Appendix: Monetary Policy Drivers of Bond and Equity Risks

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This document is a supplemental appendix to Campbell, Pflueger, and Viceira (2015). The contents of this appendix are as follows:

1. Section A provides details for the model solution. It discusses equilibrium selection criteria and provides the details of the numerical solution of bond and stock returns.

2. Section B provides additional model details. It re-derives the log-linearized Phillips curve following the framework of Smets and Wouters (2003). It also approximates log habit as a distributed lag of log consumption and provides additional details on how the preferences in this paper compare to Wachter (2006).

3. Section C shows robustness to allowing for a small probability of regime switches.

4. Section D provides additional empirical results. It calibrates the model with an additional regime break in 1987 to coincide with Alan Greenspan's appointment as Fed chairman. It also shows that estimated monetary policy rules are robust to excluding the financial crisis and to using a real time measure of the output gap. Section D also shows that the bond-stock correlation has the opposite sign from the inflation-output gap correlation in all three subperiods.

A Model Solution

We solve the model with an additional shock v_t, driving a wedge between the output gap and consumption and trend consumption growth g

$$c_t = gt + \tau \left(x_t + (1-\phi)[x_{t-1} + x_{t-2} + ...]\right) + v_t. \tag{1}$$

The error term $v_t \sim N(0, \sigma_v^2)$ is conditionally homoskedastic white noise uncorrelated with current or lagged consumption, or any other information variables known in advance. If there are shocks to potential output, which increase consumption relative to the output gap, the shock v_t would capture such shocks. We set $\sigma_v^2 = 0$ for all equilibria in the main paper.

It follows that consumption satisfies

$$c_t = g + c_{t-1} + \tau \left(x_t - \phi x_{t-1}\right) + v_t - v_{t-1}. \tag{2}$$

The Euler equation becomes

$$x_t = \underbrace{\frac{-ln\delta + \gamma g - \frac{\gamma^2\sigma_c^2}{2}\left(1+\lambda(\bar{s})\right)^2}{\gamma(\tau\phi - \theta_1)}}_{m_x} - \underbrace{\frac{1}{\gamma(\tau\phi-\theta_1)}}_{\psi} r_t + \underbrace{\frac{\tau}{\tau\phi-\theta_1}}_{\rho^{x+}} E_t x_{t+1} + \underbrace{\frac{\theta_2}{\tau\phi-\theta_1}}_{\rho^{x-}} x_{t-1}$$

$$+ \underbrace{-\frac{v_t}{\tau\phi-\theta_1}}_{u_t^{IS}}. \quad (3)$$

The dynamics of the log surplus consumption ratio can be written as

$$\hat{s}_t = s_t - \bar{s}, \quad (4)$$

$$\hat{s}_t = \theta_0 \hat{s}_{t-1} + \theta_1 \hat{x}_{t-1} + \theta_2 \hat{x}_{t-2} + \lambda(\hat{s}_{t-1} + \bar{s}, \bar{S}) Q_M u_t, \quad (5)$$

$$Q_M = \tau e_1 Q - (\tau\phi - \theta_1) e_1. \quad (6)$$

It then follows that the volatility of consumption surprises is given by

$$\sigma_c^2 = Q_M \Sigma_u Q_M'. \quad (7)$$

We assume that x_t is the demeaned output gap, i.e. it has mean zero. The steady state real short-term interest rate at $x_t = 0$ and $s_t = \bar{s}$ is then exactly as in Campbell and Cochrane (1999)

$$\bar{r} = \gamma g - 0.5\gamma^2\sigma_c^2/\bar{S}^2 - log(\delta). \quad (8)$$

The steady-state surplus consumption ratio and the sensitivity function λ are given by

$$\bar{S} = \sigma_c \sqrt{\frac{\gamma}{1-\theta_0}}, \quad (9)$$

$$\bar{s} = log(\bar{S}), \quad (10)$$

$$s_{max} = \bar{s} + 0.5(1 - \bar{S}^2), \quad (11)$$

$$\lambda(\hat{s}_t, \bar{S}) = \lambda_0\sqrt{1 - 2\hat{s}_t} - 1, \hat{s}_t \leq s_{max} - \bar{s}, \quad (12)$$

$$\lambda(\hat{s}_t, \bar{S}) = 0, \hat{s}_t \geq s_{max} - \bar{s} \quad (13)$$

$$\lambda_0 = \frac{1}{\bar{S}}. \quad (14)$$

A.1 Macroeconomic Dynamics

Let π_t^* denote the central bank's inflation target at time t. We solve the model in terms of the output gap x_t and inflation and nominal interest rate gaps:

$$\hat{\pi}_t = \pi_t - \pi_t^*, \quad (15)$$

$$\hat{i}_t = i_t - \pi_t^*. \quad (16)$$

Denote the vector of state variables by:

$$\hat{Y}_t = [x_t, \hat{\pi}_t, \hat{i}_t]'. \tag{17}$$

We can re-write the model dynamics in terms of the state vector $\hat{Y}_t$:

$$\begin{aligned}
x_t &= \rho^{x-} x_{t-1} + \rho^{x+} E_t x_{t+1} - \psi \left(\hat{i}_t - E_t \hat{\pi}_{t+1} \right) + u_t^{IS}, && (18)\\
\hat{\pi}_t &= \rho^{\pi} \hat{\pi}_{t-1} + (1-\rho^{\pi}) E_t \hat{\pi}_{t+1} + \lambda x_t - \rho^{\pi} u_t^* + u_t^{PC}, && (19)\\
i_t &= \rho^i i_{t-1} + (1-\rho^i) \left[\gamma^x x_t + \gamma^{\pi} (\pi_t - \pi_t^*) + \pi_t^* \right] + u_t^{MP} && (20)\\
&= \rho^i (i_{t-1} - \pi_{t-1}^*) + (1-\rho^i) \left[\gamma^x x_t + \gamma^{\pi} (\pi_t - \pi_t^*) \right] + \pi_t^* - \rho^i u_t^* + u_t^{MP} && (21)\\
\pi_t^* - \pi_{t-1}^* &= u_t^*. && (22)
\end{aligned}$$

The fundamental shocks are assumed to be independent and iid with variance-covariance matrix:

$$E_{t1}[u_t u_t'] = \Sigma_u = \begin{bmatrix} \left(\sigma^{IS}\right)^2 & 0 & 0 & 0 \\ 0 & \left(\sigma^{PC}\right)^2 & 0 & 0 \\ 0 & 0 & \left(\sigma^{MP}\right)^2 & 0 \\ 0 & 0 & 0 & (\sigma^*)^2 \end{bmatrix}. \tag{23}$$

We can write the model as:

$$0 = F E_t \hat{Y}_{t+1} + G \hat{Y}_t + H \hat{Y}_{t-1} + M u_t. \tag{24}$$

where

$$F = \begin{bmatrix} \rho^{x+} & \psi & 0 \\ 0 & (1-\rho^{\pi}) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \tag{25}$$

$$G = \begin{bmatrix} -1 & 0 & -\psi \\ \lambda & -1 & 0 \\ (1-\rho^i)\gamma^x & (1-\rho^i)\gamma^{\pi} & -1 \end{bmatrix}, \tag{26}$$

$$H = \begin{bmatrix} \rho^{x-} & 0 & 0 \\ 0 & \rho^{\pi} & 0 \\ 0 & 0 & \rho^i \end{bmatrix}, \tag{27}$$

$$M = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -\rho^{\pi} \\ 0 & 0 & 1 & -\rho^i \end{bmatrix}. \tag{28}$$

We focus on solutions of the form:

$$\hat{Y}_t = P \hat{Y}_{t-1} + Q u_t. \tag{29}$$

Additional solutions, such as solutions depending on two lags of state variables, may exist, see e.g. Evans and McGough (2005). P has to satisfy:

$$F P^2 + G P + H = 0. \tag{30}$$

Following Uhlig (1999), we first solve for the generalized eigenvectors and eigenvalues of Ξ with respect to Δ, where:

$$\Xi = \begin{bmatrix} -G & -H \\ I_3 & 0_3 \end{bmatrix}, \tag{31}$$

$$\Delta = \begin{bmatrix} F & 0_3 \\ 0_3 & I_3 \end{bmatrix}. \tag{32}$$

For three generalized eigenvalues $\lambda_1, \lambda_2, \lambda_3$ with generalized eigenvectors $[\lambda_1 z_1', z_1']'$, $[\lambda_2 z_2', z_2']'$, $[\lambda_3 z_3', z_3']'$, a solution is given by

$$P = \Omega \Lambda \Omega^{-1}, \tag{33}$$

where $\Lambda = diag(\lambda_1, \lambda_2, \lambda_3)$ and $\Omega = [z_1, z_2, z_3]$. Generalized eigenvalues are stable if their absolute value is < 1.

The matrix Q has to satisfy

$$Q = -[FP + G]^{-1} M \tag{34}$$

As long as we focus on solutions of the form (29) and the matrix of lagged terms H is non-singular, the solution cannot contain arbitrary random variables, or 'sunspots'. If we were to allow for more complicated solution forms, where $\hat{Y}_t$ can depend on two lags of itself as well as current and lagged shocks, sunspot solutions may be possible (Evans and McGough, 2005).

To see that solutions of the form (29) do not allow for sunspots, suppose the contrary. Assume that for some vector of random variables ϵ_t uncorrelated with $\hat{Y}_{t-1}$ and u_t:

$$\hat{Y}_t = P\hat{Y}_{t-1} + Qu_t + \epsilon_t. \tag{35}$$

The expression (35) corresponds to the definition of sunspot equilibria, see e.g. Cho and Moreno (2011). Then substituting (35) into (24) gives the same conditions for P and M as before and:

$$(FP + G)\epsilon_t \equiv 0. \tag{36}$$

But from (30), $(FP+G) \times P = -H$ is non-singular. Therefore, $FP+G$ is non-singular and $\epsilon_t \equiv 0$. This completes the proof that there are no sunspot solutions.

A.2 Equilibrium Selection and Properties

We are essentially solving a quadratic matrix equation, so picking a solution amounts to picking three out of six generalized eigenvalues. We only consider dynamically stable solutions with all eigenvalues less than 1 in absolute value, yielding non-explosive solutions for the output gap, inflation, and the real interest rate. When there are only four generalized eigenvalues with absolute values less than 1, there exists a unique dynamically stable solution. For the period 1 calibration, we have $\gamma^{\pi} < 1$ and there exist multiple real-valued, dynamically stable solutions. The period 2 and 3 calibrations have unique dynamically stable solutions.

We only consider solutions that are real-valued, and have finite entries for Q. We also require the diagonal entries of Q to be positive. This requirement means that the immediate impact of a positive IS shock on the output gap is positive rather than negative.

We apply multiple equilibrium selection criteria, which have been proposed in the literature, to rule out unreasonable solutions. These different equilibrium refinements are not identical, but coincide in many cases. Therefore, there exists a unique solution satisfying all criteria for a large part of our parameter space.

McCallum (1983) proposes to pick the minimum state variable solution. This solution has a minimum set of state variables and satisfies a continuity criterion. Unfortunately, Uhlig (1999) points out that implementing this criterion directly can be computationally demanding. We therefore follow Uhlig (1999) in picking the solution with the minimum absolute eigenvalues, which under certain conditions coincides with the minimum state variable solution (McCallum 2004).

We also require that our solution is locally E-stable (Evans 1985, 1986, Evans and Honkapohja 1994) as a plausible necessary, but not sufficient, condition. Local E-stability intuitively requires that the solution is learnable. If agents expectations deviate slightly from equilibrium dynamics, the system will return to an E-stable equilibrium under a simple revision rule.

