Online Appendix to

"Endowments, Factor Prices, and Skill-Biased Technology: Importing Development Accounting into HOV"

Appendix A. A Helpman and Krugman (1985) Model

This section shows that our estimating equations can also be generated using a model of monopolistic competition, increasing returns to scale, and costless trade.

Appendix A.1. Preferences, Endowments and Technology

Let $i,j = 1,\ldots,N$ index countries, let $g = 1,\ldots,G$ index goods (or industries), and let $\omega \in \Omega_{gi}$ index varieties of a horizontally differentiated good g produced in country i as in Krugman (1980). Preferences are internationally identical and homothetic with the nested structure:

$$U = \prod_{g=1}^{G} (U_g)^{\eta_g} \quad \text{and} \quad U_g = \left(\sum_{i=1}^{N} \int_{\omega \in \Omega_{gi}} x_{gi}(\omega)^{\frac{\rho_g - 1}{\rho_g}} d\omega \right)^{\frac{\rho_g}{\rho_g - 1}},$$

where $\rho_g > 1$ is the elasticitiy of substitution, $\eta_g > 0$, $\Sigma_g \eta_g = 1$, and $x_{gi}(\omega)$ is a quantity consumed. Let $p_{gi}(\omega)$ be the corresponding price. We assume that trade is costless so that all consumers worldwide face the same price $p_{gi}(\omega)$. Then the price index associated with U_g is $P_g = \left(\sum_{i=1}^{N} \int_{\omega \in \Omega_{gi}} p_{gi}(\omega)^{1-\rho_g} d\omega \right)^{\frac{1}{1-\rho_g}}$.

Let $f = 1,\ldots,K$ index primary factors such as labor. V_{fi} is country i's exogenous endowment of factor f and w_{fi} is its price. Let $\mathbf{w}_i = (w_{1i},\ldots,w_{Ki})$. We assume that factors are mobile across industries within a country and immobile across countries.

Turning to technology, a firm uses both primary factors and intermediate inputs of goods $h = 1,\ldots,G$. The production function is Cobb-Douglas in (*a*) an index of primary factors and (*b*) CES indexes of each of the G intermediate goods. This results in a unit cost function for $\omega \in \Omega_{gi}$ of the form

$$\phi_{gi}(\mathbf{w}_i,\mathbf{p}) = \left[c_{gi}(\mathbf{w}_i) \right]^{\gamma_{g0}} \prod_{h=1}^{G} P_{gh}^{\gamma_{gh}} \tag{36}$$

where

$$P_{gh} = \left(\sum_{j=1}^{N} \int_{\nu \in \Omega_{hj}} \alpha_{gh} \; p_{hj}(\nu)^{1-\rho_h} \, d\nu \right)^{\frac{1}{1-\rho_h}},$$

$\mathbf{p} = \{p_{hj}(\nu) : \nu \in \Omega_{hj}, \forall h,j\}$ is the set of all product prices, $\nu \in \Omega_{hj}$ indexes varieties when used as inputs, and the γ_{gh} are positive constants with $\Sigma_{h=0}^{G} \gamma_{gh} = 1$. $c_{gi}(\mathbf{w}_i)$ is a constant-returns-to-scale unit cost function associated with primary factors. P_{gh} is the unit cost function associated with the CES index of intermediate good h in the production of good g. The α_{gh} are constants that allow for empirically relevant factor intensity asymmetries.[45]

Marginal costs are $\phi_{gi}(\mathbf{w}_i,\mathbf{p})$. Per variety variable costs are $\phi_{gi}(\mathbf{w}_i,\mathbf{p})q_{gi}(\omega)$. We assume that fixed costs are proportional to marginal costs and given by $\phi_{gi}(\mathbf{w}_i,\mathbf{p})\overline{\phi}_g$ for some constant $\overline{\phi}_g > 0$.

Appendix A.2. Firm Behavior

Profits for any variety $\omega \in \Omega_{gi}$ are

$$\left[p_{gi} - \phi_{gi}(\mathbf{w}_i,\mathbf{p}) \right] q_{gi} - \phi_{gi}(\mathbf{w}_i,\mathbf{p})\overline{\phi}_g.$$

[45] We assume that a firm does not buy from itself. Since anything it bought from itself would have zero measure, we do not have to keep track of this in the expression for P_{gh}; however, we will have to keep track of this in the discussion of profit maximization below.

There are two sources of demand for $\omega \in \Omega_{gi}$: (1) Consumers in country j demand $p_{gi}{}^{-\rho_g} P_g^{\rho_g-1} \eta_g Y_j$ where Y_j is national income. (2) Downstream producers of variety $\nu \in \Omega_{hj}$ each demand $b_{ij}(g,h)\left[q_{hj}+\overline{\phi}_h\right]$ where, by Shephard's Lemma,

$$b_{ij}(g,h;\mathbf{w}_j,\mathbf{p}) = \frac{\partial \phi_{hj}(\mathbf{w}_j,\mathbf{p})}{\partial p_{gi}}.$$

$b_{ij}(g,h;\mathbf{w}_j,\mathbf{p})$ is necessarily complicated notation because we need to track the entire global supply chain. Aggregating over both final and intermediate-input demands for a typical variety $\omega \in \Omega_{gi}$ yields the following result that will be useful later:

Lemma 1 *$q_{gi} = p_{gi}^{-\rho_g}\kappa_g$ for some $\kappa_g > 0$ and all i.*

The proof appears in the appendix to this section. The last line of the proof is an expression for κ_g, from which it is apparent that κ_g is a constant from the firm's perspective.

Using lemma 1, profit maximization for $\omega \in \Omega_{gi}$ yields the standard optimal price:

$$p_{gi} = \frac{\rho_g}{\rho_g - 1}\phi_{gi}(\mathbf{w}_i,\mathbf{p}). \tag{37}$$

Zero profits for $\omega \in \Omega_{gi}$, together with this pricing rule, yield:

$$q_{gi} = (\rho_g - 1)\overline{\phi}_g\ . \tag{38}$$

Turning to factor markets, consider the demand for factor f by firm $\omega \in \Omega_{gi}$. By Shephard's Lemma this (direct) demand per unit of output is

$$d_{fgi}(\mathbf{w}_i,\mathbf{p}) = \frac{\partial \phi_{gi}(\mathbf{w}_i,\mathbf{p})}{\partial w_{fi}}.$$

Factor market clearing in country i is thus

$$V_{fi} = \sum_{g=1}^{G} n_{gi} d_{fgi}(\mathbf{w}_i,\mathbf{p})\left[q_{gi}+\overline{\phi}_g\right] \tag{39}$$

where $n_{gi} = \int_{\omega\in\Omega_{gi}} d\omega$ is the measure of identical firms producing varieties of g in country i.

