

ONLINE APPENDIX

Sales Mechanisms in Online Markets

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This online appendix contains two items. Appendix A provides a more general version of the model from Section 3. The remainder of the Appendix includes additional charts and tables that are referenced in the main text.

Appendix A. Extended Model

The model in Section 3 assumes that all buyers have the same value v . We mention in the text that allowing heterogeneity in buyer valuations would not change the main results. Here we describe this extension.

There are $n \geq 1$ potential buyers, indexed by $i = 1, \dots, n$. Buyer i has a value $v_i = v + w_i$, where v is a common value for all buyers drawn from a distribution f , and each w_i is an idiosyncratic value drawn independently from a distribution g . The distributions f and g are both log-concave. Let $w^{(1)}, \dots, w^{(n)}$ denote the order statistics of the idiosyncratic value draws, and $v^{(k)} = v + w^{(k)}$ be the k th order statistic of the buyer valuations. All buyers have a common reservation value u , and auction disutility λ .

If the seller offers a posted price p , the probability of sale is

$$Q_F(p) = \Pr[v^{(1)} - u \geq p].$$

If the seller offers an auction with reserve price r , the probability of sale is

$$Q_A(r) = \Pr[v^{(1)} - \lambda - u \geq r].$$

Therefore (as in Figure 5) the posted price demand curve is a vertical translation of the auction sales curve: $Q_F(r + \lambda) = Q_A(r)$. That is, setting a posted price of $r + \lambda$ gives an equivalent probability of sale as running an auction with reserve price r .

Next, the expected auction price given reserve price r is

$$p_A(r) = r + \mathbb{E}[\max\{v^{(2)} - \lambda - u - r, 0\} \mid v^{(1)} - \lambda - u - r \geq 0].$$

We claim that the second term is decreasing in r . This is because

$$\begin{aligned} & \mathbb{E} \left[\max \{ v^{(2)} - \lambda - u - r, 0 \} \mid v^{(1)} - \lambda - u - r \geq 0 \right] \\ &= \mathbb{E} \left[v^{(2)} - \lambda - u - r \mid v^{(2)} - \lambda - u - r \geq 0 \right] \\ & \quad \times \Pr \left[v^{(2)} - \lambda - u - r \geq 0 \mid v^{(1)} - \lambda - u - r \geq 0 \right]. \end{aligned}$$

Log-concavity of the distributions $f(v)$ and $g(w_i)$ is preserved for the distribution of $v + w_i$, and for the corresponding order statistics (Barlow and Proschan, 1966).¹ Moreover, the mean residual lifetime $\mathbb{E} [v_i \mid v_i \geq r'] - r'$ of a random variable whose distribution is log-concave is decreasing in r' . So $\mathbb{E} [v^{(2)} - \lambda - u - r \mid v^{(2)} - \lambda - u - r \geq 0]$ is decreasing in r .

It remains to be proven that $\Pr [v^{(2)} - \lambda - u - r \geq 0 \mid v^{(1)} - \lambda - u - r \geq 0]$ is also decreasing in r . Let H be the cumulative distribution function of $v_i = v + w_i$ and n the number of bidders. Then

$$\begin{aligned} \Pr [v^{(2)} - \lambda - u - r \geq 0 \mid v^{(1)} - \lambda - u - r \geq 0] &= \frac{\Pr [v^{(2)} - \lambda - u - r \geq 0]}{\Pr [v^{(1)} - \lambda - u - r \geq 0]} \\ &= 1 - \frac{nH^{n-1}(r + \lambda + u) [1 - H(r + \lambda + u)]}{[1 - H^n(r + \lambda + u)]} \\ &= 1 - \frac{nH^{n-1}(r + \lambda + u)}{1 + \sum_{k=1}^{n-1} H^k(r + \lambda + u)}. \end{aligned}$$

The last term is an increasing function of r , so $\Pr [v^{(2)} - \lambda - u - r \geq 0 \mid v^{(1)} - \lambda - u - r \geq 0]$ is decreasing in the reserve price.

With this established, it follows that the posted price demand curve is steeper than the auction demand curve. To see this, for any q define r such that $q = Q_A(r)$. Then the vertical distance between the auction demand curve and the posted price demand curve at q is $p_A(r) - (r + \lambda)$, which is decreasing in r , and hence increasing in q . Furthermore, at $q = 0$, we have $p_A(r) = r < r + \lambda$. So the auction demand curve starts below the posted price demand curve. At $q = 1$, we have $p(r) = \mathbb{E} [v + w^{(1)}] - \lambda - u$, whereas the relevant posted price is $\underline{v} + \underline{w} - u$, so provided $\lambda < \mathbb{E} [v + w^{(1)}] - (\underline{v} + \underline{w})$, the auction demand curve ends higher.

Therefore the posted price demand curve crosses the auction demand curve once from above as in Figure 5. Under this condition, the comparative statics in the main text with respect to c , u , and λ are straightforward. In particular, an increase in any of these parameters makes it more likely that the best posted price will dominate an auction with an optimal reserve price.

¹Barlow, Richard E., and Frank Proschan (1966). "Inequalities for Linear Combinations of Order Statistics from Restricted Families." *Annals of Mathematical Statistics* 37(6), 1574-1592.