Finally, we ensure uniqueness of our solution by requiring that it equals the forward solution of Cho and Moreno (2011). The forward solution is obtained by imposing a zero terminal condition. Expectations about shocks far in the future do not affect the current equilibrium. Viewed differently, if we assume that all state variables are constant from time t+T onwards, we can solve for the time t output gap, inflation gap, and interest rate gap recursively. The forward solution obtains by letting T go to infinity.

Let *vec* denote vectorization. Applying Proposition 1.3 of Fudenberg and Levine (1998, p.25) the E-stability condition translates into the requirement that the eigenvalues of the derivative

$$\frac{\partial vec([FP+G]^{-1}H)}{\partial vec(P)} \tag{37}$$

have eigenvalues with absolute values less than 1.

We implement the Cho and Moreno (2011) criterion by requiring that the following sequence $P_n, n = 0, 1, ...$ converges to P

$$P_0 = 0_{3\times3} \tag{38}$$
$$P_{n+1} = -[FP_n + G]^{-1} \times H \tag{39}$$

This sequence P_n has at most one limit and therefore this selection criterion yields a unique solution.

A.2.1 Recursion for Zero-Coupon Dividend Claims

In our calculations of asset prices, we repeatedly use the following expression for the real short rate Euler equation

$$log(\delta) - \gamma g + \gamma \hat{s}_t + \gamma\tau\phi x_t - \gamma E_t \hat{s}_{t+1} - \gamma\tau\phi E_t x_{t+1} \tag{40}$$
$$= -r^f - (e_3 - e_2 P)A^{-1}\tilde{Z}_t - \frac{\gamma}{2}(1-\theta_0)(1-2\hat{s}_t). \tag{41}$$

Let $\frac{P^d_{nt}}{D_t}$ denote the price-dividend ratio of a zero coupon claim on the dividend paid at time $t+n$. The price of a zero coupon claim for the dividend at time t is given by $\frac{P^d_{0t}}{D_t} = 1$. We now show that for $n \geq 1$, there exists a function $F_n(\tilde{Z}_t, \hat{s}_t, x_{t-1})$, such that

$$\frac{P^d_{nt}}{D_t} = exp((\gamma - \delta_{eq})v_t)F_n\left(\tilde{Z}_t, \hat{s}_t, x_{t-1}\right). \tag{42}$$

We use the following factorization for $M_{t+1}\frac{D_{t+1}}{D_t}$:

$$M_{t+1}\frac{D_{t+1}}{D_t}$$
$$= \delta exp(-\gamma(\hat{s}_{t+1} - \hat{s}_t) - (\gamma - \delta_{eq})(c_{t+1} - c_t)),$$
$$= \delta exp(-\gamma(\hat{s}_{t+1} - \hat{s}_t) - (\gamma - \delta_{eq})(g + \tau x_{t+1} - \tau\phi x_t + v_{t+1} - v_t)). \tag{43}$$

Let f_n denote the log of F_n

$$f_n = log(F_n). \tag{44}$$

For $n = 1$, we can derive f_1 explicitly:

$$\begin{aligned}
f_1(\tilde{Z}_t, \hat{s}_t, x_{t-1}) &= \underbrace{log(\delta) - (\gamma - \delta_{eq})\, g + \gamma \hat{s}_t + (\gamma - \delta_{eq})\tau\phi x_t}_{factor1init} \\
&\quad -\gamma\left(\theta_0 \hat{s}_t + \theta_1 x_t + \theta_2 x_{t-1}\right) \\
&\quad -\left(\gamma - \delta_{eq}\right)\tau e_1 P A^{-1} \tilde{Z}_t \\
&\quad +\frac{1}{2}(\gamma\lambda(\hat{s}_t) + (\gamma - \delta_{eq}))^2 \sigma_c^2, \\
&= \delta_{eq} g + \delta_{eq}\tau e_1 [P - \phi I] A^{-1}\tilde{Z}_t - r^f - (e_3 - e_2 P) A^{-1} \tilde{Z}_t \\
&\quad -\frac{\gamma}{2}(1 - \theta_0)(1 - 2\hat{s}_t) \\
&\quad +\frac{1}{2}(\gamma\lambda(\hat{s}_t) + (\gamma - \delta_{eq}))^2 \sigma_c^2. \qquad (45)
\end{aligned}$$

For $n > 1$, f_n is given by the recursion

$$\begin{aligned}
f_n(\tilde{Z}_t, \hat{s}_t, x_{t-1}) &= log\Big[E_t\Big[exp\Big(log(\delta) - (\gamma - \delta_{eq})g + \gamma\hat{s}_t \\
&\quad -\gamma\hat{s}_{t+1} - (\gamma - \delta_{eq})\tau E_t x_{t+1} + (\gamma - \delta_{eq})\tau\phi x_t \\
&\quad + \; -(\gamma - \delta_{eq})\tau e_1 A^{-1} e_1 \epsilon_{t+1} + -(\gamma - \delta_{eq})\tau e_1 A^{-1}(e_2 + e_3)\epsilon_{t+1} \\
&\quad + f_{n-1}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, x_t)\Big)\Big]\Big], \\
&= log\Big[E_t\Big[exp\Big(\delta_{eq} g + \delta_{eq}\tau e_1[P - \phi I]A^{-1}\tilde{Z}_t \\
&\quad -r^f - (e_3 - e_2 P)A^{-1}\tilde{Z}_t - \frac{\gamma}{2}(1 - \theta_0)(1 - 2\hat{s}_t) \qquad (46) \\
&\quad -(\gamma(1 + \lambda(\hat{s}_t)) - \delta_{eq})\underbrace{\tau e_1 A^{-1} e_1}_{ve_1}\epsilon_{t+1} \qquad (47) \\
&\quad -(\gamma(1 + \lambda(\hat{s}_t)) - \delta_{eq})\underbrace{\tau e_1 A^{-1}(e_2 + e_3)}_{ve_2}\epsilon_{t+1} \\
&\quad + f_{n-1}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, x_t)\Big)\Big]\Big]. \qquad (48)
\end{aligned}$$

A.2.2 Recursion for Zero Coupon Bond Prices

Let $P^{\$}_{n,t}$ and $P_{n,t}$ denote the prices of nominal and real zero coupon bonds. The log yields on n-period nominal and real bonds are then given by

$$y^{\$}_{n,t} = -p^{\$}_{n,t}/n, \qquad (49)$$
$$y_{n,t} = -p_{n,t}/n. \qquad (50)$$

One-period bond prices are given by

$$P^{\$}_{1,t} = exp(-\hat{i}_t - \pi^*_t - r^f), \quad (51)$$
$$P_{1,t} = exp(-\hat{i}_t + E_t\hat{\pi}_{t+1} - r^f). \quad (52)$$

For $n > 1$, the prices of real and nominal zero coupon bonds with maturity n are of the form

$$P_{n,t} = exp(\gamma v_t)B_n(\tilde{Z}_t, \hat{s}_t, x_{t-1}), \quad (53)$$
$$P^{\$}_{n,t} = exp(\gamma v_t - n\pi^*_t)B^{\$}_n(\tilde{Z}_t, \hat{s}_t, x_{t-1}). \quad (54)$$

It hence follows that

$$\begin{aligned}
B^{\$}_2 &= exp\left(\underbrace{log(\delta) - \gamma g + \gamma\hat{s}_t}_{factor1bonds} + \gamma\tau\phi x_t\right) \\
&\quad \times exp\left(E_t\left(-\gamma\hat{s}_{t+1} - \gamma\tau x_{t+1} - \hat{i}_{t+1} - \hat{\pi}_{t+1} - r^f\right)\right) \\
&\quad \times E_t exp\left(\underbrace{(-\gamma\left(\lambda(\hat{s}_t) + 1\right)Q_M - (e_2 + e_3)Q - 2e_4)}_{v^{\$}}u_{t+1}\right) \\
b^{\$}_2 &= \underbrace{log(\delta) - \gamma g + \gamma\hat{s}_t}_{factor1bonds} + \gamma\tau\phi x_t - \gamma\left(\theta_0\hat{s}_t + \theta_1 x_t + \theta_2 x_{t-1}\right) \\
&\quad -(\gamma\tau e_1 + e_2 + e_3)PA^{-1}\tilde{Z}_t + \frac{1}{2}v^{\$}\Sigma_u v^{\$\prime} - r^f \quad (55) \\
&= -e_3[I + P]A^{-1}\tilde{Z}_t - 2r^f \\
&\quad + \frac{1}{2}v^{\$}\Sigma_u v^{\$\prime} - \frac{\gamma}{2}(1 - \theta_0)(1 - 2\hat{s}_t). \quad (56)
\end{aligned}$$

The two-period real bond price then solves

$$\begin{aligned}
B_2 &= exp\left(\underbrace{log(\delta) - \gamma g + \gamma\hat{s}_t}_{factor1bonds} + \gamma\tau\phi x_t\right) \\
&\quad \times exp\left(E_t\left(-\gamma\hat{s}_{t+1} - \gamma\tau x_{t+1} - \hat{i}_{t+1} + E_{t+1}\hat{\pi}_{t+2} - r^f\right)\right) \\
&\quad \times E_t exp\left(\underbrace{(-\gamma(\lambda(\hat{s}_t) + 1)Q_M - (e_3 - e_2 P)Q)}_{v}u_{t+1}\right) \\
b_2 &= (\underbrace{log(\delta) - \gamma g + \gamma\hat{s}_t}_{factor1bonds} + \gamma\tau\phi x_t - \gamma\left(\theta_0\hat{s}_t + \theta_1 x_t + \theta_2 x_{t-1}\right) \\
&\quad -(\gamma\tau e_1 + e_3 - e_2 P)PA^{-1}\tilde{Z}_t + \frac{1}{2}v\Sigma_u v' - r^f, \quad (57) \\
&= -(e_3 - e_2 P)\left[I + P\right]A^{-1}\tilde{Z}_t - 2r^f + \frac{1}{2}v\Sigma_u v' - \frac{\gamma}{2}(1 - \theta_0)(1 - 2\hat{s}_t) \quad (58)
\end{aligned}$$

For $n > 2$ the function B_n satisfies the recursion

$$\begin{aligned} B_n(\tilde{Z}_t, \hat{s}_t, x_{t-1}) &= E_t \Bigg[exp \Bigg(\underbrace{log(\delta) - \gamma g + \gamma \hat{s}_t}_{factor1bonds} \\ &\quad + \underbrace{-\gamma \hat{s}_{t+1} + \gamma \tau \phi x_t - \gamma \tau x_{t+1}}_{factor2real} + b_{n-1}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, x_t) \Bigg) \Bigg], \\ &= E_t \left[exp \left(-r^f - (e_3 - e_2 P) A^{-1} \tilde{Z}_t - \frac{\gamma}{2}(1-\theta_0)(1-2\hat{s}_t) \right.\right. \\ &\quad -\gamma(1+\lambda(\hat{s}_t)) \underbrace{\tau e_1 A^{-1} e_1}_{ve_1} \epsilon_{t+1} - \gamma(1+\lambda(\hat{s}_t)) \underbrace{\tau e_1 A^{-1}(e_2+e_3)}_{ve_2} \epsilon_{t+1} \\ &\quad \left.\left. + b_{n-1}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, x_t) \right) \right]. \end{aligned} \tag{59}$$