Appendix A.3. Equilibrium

Define $\mathbf{n}^* = \left\{n_{gi}^*\right\}_{\forall g,i}$, $\mathbf{w}^* = \left\{w_{fi}^*\right\}_{\forall f,i}$, and $\mathbf{p}^* = \{p_{gi}^*(\omega)\colon \omega \in \Omega_{gi},\ \forall g,i\}$. An equilibrium is a triplet $(\mathbf{w}^*,\mathbf{p}^*,\mathbf{n}^*)$ such that when consumers maximize utility and firms maximize profits, product markets clear internationally for each variety, factor markets clear nationally for each factor, and profits are zero. Market clearing and zero profits imply that all income is factor income ($Y_i = \Sigma_f w_{fi} V_{fi}$) and that trade is balanced. It follows from this definition of equilibrium that $(\mathbf{w}^*,\mathbf{p}^*,\mathbf{n}^*)$ is an equilibrium if it satisfies equations (37)–(39). [46]

[46] From equation (38), firm output $q_{gi} = q_g$ is independent of i. Since $q_g = p_{gi}^{-\rho_g}\kappa_g$, it follows that price $p_{gi} = p_g$ is also independent of i. As discussed in Remark 1 of the appendix to this section, this plays no role and is for expositional simplicity.

Appendix A.4. Empirical Specification

This subsection shows how the alternate model of the previous section delivers the empirical specifications in the main text. Assume that the cost function for primary factors is

$$c_{gi}(\mathbf{w}_i) = \left[\sum_f \frac{\alpha_{fg}}{\delta_{gi}} \left(\frac{w_{fi}}{\pi_{fi}} \right)^{1-\sigma} \right]^{\frac{1}{1-\sigma}} \tag{40}$$

where the α_{fg} control (exogenous) factor intensities, the π_{fi} are factor-augmenting productivity (technology) parameters, σ is the elasticity of substitution, and the δ_{gi} are Ricardian technology parameters. We will assume that cost functions satisfy equation (40) for the remainder of this appendix. When the $\delta_{gi} = 1$ for all g and i, equation (40) is a special case of Trefler's (1993) factor-augmenting technology and PFPE is straightforwardly defined as

$$\frac{w_{fi}}{\pi_{fi}} = \frac{w_{f,\text{us}}}{\pi_{f,\text{us}}} . \tag{41}$$

We consider a diversified equilibrium in which the Ricardian technology differences δ_{gi} lead to failure of PFPE.[47] It is straightforward to show that there are δ_{gi} which support a diversified equilibrium, but we will need to ensure that our empirical counterparts of δ_{gi}, $\widehat{\delta}_{gi}$, are consistent with such an equilibrium. Here we review several minor points about diversification. First, the δ_{gi} can be interpreted as differences in quality, in which case our diversification has the flavour of Schott (2004).[48] Schott provides abundant evidence of diversification in his analysis of 'product overlap' at the 10-digit HS level. Second, we can treat observed diversification as trade in varieties rather than as a function of aggregation bias. Third, country-level productivity can be loaded onto either the π_{fi} or the δ_{gi} so a normalization is needed. As in th main text, we normalize the δ_{gi} using $\delta_{g,\text{us}} = 1$ for all g and $\Sigma_g \theta_{Lgi} \delta_{gi} = 1$ $\forall i$ where θ_{Lgi} is the share of country i's total labor endowment employed in industry g.

If varieties of good g are produced both by country i and by the United States, then Shephard's lemma implies

$$d_{fgi} = \beta_{fi} d_{fg,\text{us}} / \delta_{gi} \tag{42}$$

where

$$\beta_{fi} \equiv \left(\frac{w_{fi}/\pi_{fi}}{w_{f,\text{us}}/\pi_{f,\text{us}}} \right)^{-\sigma} \left(\frac{\pi_{fi}}{\pi_{f,\text{us}}} \right)^{-1} . \tag{43}$$

Appendix A.5. The Three Estimating Equations

We now show that the above model delivers three estimating equations that are identical to those derived in the main text. Consider first the Vanek equation. Recall that $F_{fi} = \mathbf{D}_f (\mathbf{I}_{NG} - \mathbf{B})^{-1} \mathbf{T}_i$ is the factor content of trade using observed factor usage $\mathbf{D}_f$. Let $\mathbf{D}_f(\beta_f)$ be a $1 \times GN$ matrix with typical element $\beta_{fi} d_{fg,\text{us}} / \delta_{gi}$ (the right-hand side of equation 42) and define

$$F_{fi}(\beta_f) \equiv \mathbf{D}_f \left(\beta_f \right) \left[\mathbf{I}_{NG} - \mathbf{B} \right]^{-1} \mathbf{T}_i . \tag{44}$$

(We suppress the δ_{gi} as arguments.) Under our cost function assumption (equation 40), $\mathbf{D}_f(\beta_f)$ equals the data $\mathbf{D}_f$ and $F_{fi}(\beta_f)$ equals the data F_{fi}. It follows that the Vanek equation becomes

$$F_{fi}(\beta_f) = V_{fi} - s_i \sum_{j=1}^{N} V_{fj} . \tag{V}$$

[47] δ_{gi} prevents international goods price equalization from leading to international productivity adjusted factor price equalization.

[48] This is similar to calibrating 'wedges' e.g., Hottman, Redding and Weinstein (2014) infer quality as the wedge that rationalizes demand for a given set of prices and quantities.

As in the main text, because $F_{fi}(\beta_f)$ is linear in $\mathbf{D}_f(\beta_f)$, $\mathbf{D}_f(\beta_f)$ is linear in β_f, (therefore) $F_{fi}(\beta_f)$ is linear in β_f, and equation (V) can be written as a system of linear equations that uniquely solve for the vector β_f.