Figure A1: Auction Discount for Different Auction Subsamples

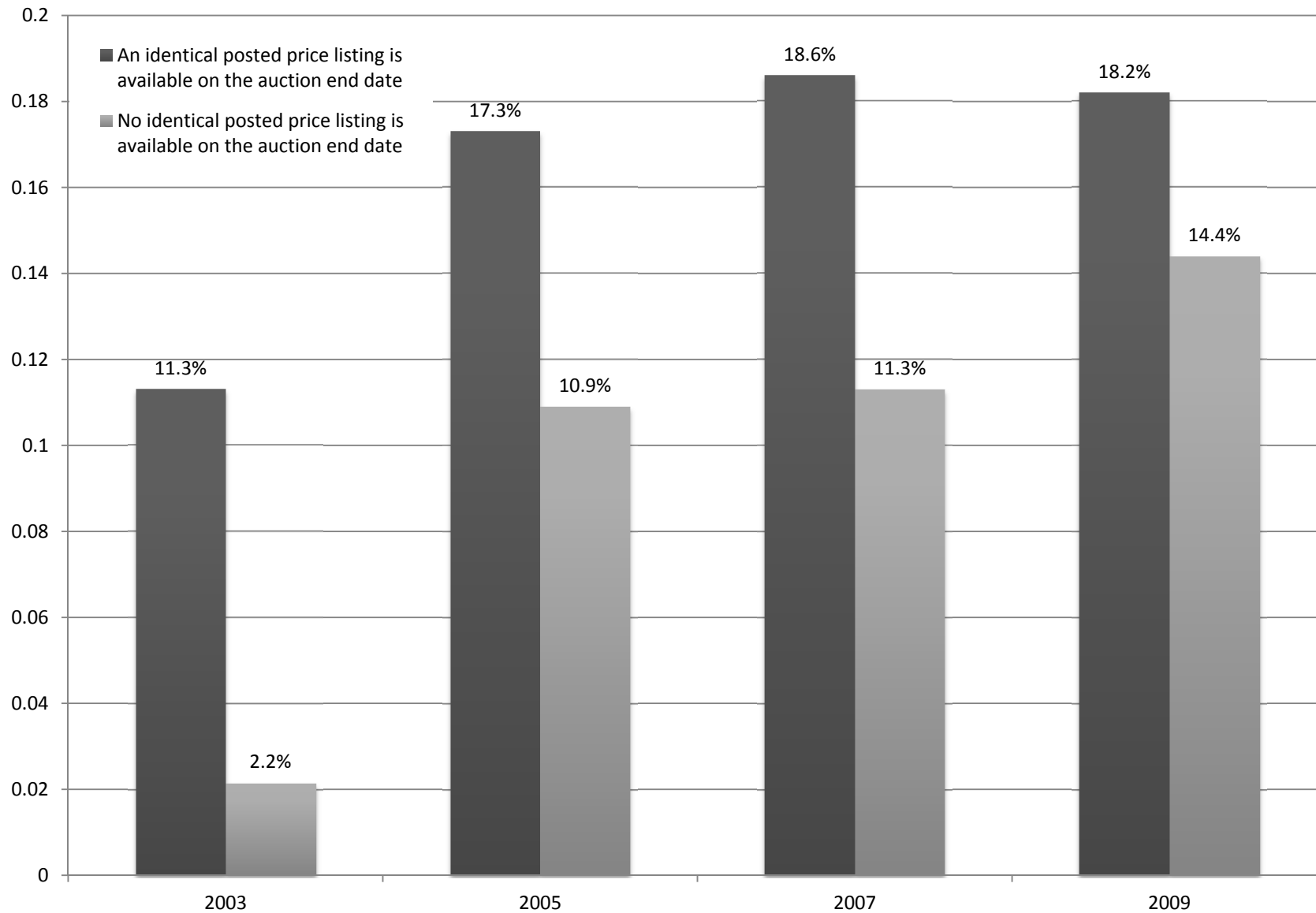


Figure presents the implied auction discounts for the two subsamples – auctions with concurrent posted price listings and auctions without – based on the results presented in Appendix Table A1(b).