The solution for nominal bond prices is similar with

$$\begin{aligned} B_n^{\$}(\tilde{Z}_t, \hat{s}_t, x_{t-1}) &= E_t \left[exp \left(log(\delta) - \gamma g + \gamma \hat{s}_t + \gamma \tau \phi x_t \right.\right. \\ &\quad \left.\left. + -\gamma \hat{s}_{t+1} - \gamma \tau x_{t+1} - \hat{\pi}_{t+1} - n u^*_{t+1} + b^{\$}_{n-1}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, B^{\$} x_t) \right) \right]. \end{aligned} \tag{60}$$

Now, in order to evaluate (65) numerically, it is useful to split up u^*_{t+1} into a component that is spanned by the normalized shocks ϵ_{t+1} and one component that is orthogonal $\epsilon^{\perp}_{t+1} \sim N(0,1)$. Hence, we need a vector v^* such that

$$u^*_{t+1} = v^* \epsilon_{t+1} + \sigma^{\perp} \epsilon^{\perp}_{t+1}, \tag{61}$$

$$E_t \left(u^*_{t+1} \epsilon^{\perp}_{t+1} \right) = 0. \tag{62}$$

Now, the distribution of u^*_{t+1} conditional on ϵ_{t+1} is normal with

$$u^*_{t+1} | \epsilon_{t+1} \sim N \left(\underbrace{(AQ\Sigma_u e_4')}_{v^*} \epsilon_{t+1}, \underbrace{(\sigma^*)^2 - (AQ\Sigma_u e_4')'(AQ\Sigma_u e_4')}_{(\sigma^{\perp})^2} \right). \tag{63}$$

The nominal bond price iteration therefore simplifies

$$
\begin{aligned}
B^{\$}_n(\tilde{Z}_t, \hat{s}_t, x_{t-1}) &= E_t\left[exp\left(\underbrace{log(\delta) - \gamma g + \gamma \hat{s}_t}_{factor1bonds} + \frac{n^2}{2}(\sigma^{\perp})^2\right.\right. && (64)\\
&\quad \left.\left.+ -\gamma \hat{s}_{t+1} - \gamma\tau x_{t+1} + \gamma\tau\phi x_t - \hat{\pi}_{t+1} - nv^*\epsilon_{t+1} + b^{\$}_{n-1}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, B^{\$}x_t)\right)\right].\\
&= E_t\left[exp\left(\underbrace{log(\delta) - \gamma g + \gamma \hat{s}_t}_{factor1bonds} + \frac{n^2}{2}(\sigma^{\perp})^2\right.\right.\\
&\quad + -\gamma \hat{s}_{t+1} - \gamma\tau E_t x_{t+1} - E_t\hat{\pi}_{t+1} + \gamma\tau\phi x_t\\
&\quad +(-\gamma\tau e_1 A^{-1} - e_2 A^{-1})e_1\epsilon_{t+1} - nv^* e_1\epsilon_{t+1}\\
&\quad +(-\gamma\tau e_1 A^{-1} - e_2 A^{-1}))(e_2 + e_3)\epsilon_{t+1} - nv^*(e_2 + e_3)\epsilon_{t+1}\\
&\quad \left.\left.+b^{\$}_{n-1}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, B^{\$}x_t)\right)\right].\\
&= E_t\left[exp\left(-r^f - e_3 A^{-1}\tilde{Z}_t - \frac{\gamma}{2}(1-\theta_0)(1-2\hat{s}_t)\right.\right.\\
&\quad +(-\gamma(1+\lambda(\hat{s}_t))ve_1 - \underbrace{e_2 A^{-1} e_1}_{vpi_1})\epsilon_{t+1} - nv^* e_1\epsilon_{t+1}\\
&\quad +(-\gamma(1+\lambda(\hat{s}_t))ve_2 - \underbrace{e_2 A^{-1}(e_2 + e_3)}_{vpi_2})\epsilon_{t+1} - nv^*(e_2 + e_3)\epsilon_{t+1}\\
&\quad \left.\left.+\frac{n^2}{2}(\sigma^{\perp})^2 + b^{\$}_{n-1}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, B^{\$}x_t)\right)\right] && (65)
\end{aligned}
$$

A.2.3 Scaled State Vector

For our numerical asset pricing solution, it is convenient to scale the state vector $\hat{Y}_t$ so that innovations to the sclaed state vector are independent standard normal. The dynamics for $\hat{Y}_t$ are given by:

$$
\begin{aligned}
\hat{Y}_t &= P\hat{Y}_{t-1} + Qu_t, && (66)\\
\hat{Y}_t &= Y_t - \bar{Y}, && (67)\\
&= [x_t, \pi_t - \pi^*_t, i_t - \pi^*_t - r^f], && (68)\\
E[u_t u_t'] &= diag(\sigma^2_u) = \Sigma_u. && (69)
\end{aligned}
$$

The dynamics for consumption and surplus consumption follow

$$
\begin{aligned}
c_t &= g + c_{t-1} + \tau\left(x_t - \phi x_{t-1}\right) + v_t - v_{t-1}, && (70)\\
s_{t+1} &= (1-\theta_0)\bar{s} + \theta_0 s_t + \theta_1 x_t + \theta_2 x_{t-1} + \lambda(s_t)\varepsilon_{c,t+1} && (71)\\
\varepsilon_{c,t+1} &= c_{t+1} - E_t c_{t+1} = \tau\left(x_{t+1} - E_t x_{t+1}\right) + v_{t+1}. && (72)
\end{aligned}
$$

The process $\tilde{Z}_t$ scales and rotates the variables in $\hat{Y}_t$ such that shocks to $\tilde{Z}_t$ are independent standard normal. Moreover, the first element of $\tilde{Z}_t$ is conditionally perfectly correlated with consumption (and hence SDF) surprises.

$$\tilde{Z}_t = A\hat{Y}_t, \tag{73}$$
$$\tilde{Z}_t = \tilde{P}\tilde{Z}_{t-1} + \epsilon_t, \tag{74}$$
$$\tilde{P} = APA^{-1}, \tag{75}$$
$$\epsilon_{t+1} = AQu_{t+1}. \tag{76}$$

We therefore solve for a matrix A with the following two properties:

$$\sigma_c e_1 AQ = Q_M, \tag{77}$$
$$AQ\Sigma_u Q'A' = I_{3\times 3}. \tag{78}$$

Provided that $\sigma^{IS} = 0$, we can find a vector v_c such that

$$v_c \times Q = Q_M. \tag{79}$$

Next, we pick an orthonormal matrix X, such that

$$e_1 X = v_c \left(Q\Sigma_u Q'\right)^{1/2} / \sigma_c. \tag{80}$$

Then, define

$$A = X \left(Q\Sigma_u Q'\right)^{-1/2}. \tag{81}$$

The matrix A satisfies conditions (77) and (78).

The unconditional variance-covariance matrix of $\tilde{Z}$ is determined by

$$Var(\tilde{Z}) = \tilde{P}Var(\tilde{Z})\tilde{P}' + I_4. \tag{82}$$

We can use linear algebra to solve for $Var\tilde{Z}$. We use $Std(\tilde{Z})$ to denote the vector of unconditional standard deviations of $\tilde{Z}$.

A.2.4 Numerical Implementation

We need to evaluate the expectational terms in expressions (48), (59) and (65). We evaluate the functions F_1, B_2 and $B_2^{\$}$ along a grid. We then use the knowledge of F_{n-1}, $B_{n-1}^{\$}$ and B_{n-1} along this grid combined with loglinear interpolation.

We start by constructing a three-dimensional grid for $\tilde{Z}_t$. Let N denote the number of grid points along each dimension and m the width of the grid as a multiple of

the unconditional standard deviation of $\tilde{Z}_{t,k}$. Our baseline solution sets $N = 2$ and $m = 2$. For a 3-tuple
$n = (n_1, n_2, n_3) \in \{1, 2, ..., N\}^3$ the corresponding grid point is given by $z_n = (z_{n,1}, z_{n,2}, z_{n,3})$ with

$$z_{n,k} = -m \cdot std(\tilde{Z}_k) \quad (83)$$

$$+(n-1) \cdot \frac{2m \cdot std(\tilde{Z}_k)}{N-1}. \quad (84)$$

First, we compute expectations conditional on $e_2\tilde{Z}_{t+1}$ and $e_3\tilde{Z}_{t+1}$, integrating over $e_1\tilde{Z}_{t+1}$. We use 40-point Gauss-Legendre quadrature as in Wachter (2006) for this first integration step. We bound the integral at -8 and +8 standard deviations of $e_1\epsilon_{t+1}$. Second, we compute the time-t expectation by integrating over $e_2\tilde{Z}_{t+1}$ and $e_2\tilde{Z}_{t+1}$. We do this using 10-point Gauss-Legendre quadrature along each dimension and again bound the integral along each dimension at -8 and +8. We use that ϵ_{t+1} is independent and standard normal for computing the probability density functions.

A.3 Risk-Neutral Returns

In order to better understand the role of risk-premia, we compute bond and stock returns when there are no risk premia (i.e. expected excess returns are constant). Note that risk-neutrality is not consistent with the Euler equation. Consumption dynamics therefore rely crucially on risk-aversion. However, it is still instructive to compute bond and stock returns without risk premia, holding constant consumption dynamics.

For this subsection, we use the superscript ^{rn} to refer to risk-neutral quantities. We denote log returns on nominal and real zero coupon bonds by $r^{\$}_{n-1,t+1}$ and $r_{n-1,t+1}$.

Unexpected nominal bond returns with no risk premia are simply given by the innovation to expected nominal bond yields

$$r^{\$,rn}_{n-1,t+1} - E_t r^{\$,rn}_{n-1,t+1} = -(E_{t+1} - E_t) \sum_{j=1}^{n-1} \left(\hat{i}_{t+j} + \pi^*_{t+j} \right) \quad (85)$$

$$= -(n-1)u^*_{t+1} - e_3 \sum_{j=1}^{n-1} P^{j-1} Q u_{t+1} \quad (86)$$

$$= -(n-1)u^*_{t+1} - e_3 \left[I - P^{n-1} \right] [I - P]^{-1} A^{-1} \varepsilon_{t+1}. \quad (87)$$

Unexpected real bond returns with no risk premia are similarly given by

$$rrn_{n-1,t+1} - E_t r^{rn}_{n-1,t+1} = -(e_3 - e_2 P) \left[I - P^{n-1} \right] [I - P]^{-1} A^{-1} \varepsilon_{t+1}. \quad (88)$$

Now, we compute unexpected returns without risk premia using the Campbell-Shiller loglinearization. For this, define the loglinearization constant

$$\rho \;=\; \frac{1}{1+1/exp\left(\overline{p-d}\right)}. \tag{89}$$

We obtain the numerical value for ρ from simulated model price-dividend ratios as reported in the main paper.

