	-0.186	0.068	0.035	0.038	-0.408	-0.099	0.037
(6) log yield spread	-0.002	-0.742	0.005	-0.017	-0.408	0.059	-0.183	0.180
(7) log credit spread	-0.047	0.265	0.024	-0.031	-0.099	-0.183	0.765	-0.111
(8) EVAR	-0.690	-0.232	0.617	0.119	0.037	0.180	-0.111	0.765

Panel C2: Residual correlation matrix (unscaled)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) log stock excess returns	26.162	0.035	-0.871	0.032	-0.057	-0.028	-0.026	-0.690
(2) log bond excess returns	0.035	6.326	-0.051	0.022	-0.178	-0.739	0.264	-0.217
(3) log dividend yield	-0.871	-0.051	30.721	-0.013	0.081	0.032	0.009	0.610
(4) log inflation	0.032	0.022	-0.013	1.882	0.040	-0.038	-0.014	0.121
(5) log short rate	-0.057	-0.178	0.081	0.040	0.051	-0.426	-0.096	0.022
(6) log yield spread	-0.028	-0.739	0.032	-0.038	-0.426	0.077	-0.185	0.165
(7) log credit spread	-0.026	0.264	0.009	-0.014	-0.096	-0.185	0.995	-0.084
(8) EVAR	-0.690	-0.217	0.610	0.121	0.022	0.165	-0.084	1.073

Table H6. Estimates of VAR(1) Model with Stochastic Volatility (United Kingdom)

Panel A: Forecasting Monthly Realized Variance (RVAR)

Intercept	log stock excess returns	log bond excess returns	log dividend yield	log inflation	log short rate	log yield spread	log credit spread	RVAR	Rsq
-0.001	-0.016	0.011	0.000	0.059	-0.045	-0.231	0.149	0.362	0.305
-0.139	-2.045	1.470	-0.323	1.687	-0.229	-0.644	3.643	2.862	

Panel B: VAR Estimates

Second Stage	Coefficients on lagged variables									
	Intercept	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	Rsq
(1) log stock excess returns	0.139	-0.011	0.227	0.037	-0.298	-1.661	-0.936	-0.595	0.138	0.042
	(2.588)	(-0.151)	(2.039)	(2.608)	(-0.491)	(-1.051)	(-0.278)	(-1.106)	(0.079)	
(2) log bond excess returns	0.023	0.038	-0.037	0.007	-0.085	0.202	2.167	-0.782	3.033	0.081
	(1.141)	(0.826)	(-0.474)	(1.410)	(-0.280)	(0.280)	(1.540)	(-2.711)	(2.832)	
(3) log dividend yield	-0.100	0.003	-0.209	0.971	0.138	0.422	-0.911	0.394	-0.631	0.952
	(-1.728)	(0.043)	(-1.746)	(62.577)	(0.226)	(0.254)	(-0.258)	(0.636)	(-0.316)	
(4) log inflation	0.007	-0.005	-0.001	0.001	0.113	0.147	-0.244	0.057	-0.289	0.088
	(1.661)	(-0.798)	(-0.063)	(1.171)	(2.228)	(1.374)	(-0.995)	(1.379)	(-2.137)	
(5) log short rate	0.000	-0.001	-0.002	0.000	0.004	1.000	0.021	0.002	-0.029	0.996
	(-0.625)	(-1.787)	(-2.490)	(-0.903)	(1.343)	(161.178)	(1.325)	(0.829)	(-3.537)	
(6) log yield spread	0.000	0.000	0.001	0.000	-0.003	-0.009	0.954	0.006	0.005	0.956
	(-0.027)	(0.872)	(1.352)	(0.017)	(-0.703)	(-1.188)	(51.926)	(2.017)	(0.467)	
(7) log credit spread	0.002	-0.005	-0.002	0.000	-0.015	0.055	0.053	0.853	0.305	0.916
	(0.619)	(-1.106)	(-0.263)	(0.558)	(-0.474)	(0.781)	(0.279)	(17.814)	(1.889)	
(8) EVAR	-0.002	0.000	-0.005	-0.001	0.007	0.009	-0.181	0.132	0.405	0.509
	(-0.770)	(-0.069)	(-1.031)	(-1.189)	(0.410)	(0.098)	(-1.015)	(4.809)	(3.341)	

Panel C1: Residual correlation matrix (scaled)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) log stock excess returns	15.321	0.163	-0.909	-0.022	-0.087	-0.080	-0.069	-0.572
(2) log bond excess returns	0.163	7.155	-0.150	-0.071	-0.269	-0.458	0.300	0.091
(3) log dividend yield	-0.909	-0.150	17.178	0.071	0.072	0.088	0.075	0.515
(4) log inflation	-0.022	-0.071	0.071	1.398	0.073	0.057	-0.083	0.041
(5) log short rate	-0.087	-0.269	0.072	0.073	0.071	-0.562	-0.076	-0.045
(6) log yield spread	-0.080	-0.458	0.088	0.057	-0.562	0.094	-0.396	-0.173
(7) log credit spread	-0.069	0.300	0.075	-0.083	-0.076	-0.396	0.927	0.536
(8) EVAR	-0.572	0.091	0.515	0.041	-0.045	-0.173	0.536	0.823