Figure A2: Distribution of Number of Unique Bidders per Auction

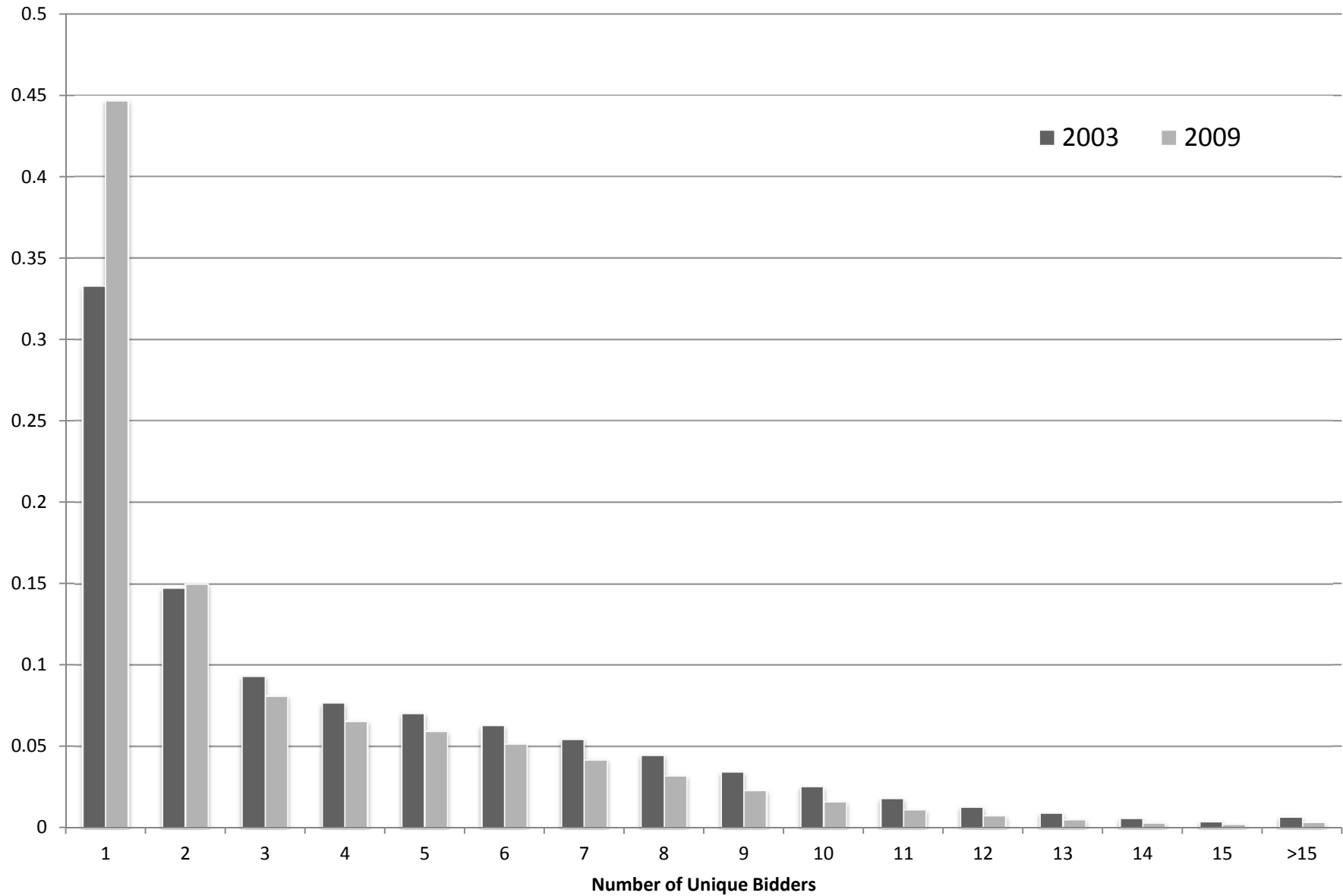


Figure presents the distribution of the number of unique bidders of the successful auction listings from the 2003 and 2009 experiment data.