Ignoring return surprises due to changes in expected stock returns, the loglinear decomposition by Campbell (1991) gives us

$$\begin{aligned} r^{e,rn}_{t+1} - E_t r^{e,rn}_{t+1} &= (E_{t+1}-E_t)\sum_{j=0}^{\infty}\rho^j \Delta d_{t+1+j} - (E_{t+1}-E_t)\sum_{j=1}^{\infty}\rho^j r_{t+j}, \\ &= (1-\rho)\,v_{t+1} + \delta\tau(1-\rho\phi)e_1\left[I-\rho P\right]^{-1}A^{-1}\varepsilon_{t+1} - \rho(e_3-e_2P)\left[I-\rho P\right]^{-1}A^{-1}\varepsilon_{t+1}. \end{aligned} \tag{90}$$

We can similarly compute bond yields and equity dividend yields using Campbell-Shiller loglinearizations while imposing constant expected excess returns. With constant expected equity excess returns, we obtain the following expression for the log dividend yield:

$$(d-p)^{rn}_t \;=\; -E_t\sum_{j=0}^{\infty}\rho^j\Delta d_{t+1+j} + E_t\sum_{j=0}^{\infty}\rho^j r_{t+1+j}. \tag{91}$$

First, we derive an expression for the discounted sum of future expected real rates

$$E_t\sum_{j=0}^{\infty}\rho^j r_{t+1+j} \;=\; \sum_{j=0}^{\infty}\rho^j(e_3-e_2P)P^{j+1}\hat{Y}_t, \tag{92}$$

$$\;=\; (e_3-e_2P)P[I-\rho P]^{-1}\hat{Y}_t. \tag{93}$$

Next,

$$-E_t\sum_{j=0}^{\infty}\rho^j\Delta d_{t+1+j} \tag{94}$$

$$= -\delta E_t\sum_{j=0}^{\infty}\rho^j\left(\tau(x_{t+1+j}-\phi x_{t+j}) + v_{t+1+j} - v_{t+j}\right) \tag{95}$$

$$= \delta v_t + \phi\delta x_t - \delta\tau(1-\rho\phi)E_t\sum_{j=0}^{\infty}\rho^j x_{t+1+j}, \tag{96}$$

$$= \delta v_t + \phi\delta x_t - \delta\tau(1-\rho\phi)e_1P\left[I-\rho P\right]^{-1}\hat{Y}_t. \tag{97}$$

It then follows that the no-risk-premium log dividend yield can be approximated as

$$(d-p)^{rn}_t \;=\; \delta v_t + V^{eq}\hat{Y}_t, \tag{98}$$

$$V^{eq} \;=\; \phi\delta e_1 + \left((e_3-e_2P) - \delta\tau(1-\rho\phi)e_1\right)P\left[I-\rho P\right]^{-1}. \tag{99}$$

Next, nominal bond yields with no risk premia are simply given by the average of expected future nominal short rates, as under the expectations hypothesis

$$y_{n,t}^{\$,rn} = \frac{1}{n}E_t\sum_{j=0}^{n-1}\left(\pi_{t+j}^{*}+\hat{i}_{t+j}\right), \tag{100}$$

$$= \pi_t^{*}+\frac{1}{n}\sum_{j=0}^{n-1}e_3P^j\hat{Y}_t, \tag{101}$$

$$= \pi_t^{*}+V^{nom}\hat{Y}_t, \tag{102}$$

$$V^{nom} = \frac{1}{n}e_3\left[I-P^n\right]\left[I-P\right]^{-1}. \tag{103}$$

Real bond yields without risk premia are given by

$$y_{n,t}^{rn} = \frac{1}{n}E_t\sum_{j=0}^{n-1}\left(\hat{i}_{t+j}-E_{t+j}\hat{\pi}_{t+j+1}\right), \tag{104}$$

$$= V^{real}\hat{Y}_t, \tag{105}$$

$$V^{real} = \frac{1}{n}\left(e_3-e_2P\right)\left[I-P^n\right]\left[I-P\right]^{-1}. \tag{106}$$

We compute the risk premium components of dividend yields and bond yields as the difference between total and risk-neutral quantities:

$$(d-p)_t^{rp} = (d-p)_t-(d-p)_t^{rn}, \tag{107}$$

$$y_{n,t}^{\$,rp} = y_{n,t}^{\$}-y_{n,t}^{\$,rn}. \tag{108}$$

A.4 A Note on Units

Empirical yields and returns are in annualized percent units. Log real dividends and the log output gap are in natural percent units. Our empirical units are analogous to those used by CGG. Our empirical coefficients in Table 3 in the main paper can therefore be compared directly to those in CGG.

We solve the model in natural units and subsequently report scaled parameters and model moments reflecting our choice of empirical units. Let a superscript ^{c} denote natural units used for solving the calibrated model. Values with no superscript denote the parameters and variables corresponding to empirical units.

Our quantities in empirical units are related to quantities in calibration units according to: $x_t = 100x_t^c$, $i_t = 400i_t^c$, $\pi_t = 400\pi_t^c$, and $y_t^{\$,n} = 400y_t^{\$,n}$ and $\pi_t^{*} = 400\pi_t^{*}$. We can therefore write the model as:

$$
\begin{aligned}
x_t &= \rho^{x-,c}x_{t-1} + \rho^{x+,c}E_t x_{t+1} - \frac{\psi^c}{4}\left(i_t - E_t\pi_{t+1}\right) + 100 \times u_t^{IS,c} && (109)\\
\pi_t &= \rho^{\pi,c}\pi_{t-1} + (1-\rho^{\pi,c})E_t\pi_{t+1} + 4\lambda^c x_t + 400 \times u_t^{PC,c} && (110)\\
i_t &= \rho^{i,c} i_{t-1} + (1-\rho^{i,c})\left[4\gamma^{x,c}x_t + \gamma^{\pi,c}(\pi_t - \pi_t^*) + \pi_t^*\right] + 400 u_t^{MP,c} && (111)\\
\pi_t^* &= \pi_{t-1}^* + 400 u_t^* && (112)
\end{aligned}
$$

Equations (109) through (112) imply relations between the empirical and calibration parameters:

$$
\begin{aligned}
\rho^{x-} &= \rho^{x-,c}, \rho^{x+} = \rho^{x+,c}, \psi = \frac{\psi^c}{4} && (113)\\
\rho^{\pi} &= \rho^{\pi,c}, \lambda = 4\lambda^c && (114)\\
\rho^{i} &= \rho^{i,c}, \gamma^x = 4\gamma^{x,c}, \gamma^\pi = \gamma^{\pi,c} && (115)\\
\bar{\sigma}^{IS} &= 100\bar{\sigma}^{IS,c}, \bar{\sigma}^{PC} = 400\bar{\sigma}^{PC,c}, \bar{\sigma}^{MP} = 400\bar{\sigma}^{MP,c}, \bar{\sigma}^* = 400\bar{\sigma}^* && (116)
\end{aligned}
$$

Yogo (2004) scales interest rates and inflation to quarterly units. Our calibrated values for ψ^c in natural units can therefore be compared directly to the estimated values in Yogo (2004). We report the value ψ^c corresponding to natural units rather than ψ corresponding to empirical units throughout the paper for comparability with values in the literature.

We report the persistence parameter θ_0 and ϕ in annualized units for comparability with Campbell and Cochrane (1999) and Wachter (2006). That is, $\theta_0 = (\theta_0^c)^4$. All other parameters ($\tau$, θ_1, θ_2) are reported in calibration units.

A.5 Consumption Variance Ratios

This subsection derives expressions for variance ratios for consumption innovations. The volatility of consumption surprises is given by

$$
\sigma_c^2 = Q_M \Sigma_u Q_M'. \tag{117}
$$

We can use (2) to compute the variance of consumption at different horizons.

$$\begin{aligned}
Var_t\left(c_{t+1}\right) &= Q_M \Sigma_u Q_M' = \sigma_c^2, & (118)\\
Var_t\left(c_{t+2}\right) &= Var_t\left(\tau\left(x_{t+2} + (1-\phi)x_{t+1} - \phi x_t\right) + v_{t+2}\right)\\
&= Var_t\left(\tau e_1 Q u_{t+2} + \tau e_1 P Q u_{t+1} + \tau(1-\phi) e_1 Q u_{t+1} - (\tau\phi - \theta_1) e_1 u_{t+2}\right)\\
&= \sigma_c^2 + (\tau e_1 P Q + \tau(1-\phi) e_1 Q)\Sigma_u(\tau e_1 P Q + \tau(1-\phi)e_1 Q)' & (119)\\
Var_t\left(c_{t+3}\right) &= Var_t\left(\tau\left(x_{t+3} + (1-\phi)\left(x_{t+2} + x_{t+1}\right) - \phi x_t\right) + v_{t+3}\right)\\
&= Var_t((\tau e_1 Q - \left(\tau\phi - \theta_1\right) e_1) u_{t+3}\\
&\quad + \left(\tau e_1 P Q + \tau\left(1-\phi\right) e_1 Q\right) u_{t+2}\\
&\quad + \left(\tau e_1 P^2 Q + \tau\left(1-\phi\right) e_1 P Q + \tau\left(1-\phi\right) e_1 Q\right) u_{t+1}) & (120)\\
&= \sigma_c^2 + \left(\tau e_1 P Q + \tau\left(1-\phi\right) e_1 Q\right)\Sigma_u\left(\tau e_1 P Q + \tau\left(1-\phi\right) e_1 Q\right)'\\
&\quad + \left(\tau e_1 P^2 Q + \tau\left(1-\phi\right) e_1 P Q + \tau\left(1-\phi\right) e_1 Q\right)\Sigma_u\\
&\quad \left(\tau e_1 P^2 Q + \tau\left(1-\phi\right) e_1 P Q + \tau\left(1-\phi\right) e_1 Q\right)' & (121)
\end{aligned}$$

In order to derive the expression of conditional variance $Var_t\left(c_{t+k}\right)$ for any $k>0$, it is useful to realize the recursion rule of $c_{t+k} - E_t c_{t+k}$:

$$\begin{aligned}
c_{t+k} - E_t c_{t+k} &= g + c_{t+k-1} + \tau\left(x_{t+k} - \phi x_{t+k-1}\right) + v_{t+k} - v_{t+k-1}\\
&\quad -g - E_t c_{t+k-1} - \tau\left(E_t x_{t+k} - \phi E_t x_{t+k-1}\right) - E_t v_{t+k} + E_t v_{t+k-1} & (122)\\
c_{t+k-1} - E_t c_{t+k-1} &= g + c_{t+k-2} + \tau\left(x_{t+k-1} - \phi x_{t+k-2}\right) + v_{t+k-1} - v_{t+k-2}\\
&\quad -g - E_t c_{t+k-2} - \tau\left(E_t x_{t+k-1} - \phi E_t x_{t+k-2}\right) - E_t v_{t+k-1} + E_t v_{t+k-2} & (123)
\end{aligned}$$

Given this recursion, we can derive the formula for conditional variance as follows:

$$\begin{aligned}
Var_t\left(c_{t+k}\right) &= Var_t\left(c_{t+k} - E_t c_{t+k}\right)\\
&= Var_t\left(\tau x_{t+k} + \tau\left(1-\phi\right)\Sigma_{j=1}^{k-1} x_{t+k-j} - \tau\phi x_t + v_{t+k} - v_t\right)\\
&= Var_t(\tau e_1 Q u_{t+k} + \tau\Sigma_{l=1}^{k-1} e_1 P^l Q u_{t+k-l} +\\
&\quad \tau\left(1-\phi\right)\Sigma_{j=1}^{k-1}\Sigma_{i=0}^{j-1} e_1 P^i Q u_{t+k-i} + v_{t+k})\\
&= Var_t(\tau e_1 Q u_{t+k} +\\
&\quad \Sigma_{j=1}^{k-1}\left(\tau e_1 P^j Q + \tau\left(1-\phi\right)\Sigma_{i=0}^{j-1} e_1 P^i Q\right) u_{t+k-i} - \left(\tau\phi - \theta_1\right) e_1 u_{t+k}) & (124)
\end{aligned}$$

$$\begin{aligned}
Var_t\left(c_{t+k}\right) &= Var_t((\tau e_1 Q - \left(\tau\phi - \theta_1\right)) u_{t+k} +\\
&\quad \Sigma_{j=1}^{k-1}\left(\tau e_1 P^j Q + \tau\left(1-\phi\right)\Sigma_{i=0}^{j-1} e_1 P^i Q\right) u_{t+k-j})\\
&= \sigma_c^2 + \Sigma_{j=1}^{k-1}(\left(\tau e_1 P^j Q + \tau\left(1-\phi\right)\Sigma_{i=0}^{j-1} e_1 P^i Q\right)\Sigma_u\\
&\quad \left(\tau e_1 P^j Q + \tau\left(1-\phi\right)\Sigma_{i=0}^{j-1} e_1 P^i Q\right)')\\
&= \sigma_c^2 + \Sigma_{j=1}^{k-1}(\left(\tau e_1 P^j Q + \tau\left(1-\phi\right) e_1\left(I-P\right)^{-1}\left(I-P^j\right) Q\right)\Sigma_u\\
&\quad \left(\tau e_1 P^j Q + \tau\left(1-\phi\right) e_1\left(I-P\right)^{-1}\left(I-P^j\right) Q\right)') & (125)
\end{aligned}$$

This allows us to calculate the variance ratio:

$$V_t(k) = \frac{Var_t(c_{t+k})}{kVar_t(c_{t+1})}. \tag{126}$$

B Additional Model Details

B.1 Deriving the Log-Linearized Phillips Curve

This section derives the loglinearized forward- and backward-looking Phillips curve (equation (2) in the main paper). Firms set prices to maximize future expected profits discounted at the investors' stochastic discount factor. Similarly to us, Smets and Wouters (2003) use difference habit preferences, so we can naturally extend their derivation to our framework. The main difference in our model is that we allow for steady-state growth in consumption. All expressions remain the same if we replace the pure time discount rate by a growth-adjusted discount rate.