Turning to the Wage equation, substitute factor demands (equation 42) into the factor-market clearing condition (equation 39) and solve for productivity adjusted wages to obtain

$$\frac{w_{fi}/\pi_{fi}}{w_{f,\text{us}}/\pi_{f,\text{us}}} = \left[\frac{\pi_{f,\text{us}}V_{f,\text{us}}}{\pi_{fi}V_{fi}}\right]^{1/\sigma}\left(\sum_{g=1}^{G}\frac{d_{fg,\text{us}}Q_{gi}}{\delta_{gi}V_{f\text{us}}}\right)^{1/\sigma} . \tag{45}$$

See the appendix for a proof. Rearranging this equation yields the “Wage Equation":

$$W_{fi}(\mathbf{D}_f, \mathbf{Q}, V_{fi}, \delta) \equiv \left[\sum_{g=1}^{G}\frac{d_{fg,\text{us}}Q_{gi}}{\delta_{gi}V_{fi}}\right]^{-1} = \beta_{fi} \tag{W}$$

where $\delta \equiv \{\delta_{gi}\}_{\forall g,i}$ and $W_{fi}()$ is a function.

Turning to the third and last equation, the Techniques equation, we aggregate equation (42) up to the same level as the Vanek and Wage equations, namely, at the factor-country level. Specifically, taking the employment-weighted average of equation (42) yields $\sum_g \theta_{fgi}d_{fgi}/d_{fg,\text{us}} = \sum_g \theta_{fgi}\beta_{fi}/\delta_{gi}$ where θ_{fgi} is the share of V_{fi} that is employed in industry g. The θ_{fgi} are data and satisfy $\sum_g \theta_{fgi} = 1$. Rearranging to isolate β_{fi} yields

$$T_{fi}(\mathbf{D}_f, \delta) \equiv \frac{\sum_{g=1}^{G}\theta_{fgi}\left(d_{fgi}/d_{fg,\text{us}}\right)}{\sum_{g=1}^{G}\theta_{fgi}/\delta_{gi}} = \beta_{fi} \tag{T}$$

where $T_{fi}()$ is a function.

While the interpretation of δ_{gi} is different than in the main text, our strategy to calibrate it remains the same. A in the main text, we use the normalizations $\delta_{g,\text{us}} = 1$ and $\sum_g \theta_{Lgi}\delta_{gi} = 1$. From equation (42), $\delta_{gi} = (d_{fg,\text{us}}/d_{fgi})\beta_{fi}$. Hence, $\delta_{gi} = \delta_{gi}/\sum_g \theta_{Lgi}\delta_{gi} = (d_{fg,\text{us}}/d_{fgi})/\sum_g \theta_{Lgi}(d_{fg,\text{us}}/d_{fgi})$. This establishes that we can calibrate the δ_{gi} using data on factor usages d_{fgi}.[49] Note that since the calibrated δ_{gi} satisfy equation (13), they are consistent with a diversified equilibrium. As in the main text, this calibration of δ_{gi} depends on f and so is not unique. As in the main text, we work with the geometric mean of the two: $\delta_{gi} = [(d_{Ug,\text{us}}/d_{Ugi})\beta_{Ui}]^{1/2}[(d_{Sg,\text{us}}/d_{Sgi})\beta_{Si}]^{1/2}$. This yields

$$\widehat{\delta}_{gi} \equiv \frac{(d_{Ug,\text{us}}/d_{Ugi})^{1/2}(d_{Sg,\text{us}}/d_{Sgi})^{1/2}}{\sum_{g'=1}^{G}\theta_{Lg'i}(d_{Ug',\text{us}}/d_{Ug'i})^{1/2}(d_{Sg',\text{us}}/d_{Sg'i})^{1/2}} . \tag{46}$$

[49] Intuitively, a Ricardian technology difference $\delta_{gi}/\delta_{g,\text{us}}$ is the average difference in input requirements $d_{fg,\text{us}}/d_{fgi}$ after purging them of their factor-augmenting productivity and wage components β_{fi}.

Appendix to Appendix A

Proof of Lemma 1 We start with a preliminary result involving change of indexes. From equation (36), $\phi_{hj}(\mathbf{w}_j,\mathbf{p}) = \left[c_{hj}(\mathbf{w}_j)\right]^{\gamma_{h0}} \prod_{g=1}^{G} P_{hg}^{\gamma_{hg}}$ where $P_{hg} = \left(\sum_{i=1}^{N} \int_{\omega\in\Omega_{gi}} \alpha_{hg}\ p_{gi}(\omega)^{1-\rho_g}\ d\omega\right)^{\frac{1}{1-\rho_g}}$. Also, note that $\partial P_{hg'}^{\gamma_{hg'}}/\partial p_{gi}(\omega) = 0$ for $g' \neq g$ and $\partial P_{hg}^{\gamma_{hg}}/\partial p_{gi}(\omega) = \gamma_{hg} P_{hg}^{\gamma_{hg}-1+\rho_g}\ \alpha_{hg}\ p_{gi}(\omega)^{-\rho_g}$. Hence

$$\begin{aligned} b_{ij}(g,h,\mathbf{w}_j,\mathbf{p}) &= \partial\phi_{hj}(\mathbf{w}_j,\mathbf{p})/\partial p_{gi}(\omega) \\ &= c_{hj}^{\gamma_{g0}} \prod_{g'\neq g} P_{hg'}^{\gamma_{hg'}} \left[\gamma_{hg} P_{hg}^{\gamma_{hg}-1+\rho_g} \alpha_{hg} p_{gi}(\omega)^{-\rho_g}\right] \\ &= \phi_{hj}(\mathbf{w}_j,\mathbf{p})\gamma_{hg} P_{hg}^{-1+\rho_g} \alpha_{hg} p_{gi}(\omega)^{-\rho_g}\ . \end{aligned} \tag{47}$$

As explained in section (Appendix A.2), demand for variety $\omega \in \Omega_{gi}$ is the sum of demands for final goods and intermediate inputs: $q_{gi}(\omega) = p_{gi}(\omega)^{-\rho_g} P_g^{\rho_g-1} \eta_g \Sigma_{j=1}^N Y_j + \Sigma_{h=1}^G \Sigma_{j=1}^N \int_{\nu\in\Omega_{hj}} b_{ij}(g,h,\mathbf{w}_j,\mathbf{p}) \left[q_{hj}(\nu) + \overline{\phi}_h\right] d\nu$. Substituting in equation (47), the lemma follows with $\kappa_g \equiv P_g^{\rho_g-1}\eta_g \Sigma_{j=1}^N Y_j + \Sigma_{h=1}^G \Sigma_{j=1}^N \int_{\nu\in\Omega_{hj}} \phi_{hj}(\mathbf{w}_j,\mathbf{p})\gamma_{hg} P_{hg}^{-1+\rho_g} \alpha_{hg} \left[q_{hj}(\nu) + \overline{\phi}_h\right] d\nu$. ■