Figure A3: Distribution of Auction Reserve Prices

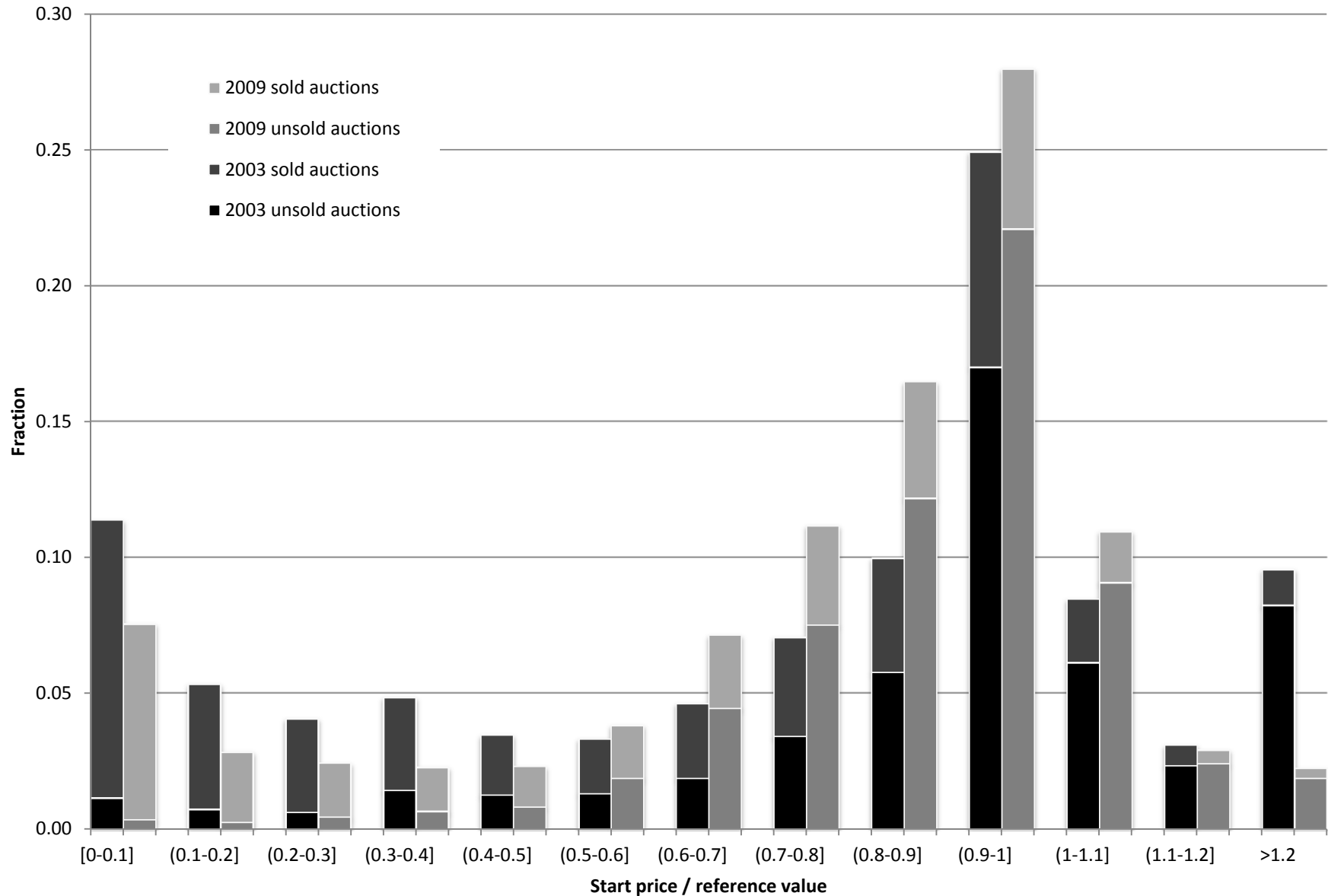


Figure presents the distribution of the normalized reserve prices of the auction listings from the 2003 and 2009 experiment data. Each column is divided in two separate groups, corresponding to the auction listings that did not sell (bottom), and those that resulted in a sale (top).