Panel C2: Residual correlation matrix (unscaled)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) log stock excess returns	18.215	0.088	-0.904	-0.018	-0.058	-0.086	-0.076	-0.539
(2) log bond excess returns	0.088	8.436	-0.092	-0.032	-0.260	-0.432	0.212	0.058
(3) log dividend yield	-0.904	-0.092	20.521	0.087	0.050	0.106	0.059	0.457
(4) log inflation	-0.018	-0.032	0.087	1.669	0.085	0.054	-0.138	-0.010
(5) log short rate	-0.058	-0.260	0.050	0.085	0.081	-0.557	-0.087	-0.055
(6) log yield spread	-0.086	-0.432	0.106	0.054	-0.557	0.109	-0.382	-0.135
(7) log credit spread	-0.076	0.212	0.059	-0.138	-0.087	-0.382	1.168	0.555
(8) EVAR	-0.539	0.058	0.457	-0.010	-0.055	-0.135	0.555	1.138

Table H7. Estimates of VAR(1) Model with Stochastic Volatility (United States)

Panel A: Forecasting Monthly Realized Variance (RVAR)

Intercept	log stock excess returns	log bond excess returns	log dividend yield	log inflation	log short rate	log yield spread	log credit spread	RVAR	Rsq
-0.016	-0.020	-0.005	-0.003	-0.052	0.600	1.069	0.352	0.344	0.240
-2.485	-3.661	-0.280	-2.843	-1.051	1.683	1.455	2.895	1.955	

Panel B: VAR Estimates

Second Stage	Coefficients on lagged variables									
	Intercept	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	Rsq
(1) log stock excess returns	0.077	-0.110	0.037	0.016	-0.716	-1.537	-2.546	0.701	-3.295	0.054
	(1.288)	(-1.299)	(0.288)	(1.305)	(-1.108)	(-0.650)	(-0.597)	(0.507)	(-1.620)	
(2) log bond excess returns	-0.005	-0.063	0.046	-0.001	-0.615	1.052	2.936	-0.259	0.314	0.074
	(-0.193)	(-1.871)	(0.790)	(-0.200)	(-2.002)	(1.099)	(1.730)	(-0.440)	(0.357)	
(3) log dividend yield	-0.044	0.071	-0.023	0.988	0.951	-0.585	1.743	-1.179	2.065	0.981
	(-0.711)	(0.824)	(-0.177)	(80.168)	(1.468)	(-0.240)	(0.399)	(-0.788)	(0.966)	
(4) log inflation	-0.008	-0.008	-0.014	-0.002	0.384	0.508	0.511	0.278	-0.418	0.291
	(-1.594)	(-1.260)	(-1.771)	(-1.582)	(6.756)	(2.759)	(1.648)	(2.263)	(-2.002)	
(5) log short rate	-0.002	-0.002	0.000	0.000	-0.001	1.074	0.227	0.030	-0.069	0.968
	(-4.119)	(-2.735)	(0.131)	(-3.672)	(-0.103)	(59.545)	(6.074)	(2.611)	(-3.592)	
(6) log yield spread	0.002	0.003	-0.001	0.000	0.008	-0.095	0.731	-0.030	0.072	0.793
	(3.443)	(3.299)	(-0.659)	(3.189)	(1.092)	(-3.813)	(15.620)	(-1.835)	(2.643)	
(7) log credit spread	0.005	0.000	0.003	0.001	-0.012	-0.169	-0.325	0.829	0.191	0.943
	(2.476)	(-0.124)	(1.491)	(2.159)	(-0.821)	(-2.224)	(-2.542)	(14.742)	(1.964)	
(8) EVAR	-0.014	0.005	0.001	-0.003	-0.004	0.481	0.793	0.262	0.507	0.488
	(-2.708)	(0.930)	(0.103)	(-2.803)	(-0.134)	(2.385)	(2.151)	(2.270)	(2.331)	

Panel C1: Residual correlation matrix (scaled)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) log stock excess returns	14.892	-0.035	-0.964	-0.053	-0.023	0.025	-0.159	-0.664
(2) log bond excess returns	-0.035	6.150	0.014	-0.133	0.018	-0.473	0.089	0.003
(3) log dividend yield	-0.964	0.014	15.242	0.065	0.043	-0.031	0.152	0.637
(4) log inflation	-0.053	-0.133	0.065	0.952	-0.089	0.135	-0.296	-0.169
(5) log short rate	-0.023	0.018	0.043	-0.089	0.127	-0.874	0.068	0.050
(6) log yield spread	0.025	-0.473	-0.031	0.135	-0.874	0.154	-0.102	-0.032
(7) log credit spread	-0.159	0.089	0.152	-0.296	0.068	-0.102	0.298	0.463
(8) EVAR	-0.664	0.003	0.637	-0.169	0.050	-0.032	0.463	0.805

Panel C2: Residual correlation matrix (unscaled)

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
(1) log stock excess returns	18.864	-0.026	-0.967	-0.032	-0.007	0.009	-0.153	-0.625
(2) log bond excess returns	-0.026	7.668	0.002	-0.220	0.095	-0.527	0.157	0.052
(3) log dividend yield	-0.967	0.002	19.352	0.049	0.020	-0.008	0.144	0.595
(4) log inflation	-0.032	-0.220	0.049	1.236	-0.144	0.217	-0.420	-0.283
(5) log short rate	-0.007	0.095	0.020	-0.144	0.157	-0.880	0.138	0.090
(6) log yield spread	0.009	-0.527	-0.008	0.217	-0.880	0.196	-0.180	-0.085
(7) log credit spread	-0.153	0.157	0.144	-0.420	0.138	-0.180	0.438	0.570
(8) EVAR	-0.625	0.052	0.595	-0.283	0.090	-0.085	0.570	1.153