Proof of Equation** (42)**: By Shephard's lemma, $d_{fgi} = \partial\phi_{gi}/\partial w_{fi}$. Hence, $d_{fgi} = \left[\partial\phi_{gi}/\partial c_{gi}\right]\left[\partial c_{gi}/\partial w_{fi}\right] = \left[\gamma_{g0} c_{gi}^{-1} \phi_{gi}\right] \left[\alpha_{fg}\left(w_{fi}\right)^{-\sigma}\left(\pi_{fi}\right)^{\sigma-1}\left(\delta_{gi}\right)^{-1}\left(c_{gi}\right)^{\sigma}\right]$. Recall that at the end of section Appendix A.3 we established that $p_{gi} = p_g$. Hence from equation (37), $1 = p_{gi}/p_{g,\text{us}} = \phi_{gi}(\mathbf{w}_i,\mathbf{p})/\phi_{g,\text{us}}(\mathbf{w}_{\text{us}},\mathbf{p}) = \left(c_{gi}^{\gamma_{g0}} \prod_{h=1}^G P_{gh}^{\gamma_{gh}}\right) / \left(c_{g,\text{us}}^{\gamma_{g0}} \prod_{h=1}^G P_{gh}^{\gamma_{gh}}\right) = \left(c_{gi}/c_{g,\text{us}}\right)^{\gamma_{g0}}$ or $c_{gi} = c_{g,\text{us}}$ and $\phi_{gi}(\mathbf{w}_i,\mathbf{p}) = \phi_{g,\text{us}}(\mathbf{w}_{\text{us}},\mathbf{p})$. Hence, $d_{fgi}/d_{fg,\text{us}} = (w_{fi}/w_{f,\text{us}})^{-\sigma}(\pi_{fi}/\pi_{f,\text{us}})^{\sigma-1}(\delta_{gi}/\delta_{g,\text{us}})^{-1}$. Equation (42) follows with the normalization $\delta_{g,\text{us}} = 1$. ■

Remark 1 *If $p_{gi} \neq p_{g,us}$ then we get $d_{fgi} = d_{fg,us}(w_{fi}/w_{f,us})^{-\sigma}(\pi_{fi}/\pi_{f,us})^{\sigma-1}(\delta'_{gi}/\delta'_{g,us})^{-1}$ where $\delta'_{gi} = \delta_{gi}/p_{gi}$. That is, this leads to a reinterpretation of the δ_{gi}, but does not otherwise affect anything in the paper.*

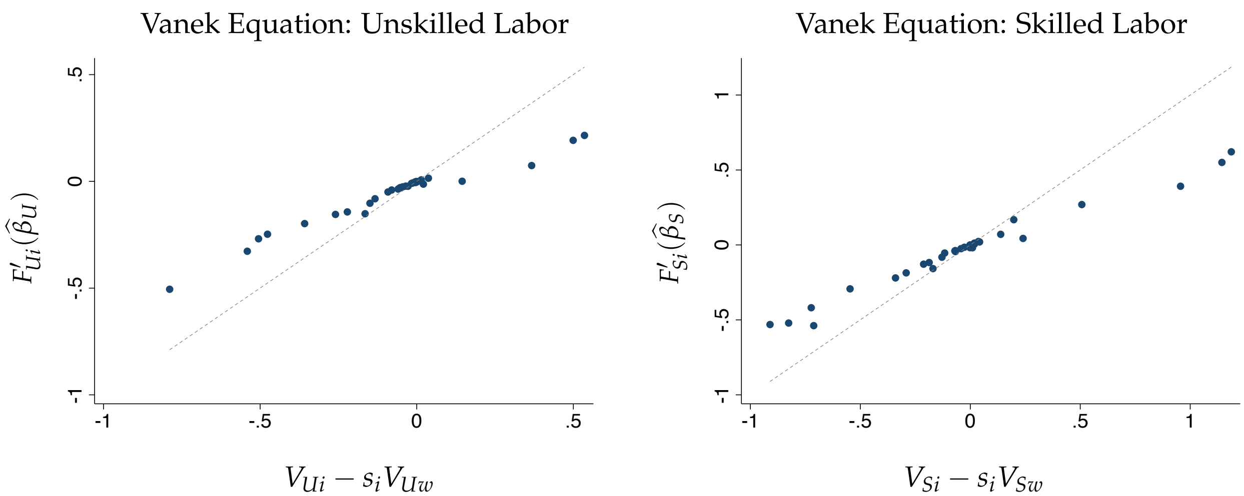
Proof of Equation** (W)**: Recall that $Q_{gi} = n_{gi}(q_g + \overline{\phi}_g)$. Plugging this into the factor-market clearing equation (39) yields $V_{fi} = \Sigma_g d_{fgi} Q_{gi}$. Substituting in $d_{fgi} = d_{fg,\text{us}}\beta_{fi}/\delta_{gi}$ (equation 13) into this expression delivers $V_{fi} = \beta_{fi}\Sigma_g d_{fg,\text{us}} Q_{gi}/\delta_{gi}$. Equation ($W$) follows from a simple re-arrangement. ■

Appendix B. Additional Empirical Results

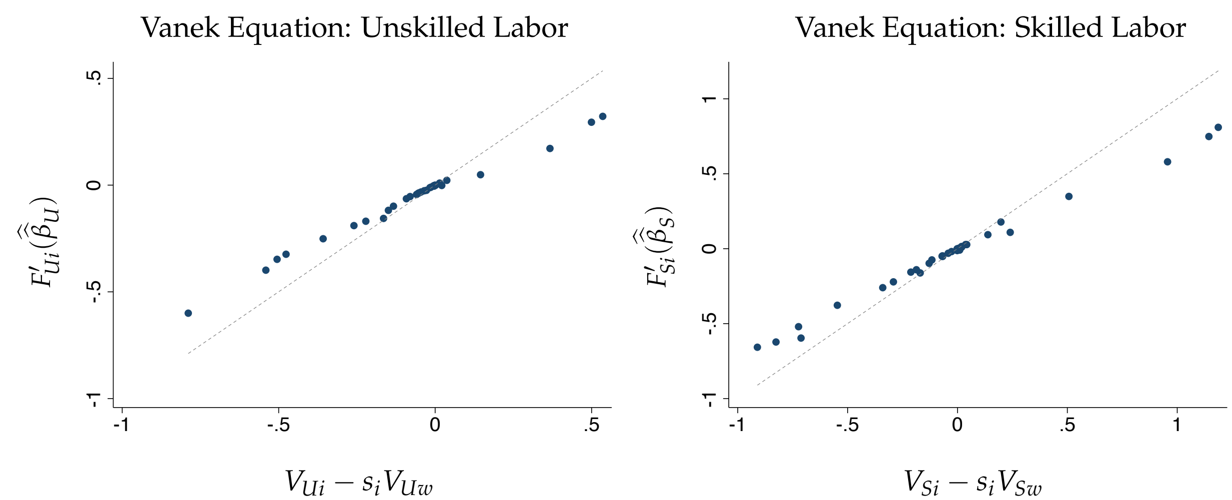
This section contains additional empirical results. The top row of figure A1 presents the same results for the Vanek equation as in figures 3 and 6 except that the observations for China and the United States are removed for visual exposition. The top row of figure A2 presents results for the Vanek equation placing all weight on the Wage equation, and then bottom row of figure A2 presents results for the Vanek equation placing all weight on the Techniques equation. Figure A3 presents results for the Wage equation, Techniques equation, Vanek equation, and disaggregated techniques equation when we use three types of labor (high skilled, medium skilled, and low skilled). Table A1 presents test statistics associated with these three types of labor analogous to Table 1. Figure A4 presents results for the Wage equation, Techniques equation, Vanek equation, and disaggregated techniques equation for capital and labor. Table A2 presents test statistics associated with this specification.