This section follows the notation of Smets and Wouters (2003). For standard algebra, see also Walsh (2010, Chapter 8.6.1). Intermediate good producer j faces downward-sloping demand in terms of his own price p_t^j relative to the aggregate price level P_t

$$y_t^j = \left(p_t^j/P_t\right)^{-(1+\lambda_{p,t})/\lambda_{p,t}} Y_t, \tag{127}$$

where Y_t is aggregate output. The stochastic parameter $\lambda_{p,t}$ represents a markup shock. Aggregate output and prices are given by

$$P_t = \left[\int_0^1 (p_t^j)^{-1/\lambda_{p,t}} dj\right]^{-\lambda_{p,t}}, \tag{128}$$

$$Y_t = \left[\int_0^1 (y_t^j)^{1/(1+\lambda_{p,t})} dj\right]^{1+\lambda_{p,t}} \tag{129}$$

Nominal intermediate firm profits are given by

$$\pi_t^j = (p_t^j - MC_t)\left(\frac{p_t^j}{P_t}\right)^{-(1+\lambda_{p,t})/\lambda_{p,t}} Y_t - MC_t\Phi, \tag{130}$$

where MC_t is the nominal marginal cost of production and Φ is a fixed cost. Taking the first order condition of (130) shows that in a flexible-price equilibrium the optimal price at time t is

$$p_t^* = (1+\lambda_{p,t})MC_t. \tag{131}$$

We assume that firms get the chance to update prices in every period with fixed probability ξ_p, as in Calvo (1983). When firms cannot update prices, their prices are partially indexed to lagged inflation (Smets and Wouters 2003) with indexation parameter γ_p. The time $t+i$ nominal price of a firm that last re-set its price at time t to $\tilde{p}_t^j$ is given by

$$\tilde{p}_t^j \left(P_{t-1+i}/P_{t-1}\right)^{\gamma_p}. \tag{132}$$

The law of motion for the price level is given by

$$P_t^{-1/\lambda_{p,t}} = \xi_p \left(P_{t-1}\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_p}\right)^{-1/\lambda_{p,t}} + (1-\xi_p)(\tilde{p}_t^j)^{-1/\lambda_{p,t}}. \tag{133}$$

A firm j that has the ability to re-set its price at time t therefore maximizes the expected value of future profits discounted with the consumer's stochastic discount factor. Now, we denote the SDF for pricing period $t+i$ cash flows at time t

$$\begin{aligned} M_{t\to t+i} &= \beta^i exp\left(-\gamma\left(c_{t+i}+s_{t+i}-c_t-s_t\right)\right), & (134)\\ &= \beta_g^i \lambda_{t+i}, & (135)\\ \beta_g &= \beta exp(-\gamma g), & (136)\end{aligned}$$

The factor λ_{t+i} is constant at one in the nonstochatic steady-state.

The first order condition for optimal price-setting behavior is given by

$$E_t \sum_{i=0}^{\infty} \xi_p^i M_{t\to t+i} y_{t+i}^j \left(\frac{\tilde{p}_t^j}{P_t}\left(\frac{\left(P_{t-1+i}/P_{t-1}\right)^{\gamma_p}}{P_{t+i}/P_t}\right) - (1+\lambda_{p,t+i}) mc_{t+i}\right) = 0, \tag{137}$$

where $mc_{t+i} = \frac{MC_{t+i}}{P_{t+i}}$ is the real marginal cost of production.

Now, we approximate (137) loglinearly around the nonstochastic steady-state. Rewrite (137) as:

$$\frac{\tilde{p}_t^j}{P_t} E_t\left[\sum_{i=0}^{\infty}\left(\beta_g \xi_p\right)^i \lambda_{t+i} y_{t+i}^j \left(\frac{\left(P_{t-1+i}/P_{t-1}\right)^{\gamma_p}}{P_{t+i}/P_t}\right)\right] = E_t\left[\sum_{i=0}^{\infty}\left(\beta_g \xi_p\right)^i \lambda_{t+i} y_{t+i}^j \left(1+\lambda_{p,t+i}\right) mc_{t+i}\right] \tag{138}$$

Now, we will approximate the left-hand side. The ratios of all prices in steady state equal to one (i.e. $\tilde{p}^j/P = 1$ and $P/P = 1$). From now on, we drop the superscript j indicating the particular firm to keep the notation simple; the percentage deviations from the steady-state values are denoted by hat; moreover, $\hat{\tilde{p}}_t$ is the percentage deviation of $\frac{\tilde{p}}{P_t}$ from steady-state. Accordingly, the left-hand side can be approximated in the following way:

$$\left(1+\hat{\tilde{p}}_t\right) E_t\left[\sum_{i=0}^{\infty}\left(\beta_g \xi_p\right)^i \left(1+\hat{\lambda}_{t+i}\right) y\left(1+\hat{y}_{t+1}\right)\left(1+\gamma_p\left(\hat{p}_{t-1+i}-\hat{p}_{t-1}\right)\right)\left(1+\hat{p}_t-\hat{p}_{t+i}\right)\right] \tag{139}$$

Multiplying out and dropping the terms consisting of a product of at least two deviation variables (i.e. for instance, $\hat{y}_{t+i}\hat{\lambda}_{t+i} \approx 0$), leads to following expression:

$$E_t\left[\sum_{i=0}^{\infty}\left(\beta_g\xi_p\right)^i y\left(1+\hat{\tilde{p}}_t+\hat{\lambda}_{t+i}+\hat{y}_{t+1}+\gamma_p\left(\hat{p}_{t-1+i}-\hat{p}_{t-1}\right)+\hat{p}_t-\hat{p}_{t+i}\right)\right] \tag{140}$$

Some of the terms in the above expression do not depend on time (i.e. not indexed by i), so the expression simplifies:

$$\frac{y}{1-\beta_g\xi_p}+\frac{y\hat{\tilde{p}}_t}{1-\beta_g\xi_p}+yE_t\left[\sum_{i=0}^{\infty}\left(\beta_g\xi_p\right)^i\left(\hat{\lambda}_{t+i}+\hat{y}_{t+1}+\gamma_p\left(\hat{p}_{t-1+i}-\hat{p}_{t-1}\right)+\hat{p}_t-\hat{p}_{t+i}\right)\right] \tag{141}$$

The above expression is the linear approximation of the left-hand side of equation (138). Now, let's look at the right-hand side. Let's denote the term $(1+\lambda_{p,t+i})$ as L_{t+i}. Moreover, in the steady state the following should hold: $(1+\lambda_p)\,mc = Lmc = 1$. The right-hand side of equation (138) can be approximated as follows:

$$E_t\left[\sum_{i=0}^{\infty}\left(\beta_g\xi_p\right)^i\left(1+\hat{\lambda}_{t+i}\right)y\left(1+\hat{y}_{t+i}\right)L\left(1+\hat{l}_{t+i}\right)mc\left(1+\hat{mc}_{t+i}\right)\right] \tag{142}$$

Multiplying out and dropping the terms consisting of at least two percentage deviation terms gives:

$$E_t\left[\sum_{i=0}^{\infty}\left(\beta_g\xi_p\right)^i y\left(1+\hat{\lambda}_{t+i}+\hat{y}_{t+i}+\hat{mc}_{t+i}+\hat{l}_{t+i}\right)\right] \tag{143}$$

Again, we can simplify the expression for the not indexed terms:

$$\frac{y}{1-\beta_g\xi_p}+yE_t\left[\sum_{i=0}^{\infty}\left(\beta_g\xi_p\right)^i\left(\hat{\lambda}_{t+i}+\hat{y}_{t+i}+\hat{mc}_{t+i}+\hat{l}_{t+i}\right)\right] \tag{144}$$

The above expression is the linear approximation of the right-hand side of equation (138). Now, if we plug in (141) and (144) into (138), the first term on both sides cancels. Then we can divide both sides by y and subtract $E_t\left[\sum_{i=0}^{\infty}\left(\beta_g\xi_p\right)^i\left(\hat{\lambda}_{t+i}+\hat{y}_{t+i}\right)\right]$. This gives following equation:

$$\frac{\hat{\tilde{p}}_t}{1-\beta_g\xi_p}+E_t\left[\sum_{i=0}^{\infty}\left(\beta_g\xi_p\right)^i\left(\gamma_p\left(\hat{p}_{t-1+i}-\hat{p}_{t-1}\right)+\hat{p}_t-\hat{p}_{t+i}\right)\right]=E_t\left[\sum_{i=0}^{\infty}\left(\beta_g\xi_p\right)^i\left(\hat{mc}_{t+i}+\hat{l}_{t+i}\right)\right] \tag{145}$$

The terms $\gamma_p \hat{p}_{t-1}$ and $\hat{p}_t$ are not indexed by index i and their sums can be expressed explicitly:

$$\frac{\hat{\tilde{p}}_t}{1-\beta_g \xi_p} - \frac{\gamma_p \hat{p}_{t-1}}{1-\beta_g \xi_p} + \frac{\hat{p}_t}{1-\beta_g \xi_p} + E_t \left[\sum_{i=0}^{\infty} \left(\beta_g \xi_p\right)^i \left(\gamma_p \hat{p}_{t-1+i} - \hat{p}_{t+i}\right) \right] = E_t \left[\sum_{i=0}^{\infty} \left(\beta_g \xi_p\right)^i \left(\hat{mc}_{t+i} + \hat{l}_{t+i}\right) \right] \quad (146)$$

We solve for $\hat{\tilde{p}}_t + \hat{p}_t$:

$$\hat{\tilde{p}}_t + \hat{p}_t = \left(1-\beta_g \xi_p\right) E_t \left[\sum_{i=0}^{\infty} \left(\beta_g \xi_p\right)^i \left(\hat{mc}_{t+i} + \hat{l}_{t+i} - \gamma_p \hat{p}_{t-1+i} + \hat{p}_{t+i}\right) \right] + \gamma_p \hat{p}_{t-1} \quad (147)$$

Now, we shift the whole expression by one period forward. The forward-shifted expression will be used later to plug in and simplify.