Figure A4: Distribution of (Normalized) Auction Sale Prices

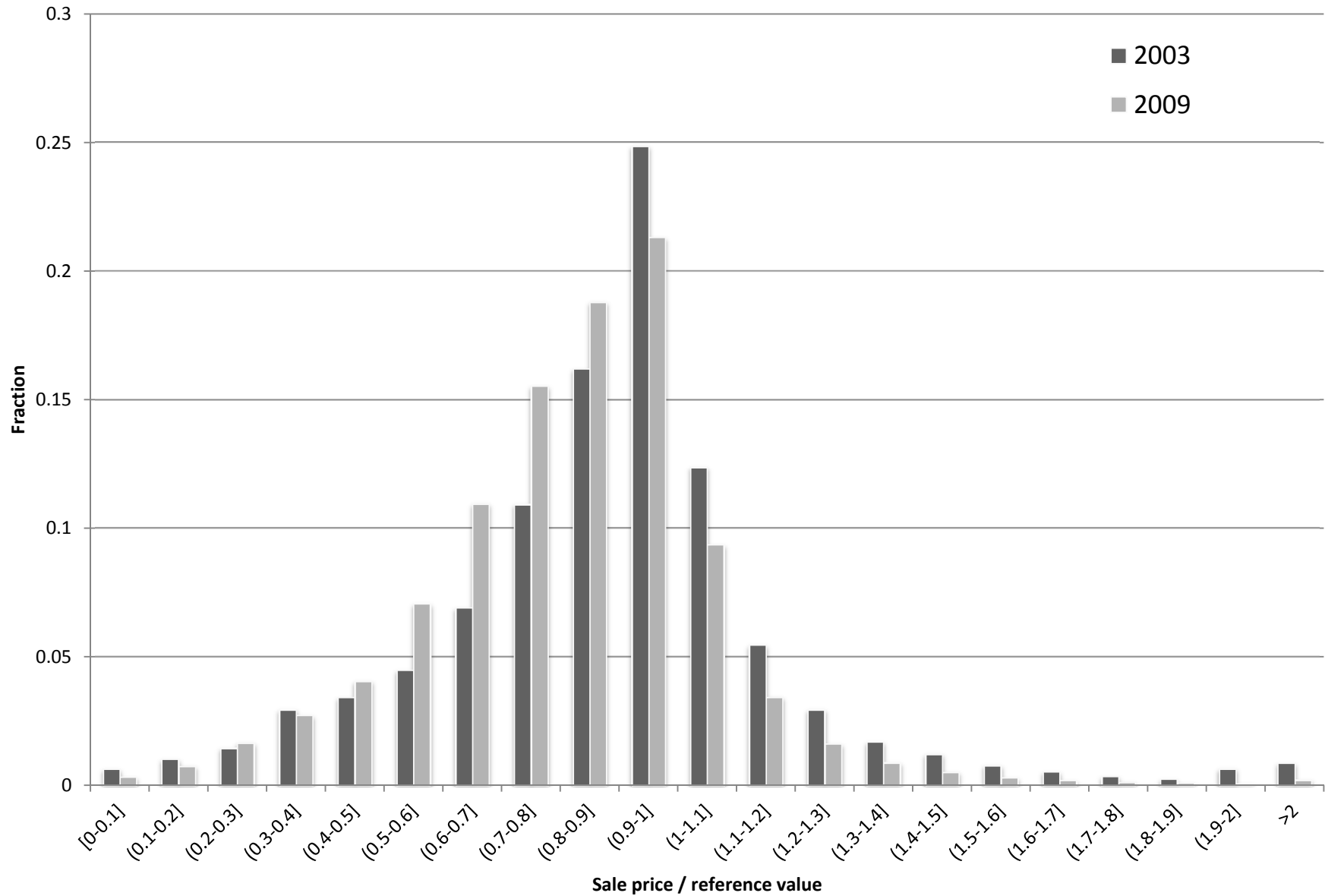


Figure presents the distribution of the normalized sale prices of the successful auction listings from the 2003 and 2009 experiment data.

Figure A5: Estimation Results that give rise to the Auction Demand Curves

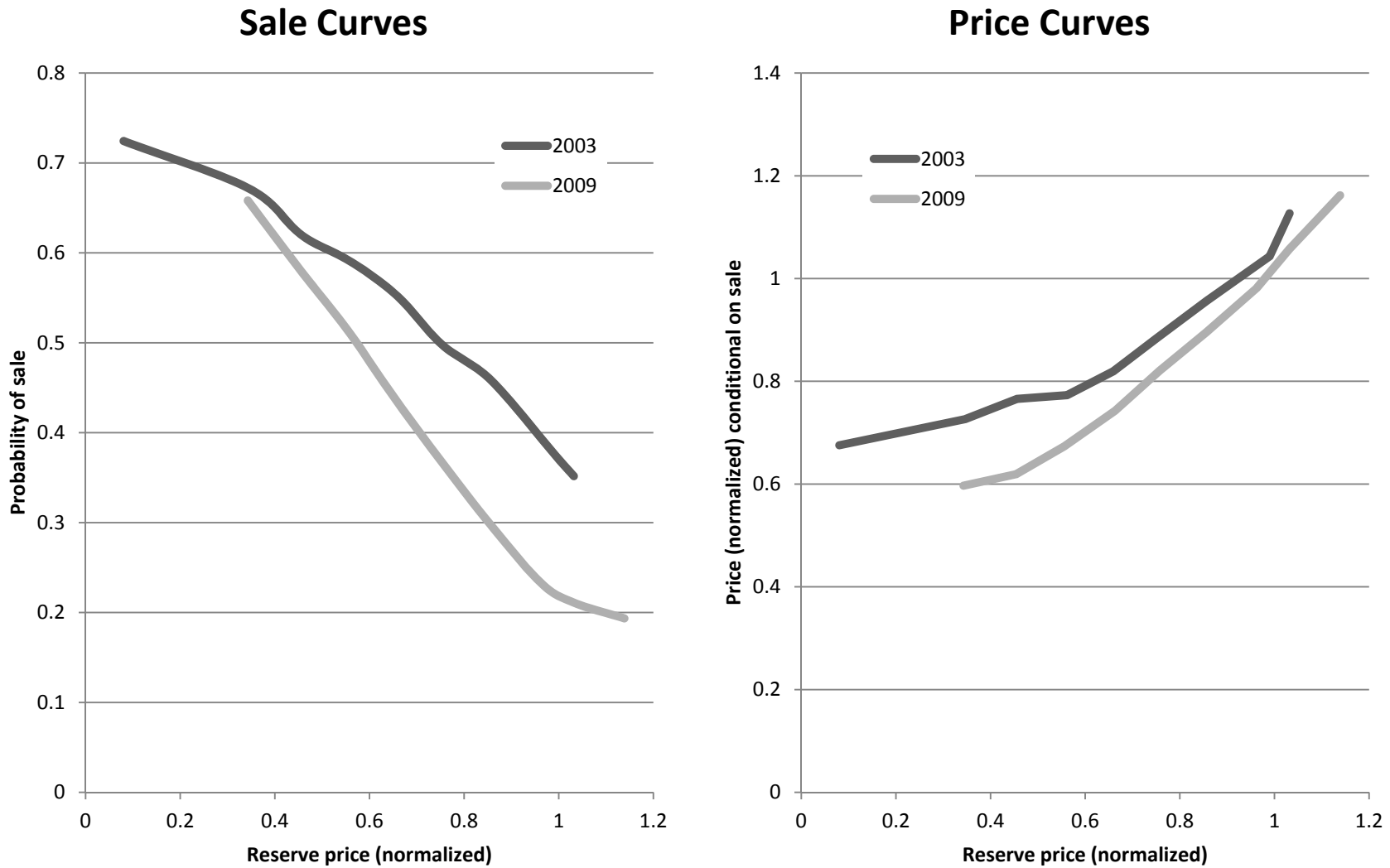


Figure presents a graphical version of the estimation results reported in Appendix Table A2.