Figure A1: The Vanek Equation with Outliers (US and China) not Displayed

Panel A. Two-Equation Approach ($\widehat{\beta}_f$, Equations W and T)



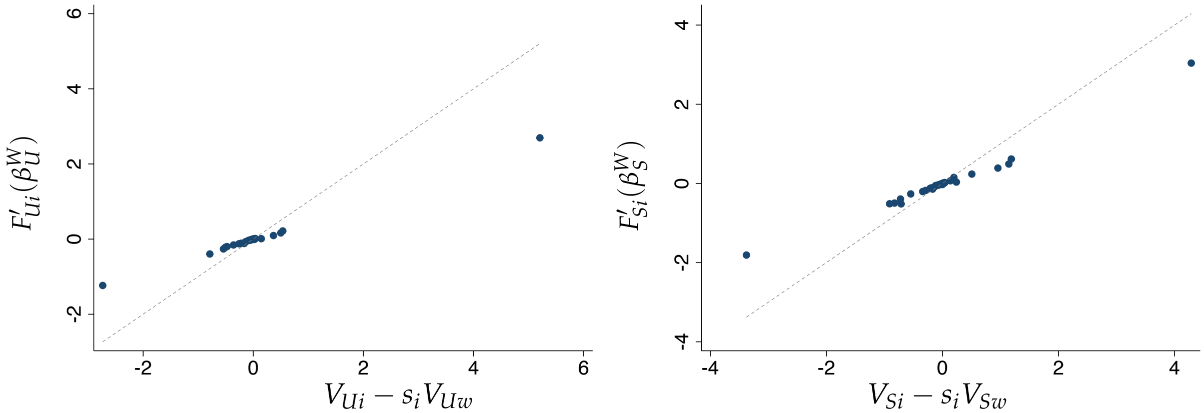
Panel B. Three-Equation Approach ($\widehat{\widehat{\beta}}_f$, Equations W, T and V)



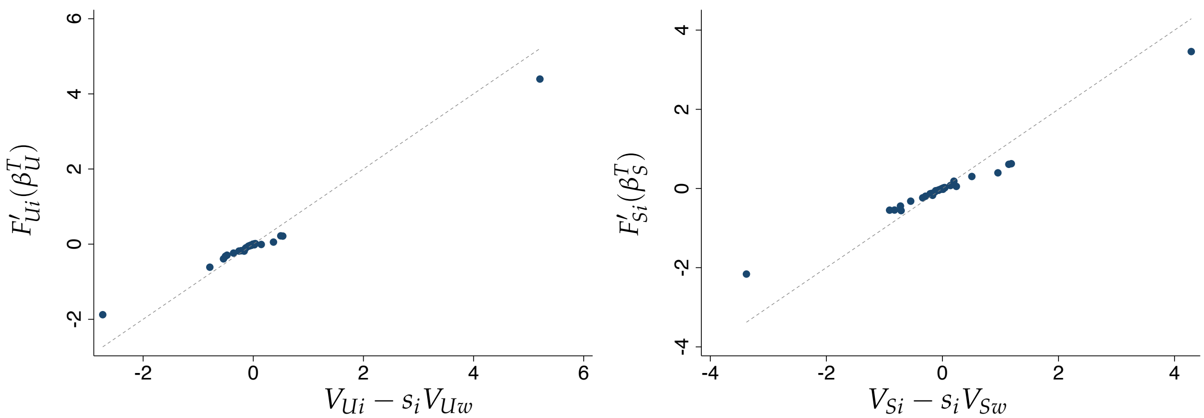
Notes: These plots are the same as those appearing in the main text except that the two outliers (China and United States) are not displayed so as to 'unpack' the remaining observations. Panel A corresponds to the bottom row of figure 3. It plots $V_{fi} - s_i V_{fw}$ against the Government Services adjusted factor content of trade (evaluated at the two-equation estimate of β_f, $\widehat{\beta}_f$). Panel B corresponds to the unskilled and skilled figures in the bottom row of figure 6. It plots $V_{fi} - s_i V_{fw}$ against the Government Services adjusted factor content of trade (evaluated at the three-equation estimate of β_f, $\widehat{\widehat{\beta}}_f$). The left panels are for unskilled labor and the right panels are for skilled labor. All lines are 45° lines.