$$\hat{\tilde{p}}_{t+1} + \hat{p}_{t+1} = \left(1-\beta_g \xi_p\right) E_{t+1} \left[\sum_{i=0}^{\infty} \left(\beta_g \xi_p\right)^i \left(\hat{mc}_{t+i+1} + \hat{l}_{t+i+1} - \gamma_p \hat{p}_{t+i} + \hat{p}_{t+i+1}\right) \right] + \gamma_p \hat{p}_t \quad (148)$$

Now, let's go back to the current expression (147) and and pull out of the sum the terms with index $i = 0$:

$$\begin{aligned} \hat{\tilde{p}}_t + \hat{p}_t &= \left(1-\beta_g \xi_p\right)\left(\hat{mc}_t + \hat{l}_t\right) - \left(1-\beta_g \xi_p\right)\gamma_p \hat{p}_{t-1} + \left(1-\beta_g \xi_p\right)\hat{p}_t + \gamma_p \hat{p}_{t-1} + \\ &+ \left(1-\beta_g \xi_p\right)\beta_g \xi_p E_t \left[\sum_{i=0}^{\infty} \left(\beta_g \xi_p\right)^i \left(\hat{mc}_{t+i+1} + \hat{l}_{t+i+1} - \gamma_p \hat{p}_{t+i} + \hat{p}_{t+i+1}\right) \right] \end{aligned}$$

Now, we rearrange the terms and add and subtract $\beta_g \xi_p \gamma_p \hat{p}_t$ in order to get a term proportional to $\hat{\tilde{p}}_{t+1} + \hat{p}_{t+1}$ on the right-hand side:

$$\begin{aligned} \hat{\tilde{p}}_t + \hat{p}_t &= \left(1-\beta_g \xi_p\right)\left(\hat{mc}_t + \hat{l}_t\right) + \beta_g \xi_p \gamma_p \hat{p}_{t-1} + \left(1-\beta_g \xi_p\right)\hat{p}_t - \beta_g \xi_p \gamma_p \hat{p}_t + \\ &+ \beta_g \xi_p \left(\left(1-\beta_g \xi_p\right) E_t \left[\sum_{i=0}^{\infty} \left(\beta_g \xi_p\right)^i \left(\hat{mc}_{t+i+1} + \hat{l}_{t+i+1} - \gamma_p \hat{p}_{t+i} + \hat{p}_{t+i+1}\right) \right] + \beta_g \xi_p \gamma_p \hat{p}_t \right) \end{aligned} \quad (149)$$

We can see that the last term is $\beta_g \xi_p \left(E_t \left[\hat{\tilde{p}}_{t+1} + \hat{p}_{t+1}\right]\right)$:

$$\hat{\tilde{p}}_t + \hat{p}_t = \left(1-\beta_g \xi_p\right)\left(\hat{mc}_t + \hat{l}_t\right) + \beta_g \xi_p \gamma_p \hat{p}_{t-1} + \left(1-\beta_g \xi_p\right)\hat{p}_t - \beta_g \xi_p \gamma_p \hat{p}_t + \beta_g \xi_p \left(E_t \left[\hat{\tilde{p}}_{t+1} + \hat{p}_{t+1}\right]\right) \quad (150)$$

Now we should recall that the percentage deviation of the inflation from steady state is $\hat{\pi}_t = \hat{p}_t - \hat{p}_{t-1}$.

$$\hat{\tilde{p}}_t = \left(1-\beta_g \xi_p\right)\left(\hat{mc}_t + \hat{l}_t\right) - \beta_g \xi_p \gamma_p \hat{\pi}_t + \beta_g \xi_p E_t \left[\hat{\pi}_{t+1}\right] + \beta_g \xi_p E_t \left[\hat{\tilde{p}}_{t+1}\right] \quad (151)$$

Loglinearizing the law of motion for the price level (133) gives:

$$\xi_p \hat{\pi}_t = \xi_p \gamma_p \hat{\pi}_{t-1} + (1-\xi_p)(\hat{\tilde{p}}_t). \tag{152}$$

The loglinear approximation to the law of motion implies that

$$\hat{\tilde{p}}_t = \frac{\xi_p}{1-\xi_p}\left(\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1}\right) \tag{153}$$

Now we can use above expression and plug it into equation (151). We will do it for $\hat{\tilde{p}}_t$ on the left-hand side and for $\hat{\tilde{p}}_{t+1}$ on the right-hand side. We also multiply by $\frac{1-\xi_p}{\xi_p}$ to get:

$$\begin{aligned}\left(\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1}\right) &= \frac{(1-\xi_p)}{\xi_p}\left(1-\beta_g \xi_p\right)\left(\hat{mc}_t + \hat{l}_t\right) - \left(1-\xi_p\right)\beta\gamma_p\hat{\pi}_t + \left(1-\xi_p\right)\beta_g E_t\left[\hat{\pi}_{t+1}\right] + \\ &\quad + \beta_g \xi_p\left(E_t\left[\hat{\pi}_{t+1}\right] - \gamma_p \hat{\pi}_t\right)\end{aligned} \tag{154}$$

Now we can rearrange the terms:

$$\left(\hat{\pi}_t - \gamma_p \hat{\pi}_{t-1}\right) = \frac{(1-\xi_p)}{\xi_p}\left(1-\beta_g \xi_p\right)\left(\hat{mc}_t + \hat{l}_t\right) - \beta_g\gamma_p\hat{\pi}_t + \beta_g E_t\left[\hat{\pi}_{t+1}\right] \tag{155}$$

We rearrange again:

$$\left(1+\beta\gamma_p\right)\hat{\pi}_t = \frac{(1-\xi_p)}{\xi_p}\left(1-\beta_g \xi_p\right)\left(\hat{mc}_t + \hat{l}_t\right) + \gamma_p\hat{\pi}_{t-1} + \beta_g E_t\left[\hat{\pi}_{t+1}\right] \tag{156}$$

We divide by $\left(1+\beta_g\right)$ and get the expression for the New Keynesian Phillips curve:

$$\hat{\pi}_t = \frac{\beta_g}{1+\beta_g\gamma_p}E_t\left[\hat{\pi}_{t+1}\right] + \frac{\gamma_p}{1+\beta_g\gamma_p}\hat{\pi}_{t-1} + \frac{1}{1+\beta\gamma_p}\frac{\left(1-\beta_g\xi_p\right)\left(1-\xi_p\right)}{\xi_p}\left(\hat{mc}_t + \hat{l}_t\right) \tag{157}$$

Now, if we follow Romer (2006, Chapter 6.6) in assuming that wages and marginal costs increase in the output gap, the loglinearized Phillips curve (157) takes the form of equation (2) in the paper. In the case with labor market imperfections, firms may pay wages above the market-clearing level and wages may be given by a “real-wage function” rather than the elasticity of labor supply. Markup shocks $\hat{l}_t$, productivity shocks, and labor supply shocks, can drive a wedge between marginal costs and the output gap and therefore act as Phillips Curve shocks. The only difference between (157) and the corresponding expression in Smets and Wouters (2003) is that we need to replace the pure discount rate β by a different constant close to one, the growth-adjusted discount rate $\beta_g = \beta exp(-\gamma g)$.

B.2 Loglinear Habit Dynamics Around Steady State

We now show that a loglinear approximation around the nonstochastic steady-state yields approximate log habit dynamics of the form

$$h_{t+1} = \sum_{j=0}^{\infty} a_j c_{t-j}. \tag{158}$$

Approximating

$$\hat{s}_t = \left(1 - \frac{1}{\bar{S}}\right)(h_t - c_t - h), \tag{159}$$

we get loglinear approximate dynamics for log-habit h_t (ignoring constants):

$$h_{t+1} \approx \theta_0 h_t + (1-\theta_0)c_t + \frac{\theta_1}{\tau(1-\frac{1}{\bar{S}})}x_t + \frac{\theta_2}{\tau(1-\frac{1}{\bar{S}})}x_{t-1}, \tag{160}$$

$$= (1-\theta_0)\sum_{j=0}^{\infty}\theta_0^j c_{t-j} - \frac{\theta_1}{\tau(\frac{1}{\bar{S}}-1)}\left(c_t - (1-\phi)\sum_{j=1}^{\infty}\phi^j c_{t-j}\right) \tag{161}$$

$$-\frac{\theta_2}{\tau(\frac{1}{\bar{S}}-1)}\left(c_{t-1} - (1-\phi)\sum_{j=1}^{\infty}\phi^j c_{t-1-j}\right). \tag{162}$$

Note that if $\theta_1 = \theta_2 = 0$, the loglinear approximation to surplus consumption takes the form

$$\hat{s}_t \approx \left(\frac{1}{\bar{S}} - 1\right)\left(c_t - (1-\theta_0)\sum_{j=0}^{\infty}\theta_0^j c_{t-1-j} + h\right). \tag{163}$$

This is essentially the same functional form as for the output gap (equation (9) in the main paper)

$$x_t = \tau^{-1}\left(c_t - (1-\phi)\sum_{j=0}^{\infty}\phi^j c_{t-1-j}\right). \tag{164}$$

With $\theta_0 > \phi$, the approximate surplus consumption ratio (163) is proportional to consumption in excess of a long-term moving average, whereas the output gap (164) is proportional to consumption in excess of a more medium-term moving average.

Positive values for $\theta_1 > 0$ and $\theta_2 > 0$ decrease a_0 and a_1 in (158) while increasing the weight on medium-term lags. With $\phi < \theta_0$, a_j converges to $(1-\theta_0)\theta_0^j$ as j goes to infinity. Hence, approximate log habit dynamics load onto long lags of consumption exactly as in Campbell and Cochrane (1999).

B.3 Comparison with Wachter (2006) Preferences

Wachter (2006) uses a slightly different modification for Campbell and Cochrane (1999) preferences to generate time-varying bond risk premia. She assumes iid homoskedastic consumption growth and specifies the following dynamics for the surplus consumption ratio

$$s_{t+1} = (1-\phi)\bar{s} + \phi s_t + \lambda(s_t, f_{\bar{S}}(\phi + b/\gamma))\varepsilon_{c,t+1} \tag{165}$$

Here, b is a constant determining the dynamics of short-term real interest rates and we denote the function

$$f_{\bar{S}}(\theta) \quad = \quad \sigma_c \sqrt{\frac{\gamma}{1-\theta}}, \tag{166}$$

so $f_{\bar{S}}(\phi)$ gives the steady-state surplus consumption ratio in the Campbell-Cochrane model. Now, define

$$\theta_0 \quad = \quad \phi + b/\gamma. \tag{167}$$

Then, we can re-write the dynamics (165) as

$$s_{t+1} \quad = \quad (1-\theta_0)\bar{s} + \theta_0 s_t - \frac{b}{\gamma} s_t + \lambda(s_t, f_{\bar{S}}(\theta_0))\varepsilon_{c,t+1} \tag{168}$$

This compares to our model for surplus consumption dynamics

$$s_{t+1} \quad = \quad (1-\theta_0)\bar{s} + \theta_0 s_t + \theta_1 x_t + \theta_2 x_{t-1} + \lambda(s_t, f_{\bar{S}}(\theta_0))\varepsilon_{c,t+1} \tag{169}$$

Therefore, the difference between the two types of preferences is only that we replace a term

$$\frac{-b}{\gamma} s_t \tag{170}$$

by

$$\theta_1 x_t + \theta_2 x_{t-1}. \tag{171}$$

If the output gap is closely related to consumption in excess of habit, as is the case in our model, these two variants for obtaining time-varying real interest rates are also very closely related.