Table A1(a): Auction Success and Auction Discount over Time

	Dependent variable: 1 if item purchased				Dependent variable: log(sale price)			
	2003	2005	2007	2009	2003	2005	2007	2009
Auction Indicator	0.0813 (0.0010)	0.0437 (0.0006)	0.0583 (0.0005)	0.115 (0.0005)	-0.0467 (0.0013)	-0.138 (0.0008)	-0.144 (0.0006)	-0.165 (0.0005)
Constant	0.403 (0.0008)	0.373 (0.0004)	0.397 (0.0003)	0.25 (0.0004)	3.099 (0.0010)	3.078 (0.0006)	3.343 (0.0004)	2.998 (0.0004)
Fixed Effects	Experiment	Experiment	Experiment	Experiment	Experiment	Experiment	Experiment	Experiment
No. of obs. (listings)	1,168,033	4,064,319	5,456,389	5,924,448	528,230	1,635,185	2,340,739	1,963,350

Table presents the results from estimating equations (2) and (3) in the main text for the years 2003, 2005, 2007, and 2009. The graphical representation of the above results is presented in Figure 7 of the paper. The four leftmost columns report the results from a linear probability regression of sale on a dummy for whether the listing is an auction listing, and they include experiment fixed effects. The four columns to the right include only successful listings, i.e. listings that resulted in a sale. The specification is similar to the other columns, with the dependent variable being the (log) sale price.

Table A1(b): Auction Success and Auction Discount for Different Auction Subsamples

A. Auctions with a concurrent posted price listing of the same item:

	Dependent variable: 1 if item purchased				Dependent variable: log(sale price)			
	2003	2005	2007	2009	2003	2005	2007	2009
Auction Indicator	0.0959 (0.0015)	0.0319 (0.0007)	0.0649 (0.0006)	0.11 (0.0006)	-0.113 (0.0020)	-0.173 (0.0011)	-0.186 (0.0009)	-0.182 (0.0005)
Constant	0.433 (0.0007)	0.374 (0.0004)	0.401 (0.0003)	0.248 (0.0004)	3.183 (0.0011)	3.077 (0.0006)	3.411 (0.0004)	3.017 (0.0004)
Fixed Effects	Experiment	Experiment	Experiment	Experiment	Experiment	Experiment	Experiment	Experiment
No. of obs. (listings)	623,472	2,670,128	3,664,597	4,579,688	285,553	1,040,252	1,548,565	1,447,246

B. Auctions without a concurrent posted price listing of the same item:

	Dependent variable: 1 if item purchased				Dependent variable: log(sale price)			
	2003	2005	2007	2009	2003	2005	2007	2009
Auction Indicator	0.0707 (0.0012)	0.0494 (0.0007)	0.0553 (0.0006)	0.111 (0.0007)	-0.0215 (0.0014)	-0.109 (0.0010)	-0.113 (0.0007)	-0.144 (0.0008)
Constant	0.385 (0.0008)	0.373 (0.0004)	0.376 (0.0003)	0.266 (0.0004)	2.993 (0.0010)	3.035 (0.0006)	3.201 (0.0003)	3.038 (0.0005)
Fixed Effects	Experiment	Experiment	Experiment	Experiment	Experiment	Experiment	Experiment	Experiment
No. of obs. (listings)	964,049	2,726,061	4,256,290	3,083,239	405,648	1,085,119	1,698,714	969,727

Table is similar to Appendix Table A1(a), but estimates the regressions separately for two subsamples of the auction listings. The top panel includes auction listings for which there is a posted price listing belonging to the same experiment that is active on the auction end date. The bottom panel includes auction listings that do not have such a concurrent posted price listing.

Table A2: Estimation Results that give rise to the Auction Demand Curves

Dependent Var.	2003 Auction Sample		2009 Auction Sample	
	1 if purchased	Normalized sale price, conditional on sale	1 if purchased	Normalized sale price, conditional on sale
Constant	0.724 (0.005)	0.676 (0.006)	0.832 (0.002)	0.623 (0.002)
Normalized start price is <0.3	Omitted	Omitted	Omitted	Omitted
Normalized start price in (0.3-0.4]	-0.053 (0.007)	0.0508 (0.007)	-0.174 (0.003)	-0.026 (0.002)
Normalized start price in (0.4-0.5]	-0.104 (0.007)	0.0902 (0.016)	-0.251 (0.004)	-0.00356 (0.003)
Normalized start price in (0.5-0.6]	-0.135 (0.008)	0.0974 (0.008)	-0.32 (0.003)	0.0505 (0.002)
Normalized start price in (0.6-0.7]	-0.173 (0.011)	0.144 (0.008)	-0.401 (0.003)	0.12 (0.002)
Normalized start price in (0.7-0.8]	-0.226 (0.007)	0.209 (0.008)	-0.468 (0.002)	0.198 (0.002)
Normalized start price in (0.8-0.9]	-0.266 (0.007)	0.281 (0.008)	-0.534 (0.002)	0.272 (0.002)
Normalized start price in (0.9-1]	-0.348 (0.007)	0.368 (0.008)	-0.599 (0.002)	0.358 (0.002)
Normalized start price in (1-1.1]	-0.373 (0.008)	0.451 (0.008)	-0.621 (0.003)	0.434 (0.002)
Normalized start price in (1.1-1.2]	-0.391 (0.009)	0.56 (0.009)	-0.638 (0.006)	0.539 (0.003)
Normalized start price is >1.2	-0.399 (0.008)	0.945 (0.012)	-0.673 (0.007)	0.741 (0.006)
Fixed Effects	Experiment	Experiment	Experiment	Experiment
No. of observations (listings)	748,545	365,266	4,185,969	1,509,727