Figure A2: Performance of the Vanek Equation Using β_{fi}^{W} and β_{fi}^{T}

Productivity Calibrated to the Wage Equation: β_f^W

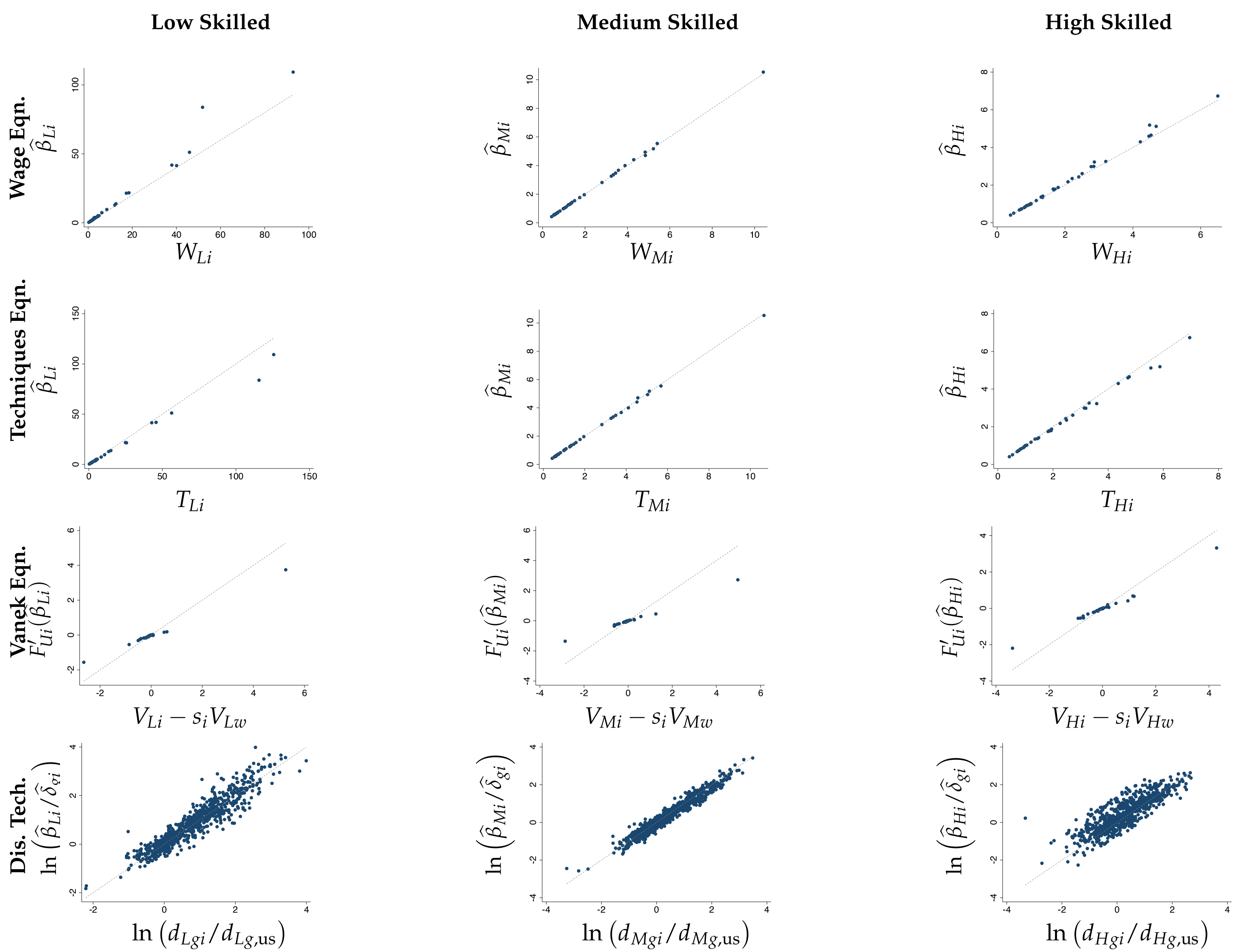


Productivity Calibrated to the Techniques Equation: β_f^T



Notes: Each panel plots $V_{fi} - s_i V_{fw}$ against the factor content of trade $F'_{fi}(\beta_f)$ from equation (21) i.e., adjusted for nontradable Government Services. In the top row, the β_f that makes the Wage equation fit perfectly (β_f^W) is plugged into the Vanek equation. This yields a very good fit of the Vanek equation for both unskilled labor (left panel) and skilled labor (right panel). In the bottom row, the β_f that makes the Techniques equation fit perfectly (β_f^T) is plugged into the Vanek equation. This again yields a very good fit of the Vanek equation. Each point is a country and all lines are 45° lines.

Figure A3: Three Types of Labor



Notes: The left-hand column plots are for least skilled labor, the middle column plot is for medium skilled labor, and the right column plots are for high skilled labor. The top row is the Wage equation (W), the second row is the Techniques equation (T), and the third row is the Vanek equation (V), and the fourth row are relative factor demands from equation (13), namely, $\ln(d_{fgi}/d_{fg,\text{us}}) = \ln(\beta_{fi}/\delta_{gi})$. All equations are evaluated at the two-equation estimates of the β_{fi} and calibrated values of the δ_{gi}. Each observation is a factor and country except for the bottom row which is factor-country-industry specific. The 45° line is displayed in each panel.