Wachter (2006) chooses a positive value for b to obtain a negative covariance between real interest rates and surplus consumption and a positive real bond beta. The relation in our model between interest rate cyclicality and θ_1 and θ_2 is more complicated. It depends on the endogenous output gap dynamics and the relative values of θ_1 and θ_2. As we see in the calibration, we can obtain both positive and negative nominal and real bond betas with $\theta_1 > 0$ and $\theta_2 > 0$.

There are two reasons for us to choose our specification for surplus consumption ratio dynamics rather than Wachter's (2006). First, our specification avoids sunspot fluctuations in macreconomic equilibrium dynamics by including an additional lag term. Second, it generates dynamics for consumption, the output gap, inflation, and interest rates that are conditionally homoskedastic, consistent with no strong heteroskedasticity in macroeconomic data.

C Additional Calibration Features and Robustness

C.1 Solution with Regime Switches

For robustness, we consider a simple regime-switching modification of our baseline model. Suppose that with probability q, the system switches from state a to states b or c, each with equal probability. Let X_t denote the regime in period t. If $X_t \in \{a, b, c\}$, we assume that the state vector at time $t+1$ $\hat{Y}_{t+1}$ satisfies

$$\hat{Y}_{t+1} = P_{X_t}\hat{Y}_t + Q_{X_t}u_{t+1}. \tag{172}$$

Regime X_{t+1} is realized after $\hat{Y}_{t+1}$, so at time t the transition matrix from time t to time $t+1$ is known.

Next, we solve for asset prices in the regime-switching model. In states b and c, the solutions for zero coupon bond prices and zero coupon dividend claims are unchanged, because those are absorbing states. In state a, we have to modify the solution recursions. We start with equity zero coupon dividend claims. The one-period claim $f_{a,1}$ is unchanged. For $n > 1$, $f_{1,n}$ is given by the recursion

$$\begin{aligned} f_{a,n} = \; & log\Big[E_t\Big[exp\Big(\delta_{eq}g + \delta_{eq}\tau e_1[P-\phi I]A^{-1}\tilde{Z}_t \\ & -r^f - (e_3 - e_2P)A^{-1}\tilde{Z}_t - \frac{\gamma}{2}(1-\theta_0)(1-2\hat{s}_t) \qquad (173) \\ & -(\gamma(1+\lambda(\hat{s}_t)) - \delta_{eq})\underbrace{\tau e_1 A^{-1} e_1}_{ve_1}\epsilon_{t+1} \qquad (174) \\ & -(\gamma(1+\lambda(\hat{s}_t)) - \delta_{eq})\underbrace{\tau e_1 A^{-1}(e_2+e_3)}_{ve_2}\epsilon_{t+1} \\ & +\bar{f}_{a,n-1}\Big)\Big]\Big]. \qquad (175) \end{aligned}$$

$$\bar{f}_{a,n} = log\Big((1-q)F_{a,n-1}(\tilde{Z}_{t+1}, \hat{s}_{t+1}, x_t) + \frac{q}{2}\Big(F_{b,n-1} + F_{c,n-1})(\tilde{Z}_{t+1}, \hat{s}_{t+1}, x_t)\Big)\Big).$$

Similarly, the recursions for real and nominal bonds remain unchanged, except for replacing b_{n-1} and $b^{\$}_{n-1}$ by

$$\bar{b}_{a,n-1} = log\left((1-q)B_{a,n-1}) + \frac{q}{2}\left(B_{b,n-1} + B_{c,n-1}\right)\right), \tag{176}$$

$$\bar{b}^{\$}_{a,n-1} = log\left((1-q)B^{\$}_{a,n-1} + \frac{q}{2}\left(B^{\$}_{b,n-1} + B^{\$}_{c,n-1}\right)\right). \tag{177}$$

We first solve numerically for asset prices in regimes b and c. We then use loglinear interpolation to compute regime b and c asset prices along grid points for the grid chosen for regime b. We then solve for regime a asset prices.

Table A.1 is analogous to Table 5 in the main paper, except that we allow for a regime switching probability of $q = 1\%$. This regime switching probability corresponds to a regime half-life of 20 years, consistent with our empirical regimes lasting between between 10 and 24 years. All asset pricing properties shown in Table A.1 are almost indistinguishable from Table 5 in the main paper, indicating that we do not lose much by formally modeling regimes as lasting an infinite amount of time in our baseline model.

D Additional Empirical Results

This section reports additional empirical results.

D.1 Additional Break in 1987

Next, we split the second subperiod 1977.Q2-2000.Q4 into two periods, according to Alan Greenspan's appointment as Fed chairman. We choose the pre-1987 monetary policy parameters to minimize the distance between empirical and model regressions of the Federal Funds rate on the output gap, inflation, and the lagged Federal Funds rate. We force the post-1987 monetary policy parameters to be identical to those for our full subperiod 3 to capture the notion that the monetary policy rule changed with Fed chairmen. We subsequently choose the volatilities of shocks to minimize the distance between empirical and model second asset pricing moments and standard deviations of VAR(1) residuals.

Table A.2 reports the parameter values. All non-reported parameters are as in Table 4 in the main paper. The pre-1987 period is characterized by an even stronger inflation reaction coefficient and a smaller output gap coefficient than the full period 2, further driving up bond betas relative to the full period 2. The pre-1987 period is characterized by very volatile monetary policy shocks, consistent with a volatile Federal Funds rate during this period.

The volatilities of shocks in the 1987-2000 sample are a convex combination of those for our full subperiods 2 and 3. Phillips curve shocks are as volatile as in the full subperiod 2. However, monetary policy shocks are not volatile, similarly to the full subperiod 3. The inflation target volatility is between the values for our full subsamples 2 and 3, but closer to the subsample 2 value.

Table A.3 is analogous to Table 5 in the main paper. It shows that we can fit the empirical Taylor rule regressions and asset pricing moments for the pre-1987 and post-1987 subsamples. The beta of nominal bonds was strongly positive before 1987, but slightly negative during 1987-2000 sample.

Overall, if we force the monetary policy rule to be constaFnt from 1987 onwards, the model relies on a decrease in the volatility of PC shocks and an increase in persistent monetary policy shocks to explain the substantial decline in nominal bond betas after 2000.

D.2 Allowing for Non-Zero Correlation Between Monetary Policy Shocks and Inflation Target Shocks

We now investigate whether allowing for a non-zero correlation between monetary policy and inflation target shocks can help us generate more volatile bond returns. When the central bank is less credible in controlling inflation, it might not be able to generate an independent inflation target shock simply by communicating the new target. In our main calibration, we capture this through reduced inflation target volatility in subperiods 1 and 2. An alternative way of capturing this notion might be through allowing a negative correlation between monetary policy and inflation target shocks. If the central bank has to raise the policy rate first in order to credibly signal its commitment to a lower inflation target, this might act similarly to a reduced overall inflation target volatility in periods 1 and 2.

We choose parameters by first optimizing over the volatility of shocks and the monetary policy-inflation target correlation, while holding all other parameters constant. Next, we reoptimize over the monetary policy parameters to match the empirical Taylor rule regression as closely as possible for all three subperiods.

Table A.4 shows the resulting parameter values. The MP-inflation target correlation is highly negative for the first two subperiods, but close to zero for the third subperiod. At the same time, the inflation target volatility is smaller than in our main calibration and roughly constant across subperiods.

Tablle A.5 compares empirical and model moments for this alternative calibration. The model fits broad change in bond betas across subperiods, similarly to our main calibration. This alternative calibration generates more volatile bond returns, especially in the second subperiod. However, this increased volatility comes at the expense of significantly more volatility in the output gap and stock returns than in our baseline calibration. We conclude that allowing for a negative correlation between inflation target shocks and monetary policy shocks acts similarly to a lower overall inflation target volatility on betas.

D.3 Robustness of Monetary Policy Rule

Next, we estimate empirical monetary policy regressions for pre- and post-crisis samples. Table A.6 is analogous to Table 3 in the main paper, except that it splits superiod 3 (2001.Q-2011.Q4) into a pre-crisis subsample (2001.Q1-2008.Q2) and a post-crisis subsample (2008.Q3-2011.Q4). The estimates for the pre-crisis subsample are in line with the estimates for the full subperiod 3. The Federal Funds rate is persistent and responds strongly to the output gap. The estimated naive inflation reaction coefficient $\hat{\gamma}^{\pi}$ is even below one, but a two-standard deviation interval includes one. The estimated monetary policy rule for the post-crisis period has all coefficients very close to zero. This is not surprising, since the Federal Funds rate was stuck at the zero lower bound at this time.

Table A.7 reports monetary policy rule estimates using a real-time measure of the output gap. We are grateful to Athanasios Orphanides for providing this real time measure to us. Table A.7 is analogous to Table 3 in the main paper, except that it uses this alternative measure for the output gap. Subperiod 2 is again characterized by a naive inflation gap coefficient $\hat{\gamma}^{\pi}$ above one and a small naive output gap reaction coefficient $\hat{\gamma}^{x}$. The estimated persistence parameter increased substantially from period 2 to period 3. With the real time output gap, the estimated naive estimated output gap coefficient $\hat{\gamma}^{x}$ for the most recent subperiod is negative. However, we have to keep in mind that the identification of regime break dates is necessarily imprecise. If we use a break date of 1997 instead, which is when bond betas turned negative, we again estimate a large and positive naive output gap coefficient $\hat{\gamma}^{x}$.

D.4 Macroeconomic Correlations

If changes in bond risks are driven by macroeconomic factors, then changes in bond risks should be reflected in changing macroeconomic correlations. Lower than expected inflation raises nominal bond prices, all else equal, so the inflation-output correlation should typically take the opposite sign from the bond-stock correlation.

Table A.8 compares sub-sample correlations of asset prices and macroeconomic variables. The empirical output gap is highly persistent and it is therefore unsurprising that three year equity excess returns are more strongly correlated with the output gap than highly volatile quarterly stock returns. We therefore use quarterly overlapping three year bond and stock excess returns for our comparison of asset return correlations and macroeconomic correlations. Table A.8 confirms our intuition that bond excess returns should at least partly reflect news about inflation and that equity excess returns should reflect the business cycle. In each sub period, empirical bond excess returns are negatively correlation with inflation and equity excess returns are positively correlated with the output gap.

Table A.8 confirms that the changes in the bond-stock comovement documented in Figure 1 and Table 5 are robust to using three year returns instead of daily or quarterly returns. The correlation between three year stock returns and three year bond returns was positive and significant in the first sub-period, increased in the second sub period, and became negative and significant in the last sub period.

The bond-output gap, inflation-stock, and inflation-output gap correlations confirm our intuition that changing bond risks are related to the prevalence of inflationary recessions versus deflationary recessions during different regimes. The bond-output gap correlation typically has the same sign as the bond-stock correlation, while the inflation-stock return correlation and the inflation-output correlation has the opposite sign. The only exception to this pattern is the first sub period bond-output gap correlation, which takes a negative, but small and insignificant, value.