Table presents estimation results that give rise to the auction demand curves reported in the paper. The analysis is based on estimating equations (4) and (5) in the paper, using only auction listings. The right-hand-side variable is the normalized auction start price, categorized in 11 bins, as well as experiment fixed effects. The sample for the second and fourth columns is restricted to successful listings only. Appendix Figure A5 provides a graphical presentation of these results. In all specifications, we weight each experiment by its total number of (auction and posted price) listings to make results comparable (in terms of compositional differences) between auctions and posted price demand curves.

Table A3: Estimation Results that give rise to the Posted Price Demand Curves

	2003 Posted Price Sample	2009 Posted Price Sample
Dependent Var.	1 if purchased	1 if purchased
Constant	0.452 (0.005)	0.521 (0.010)
Normalized posted price is <0.7	Omitted	Omitted
Normalized posted price in (0.7-0.8]	0.0299 (0.010)	-0.0475 (0.011)
Normalized posted price in (0.8-0.85]	0.0566 (0.011)	-0.083 (0.011)
Normalized posted price in (0.85-0.9]	0.0305 (0.008)	-0.108 (0.011)
Normalized posted price in (0.9-0.95]	0.0021 (0.007)	-0.143 (0.010)
Normalized posted price in (0.95-1.05]	-0.0392 (0.005)	-0.219 (0.010)
Normalized posted price in (1.05-1.1]	-0.0762 (0.007)	-0.24 (0.010)
Normalized start price is >1.1	-0.119 (0.007)	-0.232 (0.010)
Fixed Effects	Experiment	Experiment
No. of observations (listings)	419,488	1,738,479

Table presents estimation results that give rise to the posted price demand curves shown in Figure 8 of the paper. The analysis is based on estimating equation (6) in the paper, using only posted price listings. The right-hand-side variable is the normalized posted price, categorized in 8 bins, as well experiment fixed effects. In all specifications, we weight each experiment by its total number of (auction and posted price) listings to make results comparable (in terms of compositional differences) between auctions and posted price demand curves.

Table A4: Classifying Categories using Principal Component Analysis

Category	Share of listings classified in eBay catalog	Share of sales from multi-unit listings	Share of "new" listings	Share of duplicate listings	Score (Principal Component Analysis)
Collectibles	0.0001	0.17	0.08	0.60	-1.73
Fan Shop	0.0001	0.14	0.10	0.61	-1.62
Toys	0.0008	0.27	0.13	0.65	-0.83
Clothing	0.0000	0.17	0.22	0.70	-0.36
Jewelry	0.0001	0.21	0.10	0.84	-0.04
Crafts	0.0004	0.42	0.13	0.78	0.39
Books	0.6591	0.22	0.16	0.74	0.47
Sport	0.0143	0.34	0.28	0.74	0.72
Music	0.4649	0.16	0.38	0.73	1.20
Home	0.0028	0.51	0.20	0.82	1.23
Electronics	0.0535	0.57	0.23	0.81	1.64
Health	0.0007	0.54	0.27	0.82	1.68
PCs	0.0222	0.61	0.25	0.79	1.69
Phones	0.0508	0.61	0.22	0.87	2.05
DVDs	0.7848	0.35	0.50	0.79	3.04

Using 2009 listings, the 33 main eBay categories are ranked from most idiosyncratic to least idiosyncratic according to their scores from a principal component analysis that includes the following variables: share of items that are classified in the eBay catalog, share of sales from multi-unit listings, share of listings with the word "new" in the title, share of duplicate listings. The table reports the 15 largest categories, with their corresponding scores. As described in the main text, we use this ranking to select five "idiosyncratic" categories (Collectibles, Fan Shop, Toys, Jewelry, and Clothing) and five "commodity" categories (Personal Computers, Phones, DVDs, Health, and Electronics).