Table A1: Test Statistics for the Fit of the Vanek Equation

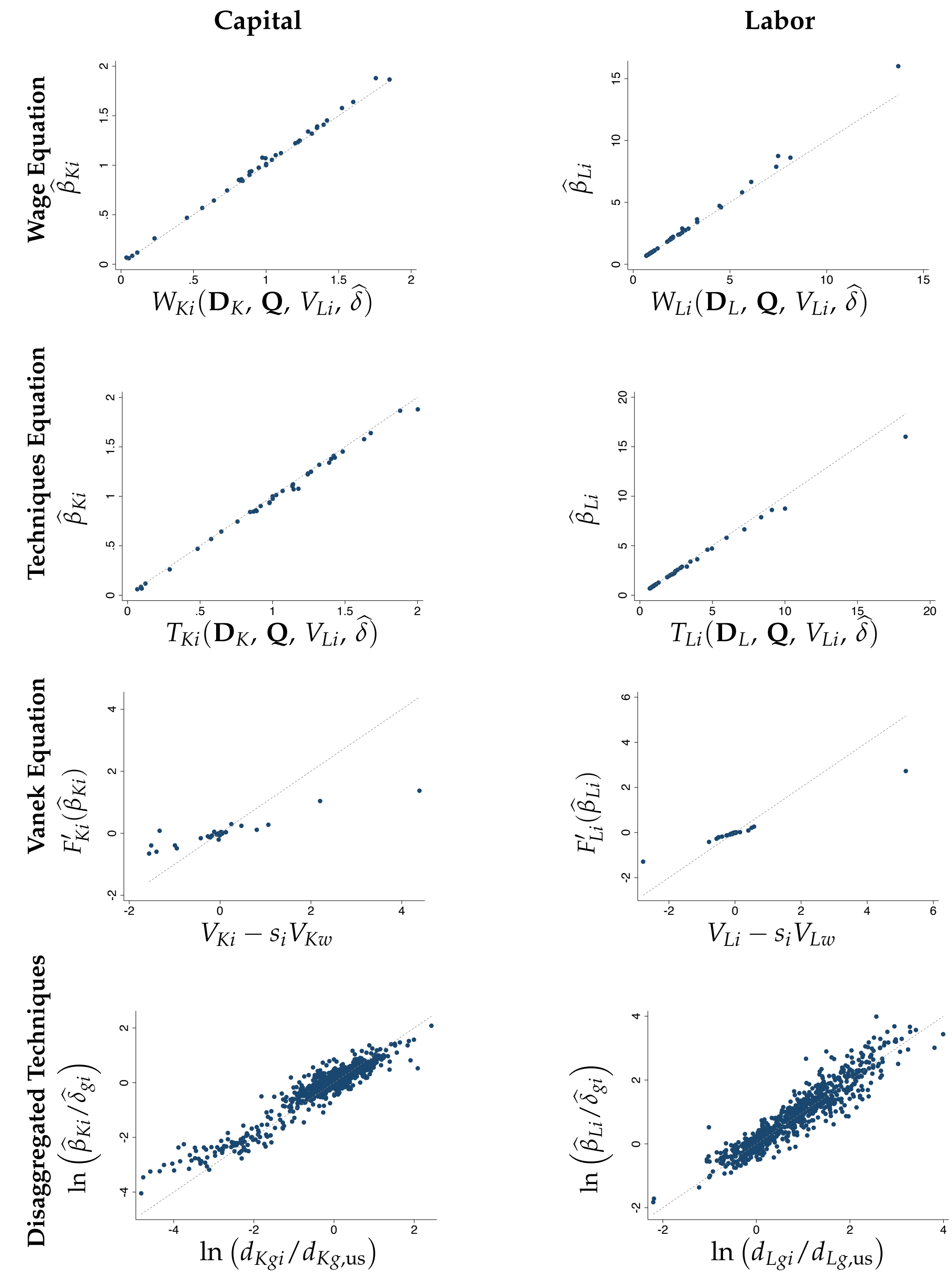
Factor Content of Trade	Least Skilled Labor: Rank Corr.	Variance Ratio	Sign Test	Slope Test
	(1)	(2)	(3)	(4)
1. F_{fi}	0.980	0.040	0.974	0.199
2. $F_{fi}(\widehat{\beta}_f)$	0.983	0.120	0.974	0.344
3. $F'_{fi}(\widehat{\beta}_f)$	0.962	0.463	0.921	0.677
4. $F_{fi}(\widehat{\beta}_f)$	0.982	0.162	0.974	0.400
5. $F'_{fi}[\iota]$	0.493	0.000	0.002	0.005
6. $F'_{fi}(\widehat{\widehat{\beta}}_f)$	0.998	0.618	0.974	0.784

Factor Content of Trade	Medium Skilled Labor: Rank Corr.	Variance Ratio	Sign Test	Slope Test
	(1)	(2)	(3)	(4)
1. F_{fi}	0.972	0.033	0.895	0.181
2. $F_{fi}(\widehat{\beta}_f)$	0.968	0.035	0.895	0.185
3. $F'_{fi}(\widehat{\beta}_f)$	0.995	0.271	1.000	0.518
4. $F_{fi}(\widehat{\beta}_f)$	0.964	0.072	0.921	0.264
5. $F'_{fi}[\iota]$	0.405	0.005	0.553	0.064
6. $F'_{fi}(\widehat{\widehat{\beta}}_f)$	0.998	0.462	1.000	0.679

Factor Content of Trade	High Skilled Labor: Rank Corr.	Variance Ratio	Sign Test	Slope Test
	(1)	(2)	(3)	(4)
1. F_{fi}	0.974	0.016	0.947	0.127
2. $F_{fi}(\widehat{\beta}_f)$	0.941	0.044	0.842	0.202
3. $F'_{fi}(\widehat{\beta}_f)$	0.992	0.496	0.947	0.699
4. $F_{fi}(\widehat{\beta}_f)$	0.902	0.206	0.842	0.421
5. $F'_{fi}[\iota]$	0.179	0.036	0.500	0.154
6. $F'_{fi}(\widehat{\widehat{\beta}}_f)$	0.995	0.642	0.974	0.799

Notes: These tables presents test statistics for the fit of the Vanek equation (V) for different specifications of the factor content of trade. The top six rows are for least skilled labor. In row 1, the actual factor content of trade is used. In row 2, the factor content of trade is calculated using $\widehat{\beta}_f$ (the two-equation estimate of β_f) and equation (16). In row 3, the factor content of trade is adjusted for nontraded Government Services using equation (21). In row 4, the nontraded Government Services adjustment is put on the right-hand side of the Vanek equation as in part 1 of lemma 2 and as in Davis and Weinstein (2001). In row 5, the factor content of trade is again adjusted for nontraded Government Services using equation (21), but all elements of the vector $\widehat{\beta}_f$ are set to 1. In row 6, the factor content of trade is again adjusted for nontraded Government Services using equation (21), but the estimate of β_f is from the three-equation approach. The middle six rows repeat this exercise for medium skilled labor. The bottom six rows repeat it for high skilled labor. 'Rank Corr.' is the rank or Spearman correlation between the factor content of trade and $V_{fi} - s_i V_{fw}$. 'Variance Ratio' is the variance of the factor content of trade divided by the variance of $V_{fi} - s_i V_{fw}$. 'Sign Test' is the proportion of observations for which the factor content of trade and $V_{fi} - s_i V_{fw}$ have the same sign. 'Slope Test' is the OLS slope estimate from a regression of the factor content of trade on $V_{fi} - s_i V_{fw}$.

Figure A4: Capital and Labor



Notes: The left-hand side plots are for capital, the right-hand side plots are for labor. The top panels are the Wage equation (W). The upper middle panels are the Techniques equations (T). The lower middle panels are the Vanek equations (V). The bottom panels are relative factor demands from equation (13), namely, $\ln(d_{fgi}/d_{fg,\text{us}}) = \ln(\beta_{fi}/\delta_{gi})$. All equations are evaluated at the estimated values $\widehat{\beta}_{fi}$ and calibrated values $\widehat{\delta}_{gi}$. In the top and middle panels, each observation is a factor and country (f,i) while in the bottom panels each observation is a factor, industry and country (f,g,i). The 45° line is displayed in each panel.