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Table A.1: Model and Empirical Moments with Regime Switches

Panel A: Estimated MP Rule – Fed Funds onto Output Gap, Infl. and Lag. Fed Funds

	60.Q2-77.Q1		77.Q2-00.Q4		01.Q1-11.Q4	
	Empirical	Model	Empirical	Model	Empirical	Model
Output Gap	0.17**	0.15	0.03	-0.10	0.04	0.06
Inflation	0.21**	0.28	0.41*	0.36	0.21**	0.16
Lagged Fed Funds	0.69**	0.71	0.66*	0.63	0.83**	0.84

Panel B: Subperiod Second Moments

	60.Q2-77.Q1		77.Q2-00.Q4		01.Q1-11.Q4	
Std. Asset Returns	Empirical	Model	Empirical	Model	Empirical	Model
Std. Eq. Ret.	18.35	16.77	15.68	16.53	20.34	14.91
Std. Nom. Bond Ret.	4.92	1.42	8.11	2.61	5.92	3.57
Nominal Bond Beta	0.07**	0.05	0.12	0.12	-0.18**	-0.17
Std. Real Bond Ret.		4.59		2.16	4.27	2.08
Real Bond Beta		-0.25		0.05	-0.08	-0.06
Std. VAR(1) Residuals						
Output Gap	0.83	1.23	0.76	1.06	0.67	0.97
Inflation	1.05	0.79	1.04	0.33	0.86	0.26
Fed Funds Rate	0.90	0.75	1.55	1.52	0.47	0.57
Log Nominal Yield	0.47	0.15	0.77	0.27	0.56	0.38

This table is analogous to Table 5 in the main paper, except that it allows for a probability of regime switches, as described in Section C.1. The probability of switching out of the current regime is set to 1% per quarter, corresponding to a regime half-life of 20 years.

Table A.2: Parameter Choices: Additional Break in 1987

Monetary Policy Rule		77.Q2-00.Q4	77.Q2-87.Q2	87.Q3-00.Q4
Output Weight	γ^x	0.28	0.23	0.84
Inflation Weight	γ^π	1.61	1.91	1.60
Persistence MP	ρ^i	0.64	0.54	0.82
Std. Shocks				
Std. PC		0.49	0.64	0.34
Std. MP		1.60	2.34	0.52
Std. Infl. Target		0.13	0.05	0.17

Panel B: Implied Parameters

		77.Q2-00.Q4	77.Q2-87.Q2	87.Q3-00.Q4
Discount Rate	β	0.85	0.88	0.88
IS Curve lag Coefficient	ρ^{x-}	0.02	0.02	0.02
IS Curve Forward Coefficient	ρ^{x+}	1.10	1.10	1.10
IS Curve Real Rate Slope	ψ	0.41	0.41	0.41
Steady-State Surplus Cons. Ratio	$\bar{S}$	0.12	0.14	0.09
Log Max. Surplus Cons. Ratio	s_{max}	-1.65	-1.49	-1.88
Max Surplus Cons. Ratio	S_{max}	0.19	0.23	0.15
Std. Cons. Innovation	σ_c	2.51	2.70	1.81
Twelve-Quarter Cons. VR		0.70	1.10	0.77
AR(1) Coefficient Output Gap		0.89	0.94	0.91

Table A.3: Additional Break in 1987: Model and Empirical Moments

Panel A: Estimated MP Rule – Fed Funds onto Output Gap, Infl. and Lag. Fed Funds

	77.Q2-00.Q4		77.Q2-87.Q2		87.Q3-00.Q4	
	Empirical	Model	Empirical	Model	Empirical	Model
Output	0.03	-0.03	-0.11	-0.11	0.17**	0.16
Inflation	0.41*	0.33	0.44*	0.35	0.25*	0.34
Lagged Fed Funds	0.66*	0.66	0.50*	0.52	0.80**	0.66

Panel B: Subperiod Second Moments

	77.Q2-00.Q4		77.Q2-87.Q2		87.Q3-00.Q4	
Std. Asset Returns	Empirical	Model	Empirical	Model	Empirical	Model
Std. Eq. Ret.	15.68	18.14	15.63	21.16	15.86	13.55
Std. Nom. Bond Ret.	8.11	2.93	10.51	4.24	5.74	1.55
Nominal Bond Beta	0.12	0.12	0.32	0.18	-0.03	-0.03
Std. Real Bond Ret.		2.13		2.55		2.09
Real Bond Beta		0.03		0.06		-0.14
Std. VAR(1) Residuals						
Output Gap	0.76	1.40	0.95	1.96	0.50	0.65
Inflation	1.04	0.45	1.33	0.59	0.61	0.33
Fed Funds Rate	1.55	1.56	2.17	2.29	0.46	0.48
Log Nominal Yield	0.77	0.30	0.96	0.44	0.58	0.16

This table reports average model moments from 5 simulations of length 10000. * and ** denote significance at the 5% and 1% levels. We use Newey-West standard erros with 2 lags for the nominal bond beta and Newey-West standard errors with 6 lags for the empirical Taylor rule estimation in the bottom panel.

Table A.4: Parameter Choices with Correlation (MP Shocks, Infl. Tgt. Shocks)

Monetary Policy Rule		77.Q2-00.Q4	77.Q2-87.Q2	87.Q3-00.Q4
Output Weight	γ^x	0.48	0.11	0.72
Inflation Weight	γ^π	0.75	1.21	1.43
Persistence MP	ρ^i	0.53	0.90	0.77
Std. Shocks				
Std. PC		0.68	0.09	0.29
Std. MP		0.77	1.82	0.66
Std. Infl. Target		0.22	0.27	0.27
Corr(Infl. Tgt., MP Shocks)		-0.80	-0.88	-0.12

Panel B: Implied Parameters

		77.Q2-00.Q4	77.Q2-87.Q2	87.Q3-00.Q4
Discount Rate	β	0.85	0.85	0.85
IS Curve lag Coefficient	ρ^{x-}	0.02	0.02	0.02
IS Curve Forward Coefficient	ρ^{x+}	1.10	1.10	1.10
IS Curve Real Rate Slope	ψ	0.41	0.41	0.41
Steady-State Surplus Cons. Ratio	$\bar{S}$	0.08	0.21	0.06
Log Max. Surplus Cons. Ratio	s_{max}	-2.00	-1.10	-2.25
Max Surplus Cons. Ratio	S_{max}	0.13	0.33	0.11
Std. Cons. Innovation	σ_c	1.75	4.38	1.38
Twelve-Quarter Cons. VR		1.06	0.37	0.81
AR(1) Coefficient Output Gap		0.94	0.81	0.91

Table A.5: Model and Empirical Moments with Correlation (MP Shocks, Infl. Tgt. Shocks)

Panel A: Estimated MP Rule – Fed Funds onto Output Gap, Infl. and Lag. Fed Funds

	77.Q2-00.Q4		77.Q2-87.Q2		87.Q3-00.Q4	
	Empirical	Model	Empirical	Model	Empirical	Model
Output Gap	0.17**	0.16	0.03	-0.38	0.04	0.05
Inflation	0.21**	0.28	0.41*	0.31	0.21**	0.20
Lagged Fed Funds	0.69**	0.72	0.66*	0.68	0.83**	0.80

Panel B: Subperiod Second Moments

	77.Q2-00.Q4		77.Q2-87.Q2		87.Q3-00.Q4	
Std. Asset Returns	Empirical	Model	Empirical	Model	Empirical	Model
Std. Eq. Ret.	18.35	14.60	15.68	25.08	20.34	14.62
Std. Nom. Bond Ret.	4.92	1.40	8.11	7.10	5.92	2.29
Nominal Bond Beta	0.07**	0.00	0.12	0.27	-0.18**	-0.07
Std. Real Bond Ret.		4.31		11.44	4.27	1.92
Real Bond Beta		-0.27		0.44	-0.08	-0.06
Std. VAR(1) Residuals						
Output Gap	0.83	0.98	0.76	2.49	0.67	0.78
Inflation	1.05	0.66	1.04	0.13	0.86	0.28
Fed Funds Rate	0.90	0.66	1.55	1.80	0.47	0.61
Log Nominal Yield	0.47	0.15	0.77	0.72	0.56	0.24

This table reports average model moments from 5 simulations of length 10000. * and ** denote significance at the 5% and 1% levels. We use Newey-West standard erros with 2 lags for the nominal bond beta and Newey-West standard errors with 6 lags for the empirical Taylor rule estimation in the bottom panel.

Table A.6: Empirical Monetary Policy Function Crisis Sample

Fed Funds i_t	00.Q1-11.Q4	00.Q1-08.Q2	08.Q3-11.Q4
Output Gap	0.04	0.41**	-0.02
	(0.03)	(0.10)	(0.03)
Inflation	0.21**	0.23**	-0.00
	(0.07)	(0.05)	(0.01)
Lagged Fed Funds	0.83**	0.73**	0.01
	(0.08)	(0.09)	(0.06)
Constant	-0.12	0.18	-0.02
	(0.22)	(0.19)	(0.27)
R^2	0.94	0.93	0.14
Implied $\hat{\gamma}^x$	0.22	1.52**	-0.02
	(0.12)	(0.53)	(0.04)
Implied $\hat{\gamma}^\pi$	1.19**	0.87**	0.00
	(0.68)	(0.25)	(0.01)
Implied $\hat{\rho}^i$	0.83**	0.73**	0.01
	(0.08)	(0.09)	(0.06)

This table estimates the monetary policy rule before and after the Lehman brothers bankruptcy in 2008.Q3. All variables and test specifications are described in Table 3 in the main text.

Table A.7: Estimating the Monetary Policy Function - Real-Time Output Gap from Orphanides

Fed Funds i_t	60.Q2-77.Q1	77.Q2-00.Q4	01.Q1-11.Q4	97.Q1-11.Q4
Output Gap	0.11**	0.03	-0.02	0.07
	(0.03)	(0.04)	(0.04)	(0.05)
Inflation	0.21*	0.42*	0.26**	0.15*
	(0.08)	(0.16)	(0.08)	(0.07)
Lagged Fed Funds Rate	0.70**	0.67**	0.89**	0.88**
	(0.06)	(0.13)	(0.07)	(0.07)
Constant	1.23**	1.12	-0.51	0.04
	(0.25)	(0.58)	(0.25)	(0.28)
R^2	0.85	0.77	0.94	0.95
Implied $\hat{\gamma}^x$	0.36**	0.10	-0.20	0.61**
	(0.10)	(0.15)	(0.40)	(0.17)
Implied $\hat{\gamma}^\pi$	0.71**	1.27**	2.22**	1.31**
	(0.19)	(0.15)	(1.55)	(1.02)
Implied $\hat{\rho}^i$	0.85**	0.58**	0.56**	0.89**
	(0.04)	(0.11)	(0.15)	(0.06)

This table is identical to Table 4 in the main text, but it uses a different measure of the output gap. We are grateful to Athanasios Orphanides for sharing his real-time output gap data with us.

Table A.8: Sub-Period Correlations of Bond Returns, Stock Returns, Output Gap, and Inflation

60.Q2-77.Q1	Bond Excess Returns	Stock Excess Returns	Output Gap	Inflation
Bond Excess Returns	1			
Stock Excess Returns	0.28*	1		
Output Gap	-0.14	0.41*	1.00	
Inflation	-0.34*	-0.73*	-0.13	1.00

77.Q2-00.Q4	Bond Excess Returns	Stock Excess Returns	Output Gap	Inflation
Bond Excess Returns	1			
Stock Excess Returns	0.36*	1		
Output Gap	0.22*	0.40*	1	
Inflation	-0.63*	-0.42*	-0.18	1

01.Q1-11.Q4	Bond Excess Returns	Stock Excess Returns	Output Gap	Inflation
Bond Excess Returns	1			
Stock Excess Returns	-0.78*	1		
Output Gap	-0.56*	0.39*	1	
Inflation	-0.36*	0.45*	0.57*	1

Quarterly overlapping 3 year log equity returns in excess of log three month T-bill, 3 year log excess return on 5 year nominal bond in excess of three month log T-bill. Quarterly inflation and output as in Table 1. We report correlations of log excess returns from time $t-12$ to t and macroeconomic variables as of quarter t. * and ** denote significance at the 5% and 1% level. Significance levels not adjusted for time series dependence.