Table A5(a): Auction Demand Curve Estimates for “Commodity” Categories

Dependent Var.	2003 Auction Sample ("Commodity" categories only)		2009 Auction Sample ("Commodity" categories only)	
	1 if purchased	Normalized sale price, conditional on sale	1 if purchased	Normalized sale price, conditional on sale
Constant	0.799 (0.007)	0.834 (0.005)	0.841 (0.004)	0.666 (0.002)
Normalized start price is <0.3	Omitted	Omitted	Omitted	Omitted
Normalized start price in (0.3-0.4]	0.0801 (0.012)	-0.000402 (0.008)	-0.176 (0.005)	-0.0178 (0.004)
Normalized start price in (0.4-0.5]	0.014 (0.012)	-0.0161 (0.009)	-0.2 (0.006)	-0.00222 (0.004)
Normalized start price in (0.5-0.7]	-0.146 (0.011)	-0.0405 (0.007)	-0.336 (0.005)	0.0773 (0.003)
Normalized start price in (0.7-0.8]	-0.243 (0.012)	0.0442 (0.007)	-0.444 (0.005)	0.168 (0.003)
Normalized start price in (0.8-0.9]	-0.296 (0.012)	0.104 (0.008)	-0.517 (0.005)	0.228 (0.003)
Normalized start price in (0.9-1]	-0.408 (0.010)	0.185 (0.007)	-0.588 (0.006)	0.308 (0.003)
Normalized start price in (1-1.1]	-0.43 (0.013)	0.259 (0.008)	-0.623 (0.006)	0.376 (0.004)
Normalized start price in (1.1-1.2]	-0.454 (0.017)	0.361 (0.009)	-0.655 (0.006)	0.488 (0.004)
Normalized start price is >1.2	-0.433 (0.018)	0.671 (0.019)	-0.703 (0.007)	0.674 (0.007)
Fixed Effects	Experiment	Experiment	Experiment	Experiment
No. of observations (listings)	152,905	88,659	1,062,388	432,147

Table is similar to Appendix Table A2, except that the sample only includes products that are in the top five “Commodity” categories (see Appendix Table A4).

Table A5(b): Auction Demand Curve Estimates for “Idiosyncratic” Categories

Dependent Var.	2003 Auction Sample ("Idiosyncratic" categories only)		2009 Auction Sample ("Idiosyncratic" categories only)	
	1 if purchased	Normalized sale price, conditional on sale	1 if purchased	Normalized sale price, conditional on sale
Constant	0.634 (0.010)	0.726 (0.011)	0.793 (0.003)	0.563 (0.003)
Normalized start price is <0.3	Omitted	Omitted	Omitted	Omitted
Normalized start price in (0.3-0.4]	-0.0318 (0.015)	0.0167 (0.016)	-0.119 (0.006)	-0.0175 (0.004)
Normalized start price in (0.4-0.5]	-0.0378 (0.016)	-0.00918 (0.016)	-0.27 (0.005)	0.0262 (0.004)
Normalized start price in (0.5-0.7]	-0.126 (0.012)	0.0555 (0.012)	-0.381 (0.004)	0.127 (0.004)
Normalized start price in (0.7-0.8]	-0.197 (0.012)	0.15 (0.012)	-0.481 (0.004)	0.238 (0.003)
Normalized start price in (0.8-0.9]	-0.227 (0.012)	0.234 (0.012)	-0.537 (0.004)	0.32 (0.003)
Normalized start price in (0.9-1]	-0.294 (0.012)	0.304 (0.012)	-0.585 (0.004)	0.414 (0.003)
Normalized start price in (1-1.1]	-0.319 (0.013)	0.398 (0.013)	-0.609 (0.004)	0.494 (0.004)
Normalized start price in (1.1-1.2]	-0.329 (0.014)	0.511 (0.014)	-0.634 (0.005)	0.587 (0.004)
Normalized start price is >1.2	-0.377 (0.013)	0.93 (0.018)	-0.634 (0.005)	0.776 (0.007)
Fixed Effects	Experiment	Experiment	Experiment	Experiment
No. of observations (listings)	258,606	94,872	1,539,602	462,941

Table is similar to Appendix Table A2, except that the sample only includes products that are in the top five “Idiosyncratic” categories (see Appendix Table A4).

Table A5(c): Posted Price Demand Estimates for “Commodity” and “Idiosyncratic” Categories

Dependent Var.	"Commodity" categories		"Idiosyncratic" categories	
	2003 Posted Price Sample 1 if purchased	2009 Posted Price Sample 1 if purchased	2003 Posted Price Sample 1 if purchased	2009 Posted Price Sample 1 if purchased
Constant	0.531 (0.011)	0.661 (0.020)	0.382 (0.008)	0.465 (0.013)
Normalized posted price is <0.7	Omitted	Omitted	Omitted	Omitted
Normalized posted price in (0.7-0.8]	0.0286 (0.022)	-0.0455 (0.025)	0.0478 (0.017)	-0.0465 (0.016)
Normalized posted price in (0.8-0.85]	0.0584 (0.016)	-0.145 (0.022)	0.0646 (0.012)	-0.0724 (0.014)
Normalized posted price in (0.85-0.9]	0.0488 (0.014)	-0.221 (0.021)	0.0237 (0.013)	-0.107 (0.014)
Normalized posted price in (0.9-0.95]	-0.0213 (0.012)	-0.324 (0.021)	-0.0362 (0.009)	-0.168 (0.014)
Normalized posted price in (0.95-1.05]	-0.0522 (0.015)	-0.342 (0.022)	-0.0573 (0.012)	-0.175 (0.014)
Normalized posted price is >1.05	-0.103 (0.014)	-0.326 (0.021)	-0.0789 (0.010)	-0.178 (0.014)
Fixed Effects	Experiment	Experiment	Experiment	Experiment
No. of observations (listings)	96,440	423,517	143,335	531,102

Table is similar to Appendix Table A3, except that the sample includes products that are in either the top five “Commodity” categories (left) or “Idiosyncratic” categories (right). See Appendix Table A4 for more details about category classification.