Table A2: Test Statistics for the Fit of the Vanek Equation: Capital and Labor

Factor Content of Trade	Capital Rank Corr. (1)	Capital Variance Ratio (2)	Capital Sign Test (3)	Capital Slope Test (4)	Labor Rank Corr. (5)	Labor Variance Ratio (6)	Labor Sign Test (7)	Labor Slope Test (8)
1. F_{fi}	0.845	0.059	0.789	0.228	0.948	0.036	0.816	0.189
2. $F_{fi}(\widehat{\beta}_f)$	0.741	0.093	0.763	0.187	0.933	0.065	0.842	0.253
3. $F'_{fi}(\widehat{\beta}_f)$	0.816	0.126	0.868	0.332	0.957	0.261	0.868	0.509
4. $F_{fi}(\widehat{\beta}_f)$	0.685	0.124	0.711	0.246	0.927	0.107	0.868	0.324
5. $F'_{fi}[\iota]$	-0.027	0.020	0.553	-0.052	0.074	0.000	0.447	0.006
6. $F'_{fi}(\widehat{\widehat{\beta}}_f)$	0.973	0.315	0.947	0.554	0.980	0.453	0.947	0.673

Notes: This table presents test statistics for the fit of the Vanek equation (V) for different specifications of the factor content of trade. In row 1, the actual factor content of trade is used. In row 2, the factor content of trade is calculated using $\widehat{\beta}_f$ (the two-equation estimate of β_f) and equation (16). In row 3, the factor content of trade is adjusted for nontraded Government Services using equation (21). In row 4, the nontraded Government Services adjustment is put on the right-hand side of the Vanek equation as in part 1 of lemma 2 and as in Davis and Weinstein (2001). In row 5, the factor content of trade is again adjusted for nontraded Government Services using equation (21), but all elements of the vector $\widehat{\beta}_f$ are set to 1. In row 6, the factor content of trade is again adjusted for nontraded Government Services using equation (21), but the estimate of β_f is from the three-equation approach. 'Rank Corr.' is the rank or Spearman correlation between the factor content of trade and $V_{fi} - s_i V_{fw}$. 'Variance Ratio' is the variance of the factor content of trade divided by the variance of $V_{fi} - s_i V_{fw}$. 'Sign Test' is the proportion of observations for which the factor content of trade and $V_{fi} - s_i V_{fw}$ have the same sign. 'Slope Test' is the OLS slope estimate from a regression of the factor content of trade on $V_{fi} - s_i V_{fw}$.

Appendix C. Small Changes in β_f^{VT}

We establish here that when one places the unknown productivity parameters on the 'right-hand side' as in equation (VT), the performance of the Vanek equation is extremely sensitive to small changes in the vector β_f.[50] Start by defining the *predicted* factor content of trade using this approach as $F^*_{fi}(\beta_f) \equiv \beta_{fi}^{-1} V_{fi} - s_i \sum_j \beta_{fj}^{-1} V_{fj}$. Trefler (1993a) derives a model in which F^{us}_{fi} is the measured factor content of trade with no Ricardian productivity differences and all techniques set equal to their US values, and $F^*_{fi}(\beta_f)$ is its predicted value (the 'predicted factor content'). See footnote 32. We now show that the relationship between F^{us}_{fi} and $F^*_{fi}(\beta_f)$ is extremely sensitive to differences in β_f even if those differences are very small.

Start by defining two productivity vectors for unskilled labor: β_U^W and β_U^{VT}–the latter of which makes the Vanek equation fit perfectly with productivity terms on the right-hand side such that $F^{us}_{Ui} = F^*_{Ui}(\beta_U^{VT})$–and define the (small) difference between the two as $\varepsilon_U = \left(\beta_U^W\right)^{-1} - \left(\beta_U^{VT}\right)^{-1}$. Because $F^*_{Ui}(\beta_U)$ is linear in its arguments, $F^*_{Ui}(\beta_U^W) - F^*_{Ui}(\beta_U^{VT}) = F^*_{Ui}(\varepsilon_U)$ or

$$F^*_{Ui}(\beta_U^W) - F^*_{Ui}(\beta_U^{VT}) = \varepsilon_{Ui} V_{Ui} - s_i \sum_{j=1}^{N} \varepsilon_{Uj} V_{Uj}$$

where ε_{Ui} is the ith element of ε_U. Now consider the variance of the right-hand side. Suppose that the ε_{Ui} are purely random variables with mean 0 and small variance $\sigma^2_{\varepsilon U} \approx 0.02$.[51] Then the right-hand side is 0 on average. Its variance is $\sigma^2_{\varepsilon U} \mathrm{var}[V_{Ui} - s_i \sum_j V_{Uj}]$. Let σ^2_{FU} and σ^2_{VU} be the variances of F^{us}_{fi} and $V_{Ui} - s_i \sum_j V_{Uj}$, respectively, where the variation is across observations i. Because missing trade is so severe, the variance ratio is $\sigma^2_{FU}/\sigma^2_{VU} = 0.0001$. Hence the variance of the right-hand side is $\sigma^2_{\varepsilon U}\sigma^2_{VU} = \sigma^2_{\varepsilon U}\sigma^2_{FU}/0.0001 = 200\sigma^2_{FU}$! Thus, even though $\sigma^2_{\varepsilon U}$ is small, the right-hand side has a large variance relative to the variance of what is to be explained (σ^2_{FU}). Restated, $F^*_{Ui}(\beta_U^W)$ and $F^*_{Ui}(\beta_U^{VT})$ may be equal on average, but because of missing trade, there is a large variance between the two. Right-hand side approaches are like drunk dart players: Every dart completely misses the dartboard, but if you average them you get a bull's-eye.

This establishes that the function $F^*_{fi}(\beta_f)$ is very sensitive to the choice of β_f. It thus explains the discrepancy in results between the approaches of Trefler (1993a) and Gabaix (1997b). Even though they generate similar values of the β_f, they generate very different predictions for the Vanek equation. Similarly, very large differences in the measured factor content of trade can result in very similar differences in β_f. This helps to explain the famous result of Gabaix (1997b) in which he shows that setting the measured factor content of trade to zero or setting it equal to its additive inverse affects the resulting values of β_{fi} very little. This reason for the substantial disagreements regarding the performance of RHS approaches (Trefler, 1993a, Gabaix, 1997b) is new to the literature and serves as a caveat for interpreting results.

[50] This is an additional reason to place the β_f terms on the 'left-hand side' as we do in our main analysis. Note that in figure A2, the performance of the Vanek equation is *not* sensitive to the small differences between β_f^W and β_f^T.

[51] This is the variance of the deviations between $(\beta_{Ui}^W)^{-1}$ and $(\beta_{Ui}^{VT})^{-1}$